Enhancing Financial Risk Modelling For A Regional Development Firm: Navigating The SME Environment

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HORIZON RUIMTE VOOR ONDERNEMEN IN FLEVOLAND

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Management Summary

This thesis enhances financial risk modelling for Horizon Flevoland, specifically targeting the 'Groei Fonds', which financially supports SMEs in their scale-up phase. The objective was to develop a new robust and comprehensible financial risk model for loans and direct investments, improving the current model that was perceived to be inaccurate, opaque, and unintuitive.

We employed a blend of qualitative and quantitative research methodologies. We started with a literature review to identify potential methods for modelling financial risk of loans and direct investments. Next, we categorised 'Groei Fonds' companies using qualitative assessments, allowing us to adjust the models to make them applicable to Horizon's operations. We then adjusted these methods to suit the specific characteristics of these companies.

Our research identified state-of-the-art methods for credit and market and liquidity risk modelling, which we tailored to meet Horizon Flevoland's specific needs. We developed a Monte Carlo Value-at-Risk (VaR) model for both credit risk and market and liquidity risk. This method was selected due to its adaptability and robustness in handling the uncertainties and specific characteristics of the 'Groei Fonds' participants. By incorporating expert judgment in the Probability of Default calculation and adjusting the model parameters to reflect the unique financial profiles and risks associated with SMEs, we have tailored the Monte Carlo VaR model to Horizon's operational context.

The proposed VaR model is designed for implementation within Horizon Flevoland's existing framework for financial risk assessment. The model is unique as it applies to a data-limited environment, a situation not uncommon in early-stage companies. Besides, the dynamic capabilities of the models are particularly valuable as they enable dynamic validation processes and continuous improvement. When we compare the financial risk assessment before the introduction of the models, the lack of adequate data and transparency in the model left decision-makers grappling with a figurative "black box" where the factors influencing risk assessments remained unclear. Using the two VaR models, we enhance financial risk assessment and support better-informed decision-making for loans and direct investments.

The development of these two models represents a significant advancement in financial risk management at Horizon, promising to shed light on previously opaque financial risk areas. The clarification in the financial area mainly concerns qualitative and quantitative factors that influence the counterparty's Probability of Default. In addition, a more intuitive model has also been developed for market and liquidity risk. Experts indicate that this transition promises continuous improvement, enabling the model to adapt to challenges and opportunities, thereby ensuring Horizon's agility and resilience in risk management.

Future research should focus on improving financial risk models for SMEs by addressing current limitations. Key areas include:

- **Data Specificity**: Researchers should gather SME-specific data and performance metrics for loans and direct investments to test and enhance model accuracy.
- **Empirical Validation**: Researchers should validate models with historical data to ensure robustness and practical accuracy across different economic conditions and SME segments.

- **SME Heterogeneity**: Researchers should develop sector-specific and size-specific models to better account for the diverse operational characteristics and risk profiles within the SME sector.

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Contents

Acronyms

- **BWM**: Best Worst Method
- **CAPM**: Capital Asset Pricing Model
- **CLA**: Convertible Loan Agreement
- **CVaR**: Conditional Value-at-Risk
- **EAD**: Exposure at Default
- **EBIT**: Earnings Before Interest and Taxes
- **ES**: Expected Shortfall
- **EWMA**: Exponentially Weighted Moving Average
- **GARCH** Generalized Autoregressive Conditional Heteroskedasticity
- **HML**: High minus Low
- **IFRS**: International Financial Reporting Standards
- **IPO**: Initial Public Offering
- **IRBA**: Internal Ratings Based Approach
- **LGD**: Loss Given Default
- **MCDM**: Multi-Criteria Decision-Making
- **NASDAQ**: National Association of Securities Dealers Automated Quotations
- **NYSE**: New York Stock Exchange
- **OECD**: Organisation for Economic Co-operation and Development
- **PE**: Private Equity
- **PD**: Probability of Default
- **PDF**: Probability Density Function
- **RMW**: Robust minus Weak
- **RR**: Recovery Rate
- **SA**: Standardised Approach
- **SMB**: Small minus Big
- **SME**: Small and Medium-sized Enterprises
- **S&P**: Standard & Poor's
- **TRL**: Technology Readiness Level
- **VaR**: Value-at-Risk

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1 Introduction

In this introductory chapter, we get to learn about the host company of the thesis, Horizon. We then present how Horizon perceives the problem and why it considers it significant. Finally, we motivate the relevance of the perceived problem statement, emphasising the need for accurate risk quantification for effective fund management.

1.1 Company Description

Horizon Flevoland B.V., situated in Lelystad, is a regional non-profit development firm in the Flevoland province of the Netherlands. Horizon emerged in 2019 from the merger of Ontwikkelingsmaatschappij Flevoland with Beheer Flevoland Participaties. As a subsidiary of the province of Flevoland, Horizon promotes the regional economy. Horizon operates through three business units: Business Development, International, and Capital. Horizon focuses on key societal transitions, including Energy, Food, and Circularity, as well as initiatives that enable these transitions through new innovative products and services. Horizon Flevoland aims to assist Small and Medium-sized Enterprises (SMEs) with the ambition and potential for growth, which, in turn, stimulates employment and/or contributes to sustainability in the province. The province of Flevoland holds all of the company's shares. By providing loans and making direct investments, Horizon actively stimulates employment and sustainability in the region, thereby playing a key role in shaping the region's future.

The Capital team, where this thesis is executed, manages the funds in which the province is a major shareholder. Horizon provides financing to companies in Flevoland that are unable to meet their growth capital needs in the market due to associated risks. This financing takes the form of loans, direct investments, or convertible loans. When providing loans, Horizon offers the necessary capital and receives interest and the principal amount back. Through direct investments, Horizon takes an equity stake in the targeted company with a long-term exit strategy. Convertible loans give Horizon the option to convert outstanding debt into equity capital. The Capital team is, among others, responsible for:

- Assessing and analysing companies requesting capital
- Ensuring the revolvability[1] of the managed funds
- Reporting the performance of managed funds to the shareholder

Horizon's active investment strategy aims to drive the regional economy by investing only in companies that create employment opportunities in Flevoland and/or contribute to the province's sustainability. Horizon supports these companies through three main methods: providing loans, purchasing equity stakes, or a combination of both.

1.2 Problem Description

Horizon manages several funds to help drive the economy in the province of Flevoland, including the 'TMI Proof of Concept Fonds', 'Techno Fonds', 'MKB Fonds', and 'Groei Fonds'. This research focuses on the 'Groei Fonds', which has €16 million at its disposal and invests in companies in the scale-up phase, a pivotal developmental stage for SMEs. These companies have proven scalable business models and aim to expand their activities to achieve growth.

[1]Revolving funds finance societal goals. Revolvability means that funds are allocated to a pool, a portion is received back in the form of interest payments and/or return on equity, and that money is used for the same policy objective.

During the first meeting with Horizon, it became clear that there was dissatisfaction with the current method of measuring financial risk in their portfolio. Recently, Horizon's shareholder introduced a tool to help identify and quantify the financial risk of outstanding loans and direct investments. Provided by an external firm, this tool relies heavily on assumptions suitable for the banking sector. For example, it assumes a certain maturity and predictability in the performance of portfolio companies, which is not the case with Horizon's predominantly scale-up portfolio. Allegedly, the model was considered inaccurate as a consequence of this. Additionally, the model was found counterintuitive because the calculations from input parameters to risk quantification were opaque, preventing Horizon from understanding the underlying processes and explaining the results.

Accurate risk quantification of individual companies in the fund and the financial risk of the fund as a whole is crucial for Horizon. Horizon's shareholder benefits from precise risk assessments due to stringent requirements for the fund's revolvability and risk profile. The shareholder demands 100% revolvability, meaning the initial investment must retain its value over time. This requirement is common among regional development firms in the Netherlands, where provinces have a majority stake. Practically, this means the fund must closely monitor its risk profile and maintain a comprehensive understanding of potential losses. Ultimately, Horizon is expected to keep the risk profile below a 25% threshold. However, the shareholder does not specify the nature of this 25% risk, leaving its interpretation to Horizon.

At the moment, Horizon is dissatisfied with the current risk quantification model for individual companies and the fund overall. This is because the model that measures risk, using the Value-at-risk (VaR) metric, is highly, allegedly, inaccurate. The current tool calculates the VaR for individual companies, whether through loans or equity capital, but the parameters used in determining VaR are problematic. Some example parameters for calculating the VaR of loans include Probability of Default (PD), Loss Given Default (LGD), and Exposure At Default (EAD). For equity capital, VaR is calculated using the initial investment, expected return, and its volatility. The tool suggests a high degree of certainty in outcomes, which contradicts the volatile returns inherent to scale-up companies. These uncertainties and risks cause a poor approximation of returns. Inherently, the corresponding VaR metric for direct investments, which is determined from the projected returns, can be poorly determined. For loans, there is a similar problem, namely the estimation of the parameters is subject to risk and uncertainty. This research aims to identify methods and parameters for a more accurate assessment of the risk profile of 'Groei Fonds' companies and the fund as a whole. Additionally, a method will be chosen and a new model will be developed to help Horizon interpret and use the results effectively.

The primary challenge in improving risk quantification parameters for the 'Groei Fonds' is the lack of data. Scale-up companies typically lack the quantitative data necessary for accurate VaR calculations, forcing Horizon to rely on approximations and subjective judgments. While Horizon currently uses a model to calculate VaR, it is considered counterintuitive, impractical, and inaccurate.

1.3 Research Motivation

Horizon's motivation to resolve the discrepancy between norm and reality is twofold (Heerkens and Winden, 2023). The norm is the situation where VaR is, allegedly, more accurately calculated and therefore more meaningful. However, the current reality portrays VaR as poorly calculated and lacking decisiveness as a parameter. Horizon aims to thoroughly explore potential methodologies that could aid in quantifying the risk associated with (potential) portfolio companies. By doing so, they seek to attain a comprehensive understanding of relevant parties at the outset of a potential investment (loan and/or equity). This comprehensive understanding enables risk minimisation at the onset, resulting in a mutually beneficial situation for both Horizon's shareholder and Horizon itself. Another significant motivating factor for this research is Horizon's shareholder, who desires increased insight into the risk profiles of portfolio companies and the fund. Therefore, Horizon must embark on these research endeavors as it is requested by the shareholder.

2 Problem Approach

In this chapter, we identify the core problem from the outlined problem description. We then define the methodological framework for addressing this core problem, review the previous research, determine the research questions, and define the problem-solving approach for these research questions.

2.1 Core Problem Identification

As discussed in Section 1.2, there exists a significant discrepancy between the norm and reality, particularly concerning Horizon's desire for Value-at-Risk (VaR) calculations to play a more decisive role in comprehending investment risk. Heerkens and van Winden describe a discrepancy between norm and reality as the starting point of finding the core problem (Heerkens and Winden, 2023). They suggest finding the underlying drivers of this discrepancy, also known as an action problem.

In Horizon's case, the action problem revolves around the need for more accurately calculated VaR. The current VaR calculations are deemed inadequate by Horizon, leading to a lack of meaningfulness. This inadequacy comes from a lack of understanding of the VaR parameters and a mismatch between the perceived outcomes of the VaR and the actual outcomes of the VaR. Key VaR parameters such as PD, EAD, LGD, and interest rates for loans and expected returns and volatility for equity investments contribute to this discrepancy. Furthermore, the lack of transparency in the VaR model's calculation process, provided by an external party, exacerbates the problem. Gaining insight into the calculation steps is key in identifying possible mismatches with the risk profiling applicable to Horizon. Therefore, our initial focus is on identifying the deficiencies in the current model and its lack of transparency, followed by exploring methods to measure risk in Horizon's cases (loans and equity) and evaluating potential improvements.

Examining the current application of the VaR model reveals its use by Horizon both prior to and during financing decisions (loan and direct investment). This has important implications for this thesis, as the identified risk modelling methods should be applicable in both pre-financing and post-financing scenarios. Typically, financial risk modelling distinguishes between these situations. Therefore, we possibly require necessitating adjustments to the chosen modelling approach. If required, a separate section will address these adaptations.

In summary, two main issues emerge: perceived inaccuracies in the VaR model outcomes and the lack of comprehensibility of the model itself. For this thesis, we prioritise addressing the inaccuracies in VaR outcomes. While improving model comprehensibility may be a secondary goal, our core problem and primary focus remain on enhancing the accuracy of VaR calculations.

2.2 Previous Research

Previous research has explored VaR calculation methods for loans and direct investments. In the case of loans, several academic articles propose models for credit risk scoring, a method used to calculate VaR. For instance, Florez-Lopez (2010) conducts a comparative analysis in credit risk scoring, where credit risk scoring is one method to calculate the VaR metric. Cipovová and Dlasková (2016) compare other methods used for credit risk management to accelerate lending activities by banks. However, these methods heavily rely on data, which is currently lacking at Horizon. The gap in the literature is the lack of models that assess credit risk specifically for scale-up companies, a phase where little data is available. Similarly, the literature on direct investments lacks specific adaptations for scale-up companies due to data limitations. Because there is no literature available on scale-up adaptations to models specifically, we have to identify the categorisation that we might apply to Horizon portfolio companies. In some ways, Horizon's operations are comparable to private equity. More specifically, Buchner (2017) zooms in on risk management methods for private equity funds, a situation considered comparable to Horizon. Buchner identifies market risk, liquidity risk, and cash flow risk for these private equity funds as the most important dynamic risk measures. In the paper, they use VaR to quantify the risk. Unfortunately, the paper is unable to explain how to deal with a highly uncertain environment, and how to calculate the VaR metric then (Buchner, 2017). A forward look at this issue is presented in the article by Soares et al. (2011), where an integrated form of qualitative and quantitative indicators is used to interpret market risk. Some qualitative indicators could be used to complete a model with a lack of quantitative data. Venczel, Berényi, and Hriczó (2024) describe and summarise possible scientific sources that could be of use in this search for qualitative indicators. Although studies such as Buchner (2017) and Soares et al. (2011) discuss risk management methods for private equity funds, which share similarities with Horizon's operations, they do not address VaR calculation in highly uncertain environments or with limited quantitative data. Thus, our research aims to fill this gap by identifying suitable qualitative indicators to complement quantitative models for risk assessment in Horizon's portfolio companies.

2.3 Research Questions

The emphasis for Horizon, as outlined in the previous section, is on the alleged inaccuracy of the VaR model. Various factors could explain the problems with the VaR model. However, the crucial aspect of improving the model is identifying the most suitable model with the appropriate parameters for Horizon's specific context. In addition, the model should provide the risk insights Horizon is looking for. Some details influence the choice of model. Firstly, the availability and type of data determine the options for financial risk modelling. Secondly, Horizon's portfolio consists entirely of non-public companies, which means that much of the required data is not publicly accessible. Thirdly, the companies that are part of Horizon's portfolio are in the scale-up phase, which entails a corresponding level of risk. These components, among others, significantly impact the models that could be employed.

Practically, we begin by exploring Horizon's possibilities to model their financial risk, in loans and equity. Next, we consider the specific environment in which this model should be applied, including the three points made earlier in this section. Hence, we formulate the following primary research question:

What method is best suited for financial risk modelling of loans and direct investments in Horizon's portfolio companies?

To structure our approach to this research question, we develop sub-questions that collectively address the main question. These sub-questions are listed below, along with brief descriptions of their context:

1. What are State-of-the-art methods for credit risk modelling?
First, we need to identify the range of methods that might be applied to credit risk modelling, specifically for loans, which is one of the two instruments Horizon uses to allocate its funds. This involves identifying the necessary data for each method concerning loans to ensure their viability. Additionally, background information on recent developments in credit risk modelling will be provided.

2. What are State-of-the-art methods for market and liquidity risk modelling?

Second, we need to identify the range of methods that might be applied to equity risk modelling. This is the financial risk modelling of the direct investments, the other instrument Horizon uses. Liquidity risk will also be considered, given that we are dealing with private firms. Current literature suggests that this has implications, mostly as a discount on shares. This sub-question will also involve identifying the necessary data for each method concerning equity to ensure their viability.

3. How to categorise Horizon's portfolio companies and how to deal with this environment in credit, market, and liquidity risk modelling?
Third, we need to categorise Horizon's portfolio companies to understand their environment. Following this, we can determine how to address this specific environment in credit, market, and liquidity risk modelling to ensure the proposed methods apply to Horizon.

4. What defines 'Groei Fonds' participants and what are their characteristics?
Fourth, we need to define the characteristics of 'Groei Fonds' participants regarding assessable investment indicators. We identify these indicators and characterise the participants to get an overview of the pool of participants in the 'Groei Fonds'. Consequently, we can evaluate these indicators and see how they could enhance the development of the risk modelling method.

2.4 Problem-Solving Approach

The problem-solving approach to answering the research question involves the following:

1. Qualitative research
The qualitative research consists of a twofold literature study. We conduct a literature review to answer sub-questions 1 and 2, identifying methods that could be used for loan risk and equity risk modelling, respectively. We dedicate one chapter to each sub-question. Sub-question 3 requires qualitative research to obtain an answer as well. We need to find a categorisation for 'Groei Fonds' companies based on the literature so that we can make any necessary modifications to the models correctly.

2. Quantitative research
The quantitative adjustments that have to be made to the chosen method are part of the quantitative research. We use data analysis to assist in answering sub-question 4, analysing data about 'Groei Fonds' participants to effectively model a risk assessment method specific to Horizon. 'Groei Fonds' participants have distinct characteristics and bear a certain risk. We incorporate this factor into the chosen model, to make the quantitative part of the method as accurate as possible.

2.5 Thesis Outline

We provide an outline of the thesis to give the reader a clear understanding of its structure. In Chapter 3, we answer sub-question 1 employing a literature review on risks involved with loans. First, we define credit risk and provide some background information on the developments within credit risk. Second, we identify possible methods to evaluate credit risk for Horizon. Third, we identify necessary model adaptations to make the model fit Horizon's case. In this last part of Chapter 3, we also answer sub-question 3 on how to categorise portfolio companies. In Chapter 4, we answer sub-question 2 employing a literature review on risks involved with direct investments. First, we define market and liquidity risk and provide some background information on the developments in this field. Second, we identify possible methods to evaluate market and liquidity risk for Horizon. Third, we discuss whether model adjustments are necessary and we review the integration of liquidity risk. In Chapter 5, we discuss the methodology and relevant considerations regarding topics pertinent to our research. Also, we perform a data analysis that answers sub-question 4. Next to this, we decide on the models to use for credit risk and market and liquidity risk. We finalise the chapter by constructing the parameters of the chosen models. In Chapter 6, we construct the models for the two risk categories. We integrate the parameters constructed in Chapter 5 to build the models. We also discuss the approach in the modelling of dependency within loans and within direct investments. In Chapter 7, we validate the outcomes of both models. We do this through a sensitivity analysis and expert review. In Chapter 8, we formulate our conclusions and reflect on these conclusions.

3 Literature Review: Credit Risk Modelling

In this chapter, we explore methods to model financial risks associated with loans by means of a literature review. First, we outline the risks involved in providing loans to counterparties. Then, we discuss the relevance of this topic and the influence of regulations. Following this, we identify a range of methods that might be applied to credit risk modelling, considering the necessary data for each method to ensure applicability in Horizon's environment with limited data.

3.1 Credit Risk

The first of the two types of financing Horizon provides is lending. A loan is a form of credit where a sum of money is lent to another party in exchange for future principal repayment. In addition, a fee is paid for the risk of the borrower not being able to repay the loan. This is also known as interest and can be seen as compensation for the lender's risk and the time value of money. Companies often approach Horizon for loans when traditional banks decline due to perceived risks, a phenomenon largely influenced by Basel II regulations' impact on small and medium enterprises (SMEs). Basel II introduced new credit risk management techniques to make banks more resilient against economic downturns. Altman and Sabato (2005) highlighted Basel II's introduction as a risk for SMEs due to heightened capital requirements for banks financing them, potentially discouraging such investments crucial for the economy. Butera and Faff (2006) demonstrate that minimum required capital standards may improve banks' stability, but is likely to come at a cost in terms of imposing restrictions on how banks conduct their normal business activity and thereby may lead to inefficiencies. However, the regulatory component in the financing market is something Horizon has to deal with. Horizon is financing the more risky businesses within SMEs, which emphasises the importance of being able to identify financial risk correctly.

The financial risk involved in issuing a loan is also known as credit risk. Credit risk entails the likelihood of facing financial loss due to a borrower's inability to repay a loan. In essence, it pertains to the potential that a lender may not receive the owed principal and interest, leading to disruptions in cash flows and increased expenses associated with collection efforts. When a principal or interest payment is not paid, we call this a default.

Navigating the uncertainties of potential defaults by individuals or entities can be challenging. However, a comprehensive assessment and effective credit risk management can significantly reduce the impact of such situations. Interest payments received by lenders or investors serve as compensation for assuming inherent credit risk. While predicting a borrower's ability to meet obligations remains uncertain, certain metrics help gauge credit risk. Qualitative metrics, like management quality, business case viability, board experience, and sector health, play a crucial role. The literature presents these factors, amongst others, as explanatory variables that have predicting value in assessing the creditworthiness of companies. These indicators can be systematically graded on a predefined scale, forming the basis for expressing credit risk. Angilella and Mazzù (2015) explains a multi-criteria model, where development risk, technological risk, market risk, production risk, and innovation indicators form the core of the model. These criteria are sub-defined in indicators that represent this criterion and are assessed using scores on a pre-defined scale. The credit risk associated with the business being assessed is scaled to a weighted average. In this way, credit risk is then assessed on a relative scale. To summarise, integrating qualitative metrics into credit risk assessment and a structured grading system facilitates the creation of a robust credit score. But obviously, lending is not solely based on qualitative information; that would be incomplete.

Besides these qualitative metrics, the literature also presents thorough research on quantitative metrics. Example figures are Probability of Default (PD), Loss Given Default (LGD), Exposure At Default (EAD), and Value-at-risk (VaR) for single investments. More specifically, PD is the likelihood that a borrower will fail to pay back a loan. LGD is the estimated amount of money that is lost during a default case. EAD is the total loss exposure at the time of default. At the individual and portfolio level, VaR can be calculated, provided concentration risk is considered, as defaults tend to be correlated and not independent. Volatility, another quantitative risk measure, indicates the fluctuations in returns on the loan portfolio.

3.2 Basel

Before delving into specific methods used in credit risk modelling, it is important to provide some historical context for measuring credit risk. While credit risk has existed since the advent of cash, its quantification is a relatively recent development. The motivation to develop credit risk models arose in 1974 in the aftermath of serious disturbances in international currency and banking markets. The most notable event at that time was the failure of Bankhaus Herstatt in West Germany. In response, the Basel Committee was established to address gaps in international supervisory coverage, ensuring that (i) no banking establishment would escape supervision and (ii) supervision would be adequate and consistent across member jurisdictions (BIS, 2023). With the foundations for the supervision of internationally active banks laid in the years after its establishment, capital adequacy became the main focus of the Basel Committee. The first significant outcome was the Basel Capital Accord, or Basel I, which called for a minimum ratio of capital to risk-weighted assets of 8% by 1992 (BIS, 2023). The committee allocated a specific amount of risk to each debt instrument, and it was up to the member jurisdictions to make sure they met the 8% threshold of capital to risk-weighted assets.

Basel I focused on minimum capital requirements to address credit risk, Basel II preserved this focus and extended it. Basel II has three main pillars: minimum capital requirements, market discipline, and regulatory supervision. Building on Basel I, Basel II provided guidelines for the calculation of minimum regulatory capital ratios and confirmed the requirement that banks maintain a capital reserve equal to at least 8% of their risk-weighted assets (Chen, 2023). Regulatory supervision, constituting the second pillar of Basel II, establishes a structured framework for national regulatory bodies to manage diverse risk categories effectively. This comprehensive approach encompasses the oversight of systemic risk, liquidity risk, and legal risks within the financial system. The market discipline pillar of Basel II mandates the implementation of diverse disclosure requirements on banks' risk exposures, risk assessment procedures, and capital adequacy. This aspect aims to enhance transparency regarding the robustness of a bank's operational strategies, enabling investors and other stakeholders to make informed comparisons among banks. Basel II has been widely regarded as complex and inadequate since the occurrence of the subprime mortgage meltdown in 2007. The financial crisis following this event signaled to the Basel Committee that Basel II had failed.

In Basel III, leverage and capitalisation stood central in the new regulations imposed on member jurisdictions, as they were major reasons for the financial crisis of 2007. Basel III is an effort to improve the banking system's ability to deal with financial distress and to promote transparency. The most important imposed regulation is the additional reserves, also known as countercyclical capital buffers. These buffers can be imposed on banks during periods of economic growth. This way, banks should be more resilient during economic contractions, such as a recession, when they possibly face greater losses.

Implementing Basel III has proven challenging, but the overarching goal of these regulations is to ensure that prominent banks do not take excessive risks and maintain adequate buffer capital to safeguard the financial system's health. However, one consequence of placing limits on banks' ability to pursue their policies is that it affects the investment decisions they make. Marek and Stein (2022) investigated the impact of Basel III on banks' lending behavior and found that in Germany, lending to non-financial SMEs decreased under the standardised approach (SA). In contrast, banks large enough to use an internal ratings-based approach (IRBA) did not experience a decline in lending volumes. Fisera, Horvath, and Melecky (2019) drew similar conclusions on the impacts of Basel III on lending behaviour to SMEs in developing economies. Namely, Basel III harmed lending volume in these developing economies. These findings highlight the importance of regional investment firms, like Horizon, which provide financing to firms unable to secure it from banks due to their risk profiles. Although Horizon is a non-profit organisation, it needs to quantify risk to maintain its risk profile below a 25% threshold. The shareholder mandates a comprehensive understanding and quantification of potential losses on investments but does not specify the nature of this 25% risk, leaving its interpretation to Horizon.

In essence, the observed challenges in lending behavior due to regulatory frameworks such as Basel underscore the pivotal role of regional investment firms like Horizon. The intentional embrace of a higher-risk portfolio necessitates a thorough credit risk quantification approach, especially since the risk level has to be kept under this 25% threshold. In the current Value-at-Risk models, given a confidence level of 95%, this 25% threshold is defined as:

$$\text{Risk} = \frac{\text{5-Year VaR}}{\text{Total invested amount}}$$

$$\begin{aligned} \text{Risk}: &\quad \text{Risk (in Percentages)} \\ \text{5-Year VaR}: &\quad \text{5-Year Value at Risk (in Euros)} \\ \text{Total invested amount}: &\quad \text{Total invested amount (in Euros)} \end{aligned}$$

3.3 Credit Risk Modelling

Having established an understanding of credit risk and the background and impact of credit risk regulation, we can now identify potential methods for Horizon to model credit risk. Data availability within Horizon is limited, given that the 'Groei Fonds' has only existed for a few years. As a result, a selection process takes place in advance, based on data availability within Horizon. Additionally, the methods are narrowed down based on their applicability to Horizon. Appendix A is entirely devoted to naming the inapplicable methods. Given this, we have identified three categories of methods that meet the criteria:

- Accounting-based Modelling
  - Financial Statement Analysis
  - Altman Z-score, Zeta Model And O-score
- Quantitative Modelling
  - Discriminant Analysis Model
  - Value-at-Risk Model
- Qualitative Modelling
  - Expert Judgment

It is possible that the identified method may not apply to both pre-loan and during-loan risk modelling, as discussed in Section 2.1, Core Problem Identification. In such cases, Chapter 5, Methodology, will include a section on adapting the chosen method to suit both risk modelling scenarios. We decide this so as not to exclude any method in advance and to maintain as broad a picture of solutions as possible.

3.3.1 Financial Statement Analysis

The first method we explain is the Financial Statement Analysis (FSA). The FSA is a fundamental tool in the toolkit of financial analysts to get an impression of the financial health and stability of a firm. Before using quantitative and data-driven models, financial institutions relied virtually exclusively on subjective analysis to assess credit risk (Altman and Saunders, 1997). These institutions used, at that time, various borrower characteristics, like borrower character (reputation), capital (leverage), capacity (volatility of earnings), and collateral, to assess credit risk. The assessment of these four characteristics is sometimes also referred to as the 4 Cs model. However, as Somerville and Taffler (1995) demonstrated, bankers tended to be overly subjective in their assessments, generally exhibiting excessive pessimism about credit ratings.

Subsequently, this subjective approach evolved into an accounting-based credit-scoring system. At its core, the accounting-based credit-scoring system involves a review and interpretation of financial statements to extract profound insights into a company's performance and financial standing. By leveraging well-established financial ratios, liquidity metrics, and profitability indicators, this method enables a quantitative assessment of a company's operational efficiency, solvency, and holistic financial stability. The theoretical underpinning of credit scoring is firmly grounded in fundamental accounting principles, and empirical methodologies, collectively forming a systematic and structured framework for comprehensive analysis. Numerous metrics have been applied to different types of credit-scoring systems, e.g. profitability, liquidity, solvency, bank loans, and leverage (Emel et al., 2003; Cramer, 2004; Bensic, Sarlija, and Zekic-Susac, 2005). According to (Abdou and Pointon, 2011), there is no optimal amount of variables to include in the analysis.

For this method, a comprehensive overview of the company's financials is required. The necessary data includes the income statement, balance sheet, and cash flow statement. All available information is utilised in the analysis of the financials.

3.3.2 Altman Z-score, Zeta Model, and O-score

Altman (1968) used accounting ratios and metrics to develop an analytical formula to assess credit risk. The Altman Z-score is a financial metric designed to assess the likelihood of a company facing bankruptcy within two years. This multivariate formula integrates various financial ratios to provide a comprehensive evaluation of a company's financial health, aiding investors, creditors, and analysts in gauging its creditworthiness. The analytical formula consists of a profitability ratio, liquidity ratio, solvency ratio, and activity ratio. With this analytical formula, Altman (1968) aimed to demonstrate that failed and non-failed firms have dissimilar ratios, but not that these indicators have predictive power. The Z-score derived from this formula can be evaluated by hand of the zones of discrimination, and indicate the level of financial distress in the evaluated company. The model was initially designed to assess manufacturing firms on credit risk. Also, the data sample consisted of relatively small enterprises. The model has a forecasting accuracy of 72%, two years prior to the bankruptcy occurrence.

The original Altman Z-score bankruptcy model is defined as follows:

$$Z = 1.2 \times X_1 + 1.4 \times X_2 + 3.3 \times X_3 + 0.6 \times X_4 + 1 \times X_5 \tag{1}$$

where:

$$X_1 = \frac{\text{Working Capital}}{\text{Total Assets}}$$
$$X_2 = \frac{\text{Retained Earnings}}{\text{Total Assets}}$$
$$X_3 = \frac{\text{EBIT}}{\text{Total Assets}}$$
$$X_4 = \frac{\text{Market Value of Equity}}{\text{Total Liabilities}}$$
$$X_5 = \frac{\text{Sales}}{\text{Total Assets}}$$

The zones of discrimination are defined as follows:

- $Z > 2.99$ – "Safe" zone
- $1.81 < Z < 2.99$ – "Grey" zone
- $Z < 1.81$ – "Distress" zone

In 1977, the existing model was altered and improved in terms of accuracy by Altman, Haldeman, and Narayanan (1977) based upon a new dataset from 1969 to 1975. This model uses the same parameters as the model from Altman (1968), but now the coefficients of the parameters have been improved significantly. Therefore, increasing the accuracy of the model significantly. This improvement was further achieved by incorporating continuous variables in the Zeta model, allowing for a more nuanced and granular assessment of creditworthiness The forecasting ability of this model two years prior to bankruptcy is 84.9%. Similar to the Z-score model, the Zeta model is designed for retailers and manufacturers

Lastly, we investigate the Ohlson O-score, a more extensive variant of the original Z-score. In 1980, this new model was presented by Ohlson (1980). This bankruptcy forecasting model has proven to be even more accurate than the Zeta model from Altman, Haldeman, and Narayanan (1977). The model achieved a 96.3% accuracy, a superior number compared to all accounting-based methods. The inclusion of more variables resulted in a more accurate prediction of bankruptcy cases. The analytical formula of the O-score is given by:

$$O\text{-score} = -1.32 + 0.407X_1 + 6.03X_2 + 3.41X_3 + 0.1X_4 + 0.988X_5 + 0.045X_6 + 0.2X_7 + 0.998X_8 \tag{2}$$

where:

$$X_1 = \frac{\text{Working Capital}}{\text{Total Assets}}$$
$$X_2 = \frac{\text{Retained Earnings}}{\text{Total Assets}}$$
$$X_3 = \frac{\text{EBIT}}{\text{Total Assets}}$$
$$X_4 = \frac{\text{Market Value of Equity}}{\text{Book Value of Total Debt}}$$
$$X_5 = \frac{\text{Sales}}{\text{Total Assets}}$$
$$X_6 = \frac{\text{Total Market Value of Equity}}{\text{Total Liabilities}}$$
$$X_7 = \frac{\text{Retained Earnings}}{\text{Total Assets} - \text{Current Liabilities}}$$
$$X_8 = \frac{\text{Book Value of Equity - Preferred Stock}}{\text{Total Liabilities}}$$

The zones of discrimination are defined as follows:

- $Z > 0.5$ – "Safe" zone
- $-0.5 < Z < 0.5$ – "Grey" zone
- $Z < -0.5$ – "Distress" zone

Based on these analytical formulas, we can identify the necessary data quickly, namely all the named variables in Formulas 1 & 2.

3.3.3 Discriminant Analysis Model

Discriminant Analysis, a widely utilised statistical technique in finance, economics, and various academic disciplines, aims to differentiate between multiple groups by utilising a set of predictor variables. The primary objective of Discriminant Analysis is to identify the combination of predictors that optimally distinguishes these predefined groups. This method is particularly valuable for categorising observations into well-defined classes or identifying the pivotal variables contributing to group distinctions. In the methods from Subsection 3.3.2, these distinctions were bankruptcy and non-bankruptcy cases.

This analytical model, deeply rooted in statistics, relies on the assumption of multivariate normality within each group. The fundamental strategy involves maximising the variance between groups while minimising the variance within each group, effectively resulting in the creation of a discriminatory function. Derived from the combination of predictor variables, this discriminatory function serves as the foundational element for the classification of observations into their respective groups. The relationship between the predictor variables can be modelled linearly, or quadratically, depending on the data set. Generally, a quadratic model is chosen when dealing with small sample sizes and/or unequal variances among groups. A linear model is mostly preferred if normality can be assumed and the sample size is larger (W. Wu et al., 1996). In essence, Discriminant Analysis provides a robust framework for exploring and understanding the inherent differences among distinct groups based on a comprehensive set of predictors. These predictor variables can be used to distinguish between so-called good customers and bad customers, indicating creditworthiness.

The linear discriminant function for i variables could be expressed as:

$$D(X) = b_1 \cdot X_1 + b_2 \cdot X_2 + \ldots + b_i \cdot X_i + b_0 \tag{3}$$

With the following terms:

$D(X)$ is the discriminant score for the observation.

$X_1, X_2, \ldots, X_i$ are the values of the predictors for the observation.

$b_1, b_2, \ldots, b_i$ are the coefficients associated with $X_1, X_2, \ldots, X_i$.

b_0 is the intercept.

We find the data required to apply this method is ambiguous, as it depends on the availability of various factors. Essentially, all predictor variables we want to test can consist of the data pool, making the model adaptable to the available data.

3.3.4 Value-at-Risk Model

Value-at-Risk (VaR) has become the standard measure used by financial analysts to quantify financial risk. While it is primarily applied to measure market risk, it can also be used to express credit risk. Generally, VaR is defined as the maximum potential loss in value of a portfolio of financial instruments with a given probability over a certain horizon (Buchner, 2017). Modelling credit loss in portfolios can also be done with the Value-at-Risk model. Intuitively, this is visualised in Figure 1.

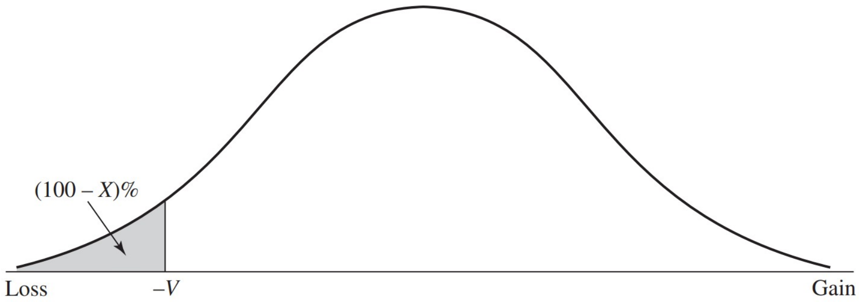


Figure 1: Value-at-Risk from the Probability Distribution of the gain in the portfolio value. Losses are negative gains; confidence level is X%; VaR level is -V (Hull, 2006)

The VaR model relies on robust theoretical foundations, drawing from statistical and mathematical principles. It incorporates key elements such as historical data, parametric assumptions, and Monte Carlo simulation to estimate the potential impact of credit events on a portfolio. The versatility of the VaR model allows it to adapt to the complexities inherent in credit risk scenarios. For example, whether the normality assumption holds for the evaluated portfolio and how the model needs to be adjusted. Or, more generally, how to evaluate the impact of distributions on the Value-at-risk method (Olson and D. Wu, 2013).

In the literature, three different methods exist for calculating VaR: Parametric, Historical, and Monte Carlo. These methods are briefly explained below:

Parametric Method
The parametric Value-at-Risk (VaR) model, sometimes referred to as the variance-covariance model, stands out as the predominant approach in financial risk management due to its widespread use and practical convenience. The model is favoured due to its ability to generate reliable estimates with minimal data requirements. Unlike other models, the parametric VaR model does not need an entire dataset for its calculations. Instead, it relies on key parameters, such as the mean and standard deviation, to make predictions. The Z-score we use in this method is a statistical measurement that describes a value's relationship to the mean of a group of values. More specifically, it is a statistical measurement that describes a value's relationship to the mean of a group of values, measured in terms of standard deviations from the mean. Once these parameters are input into the model, it assumes a specific distribution, often the normal distribution, for the analysed parameters. The VaR is then computed by considering both the distribution characteristics and the variance. The popularity of the parametric model is grounded in its simplicity and effectiveness. It provides reasonably accurate results with a streamlined approach, making it particularly valuable for assessing risk in traditional assets like stocks and bonds. To use the method for credit risk modelling, we simplify the parametric method described by, among others, Khindanova, Rachev, and Schwartz (2001).

The Value-at-Risk of a single instrument is calculated using the following formula, assuming normality:

$$\text{-VaR}_\alpha = \mu - \text{Portfolio Value} \times \text{Z-score}_\alpha \times \sigma \tag{4}$$

The variables are defined as follows:

VaR_α :	The Value at Risk, representing the potential loss within a specified confidence level.
μ :	The mean return realised with this instrument.
Z-score :	The Z-score associated with the desired confidence level.
σ :	The standard deviation of the investment's returns over the specified time period.
Portfolio Value :	The total value of the portfolio.

The VaR parameters for a portfolio consisting of two securities are constructed as follows:

$$\mu_p = w_1\mu_1 + w_2\mu_2 \tag{5}$$

In both formulas, the weights w_1 and w_2 represent the proportion of the portfolio invested in securities 1 and 2, respectively. ρ_{12} is the correlation between their returns.

$$\sigma_p = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho_{12}} \tag{6}$$

The Value-at-Risk (VaR) for a portfolio with two securities, assuming normality, is given by:

$$\text{-VaR} = \mu_p - \text{Z-score}_\alpha \cdot \sigma_p \tag{7}$$

However, analysing these formulas reveals that intersections with credit risk can be difficult to understand. This complexity arises because specific credit events should influence the creditworthiness of the counterparty, rather than necessarily impacting the lender's portfolio. Again, this has to do with the fact that this model initially was designed to capture market risk. Despite this, the model is still widely applied to quantify risk for a portfolio.

This Parametric method needs the following set of data:

- A list of historical daily volatilities for the specified time period.
- A list of historical returns of the instrument for the specified time period.
- The portfolio value of the instrument at the point of measurement.
- A chosen confidence level and the Z-score associated with it, drawn from the normal distribution.

Historical Method
The next method we consider is the Historical method. As described by Hull (2006), this non-parametric approach relies on historical returns that are indicative of financial outcomes (gains and losses). The historical method involves using historical data on financial outcomes or changes in credit ratings to simulate the distribution of future financial outcomes. It is a very simple method and is easy to implement. The method consists of the following step-by-step approach, assuming a 170,000 euro portfolio:

1. Select a confidence level, say 95%.
2. Select a lookback window, say 100 days.
3. Collect daily gains or losses on your credit portfolio for each day.
4. Arrange all these daily values in ascending order, with the greatest losses on top.
5. The corresponding level of significance is 100 - 95 = 5%, which implies taking the 5% of 100 = 5th return from the left, e.g. -2.1%. Therefore, the Value at Risk = -2.1% of 170,000 = -3,570 euro. Statistically, there is a 5% chance that the daily

loss will be more than the Value at Risk.

The Historical method is not bound to any specific distribution, making it flexible. However, it is challenging to implement in the context of credit risk. This difficulty arises because modelling returns is complicated by the nature of loans, which typically repay the principal on selected dates. Therefore, returns are, to a large extent, pre-determined and historical modelling might be rather inefficient (F. Stambaugh, 1996).

This Historical method needs the following set of data:

- A lookback period in days.
- A list of historical returns of the instrument for the specified time period.
- A chosen confidence level.

Monte Carlo Method
The Monte Carlo method, like the Parametric method, ultimately aims to arrive at the VaR metric. However, unlike the Parametric method, the Monte Carlo method is non-parametric, meaning it does not depend on the underlying probability distribution of the data. Instead, it uses random samples drawn from a chosen distribution, often Gaussian, to generate returns. These returns are then arranged in ascending order. By hand of this, the VaR is calculated at a specified confidence level, as done previously in the parametric method.

In practice, many practitioner methods are complemented with the use of Monte Carlo simulations. Monte Carlo simulations are particularly useful for capturing a broader range of potential scenarios in credit portfolios. For example, in the CreditMetrics method (J.P. Morgan, 1997), Monte Carlo simulation is used to generate scenarios for possible future ratings of the specific instrument (Mišanková et al., 2014). By employing this method, numerous future values of the instrument are calculated and their distribution can be derived. This distribution is then used to estimate the VaR metric in the end. The following formula calculates the VaR for one instrument:

$$\mathrm{VaR}_\alpha = \mathrm{PD} \times \mathrm{LGD} \times \mathrm{EAD} \times \Phi^{-1}(\alpha) \tag{8}$$

The components of the formula are the following:

VaR_α :	The Credit Value at Risk, representing the potential loss due to credit risk.
PD :	Probability of Default, indicating the likelihood that a borrower will default within a specified time frame.
LGD :	Loss Given Default, representing the proportion of exposure that is lost if a borrower defaults.
EAD :	Exposure at Default, the amount of exposure a lender faces when a borrower defaults.
$\Phi^{-1}(\alpha)$:	The inverse cumulative distribution function of a standard normal distribution.
α :	The confidence level, indicating the level of certainty associated with the VaR estimate.

When this method requires an extension to a portfolio VaR, this can be done with the incorporation of the correlation factors between the loans. This results in the following formula:

$$\begin{aligned}\mathrm{VaR}_\alpha = &\sum_{i=1}^{n} \mathrm{PD}_i \times \mathrm{LGD}_i \times \mathrm{EAD}_i \times \Phi^{-1}(\alpha) \\ &+ \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} \mathrm{Corr}_{i,j} \times \mathrm{PD}_i \times \mathrm{LGD}_i \times \mathrm{EAD}_i \\ &\times \mathrm{PD}_j \times \mathrm{LGD}_j \times \mathrm{EAD}_j \times \Phi^{-1}(\alpha)\end{aligned} \tag{9}$$

With the parameters being the following:

$\mathrm{VaR}_{\mathrm{portfolio}}$:	The Value at Risk for the entire credit portfolio.
n :	The number of instruments or counterparties in the portfolio.
$\mathrm{PD}_i, \mathrm{LGD}_i, \mathrm{EAD}_i$:	Probability of Default, Loss Given Default, and Exposure at Default for the i-th counterparty.
$\mathrm{Corr}_{i,j}$:	Correlation coefficient between the credit risks associated with the i-th and j-th counterparties.
α :	The confidence level, indicating the level of certainty associated with the VaR estimate.

This Monte Carlo method needs the following set of data:

- A credit rating system that allocates a Probability of Default to a specific credit rating of each counterparty.
- An estimation of Loss Given Default attached to each counterparty.
- An estimation of Exposure at Default attached to each counterparty.
- A chosen confidence level and the Z-score associated with it, drawn from the normal distribution.
- A correlation factor drawn that represents the correlation between defaults of different credit ratings.

3.3.5 Expert Judgment

Expert judgment in assessing credit risk involves incorporating subjective insights and qualitative evaluations provided by experienced professionals with industry-specific knowledge and expertise. This method complements quantitative models, enabling a more thorough and nuanced comprehension of creditworthiness. It takes into account factors extending beyond numerical data, including reputation, management quality, and the behavioural aspects of investment parties. These factors are often used in a multicriteria credit scoring model (Roy and Shaw, 2021). In a multicriteria credit scoring model, an evaluation is made of the creditworthiness of the analysed firm through a score dependent on financial and/or non-financial variables. The significance of the tested variables mostly derives from statistical learning. Examples of methods employed in these instances include linear regression, discriminant analysis, and probit models (Li et al., 2016). When the variables are statistically significant, they can be integrated into a framework that provides a grip on assessing creditworthiness. An example of such a framework is designed by Roy and Shaw (2021), as presented in Figure 2.

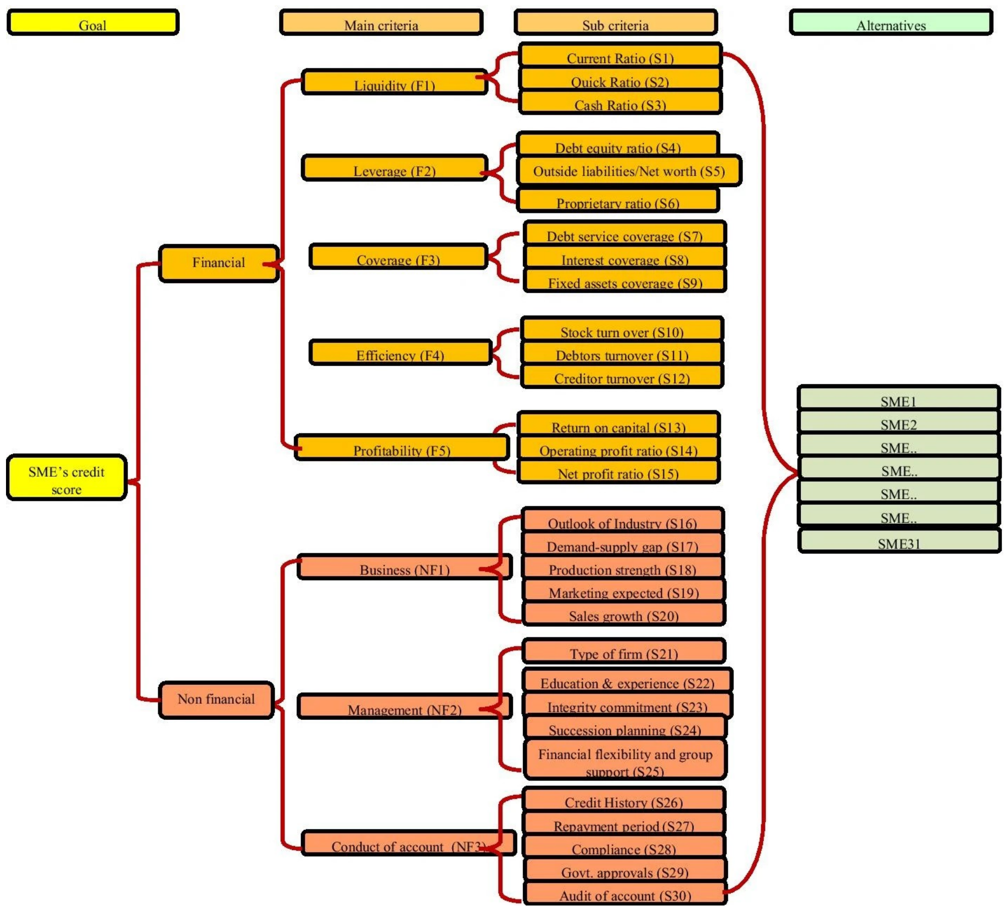


Figure 2: Decision layers of a multi-criteria credit scoring model (Roy and Shaw, 2021)

In addition to the factors in this multicriteria framework, other factors are considered by experienced professionals. Moro and Fink (2013) emphasise the importance of the trust variable, which is defined as a combination of factors, namely ability, benevolence, and integrity. The extent to which the borrower scores high on these factors influences their access to credit, and subsequently, the lender's decision-making process. Other relevant factors are the Management Board, Staff, Human resources management, Vision and Values, Processes and Technology, Innovation, Customers, Image of the company, Networks, and Transparency (Serrano-Cinca and Gutiérrez-Nieto, 2013).

Chi and Zhang (2017) have developed a similar multi-criteria credit rating model. They focus especially on small enterprises and use financial and non-financial factors to predict bankruptcy cases in China. They calculate Entropy Weights, an indicator of relative importance, for each individual factor. They were able to predict bankruptcy cases correctly 86.3% of the time. Figure 3 outlines all the evaluated factors.

(a) No.	(b) 1st Level Criteria	(c) 2nd Level Criteria	(d) 3rd Level Criteria	(e) Index	(f) Entropy Weighting
1	Repayment Ability	Internal Financial Factors	C_1 Solvency	X_5 Main business revenue/cash ratio	0.0418
2				X_{10} Super quick ratio	0.0394
3				X_{12} Net assets/year end loan balance ratio	0.0882
4				X_{13} Fixed capital ratio	0.0056
5				X_{15} Long term asset appropriation rate	0.1390
6				X_{20} EBITDA/current liabilities ratio	0.1284
7			C_2 Profitability	X_{24} Return on total assets	0.0674
8				X_{27} Gross margins	0.0667
9			C_3 Operating Capacity	X_{39} Shareholders' equity turnover rate	0.0708
10			C_4 Growth Capacity	X_{47} Capital accumulation rate	0.0006
11		C_5 External Macroeconomic Conditions		X_{49} Industry sentiment index	0.0386
12				X_{52} Consumer Price Index	0.0697
13				X_{54} Engel coefficient	0.0437
14		C_6 Internal Non-financial factors		X_{61} Sales reach	0.0467
15				X_{63} Ratio of loans repaid	0.0336
16	Repayment Ability	C_7 Legal Person Situation		X_{66} Legal person credit card history	0.0499
17				X_{68} Residence status	0.0236
18				X_{69} Time spent in current residence	0.0261
19				X_{74} Time in current position	0.0016
20		C_8 Enterprise Situation		X_{76} Credit received in past 3 years	0.0003
21		C_9 Commercial Reputation		X_{80} No. of breaches of contract	0.0007
22		C_{10} Collateral Guarantee Factor		X_{81} Collateral Guarantee Rating	0.0174

Figure 3: The small enterprise index system and the comparison with the five Cs of credit (Chi and Zhang, 2017)

Assigning coefficients and scores within such a scoring model is a tricky proposition. The applicability of different factors varies across different environments and is often sector-, demographic-, and company-specific. Depending on the interpretation and experience of the assessing professional, a coefficient may be established. No literature was found to explain the bias in this formation. The only relevant studies were performed by Serrano-Cinca and Gutiérrez-Nieto (2013) and Roy and Shaw (2021). According to Serrano-Cinca and Gutiérrez-Nieto (2013), the most important is the coherence of decisions made about prioritising factors. A widely applied method for this is the Best-Worst Method (BWM), a systematic approach of assigning coefficients to the individual factors (Roy and Shaw, 2021).

The necessary data depends on the type of model applied to assess a company's creditworthiness. Typically, available data is combined and used for the assessment. Examples of financial and non-financial ratios considered valuable by literature are shown in Figures 3 & 4.

3.4 SME Model Adjustments

In the "Groeifonds", as introduced earlier, we deal with a specific type of company, namely the scale-up. According to the Organisation for Economic Co-operation and Development (OECD), a scale-up is defined as an enterprise with average annualised growth greater than 20% per annum, over a three-year period, and with ten or more employees at the beginning of the observation period. Growth is measured by the number of employees and turnover (OECD, 2007).

As explained in Section 2.1 Core Problem Identification, categorising Horizon portfolio companies is necessary. Given that scale-ups in financial risk modelling remain underexplored in the literature, we opt to categorise them as Small and Medium-sized Enterprises (SMEs). Although scale-ups differ significantly from the generic SME definition, it represents the closest categorisation to ensure comprehensive research in the literature.

In the context of this thesis, Small and Medium-sized Enterprises (SMEs) are defined based on criteria outlined by the OECD. According to the OECD, an SME typically falls within certain thresholds regarding its size, turnover, and number of employees. While specific definitions may vary by country and industry, SMEs are generally characterised by their relatively small scale of operations compared to larger corporations. According to the European Commission (2003), SMEs are defined as follows:

Micro enterprises: Firms with fewer than 10 employees and an annual turnover or balance sheet total not exceeding €2 million.
Small enterprises: Firms with fewer than 50 employees and an annual turnover or balance sheet total not exceeding €10 million.
Medium-sized enterprises: Firms with fewer than 250 employees and an annual turnover not exceeding €50 million or a balance sheet total not exceeding €43 million.

These thresholds may vary depending on the industry and specific regulations of each country. However, the fundamental characteristic of SMEs is their relatively small scale of operations compared to larger corporations.

In the realm of credit risk modelling for SMEs, several key risks exert great influence (Falkner and Hiebl, 2015; Woschke, Haase, and Kratzer, 2017; Gupta et al., 2014; Rathnasiri, 2014; Korpysa, 2020):

- **Limited Access To Financial Resources**: SMEs struggle to access sufficient funds, affecting their ability to manage economic downturns and meet debt obligations.
- **Volatility in Cash Flows**: SMEs experience irregular cash flow patterns due to market fluctuations, making their creditworthiness assessment challenging.
- **Lack of Diversification**: SMEs operate in narrow markets, lacking diversified revenue sources and facing heightened sector-specific risks.
- **Informal Reporting Practices**: SMEs maintain less rigorous financial reporting, leading to opaque financial statements and data availability issues for credit risk assessment.
- **Entrepreneurial Management**: SMEs prioritise growth, potentially compromising financial stability through aggressive strategies, elevating credit risk.

We use these key risks to define the SME environment. When we look back on the methods that we identified in Section 3.3 Credit Risk Modelling, the following methods have to be re-evaluated considering the SME environment:

- **All Accounting-based Models**
- **Value-at-Risk Model**

Firstly, we discuss the accounting models: Financial Statement Analysis, Altman Z-score, Zeta Model, and O-score. In essence, the SME-specific effect on accounting-based methods is quite straightforward. This is due to the simplified form of financial reporting allowed for SMEs, in contrast to companies required to adhere to the International Financial Reporting Standards (IFRS) (Perera and Chand, 2015). Consequently, the analysed company may not report some financial numbers necessary for these accounting-based models. Another aspect that has to be considered is the applicability of the accounting-based models. All models discussed were designed to have explanatory value in specific types of companies or industries, e.g. Altman Z-score for manufacturing companies. Therefore, the models have to be evaluated for their applicability to SMEs. Given these two factors, the current models are no longer suitable for assessing the credit risk of SMEs. Models might be redeveloped to apply to the SME environment, but these accounting-based analytical formulas do not exist yet.What does exist are default predictions that offer insight into the explanatory value of various financial numbers. However, it's crucial to note that the effectiveness of any model developed is contingent upon the quality of the data it utilises.

Secondly, we discuss the Value-at-Risk model. Fundamentally, the analytical formula of this method remains unchanged when considering the SME environment. However, the frameworks of the method undergo alteration, particularly in the calculation of the Probability of Default (PD). It is justified that the PD assigned to the evaluated SME should be derived from a representative pool of SMEs. Additionally, all parameter calculations remain unaffected. The adjustment required in the PD calculation involves moving away from the original calculation based on credit ratings. Gabbi, Giammarino, and Matthias (2020) propose an approach to incorporate soft information into the PD calculation, employing a regression analysis on an SME sample in Italy. Other methods identified in Section 3.3 Credit Risk Modelling could also be applied in the PD calculation.

4 Literature Review: Market Risk and Liquidity Risk Modelling

Chapter 4 constitutes the second part of the literature review. Here, we aim to identify methods for modelling market risk and liquidity risk associated with a direct investment, which is another financing instrument utilised by Horizon. We identify two underlying risk types in these direct investments, namely market risk and liquidity risk. We define these two types of risk. We define these two types of risk and subsequently identify methods to capture both types of risk. As in Chapter 3, we discuss the data required for these methods, considering the data-scarce environment within Horizon.

4.1 Market Risk

A direct investment is the acquisition of a controlling interest in a business. This investment provides capital funding in exchange for an equity portion of a company. Horizon's choice to acquire share capital often stems from how Horizon wants to be involved in the company. In addition, it also depends on liquidity within the fund. For example, if few financial resources are available within the fund, then a loan is preferred, given that the repayments contribute to liquidity within the fund. But, if there is more room in the financial resources, then a direct investment might be preferred.

Any direct investment is subject to the overall systematic risk of the financial markets, also known as market risk. Market risk encompasses the inherent risk in the overall performance of investments in financial markets, including interest rate risk, equity price risk, foreign exchange risk, and commodity risk (Nickolas, 2022). Various models have been trying to capture this market risk factor. One of the most prominent methods is developed by Fama and French (1993), which was later developed into a five-factor model also designed by Fama and French (2014). In essence, this model captures excess returns of a portfolio in various variables like, amongst others, Market risk, Small minus Big (SMB), High minus Low (HML), and Robust minus Weak (RMW). The level of market risk is determined by the market risk premium, representing the difference between the expected market return and the risk-free rate. Another indicator of market risk is β, as per the Capital Asset Pricing Model (CAPM) (Sharpe, 1964):

$$\beta = \frac{\text{Cov}(R_i, R_m)}{\text{Var}(R_m)} \tag{10}$$

Where:

β :	Beta of the asset
$\text{Cov}(R_i, R_m)$:	Covariance between returns of the asset R_i and market returns R_m
$\text{Var}(R_m)$:	Variance of the market returns R_m

However, applying both methods in the context of private firms poses challenges due to the limited availability of financial data necessary for these analytical formulas. Share prices of private firms, crucial for calculating β, can be estimated through discounted cash flow analysis or comparable company analysis. Yet, these estimates are seldom market-validated. The most reliable validation typically arises from the fair value of the firm's shares established during fundraising rounds. Unfortunately, fundraising rounds occur infrequently. Therefore, we face different aspects of market risk than publicly traded shares. As we face systematic risk, this market risk cannot be eliminated by a diversified portfolio.

The specific market risk that Horizon is exposed to is not the market risk involved with publicly traded shares. To some extent, Horizon operates similarly as a private equity firm. Horizon operates in the segments of mainly SMEs that cannot acquire financing through the debt and regular equity markets. Most of the time the inaccessibility to financing is due to the risk involved with the businesses. However, the aim of Horizon's financing practices does not correspond to the financing practices of private equity funds. Mainly, since the profit motive of private equity funds naturally lies in producing superior returns, whereas with Horizon these lie mainly in sustaining the fund and ensuring revolvability. The similarity lies in managing the market risk involved in private equity investing, which justifies the comparison between Horizon and private equity. Significant differences exist when compared to publicly traded shares on a stock exchange. For instance, valuing private companies is typically more complicated due to the lack of market data, as their share prices are not directly determined by supply and demand. Additionally, private companies generally disclose less information than public companies because they are not subject to the same level of disclosure requirements. This makes measuring market risk for direct investments more challenging.

These factors significantly influence the environment in which Horizon operates. The limited availability of market information and the complexities of valuation techniques make it crucial to accurately identify and quantify market risk. The inherently risky nature of the businesses in which Horizon invests further emphasises the importance of this task.

4.2 Liquidity Risk

Besides the implications of market risk, there is another risk factor that comes into play when investing directly in private companies: liquidity risk. In the context of financial markets, liquidity pertains to the ease with which an asset or security can be bought or sold. Essentially, it characterises the speed at which an asset can be converted into cash. In times of illiquidity, the asset holds its value, but the absence of buyers hinders its conversion into cash, often requiring a substantial discount for transactions. Franzoni, Nowak, and Phalippou (2012) find that the liquidity risk premium implies a discount of roughly 10% in the valuation of the typical investment, based upon a data set of 652 Private Equity (PE) houses. Acharya and Pedersen (2005) point out that liquidity varies over time, affecting the rate of return on the asset class. This has major implications, especially since direct investments are often accompanied by an exit strategy. It is crucial to include this inherent risk in the evaluation of direct investments, given the significant potential for downward valuation adjustments. As liquidity risk is mainly a risk at the proximity of the exit moment, liquidity risk is also referred to as exit risk (Cumming, Fleming, and Schwienbacher, 2005).

Different measurement parameters of liquidity exist in financial markets. Amihud (2002) examines the average ratio of daily return to dollar trading volume on that day, where the percentage price change per dollar of daily volume is interpreted as the daily price impact of order flow. Pástor and R. F. Stambaugh (2003) use a complex regression procedure involving daily firm returns and signed dollar volume to measure price reversals, both at the firm and market levels. Price reversals are viewed as reflecting illiquidity. These indicators of liquidity may only be applied to publicly traded shares.

While these concepts provide an indication of market liquidity, they do not directly address the impact on price realisation in the private equity market. Therefore, it is important to evaluate the impact of liquidity. Al Janabi, Ferrer, and Shahzad (2019) investigated this and incorporated liquidity risk within market risk modelling using a GARCH(1,1) model. This method of modelling liquidity risk is calculation-intensive and requires numerous asset price data points.

Cumming, Fleming, and Schwienbacher (2005) measure liquidity in the private equity market by the number of IPOs per year on the NASDAQ, and NYSE. This is the only study that focuses on non-publicly traded companies, namely private equity. As concluded earlier, there is indeed a price impact if liquidity risk is observed by the investor. In terms of liquidity risk modelling, liquidity risk is primarily considered an additional risk factor, necessitating additional compensation for the risk incurred, known as the liquidity risk premium (Liu, 2006).

4.3 Risk-Return Trade-off

Market risk comes into play in modern portfolio theory in financial markets, where higher returns are achievable only by exposing oneself to higher risk. The pioneering work of Markowitz (1952) laid the groundwork for this several decades ago. Markowitz's approach to portfolio construction aims to achieve optimal diversification, minimising unsystematic risk while maximising the portfolio's risk-adjusted return. Theoretically, this approach makes a lot of sense and ensures a very consistent way of portfolio construction. Actively deciding on the risk taken on direct investments is very important for Horizon. Practically, it is difficult for Horizon to actively implement this in their investment policy. Mainly because the region in which they operate is very young and the scale of financing requests is not so large that it can provide Horizon with major diversification effects in the "Groeifonds". Therefore, we decide to ignore diversification effects at the quantitative level in market risk modelling. Although we disregard quantitative diversification effects, it is worthwhile to use the concept of diversification at a qualitative level. In addition, it is worth going deeper into the expected returns in the private equity environment and the typical risks of these investments.

The risk assumed through direct investment comes mainly from the type of business Horizon invests in. As mentioned in Section 1.2, "Groeifonds" companies are characterised by their scale-up phase, which inherently carries a higher risk profile. These companies are often still seeking market traction, mainly because their revenue models have not yet been validated. A company's stage of development is often indicated by the Technology Readiness Level (TRL), where a typical "Groeifonds" company is at TRL stage 8 (Rijksdienst voor Ondernemend Nederland, 2022). This means the business is as good as its final shape and the business model is completely clear. In addition, the technological workings have been tested and the company complies with laws and regulations. The risk Horizon takes by investing in these companies is inherent to the stage of development the company is in.

In the best-case scenario, returns would be optimal at a relatively high level of risk. However, this is neither a Horizon objective nor a feasible scenario. Horizon accepts a market-based return for the relatively high level of risk. Additionally, the limited scope for diversification means that having a risk-adjusted optimal portfolio is unachievable.

4.4 Market & Liquidity Risk Modelling

Now that we understand the definition of market and liquidity risk and know the risk-return trade-off that Horizon deals with, we can proceed with identifying possible methods to model market and liquidity risk. As we identified in Section 4.1

Market Risk, liquidity risk is an additional risk factor to market risk. Therefore, we aim to identify market risk modelling methods and after that adjust the model for the adoption of liquidity risk. We narrow the methods down based on the data availability. Appendix B summarises the methods that are considered out of scope due to inapplicability. Given this, we find two different methods that apply to Horizon:

- **Value-at-Risk**
- **Expected Shortfall**

The potential situation might arise where the identified method is not applicable in both risk modelling before and during a direct investment, as we addressed in Section 2.1 Core problem identification. In this particular case, a section in Chapter 5 Methodology is devoted to adapting the chosen method to enable both risk modelling situations. We decided to do this to avoid excluding any methods in advance and to maintain as wide a view of solutions as possible.

Before delving into the two identified methods, we dedicate a subsection to volatility. This is because volatility is a relative measure of risk and is frequently used in market risk as a parameter in the Value-at-Risk method. Given the various ways of measuring volatility, it is necessary to outline the background of this parameter.

4.4.1 Volatility

Volatility measures the dispersion of asset prices or returns from their average value over a specific time period. The perception that volatile assets carry higher risk compared to less volatile ones stems from the anticipation of their prices being less predictable. In essence, there are two kinds of volatility: historical volatility and implied volatility. Historical volatility is the measure of past price fluctuations of the assets. Implied volatility is a measure of the expected volatility of a financial asset's price in the future, as implied by the prices of its options contracts. Given the low disclosure and standardisation of option contracts in direct investment, we exclude the application of implied volatility. Therefore, we evaluate only historical volatility. There are several ways to calculate historical volatility, including simple volatility, the Exponentially Weighted Moving Average (EWMA), and Generalised Autoregressive Conditional Heteroskedasticity (GARCH).

Simple Volatility

According to the method of simple volatility, volatility is defined as the statistical relationship between the differences in returns for a given instrument over a given time period. The importance of this method lies in assessing how instability can influence investors' expectations regarding the potential extent of market changes, providing valuable insight for making price forecasts and executing trades. In other words, investors use volatility to gauge the market's status and act accordingly. The formula for calculating simple volatility is given by:

$$\text{Volatility} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(r_i - \bar{r})^2} \tag{11}$$

Where:

n :	is the number of historical data points considered.
r_i :	represents the returns at each data point.
$\bar{r}$:	denotes the average return over the historical period.

This approach applies to both individual financial instruments and portfolios. For individual financial instruments, the method involves determining the volatility specific to that instrument. In the context of a portfolio, the method calculates the aggregate volatility by considering all instruments within the portfolio.

Exponentially Weighted Moving Average (EWMA)
Exponentially Weighted Moving Average (EWMA) is a technique employed to estimate volatility in financial markets, first, introduced by Roberts (1959). Unlike the simple volatility method, EWMA assigns more weight to recent data points, rendering it more sensitive to recent market fluctuations. This approach proves invaluable in gauging market instability and its potential ramifications on future price movements, thereby assisting investors in making well-informed decisions.
The formula for computing EWMA volatility is:

$$\sigma_n^2(\text{EWMA}) = \lambda\sigma_{n-1}^2 + (1-\lambda)u_{n-1}^2 \tag{12}$$

Where:

$\sigma^2_{n\text{EWMA}}$:	represents the exponentially weighted moving average volatility.
λ :	denotes the smoothing parameter, determining the weight accorded to recent observations.
σ_n^2 :	Variance at time n.
σ_{n-1}^2 :	Variance at time $n-1$.
u_{n-1}^2 :	Squared return at time $n-1$.

Similar to the simple volatility method, EWMA applies to both individual financial instruments and portfolios. It furnishes a volatility measure tailored to recent market dynamics, facilitating investors in evaluating market conditions and adjusting their strategies accordingly.

Generalised Autoregressive Conditional Heteroskedasticity (GARCH)

Generalized Autoregressive Conditional Heteroskedasticity (GARCH) is a widely-used statistical model employed for estimating volatility in financial markets, first introduced by Bollerslev (1986). Unlike simple volatility, GARCH accounts for the autocorrelation and time-varying nature of volatility, as EWMA does. This sophisticated model is particularly valuable in capturing the complex dynamics of financial markets and providing accurate forecasts of future volatility.
The formula for GARCH volatility is:

$$\sigma_t^2 = \omega + \alpha_1\epsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2 \tag{13}$$

Where:

σ_t^2 :	represents the conditional variance of returns at time t.
ω :	denotes the constant term.
α_1 :	represents the coefficient of the lagged squared error term ϵ_{t-1}^2.
β_1 :	represents the coefficient of the lagged conditional variance σ_{t-1}^2.

GARCH models are widely applied to both individual financial assets and portfolios. By accounting for the persistence and time-varying nature of volatility, GARCH offers enhanced insights into market dynamics and aids investors in risk management and decision-making processes. As can be observed from the formula, the outcome is generally the conditional variance. To align the outcome from GARCH with the outcomes from the EWMA and simple volatility, we simply take the square root of the variance to get the standard deviation.

4.4.2 Value-at-Risk

Having discussed the three different methods of calculating the volatility metric, we now explain the method of Value-at-Risk. In Subsection 3.3.4 we already explained the method in the context of credit risk. Although we do not use the VaR method to model credit risk, we do use it to model market risk. Essentially, the intuition behind the model remains the same, based on the assumption of normally distributed returns. We identify the same three types of modelling approaches as discussed in Chapter 3, Literature Review: Credit Risk Modelling, namely:

- Parametric Method
- Historical Method
- Monte Carlo Method

Since we have already discussed the essence of the three types of VaR in Chapter 3, we briefly repeat the formula and any change in the definition of parameters.

Parametric Method
The Value-at-Risk of a single instrument is calculated using the following formula, assuming normality:

$$\text{-VaR}_\alpha = \mu - \text{Portfolio Value} \times \text{Z-score}_\alpha \times \sigma \tag{14}$$

The variables are defined as follows:

VaR_α :	The Value at Risk, representing the potential loss within a specified confidence level.
μ :	The mean return realised with this instrument.
Z-score :	The Z-score associated with the desired confidence level.
σ :	The standard deviation of the investment's returns over the specified time period.
Portfolio Value :	The total value of the portfolio.

The VaR parameters for a portfolio consisting of two securities are constructed as follows:

$$\mu_p = w_1\mu_1 + w_2\mu_2 \tag{15}$$

In both formulas, the weights w_1 and w_2 represent the proportion of the portfolio invested in securities 1 and 2. ρ_{12} is the correlation between their returns.

$$\sigma_p = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho_{12}} \tag{16}$$

The Value-at-Risk (VaR) for a portfolio with two securities, assuming normality, is given by:

$$\text{-VaR} = (\mu_p - \text{Z-score}_\alpha \cdot \sigma_p) \tag{17}$$

One distinguished change in the model should be observed, namely the meaning of the volatility variable. In credit risk, this variable indicates the change in the value of the credit instrument or credit portfolio. In market risk, volatility is the fluctuation in asset price or portfolio value.

This Parametric method needs the following set of data:

- A list of historical daily volatilities for the specified time period.
- A list of historical returns of the instrument for the specified time period.
- The portfolio value of the instrument at the point of measurement.
- A chosen confidence level and the Z-score associated with it, drawn from the normal distribution.

Historical Method
The method consists of the following step-by-step approach, assuming a 170,000 euro portfolio:

1. Select a confidence level, say 95%.
2. Select a lookback window, say 100 days.
3. Collect daily gains or losses on your portfolio for each day.
4. Arrange all these daily values in ascending order, with the greatest losses on top.
5. The corresponding level of significance is 100 - 95 = 5%, which implies taking the 5% of 100 = 5th return from the left, e.g. -2.1%. Therefore, the Value-at-Risk = -2.1% of 170,000 = -3,570 euro. Statistically, there is a 5% chance that the daily loss will be more than the Value-at-Risk.

In essence, nothing changes to this method in comparison to credit risk modelling.

This Historical method needs the following set of data:

- A lookback period in days.
- A list of historical returns of the instrument for the specified time period.
- A chosen confidence level.

Monte Carlo Method
The Monte Carlo method is the final technique we explore, employed for estimating the VaR metric. This method differs significantly from the one applied in the credit risk context because we focused on the CreditMetrics approach for credit risk. This method does not apply to market risk, and thus the application of Monte Carlo is quite different. Unlike an analytical formula, it involves a step-by-step approach similar to Historical Simulation:

1. Select a confidence level α, say 95%.
2. Establish the asset value.
3. Simulate percentual returns from the normal distribution, with a chosen mean and standard deviation.
4. For each simulation calculate the euro return, based on the asset value and simulated return.
5. We sort the euro returns from low to high in ascending order.
6. We take the $1 - \alpha$ percentile from this sorted list, this is the α-VaR.

When this method requires an extension to a portfolio VaR, it depends on the underlying assumptions of how to calculate the portfolio VaR. The construction can be done with the incorporation of the covariance factors, not assuming independence. The construction can also be done assuming independence. Then, we simply sum the corresponding losses over each asset and apply the same sorting principle. For now, we choose to explain the method of assuming independence. However, should this model be chosen, we will discuss it in a separate section in Chapter 6 Model

Construction. Knowing this, we can take the following steps to calculate the portfolio VaR:

1. Select a confidence level α, say 95%, which should be the same for all the individual VaRs.
2. Take the ordered list of returns from each asset in ascending order.
3. Sum the corresponding returns over all assets to obtain the overall portfolio return.
4. We order the list of portfolio returns in ascending order.
5. We take the $1-\alpha$ percentile from this sorted list, this is the α-VaR.

The Monte Carlo method needs the following set of data:

- The individual assets values.
- The corresponding weights in the portfolio.
- List of generated asset returns.
- A chosen confidence level.

4.4.3 Expected Shortfall

Expected Shortfall (ES), also known as Conditional Value-at-Risk (CVaR), is a risk measure that provides insights into the tail behavior of the distribution of losses. Unlike VaR, which assesses the probability of experiencing losses beyond a specific threshold, ES evaluates the average magnitude of losses occurring beyond the VaR level. Essentially, ES quantifies the expected loss given that the loss exceeds the VaR threshold.
The computation of ES involves two primary steps: first, identifying the VaR threshold, and second, calculating the average of losses surpassing this threshold. ES can be expressed mathematically as the conditional expectation of losses exceeding the VaR level. For individual assets, the calculation of ES follows a similar principle. However, instead of integrating the entire loss distribution, the focus is on the loss distribution of the specific asset. The formula for computing ES for an individual asset is as follows:

$$\mathrm{ES}_\alpha = \frac{1}{1-\alpha}\int_{-\infty}^{-\mathrm{VaR}_\alpha} x \cdot p(x)\,dx \tag{18}$$

Where:

ES_α :	The Expected Shortfall at the confidence level α for the individual asset i.
$p(x)$:	The probability density function (pdf) of the asset's loss distribution.
x :	The potential loss value.

For portfolios, the calculation of ES involves aggregating the losses across all assets within the portfolio. The formula for calculating ES for a portfolio is given by:

$$\mathrm{ES}_\alpha^P = \frac{1}{1-\alpha}\int_{-\infty}^{-\mathrm{VaR}_\alpha^P} x \cdot p^P(x)\,dx \tag{19}$$

Where:

ES_{α}^{P} :	The Expected Shortfall at the confidence level α for the portfolio.
x :	The loss of the portfolio.
$p^{P}(x)$:	The probability density function of the loss distribution of the portfolio.

The computation of ES is rather complex compared to the previously introduced methods. Additionally, it requires an approximation of the probability density function of the loss distribution, which is challenging due to the absence of data within Horizon. Nevertheless, the computation of ES provides valuable insights into the tail risk of both individual assets and portfolios, aiding investors in making informed decisions regarding risk management and portfolio optimisation.

4.5 Model Adjustments

With regards to model adjustments, we have the model adjustments to the SME environment and the adjustments to the liquidity risk. We apply the same rationale as in Chapter 3, where we categorise the Horizon portfolio companies as SMEs. We discuss the model adjustments in this section.

4.5.1 SME Environment

In exploring specific market risk modelling changes for SMEs, it is necessary to delve into the unique challenges and risk factors these businesses encounter within the market landscape. Market risk encompasses various factors that can lead to financial losses, impacting SMEs' stability and growth prospects. Within this context, key risk factors for SMEs in market risk modelling include (Gherghina et al., 2020):

- **Local Economic Conditions:** SMEs' performance is directly tied to local economic conditions, with changes directly impacting revenue streams and subsequently equity prices.

Certainly, numerous other risk factors affect businesses. However, this particular risk factor directly influences the market risk being evaluated. Conversely, other risk factors do not have a direct impact on market risk. Local economic conditions represent a dependency that SMEs must contend with. Due to the specific nature of this characteristic, quantifying it as a risk factor becomes challenging. As a result, its impact on market risk evaluation is complex. Therefore, we consider this specific risk factor to be out of scope. This also means no adaptation to the identified market risk modelling methods is needed.

4.5.2 Liquidity Risk

As we concluded earlier in Section 4.2 Liquidity Risk, the liquidity risk premium is the form in which liquidity risk materialises. Vette et al. (2023) observe that market liquidity risk has increased as the volatility has increased in financial markets, mainly due to the unpredictability of the monetary policy. The numerical impact it has on the market risk-measuring techniques is also dynamic. In other words, liquidity risk varies over time. Given this, the volatility measurement does not need a liquidity adjustment as the market price already incorporates liquidity. The only method that needs direct adjustments is the VaR method. Inherently, this changes the ES method, due to its dependency on VaR.

Extensive research has been conducted on quantifying the liquidity risk premium for publicly traded companies. Wang (2017) calculated the cost of liquidity to be 0.16%

for large-cap funds, based on a sample of companies from Sweden. However, the sample of small-cap funds had a liquidity cost of 8.61%, based on a sample of Swedish companies. These premia were calculated over a 90-day period. It is unclear which year this data comes from. Nadauld et al. (2019) find that for the researched funds, the typical transaction is at a discount of 5% if adjusted for liquidity risk. Hibbert et al. (2009) report a liquidity premium of 0% to 1.5%. Other researchers, such as Anson (2017), discovered that negative risk premiums appear in financial markets as well. In other words, an investment is awarded a premium if it is illiquid. All these results come from publicly traded assets, a type of asset that is less subject to liquidity risk influences and thus also to the premium paid for this liquidity (Anson, 2017).

The only traceable literary work produced on quantifying liquidity risk in non-publicly traded shares is done by Anson (2017) in private equity. He reports a significantly higher liquidity risk premium of 3.5%, compared to the average of the reported ones with publicly traded shares.

5 Methodology

In the literature reviews of Chapters 3 and 4, we identified the possible methods we can apply to research questions 1 and 2, respectively. Additionally, we addressed the follow-up research question on categorising the company environment, thus answering the second part of the research question concerning model adjustments. This chapter discusses relevant considerations regarding topics pertinent to our research. We then conduct a data analysis to characterise typical "Groeifonds" companies. Subsequently, we determine the modelling approach for credit risk, as well as for market and liquidity risk. Finally, we delve into the parameter construction of the corresponding model and implement necessary model adjustments, as discussed in Sections 3.4 and 4.5.

5.1 Considerations and Assumptions

In this section, we discuss three crucial considerations for scoping, as they are not directly elaborated on in the research questions. These considerations are the categorisation of convertible loans, the application of the proposed financial risk modelling approaches before and after the financing decision, and addressing the interdependency between loans and direct investments.

5.1.1 Convertible Loans

We have discussed two types of financing, namely loans and direct investments. In practice, Horizon employs another financial instrument for providing financing: the convertible loan. We discuss the categorisation of this debt instrument as it exhibits characteristics of both debt and equity. A convertible loan is a debt instrument that can be converted into equity. In practice, the convertible loan is often referred to as the Convertible Loan Agreement (CLA). A CLA involves providing a loan that can be converted into shares or share certificates at a later date, typically triggered by a conversion event. When the loan is converted into shares at such an event, the company issues shares to the lender, who offsets the consideration against their loan, plus interest.

The CLA exhibits various forms. The most common CLA involves conversion either at the end of the term or upon a Qualified Financing event at a valuation to be determined (or at a set valuation). However, convertible loans that are non-repayable (and thus always convert) are increasingly prevalent. Additionally, there is a rising trend of convertible loans that convert at a predetermined price per share, with or without a mechanism to adjust the price.

It is important for Horizon to categorise convertible loans as they should also be included in the financial risk analysis. The use of a convertible loan for Horizon mainly arises from the difficulty in making a valuation at the start of financing. According to Imdieke and Weygandt (1969), to classify convertible debt properly, one must predict the likelihood and timing of conversion. Unfortunately, these factors are challenging to determine at the start of financing. In practice, this instrument, which may convert into equity, is treated as debt and/or equity depending on accounting practices. In this research, we treat these convertibles as equity, since the likelihood of conversion is rather high at Horizon.

5.1.2 Pre- and Post-Financial Risk Assessment

In Literature Review Chapters 3 and 4, there is no active distinction between the pre-financial and post-financial risk assessment. All the methods mentioned can be used for both purposes. One example is mentioned in Chapter 3, namely credit scoring. This methodology is applicable both before a loan origination and during a loan but is almost always used in practice before origination. In the case of Horizon, the chosen method for financial risk modelling must fulfil both dimensions. This is because Horizon reassesses the targeted company's financial health periodically to reevaluate financial risk. Although this topic is not directly mentioned in the literature review, it is crucial to make a consistent financial risk assessment. The applicability of all identified methods to both dimensions positively influences the impact of this research.

5.1.3 Dependency Between Loans and Direct Investments

For all the identified methods in Literature Review Chapters 3 & 4, we describe possible extensions to a portfolio evaluation level. In these portfolio extensions, there are two approaches to dealing with the interdependency of variables: accounting for it with a covariance/correlation factor or assuming independence. Here, we refer to dependencies within loans and direct investments; we do not model possible dependencies between a loan and a direct investment.

To determine the most appropriate approach, it is essential to understand the dependencies of all loans and direct investments. Additionally, it is essential to consider whether any form of dependency modelling is both feasible and necessary, given the limited data. For both loans and direct investments, we review the application of dependency modelling in Section 6.3 Dependency Modelling. We take this approach since the chosen modelling approach is essential for choosing the incorporation of dependency modelling. This modelling approach will be determined in Section 5.3 Model Decision.

5.2 Data Analysis

To answer research question 4, we must analyse the existing data about the "Groei Fonds". After this analysis, we can then use the results to ensure the applicability of the developed model. In other words, we ensure that the model uses only inputs that are available to Horizon.

As mentioned in Section 1.2, the data constraint is one of the significant challenges in this research. The "Groei Fonds" consists of only five companies from which only a sparse amount of data is available, at least publicly available data. This information can be seen in Tables 1 & 2, where Table 1 is about non-financial information and Table 2 is about financial information. The data in Table 2 is processed financial information from the companies' balance sheets.

Company Name	Year of establishment	Sector	Employees
Company A	2015	Maritime	10 to 19
Company B	2017	Agricultural	15
Company C	2013	Technology	24
Company D	2013	Construction	33
Company E	2007	Pharmaceutical	45

Table 1: Groei Fonds companies' non-financial ratios

The names of the companies are kept confidential, however, this poses no issue as the companies' names are irrelevant in analysing the companies' characteristics. A closer look at Table 1, reveals that the companies fall into either the micro-enterprise or small-enterprise category, as defined in Subsection 3.3.4. This classification is based on the number of employees working for the respective companies. Additionally, most companies have existed for a considerable time. We know that the Technology Readiness Level (TRL) stage 8 is common for typical "Groei Fonds" companies, indicating that some time has been spent developing the technology.

Company Name	Year	Current Ratio	Quick Ratio	Cash Ratio	Debt-to-Equity Ratio	Proprietary Ratio
Company A	2021	0.62	0.29	0.00	−4.37	−0.30
	2022	0.82	0.28	0.00	−6.31	−0.19
Company B	2020	0.96	0.85	0.00	334.42	0.00
	2021	0.55	0.24	0.00	−2.50	−0.67
Company C	2021	2.00	2.00	0.00	3.66	0.21
	2022	0.55	0.55	0.00	−9.44	−0.12
Company D	2020	1.48	0.82	0.11	−4.55	−0.26
	2021	0.83	0.42	0.11	−4.55	−0.27
Company E	2021	1.7	1.49	0.21	0.55	0.65
	2022	1.38	1.29	0.19	0.53	0.65

Table 2: Groei Fonds companies' financial ratios

From Table 2, we gain an impression of the financials of the analysed companies. In general, we derive three categories of ratios: liquidity, leverage, and coverage. The current, quick, and cash ratios are liquidity ratios. The debt-to-equity ratio is a leverage ratio and the proprietary ratio is a solvency ratio. All these ratios can be derived individually from the financial statements published by the five companies. Unfortunately, these are also the only financial ratios that can be calculated.

Based on Table 2, we can say a few things about the "Groei Fonds" companies. First, we note that, in most cases, the companies' liquidity ratio is insufficient. The most representative ratio in this case is the Cash Ratio, the degree to which cash is sufficient to meet short-term obligations. Since most ratios are lower than one, this means that the cash on hand is not sufficient to meet short-term obligations. This can become a problematic situation as there are simply no financial resources to pay off the debts. Second, we note that incredibly high or negative Debt-to-Equity ratios dominate the Table. A reason for this could be that a lot of debt is often incurred at the establishment of a company, as the owner finds it difficult to relinquish his shares. Negative equity on a balance sheet occurs when a company's liabilities exceed its assets. This situation is common in start-ups, which often operate at a loss during their early years as they invest heavily in growth and development. These losses reduce retained earnings, a key component of equity. Additionally, start-ups may incur significant debt to finance their operations, increasing their liabilities. If the value of their assets, such as investments or inventory, decreases, the total assets diminish. Consequently, the equity becomes negative, indicating that the company's obligations surpass its resources.

Besides the publicly available data, Horizon has a small amount of data provided by "Groei Fonds" companies. In contrast, this information is not publicly available and is confidential. Part of this non-public data is substantive reports on research and development, and cash flow forecasts. Additionally, there is more detailed information on financial statements, contributing to a more complete flow of information. This includes a more detailed balance sheet and an income statement. However, the information is not as extensive or professional as that for mature and/or listed companies.

In summary, the information available to Horizon comprises limited financial information and somewhat more detailed non-financial information. Considering the identified methods in both literature reviews, particularly the data they require, it is evident that Horizon faces a significant data constraint.

5.3 Model Decision

As outlined in Section 1.2, the main issue for Horizon is to more accurately quantify the risk associated with financing. Therefore, it is desirable to apply a method that places risk in a relative framework, making it interpretable. In this section, we elaborate on the chosen methods for financial risk modelling.

5.3.1 Credit Risk

Regarding the credit risk model, the primary factor influencing our choice is the data constraint, as elaborated in Section 5.2 Data Analysis. From this, we conclude that accounting-based methods are overly dependent on data availability and completeness. The same limitation applies to the discriminant analysis model, as there is insufficient data to establish a predictive model like discriminant analysis. Consequently, these methods are not sufficient for a comprehensive assessment of financial risk. Therefore, we seek a more complete method that ensures applicability within Horizon and allows for necessary adjustments to achieve a more appropriate risk assessment. These attributes are most prominent in the widely used Monte Carlo Value-at-Risk model. Additionally, this type of model is currently applied to risk modelling within Horizon. The Probability of Default adaptation allows for expert judgment to come into play and add the explanatory value of qualitative data. As per Gabbi, Giammarino, and Matthias (2020), we apply this combination of identified methods for credit risk modelling.

5.3.2 Market Risk & Liquidity Risk

We have two options for the Market and Liquidity Risk model: Value-at-Risk (VaR) and Expected Shortfall (ES). The models differ from each other in the way they calculate the risk. Value-at-Risk (VaR) determines the maximum potential loss within a given confidence level, while Expected Shortfall (ES) calculates the average magnitude of losses beyond the VaR threshold. ES proves particularly valuable in understanding the tail risk associated with losses. However, for the specific context of Horizon, where quantifying the maximum potential loss with a certain confidence level holds greater significance than assessing tail risk, we opt for the VaR approach for this risk category. Specifically, we choose the Monte Carlo method. This choice is due to the unavailability of return data, rendering the Historical Method inapplicable. Additionally, the Parametric Method is simple but static, whereas the Monte Carlo VaR method allows for practitioner methods. The more comprehensive nature of the Monte Carlo VaR method makes it preferable to other VaR methods. Therefore, we proceed with Monte Carlo VaR for Market and Liquidity Risk.

5.4 Parameter Construction

In this section, we build upon the modelling decisions from the previous subsection. We concluded that the Monte Carlo Value-at-Risk modelling approach is best suited for evaluating both credit risk and market and liquidity risk. We follow up on this decision by constructing the parameters necessary to apply this method.

5.4.1 Value-at-Risk (Credit Risk)

In this subsection, we construct the individual components of the Monte Carlo Value-at-Risk, namely the PD, LGD, EAD, and the cumulative inverse normal distribution. It is important to note that all the parameters we cover are dynamic. This means that the values used change over time and should be updated frequently to ensure their applicability and explanatory value. In Figure 4, we illustrate the steps from the simulation of defaults to the calculation of VaR. It is an easy way of identifying how each parameter contributes to the calculation of VaR.

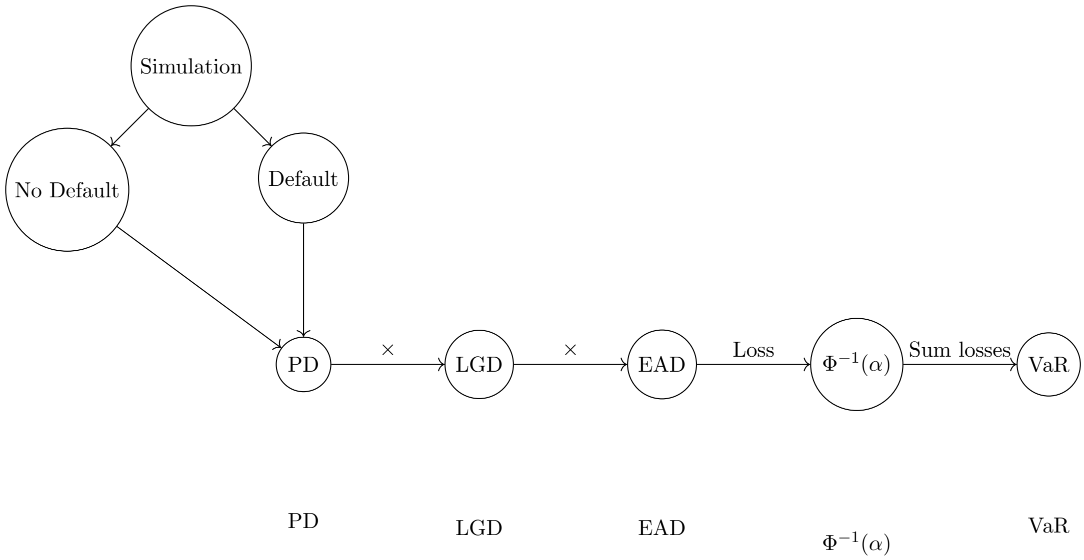


Figure 4: Relationship between Rand (random number), Default/No Default, PD, LGD, EAD, $\Phi^{-1}(\alpha)$, and VaR

Probability of Default (PD)

As per Subsection 5.3.1, we decide on integrating expert judgment in Monte Carlo VaR to apply the qualitative component in assessing credit risk, following the method of Gabbi, Giammarino, and Matthias (2020). For this method, we use a set of qualitative features derived from the literature review, where they have proven their explanatory value. For this feature identification step, we use Roy and Shaw (2021), Chi and Zhang (2017), Gabbi, Giammarino, and Matthias (2020), and Moro and Fink (2013). In deciding which features to apply in the PD model, we look at applicability, unambiguity, measurability, and consistency. All these factors are tested against expert opinion within Horizon. An essential part of applying these features is to ensure uniformity and objectivity in their measurable definitions. To achieve this, the definitions are taken from the supplementary documents provided with the papers from which the features are derived. The set of chosen features is presented in Figure 5.

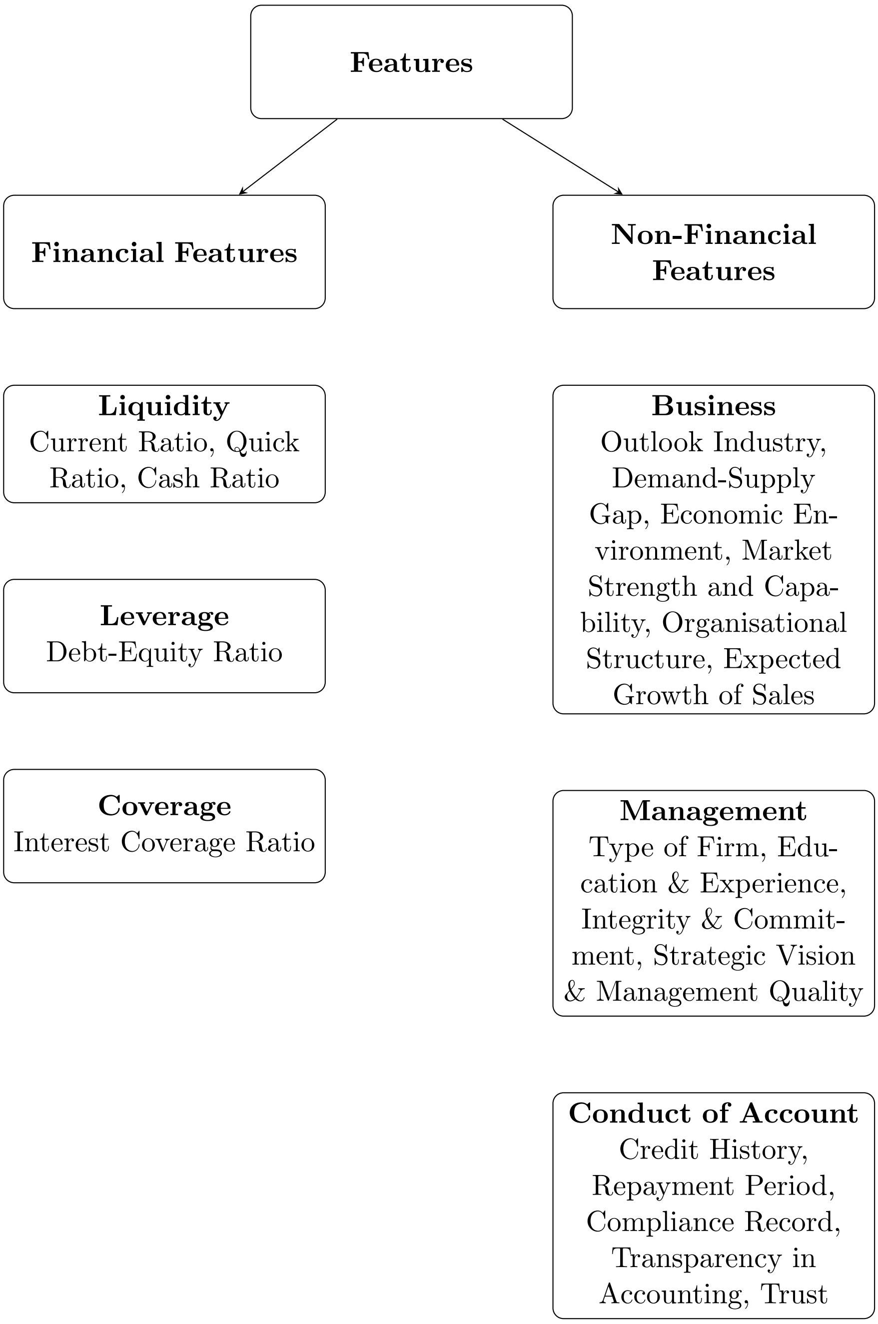


Figure 5: Classification of all features that form the basis of assessing creditworthiness

Now that we have identified the features, we can begin assessing these features and how they come together in an appraisal of creditworthiness. All features are rated on a Likert scale from 0 to 4, as applied by Roy and Shaw (2021). The score is then adjusted based on the weight assigned to each feature and subsequently based on the weight of the feature category. For assigning weights to each feature and each feature category, we apply the Best Worst Method (BWM) as proposed by Rezaei (2015). BWM is a multi-criteria decision-making (MCDM) method, which supports finding the importance (weight) of each feature that is used in modelling the PD. To ensure consistency, we use expert opinion ratings for individual features from the same literature that initially identified the characteristics. For the weights corresponding to the feature categories, we use expert opinion from Horizon. Finally, a score is produced, which is then normalised on a scale of 0 to 1.

To derive a Probability of Default (PD), we need a mathematical transformation of this score. Given that we already have a normalised score, we can put it into a mathematical function. One effective method for modelling PDs is a logistic function. The logistic function is expressed as follows:

$$P(x) = \frac{1}{1 + e^{-ax}} \tag{20}$$

Where:

$P(x)$:	Probability as a function of score x
x :	Score
a :	Constant that affects the steepness of the curve

The logistic function is particularly suitable because it produces values on a 0 to 1 scale, allowing for probability modelling. Additionally, it is monotonically increasing, making it straightforward to derive the PD, provided the function accurately models defaults. Moreover, the function's steepness parameter allows for the modelling of different time horizons, such as 1-year PD, 5-year PD, or 10-year PD. This flexibility is crucial for our analysis. Other methods, such as the exponential function or directly interpreting the scores, have also been evaluated. However, the exponential function does not provide the beneficial S-shaped curve for modelling PDs, and directly interpreting the scores is unsuitable due to their lack of explanatory value, offering only a relative scale indication. Therefore, we proceed with the logistic function.

To construct the analytical function, we fit the logistic function to global default rates. We base this fitting on credit ratings, assuming that a credit rating corresponds to a level on the scoring scale. In the model, we establish intervals corresponding to credit ratings. We distinguish nine credit ratings: AAA, AA, A, BBB, BB, B, CCC, CC, and C, following the terminology of Standard & Poor's (S&P). The highest credit rating corresponds to an AAA rating, and the worst score (non-default) corresponds to a C/CCC rating. These ratings are mapped to an interval between 0 and 1. For this model, we assume that each credit rating tranche is of equal size. This implies that we have 1/9 as the size for each credit rating. Given this, the AAA rating corresponds with a score between 8/9 and 1. Alternatively, the CCC/C rating corresponds with a score between 0 and 3/9.

We make a distinction between the 1-year, 5-year, and 10-year PD. We do this since they align with the evaluation criteria of Horizon. Therefore, we formulate three formulas, where each formula aligns with the changes in PD corresponding to the time horizon. For this, we use the Global Average Cumulative Default Rates over the years 1981 to 2018 collected by S&P, since these are the most recent numbers and, most importantly, not behind a paywall (Vazza et al., 2018). Ideally, a PD dataset corresponding to the target group of this study, namely SMEs, would be used for the logistic function. Due to data limitations, it is not possible to find a more suitable dataset for this purpose. As global default rates are publicly available and cover a wide range of companies, we see this as the best alternative solution. When we fit this set of data to the logistic formula, for the 10-year PD we find:

$$S(x) = \frac{1}{1 + e^{-3.706x}} \tag{21}$$

For the 5-year PD we find:

$$T(x) = \frac{1}{1 + e^{-4.373x}} \tag{22}$$

For the 1-year PD we find:

$$U(x) = \frac{1}{1 + e^{-7.628x}} \tag{23}$$

Based on these formulas we can estimate the outcome of each function. From these outcomes, we take the average of all values where x falls between the boundaries mentioned in Table 3. With a small transformation of 1 − (Average value of $S(x)$), we arrive at the 10-year PD. Doing similar calculations for T(x) and U(x) lead to the 5-year PD and 1-year PD, respectively.

Rating	Lower boundary	Upper boundary	10-year PD	5-year PD	1-year PD
AAA	0.889	1	0.029	0.016	0.001
AA	0.778	0.889	0.044	0.026	0.002
A	0.667	0.778	0.065	0.042	0.004
BBB	0.556	0.667	0.095	0.065	0.010
BB	0.444	0.556	0.136	0.102	0.022
B	0.333	0.444	0.191	0.155	0.050
CCC and below	0	0.333	0.356	0.334	0.246

Table 3: The Probability of Default values and how they correspond with credit ratings

Loss Given Default (LGD)

The LGD, as noted earlier in Subsection 3.3.4, represents the proportion of exposure lost if a borrower defaults. Consequently, LGD is frequently expressed as a percentage. For modelling LGD, it is crucial to identify the factors that influence it. Maintaining consistency in selecting these factors is of unprecedented importance for building a generalisable model. A consistent approach ensures uniform credit risk assessment for every loan.

According to a literature study on LGD, the one factor proven to directly reduce LGD is collateral (Tanoue, Kawada, and Yamashita, 2017). Analyses have also examined the relationship between LGD and factors such as creditworthiness, company size, and business cycles. However, the effectiveness of these analyses varies significantly depending on the method used. Given these results, we only adopt the proven factor to determine the LGD. This means that for each loan, we need to calculate the effect of collateral on the LGD. Practitioners frequently utilise the Recovery Rate (RR) for this, which is the percentage of the total outstanding loan that can be recovered after a default. LGD and RR are related as follows:

$$\text{LGD} = 1 - \text{RR} \tag{24}$$

Most studies that approximate the RR focus on banks. Under the IRBA, the RR is set at 55%, implying an LGD of 45% (Habachi, Benbachir, and McMillan, 2019). Generalising this number across all of Horizon's loans would be inaccurate since not all contracts are the same. Specifically, not all loan agreements grant Horizon the same rights to acquire and liquidate the collateral of the borrowing party. Therefore, we need a loan-specific LGD, calculated via the following definition:

$$\text{RR} = \text{Value of collateral} \ / \ \text{Value of loan outstanding} \tag{25}$$

Exposure at Default (EAD)

Exposure at Default represents the highest potential loss a lender could face in the event of borrower default. It serves as a risk assessment indicator, enabling lenders to evaluate their position. For credit risk modelling, the highest potential loss for the lender is the total remaining value of the outstanding loan plus the interest on the outstanding amount. The applicable interest rate is the contractual interest rate paid by the borrower. The EAD value is loan-specific and should be determined for each loan. We calculate the EAD as follows:

$$\text{EAD} = \text{Remaining principal amount} + (\text{Interest rate} * \text{Remaining principal amount}) \tag{26}$$

Inverse Cumulative Normal Distribution

The final component of the formula is the value drawn from the inverse cumulative normal distribution, corresponding to the determined confidence level. As per Subsection 3.3.4, we assume that returns are normally distributed in this modelling approach. However, the actual distribution of returns might not always reflect the normal distribution, especially not during times of financial distress. Nevertheless, applying the normal distribution, a well-behaved distribution, eases the application of the Monte Carlo Simulation. Given this assumption, we proceed with employing the inverse cumulative normal distribution function. This statistical approach is crucial for determining the critical value corresponding to the specified confidence level. This critical threshold indicates the point beyond which potential losses are classified as extreme, providing crucial insights within the analysed context of VaR. For this approach, we use a confidence level of 95%, as required by the shareholder.

5.4.2 Value-at-Risk (Market & Liquidity Risk)

In this subsection, we construct the individual components of the Monte Carlo Value-at-Risk, namely the forecasted returns, the standard deviation of these returns, the asset value, and the cumulative inverse normal distribution. Like the credit risk model, all the parameters we cover are dynamic. By this, we mean that the used numbers change over time and should be updated frequently to ensure applicability and explanatory value. In Figure 6, we visualise how all parameters contribute to calculating the VaR metric for market and liquidity risk. Notably, the application of the inverse cumulative normal distribution differs between the credit risk model and the market and liquidity risk model. Within credit risk, the Z-score is used to build the confidence level, involving nothing more than multiplying the Z-score by the realised loss. Within market and liquidity risk, the inverse cumulative normal distribution is used to simulate future returns, with the confidence level indicating the corresponding returns as VaR. Here, it is not a multiplication but a percentile of the ordered list of simulated returns.

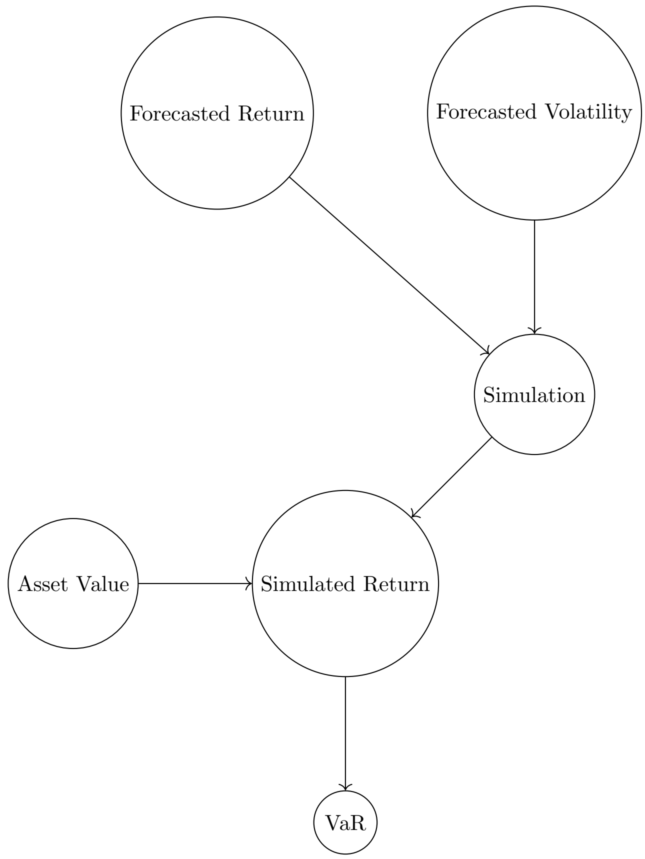


Figure 6: Relationship between Forecasted Returns, Volatility, Asset Value, Simulated Returns, and VaR

Asset Value

The first parameter we define is the asset value. As detailed in Subsection 4.4.2, the asset value is the current value of the investment. At the start of an investment, this value is the initial amount invested, expressed in euros. This parameter serves as the starting point for calculating the VaR metric. The dynamism of this parameter is critical because changes in the investment's value directly affect the VaR measurement. Over time, the investment value can fluctuate both positively and negatively, impacting the VaR calculation. Therefore, it is crucial to use the most recent valuation of the position when calculating VaR. The most logical and straightforward time to update this valuation is following a new investment round, reflecting the market's valuation of the asset at that time.

Forecasted Return

For every investment made by Horizon, a return is envisaged. This forecasted return is substantiated in the investment proposal by Horizon, based on information provided by the company seeking financing. Ideally, historical data on returns would also be incorporated into this analysis. However, due to the unavailability of such data, we proceed with an estimation based on the investment proposal. This document, drafted by Horizon, records and substantiates the investment considerations and decisions. The most important use of the forecasted return is for simulating purposes. The forecasted return is primarily used for simulation purposes. We assume the same distribution of returns as in the Credit Risk model, namely the normal distribution. The application of the normal distribution uses the same rationale as in Subsection 5.4.1. We simulate the returns based on the normal distribution with the forecasted returns as the mean and the forecasted volatility as the standard deviation. The number of simulations can be determined in the model itself, as the number of simulations increases, so does the reliability of the outcomes. We measure this parameter in percentages.

Forecasted Volatility
The forecasted volatility is the other input parameter for simulating the returns of the asset. More specifically, it is the standard deviation of the return distribution, the normal distribution in our case. As with the forecasted returns, most ideally we use historical data to estimate this parameter. Due to the absence of data, we use an estimate that is in line with the investment proposal, where the estimation of the forecasted volatility parameter is based on the perceived riskiness of the investment. Expert judgment is the most influencing factor in estimating the riskiness of the investment, and thus the forecasted volatility. We measure this parameter in percentages.

Simulated Return
After identifying the forecasted return and volatility of the asset, we proceed to simulate returns based on these input parameters. Essentially, we use the normal distribution to generate all these returns. Analytically, we do the following to calculate the return for each simulation trial:

$$\text{Simulated Return} = \text{Asset Value } \times \Phi^{-1}(U; \text{Forecasted Return}, \text{Forecasted Volatility}) \tag{27}$$

Where:

U : Uniform value drawn between 0 and 1.

We perform this calculation a fixed number of times, equal to the number of simulations. The resulting simulated returns are compiled into a list and sorted in ascending order, from lowest to highest returns. The simulated returns are measured in euros.

6 Model Construction

In this chapter, we outline the construction of models for the two risk categories. In the previous chapter, we established all individual parameters for both models. Now, we proceed to construct the models using these parameters. Essentially, we apply the Monte Carlo method to the parameters to simulate outcomes, based on which we evaluate the Value-at-Risk (VaR). These models are constructed using Excel.

6.1 Value-at-Risk Model (Credit Risk)

In Subsection 5.4.1, we explained how the parameters are constructed. In this section, we build on that and explain how we calculate the VaR for individual loans. Essentially, we discuss how all the calculated parameters come together to form the VaR metric. In addition, we also make the translation from individual VaR to portfolio VaR. As we identified in Subsection 3.3.4, we calculate the Monte Carlo VaR through Equation 8. We use the PD, LGD, and EAD as input for the model.

Essentially, for each simulation, we extract a random number from the uniform distribution U(0,1). For each number, we then test whether it is in the interval of the PD, if so then a default occurs in that particular case and a loss occurs, a loss corresponding to LGD and EAD. Testing the PD interval is quite simple, e.g. if the PD is 2.5%, a default occurs if the random number falls between 0.00 and 0.025, and in the other cases no default occurs. In the first scenario mentioned, the corresponding loss is calculated for that particular simulation. In the case of no default, the second scenario mentioned, the loss is 0, because the borrowing party can meet the payment obligation. As illustrated in Figure 4, these two tested scenarios simulate the consequences of a default and a non-default case. Mathematically, we model the consequence of this default simulation as a 0 or a 1, for a non-default and default, respectively. This implies that if there is no default taking place, we multiply the PD with 0 in Equation 8, so that the loss equals 0. In case of a default, we multiply the PD with 1 in Equation 8, so that the loss occurs and is aggregated in the simulation.

Before running the simulation, we determine the number of simulations. As the number of simulations increases, the outcomes converge. Increasing the number of simulations enhances accuracy by capturing a broader range of scenarios, thus improving reliability. In this case, losses over these simulations are aggregated and consequently divided by the number of simulations. Through this, we arrive at the simulated average loss. To transform this into the VaR metric, we multiply the average loss by the inverse normal distribution value corresponding to the 95% confidence level, assuming that losses are normally distributed (as per Subsection 5.4.1). Ultimately, we arrive at the VaR outcome using Equation 8.

The step-by-step approach explained above should be repeated for each loan to calculate the VaR metric for all loans individually. We take this as a starting point for calculating the portfolio VaR. As explained in Subsection 5.1.3, we assume that loans are independent of each other, and therefore, no adjustment for correlation is needed. However, this assumption might underestimate risk. While this simplifies our risk estimation, it is necessary due to the difficulty of accurately estimating correlation factors in a data-scarce environment. Given this assumption, we calculate the portfolio VaR by summing the individual VaRs of the loans. Analytically, this is represented by the following formula, where each i is a loan (Gordy, 2003):

$$\mathrm{VaR}_\alpha = \sum_{i=1}^{n} PD_i \times LGD_i \times EAD_i \times \Phi^{-1}(\alpha) \tag{28}$$

6.2 Value-at-Risk Model (Market Risk & Liquidity Risk)

In Subsection 5.4.2, we explained how the parameters for the Market and Liquidity risk model are constructed. This section builds on that and explains how we calculate the VaR for individual direct investments. We do this specifically over three time horizons, namely one year, five years, and ten years.

The Monte Carlo simulation component simulates asset values to obtain normally distributed returns, as mentioned in Section 5.4.2. These returns reflect the asset performance. Formula 27 gives the input for modeling asset performance. We take a randomly generated percentile of the normal distribution with mean Forecasted Return and with standard deviation Forecasted Volatility. This process derives a simulated return for each generated percentile, with the number of returns equalling the number of simulations. We order the generated returns in ascending order, so from low to high, and we take the 1 - α percentile of this list, where α is the defined confidence level. This confidence level is used to model the certainty with which we can say the expected loss does not exceed the found loss value. For example, if we have 10,000 simulations and we want the 95% confidence level, then we take the $(1 - 0.95) * 10,000 = 500$th number in the ascending list. Number 500 in this list of returns is then equal to the 95%-VaR of that simulation.

This step-by-step approach should be repeated for each direct investment so that we have all VaR metrics in place. For calculating the portfolio VaR, we take a step back to the ordered lists of returns per asset. We take this as a starting point for calculating the VaR metric for all direct investments, the portfolio VaR. As per our assumption, which is elaborated on in Subsection 5.1.3, we evaluate the portfolio as if all the direct investments are independent. This means that we can simply sum all corresponding simulated returns, e.g. we sum all generated results of all assets of the first item in the sorted return list. As we sum all values of the sorted list, we create a new list in which we paste these summed results. Consequently, we sort this list again in ascending order, as the order might have changed due to the summations. Lastly, we take the pre-defined confidence level α from this list to arrive at the portfolio α-VaR. We do this by taking the 1 - α percentile of the newly ordered list, this value is the portfolio VaR.

We calculate the VaR metric for three different time horizons: one year, five years, and ten years. Each scenario is simulated individually with its corresponding parameters. The input parameters for each modelling approach are based on the one-year time horizon. We assume that volatility increases proportionally to the square root of time, a common method for modelling volatility over time. This means the following for evaluating the five-year and ten-year horizon, as described by Hull (2006):

$$\text{T-year Volatility} = \text{one-year Volatility} \times \sqrt{T} \tag{29}$$

In this modelling approach, we simplify the modelling of the Forecasted Return in the same way. While this may not be the most widely used method, it is the most appropriate for our approach given the time horizon of modelling is rather long. Another method is to increase the predicted returns linearly, as in the Geometric Brownian Motion. However, this would result in the simulated returns becoming huge over five-year and ten-year time horizons. This could lead to overly optimistic VaR outcomes since the returns would grow faster than the inherent risk does. We take the most conservative approach and model the Forecasted Return as follows:

$$\text{T-year Forecasted Return} = \text{one-year Forecasted Return} \times \sqrt{T} \tag{30}$$

Besides the market risk, we also found in the literature that liquidity risk should be included in the evaluation of VaR. We learned that liquidity risk occurs at the time an exit is to be realised. We found several benchmarks that quantified liquidity risk. In Horizon's business, at the beginning of a direct investment, it is impossible to say for what term they want to hold the investment, except that it is for the long term. Therefore, we choose to define a representative quantification of liquidity risk and subtract it from the simulated return of each time horizon. With this, we choose the most complete situation possible for each scenario. The benchmark we take for this is the 3.5% discount as defined by Anson (2017). We opt for this approach since it is the most relevant work produced on quantifying liquidity risk for non-publicly traded shares.

6.3 Dependency Modelling

In this section, we determine our approach concerning dependency modelling. As introduced in subsection 5.1.3, we have to account for possible correlation effects within loans and within direct investments. We substantiate our approach for Credit Risk and Market and Liquidity Risk in the subsections below.

6.3.1 Approach Credit Risk

Regarding the credit risk model, we need to choose an approach to deal with dependency between loans that fits the chosen modelling approach. The most problematic is that there is no historical data regarding Loss Given Default and Probability of Default. This means there is no way to measure the dependence between variables. This forces us to make an assumption about the dependence between these variables.

Due to the unavailability of data on dependency modelling, we cannot provide meaningful insights into the actual dependency between variables. Therefore, we assume independence between loans. This assumption implies a correlation of 0, though the real correlation might differ. The most significant consequence of this correlation is its economic impact on the risk metric. When loans are correlated, the financial risk increases because the financial risks reinforce each other. Quantitatively, we cannot assess the economic impact of the correlation without knowing its value. Qualitatively, we observe diversification in the active sectors of portfolio companies, as detailed in Section 5.2. This diversification helps reduce the correlation and, consequently, its economic impact.

However, it is important to note that the assumption of independence is typically overly optimistic. In practice, loans and other financial instruments often exhibit some degree of correlation. This can be due to various macroeconomic factors, sectoral dependencies, or geographical concentrations, which can cause the financial risks to be interconnected rather than independent. For instance, when multiple loans are given to companies in the same industry or region, a downturn in that industry or region can lead to a simultaneous increase in defaults. This demographic dependency has a huge impact on Horizon because it focuses only on the province of Flevoland. In turn, diversification helps spread risk. A representative dataset could help quantitatively assess correlation and its impact. This is an important focus for future research.

6.3.2 Approach Market & Liquidity Risk

Regarding the market and liquidity risk model, we need to choose an approach to deal with dependency between direct investments that fits the chosen modelling approach. One way to evaluate this is to calculate the correlation over produced returns for five direct investments. However, we do not have data regarding returns. Again, a representative dataset of returns would help in modelling dependency. This is therefore also an important focus for future research. We use the same reasoning for substantiating the independence assumption as in the previous subsection since the diversifying attenuation effects also apply here.

7 Model Assessment

In the preceding Chapter 6, we constructed two models: one for assessing credit risk and another for market and liquidity risk. With these models now in place, it's prudent to evaluate their outcomes. While constraints prevent us from testing the models against historical data, we can still conduct a sensitivity analysis and garner insights from expert reviews. Through this approach, we aim to assess the model's feasibility, applicability, and reliability. In this chapter, we validate and assess the models based on a sensitivity analysis and expert feedback.

7.1 Sensitivity Analysis Credit Risk

A sensitivity analysis evaluates how variations in independent variables affect a dependent variable within a given model or system. This analysis is crucial for determining the robustness of the model's outcomes to changes in input parameters, thereby informing decision-making and risk management. By systematically altering inputs and observing resultant output changes, a sensitivity analysis illuminates the stability and reliability of a model's predictions. It helps identify influential parameters on the VaR metric and potential risks. We perform a sensitivity analysis for both our models, part of which is a sanity check for both models. For the credit risk model, we create a scenario in which the simulation of the PD is fixed at 10,000 simulations, and the confidence level is set at 95%. We elaborate on the input parameters in the subsections below. The output parameter in the model is the VaR metric.

7.1.1 Setup for Credit Risk Model

The independent input parameters that form the basis of the VaR metric in the credit risk model are PD, LGD, and EAD. Since we are testing three parameters and need an axis for each parameter, we are dealing with a three-dimensional matrix for plotting the VaR outcomes. Excel is the modelling environment we chose for both models. The worksheet of Excel is only two-dimensional, which poses an issue for our sensitivity analysis, where there are three input parameters to test. To solve this, we evaluate each EAD for all possible PD and LGD combinations and plot the VaR for all these combinations in a three-dimensional array. To clarify, we construct one three-dimensional array for each EAD. In this three-dimensional array, we plot the LGD on the x-axis, the PD on the y-axis, and the VaR metric on the z-axis. We segment the input parameter data into the following tranches:

PD	LGD	EAD
0.077	0	50,000.00
0.183	10	150,000.00
0.971	20	250,000.00
1.597	30	350,000.00
2.608	40	450,000.00
4.436	50	550,000.00
6.544	60	650,000.00
13.609	70	750,000.00
19.136	80	850,000.00
33.377	90	950,000.00
35.603	100	1,050,000.00

Table 4: The evaluated tranches for PD (in Percentages), LGD (in Percentages), and EAD (in Euros)

For each EAD, one array is constructed that plots all PD tranches, all LGD tranches, and the corresponding VaR outcomes. The tranches are chosen to match Horizon's assessment cases.

Besides these three input parameters, we apply the inverse cumulative normal distribution as a multiplication factor in the calculation of the VaR metric. We do not perform a sensitivity analysis on this, since the effect on the outcomes is known for this specific influential parameter. The number of simulations, however, is a crucial parameter to perform a sensitivity analysis on. It is important to get an impression of the convergence behaviour of the simulation. We test the convergence behaviour of the VaR from 10 to 1,000,000 simulations. We fix the other parameters, with the following values:

PD	LGD	EAD		α
0.13609	80%	€	650,000.00	95%

Table 5: Fixed values for PD, LGD, EAD, and α

7.1.2 Sanity Check Credit Risk Model

In total, we evaluate 11 PD tranches, 11 LGD tranches, and 11 EAD tranches, which means that we produce $11 * 11 * 11 = 1331$ VaR outcomes to evaluate. We analyse these outcomes by means of 11 3D graphs that visualise the relation between the input parameters. All these graphs can be found in Appendix D. For illustrational purposes, we highlight the following example in Figure 7:

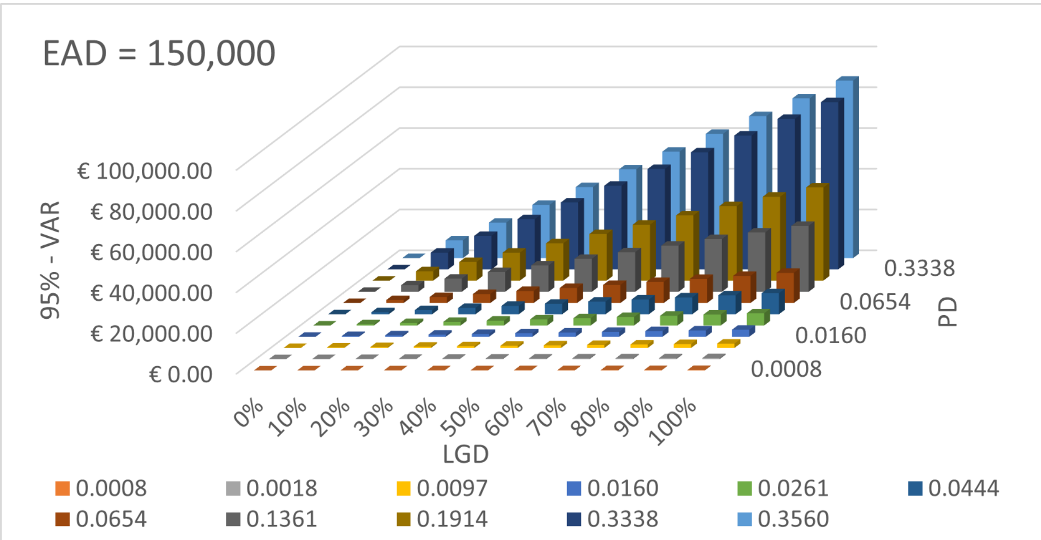


Figure 7: 3D graph representing the sensitivity of VaR based on the LGD and PD with a fixed EAD at 150,000

We can derive from this 3D model that the outcomes of the VaR metric seem to follow the analytical formula of the VaR metric. This confirms the model's approach in calculating the VaR metric. We can derive this from the linear increase of the graphs when moving over the LGD-axis. The same holds for the PD-axis: the VaR outcomes increase approximately linearly. However, it is only an approximation since we did not divide the PD tranches equally, which influences the outcomes in the 3D model in Figure 7. We take unequal tranches since all these values are PDs that roll out of the PD assessment in the model, therefore these are the only ones relevant.

The essential takeaway from this sensitivity analysis is that each parameter contributes equally to the VaR outcome. Though, it must be said that in terms of modelling importance, there is one parameter that requires more attention. The PD parameter is the only parameter that affects the actual modelling of defaults, which makes it more important than LGD and EAD. This is because the LGD and EAD only come into play in calculating the losses if the simulation produces a default, which is based on the PD parameter. The PD parameter thus plays an earlier role in the process. The other two can be seen as consequence parameters. Due to this, we want to emphasise the importance of reliable PD modelling. We do this with the extensive approach taken, as per Subsection 5.4.1.

7.1.3 Outcomes Number of Simulations Credit Risk Model

To assess the convergence behaviour of our model to the true VaR, we conduct a sensitivity analysis on the number of simulations. We conduct the sensitivity analysis with fixed input parameters: PD, LGD, EAD, and α, and with these parameters, we constantly rerun the model for a different number of simulations. In Table 6, we summarise the VaR output per number of simulations ran. The results from this are visualised in Figure 8.

Number of Simulations	VaR Value (€)
10	85,532.39
100	111,192.11
500	106,060.16
1000	118,034.70
1500	110,621.89
2000	113,330.41
3000	122,596.42
4000	109,267.63
5000	120,771.73
7500	114,955.53
9000	115,563.76
10000	112,389.56
20000	116,922.78
50000	116,939.88
100000	115,528.60
300000	115,913.49
500000	116,570.38
750000	116,384.49
1000000	116,647.36

Table 6: Resulting VaR values at different simulation counts

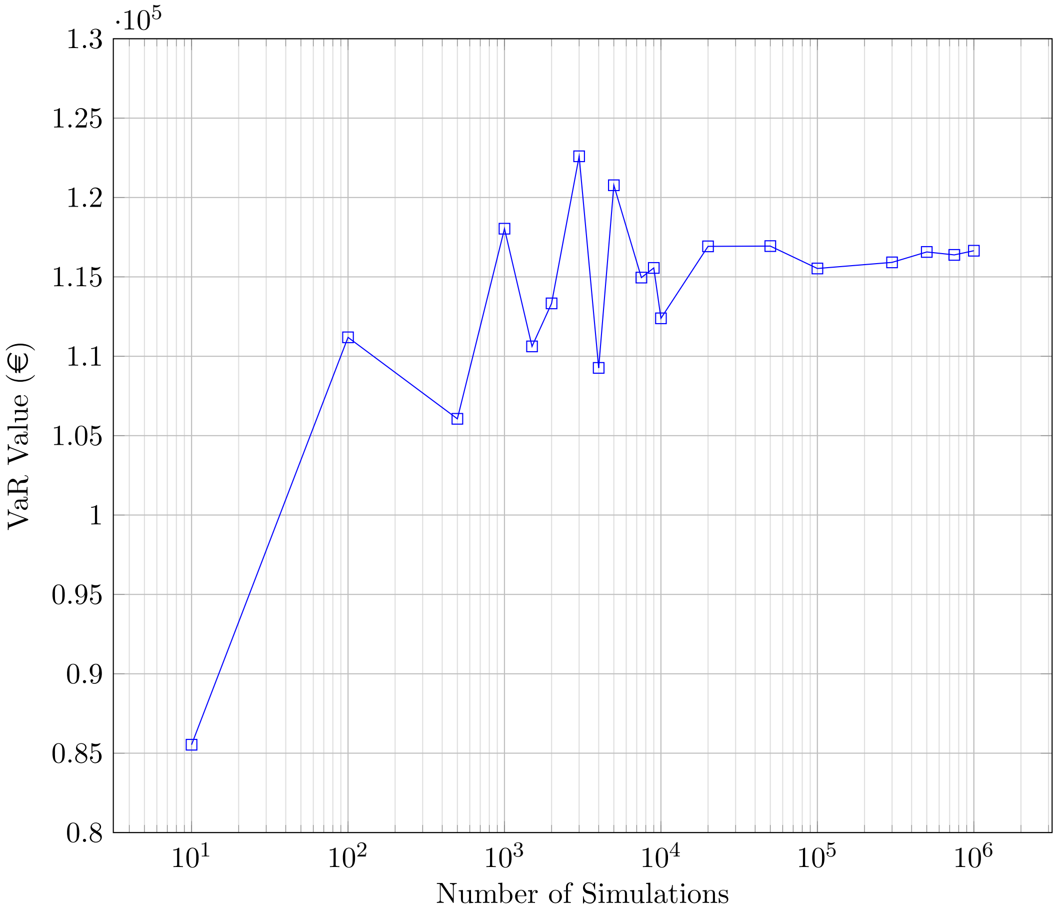


Figure 8: VaR values convergence with increasing Number of Simulations

We can observe the converging behaviour of our model toward a specific VaR value, which confirms that our model performs as intended. We can also derive from this that a substantial number of simulations is necessary to acquire the true VaR. As this is a trade-off between model reliability and model running time, it is up to Horizon to find the sweet spot. 20,000 simulations should provide rather reliable results at a running time of around 15 seconds.

7.2 Sensitivity Analysis Market & Liquidity Risk

For the market and liquidity risk model, we conduct a sensitivity analysis as well. We create a scenario in which the simulation is fixed at 10,000 simulations, and the confidence level is set at 95%. The independent input parameters for this model are the Rate of Return and Volatility. We elaborate on these input parameters in the subsection below. The dependent output parameter in the model is the VaR metric. 10,000 simulations are run since they offer the best balance between model running time and model reliability.

7.2.1 Setup for Market & Liquidity Risk Model

The independent input parameters that form the basis of the VaR metric in the market and liquidity risk model are Rate of Return and Volatility. The dimensionality issues we had to deal with within credit risk do not apply to this case, we can proceed with modelling this two-dimensional environment.

Rate of Return	Volatility
4%	4%
5%	5%
6%	6%
7%	7%
8%	8%
9%	9%
10%	10%
11%	11%
12%	12%
13%	13%
14%	14%
15%	15%
16%	16%
17%	17%
18%	18%
19%	19%
20%	20%
21%	21%
22%	22%
23%	23%
24%	24%
25%	25%

Table 7: The evaluated tranches for Rate of Return and Volatility

One array is constructed that plots all Rate of Return tranches, all Volatility tranches, and the corresponding VaR outcomes. The tranches are chosen to match Horizon's assessment cases. The Volatility does not exceed 25% as the shareholder requires Horizon to keep its risk under 25%.

Besides these two input parameters, there is another crucial parameter to perform a sensitivity analysis on. The number of simulations is this specific parameter. It is important to get an impression of the convergence behaviour of the simulation. We test the convergence behaviour of the VaR from 10 to 1,000,000 simulations. We fix the other parameters, with the following values:

Rate of Return	Volatility	α
9%	16%	95%

Table 8: Set values for Rate of Return, Volatility, and α for which we test convergence behaviour of the model

7.2.2 Sanity Check Market & Liquidity Risk Model

In total, we evaluate 22 Rate of Return tranches, and 22 Volatility tranches, which means that we produce $25 * 25 = 625$ VaR outcomes. However, we do not evaluate all 625 of these outcomes. We evaluate only those cases where returns are less than or equal to Volatility. We make the decision based on the fundamental finance principle tied to risk and reward, as briefly touched upon in Section 4.3. This principle suggests that higher rewards always go hand in hand with higher risk, and vice versa. Therefore, we eliminate all the outcomes where returns are less than or equal to Volatility. After we narrow the set of VaR outcomes to the explained restriction, we find that we end up with 253 VaR outcomes. We analyse these outcomes through a 3D graph that visualises the relation between the input parameters. The visual representation of the sensitivity analysis can be found in Figure 9.

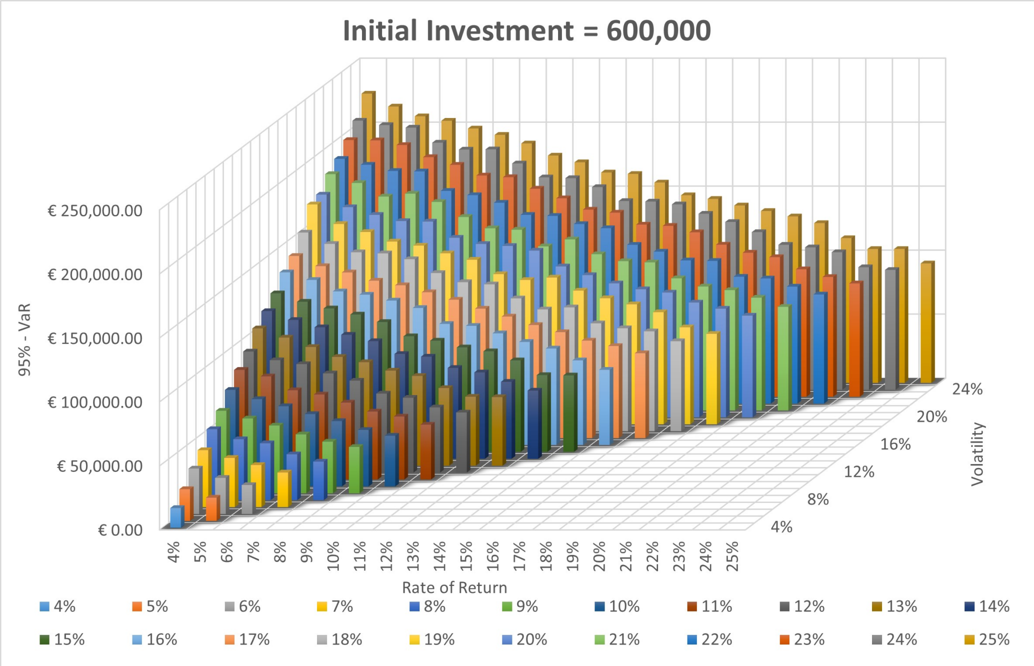
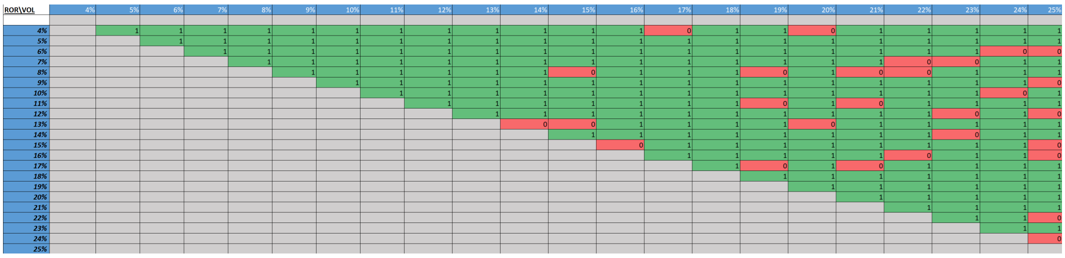


Figure 9: 3D graph representing the sensitivity of VaR based on the Rate of Return and Volatility at an initial investment of 600,000

From Figure 9, we infer that the model behaves as expected. We can say this based on the growth of VaR on the Volatility axis and the decline of VaR on the Rate of Return axis. Regarding the course of the Volatility axis, recall that we simulate normally distributed returns based on the Rate of Return as the mean and the Volatility as the standard deviation of this distribution. We observe that as the Volatility increases and the Rate of Return is fixed at some value, we deal with a higher VaR value. Intuitively this makes sense because the increase in Volatility causes an increase in the distribution's spread. This means that the range of potential returns is wider implying a higher probability of extreme losses and extreme returns. Given that the VaR measures potential losses at a specific confidence level, we can derive that the VaR must increase as well. Looking specifically at the type of relationship between VaR and Volatility, the graph insinuates a linearly increasing relationship. However, we cannot say this with certainty because the VaR is taken from a distribution and not from an analytical formula. As a result, there is always some slack in the final VaR value.

Regarding the behaviour of VaR on the Rate of Return axis, this observation makes sense as well. The Rate of Return, which is the mean of the return distribution, forms the distribution's center. As the Rate of Return increases, the distribution's center shifts to the right. Given that the Volatility is kept constant, the range of potential return also shifts to the right, implying a lower probability of extreme negative returns. Based on this, it is evident that the VaR should decrease since the loss potential decreases. We observe a decreasing relationship with VaR for Rate of Return. Using the same reasoning as the Volatility case, we cannot say the relationship is linear.

Knowing that both Rate of Return and Volatility have a positive relationship with the VaR, it is also interesting to evaluate which of the two has a stronger impact on the VaR. To evaluate this, we construct a matrix of the percentual changes over the rows and a matrix over the percentual changes of the columns. Through these matrices, we can assess whether a percentual increase in Volatility is of greater impact on the VaR than the percentual increase in Rate of Return. We can compare these two matrices directly since we make use of the same measurement units for both variables, namely percentages. By comparing the incremental changes in VaR for Rate of Return and Volatility individually, we can get an impression of which variable causes a greater change in VaR. We make this comparison by checking which incremental change in the variable is larger, we check this in Figure 10.



ROR\VOL	4%	5%	6%	7%	8%	9%	10%	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	21%	22%	23%	24%	25%
4%		1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	1	1	1	1	1
5%			1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
6%				1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
7%					1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1
8%						1	1	1	1	1	1	0	1	1	1	0	1	0	0	1	1	1
9%							1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
10%								1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
11%									1	1	1	1	1	1	1	0	1	0	1	1	1	1
12%										1	1	1	1	1	1	1	1	1	1	0	1	0
13%											0	0	1	1	1	1	0	1	1	1	1	1
14%												1	1	1	1	1	1	1	1	0	1	1
15%													0	1	1	1	1	1	1	1	1	0
16%														1	1	1	1	1	0	1	1	0
17%															1	0	1	0	1	1	1	1
18%																1	1	1	1	1	1	1
19%																	1	1	1	1	1	1
20%																		1	1	1	1	1
21%																			1	1	1	1
22%																				1	1	0
23%																					1	1
24%																						0
25%																						

Figure 10: Comparison of the percentual change over column (Rate of Return) and row (Volatility)

In Figure 10, we fill the result cells with a 1 or a 0. We fill the cell with a 1 if an increase of VaR in the incremental column values is higher than in the incremental row values. For 0, we have the exact opposite case. We fill the cell with a 0 if an increase of VaR in the incremental row values is higher than in the incremental column values. To clarify, we explain this approach using an example: we measure the increase in VaR based on an increase in Rate of Return from 4% to 5%, with Volatility remaining the same. We do the same for the increase in Volatility from 4% to 5%, with the Rate of Return remaining the same. Then, we calculate the percentage increase for both cases. After that, we construct a resulting matrix that presents the highest percentage increase. If the increase of VaR was the highest by a column increment, thus the Volatility, we note a 1. If the increase of VaR was the highest by a row increment, thus the Rate of Return, we note a 0.

Based on the resulting Figure 10, we identify a majority of 1's. This result implies that Volatility causes a greater percentual change in VaR than the Rate of Return. At least, this is the case for the majority. We cannot conclude that Volatility structurally has a stronger impact on VaR than the Rate of Return.

7.2.3 Outcomes Number of Simulations Market & Liquidity Risk Model

To assess the convergence behaviour of our model to the true VaR, we conduct a sensitivity analysis on the number of simulations. We conduct the sensitivity analysis with fixed input parameters: Rate of Return, Volatility, and α, and with these parameters, we constantly rerun the model for a different number of simulations. In Table 9, we summarise the VaR output per number of simulations ran. The results from this are visualised in Figure 11.

Number of Simulations	VaR Value (€)
10	103,865.79
100	102,220.30
500	103,332.15
1000	105,303.25
1500	110,887.78
2000	102,195.85
3000	103,091.15
4000	103,537.58
5000	103,879.04
7500	102,300.33
9000	101,229.96
10000	106,867.17
20000	98,843.92
50000	102,584.34
100000	106,567.91
300000	101,810.01
500000	102,341.13
750000	104,315.96
1000000	101,541.28

Table 9: VaR values behaviour for increasing Number of Simulations

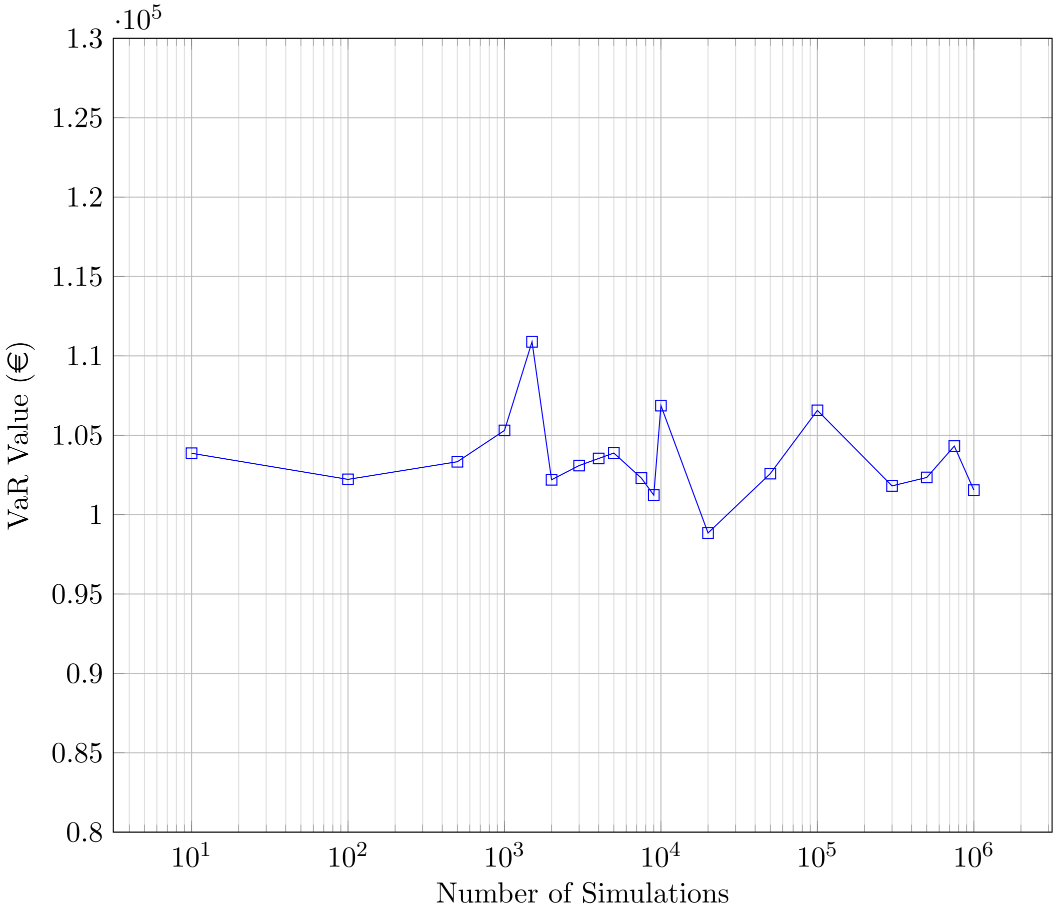


Figure 11: VaR values with increasing Number of Simulations

When observing the pattern of our model, we cannot say that the model is particularly converging to a specific VaR value. This is also not very miraculous since the calculation of the VaR is taking a percentile of generated returns. As the number of simulations increases, there is also a greater chance of generating huge outliers, which cause radical differences in the list of sorted returns and thus in the VaR. Of course, as the number of simulations increases, the simulated returns come closer to a normal distribution. However, since we use a small percentile of these simulated returns for calculating our VaR, the VaR itself does not converge to a specific value.

In the sections above, we gain insights into the behaviour of our constructed models. All of the tests we perform are done based on quantitative information. In the next section, we turn our attention to the qualitative and more practical side of the models. We do this by hand of semi-structured interviews.

7.3 Expert Review

In this section, we discuss the conducted semi-structured interviews with industry experts. In the sensitivity analysis, we highlighted the quantitative part of the model, mostly concerning the outcome variable VaR. In the semi-structured interviews, we shift the focus to the qualitative parts of the model. By this, we mean the intuitiveness of the model, the understandability of the model, and the reasonability of the model. Also, we reflect on the features used for modelling the PD in the credit risk model. This expert review aims to validate the outcomes of the credit and market and liquidity risk models as much as possible. Ideally, we would use historical data to validate the models. However, this data is unavailable, and expert opinion from practitioners is the most appropriate way to get feedback on the model.

7.3.1 Setup for Semi-structured Interviews

The setup for the conducted interview consists of the following:

1. **Interview Type**: We conducted semi-structured interviews, which allowed for flexible yet focused discussions, enabling in-depth exploration of specific aspects of the model while also accommodating any emergent themes or insights.

2. **Participants**: We interviewed industry experts with practical experience in credit risk, market risk, and liquidity risk, ensuring that the feedback was grounded in real-world application.

3. **Structure**: We guided the interviews with a predefined set of questions aimed at covering key areas such as model intuitiveness, understandability, reasonability, and the appropriateness of features used in PD modelling.

4. **Duration and Format**: Each interview was conducted in one hour, allowing sufficient time for detailed discussions while maintaining focus on the key aims. We took notes to capture the feedback for subsequent analysis accurately.

We conducted the semi-structured interviews using a predefined set of questions focusing on key areas. In Appendix E, we present this predefined set of questions. To elaborate on the aims for the semi-structured interviews, we set the following primary aims:

1. **Validate the Model's Reasonability and Practical Relevance**: Assess the reliability and real-world applicability of the developed credit risk, market risk, and liquidity risk models.

2. **Evaluate Model Intuitiveness and Accessibility**: Gather feedback on how intuitive and user-friendly the model is, ensuring it can be easily understood and utilised by practitioners.

3. **Assess Model Understandability**: Reflect on how well the model's processes and outcomes are understood by the experts.

4. **Review the Appropriateness of Features Used in PD Modelling**: Determine if the features used for modelling the Probability of Default (PD) in the credit risk model are appropriate and relevant.

7.3.2 Results

In this sub-section, we present the results of the semi-structured interviews. We discuss the four aims individually and present the feedback consisting of a summary of feedback, positive aspects, and areas for improvement. Textually, the interview feedback is an integration of four different interviews. Initially, we invited five experts for an interview, of which four ended up participating in an interview, representing an 80% response rate. We select the experts based on the following requirements:

- Has minimally four years of experience as an investment manager or role with similar specifications.
- Has experience in using financial risk models.
- Has an in-depth understanding of financial instruments, market structures, and risk management principles.

We recorded each interview, with the consent of the participants, and took notes based on the answers to the questions asked. The proposal for conducting the interviews and dealing with any ethical issues was approved by the University of Twente's ethics committee.

Model Reasonability

Feedback Summary The reasonability of the model was generally viewed positively, although practical validation is still required.

Positive Aspects

- One expert mentioned that the model appeared to be a working document, implying its functionality and reliability in its current state.

Areas for Improvement

- One expert highlighted the importance of verifying the model's calculations and expressed concern over the weighting factors used for the Probability of Default (PD).
- Suggestions included developing an index for the value ranges of parameters and improving the correlation modelling between loans and participations, acknowledging that correlations might exist due to sector-specific risks.
- It was noted that the financial parameters might change significantly over time, suggesting the need for dynamic validation and adjustment of the model.

Model Intuitiveness and Accessibility

Feedback Summary The experts generally found the model intuitive and accessible. One expert mentioned that the model had an intuitive feel, especially the "cockpit" interface which facilitated ease of use. Another expert, while noting the complexity of the subject, also found the model process easy to follow due to prior involvement.

Positive Aspects

- The cockpit interface was highlighted as particularly intuitive.
- The general layout and structure were found user-friendly by those familiar with financial models.

Areas for Improvement

- One expert suggested adding a summary document to make the model more accessible, particularly for stakeholders who may not be deeply involved in the details.
- It was recommended to include an index or a clear indicator for the value ranges of all parameters to enhance user guidance.
- Another expert noted that the model should not overwhelm users with unnecessary information, emphasising that only the most crucial metrics, such as VaR and Achieved Risk, should be prominently displayed.

Model Understandability

Feedback Summary The experts had mixed views on the understandability of the model. While some aspects were clear, other parts needed better documentation and explanation.

Positive Aspects

- The steps taken in the model were clear to experts who were already involved in the process.
- The equity valuation part was straightforward for one of the experts.

Areas for Improvement

- One expert suggested that integrating the models within teams and possibly using Python could enhance clarity and understanding.
- The liquidity risk component of the market risk model required additional explanation on its exact functionality and the appropriate circumstances for its use.
- A comprehensive user guide, written in simple language, was recommended to help users navigate the model more effectively.

Appropriateness of Features Used in PD Modelling

Feedback Summary Experts provided valuable insights into the relevance and importance of the features used in PD modelling.

Positive Aspects

- The selection of features for PD modelling was generally seen as relevant.

Areas for Improvement

- One expert suggested that non-financial factors, such as business factors, should be weighted more heavily, as they often provide more insight than financial ratios alone.
- The need for clearer definitions of the parameters was emphasised, as some features were seen as too cryptic or not practically aligned.
- It was noted that the organisational structure parameter plays a significant role but also changes significantly over time, necessitating a more flexible approach.

7.3.3 Addressing Areas for Improvement

The expert feedback provided valuable insights into the strengths and areas for improvement in the model. While it is not feasible to incorporate every suggestion within the scope of this thesis, the following approach has been taken to address the feedback:

Prioritised Improvements Based on the feedback, the following key improvements were prioritised and implemented:

- **Summary Document**: A summary document has been added to enhance the accessibility of the model, particularly for stakeholders less involved in the details.
- **Clear Indicator for Parameter Ranges**: An index indicating the value ranges of all parameters has been included to guide users.
- **Dynamic Validation and Adjustment**: Both risk models have been adjusted to be suitable for dynamic validation. It is accounted for in the form of a possibility to save the simulated results within the same document.
- **Non-financial Factors in PD Modelling**: The importance of non-financial factors in PD modelling have been enhanced. The influence of financial factors has declined; instead, there is now a greater influence of non-financial factors.

Future Work Several suggestions were identified as valuable for future research and development:

- **Family Business Considerations**: The unique characteristics of family businesses suggest a need for distinct modelling approaches, which could be a focus for future studies.

Justification of Non-implemented Suggestions Due to time constraints and the scope of this thesis, the following suggestions were not implemented:

- **Integration of Models within Digital Environment**: While integrating the model within Microsoft Teams by using Python could enhance clarity, this requires a collaborative approach and resources beyond the current project's scope.

This area for improvement highlights opportunities for further refinement and development of the model. Especially, on simplifying and enhancing the adoption of the designed risk models. The feedback received has been invaluable in understanding the model's current limitations and potential enhancements, providing a clear direction for future research and practical application.

8 Conclusions and Discussion

This research aimed to find, build and implement an approach to modelling financial risk for loans and direct investments that is tailored to Horizon's operations. We started this research by exploring the methods for financial risk modelling through a literature review. We performed this literature review for loans and direct investments. Given the current data-scarce environment at Horizon and the focus on typical scale-up companies, we had to initiate some adjustments to the identified methods. After that, we decided that the Monte Carlo Value-at-Risk method would be best suited for financial risk modelling of loans and direct investments. Consequently, we started constructing the necessary parameters. After constructing the parameters, we integrated them into the model environment where the input was converted into a VaR outcome. Once the model was finalised, we conducted two validation steps: sensitivity analysis and semi-structured interviews.

In this last chapter, we reflect on the key findings and discuss their implications. We reflect on this by answering the main research question. At the start of this research, we formulated the following main research question:

What method is best suited for financial risk modelling of loans and direct investments in Horizon's portfolio companies?

In Section 8.1 we provide an answer to this research question. In Section 8.2 we discuss the results from this research. In Section 8.3 we reflect on the limitations of this research and outline future research topics. Lastly, Section 8.4 outlines the contributions of this research in terms of theory and practice.

8.1 Conclusion on the Research Questions

In this research, we address the core problem that Horizon currently has a perceived inaccurate financial risk model. This sense of inaccuracy stems from little transparency in the calculations which gives Horizon no way to understand the underlying calculations. Also, the mismatch between expected outcomes and actual outcomes reinforces the sense of inadequacy. Due to this, Horizon decided that a new model would be necessary. As a consequence, we came up with the main research question.

To answer this research question, we first divided the main research question into smaller sub-questions. For each sub-question, the answers were the following:

1. What are State-of-the-art methods for credit risk modelling?
In Chapter 3, we answered this sub-question by conducting a literature review on the state-of-the-art methods for credit risk modelling. We found seven different models, namely Financial Statement Analysis, Altman Z-score, Zeta Model, O-score, Discriminant Analysis Model, Value-at-Risk, and Expert Judgment. We excluded some methods from our search, as we focussed on typical "Groei Fonds" companies and some methods did not apply to this focus. The excluded methods can be found in Appendix A. In Subsection 5.3.1, we decide to proceed with the Monte Carlo Value-at-Risk model and incorporate expert judgment in the Probability of Default calculation.

2. What are State-of-the-art methods for market and liquidity risk modelling?
In Chapter 4, we answered this sub-question by conducting a literature review on the state-of-the-art methods for market and liquidity risk modelling. We found two different models, namely Value-at-Risk, and Expected Shortfall. We excluded one

method from our search, as we focussed on typical "Groei Fonds" companies and some methods did not apply to this focus. The excluded methods can be found in Appendix B. In Subsection 5.3.2, we decide to proceed with the Monte Carlo Value-at-Risk model.

3. How to categorise Horizon's portfolio companies and how to deal with this environment in credit, market, and liquidity risk modelling?
The crux of this question stems from Horizon's interest in properly classifying their portfolio companies, mainly because it is essential for financial risk modelling. In Section 3.4, we find that the scale-up environment is an unexplored part of the financial risk modelling literature. Therefore, we opted to categorise portfolio companies as SMEs, as it represented the closest definition. Based on this, we identified key risks that should be integrated into the models. For credit risk, we outline these and explain the according model adjustments in Section 3.4. For market and liquidity risk, we outline key risks and model adjustments in Section 4.5.

4. What defines 'Groei Fonds' participants and what are their characteristics?
Employing the data analysis in Section 5.2, we define what typical "Groei Fonds" participants are. We do this based on financial and non-financial data. Within the financial context, most "Groei Fonds" participants are generally illiquid and have high or negative debt-to-equity ratios. Within the non-financial context, the analysed companies fall into either the micro-enterprise or small-enterprise classification. Also, most companies have existed for quite some time. Inherent to the development stage of these companies, relatively little corporate information is available compared to listed companies.

With the insights from addressing the sub-questions, we can now answer the main research question: "What method is best suited for financial risk modelling of loans and direct investments in Horizon's portfolio companies?"

We found that the Monte Carlo Value-at-Risk (VaR) method is the most suitable approach for Horizon's needs. This method was selected due to its adaptability and robustness in handling the uncertainties and specific characteristics of the 'Groei Fonds' participants. By incorporating expert judgment in the Probability of Default calculation and adjusting the model parameters to reflect the unique financial profiles and risks associated with SMEs, we have tailored the Monte Carlo VaR model to Horizon's operational context.

8.2 Policy Recommendations

The incorporation of the Monte Carlo VaR method, reinforced by qualitative adjustments and expert insights, tackles the fundamental limitations of Horizon's previous financial risk model. This approach provides a more transparent, reliable, and contextually relevant framework for assessing the financial risks associated with loans and direct investments in Horizon's portfolio. Importantly, this updated model not only aligns with the unique risk profiles and characteristics of 'Groei Fonds' participants but also improves the clarity of Horizon's financial risk assessments, thereby enabling more informed decision-making and risk management practices.

Moreover, our research addresses a crucial gap in the existing literature by focusing on the specific context of SMEs and offering practical guidance for financial risk modelling in environments with limited data availability. It specifically addresses a significant challenge in informed decision-making. Previously, the lack of adequate data and transparency in the model left decision-makers grappling with a figurative "black box" where the factors influencing risk assessments remained unclear. By adopting the Monte Carlo VaR method alongside qualitative adjustments and expert insights, a substantial transformation occurs. Instead of a static model, there emerges a dynamic, adaptable framework capable of incorporating new experiences and insights as they arise. This shift from opacity to adaptability not only improves transparency and understanding but also empowers decision-makers with the flexibility to respond to evolving circumstances.

This development represents a substantial progression in financial risk management at Horizon, promising to illuminate previously opaque areas. As experts have indicated, this transition holds the promise of continuous improvement, with the model adapting to emerging challenges and opportunities. This ensures that Horizon maintains its agility and resilience in managing risks effectively.

8.3 Discussion

In this section, we delve deeper into the findings presented in this thesis and explore their broader implications. We discuss the limitations and theoretical and practical contributions of this research and highlight how it advances our understanding of financial risk modelling for loans and direct investments, especially in data-scarce environments. Finally, we outline potential avenues for future research to address the identified limitations and further refine the models developed in this study.

8.3.1 Limitations

To list the limitations of this study, we outline several areas where necessary assumptions and simplifications had to be made, one of the main reasons being the scarcity of data:

- **Data Limitations**:
  - We used global default rates for the Probability of Default (PD) modelling, which may not accurately reflect SMEs. This reliance on broader datasets slightly compromises the model's reliability due to constraints in data availability.
  - We used a logistic function to derive an analytical function from the PD scores. There is an acknowledged error margin in fitting the PD on the logistic function, as we were not able to achieve a mean squared error of 0. This implies an error margin in the translation to PD values from the global default rates.
- **Model Assumptions and Simplifications**:
  - We made certain assumptions and simplifications to manage the modelling process. Both models make use of the normality assumption, which is only an approximation of how the returns are distributed. Besides this, we also assume that loans are independent from each other and that each direct investment is independent from another direct investment. While we provide a qualitative argument for limited impact, it is uncertain what the actual economic impact would be if true dependency were measured. A representative dataset is crucial in determining the true dependency within loans and direct investments.
  - We constructed both models using the SME categorisation of portfolio companies. This simplification was necessary as there was no literature available about financial risk modelling for scale-ups specifically.
- **Limited Validation and Review**:

  - While we conducted sensitivity analysis, sanity checks and expert reviews, the validation process could have been more extensive. Broader validation with more diverse expert inputs and real-world testing could further refine the model's accuracy and reliability. Validating the models with historical data is essential to get insights into the accuracy of the models.

- **Heterogeneity within SMEs**:
  - SMEs are highly heterogeneous, with significant variations in size, sector, and operational characteristics. This diversity makes it challenging to create a one-size-fits-all model, and the results may not be uniformly applicable across different types of SMEs.

8.3.2 Theoretical and Practical Contributions

This research makes significant theoretical and practical contributions to the field of financial risk modelling for SMEs, particularly in environments with limited data.

Theoretical Contributions
The primary theoretical contribution of this study is the adaptation of the Monte Carlo Value-at-Risk (VaR) method to model financial risks in data-scarce contexts. By integrating qualitative adjustments and expert inputs, this research addresses the specific financial risk profiles of SMEs. This model fills a critical gap by providing a robust framework for assessing the PD and other risk factors in SMEs, which often lack the extensive financial histories of larger enterprises.

Furthermore, this study advances the understanding of financial risk modelling by highlighting the importance of non-financial factors in PD calculations. Traditional models predominantly focus on financial ratios and historical data; however, this research demonstrates that incorporating non-financial elements, such as business characteristics and market conditions, can enhance the relevance and comprehensibility of risk assessments for SMEs.

Practical Contributions
Practically, this research offers a tailored risk assessment tool for Horizon, improving decision-making and risk management for loans and direct investments in their portfolio companies. The Monte Carlo VaR model, customised with specific adjustments for SMEs, provides a more transparent and contextually appropriate means of evaluating financial risks. This allows Horizon to make more informed and reliable investment decisions.

Additionally, the research includes the development of a dynamic validation process, enabling ongoing refinement and adjustment of the model as new data becomes available. The adaptability is crucial for maintaining the model's applicability and reliability over time, particularly as the financial and non-financial situation of SMEs can change rapidly.

In summary, this research contributes both to the academic field of financial risk modelling and to practical risk management strategies for SMEs, offering a comprehensive and adaptable framework that can be utilised in various data-scarce contexts.

8.3.3 Future Research

Future research should directly address the limitations identified in this study to enhance the reliability and applicability of financial risk models for SMEs.

Improved Data Specificity
The reliance on global default rates for Probability of Default (PD) modelling highlighted a gap due to the scarcity of SME-specific data. Future research should focus on collecting and incorporating more granular data specific to SMEs. This will improve the accuracy of PD models by better reflecting the unique characteristics of these enterprises.

Another topic on which data should be collected is the performance of loans and direct investments. For now, we assumed that loans are independent and direct investments are independent. However, this assumption was necessary due to a data constraint. Most probably, there is some form of dependency as the focus on financing is demographically centered around the province of Flevoland. With the collection and incorporation of historical data, the dependency could be derived which would improve the accuracy and reliability of the financial risk models.

Enhanced Empirical Validation
Given the validation and review conducted in this study, future research should aim to validate the models extensively using historical data. This will help in refining the models, ensuring their robustness, and providing deeper insights into their practical accuracy and applicability across various economic conditions and SME segments.

Addressing Heterogeneity within SMEs
The diverse nature of SMEs poses a significant challenge for creating universal models. Future research should investigate sector-specific and size-specific models to account for the variations in operational characteristics and risk profiles within the SME sector. This approach will lead to more tailored and precise risk assessments.

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Appendix A

Concerning credit risk modelling, we exclude the following methods from our research:

- **Machine Learning Models:**
  - Logistic Regression
  - Random Forests and Gradient Boosting Machines
  - Neural Networks
- **Market-Based Models:**
  - Incorporating market indicators and economic factors
- **Stress Testing:**
  - Simulating impact of adverse economic conditions
- **Dynamic Models:**
  - Accounting for changes in credit risk over time
- **External Ratings:**
  - Incorporating external credit ratings

Appendix B

Concerning market and liquidity risk modelling, we exclude the following method from our research:

- **Beta (CAPM):**
  - Beta (β) is a measure of the volatility

Appendix C

Feature	**Feature Definition**
Current Ratio	Current Ratio = Current Assets / Current Liabilities
Quick Ratio	Quick Ratio = Quick Assets / Current Liabilities
Cash Ratio	Cash Ratio = (Cash + Cash Equivalent) / Current Liabilities
Debt-Equity Ratio	Debt Equity Ratio = Total Debt / Total Equity
Interest Coverage Ratio	ICR = (Profit after Taxes + Depreciation + Interest on Loan) / Interest repayment
Outlook Industry	Worldwide arrangement with respect to government, geopolitical conditions, etc. Both household and worldwide economic developments play a crucial part in the operations of SMEs.
Demand-Supply Gap	The shortage of raw materials influences the prospect of SMEs. The demand impacts the performance of SMEs straightforwardly.
Economic Environment	Interest rate levels set by ECB. It serves as an indicator of the economic tide and where the cost of financing might go.
Market Strength and Capability	Market direction may act as a catalyst to build marketing efficiency through innovative marketing capabilities. Strong branding and innovation significantly boost marketing performance.
Organisational Structure	The effectiveness of the organisation is reflected in the responsibilities within the organisation and the extent to which these are well-managed.
Expected Growth of Sales	Diversification in sales in terms of geographic regions and product range impacts profitability and sustainability positively. Exporting incentivises the firm from various sectors and causes successful operations.
Type of Firm	Whether a firm is family-controlled or professionally managed can be examined from its constitutional deeds. The impact of family association in administration also affects the firm's performance.
Education & Experience	The relevant qualifications of key personnel in management play a vital role in SME performance. Engaging a professional advisor/consultant positively impacts the performance and helps reduce sole dependency on management.
Integrity & Commitment	The market reputation of the individual behind the firm affects the financial soundness of the creditor. A legitimate assessment of market reputation plays a critical role in the credit scoring of the firm.
Strategic Vision & Management Quality	Having long-term plans and highlighting values, purposes, and goals. Combined with the management's ability to oversee all activities and tasks necessary to maintain a desired level of excellence.
Credit History	Indicates the record of the firm and its promoters in the timely repayment of debt obligations. Credit history should include the number of advances, instalments, and the repayment track period of all types of loans granted to the firm.

Repayment Period	A short-term loan is less risky and more manageable. The repayment of loans should align with the firm's cash flow and coverage ratio. Prolonged tenure is said to be more difficult, keeping all other parameters equal.
Compliance Record	A regular firm in all the statutory regulations, such as paying tax, filing returns in time, and submitting documents, is considered compliant.
Transparency in Accounting	Audit of accounts not only ensures administrative compliance but also improves practices within the firm for smooth bookkeeping and financial administration. An auditor guides the administration on appropriate monetary planning and other consultancies.
Trust	The extent to which the counterparty has kept their word and complied with arrangements made.

Appendix D

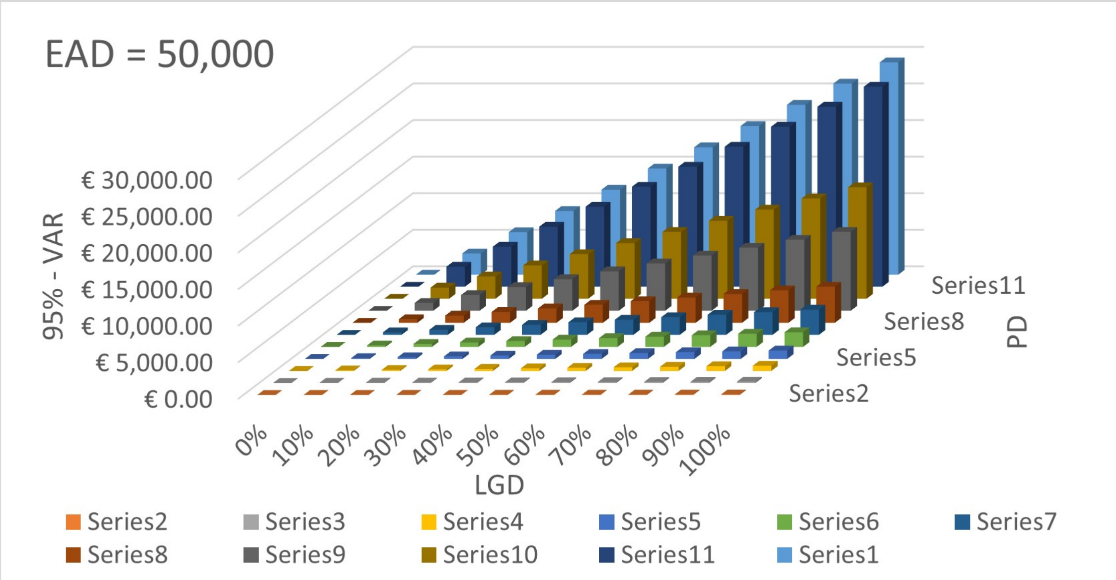

Figure D.1: 3D graph representing the sensitivity of VaR based on the LGD and PD with a fixed EAD at 50,000

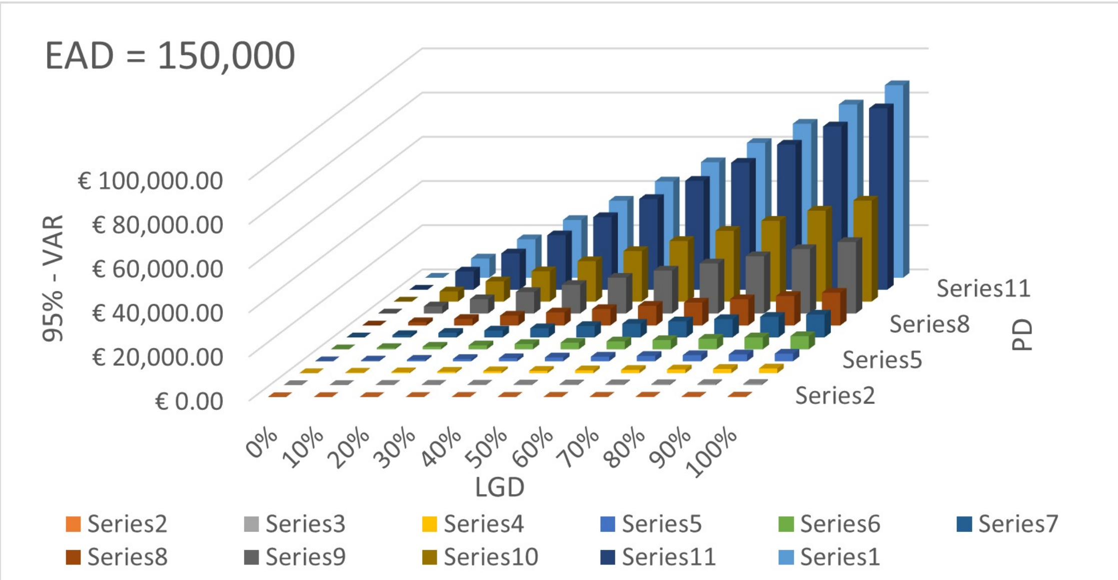

Figure D.2: 3D graph representing the sensitivity of VaR based on the LGD and PD with a fixed EAD at 150,000

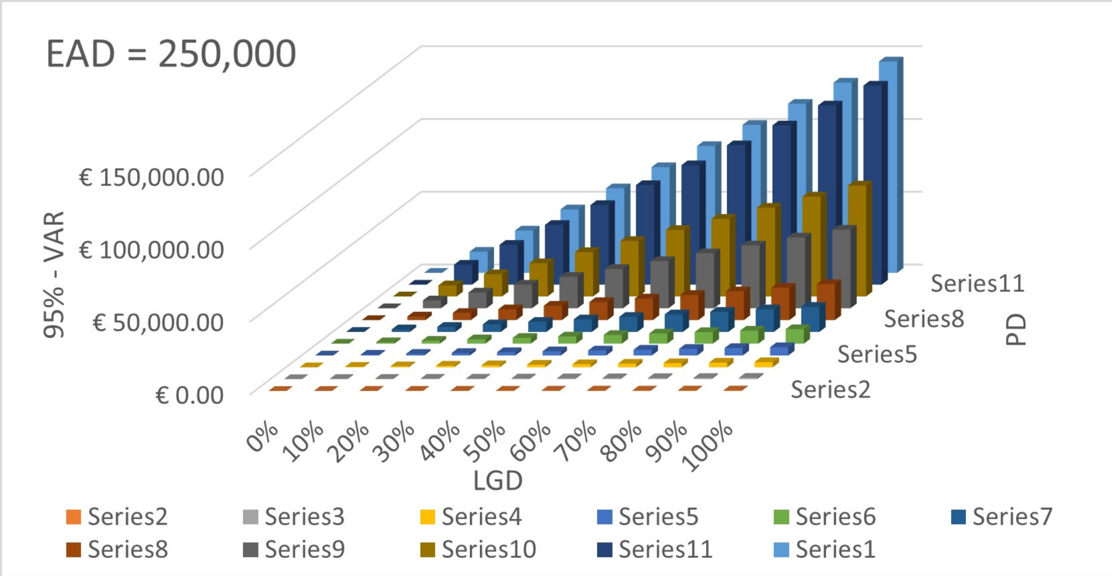

Figure D.3: 3D graph representing the sensitivity of VaR based on the LGD and PD with a fixed EAD at 250,000

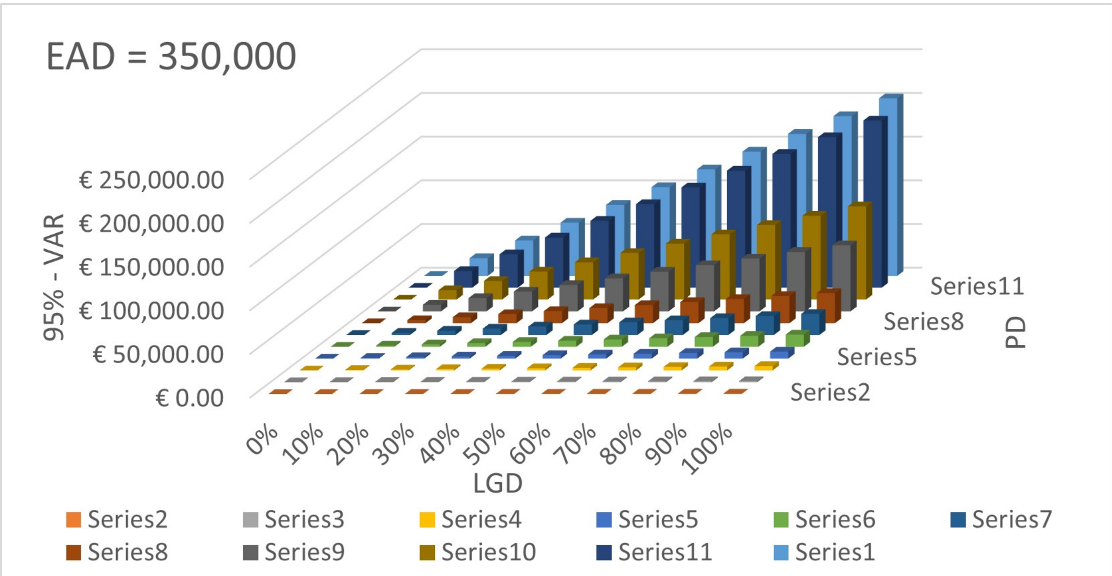

Figure D.4: 3D graph representing the sensitivity of VaR based on the LGD and PD with a fixed EAD at 350,000

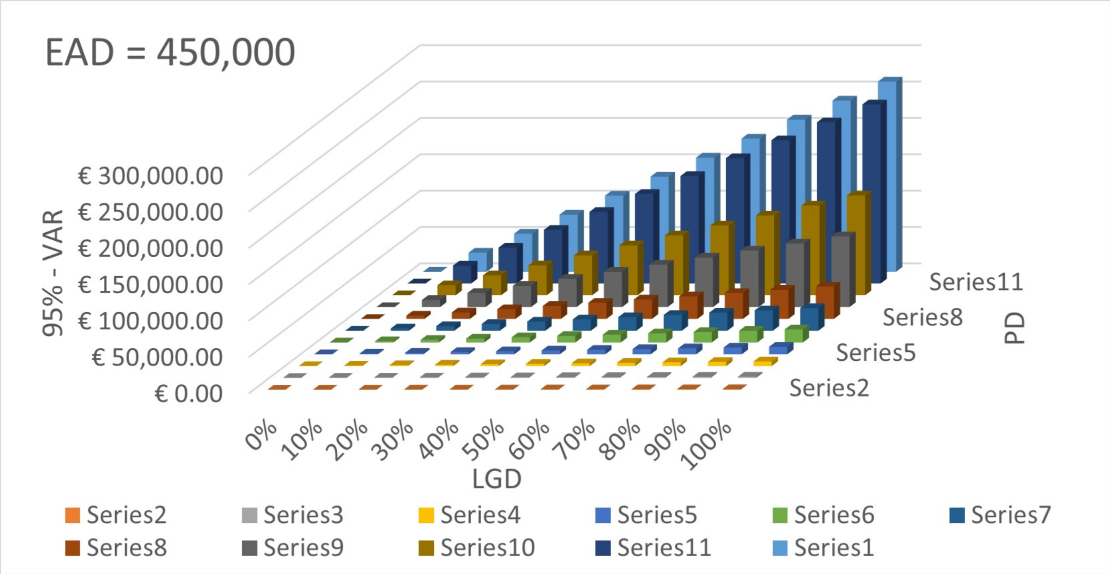

Figure D.5: 3D graph representing the sensitivity of VaR based on the LGD and PD with a fixed EAD at 450,000

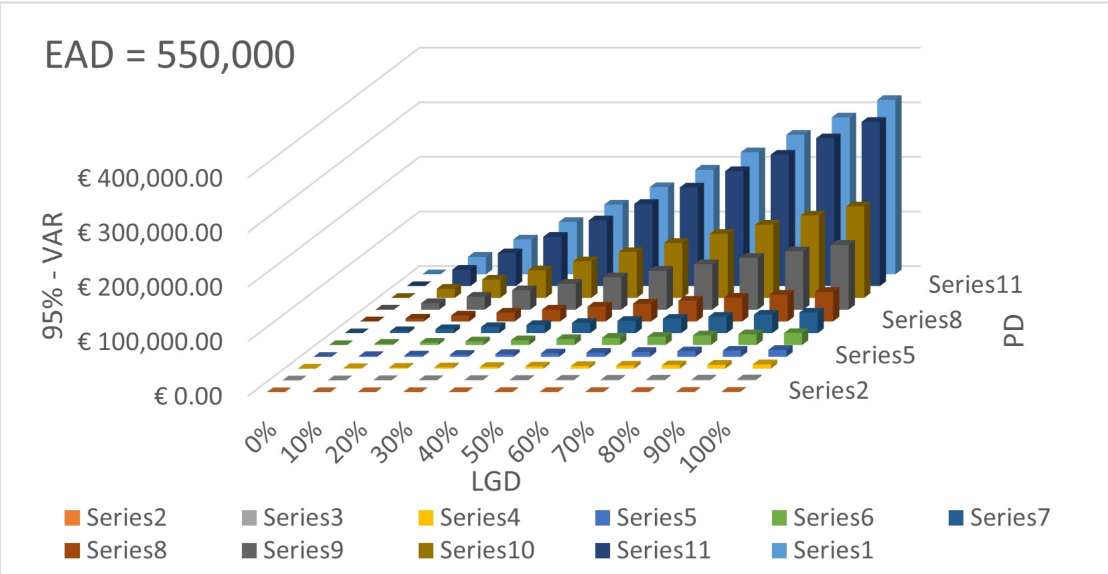

Figure D.6: 3D graph representing the sensitivity of VaR based on the LGD and PD with a fixed EAD at 550,000

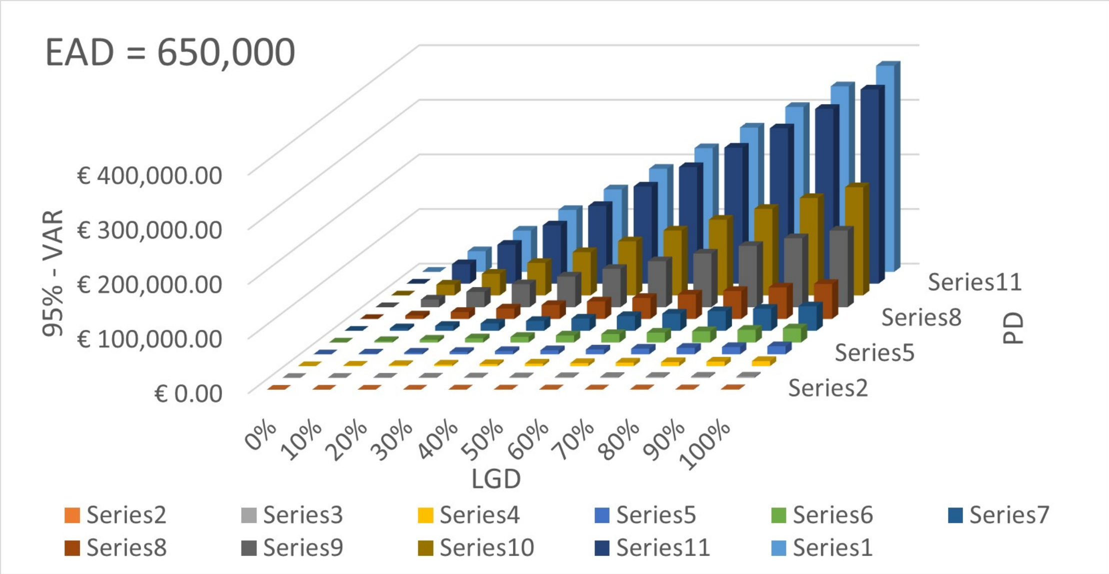

Figure D.7: 3D graph representing the sensitivity of VaR based on the LGD and PD with a fixed EAD at 650,000

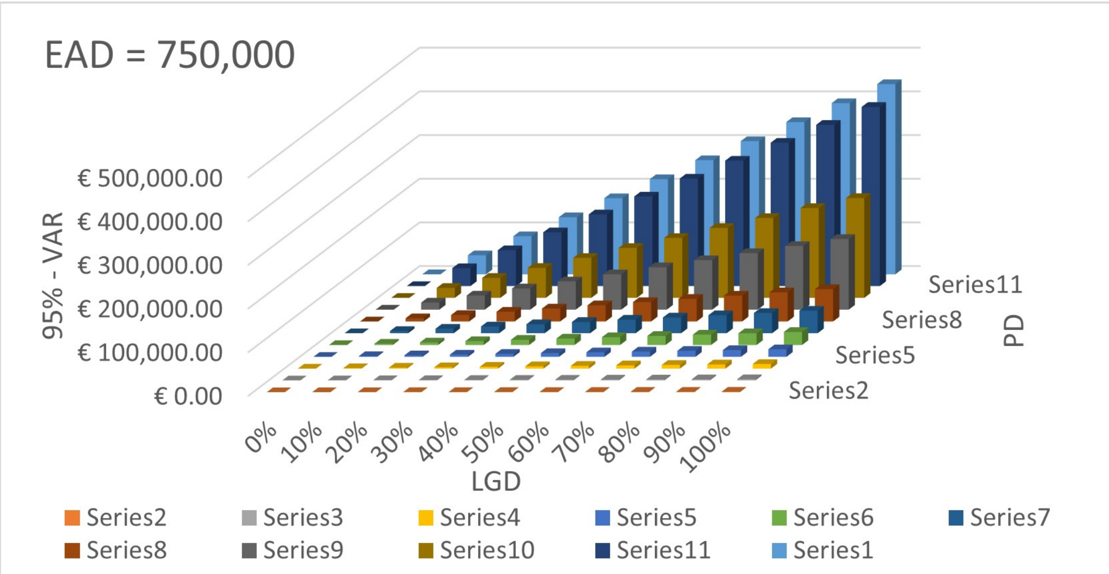

Figure D.8: 3D graph representing the sensitivity of VaR based on the LGD and PD with a fixed EAD at 750,000

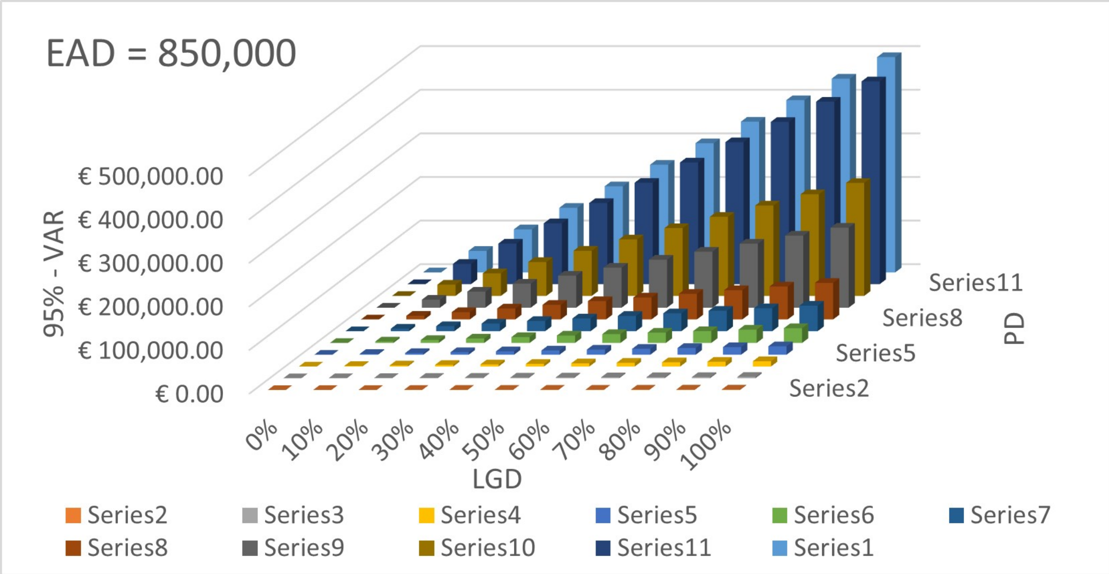

Figure D.9: 3D graph representing the sensitivity of VaR based on the LGD and PD with a fixed EAD at 850,000

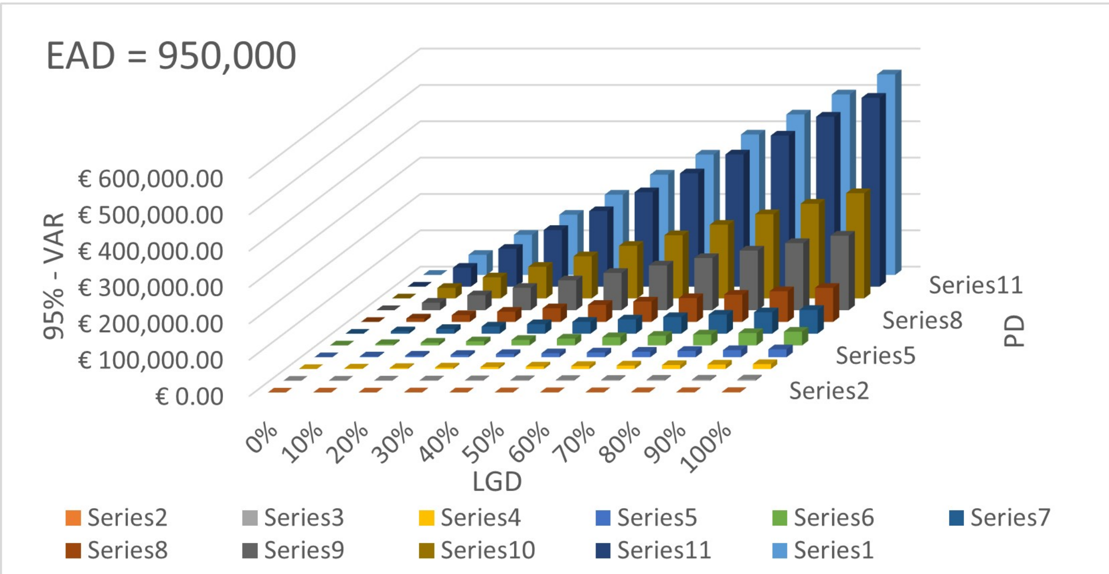

Figure D.10: 3D graph representing the sensitivity of VaR based on the LGD and PD with a fixed EAD at 950,000

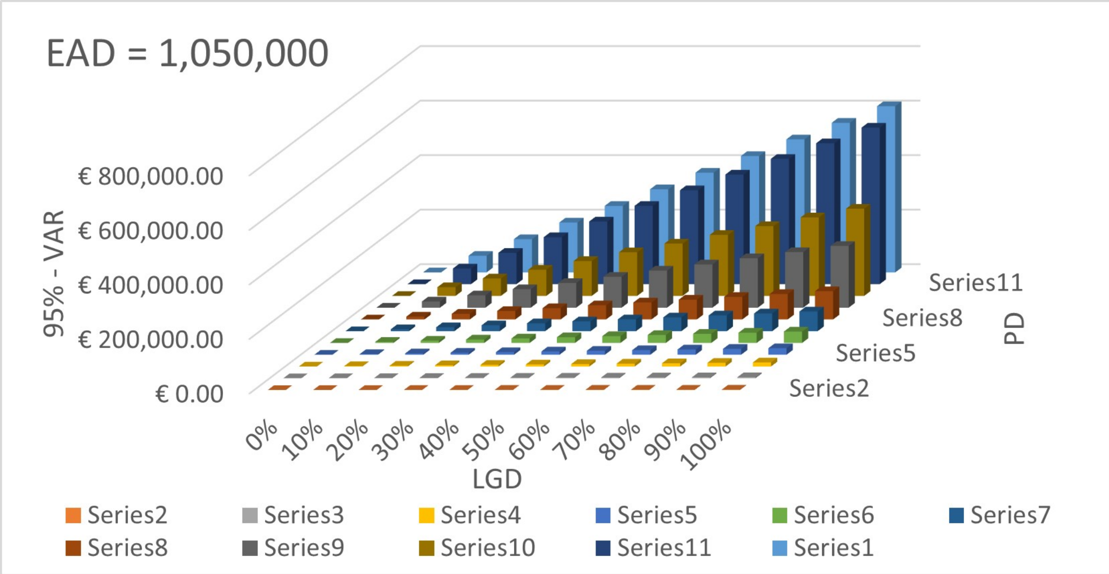

Figure D.11: 3D graph representing the sensitivity of VaR based on the LGD and PD with a fixed EAD at 1,050,000

Appendix E

General Introduction

1. Could you briefly introduce yourself and explain your role within your organisation?

2. Do you have experience with using financial risk models?

Model Specifications

3. How important do you think financial risk management is for your organisation from your perspective?

4. In your opinion, what are the main risks that Horizon faces?

Accuracy of Calculations (to be answered for both models)

5. Were you able to verify the calculations in the model? If so, were there any errors or inaccuracies?

6. Are there specific aspects of the calculations you would like to examine further? Or do specific aspects require clarification?

7. How do you view the modelling of correlation between loans and between participations? What would be your approximation for a correlation factor? (We currently assume the correlation is 0.)

Clarity of Steps Taken in the Model (to be answered for both models)

8. Did you find the steps taken in the model clear and well-documented?

9. Are there areas that you think could be better explained or documented?

Intuitiveness and Usability of the Model (to be answered for both models)

10. How would you rate the usability of the model?

11. Do you have any suggestions to make the model more intuitive for users? The model requires the user to have some financial knowledge and understanding of the role of the model.

Relevance of the Modelled Parameters for Credit Risk Specifically

12. Do you think the modelled parameters are relevant to Horizon's credit risk? Here, I refer to the features used for scoring the Probability of Default.

13. Are there specific features that you believe are more important than others? If so, why? This question pertains to the weights assigned to individual features within the Probability of Default.

General Feedback

14. Are there any other comments or suggestions you would like to share regarding the models for credit risk and market and liquidity risk?

15. Would you consider using these models in practice? Why or why not?

Closing

16. Thank you for your time and valuable input. Do you have any follow-up questions or comments before we conclude?