Multi-Source Direction of Arrival Estimation with Single-Bit Quantization

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***Abstract*—With the combination of the antenna arrays and signal processing, the position of a Radio Frequency Identification(RFID) tag can be estimated. This can be done by applying different types of Direction Of Arrival (DOA) estimating algorithms. This paper studies the effect of the single-bit quantization on the DOA estimation of two sources. Previous research has addressed ways to reconstruct covariance matrices and the effect of single-bit quantitation for a single source, this study focuses on estimating the DOA of two sources by applying two types of algorithms, which are Classical Beamforming and Multiple Signal Classification (MUSIC). The comparison of these two algorithms revealed that Classical Beamforming showed robust performance for both the original signals and quantized signals once the SNR reached -10[dB]. In contrast, MUSIC had an unexpected degradation of accuracy with SNR higher than 10[dB]. With this analysis, further research is still required to overcome the degradation of the accuracy when MUSIC is applied to the quantized signal with high SNR.**

I. Introduction

In daily life, people generally consider the Ultra High Frequency(UHF) Radio Frequency IDentification (RFID) tag as a medium that stores a certain item's data or information, which can be read through the tag reader[1]. However, it is possible not only to store and read the tags but also to calculate their positions with an appropriate antenna system and signal processing. This can be done by utilizing multiple arrays which are also known as phased arrays combined with the direction of arrival (DOA) estimation algorithms[2]. Each tag backscatters a signal when the reader tries to read the tag, and each phased array can estimate which direction this backscattered signal came from. With multiple phase arrays estimating the DOAs, the tag's position is estimated. The output signal of each antenna is quantized which allows the post-processing of the signal in the digital domain. Single-bit analog to digital converters (ADC) can be utilized in the system, which is more cost-efficient than the high-resolution ADC and reduces power consumption. Despite the advantages of single-bit quantization, the errors due to quantization are inevitable. The effect of single-bit quantization on the DOA of a single UHF RFID tag has been studied along the way to compensate for the error caused by quantization [3]. This was done by finding the relation of the correlation coefficients before and after the quantization and it was also validated with experiments in. Whereas the way to reconstruct the original covariance matrix from the quantized covariance matrix based on arcsine law was proposed in [4]. Additionally, it was also analyzed that covariance matrix-based estimation algorithms can be straightly applied to the quantized covariance matrix, especially for relatively low SNR [5].

The goal of this research is to investigate the effect of single-bit quantization on DOA estimation of multiple sources, and further compare the performances of two algorithms in a given situation, which are Classical beamforming and MUltiple SIgnal Classification (MUSIC). Ultimately, this is required because not only single tag of an item is tracked, but the positions of several items must be estimated at the same time. During the research, only two sources were dealt with to investigate the effect, however, the results can be extended to more than two sources allowing a single-phased array estimating the DOA of multiple sources at once. First, the simulation environment and the algorithms used will be explained. Then the general problem of estimating single-source DOA will be illustrated to be compared with two-source DOA estimation performance. This will be followed by a thorough analysis of the two-source DOA estimation. Lastly, a performance comparison of two different DOA estimation algorithms in the given situation will be presented.

II. Background

A. Simulation model

DOA estimation of the tags was made using a Uniform Linear Array (ULA), in which the antennas are placed at an equal distance from each other. One antenna is set as the reference antenna. This represents that the signal received at this antenna had no delay and the other antennas would receive the delayed signal relative to the reference signal. Each antenna in ULA had an equal spacing of half the wavelength ($\lambda/2$) [6].In the signal model, far-field assumptions were made so that the wavefront is assumed to be planar. Fig. 1 demonstrates a single signal impinging on the antenna array.

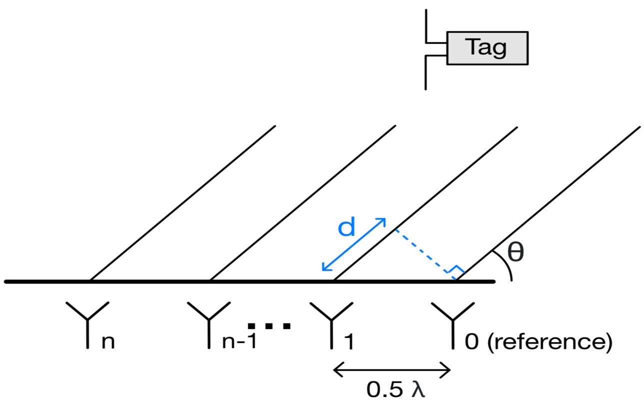


Fig. 1: Basic model of single source DOA estimation

When the signal is reached at the phased array with the angle θ, due to the distance between each antenna, the signal has to propagate extra distances to reach the other antennas compared to the reference antenna. In Fig. 1, distance d is the extra distance that the signal has to propagate compared to the signal that reaches the reference antenna. This extra distance can be expressed with simple trigonometry:

$$d = \frac{\lambda}{2} \cdot cos(\theta). \tag{1}$$

Then the sampled complex signal $\mathbf{s}[l]$ with frequency of f which is represented as:

$$\mathbf{s}[l] = exp(j \cdot 2\pi \cdot f \cdot t_l), \qquad l = [0, 1 \cdots, L], \tag{2}$$

would take an extra d/c amount of time to reach the next antennas with c as the speed of light. Then the single source signal at the nth antenna at the time index of l can be expressed and was simplified with the relation $c = \lambda \cdot f$:

$$\begin{aligned} x_n[l] &= s[l] \cdot exp(j \cdot 2\pi \cdot f \cdot n \cdot \frac{d}{c}) \\ &= s[l] \cdot exp(j \cdot \pi \cdot n \cdot cos(\theta)). \end{aligned} \tag{3}$$

This expression can be extended to express the signal of M sources impinging on the antenna array with the Additive white Gaussian noise $AWGN_n[l]$ added:

$$\begin{aligned} x_n[l] = s[l]\{&exp(j\pi n cos(\theta_1)) + exp(j\pi n cos(\theta_2)) \\ &+ \cdots + exp(j\pi n cos(\theta_m))\} + AWGN_n[l]. \end{aligned} \tag{4}$$

Then these n signals are combined into a single matrix $\mathbf{X}$, with each row corresponding to each antenna output:

$$\mathbf{X} = \begin{bmatrix} \mathbf{x}_0[l] \\ \mathbf{x}_1[l] \\ \vdots \\ \mathbf{x}_{N-1}[l] \\ \mathbf{x}_N[l] \end{bmatrix}, \qquad l = [0, 1 \cdots, L]. \tag{5}$$

B. DOA estimation algorithms

The DOAs of the signals impinging on the array will be estimated with the signal model (5). In this research, two methods, namely Classical beamforming and MUSIC, were taken into account to investigate how the quantization of the signal would affect the DOA estimation accuracy and also find out which method is more suitable for two-source quantized signals. Comparing these two types of methods is expected to give insightful results since the way how each algorithm estimates DOA is distinctly different.

Classical Beamforming involves multiplying $\mathbf{X}$ with a set of weights $\mathbf{W}$ to the $\mathbf{X}$, which is expressed as:

$$\mathbf{W} = [\mathbf{w}_0 \quad \mathbf{w}_1 \quad \cdots \quad \mathbf{w}_{180}]^H, \tag{6}$$

$$\mathbf{w}_\Theta = \begin{bmatrix} exp(j \cdot \pi \cdot 0 \cdot cos(\Theta)) \\ exp(j \cdot \pi \cdot 1 \cdot cos(\Theta)) \\ \vdots \\ exp(j \cdot \pi \cdot N \cdot cos(\Theta)) \end{bmatrix}. \tag{7}$$

The multiplication of the matrices would yield a delayed sum of the signals of all N antennas at each row. Then spatial power Spectrum can be constructed based on these multiplied matrices:

$$P_{beamforming}(\Theta) = power(\mathbf{W} \cdot \mathbf{X}). \tag{8}$$

Beamforming searches for a Θ that maximizes the power of the delayed and summed signal which is considered to be the angle of arrival [7]. This is because when delays are properly removed from the signals, the signals would be constructively added to each other and would be amplified the most. The spatial spectrum for a single source coming from angle $\Theta = 30$ can be shown in Fig. 2. In the example, $N = 15$ antennas were used with an SNR of $-5[dB]$.

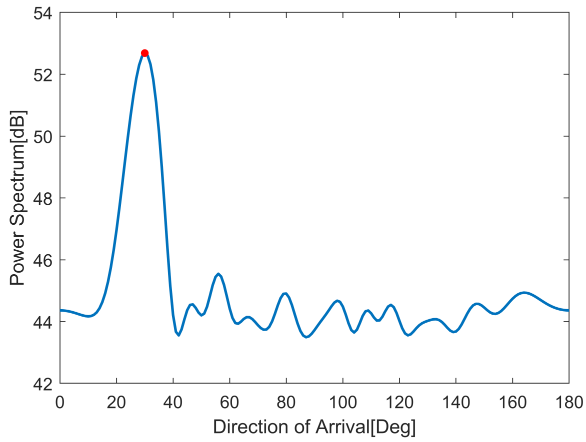


Fig. 2: Spatial power spectrum with Beamforming. The peak corresponds to the true angle of arrival $\theta = 30$.

On the other hand, MUSIC takes the covariance matrix of the signal which is calculated as:

$$\mathbf{R} = E\{\mathbf{X} \cdot \mathbf{X}^{\mathrm{H}}\}. \tag{9}$$

Then MUSIC algorithm computes the eigen decomposition of the covariance matrix $\mathbf{R}$ to obtain the eigenvalues and the corresponding eigenvectors to sort the noise subspace from the signal subspace. If there are M sources impinging on the array, the first largest M eigenvalues and the corresponding eigenvectors are the signal subspace, and the rest of the eigenvectors are the noise subspace $\mathbf{U}_n$. Finally, MUSIC constructs the power spectrum as:

$$P_{MUSIC}(\Theta) = \frac{1}{w_\Theta^H \mathbf{U}_n \mathbf{U}_n^H w_\Theta}. \tag{10}$$

This implies that the angle that is least likely to be influenced by the noise will maximize the $P_{MUSIC}(\Theta)$ concluding that the angle is the DOA [7]. The same parameters were used but MUSIC is applied to create the spectrum shown in Fig. 3.

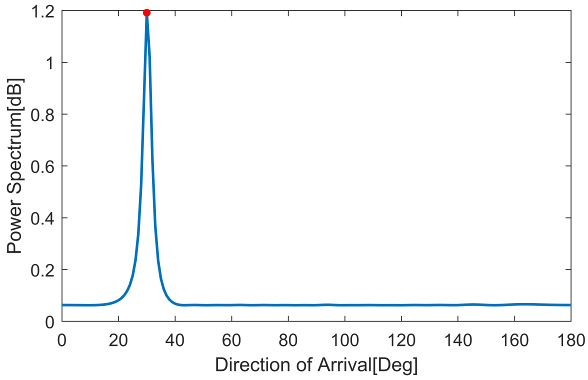


Fig. 3: MUSIC spectrum, with its peak located at the true angle of arrival $\theta = 30$

When applying these algorithms to estimate the DOA of two sources, the same procedures can be taken. However, each algorithm seeks the two peaks that correspond to the two angles θ_1 and θ_2, as shown in the Fig. 4.

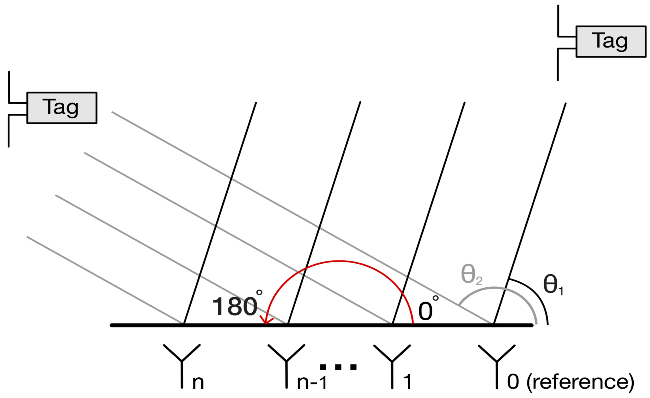


Fig. 4: Setup of the two source DOA estimation

C. *Quantization*

Before DOA estimation algorithms are applied to the set of signals $\mathbf{X}$, each signal, $x_n[l]$ is quantized into either 1 or -1:

$$Q(x) = \begin{cases} 1 & \text{if } \ x \geq 0 \\ -1 & \text{if } \ x < 0 \end{cases} \tag{11}$$

Since the DOA estimation focuses on the phase, not the information itself, low-resolution quantization is not expected to significantly degrade the performance of the estimation algorithms. Moreover, it is expected to reduce the overall cost needed to set up the DOA estimation system by adopting low-resolution ADCs. Nevertheless, when estimating the DOA for more than one source, additional error is expected to be introduced due to the superposition of numerous signals. Therefore in the following section, the influence of the single-bit quantization on the performance of the DOA estimation algorithms described above will be investigated.

III. Simulations and Analysis

Throughout the simulations, root mean square error (RMSE) was used to evaluate the performance of the DOA estimation accuracy. RMSE was obtained from over 6000 noise realizations for each specific parameter.

A. *Effect of Quantization*

1) *Single-source DOA estimation*

Before diving into the simulation of two-source DOA estimation, the effect of quantization on the single-source DOA estimation was first examined. This was done so that it can be utilized as reference data when investigating the effect of quantization on DOA estimation with two sources.

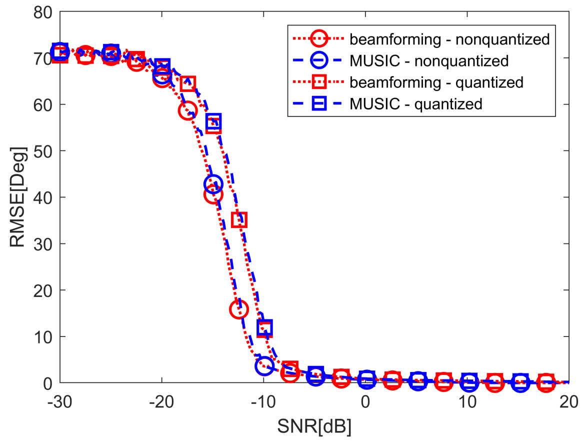


Fig. 5: RMSE plotted against SNR for Beamforming and MUSIC applied to single-source signal impinging on the antenna array from an angle $\theta = 30$

Blue plots on Fig. 5 correspond to the RMSE versus SNR when MUSIC was applied to signal with and without quantization, while the red plots represent the result of beamforming. It was observed that both algorithms resulted in nearly identical performance with their graphs overlapping each other. Both the original signal and quantized signal showed an improved accuracy with increasing SNR. However, the quantized signal required an average of 2[dB] higher SNR to achieve the same level of precision as the original signal. Additionally, the effect of quantization was investigated by varying the true angles of the source.

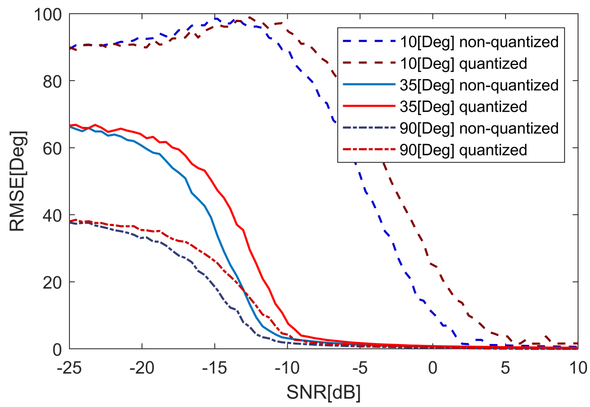


Fig. 6: RMSE comparison between quantized and non-quantized signal with angles $\theta = 10, 35, 90$.

The six plots in Fig. 6 are three sets of non-quantized and quantized signal's RMSE obtained throughout three different angles. Regardless of the actual angle, each pair showed a similar pattern where the DOA estimation of the quantized signal required approximately 2[dB] higher than those of the single that is not quantized to achieve the same level of precision. Furthermore, the overall level of the error increased as the true angle was higher or lower than 90[Deg].

Based on these results, this difference can be seen as the effect of the quantization. A similar effect of quantization is expected when the algorithms are applied to the two-source signal, in which a quantized signal requires a higher SNR to achieve a similar performance as the original signal.

2) Two-source DOA estimation

With all the simulation parameters set and the effect of quantization on single-source DOA estimation considered, the effect of quantization on two sources was eventually analyzed. With anything unknown yet, the two angles of each source to be estimated were first set with enough space in between DOA, which were 10[Deg] and 120[Deg]. Moreover, considering that the number of antennas is proportional to the directivity of the antenna array, the number of antennas in the array was set to 16[6].

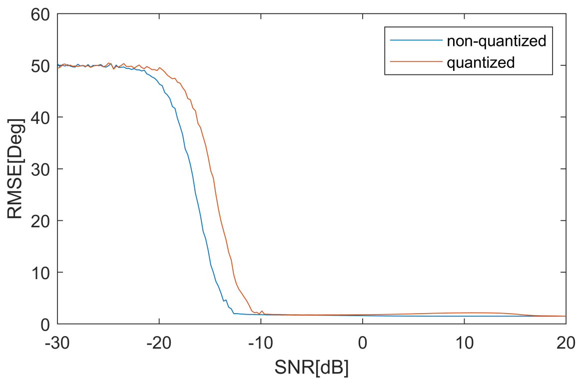


Fig. 7: RMSE versus SNR with the two angles of the source set as $\theta_1 = 10, \theta_2 = 120$ estimated with Beamforming

Two plots in Fig. 7 were obtained by applying Beamforming to the original signal and quantized signal by changing the SNR of the signals. RMSE decreased as the SNR of the signal increased and RMSE converged to approximately 2[Deg]. Especially for SNR below -10[dB], the RMSE of the quantized signal required a higher SNR approximately 2[dB] to achieve the same performance similar to the case of single-source DOA estimation.

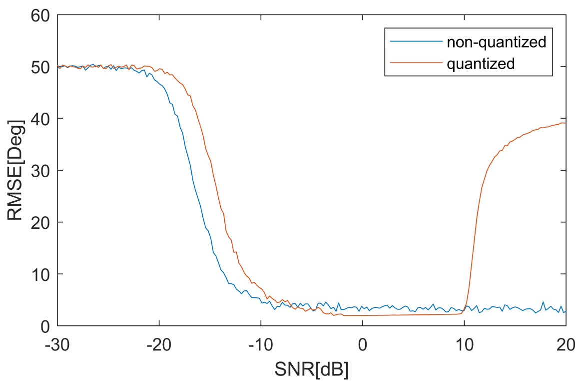


Fig. 8: RMSE versus SNR with the two angles of the source set as $\theta_1 = 10, \theta_2 = 120$ estimated with MUSIC

The plots in Fig. 8 were also obtained the same way as obtained with beamforming, instead MUSIC was applied. It can be seen that the RMSE for the quantized signal settles down to an error of 2[Deg] when the non-quantized signal converges to an RMSE of 3[Deg] requiring less SNR, approximately 2[dB]. However, a sudden increase of RMSE was observed after SNR exceeded 10[dB] in the case of quantized signal. This phenomenon was not present when MUSIC was applied to the non-quantized signal or beamforming was applied to the quantized signal. It can be deduced that this significant error was caused by the combination of both signal quantization and also characteristics of the MUSIC algorithm itself.

The power spectrum of MUSIC of both non-quantized and quantized signals was investigated to search for abnormal behavior. The spectrum was obtained with the SNR set to 20[dB] and the corresponding spectrum is shown in Fig. 9. The power spectrum of the non-quantized signal had two spikes at the specified DOAs, whereas the spectrum of the quantized signal displayed two additional spikes at irrelevant DOA angles. These irrelevant spikes frequently had values larger than those at the desired DOA angle causing inaccuracy of the DOA estimation with MUSIC.

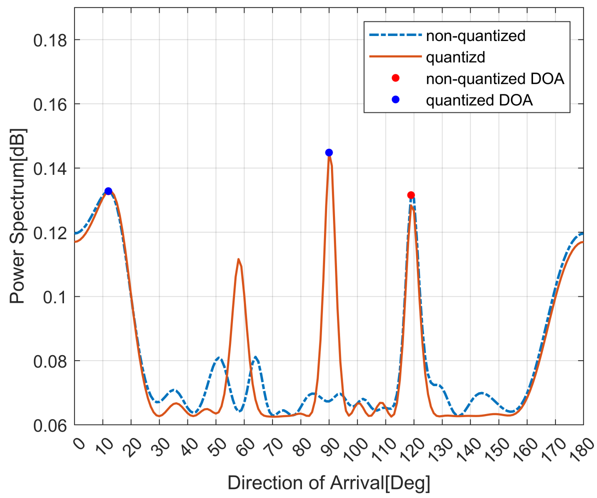


Fig. 9: Music Power spectrum of non-quantized and quantized signal at 20[dB], showing wrong estimation (12[Deg],90[Deg]) and correct estimation(10[Deg],120[Deg])

The irrelevant spikes which are depicted in Fig. 9 gradually faded away and eventually resembled the non-quantized signal's spectrum upon decreasing the SNR from 20[dB] to the SNR level of 5[dB] where MUSIC was able to correctly estimate the DOAs. Fig. 10 shows that irrelevant peaks vanished from the spectrum when the SNR was lowered to 5[dB]. It can be deduced that the tendency of MUSIC to detect more sources than the given number of sources is causing the error in estimating the DOAs when the signals from the two sources are superposition and quantized.

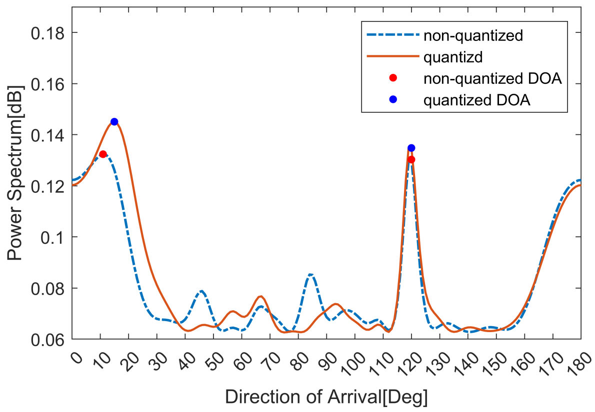
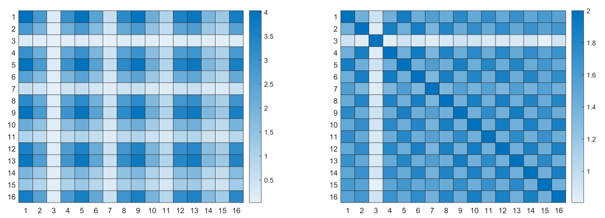


Fig. 10: Music Power spectrum of non-quantized and quantized signal, SNR lowered to 5[dB] with irrelevant peaks diminished, leading to better estimation

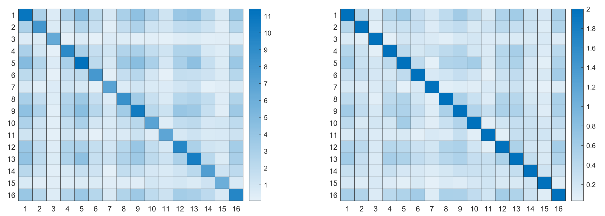
Given that the quantization maps each signal to either 1 or -1, this process is expected to distort both the amplitude and the phase information that the signal contains, and further lead to the incorrect estimation of the covariance matrix. To investigate this effect, covariance matrices with different SNR levels were examined by visualizing them as a heatmap of their magnitudes. Both the quantized signal and non-quantized signal's covariance matrix heatmap were plotted at two different SNR levels which are 20[dB] and -5[dB]. As shown in Fig. 11 two heatmaps showed distinct patterns demonstrating the inaccuracy of MUSIC at SNR of 20[dB].



(a) non-quantized (b) quantized

Fig. 11: Comparison of covariance matrix's heatmap at SNR of 20[dB]

Compared to the previous heatmaps plotted at an SNR of 20[dB], heatmaps plotted at an SNR of -5[dB], where performance for quantized and non-quantized signals show similar levels, shown in Fig. 12 shows highly similar patterns.



(a) non-quantized (b) quantized

Fig. 12: Comparison of covariance matrix's heatmap at SNR of -5[dB]

Further investigations were conducted on whether different combinations of true angles with SNR of 20[dB] would also affect the DOA estimation. Some angle combinations such as 10[Deg] and 89[Deg] showed a high level of accuracy even though SNR stayed at 20[dB]. This was observed by fixing one source's angle as 10[Deg] and varying the second source's angle. Fig. 13 shows the RMSE obtained with the increasing second source's angle. It can be seen that the RMSE tends to decrease as the angles get close to 90[Deg] and then increase again. However, overall accuracy was poor for all angles except 89[Deg] and 90[Deg] with RMSE lower than 2[Deg].

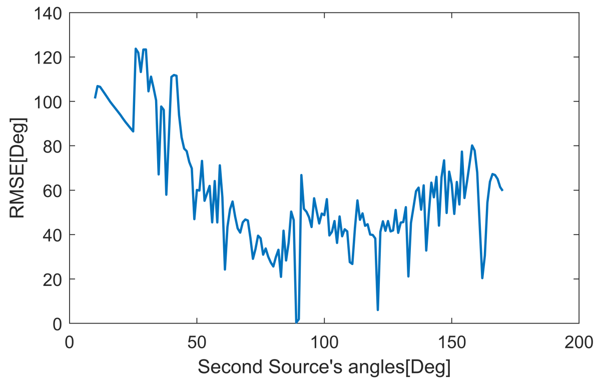


Fig. 13: RMSE plotted against different angles of the second source with SNR of 20[dB].

As the slight difference in angles resulted in different levels of accuracy, RMSE was obtained by sweeping both sources' angles of arrival from 10[Deg] to 170[Deg] with 2[Deg] increments, with the SNR fixed to 20[dB]. The errors were visualized with a heatmap where darker colors indicated higher errors while bright colors indicated low errors. The error heatmap illustrated in Fig. 14 shows a unique pattern but it was not able to find a clear correlation between two angles, and it exhibited low accuracy throughout all the angle combinations.

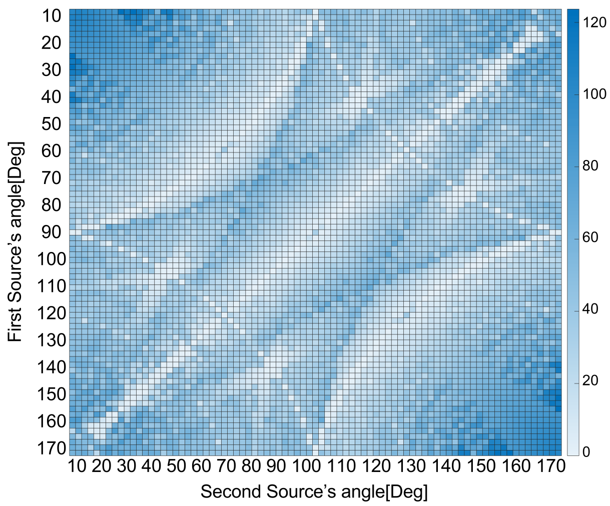


Fig. 14: Heatmap of error made with MUSIC applied to various angle combinations with SNR 20[dB]

Conversely, when Beamforming was utilized to obtain the errors with various angle combinations as depicted in Fig. 15, it was evident that the error increased as the difference between two angles decreased. Additionally, it showed a high error at the corners of the heatmap which corresponds to the angles placed near 0[Deg] or 180[Deg]. This is because when the peaks are placed at either the left or right edge of the spectrum plot, they can not be distinguished when algorithms search for the local maxima. Besides these specific cases, Beamforming demonstrated highly accurate performance.

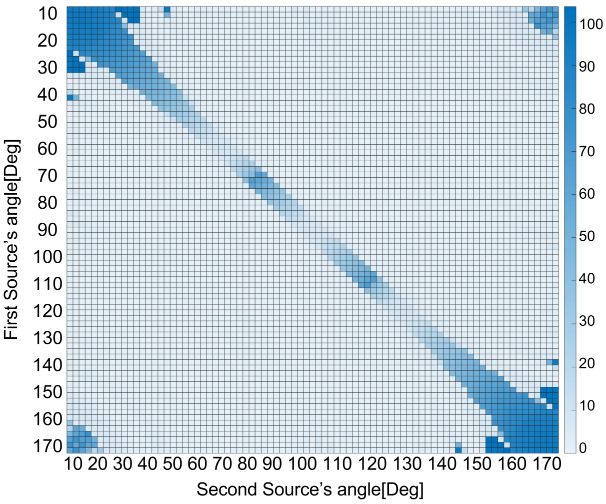


Fig. 15: Heatmap of error made when Beamforming applied to various angle combinations with SNR 20[dB]

While both algorithms showed highly similar performance in the estimation of the single source DOA, simulations demonstrated that beamforming was more effective for estimating DOAs of two sources with quantized signals. Direct comparison of both algorithms applied to the quantized signals shown in Fig. 16 confirms that beamforming showed a consistent accuracy once the SNR of the signal reached -10[dB]. On the other hand, MUSIC achieved accurate estimation from -2[dB], higher than that of the Beamforming, with experiencing significant degradation of accuracy for signals with SNR higher than 10[dB]. Beamforming is believed to achieve its performance because it searches for the angle that results in the maximum power of the delayed and summed signals rather than relying on the correlation across all signals received at the antenna.

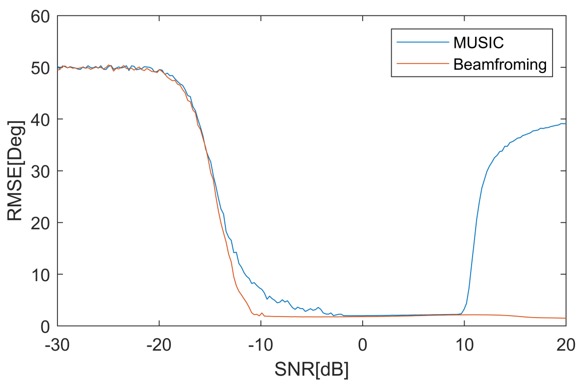


Fig. 16: Accuracy Comparison of Beamforming and MUSIC for angles of two sources given as 10[Deg] and 120[Deg]

IV. Discussion

The paper has analyzed the effect of the quantization on the DOA estimation for two sources impinging on the antenna array. The insights gained in this study can be used to find possible solutions to overcome the observed errors and also optimize the system to improve the accuracy of the DOA estimation.

The analysis involved investigating how the quantization caused errors in the process of estimating the DOA of two sources. It was found that the covariance matrix is not properly estimated when the signal is quantized. To prevent this estimation error, the reconstruction of the non-quantized signal's covariance stated in [4] can be applied to the covariance matrix of the quantized signal. Then the effect of the quantization can be further investigated whether the dramatic increase of the error is resolved for signals with SNR higher than 10[dB]. The other possible option is the preprocessing of the received data before the quantization. Comparing the performance of the MUSIC applied to the single source signal, it was observed that the degradation of the accuracy was not present once the RMSE value converged to approximately 2[Deg]. Therefore the Blind Source Separation(BSS) algorithm for the complex signal as described in [8], can be applied to the original signal received at the antenna to remove the error that occurs after the signal contains two DOA information. After the sources are separated and quantized, the MUSIC algorithms can then be applied to the separated signals to estimate each of the DOAs that the signal is coming from.

As the number of sources grows, the number of antennas must also increase to precisely distinguish the angle of arrivals of multiple sources. Even the two-source DOA estimation requires more than 15 antennas to have sufficient accuracy, more antennas are expected to estimate the DOA of numerous sources. Increasing the number of antennas in each antenna array is not only cost-inefficient but also results in significant growth in the size of each antenna array module. The geometry of the antenna array can be modified in a layout that consists of two ULA antenna subarrays known as the coprime array. This layout would introduce unique delays between the signals resulting in a virtual array [9]. The coprime array would allow the system to achieve the effect of receiving signals with more antennas than those that are physically placed in the limited space, resolving both cost and size problems. Consequently, the use of a coprime antenna can be considered for multi-source DOA estimation.

Lastly, simulations used only the simple sinusoidal signal waves as the signal received at the antenna array, with identical frequencies and amplitudes. However, in real-world scenarios, the signals that are transmitted and received can be modulated. Consequently, further research on the effect of quantization for multi-source DOA is necessary.

V. Conclussion

This paper has analyzed the effect of single-bit quantization on the DOA estimation of two sources using beamforming and MUSIC algorithms. While both algorithms exhibited similar performance in estimating single source DOA when signals were quantized, significant errors occurred when applying MUSIC, especially for signals with SNR higher than 10[dB]. Beamforming, however, showed a robust performance throughout the SNR once the SNR exceeded -10[dB] regardless of any combinations of the angle of arrival. Beamforming resulted in an error only when two angles were closely placed to each other, whereas no clear correlation could be identified for MUSIC. However, further research is necessary to investigate the solutions to overcome these errors and to consider a more practical environment, such as using more complicated signals and the use of different types of antenna arrays, to thoroughly analyze the effect of the quantization on the multi-source DOA estimation, and further optimize the DOA estimation system.

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Appendix

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%% Matlab code to generate the signal model and estimate DOA with beamforming and
   MUSIC

%%Transmitted signal
clear
c = 3*10^8;                                     %light speed
f = 100;                                        %frequency of the signal
lamda  = c/f;                                   %wave length
t = linspace(0,0.06,400);                       %time samples
s = exp(1i*2*pi*f*t);                           %trasnmitted signal

%%Receved signals
Nsig = 2;                                       %number of signals
SNR = -5;                                       %SNR of each signals
angle = [10 120];                               %true angle of arrival of two sources
antenna_num = 16;                               %antenna numbers of antenna array

x1 = signal_model(s,angle(1),antenna_num);      %signal model of first source
x2 = signal_model(s,angle(2),antenna_num);      %signal model of second source
temp_x = x1 + x2;                               %two signal model superpositioned
x = awgn(temp_x,SNR,"measured");                %noise added to all the signals

%%DOA estimations
%Beamforming
angle_scan = linspace(0,180,181);               %sets of angles to scan
angle_scan_rad = angle_scan*pi/180;
k_vector = (1:antenna_num)-1;                   %antenna number dependant delays
w_matrix = exp(1i*pi*k_vector'*(cos(angle_scan_rad)));%weight matrix
beam_DOA  = beamforming(x,w_matrix,angle_scan,Nsig)

%MUSIC
cov_mat = (x*x')/length(s);                     %covariance matrix
music_DOA = sort(90-musicdoa(cov_mat,Nsig))     %compensation for different angle
                                                defining
%%Functions
%Singnal model
function x = signal_model(s,aoa_degree,antenna_num)
    aoa = (aoa_degree*pi) /180;
    x = zeros(antenna_num,length(s));

    for k = 1:antenna_num
        x(k,:) = s*exp(1i*pi*(k-1)*cos(aoa));
    end
end

%Beamforming
function DOA = beamforming(x,w_matrix,angle_scan,nsig)
    switch nsig
        %DOA estimation for single_source
        case 1
            matrix_est_power = mean(abs(w_matrix'*x).^2, 2);
            [~,I] = max(matrix_est_power);
            DOA = angle_scan(I);
        %DOA estimation for double_source
        case 2
            matrix_est_power = mean(abs(w_matrix'*x).^2, 2);
            [~,loc] = findpeaks(matrix_est_power,'SortStr','descend');
            DOA = sort([angle_scan(loc(1)) angle_scan(loc(2))]);
    end
end
```