

MSC - INDUSTRIAL ENGINEERING AND MANAGEMENT MASTER THESIS

A Stochastic Bilevel Model to Study the Market Power of a Strategic Wind Power Producer in the Norwegian Auction-Based Intraday Market

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Preface

This master's thesis was written by Wessel ter Laak as the final step of the master Industrial Engineering & Management at the University of Twente (UT), The Netherlands. During the course of the spring semester in 2024, I was given the opportunity to write my thesis and conduct research at the Industrial Economics and Technology Management department at the Norwegian University of Science and Technology (NTNU), where I also participated in an Erasmus exchange program the semester prior.

The research of the thesis focuses on studying the market power of a strategic wind power producer in the Norwegian electricity market. During the research, I received help through weekly meetings with Wouter Koks and Ehsan Nokandi, two researchers who specialize in electricity markets at NTNU, whom I would like to thank for their valuable input and discussions, and the time they invested in the project. I also would like to thank my NTNU supervisor Prof. Pedro Crespo del Granado, and my UT supervisor Prof. Alessio Trivella for their contributions to my thesis.

I hope you enjoy reading my master's thesis!

Wessel ter Laak

Trondheim, 21st August 2024

Abstract

The introduction of renewable energy sources to the energy mix, such as wind power, has implications for electricity markets due to its unpredictable nature. As the penetration of wind power continues to increase, the design and structure of electricity markets are put under consideration in an attempt to deal with this trend. The structure of the intraday market in Norway, in which market participants are able to re-adjust their commitment, is planned to be reformed. Multiple auctionbased markets will be introduced, and the effect of this is not entirely clear. The impact of this new structure on the ability of market players, in this case, a wind power producer, to strategically influence the market will be studied in this thesis. In short, the aim of this study is to investigate under what conditions a wind power producer will be able to exercise market power. To this end, a bilevel model is developed, in which the upper level represents the profit-maximizing behaviour of the wind power producer through a multi-stage stochastic program, adjusting its bids based on uncertainty in wind forecasts and bids and offers by other market participants. The lower-level models represent the market clearing process, in which supply and demand are balanced to maximize social welfare. The balancing market, used to maintain the real-time balance between supply and demand, is also included. The prices for balancing electricity are also represented by uncertain parameters. The results demonstrate that the wind power producer is able to increase its profit by utilising strategic offering. The consequences for the market are relatively minor, as the wind power producer does not try to influence the equilibrium point to its advantage, but rather offers a higher quantity at a less favourable price. This strategy is employed in order to take advantage of expected differences in the market prices between the sequential markets, resulting in a higher trading volume and relatively large swings in its commitment to the market. Thus, although the market price changes due to this strategy, this is not necessarily due to the strategic producer exercising market power, as this would be less profitable. It does result in a lower social welfare, as the wind power producer chooses not to offer its quantity against marginal costs anymore. Ultimately, this research contributes to a better understanding of the impact of strategic offering by a wind power producer on the new proposed intraday structure, as well as the strategic interactions between multiple auction-based intraday gates.

Definitions

The following abbrevations and corresponding definitions can be found in the thesis:

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1 Introduction

Over the past years, energy from renewable sources such as wind energy has become increasingly relevant to replace conventional power producers and ultimately reduce greenhouse gas emissions. The wind power industry in Norway has grown steadily over the past few decades, and primary energy from wind is expected to grow 9% year-on-year until 2050. In that year, renewable energy will represent 76% of the domestic energy mix (DNV, [2023\)](#page-58-1). The introduction of wind energy to the energy pool poses challenges for energy system operators due to its uncertain and stochastic nature, which exposes wind power producers (WPP) to risks in a competitive environment. Governments have supported wind power producers through market incentives to get a competitive advantage over conventional power producers to reduce this risk. Since the costs of wind power has been decreasing rapidly, wind power producers are more and more encouraged to participate on the energy market and compete with the other market participants under the same conditions (Zugno et al., [2013\)](#page-59-1).

Intraday markets are a useful mechanism that allows generation units and consumers to adjust dayahead positions based on improved forecasts. These markets are particularly important for wind power generators, as wind power is intermittent and poorly predictable (Karanfil & Li, [2017\)](#page-58-2). Since the amount of renewable intermittent production is increasing, trading in these intraday markets is of increasing interest, as it becomes ever more challenging to be in balance after the day-ahead market closes for market participants ('Intraday Market', [2024\)](#page-58-3).

Planned change of intraday trading mechanism

To this end, Nordpool, who operates the electricity market across many European countries, is planning to reform the functioning and structure of its intraday market ('Nord Pool - About the Intraday Auctions (IDAs)', [2024\)](#page-58-4). Norway is one of the countries of which the market is ran by NordPool. An auctionbased intraday market will be introduced in addition to the current, existing continuous market, which is already adopted by countries such as Portugal, Spain, and Italy. The auction-based intraday market will feature three gate-closures, in which generators and consumers can adjust their day-ahead position by submitting bids and offers. A more detailed explanation is given in Chapter [2.](#page-10-0)

There are some concerns about the consequences of implementing this new structure. The switch can lead to challenges for all market participants, such as wind power producers, to set up bidding strategies. It can also be difficult for policy makers to understand how market prices can be affected by strategic players, which is unclear at the moment. At the Industrial Economy and Technology Management Department of the Norwegian University of Science and Technology (NTNU), a project is ongoing that aims to understand the consequences of this planned change deeply. The PowerDig project aims to propose designs for the electricity market that are able to handle a higher share of renewable energy sources, as well as address shortcomings in existing electricity markets (PowerDig, [2024\)](#page-59-2). The project is led by NTNU in collaboration with SINTEF, NHH, Statnett and Statkraft and is funded by the Norwegian government ('Digitalization of short-term resource allocation in power markets', [2021\)](#page-58-5). The thesis assignment is defined based on this project. The research at NTNU takes place at the department of Industrial Economics and Technology Management, which conducts research and has strong competence on optimization topics related to energy and power systems, energy markets, energy grids, energy transition, etc.

As said before, the effect that the new intraday trading mechanism and structure has from both a system operator and market participant perspective are not entirely clear. One of the concerns revolves around the ability of market participants to strategically influence market outcomes. This thesis assignment tries to approach this concern from the perspective of a large wind power producer located in Norway. This wind power producer tries to optimize its bidding strategy through strategic offering in the proposed intraday structure. The organization wants to find out under which conditions or in which cases the chosen wind power producer can exercise its market power and to what extent the wind power producer can influence the market price through strategic offering.

Objective and main research questions

The research aims to view the sequential energy market from the perspective of a large wind power producer. This wind power producer aims to strategically optimize its profit, taking the competitive behavior of other market participants and the regular electricity market mechanisms, such as the clearing of the market with the aim of maximizing social welfare, into account when making its decisions.

The research aims to investigate what the role of a strategic producer in the newly proposed electricity market set up of Norway is and how this can lead to financial benefits and different market outcomes. To accurately represent the decisions that the strategic wind power producer faces over the course of the auction-based intraday and balancing markets, a stochastic model will be used that reflects real life uncertainty and correlation between the sequential markets, for example for the case of wind power production and market prices. The following research questions are answered in the thesis. A more extensive description of research questions per category are explained in Appendix [A.](#page-60-1)

- 1. What literature already exists on strategic offerings in electricity markets, and how does the current research distinguish itself from and contribute towards the existing literature?
- 2. How can an optimization model be developed that determines the optimal offering strategy of a wind power producer in the new market structure?
- 3. Under which conditions or in which cases can a wind power producer exercise its market power in the new market structure?
- 4. To what extent can strategic behaviour affect the wind power producer's profit?
- 5. To what extent can the strategic behavior of the wind power producer change the intraday market equilibrium and social welfare?

Model

The optimal bidding strategy of a wind power producer is modelled using a stochastic bilevel model. Bilevel models are used in processes that involve hierarchical decisions with two or more levels, and are especially suitable to model strategic behaviour in electricity markets. The higher level represents the profit maximization objective of the strategic wind power producer, while the lower levels represent the sequential clearing of the intraday markets. The bilevel is then converted to a single-level mathematical program with equilibrium constraints (MPEC) using its Karush-Kuhn-Tucker (KKT) optimality conditions. Afterwards, it is linearalized into a solvable Mixed-Integer Linear Problem (MILP). Uncertainty in the model is represtented by stochastic parameters related to the wind forecast, balancing prices, and bids and offers from other market participants.

Experiments

In order to answer the research questions, some experiments will be conducted. For this purpose, a dataset with real bids and offers will be used to simulate the market clearing. Furthermore, a scenario tree is constructed to accurately represent the chosen stochastic parameters. An instance in which the wind power producer behaves strategically will be compared to one with competitive behaviour. The expected profits and the obtained market prices will be analyzed to draw conclusions on the effect of strategic behaviour on the producer's profit and the market outcome.

Structure of the thesis

After introducing the topic of the thesis, the general outline of the thesis will be given. Chapter 2 provides more extensive information of the context needed for this research. It explains the functioning of relevant parts of the electricity markets, and it introduces the modelling concepts that will be used. Furthermore, this chapter will also include the literature review and the contribution to existing research. The specific model that is studied will be described in Chapter 3. The identification of the strategic player within the market, and the assumptions that were made are also mentioned. Chapter 4 details the methodological approach taken, which includes the step-by-step procedure and the final formulation of the bilevel model. In Chapter 5, the focus will shift to the practical aspect by conducting a case study. The input data that was used, as well as the considered scenarios will be elaborated upon. Then, Chapter 6 is used to present the findings of the case study and discuss the results. Finally, Chapter 7 concludes the thesis by summarizing the most important points, discussing the implications of the findings, and suggesting areas for research in the future.

2 Background

This chapter serves as a foundation in understanding the main aspects of the research. A more detailed explanation of electricity markets will be given, with special focus on continuous and auction-based intraday markets, and how the new intraday structure will look like. A literature review is conducted to evaluate existing literature on the same topic, such that a research gap can be identified, along with the contributions the research brings. Additionally, the theoretical concepts that will be used in the methodology will be explained.

2.1 Electricity markets

NordPool is the leading power-exchange market in Europe, operating electricity markets across many European countries. At first, NordPool only served the Nordic region, but later expanded to the Baltics and countries in Western Europe as well. As with any power-exchange market, it facilitates the buying and selling of power between generators and consumers. Due to NordPool's integrated network, power is also able to cross national borders, which promotes efficiency and security of power supply ('NordPool - About us', [2024\)](#page-58-6). The individual electricity markets composing the NordPool power-exchange are now elaborated upon.

Day-ahead market

In electricity markets, the day-ahead stage is used by market participants to submit production and consumption offers, consisting of an energy volume and a price, for a specific delivery period on the following day. Based on the various offers submitted by the participants, supply and demand curves are made. These curves are used to determine the market equilibrium point and thus the market price that all the participants either receive or pay for their energy. The day-ahead market usually accounts for the vast majority of trades (Lin & H. Magnago, [2017\)](#page-58-7). In the NordPool area in 2015, 93% of the consumption was traded in the day-ahead market ('Norway and the European power market', [2016\)](#page-59-3).

Intraday markets

The intraday market works slightly differently compared to the day-ahead market, and allows for market participants to submit bids and offers to adjust their day-ahead position. Producers who have an uncertain generation forecast, such as wind producers, can utilize this market to avoid having an imbalance during real-time energy delivery. Intraday markets in Europe are either continuous or auctionbased. In a continuous market, trades can be settled whenever a market participant accepts an offer of another market participant, which means the price varies from trade to trade. An advantage of this mechanism is that it allows market participants to trade whenever they expect benefits from trading. In contrast, auction-based intraday markets are cleared at discrete times with pre-defined gate closures. The price is then determined at the point where the supply and demand curve meet if there is no congestion (Scharff & Amelin, [2016\)](#page-59-0).

Furthermore, an auction can be characterized as a pay-as-clear pricing system, in which all participants pay or receive the same market-clearing price, regardless of their individual bids. This system incentives participants to bid their marginal costs, as they will pay or receive the uniform clearing price anyway. In contrast, in a continuous market, a participant's profit depends on their ability to find and fulfill attractive orders. Participants receive or pay the price of their bid if it is matched by another participant. This pricing system is known as pay-as-bid and, combined with the first-come, first-served principle of continuous trading, can lead to an inefficient allocation of scarce transmission capacities (Bindu et al., [2023\)](#page-58-8).

This may be because capacity is allocated to those participants who bid first. The capacity is then used for lower-value transactions as opposed to higher-value transactions.

Figure 1: Clearing of an intraday auction and continuous trading (Kuppelwieser & Wozabal, [2021\)](#page-58-9)

Figure [1](#page-11-0) shows how trading happens in an auction-based and a continuous intraday. The yellow marker in the auction clearing instance signifies the uniform clearing price, while the yellow markets on the right represent price ticks. These are instances when orders were cleared, representing a match between a newly placed order and an order in the order book generating a trade (Kuppelwieser & Wozabal, [2021\)](#page-58-9).

Proposed implementation of auction-based intraday market

Figure 2: Proposed intraday market model in Norway

Figure [2](#page-11-1) shows the proposed implementation of the new auction-based intraday market model for energy markets controlled by NordPool, which consists mainly of Scandinavian and Western European countries, including Norway. This intraday model shows three closing gates at which the bids and offers are settled. All the bidding zones in Norway use a 60-minute Market Time Unit (MTU), which means that market participants bid energy for a one-hour period. The way this mechanism works is that market participants are able to participate in the first and second intraday auction if they want to make adjustments for the delivery period ranging from 00:00 to 24:00 the following day. The third intraday gate allows adjustments to be made for the delivery period ranging from 12:00 to 24:00 on the same day ('Nord Pool - About the Intraday Auctions (IDAs)', [2024\)](#page-58-4). After the closure of the gates and the clearing of the market, the TSO takes control of the market through the balancing mechanism.

Balancing market and imbalance settlement

In the balancing market, the transmission system operator (TSO) is responsible for making sure the energy system is in balance and that the real-time demand is met by an equal amount of energy generation.

For various reasons, market participants that have made generation or consumption commitments in the day-ahead and intraday markets, may not always meet these commitments during the time of actual energy delivery. A balancing system that uses up- and down-regulation allows the TSO to make sure these imbalances are dealt with by activating reserve capacity that market participants have made available to be used in case of deviations. These reserve capacities must be available at short notice, and are activated based on the merit order (cheaper options are activated first). Participants supplying reserve capacity are paid a balancing settlement price, which is usually higher than the price they would get in the other energy markets (Kirschen & Strbac, [2004\)](#page-58-10).

The balancing market in electricity markets play an important role in ensuring stability of the power system network. It manages real-time imbalances between generation and consumption. The balancing market uses a pricing system that is designed to incentivize certain behaviour of the market participants, which can have a significant impact on this. In balancing markets, one of primarily two pricing systems are usually employed, namely the (a) two-price system, and the (b) one-price system. They are visualized in Table [1](#page-12-0) and elaborated in Scharff and Amelin [\(2016\)](#page-59-0).

In the two-price system, deviations from commitments made in previous markets are penalized, such that the need for balancing is discouraged. In this case, market participants do not profit excessively from deviations that could have been managed in previous markets. The one-price system allows participants to profit from real-time deviations, increasing the chance for imbalances, and creating opportunities for strategic trading. NordPool uses a two-price system.

Table 1: Imbalance settlement pricing scheme (Scharff & Amelin, [2016\)](#page-59-0)

Market equilibrium

Every supplier and consumer of energy submits bids to the energy market in which it participates, which consists of an energy quantity and corresponding price at which it is willing to sell and purchase, respectively, this quantity. This must be done before the market closes, after which the market clearing process can start. The *equilibrium price* or *market clearing price* is at the point where the amount of energy that generators want to provide is equal to the amount that the consumers wish to obtain (Kirschen & Strbac, [2004\)](#page-58-10). Energy will be traded as long as the price that consumers are willing to pay for a certain quantity exceeds the price of the suppliers are willing to sell their energy at. Figure [1](#page-11-0) shows an example of a supply and demand curve on the left, including the corresponding equilibrium point. The area under the curve shows the situation in which offers are being accepted, while the part on the right side of the equilibrium shows offers from both consumers and suppliers that are not fulfilled. On the intraday market, both suppliers and consumers can buy and sell energy. This is because they sometimes have to adjust their position (either up or down) based on new insights (Scharff & Amelin, [2016\)](#page-59-0).

Market competition and market power

An energy market experiences *perfect competition* if all market participants are price-takers (Gabriel et al., [2013\)](#page-58-11). Under the perfect competition or price-taker assumption, the optimal supply curve for producers of energy is the marginal cost of generation (Morales et al., [2014\)](#page-58-12). Defining the bidding strategy purely based on the marginal costs means demand is met at the lowest possible costs. This is a very desirable goal from a global perspective because it ensures that the marginal cost of production is equal to the marginal value of the goods to the consumers, such that the consumers pay the lowest price. The market price in this case is a variable over which the market participants have no control (Kirschen & Strbac, [2004\)](#page-58-10).

If any of the market participants try to predict what consequences their actions have on the market price, then the market experiences imperfect competition (Gabriel et al., [2013\)](#page-58-11). This is called pricemaking. In a market of imperfect competition, each participant then must consider how the quantity it produces might affect the price, or how the price it chooses might affect the quantity it sells. In energy markets, some producers and consumers control a share of the market that is large enough to enable them to exert *market power*. These participants are referred to as strategic players. They can manipulate the market price by e.g. withholding quantity or by increasing the asking price (Kirschen & Strbac, [2004\)](#page-58-10).

2.2 Literature review and contribution

In this section, we will conduct a comprehensive review of the most relevant research works in the literature, to identify existing research gaps, and try to highlight areas where further investigation is needed. Subsequently, we will then discuss how this thesis intends to address these gaps, thereby contributing to the advancement of the field.

2.2.1 Literature review

Optimal strategy in the electricity market is an important area that has attracted significant attention in the literature. The multistage stochastic approach has been extensively used in this area to capture the inherent uncertainties in the problem and sequential decision-making stages.

Pousinho et al. [\(2012\)](#page-59-4) propose a stochastic model for optimal bidding strategy of a wind power producer in the market taking into account the uncertainties of the wind generation and market price. Morales et al. [\(2010\)](#page-58-13) propose a similar model, but also considers the uncertainty in the balancing of energy. Three optimal bidding strategy models in day-ahead markets for wind power producers were suggested by Zhang et al. [\(2012\)](#page-59-5). They proposed three different bidding strategies, i.e., the expected profitmaximization strategy, the chance-constrained programming-based strategy, and the multi-objective bidding strategy, and conducted a comparison among these three. They found that the optimal bidding strategy was dependent on the preferred behaviour of the wind power producer.

The focus of the mentioned papers was on the day-ahead and balancing market, and intraday trading was neglected in their work. A three-stage stochastic model was proposed by Heydarian-Forushani et al. [\(2014\)](#page-58-14) for the optimal bidding strategy of a wind power producer in the day-ahead market, considering the possibility of buying flexibility at the intraday stage. Wozabal and Rameseder [\(2020\)](#page-59-6) study the optimal bidding strategy of a virtual power plant comprising wind power units in the Spanish day-ahead and intraday markets. It proposed a multistage stochastic model that incorporates the dayahead market and six auction-based intraday markets, using a Markov decision process for modeling. Another multistage stochastic model was proposed by Silva et al. [\(2022\)](#page-59-7) to investigate the optimal bidding strategy of a virtual power plant in the day-ahead market, considering subsequent intraday and balancing markets. The study accounted for uncertainties in wind and solar generation, as well as market prices, and compared the profit of participating in the intraday market with that of participating only in the day-ahead and balancing markets. In the mentioned work, the studied players were considered price takers, meaning they could not affect market prices through their market power. The primary goal of this thesis is to study how the strategic behaviour of a market participant, specifically a wind power producer, can influence market operations. However, by assuming that market participants are price takers, the ability to influence the market is overlooked in the mentioned papers.

Therefore, scientific papers that consider market participants as price makers will also be reviewed. Rintakäki et al. [\(2020\)](#page-59-8) study the strategic behaviour of flexible producers, such as hydropower and gas-fired generators in the day-ahead and intraday market. A bilevel model is proposed, in which the upper level represents the profit-maximization of the strategic producer, and the lower-levels clear the day-ahead and intraday markets sequentially. Tsimopoulos and Georgiadis [\(2019\)](#page-59-9) also set up a bilevel problem for the day-ahead and balancing markets to analyze the bidding strategies of conventional producers. In this case, an energy market with high penetration of wind power is considered with plausible wind power production scenarios. The lower-level problems model the market clearing through a two-stage stochastic program. Ruiz and Conejo [\(2009\)](#page-59-10) also consider a strategic power producer. A multiperiod network-constrained market-clearing algorithm is considered using bilevel optimization. It also considers uncertainty for bids and offers using a set of scenarios. The impact of large-scale wind power integration on electricity market equilibria is examined by S. J. Kazempour and Zareipour [\(2014\)](#page-58-15). The day-ahead market is modelled using a stochastic bilevel approach, in which multiple producers behave strategically. Wind power uncertainty is included through plausible scenarios. Some papers also investigate the strategic behaviour from customers in various electricity market structures. Examples of this are Tavakkoli et al. [\(2022\)](#page-59-11), S. J. Kazempour et al. [\(2015\)](#page-58-16), and Daraeepour et al. [\(2016\)](#page-58-17). Again, bilevel optimization is used to model the hierarchical relationship between the strategic participant and the market clearing.

While the aforementioned papers focused on other types of market participants, the strategic behavior of wind power producers has also been the subject of some research in the literature. Dadashi et al. [\(2022\)](#page-58-18) addresses the challenges of wind power participation in electricity markets due to its uncertain nature. A bilevel framework is used to model optimal offers in the day-ahead and real-time market by a wind power producer that is able to utilize a battery energy storage system. Zugno et al. [\(2013\)](#page-59-1) considers a wind power producer, being a price-taker in the day-ahead market, and a price-maker in the balancing market. The problem is formulated as a mathematical program with equilibrium constraints (MPEC), and transformed into a solvable MILP, also using bilevel optimization. A strategic wind power producer is also studied by Dai and Qiao [\(2015\)](#page-58-19) over concerns that their bidding behaviour will have an impact on market price. The short-term electricity market is modelled, and uncertainties of demand, wind power production, and bidding strategies of the strategic conventional power producers were taken into account through scenarios.

As discussed, the bidding strategy of strategic market players, specifically wind power producers, has been the topic of some papers. These studies aim to understand how a price-maker player behaves in electricity markets. Most of the reviewed papers focus on strategic behavior in day-ahead markets, occasionally considering intraday and balancing markets as subsequent stages. However, to the best of our knowledge, there is no research specifically on the strategic behavior of market players in the intraday market. Additionally, no paper has used real market data to test these models; they have only employed abstract data.

2.2.2 Research gap and contribution

Identification of gaps

Based on the literature review the conclusion can be drawn that, to the authors best knowledge, no current literature exists that studies a wind producer as a price-maker in a multi-stage auction-based intraday market and a balancing market. Many scientific publications propose bilevel models to study strategic offerings by energy market participants, but almost always neglect the intraday market, so it is unclear how strategic producers can exercise market power in intraday markets. Furthermore, no scientific paper has been found that uses real-life data for bids and offers in the intraday market, especially not on a country scale that models all the individual bidding zones that interact with each other through a transmission network within that country. This is one of the unique aspects of the research at hand. The goal of this research is to extend the existing literature by introducing a bilevel model of the Norwegian electricity market with an upper level that maximizes profit, and multiple lower levels that clear the auction-based intraday markets. Then, the optimal offering strategy of an ordinary wind producer in the energy pool of Norway can be studied.

Contribution of thesis

The proposed research is unique in its setup, which is why it can be considered a contribution to existing literature, as modelling an intraday structure like this, in combination with the chosen stochastic parameters, hasn't been done before. In short, the contributions of this research to the existing literature are as follows:

- 1. Develop a multi-stage stochastic bi-level optimization model to determine an optimal offering strategy for a wind producer in a multi-gate intraday and balancing market
- 2. Investigate the market clearing of all the bidding zones composing the Norwegian electricity network by allowing interzonal transmission using real market data with bids and offers
- 3. Answer the question under which condition or in which cases can a wind producer exercise its market power in this new energy market structure

2.3 Theory

This section discusses the optimization techniques that are used to model strategic behaviour under uncertainty in the electricity market. First, stochastic optimization for multiple stages will be described, as this is the first step in modelling an optimization problem under uncertainty. Then, bilevel optimization is discussed, which allows the modelling of hierarchical decision-making between a leader and a follower. After this, Mathematical Programs with Equilibrium Constraints (MPEC) will be introduced, which is a bilevel problem that is characterized by equilibrium conditions.

2.3.1 Multi-stage stochastic optimization

Many problems in industry nowadays can be solved with the use of mathematical models. A mathematical model comprises a set of variables and the relationships that define key aspects of a particular problem. The model consists of a mathematical structure represented by decision variables that aims at optimizing an objective function subject to constraints that limit the possible values of the decision variables. These models can be either deterministic, where parameters are assumed to be known, or stochastic, where parameters are uncertain (Rader JR., [2010\)](#page-59-12). The general form of a deterministic optimization problem is given below (Morales et al., [2014\)](#page-58-12):

$$
\min_{x \in \mathcal{X}} f(x) \tag{1}
$$

x s.t. $h(x) = 0$ (2)

$$
g(x) \le 0 \tag{3}
$$

The problem includes the following elements (Morales et al., [2014\)](#page-58-12):

- $x \in \mathbb{R}^n$: Decision variable vector with *n* elements.
- $f(x): \mathbb{R}^n \to \mathbb{R}$: Objective function, mapping x to a real value representing cost (for minimization) or benefit (for maximization).
- $h(x) : \mathbb{R}^n \to \mathbb{R}^m$ and $g(x) : \mathbb{R}^n \to \mathbb{R}^l$: Constraint functions defining *m* equality constraints $h(x) = 0$ and *l* inequality constraints $g(x) \leq 0$.

The equalities (2) and inequalities (3) together define the feasibility region of the problem, and a decision *x* is called *feasible* if it satisfies these constraints. The aim of the optimization problem is to determine the decision that yields the lowest (or highest) value of the objective function, among the set of feasible decisions (Morales et al., [2014\)](#page-58-12).

Since the optimization problem at hand includes uncertainty, stochastic optimization will be used, including multiple decision stages to model the participation of the strategic producer in the sequential energy market. The basic idea of multi-stage stochastic programming is that optimal decisions that are being taken should only be based on data available at the time the decisions are made and scenarios for the future, and thus should not depend on any future observations (Shapiro & Philpott, [2007\)](#page-59-13).

Stochastic linear programs are linear programs that contains some uncertain problem data. *Recourse programs* are linear programs in which some of the decisions are allowed to be taken after uncertainty is revealed. In this case, data uncertainty refers to data that is represented as random variables with a corresponding probabilistic description (Birge & Louveaux, [2011\)](#page-58-20). Recourse programs are classified by their number of stages, each one indicating a decision moment in time. The number of stages depends on how decisions are sequentially made in relation to how the uncertain input information is revealed over time (Gassmann & Prékopa, [2005\)](#page-58-21).

A two-stage stochastic programming problem is the simplest recourse problem, and its sequence of decisions and events is as follows. This sequence can be extended for multistage stochastic programming problems (more than two stages) as well (Morales et al., [2014\)](#page-58-12).

- 1. First stage decisions, *x*, are made. This is done before the realization of uncertain parameters and thus does not depend on them.
- 2. The outcome λ_{ω} of random parameter vector λ is known.
- 3. Second-stage decisions, $y(x, \omega)$, are made, and thus depend on each plausible value of the random parameters

A generic example of a two-stage stochastic linear program is given below (Morales et al., [2014\)](#page-58-12):

$$
\min_{x,y} f(x,y) \tag{4}
$$

$$
s.t. \quad h(x, y) = 0 \tag{5}
$$

$$
g(x, y) \le 0 \tag{6}
$$

In this example, the objective function $f(x, y)$ represents the total cost which is the sum of the first-stage cost and the expected second-stage cost. It can be expressed as:

$$
f(x, y) = c^{\top} x + \mathbb{E}_{\omega} \{Q(x, \omega)\}\
$$

where $c^\top x$ is the cost associated with the first-stage decisions, and $\mathbb{E}_\omega\{Q(x,\omega)\}$ is the expected cost of the second-stage decisions given the first-stage decisions x and the realization of the unknown parameters ω . Furthermore, the equality constraints $h(x, y) = 0$ and inequality constraints $g(x, y) \le 0$ are constraints that need to be satisfied by both the first-stage decisions *x* and second-stage decisions *y*(*x*,*ω*) (Morales et al., [2014\)](#page-58-12).

2.3.2 Karush-Kuhn-Tucker conditions

The optimality conditions of any optimization problem can be used to reformulate the problem in such a way that these conditions are incorporated into a larger model. This approach is only feasible if the problem is convex, such that any stationary point found is also a global optimum. If this is not the case, the feasible set of solutions is enlarged by local optimal solutions and stationary points, which can lead to solutions that are not globally optimal (Dempe et al., [2015\)](#page-58-22). The KKT optimality conditions consists of four categories, namely: stationarity condition, primal feasibility, dual feasibility and complementary slackness.

Lagrange multiplier

The Lagrangian function for a generic optimization problem can be defined as follows (Morales et al., [2014\)](#page-58-12):

$$
\mathcal{L}(x,\lambda,\mu) = f(x) + \lambda^{T}h(x) + \mu^{T}g(x)
$$
\n(7)

Here, λ^T are the Lagrange multipliers for the equality constraints $h(x)$, and μ^T are the multipliers for the inequality constraints $g(x)$. The following KKT conditions are necessary and sufficient for optimality of the optimization problem as mentioned in Morales et al. [\(2014\)](#page-58-12).

$$
\nabla_x f(x) + \lambda^T \nabla_x h(x) + \mu^T \nabla_x g(x) = 0
$$
\n(8)

$$
h(x) = 0 \tag{9}
$$

$$
g(x) \le 0 \tag{10}
$$

$$
\mu \ge 0 \tag{11}
$$

$$
\mu^T g(x) = 0 \tag{12}
$$

Equation (8) represents the stationarity condition. The stationarity condition is obtained by differentiating the Lagrangian function (1) with regards to the decision variables. In bilevel problems, which will be explained later, the stationarity condition is obtained by differentiating the Lagrangian function (7) of the lower-level problems with regards to the decision variables of the upper level problem. Constraints (9) and (10) ensures feasibility of the primal problem, which is the original optimization problem, and equation (11) of the dual problem. Equation (12) represents the complementary slackness condition. The scalar product on the left-hand side of the complementary slackness equation is the sum of non-positive terms only, which implies that the element-by-element product between μ ^{*i*} and *gⁱ* (*x*) is equal to 0. Lastly, equations (10-12) can be combined into a nonlinear constraint using perpendicularity, and can be written as follows (Morales et al., [2014\)](#page-58-12):

$$
0 \ge g(x) \perp \mu \ge 0 \tag{13}
$$

2.3.3 Mathematical Program with Equilibrium Constraints

A mathematical program with equilibrium constraints (MPEC) can be defined as an optimization problem that includes constraints used to model the equilibrium nature of the corresponding optimization problem. It is widely used in energy markets, due to the equilibrium aspect of the market. The constraints of the MPEC include other interrelated optimization problems, such as the clearing of the energy market. A schematic representation of an MPEC can be found in Figure [3](#page-18-2) (Gabriel et al., [2013\)](#page-58-11). The objective function would consist of maximizing the profit of the producer through its chosen bidding strategy, while the constraining optimization problems would relate to the market clearing optimization problems in the two auction-based intraday markets.

Figure 3: Mathematical Program with Equilibrium Constraints (MPEC) (Gabriel et al., [2013\)](#page-58-11)

2.3.4 Bilevel optimization

Optimization problems constrained by complementarity and other optimization problems are known as bilevel problems, and they belong to the field of study called hierarchical optimization (Gabriel et al., [2013\)](#page-58-11). Ever since the first bilevel formulation in 1934 by Henrich Freiherr von Stackelberg, there has been an increase in applications of bilevel optimization. Formulated as a hierarchical game (also called a Stackelberg game), two decision makers act in this problem, in which one acts as a socalled leader and one acts as a so-called follower. The leader aims to optimize its objective function subject to the conditions composed by optimal decisions of the follower. The choices of the leader influences the feasible set and the objective function of the follower's problem, which has a significant impact on the leader's objective value. Neither player can dominate the other one completely. A bilevel optimization problem is a leader's problem, which is formulated mathematically using the solution set of the follower's problem (Dempe et al., [2015\)](#page-58-22).

Bilevel models allow us to represent a sequential decision-making process as opposed to single-level models where all decisions are considered to be taken immediately, which can be a gross simplification of reality and distort model outcomes (Dempe & Zemkoho, [2020\)](#page-58-23). These type of models are common practice when dealing with markets in which a player has market power, because market agents have to make decisions in anticipation of the equilibrium of the market. Specifically speaking about electricity markets, anticipating the market equilibrium and thus estimating the market-clearing prices is important as it can be used to establish a strategy of a market agent. In this case, the player can affect the market equilibrium by its choices, which won't happen in a market under perfect competition. Such strategy can generally be formulated as an optimization problem, for example to maximize profit, that is constrained by the market equilibrium. In turn, the market equilibrium can also be formulated as an optimization problem. The resulting problem is an optimization problem (agent decision-making) subject to another optimization problem (market equilibrium), or an upper-level subject to a (collection of) lower-level(s). An explicit hierarchy is assumed in these type of optimization problems (Gabriel et al., [2013\)](#page-58-11).

General bilevel formulation

su

The general formulation of a bilevel optimization problem can be written as follows (Morales et al., [2014\)](#page-58-12):

Minimize
$$
f^U(x, y)
$$
 (14)

subject to $g^U(x, y) \le 0$ (15)

$$
h^U(x, y) = 0 \tag{16}
$$

$$
y \in \arg\min_{z} \{ f^{L}(x, z) \mid h^{L}(x, z) = 0, g^{L}(x, z) \le 0 \}
$$
\n(17)

The objective function $f^U(x, y)$ is the objective function of the upper level, depending on both x and y . $g^U(x, y) \le 0$ and $h^U(x, y) = 0$ are the inequality and equality constraints, respectively, for the upperlevel problem. For a given *x*, the variable *y* is chosen to minimize $f^L(x, z)$, subject to $h^L(x, z) = 0$ and $g^L(x, z) \leq 0$, which are the equality and inequality constraints of the lower level, respectively. The optimality condition in equation (17) means that *y* is the optimal solution of the lower-level problem for a given *x*.

Under the assumption that KKT conditions are necessary and sufficient for optimality in the lower-level problem, they can be used to replace equation (17) if it is convex. The bilevel problem can then be formulated as follows (Morales et al., [2014\)](#page-58-12):

Minimize
$$
f^{U}(x, y, \lambda, \mu)
$$
 (18)

$$
bject to \t hU(x, y) = 0 \t(19)
$$

$$
g^U(x, y) \le 0 \tag{20}
$$

$$
\nabla_y f^L(x, y) + \lambda^T \nabla_y h^L(x, y) + \mu^T \nabla_y g^L(x, y) = 0
$$
\n(21)

$$
h^L(x, y) = 0\tag{22}
$$

$$
g^L(x, y) \le 0 \tag{23}
$$

$$
\mu \ge 0 \tag{24}
$$

$$
\mu^T g^L(x, y) = 0 \tag{25}
$$

Fortuny-Amat and McCarl [\(1981\)](#page-58-24) presents a widely used method for solving MPECs, of which an example is given below. This method utilizes the so-called *big M* reformulation of the complementary slackness equations, as shown in (25). The reformulation is shown below:

- $\mu_i \le (1 z_i) M_{2i}, \quad \forall i$ (27)
- $z_i \in \{0, 1\}, \quad \forall i$ (28)

3 Problem description

This chapter describes the specific model that is studied. The model considers a strategic wind power producer, whose decisions may or may not impact market outcomes. The specific setup of the eventual bilevel model is elaborated upon here by providing a detailed modelling approach, such as which of the sequential markets and auction-based intraday gates will be included. Additionally, this chapter outlines the key modelling assumptions that were made in the process. Ultimately, the main purpose is to set the stage for the formulation of the mathematical model.

3.1 Market design

Electricity markets considered

The Norwegian sequential electricity market consists of three different markets, namely the day-ahead, intraday, and balancing market. The main focus of the thesis is on the intraday market. The considered model in this thesis includes the balancing market but neglects the day-ahead market. This is because of the following reasons:

- Including the day-ahead market would vastly increase the runtime of the model, as it gives the strategic producer the opportunity to try to exercise market power there as well. In addition, another market clearing has to take place, which would include bids and offers from many market participants due to the high liquidity of day-ahead markets compared to intraday markets. This would make the model more computationally intensive to solve to optimality.
- The aim is to model strategic behaviour in specifically the new auction-based intraday market. By considering the day-ahead decision, the problem would turn into optimal bidding of a wind power producer in the day-ahead market, considering the presence of the intraday and balancing markets.

Instead, the day-ahead market will be represented through a fixed parameter that represents the dayahead commitment of the strategic producer. The balancing market is chosen to be included because no market clearing takes place, which means that it won't impact the computational intensity significantly. Furthermore, predicting the balancing prices can significantly impact the offering strategy, as the wind power producer can opt to withhold quantity in the intraday market to benefit from favourable balancing prices.

Number of gates considered

As mentioned before, the proposed auction-based intraday market has three sequential gates. Only two of the three proposed gates will be considered. One of the reasons is that, since the dataset contains bids and offers of a continuous intraday market, the data will have to be converted to a multi-gate auction-based market. Having three gates instead of two further complicates this process. Additionally, three gates also makes the model more difficult to solve as another lower level is required to represent the third gate. The main purpose of the research is to model a multi-gate auction-based intraday market, so two gates is considered sufficient for this purpose, because this still allows an analysis of the consequences of this when compared to just one gate. Furthermore, based on a study of the historical dataset, as shown in Figure [4,](#page-22-0) almost all trades happen as close as possible to the delivery hour. The expectation is thus that the transaction in the first gate can be neglected in comparison with the next two gates. Therefore, only the second and third gate will be modelled.

Figure 4: Hours ahead of delivery bid/offer was submitted

Bidding zones considered

As mentioned before, the NordPool network consists of many European countries, which are all divided into bidding zones. Between these zones, transmission can take place, depending on the available line capacity. This is an important consideration in the model. Figure [5](#page-22-1) shows an example of the connections between bidding zones within the NordPool region. As you can see, Norway has five bidding zones (NO1 - NO5). In this network, all bids and offers are aggregated and power will flow from bidding areas where the offered price is lower towards bidding areas where the demand and offering price is higher. Since there are capacity restrictions on the transmission grid between these zones, congestion will lead to bidding areas having different prices. If there was no congestion, the price would be the same across all zones ('NordPool - Price calculation', [2024\)](#page-59-14)

Figure 5: Bidding zones in the Nordic electricity markets ('NordPool - Price calculation', [2024\)](#page-59-14)

Transmission constraints are considered in the mathematical model to give a more accurate representation of how the market clearing process works in real life. Only domestic flows between bidding zones within Norway are considered by modelling the market clearing of the five Norwegian zones (NO1 - NO5), since it reduces modelling complexity and bids and offers from other bidding zones are not available. An analysis made of the transmission capacities on the NordPool website shows that 54.6% of the transmission capacity related to the Norwegian transmission network is used for interzonal transmission within Norway, whereas the other 45.4% of the transmission capacity is available for transmission to other countries (Sweden, Denmark, and the Netherlands) ('Nordpool - Market data', [2024\)](#page-59-15). The available transmission capacity for the intraday is determined after the day-ahead auction has taken place and it is known how much capacity is taken up already. The available capacities between these zones are taken from the marketdata page on the NordPool website ('Nordpool - Market data', [2024\)](#page-59-15) and are shown in Table [2.](#page-23-1) The specific line capacities used in the model will be explained in chapter [5.](#page-37-0)

From/To	NO ₁	NO ₂	NO ₃	NO4	NO5
NO ₁	$\mathbf{0}$	2200	500		600
NO ₂	3500	$\mathbf{\Omega}$	0	O)	500
N _O 3	500	$\mathbf{0}$	0	400	800
NO4	$\mathbf{0}$	$\mathbf{\Omega}$	1200	Ω	O)
NO ₅	3900	600	500	0	

Table 2: Transmission capacities between bidding areas in Norway (MW)

Strategic producer considered

A large wind power producer located in Norway is considered as the strategic player for which the optimal offering strategy will be determined. A list of wind farms owned by wind power producers and the energy mix of European countries, including Norway, can be found on ENTSO-E, [2024.](#page-58-25) It is listed for each bidding zone seperately, as each power generator participates in the bidding zone in which it is physically located. Table [3](#page-23-2) summarizes the most important aspects. For selecting a bidding zone and a corresponding wind farm owned by a wind power producer, the following points are considered:

- Capacity of the wind power producer's farm
- Capacity of the wind power producer's farm in its bidding zone as a percentage of the total power capacity in that zone

• Average market price of the bidding zone in which the wind power producer is located

Table 3: Power and wind capacity data for all bidding zones in Norway

Preferably, the wind penetration and the zone's average market price are high as this makes it more likely that the wind power producer can exercise market power. NO1 has wind capacity, but no specific wind farms are mentioned on ENTSO-E, [2024,](#page-58-25) while NO5 has no wind capacity at all. NO2 has the highest average market price $(\in 75.08)$, but very low penetration. Both NO3 and NO4 have similar market prices (\in 40.39 and \in 35.03, respectively), although not very high. The wind penetration is the highest in these zones however. Ultimately, NO3 is chosen.

3.2 Modelling approach

This problem can be formulated as a multi-stage optimization problem, where at each stage the bidding strategy of the chosen wind producer is determined. To account for the uncertainty faced by the wind power producer, stochasticity must be introduced, which is done by including scenarios for wind power

generation, market prices, and other players' bids and offers. To accurately model how the wind power producer can strategically influence the market price, the objective of the system operator should also be considered, which is to maximize social welfare by clearing the market and determining a market equilibrium. This type of optimization can be characterized as a bilevel optimization problem.

Figure 6: Decision process as faced by strategic producer

The decision process as faced by the strategic wind producer is shown in Figure [6.](#page-24-1) It starts at the first intraday gate, meaning the day-ahead market has already cleared. The producer receives an updated wind forecast, and also makes a prediction for the intraday and balancing stages. Using this information, an offer is made in the first intraday market, after which it is cleared and the same sequence happens in the second intraday gate. The decision in the balancing stage is forced, because the wind producer is forced to use up- or downregulation to meet its previously made commitments. Of course, the wind producer predicts the balancing settlement prices along the way and can use this information to adjust its position accordingly. Figure [7](#page-24-2) shows a schematical representation of the proposed bilevel optimization model. The problem has an upper level, which is the profit maximization objective of the strategic wind producer, and two lower levels that aim to maximize the social welfare by clearing the energy market.

Figure 7: Proposed bilevel model with upper level and lower levels

3.3 Uncertainty considered

Figure [6](#page-24-1) and the corresponding explanation of the graph mentions the uncertainty the strategic producer has to deal with. The inclusion of these parameters aligns with the objective of the thesis, and provides a realistic setting of the uncertainty that the wind power producer faces in this particular context. The following stochastic parameters will be included in the model:

• **Forecast of wind power production**

As mentioned before, wind power is inherently hard to predict due to uncertainty of future wind speeds. The accuracy of wind power production directly impacts the strategic's producer bidding strategy, as an overestimation can result in expensive upregulation, while an underestimation can result in lost revenue in the intraday market. The stochastic modelling of wind power forecasts allows for the analysis of various scenarios of an optimal offering strategy under realistic uncertainty that is also seen in real-life, and with which wind power producers have to deal when making bidding decisions.

• **Up- and downregulation prices in the balancing market**

Prices to up- and downregulate reflect the cost of correcting deviations from earlier commitments. The strategic producer must anticipate potential imbalances and consider the potential balancing costs when submitting bids to the intraday market. The strategic producer does not know in advance what these balancing prices will be, and thus must adjust its offering strategy based on this. By accounting for uncertainty in balancing prices, the model can optimize the strategic producer's profit by deciding whether it is best to sell energy in the intraday market or withhold it for potential balancing actions. Therefore, stochasticity for both up- and downregulation prices will be included. Since the NordPool markets use a two-price system for balancing prices, this system will also be used in this research.

• **Bids and offers by competitive market participants**

The bids and offers by competitive market participants directly impact the market price and thus the strategic's producer ability to influence market outcomes. Its decisions will depend on the actions of others, which makes it necessary to model this uncertainty as the assumption is that the strategic producer has no perfect knowledge on these bids and offers.

3.4 Modelling assumptions

While modelling a complex real-world situation where not all information is known, it is important to simplify the model to some extent by making certain assumptions. This is necessary step in creating a mathematical model, as it provides clarity about the scope and limits of the model. The following assumptions are worth mentioning:

Perfect competition

All other market participants are assumed to behave competitively in the sequential market, meaning they are considered price-takers. They maximize their own interests without strategically manipulating the market and its prices. For competitive participants this usually means that they offer their true costs of generation, also called the marginal costs.

Strategic behaviour

The chosen wind power producer acts in a strategic manner with the aim of maximizing its profits across all considered markets. This allows for a study on how the strategic producer's actions can influence the outcomes of the markets through bidding strategies.

Quantity offered by strategic producer

The quantity offered can take on any value. The idea is that it will not offer a high quantity anyway, as this may result in a lower profit since there are more risks associated. Also, all the quantity should be offered at one price, and no block bids are allowed.

Strategic producer's offer across multiple bidding scenarios

When considering uncertainty with regards to the bids and offers submitted by competitive market participants, the strategic producer has to make only one decision that is optimal for all considered scenarios combined. Since the assumption is that the strategic producer does not know for sure what other participants submitted and thus what the market price will be, a price and quantity offer should be submitted that takes this uncertainty into account. Besides the price and quantity offered, this will also apply to the direction of the offer, such that an offer consisting of a price and quantity cannot result in upregulation in one scenario and downregulation in another scenario.

Ability to buy deficit or sell excess energy in the balancing market

There are no limits on the capacity available for either purchasing (upregulation) or selling (downregulation) to trade into balance for the strategic producer. This means that the producer can strategically choose to under- or overpromise its energy production in order to get a more favourable price in the balancing market.

4 Methodology

To study the optimal offering strategy by our wind power producer, an optimization model will be developed. First, the foundation of the bilevel model will be given schematically, after which the individual components will be constructed, such as the higher and lower levels. Then, the bilevel model will be set up, in which these levels are incorporated. This chapter will also discuss how transmission is included in the model and how the optimization model is linearalized to make it suitable for solving.

4.1 Sequence of decisions and events

A schematic overview of the sequence of decisions and events as to be modelled is shown in Figure [8.](#page-27-3) The strategic wind power producer submits its offer for the first intraday, which includes a price and quantity. This information is then used by the market operator to clear the first intraday market, along with the offers from the competitive participants.

When the market is cleared, the strategic producer knows how much power generation has been committed to the market. It can then adjust its position for the second intraday market based on new information, by again submitting a price and quantity. The market is then cleared again by the market operator. Any deviation is taken care of by the balancing settlement phase.

Figure 8: Sequence of decisions and events

4.2 Bilevel model

The formulation of the proposed bilevel optimization model will now be given. The sets and incides are used to represent a category or group involved in the mathematical problem. The parameters are constants within the model, represented by (un)certain data or fixed values. The decision variables are the core part of the optimization model, and represent the actions that the model aims to optimize. First, the sets and indices, as well as the parameters and decision variables present in the model will be given.

Sets and indices

- $n \in N$ Node
- $l \in L$ Line
- $k \in K$ Competitive participant offering upregulation in ID1
- *z* ∈ *Z* Competitive participant offering downregulation in ID1
- $x \in X$ Competitive participant offering upregulation in ID2
- $c \in C$ Competitive participant offering downregulation in ID2

Parameters

- *π^s* Probability of scenario s
- *W^s* Wind forecast in scenario s

 $G_{n,k,s}^{\max, \text{up}, \text{id}1}$ *n*,*k*,*s G* max,down,id1 *n*,*z*,*s* $G_{n,r,s}^{\max, \text{up}, \text{id2}}$ *G* max,down,id2 *n*,*c*,*s* $P_{\substack{\text{n k s}}$ *P* down,id1 n,z,s $P_{\text{n} \text{ y s}}^{\text{up}, \text{id2}}$ *P* down,id2 n,c,s *γ* up/down *s G Yl*,*ⁿ* $F_{\rm l}^{\rm da}$ *F* min,max *Λ*

Upregulation quantity offer of competitive participant k in zone n in scenario s in ID1 Downregulation quantity offer of competitive participant z in zone n in scenario s in ID1 Upregulation quantity offer of competitive participant x in zone n in scenarios s in ID2 Downregulation quantity offer of competitive participant c in zone n in scenario s in ID2 Upregulation price offer of competitive participant k in zone n in scenario s in ID1 Downregulation price offer of competitive participant z in zone n in scenario s in ID1 Upregulation price offer of competitive participant x in zone n in scenario s in ID2 Downregulation price offer of competitive participant c in zone n in scenario s in ID2 Up/down-regulation balancing price Commitment to day-ahead market of strategic producer

Incidence matrix of transmission line l and node n

Transmission flow on line l in the day-ahead market

^l Minimum/maximum transmission capacity on line l

Minimum/maximum intraday market price in zone of wind power producer

Decision variables

Interzonal transmission

Figure 9: Transmission lines

Now, some attention is given towards the interzonal transmission network and the way this will be included. Figure [9](#page-29-0) shows a simplified version of the transmission network in Norway, including the individual zones and the interzonal transmission lines between these zones ('Nordpool - Market data', [2024\)](#page-59-15). As shown in the graph, there aren't transmission lines for every combination of zones. The incidence matrix *Yl*,*ⁿ* as shown in Table [4](#page-30-0) describes the relationship between the bidding zones and the transmission lines. This parameter has indices (l, n) , which are described in the following way:

$Y_{l,n}$				$n=1$ $n=2$ $n=3$ $n=4$ $n=5$	
$l=1$		-1	$\mathbf{0}$		
$l=2$	1.	0	-1	0	0
$l=3$		0	O	0	-1
$l = 4$	0		O	O	-1
$l = 5$	0	0	1	-1	0
$l = 6$		0		$\mathbf{0}$	-1

Table 4: Incidence matrix for transmission network

Upper level: optimal offering strategy

$$
\text{Maximize} \quad \sum_{s} \sum_{n} \pi_{s} \left[(g_s^{\text{up},\text{id}1} - g_s^{\text{down},\text{id}1}) \lambda_{n,s}^{\text{id1}} + (g_s^{\text{up},\text{id}2} - g_s^{\text{down},\text{id}2}) \lambda_{n,s}^{\text{id2}} + (g_s^{\text{exc}} \gamma_s^{\text{down}} - g_s^{\text{def}} \gamma_s^{\text{up}}) - \epsilon \sum_{l} t_{l,s} \right] \tag{29}
$$

s.t.
$$
G^{da} + g_s^{id1,up} - g_s^{id1,down} + g_s^{id2,up} - g_s^{id1,down} + g_s^{exc} - g_s^{def} = W_s
$$
 $\forall s$ (30)

$$
g_s^{\text{up},\text{id1}} \le M x^{\text{id1}} \quad \forall s \tag{31}
$$

$$
g_s^{\text{down},\text{id1}} \le M(1 - x^{\text{id1}}) \quad \forall s \tag{32}
$$

$$
g_s^{\text{up}, \text{id2}} \le M x_s^{\text{id2}} \quad \forall s
$$
\n
$$
\text{(33)}
$$
\n
$$
\text{down } \text{id2} \le M x_s^{\text{id2}} \quad \text{id2} \tag{34}
$$

$$
g_s^{\text{down},\text{id2}} \le M(1 - x_s^{\text{id2}}) \quad \forall s \tag{34}
$$

$$
g_s^{\text{exc}} \le M x_s^{\text{bal}} \quad \forall s \tag{35}
$$

$$
g_s^{\text{def}} \le M(1 - x_s^{\text{bal}}) \quad \forall s
$$
\n
$$
\text{(36)}
$$
\n
$$
\text{(37)}
$$

$$
p_s^{\text{id2}} = p_{s'}^{\text{id2}} \quad \forall s, s' \tag{37}
$$

$$
q^{\text{id2}} = q^{\text{id2}} \quad \forall s, s' \tag{38}
$$

$$
q_s^{\rm id2} = q_{s'}^{\rm id2} \quad \forall s, s' \tag{38}
$$

The upper level represents the profit maximization of the strategic producer. The objective function (29) aims to maximize the average profit across the intraday and balancing markets, given multiple scenarios. The objective function also include penalty functions for the transmission, which are elaborated upon in Appendix [C.](#page-64-0) The transmission value *tl*,*^s* is multiplied by a small value *ε* to introduce the penalty. Constraint (30) ensures that the strategic producer's commitments are always equal to the real-time wind power production. Furthermore, constraints (31-36) ensure that the strategic producer can always only up- or downregulate, or have an excess or deficit in the balancing stage. Constraints (37-38) represent non-anticipativity, which ensures that decisions in the second stage are consistent all scenarios with the same history up to that point.

Lower level 1: market clearing intraday 1

$$
\forall s \n\begin{cases}\n\text{Minimize} & \sum_{n} \sum_{k} P_{n,k,s}^{\text{up},\text{id}1} \sin(3\theta) - \sum_{n} \sum_{z} P_{n,z,s}^{\text{down},\text{id}1} \sin(3\theta) \sin(3\theta) - \sum_{n} \sum_{z} P_{n,z,s}^{\text{down},\text{id}1} \sin(3\theta) \sin(3\theta) \\
\text{s.t.} & \sum_{k} g_{n,k,s}^{\text{comp},\text{up},\text{id}1} - \sum_{z} g_{n,z,s}^{\text{comp},\text{down},\text{id}1} + g_{s}^{\text{up},\text{id}1} - g_{s}^{\text{down},\text{id}1} - \sum_{l} Y_{l,n} f_{l,s}^{\text{id}1} = 0 \quad : \lambda_{n,s}^{\text{id}1} \quad \forall n \text{ (40)} \\
& 0 \leq g_{n,k,s}^{\text{comp},\text{up},\text{up},\text{id}1} \leq G_{n,k,s}^{\text{max},\text{up},\text{val}1} \quad : \underline{\mu}_{n,k,s}^{\text{id}1}, \overline{\mu}_{n,k,s}^{\text{id}1} \quad \forall n, k \text{ (41)} \\
& 0 \leq g_{n,z,s}^{\text{comp},\text{down},\text{id}1} \leq G_{n,z,s}^{\text{max},\text{down},\text{id}1} \quad : \underline{\mu}_{n,z,s}^{\text{id}1}, \overline{\mu}_{n,z,s}^{\text{id}1} \quad \forall n, z \text{ (42)} \\
& 0 \leq g_{s}^{\text{up},\text{id}1} + g_{s}^{\text{down},\text{id}1} \leq q^{\text{id}1} \quad : \underline{\mu}_{s}^{\text{id}1}, \overline{\mu}_{s}^{\text{id}1} \text{ (43)} \\
& F_{l}^{\text{min}} \leq F_{l}^{\text{da}} + f_{l,s}^{\text{id}} \leq F_{l}^{\text{max}} \quad : \underline{\alpha}_{l,s}^{\text{id}1}, \overline{\alpha}_{l,s}^{\text{id}1} \quad \forall l \text{ (44)}\n\end{cases}
$$

Lower level 2: market clearing intraday 2

$$
\left(\text{Minimize } \sum_{n} \sum_{x} P_{n,x,s}^{\text{up},\text{id2}} g_{n,x,s}^{\text{comp},\text{up},\text{id2}} - \sum_{n} \sum_{c} P_{n,c,s}^{\text{down},\text{id2}} g_{n,c,s}^{\text{comp},\text{down},\text{id2}} + (g_s^{\text{up},\text{id2}} - g_s^{\text{down},\text{id2}}) p_s^{\text{id2}} \right) (45)
$$

$$
\sum_{n} \sum_{x} P_{n,x,s}^{1} g_{n,x,s}^{1} g_{n,x,s}^{1} = \sum_{n} \sum_{c} P_{n,c,s}^{1} g_{n,c,s}^{1} g_{n,c,s}^{1} + (g_{s}^{1})^{1} g_{s}^{1} - g_{s}^{1} g_{s}^{1} (45)
$$
\n
$$
\text{s.t. } \sum_{x} g_{n,x,s}^{\text{comp,up,id2}} - \sum_{c} g_{n,c,s}^{\text{comp,down,id2}} + g_{s}^{\text{up,id2}} - g_{s}^{\text{down,id2}} - \sum_{l} Y_{l,n} f_{l,s}^{\text{id2}} = 0 \quad : \lambda_{n,s}^{\text{id2}} \quad \forall n \text{ (46)}
$$
\n
$$
0 < g^{\text{comp,up,id2}} < G^{\text{max,up,id2}} \quad : \mu^{\text{id2}} \quad \forall n, x \text{ (47)}
$$

$$
0 \leq g_{n,x,s}^{\text{comp,up,id2}} \leq G_{n,x,s}^{\text{max,up,id2}} \quad : \underline{\mu}_{n,x,s}^{\text{id2}}, \overline{\mu}_{n,x,s}^{\text{id2}} \quad \forall n, x \tag{47}
$$
\n
$$
0 \leq g_{n,c,s}^{\text{comp,down,id2}} \leq G_{n,c,s}^{\text{max,down,id2}} \quad : \underline{\mu}_{n,c,s}^{\text{id2}}, \overline{\mu}_{n,c,s}^{\text{id2}} \quad \forall n, c \tag{48}
$$

$$
0 \le g_s^{\text{up}, \text{id}2} + g_s^{\text{down}, \text{id}2} \le q_s^{\text{id}2} : \mu_s^{\text{id}2}, \overline{\mu}_s^{\text{id}2} \tag{49}
$$

$$
F_l^{\min} \le F_l^{\text{da}} + f_{l,s}^{\text{id}1} + f_{l,s}^{\text{id}2} \le F_l^{\max} \quad : \underline{\alpha}_{l,s}^{\text{id}2}, \overline{\alpha}_{l,s}^{\text{id}2} \quad \forall l \tag{50}
$$

The objective functions (39, 45) aim to minimize the total costs of generation in the intraday market based on the prices and quantity offered by up- and downregulators. The balance constraints (40, 46) ensure that the upregulation and the downregulation in a particular zone is balanced, while also taking in- and outflow to other zones into account. Note that only the balance constraint for the wind power producer's zone is displayed. The balance constraint in other zones merely consists of the bids and offers of competitive participants. Equations (41, 42, 47, 48) limit the up- and downregulation of the competitive participants to be between 0 and the offered up- or downregulation quantity. Equations (43, 49) limit the up- or downregulation from the strategic wind power producer to be between 0 and the offered quantity. Lastly, equations (44, 50) refer to the transmission constraints. The cumulative power flow between two zones in the day-ahead and intraday stages should be limited by the transmission capacities of the lines between the zones. This applies to both ways (to and from a zone). The lower level consists of both primal and dual variables, and the dual variables are defined following the colon in each constraint.

Reformulation to MPEC

∀s

 $\begin{array}{c} \hline \rule{0pt}{2.5ex} \$

In order to convert the bilevel program to a single optimization problem such that it can be solved, it must be reformulated as a mathematical program with equilibrium constraints (MPEC). This can be achieved by replacing the lower-level problems by their corresponding Karush-Kuhn-Tucker (KKT) conditions. In any problem represented by KKT conditions, the following conditions must hold (Morales et al., [2014\)](#page-58-12):

- Stationarity
- Primal feasibility
- Dual feasibility
- Complementary slackness

To find the stationarity conditions, we first convert the lower-level problems to their Lagrangian function, and taking the gradient with respect to every primal decision variable. Then, the partial derivatives are set to zero to ensure that the solution is at a stationarity point of Lagrangian. The primal feasibility ensure that the decision variables ensure the problem's constraints. The dual feasibility condition requires that the associated lagrange multipliers of the inequality constraints should be non-negative. The complementary slackness condition states that at an optimal solution, the product of each inequality constraints and its corresponding lagrange multiplier must be equal to zero.

The Lagrangian multiplier of the objective function in the market clearing of ID1 is given by:

$$
L = \sum_{n} \sum_{k} \sum_{s} P_{n,k,s}^{\text{up},\text{id1}} g_{n,k,s}^{\text{comp},\text{up},\text{id1}} - \sum_{n} \sum_{z} \sum_{s} P_{n,z,s}^{\text{down},\text{id1}} g_{n,z,s}^{\text{comp},\text{down},\text{id1}} + p^{\text{id1}} (g_s^{\text{up},\text{id1}} - g_s^{\text{down},\text{id1}}) + \sum_{n} \sum_{s} \lambda_{n,s}^{\text{id1}} \left(\sum_{k} g_{n,k,s}^{\text{comp},\text{up},\text{id1}} - \sum_{z} g_{n,z,s}^{\text{comp},\text{down},\text{id1}} + g_s^{\text{up},\text{id1}} - g_s^{\text{down},\text{id1}} - \sum_{l} Y_{l,n} f_{l,s}^{\text{id1}} \right) + \sum_{n} \sum_{s} \left(\sum_{k} \mu_{n,k,s}^{\text{id1}} (g_{n,k,s}^{\text{comp},\text{up},\text{val1}} - G_{n,k,s}^{\text{max},\text{up},\text{id1}}) - \sum_{k} \overline{\mu}_{n,k,s}^{\text{id1}} (g_{n,k,s}^{\text{comp},\text{up},\text{id1}}) \right) + \sum_{n} \sum_{s} \left(\sum_{z} \mu_{n,z,s}^{\text{id1}} (g_{n,z,s}^{\text{comp},\text{down},\text{id1}} - G_{n,z,s}^{\text{max},\text{down},\text{id1}}) - \sum_{z} \overline{\mu}_{n,z,s}^{\text{id1}} (g_{n,z,s}^{\text{comp},\text{down},\text{id1}}) \right) + \sum_{s} \left(\underline{\mu}_{s}^{\text{id1}} (g_{s}^{\text{up},\text{id1}} + g_{s}^{\text{down},\text{id1}} - q^{\text{id1}}) - \overline{\mu}_{s}^{\text{id1}} (g_{s}^{\text{up},\text{id1}} - g_{s}^{\text{down},\text{id1}}) \right) + \sum_{s} \sum_{s} \left(\underline{\alpha}_{s}^{\text{id1}} (F_{l}^{\text{da}} + f_{l,s}^{\text{id1}} - F_{l}^{\text{max}}) + \overline{\alpha}_{l,s}^{\text{
$$

In order to find the stationarity conditions, the partial derivative of the Lagrangian function with respect to every primal decision variable must be taken, and setting them to equal to zero to obtain optimality. Equations (53-54) are used in case the strategic producer is determining the market price, and are thus only used in one specific zone. A separate formula for upregulation (53) and downregulation (54) is required because if the strategic producer makes a bid to upregulate and this order is fully fulfilled according to the market clearing, then the market price should be equal or higher than the strategic producer's offer price, whereas in the case of downregulation, the market price should be equal or lower than the strategic producer's offer price.

$$
\frac{\partial L}{\partial g_{n,k,s}^{\text{comp,up,id1}}} = P_{n,k,s}^{\text{up,id1}} - \lambda_{n,s}^{\text{id1}} + \underline{\mu}_{n,k,s}^{\text{id1}} - \overline{\mu}_{n,k,s}^{\text{id1}} \quad \forall n,k,s
$$
\n(51)

$$
\frac{\partial L}{\partial g_{n,z,s}^{\text{comp,down},\text{id}1}} = -P_{n,z,s}^{\text{down},\text{id}1} + \lambda_{n,s}^{\text{id}1} + \underline{\mu}_{n,z,s}^{\text{id}1} - \overline{\mu}_{n,z,s}^{\text{id}1} \quad \forall n,z,s
$$
(52)

$$
\frac{\partial L}{\partial g_s^{\text{up},\text{idl}}} = p^{\text{id}1} - \lambda_{n,s}^{\text{id}1} + \underline{\mu}_s^{\text{id}1} - \overline{\mu}_s^{\text{id}1} \quad \forall s
$$
\n(53)

$$
\frac{\partial L}{\partial g_s^{\text{down},\text{id}1}} = -p^{\text{id}1} + \lambda_{n,s}^{\text{id}1} + \underline{\mu}_s^{\text{id}1} - \overline{\mu}_s^{\text{id}1} \quad \forall s
$$
\n(54)

$$
\frac{\partial L}{\partial f_{l,s}^{\text{id}1}} = -\sum_{n} Y_{n,l} \lambda_{n,s}^{\text{id}1} + \underline{\alpha}_{l,s}^{\text{id}1} - \overline{\alpha}_{l,s}^{\text{id}1} \quad \forall l,s
$$
\n(55)

Deriving the market price

The stationarity conditions (51-55) are used to determine the market price based on the market clearing in the lower level. In electricity markets, the market price is determined in an equilibrium stage by the participant (either an up- or downregulator), whose quantity offer is only partially fulfilled. Obviously, there can only be one such participant at most in a market clearing. This is identified by the dual variables μ and $\bar{\mu}$ for the lower and upper bound of a quantity offer, respectively, as shown above. If the offer \vec{r} a participant is either not or fully fulfilled, then these dual variables can take on any value and the market price is not forced to be equal to the price offer of this participant. If the offer is partially fulfilled, then these dual variables are forced to be zero, which sets the market price equal to the price offer. This applies to any participant, whether it is our strategic producer or a regular competitive participant. The formulas for the upregulators and downregulators are shown in equations (56-57), respectively. A different equation is required for each side, because if an upregulator is fully fulfilled, the market price will be higher than this, while if a downregulator is fully fulfilled, the market price should be lower than the price offer of this participant.

$$
p - \lambda + \mu - \overline{\mu} = 0 \tag{56}
$$

$$
-p + \lambda + \mu - \overline{\mu} = 0 \tag{57}
$$

The stationarity conditions also include the transmission constraints within the market. When transmission capacity is unlimited, which is indicated by dual variables $\overline{\alpha}_l$ and $\underline{\alpha}_l$, as shown in equation (58), the market price in the corresponding zones should equalize. This is because, under unlimited transmission, electricity can flow freely between these zones, effectively creating a single market in which there is only one price-determining participant. In such cases, the dual variables $\overline{\alpha}_l$ and $\underline{\alpha}_l$ should be zero, as there is no limitation on the transmission. The incidence matrix *Yn*,*^l* then ensures that the market prices for the zones *n* connected by the transmission line *l* are equal.

In contrast, when transmission is limited, the market prices in different zones are not forced to be equal. The dual variables $\overline{\alpha}_l$ and $\underline{\alpha}_l$ can take on any value and the model then determines the market price for each zone individually. This relationship is captured in the following stationarity equation, which is derived from the optimization problem's KKT conditions:

$$
-\sum_{n} Y_{n,l} \lambda_n + \underline{\alpha}_l - \overline{\alpha}_l \quad \forall l \tag{58}
$$

Here, $\sum_n Y_{n,l} \lambda_n$ represents the combined impact of the market prices at all zones *n* connected by line *l*, where *Yn*,*^l* is the incidence matrix that links zone *n* with line *l*. When transmission is unlimited, the dual variables $\overline{\alpha}_l$ and $\underline{\alpha}_l$ are zero, enforcing the market prices λ_n to be equal across the connected zones. When transmission is constrained, the dual variables $\overline{\alpha}_l$ and $\underline{\alpha}_l$ are non-zero, allowing for different prices in the connected zones. A similar technique is applied in Rintakäki et al. [\(2020\)](#page-59-8).

MPEC formulation

As mentioned before, in order to solve the bilevel program as a single level optimization problem, the problem is reformulated as a single level MPEC by replacing the lower-level problems by their KKT conditions as is described above for this particular problem. The resulting MPEC formulation is then as follows:

$$
q_s^{\text{id2}} = q_{s'}^{\text{id2}} \quad \forall s, s' \tag{92}
$$

$$
p_s^{\rm id2} = p_{s'}^{\rm id2} \quad \forall s, s' \tag{91}
$$

$$
(F_l^{\text{da}} + f_{l,s}^{\text{id}1} + f_{l,s}^{\text{id}2} - F_l^{\text{max}}) \perp \overline{\alpha}_{l,s}^{\text{id}2} \ge 0 \quad \forall l, s
$$
 (90)

$$
0 \le (F_l^{\min} - F_l^{\text{da}} - f_{l,s}^{\text{id1}} - f_{l,s}^{\text{id2}}) \perp \underline{\alpha}_{l,s}^{\text{id2}} \ge 0 \quad \forall l, s
$$

\n
$$
0 \le (F_l^{\text{da}} + f_{l,s}^{\text{id1}} + f_{l,s}^{\text{id2}} - F_l^{\max}) \perp \overline{\alpha}_{l,s}^{\text{id2}} \ge 0 \quad \forall l, s
$$

\n(90)

$$
0 \le (g_s^{\text{up}, \text{id2}} + g_s^{\text{down}, \text{id2}}) \perp \overline{\mu}_s^{\text{id2}} \ge 0 \quad \forall s \tag{88}
$$

$$
0 \le (q_s^{\text{id2}} - g_s^{\text{up}, \text{id2}} - g_s^{\text{down}, \text{id2}}) \perp \underline{\mu}_s^{\text{id2}} \ge 0 \quad \forall s
$$

$$
0 \leq g_{n,c,s}^{\text{comp,down,id2}} \perp \overline{\mu}_{n,c,s}^{\text{id2}} \geq 0 \quad \forall n,c,s
$$
\n(86)

$$
0 \leq (G_{n,c,s}^{\max, \text{down}, \text{id2}} - g_{n,c,s}^{\text{comp}, \text{down}, \text{id2}}) \perp \underline{\mu}_{n,c,s}^{\text{id2}} \geq 0 \quad \forall n, c, s
$$
\n
$$
(85)
$$

$$
\leq g_{n,x,s}^{\text{comp,up,id2}} \perp \overline{\mu}_{n,x,s}^{\text{id2}} \geq 0 \quad \forall n,x,s \tag{84}
$$

$$
0 \leq (G_{n,x,s}^{\max,\text{up},\text{id2}} - g_{n,x,s}^{\text{comp},\text{up},\text{id2}}) \perp \underline{\mu}_{n,x,s}^{\text{id2}} \geq 0 \quad \forall n, x, s
$$
\n(83)

$$
\sum_{x} g_{n,x,s}^{\text{comp,up,id2}} + g_s^{\text{up,id2}} - \sum_{c} g_{n,c,s}^{\text{comp,down,id2}} - g_s^{\text{down,id2}} - \sum_{l} Y_{l,n} f_{l,s}^{\text{id2}} = 0 \quad \forall n, s \tag{82}
$$

0 ≤ *g*

$$
-p_s^{\text{id2}} + \lambda_{n,s}^{\text{id2}} + \underline{\mu}_s^{\text{id2}} - \overline{\mu}_s^{\text{id2}} = 0 \quad \forall s \tag{81}
$$

$$
p_s^{\text{id2}} - \lambda_{n,s} - \underline{\mu}_s^{\text{id2}} - \overline{\mu}_s^{\text{id2}} = 0 \quad \forall s
$$
\n(80)

$$
\sum_{c,s}^{\text{own},\text{id2}} + \lambda_{n,s}^{\text{id2}} + \underline{\mu}_{n,c,s}^{\text{id2}} - \overline{\mu}_{n,c,s}^{\text{id2}} = 0 \quad \forall n, c, s
$$
\n(79)

$$
-P_{n,c,s}^{\text{down},id2} + \lambda_{n,s}^{\text{id2}} + \underline{\mu}_{n,c,s}^{\text{id2}} - \overline{\mu}_{n,c,s}^{\text{id2}} = 0 \quad \forall n, c, s
$$
 (79)

$$
\mu_{n,x,s}^{\text{up},\text{id2}} - \lambda_{n,s}^{\text{id2}} + \underline{\mu}_{n,x,s}^{\text{id2}} - \overline{\mu}_{n,x,s}^{\text{id2}} = 0 \quad \forall n, x, s
$$
\n(78)

$$
0 \le (F_l^{da} + f_{l,s}^{ad} - F_l^{max}) \perp \overline{\alpha}_{l,s}^{da} \ge 0 \quad \forall l, s
$$

\n
$$
P_{n, s, s}^{np, id2} - \lambda_{n, s}^{id2} + \mu^{id2} - \overline{\mu}_{n, s, s}^{id2} = 0 \quad \forall n, x, s
$$
\n(77)

$$
0 \leq (F_l^{da} + f_{l,s}^{id1} - F_l^{max}) \perp \overline{\alpha}_{l,s}^{id1} \geq 0 \quad \forall l,s
$$
\n(77)

$$
0 \leq (g_s^{\min} - F_l^{\text{da}} - f_{l,s}^{\text{id1}}) \perp \underline{\alpha}_{l,s}^{\text{id1}} \geq 0 \quad \forall l, s
$$
 (76)

$$
0 \le (g_s^{\text{up}, \text{id}} + g_s^{\text{down}, \text{id}}) \perp \overline{\mu}_s^{\text{id}} \ge 0 \quad \forall s
$$
 (75)

$$
0 \le (q^{id1} - g_s^{up,id1} - g_s^{down,id1}) \perp \underline{\mu}_s^{id1} \ge 0 \quad \forall s
$$
\n
$$
0 \le (\text{ } |m|^{id1} \quad \text{down}^{id1}) \perp \underline{\mu}_s^{id1} > 0 \quad \forall s
$$
\n
$$
(74)
$$

$$
0 \leq \varepsilon_{n,z,s} \qquad \qquad + \mu_{n,z,s} \leq 0 \quad \text{and} \quad \varepsilon_{n,z,s} \leq 0 \tag{75}
$$
\n
$$
0 \leq (q^{\text{id}1} - g_s^{\text{up},\text{id}1} - g_s^{\text{down},\text{id}1}) \perp \underline{\mu}_s^{\text{id}1} \geq 0 \quad \forall s \tag{74}
$$

$$
0 \le g_{n,z,s}^{\text{comp,down,id1}} \perp \overline{\mu}_{n,z,s}^{\text{id1}} \ge 0 \quad \forall n,z,s
$$
\n
$$
(73)
$$

$$
0 \leq g_{n,z,s}^{\text{comp,down,id1}} \perp \overline{\mu}_{n,z,s}^{\text{id1}} \geq 0 \quad \forall n,z,s
$$
\n
$$
(73)
$$

$$
0 \leq g_{n,z,s}^{\text{comp,down,idl}} \perp \overline{\mu}_{n,z,s}^{\text{id1}} \geq 0 \quad \forall n,z,s
$$
\n(73)

$$
\sum_{n,z,s} \left(\sigma_{n,z,s} - \sigma_{n,z,s} \right) + \underline{\mu}_{n,z,s} \leq 0 \quad \forall n,z,s \tag{72}
$$
\n
$$
0 \leq g_{n,z,s}^{\text{comp,down,idl}} \perp \overline{\mu}_{n,z,s}^{\text{id}} \geq 0 \quad \forall n,z,s \tag{73}
$$

$$
0 \leq (G_{n,z,s}^{\max, \text{down}, \text{id1}} - g_{n,z,s}^{\text{comp}, \text{down}, \text{id1}}) \perp \underline{\mu}_{n,z,s}^{\text{id1}} \geq 0 \quad \forall n,z,s
$$
\n
$$
(72)
$$

$$
0 \leq \mathcal{S}_{n,k,s} \qquad \qquad + \mu_{n,k,s} \leq 0 \qquad \text{with } n, s \tag{71}
$$
\n
$$
\max, \text{down}, \text{id1} \qquad \text{g} \text{comp}, \text{down}, \text{id1} \qquad | \quad \text{d} \cdot 1 \qquad \text{if } n, s \tag{72}
$$

$$
0 \leq g_{n,k,s}^{\text{comp},\text{up},\text{up}} \perp \overline{\mu}_{n,k,s}^{\text{up}} \geq 0 \quad \forall n,k,s
$$
\n
$$
f_{n,k,s}^{\text{comp},\text{up},\text{up}} \geq 0 \quad \forall n,k,s
$$
\n
$$
f_{n,k,s}^{\text{comp},\text{up}} \geq 0 \quad \forall n,k,s
$$
\n
$$
f_{n,k,s}^{\text{comp},\text{up}} \geq 0 \quad \forall n,k,s
$$
\n
$$
f_{n,k,s}^{\text{comp},\text{up}} \geq 0 \quad \forall n,k,s
$$
\n
$$
f_{n,k,s}^{\text{comp},\text{up}} \geq 0 \quad \forall n,k,s
$$
\n
$$
f_{n,k,s}^{\text{comp},\text{up}} \geq 0 \quad \forall n,k,s
$$
\n
$$
f_{n,k,s}^{\text{comp},\text{up}} \geq 0 \quad \forall n,k,s
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\n
$$
f_{n,k,s}^{\text{comp},\text{up}} \geq 0 \quad \forall n,k,s
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f_{n,k,s}^{\text{comp},\text{up}} \geq 0 \quad \forall n,k,s
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f_{n,k,s}^{\text{comp},\text{up}} \geq 0 \quad \forall n,k,s
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$$
f_{n,k,s}^{\text{comp},\text{up}} \geq 0 \quad \forall n,k,s
$$
\n
$$
f_{n,k,s}^{\text{comp},\text{up}} \geq 0 \quad \forall n,k,s
$$
\n
$$
f_{n,k,s}^{\text{comp},\text{up}} \geq 0 \quad \forall n,k,s
$$
\n
$$
f_{n,k,s}^{\text{comp},\text{up}} \geq 0 \quad \forall n,k,s
$$
\n
$$
f_{n,k,s}^{\text{comp},\text{up}} \geq 0 \quad \forall n,k,s
$$
\n
$$
f_{n,k,s}^{\text{comp},\text{up}} \geq 0 \quad \forall n,k,s
$$

$$
0 \le g_{n,k,s}^{\text{comp,up,1d}} \perp \overline{\mu}_{n,k,s}^{\text{ld}} \ge 0 \quad \forall n,k,s \tag{71}
$$

$$
0 \le g_{n,k,s}^{\text{comp,up,idl}} \perp \overline{\mu}_{n,k,s}^{\text{id}} \ge 0 \quad \forall n,k,s
$$
\n
$$
(71)
$$

$$
0 \le g_{n,k,s}^{\text{comp,up,id1}} \perp \overline{\mu}_{n,k,s}^{\text{id1}} \ge 0 \quad \forall n,k,s
$$
\n
$$
(71)
$$

$$
0 \leq g_{n,k,s}^{\text{comp,up,id1}} \perp \overline{\mu}_{n,k,s}^{\text{id1}} \geq 0 \quad \forall n,k,s
$$
\n
$$
(71)
$$

$$
0 \leq (\sigma_{n,k,s} - \sigma_{n,k,s} - \mu_{n,k,s} \leq 0 \quad \forall n, k, s
$$
\n
$$
0 \leq g \sup_{n} \sup_{k=1}^{n} \mu_{n,k,s} \geq 0 \quad \forall n, k, s
$$
\n(71)

$$
0 \leq (G_{n,k,s}^{\max,\text{up},\text{id1}} - g_{n,k,s}^{\text{comp},\text{up},\text{id1}}) \perp \underline{\mu}_{n,k,s}^{\text{id1}} \geq 0 \quad \forall n,k,s
$$
\n
$$
(70)
$$

$$
0 \leq (G_{n,k,s}^{\max,up,id1} - g_{n,k,s}^{\text{comp},up,id1}) \perp \mu^{\text{id}1} \geq 0 \quad \forall n,k,s
$$
 (70)

$$
\begin{array}{cccc}\n0 & \delta_s & \sum_{z} \delta_{n,z,s} & \delta_s & \sum_{l} \epsilon_{l,n,l,s} & \delta_{l,n,l,s} \\
0 & \delta_s & \sum_{l} \epsilon_{l,n,l,s} & \delta_{l,n,l,s} & \delta_{l,n,l,s} \\
0 & \delta_{l,n,l,s} & \delta_{l,n,l,s} & \delta_{l,n,l,s} & \delta_{l,n,l,s} & \delta_{l,n,l,s} \\
0 & \delta_{l,n,l,s} & \delta_{l,n,l,s} & \delta_{l,n,l,s} & \delta_{l,n,l,s} & \delta_{l,n,l,s} \\
0 & \delta_{l,n,l,s} & \delta_{l,n,l,s} & \delta_{l,n,l,s} & \delta_{l,n,l,s} & \delta_{l,n,l,s} & \delta_{l,n,l,s} \\
0 & \delta_{l,n,l,s} \\
0 & \delta_{l,n,l,s} \\
0 & \delta_{l,n,l,s} & \delta_{l,n,l,s}
$$

$$
+g_s = \sum_{\mathbf{z}} g_{n,\mathbf{z},s} - g_s = \sum_{l} \mathbf{I}_{l,n} \mathbf{I}_{l,s} = 0 \quad \forall n, s \tag{09}
$$

$$
{}_{s}^{p,\text{up,var}} + g_{s}^{\text{up,var}} - \sum_{z} g_{n,z,s}^{\text{comp,down,var}} - g_{s}^{\text{down,var}} - \sum_{l} Y_{l,n} f_{l,s}^{\text{var}} = 0 \quad \forall n, s
$$
 (69)

$$
\sum_{s,s}^{(m,p,\text{up},\text{all})} + g_s^{\text{up},\text{all}} - \sum_{s} g_{n,s,s}^{\text{comp},\text{down},\text{all}} - g_s^{\text{down},\text{all}} - \sum_{l} Y_{l,n} f_{l,s}^{\text{all}} = 0 \quad \forall n, s
$$
(69)

$$
\sum_{k,s}^{mp,up,id1} + g_s^{up,id1} - \sum_z g_{n,z,s}^{\text{comp,down},id1} - g_s^{\text{down},id1} - \sum_l Y_{l,n} f_{l,s}^{\text{id1}} = 0 \quad \forall n, s \tag{69}
$$

$$
-p_s^{2} + \lambda_{n,s}^{2} + \underline{\mu}_s^{2} - \mu_s = 0 \quad \forall s
$$
\n
$$
g_{n,k,s}^{\text{comp,up,id1}} + g_s^{\text{up,id1}} - \sum g_{n,s,s}^{\text{comp,down,id1}} - g_s^{\text{down,id1}} - \sum Y_{l,n} f_{l,s}^{\text{id1}} = 0 \quad \forall n, s
$$
\n(69)

$$
-p_s^{\text{id}1} + \lambda_{n,s}^{\text{id}1} + \mu_s^{\text{id}1} - \overline{\mu}_s^{\text{id}1} = 0 \quad \forall s
$$
\n
$$
p, \text{up}, \text{id}1 \perp \text{gup}, \text{up}1 \perp \sum_{\alpha} \text{comp}, \text{down}, \text{id}1 \perp \text{gdown}, \text{id}1 \perp \sum_{\alpha} \text{gup}, \text{
$$

$$
-p_s^{\text{id}1} + \lambda_{n,s}^{\text{id}1} + \underline{\mu}_s^{\text{id}1} - \overline{\mu}_s^{\text{id}1} = 0 \quad \forall s
$$
 (68)

 $(g_s^{\text{up},\text{id} 1}-g_s^{\text{down},\text{id} 1})\lambda_{n,s}^{\text{id} 1}+(g_s^{\text{up},\text{id} 2}-g_s^{\text{down},\text{id} 2})\lambda_{n,s}^{\text{id} 2}+(g_s^{\text{exc}}\gamma_s^{\text{down}}-g_s^{\text{def}}\gamma_s^{\text{up}})-\epsilon\sum$

s.t. $G^{da} + g_s^{up, id1} - g_s^{down, id1} + g_s^{up, id2} - g_s^{down, id2} + g_s^{exc} - g_s^{def} = W_s \quad \forall s$ (60)

Maximize \sum

s πs Г

 ∇ *k*

$$
P_s - \lambda_{n,s} = \underline{\mu}_s - \mu_s = 0 \quad \text{vs}
$$

$$
-p_s^{\text{id}1} + \lambda_{n,s}^{\text{id}1} + \underline{\mu}_s^{\text{id}1} - \overline{\mu}_s^{\text{id}1} = 0 \quad \forall s
$$
 (68)

$$
p_s^{\text{id}1} - \lambda_{n,s}^{\text{id}1} - \mu_s^{\text{id}1} - \overline{\mu}_s^{\text{id}1} = 0 \quad \forall s
$$
 (67)

$$
p_s^{\text{id1}} - \lambda_{n_s}^{\text{id1}} - \mu_s^{\text{id1}} - \overline{\mu}_s^{\text{id1}} = 0 \quad \forall s
$$
 (67)

$$
P_{n,z,s}^{\text{down},\text{id1}} + \lambda_{n,s}^{\text{id1}} + \underline{\mu}_{n,z,s}^{\text{id1}} - \overline{\mu}_{n,z,s}^{\text{id1}} = 0 \quad \forall n, z, s
$$
 (66)

$$
P_{n,k,s}^{P_{n,k,s}} - \lambda_{n,s}^{m} + \underline{\mu}_{n,k,s}^{m} - \mu_{n,k,s}^{m} = 0 \quad \forall n, k, s
$$
\n
$$
-P_{n,s}^{\text{down},\text{old}} + \lambda_{n,s}^{\text{ld}} + \mu_{n,s}^{\text{ld}} - \overline{\mu}_{n,s}^{\text{ld}} = 0 \quad \forall n, z, s
$$
\n(66)

$$
P_{n,k,s}^{\text{up},\text{idl}} - \lambda_{n,s}^{\text{idl}} + \underline{\mu}_{n,k,s}^{\text{idl}} - \overline{\mu}_{n,k,s}^{\text{idl}} = 0 \quad \forall n,k,s
$$
 (65)

$$
g_s^{\text{down},\text{u2}} \le M(1 - x_s^{\text{u2}}) \quad \forall s
$$
\n
$$
P_{n,k,s}^{\text{up},\text{id1}} - \lambda_{n,s}^{\text{id1}} + \mu_{n,k,s}^{\text{id1}} - \overline{\mu}_{n,k,s}^{\text{id1}} = 0 \quad \forall n,k,s
$$
\n(65)

$$
g_s^{\text{down,id2}} \le M(1 - x_s^{\text{id2}}) \quad \forall s
$$
\n
$$
p_{\text{up,id1}}^{\text{up,id1}} - 2_{\text{id1}}^{\text{id1}} - \overline{n}_{\text{id1}}^{\text{id1}} = 0 \quad \forall n \, k \, s
$$
\n
$$
(65)
$$

$$
g_s^{\text{down},\text{id2}} \le M(1 - x_s^{\text{id2}}) \quad \forall s \tag{64}
$$

$$
g_s^{\text{down},\text{id2}} \le M(1 - x_s^{\text{id2}}) \quad \forall s
$$

$$
g_s^{\text{down},\text{id2}} \le M(1 - x_s^{\text{id2}}) \quad \forall s
$$
\n
$$
\lim_{s \to 1} \dots \tag{64}
$$

$$
g_s^{\text{down},\text{id2}} \le M(1 - x_s^{\text{id2}}) \quad \forall s
$$
\n
$$
\lim_{s \to 1} \lim_{s \to
$$

$$
g_s^{\text{down}, \text{id2}} \le M(1 - x_s^{\text{id2}}) \quad \forall s \tag{64}
$$

$$
g_s^{\text{down},\text{id2}} \le M(1 - x_s^{\text{id2}}) \quad \forall s
$$
\n
$$
g_s^{\text{down},\text{id2}} \le M(1 - x_s^{\text{id2}}) \quad \forall s
$$
\n
$$
(64)
$$

$$
g_s^{\text{down},\text{id2}} \le M(1 - x_s^{\text{id2}}) \quad \forall s
$$
\n
$$
\text{up}, \text{old} \quad \text{old} \quad \text{old} \quad \text{old} \tag{64}
$$

$$
g_s^{\text{down}, \text{id2}} \le M(1 - x_s^{\text{id2}}) \quad \forall s
$$
\n
$$
\lim_{s \to 1} \lim_{s \to
$$

$$
g_s^{\text{down},\text{id}2} \le M(1 - x_s^{\text{id}2}) \quad \forall s
$$
\n(64)

\nand

\n
$$
g_s^{\text{old}} = \text{id}1 \quad \text{odd} \quad \text{odd} \quad \text{odd}
$$

$$
g_s^{\text{down},\text{id2}} \le M(1 - x_s^{\text{id2}}) \quad \forall s
$$
\n
$$
\text{un\ddot{}}_{\text{ucl}}^{\text{id1}} \quad \text{d}^{1} = \text{id}^{1} \quad \text{and} \quad \text{d}^{1} = \text{id}^{1
$$

$$
g_s^{\text{down}, \text{id2}} \le M x_s^{\text{d2}} \quad \forall s
$$
\n
$$
g_c^{\text{down}, \text{id2}} \le M(1 - x_c^{\text{id2}}) \quad \forall s
$$
\n
$$
(6)
$$

$$
g_s^{\text{up},\text{id2}} \le M x_s^{\text{id2}} \quad \forall s \tag{63}
$$

$$
g_s^{\text{up},\text{id2}} \le M x_\text{c}^{\text{id2}} \quad \forall s \tag{63}
$$

$$
e^{\text{up}, \text{id2}} < M x^{\text{id2}} \quad \forall s \tag{63}
$$

$$
g_S^{\text{down,idl}} \le M(1 - x^{\text{id}1}) \quad \forall s
$$
\n
$$
g_S^{\text{up,idd2}} \le Mx^{\text{id}2} \quad \forall s
$$
\n(62)

$$
\sigma^{\text{up}, \text{id2}} < M \mathbf{v}^{\text{id2}} \quad \forall \mathbf{s} \tag{63}
$$

$$
\sigma^{\text{up}, \text{id2}} < M r^{\text{id2}} \quad \forall s \tag{63}
$$

$$
e^{\text{up}, \text{id2}} < Mx^{\text{id2}} \quad \forall s \tag{63}
$$

$$
g_s^{\text{up}, \text{id2}} \le M x_s^{\text{id2}} \quad \forall s \tag{63}
$$

$$
g_s^{\text{up},\text{id2}} \le M x_s^{\text{id2}} \quad \forall s \tag{63}
$$

$$
g_s^{\text{up},\text{id2}} \le M x_s^{\text{id2}} \quad \forall s \tag{63}
$$

$$
g_s^{\text{up},\text{id2}} \le M x_s^{\text{id2}} \quad \forall s \tag{63}
$$

$$
g_s^{\text{up},\text{max}} \le M x_s^{\text{max}} \qquad \forall s \tag{63}
$$

$$
g_s^{1/2} \le Mx_s \qquad \forall s \tag{63}
$$

down_id2 $\ge M(1 \quad \text{odd})$

$$
g_s^{\text{F}} = M x_s^{\text{F}} \qquad \text{vs}
$$
\n
$$
g_s^{\text{Dwyn}} = \frac{1}{2} M x_s^{\text{F}} \qquad \text{vs}
$$
\n
$$
g_s^{\text{Dwyn}} = \frac{1}{2} M x_s^{\text{F}} \qquad \text{vs}
$$
\n
$$
g_s^{\text{F}} = \frac{1}{2} M x_s^{\text{F}} \qquad \text{vs}
$$
\n
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g_s^{\text{F}} = \frac{1}{2} M x_s^{\text{F}} \qquad \text{vs}
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g_s^{\text{F}} = \frac{1}{2} M x_s^{\text{F}} \qquad \text{vs}
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g_s^{\text{F}} = \frac{1}{2} M x_s^{\text{F}} \qquad \text{vs}
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g_s^{\text{F}} = \frac{1}{2} M x_s^{\text{F}} \qquad \text{vs}
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g_s^{\text{F}} = \frac{1}{2} M x_s^{\text{F}} \qquad \text{vs}
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g_s^{\text{F}} = \frac{1}{2} M x_s^{\text{F}} \qquad \text{vs}
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g_s^{\text{F}} = \frac{1}{2} M x_s^{\text{F}} \qquad \text{vs}
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g_s^{\text{F}} = \frac{1}{2} M x_s^{\text{F}} \qquad \text{vs}
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g_s^{\text{F}} = \frac{1}{2} M x_s^{\text{F}} \qquad \text{vs}
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g_s^{\text{F}} = \frac{1}{2} M x_s^{\text{F}} \qquad \text{vs}
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g_s^{\text{F}} = \frac{1}{2} M x_s^{\text{F}} \qquad \text{vs}
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$$
g_s^{\text{F}} = \frac{1}{2} M x_s^{\text{F}} \qquad \text{vs}
$$
\n
$$
g_s^{\text{F}} = \frac{1}{2} M x_s^{\text{F}} \qquad \text{vs}
$$
\n
$$
g_s^{\text{F}} = \frac{1}{2} M x_s^{\text{F}} \qquad \text{vs}
$$

$$
\mathcal{S}_s^* \geq M \mathcal{A}_s \tag{0.3}
$$

lown_id2 $\mathcal{A}_d(1, \ldots, \text{id}^2)$

$$
\sigma_s^{\text{up},\text{ia2}} \le M x_s^{\text{ia2}} \quad \forall s \tag{63}
$$

$$
r_s^{\text{up}, \text{id2}} \le M x_s^{\text{id2}} \quad \forall s \tag{63}
$$

$$
g_s^{\text{up},\text{id2}} \le M x_s^{\text{id2}} \quad \forall s \tag{63}
$$

$$
g^{\text{up},\text{id}2} \le Mx^{\text{id}2} \quad \forall s \tag{63}
$$

$$
e^{\text{up},\text{id}2} < Mx^{\text{id}2} \quad \forall s \tag{63}
$$

$$
\sigma^{\text{up},\text{id2}} < M x^{\text{id2}} \quad \forall s \tag{63}
$$

$$
\sigma^{up,id2} < M r^{id2} \quad \forall s \tag{63}
$$

$$
\leq M(1 - \lambda) \quad \text{vs}
$$
\n
$$
\sigma^{\text{up}, \text{id2}} < M \, \text{v}^{\text{id2}} \quad \forall s \tag{63}
$$

 $g_s^{\text{up}, \text{id}1} \leq M x^{\text{id}1}$ $\forall s$ (61)

l tl,*s* ٦

(59)

$$
e^{\text{up}, \text{id2}} < M x^{\text{id2}} \quad \forall s \tag{63}
$$

Reformulate MPEC as an MILP

The model as formulated as an MPEC is considered nonlinear. The objective function (59) as well as the constraints (70-77) and (83-90) all contain bilinear terms. In order to reformulate the MPEC as an MILP, such that it can be solved efficiently using off-the-shelf optimization software, these terms must be dealt with.

Linearization of complementary constraints

The complementary constraints (70-77) and (83-90) are nonlinear. Complementary constraints, such as $0 \leq (G_{n,k,s}^{\max, \text{up}, \text{id}1})$ $\max_{n,k,s}$ *m*_{*n*},*k*_{,*s*}</sub> *m*_{*n*},*k*_{,*s*} *m*_{*n*},*k*_{,*s*} $\lim_{n,k,s}$ (*n*,*k*,*s*) $\perp \underline{\mu}_{n,k}$ $\sum_{n,k,s}$ ≥ 0 can be linearized in the following way:

$$
(G_{n,k,s}^{\max,up,id1} - g_{n,k,s}^{\text{comp},up,id1}) \ge 0
$$
\n(93)

$$
\mu_{n,k,s}^{\text{id1}} \ge 0\tag{94}
$$

$$
(G_{n,k,s}^{\max,up,id1} - g_{n,k,s}^{\text{comp,up,id1}}) \le \underline{v}_{n,k,s}^{\text{id1}} M \tag{95}
$$

$$
\underline{\mu}_{n,k,s}^{\text{id}1} \le (1 - \underline{\nu}_{n,k,s}^{\text{id}1})M \tag{96}
$$

$$
\underline{v}_{n,k,s}^{\text{id1}} \in \{0,1\} \tag{97}
$$

M is chosen to be a sufficiently large constant. This linearalization is done for all complementary constraints. The MILP model is shown in Appendix [B.](#page-62-0)

Discretization of objective function

The objective function also contains four bilinear terms. One example is *g*^{up,id1}λ^{id1}. In most scientific papers mentioned in Chapter [2.2,](#page-14-0) linearization of products in the objective function are linearized by rewriting the bilinear term using some of the KKT conditions and the strong duality theorem, which states that if the primal has an optimal solution, then the dual has an optimal solution too, and the two objective values are equal (Boyd & Vandenberghe, [2004\)](#page-58-26). The objective functions of the primal and dual are equal, and the bilinear term can be rewritten. An example can be found in Ruiz and Conejo [\(2009\)](#page-59-10) and J. Kazempour [\(2021\)](#page-58-27).These examples show that the objective function of the dual problem must solely contain linear terms. Since the lower levels contain the variables $q^{\text{id}1}$ and $q^{\text{id}2}_{s}$, the strong duality theorem can not be used in this case.

An alternative method, called binary expansion, is proposed by Barroso et al. [\(2006\)](#page-58-28) and implemented by Rintakäki et al. [\(2020\)](#page-59-8). The implementation in the model is shown below for the bilinear term *g* up,id1*λ* id1 .

$$
g^{\text{up}, \text{id1}} = \sum_{j} 2^{j-1} h_j^{\text{up}, \text{id1}} \tag{98}
$$

$$
-M(1-h_j^{\text{up},\text{id1}}) \le \lambda^{\text{id1}} - \hat{h}_j^{\text{up},\text{id1}} \le M(1-h_j^{\text{up},\text{id1}}) \quad \forall j
$$
\n(99)

$$
\Lambda^{\min}h_j^{\text{up},\text{id1}} \le \hat{h}_j^{\text{up},\text{id1}} \le \Lambda^{\max}h_j^{\text{up},\text{id1}} \quad \forall j \tag{100}
$$

$$
\nu^{\text{up},\text{idl}} = \sum_{j} 2^{j-1} \hat{h}_j^{\text{up},\text{idl}}
$$
 (101)

The term $g^{\rm id1,up}$ is represented by multiple binary variables in the form of $h^{\rm id1,up}_i$ $j_j^{\text{ta1},\text{up}}$ through constraint (98), where j is chosen to be a sufficiently large number. Then, for every binary variable where $h_j^{\rm id1,up}=1,$ the market price is set equal to $\hat{h}^{\text{id}1,\text{up}}_{i}$ ^{id1,up} in constraint (99). The variable $v^{id1,up}$ is then represented by all the summed up values of $\hat{h}^{\text{id}1,\text{up}}_{i}$ j ^{(d1,up} in constraint (101), which equals the value of bilinear term $g^{id1,up}\lambda^{id1}$. Note that constraint (100) restricts the market price between a lower and upper bound. These boundaries are chosen in such a way that they don't affect the overall solution of the model, but they significantly reduce the computation time since much less potential market price values have to be explored.

After applying the binary expansion technique to all bilinear terms, the objective function will look as follows:

$$
\text{Maximize} \quad \sum_{s} \pi_s \left[\nu_s^{\text{up}, \text{id}1} - \nu_s^{\text{down}, \text{id}1} + \nu_s^{\text{up}, \text{id}2} - \nu_s^{\text{down}, \text{id}2} + \left(g_s^{\text{exc}} \gamma_s^{\text{down}} - g_s^{\text{def}} \gamma_s^{\text{up}} \right) \right]
$$

Last, some necessary adjustments to the mathematical model had to be done that require a bit more explanation. This is done in Appendix [C.](#page-64-0) This includes introducing penalty functions and an adjustment to the stationarity condition to ensure that a correct market price is derived, depending on whether the strategic producer intends to up- or downregulate. In the next chapter, the case study will be discussed. In this chapter, the input data for the model, as well as the scenarios will be discussed. Furthermore, the setup of the experiments will also be elaborated upon.

5 Case study

5.1 Input dataset

In order to accurately model a market clearing in the intraday market, real market data has been used with bids and offers that were submitted by market participants to the intraday market. The offering strategy of the strategic producer is highly dependent on this, as it determines the market conditions and thus the way to optimize its own profit. The historical data originates from 2021 of the NordPool market when the intraday market was continuous. The data has to be converted to make it suitable for the purpose of the research.

The dataset consists of submitted intraday bids in all five market zones in Norway, and each bid includes a quantity and corresponding price. Since the data is based on the intraday market, the bids refer to upor downregulation. The delivery hour for which the bid was submitted is also included in the dataset. The delivery hour is used to categorize the bids by delivery hour, since these bids and offers are the ones competing with each other and are used to determine a market equilibrium for each corresponding zone. Instead of the bids and offers directly trading with each other continuously over a certain time period, as is the case in a continuous market from which the dataset is derived, these bids and offers will now be cleared together through an auction.

Data pre-processing

The data must be pre-processed to make it suitable as input data. The dataset contains all bids and offers that were ever submitted to the market, in which the columns referring to the OrderType and State of the order are of interest. Only limit orders with an "active" status are considered, while bids and offers that don't satisfy this criteria are removed from the dataset. The dataset contains many bids and offers that have either been modified or deleted, and are thus not taken into consideration.

Furthermore, in order to reduce the computational intensity of running the model, bids and offers can be aggregated. For all bids and offers that share the same bidding zone, delivery hour, and intraday, offers can be grouped if they share similar characteristics. The mathematical formulation shows that every single competitive offer results in many constraints and binary variables due to the KKT conditions. To this end, bids and offers that share exactly the same price are aggregated.

Data validation

After pre-processing, the dataset is validated by comparing the intraday price to the corresponding day-ahead price of every hour in 2021. For this purpose, a modified version of the MILP model from Appendix [B](#page-62-0) is used to clear the market based on the competitive bids and offers, in which the strategic player is neglected. Instead of using a profit maximization objective, the objective is equal to the one as formulated in the market clearing model in Chapter [4.](#page-27-0) Using this model allows for an easier determination of the market prices due to the optimality conditions, as opposed to a regular market clearing model. Then, every hour of the 2021 dataset is ran separately in this model and the market prices as determined by the model are derived and analyzed. In order to compare day-ahead and intraday prices and identify the correlation, the Pearson Correlation Coefficient (r) and Mean Absolute Percentage Error (MAPE) will be used, which are characterized by the following formulas:

$$
r = \frac{\sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n} (X_i - \bar{X})^2 \sum_{i=1}^{n} (Y_i - \bar{Y})^2}}
$$
 MAPE = $\frac{100\%}{n} \sum_{i=1}^{n} \left| \frac{A_i - F_i}{A_i} \right|$

To this end, the following values are obtained for each zone:

Zone	Average DA MP (ϵ) Average ID MP (ϵ)		r	MAPE
NO ₁	74.67	77.28	0.953	13.2 %
NO ₂	75.08	78.14	0.959	12.6%
N _O 3	41.07	34.58	0.877	23.8%
N _O 4	35.03	34.91	0.856	31.6 %
N _O ₅	74.39	77.05	0.954	11.9 %

Table 5: Comparison of DA and ID prices

As shown in Table [5,](#page-38-1) the day-ahead and intraday prices have strong correlation, although there are clear differences per zone. Values of *r* between 0.7 and 1 indicate a strong positive correlation. When the day-ahead price increases, the intraday price tends to increase as well. According to MAPE, percentages between 10% and 20% indicate moderate accuracy. It measures the average magnitude of error of the intraday price compared to the day-ahead price, and indicates to what extent intraday prices are off from day-ahead prices, either positive or negative. As will be explained later, day-ahead and intraday prices are similar in most cases. Large deviations in the dataset at hand can occur due to the lack of international transmission and/or the fact that the market prices are derived from an auction-based model while the data is based on a continuous market. Since the actual intraday prices of 2021 are not available, the accuracy of the correlations and errors as obtained in Table [5](#page-38-1) is not entirely known.

Splitting up the data

Since two-auction-based intraday gates are included in the model, the bids and offers must be divided. There are several ways to do this. First of all, it is important to analyze the correlation between different gates in an intraday market and review to what extent the same market prices among these gates is obtained. This will help choose a division technique that best reflects the expected outcome of the different gates when the proposed intraday structure is implemented. For this purpose, an analysis made based on intraday prices in the Spanish and Portuguese electricity market can be used. Figure [10](#page-38-0) shows how the prices of the six auction-based intraday gates in Spain compare to each other over multiple time periods (hours), ranging from May 24th, 2024 until June 3rd, 2024. The data is derived from 'OMIE - Market results - European Intraday (IDAs) - Intraday price' [\(2024\)](#page-59-16) and the six gates close at 15:00, 17:50, 21:50, 1:50, 4:50, and 9:50, respectively (OMIE, [2024\)](#page-59-17).

Figure 10: Intraday prices compared in the multi-gate Spanish market

Figure [10](#page-38-0) illustrates that the intraday prices in a multi-gate structure are strongly correlated. To this end, the correlation should be maintained when splitting up the data for the research at hand. It is therefore desirable that the market prices obtained when running a market clearing model with the bids and offers are in the same range as the market prices obtained in both the first and second intraday after the data has been split up. Putting an arbitrary number of bids and offers in the first intraday gate, and the leftover bids in the second gate may not yield the desired correlation, as this method could introduce artificial differences in the market conditions. There is a risk of grouping certain bids and offers and thus certain behaviour into one session, which can skew the results of the market clearing in both sessions. Alternatively, the quantity of every single bid and offer can be divided. Based on a certain percentage, part of the quantity will be offered in first intraday gate and the other part in the second one. The percentage is determined by analyzing the total quantity traded during each hour, in time periods correlating with the implementation of the first and second intraday, as shown in Table [7.](#page-41-0) The percentage of volume before and after the cut-off point of the first intraday is used to divide the quantities of bids and offers for that specific hour. The offering price will remain the same. This is a more appropriate method that also better reflects a real-life situation for the following reasons:

- Each intraday will maintain the same number of bids and offers. This is more realistic, as it is expected that the number of intraday gates does not necessarily decrease the number of bids and offers as market participants can utilize each of these opportunities to adjust their position
- Even though the bids and offers remain the same, the total quantity of these bids and offers per intraday decreases. This is because the adjust each participant makes per intraday is smaller if there are more intraday markets, as e.g. the wind forecast error compared to the previous time a commitment was made is smaller.

Choosing representative hours

Now that both intraday markets have a set of bids and offers, the hours that will be used in the experiment can be chosen. Figure [11](#page-40-1) shows the day-ahead price as a baseline, and the corresponding intraday and balancing prices in comparison. The underlying data in this graph has been taken from the NordPool website ('Nordpool - Market data', [2024\)](#page-59-15). Since the intraday prices here are still based on a continuous market, and there is not one single price per hour, the volume weighted average price (VWAP) is used. Figure [11](#page-40-1) shows that the day-ahead price and intraday prices are correlated and to an extent pretty similar in most cases. This is something that should be taken into account when choosing representative time periods.

For this purpose, a market clearing model for both intraday markets using the new data is ran to obtain market prices. Not every hour of the 2021 data is suitable for experimenting. This is due to a variety of reasons, such as a lack of bids and offers, unlogical intraday prices or intraday prices that deviate excessively from their day-ahead price. These hours will be excluded from consideration.

Figure 11: Comparison of day-ahead, intraday, and balancing prices in zone NO3

Multiple hours in two distinct periods in the year will be used as input for the optimization model. One of these periods will be characterized by lower market prices, and the other will be focused on higher market prices, in order to retrieve results for hours that are clearly different from each other. In electricity markets, there is usually an obvious difference in electricity prices between seasons and the effect that each of these instances has on the ability of the strategic producer to exercise market power is of interest to investigate. The chosen hours are listed in Table [6.](#page-40-2)

2021 date	2021 hour	Approximate market price (\in / MW)
March 20th	1891, 1894, 1896	20
	September 24th 6398, 6399, 6406	50

Table 6: Hours that are used in the experiments

Transmission capacities

In order to accurately model specific hours of the 2021 dataset, corresponding transmission capacities are also required. In the day-ahead market, the line capacities are known, and these are utilized by the bids and offers submitted in that market. The intraday capacities are what is still leftover after the transmission flows from the day-ahead market are substracted from the available line capacities.

The transmission capacities of the intraday stage for the dataset used in this research are derived from ENTSO-E, which is a platform for data transparency related to electricity markets in Europe (ENTSO-E, [2024\)](#page-58-25). These intraday capacities relate directly to every hour in the dataset and can thus be used for the purpose of the research.

5.2 Scenario generation

Prediction of wind generation

To simulate and study the offering strategy of a wind producer under uncertainty, scenarios for wind power forecasts must be generated. There is rich literature on how wind forecasts evolve over time for different look-ahead horizons. Generally, it is evident that the uncertainty in wind forecasts decreases closer to delivery time. A study conducted by the European Commission mentions the forecasts error

for different horizons based on French wind onshore production projections. The figure can be found in Ehrenmann et al. [\(2018\)](#page-58-29) and shows the forecast error from day-ahead stage to intraday stage (until 1 hour before delivery) for both a 16-hour and 24-hour period. It can be seen that the forecast error gradually decreases over time, before going to 0% as the real-time delivery occurs. This data can be used for the purposes of the research at hand.

The value of the mean deviation *d* for infinite normal samples is given by

$$
d = \sqrt{\frac{2}{\pi}}\sigma\tag{102}
$$

so that

$$
\frac{d}{\sigma} = \sqrt{\frac{2}{\pi}} = 0.7978845608... \tag{103}
$$

which is the ratio of the mean deviation to the standard deviation (Geary, [1935\)](#page-58-30). The wind forecast errors as shown in the figure in Ehrenmann et al. [\(2018\)](#page-58-29) refer to the mean deviation *d*. This way, a standard deviation for every forecast error can be obtained, such that a normal distribution can be used to generate wind power predictions for different look-ahead horizons.

Wind forecast in relation to the intraday gates

The forecast errors that are used in generating wind power forecasts reflects the time between the different gates, as well as the actual delivery moment. As mentioned before, only gate ID2 and ID3 as shown in Figure [2](#page-11-1) are considered. Therefore, the hours between the gates are shown in Table [7](#page-41-0) and converted to standard deviations. It is obvious that in the final hours before delivery the wind prediction can still change significantly. Furthermore, it is assumed that all hours considered in the experiments are in the second part of the day, such that they would experience three gates in the proposed intraday structure and thus gates ID2 and ID3 in this research. The hours to delivery after the closing of the final gate is specific for every hour, ranging from 2 to 14 hours, but an average is used instead.

		Hours Hours to delivery Forecast error (%)	
Day-ahead - ID1	12	32	5%
$ID1 - ID2$	12	20	5%
ID2 - Delivery			12%

Table 7: Forecast error to be used for each stage

The forecast errors in Table [7](#page-41-0) are converted to their corresponding standard deviation using equation (103). A normal distribution is used to generate an arbitrary (1000) number of scenarios. For the wind forecast scenarios in the second intraday, the deterministic instances as explained in subsection [5.3](#page-43-0) are used as mean. For the wind forecast scenarios in the balancing stage, the generated wind forecast values for the second intraday are used as mean to again generate 1000 scenarios for each scenario in the second intraday again, resulting in 1.000.000 wind forecast scenarios in total. This is done to capture the relationship of the wind forecast between the two stages. The wind forecast scenario tree is reduced using K-means clustering, such that both the second intraday and the balancing stage have a limited number of scenarios, which also includes a weight factor to signify the importance of each scenario. The exact number of reduced scenarios is later elaborated upon.

Balancing prices

The strategic producer does not have knowledge about the up- and downregulation prices in the balancing market. Therefore, an analysis will be made to properly include this uncertainty through a stochastic parameter into the model. The actual balancing prices for the 2021 dataset are not known, which is why balancing data of 2024 must be used. First of all, the correlation between day-ahead prices, intraday prices, and balancing prices will be studied. This is relevant, as any potential correlation can help with creating realistic balancing scenarios. Price data from these three markets is derived from the NordPool website for a random week in 2024 for multiple bidding zones. An example is given in Figure [11.](#page-40-1)

In Figure [11](#page-40-1) the day-ahead price is used as baseline. The deviation of the intraday price compared to the day-ahead price, as well as up- and downregulating prices compared to the day-ahead price, is shown. The graph shows that there is relatively strong correlation between the day-ahead price and the intraday price, deviating not too much in most cases. Furthermore, in line with the two-price system as mentioned before, either the up- or downregulation price (or both) is almost always equal to the day-ahead price. The intraday price is much less correlated with the balancing prices. Table [8](#page-42-0) shows the possible four cases that occur with regards to the comparison between day-ahead and balancing prices.

	Upregulation price			
		Equal to DA Higher than DA		
Downregulation price	Equal to DA	22.0%	39.7%	
	Lower than DA	36.2%	2.1%	

Table 8: Balancing prices compared to day-ahead prices

These percentages will be used to create balancing prices. Since the day-ahead price for the 2021 dataset is known, scenarios can be generated based on the knowledge obtained of the correlation between day-ahead and balancing prices. Another aspect that is neglected in this case is the possible correlation between deviation in wind forecast and balancing prices. One might assume that in case of a large prediction error, there is a need for up- or downregulation to compensate for this. The NordPool website ('Nordpool - Market data', [2024\)](#page-59-15) includes data on wind power predictions and production. Table [9](#page-42-1) shows that this assumption is not necessarily true. The analysis is based on zones NO1 until NO4, since NO5 has no wind power. There was roughly an equal amount of upregulation instances in the balancing market, regardless of the direction of the error of the wind power prediction.

Table 9: Regulation requirements based on wind production forecasts

The analysis as done on historical balancing prices from NO3 is used to create scenarios for balancing prices. In total, 672 balancing prices and their corresponding day-ahead price are included. In the balancing price scenarios, the probability of each combination of upregulation and downregulation price occuring corresponds to those as mentioned in Table [8.](#page-42-0) That means, in 22% of the scenarios, both balancing prices will be equal to the day-ahead price, etc. The prices for upregulating and downregulating (when these are distinctive from the day-ahead price) will be derived from the data analysis.

The deviation of the upregulation and downregulation prices from their corresponding day-ahead price will be normalized and adjusted for the day-ahead price of the specific hour that is ran in the optimization model. K-means clustering is used to reduce the number of upregulation and downregulation scenarios seperately. In the end, this will decide the composition of the upregulation and downregulation scenarios and how the 39.7% and 36.2% that relate to these instances are filled in. The instance in which both upregulation and downregulation are unequal to the day-ahead price is neglected to reduce the number of scenarios due to its low significance. Then, the percentages of the other three instances are adjusted to make up the 100% together. For illustration, a simple example with five scenarios is shown in Table [10.](#page-43-2) In this case, instance (2), in which the upregulation price is higher than the day-ahead price, and the downregulation price is equal to the day-ahead price, was reduced to two scenarios. As you can see, these two scenarios add up together to the 36.2% as mentioned in Table [8.](#page-42-0) The exact number of scenarios for each instance is later elaborated upon.

Instance		Upregulation Price Downregulation Price Scenario weight Instance weight		
	26.2	26.2	22.0%	22.0%
	30.6	26.2	20.4%	
2	32.3	26.2	15.8%	36.2%
3	26.2	22.8	18.5%	
	26.2	20.4	21.2%	39.7%

Table 10: Balancing prices scenarios example

Bidding scenarios

Figure 12: Example of scenario generation for data of bids and offers in the intraday market

It is assumed that the strategic producer, when bidding to the market, does not exactly know what bids and offers the competitive participants are going to submit, although it has an estimation. Therefore, scenarios will be created based on the bids and offers for each specific zone to create a bidding curve of each zone separately, as well as for each delivery hour. An example is illustrated in Figure [12.](#page-43-1) Every bid and offer in the data will be decreased or increased by a set percentage to generate scenarios and alter the quantity and price of each individual bid. Two bidding scenarios are considered per intraday. For the first scenario, all offer prices are multiplied by a random value between 0.975 and 1, and for the second scenario all offer prices are multiplied by a random value between 1 and 1.025. This is done for both intraday one and two seperately. The same is done for the offer quantities, except a random value between 0.75 and 1 for scenario one, and a random value between 1 and 1.25 for scenario two are used, respectively.

5.3 Scenario tree

As mentioned before, this research considers uncertainties of wind production, bids and offers, and balancing prices. Two distinct scenario trees, one for the second intraday stage and one for the balancing stage are generated, as visualized in Figure [13.](#page-44-0) The scenario tree explains how uncertainty unfolds across the stages of the stochastic program. When the strategic producer is in the first intraday, no information with regards to the uncertain parameter has been revealed yet. At the second intraday, a new wind production forecast is known, as well as the bids and offers from the first intraday. The same updated information is known at the balancing stage, except now the strategic producer also has information on the balancing prices. For each stochastic parameter, a set of scenarios is generated. These are then combined to generate two scenario trees. The wind production parameters in both stages are generated in such a way that they are dependent on each other. That is, a high wind production scenario in ID2 will also result in higher wind production scenarios in the balancing stage. The two scenario trees are connected, such that every scenario path of the ID2 scenario tree serves as a starting point for every scenario path in the BAL scenario tree. This way, a decision taken at any stage will also account for all uncertainty ahead.

Figure 13: Scenario tree

Deterministic wind forecast in ID1

Deterministic deviation values are considered at the first intraday stage. Upon reaching the first intraday gate, the wind power producer's prediction of actual generation is updated. This new estimation, while usually more accurate, still deviates from the day-ahead prediction. Two cases with deterministic values represent this characteristic to account for the deviations from the day-ahead wind forecast.

Figure 14: Deterministic scenarios wind forecast in ID1

Figure [14](#page-44-1) shows the two deterministic instances, one representing an increase in expected production, and another one representing a decrease compared to the forecast in the day-ahead market. These two deviations are based on the values as discussed in the wind prediction subsection, reflecting the wind forecast error that occurs between the day-ahead stage and the first intraday. The day-ahead forecast and commitment of 115 MW should then be adjusted for based on these two deterministic instances. In order to do this, the optimization model will be ran separately for both the instances in which the wind forecast in the first intraday increases and decreases. For the increase in forecast, the deviation from the day-ahead forecast and commitment is set equal to the lower instance, and in the second one it is set equal to the higher instance. The scenario tree and the wind forecasts for the following stages then follows from the deterministic value in each of the two deterministic instances. Afterwards, the results for both deterministic instances are displayed separately. In Chapter [6,](#page-48-0) these instances will be referred to as 'Negative' and 'Positive' ID1 forecast directions.

Non-anticipativity constraints

The non-anticipativity constraints ensure that decisions made at different points in time do not depend on unknown information at that time. For the scenarios as mentioned above, this means that for example the price, quantity, as well as the direction of the offer (to upregulate or downregulate) across all bidding scenarios should be the same. This allows the modelling of uncertainty as the wind power producer doesn't known the exact behaviour of the other participants, and it can only submit one price and quantity based on multiple possible outcomes. The decision in the first intraday is not scenariodependent, as no uncertainty is revealed at this stage yet. The decisions in the second stage are the same for every single branch of the scenario tree on the left in Figure [13.](#page-44-0)

5.4 Experimentation plan

Competitive wind power producer

In order to study the effects of the behaviour of our strategic wind power producer, it must be compared to the situation in which it would behave just like all the other market participants, namely competitively. A competitive participant bids its marginal costs, which includes expenses such as fuel costs and variable operating costs. For a wind power producer, the marginal costs and thus its bid to the market is typically zero. This is because once a wind farm is in operation, additional energy costs are basically zero. Also, wind energy has to be sold anyway, so the producer preferably sells all of its energy output. To simulate competitive behaviour, the bidding prices of the wind power producer will be set to zero when upregulating, and to the same price point of the most expensive downregulator when downregulating. It will offer a quantity that is equal to the wind forecast deviation compared to the previous stage. Then, the market price and the wind power producer's profit will be evaluated and compared to strategic offering. In short, the following two instances are experimented with:

- Case 1: Producer is strategic in both intraday markets
- Case 2: Producer is competitive in both intraday markets

Different levels of wind production forecast deviation

In addition, a series of experiments will be conducted to analyse the results amidst varying levels of wind forecast deviation. A sensitivity analysis on the amount of forecast error and the effect this has on the evaluations metrics will be performed. The main objective is to observe how changes in wind forecast deviation and the amount of uncertainty influences the strategic behaviour of the wind power producer. To this end, the wind forecast error will be decreased and increased, and the strategic behaviour and corresponding market prices will be evaluated.

5.5 Experimentation settings

Day-ahead bid

A starting point must be determined that shall be used as bid in the day-ahead market of the wind power

producer. The day-ahead market is not included in the model, but a starting point from which the wind power producer will make adjustments in the intraday markets is needed. For this, the capacity factor can be used. The capacity factor is defined as the average output over a period of time of a particular technology divided by its output if it had operated at full (rated) capacity over that time period. The average capacity factor for onshore wind farms (which is the only type of farms currently installed in Norway), is 37.4% ('Wind Europe', [2024\)](#page-59-18). This percentage shall be used to determine a realistic day-ahead wind forecast and bid based on the total capacity of the wind farm.

Day-ahead market price

The day-ahead market price that is used as input to determine the balancing prices is based on the chosen delivery hour of which the bids and offers are used from the 2021 data. Furthermore, the dayahead price should be reasonably aligned with the market prices obtained in both intraday markets, as it determines the balancing prices, and if there's a lack of correlation between intraday and balancing prices, then the obtained strategy will be non-logical.

Number of scenarios

Generally, the number of scenarios should be high enough to capture various possible scenario paths. The computational complexity can exponentially increase with more scenarios, and the optimization model developed during this research has proven to be computationally very intensive. Therefore, the number of scenarios is chosen to be limited as much as possible in order to reach an optimal solution every time. Furthermore, the scenarios of the stochastic parameters are first reduced individually. This is done instead of reducing the scenario tree as a whole, since the three stochastic parameters have no correlation anyway. Then, all possible combinations are made into a scenario tree, such that every possible combination is captured.

The number of scenarios are shown in Table [11.](#page-46-1) Two wind forecast scenarios are used in every stage, for a total of four wind forecast scenarios in the balancing stage. For the balancing prices, one scenario consists of the instance in which the day-ahead price is equal to the upregulation and downregulation price. One scenario is used for the instance in which the upregulation price is higher than the day-ahead price, and one scenario is used for the instance in which the downregulation price is lower than the day-ahead price. Also, two scenarios for both the bids and offers in ID1 and ID2 are used.

In total, 48 scenarios were used, which is the bare minimum number of scenarios that still makes all parameters to be stochastic. Various attempts at running the model with more scenarios resulted in extremely high solving times, if an optimal solution could even be found. Using less scenarios for any of the parameters would turn them into deterministic parameters as there would be no uncertainty anymore. The model is tested with different random number seeds for the stochastic parameters and the model produces a similar objective value each time.

Stochastic parameter Number of scenarios	
Bids and offers ID1	2
Wind production ID2	2.
Bids and offers ID2	2.
Wind production BAL	2.
Balancing prices	З
Total scenarios	

Table 11: Number of scenarios per stochastic parameter and total

5.6 Evaluation metrics

In order to determine the market power of the chosen strategic wind power producer, the following evaluation metrics are analyzed, including the underlying reasoning:

• Average expected profit

The average expected profit is the anticipated profit the strategic producer expects to earn over

different scenarios. It can be used to evaluate the effectiveness of his strategic offering. A high profit can indicate that the producer has some amount of market power.

• Market price

If the strategic producer's actions directly influence the market price, then this can indicate market power. The market price can be compared to the market price in a market under perfect competition to highlight the influence of strategic offering on these prices.

• Offering price

By examining the offering prices, it can be revealed whether the strategic producer is engaging in strategic offering and trying to influence the market. If offering prices are significantly deviated from the producer's marginal costs, then this can indicate an attempt to increase market prices to obtain a higher profit.

• Offering quantity

The offering quantity can be used to determine whether the strategic producer is withholding quantity in certain markets, in order to get a better price in another one. In accordance with the predicted wind forecast, the producer may choose to deviate from this and not try to trade into balance by trying to take advantage of favorable balancing prices. Furthermore, the eventual accepted quantity can be used to detect certain bidding behaviour by the strategic wind power producer.

• Social welfare

The social welfare can help determine if resources are optimally allocated such that the benefits for society are maximized. It does this by combining consumer and producer surplus. It is thus of interest as it can indicate to what extent the market is economically inefficient if it is influenced by one of the market participants.

6 Results and discussion

This chapter discusses the results of the experiments that were conducted, as explained in chapter [5.](#page-37-0) First, the increase in profit that the wind power producer is able to obtain when behaving strategically is shown. Then, the market power aspect of the results is elaborated upon. A detailed explanation of how the wind power producer's behaviour influences the market will be given, along with the impact on social welfare. Furthermore, trading volumes and different instances of wind forecast uncertainties will also be reviewed. The optimizations were performed using the Gurobi Optimizer version 11.0.0 in Python 3.12.1. This was done on a computer utilizing an Intel 13th Gen i7-13700H with 14 physical cores, clocking at 2.4 GHz with a boost up to 5GHz. The algorithm converges to an optimal solution, depending on the instance that is being ran, after roughly 30-45 minutes. The model size as solved in Gurobi is shown in Table [12.](#page-48-1)

	Size
Rows	76 941
Columns	61 584
Non-zeros	169 506
Continuous	37 248
Integer	24 336
Binaries	24 336

Table 12: Model size in Gurobi

Financial benefit from strategic behaviour

First, the extent to which the wind power producer can financially benefit from strategic offering is discussed. This is compared to an instance in which the wind power producer behaves competitively and tries not to influence the market. In this case, the wind power producer "trades into balance" by offering a quantity to the market that is in line with the wind power forecast at that stage for a price that is equal to its marginal costs. The same optimization model is used for this, except the wind power producer does not determine an optimal bid to offer to the market, but this bid is instead based on the new wind forecast. The actual wind production realization in the balancing stage, and the deviation of this compared to the producer's commitment in the second intraday, is dealt with in the balancing market using the balancing prices according to the scenario tree. For this, the forecast errors as mentioned in Table [7](#page-41-0) are used.

Hour	Direction of ID1 forecast	Competitive (\in)	Strategic (\in)
1891	Negative	-163.51	-129.74
1891	Positive	107.52	143.38
1894	Negative	-123.40	-96.04
1894	Positive	81.64	112.64
1896	Negative	-115.45	-94.75
1896	Positive	76.01	98.07

Table 13: Expected profit for strategic and competitive behaviour for low market price hours

Table 14: Expected profit for strategic and competitive behaviour for high market price hours

The results are shown in Table [13](#page-48-2) and Table [14,](#page-48-2) in which the expected profits are directly derived from

the objective values as determined by the model. The wind power producer can achieve better profits if it behaves strategically, and the results show that the average profit expected increase is around 24%. Analyzing the data, this is mainly because of trading quantities between the markets, which will be explained later in more detail. The market prices obtained in both intradays, as well as the balancing prices according to the scenario tree, are all relatively similar. Furthermore, the volume to trade into balance is relatively low, which means that no huge profits, either positive or negative, are expected anyway. The variety in profits under strategic behaviour for different hours is mainly due to differences in intraday prices for each hour.

Exercising market power

The wind power producer's ability to exercise market power is determined by the extent to which it can change the market equilibrium in its favor. Therefore, a comparison will be made of the market prices under both behaviours of the wind power producer, and how much it deviates, either positively or negatively.

Hour	Direction of ID1 forecast	Avg market price (\in / MW)	Average deviation (\in / MW)
6398	Negative	53.10	0.078
6398	Positive	53.10	0.078
6399	Negative	52.79	0.108
6399	Positive	52.79	0.099
6406	Negative	49.46	0.060
6406	Positive	49.46	0.026

Table 15: Deviation in average market price for low market price hours

Table 16: Deviation in average market price for high market price hours

In Table [15](#page-49-0) and Table [16,](#page-49-0) the market price under competitive behaviour and the average deviation in market price under strategic behaviour is shown. The values are averaged over both intradays. The absolute deviation is shown, as the market price can be changed in both directions, dependent on whether the strategic producer is up- or downregulating. It is important to note that the presence of market participants already by itself can result in different market prices, as their bids and offers are what ultimately determine the market price. Thus, a deviation in market price does not necessarily imply market influence through strategic behaviour. For influencing the market price to its advantage, the following two instances are identified:

- In the case of upregulation, the wind power producer will try to increase the market price to sell energy at a higher price
- In the case of downregulation, the wind power producer will try to decrease the market price to buy energy at a lower price

To this end, the relationship between whether the strategic producer intends to up- or downregulate and the direction of the deviation in market price compared to the market prices under competitive behaviour will be analyzed.

Table [17](#page-50-1) shows that in most cases, the market prices remains unchanged. This mostly happens when the strategic wind power producer neither upregulates nor downregulates. In some cases, the market price can change, even if no action takes place. This is due to the fact that the market price used as benchmark is based on the presence of the competitive wind power producer, and if he decides not

		Upregulation Downregulation	No action
Market price increased	0%	11.9%	3.9%
Market price decreased	16.7%	0%	2.4%
Market price remained equal	10.8%	10.9%	43.5%

Table 17: Effect of strategic producer's actions on the market price as determined in the competitive behaviour instance

to participate under strategic behaviour, the market price can change. Furthermore, Table [17](#page-50-1) shows that in no observed scenario does the market price change in the strategic wind power producer's favor. Instead, many cases are observed where the market price changes in his disadvantage. When upregulating, the market price decreases, and when downregulating, the market price increases. A detailed analysis of this will be given in the following subsection.

Visualizing market influence

As mentioned in the previous section, the wind power producer can influence the market price under certain conditions and in multiple ways. An example based on hour 1896 - negative is used to illustrate this. In this subsection, abstract merit order curves that share the same behaviour as the actual curves are used to explain this in order to better visualize and elaborate the wind power producer's different behaviours. The actual bidding curves of this particular hour can be found in Appendix [D.](#page-66-0)

Figure 15: WPP is absent Figure 16: WPP is behaving competitively

Figure [15](#page-50-0) shows the offering curve without the wind power producer participating. This is the starting point, from which the wind power producer will bid into the market. The marginal producer in this case is an upregulator. Figure [16](#page-50-0) shows what this curve would look like if the wind power producer offered its predicted increase in wind power production to the market at marginal costs. Due to this cheap offer to sell energy, the market price, determined by the equilibrium point, decreases, resulting in one of the downregulators being the marginal producer.

If the wind power producer is behaving strategically, it will not offer its marginal costs, but instead submit a price and quantity that optimizes its expected profit. Intending to upregulate, it has two options to potentially influence the market price. In Figure [17,](#page-51-0) the wind power producer will offer a quantity at the same price as the marginal producer in Figure [15,](#page-50-0) essentially replacing this participant. The wind power producer's bid is prioritized due to the profit maximization objective of the optimization model. The curve shows that the market price increases in the wind power producer's favor compared to its behaviour in Figure [16.](#page-50-0) The curve also shows it will only be able to sell a relatively small quantity. Instead, it can opt to choose for the strategy as shown in Figure [18.](#page-51-0) In this case, it will offer a much higher quantity at the same price as the most expensive upregulator in Figure [16.](#page-50-0) It is obvious that the wind power producer will be able to sell a significant higher quantity at a lower price, as this makes his energy more attractive to the market. This is due to the inherent nature of the merit order curve's shape, in which the bidding and offering curves are sorted on price, and in which the bids and offers close to the equilibrium point are usually smaller in volume and closer in price to the surround bids and offers. This makes profiting from exercising market power to your advantage challenging.

upregulation downregulatio $\overline{7}$ WPP 60 (ϵ / N) 40 Price $\overline{3}$ $\overline{20}$ 10 \mathbf{o} $\overline{20}$ 40 60
Quantity (MW) $\overline{\mathbf{8}}$ 100

Figure 17: WPP is influencing the market price to its advantage

Figure 18: WPP is influencing the market price to its disadvantage

The same reasoning can be applied to an instance of downregulating. Offering at a price close to the original equilibrium point may only yield so much accepted downregulation. Therefore, it is more profitable for the wind power producer to offer a higher quantity at a less favorable price. The reason that the wind power producer tends to prefer trading higher quantities of energy, when it is not bound to offer a quantity in line with the updated wind forecast, is because it can take advantage of more favourable prices in upcoming stages this way. That is, taking losses in one particular stage in order to achieve higher profits in another stage. The extent to which this represents reality is questionable, as this strategy would be considered high risk due to the uncertain nature of electricity markets, although the same can be said for the strategy in Figure [16.](#page-50-0)

 $\overline{8}$

Impact on social welfare

The social welfare in the context of electricity markets is defined as the sum of consumer and producer surplus. It is the difference between what participants are willing to pay or receive and what they eventually pay and receive. The presence of a wind producer should normally yield higher social welfare, as it tends to offer its energy for marginal costs, which are basically zero. For all six hours, the social welfare is calculated for three different types of behaviour. The results are shown in Figure [19](#page-51-1) and Figure [20.](#page-51-1) The values are aggregated for both intraday gates. Hour 6406 is excluded from the chart as the social welfare is particularly high due to the size of the total trading volume, but similar results are achieved in this hour.

Figure 19: Comparison of social welfare for different behaviours for negative deviation instance

Figure 20: Comparison of social welfare for different behaviours for positive deviation instance

Figure [19](#page-51-1) and Figure [20](#page-51-1) show that, for every hour, the competitive behaviour instance yields the highest social welfare. This is unsurprising, as it offers energy at favorable prices to the market. In the case

of downregulating, the price offered by the wind power producer is assumed to be equal to the most expensive downregulator. Thus, society as a whole benefits the most when the wind power producer behaves competitively by just offering its forecast deviation at marginal costs. The social welfare when comparing absent and strategic behaviour is roughly equal, albeit slightly higher in the strategic case for all hours. This is most likely because the wind power producer usually just replaces another participant by offering a similar price, which means the social welfare shouldn't change drastically. Offering a larger upregulation quantity in the market at a unique price point that is below the equilibrium point can increase the social welfare, but this effect is mitigated by the instances where the wind power producer downregulates, as the same effect happens, just in the other direction, decreasing the social welfare.

Figure 21: Impact of strategic offering on social welfare

When the wind power producer offers energy at a strategic price, the social welfare decreases. This is illustrated in Figure [21.](#page-52-0) The area under the curve is now much smaller, as the cheap producer has disappeared and may now be trading at higher prices. Furthermore, this causes the supply curve to shift towards the left, creating a new (higher) equilibrium point. Even though the strategic producer may now be trading higher quantities, since this is usually at a price close to the equilibrium point, this does not make up for the disappearance of the area under the curve that was present when it was behaving competitively.

Trading volumes

The total traded volume has also been analyzed. When the wind power producer behaves competitively and trades into balance, the volume offered to the market at marginal costs will be relatively low, as it only consists of the deviation in the wind forecast. Therefore, comparing this to the strategic behaviour instance is of relevance, as it signifies the difference in how the wind power producer views the market under different behaviours. The comparison will be done separately for the three markets considered, as it will also show how the balancing market is used when comparing competitive with strategic behaviour, as well as potential differences in volume between the two auction-based gates.

Figure 22: Volume traded by WPP in each market for competitive and strategic behaviour in negative deviation instance

Figure 23: Volume traded by WPP in each market for competitive and strategic behaviour in positive deviation instance

The trading volumes are shown in Figure [22](#page-53-0) and Figure [23,](#page-53-0) averaged over all hours. The trading volume under competitive behaviour increases in later markets as there is more deviation in wind power prediction closer to delivery time. It clearly shows that the up- or downregulation under strategic behaviour is significantly larger than if the wind power producer would trade into balance. As mentioned before, this is not surprising as it tries to take advantage of the other markets, which can create big swings in the wind power producer's commitment to the market at each stage. The graph also illustrates that the second gate has much more trading volume than the first gate. The second gate may have higher liquidity due to the way the bids and offers are split up into the two gates. This results in bids with higher quantity, making it easier for the wind power producer to also up- or downregulate more. Furthermore, the excess and/or deficit in the balancing market also increases, as in some cases it may be more profitable to utilize this market.

Figure 24: Total volume traded for each hour for both competitive and strategic behaviour for negative deviation instance

Figure 25: Total volume traded for each hour for both competitive and strategic behaviour for positive deviation instance

Figure [24](#page-53-1) and [25](#page-53-1) show the total volume traded in both intraday gates combined, considering the bids and offers of all participants that participate in up- and downregulating. The graph again compares competitive and strategic behaviour of the wind power producer. Behaving strategically, the wind power

producer has a small impact on the total volume traded. This further amplifies the argument that it prioritizes quantity offered over the associated price as it aims to maximize its profit.

Different levels of wind production forecast deviation

This experiment examines the impact of deviations in wind forecasts on profit and market influence. To this end, the chosen wind forecast error will be alternated to review the impact on the market relative to the uncertainty that the wind power producer experiences. One hour of the low market price period and one hour of the high market price period are ran. The average profit for an instance with small uncertainty, as well as an instance with large uncertainty are compared to the benchmark instance as used in the other experiments.

Table 18: Expected profit of strategic and competitive behaviour for different wind power production forecast deviations

Table 19: Average market price under competitive behaviour and average absolute deviation of market price under strategic behaviour compared to competitive behaviour for different wind power production forecast deviations

Table [18](#page-54-0) shows that a small wind power production forecast uncertainty level results in a higher profit when comparing it to the benchmark case. Subsequently, a large uncertainty level results in a lower profit. The results thus indicate that there is a relation between the uncertainty the strategic wind power producer experiences with regards to forecasting their production and the corresponding expected profit. More uncertainty results in more deviation in wind production forecast between the sequential stages of the electricity market, which means the wind power producer has to correct more

in each stage. This also results in a lower profit under competitive behaviour. The reason that a smaller uncertainty results in a higher profit, is because all wind power producers can bid any quantity to the market. There are no limitations to this. Since less uncertainty results in less absolute deviation in the wind forecast, this allows the strategic wind power producer to take better advantage of price differences between markets, as it has to take the same decision with regards to price and quantity, but over multiple wind forecasts scenarios that have much less variation. Furthermore, the change in market price is shown in Table [19.](#page-54-1) Again, every time the market price deviates from the competitive case, it decreases when the wind power producer is an upregulator, and it increases when the wind power producer is a downregulator. It does not seem like there is a direct correlation between the amount of uncertainty and the deviation in market price.

7 Conclusion

Concluding the thesis

This master thesis provides an investigation into the strategic behaviour of a wind power producer participating in a multi-gate auction-based intraday market structure, as well as a balancing market to correct deviations. The study considers two intraday gates and a balancing market. It introduces a bilevel, three-stage stochastic model, where the two lower-level problems represent the clearing of each intraday gate. Through the development of a bilevel model, this research has examined how and to what extent a wind power producer is able to influence market outcomes by strategically adjusting bids in the intraday market, while also taking the corrective role of the balancing market into account. The aim was to analyze the impact of this behaviour on profits, market equilibriums, and social welfare. The practical application of the model has been demonstrated by means of a case study conducted on the Norwegian intraday market using real data for bids and offers.

The case study finds that the wind power producer is able to obtain increased profits when comparing strategic behaviour, in which any quantity can be offered at any price, with competitive behaviour, in which case only a forecast deviation is offered at marginal costs. Looking into the behaviour the wind power producer employs during different hours and scenarios, it shows that he is able to utilize a difference in equilibrium point, and thus market price, between the different auction-based gates to its advantage. This is done by offering large quantities to the market, e.g. downregulating in the first gate if a higher price is expected in the second gate. The trading volumes under competitive behaviour are merely based on the wind forecast deviation from the previous stage. Under strategic behaviour, an increase in trading volume of 229.17% in ID1, 485.36% in ID2, and 228.44% in the balancing market is observed. A quantity is offered at a price point that ensures this quantity has a higher chance of getting accepted, which is usually close to the equilibrium point. The gain in profit is relatively small, which is mainly because there is not that much variation in market price between the two gates, reducing the potential for significant gains.

The impact on the market equilibrium of this strategy is apparent, although minor. In many scenarios, there is no change in the market price as the wind power producer just 'replaces' the offer of another participant. In other cases, the market price changes in such a way that the wind power producer does not benefit from this (e.g. an increase in market price when up-regulating), because the potential for offering larger quantities is much smaller. Instead, the market price goes down and a lower price per electricity is received. A deviation in market price ranging from $0.01 \in /MW$ to $0.10 \in /MW$ is observed, depending on the hour. Furthermore, a clear difference in social welfare is observed. The wind power producer, behaving competitively, offers energy at low prices to the market, which is replaced by higher prices (when upregulating). This results in lower social welfare. On average, the social welfare decreases by 9.91%.

In general, the wind power producer's ability to influence the market price is also highly dependent on the way, as well as to what extent uncertainty for bids and offers is incorporated in the model. The results show that, under multiple bidding scenarios, and depending on whether the strategic producer wants to up- or downregulate, it will usually select a price offer that is optimal for e.g. the lowest price scenario in case of upregulation, and exactly equal to a specific price in this scenario. This is because offering a quantity at a higher price can result in the offer for the lower price to not be accepted anymore. Since this parameter is stochastic, the strategic producer has to optimize for a decision across all scenarios. It is unclear to what extent perfect information, meaning this parameter is deterministic and the strategic producer has no uncertainty on bids and offers, influences the results, since this is excluded from the research.

Concluding, the wind power producer has the potential to exercise market power, as it can offer at a price that changes the equilibrium point in its favor. It chooses not to do so, since the quantity to be sold at this price point is relatively low and more profit can be obtained by up- or downregulating more energy and benefiting from better prices in other markets.

Future research

The most important shortcomings of the model and areas for future research will be briefly mentioned. This can be used to extent the research at hand to other, comparable cases.

- The model could be extended to represent the entire new structure of the intraday market, including the third gate, as well as the existing continuous market. Modelling the continuous market alongside the auction-based market poses challenges however due to the nature of how trading happens there.
- The used stochastic parameters are uncorrelated. One can imagine that there is a certain level of correlation between wind forecasts and balancing prices. In periods with high wind power production, an oversupply of energy can occur which increases the need for downregulation to reduce production. On the other hand, low wind power production can create a supply deficit which increases the electricity price and the need for upregulation by other participants to compensate for this. This correlation is becoming more important when the wind power penetration in the electricity market is increasing. This effect is even further amplified when wind producers are located at nearby locations, which is the case in Norway.
- The dataset that was used is based on a continuous intraday market, and does thus not fully represent trading behaviour of market participants in a (multi-gate) auction-based intraday market. It is therefore somewhat questionable to what extent these bids and offers can be used in the context of modelling the new intraday structure, as it is based on a differently structured intraday market. The expected liquidity of a multi-gate intraday market, and how this compares.

Furthermore, another interesting aspect is that, due to the intermittent and non-dispatchable nature of wind power, produced energy must be used at the same time as when it is generated. The ability to store energy can offer advantages for wind power producers, as it helps mitigate the effects of this. During periods of high wind where excess energy is generated, electricity can be stored and used in periods with low wind speeds. This also reduces the generator's potential imbalance and thus the need for up- and downregulation, which is one of the main concerns with regards to the increasing share of renewables in the energy mix. The model can be extended to include this possibility, such that the advantages from a producer's, as well as from a market operator's perspective can be analyzed.

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Appendix

A Research questions

This chapter defines the research questions that are setup. The research questions will be used as a guideline to the research and will be answered during the research.

I Literature

The first step of the research is to conduct a literature review. It is important to get a good understanding of the characteristics of the Norwegian energy market. Some research questions are formulated that relate to the functioning of the market, as well as questions specifically about the assignment as mentioned above:

- What are the key components of the Norwegian energy market, how do they work and how are they linked to each other?
- How is the balance between energy supply and demand managed in the Norwegian sequential energy markets, and how are the markets cleared to determine an equilibrium?
- How does (strategic) bidding in sequential energy markets work, and what factors influence this?
- How do the dynamics of price makers and price takers influence strategic bidding behaviour, and what implications does this have for market power and competitiveness?

Furthermore, literature about methodology for the research problem must be reviewed. These methods will be used to set up the optimization problem such that the main research question can be answered. Also, a comparative analysis will be done to review what literature on comparable problems is already available and may be of use for the research at hand:

- What is stochastic optimization, and how can this be used for the formulated assignment?
- What literature already exists on bilevel optimization in energy markets, and how can this be of use for our research?
- What methodology is used to solve similar or relatable problems, and how can this be of relevance to the research at hand?
- What will be the contribution of the research to existing literature and how does it distinguish from what has already been done?

II Methodology

The methodology chapter covers the approach to solving the problem. It outlines how data that will be used as input is collected, what methods are used and how these will be set up. The nature of the research is such that it involves several single level optimization problems that will be combined into a bilevel model. The following research questions will be answered:

- What data is necessary for the proposed optimization problems, and how will this be retrieved or generated?
- How can existing optimization techniques be used to formulate an optimization problem that maximizes social welfare by clearing the intraday stage of the Norwegian energy market?
- How can existing optimization techniques be used to formulate a three-stage stochastic optimization problem for a two-gate discrete intraday and balancing market in Norway that maximizes the profit for a strategic wind producer?

• How can bilevel optimization be used to formulate a hierarchical optimization problem in which there is a higher level (maximizing the wind power producer's profit) and multiple lower levels (maximizing social welfare)?

III Numerical experiments

This section covers the aspect of the research where the optimization models are run. An experimentation plan will be drafted that outlines what kind of experiments will be conducted on the model such that the market power can be measured, and the main research question can be answered. The following questions are used:

- How can an experimentation plan be set up that uses the gathered data to verify and run the optimization problems with the aim of answering the main research question?
- How can market power through strategic bidding be measured, and how is this done in related literature?

IV Insights and recommendations

After running the optimization model, the main research question can be answered. The results will be analysed, and conclusions can be drawn for this. Also, possible extensions that were not investigated due to lack of time or being out of scope for this research will be mentioned:

- How and to what extent can a wind producer use strategic bidding to exercise market power and influence the market price in the Norwegian energy market?
- What are possible extensions of the research that are interesting to investigate in the future?

B MILP model

Maximize
$$
\sum_{s} \pi_{s} \left[(g_s^{\text{up},\text{old}} - g_s^{\text{down},\text{old}}) \lambda_{\text{in},s}^{\text{old}} + (g_s^{\text{up},\text{old}} - g_s^{\text{down},\text{old}}) \lambda_{\text{in},s}^{\text{old}} + (g_s^{\text{new}} - g_s^{\text{dev}}) \lambda_{\text{in},s}^{\text{old}} + (g_s^{\text{new}} - g_s^{\text{dev}}) \lambda_{\text{in},s}^{\text{old}} + (g_s^{\text{new}} - g_s^{\text{dev}}) \lambda_{\text{in},s}^{\text{old}}} \lambda_{\text{in},s}^{\text{old}} + (g_s^{\text{new}} - g_s^{\text{dev}}) \lambda_{\text{in},s}^{\text{old}}} \lambda_{\text{in},s}^{\text{old}} + (g_s^{\text{new}} - g_s^{\text{dev}}) \lambda_{\text{in},s}^{\text{old}}} \leq M \lambda^{\text{old}} \quad \forall s
$$
\n
$$
g_s^{\text{up},\text{old}} - \lambda_{\text{in},s}^{\text{old}} + \sum_{\text{in},s}^{\text{old}} \lambda_{\text{in},s}^{\text{old}} = M_{\text{in},s}^{\text{old}} - \lambda_{\text{in},s}^{\text{old}} + \sum_{\text{in},s}^{\text{old}} \lambda_{\text{in},s}^{\text{old}} = 0 \quad \forall n, k, s
$$
\n
$$
P_{n,k,s}^{\text{up},\text{old}} - \lambda_{\text{in},s}^{\text{old}} + \sum_{\text{in},s}^{\text{old}} \lambda_{\text{in},s}^{\text{old}} = \overline{D}_{n,k,s}^{\text{old}} = 0 \quad \forall n, k, s
$$
\n
$$
P_{n,k,s}^{\text{sum},\text{old}} - \lambda_{\text{in},s}^{\text{sum},\text{old}} + \mu_{\text{in},s}^{\text{old}} - \overline{D}_{\text{in},s}^{\text{old}} = 0 \quad \forall n, k, s
$$
\n
$$
P_{n,k,s}^{\text{sum},\text{old}} - \sum_{\text{in},s} \sum_{\text{in},s}^{\text{comp},\text{poly},\text{old}} - \sum_{\text
$$

$$
P_{n,x,s}^{un,j,d2} - \lambda_{n,s}^{i,d2} + \mu_{n,x,s}^{i,d2} - \overline{\mu}_{n,x,s}^{i,d2} = 0 \forall n, x, s
$$
\n
$$
-P_{n,c,s}^{down,j,d2} + \lambda_{n,s}^{i,d2} + \mu_{n,c,s}^{i,d2} - \overline{\mu}_{n,c,s}^{i,d2} = 0 \forall n, c, s
$$
\n
$$
p_s^{i,d2} - \lambda_{n,s} - \mu_{st}^{i,d2} - \overline{\mu}_{st}^{i,d2} = 0 \forall s
$$
\n
$$
-p_s^{i,d2} + \lambda_{n,s}^{i,d2} + \mu_{st}^{i,d2} - \overline{\mu}_{st}^{i,d2} = 0 \forall s
$$
\n
$$
\sum_{x} g_{n,x,s}^{\text{comp,up},id2} - \sum_{g} g_{n,x,s}^{\text{comp,down},id2} - g_{g}^{\text{down},id2} - \sum_{l} Y_{l,n} f_{l,s}^{i,d2} = 0 \forall n, s
$$
\n
$$
G_{n,x,s}^{\text{max,up},id2} - g_{n,x,s}^{\text{comp,up},id2} = 20 \forall n, x, s
$$
\n
$$
G_{n,x,s}^{\text{max,up},id2} - g_{n,x,s}^{\text{comp,up},id2} \le \underline{\nu}_{n,x,s}^{i,d} M \forall n, x, s
$$
\n
$$
0 \le \underline{\mu}_{n,x,s}^{i,d2} \le (1 - \underline{\nu}_{n,x,s}^{i,d}) M \forall n, x, s
$$
\n
$$
g_{n,x,s}^{\text{comp,up},id2} = g_{n,c,s}^{\text{comp,down},id2} \ge 0 \forall n, c, s
$$
\n
$$
G_{n,c,s}^{\text{max,down},id2} - g_{n,c,s}^{\text{comp,down},id2} \ge 0 \forall n, c, s
$$
\n
$$
G_{n,c,s}^{\text{max,down},id2} - g_{n,c,s}^{\text{comp,down},id2} \le 0 \forall n, c, s
$$
\n
$$
G_{n,c,s}^{\text{comp,down},id2} - g_{n,c,s}^{\text{down},id2} \le 0 \forall n, c, s
$$
\n<math display="</math>

C Mathematical model adjustments

While setting up the mathematical model, often issues were encountered that required some minor adjustments. These will be explained to give some more context to the mathematical model

Constraints that determine the market price for the strategic producer

The stationarity conditions are used to determine the market based on price offers, including those from the strategic producer. Since the strategic producer can be either a upregulator or a downregulator, only one of these formulas must be used, dependent on the situation. A slack variable is introduced in these equations with an infinite lower and upper bound. This allows either formula to be turned off. The updated constraints are shown below.

$$
p - \lambda + \mu - \overline{\mu} + \mathbf{s}^{\text{up}} = 0 \tag{104}
$$

$$
-p + \lambda + \mu - \overline{\mu} + \mathbf{s}^{\text{down}} = 0 \tag{105}
$$

$$
-(1 - x^{\text{id}1})M \le \mathbf{s}^{\text{up}} \le (1 - x^{\text{id}1})M\tag{106}
$$

$$
-x^{\text{id}1}M \le \mathbf{s}^{\text{down}} \le x^{\text{id}1}M\tag{107}
$$

Penalty function for transmission

A penalty function isintroduced for the transmission between the zones. If there is no penalty associated with the transmission values, then there is no disadvantage in transmitting up to the maximum limit of the transmission lines and essentially transmitting around energy all across the network. To avoid this, a small penalty value is included such that only essential transmission takes place.

$$
\text{Transmission penalty value}_{l,s} = \left[\left| f_{l,s}^{\text{id}1} \right| + \left| f_{l,s}^{\text{id}2} \right| \right] \quad \forall l,s \tag{108}
$$

Lack of marginal producer

In some specific instances, it can occur that there is no marginal producer. An example is given in the table below. As shown in the table, the quantity offers of every up- and downregulator is either not or fully fulfilled, and there is no marginal producer whose order is partly fulfilled. In that case, the market price would be the marginal producer's price offer. In this instance, the formulas used for setting the market price do not work. The feasible range for the market price in this case would be as follows:

- Equal or higher than the price offer of every upregulator whose offer is fully fufilled (47)
- Equal or lower than the price offer of every upregulator whose offer is not fulfilled (65)
- Equal or higher than the price offer of every downregulator whose offer is not fulfilled (51)
- Equal or lower than the price offer of every downregulator whose offer is fully fulfilled (69)

Thus, the feasible region ranges from 51 to 65. Since the objective function aims to maximize the profit of the strategic producer, and because the producer is upregulating in this case, the model will automatically use 65 as market price as this is more profitable. The strategic producer may offer a quantity that allows this scenario to happen such that it can take advantage of this error in the model.

Table 20: Market clearing example

A workaround has been found by also including the quantity in the bidding scenarios, such that there are different quantities offered across multiple bidding scenarios. Since the strategic producer should only offer one quantity, this situation is less likely to occur. Furthermore, a function is included to calculate the number of scenarios in which this error occurs.

D Merit order curves

Figure 27: WPP is competitive (1896 - negative)

Figure 28: WPP is influencing the market price to its advantage (1896 - negative)

Figure 29: WPP is influencing the market price to its disadvantage (1896 - negative)