Quantum computing for portfolio optimization

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1 INTRODUCTION

In the early stages of computing, co-founder, and former CEO of Intel Gordon Moore predicted that the number of transistors on a microchip would double every two years [90], thereby hinting toward an exponential increase in computing power every couple of years. To date, Moore's law has had a relative high degree of accuracy, however, it could be challenged by the laws of physics in the near future [102], as Heisenberg's uncertainty principle will interfere with the increased miniaturization of computing components [12], therefore contradicting Moore's law. As there are many assumptions that classical computers will reach their limit [15], quantum computing has garnered more attention in recent years. The need for computing architectures, especially quantum computers, that cater towards this necessity of constant improvements in computational power is persistently being emphasized by studies showing that there is an increased amount of attention and funding going towards projects in this industry [112, 107]. Actors in the financial industry such as JP Morgan Chase & Co, and Morgan Stanley are investing in quantum computing as they can see, and justify the great potential it can have on their operations [69, 86]

Quantum computing exploits the use of quantum mechanics, giving it the ability to compute complex mathematical problems faster than traditional computers in theory [116]. A company equipped with a quantum computer would gain a substantial competitive advantage over rivals, which is a key reason why some companies invest heavily in quantum computing [20].

In this research, the relationship between quantum computing and portfolio optimization will be explored. Additionally, the manner in which quantum computing and portfolio optimization are currently described in the literature will be examined by a systematic literature review. Subsequently, comprehensive research findings in corporate white papers are reviewed and related to the findings from the systematic literature review.

Currently, there is lack of literature that shows a congruent structure and relation between the development and implementation of quantum computing for portfolio optimization in academic and corporate settings, therefore this research is performed. As a result, the main research question is characterized as follows; *"how can quantum computing effectively be applied to address the challenges of portfolio optimization considering existing theories, practical use cases, and corporate whitepapers in the financial industry"*.

This study contributes to the field of literature by synthesizing a comprehensive review and analysis of the existing literature on quantum computing, specifically in the context of portfolio optimization. Additionally, a document analysis based on up-to-date corporate whitepapers is performed. By synthesizing these insights from academic and corporate sources, this research offers a clear overview of the current knowledge on the subject of portfolio optimization and quantum computing,

2 LITERATURE REVIEW

The following literature review gives insight into the components of quantum computing that are valuable towards this research, along with the current theoretical framework regarding quantum portfolio optimization.

2.1 Quantum computing theory

Classical computing works through bits in a binary format, these bits can have two possible values, of which are either '0' or '1' [109]. These bits are the smallest notation in which data is stored on a computer and are often represented by a certain value such as 'true/false' or 'yes/no' [108]. In classical computing, a bit can only be in one of the two states at a time [108]. Quantum computing works through 'qubits', which are bits that exist in a superposition of both '0' and '1' until they are observed [109, 110]. Following will be the most important subjects discussed.

Superposition and qubits

Quantum computing is represented by qubits, which are bits that can be present in different states at the same time, this state is called superposition [85, 86]. However, the moment this state is measured, it will shift towards a definite, observable state of either '0' or '1'. A visual representation of how superposition works, and how qubits can be represented may help to give insight, figure 1 illustrates a simplified version of this.

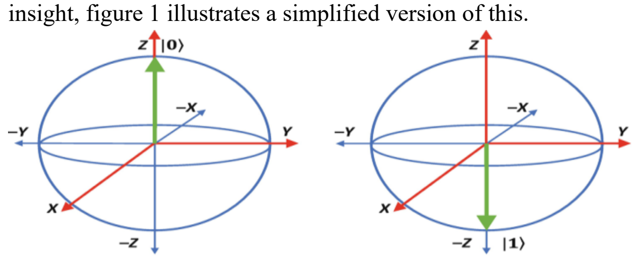

Figure 1, representation of qubit positions in a Bloch sphere when observed [70]

Figure 1 is a representation of qubit positions on a Bloch sphere. Following the green arrow, the two possible positions of an observed qubit are characterized by the state of '1' or '0'. For actual superposition, it must be envisioned that the green arrow is pointing in a direction that is not aligned with either '1' or '0'.

Quantum entanglement
Quantum entanglement is a key subject enabling the exploration of multiple solutions simultaneously. Quantum entanglement is when two or more qubits are placed in entangled states [109, 110], meaning that despite the qubits being physically separated, they will still influence the outcome of measurements performed on each other [109, 110]. When measuring these entangled qubits, there will always be a correlation between the outcomes that they give [23], such a correlation can be depicted by an entangled pair of qubits. Qubit entanglement among other factors enables the exploitation of quantum operations to increase the probability of desired outcomes and decreasing undesired ones [109]. Figure 2 shows a representation of how entanglement can be interpreted in a simpler format.

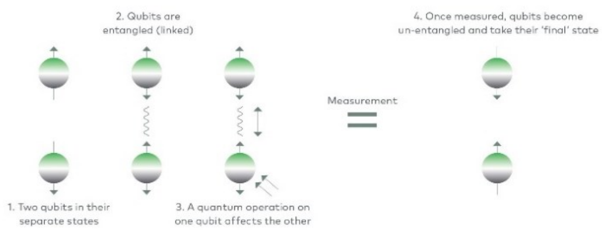

Figure 2, entangled qubits [19]

Quantum decoherence
For quantum computing systems to work properly, they should be isolated from any outside interference [86]. If any outside factor interferes with the qubits, then the state of the qubit can collapse [86]. Examples of such interferences can be small changes in temperature, stray electric or magnetic fields [109].

In general, the measurement of qubits is probabilistic [96], meaning that multiple measurements have to be done over time to achieve a more desired output [96], where this is generally the highest average of the results given from the outputs *(e.g. results with the highest chance of occurrence)*. Furthermore, to achieve these results the state of the quantum system is often manipulated in such a way that the desired result has the highest likelihood of occurring [96, 95], this is further mentioned in 2.3

2.2 Insights into portfolio optimization
Portfolio optimization constitutes the act of maximizing gains while minimizing risk [79]. A financial portfolio is characterized by a collection of investments in assets such as stocks, bonds, commodities, cash, and ETFs [117]. The objective of the investor is different when observing the initial objectives [57], where the amount of risk an investor should take is related towards the degree of potential gains, this trade-off should be favorable to undertake an investment. One of the cornerstones of portfolio optimization is 'modern-portfolio theory' (MPT) [84], developed by Harry Markowitz [84], with the aim of creating an efficient portfolio that maximizes gains and minimizes risk [84]. The ideal trade-off between risk and reward can be visualized on a graph called the 'efficient frontier', see figure 3. Many factors can influence expected risk and return, these influences often appear in the form of added variables or constraints in the calculation of most efficient portfolios *(e.g. budget constraints, investor preferences, regulatory requirements, liquidity needs).* Both in classic and quantum computing methods, these variables and constraints are each integrated in models/algorithms adapted for the computing methods, this is further explained in 2.3. The capital market line (CML) represents portfolios that optimize the risk and return relationship, defined by the 'Sharpe ratio' and risk-free rate [46]. The Sharpe-ratio is a measure of risk adjusted return, mostly used as a performance measure for optimization models [61, 46]. The formula for the Sharpe-ratio is as follows:

$$\text{Sharpe-ratio} = (Rm - Rf) / \sigma p \qquad (2)$$

Where Rm is the expected return of a portfolio based on the market, Rf is the risk-free rate, and σp is the standard deviation of returns of the portfolio [8].

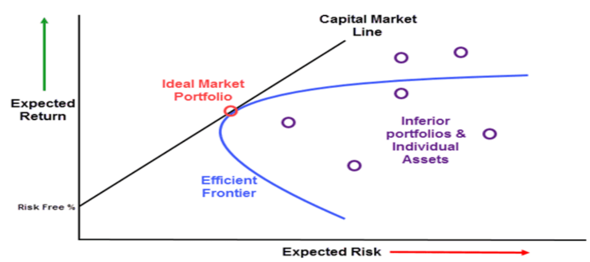

Figure 3, Efficient frontier example [15]

Approaches toward portfolios are mainly determined by pre-defined objectives [1], these objectives are reflected in the asset mix and risk and reward trade-off over a predetermined amount of time [1]. Dependent on these objectives, the efficient frontier changes accordingly. Portfolio optimization can be approached in multiple ways, classical approaches, and intelligent approaches [51]. Classical approaches are based on traditional financial theories such as MPT, or Capital Asset Pricing Model (CAPM). 'Intelligent approaches' are characterized by their machine learning capabilities and ability to learn from historical data [51]. These intelligent approaches mainly include Bayesian, support vector machine, neural network, reinforcement learning, and evolutionary-based approaches [51]. For quantum computing, most often it is observed that classical and intelligent functions are altered in a way to fit certain quantum algorithms.

2.3 Quantum Portfolio Optimization Methods
The main goals of applying quantum mechanics towards the use of optimization problems is the greater speed and accuracy it can provide [57]. Portfolio optimization's main function should be to construct a portfolio of assets that maximizes returns and minimizes risk [79]. The next part only gives insight into the broader quantum methods to lay the foundation of what is to be specified in part 4 'findings'.

Quantum hardware for finance
Solving quantum computational problems is facilitated through the use of quantum hardware [5, 57], where this hardware enables the solving of quantum problems not feasible on 'classical' hardware [5]. Quantum hardware mainly consists of two recognized types: gate-based quantum computers, and quantum annealers [5]. Quantum simulators can also be seen as a way to model the behavior of quantum systems [57], which is the simulation of quantum hardware on a classical computer [57], mostly used to theorize future quantum hardware possibilities in problem solving methods [57]. Current quantum hardware is also called 'noisy intermediate-scale quantum' (NISQ) devices, this characterizes the fact that current quantum hardware is still underpowered and prone to errors [57].

Quantum annealers are mostly used for optimization problems [5], which work through leveraging quantum mechanics principles to solve certain problems [40, 75]. The annealing process involves qubits in a superposition, which are influenced via biases (e.g. magnetic forces) and couplers to achieve different probabilities of finding a certain state of the qubit(s), either in the '0' or '1'state [40, 75]. Couplers serve the purpose of creating interaction, or entanglement, between qubits so that desired outcomes are achievable [40. 75]. In short, quantum annealers gradually change the form of a particle from its initial state to fit a desired functional form [96], this desired form in a quantum annealer is either a minimum or maximum state and therefore also the solution to the problem statement *(think of min/max size/cost/distance or risk from a set of solutions.)*

Gate-based quantum computers have many different physical realizations [96], however, they all work according to the same fundamental principles. A gate-based quantum computer can be depicted as: *"quantum computers that operate using qubits in a superposition state, manipulated by quantum gates to perform specific computations for a desired classical result, where error correction techniques ensure greater reliability of results"* [96, 5, 57]. Gates in classical computers are switches that at discrete time intervals generate a pulse of electricity corresponding with either '0' or '1' [96]. Quantum gates are an extension on this principle, where they are physical devices made out of some material that manipulate the quantum state of qubits [96].

On these quantum hardware, certain mathematical and computational models are applied, each differing in their objective function and problem formulation. Models such as QUBO or the Ising model *(for a quantum annealer)* are often taken as the base and adapted upon to fit certain algorithms to optimize a variety of problems [96, 95], where problems for gate-based quantum computing are often reformulated to fit certain developed types of quantum gates, and differing numbers of qubits to best fit an objective function [96].

Quantum algorithms
Quantum algorithms are specialized algorithms that run on quantum computers [41]. Quantum algorithms form the basis of quantum computing applications, where algorithms are adapted and tailored to find solution for specific problems, from optimization to machine learning and Monte Carlo [57]. Considering quantum algorithms, there are countless to name, each having their specific application towards certain problems. When analyzing the literature available, many reports either took inspiration from foundational algorithms/models and adapted upon them to fit specific problems or found ways to optimize existing quantum algorithms/models. Most commonly, foundational algorithms such as QUBO, the Ising Model, Grover's algorithm, Shor's algorithm, or Harrows-Hassidim-Lloyd (HHL) algorithm, to name a few, are taken and made to fit certain methodologies and problems *(e.g. optimization for portfolio risk or Monte Carlo for derivative pricing)* [5, 57].

Machine learning
Quantum machine learning is a certain methodology that makes use of quantum algorithms to enhance traditional machine learning techniques to be used for things such as classification, clustering, regression, quantum neural networks, reinforced learning, generative models, dimensionality reduction, and other novel uses [57, 101]. As for portfolio optimization/finance, quantum machine learning has its potential use in big datasets for anomaly/fraud detection, asset pricing, financial forecasting, credit scoring, stock selection, and metrics that capture a market's forecast of likely movement [57, 101]

Stochastic modeling (Monte Carlo)
Stochastic modeling tries to find the probability of various outcomes under different conditions using random variables [57, 72]. A key characteristic that makes stochastic modeling separate is that it inherently incorporates uncertainty into the analysis *(which is often characterized by the term 'fuzzy' in literature)* [72]. In the realm of quantum stochastic modeling for finance, quantum algorithms are often related towards a Monte Carlo type integration (MCI) [57, 5], where sampling from a probability distribution is traditionally utilized to approximate solutions for a desired problem statement [5]. Problem statements in stochastic modeling are found in the form of estimations of probabilities or expectations *(e.g. estimation of risk measures, pricing of derivatives, or expected payoff of a financial derivative at a future time)* [5, 41, 57]. In quantum Monte Carlo Integration (QMC), a quantum speedup is most often achieved through the use of the Quantum Amplitude Estimation algorithm (QAE) [57], an algorithm that aims to estimate the probability of a specific outcome in a quantum system. Compared to classical MCI, where samples are considered as classical queries [57], and thus the key to giving a desired result, QMC using QAE requires significantly less queries to achieve a result, thereby embodying a quantum speedup in theory [57]. Even though, the use of QAE for QMC

is most often considered, other algorithms for QMC exist. Examples of quantum algorithms used for Monte Carlo in finance are HHL, qPCA, QPA, and QPE [5].

Quantum Optimization

Optimization is the most prevalent methodology in quantum computing for finance. Actual problem statements can be distinguished between two different general groups [57]. NP-hard problems are seen as problems that are currently not solvable efficiently [57, 1], and therefore present a great challenge for both classical and quantum hardware, where quantum hardware is able to tackle NP-hard problems more efficiently than classical algorithms, it still cannot solve it most efficiently [57, 1]. Besides that, there are problems that are not NP-hard, which can be solved efficiently and have a great body of literature encompassing how to solve them efficiently [57]. Ultimately, NP-hard problems are not specific to optimization problems but can also be formed for other methodologies.

Types of quantum optimization problems can also be grouped in broad terms; three main groups can be recognized. 'Combinatorial optimization' is *"the act of trying to find the combination of values of variables that optimizes an index from among many other options"*, often using discrete or integer optimization for quantum algorithms [57]. Next to that, (non) convex optimization problems encompass *"the process of minimizing a convex objective function subject to convex constraints"* [87], where the minimum of this function conveys the desired result for the problem [87]. Lastly, Large-scale optimization problems are characterized by a significant number of variables and constraints that currently may prove to be too hard to solve for NISQ hardware [57], where it is suggested that to compensate for this lack of computing power, a hybrid between classical and quantum computing is to be realized [57], where the problem is to be subdivided into subproblems that are either solved/optimized on a quantum computer and classical computer [57], multiple reports exist on this hybrid between quantum and classical computing for optimization.

Financial application for quantum optimization algorithms mainly includes portfolio optimization, swap netting *(financial consolidation of payments or obligations to reduce risk and create better operational efficiency [55]),* predicting financial crashes, identifying creditworthiness, optimal arbitrage *(buying and selling financial assets in different markets for a profit)* [57]. Most common algorithms for quantum optimization problems and quantum portfolio optimization include quantum annealing, QAOA, VQE, VarQITE, QTS, QUBO, QIPM, HHL, and other novel variations of these algorithms.

The next part explains the results found in the initial literature search, with its process being explained in part 3 'methodology', consisting of a comprehensive overview of quantum portfolio optimization methods from academic literature.

2.4 Quantum computing in finance review

As mentioned in part 2.3, quantum computing follows certain objective functions, algorithms, in certain methodologies, on quantum hardware. To visualize this process, figure 6 *(see appendix),* inspired by Alabereti et al (2022) shows this process. In the next part, a total of 57 papers, that were summarized for use in table 7, are analyzed and taken as a representative sample for current views on Quantum Computing in finance, specifically portfolio optimization.

Algorithms used

To first put into perspective usage of quantum algorithms *(Full-quantum algorithms, heuristics, metaheuristics)* for PO problems, chart 1 is made.

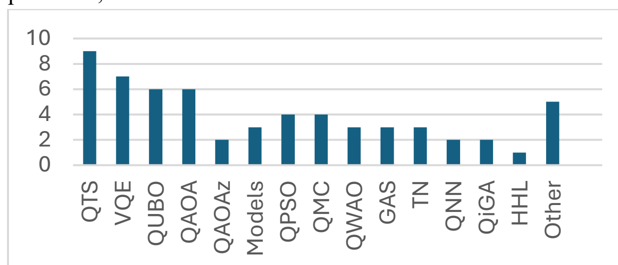

Chart 1, Frequency of Quantum-algorithms used

In most of the literature, base algorithms such as VQE, or QTS were improved upon via certain proposed methods *(e.g. parameter optimization, or optimization of the classical part of the algorithm, as it is a metaheuristic)*. Some papers did not use any algorithms for problem solving as they were surveys or literature reviews. Lastly, 'models' signify instances where solving a problem involves a conglomerate of methods put together into one to solve a particular problem *(e.g. the use of DDQCL on QCBMs model [6])*

Furthermore, in more than half of the papers, QUBO is used as a 'format' to both formulate certain problems and as a solver, this dual purpose can understandably create some confusion, QUBO can only be applied to combinatorial problem classes. Next to that, QAOAz is the successor of QAOA, which is found in more recent papers as it offers greater flexibility and exploration of the solution space. Additionally, QTS showed predominant use, this was mainly because different works in this literature pool sought to improve on other works that used QTS. Lastly, certain algorithms are also often used to optimize certain sub-parts of a calculation *(e.g. the use of VQE for parameter optimization, or the use of VQE to generate an optimized asset pool for a PO problem).* Results showed that the use of this method provided better and more efficient results on average.

In multiple papers, quantum algorithms were put to the test against classical algorithms, where in the remaining they were put to the test against other quantum algorithms. For the sake of putting into perspective quantum speedup, a comparison against classical methods is a pre-requisite. As [49] mentions for one of the pre-requisites to fully assess quantum speedup, "*The quantum algorithm should have a plausible case for asymptotic quantum speedup",* indicating that a comparison between classical and quantum is a need to estimate practicality. Classical algorithms that were benchmarked against were predominantly; Brute-force, Genetic Algorithms, SMA [24], SRO [24], MVO [24], and the non-quantum counterparts of the algorithm *(e.g. PSO against QPSO [52]).*

Use of constraints and different problem sizes

It is natural to assume that conditions under which the optimal portfolio is formulated represent that of a real situation, therefore, the use of constraints and different problem sizes in the formulation of a PO problem is important, as this seeks to fill in the gap between theoretical and practical models. Furthermore, as investor preferences are different, certain constraints or changes to the formulation of the PO problem can be added. The greater part of the papers in this review incorporate the use of different constraints to achieve a higher degree of practicality, however, this is often at the cost of added complexity to solving the problem, thereby necessitating more computational resources.

In the case of the 57 reviewed papers, as problem sizes increased, the performance and accuracy of results of quantum algorithms increased overall [6, 41, 64, 92]. Some papers mentioned a decreasing trend in the ability to solve larger problem sizes [81] this may have been due to increased noise, error rates, and qubit connectivity of current NISQ devices in this paper, thereby also stressing the importance of error and noise reduction methods in current NISQ devices.

As for constraints, it was perceived that as more constraints were added for better representativeness to real-world situations, results tended to be closer to optimal for the objective function [92, 74, 82, 88, 104, 111]. However, added constraints were proven to be cause for additional computational power needed, thereby also increasing solving times slightly [37]. Sometimes constraints were neglected by the algorithm to find more adequate results [74, 88, 78], this can mainly be traced back to soft-constraints being applied instead of hard-constraints., meaning that solvers are allowed some tolerance in adhering to set constraints, and thereby given more room in the search space. Hard constrained optimizers are easier to optimize as their landscape is easier to quantify and has more direct parameters, therefore creating a straighter road to the solution so to say, whilst soft constrained optimizers have a more challenging landscape due to their increased flexibility, allowing for a broader range of possible solutions [14].

Quantum versus Classical performances

A couple of preliminary things ought to be mentioned. First of all, finding an optimal solution to an objective function does not directly imply better performances, as both methods may have found the optimal solution. It is only when the problem instances grow to a size or format *(e.g.in non-convex optimization problems [88, 25])*, where it is infeasible for classical methods to solve, that measures in optimality of solutions are relevant. In situations where both methods should be able to find the optimal solution, the two most looked at measures are that of 'time-to-solution' and whether the method can actually find that optimal solution. Furthermore, there are some instances where the optimal solution is not known. In such a situation, benchmarks are performed by comparing results of each method against each other, or against a baseline solution that is known to be 'good'.

Lastly, it is very important to mention the difference between tests performed on simulated/digital and real quantum hardware, where simulated/digital environments allow researchers to test algorithms and obtain theoretical performance measures in environments without most of the constraints of NISQ hardware *(e.g. noise, errors, decoherence, qubit limitations, gate limitations, qubit connectivity, to name a few),* it tries to simulate a close to idealized environment for potential performances of future realized and fully working quantum computers, as current quantum devices cannot perform on that level yet. However, simulations are performed on classical devices, thereby still being limited in their computational abilities. Nevertheless, in the 57 papers, some experiments are done on real-quantum hardware, but in general, simulated/digital hardware is used for benchmarking.

The following charts will give a good representation of NISQ, Classical, and simulated/digital performances against each other *(where they show percentages of which method showed better performances than the one that is compared with),* indicating which method is better 40/57 available papers are used.

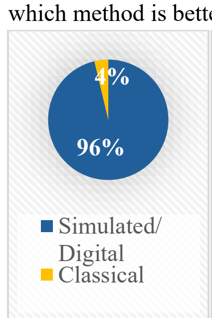

Chart 2: Simulated/Digital vs Classical N = 27

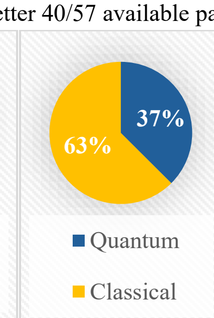

Chart 3: Quantum vs Classical N = 8

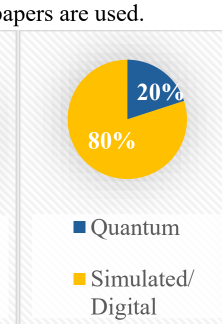

Chart 4: Quantum vs Simulated /Digital N = 5

17 papers could not be used for comparisons due to multiple reasons; some papers only acknowledged simulated versus simulated results. Furthermore, some only mentioned performance benchmarking against previous works that were further build upon, only benchmarking against the previous iteration of the paper. Lastly, several papers either reported

similar performances across methods, remained neutral, or were unclear about the differences between them.

Looking at the charts, simulated hardware outperforms classical methods 96% of the time, where the only outlier mentioned that the classical method *(Frieze-Kannan-Vempala)* outperformed the simulated hardware, where the proposed model was not well-suited for the quantum method due to its reliance on high-rank and high-condition number matrices, which led to poorer performance compared to classical methods like FKV, showing in the numerical results from the test *(high error rates, high noise, longer time-to-solve).*

Furthermore, the current limitations of real NISQ hardware can be traced back into the poor performances mentioned in most of the papers that utilize them. With only 37% and 20% of used papers linking better performances to real quantum devices. In the greater part of these instances, the only better performances were perceived via the most recent devices on the market, which are IonQ's Trapped-Ion Device 'Aria-1' [8], 'D-Wave 2000Q' [91, 119, 120] and 'D-Wave Advantage' [120]. However, problem sizes were limited due to the increased noise and error rates occurring in NISQ devices. Chart 2 gives a great indication in regard to a future outlook on the use of real quantum devices. For a detailed view into the results found in the above paragraph, see table 7 and Table 8 in the appendix.

Challenges and limitations
As the name 'NISQ' suggests, current quantum computers perceive multiple challenges and limitations. Looking at the studied papers, a couple of things can be said on this topic.

Noise and errors in simulated devices
As the use of simulated devices aims to show the full potential of quantum computing, nevertheless, there are still papers considering the simulated implementation of noise and error to test their mitigation methods on. These studies investigate a more 'realistic' scenario, where the inherent challenges of NISQ hardware are put to test using various error mitigation strategies.

Error, Noise, local minima/maximums, resource requirements.
One of the main issues addressed was the importance of error mitigation techniques, as multiple papers found that the quantum algorithms used were prone to errors, which could be due to a multitude of reasons *(e.g. Hamiltonian simulation error, or higher errors perceived due to increased distances between qubit connections [16, 22])*, they suggested or implemented the use of error mitigation techniques to solve this issue [11, 72, 77]. Results using error mitigation techniques showed great improvements in error rates, and thereby superior solution quality and efficiency of the computational processes [29, 51, 80]. However, error mitigation techniques were proven to be cause for additional computational overhead [41, 52]. Real-quantum hardware was found to be significantly more prone to error and noise.

Another difficult hurdle to overcome was the convergence of the algorithms to local optima. As most of the used problem types *(e.g. non-convex and combinatorial problems)* are cause for there to be many suboptimal solutions, the algorithms were prone to finding these suboptimal solutions and become stuck, thereby not recognizing the global optimal solution [18, 29, 82, 111], or for the algorithm to recognize it and move away from it. Multiple papers introduced measures that helped the algorithms to avoid these local solutions [18, 66, 74, 76, 82, 94]

Considering resource requirements, there was a relation seen between the complexity of the problem and the computational resources needed. However, it was mentioned that as complexity increased for classical methods, their time-to-solve would grow exponentially [24], whereas quantum methods showed a linear trend in increased complexity time-to-solve [24].

3 METHODOLOGY AND RESEARCH DESIGN

3.1 Research protocol and data gathering

The methodology part pertains information on exactly how the main research question is answered. Considering the current structure and layout of the research paper, a systematic literature review was chosen. A systematic literature review is characterized by its nature to identify, select, and critically appraise papers to be able to answer formulated research questions [26]. This research is meant to give perspective on the current, and of best quality, literature.

One important factor in a systematic literature review is bias, more specifically the lack of a bias. As systematic reviews and meta-analyses are susceptible to a multitude of biases, this ought to be minimized [39]. This research will follow the PRISMA 2020 flow diagram to ensure that up to date, unbiased, and high-quality articles are chosen. The PRISMA flow diagram aims to enhance the transparency and reproducibility of systematic reviews. It assists in finding quality papers by going through a process/flow chart that gives a predefined protocol.
Three databases are used to synthesize the primary and final pool of sources after they have gone through the process of screening and selection. These databases are the 'Scopus database' the 'Web of Science', and the 'ArXiv' database.

There are many papers discussing quantum computing, and many papers discussing portfolio optimization, however, the link between these two is found by searching for certain keywords in the databases of Scopus and Web of Science and ArXiv. Before the first search of literature, keywords had to be identified, after searching through the results these keywords gave, a secondary search for new terms based upon these results was issued. Table 1 in the appendix shows the formed keywords.

These keywords on their own will result in too broad of a search, therefore combinations of these keywords are searched for in a Boolean manner. A Boolean approach uses logical operators such as AND, OR, NOT. By using these logical operators certain keywords can be put together more effectively. Furthermore, truncation symbols may be used to get broader results when needed, where truncation symbols ensure that all variations of a word can be looked for *(e.g. comput* can mean "computing", or "computer", or "computation" etcetera).* The combinations used both on Scopus and Web of Science can be seen in 'table 2' in the appendix, they were not used on Arxiv.

3.2 Searching for relevant studies, initial search

Following the Prisma 2020 flow chart, certain inclusion and exclusion criteria need to be stated. Particular search filters can be applied to find more relevant papers. First of all, considering the Gartner hype cycle for data security measures, specifically on quantum computing, it appeared first on the model in 2011 with a mainstream adoption expectation of more than ten years [62]. For the 2023 Gartner model, the expected plateau will be in two to five years [100]. Next to that, around the year 2011 was when the first commercial quantum processors went mainstream and could be tested on [21]. Furthermore, this time marks the start of the physical process to quantum supremacy [21]. Therefore, research from before the year of 2011 will be filtered out during the performed searches and results from the time span of 2011-2024 will be used. However, in the end, all papers *(except one outlier)* used in both the searches surprisingly proved to be from the period 2018-2024 as substantially more papers were uploaded in that period on this topic. Besides, the papers before 2018 were ultimately filtered out due to full-text analyses showing they all were irrelevant. Furthermore, the language in which papers will be searched is 'English'. The tables showing the inclusion and exclusion criteria can be found in the appendix. abstract, title, and full-text screening was performed after literature was collected, leaving 57 papers to be used. Following this rigorous selection process, the PRISMA 2020 flow diagram is shown in the appendix *(figure 4).*

Subsequently, these findings are uploaded, summarized and classified into different groups in the Endnote X9 software. Furthermore, as some papers in the final pool of literature are considered white papers, they will be added to the final pool of the 'white paper literature search', only if they are not cause for duplicate papers in that pool. Lastly, some papers were ultimately not used as they were either predecessors of other works, showed limited use in furthering the scope/quality of the thesis, or ultimately proved to be non-relevant to this thesis. Ultimately, the most important used papers were synthesized into a matrix (Table 7) to create a clear overview.

3.3 Use of corporate white papers

White papers are used to ensure the inclusion of practical, up-to-date, and real-world insights into current industry applications of quantum computing for portfolio optimization.

The following steps were taken in the research. First, a layout of current companies and start-ups working on quantum computing for the finance industry was mapped out. Subsequently, websites of these corporations were analyzed, as they contain papers that are valuable to gather insights from. After an initial pool is collected and uploaded to the Endnote X9 software, they were included or excluded based upon the named criteria in part 3.2, criteria that does not apply to these papers are not used. Additionally, a final search is done on the databases of ArXiv, IEEE Xplore, and online libraries to gather additional papers, as ArXiv and IEEE Xplore are great options to find white papers from companies. Lastly, as some search inquiries from the first systematic literature review included some white papers, those that are no duplicates will be added to the final pool of the white paper research. To fully map out this process, a second PRISMA 2020 flow diagram was made, however, this one is altered to better fit this kind of search, see appendix *(figure 5).*

To map out companies and startups in the field of quantum computing, resources such as 'The Quantum Insider', 'Quantum Computing Report' (QCR), and 'The Quantum Economic Development Consortium' (QEDC) were used. Furthermore, as some financial companies are not directly related towards quantum computing, but do take effort in research on the subject, additional searches are done on these companies on various financial outlets and other sources. After that, the companies are screened based on whether they convey any valuable information regarding quantum computing and finance, those that do not are excluded from the final pool, the remaining amount are further researched, see figure 5.

4 White paper findings

Table 6 and 9 in the appendix give a full overview of papers used and their contents in this following part. Next, a total of 25 white papers are analyzed and taken as a representative sample.

Algorithms used

To first put into perspective usage of quantum algorithms *(Full-quantum algorithms, heuristics, metaheuristics)* for PO problems in the 25 whitepapers, chart 5 is made.

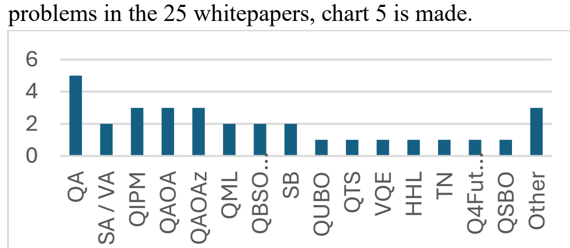

Chart 5, Frequency of Quantum-algorithms used

As can be deterred from chart 5, Quantum Annealing (QA) is the most used method, a stark difference as compared to that of the initial literature search. In the use of the whitepapers, quantum annealing is mostly specialized under D-Wave's devices *(including QBSOLV)*, as they have pioneered and extensively commercialized this approach, thereby signifying the companies' prevalence in this industry. Both D-Wave and collaborating companies experiment on D-Wave's devices in multiple whitepapers. Furthermore, a noticeable difference with the initial 57 papers, is the near total absence of Quantum Tabu Search (QTS), and the Variational Quantum Eigensolver (VQE). This may have been due to the VQE's primary use in gate-based quantum computing, of there is significantly less whitepapers on due to its specialized applicability, less 'practical' use in NISQ hardware, and the abundance of whitepapers experimenting on Quantum Annealers. As for QTS, it is a more recent algorithm, could be overshadowed by more 'practical' approaches, and may have had an unreasonable representation in the first literature search *(as was mentioned there)*.

Use of constraints and different problem sizes

Looking at the use of constraints, it can be said that the findings are mostly in line with those of the first literature search, showing that as more constraints were added, performances in regard to practical usage increased, or were generally very positive [2, 25, 83, 98, 99]. It was perceived that as more constraints were added, that computational resources needed also increased [25, 27]. In the initial literature review it was found that some models did not adhere to set constraints, two instances were found where this was the case in the whitepapers, this was for a real NISQ device, and D-Wave QBSOLV *(simulated solver)* [2, 43]. The possibility of constraints not being adhered to was also questioned and tested in some additional whitepapers [59, 99]. Findings that were contrary of those in the first literature search were sparse, however, two whitepapers managed to find opposing results. In the first, it was found that as less constraints were added, that only then quantum advantage showed over classical solutions [73]. As constraints are often a result of investor preferences, disobeying these constraints may have led to 'better performances', but not in the eyes of the investor. Furthermore, in one paper it was found that hard constraints performed better in the same model than soft constraints, thereby contradicting the findings in the first literature search. The reason for this contradiction may have been due to the initial paper in that search not adequately incorporating hard constraints *(as this often proves to be difficult),* which in the case of the white paper was done, where a method to better incorporate hard constraints was performed.

As for problem sizes, the findings were the same as the initial literature review, where performance of the quantum devices, mostly theoretically on simulations, showed to increase performances overall [98, 83, 73, 114]. Furthermore, computational resources needed were also found to increase as problem sizes increased [114]. One paper did find contrary findings to those in the initial literature research, where this paper mentioned that quantum annealing struggled with larger problem sizes, as it was difficult to embed larger problem sizes into the system [36]. However, the device that the problem size was scaled on was the physical D-Wave Advantage, a NISQ device. Whereas real NISQ devices still show issues regarding larger problem sizes, this result was natural for them to find.

Quantum versus Classical performances

The importance of the division between tests performed via simulated/digital devices and real NISQ devices has to be stressed. Where NISQ devices still show varying limitations and challenges in their computational abilities *(e.g. noise, errors, decoherence, qubit limitations, gate limitations, qubit connectivity, to name a few)*, and simulated/digital devices aim to produce a more idealized/theorized environment of testing. The following charts give a representation of NISQ, Classical and simulated/digital performances against each other.

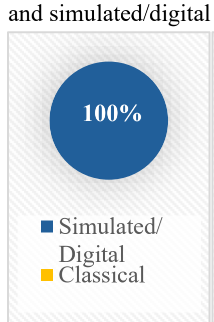

Chart 6: Simulated/Digital vs Classical N = 15

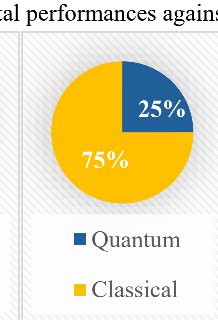

Chart 7: Quantum vs Classical N = 8

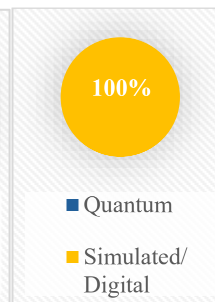

Chart 8: Quantum vs Simulated /Digital N = 4

5 papers could not be used for comparison as they included either simulated versus simulated results, and papers that where either unclear on their standpoint, showed similar performances between methods, or were indifferent. Looking at the charts, a lot of interesting conclusions and comparisons can be made. Firstly, the dominance of simulated/digital quantum methods compared to classical ones were shown in chart 6, with simulated methods clearly being superior to classical ones. Experiments performed in the whitepapers showed that simulated methods had greater efficiency, time-to-solve, error rates, practical implementation, and quality of solutions. It is clear that the if the future of quantum computing follows this given, theoretical, outlook, it would mean substantial advancements in optimization and problem-solving capabilities.

As for the comparison between NISQ devices and simulated devices, the same trend followed in the initial literature search is perceived in the white papers: simulated/digital devices consistently outperformed NISQ devices. This outcome is to be expected, as simulated/digital devices can account for some of the current limitations of NISQ hardware.

A very noticeable difference in the comparison between quantum and classical methods was that in 25% of the instances, PO problem solving on real NISQ devices outperformed classical methods. This case was close to the same for the initial literature search, however, there it was mentioned that problem sizes were downsized to compensate for the lack of NISQ hardware to solve large problem sizes. However, in the case of the two papers that outperformed the classical methods, the objective problems and data pools were of more practical use. These two papers are amongst the most relevant in the benchmarking of current NISQ hardware. Nevertheless, they still did not show the full potential of quantum computing.

In the first paper, the D-Wave advantage 6.2 system was used [97], it has 5610 qubits, however, these cannot be used to their full potential due to the limitations in qubit connectivity, coherence times, embedding difficulty, and calibration difficulties, meaning that only a certain small amount of those 5610 qubits can be used close to their potential. Nevertheless, the quantum method performed on the D-Wave Advantage with the Q4FuturePOP algorithm showed better results than industry experts at Welzia Management Company were able to achieve [97]. The experiment performed involved the use of 53 daily values of different assets spanning over a period of 13 years, the dataset is split up in 6 different combinations of periods and asset counts, with periods ranging from 12 to 28 months [97]. The quantum method offered better solutions in more than half of the instances considering either risk measures or expected return measures [97]. Additional information is found in table 6.

The second paper considered the use of the IONQ's trapped ion device 'AQTION' for Quantum Monte Carlo compared to traditional Monte Carlo on 5 asset portfolios, with 1000-euro budgets, over a longer period, and for three different market scenarios *(stable, bearish, bullish)* [106]. The device showed better performances with QMC than traditional Monte Carlo in terms of error reduction and efficiency [106]. QMC had smaller estimation errors and provided more efficient and accurate means of estimating asset values under stable and bullish market conditions, as queries increased, the QMC achieved less errors compared to normal MC [106]. Quantum speedup was achieved according to the paper [106]. Nevertheless, in the multitude of papers from both the initial literature research, and the white paper research, it was found that current NISQ hardware still has multiple limitations, where better performances compared to classical methods are predominantly not linked to each other.

Challenges and limitations

Error, Noise, local minima/maximums, resource requirements

Multiple whitepapers acknowledged the importance of error mitigation methods [49, 50]. These whitepapers implemented error mitigation techniques *(e.g. a self-error reduction technique [49])* to try and show the practicality of it, and its use for more accurate results, which were achieved [49, 50]. Furthermore, it was found that error mitigation techniques were cause for additional computational overhead, thereby decreasing time-to-solution [50]. Unfortunately, no whitepapers were found to specifically operate without error reduction techniques.

As mentioned in 2.4; "a difficult hurdle to overcome was the convergence of the algorithms to local optima. As most of the used problem types *(e.g. non-convex and combinatorial problems)* are cause for there to be many suboptimal solutions". The same was the case for some of the whitepapers, where convergence to local optima was perceived [28, 27], however, it was mentioned in one of the papers that these local minima could easily be avoided through various methods [27]

Considering resource requirements, it follows the trend of the initial literature research, with a direct relation seen between the complexity of the problem and its inherent use of computational resources [28, 27, 45]. In one of the papers it was mentioned that computational resources needed for quantum computing can be anticipated as it follows a linear scheme, on the contrary, classical computing follows an exponential line in computational needs for larger problems [28]. Furthermore, as greater parameter precision was introduced to offer better precision values for more accurate/optimal results, it showed to be cause for greater computational overhead [27]. Next to that, it was found that increased repetitions of the quantum circuit resulted in a higher probability of finding the optimal solution, however, it is definite cause for additional computational overhead [45]. Lastly, one paper showed that the involvement of methods such as QCL enhanced QPE *(which were specific to the HHL algorithms used in that instance),* and qubit reset and reuse techniques offer more efficiency and thereby less computational overhead, signifying the potential, and the need, for these methods in current NISQ hardware [121]

5 Discussion

Conclusion

What was found in initial literature review was that the most used algorithms included the VQE, QAOA, and QTS. Furthermore, adding real-world constraints improved the accuracy of results, and the likeness to investor preferences. However, coming at the cost of added complexity and computational resources. NISQ devices showed limitations in solving the problems due to increased error rates and noise. Comparing quantum and classical methods showed that in most cases; simulated methods outperformed classical (96%) and quantum methods (80%) based on time-to-solution and accuracy of results. Classical methods outperformed real NISQ devices (63%). As for challenges and limitations, both simulated and quantum devices faced noise and error challenges. Furthermore, a general challenge for certain problem types was the convergence of the algorithm toward a local optimum, thereby disregarding global optima. Certain efforts such as error/noise mitigation methods showed to increase performances, but at the cost of complexity to the problem and additional computational resources needed, thereby resulting in higher time-to-solution.

In the whitepaper search it was found that most used algorithms were Quantum annealing and its variations such as SA, VA, and QBSOLV, along with moderate use of QAOA and QAOAz algorithms. The reason for this representation in whitepapers is because of D-Wave's prevalence via their own works and collaborations with other companies in the literature availlable. Gate-based quantum computers are of less frequency in whitepapers due to its specialized applicability, less 'practical' use cases in NISQ hardware, and overall smaller development compared to quantum annealers. Therefore, the almost complete absence of VQE can be attributed to these named reasons, as VQE is primarily used on gate-based quantum computers. Most papers considering constraints showed that adding more constraints improved the practical relevance and accuracy of results on quantum methods. However, it was also shown to be cause for additional computational resources needed, thereby increasing time-to-solution. There were also some instances where constraints used were not adhered to, this was the case for a real NISQ device, and D-Wave QBSOLV *(simulated solver).* Furthermore, Problem sizes were found to have a positive relation with the number of computational resources needed. Simulated methods showed superior performances as compared to classical and quantum methods on NISQ devices, with 100% of the papers used *(n = 15)* showing the superiority of simulated devices versus classical ones. As for the comparison between quantum and classical methods, 25% *(n = 2)* of the quantum methods showed improved performances over classical methods in practice. These papers were especially interesting as they utilized the most up-to-date quantum devices the industry currently has to offer *(D-Wave Advantage, and IonQ's trapped-ion device AQTION),* showing that for impressive datasets and problem sizes *(relative to what NISQ devices should be capable of performing),* the real quantum hardware outperformed classical solutions. Lastly, the whitepapers showed that the importance of error mitigation techniques was acknowledged, and whitepapers that implemented it showed more accurate results. Two papers recognized the convergence of used algorithms to local optima.

The comparison between the initial literature research and whitepaper search showed that both follow the same trends in; acknowledging the current limitations of NISQ hardware, as shown in both searches, where the general format regarding time-to-solve, performance, accuracy, followed simulation > classical > NISQ devices. Findings in this paper showed that academic literature and the experiments performed in those papers differ marginally from findings in the whitepapers. However, generally it can be assumed that there is a common trend followed in both types of literature. The current limitations of real NISQ-devices are highlighted, and it is shown that even though current NISQ-devices have their limitations, they could still offer some practical significance in finance. However, actual effective widespread application of quantum computers is not something that is likely to be realized in the near future. Hybrid devices may offer a middle ground during the development of real quantum-devices.

Practical applications

As far as practical applications go, this paper can be used for giving insight into current industry applications regarding the development level, use cases, and a more detailed view into the link between theoretical insights and current practical applications/company-findings on quantum computing for finance, specifically portfolio optimization. Furthermore, this paper can be used to give a clear view of benchmarked performances of quantum methods against each other and classical ones, along with current limitations and challenges regarding quantum devices, specifically NISQ devices. Next to that, industry trends in the use of certain algorithms are identified, along with an indication of current problem sizes able to be solved *(mostly mentioned in table 6 and 7).*

Theoretical implications

As for theoretical implications, this paper does not challenge existing theories, it rather tries to validate existing theories through comparing theoretical implementations, use cases, current industry applications, and company specific research from online databases and whitepapers.

Limitations

Publication bias is accounted for by performing two different literature searches. Limitations of this study include the small likelihood of the data pool used for both searches not being representative, however the chance of this being true is small as multiple measures have been taken during the gathering of the papers via the Prisma 2020 format to ensure reduced bias in the literature search. the only potential real source of bias that can be found is the misrepresentation in the actual prevalence of quantum annealers in the whitepaper literature. However, this can be justified to a degree by the efforts made from D-Wave to generate a lot of literature through their own research and collaborations made with other companies.

Future research

Suggested areas which a follow up paper could address is the use of added literature, as this paper includes limited, but high quality, number of papers, where multiple papers have been taken out of the final literature pool because of multiple valid reasons discussed in the PRISMA 2020 flow charts. Introducing additional search terms could bring to light more quality papers to the research Furthermore, an additional topic which could be further addressed and delved into in future research is the addition of more literature on real gate-based quantum computers addressing optimization problems in finance.

REFERENCES

1 Abbas, A., Ambainis, A., Augustino, B., Bärtschi, A., Buhrman, H., Coffrin, C., Cortiana, G., Dunjko, V., Egger, D. J., Elmegreen, B. G., Franco, N., Fratini, F., Fuller, B., Gacon, J., Gonciulea, C., Gribling, S., Gupta, S., Hadfield, S., Heese, R., Kircher, G., Kleinert, T., Koch, T., Korpas, G., Lenk, S., Marecek, J., Markov, V., Mazzola, G., Mensa, S., Mohseni, N., Nannicini, G., O'Meara, C., Peña Tapia, E., Pokutta, S., Proissl, M., Rebentrost, P., Sahin, E., Symons, B. C. B., Tornow, S., Valls, V., Woerner, S., Wolf-Bauwens, M. L., Yard, J., Yarkoni, S., Zechiel, D., Zhuk, S., & Zoufal, C. (2023). Quantum Optimization: Potential, Challenges, and the Path Forward. arXiv:2312.02279. Retrieved December 01, 2023, from https://ui.adsabs.harvard.edu/abs/2023arXiv231202279A

2 Aguilera, E., de Jong, J., Phillipson, F., Taamallah, S., & Vos, M. (2024). Multi-Objective Portfolio Optimization Using a Quantum Annealer. *Mathematics, 12*(9), 1291. https://www.mdpi.com/2227-7390/12/9/1291

3 Alaminos, D., Salas, M. B., & Fernández-Gámez, M. A. (2022, Aug). FORECASTING STOCK MARKET CRASHES VIA REAL-TIME RECESSION PROBABILITIES: A QUANTUM COMPUTING APPROACH. *Fractals-Complex Geometry Patterns and Scaling in Nature and Society, 30*(05), Article 2240162. https://doi.org/10.1142/s0218348x22401624

4 Alaminos, D., Salas, M. B., & Fernández-Gámez, M. A. (2023, Jun). Quantum Monte Carlo simulations for estimating FOREX markets: a speculative attacks experience. *Humanities & Social Sciences Communications, 10*(1), Article 353. https://doi.org/10.1057/s41599-023-01836-2

5 Albareti, F. D., Ankenbrand, T., Bieri, D., Hänggi, E., Lötscher, D., Stettler, S., & Schöngens, M. (2022). A Structured Survey of Quantum Computing for the Financial Industry. arXiv:2204.10026. Retrieved April 01, 2022, from https://ui.adsabs.harvard.edu/abs/2022arXiv220410026A

6 Alcazar, J., Leyton-Ortega, V., & Perdomo-Ortiz, A. (2020, Sep). Classical versus quantum models in machine learning: insights from a finance application. *Machine Learning-Science and Technology, 1*(3), Article 035003. https://doi.org/10.1088/2632-2153/ab9009

7 Alcazar, J., Vakili, M. G., Kalayci, C. B., & Perdomo-Ortiz, A. (2024, Mar). Enhancing combinatorial optimization with classical and quantum generative models. *Nature Communications, 15*(1), Article 2761. https://doi.org/10.1038/s41467-024-46959-5

8 Alessandroni, E., Ramos-Calderer, S., Roth, I., Traversi, E., & Aolita, L. (2023). Alleviating the quantum Big-M problem. arXiv:2307.10379. Retrieved July 01, 2023, from https://ui.adsabs.harvard.edu/abs/2023arXiv230710379A

9 Alkhafaji, M. A., Ameer, S. A., Alawadi, A. H., & Sharif, H. (2023). Quantum Chameleon Swarm with Fuzzy Decision Making Tool for Financial Risk Management. 6th Iraqi International Conference on Engineering Technology and its Applications, IICETA 2023,

10 Arcuri, M. C., Gandolfi, G., & Laurini, F. (2023, 2023/06/01). Robust portfolio optimization for banking foundations: a CVaR approach for asset allocation with mandatory constraints. *Central European Journal of Operations Research, 31*(2), 557-581. https://doi.org/10.1007/s10100-022-00821-5

11 Arrazola, J. M., Delgado, A., Bardhan, B. R., & Lloyd, S. (2020, Aug). Quantum-inspired algorithms in practice. *Quantum, 4*, Article 307. https://doi.org/10.22331/q-2020-08-13-307

12 Atkinson, D., & Peijnenburg, J. (2022). How Certain is Heisenberg's Uncertainty Principle? *HOPOS: The Journal of the International Society for the History of Philosophy of Science, 12*(1), 1-21. https://doi.org/10.1086/716930

13 Ayodele, M., Allmendinger, R., López-Ibáñez, M., & Parizy, M. (2022). A Study of Scalarisation Techniques for Multi-Objective QUBO Solving. arXiv:2210.11321. Retrieved October 01, 2022, from https://ui.adsabs.harvard.edu/abs/2022arXiv221011321A

14 Baker, J. S., & Radha, S. K. (2022). Wasserstein Solution Quality and the Quantum Approximate Optimization Algorithm: A Portfolio Optimization Case Study. arXiv:2202.06782. Retrieved February 01, 2022, from https://ui.adsabs.harvard.edu/abs/2022arXiv220206782B

15 Barde, N., Thakur, D., Bardapurkar, P., & Dalvi, S. (2011, 07/01). Consequences and Limitations of Conventional Computers and their Solutions through Quantum Computers. *Leonardo Electronic Journal of Practices and Technologies, 10*, 161-171.

16 Bärtschi, A., & Eidenbenz, S. (2020, Oct 12-16). Grover Mixers for QAOA: Shifting Complexity from Mixer Design to State Preparation. [Ieee international conference on quantum computing and engineering (qce20)]. IEEE International Conference on Quantum Computing and Engineering (QCE), Electr Network.

17 Bennett, T., & Wang, J. B. (2021). Quantum optimisation via maximally amplified states. arXiv:2111.00796. Retrieved November 01, 2021, from https://ui.adsabs.harvard.edu/abs/2021arXiv211100796B

18 Bouchti, A. E., Tribis, Y., Nahhal, T., & Okar, C. (2018). Forecasting financial risk using quantum neural networks. 2018 13th International Conference on Digital Information Management, ICDIM 2018,

19 Bouquet, T., Hmyene, M., Porcher, F., Pugliese, L., & Zeroual, J. (2021). Approximating Optimal Asset Allocations using Simulated Bifurcation. arXiv:2108.03092. Retrieved August 01, 2021, from https://ui.adsabs.harvard.edu/abs/2021arXiv210803092B

20 Bova, F. a. G., Avi and Melko, Roger. (February 2022). Quantum Economic Advantage. *NBER*. https://ssrn.com/abstract=4028340

21 BTQ. (2024). Quantum computing: a timeline. https://www.btq.com/blog/quantum-computing-a-timeline

22 Buonaiuto, G., Gargiulo, F., De Pietro, G., Esposito, M., & Pota, M. (2023, 2023/11/08). Best practices for portfolio optimization by quantum computing, experimented on real quantum devices. *Scientific Reports, 13*(1), 19434. https://doi.org/10.1038/s41598-023-45392-w

23 Caltech Science Exchange. (2024). *What is entanglement and why is it important?* Caltech. https://scienceexchange.caltech.edu/topics/quantum-science-explained/entanglement#:~:text=When%20two%20particles%2C%20such%20as,from%20the%20connection%20between%20particles.

24 Carrascal, G., Hernamperez, P., Botella, G., & Barrio, A. D. (2024). Backtesting Quantum Computing Algorithms for Portfolio Optimization [Article]. *IEEE Transactions on Quantum Engineering, 5*, 1-20, Article 3100220. https://doi.org/10.1109/TQE.2023.3337328

25 Certo, S., Dung Pham, A., & Beaulieu, D. (2022). Comparing Classical-Quantum Portfolio Optimization with Enhanced Constraints. arXiv:2203.04912. Retrieved March 01, 2022, from https://ui.adsabs.harvard.edu/abs/2022arXiv220304912C

26 Charles Sturt University. (2024). *Systematic literature review meaning*. Retrieved 30/07/2024 from https://libguides.csu.edu.au/review/Systematic

27 Chen, B., Wu, H., Yuan, H., Wu, L., & Li, X. (2023). Quasi-binary encoding based quantum alternating operator ansatz. arXiv:2304.06915. Retrieved April 01, 2023, from https://ui.adsabs.harvard.edu/abs/2023arXiv230406915C

28 Chen, C.-C., Chung, S.-L., & Goan, H.-S. (2023). Black-Litterman Portfolio Optimization with Noisy Intermediate-Scale Quantum Computers. arXiv:2312.00892. Retrieved December 01, 2023, from https://ui.adsabs.harvard.edu/abs/2023arXiv231200892C

29 Chimprang, N., & Tansuchat, R. (2022). An Application of Quantum Optimization with Fuzzy Inference System for Stock Index Futures Forecasting. In *Studies in Systems, Decision and Control* (Vol. 429, pp. 393-410). https://doi.org/10.1007/978-3-030-97273-8_27

30 Chou, Y. H., Jiang, Y. C., Hsu, Y. R., Kuo, S. Y., & Kuo, S. Y. (2022, Aug). A Weighted Portfolio Optimization Model Based on the Trend Ratio, Emotion Index, and ANGQTS. *Ieee Transactions on Emerging Topics in Computational Intelligence, 6*(4), 867-882. https://doi.org/10.1109/tetci.2021.3118041

31 Chou, Y. H., Jiang, Y. C., & Kuo, S. Y. (2021). Portfolio Optimization in Both Long and Short Selling Trading Using Trend Ratios and Quantum-Inspired Evolutionary Algorithms. *Ieee Access, 9*, 152115-152130. https://doi.org/10.1109/access.2021.3126652

32 Chou, Y. H., Kuo, S. Y., & Jiang, Y. C. (2019, Aug). A Novel Portfolio Optimization Model Based on Trend Ratio and Evolutionary Computation. *Ieee Transactions on Emerging Topics in Computational Intelligence, 3*(4), 337-350. https://doi.org/10.1109/tetci.2018.2868939

33 Cohen, J., & Alexander, C. (2020). Picking Efficient Portfolios from 3,171 US Common Stocks with New Quantum and Classical Solvers. arXiv:2011.01308. Retrieved October 01, 2020, from https://ui.adsabs.harvard.edu/abs/2020arXiv201101308C

34 Cohen, J., Khan, A., & Alexander, C. (2020). Portfolio Optimization of 40 Stocks Using the DWave Quantum Annealer. arXiv:2007.01430. Retrieved July 01, 2020, from https://ui.adsabs.harvard.edu/abs/2020arXiv200701430C

35 Cohen, J., Khan, A., & Alexander, C. (2020). Portfolio Optimization of 60 Stocks Using Classical and Quantum Algorithms. arXiv:2008.08669. Retrieved August 01, 2020, from https://ui.adsabs.harvard.edu/abs/2020arXiv200808669C

36 Dalzell, A. M., Clader, B. D., Salton, G., Berta, M., Lin, C. Y.-Y., Bader, D. A., Stamatopoulos, N., Schuetz, M. J. A., Brandão, F. G. S. L., Katzgraber, H. G., & Zeng, W. J. (2023, November 01, 2023). End-To-End Resource Analysis for Quantum Interior-Point Methods and Portfolio Optimization. *PRX Quantum, 4*, 040325. https://doi.org/10.1103/PRXQuantum.4.040325

37 Dalzell, A. M., McArdle, S., Berta, M., Bienias, P., Chen, C.-F., Gilyén, A., Hann, C. T., Kastoryano, M. J., Khabiboulline, E. T., Kubica, A., Salton, G., Wang, S., & Brandão, F. G. S. L. (2023). Quantum algorithms: A survey of applications and end-to-end complexities. arXiv:2310.03011. Retrieved October 01, 2023, from https://ui.adsabs.harvard.edu/abs/2023arXiv231003011D

38 de Pedro, L., Murillo, R., de Vergara, J. E. L., López-Buedo, S., & Gómez-Arribas, F. J. (2023, Oct). VaR Estimation with Quantum Computing Noise Correction Using Neural Networks. *Mathematics, 11*(20), Article 4355. https://doi.org/10.3390/math11204355

39 Drucker, A. M., Fleming, P., & Chan, A. W. (2016, Nov). Research Techniques Made Simple: Assessing Risk of Bias in Systematic Reviews. *J Invest Dermatol, 136*(11), e109-e114. https://doi.org/10.1016/j.jid.2016.08.021

40 D-Wave. (2024). *What is quantum annealing?* https://docs.dwavesys.com/docs/latest/c_gs_2.html

41 Egger, D. J., Gambella, C., Marecek, J., McFaddin, S., Mevissen, M., Raymond, R., Simonetto, A., Woerner, S., & Yndurain, E. (2020). Quantum Computing for Finance: State of the Art and Future Prospects. arXiv:2006.14510. Retrieved June 01, 2020, from https://ui.adsabs.harvard.edu/abs/2020arXiv200614510E

42 Egger, D. J., Gambella, C., Marecek, J., McFaddin, S., Mevissen, M., Raymond, R., Simonetto, A., Woerner, S., & Yndurain, E. (2020). Quantum Computing for Finance: State of the Art and Future Prospects. arXiv:2006.14510. Retrieved June 01, 2020, from https://ui.adsabs.harvard.edu/abs/2020arXiv200614510E

43 Elsokkary, N., Khan, F. S., La Torre, D., Humble, T., & Gottlieb, J. (2017). *Financial Portfolio Management using D-Wave Quantum Optimizer: The Case of Abu Dhabi Securities Exchange* Conference: 2017 IEEE High-performance Extreme Computing (HPEC) - Waltham, Massachusetts, United States of America - 9/12/2017 12:00:00 PM-, United States. https://www.osti.gov/biblio/1423041

44 Fernandes, L., Kulkarni, M., & Pande, M. B. (2023). A Systematic Literature Review of Classical and Quantum Machine Learning Approaches for Mutual Fund Portfolio Optimization. 2023 IEEE Pune Section International Conference, PuneCon 2023,

45 Gacon, J., Zoufal, C., & Woerner, S. (2020). Quantum-Enhanced Simulation-Based Optimization. Proceedings - IEEE International Conference on Quantum Computing and Engineering, QCE 2020,

46 Ganti, A. (2022, June 29, 2024). *Understanding capital market line*. https://www.investopedia.com/terms/c/cml.asp

47 Gilliam, A., Woerner, S., & Gonciulea, C. (2021, Apr). Grover Adaptive Search for Constrained Polynomial Binary Optimization. *Quantum, 5*, Article 428. https://doi.org/10.22331/q-2021-04-08-428

48 Giron, M., Korpas, G., Parvaiz, W., Malik, P., & Aspman, J. (2023). Approaching Collateral Optimization for NISQ and Quantum-Inspired Computing. arXiv:2305.16395. Retrieved May 01, 2023, from https://ui.adsabs.harvard.edu/abs/2023arXiv230516395G

49 Goldman Sachs, A. (2023). *A detailed end-to-end assessment of quantum algorithm for portfolio optimization*. https://aws.amazon.com/blogs/quantum-computing/a-detailed-end-to-end-assessment-of-a-quantum-algorithm-for-portfolio-optimization-released-by-goldman-sachs-and-aws/

50 Gomez Cadavid, A., Montalban, I., Dalal, A., Solano, E., & Hegade, N. N. (2023). Efficient DCQO Algorithm within the Impulse Regime for Portfolio Optimization. arXiv:2308.15475. Retrieved August 01, 2023, from https://ui.adsabs.harvard.edu/abs/2023arXiv230815475G

51 Gunjan, A., & Bhattacharyya, S. (2023, 2023/05/01). A brief review of portfolio optimization techniques. *Artificial Intelligence Review, 56*(5), 3847-3886. https://doi.org/10.1007/s10462-022-10273-7

52 Gunjan, A., & Bhattacharyya, S. (2024, 2024 Mar). Quantum-inspired meta-heuristic approaches for a constrained portfolio optimization problem. *Evolutionary Intelligence*. https://doi.org/10.1007/s12065-024-00929-4

53 Gunjan, A., Bhattacharyya, S., & Hassanien, A. E. (2023). Portfolio Optimization Using Quantum-Inspired Modified Genetic Algorithm. Smart Innovation, Systems and Technologies,

54 Guo, Z., Song, T., & Lin, G. (2024, 2024/07/09). Portfolio optimization based on quantum linear algorithm. *Physica Scripta, 99*(8), 085107. https://doi.org/10.1088/1402-4896/ad5c1d

55 Hargrave, M. (2024). *Netting: definition, how it works, types, benefits, and example*. https://www.investopedia.com/terms/n/netting.asp

56 He, G., & Lu, X. L. (2021, Jun). An improved QPSO algorithm and its application in fuzzy portfolio model with constraints. *Soft Computing, 25*(12), 7695-7706. https://doi.org/10.1007/s00500-021-05688-3

57 Herman, D., Googin, C., Liu, X., Galda, A., Safro, I., Sun, Y., Pistoia, M., & Alexeev, Y. (2022). A Survey of

Quantum Computing for Finance. arXiv:2201.02773. Retrieved January 01, 2022, from https://ui.adsabs.harvard.edu/abs/2022arXiv220102773H

58 Hodson, M., Ruck, B., Ong, H., Garvin, D., & Dulman, S. (2019). Portfolio rebalancing experiments using the Quantum Alternating Operator Ansatz. arXiv:1911.05296. Retrieved November 01, 2019, from https://ui.adsabs.harvard.edu/abs/2019arXiv191105296H

59 Hughes, A. G., Baker, J. S., & Radha, S. K. (2023). A Quantum-Inspired Binary Optimization Algorithm for Representative Selection. arXiv:2301.01836. Retrieved January 01, 2023, from https://ui.adsabs.harvard.edu/abs/2023arXiv230101836H

60 Huot, C., Kea, K., Kim, T.-K., & Han, Y. (2024). Empirical Analysis of Quantum Approximate Optimization Algorithm for Knapsack-based Financial Portfolio Optimization. arXiv:2402.07123. Retrieved February 01, 2024, from https://ui.adsabs.harvard.edu/abs/2024arXiv240207123H

61 J, Fernando. (2024). *Sharpe ratio: definition, formula, and examples*. https://www.investopedia.com/terms/s/sharperatio.asp

62 Jack Fenn, H. L. (2011). *Gartner hype cycle*. https://www.gartner.com/en/documents/1754719

63 Jain, N., & Girish Chandra, M. (2023). Exploring the synergistic potential of quantum annealing and gate model computing for portfolio optimization. arXiv:2305.01480. Retrieved May 01, 2023, from https://ui.adsabs.harvard.edu/abs/2023arXiv230501480J

64 Jain, N., Khandelwal, A., & Girish Chandra, M. (2023). Efficient and Flexible Annealer-Gate Hybrid Model for Solving Large-Scale Portfolio Optimization. Proceedings - 2023 IEEE International Conference on Quantum Computing and Engineering, QCE 2023,

65 Javier Alcazar, M. G. V., Can B. Kalayci,, and Alejandro Perdomo-Ortiz. (2022). *GEO: Enhancing Combinatorial Optimization with Classical and Quantum Generative Models*.

66 Jiang, Y. C., Cheam, X. J., Chen, C. Y., Kuo, S. Y., Chou, Y. H., & Ieee. (2018, Oct 07-10). A Novel Portfolio Optimization with Short Selling Using GNQTS and Trend Ratio.*IEEE International Conference on Systems Man and Cybernetics Conference Proceedings* [2018 ieee international conference on systems, man, and cybernetics (smc)]. IEEE International Conference on Systems, Man, and Cybernetics (SMC), IEEE Syst Man & Cybernet Soc, Miyazaki, JAPAN.

67 Jiang, Y. C., Lai, Y. T., Chen, P. C., Chang, Y. Y., Wu, K. M., Kuo, S. Y., Chou, Y. H., Kuo, S. Y., & Ieee. (2023, Jul 02-05). Quantum-inspired Computing: Entanglement-enhanced Technique for Short Portfolio in Global Markets.*IEEE International Conference on Nanotechnology* [2023 ieee 23rd international conference on nanotechnology, nano]. IEEE 23rd International Conference on Nanotechnology (NANO), Jeju, SOUTH KOREA.

68 Jiang, Y. C., Yeoh, W. L., Kuo, S. Y., Chou, Y. H., & Ieee. (2018, Oct 07-10). Portfolio Optimization considering Diversified Investment Methods using GNQTS and Trend Ratio.*IEEE International Conference on Systems Man and Cybernetics Conference Proceedings* [2018 ieee international conference on systems, man, and cybernetics (smc)]. IEEE International Conference on Systems, Man, and Cybernetics (SMC), IEEE Syst Man & Cybernet Soc, Miyazaki, JAPAN.

69 JP. Morgan. (2024). *Applied research JP Morgan*. https://www.jpmorgan.com/technology/applied-research

70 Kasirajan, V. (2021). The Quantum Superposition Principle and Bloch Sphere Representation. In V. Kasirajan (Ed.), *Fundamentals of Quantum Computing: Theory and Practice* (pp. 75-104). Springer International Publishing. https://doi.org/10.1007/978-3-030-63689-0_3

71 Kaushik, N., Raj, A., Srivastava, M., Ansari, M. S., Pushpalatha, M., Gayathri, M., Kavisankar, L., Deshpande, S., & Venkatraman, R. (2023). Financial Portfolio Optimization: A QAOA and VQE Formulation for Sharpe Ratio Maximization. Proceedings of the 2023 6th International Conference on Recent Trends in Advance Computing, ICRTAC 2023,

72 Kenton, W. (2021). *Stochastic modeling: definition, advantage, and who uses it*. https://www.investopedia.com/terms/s/stochastic-modeling.asp

73 Kerenidis, I., Prakash, A., & Szilágyi, D. (2019). Quantum Algorithms for Portfolio Optimization. arXiv:1908.08040. Retrieved August 01, 2019, from https://ui.adsabs.harvard.edu/abs/2019arXiv190808040K

74 Khan, A. T., Cao, X. W., Li, S., Hu, B., & Katsikis, V. N. (2021, May). Quantum beetle antennae search: a novel technique for the constrained portfolio optimization problem. *Science China-Information Sciences, 64*(5), Article 152204. https://doi.org/10.1007/s11432-020-2894-9

75 Krishnan, A. (2023). Quantum annealing for absolute beginners. https://medium.com/@anjanakrishnan3100/quantum-annealing-for-absolute-beginners-d38c51ac2b3f

76 Kuo, S. Y., Cheam, X. J., Jiang, Y. C., Lai, Y. T., Chang, K. N., Chou, Y. H., & Ieee. (2019, Oct 06-09). Portfolio Optimization Model using ANQTS with Trend Ratio on Quadratic Regression.*IEEE International Conference on Systems Man and Cybernetics Conference Proceedings* [2019 ieee international conference on systems, man and cybernetics (smc)]. IEEE International Conference on Systems, Man and Cybernetics (SMC), Bari, ITALY.

77 Kuo, S. Y., Lai, Y. T., Jiang, Y. C., Chang, M. H., Wu, K. M., Chen, P. C., Chang, Y. Y., Tong, Y. F., Chou, Y. H., & Acm. (2023, Jul 15-19). Entanglement Local Search-Assisted Quantum-Inspired Optimization for Portfolio Optimization in G20 Markets. [Proceedings of the 2023 genetic and evolutionary computation conference companion, gecco 2023 companion]. Genetic and Evolutionary Computation Conference (GECCO), Lisbon, PORTUGAL.

78 Lang, J., Zielinski, S., & Feld, S. (2022, Dec). Strategic Portfolio Optimization Using Simulated, Digital, and Quantum Annealing. *Applied Sciences-Basel, 12*(23), Article 12288. https://doi.org/10.3390/app122312288

79 Li, F. e. a. (2021). Portfolio optimization [Computational Optimization Open Textbook]. https://optimization.cbe.cornell.edu/index.php?title=Portfolio_optimization

80 Li, Q. H., Wu, H., Qian, W. Z., Li, X. Y., Zhu, Q. S., & Yang, S. (2022, Jul 15-20). Portfolio Optimization Based on Quantum HHL Algorithm.*Lecture Notes in Computer Science* [Artificial intelligence and security, icais 2022, pt ii]. 8th International Conference on Artificial Intelligence and Security (ICAIS), Qinghai, PEOPLES R CHINA.

81 Liu, C.-Y., & Goan, H.-S. (2022). Hybrid Gate-Based and Annealing Quantum Computing for Large-Size Ising Problems. arXiv:2208.03283. Retrieved August 01, 2022, from https://ui.adsabs.harvard.edu/abs/2022arXiv220803283L

82 Lu, X. L., & He, G. (2021, Feb). QPSO algorithm based on Levy flight and its application in fuzzy portfolio. *Applied Soft Computing, 99*, Article 106894. https://doi.org/10.1016/j.asoc.2020.106894

83 M. Esencan, C. F., A. Ho, T. Kumar, C. Unlu. (2023). *Improved and large-scale portfolio optimization using vector annealing*. https://icosacomputing.com/wp-content/uploads/2023/10/icosa-whitepaper-1.pdf

84 M, Zandjirdar. (2020). Overview of PO models. *Advances in Mathematical finance & applications, 5*(4). https://sanad.iau.ir/journal/amfa/Article/674941?jid=674941

85 Maiyahi, S. S. A. (2019). *Efficient frontier curve*. https://www.linkedin.com/pulse/efficient-frontier-curve-sultan-saif-al-maiyahi

86 Management, Morgan Stanley Investment. (2020). *Quantum Computing*. https://www.morganstanley.com/im/en-us/individual-investor/insights/articles/quantum-computing.html

87 Mathworks. (2024). *What is convex optimization?* Retrieved 30/07/2024 from https://www.mathworks.com/discovery/convex-optimization.html

88 Mattesi, M., Asproni, L., Mattia, C., Tufano, S., Ranieri, G., Caputo, D., & Corbelletto, D. (2023). Diversifying Investments and Maximizing Sharpe Ratio: a novel QUBO formulation. arXiv:2302.12291. Retrieved February 01, 2023, from https://ui.adsabs.harvard.edu/abs/2023arXiv230212291M

89 Mironowicz, P., Shenoy H., A., Mandarino, A., Yilmaz, A. E., & Ankenbrand, T. (2024). Applications of Quantum Machine Learning for Quantitative Finance. arXiv:2405.10119. Retrieved May 01, 2024, from https://ui.adsabs.harvard.edu/abs/2024arXiv240510119M

90 Moore, G. (1965). Moore’s law. *Electronics Magazine, 38*(8), 114.

91 Mugel, S., Abad, M., Bermejo, M., Sánchez, J., Lizaso, E., & Orús, R. (2021, Oct). Hybrid quantum investment optimization with minimal holding period. *Scientific Reports, 11*(1), Article 19587. https://doi.org/10.1038/s41598-021-98297-x

92 Mugel, S., Kuchkovsky, C., Sánchez, E., Fernández-Lorenzo, S., Luis-Hita, J., Lizaso, E., & Orús, R. (2022, Jan). Dynamic portfolio optimization with real datasets using quantum processors and quantum-inspired tensor networks. *Physical Review Research, 4*(1), Article 013006. https://doi.org/10.1103/PhysRevResearch.4.013006

93 Mugel, S., Lizaso, E., & Orús, R. (2022). Use Cases of Quantum Optimization for Finance. In *Studies in Systems, Decision and Control* (Vol. 429, pp. 211-220). https://doi.org/10.1007/978-3-030-97273-8_15

94 Naik, A., Yeniaras, E., Hellstern, G., Prasad, G., & Lalta Prasad Vishwakarma, S. K. (2023). From Portfolio Optimization to Quantum Blockchain and Security: A Systematic Review of Quantum Computing in Finance. arXiv:2307.01155. Retrieved June 01, 2023, from

https://ui.adsabs.harvard.edu/abs/2023arXiv230701155N

95 NOREA. (2024). Quantum annealing explained. https://www.norea.nl/uploads/bfile/b2f8c1cd-f550-427e-90ac-535ee9e79af8

96 NOREA. (2024). Quantum computing explained. https://www.norea.nl/uploads/bfile/b90d6290-5f15-4736-9cb8-4a001d1539a8

97 Osaba, E., Gelabert, G., Villar-Rodriguez, E., Asla, A., & Oregi, I. (2023). A Quantum Computing-based System for Portfolio Optimization using Future Asset Values and Automatic Reduction of the Investment Universe. arXiv:2309.12627. Retrieved September 01, 2023, from https://ui.adsabs.harvard.edu/abs/2023arXiv230912627O

98 Palmer, S., Karagiannis, K., Florence, A., Rodriguez, A., Orus, R., Naik, H., & Mugel, S. (2022). *Financial Index Tracking via Quantum Computing with Cardinality Constraints*. https://ui.adsabs.harvard.edu/abs/2022arXiv220811380P

99 Palmer, S., Sahin, S., Hernandez, R., Mugel, S., & Orus, R. (2021). Quantum Portfolio Optimization with Investment Bands and Target Volatility. arXiv:2106.06735. Retrieved June 01, 2021, from https://ui.adsabs.harvard.edu/abs/2021arXiv210606735P

100 Perri, L. (2023). *What's New in the 2023 Gartner Hype Cycle for Emerging Technologies*. https://www.gartner.com/en/articles/what-s-new-in-the-2023-gartner-hype-cycle-for-emerging-technologies

101 Pistoia, M., Farhan Ahmad, S., Ajagekar, A., Buts, A., Chakrabarti, S., Herman, D., Hu, S., Jena, A., Minssen, P., Niroula, P., Rattew, A., Sun, Y., & Yalovetzky, R. (2021). Quantum Machine Learning for Finance. arXiv:2109.04298. Retrieved September 01, 2021, from https://ui.adsabs.harvard.edu/abs/2021arXiv210904298P

102 Powell, J. R. (2008). The Quantum Limit to Moore's Law. *Proceedings of the IEEE, 96*(8), 1247-1248. https://doi.org/10.1109/JPROC.2008.925411

103 Qu, D. K., Matwiejew, E., Wang, K. K., Wang, J. B., & Xue, P. (2024, Apr). Experimental implementation of quantum-walk-based portfolio optimization. *Quantum Science and Technology, 9*(2), Article 025014. https://doi.org/10.1088/2058-9565/ad27e9

104 Ramaiah, K., & Soundarabai, P. B. (2024, 2024 Feb). A constrained multi-period portfolio optimization model based on quantum-inspired optimization. *Multimedia Tools and Applications*. https://doi.org/10.1007/s11042-024-18597-y

105 Rounds, M., Rosenberg, G. (2018). *Long-Short Minimum Risk Parity Optimization Using a Quantum or Digital Annealer*.

106 Sanz-Fernandez, C., Hernandez, R., Marciniak, C. D., Pogorelov, I., Monz, T., Benfenati, F., Mugel, S., & Orus, R. (2021). Quantum portfolio value forecasting. arXiv:2111.14970. Retrieved November 01, 2021, from https://ui.adsabs.harvard.edu/abs/2021arXiv211114970S

107 Shalf, J. (2020, 01/20). The future of computing beyond Moore's Law. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences, 378*, 20190061. https://doi.org/10.1098/rsta.2019.0061

108 Sheldon, R. (2022). *bit (binary digit)*. https://www.techtarget.com/whatis/definition/bit-binary-digit#:~:text=A%20bit%20(binary%20digit)%20is,%2Foff%20or%20true%2Ffalse

109 Singh Gill, S., Kumar, A., Singh, H., Singh, M., Kaur, K., Usman, M., & Buyya, R. (2020). Quantum Computing: A Taxonomy, Systematic Review and Future Directions. arXiv:2010.15559. Retrieved September 01, 2020, from https://ui.adsabs.harvard.edu/abs/2020arXiv201015559S

110 Singh, J., & Bhangu, K. S. (2023, 2023/01/01). Contemporary Quantum Computing Use Cases: Taxonomy, Review and Challenges. *Archives of Computational Methods in Engineering, 30*(1), 615-638. https://doi.org/10.1007/s11831-022-09809-5

111 Slate, N., Matwiejew, E., Marsh, S., & Wang, J. B. (2021, Jul). Quantum walk-based portfolio optimisation. *Quantum, 5*. https://doi.org/10.22331/q-2021-07-28-513

112 Smith, C. S. (2024 May 9). Top 10 quantum computing companies making change. Forbes. https://www.forbes.com/sites/technology/article/top-quantum-computing-companies/?sh=7243cd83a941

113 Stackpole, B. (2024). *Quantum computing: what leaders need to know*. https://mitsloan.mit.edu/ideas-made-to-matter/quantum-computing-what-leaders-need-to-know-now

114 Steinhauer, K., Fukadai, T., & Yoshida, S. (2020). Solving the Optimal Trading Trajectory Problem Using Simulated Bifurcation. arXiv:2009.08412. Retrieved September 01, 2020, from https://ui.adsabs.harvard.edu/abs/2020arXiv200908412S

115 Stuart Robertson, D. J. (2022). Quantum Computing — New Paradigm or False Dawn? https://www.lek.com/insights/ei/quantum-computing-new-paradigm-or-false-dawn

116 Surya Teja Marella, H. S. K. P. (2022). Introduction to quantum computing. In *Quantum computing and Communications*. https://books.google.nl/books?hl=nl&lr=&id=KF5iEAAAQBAJ&oi=fnd&pg=PA61&dq=advantages+of+quantum+computing&ots=R2UKAAVQDV&sig=J8-TfmwerFfHaKLs90lUKJEw9nw&redir_esc=y#v=onepage&q&f=false

117 Tardi, C. (2024, 06/06/2024). *Financial portfolio: what is it, and how to create and manage one*. https://www.investopedia.com/terms/p/portfolio.asp

118 Tripathy, S. S., Koul, N., & Patel, H. (2022). Comparative Study between Quantum and Classical Methods: Few Observations from Portfolio Optimization Problem. 2022 International Conference on Trends in Quantum Computing and Emerging Business Technologies, TQCEBT 2022,

119 Venturelli, D., & Kondratyev, A. (2019, May). Reverse quantum annealing approach to portfolio optimization problems. *Quantum Machine Intelligence, 1*(1-2), 17-30. https://doi.org/10.1007/s42484-019-00001-w

120 Xu, H. J., Dasgupta, S., Pothen, A., & Banerjee, A. (2023, Mar). Dynamic Asset Allocation with Expected Shortfall via Quantum Annealing. *Entropy, 25*(3), Article 541. https://doi.org/10.3390/e25030541

121 Yalovetzky, R. (2021). *NISQ-HHL Portfolio optimization for near-term quantum hardware*. https://www.jpmorgan.com/content/dam/jpm/cib/complex/content/technology/publications/white-paper-nisq-hhl-portfolio-optimization-for-near-team-quantum-hardward.pdf

Appendix

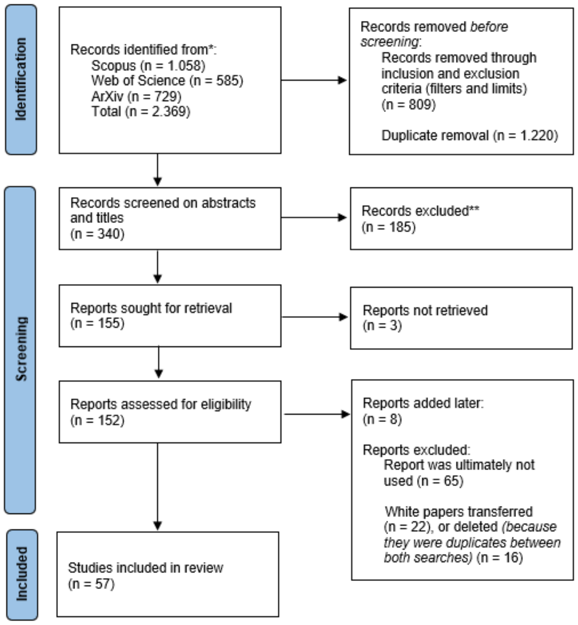

Figure 4, PRISMA 2020 Flowchart

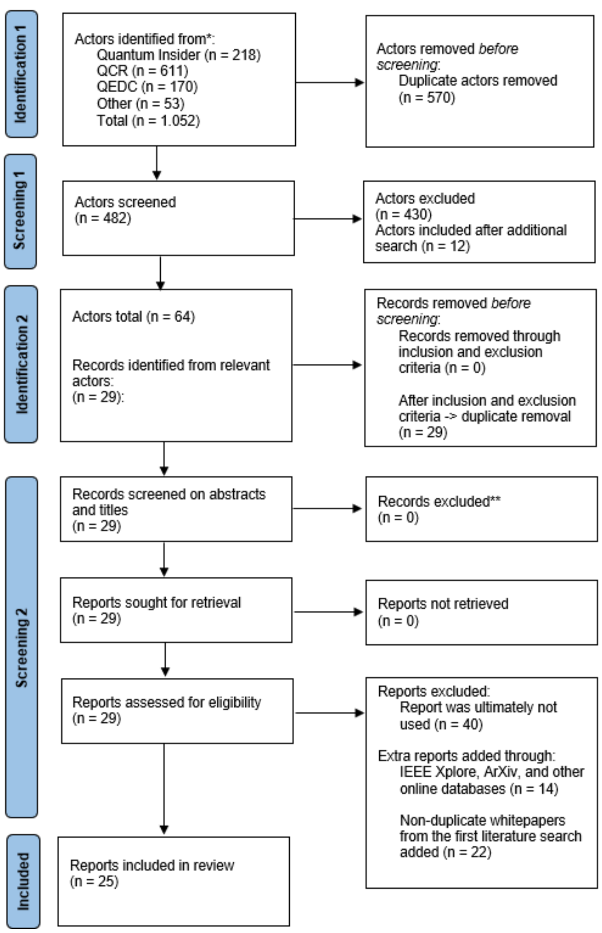

Figure 5, PRISMA 2020 adjusted flow chart for corporate white papers

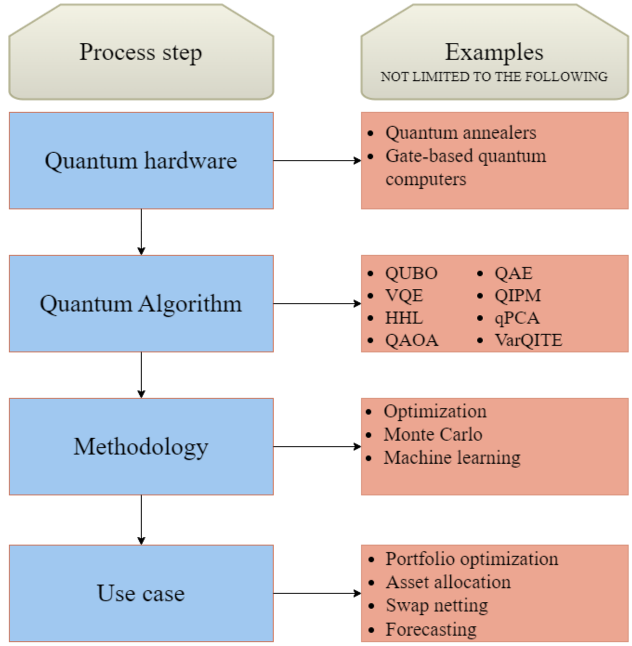

Figure 6, visualization of main quantum computing process

	Keywords
Initial search	"portfolio", "optimization", "quantum", "computing", "quantum optimization", "analysis", "methods", "simulation", "investment"
Secondary search	"quantum algorithm", "quantum finance", "quantum annealing", "financial optimization", "financial modeling", "portfolio management", "risk management", "optimization model", "optimization techniques", "asset allocation". "QUBO", "eigensolver", "forecasting",

Table 1; Keywords

Criteria	**Reason for inclusion**
Studies from the timeframe of 2011-2024	2011 was when quantum computing first appeared on the Gartner hype cycle and marks the first physical step towards quantum supremacy, therefore making room to (dis)prove previous articles.
Literature containing the named combinations of keywords from table 2 in either the article title, abstract, or keywords	Ensuring that keyword combinations made in table 2 are included in the chosen literature

Table 3: Inclusion Criteria

Criteria	Reason for exclusion
Literature not containing the named keyword combination from table 2 in the title, abstract, or keywords	Ensuring that keyword combinations made are included in the chosen literature
Exclude literature not published in the English language	Narrows down the results and facilitates consistent understanding of literature
Exclude literature made before 2011	Literature before 2011 has an increased risk of giving out wrongful information as the field of quantum computing has rapidly evolved after that timeframe
Exclude unfinished literature	Literature ought to be finished, as unfinished literature poses the risk of non-representative findings
Duplicate papers (papers that are identical either on different databases, or in the same one)	Duplicate papers ought to be excluded as they serve no additional purpose

Table 4: Exclusion Criteria

Comparison made	Papers
Simulated/digital versus Classical	Simulated: [4], [6], [7], [9], [13], [17], [18], [24], [29], [30], [41], [51], [52], [53], [63], [64], [66], [71], [74], [77], [80], [82], [88], [92], [104], [118] Classical: [11]
Quantum versus Classical	Quantum: [8], [91], [119] Classical: [14], [22], [24], [44], [47]
Simulated/Digital versus Quantum	Quantum: [120] Simulated: [24], [60], [78], [81]
Simulated vs Simulated	[3], [56], [103], [111]
Indifferent, Unclear, or Similar Performances	[37], [38], [48], [67], [68], [76]

Table 8, An insight into each academic paper's findings

Comparison made	Papers
Simulated/digital versus Classical	Simulated: [105], [2], [19], [28], [45], [49], [83], [73], [121], [99], [58], [114], [7], [33], [36] Classical: N/A
Quantum versus Classical	Quantum: [97], [106] Classical: [2], [25], [28], [34], [35], [36]
Simulated/Digital versus Quantum	Quantum: N/A Simulated: [2], [33], [35], [36]
Simulated vs Simulated	[27]
Indifferent, Unclear, or Similar Performances	[43], [50], [59], [121] , [98]

Table 9, An insight into each white paper's findings

	Prompts	Initial results (ArXiv Search in ‘all fields’)	Results with exclusion and inclusion criteria (not accounting for duplicates) (for Arxiv, this is done manually, along with direct observation of potential use for this research)
Keyword combinations	1. Quantum AND comput* AND portfolio AND optim* 2. Portfolio AND optim* AND quantum 3. (Quantum AND optim* AND portfolio) AND (invest* OR algorithm) 4. Quantum AND simulation AND portfolio 5. Quantum AND portfolio AND optim* AND algorithm 6. Quantum AND machine AND learning AND portfolio 7. Quantum AND algorithm AND finan* AND portfolio 8. (Quantum AND portfolio AND optim*) AND (methods OR techniques) 9. Quantum AND Portfolio AND management AND optim* 10. (Quantum AND risk AND forecast*) AND (finan* OR management) 11. Quantum AND portfolio AND asset AND allocation 12. (Quantum AND optim* AND portfolio) AND (techniques OR risk OR model*) 13. (Quantum AND methods AND portfolio) AND (optim* OR finan* OR model) 14. Quantum AND QUBO AND portfolio 15. Quantum AND eigensolver AND portfolio 16. Quantum AND forecast* AND portfolio And for ArXiv, these additional searches were done: 17. Quantum AND finan* AND model* AND optim* 18. Quantum AND finan* AND optim* AND algorithm	1. Scopus: n = 93 WoS: n = 51 Arxiv: n = 29 2. Scopus: n = 159 WoS: n = 100 Arxiv: n = 97 3. Scopus: n = 123 WoS: n = 79 Arxiv: n = 82 4. Scopus: n = 50 WoS: n = 22 Arxiv: n = 29 5. Scopus: n = 103 WoS: n = 69 Arxiv: n = 73 6. Scopus: n = 37 WoS: n = 14 Arxiv: n = 19 7. Scopus: n = 88 WoS: n = 44 Arxiv: n = 45 8. Scopus: n = 82 WoS: n = 48 Arxiv: n = 51 9. Scopus: n = 35 WoS: n = 12 Arxiv: n = 27 10. Scopus: n = 45 WoS: n = 13 Arxiv: n = 4 11. Scopus: n = 8 WoS: n = 5 Arxiv: n = 10 12. Scopus: n = 117 WoS: n = 66 Arxiv: n = 83 13. Scopus: n = 90 WoS: n = 47 Arxiv: n = 40 14. Scopus: n = 12 WoS: n = 9 Arxiv: n = 11 15. Scopus: n = 6 WoS: n = 3 Arxiv: n = 6 16. Scopus: n = 10	1. Scopus: n = 81 WoS: n = 49 Arxiv: n = 15 2. Scopus: n = 134 WoS: n = 95 Arxiv: n = 35 3. Scopus: n = 105 WoS: n = 78 Arxiv: n = 18 4. Scopus: n = 37 WoS: n = 20 Arxiv: n = 5 5. Scopus: n = 86 WoS: n = 68 Arxiv: n = 4 6. Scopus: n = 30 WoS: n = 14 Arxiv: n = 2 7. Scopus: n = 79 WoS: n = 42 Arxiv: n = 0 8. Scopus: n = 71 WoS: n = 48 Arxiv: n = 1 9. Scopus: n = 27 WoS: n = 12 Arxiv: n = 2 10. Scopus: n = 37 WoS: n = 12 Arxiv: n = 1 11. Scopus: n = 6 WoS: n = 4 Arxiv: n = 1 12. Scopus: n = 97 WoS: n = 63 Arxiv: n = 2 13. Scopus: n = 73 WoS: n = 43 Arxiv: n = 0 14. Scopus: n = 12 WoS: n = 9 Arxiv: n = 0 15. Scopus: n = 6 WoS: n = 3 Arxiv: n = 1 16. Scopus: n = 10

		WoS: n = 3 Arxiv: n = 1 17. Arxiv: n = 72 18. Arxiv: n = 38 Total: n = 2.369 Scopus: n = 1.058 WoS: n = 585 Arxiv: n = 729	WoS: n = 3 Arxiv: n = 1 17. Arxiv: n = 14 18. Arxiv: n = 4 Total: n = 1.560 Scopus: n = 891 WoS: n = 563 Arxiv: n = 106

Table 2: keyword combinations and search results

Table 5, search results in different stages for initial literature search

	Results with inclusion and exclusion criteria (not accounting for duplicates)	**Results with inclusion and exclusion criteria (accounting for duplicate literature)**	**Results with inclusion and exclusion criteria (accounting for duplicate literature and abstract, title, full-text screening, articles that were added later, articles not used, and white papers transferred/deleted:)**
Results	Articles with inclusion and exclusion criteria, not accounting for duplicates: Total: n = 1.560 Scopus: n = 891 WoS: n = 563 Arxiv: n = 106	Articles with inclusion and exclusion criteria, accounting for duplicates: Total: n = 340 ------------------------------------- Duplicates excluded in the same database: Total: n = 1.127 Scopus: n = 686 WoS: n = 438 Arxiv: n = 3 ------------------------------------- Duplicates excluded among all databases together: Total: n = 93 ------------------------------------- Total duplicates: n = 1.220	Articles with inclusion and exclusion criteria accounting for duplicate literature, abstract, title, full-text screening, papers that were added later, papers not used, and white papers transferred/deleted: Total: n = 340 Excluded: n = 185 Reports added (n = 8) Reports not used (n = 65) White papers transferred (n = 22) White papers deleted because they were duplicate (n = 16) Total end number of reports (n = 57)

Table 6, Insight into White Paper findings (Table 7 starts on page 57)

Paper (25) (Actors involved, or whom the authors are affiliated to) (Authors) (Year)	Challenge addressed / Introduction	Main findings/purpose	Quantum hardware, Quantum algorithm, Methodology, Use case	Additional specifics
[105] Long-short minimum risk parity optimization using a quantum digital annealer **(1Qbit)** **(Gili Rosenberg and Maxwell Rounds., 2018)**	This white paper is issued by 1Qbit and entails a novel approach to PO which addresses the issue that many weight allocation strategies result in long-term portfolio positions. The proposed strategy is one where directions (long or short positions) are assigned to each weight allocation so that the variance of the portfolio is either minimized or maximized. Furthermore, this proposed problem formulation is then shown to be applicable towards real quantum annealers of D-Wave Systems, and the Digital Annealer of Fujitsu Next to that, back tested results are shown for the problem formulation on three datasets using a tabu solver.	Objective: - Propose a weight allocation strategy where a direction is assigned to each weight encompassing either a short or long position, this is to ultimately reduce volatility and improve risk-adjusted returns for portfolios compared to traditional methods. - Furthermore, the proposed method is back tested on three datasets using a multi-start tabu 1-opt search with 100 starts (to act as a stand-in for the quantum digital annealer) and a sliding window mechanism of three months, portfolios were rebalanced on the first day of the month - To collect statistical data to run the algorithm on, bootstrapping is used with 25 samples Datasets: - Dataset 1 specifics: a portfolio for a commodity trading advisor (CTA), consisting of 38 futures contracts, including stocks and bond of different countries, as well as commodities such as oil, wheat, and gold - Dataset 2 specifics: Dow Jones Industrial Average, consisting of 30 large-cap US stocks - Dataset 3 specifics: nine S&P 500 sector ETFs Evaluation: - Show the performance acquired by the proposed formulation, it is applied on different methods (inverse variance parity, equal weighting, minimum variance, hierarchical risk parity, and quantum hierarchical risk-parity) used to show its improved efficiency and performance Results: - With the weighting methods used, it can concluded that the proposed method would outperform traditional methods in a risk-parity situation for PO, - "Our results suggest that by utilizing intelligent shorting, this method is able to reduce the volatility of long-only strategies, leading to shorter maximum drawdowns and higher Sharpe ratios, albeit with a higher turnover." (p. 1)	Quantum system: Algorithms used: Tabu Solver (TS) Methodology: Optimization Use case: Portfolio optimization	

[2] Multi-Objective Portfolio Optimization Using a Quantum Annealer (Rabobank, School of business economics Maastricht) (Aguilera er al., 2024)	"In this study, the portfolio optimization problem is explored, using a combination of classical and quantum computing techniques" (p.1) Furthermore, "In this paper, a specific problem is introduced, where a portfolio of loans needs to be optimized for 2030, considering 'Return on Capital' and 'Concentration Risk' objectives, as well as a carbon footprint constraint. This paper introduces the formulation of the problem and how it can be optimized using quantum computing, using a reformulation of the problem as a quadratic unconstrained binary optimization (QUBO)" (p.1)	Objective(s): - Using QUBO on simulated and physical quantum annealing, the paper sought to optimize a multi-objective portfolio optimization problem specialized for two made QUBO formulations (QUBO1 and QUBO2) from a real financial case considering the next variables: the return per asset, outstanding amount per asset, regulatory capital per asset, lower and upper bound outstanding amounts per asset, and an emission intensity/reduction per asset (p. 7) - The two QUBO models were then subsequently experimented upon using, where a classical benchmark is used a baseline to compare results with Results: - The results after putting in the data in both simulated and physical annealing were compared to a classical convex optimization approach, where the classical approach yielded less portfolios that fit emission constraints and was increasingly slower than QUBO2 (not QUBO1) using a higher number of assets. - For QUBO1, simulated annealing on QUBO1 showed better performance than random sampling, meeting constraints more effectively and producing solutions closer to the Pareto frontier. - For QUBO2, The second QUBO formulation outperformed QUBO1 in finding solutions near the Pareto frontier, with simulated annealing results suggesting potential advantages over classical methods. - Quantum computing, particularly quantum annealing, demonstrates potential in solving complex portfolio optimization problems by generating multiple viable solutions. - The quantum annealer showed a broader range of solutions compared to the simulated annealing results but struggled to match the classical benchmark closely. - The quantum annealing approach yielded fewer solutions near the Pareto frontier compared to simulated annealing and had limited success in meeting emission constraints. Purpose: To give insight into a novel way to use QUBO on a quantum annealer for multi-objective portfolio optimization	Quantum hardware: Simulated / physical annealing Quantum algorithm: QUBO Methodology: Optimization Use case: Multi-objective portfolio optimization	Solving of multi-objective portfolio optimization problem by deducting a specific real-world case into a QUBO problem formulation for a quantum annealer (p.3) Next to that, a specific variant of multi-objective optimization is used that aims to find the most efficient pareto frontier of a combination of return, diversification, and carbon equivalent emissions (CO2e)" (p.3), pareto frontier meaning a line of portfolios on a graph with Y = ROC and X = diversification, where no portfolio can be improved without worsening another part of it
[19] Approximating Optimal Asset Allocations using	A problem acknowledged by this paper is the lack of practical application by existing algorithms when datasets exceed 100 elements, therefore, simulated	Objective(s) - Analyze and apply simulated bifurcation to a PO problem for optimal asset-allocation following the Ising-problem formulation equivalent to the Markowitz model for maximizing risk-adjusted returns. - To test the usefulness of the proposed simulated bifurcation algorithm, a dataset is made from	Quantum system: Simulated Bifurcation in PYTHON Algorithms used: Simulated bifurcation	Simulated bifurcation = a method of optimization where solutions to simpler problems are modified to

Simulated Bifurcation **(NICS; CentraleSupélec; Université Paris-Saclay)** **(Thomas Bouquet et al., 2021)**	bifurcation is mentioned as the potential solver of this problem in this paper The objective of the study is to analyze and apply simulated bifurcation to the PO problem of optimal asset allocation (maximizing risk-adjusted returns over given time horizon)	historical data from YAHOO! Finance and used in a particular case with one-bit weights whilst looking for the optimal subset of assets. - The results obtained by the simulated bifurcation will be compared to a brute-force algorithm Dataset specification: - Closing prices of 441 assets belonging to the S&P500 index during the period of 02/2003 – 02/2021 on the New York Stock Exchange. Daily returns are calculated and used to estimate the covariance matrix Results: - Applying the one-bit weights simulated bifurcation method to the complete dataset shows that the algorithm runs the computation in about 5 seconds and selects 120 out of 441 assets - The performance of the selected portfolio by simulated bifurcation is significantly better than the one chosen via brute-force, indicating better risk-awareness - The simulated bifurcation algorithm has great eye for diversification of assets to reduce correlation/spread risk. - As numbers of assets increased, the simulated bifurcation showed greater degrees of accuracy in approximating the weights for each asset, 138 out of 150 simulations the algorithm could return the optimal allocation of weights. - For a problem with 4 assets and 5 bits per asset, the simulated bifurcation showed 90.4% Hamming accuracy (which basically is a measure of accuracy for algorithms) - Figure 7 in the paper gives a representation of a time efficiency comparison between brute-force (classical) and simulated bifurcation. This figure shows that after a certain point in a dataset, the complexity of solving a problem becomes exponentially more time consuming for brute-force, however, simulated bifurcation does not show this and thus has a superior ability to compute problem if they become exponentially more complex - Important notes: - It is impossible to proof optimality of the found portfolios, therefore methods can only be compared to each other. - In the computational tests for simulated bifurcation *(to help give an indication of the amount of assets and bits needed to be used in the actual*	Methodology: Optimization Use case: Portfolio optimization (optimal asset-allocation)	converge to an optimal solution Covariance matrix = a matrix giving insight into the covariance, or relationship between assets, in finance this is used to show correlation degree between assets.

			benchmarking), each assets can be represented by differing numbers of bits, more bits means better accuracy, however, as more bits also means more complexity to the calculation of the objective function, a consideration has to be made between number of bits and number of assets for this to work *(or in other words a balance between accuracy and computation time needs to be found),* this principle is also shown in table 6.2.1, as some combinations of number of assets and bits are computationally intractable. Ultimately, this test showed that lower bit values showed best accuracy toward the results obtained by brute-force strategies.		

[25] Comparing Classical-Quantum Portfolio Optimization with Enhanced Constraints **(Deloitte Consulting, Salvatore)** **(Certo et al., 2022)**	In this paper, quantum advantage is put to the test in a portfolio optimization perspective, where a quantum annealer is used along with some algorithms against classical methods More specifically, this paper employs several real-world constraints on the quantum annealer, thereby adding to the complexity of the problem to be solved. Furthermore, diverse traditional and new constraints are used both on state-of-the art classical algorithms and quantum algorithms The state-of-the art algorithms are solved using the d-Wave's quantum processor.	Objective(s): - Compare state-of-the art algorithms toward algorithms used on a quantum annealer - Map the Markowitz problem into a QUBO format to solve on an annealer. - Employ a variety of new and traditional constraints to increase the complexity of the problem to be solved and give greater insight to the difference between classical and current hybrid solutions in the static PO model. - Constraints used interchangeably are: minimum and maximum sector bands *(proportion of each industry sector is invested in)*, 2 types of balance sheet constraints (constraints based on mostly balance sheet ratios e.g. current ratio) first of which is a min current ratio constraint and the second being that the entire portfolio should have a minimum average, cardinality constraint of Limited Asset Markowitz (LAM), full budget must be used (budget constraint), and an asset must not be more than 2,5% of the portfolio. - For the real dataset test, only the last two mentioned constraints were used. And one last example with a volatility constraint added for CQM. The authors leave the combination of all types of other constraints for further research. Dataset: - Full S&P 500 Results: - For **specifically the use of min and max sector constraints,** the optimization model was run on the entire S&P 500 with quantum annealing. Results showed tighter investments bands, more flexibility, and the hybrid solver was able to satisfy all constraints. - The CQM model significantly outperformed the BQM model, but for higher values of q (above 25), BQM outperformed CQM. (q is the risk appetite level of the investor) - The classical solution found the efficient frontier with minimal effort, even with multiple real-world constraints - The CPLE solver outperformed all other in Sharpe ratios. - "Many have proposed portfolio optimization as a prime candidate for quantum advantage; however, the real-world constraints we have discussed thus far show that at least in the static integer-valued case, it is unlikely to outperform classical solutions." (p. 5), although this is mentioned, the problem solved is still convex, thereby not fully giving way to the	Quantum system: D-Wave's hybrid models (binary quadratic model (BQM) and constrained Quadratic Model (CQM)), and CPLEX for classical optimizing Algorithms used: Classical and quantum-annealing algorithms Methodology: Optimization Use case: Portfolio optimization	Current ratio = a ratio giving insight into how well a company is able to fulfill short-term obligations, thus a measure of liquidity. LAM = a constraint that ensures assets in a portfolio are limited (which may be due to several reasons such as limiting transaction costs)

advantages of quantum computing, if the problem were non-convex, the authors mentioned that QA may have an advantage, but they also question whether a real-world scenario with a non-convex constraint will actually be used.

- Sharpe ratios for various constraints mentioned:

	Q=1	Sector constraint	Local CR	Global CR	Cardinality
BQM	3.25	2.79	1.86	2.60	1.67
CQM	3.88	3.81	3.41	3.32	3.40
CPLEX	3.88	3.81	3.41	3.73	3.70

-

Important information:

- Although
- Constraints are mostly formed as penalty terms in the formulation of the objective function.
- The problem in this paper follows that of the Markowitz's modern portfolio theory of maximizing returns for a given level of risk.
- "As current QA do not have the number of qubits nor the required connectivity between them to implement large-scale models directly on annealers, we explore the use of D-Wave's hybrid models" (p. 2)
- "While gate-based machines in the Noisy Intermediate Scale Quantum (NISQ) era struggle to find appropriate feasible applications, quantum annealers have less constraints and appear to be the most promising in near-term industrial implementations" (p. 1)

[28] Black-Litterman Portfolio Optimization with Noisy Intermediate-Scale Quantum Computers **(Chi-Chun Chen et al., 2023)**	In this paper, the practical applications of NISQ algorithms are used in the enhancement of the Black-Litterman PO model. As proof of concept, a 12-qubit example of selecting 6 assets out of a 12-asset pool is used, where the approach involves predicting investor views with Quantum Machine Learning (QML), and addressing the optimization problem using the Variational Quantum Eigensolver (VQE)	Objective(s): - Formulate a Black-Litterman PO problem and estimate the investors 'view via QML, and solve the QUBO formulation via VQE, or QAOA. Optimize the parameters using Sequential Least Squares Programming (SLSQP) - Formulate the Po problem into a QUBO format, where the aim of the formula is to maximize return while minimizing risk with a budget constraint and penalty terms. - Find the investors' view in the formula with Quantum Machine learning, and the market implied return with data from the market, both are specific to the Black-Litterman approach. - Approach the quantification of the investors' view via 4 quantum machine learning methods (QSVM, QNN, SVM, NN) - Demonstrate a 12 and 16 qubit case that shows the capability of obtaining solutions with good back testing performance. Data for the back test: - Time period 2008/01/01 to 2021/12/31 (split up in 9 time segments) with a 260 week training period and 52 week testing period.12 Individual stocks from S&P 500. VQE was used with p = 4 repetitions of the circuit, and QAOA with p = 8. Tests are compared to the approximation ratio, which is a ratio between 'a good solution' and that found through the test either via VQE or QAOA. Results: Investors' view performances: - Specifically looking at the estimation of investors' view, the following could be said: QSVM ≈ SVM > NN > QNN in terms of testing accuracy, and QSVM was also much faster to train than QNN. Optimization test of BL-PO: - Considering the BL-PO test, VQE had an approximation ratio of at least 0.9 and mean 0.96 - Variances via VQE were close to zero (so low risk), and those form QAOA are large. - Tests were still proven to be susceptible to finding local minima instead of global minima. - VQE heuristic ansatz should be preferred over QAOA - Looking at the given figures depicting approximation ratios and variances, VQE outperforms QOA significantly, with QAOA having greatly varying and worse results. Back testing performance with investors' view from QSVM:	Quantum system: Algorithms used: VQE, QAOA, and QML Methodology: Optimization Use case: Black-Litterman Portfolio optimization	Black-Litterman PO = a PO approach that combines elements of modern portfolio theory with investor views to improve the Markowitz mean-variance model. Investors' view = the objectification of the investors' view on the assets, which will either be bullish or bearish.

		- The BL-PO model outperforms Modern Portfolio Theory in pure returns and certainty-equivalent return over a long continuous back testing period. - VQE/QOA find high approximation ratios close to the optimal solutions, and sometime even outperform exact solution in the approximation ratio. - There is balance problem found between balancing out computational cost and preciseness of the solution. - **The ability to perform well without exact solutions suggests efficiency gains in quantum optimization methods.** - "The solutions obtained from VQE exhibit a high approximation ratio behavior, and consistently outperform several common portfolio models in back testing over a long period of time." (p. 1) - "The scale of real quantum device today are not able to solve discrete portfolio optimization problems beyond classical computer limit (and quantum computers cannot be efficiently simulated classically)" (p. 2) Important notes: - The computational resources needed for quantum computing can be anticipated as it follows a linear scheme, on the contrary, classical computing follows a exponential line when it comes to computational resources needed for larger problems. - QSVM was used for investors' view approximation.		
[27] Quasi-binary encoding based quantum alternating operator ansatz **(CCB Fintech)** **(Bingren Chen et al., 2023)**	In this paper, a quasi-binary encoding based algorithm is proposed for solving specific quadratic optimization model in the QAOAz framework. Three constraints are imposed on the model: Discrete constraint, bound constraint, sum constraint In some parts of the given objective function for QAOA, ideas such as CVaR-QAOA and parameter scheduling are used to optimize the solution quality.	Objective: - Form a quasi-binary encoding based QAOAz to solve quadratic optimization problems (based on the Markowitz model for PO) with integer variables in a hard constraint way. - Make use of parameter scheduling techniques and CVaR-QAOAz to enhance solution quality - Use 4 methods for optimal parameter scheduling: **1:** Sample20: 20 random parameters are chosen for the training process, and over 1000 iteration in COBYLA, the best option will be chosen **2:** Optimized Linear Schedule (OLS) **3:** Iterative Optimized Linear Schedule (IOLS) **4:** Iterative QAOA (IQAOA) - Make use of COBYLA as the classical optimizer to finds the best parameter. - Lastly, perform experimental test with the CVaR-QAOAz and Normal-QAOAz on two instances to show performance differences for their use in the broader QB-QAOAz framework: 1: Selecting 6 stocks with a total of 18 qubits required for the experiment, and different simulations are conducted on P = 1, p = 2, p = 4, p = 8 and p = 16 *(p represents the*	Quantum system: Qiskit (simulator) Algorithms used: QB-QAOAz Methodology: Optimization Use case: Portfolio optimization	Parameter scheduling = adjusting parameters of an algorithm over time to improve its performance Quasi-Binary approach = a way to simplify the problem representation so that quantum hardware can be leveraged more effectively, with the aim to reduce resource requirements and better performance.

	Lastly, a numerical simulation will be used on a PO case to show the performance of the given algorithm	*depth of the quantum circuit, the number of iterations, so in simple terms the complexity)* and with α = 0.5 *(with upper and lower bound being -1/+1) (which signifies the precision of the parameters, more precision = better results on average, but also more computational resources needed)* using all 4 parameter methods. 2: general stock pools from the Chinese Shenzhen and Shanghai Stock Exchange. 4-8 stocks are randomly selected from 4836 stocks. α = 0.05, 320 experiments on each of the four parameter scheduling methods and five different depths (p = 1, 2, 4, 8, 16) - Lastly, a method to increase precision of the instances is proposed for QB-QAOAz, first QB-QAOAz is used with CVaR-QAOAz and IQAOA scheduling method, and then the course solution it gives is optimized via increasing α exponentially via an iterative method (with the purpose of finding a better solution with fewer qubits needed) Dataset specifications: - Six NASDAQ stocks with historical return rates of these stocks as the input data, Three constraints are imposed on the model: 1. Discrete constraint, the variables are required to be integers 2. bound constraint, variables ought to be greater than or equal to a certain constraint and less than or equal to another integer 3. sum constraint, the sum of all variables should be a given integer Results: Result for instance 1: - Results showed that CVaR-QAOAz outperformed the normal-QAOAz significantly, where CVaR-QAOAz is also superior to brute-force (classical) when p exceeded 2. - As for the parameter optimization, IQAOA could not show its proposed superiority over the other parameter scheduling methods, furthermore, IQAOA and IOLS often fell into local optima. In most cases, as p got higher, the performances decreased due to high parameter count. - IOLS performance increased with circuit dept, furthermore, IQAOA performed better under CVaR-QAOAz than Normal-QAOAz, final recommendation was to use CVaR-QAOAz with IOLS or IQAOA with p above 8 **to achieve an approximation ratio of 0.99.** Instance 2: - CVaR-QAOA showed an approximation ratio between 0.973 and 0.997.		

		- The approximation ratio of the parameter scheduling methods increased as circuit dept increased, with less errors - For the parameter scheduling methods, IQAOA performed the best. - Overall, for the two instances, it was still observed that the precision of results was too coarse for business application. Iterative QB-QAOAz method: - The iterative method for QB-QAOAz with CVaR-QAOAz and IQAOA showed significant improvements in the quality of solutions, and the probability of finding the optimal solution increased (all whilst keeping the same number of low qubits) - "If we increase the precision, for example, by setting α to one-thousandth, then the total number of qubits required in Instance 1 is 96, which already exceeds the computational limit of most quantum computers and simulators. " (p. 15) Important notes: - To address the limitations of current (2023) quantum hardware, an iterative method will be used where the solution of the experiment will be improved through multiple few-qubit experiments, and parameters will slowly become more precise over the iterative process. - no penalty terms are used in the objective function.		
[43] Financial Portfolio Management using D-Wave Quantum Optimizer: The Case of Abu Dhabi Securities Exchange **(UT-Batelle LLC with non-exclusive contract with U.S. Department of Energy)**	Based on a formulation of the Markowitz's mean-variance model, where it is formulated as a QUBO problem including expected return, volatility, penalization terms, and according to weights for each criterion, to be solved via a D-Wave quantum optimizer	Objective(s): - Formulate the mean0variance Markowitz model in a QUBO formulation, and solve it via a D-Wave quantum optimizer - Solve the given problem on MATLAB (mathematical software) via the genetic algorithm (classical approach) - Compare the results from the MATLAB experiment and those fo the D-Wave quantum optimizer Data specifics: - 63 to 68 securities from the Abu Dhabi Securities Exchange, with weekly closing prices over the period 01/12/2015 to 30/11/2016 and a covariance matrix and matrix for expected returns was made. Total budget = 100 USD Results: - The QBSOLV produced portfolios that exceeded the budget (121.176 USD instead of the budget 100 USD) in order to fit the QUBO model - The choice of exceeding the budget has clearly ignored the influence of the co-variance matrix to	Quantum system: D-Wave QBSOLV (simulated solver) Algorithms used: D-Wave QBSOLV Methodology: Optimization Use case: Portfolio optimization	

(Nada Elsokkary et al., 2021)		minimize risk, so the portfolio was not diversified to spread risk - Longer annealing times showed slightly improved results with portfolios similar to lower annealing times, but with lower cost portfolios - Compared to the classical solution, QBSOLV found portfolios in good agreement with those found in the MATLAB-derived solution. Important notes: - This paper leaves a lot of additional, sometimes needed information, out of the picture, it mostly states the core findings and pre-requisites of the research		
[45] Quantum-Enhanced Simulation-Based Optimization **(IBM Research and ETH Zurich)** **(Gacon J et al., 2020)**	In this paper, a quantum-enhanced algorithm (QAE) for simulation-based optimization is introduced to optimize simulation based optimization and form the Quantum-Enhanced Simulation Based Optimization Algorithm (QSBO), where it is applied towards a PO problem with Value-at-Risk constraint and inventory management The algorithm is proposed for continuous and discrete decision variables..	Objective(s): - Formulate a Simulation based PO problem including Value-at-Risk or inventory management and solve it via the QSBO algorithm. - Optimize SBO with QAE to accelerate the estimation of values, specifically, use QAE in QSBO to enhance the precision and efficiency of evaluation the objective functions. - Use an adapted version of VQE (for discrete optimization problems) to optimize the decision variable y* (which is part of the objective QUBO function) (to optimize y* means to get better results for the eventual calculation of the QUBO function) - Apply the algorithm to small instances of practically relevant problems, from inventory management and finance to PO with VaR based objective function. Dataset specifications for PO problem: - A two-asset portfolio, where 13 qubits are used for the VaR estimation, and 12 qubits for the expectation value x, with a risk appetite of 0.09, and $\alpha = 0.05$ (simply put, precision level of the parameters) Results (objective function is to minimize risk): Newsvendor problem: - The most optimal solution was found accurately, looking at the graph depicting the given solutions, it can clearly be seen that all results are estimated accurately, and the optimal solution is found. Portfolio optimization: - The algorithm identified he optimal solution with a 90% probability, showing that with 90% certainty, the first out of the two possible assets maximizes the portfolio. - The results show that the proposed algorithm is able to compute PO problems accurately. Overall: - Increasing the number of repetitions of the algorithm leads to more parameters, thereby more search space,	Quantum system: Qiskit (simulator) And for the classical part of the algorithm, COBYLA is used Algorithms used: Quantum Amplitude Estimation (QAE), Quantum-enhance simulation based optimization (QSBO) Methodology: Use case:	Newsvendor problem = a problem that involves determining the optimal number of newspaper batches to purchase to balance the cost of leftover newspapers and the lost income from unmet demand. The goal is to minimize the expected cost function, which accounts for both overage and opportunity costs.

		but at the expense of computational resources needed as the problem becomes more complex. - "Quantum Amplitude Estimation (QAE) is a quantum algorithm that provides a quadratic speedup over classical Monte Carlo simulation, i.e., its estimation error scales as O(M−1)." (p. 1) - For all experiments, the optimal solution was found with high probabilities. - The algorithm shoed great capabilities in solving inventory management and PO problems with both continuous and discrete variables - "The algorithm offers a quadratic speedup for the evaluation of the objective function compared to classical Monte Carlo simulation." (p. 7) Important notes: - QAE is commonly used for estimating parameters and optimizing them (ultimately reducing circuit complexity and depth), in the case of this paper it is used to estimate expected values of functions related to the objective function. This paper aims to use QAE for a quadratic speedup of the normal SBO.		
[49] **A detailed end-to-end assessment of quantum algorithm for portfolio optimization** **(Goldman Sachs and AWS)** **(Alexander Dalzell et al., 2023)**	In this paper, a detailed explanation is given towards the use of a quantum algorithm for portfolio optimization. This paper is inspired by the "End-To-End Resource Analysis for Quantum Interior-Point Methods and Portfolio Optimization" Issues addressed are: 1: to determine the practicality of a quantum algorithm 2: the PO model itself 3: Quantum interior point methods 4: Resource estimate for QIPM	Points to determine the practicality of a quantum algorithm: - The quantum algorithm produces a classical output that allows for benchmarking via classical methods - The quantum algorithm relies on a reasonable input model, as some models (mostly for QML) were thought to offer significant advantages over classical methods until it was pointed out that they did not because they used unreasonable assumptions about the input model. - The quantum algorithm has a plausible case for asymptotic speedup, meaning that it is used on a case that shows the quantum algorithm to outperform a classical counterpart on a sufficiently large size instance, as that is where quantum advantage is found. - The instance size, or the tipping point where the quantum algorithm outperforms the classical one must be of commercial use, if it outperforms a classical algorithm at a point where it is of no commercial use, the quantum algorithm may as well not be used. QIPM for PO model: - PO aims to maximize returns while minimizing risk of a fixed investment budget. QIPM tries to achieve this by using quantum computing methods to specific computational processes in the classical algorithm. In particular, QIPM improves on classical interior point techniques by employing quantum algorithms to solve linear problems, quantum random access	Quantum system: Amazon Braket Algorithms/method used: Quantum Interior Point Method (QIPM) Methodology: Optimization Use case: Portfolio optimization	

		memory (QRAM) to rapidly access data, and quantum state tomography to transform quantum states into classical information. Challenges for QIPM: - Error management: errors can affect accuracy, however, the IPM's design allows for self-error correction. - Limitation of current NISQ hardware: e.g. limited qubits and frequency of errors and noise interferences. - Dependency on parameters Resource estimation for the QIPM: - The estimate to encode a PO problem with 100 assets is around 8 million qubits, far from what is currently feasible on quantum hardware. - Quantum gates needed for n = 100 (or more specifically T-gates for QIPM) is approximately 7 x 10^29, far from currently feasible - T-Depth (or depth of the circuit / number of layers of T-gates in parallel) for n = 100 is 2 x 10^24, which is very computationally demanding and currently not realizable. - Currently, the estimation for QIPM runtime is in the millions of years for bigger PO problems. Results/findings: - Simulations suggest that QIPM may theoretically offer speedups, but current implementation do not show a clear advantage over classical algorithms for problem size between n = 10 and n = 120., practicality for larger problems remains uncertain. - Even when algorithms present promising advantages, further inspection on it can reveal a drastically different picture due to multiple factors (e.g. assumptions made for the algorithm are not realistic) - QIPM showed great data cost and computation time, needing significant QRAM to operate. - Currently, QRAM is not practical, it is suggested that to improve its practicality, dedicated QRAM hardware ought to be made that can leverage the special aspects of QRAM more efficiently. And this applies to all algorithms making use of QRAM.		
[50] Efficient DCQO Algorithm within the Impulse Regime for Portfolio	In this paper, a digital quantum algorithm is proposed for portfolio optimization using the digitized-counterdiabetic quantum optimization (DCQO) algorithm.	Objective(s): - Form a fast, purely-quantum digitized-counterdiabatic quantum optimization protocol (DCQO) relying on the concept of the impulse regime, along with a hybrid version (H-DCQO) - Experiment with these models on a 20-asset PO problem on the IonQ quantum computers.	Quantum system: IonQ trapped-ion quantum computer Algorithms/model used: DCQO Methodology:	Impulse regime = an approach that reduces circuit depth and enhances solution accuracy. In this paper it is sued as an alternative to

Optimization **(Kipu Quantum and University of the Basque Country Department of Physics)** **(Alejandro Gomez Cadavid et al., 2023)**	The DCQO is applied to a real-case scenario of PO with 20 assets, using purely quantum and hybrid-quantum paradigms. It is performed using up to 20 qubits on the IonQ trapped-ion quantum computer. The DCQO is benchmarked against the standard Quantum Approximate Optimization Algorithm (QAOA) and finite-time digitized-adiabetic algorithms. Note: this paper mostly compares the proposed quantum algorithms to each other, not directly mentioned any classical algorithms (only for the hybrid model for optimization), but it can generally be deducted by the results that promising results are shown from the experiments.	- Integrate adiabatic quantum optimization and counter diabetic protocols in DCQO to address the PO problem more efficiently - Convert proposed Markowitz PO model in this paper *(reformulated with single-time step modality of this problem with Boolean asset investment) (this is mainly to simplify the problem and make it more efficient to solve)* to a Hamiltonian formulation to be able to make it solvable via DCQO - Test the DCQO and h-DCQO to each other, QAOA, and other digitized adiabatic protocols. - Results are put into perspective via the approximation ratio of the average energy needed for a solution compared to the actual energy used. Data specifics: - 20 assets, with historical data from 06/06/2022 to 01/01/2023, budget is number of asset / 2. Results: DCQO: - Implementing CD protocols in the DCQO improved performances 2x in terms of approximation ratio compared to non-CD usage. - For the 20-asset problem on a simulator, the DCQO proved to be more efficient than compared methods, showing an average approximation ratio of 0.54 - Implementing DCQO on IonQ's 25-qubit device showed that the AR ratio could be 0.50 with error mitigation methods, similar to the simulated results. h-DCQO: - A five-layer (more complex, thus accurate results) QAOA performed similarly to a one-layer h-DCQO, showing that h-DCQO is more efficient. - For the PO problem, h-DCQO achieved an AR ratio of 0.72, showing the closest likeness to the desired solution out of all the tests. - When executed on IonQ's device with error mitigation techniques, h-DCQO showed an approximation ratio of 0.58, which is lower than the simulated test. Overall: - The two methods are effective for both portfolio optimization and other combinatorial problems, demonstrating their general utility. - "we achieved a substantial reduction in the circuit complexity while maintaining a similar solution accuracy" (p. 7), referring to the methods used to lower circuit complexity such as CD protocols. - "We obtain a significant reduction in the circuit depth by factors of 2.5 to 40, while minimizing the dependence on the classical optimization subroutine." (P. 1)	Optimization Use case: Portfolio optimization	suing methods like QAOA. Single time-step modality = means solving the problem in a single point in time, as opposed to multiple time steps or stages. Basically, meaning that the proposed model only has to solve the formulation once and give asset allocation in a portfolio one time. Boolean asset investment = a way of simplifying the inclusion, or exclusion, of an asset to a binary format, thereby simplifying the optimization problem to a series of yes/no decisions for each asset Counterdiabetic protocols (CD) = a set of techniques used in quantum computing to enhance the performance of quantum algorithms, particularly those involving quantum optimization and quantum annealing.

		- “Besides portfolio optimization, the proposed method is applicable to a large class of combinatorial optimization problems.”(p. 1) Important notes: - Classical optimization for the hybrid algorithms was done via COBYLA. - Multiple additional methods are used on DCQO and h-DCQO to optimize its efficiency and performance, these methods are not relevant to be explained but the following are employed: 1. On DCQO: impulse regime, selective trotter steps, gate reduction strategy, threshold alignment, and critical point focus 2. H-DCQO: simplified ansatz method, parameter reduction, variational optimization following variation quantum algorithms (as these are also hybrid quantum-classical), and layer count optimization. - The DCQO is a purely quantum optimizer, and h-DCQO is a hybrid version employing classical methods also. - The paper leverages adiabatic quantum optimization and counterdiabatic protocols to address the portfolio optimization problem more efficiently, thereby reducing circuit depth and increasing accuracy		
[59] **A Quantum-Inspired Binary Optimization Algorithm for Representative Selection** **(Agnostiq Inc)** **(Anna G. Hughes et al., 2023)**	In this paper, a selector algorithm is proposed: a method for selecting the most representative subset of data from a larger dataset. The proposed dataset includes datapoints that meet two requirements: 1: The data is maximally close to neighboring data 2: The data is maximally far from more distant data points This is ti make sure data selected is as diversified as possible.	Objective(s): - Form a unsupervised representative selector system/algorithm for selecting them sot representative subset of data from a data pool, where the algorithm meets two requirements: 1: The data is maximally close to neighboring data 2: The data is maximally far from more distant data points - Formulate the cost function as a QUBO problem aimed to be solved via multiple metaheuristics, where the selector algorithms pick out unique and representative data points by finding low-cost solutions to this QUBO function on quantum annealer. - Show two use cases for the selector algorithm: 1: approximately reconstructing the NASDAQ 100 index using a subset of stocks, comparing how close the return of the selected stocks are to those to the NASDAQ 100 2: diversifying a portfolio of cryptocurrencies - For case 2, compare the performance of the algorithm using two quantum annealers provided by D-Wave. - Also do experiments with synthetic data Dataset specifications (Synthetic data): - One dataset containing simple and obviously clustered data, and another dataset containing time	Quantum system: D-Wave QBSOLV for NASDAQ 100 problem D-Wave Advantage (over 5000 qubits) and D-Wave 2000Q (2048 qubits) for crypto problem. Algorithms used: Selector algorithm Methodology: Optimization Use case: Portfolio optimization	

		series data; data ordered in a chronologically ordered sequence. Dataset specifications (use cases): Reconstructing NASDAQ 100 with a classical QUBO solver: - 102 stocks, performed on D-Wave QBSOLV, daily returns of each stock are considered, historical data from 2021/02/01 to 2022/02/01 (253 days), stocks are equally weighted. Diversifying crypto portfolios with quantum annealers: - Input data from daily returns of cryptos from Crescent Crypto Market Index in the period 2021/04/01 to 2021/11/11 (seven months), annealing times were changed to find different solutions, constraint satisfaction was tested, and solution quality is compared. D-Wave Advantage and D-Wave 2000Q were used. Constraint tested: whether the selector keeps to the max of 3 cryptos. Results: For synthetic data: - The selector algorithm successfully selected representative points from the clustered data points - The selector algorithm was able to select representative data even as noise increased. - The algorithm demonstrated robustness in selecting representative points of data from both clearly and loosely clustered data, showcasing its practical application. - he algorithm maintained high accuracy in distinguishing between clusters at low noise levels, with 100% accuracy. As noise increased, accuracy dropped, but was still better than random picking. For use cases: Reconstructing NASDAQ 100 with a classical QUBO solver *(objective: use the selector algorithm to find assets that closely relate to the returns from the NASDAQ 100 index)*: - The selector algorithm found two stocks that approximated NASDAQ 100 closely, and the stock chosen proved to be competitive, meaning they performed well compared to other possible choices. - As more stocks were selected, e.g. 40, the selector achieved a reproduction of the NASDAQ 100 (concluded from mean-square-error score) - Accuracy increased with increased number of stocks. Diversifying crypto portfolios with quantum annealers *(objective: Use the selector algorithm to choose a subset of cryptocurrencies, optimizing the cost function on each quantum annealer):* D-Wave 2000Q: - Succeeded in selecting exactly 3 cryptocurrencies in only 16% of the trials		

		- Average cost function value of 4.02, within the lowest 4% of possible values, meaning that it can find good performing cryptos, but with room for improvement. D-Wave Advantage: - Achieved a success rate of over 85% in selecting 3 cryptocurrencies. - Average cost function value of 0.32, within the lowest 0.03% of values, meaning that it can find cryptos that are among the very best compared to all possible solutions, suggesting significantly better performance in PO. Overall findings: - Average annealing times were between 20-990 microseconds, but annealing times were significantly better for D-Wave advantage than for 1000Q - Longer annealing times improved the percentage of solutions meeting the constraints. - D-Wave 2000Q falls short of D-Wave Advantage - Both devices are able to select solutions with lower cost function values compared to the average of all possible solutions, however, D-Wave Advantage finds better solutions. Overall conclusions from all tests: - "Overall, we saw clear improvement between the newer Advantage QPU and the earlier 2000Q QPU, providing meaningful solutions to the combinatorial optimization problem." (p. 9)		
[83] Improved and large-scale portfolio optimization using vector annealing **(Icosa Computing; NEC M)** **(Esencan et al., 2023)**	In this paper form Icosa computing and NEC, a quantative comparison between NEC's Vector Annealing (VA) solution against the simulated annealing algorithm is performed via a financial PO problem.	Objective(s): - Propose a SA algorithm, solving a QUBO formulation of Markowitz's Modern Portfolio Theory. - Tune the parameterization of both VA and SA, and compare results with non-optimized parameterization for SA and VA. - Compare VA and SA performance via subtracting both performances from each other to give perspective in the difference between both. - Employ a four-step process in testing SA approaches: 1: obtain stock data from IEQ's platform, or from Yahoo Finance 2: using a tunable finance model, deconstruct and formulate the original problem in a discrete problem suitable for SA and VA 3: use both SA and VA for finding a candidate solution to the formulated problem 4: consider the candidate with the lowest energy state as the optimal solution. - Data specifics:	Quantum system: N/A Algorithms used: Simulated annealing (SA) and NEC's vector annealing (VA) Methodology: Optimization Use case: Portfolio optimization	

		- A problem with differing numbers of linear variables, markets, stock numbers, granularity, with historical data as training periods from the used markets. (see figure X) - S&P 500 period was between 3/12/2018 and 8/1/2019 with 486 stocks due to some missing data - US test was from the stock period between 3/18/2022 and 3.2.2023, and second test for data between 3/18/2022 and 5/16/2022. - For international test one, the period was 3/18/2022 and 5/4/2022 (with 17,833 equities traded in France, Germany, U.K., and U.S.), and second test period being 3/17/2022 and 4/1/2022 (for 25,034 equities traded in Canada, France, Germany, Japan, Turkey, U.K., and the U.S.) Results: - Va constantly performed better than SA, producing better quality solutions - the energy gap between SA and VA grew as number of variables grew, showing that VA has a scaling advantage. - Looking at the results, and the graph in figure 1, it can be said that both SA and VA perform better after tuning the parameters. - “We found that Vector Annealing generally outperformed Simulated Annealing in terms of solution quality and that its advantage over SA scales with problem size.” (P. 1) - NEC’s VA is able to compute very large numbers of variables with complex, real-world constraints. - “NEC Vector Annealing greatly reduces the computational complexity associated with traditional Simulated Annealers and accelerates the narrowing down of the candidate solutions by a factor of up to 300 times at problem sizes beyond the capabilities of conventional methods.” (p. 1) Important notes: - Actual financial returns are disregarded as this paper is only interested in performance difference between VA and SA. - It is mentioned that the SA and VA need finetuning for it to perform to a certain standard, but ‘this is out of the scope of this paper’ (p. 3)		
[73] Quantum Algorithms for Portfolio Optimization	This paper mentions it to develop the first quantum algorithm for constrained PO and test it on a PO instance	Objective(s): - Design and analyze a quantum algorithm for the general constrained portfolio optimization problem, making it applicable to a PO problem with an arbitrary number of positivity and budget constraints. - Reduce the objective PO problem to a second order cone program (SOCP) for broader applicability *(to classical interior point methods (IPM) and certain*	Quantum system: N/A Algorithms used: Quantum version of interior point methods.	Second Order Cone Programs (SOCPs) = a convex optimization problem that generalizes linear and quadratic

(CNRS, IRIF, Université Paris Diderot) **(Anupam Prakash et al., 2019)**	Furthermore, some experiments are done to bound the problem-dependent factors arising in the running time of the quantum computer, comparing computing times with classical algorithms	*quantum algorithms),* efficiency, generalization, flexibility, and stability/robustness of results. - Conduct an experiment with the proposed quantum model on dataset, compare the results with classical IPM. Dataset specifications: - Historical data from the S&P 500 stock for a period of 9 years (2007-2016), 50 companies are sampled for their stock performance in the first 100 days. Results: - The quantum algorithm shows similar performance to the classical algorithms in terms of convergence. - The quantum algorithm offers significant speedup compared to the classical methods - Running time of the algorithm scale more favorably than that of its classical counterparts, indication quadratic speedup over classical algorithms. - The quantum advantage showed to be more pronounced when the number of assets is large, and constraint numbers are low. - "We obtain a polynomial speedup over the classical algorithms, and we provide experimental results to demonstrate the potential of these advantages in practice" (p. 1) - "The experiments suggest that this parameter κ in indeed bounded and that our algorithm achieves a speedup over the corresponding classical algorithm" (p. 4) Important notes: - The goal of the quantum IPM is to significantly outperform classical approaches, especially for big matrices and high-dimensional problems, by utilizing quantum linear systems solvers and QRAM.	Methodology: Optimization Use case: Portfolio optimization	programming, basically making it useful to optimize multiple objective problems better as it is flexible (meaning it can be formulated towards many types of problems, e.g. max return, min risk), and it can handle complex constraints (also common in portfolio optimization)
[121] NISQ-HHL Portfolio optimization for near-term quantum hardware **(JP Morgan Chase)** **(Dylan Herman et al., 2021)**	As multiple components of current HHL are unsuitable to be applied to NISQ hardware, this paper introduces the NISQ-HHL, which is the first hybrid formulation of HHL suitable for small-scale PO instances. The NISQ-HHL is used in an experiment on a real quantum device to show its effectiveness	Objective(s): - Propose the NISQ-HHL formulation, where HHL is improved via mid-circuit measurements, Quantum Conditional Logic (QCL) enhanced QPE (which is the standard method used in HHL), and qubit reset and reuse *(which ensure fewer qubit needs for calculations, and reduced requirements for qubit connectivity, thereby making it more efficient)* - Furthermore, make use of a new efficient procedure to scale the matrixes used (e.g. covariance matrix) for better accuracy of end results. - Experiment with the NISQ-HHL on a real quantum computer with a 2-asset PO problem form the S&P 500. - Formulate the Markowitz's mean-variance model as a Quantum Linear Systsems Problem (QLSP). As the HHL algorithm is designed to solve such a problem.	Quantum system: Real quantum hardware (Trapped-Ion Honeywell H1system), and for certain comparison simulated hardware. Algorithms used: NISQ-HHL Methodology: Optimization Use case: Portfolio optimization	Fidelity = a measure of how close probability distributions are to each other, thereby signifying degree of accuracy. Ancillary qubits = qubits that are not mpart of the main computational qubits that directly represent the problem's fdata, but rather

	This paper proposes to make HHL more scalable.	- Test the difference between the use of QCL enhance QPE, and standalone QPE for estimating eigenvalues. - Experiment with NISQ-HHL on two further 6-asset and 14-asset PO problems with a simulator and decipher its performance against uniformly controlled rotations (which are employed in the traditional HHL algorithm for eigenvalue estimation) Dataset specifications: - Two PO problems with 6 and 14 assets from the S&P 500 index formed as a QLSP problem. 6 ancillary qubits used in both cases to increase efficiency. Results: QCL-QPE method compared to standalone QPE: - QCL enhanced QPE uses less qubits for the same problem instance than standalone QPE, thereby showing increased efficiency. Furthermore, as number of bits increase (complexity), the number of qubits stays the same for QCL-QPE as opposed to standalone QPE. - Results on the real quantum hardware shows that the fidelity of QCL-QPE is better than standalone QPE. NISQ-HHL performance *(For the 6-asset and 14-asset PO problem it was found that the circuits were very deep, making real hardware execution infeasible, therefore simulation was used for analysis)(for the 2-asset problem, the Honeywell quantum computer was used)*: - NISQ-HHL circuits demonstrated reduced depth and improved precision in rotations, leading to better performance. - 14 qubits total were needed for the 6 asset problem, and 16 qubits total for the 14 asset problem. - For the experiment, the results showed high inner product values being found (close to 1), meaning that the algorithm is accurately solving the problems. - The algorithm showed better performance for the larger 14-asset problem, thereby showing its increased performance as complexity increases. - Compared to the uniformly controlled rotation in the normal HHL algorithm, NISQ-HHL performed better in terms of efficiency, using less rotations *(4 instead of 64 for 6-asset PO, and 5 compared to 64 in the 14-asset PO),* and having lower circuit depth *(1,877 for the 6-asset PO instead of 12,911, and 6,514 for the 14-asset PO instead of 11.786 for the uniformly controlled rotations),* thereby showcasing that the NISQ-HHL can facilitate a lessening in the computational resources needed for HHL. - Accuracy of NISQ-HHL was also perceived to be higher than with the uniformly controlled rotations. - NISQ-HHL demonstrated superior performance in terms of fewer controlled rotations and reduced		qubits that are used in quantum computation to facilitate efficiency and reliability of the quantum algorithm, which they are also used for in this paper.

		- circuit depth while maintaining high accuracy in the inner product values. - NISQ-HHL was successfully implemented on the Honeywell System Model H1 to solve a portfolio optimization problem involving two S&P 500 assets.		
[98] Financial Index Tracking via Quantum Computing with Cardinality Constraints (Multiverse Computing; Quantum Computing Services; Advanced Analytics; Donostia International Physics Center; Ikerbasque Foundation for Science) **(Samuel Palmer et al., 2022)**	In this paper, it is demonstrated how non-linear cardinality constraints can be applied in real-world asset management to quantum PO. Furthermore, the methodology is applied to a practical problem of enhanced index trading.	Objective(s): - Propose a quantum model based on quantum annealing for solving of a cardinality-constrained Markowitz PO problem. - Form the PO problem as a QUBO formulation to be solved via the model. - Experiment with the model on a proposed PO problem with different problem sizes and qubit numbers used (400 – 3000), where the objective is to replicate the behaviors of a larger financial index of assets using a smaller sub-set of assets (index tracking), where error is tracked by measuring, he deviation of the solution forms the index. Dataset specifications: - Historical data consist of the daily returns from the Nasdaq 100 and S&P 500, the period form when this data is taken covers the period JUN/01/2021 to MAY/28/2022. A single asset may have a max holding of 20% in the portfolio. Tests are performed using different problem sizes and differing numbers of qubits. Results: - It is observed that the success rate of finding feasible portfolios is very high, close to 100% for the model, indicating that the cardinality-constrained model is extremely effective and reliable. - As the number of assets in the cardinality-constraint increased, the distribution of errors improved, meaning more accurate results. - The most optimal portfolio found had extremely low tracking error, almost completely tracking the given indexes, this was done for both a cardinality constraint of 25 and 75. - Smaller portfolios showed less ability to track the index to a high degree, but still performed well - As for the S&P 500 index, the model yielded good tracking results, with minimal volatility errors, and a	Quantum system: Quantum Annealer (D-Wave LEAP Hybrid solver) Algorithms used: Quantum Annealing Methodology: Optimization Use case: Optimizing a portfolio for index tracking.	Reason for cardinality-constraints: the decision to use these constraints can be driven by reducing management costs, transaction costs, or portfolio complexity, or by other investor preferences.

		- low median relative error, indicating good overall tracking performance. - For the experiment, using a cardinality-constraint of 50 assets, the proposed model performed - For enhanced index trading, the method was able to "construct smaller portfolios that significantly outperform the risk profile of the target index whilst retaining high degrees of tracking" (p. 1) - Overall, the model showed that it is possible to successfully use quantum optimization in the tracking of financial indexes. Important notes: - Introducing the cardinality-constraint makes the PO problem a non-convex problem. - Cardinality constrained PO problems are very complex to solve, as it limits the number of assets a portfolio can use to solve the target objective. - "Previous work involving cardinality-constraint optimization has primarily relied on the use of heuristic algorithms such as genetic algorithms, or classical approximations, which do not scale well for large portfolios and are not practically reliable" (p. 2)		
[97] A Quantum Computing-based System for Portfolio Optimization using Future Asset Values and Automatic Reduction of the Investment Universe (TECNALIA BRTA; Serikat) (Eneko Osaba et al., 2023)	This paper entails a quantum computing-based system for portfolio optimization with future asset values and automatic universe reduction (Q4FuturePOP) This system proposes the following innovations: 1: the tool is developed for working with future prediction of assets, instead of historical values 2: The tool includes an automatic universe reduction module to improve efficiency. Lastly, a brief preliminary performance review is discussed considering the system.	Objective(s): - Develop a Quantum Computing-based system (Q4FuturePOP) that optimizes asset-allocation with the objectives of maximizing expected returns and minimizing the financial risk. This system follows the Markowitz POP formulation - Using future projected values *(meaning that calculations are made via projected values of assets instead of historical data, and weights chosen for the assets are based on future predictions of the stock),* and automatic universe reduction *(where a representative good sub-group of the initial pool of assets is chosen and further improved upon to find the optimal asset allocation)*, reduce the complexity of the problem. - Then use the model on an experiment from the dataset below, where results are benchmarked against a historical set of portfolios obtained from Welzia Management company to serve as a baseline. - The experiment includes the data below, however, the data is split up into 6 different use cases that are 12 to 28 months long Dataset specifications: - 53 daily values of different assets from the period 01/01/2010 to 13/12/2022, this dataset is ultimately split up into 6 instances ranging from 12 to 28 months (with respectively 45, 43, 35, 38, 40, and 53 assets)	Quantum system: D-Wave Advantage 6.2 (5610 qubits) Algorithms/system used: Q4FuturePOP Methodology: Optimization Use case: Portfolio optimization	

		Results: - Results from the experiment proved to be promising, where they have been approved by experts from Welzia Management Company, thereby giving an indication as to how the industry looks at the problem (as it is usually the case that only academic results are compared with each other, giving no validation from the industry it ought to be used by) - The portfolios made by the model offered better solutions than the portfolios from the experts at Welzia Management in some cases. - Looking at table 1 that shows the results for the 6 instances in the experiment, it can be seen that for 4/6 instances the model performed better in finding higher expected returns than the experts, and 3/6 times it had better volatility or risk results. - This work shows promising results regarding the use of the Q4FuturePOP model with future value prediction and universe reduction strategy for PO optimization. Important notes: - The model consists of 3 parts, 1: A dedicated 'predicted dataset generation model' (PDG), which is used to simulate future asset prices, the PDG comes a step before the AUG, which uses the information from the PDG to find an optimal sub-set of candidates. 2: The quantum computing solver module (QCS), consisting of a QUBO problem builder and a Quantum Annealer solver to solve the QUBO formulated PO problem. 3: the Assets Universe Reduction module AUR, with the main objective to decrease the complexity of the problem by finding a representatively good sub-set of assets to use in the PO solving.		
[99] Quantum Portfolio Optimization with Investment Bands and Target Volatility (Multiverse Computing; Donostia International Physics Center; Ikerbasque	In this paper it is examined how some complex real-life constraints can be incorporated into PO problem, where it is formulated as a QUBO problem and subsequently solved the D-Wave Hybrid and D-Wave Advantage.	Objective(s): - First. Explain how to target optimal investment portfolios with a fixed volatility (risk) - Second, show how to impose investment bands in the computed portfolios - Form the PO problem based on Markowitz's Modern Portfolio Theory with investment band constraints, where the aim is to find the optimal return for a given volatility % - Form the problem as a QUBO formulation to be solved via a quantum annealer - Prove the validity of the model via an experiment by finding an optimal portfolio investment for the S&P 100 and S&P 500 with the D-Wave Advantage quantum annealer. - Constraints used: investment band constraint, target volatility constraint, and a budget constraint.	Quantum system: D-Wave Advantage (hybrid). Algorithms used: N/A Methodology: Optimization Use case: Portfolio optimization	Investment band = an imposed maximum and minimum investment for each asset.

Foundation for Science) **(Samuel Palmer et al., 2021)**		Dataset specifications: - closing prices are taken from 23/04/2021 to 23/04/2021, and covariance matrix for values of 3 months before 23/04/2021, max 10% of the portfolio may consist of one asset. Lastly, data is experimented on using different target volatilities (0.5%, 0.75%, and 1.00%) Results: S%P 100 results: - Sometimes, local minima were found, however, it is mentioned that this could be handled easily through various methods. - The S&P 100 example successfully followed volatility constraints. - As for the different target volatilities with investment bands, the found portfolios adhered to these constraints - The model demonstrated lower risks for the same return compared to random portfolios with the same return - The model demonstrated higher returns for the same level of risk as compared to random portfolios. S%P 500 results: - Target volatility constraints were met, indicating that the method is able to follow provided volatility constraints - For the different target volatility, the optimization method adhered to the specified investment bands and volatility constraints - The proposed portfolios achieved lower risk compared to random portfolios with the same levels of return - The proposed portfolios found higher returns for the same level of risk. - Compared to the S&P equally weighted index (which is also used as a benchmark), the proposed model outperformed the S&P 500 EWI, especially through favoring high-return sectors during COVID. Overall: - Both S&P500 and S&P100 quantum-optimized portfolios demonstrated improved performance over random portfolios and traditional indices, efficiently managing constraints and achieving better returns for the same or lower levels of risk. - This paper showcases the feasibility of a quantum PO model with realistic conditions on quantum computers, showing it to handle investment band and volatility constraints well, and optimize portfolios in a real-world scenario.		

		- "Our results show how practical daily constraints found in quantitative finance can be implemented in a simple way in current NISQ quantum processors, with real data, and under realistic market conditions." (p. 1) - "In combination with clustering algorithms, our methods would allow to replicate the behavior of more complex indexes, such as Nasdaq Composite or others, in turn being particularly useful to build and replicate Exchange Traded Funds (ETF)." (p. 1) Important notes: - It is also assumed that shares can only be sold in large bundles. - Short selling is not allowed. - The proposed model also allows for investment bands for specific sectors. - "To the best of our knowledge, these are the largest portfolio optimizations carried on a quantum computer and under real market conditions" (p. 4)		
[58] Portfolio rebalancing experiments using the Quantum Alternating Operator Ansatz (Rigetti Computing; Commonwealth Bank of Autralia) **(Mark Hodson et al., 2019)**	In this paper, the performance of a discrete PO problem on a gate-model of quantum computing is investigated. Furthermore, the model includes a novel problem encoding and hard constraint mixers for the Quantum Alternating Operator Ansatz (QAOAz) "In this paper we have brought together financial services and quantum software technologists to select, implement, and test a portfolio rebalancing use case using QAOA(z)" (p. 1) The characteristics of the proposed model in this paper are trading in discrete lots, non-linear trading costs, and investment constraints (all to	Objective(s): - Describe the application of QAOA and QAOAz to a PO problem with the named aspects below 1 to 6 - Experiment with the proposed QAOA and QAOAz via an experiment for PO including 1: a one-portfolio instance, and 2: portfolio rebalancing, both under different number of iteration (P) per constraint method, furthermore, compare both methods against brute-force algorithm (classical)(baseline) - Compare the use of soft, and hard investment constrained PO on the mentioned QAOA algorithms. - Incorporate the following in the model: 1: Trading in discrete lots 2: Model uncertainty into the model (thereby addressing this limitation in the traditional Markowitz model) 3: Use an investment constraint that ensures the portfolio to maintain portfolio value during rebalancing. 4: The model incorporates trading costs, assuming fixed costs for each trade (thereby reflecting a real trading scenario) 5: Representation of short, long, no position, long and short positions into the spin states *(simply put, different types of positions for an asset are included into the portfolio to maximize the optimization, however, it does increase complexity)* 6: Other constraints such as max asset holdings and min allocation sizes are used, but not detailed upon in the paper. Overall, the model aims to improve trading strategies by integrating discrete trading practices, market	Quantum system: Gate-based simulator Algorithms used: QAOAz, and QAOA Methodology: Optimization Use case: Portfolio optimization	Portfolio rebalancing = "a periodic asset management process in which traders maintain an institutional portfolio's net value, adjusting asset mix based on institutional advice and hedging risk as market conditions change." (p. 2)

	achieve better accuracy towards practical use and accuracy)	uncertainty, and trading costs into the optimization process. Data specifics: - Australian ASX.20 is used in the period 2017, the data covered 20 stocks and 252 trading days, daily returns are presented for the algorithms to work with. Data for N = 8 stocks were used in the experiments. - Number of iterations for both hard and soft constrained: p = 2,3,4. 20 runs of the algorithm are used for each instance. Results: Evaluation of QAOA, QAOAz, and brute-force for a single portfolio: - Looking at the given figure 8 *(Which shows the performance of the algorithms in solving the soft/hard constrained problems compared to brute-force)*, QAOA with hard constraints outperforms brute-force and soft-constrained QAOA in finding feasible solutions to the problem, Furthermore, QAOAz finds more low-cost feasible solutions than QAOA, it can also be said that QAOAz shows superior performance with respect to a random selection of feasible solutions. - QAOAz consistently returns feasible solutions (100% of the time) - Both QAOA and QAOAz show significant improvement in results compared to a random draw from the solution space. - Both variants of QAOA show a significant improvement over brute force methods, which validates the efficiency and effectiveness of quantum algorithms in navigating large combinatorial spaces. - Incorporating hard constraints directly into the optimization process shows better optimization results than soft constraints as penalty terms. For portfolio rebalancing with QAOA, QAOAz, and brute-force: - The QAOAz demonstrates superior performance in both maximizing returns and minimizing risk compared to the original QAOA and brute force methods. - Both QAOA variants generally perform close to optimal, but the Quantum Alternating Operator Ansatz shows more consistent and reliable results - “Experimental analysis demonstrates the potential tractability of this application on Noisy Intermediate Scale Quantum (NISQ) hardware, identifying portfolios within 5% of the optimal adjusted returns and with the optimal risk for a small eight-stock portfolio.” (p. 1) Overall:		

		- QAOAz performed best among QAOA and brute-force - Hard-constrained problems, and the subsequent method used in this paper to better incorporate hard constraints, showed to garner better results using the algorithms than soft constraints. - QAOA and QAOAz show better results than the classical counterpart, navigating larger search spaces - This study highlights the potential that quantum algorithms on NISQ hardware have, achieving portfolios within 5% of optimal adjusted return and optimal risk for an 8-asset portfolio Important notes: - Scaling the problem might prove difficult due to current NISQ hardware limitations. - Statement: "The potential for QAOA to provide guarantees on performance for problems such as MaxCUT has been demonstrated" (p. 2)		
[106] Quantum portfolio value forecasting **(Multiverse Computing; Institut Für Experimentalphysik; AQT; Ikerbasque Foundation for Science; Donostia international Physics Center)** **(Cristina Sanz-Fernández et al., 2021)**	In this paper, an algorithm is presented that efficiently estimates the intrinsic long-term value of a portfolio of asset using quantum computer, relying on quantum amplitude estimation. Two trapped ion-computers are used to experiment upon with a 5-asset portfolio PO problem.	Objective(s): - develop a quantum method to estimate the intrinsic long-term value of a portfolio of assets, and implement it with real-life data - The intrinsic-value of a portfolio is given by the Gorden-Shapiro model; therefore it is used in this paper in a modified fashion to account for both short-term and long-term growth by incorporating earnings per share and stochastic variables to better approximate asset values over a two-year period *(it is basically used for improved accuracy, creating a greater picture asset value over time horizons, flexibility, and a more precise calculation of portfolio value)* - Compare results of the QMC on an IonQ device, AQTION device, and classical Monte Carlo Dataset specifications: - 5 asset portfolios, with 1000 euros invested in each asset. Bought at market value on 2021/04/25, each with 3 scenarios *(stable, bearish, and bullish, which are accounted for using higher/lower volatility values)* Results: - Looking at the given figures, figure 1 shows how quantum results align closely with classical results, but with lower errors. Furthermore figure 2 shows that QMC achieved a decrease in error with increased amounts of queries, outperforming classical Monte Carlo in term of error reduction efficiency. - Both classical and quantum methods showed that the given portfolio was a worthwhile investment, as the intrinsic value of it was higher than the market	Quantum system: Real trapped-ion computers (IonQ, and AQTION) Algorithms used: Quantum Monte Carlo (QMC) Methodology: Optimization Use case: Portfolio optimization	

		- In the bearish market, the quantum method provided a more accurate estimation of the portfolio, as the classical portfolio overestimated the intrinsic value of the portfolio. - Quantum Monte Carlo methods demonstrated smaller estimation errors compared to classical methods, achieving a quadratic speedup in error reduction - Quantum Monte Carlo methods provide a more efficient and accurate means of estimating asset values, especially under stable or bullish market conditions. - results are consistent with classical benchmarks but result in smaller statistical errors for the same computational cost. Important notes: - Classical Monte Carlo methods in finance often take long running times to solve certain complex problems. - Furthermore, this paper gives examples of existing literature on quantum computers having similar or better results to classical algorithms. - "We choose to work with trapped ions because they provide a natural all-to-all connectivity among the qubits." (p. 1) making it simpler to implement the quantum circuit.		
[114] Solving the optimal trading trajectory using simulated bifurcation **(AlpacaJapan)** **(Kyle Steinhauer et al., 2020)**	In this paper, an optimization procedure based on simulated bifurcation (SB) is used to solve integer PO and optimal trading trajectory problems. SB is then applied to an integer PO problem, showing numerical results for up to 1000 assets.	Objective(s): - Following the mean-variance portfolio description, solve a PO problem using Simulated Bifurcation - Form the PO problem into an Ising problem to be solved via SB - Experiment with the SB on a data pool consisting of up to 1000 assets, where the objective is to find the optimal trading trajectory for a portfolio. In total, 2 experiments take place: 1: Optimal trading trajectory finding with the SB-Algorithm in different risk aversion levels *(low, moderate, and high)* 2: Optimal portfolio with the SB-Algorithm, thereby comparing results with randomly generated portfolios. 3: Finding close-to-optimal solutions for a PO instance, and the challenges that come with it. Data specifications (for the second problem): - An artificial market is created with N different assets, up to 1000. Results: Portfolio optimization problem: - For a small portfolio of 5 assets, the SB algorithm optimized the portfolio correctly, finding 5 assets are close to optimal.	Quantum system: Simulator Algorithms used: Simulated Bifurcation (SB) Methodology: Optimization Use case: Finding the optimal trading trajectory for a portfolio	

		- In an instance with added risk-free asset, the SB algorithm correctly find the optimal portfolios including the risk-free asset. - In a third scenario, where the number of assets are n = 400, the SB - For an N = 1000 assets case, the SB found the optimal solution in roughly 1 second. Optimal trading trajectory: - Looking at figure 11, it can be concluded that as max investment per asset, and asset size increased, the computational time also increased for the SB. However, when the max investment per asset was kept low (e.g. 1, 2, 4), it can be seen that there is no/minimal increase in computing time for increasing number of assets in the data pool - For a low risk aversion instance, the Sb-algorithm mainly focused on maximizing returns, ignoring risk - For moderate risk aversion, the SB-Algorithm only takes risk when returns are high, and the portfolio value was maximized. - On a small, less complex, system, the SB-Algorithm found the optimum among all 2^18 possible trajectories. For larger systems, the Sb-Algorithm found optimal or close-to-optimal results. - For high risk-aversion, the SB-Algorithm minimizes risk completely by suggesting no investment and return levels are ignored due to the risk aversion level. - The SB-algorithm effectively finds optimal or close-to-optimal asset allocation trajectories under different market conditions and risk preferences Finding close-to-optimal solutions: - Finding close-to-optimal solutions requires a lot of finetuning of parameters and other parts of the system, where ultimately the fine tuning showed increased accuracy in finding close-to-optimal solutions. - Furthermore, the proper-finetuning techniques resulted in the avoidance of finding sub-optimal solutions, and the SB-algorithm demonstrated significant computational efficiency and robustness. Overall performance findings: - In terms of scalability, the computation time increased exponentially with system size, the performance still is significantly faster than previous methods suc has branch-and-bound (classical), which took up to 4800 seconds for a 200 asset optimization, with SB performing a 256 asset Po in 4 seconds. - Performance was dependent on the parameters used in the algorithm, if incorrect parameters were used, the - The results indicate significant improvements over existing methods, however, there is still room for		

		improvements in the system, as the performance of some parts is still sub-optimal. - Overall, the SB-algorithm is efficient, faster than classical methods, can find close-to-optimal solutions efficiently, showed great performance in increased complexity problems (only after some fine-tuning of the parameters and the system), Important notes: - "This formulation has already proven to beat state of the art computation times for other NP-hard problems and is expected to show similar performance for certain portfolio optimization problems." (p. 1) - "Note that for smaller systems, different heuristic approaches, like the so called Digital Annealer, might still outperform the SB-approach" (p. 2) - "The SB-algorithm is, to our knowledge, currently the fastest way to solve a fully connected Ising problem and therefore also an ideal candidate to solve the optimal integer portfolio problem in the Ising representation." (p. 2) - An example is taken from another paper, where it showed to solve a fully connected 2000 spin problem		
[65] **GEO: Enhancing Combinatorial Optimization with Classical and Quantum Generative Models** **(Zapata Computing)** **(Javier Alcazar et al., 2022)**	"Regardless of the quantum optimization approach proposed to date, there is a need to translate the real-world problem into a polynomial unconstrained binary optimization (PUBO) expression – a task which is not necessarily straightforward and that usually results in an overhead in terms of the number of variables."(p.1), So the problem addressed in this paper is to translate real-world problems better into a PUBO format, where this aids in solving problems such as best minimum of function calls for a given budget, however, benchmarks for TN-GEO are run in the context of portfolio optimization and therefore it gives insight into an	Findings: (SOTA = state-of-the-art) - In the tests done in the paper, the TN-GEO model was used as a booster, or standalone solver for a portfolio optimization problem with cardinality constraints, where results and objectives are the following; *(objective 1 = choose portfolios which minimize risk or volatility given a specific return* *Objective 2 = choose the best portfolio given a fixed level of risk aversion).* 1. Booster: TN-GEO outperformed the classical strategies, providing more efficiency and effectiveness (p 3-5) 2. Stand-alone: TN-GEO demonstrated superior performance compared to the four tested classical algorithms in the trial, finding better solutions with fewer evaluations p (3 -5) Compared to 9 SOTA optimizers: - 67% of the time, the TN-GEO performed better or equal to the 9 other optimizers. - The TN-GEO algorithm performance is significantly better than GTS and PBILD methods, but according to a Wilcoxon signed-rank test, the null-hypothesis regarding the hypothesis that the median difference between the results of the other algorithms is rejected, meaning that there is no significant difference between the TN-GEO algorithm and the other SOTA optimizers. (p. 6)	Quantum hardware: N/A Quantum algorithm: Tensor-network Generator-Enhanced Optimization (TN-GEO) Methodology: (combinatorial) optimization Use case: Combinatorial optimization (in the case of the paper, an NP-hard version of portfolio optimization with cardinality constraints)	TN-GEO comments: TN-GEO can propose unseen candidates with lower cost function values than classical solvers, which is the first demonstration of such type of model in the context of an industrial application (p.1) Furthermore, in this study, state of the art algorithms are compared to TN-GEO in a generalized version of portfolio optimization (p.1)

	optimized way for portfolio optimization			
[34] Portfolio Optimization of 40 Stocks Using the D-Wave Quantum Annealer **(Chicago Quantum)** **(Cohen et al., 2020)**	In this paper, the use of quantum annealing for portfolio optimization in a US stock environment of 40 liquid equities. Furthermore, this problem is first addressed in a multitude of classical approaches	Objective(s): - Find the best relationship between risk and return for a portfolio in a dataset of 40 US liquid equities. - Approach the same problem using classical methods (brute force, genetic algorithm, random sampling, heuristic approaches, simulated annealer as a Monte Carlo) Results: - Classical approaches: On average, classical approaches performed worse than the quantum annealer, however, the genetic algorithms showed - Quantum annealing: In the case of quantum annealing, a couple of things stick out: First, The D-Wave quantum annealer approaches the efficient frontier in a few cases. Next to that, sometimes lower performing portfolios are suggested. Furthermore, due to the CQNS, more low-risk portfolios are chosen on the efficient frontier, making the results more conservative. - The D-Wave annealer performs well, even better that the simulated Monte Carlo methods, however, it underperforms related to the classical genetic algorithms. - The D-Wave annealer outperforms random sampling on average (showing that it is not picking randomly but better performing ones), - The completion times were fastest in the genetic (and D-Wave seeded) algorithms (3,18 ~ 3,47 seconds), followed by the D-Wave quantum annealer (3,40 seconds), however, the quantum annealer beat all other classical approaches. Important notes: - For the quantum annealing process, an optimal portfolio is seen as one which optimizes the Sharpe ratio. However, computing this in a quadratic from gives some issues in a QUBO format, therefore the Chicago Quantum Net Score (CQNS) solves this problem and can therefore be used to formulate the problem in a QUBO formulation. Genetic D-Wave seeded algorithm is the genetic algorithm that uses more optimal results acquired from the D-Wave quantum annealer as an initial starting point to achieve better end results.	Quantum hardware: D-Wave 2000Q annealer Quantum algorithm: Quantum annealing Methodology: optimization Use case: Portfolio optimization	
[35] Portfolio Optimization of 60	This paper builds upon the work of the optimization with 40 stocks. In this paper,	Objective(s): - Find the best relationship between risk and return for a portfolio in a dataset of 40 US liquid equities.	Quantum hardware: D-Wave 2000Q annealer	As this study was the follow up of the 40 stock version, it had

Stocks Using Classical and Quantum Algorithms **(Chicago Quantum)** **(Cohen et al., 2020)**	the use of quantum annealing for portfolio optimization in a US stock environment of 60 liquid equities. The main objective is to find an optimal risk and return portfolio It is investigated whether quantum annealing can scale up and find a grouping of attractive portfolios as opposed to one. Furthermore, this problem is first addressed in a multitude of classical approaches	- Approach the same problem using classical and hybrid classical/quantum methods (Fat tailed Monte Carlo, genetic algorithm, Simulated annealer, D-Wave Tabu Multistart MST2 samples, D-Wave hybrid sampler) Results/stats: - Classical solutions: 1. Fat tailed Monte Carlo analysis: 221,660 samples, the 'ideal' portfolio was found, will perform well under either large/small solution spaces, it was run twice on the sampling distribution of assets; generating the best and 2nd best answer in both 24 seconds 2. Genetic algorithm: brought out the best attributes among combining two portfolios (this is done over and over to keep generating better portfolio combinations), to find the 'most optimized' portfolio in 7 seconds, and on a D-Wave simulator 48 seconds. 3. Simulated annealer: It either finds the 'most optimal solutions' or 'good solutions' no bad portfolios, portfolio quality increased as the simulator ran longer, it found the optimal portfolio in 15 seconds on the simulator of the company where this paper is from (Chicago quantum), and the D-Wave simulator annealer did it in 11 seconds. 4. D-Wave Tabu Multistart MST2 sampler: this simulated annealer was ran on the QUBO formulation and showed the least attractive portfolios from the QUBO method, the final run took 267 seconds 5. D-Wave hybrid sampler: no valid results from this sampler using the same QUBO formulation of the problem, it does find 'good' portfolios but CQNS score attributed to it are incorrect due to applied penalties (penalties are applied to at least get some good results) - D-Wave Quantum Annealer: The quantum annealer was run repeatedly on the QUBO formulation to accumulate more valid portfolios. 3725 valid portfolios were found within the parameters, better results came from larger portfolios. - It was consistently found that the D-Wave annealer picks portfolios ahead of the efficient frontier. Against 40.000 random portfolios (to show that the annealer does not just randomly pick out portfolios), the D-Wave annealer outperforms at higher risk levels. Furthermore, Portfolios tend to be more risky than classical methods, but still efficient - "D-Wave (annealer) appears to be picking efficient portfolios, even out of a population of average	Quantum algorithm: Quantum annealing Methodology: optimization Use case: Portfolio optimization	considerable improvements and material to learn from, as shown in the results tab.

		results" (p. 14), the efficient frontier is constantly found Overall: - Comparative analysis show the best method (again), was the genetic algorithm, it found the ideal CQNS score in the least amount of time, followed by the D-Wave simulated annealer, Bespoke simulated annealer, D-Wave quantum annealer, Fat tailed Monte Carlo, and the D-Wave genetic algorithm - The D-Wave Tabu Sampler, and D-Wave Hybrid sampler were dead last due to them not finding ideal CQNS scores, bad portfolios, and long run times. - The quantum annealer comes close to the best classical algorithms used, as shown above. Important notes: - The difference with the paper considering 40 stock indexes is that this paper: a) Considers 60 stock indexes from the US market b) Quantum annealing is benchmarked against more advanced classical methods c) It is investigated whether quantum annealing can find groups of attractive portfolios as opposed to one d) Prior formulations of the Chicago Quantum Net Score are kept - "We performed our research during a time of market increases for the largest companies, and a relatively low interest rate environment. Our analysis used a risk-free rate of 1%." (p. 2) "Our model does use prior year trading history to pick its portfolios." (p. 2)		
[33] Picking Efficient Portfolios from 3,171 US Common Stocks with New Quantum and Classical Solvers **(Chicago Quantum)** **(Cohen, Jeffrey & Alexander,**	In this paper, 3.171 United States common stocks are analyzed to create an optimal portfolio based upon the Chicago Quantum Net Score (CQNS), which is used to quantify the desirability of the portfolio generated "We begin with classical solvers, then incorporate quantum annealing." (p. 1) In this work, the pool of stocks is run through a classical solver to find the most	Objective(s): - Create an optimal portfolio in 3.171 United States common stocks using quantum annealing via simulated bifurcation - Create an optimal portfolio in 3.171 United States common stocks using quantum annealing on the physical D-Wave Advantage quantum annealing computer - Compare results of both methods using CQNS Results: - The classical solvers (e.g. Monte Carlo, Genetic algorithms, simulated annealers) used to find attractive portfolios found multiple good portfolios, including the best one consisting of 134 stocks with a CQNS score of −3.14×10^-3, which suggest a relatively high attractiveness of the portfolio among the datasets - The simulated bifurcation machine showed 'good' solutions, however, it struggled with larger problem sizes.	Quantum hardware: Simulated Bifurcator and the physical D-Wave Advantage quantum annealing computer (5.760 qubits) Quantum algorithm: Quantum annealing (and results are benchmarked by CQNS) Methodology: Optimization Use case: Portfolio optimization	The CQNS is a measure/computational technique that evaluates the attractiveness of a portfolio, where the closer the value is from zero (negatively), the better or more attractive the portfolio is (at least in the case of this paper, this could change in accordance with other functions/objectives from other studies) , where the portfolio

Clark. 2020)	attractive portfolios that can be run on quantum annealers, then the best stock portfolios are taken and ran through additional solvers to find the most attractive portfolios out of the bunch	- There were some challenges with the D-Wave quantum annealer, mainly; long waiting times between runs, high chai break rates, and difficulty embedding large problem sizes - The best run with the quantum annealer had a CQNS score of -1.69 x 10^-3 - In the case of this paper, classical solver demonstrated; quicker results, better results, indicating that at the time this paper was made, classic/simulated methods outperform those run on physical ones. Still, simulated bifurcation showed the best results, thereby showing that there is great potential in real quantum hardware. Important notes: - This paper does not claim to have found the most optimal solution, rather it mentions that all solution found are 'good' solutions which measure better empirically by their stock performance than other similar methods. Lower CQNS scores indicate better portfolios in this paper		having a negative CQNS score indicates it not being optimized, but still better than most alternative portfolios. Furthermore, in the next paper it is used as a way to compensate for the shortcoming of the QUBO model in translating the Sharpe ratio into its format. Chain break rates = disruptions or failures in the chain of qubits that are connected, thus meaning that the D-Wave quantum annealer was less reliable when it comes to performance Embedding large problem sizes = the process of transferring a large and complex optimization problem into a physical system
[36] End-To-End Resource Analysis for Quantum Interior-Point Methods and Portfolio	"We study quantum interior point methods (QIPMs) for second-order cone programming (SOCP), guided by the example use case of portfolio optimization (PO). We provide a complete quantum circuit-level	Objective(s): - Develop the QIPMs for the use case of portfolio optimization (max return, min risk) - Estimate the exact resource cost of QIPM for a given PO problem with up to 120 assets, which would need up to 8 x 10^6 qubits (which is far beyond what current quantum hardware is possible of) - Put into perspective the practical quantum advantage, and the current bottlenecks, that the QIPM could have by applying it to a PO use case and benchmarking it against classical solvers.	Quantum hardware: N/A Quantum algorithm / method: Quantum Interior Point Method (QIPM) with Quantum Linear System Solver (QLSS)	(Quantum) interior point methods = finding optimal solutions to an objective problem by slowly moving to the optimal solution through multiple iterations within set parameters

Optimization **(AWS, Golman Sachs)** **(Dalzell et al., 2023)**	description of the algorithm from problem input to problem output, making several improvements to the implementation of the QIPM" (p. 1)	- Convert the PO problem as an SOCP so that it can be solved by the QIPM - Use the QLSS algorithm on QIPM to solve the SOCP converted PO problem *(QLSS = Quantum Linear System Solver, it is used because IPM (interior point methods) make use of a linear system of equation, therefore QLSS is needed to perform the step of solving linear equations in the QIPM. The linear part of the QIPM is a subroutine of the greater problem that is better solved using QLSS)* Results: - QIPM could theoretically offer quantum advantage, however, practical implications yet do not show clear improvements over classical methods, significant improvements still need to be made - Current challenges are high variability in tomography precision and the computational resources needed for problems to be solved efficiently on real quantum computers. - In the example experiment, n = 30 stocks were used, and it showed that the duality gap (between risk and return) increased exponentially for more iterations, infeasibility increased exponentially. And for scaling using various other portfolio sizes and duality gaps, the circuit becomes more sensitive to perturbations - The amount of Quantum RAM needed to perform the given PO problems was computationally infeasible at the moment. - Classical methods outperformed the QIPM, mainly due to current QRAM limitations and large constant factors. - Furthermore, compared to classical methods, QIPMs showed to be constrained in their quantum advantage by practical challenges and resource demands Important notes: - Most current quantum algorithms are hard to test whether they are practically useful, as they are mere heuristic and can only be tested on actual quantum hardware - "QIPMs structurally mirror CIPMs, and seek improvements by replacing certain subroutines with quantum primitives" (p. 2) - "The QIPM is a complex algorithm that delicately combines some purely classical steps with multiple distinct quantum subroutines" (p. 2) - Regarding the QIPM, multiple improvements are made to it before applying it towards the PO problem, for more optimal results. These improvements made are inspired by previous works from other authors.	Methodology: Optimization, and solving of Second Order Cone programs Use case: Portfolio optimization	Second Order Cone Programs (SOCPs) = a convex optimization problem that generalizes linear and quadratic programming, basically making it useful to optimize multiple objective problems better as it is flexible (meaning it can be formulated towards many types of problems, e.g. max return, min risk), and it can handle complex constraints (also common in portfolio optimization) Tomography = used for calibrating quantum gates and circuits Infeasibility = degree to how much the given solution violates given parameters or constraints Duality gap = in essence a gap that shows how optimal the solution is, the less this gap, the more optimal the solution

		The quantum component of QIPM was simulated, as mentioned, current quantum hardware cannot facilitate the problem mentioned.		

Table 7, Insight into literature synthetization process

Paper (57) **(Authors)** **(Year)**	**Challenge addressed / introduction**	**Main findings/purpose**	**Quantum hardware, Quantum algorithm, Methodology, Use case**	**Additional specifics / Explanations**
[1] **Quantum Optimization: Potential, Challenges, and the Path Forward (Abbas Et AL., 2023)**	"In this work we address the potential of quantum optimization from various angles, namely, complexity theory, problem classes and algorithmic design, execution on noisy hardware at scale, and fair benchmarking, while outlining illustrative examples form real-world cases" (p. 2)	Findings: - Complexity theory is useful, but may not always be useful for quantum advantage, therefore underscoring the need to develop and analyze quantum optimization (p.50) - The paper emphasizes the fact that there is a strong need to continue discovering new algorithms and development, as intuition gained from practical tests and new algorithms provides validation and technical advances important to optimization problems (p.50) - There should be a need to establish clear benchmarks, for a reliable interpretation of scientific insight for the broader audience (p.50) Purpose: - The purpose of this paper is mainly to give a comprehensive overview of potential challenges, and emerging research in quantum optimization. - Next to that, this paper ought to be used in this paper as a way to explain general subjects and limitations for quantum optimization	Quantum hardware: N/A Quantum algorithm: N/A Methodology: N/A Use case: N/A	N/A
[3] **FORECASTING STOCK MARKET CRASHES VIA REAL-TIME RECESSION PROBABILITIES: A QUANTUM COMPUTING APPROACH (Alaminos et al., 2022)**	The main problem addressed in this study is the inefficiency and inaccuracy of models that predict stock market crashes, where existing models, despite their high explanatory power, fail to account for time-varying risk premium and is often focused on developed	Findings: - "The methods of Quantum Support Vector Regression, Quantum Boltzmann Machines (QBMs), and Quantum Neural Networks (QNNs) have been used, and the QBMs used have obtained the highest levels of accuracy" (p-3). To test the algorithm made, the above methods have been used and adapted upon to fit the solution.	Quantum hardware: N/A Quantum algorithm/models: Support vector regression Quantum Bat algorithm (svrQBA), Quantum Boltzman Machine (QBM), Methodology:	"In order to improve the accuracy of forecasting stock market crashes models, a comparison of methodologies has been carried out in this study to predict stock market crashes via

	economies, this leads to less accurate forecasts (p. 2-3) "The literature calls for a different recession prediction model, in particular new ones that offer a more accurate to global scenes, and that make comparisons between approaches to obtain better and more accurate results." (p.2)	- Usage of the svrQBA and QBM models showed respectively an increase of 94.59% and 96.22% on average over other models (p.8), and it showed superior results over other studies, therefore optimizing the accuracy of the named quantum algorithms for predicting stock market crashes (p.13) Purpose: - This study gives new insights into a potential new model that can optimize the prediction of stock market crashes, whereby three quantum algorithms are each used to test the proposed model	Optimization Use case: Predicting stock market crashes	real-time recession probabilities and, as a result, a new model that will lead to better estimates on the likelihood of a down-turn and, therefore, a stock market crash, will occur in the future." (p.3)
[4] Quantum Monte Carlo simulations for estimating FOREX markets: a speculative attacks experience (Alaminos et al., 2023)	"In this study, we propose to apply Auxiliary-Field Quantum Monte Carlo to increase the precision of the FOREX markets models from different sample sizes to test simulations in different stress contexts." (p.1) "Our paper analyses USD/EUR and USD/JPY exchange rates in the period 2013–2021. This work compares three Monte Carlo techniques, Markov Chain Monte Carlo, Sequential Monte Carlo, and Auxiliary-Field Quantum Monte Carlo (AFQMC), with the AFQMC technique being the best performer" (p.2)	Findings: - The AFQMC has increased the accuracy of the FOREX market model over the Markov Chain Monte Carlo and Sequential Monte Carlo (classical methods) (p.3) - Through Quantum Monte Carlo Simulation, the study is able to identify possible currency movements in the foreign exchange market (p.3) - The AFQMC model is compared towards two traditional methods, specifically Markov Chain Monte Carlo and Sequential Monte Carlo, where the AFQMC technique outperforms other methods (p.19)	Quantum hardware: Simulated hardware Quantum algorithm: Auxiliary-Field Quantum Monte Carlo (AFQMC) Methodology: Quantum Monte Carlo Use case: Increase the accuracy of FOREX market models	"The present research differs from others in that it compares various Monte Carlo techniques in FOREX markets prediction. Most of the models in previous studies have been dominated by statistical techniques such as ordinary least squares, quantile regression, and recently neural network techniques" (p.3)
[5] A Structured Survey of Quantum Computing for the Financial Industry (Alabereti et al., 2022)	"This survey reviews platforms, algorithms, methodologies, and use cases of quantum computing for various applications in finance in a structured way." (p.1) "We conducted an extensive literature search and designed a multi-layered framework to enable a structured analysis of	Findings: - A morphological box showing exactly how quantum hardware, quantum algorithms, methodologies, and use cases are related. - Furthermore, each use case for certain algorithms and methodologies is elaborated upon to give insight into actual use of quantum computing for finance (e.g. Variational Quantum Eigensolver used for optimization of transaction settlement)	Quantum hardware: N/A Quantum algorithm: N/A Methodology: N/A Use case: N/A	N/A

	the available literature and the use cases described." (p.13)	- This paper serves as inspiration for figure 5. - Specific relation of quantum computing to portfolio optimization is given, and therefore helps to give further insight into quantum computing for portfolio optimization. - The paper highlights that in their literature research, NO paper was found that describes a use case for Quantum Machine Learning (p.13), which is peculiar as other papers do mention use cases for Quantum Machine Learning. Purpose: - This paper gives a great overview and visualization through e.g. a morphological box of how quantum computing can be used in the financial industry, from the current state of quantum computing to a framework for a systematic analysis of proposals for the use of quantum computing in finance. (p.1)		
[6] **Classical versus quantum models in machine learning: insights from a finance application (Alcazar et al., 2020)**	"a direct comparison of the expressive power and efficiency of classical versus quantum models for datasets originating from real-world applications is one of the key milestones towards a quantum ready era. Here, we take a first step towards addressing this challenge" (p.1) In this paper Restricted Boltzmann Machines (RBMs) (classical) are compared to Quantum Circuit Born Machines (QCBMs) (quantum) To assess the performance of the QCBMs on real-world data sets, probabilistic scenarios from portfolio optimization are taken,	Objective of the test between QCBMs and RBMs = select optimal investment portfolios whilst either maximizing returns with minimal risk, or maximizing return for a given level of risk, following the optimization goal of Markowitz. This can be done whilst imposing constraints, such as a cardinality constraint in the number of assets (p. 3) Findings: - The quantum model clearly imposed outperformance the classical machine learning model. (p. 5-6) - A scatterplot was made to better visualize the results between the QCBM and RBM models. The scatterplot shows superior performance of the QCBM model, where it wins in close to 100% of the instances (p. 5-6) - As problem size increased, the QCBM model performed increasingly better compared to the RBM model (p.5-6)	Quantum hardware: Simulated on ion-trap quantum computers Quantum algorithm/model: Differentiable Quantum Circuit Learning (DDQCL) used on the Quantum Circuit Born Machines model (QCBMs model) Methodology: Optimization / machine learning Use case: Portfolio optimization	"To date, experimental implementations of QCBMs via DDQCL have been implemented in ion trap and superconducting devices." (p.1)

	specifically data from asset subsets of the S&P500 stock market index (p.1)			
[7] **Enhancing combinatorial optimization with classical and quantum generative models (Alcazar et al., 2024)**	The focus in this paper is on Generator Enhanced Optimization (GEO), which is a framework that leverages any generative model (e.g. classical, quantum, or quantum-inspired), where in this paper is mainly focused on a quantum-inspired version of GEO named TN-GEO (p. 1) With this TN-GEO strategy, benchmarks are made in the context of the canonical cardinality-constrained portfolio optimization problem through constructing situations based on S&P 500 and other financial stock indexes. (p. 1) The aim is to show the real value that these quantum-inspired models have on industrial application. Lastly, a comparison is made between TN-GEO and state-of-the-art algorithms (p. 1)	Objective: The text highlights the need for a quantum optimization strategy that can work directly on arbitrary objective functions, thereby bypassing the translation and overhead limitations, meaning that the process of difficult optimization problems Would become more efficient and applicable to more real-world problems as, for example, the number of variables used in these calculations give current computational methods a hard time. In the experiment for cardinality-constrained portfolio optimization to compare results of TN-GEO with classical approaches, the TN-GEO is used as a standalone-solver, and as a booster to enhance existing solvers: - TN-GEO standalone: Portfolio optimization without relying on intermediate results from classical solvers using S&P 500 portfolio, with the aim to reduce risk and increase expected returns. - TN-GEO booster: use intermediate results from simulated annealing (SA)(or combined results from SA and previous algorithms) as training data for the TN-GEO, and then compare performance between classical algorithm results and TN-GEO booster Findings: - TN-GEO as booster: on average, the TN-GEO booster outperformed classical-only algorithms, and the the performance of the TN-GEO booster (compared to classical-only) increased as the number of variables increased with tests performed in the ranges of N=30 - N=100 variables. Furthermore, *"The observed performance enhancement compared with the classical-only strategy must be coming from a better exploration of the relevant search space"* (p. 4) - TN-GEO as standalone: the TN-GEO shows performance compared to the classical solvers across all scenarios (number of assets: 30;50;80;100)	Quantum hardware: Simulated hardware Quantum (inspired) algorithm: TN-GEO Methodology: Optimization Use case: (cardinality-constrained) Portfolio optimization	N/A

		Comparison with state-of-the-art algorithms (SOTA): TN-GEO was compared to SOTA algorithms and showed: - In 67% of the instances, TN-GEO either draws or outperforms the SOTAs - In all pairwise comparisons with SOAT algorithms and the TN-GEO, TN-GEO wins more than 50% of the time, every time *(null hypothesis ("there is no difference between results of SOTA and TN-GEO") rejected every time with Wilcoxon signed-rank sum tests to validate results)*		
[8] Alleviating the quantum Big-M problem (Alessandroni et al., 2023)	Quantum optimizers often need to reformulate constraints to fit the well-know QUBU format, however, current QUBO translators often fail to acknowledge the weight M of penalty terms (p. 1) Therefore, in this paper a new practical translation algorithm is proposed to outperform previous methods (p. 1) After presenting the algorithm, it is then used in portfolio optimization instances to show significant advantages in time to solution and solution quality (p.1)	Objective: Reformulating QUBO problems for quantum solvers so that they can operate more efficiently and effectively. This is mainly done by addressing "the big-M problem", which is the weights that penalties have in this algorithm, something which should be carefully optimized for optimal and efficient results according to the paper. However, the main focus for this paper on portfolio optimization is the results it has on quantum portfolio optimization Results for quantum portfolio optimization: The improved QUBO translator formulation was tested upon the Markowitz model for maximizing returns and minimizing risk, results showed: - Using MSDP when translating problems to a QUBO format shows a significant advantage over traditional penalty optimization approaches - As the complexity of the problem grows, using MSDP to reformulate problems to a QUBO format shows increasing efficiency and quality of results compared to traditional penalty optimization approaches - Using a 6-qubit trapped ion quantum computer from IonQ showed that MSDP formulations give out a superior probability of measuring the optimal solution	Quantum hardware: IonQ (company) trapped-ion device Aria-1 Quantum algorithm: QUBO (reformulation method), where formulation of optimizing penalty weight is called MSDP (Minimum Spectral Gap Differential), all in all we can call it QUBO-MSDP Methodology: (Penalty) Optimization Use case: Portfolio optimization	Spectral gap = the energy difference between optimal and suboptimal solutions, a lesser spectral gap is better as it leads to more effective and efficient results
[9] Quantum Chameleon Swarm with Fuzzy Decision Making Tool for	"This study develops a Quantum Chameleon Swarm Optimization with Fuzzy Decision Making Tool (QCSO-	Objective: presenting a novel technique that tries to optimize financial risk management, especially predicting financial distress in firms, the proposed tool (QCSO-FDMT is then benchmarked using two datasets; Australian	Quantum hardware: N/A Quantum algorithm: QCSO-FDMT	Fuzzy = a decision making criteria that is used when data is uncertain or

Financial Risk Management (Alkhafaji et al., 2023)	FDMT) for Financial Risk Management. The purpose of the QCSOFDMT system is to determine if the financial firm undergoes distress or not."(p. 1)	credit dataset, and Analecta dataset, both of which are used to test the algorithm/tool to detect financial distress/risk) Results: - Australian credit dataset: QCSO-FDMT outperformed other classical and modern machine learning models, having the highest accuracy of predicting financial distress, with a 98.98% accuracy. All other methods showed results below at least 97.10%, - Analecta dataset: QCSO-FDMT outperformed other classical and machine learning algorithms, showing a 94.44% accuracy of predicting financial distress, all other methods showed results below 93.60% To conclude, the QCSO-FDMT technique is a highly effective method to detect financial distress in companies as compared to current methods already being used.	(Quantum Chameleon Swarm Optimization (which is the algorithmic part) with Fuzzy Decision-Making Tool) Methodology: Optimization Use case: Fuzzy financial risk management	incomplete, it tries to compensate for this lack of certainty or completeness The algorithm utilizes swarm-intelligence based optimization inspired by the behavior of chameleons, thereby stating that the algorithm can take account of many things at one time, like a chameleon.
[11] Quantum-inspired algorithms in practice (Arrazola et al., 2020)	"We study the practical performance of quantum-inspired algorithms for recommendation systems and linear systems of equations. These algorithms were shown to have an exponential asymptotic speedup compared to previously known classical methods for problems involving low-rank matrices, but with complexity bounds that exhibit a hefty polynomial overhead compared to quantum algorithms" (p. 1), with the last part meaning that quantum-inspired algorithms show better results than classical options, but come at considerable additional computational costs (e.g. energy usage,	Results of Quantum-inspired Algorithms benchmarked against portfolio optimization with stocks from the S&P 500: - The quantum-inspired algorithm required substantial time to estimate coefficients and sampling, using 114.15 seconds to run the full calculation. In comparison, direct calculation methods using for instance the Frieze-Kannan-Vempala Algorithm (which is the equivalent of a classical solving method) performed these tasks much faster (0.15 seconds). Increased running time for the quantum-inspired algorithm was due to coefficient estimation and sampling, as opposed to the direct calculation method of the FKV algorithm - The quantum-inspired algorithm showed multiple errors in approximating the solution, showing multiple dis-promising statistics considering the discrepancies between approximate and real solutions. As the quantum-inspired algorithm used sampling, it was prone to more error due to sampling	Quantum hardware: N/A Quantum algorithm: Quantum-inspired algorithms Methodology: optimization Use case: Portfolio optimization	Asymptotic speedup = an increase in performance of usually an algorithm as the size of the input grows larger Recommendation systems = software algorithms and techniques designed to suggest items worth of notice to users, it provides personalized recommendations (Low) Rank = the number of independent rows or columns in a matrix which is calculated from, low rank means a

	time) than real quantum algorithms Furthermore, these quantum-inspired algorithms are benchmarked using, but not included to, portfolio optimization	noise and more estimation that needed to be done - “Quantum-inspired techniques only become advantageous for problems of extremely large dimension” (p. 18) To conclude, overall, the paper showed that quantum-inspired algorithms provide reasonable low errors and short computational times in general, but in the case of this paper (with increased rank and condition numbers), the quantum-inspired algorithms had more errors and computation times, mainly due to the way the algorithms computed the problems (which is stated above). Furthermore, direct calculation methods such as the Frieze-Kannan-Vempala (FKV) algorithm used, operated efficiently without the need for extensive sampling or coefficient estimation, As direct methods such as the FKV are tailored to exploit the low-rank structure of the dataset, it will be faster than the quantum-inspired model as the quantum-inspired model calculates differently and is tailored to this low-rank situation.		matrix which is characterized by it having less columns or rows than the minimum that is allowed (mostly to increase efficiency)
[13] **A Study of Scalarization Techniques for Multi-Objective QUBO Solving (Ayodele et al., 2022)**	“QUBO solvers are single objective solvers. To make them more efficient at solving problems with multiple objectives, a decision on how to convert such multi-objective problems to single-objective problems need to be made” (p. 1) “In this study, we compare methods of deriving scalarization weights when combining two objectives of the cardinality constrained mean-variance portfolio optimization problem into one” (p. 1),	Objective: Many optimization problems have more than one objective (e.g. the Cardinality Constrained Mean-Variance Portfolio Optimization Problem, which entails selecting assets that both maximize returns while minimizing risk). Normally, these multi-objective problems ought to be compiled into a single objective problem before solving them, so that they are pareto efficient, via quantum hardware such as Ising Machines. The objective of this paper is to derive scalarization weights so that less explored parts of the pareto frontier can be explored which normally cannot, or are usually undesirable due to certain factors (e.g. due to increased complexity, bias from the algorithm, or objective dependency of the algorithm) In this study, three methods of generating scalarization weights within the given objective for QUBO (minimizing risk, and maximizing returns) are explored, these three methods were applied to a QUBO formulation of CCMVPOP; iterative, random, and uniform Results:	Quantum hardware: Digital annealer (from Fujitsu) (Ising machine) Quantum algorithm: QUBO Methodology: Optimization Use case: Scalarization optimization for Cardinality Constrained Mean-Variance Portfolio optimization (CCMVPOP)	Cardinality constrained = a restriction/constraint on the number of assets that can included into a portfolio Scalarization (weights) = scalarization is the act of combining multiple objectives into a single function, hereby weights are assigned to each element of the combined objective function. Pareto frontier = a set of all optimal solutions where no solution can be improved without

		- The 'iterative' approach showed advantages over random and uniform methods in terms if finding diverse and high-quality solutions - The 'iterative' methods ability to explore certain regions of the pareto front not normally explored showed better trade-off solution in multi-objective scenarios (max return, min risk) - Uniform scalarization showed the most consistent and highest number of non-dominated results in multi-objective problems - "Quadratic Unconstrained Binary Optimization (QUBO) formulations of optimization problems. This is a common formulation used by hardware solvers classified as quantum or quantum-inspired machines. They have been shown to achieve a speed up compared to classical optimization algorithms implemented on general purpose computers"(p. 1) Ultimately, this study shows that attention given on scalarization methods can improve results regarding certain multi-objective problems such as portfolio optimization		negatively influencing another Uniform scalarization = distributes weights evenly across the objective Random scalarization = distributes eights randomly Iterative = distributes/adjusts weights according to desired pareto front, thereby exploring less explored regions
[14] Wasserstein Solution Quality and the Quantum Approximate Optimization Algorithm: A Portfolio Optimization Case Study (Baker, Jack S. & Radha, Santosh Kumar, 2022)	Quantum Processing Units (QPU can be very suitable for optimizing a portfolio of financial assets (p. 1) "We benchmark the success of this approach using the Quantum Approximate Optimization Algorithm (QAOA); an algorithm targeting gate-model QPUs." In this paper, the aim is to find the highest quality of solutions using the QAOA algorithm on the optimization of financial asset portfolios using QPUs	Objective: Assess the quality of results/performance of the QAOA algorithm using QPUs by solving the Mean-Variance Portfolio Optimization problem from Markowitz. These results are then to be compared to eachother. Results: - Hard constrained optimizers are easier to optimize as their landscape is easier to quantify and has more direct parameters, therefore creating a straighter road to the solution so to say, whilst soft constrained optimizers have a more challenging landscape due to their increased flexibility, allowing for a broader range of possible solutions, - The main conclusion from the paper is that QAOA algorithms show promising performance for solving MVPO problems, especially when applied to gate-model Quantum	Quantum hardware: Gate-model quantum processing units simulated on IBM, IonQ, Rigetti, and using real hardware Quantum GPU hardware (QULACS, ASPEN_10, IBMQ_Manila, IBMQ_Bogota, IBMQ_Quito, IBMQ_Belem, and IBMQ_Lima) Quantum algorithm: QAOA Methodology: Optimization Use case: Portfolio optimization	QPUs = quantum processing units, which are advanced computers using quantum mechanics to perform calculations

	To benchmark performance the variable 'η' is used, 'η' stands for "the normalized and complementary Wasserstein distance", however, the most important part about this is understanding whether 'η' from the resulting tests shows good or bad quality of results, usually a lower 'η' means better quality of results because the difference between achieved and desired probability distributions are smaller (which is more desirable) Next to that, a variable used to test quality is 'p ', which is related to "the circuit depth of the QPU system using QAOA", simply explained: the number of operations that are applied in sequence on the specific number of qubits (more operations = more complex computations = more accurate results = more computing time) Furthermore, tests are performed with differing numbers of qubits.	Processing Units (QPUs) such as those from IBM, IonQ and Rigetti. - On another note, as a big aim of the paper was to address how 'η' can be used to assess performances of GPUs on QAOA problems, specifically MVPO, it showed 'η' to be a great standardized way to evaluate performances, Among the real-quantum hardware, general observations were: - that soft-constrained problems outperformed the hard constrained problems - Compared to previous hardware, the newer hardware outperformed it, showing increased circuit depth and efficiency - Rigetti's Aspen-10 and IonQ's 11-Q showed robust performance - Significant variability in solution quality was observed across repeated runs, attributed to factors like qubit assignment differences, calibration issues, and time-varying qubit coherence times. - Current QPU benchmarks and performance are not reliably predictable based on general metrics like quantum volume (QV); application-specific benchmarks are necessary.		
[16] **Grover Mixers for QAOA: Shifting Complexity from Mixer Design to State Preparation (Bärtschi, A. Eidenbenz, S., 2020)**	"We propose GM-QAOA, a variation of the Quantum Alternating Operator Ansatz (QAOAz) that uses Grover-like selective phase shift mixing operators."(p. 1)	Objective: - "we are given a number of assets and a portfolio of short and long positions on these assets. Periodically, such a portfolio has to be rebalanced in order to maintain in order to react to market and risk changes." (p. 9)	Quantum hardware: N/A Quantum algorithm: GM-QAOAz Methodology: Optimization	Hamiltonian simulation error = The discrepancy between the evolution of a real quantum system when used over time and that of a simulator that

	"We illustrate the potential of GM-QAOA on several optimization problem classes" (p. 1)	- furthermore, the GM-QAOAz algorithm is then compared towards the QAOAz algorithm Results: - Following the discrete portfolio rebalancing problem, both algorithms show some similarities, however, GM-QAOA was able to better focus on creating an equal superposition of all feasible states, meaning can more effectively explore the solution space and create more optimal solutions - Furthermore, resulting from other tests, GM-QAOAz showed multiple strengths: it can reduce circuit complexity compared to existing mixers, and it can even, as a first in the industry, stay in the feasible space of solutions and provide transition between all states in this space whilst mixing unitaries *(mixing unitaries = operators that intend to change the amplitudes of different quantum states, with the purpose of creating a larger solution space.)* Important notes: - "GM-QAOAz works on any NP optimization problem for which it is possible to efficiently prepare an equal superposition of all feasible solutions; it is designed to perform particularly well for constraint optimization problems, where not all possible variable assignments are feasible solutions." (p. 1) - GM-QAOAz is not susceptible to Hamiltonian simulation error compared to standard mixers for QAOAz, and solutions with the same objective value are always sampled with the same amplitude	Use case: Discrete portfolio rebalancing	tries to imitate such a real system, basically meaning in this paper that the simulated system is alike to a real system when it comes to the change it perceives over time in its quantum state
[17] Quantum optimization via maximally amplified states (Bennett, Tavis Wang, Jingbo B., 2021)	"This paper presents the 'Maximum Amplification Optimisation Algorithm' (MAOA), a novel quantum algorithm designed for combinatorial optimization in the restricted circuit depth	Objective(s): *1.* Formulate MAOA mainly by using the Quantum Walk Optimization Algorithm as a way to achieve maximally amplified states in a low-convergence regime. *(basically, we want the (possibly) best solutions grouped together in a place where finding these solutions is maximized, this means that the quantum system is*	Quantum hardware: Simulated hardware Quantum algorithm: RGAS and MAOA (compared to each other, classical algorithms, and Grovers Adaptive Search (GAS)	Maximally amplified state = a state in a quantum system that can be achieved through some methods (in the case of this paper by using the Quantum Walk Optimization

	context of near-term quantum computing." (p. 1) Furthermore, another algorithm is synthesized, the 'Restricted Grover Adaptive Search' (RGAS) algorithm, which is a modification of the existing 'Grover Adaptive Search' algorithm Additionally, MAOA and RGAS are compared to each other and the QAOA algorithm Next to that, they are simulated on multiple types of problems, including a computationally demanding portfolio optimization problem	*tuned to produce a high probability of measuring the best solutions while avoiding the chaotic behavior and inefficiencies.)* 2. Formulate RGAS by placing a limit on the rotation count in GAS as current quantum devices cannot handle certain rotation amounts efficiently enough to create optimal results 3. Benchmark MAOA and RGAS against each other, classical sampling and normal Grover Adaptive Search in a portfolio optimization problem Results - Following the portfolio optimization problem (where the probability success relates to to finding the single highest return portfolio within the lowest 10% of risk), it is shown that MAOA performs best, consistently outperforming RGAS, GAS, and classical sampling. - Next to that, classical sampling comes nowhere near the speed (number of iterations / rotations) used to find the optimal solutions - Next to that, RGAs outperforms Gas, showing that GAS can currently be optimized by restricting it (as mentioned earlier, current quantum systems lack the ability to accurately and efficiently make use of unrestricted counts of rotations due to high complexity as more rotation are employed) - Furthermore, Important notes: - MAOA a opposed to RGAs and GAS had the ability to explore 2 dimensional solutions *(e.g high return and low risk),* therefore, RGAS and GAS needed to operate on such a problem translated into a 1 dimensional problem statement	Methodology: Optimization Use case: Portfolio optimization	Algorithm) to increase the probability of the most optimal solution occurring, thereby making it easier for something like the MAOA algorithm to find the most optimal solution Low convergence regime = A range, a threshold within the number of solutions is few enough that the MAOA algorithm has a significantly less likelihood of experiencing any issues in efficiency or quality of solutions when operating. Rotation count = the number of iterations applied to amplify the probability of finding the optimal solution Iteration = a single cycle or repetition of specific steps

[18] **Forecasting financial risk using quantum neural networks (Bouchti et al., 2018)**	In this paper, a novel Quantum Neural Networks are introduced for machine learning in forecasting potential financial risks in a company Furthermore, a method of training these QNNs is introduced Lastly, a new financial risk forecasting model in introduced which will be applied to forecasting risk in Moroccan companies. Afterwards, these results are then compared with Artificial Neural Networks (ANN) (classical approach) "In this work, we introduce the quantum neural networks: a hybrid quantum-classical framework with the potential of tackling high-dimensional real-world machine learning datasets on continuous variables." (p. 1)	Objective: - Develop a QNN model with features that fit toward forecasting financial risk in companies whilst at the same time having features that make it as easy as possible to model. A QNN is proposed that operates much like an ANN, however, the QNN has its functions grounded in quantum mechanics. The QNN is subsequently trained using genetic algorithms to avoid getting into local minima. Results: - The proposed QNN improved prediction efficiency of financial risk in the chosen Moroccan companies compared to classical methods (ANN) - The QNN algorithm provided good approximation results, reduced computing time, and maintained prediction accuracy over classical methods (ANN) Important notes: - The study faced limitations due to a small sample size and the exclusion of non-financial factors	Quantum hardware: Simulated hardware Quantum algorithm: QNNs Methodology: Forecasting Use case: Financial risk forecasting	"Quantum neural networks have been proposed [1], but very few of these proposals have attempted to provide an indepth method of training them. Most either do not mention how the network will be trained or simply state that they use a standard gradient descent algorithm." (p. 1) Local minima = a value that is low considering its neighbors (other groups of values), but is considered high in its own group, thereby making it an undesirable value to find with the algorithm, giving the algorithm the probability to settle for a solution that is suboptimal
[22] **Best practices for portfolio optimization by quantum computing, experimented on real quantum devices (Buonaiuto et al., 2023)**	In this paper, QUBO formulated portfolio optimization is solved using the Variational Quantum Eigensolver (VQE) Algorithm The main outcome of this work consists of finding the best hyperparameters (part of the ansatz) to set in order to find the most optimal solution using VQE, however, in this paper for portfolio optimization, only the	Objective: - Benchmark the VQE against classical algorithms - Benchmark the performance of VQE on real and simulator quantum hardware - Find the optimal investment portfolio by balancing risk and return using certain constraints such as budgets and risk aversion - Formulate the PO problem in a QUBO format, and then approximate the minimum eigenvalue (most optimal solution in this case) by using VQE	Quantum hardware: Different simulated (IBM QASM simulator) and real quantum computers (IBM Toronto, IBM Kolkata, IBM Auckland, IBMQ Toronto, IBM Geneva, IBMQ Guadalupe, IBM Hanoi, IBM Cairo, IBMQ Montreal, IBMQ Mumbai) Quantum algorithm: QUBO formulated PO optimized by VQE	Ansatz = the proposed form of the state in which an objective function is solved on a quantum computer, this state or Ansatz structure is then adjusted to optimize the solution, which is also tested for and used in the case of this paper.

	results using VQE on a portfolio optimization problem are considered Optimization problems are solved in this paper by using simulated and real quantum computers “This work presents solutions to the problem obtained on different quantum computers and with different hyperparameters settings, to find the best practices to perform PO by VQE on real quantum devices.” (p. 2) “Finally, the optimal solutions are compared among those obtained on simulators and on real quantum computers of different sizes and architectures and with the benchmark solution.”(p. 2)	Results (results shown in the paper are based on quality of the optimal solution found and algorithm convergence - For Real quantum devices (results are shown in a graph with the efficient frontier and x = volatility, and y = expected return): IBM Toronto: found the optimal solution, IBM Kolkata: found the optimal solution, IBM Auckland: found the optimal solution, IBM Geneva, IBMQ Guadalupe, IBM Hanoi, IBM Cairo, IBMQ Montreal, IBMQ Mumbai: did not find the optimal solution on efficient frontier - Less than optimal results were mainly caused by the quantum hardware not being good enough in terms of limited quantum volume and circuit depth to compute the given problem. - The classical solution (Branch-and-Bound method) did find the same optimal solution as the QUBO -VQE on different quantum hardware, and was able to solve up to 120 asset portfolios - As for simulated quantum hardware, the experiments used in the QASM quantum simulator from IBM on either noisy (which is done by importing noise from a real quantum computer) and noiseless environments using three possible optimizers from Qiskit (Cobyla, NFT, SPSA) showed that Cobyla persistently provided stable and rapid convergence to finding optimal solutions, NFT exhibited unstable and oscillatory behavior, particularly in noisy settings, and SPSA demonstrated slower convergence with increased variability. Important notes: - For the classical benchmark, the branch-and-bound method is used which is an algorithmic technique particularly useful in discrete and large solution spaces - The dataset used to benchmark VQE and other methods is as follows: the dataset is collected from Yahoo! Finance, using Yfinance (which is an	Methodology: Optimization Use case: Portfolio optimization	Quantum volume = a single number that encapsulates how well a quantum computer can handle quantum computations. Convergence = stability and consistency of the iterative process

		open-source tool) where a small selection of representative global assets are used (e.g. Apple, Netflix, Tesla) - “Results show that both the mapping of the ansatz structure on the hardware topology and the quantum volume is of pivotal importance for reaching the desired convergence. The topology of a quantum computer refers to the physical arrangement of qubits: while ansatzes connecting only the nearest qubits can be mapped efficiently, those entailing long-range connections require an overhead of gates that ultimately increases the depth of the circuit and hence foster an increase of the overall error rate during computation” (p. 11) - The VQE is a hybrid quantum-classical algorithm, whereby the quantum component is the hardware it operates on, the circuits and the ansatz it employs, and the classical component is the optimization of parameters in the quantum circuit to find more optimal solutions		

[24] Backtesting Quantum Computing Algorithms for Portfolio Optimization (Carrascal et al., 2024)	“By backtesting classical and quantum computing algorithms, we can get a sense of how these algorithms might perform in the real world. This work establishes a methodology for backtesting classical and quantum algorithms in equivalent conditions, and uses it to explore four quantum and three classical computing algorithms for portfolio optimization and compares the results” (p. 1) Furthermore, 10.000 experiments are performed under conditions that were found where quantum methods outperform classical methods. Furthermore, the Variational Quantum Eigensolver (VQE) algorithm is analyzed in detail. It is mainly tested on simulators and real quantum hardware from IBM “The main contribution of this work is to establish a reusable methodology for backtesting of quantum and classical computing algorithms for portfolio optimization” (p. 2) Lastly, the challenges involved in using real quantum computers for more than 100 qubits are discussed	Objective: - Formulate a reliable and reusable method of back testing classical and quantum algorithms for portfolio optimization - Cite the drawbacks of > 100 qubits in a quantum system - Compare different quantum and classical optimizer against each other, whilst specifically taking a look at VQE, this is executed on 27 and 127-qubit machines Results: - “Results show quantum algorithms can be competitive with classical ones, with the advantage of being able to handle a large number of assets in a reasonable time on a future larger quantum computer.” (p. 1) - First a test of VQE on IBM Athens (5 qubits) real hardware is performed on 3 assets. Herein the VQE did not find the optimal result, mainly due to it being restricted in the number of iterations it can perform, more iteration would probably mean an optimal result - Next the execution time on a real quantum computer (IBM Brisbane, IBM Cusco, and IBM Nazca which are all 127 qubit) vs classical computer was tested using VQE and, this showed that: each iteration of VQE took approximately 2 hours, newer quantum computers showed better times, classical computing time grew exponentially with increasing number of assets whilst quantum methods computing times increased on a linear scale, also the IBM QASM simulator was used and showed optimal results after 100 qubits - Furthermore, VQE was used to colve a Cvar PO problem on a 27 qubit IBM Cairo machine, this showed similar results to classical methods of solving, however the quantum method was faster - Lastly, back testing was performed using historical data from IBEX35 2016-2020, where 2016 is used for calculations going forward in year	Quantum hardware: Real quantum hardware (IBM Athens), and some simulated results via IBM simulators Quantum algorithm: Specifically VQE, but also: VQE_CvaR, GAS, QAOA. Which are benchmarked against each other and classical algorithms: Moving Average Strategy (SMA), Sharpe Ratio Optimization (SRO), Risk-Rentability Optimization (MVO) Methodology: Optimization Use case: Portfolio optimization and back testing methodologies	Back testing = a method to evaluate performance of a financial model by applying it to historical data

		2017 (results were plotted monthly and strategies were allowed to change monthly), classical algorithms used: (SMA, SRO, MVO), quantum (VQE, QAOA, VQE_CVaR, and GAS) results showed: SMA performed poorly, QAOA and VQE_CvaR had a better strategy than the rest 20%-30% of the time, QAOA and VQE_CvaR showed to be competitive algorithms with the classical ones, the main advantage perceived was that quantum algorithm perform exponentially better using a larger number of assets, where classical algorithms become unfeasible Important notes: - "It is important to make it clear that today, quantum computers do not solve the portfolio optimization problem in a novel way, and they do not reformulate the problem to make them easier to solve, instead, they solve the same optimization problem with different variable types, but in a different method." (p. 2) - The VQE is a hybrid quantum-classical algorithm, whereby the quantum component is the hardware it operates on, the circuits and the ansatz it employs, and the classical component is the optimization of parameters in the quantum circuit to find more optimal solutions - "QAOA circuits have inherently more depth, making them more prone to noise disturbances on real computers. For this reason we have chosen VQE as the main algorithm for testing on real devices during this study."(p. 8)		
[29] **An Application of Quantum Optimization with Fuzzy Inference System for Stock Index Futures Forecasting**	"In this study, we propose using a novel hybrid Wavelet Transformation-Quantum-behaved Particle Swarm Optimization-Adaptive NeuroFuzzy Inference	Objective: - Develop an new model (WT-QPSO-ANFIS) to optimize the forecasting if stock index futures in a fuzzy environment - Benchmark the WT-QPSO-ANFIS against classical methods (ANFIS model, ANN model and ARIMA	Quantum hardware: Simulated hardware Quantum algorithm model: Wavelet Transformation-Quantum-behaved	Stock index futures = contracts that obligate the buyer to purchase (or the seller to sell) a stock index at a predetermined price in the future

(Chrimprang, N. Tansuchat, R. 2022)	System (WT-QPSO-ANFIS) model to forecast stock index futures." (p. 1)	model) using 10 major daily stock index futures from 2009 - 2020 Results: - Compared to classical methods, WT-QPSO-ANFIS consistently shows better: root means square error values, mean absolute percentage error, mean absolute error, standard error of the mean, basically meaning that the WT-QPSO-ANFIS results are more optimized and precise - "The result reveals that the hybrid WT-QPSO-ANFIS model provides higher efficiency and accuracy in predicting all 11 stock index futures considered in this study compared to the conventional Sugeno-type ANFIS model, ANN model and ARIMA model" (p. 1) Important notes: - "The machine learning models generally involve complex and unintelligible rules as well as a complicated network structure. In addition, the machine learning model itself did not guarantee a global optimum solution. It easily falls to the local optimum answer that directly affects the model's predicted value accuracy."(p. 1)	Particle Swarm Optimization-Adaptive NeuroFuzzy Inference System (WT-QPSO-ANFIS) Methodology: optimization Use case: Stock index futures	
[67] **An Investigation on Quantum-Inspired Algorithms for Portfolio Optimization Across Global Markets (Chou et al., 2024)**	"This article introduces a portfolio recommendation system based on trend ratio and quantum-inspired optimization specifically designed for global cross stock markets" (p. 1)	Objective: - Develop a transparent and interpretable portfolio recommendation system based on a quantum-inspired algorithm fitted towards the trend-ratio model (trend ratio = daily expected return / daily risk) and quantum inspired optimization algorithm (ELSA-QTS) forming ELSA-QNQTS - The proposed system is used in a group of the G7 markets Results: - The first experiment using the ELSA-QNQTS compared performances between G7 markets to gather the best market, results showed great perspective into the performance and	Quantum hardware: Simulator Quantum algorithm: ELSA-QNQTS Methodology: Optimization Use case: Portfolio optimization	

		risk levels of portfolios in the G7 markets. - Furthermore, a cross-market analysis is done, where the fluctuation of stock markets in each country is put into perspective, and it shows that cross-market investments generate superior portfolios based on the ELSA-QNQTS model. - "The proposed intelligent portfolio optimization model excels at identifying strong, stable uptrends within individual markets and extends its effectiveness to cross-market analysis. Furthermore, this financial application prioritizes explainability and transparency, empowering investors to comprehend ai-generated results" (p. 1) - "Experimental results show that the proposed model has excellent capability to explore portfolios with stable uptrends within a single market and extend its effectiveness to cross markets." (p. 7)		
[30] **A Weighted Portfolio Optimization Model Based on the Trend Ratio, Emotion Index, and ANGQTS (Chou et al., 2022)**	"This paper proposes a novel weighted portfolio optimization model based on the trend ratio and emotion index to comprehensively consider the volatility of the portfolio and more accurately evaluate the performance of portfolios than the classical indicator, the Sharpe ratio" (p. 1) Furthermore, this proposed model is applied towards the US stock market, where it is benchmarked against traditional methods.	Objective: - Develop a novel weighted portfolio optimization model based on the trend-ratio and emotion index to consider the volatility (risk) of a portfolio more accurately, thereby optimizing it - This model ought to have three main contributions; it utilizes trend ratio and emotion index, it makes use of ANGQTS, and the sliding window mechanism is adopted. - Test the proposed model in the US market with Dow Jones 30, and during the covid-19 pandemic Results: - The trend ratio can better evaluate portfolios than the Sharpe ratio - ANGQTS can effectively and efficiently construct near-optimal solutions - The sliding window mitigates under and overfitting in the proposed model - Statistical tests show that ANGQTS outperforms GNQTS in **weighted** portfolio optimization	Quantum hardware: N/A Quantum algorithm: global-best guided quantum-inspired tabu search with a self-adaptive strategy and quantum-NOT gate (ANGQTS) Methodology: Optimization Use case: Portfolio optimization, specifically in short and long selling trading using the trend ratio	Emotion index = a way of quantifying emotional responses (e.g. investor sentiment) into a value that can be used when computing certain problems. Sliding window mechanism = a versatile and efficient method of processing data allowing for constant evaluation of subsets of data in larger pools, which supposedly benefits the introduced novel portfolio

		- The proposed model was applied to the US stock market Dow Jones 30 and showed better stability than the Dow Jones industry average and the Sharpe ratio during economic fluctuations - So all in all, the proposed model is more precise and stable than comparable traditional methods. Important notes: - The classical method in this paper is seen as the 'Sharpe-ratio' - The difference between ANGQTS and GNQTS is that QNQTS is more static than ANGQTS, furthermore, ANGQTS outperforms GNQTS in larger solution spaces, lastly, ANGQTS demonstrates better searchability and higher trend ratios. Thus ANGQTS has better performance and is more efficient		optimization model
[31] Portfolio Optimization in Both Long and Short Selling Trading Using Trend Ratios and Quantum-Inspired Evolutionary Algorithms (Chou et al., 2021)	"This paper utilizes the global quantum-inspired tabu search algorithm with a quantum NOT-gate (GNQTS) to effectively find the best combination of stocks. To avoid the overfitting problem, this paper employs a sliding window. Specifically, this paper combines the trend ratio, GNQTS, short selling with certificates of deposit, and sliding windows to perform the stock selection" (p. 1) "This paper uses the global-best guided quantum inspired tabu search algorithm with a quantum NOT-gate, called GNQTS" (p. 2) "This paper proposes investing simultaneously in normal trading and	Objective: - Synthesize a model incorporating; the sliding window mechanism, the trend ratio (as it is better than the Sharpe ratio), GNQTS, long and short selling positions to outperform existing models - Compare the proposed method against the Sharpe ratio and - Benchmark the proposed model on Taiwan's 50 largest market capitalization stocks from the period 2010 – 2017, where funds are distributed in the portfolio for both long- and short-term selling. Results: - Portfolios selected by the trend ratio have a lower risk than portfolios selected by the Sharpe ratio, and a higher average return. - Combining long and short selling improves performance compared to using a single trading method - Overall, the GNQTS method effectively finds stable portfolios long and short-term selling, it outperforms the Sharpe ratio in risk management and average returns. Thereby, the experiment validates the	Quantum hardware: N/A Quantum algorithm: Quantum-inspired tabu search algorithm with GNQTS Methodology: Optimization Use case: Portfolio optimization, specifically in short and long selling trading using the trend ratio	

	short selling by a trend ratio, which can further increase investment profits and spread risks." (p. 1)	fact that a broader solution space will positively influence portfolio return and risk - "The experimental results show that the trend ratio can truly derive better performance than the Sharpe ratio" (p. 15) Important notes: - This paper differentiates between long and short selling, the GNQTS is used in both of these instances.		
[32] **A Novel Portfolio Optimization Model Based on Trend Ratio and Evolutionary Computation (Chou et al., 2019)**	"This paper makes use of the quantum inspired tabu search algorithm, which is improved by an adaptive strategy, the current best-known solution, and the quantum not gate (ANQTS) to find the best portfolio in a large solution space." (p. 1) "This paper employs the sliding window to avoid the over-fitting problem." (p. 1) "In summary, this paper combines the trend ratio, ANQTS, and the sliding window to solve the problem of stock selection."(p. 1)	Objective: - Synthesize a model incorporating; the sliding window mechanism, the trend ratio (as it is better than the Sharpe ratio), ANQTS, to solve the problem of stock selection for a portfolio - Benchmark the given model on Taiwan's 50 largest market cap stocks between 2010 and 2016 and compare them to the Sharpe ratio - Benchmark trend ratio usage against the Sharpe ratio Results: - The trend ratio is more effective than the Sharpe ratio in finding optimal portfolios and single stock uptrends - Compared to to other similar quantum algorithms, ANQTS outperforms GA, GQTS, and NQTS in the same experiments in finding the portfolio solution efficiently and achieving better stability - "The experiment results show that the proposed method can find the better portfolio, and the performance is better than Taiwan 50 ETF which is recommended by the government." (p. 13) - Results from the model also showed that risk can be spread better through effective fund allocation Important notes: - In this paper, and most likely the previous two, trend ratio is a component of the model that is synthesized, to show that trend ratio is a better method to include rather than the similar Sharpe ratio, certain experiments are done, concluding in	Quantum hardware: N/A Quantum algorithm: Quantum inspired tabu search algorithm (optimized by GNQTS, adaptive strategy, current best-know solution) Methodology: Optimization Use case: Portfolio optimization (specifically stock selection)	

		all three papers that the trend ratio is better and should thus be used for the total of the model. Next to that, these 3 papers focus on generating certain models including many different aspects that will optimize a certain objective (e.g. finding an optimal portfolio including long and short selling positions), instead of fully focusing on one type of algorithm, making it so that the quantum aspect of these portfolio optimization papers is a bit toned down considering other papers. Nevertheless, what can be learned mostly from these three papers is that quantum mechanics can also aid in alleviating certain problems of lesser proportions *(e.g. giving the model the ability to handle larger amounts of data faster).* - "The best portfolio may not include the best single stock and may include a stock which has negative return. As a result, the proposed method has the ability to select the portfolio, which is in a stable uptrend, and has outstanding performance in the experiments" (p. 13)		
[37] Quantum algorithms: A survey of applications and end-to-end complexities (Dalzell et al., 2023)	As the title says, this paper is a complete survey of applications and end-to end complexities of quantum computing, 337 pages of; areas of application, quantum algorithmic primitives, and fault tolerant quantum computation. However, in this paper, only the application area of 'portfolio optimization' will be summarized	Objective(s): - Give an overview of; actual end-to-end problems solved in PO, NISQ implementations, outlook, speedup, caveats. Actual end-to-end problems solved (using the Markowitz model): - Maximize return with fixed risk parameters - Minimize risk with fixed return parameters - Optimal risk-return tradeoffs with 'risk-aversion' parameter (or an alternative formulation using the square root of the risk) In these models, certain constraints are often used, the following are recognized: - Long asset position constraints - Investment bands (the asset must be located between min or max bounds)	Quantum hardware: N/A Quantum algorithm: N/A Methodology: Optimization Use case: Portfolio optimization	

		- Turnover constraints (constraint in the degree of changing asset holding between portfolios) - Cardinality constraints (restriction on the number of assets included in a portfolio) - Sector constraints (specified min/max allocations to groups of assets) - Transaction costs (extra costs linked to changing asset holdings) Caveats: - QLSS-based approaches are often dependent on multiple specific-instance parameters, resulting in computationally increased demands (e.g. high log-depth QRAM demands, log-depth being a measure of time for QRAM to find a piece of data, simply put) - Branch-and-bound approaches do not require log-depth QRAM to acquire quantum speedup Speedup (only for QIPMs): - Speedups for using QIPMS compared to classical methods will often come from optimizing the QLSS (used for a sub-routine of QIPMs including linearity) and tomography for a linear system (at least, until current hardware can better facilitate the QIPMs) NISQ implementations (alternative approaches for quantum PO solutions): - NISQ-HHL (generalizes QIPMs to better fit current hardware specifications) - QAOA - Quantum annealing Outlook: - QIPMS (and other QLSS-based approaches) for continuous PO formulations offer the potential of quantum speedup in the future - The Branch-and-bound approach for discrete formulations has the possibility of a larger speedup than QIPMs - "In the context of Grover-like quadratic speedups in combinatorial optimization, it is unclear whether the		

		quadratic speedup is sufficient to overcome the inherently slower quantum clock speeds and overheads due to fault tolerant quantum computation for practical instance sizes." (p. 121) Important notes: - More constraint often means harder problems and more computational power needed. - Convex PO problems are easier to solve than non-convex problems (a PO problem often becomes non-convex due to its imposed constraints) - Non-convex PO problems (or its constraints) can be converted to a Mixed-Integer Program (MIP), which in essence makes it easier to solve. Furthermore, if these integer variables are encoded in binary, then it can be formulated as a QUBO problem (which is widely used for PO). Therefore, a multitude of papers will make use of this, thereby making QUBO a often reoccurring formulation in these papers.		
[38] VaR Estimation with Quantum Computing Noise Correction Using Neural Networks (de Pedro et al., 2023)	"In this paper, we present the development of a quantum computing method for calculating the value at risk (VaR) for a portfolio of assets managed by a finance institution" (p. 1) The classical Monte Carlo algorithm to calculate VaR is extended upon in a quantum manner "The resulting algorithm is suitable to be executed on real quantum computers," (p. 1), Using feedback from real quantum computers, the neural network processing is	Objective(s) - Develop a quantum neural network to extend conventional Monte Carlo for calculating Value at Risk (VaR) - Compare the results of this work with other works Results: - The quantum simulation and actual quantum computer results had discrepancies due to noise, highlighting the limitations of current quantum technology - "The results show that this approach is useful for estimating the VaR in finance institutions, particularly when dealing with a large number of assets." (p. 1) - Neural networks were used to mitigate noise in the quantum circuit by optimizing parameters. - The authors compared their work with other works, and it showed that: quantum monte carlo methods showed promising results, however,	Quantum hardware: IBM Qiskit (simulated hardware, 5 qubit) Quantum algorithm: Quantum (and neural network) optimized Monte Carlo Methodology: Monte Carlo Use case: Portfolio optimization (VaR)	This paper mainly considers optimizing classical Monte Carlo methods using, but not limited to, quantum methods.

	finetuned, as the neural network is used to mitigate noise in the quantum circuit.	are often faced with challenges related to resource requirements and circuit depth. Comparing it to the proposed method in this paper, their approach of using neural networks for quantum noise showed a promising feasible solution effectively utilizing current quantum computing resources. Important notes: - The noise affecting current quantum computers makes it almost useless to perform the posed algorithm on real quantum computers - Challenges: Grow a sufficient number of samples needed for the Quantum Monte Carlo method for increased asset sizes in portfolios, and find ‘real’ random generated samples using quantum computing, use neural networks to mitigate the noise in the quantum circuit - “A VaR estimation problem could be divided into parts and simulated partially by real quantum computers.” (p. 16)		

[41] **Quantum Computing for Finance: State of the Art and Future Prospects (Egger et al., 2020)**	This paper gives an overview off the current (2020) state of quantum computing for finance, thereby giving insight into; a survey on problem classes that are computationally challenging classically and show advantages on quantum systems, in detail described quantum algorithms, specific applications of these algorithms (simulation, optimization, Monte Carlo), and lastly a demonstrations of quantum algorithms on IBM quantum back-ends	Problems/segments recognized in financial services for quantum computing: - Banking: balancing cash with interest rates, while controlling threats (risks) related to liquidity, fraud, money laundry, and non-performing loans - Financial markets: manage geographic time-zones, immediacy needs, counter-party risk - Insurance: maximize premiums, manage threats it unplanned risks - The main reoccurring problem is risk management Problem classes for classical computing methods where quantum methods may show promising advantages: - Simulation: customer identification, financial products (e.g. Value at Risk estimates), monitor transactions, customer retention. Furthermore, in this section it is discussed how quantum amplitude estimation can provide quantum speedup over classical Monte Carlo; with current quantum methods they estimated a 30-minute runtime for calculating VaR for a one-million-asset portfolio, showing a speedup over classical methods - Optimization: Customer identification (and assessment), financial products, monitor transactions (e.g. re-balancing portfolios), customer retention Furthermore, for problem classes: convex problems *(linear programming, convex programming, semidefinite programming),* quantum methods showed the potential of significant speedups over classical methods, however, practical effectiveness is mainly determined by the specific problem instance. For problem classes: combinatorial problems (generally non-convex with discrete decision variables). "We note that, currently, there is no theoretical guarantee that variational algorithms on quantum devices can achieve significant speed-ups for QUBOs" (p. 11), however, they are appealing to study on NISQ devices as they show provable guarantees for performance. Tests performed with VQE and QAOA showed that the quantum	Quantum hardware: IBM Quantum back-ends Quantum algorithm: N/A Methodology: Optimization, Machine learning, simulation Use case: N/A	

		methods got the best results following the efficient frontier in a active investment management PO example. (however, it was mentioned that current quantum hardware cannot facilitate such results). And for a passive investment management PO problem, quantum algorithms showed performances just below classical methods, however it was mentioned that performance of quantum algorithms will increase with larger problem sizes. - Machine learning: Prediction, classifying, finding patterns (all in customer scoring/evaluation, financial product usage, transaction monitoring, customer retention methods) Furthermore, two quantum Monte Carlo methods are mentioned Variational Quantum Classification (VWC), and Quantum Kernel Estimation (QKE). Compared to classical techniques, the quantum algorithms showed improved performances in machine learning tasks, particularly in advanced feature spaces and classifiers, however, practical advantages still do not show coherently. Technical challenges in Quantum Computing: - Loading data in a quantum state is very complex compared to classical methods, increasing number of qubits in the system are cause for exponential effort increases in preparing the system - Error correction, to protect the quantum system from error, multiple mitigation techniques are used that cost significant overhead - Precision and sample complexity, many repetitions need to be made in quantum system to achieve accurate results, this has high computational costs Important notes: - Challenging problems for classical computers that are addressed are those in: asset management, investment banking, retail and corporate banking.		

[44] **A Systematic Literature Review of Classical and Quantum Machine Learning Approaches for Mutual Fund Portfolio Optimization (Fernandes et al., 2023)**	"This review paper examines literature on classical and quantum machine learning approaches for Mutual Fund PO, analyzing 44 papers from 2003 to 2023" (p. 1) "We provide an overview to the types of problems, preferred approaches, their benchmarks, deduced conclusions, and research gaps as a comprehensive survey for diverse readers." (p. 1)	Findings: - "Quantum Machine Learning (QML) PO algorithms which are an intersection of QC and ML techniques, process large datasets more efficiently, revealing hidden patterns and insights that traditional ML approaches may potentially not be able to identify" (p. 1) - Traditional ML approaches face the following problems: time constraints, high costs due to their inability to consider risk calculations at various levels - Quantum (assisted) machine learning approaches have the following benefits: provide real time solutions to market scenarios, - Quantum algorithms have successfully been implemented for portfolio optimization. - Main research gaps found were: a) The validation of quantum computer output is still a difficulty in the NISQ era of quantum technology b) Quantum linear-algebra techniques sometimes have issues being applicable towards specific linear-algebra and financial use cases due to certain constraints and prerequisites which bottleneck quantum speedup c) "No dynamic portfolio optimization framework can outperform the covariance model. ML/DL approaches require more research due to the curse of dimensionality and the DL architectures inability to improve performance of sample-based portfolios" (p. 4) - "With numerous variables and conditions that need to be considered for a Mutual Fund PO problem, classical algorithms eventually end up at the local optima and offer a non-optimal solution" (p. 4) - Currently (2023) quantum machine learning shows benefit in specific use cases in terms of solution quality and computing speed. However,	Quantum hardware: N/A Quantum algorithm: Quantum machine learning Methodology: Machine learning Use case: Portfolio optimization	Mutual funds = a portfolio of stocks, bonds, or other securities overseen by a professional fund manager. 5 main mutual fund portfolio optimization problems mentioned in the paper are: asset allocation, portfolio diversification, risk-management. Minimizing transaction costs, tax efficiency. Curse of dimensionality = common issues arising when dimensions in a problem formulation or system increase (e.g. amount of data, exponential growth of results/data etc, distinctions between near and far points blurring in high-dimensional spaces, increased computational complexity, overfitting).

		generally, papers show that many fields of research (such as machine learning) still need to experience real benefit from quantum computing Important notes: - “The existing breed of NISQ (Noisy Intermediate Scale Quantum) quantum computers have a significant potential to provide faster solutions to problems in various domains which are not just relevant for the present but also for the future” (p. 1) - The current (2023) stage of quantum technology with 50-1000 qubits that are not-fault tolerant is called ‘Noise Intermediate-scale quantum computing (NISQ) - “This paper focuses on Mutual Fund (MF) because it has seen a rise in investment in the past years and a low rate of risk in comparison to the ever-fluctuating stock market industry” - (p. 2)		
[47] Grover Adaptive Search for Constrained Polynomial Binary Optimization (Gilliam et al., 2021)	“In this paper we discuss Grover Adaptive Search (GAS) for Constrained Polynomial Binary Optimization (CPBO) problems, and in particular, Quadratic Unconstrained Binary Optimization (QUBO) problems, as a special case” (p. 1) “In this paper, we provide a framework for automatically generating efficient oracles for solving Constrained Polynomial Binary Optimization (CPBO)— a generalization of QUBO—with GAS.” (p. 1) In the analysis of this paper, there will only be focusses on the application towards	Objective(s): - Test the proposed GAS with QUBO and efficient oracles on a PO problem - In the experiment: minimize the weighted variance – portfolio return to create an optimized portfolio with budget constraints. The portfolio consist of 3 assets, no more than 7 qubits were used, and searching wads stopped after 3 iterations each time Results: - “GAS can provide a quadratic speed-up for combinatorial optimization problems compared to brute force search” (p. 1), however, this can only be performed under certain search criteria and efficient oracles - The noise in current era NISQ hardware impacted results, increasing the probability of wrong results. When the noise was not too strong, it achieved good results - QUBO with GAS on real quantum hardware consistently found the optimal solution in the given environment	Quantum hardware: Simulated hardware (Qiskit), and real hardware (IBMQ Toronto) Quantum algorithm: Grover Adaptive Search (on CPBO and QUBO) Methodology: Optimization Use case: Portfolio optimization	Oracle = a subroutine in an operation that provides information on an objective problem’s solution, this information is used to increase the probability of finding the optimal solution in the algorithm in quantum optimization cases

	portfolio optimization of the GAS for QUBO	- Besides the portfolio optimization problem, this paper managed to reduce the number of gates required for computation compared to standard quantum arithmetic approaches ("i.e. it lowers the requirements to apply GAS on real quantum hardware for practically relevant problems." (p. 7)) - Even though the quantum hardware showed promising results, it could still be said that it can not solve lager problem sizes, as the problem size used in this paper on the real hardware remains small, thereby it can also be said that the quantum hardware currently is not better than classical methods in bigger problem sizes. On the other hand, for simulations according to paper, it can be said that performances are good, but no definite conclusion can be made on the comparison with classical methods.		
[48] Approaching Collateral Optimization for NISQ and Quantum-Inspired Computing (Giron et al., 2023)	"In this study, we initially present a Mixed Integer Linear Programming (MILP) formulation for the collateral optimization problem, followed by a Quadratic Unconstrained Binary optimization (QUBO) formulation in order to pave the way towards approaching the problem in a hybrid quantum and NISQ-ready way" (p. 1) "In summary, the main objective of our paper is to present a case study on the formulation and approach of the ColOpt problem using quantum computing techniques, with the overarching aim of advancing the ongoing effort towards	Objective(s) - Study the ColOpt problem in detail - Provide a MILP formulation that is to be used as a testbed for; a QUBO version of ColOpt (making it so that quantum and quantum-inspired hardware can process it), perform small-scale experiments using that QUBO version and benchmark it to MILP - Investigate the QUBO formulations for the KnapsackProb problem, and use the best formulation for this to apply to the collateral optimization problem. Results: - "We find that while the QUBO based approaches fail to find the global optima in the small-scale experiments, they are reasonably close suggesting their potential for large instances" (p. 1) - For the KnapsackProb, classical approaches (MILP) managed to find the known optimal solutions, and for the QUBO formulation on simulated	Quantum hardware: Simulated annealing (On Fujitsu simulators, and D-Wave simulated annealer) ColOpt problem, and simulated annealing for the KnapsackProb (on ToQUBO.jl, Qiskit's QuadraticProgramToQUBO, PyQubo, and Digital Annealer). Quantum algorithm: QUBO (with MILP mapped to it in the formulation) Methodology: Optimization Use case: Collateral optimization	Collateral optimization = "the systematic allocation of financial assets to satisfy obligations or secure transactions, while simultaneously minimizing costs and optimizing the usage of available resources." (p. 1) ColOpt = an example Collateral optimization problem to solve on the given lassical and quantum methods. KnacksackProb = example knapsack

	achieving “quantum advantage” in practical applications” (p. 3)	annealing: ToQUBO.jl found the optimal solution, Qiskit found the optimal solution (through multiple runs), PyQUBO found the optimal solution (and for larger instance sizes close to optimal) Neal and Fujitsu machines consistently found optimal solution, even under penalty regimes. - For the ColOpt problem, quantum methods showed that they could not find the global optimal solution, each run found different global minima. The reason for this mentioned in the paper is probably due to a lack of runs performed in the annealing process, making it so that it could not explore sufficient search space. - The paper did mention that the solving of the problem was not fully optimized, as certain improvements can be made to obtain higher quality solutions (e.g. optimizing the annealing schedule, QUBO parameter optimization) - Classical solver showed to find optimal solution every time in the experiments, while quantum methods often fell short, there are still certain factors inhibiting it from working to its full potential in this paper on the given ColOpt and KnapsackProb problems. Important notes: - On the ColOpt problem for quantum methods, multiple penalty weights were used to make the process more efficient and give mote optimized results. - Using QUBO or Ising approaches, problem can be addressed as follows in a quadratic way: Using variation quantum algorithms (e.g. QAOA on gate-based quantum computers), using quantum annealing on adiabatic quantum computers (quantum annealers), using quantum inspired methods which can be understood under a QUBO model formulation		problem involving the optimal approach to filling a knapsack (with capacity W) with the highest possible value from a corresponding set of n items.

		“We would like to note that our paper does not aim to provide an empirical comparison between quantum and classical approaches for solving MILPs, given the limited computational resources available to us” (p. 3) - “The QUBO model can be applied to a wide range of combinatorial optimization problems that are known to be NPhard,” (p. 4) - “we utilize simulated annealing (SA), which as a metaheuristic algorithm, is quite sensitive to the problem structure and its performance can vary significantly depending on the problem instance.”(p. 12)		
[51] **A brief review of portfolio optimization techniques (Gunjan, A. & Bhattacharyya, S. 2023)**	This paper lists a brief review of portfolio optimization techniques, most techniques mentioned are non-quantum techniques. The paper makes a distinction between classical approaches and intelligent approaches. Under the list of intelligent approaches fall ‘quantum-based approaches’ In the summary of this paper, a brief list of non-quantum approaches will be mentioned (classical and intelligent approaches), after that there will be elaborated on the quantum PO part of this paper.	List of non-quantum approaches (classical and intelligent): Classical: - Markowitz mean-variance optimization, Mean Absolute Deviation, Minimax, Variance with skewness, Lower partial moments, Value-at-risk (VAR), Conditional value-at-risk (CVar). Each of these approaches will have their own advantages, disadvantages, specific uses but most notably, many of these classical approaches make an appearance in the mentioned papers as adapted versions are used for certain quantum algorithms, specifically QUBO Intelligent approaches (mostly referring to machine learning based techniques): - Bayesian approaches (e.g. Black-Litterman approach), Support vector machine-based approaches (SVR), Neural network-based approaches, reinforcement learning approaches, and evolutionary approaches. Again, most of these types of approaches can be seen back in adapted versions for quantum computing PO. Quantum Computing for PO; the following is mentioned: - “On multiple experiments, QC is shown to give better performance on complex and NP-hard problems	Quantum hardware: N/A Quantum algorithm: N/A Methodology: N/A Use case: N/A	Metaheuristic = procedures or strategies designed to generate or find god solutions to an optimization problem

		which require large solution space." (p. 23) - Quantum-inspired metaheuristic techniques are methods take advantage of the promising power that quantum computing has and those of metaheuristics, "and have shown to perform better than classical counterparts" (p. 30). Furthermore, these methods are widely used in constrained and unconstrained method (e.g. constraints in PO) - The following meta-heuristic approaches are mentioned that show promising results (however, there are more to be mentioned, as shown from the above summarized papers): Quantum-inspired Tabu search (QTS), Multi-Objective Quantum-Inspired Tabu Search (MOQTS, flexible, profitable, can optimize multiple objectives, but needs further evaluation), Quantum-Inspired Firefly algorithm with Particle Swarm Optimization (QIFAPSO, no experiments with this method to date 2023), Quantum-Inspired Tensor Networks (TN), Quantum-Inspired Acromyrmex evolutionary algorithm (QIAEA, finds efficient global optimization for complex systems, high accuracy, low error, but cannot do multiple objective scenarios, and that may be the reason it is not frequent in PO literature), Variational Quantum Eigensolver (VQE), D-Wave hybrid Quantum Annealing. - Advantage of QC approaches: "Adding qubits can increase the storage exponentially and are useful to solve very complex compute extensive problems. Faster as compared to any other methods." (p. 25) - Limitations of QC approach: "The energy required by quantum computer is much larger than traditional computers. Still there is a lot of unknowns as this is an ongoing area of research." (p. 25)		
[52] Quantum-inspired meta-heuristic approaches for a	"This paper covers and compares quantum inspired versions of four	Objective(s):	Quantum hardware: N/A	

constrained portfolio optimization problem (Gunjan, A. & Bhattacharyya, S. 2024)	popular evolutionary techniques with three benchmark datasets. Genetic algorithm, differential evolution, particle swarm optimization, ant colony optimization, and their quantum-inspired incarnations are implemented, and the results are compared" (p. 1) The experiment done on the optimization approaches were done using 10 years of stock price data from NASDAQ, Dow Jones, and BSE	- Use a genetic algorithm (GA) to solve a PO problem for the given datasets - Use Differential evolution (DE) to solve a PO problem for the given dataset - Use Particle swarm (PSO) to solve a PO problem for the given dataset - Use ant colony optimization (ACO) to solve a PO problem for the given dataset - Use the quantum inspired version of GA, DE, PSO, and ACO to solve a PO problem for the given dataset - Measure the performance of the mentioned techniques via mean error, execution time, and fitness function (minimum risk) Results: - Classical PSO showed to have lowest mean square error, root mean square error, mean absolute error, and mean absolute percentage error, basically indication that it can very closely approximate optimal solutions. - Quantum-inspired versions were faster, and often had better quality of results - "The experiments reveal that quantum-inspired ant colony optimization (QiACO) is more effective and faster than the other techniques chosen in both the classical and quantum inspired domains" (p. 23) - Further analysis of results showed: quantum-inspired approaches produce better risk values than classical approaches, Quantum PSO showed to generate the most optimal risk compared to classical methods - Results from the given tables for the experiments confirm statements made on fastness and quality of results. - Further Wilcoxon tests (to show whether made conclusion on the differences between classical and quantum methods are significant) show that almost all comparisons between classical and quantum algorithms lead to the quantum	Quantum algorithm: Quantum versions of the classical algorithms named Methodology: Optimization Use case: Portfolio optimization	

		algorithm either performing on par with classical ones, or better. - "It is observed that the quantum-inspired techniques outperform the classical counterparts." (p. 1) - "Experiments have demonstrated that these quantum-inspired versions are faster, and the results are comparable or even better than their classical counterparts "(p. 35) - "Specifically, the quantum-inspired ACO surpasses all the selected techniques in terms of speed, and its optimization results closely match those of the other selected techniques" (p. 35) Important notes: - benchmark datasets, NASDAQ (from 2012-06-23 to 2022-06-27), BSE (from 2011-05-13 to 2023-02-07) , and Dow Jones (from 2009-08-06 to 2023-05-05). - Four enhancements to the named techniques are given so that errors are minimized, they become more efficient, and quality of results are better:		
[53] Portfolio Optimization Using Quantum-Inspired Modified Genetic Algorithm (Gunjan et al., 2023)	"An effort is made to implement two different genetic versions along with their extension in the quantum-inspired space. Improvements to the popular crossover techniques, viz. (i) arithmetic and (ii) heuristic crossover are proposed to reduce computational time." (p. 665)	Objective(s): - Optimize risk and return in a PO problem for a proposed quantum genetic algorithm. - Use the following proposed classical techniques to base the QiGA upon: Arithmetic crossover, Heuristic crossover - Conduct the experiments on a dataset from the NASDAQ in the period 2012-06-28 to 2022-06-27, objective function is to find minimum risk, evaluation are done via mean square error (MSE), mean absolute error (MAE), root mean square error (RMSE), mean absolute percentage error (MAPE). Lastly, execution times are measured for the QiGA. Results: - 'It is evident from the results that the quantum-inspired version outperforms the classical counterparts	Quantum hardware: N/A Quantum algorithm: Quantum genetic algorithm (QiGA) Methodology: N/A Use case: N/A	Crossover = create new solutions to a problem by combining the features of two parent solutions, generating offspring that is closer to the optimal solution Arithmetic crossover = continuous optimization by taking a parent group of 2 and then making offspring generations as a weighted average of the parents

		as far as the minimization of portfolio risk is concerned.” (p. 671) - The QiGA with arithmetic crossover performs best overall - The classical GA algorithm is worse off on all evaluated parameters (Risk, Return, MSE, MAE, RMSE, MAPE, Mean Execution Time (MET), Total Execution Time (TET)) - QiGA with arithmetic crossover performs best on MSE, MAE, RMSE, MAPE, MET, TET - QiGA with heuristic crossover performs best on the lowest risk measure - “It is also observed that quantum-inspired versions are faster and more efficient than their classical counterparts.” (p. 672) Important notes: - “Portfolio optimization, in other words, is an iterative and computationally extensive task where a near-optimal solution is achieved through an iterative process.” (p. 665)		Heuristic crossover = choose two parents, out of which one is superior, or when combined creates a solution more specific to the objective problem
[56] **An improved QPSO algorithm and its application in fuzzy portfolio model with constraints (He, G. & Lu, X, L. 2021)**	“Aiming at the shortcomings of quantum-behaved particle swarm optimization algorithm (QPSO), an improved quantum behaved particle swarm optimization algorithm (IQPSO) is put forward, and the improved algorithm is applied in solving a kind of fuzzy portfolio selection problems” (p. 1)	Objectives: - Synthesize an improved QSPO algorithm based on the shortcoming of the QSPO algorithm - Use the other three given algorithms (QSPO, PSO-w, RQSPO) in the paper to benchmark against each other and similar metaheuristic approaches to IQSPO. Benchmarking is performed on a fuzzy PO problem with 16 different benchmarks, number of iterations: 1000-1500-2000, algorithms were run 30 times for each instance. - Compare the IQSPO with six well-know metaheuristics (Genetic algorithm, Differential evolution, bat algorithm, Cuckoo search, PSO, and QSPO), with max number of iterations 1500, and population size (assets) of 50, run 30 times Results: - For 14 of the 16 benchmarks, IQSPO was superior to the other tested algorithms (including metaheuristic	Quantum hardware: N/A Quantum algorithm: (I)QSPO Methodology: Optimization Use case: (Fuzzy) portfolio optimization	Convergency = the process where an optimization algorithm approaches the optimal/sufficientl y good solution over time iteratively

		PSO-w), showing higher accuracy and less standard deviation - Using a Wilcoxon rank-sum test, it shows that IQSPO significantly outperforms the rest of the algorithms on most of the 16 test functions. - For the comparison with other metaheuristics, IQSPO showed a better ability to search for global optima, IQSPO gets better means, more promising standard deviation, indicating more robustness and effectiveness - "IQPSO shows better calculation precision and robustness" (p. 6), "IQSPO has better mean and standard deviation across all algorithms" (p. 6) - "The experimental results on 16 benchmark functions show that IQPSO has better convergence and robustness than PSO with inertia weight, QPSO and QPSO with a hybrid probability distribution in most cases." (p. 1) - "When solving a fuzzy portfolio model, IQPSO provides comparable and superior results compared with the other metaheuristics." (p. 1) - The novel QSPO algorithm already has some advantages over the classical PSO algorithm, mainly fewer parameters needed, faster convergence speed, and strong search capability for complex problems Important notes: - Shortcomings of the QSPO algorithm are addressed in the IQSPO algorithm.		
[60] **Empirical Analysis of Quantum Approximate Optimization Algorithm for Knapsack-based Financial Portfolio Optimization (Huot et al., 2024)**	"Herein, we proposed a method that uses the knapsack-based portfolio optimization problem and incorporates the quantum computing capabilities of the quantum walk mixer with the quantum approximate optimization algorithm	Objectives: - Construct the use of quantum walks (QWS) with QAOA to enhance its performance in searching for optimal portfolio configuration. - Use the proposed QAOA model on a PO problem using 2-5 stocks from well-known companies (e.g. Apple, Amazon) from the timeframe 01-01-2018 to 01-01-2023. It was tested on: a noiseless simulator, noisy fake backend, noisy real device. Required	Quantum hardware: QASM simulator from Qiskit to give insight into the proposed QAOA algorithm, then afterwards IBM Cairo (27 qubit) is used for the given PO problem Quantum algorithm: QWM-QAOA	

	(QAOA) to address the challenges presented by the NP-hard problem." (p. 1) Furthermore, the proposed method of using QAOA for a knapsack-based PO problem is then experimented upon and results are put into perspective "Our methodology is based on the fundamental principles of mean–variance optimization, focusing on the Markowitz model." (p. 6)	qubits were different for certain stock counts but max qubits were 11 for 5 stocks, and min 7 for 2 stocks. Results: - The proposed QWM-QAOA model revealed a consistent enhancement in identifying optimal solution to the knapsack problem, approximating optimal solutions 100%-98% with 2-5 stocks. - "Our proposed method achieves efficient results in noiseless and fake device settings, ranging from100% to 98% and 98% to 80%." (p. 11) - For real devices the results showed an accuracy of 50% due to errors, indicating that there are still error performance enhancements to be made on real quantum devices. Important notes: - The proposed model and knapsack problem is based upon the Markowitz model of max return/min risk - During the optimization process, the QAOA model was optimized using a classical optimizer SHGO, and quantum walk was used to boost optimization by its ability to refine the process.	Methodology: Optimization Use case: Portfolio optimization	
[63] Exploring the synergistic potential of quantum annealing and gate model computing for portfolio optimization (Jain Naman. & Girish Chandra, M., 2023)	"In this work, we extend upon a study to use the best of both quantum annealing and gate-based quantum computing systems to enable solving large-scale optimization problems efficiently on the available hardware." (p. 1) Test are conducted on real-world dataset derived from Indian stock market, up to 64 assets are used. "We also demonstrate the effectiveness of our	Objective(s): - Form an Ising/QUBO problem formulation (as the paper mention; QUBO and ising formulations are interchangeable) and use Large System Sampling Approximation (LSSA) to divide it into smaller sub-systems. Determine the right assets for creating these sub-systems by finding the Maximum Independent Set (MIS) on a quantum annealer. Solve the smaller sub-systems independently via LSSA on a quantum annealer and then combine their solutions using Variational Quantum Eigensolver (VQE) on a gate-based quantum computer to find the optimal solution. - For the second model, change the sampling method for the sub-systems	Quantum hardware: Quantum annealer and gate-based system (D-Wave Advantage system 4.1) (VQE amplitude optimization is performed on Qiskit simulator, and parameter optimization via a classical solver COBYLA) (Python library PyQUBO was used to form the QUBO problem) Quantum algorithm: N/A Methodology: Optimization	The proposed method in the paper that this paper is based upon works using the Large System Sampling Approximation (LSSA) method, which entail dividing a larger problem in subsets of problems, to then combine the solution of those to approximate a solution to the original problem.

	approach on a range of portfolio optimization problems of different sizes." (p. 1) A QUBO formulation is made and tested on real-world stock datasets, comparing performances with previous techniques for varying numbers of assets and parameters. Lastly, the effects of different parameters on the PO problem solution quality are investigated and benchmarked against earlier works.	to MIS and random-based sampling instead of only MIS. - For the third model, use only random sampling - Benchmark the given model on a PO problem in the Indian stock market with data from 2018-2023, with n = 64 stocks, risk aversion constraints. Results: - Results from the experiment showed that both the LSSA_MIS and the LSSA_MIS_RANDOM models performed comparably to a classical D-Wave Tabu Solver, but with fewer samples needed. - Samples needed for near optimal solution: LSSA_MIS: 12 samples LSSA_MIS_RANDOM: 13 samples LSSA_RANDOM: 32 samples - "Our experimentation shows that the hybrid approach performs at par with the traditional classical optimization methods with a good approximation ratio" (p. 1) - "Our findings suggest that hybrid annealer-gate quantum computing can be a valuable tool for portfolio managers seeking to optimize their investment portfolios in the near future" (p. 1) - Scatter plots reflect the findings made in the paper. - "our findings suggest that a hybrid of annealing and gate-based quantum computing can be a promising tool for portfolio optimization,"(p. 10) Important notes: - LSSA enables the solving of greater problem sizes on available quantum hardware - "large-scale problems cannot be solved on today's (2023) quantum hardware" (p. 1) - Classical optimization methods such as Monte Carlo methods have limitation dealing with large-scale problems. - "Quantum computing methods, viz. quantum annealing [2, 3] and gate-based quantum computing can	 Use case: Portfolio optimization	This paper modifies the LSSA by introducing a modified sample step in the LSSA. This modified example is depicted as: dividing a PO problem into sub-systems of smaller sizes by selecting representative stocks of the entire market and capture the highest correlation among them. Maximum Independent Set = a way of ensuring that a subset of assets has no strongly correlated assets, as correlation is an indicator of redundancy or overlapping. For this paper MIS is mainly used to increase efficiency and effectiveness of solving large-scale optimization problems.

		potentially solve complex optimization problems more efficiently than classical methods and may provide better solutions for practical problems with many variables and constraints.” (p. 2) - “several studies show remarkable results in portfolio optimization using the above-described common methods (VQE, QAOA, QUBO, QE), these approaches require an N-qubit quantum computer to solve the problem with N assets” (p. 2) - “The proposed method is best suited for problem instances where there are grades of diversity, which is usually true in a real setting.” (p. 10) - Th text mentioned that gorver adaptive search might be better to solve the sub-sysetms instead of the imposed method.			
[64] **Efficient and Flexible Annealer-Gate Hybrid Model for Solving Large-Scale Portfolio Optimization (Jain et al., 2023)**	A two-stage approach combining quantum-annealing and gate-based quantum computing for large-scale PO problems LSSA is used and modified upon to create a more efficient and effective framework for he specific PO problem MIS is used to divide the problem in sub-systems, using a parameterized quantum circuit to combine sub-problem solutions. Experiments are performed on 128 asset simulators.	Objective(s) - Solve a QUBO formulation of the PO problem (for only long positions in an equal weighted portfolio, minimizing the objective function for various problem sizes) using a quantum annealing and gate-based quantum computing hybrid approach involving LSSA and MIS, aggregating sub-systems using quantum parameterized circuit (PQC) (in the previous paper VQE was used for that) - Experiment on the given 128 asset PO problem with different increasing numbers of sub-problems (Ns) and sub-problem sizes (Ng), the following distributions are tested upon (following Ns / Ng format): (64 / 8), (32 / 32), (32 / 64) Results: - For the experiment with 128 asset the following could be noticed: number of calls made to the quantum annealer increased as number of sub-problems increased, performance with the imposed hybrid method was increased by the imposed method involving MIS, LSSA PQC, and the framework around it.	Quantum hardware: D-Wave simulator Quantum algorithm: Hybrid quantum annealing / gate-based approach Methodology: Optimization Use case: Portfolio optimization		

		- Performance increased with the full-hybrid model as problem sizes increased to 128 assets. - ”Our results demonstrate that the proposed approach performs better with the same hardware resources” (p. 1) - “The outcomes of our research suggest that hybrid annealer-gate quantum computing can provide a practical and scalable solution to large-scale portfolio optimization problems, bridging the gap between theoretical advancements in quantum computing and real-world applications in finance” (p. 1) Important notes: - “The hardware limitations of quantum computers prevent the direct application of quantum algorithms to large-scale problems.” (p. 1) - More qubits are needed as the problem size increases		
[66] **A Novel Portfolio Optimization with Short Selling Using GNQTS and Trend Ratio (Jiang et al., 2018)**	“This paper proposes a strategy to improve the Sharpe ration denoted the trend ratio where the daily expected return is the slope of the trend line, and the risk is the difference between the trend line and the fund standardization” (p. 1) The proposed model includes doing normal trading and short selling simultaneously to increase profits and spread risk.	Objective(s): - Formulate a novel quantum model involving QTS optimized by GNQTS, whilst utilizing sliding windows to overcome over-fitting problems, and trend ratio to identify stable uptrend portfolios for normal trading, and stable downtrends for short selling. - Use the model on an experiment based upon the Taiwan top 50 ETF stocks from 2010-2017 as the training periods for the model, and 2011-2018 as the investment periods for the model. Parameters used were: initial fund of 10 million TWD, population of 10, 10000 generations with an execution number of 50. - “Use the trend ratio and GNQTS to help investors to select a potential uptrend and a downtrend portfolio, using the sliding windows to train and test, and then evaluate and change a more potential portfolio suitable for a new investment period, hoping that we can make maximum profit with low risk.” (p. 4)	Quantum hardware: N/A Quantum algorithm: Quantum-inspired Tabu Search algorithm (QTS) (improved by GNQTS) Methodology: Optimization Use case: Portfolio optimization	

		- Utilizing the sliding window mechanism, find the best training and testing period Results: - Using the sliding window on the experimental results from the model, it became clear that the best training and testing periods were month-to-month, and year-on-year month periods (comparing the same month of last and current year) - Utilizing trend ratio and GNQTS, the paper was able to find portfolios with stable up-and-down trends. Showing that it is possible to short sell and trade normally simultaneously. - Utilizing normal and short trading, the model was successfully able to simultaneously increase returns and minimize risks. - There were still some fluctuations in in the results of the experiments, but overall, the model showed promising results. - Differing period with higher/lower down/uptrends were also successfully recognized by them model. - “QTS can find the best portfolio in an extremely complicated solution space while decreasing the computational complexity” (p. 6) - “The experiment results show a promising result in which the risk is spread effectively, and the profit is maximized.” (p. 1) - “Using these methods, the experimental results show that we can find a portfolio that has better performance than the government-recommended Taiwan 50 ETF” (p. 6) Important notes: - The sliding window mechanism is used to overcome any over-fitting problems - Trend ratio is used to identify stable uptrend portfolios for normal trading, and stable downtrends for short selling. - The trend ratio can evaluate the risk of a portfolio more accurately than the Sharpe ratio		

		- QTS aims to move individuals away from the worst solution and towards best solution "in the other words, QTS finds the best solution more quickly and efficiently." (p. 1) - "This paper uses the trend ratio, GNQTS, and sliding window to select potential stocks" (p. 2) - The number of stocks in a portfolio is unrestricted in the case of this paper. - GNQTS is used to make sure the QTS algorithm does not get stuck in a local optima (which may not be the best solution) - Sliding window mechanism was also used to find the best training periods, these were proven to be month to month trading periods, and year-on-year month trading periods. Most of the 'results' part of this paper is based upon these two periods		
[67] **Quantum-inspired Computing: Entanglement-enhanced Technique for Short Portfolio in Global Markets (Jiang et al., 2023)**	"This study proposes an entanglement-based QIO to optimize the short-selling portfolio in a group of seven (G7) industrialized nations" (p. 1) "Trend-ratio is used to precisely determine the performance of a short-selling portfolio during a stable downward trend" (p. 1), this is mainly to recognize portfolios for inclusion in the model. Sliding window is used to select appropriate training and test periods for the experiment.	Objective(s): - Form a GIO based model, utilizing trend ratio to identify stable downtrend portfolios, to optimize a for a short-selling portfolio. - Experiment with the proposed model on in the G7 stock market from the period January 2013 to December 2022, selecting the 30 largest capitalization stocks. Parameters of ELSA-GNQTS: 10 individuals, 10.000 generations, 50 independent experiments, initial funds of 1 billion in local currency. Then take the best solution from the 50 experiments as benchmark. - Propose a novel Entanglement local search-assisted (ELSA) mechanism, and quantum not gate techniques, to improve Quantum Tabu Search algorithm ((GN)QTS) Results: - The best-found portfolio from the experiment can diversify risk better and achieve higher returns than other QIO algorithms. - Portfolio risk of the experiment is lower than the single-stock risk - Compared to a Sharpe ratio based ELSA-GNQTS model, the proposed trend ratio ELSA-GNQTS performed better	Quantum hardware: Simulator Quantum algorithm: Quantum inspired optimization algorithm (QIO) based ELSA - GNQTS Methodology: Optimization Use case: Portfolio optimization	

		- “Implementing the short-selling trend ratio model in the significant G7 markets broadens its applicability” (p.1) - “A steady profit short-selling portfolio can be constructed in G7 nations. The proposed ELSA technique significantly outperforms other QIO algorithms.” (p. 4), Important notes: - “Quantum search algorithms are among the applications, where the quantum computer outperforms the classical computer” (p. 1) - “Nevertheless, the current quantum computer has lower fidelity, coherence time, and fault tolerance” (p. 1) - “The QIO proves to be more effective in portfolio optimization than traditional GA” (p. 2)		
[68] **Portfolio Optimization considering Diversified Investment Methods using GNQTS and Trend Ratio (Jiang et al., 2018)**	“This paper uses the trend ratio to access the portfolio with a stable upward trend. By the portfolio trend line with initial funds” Sliding window mechanism is used to select appropriate training and test periods for the experiment. “This paper provides time deposit choice and two investment options: buying round lots only or additional odd lots, and utilizes the GNQTS to find which investment method is better under the investment periods.” (p. 2) The best portfolio among the sliding window periods is found effectively and efficiently using the GNQTS	Objective(s): - Form a GNQTS model incorporating trend ratio, 2-phase sliding window mechanism, funds standardization, time deposit, round lots and odd lots - Experiment with the proposed model on a stock selection problem for the Taiwan 50 ETF from 2010 to 2017 and 13 sliding window periods., without restrictions on the stocks (so the algorithm can choose zero or only one stock if it is the best option). The experiment is analyzed by the values: the trend ratio, daily expected return, daily risk, round lots, and odd lots. - For the algorithm: execution number is 50, 10.000 generations, and a population of 10 Results: - The experiments showed that different investment methods had their own unique suitable portfolios. - Round lots had lower risk, but also lower expected returns than odd lots, with trend ratio helping to balance return and risk for the best investment method. - The most suitable investment method varied per period in the experiment	Quantum hardware: Simulator Quantum algorithm: GNQTS Methodology: Optimization Use case: Portfolio optimization	Funds standardization = Time deposit = a bank account with interest that has a predetermined maturity date. Lot = number of units of a financial product traded on a financial market Round lots = the general trading unit on the financial exchange, which on the Taiwan stock market is 1000 shares Odd lots = an order amount less than the normal unit of trading for that asset, in the case of this paper it is less than 1000

		- Using the proposed 2-phase, sliding window, GNQTS model a higher trend ratio could be found than in a single investment situation, indicating a performance increase achieved by the proposed model. - "This paper finds that the different investment method suits the different situations and the different portfolios." (p. 6) - "The experimental results show that the proposed method can find the well-performing portfolio with higher return and lower risk in both the training and testing periods." (p. 1) Important notes: - "The trend ratio can simultaneously consider the daily expected return, daily risk and fairly compare with the different portfolios and different investment periods lengths." (p. 1)		shares in the Taiwan stock market.
[71] **Financial Portfolio Optimization: A QAOA and VQE Formulation for Sharpe Ratio Maximization (Kaushik et al., 2023)**	This paper discusses the application of QAOA and VWE for PO problems Results from the proposed approaches are compared towards each other in an experiment	Objective(s): - Transform the Markowitz model in a QUBO formulation for stocks traded on the Abu Dhabi Securities Exchange and then solved through VQE and QAOA - For the classical method of benchmarking, use the Sequential Least Squares Programming (SLQP) to form a discrete programming problem of the objective PO function, and then solve it through the classical Branch-and-Bound method. - The experiment for the quantum solvers includes 10 stocks on the Abu Dhabi Securities Exchange, which are subsequently either minimized in risk for a particular level of return for a portfolio, or maximized on returns with certain risk levels for a portfolio. Then do the same for a risk factor weight. Results: - The highest achieved Sharpe ratio on the 10-stock example was1.14, indicating that the best portfolio should give a return of 1.14 times above the risk-free rate.	Quantum hardware: D-Wave quantum optimizer QBSOLV (simulator) Quantum algorithm: QAOA, VQE Methodology: Optimization Use case: Portfolio optimization (particularly Sharpe ratio optimization)	Sharpe ratio = a ratio for the comparison between the return and risk of an investment, Sharpe-ratio is used to determine risk-adjusted performance

		- The highest Sharpe ratio for the added risk factor formulation achieved a Sharpe ratio of 1.20, this Sharpe ratio was 60 base points more than the classical approach. - “The Sharpe ratio obtained by VQE Model and QAOA Model is 1.20 and 1.21 respectively which is better than the one obtained from the classical model having a value of 1.11.” (p. 6) - The paper mentioned the potential of real-life PO problems being solved by quantum hardware as the challenges of NISQ hardware are solved. - Result of the classical method on the 10 asset portfolio: Expected returns = 34.48, expected risk = 31.15 Sharpe ratio = 1.11 - Results of VQE: Expected returns = 45.27, expected risk = 37.69, Sharpe ratio = 1.20 - Results for QAOA: Expected returns = 40.11, expected risk = 33.14, Sharpe ratio = 1.21 - “We found that Quantum algorithms are giving better results than classical solver” (p. 7) Challenges for the QUBO formulated PO problem solved via VQE and QAOA in this paper: - Restricted number of qubits available on NISQ devices. As more assets are brought into the mix, more qubits are needed to find the optimal solution. Current (2023) NISQ devices have a max of 20 qubits. - Qubit connectivity is restricted, which makes the mapping of complex problems difficult - Recision of results is decreased by errors through the noise of current NISQ devices. Quantum error correction measures ought to be imposed for higher result quality. - The complexity of encoding bigger portfolio optimization problems into the quantum hardware. Important notes: - “Quantum computing helps in faster and more accurate calculations than the classical approach, therefore it		

		can play an important role in finance and portfolio optimization." (p. 1) - "Quantum Annealing systems have been able to achieve more dependable qubits, however, these qubits encounter challenges related to low connectivity" (p. 1)		
[74] **Quantum beetle antennae search: a novel technique for the constrained portfolio optimization problem (Khan et al., 2021)**	A Quantum Beetle Antennae Search (QBAS) is formulated, where it is applied to a maximization PO problem, whilst comparing the solutions it gives towards other similar metaheuristics (GA, PSO, BAS)	Objective(s): - Formulate a quantum version of BAS named QBAS - Find the set of optimal stock allocation in a portfolio with QBAs so that it minimizes risk and maximizes mean-return. - Experiment with the proposed QBAS algorithm on different stacks of stock the Shanghai Stock Exchange 50 Index (SSE 50) to assess efficiency benchmarked on 4 given benchmark optimization functions with differing numbers of stocks (20, 50, 75, 100) obtained from the date 21March 2019 – 18 April 2019. - Apply the QBAS to real-world stock data and compare results with other meta-heuristic optimization algorithms (BAS, PSO, GA). Results: - Results with 20 stocks for QBAS compared to BAS, GA, and PSA: highest Sharpe ratio, Equality constraint is almost achieved, the best result for F(e), fastest solution time with least iterations used. - Results with 50 stocks for QBAS compared to BAS, GA, and PSA: F(e) was more optimized than the rest, Sharpe ratio was highest, equality constraint is almost followed (for all algorithms except PSO), faster computing times for finding the optimal solution. - Results with 75 stocks for QBAS compared to BAS, GA, and PSA: Highest value for F(e), Sharpe ratio is highest and comparable with GA, fastest converging times, all algorithms obey equality constraints.	Quantum hardware: Quantum-annealer D-Wave system Quantum algorithm: QBAS Methodology: Optimization Use case: Portfolio Optimization	F(e) = the given PO maximization problem Equality constraint = conditions that a found solution must satisfy, a solution must be equal to a given value in an equality constraint

		- Results with 50 stocks for QBAS compared to BAS, GA, and PSA: QBAs outsmarted the other algorithms and found the highest value for maximization function F(e), highest Sharpe ratio, fulfilling the equality constraint, faster convergence - "Results how that QBAS outperforms swarm algorithms such as Particle Swarm Optimization (PSA) and the genetic algorithm (GA) - "QBAS is powerful enough to converge to the global solution even with different initial conditions." (p. 9), and within 120 iterations, the QBAS algorithm found the optima value for the four given optimization functions. - QBAS showed to have the ability to avoid local minima, avoiding them al in 20 consecutive simulations Important information: - In a theoretical analysis the proposed QBAs formulation showed to be stable and convergent. - Constraints in the QBAs are turned into a penalty function in QBAS algorithm. - QBAS is the first quantum version of BAS - The QBAS is a metaheuristic - To the knowledge of the authors, no metaheuristic to date (2020) has been applied to address the PO problem of min risk and max mean-return. - The text mentioned that classical algorithms have a hard time considering real-world challenges in PO such as: cardinality constraints, lower/upper bounds, substantial stock size, class constraint, round-lots of constraint, computational power and time, pre-assignment constraint, and local-minima avoidance. - Current meta-heuristic approaches achieve higher efficiency and accuracy than classical approaches.		
[76] **Portfolio Optimization Model**	In the paper , the adaptive quantum inspired tabu search	Objectives: - Develop a ANQTS model incorporating a 2-phase sliding	Quantum hardware: N/A	

using ANQTS with Trend Ratio on Quadratic Regression (Kuo et al., 2019)	(ANQTS) is used together with a quadratic regression trend line, and 2-phase sliding window to search for the most optimized portfolio.	window, and a quadratic regression trend line - Experiment with the ANQTS model on stock chosen from the Taiwan 50 ETF with an investment period of 2010-2018. Model specification: 13 types of sliding window, initial fund is 10 million TQD, population is 10, 10.000 generations, 50 executions. Results: - Best sliding window periods were month to month, and year-to-year month (meaning analyzing the same month only, for every year) - The quadratic trend ratio showed to give a more specific description of the trend in the portfolio than the normal trend line. - Portfolio formed using the quadratic trend ratio show to have higher daily expected returns per unit of risk than the trend ratio, with daily risks also being lower on average for the quadratic trend ratio. - In 9 out of 13 sliding window periods, the quadratic trend ratio derived better performance than the trend line, showing stronger upward trend than the linear trend. Furthermore, compared with the Scharpe ratio, both the trend ratio and quadratic trend ratio outperform it based on upward trend. - "The experiment results show that the proposed portfolio optimization model has better performance than the Sharp ratio and trend ratio on linear regression" (p.1) - "The result shows that the proposed method is able to obtain better results." (p. 5) Important notes: - As many papers consider the shortcomings of the Sharpe ratio for PO problems, a trend line method is often approaches. However, even the trend line has some issues considering portfolio up/down trends precisely, so to achieve a precise estimation of up/down trends, a quadratic regression trend line.	Quantum algorithm: ANQTS Methodology: Optimization Use case: Portfolio optimization	

		- QTS has been proven to have beta search abilities than other metaheuristic algorithms. - "When ANQTS is stuck in the local optima, it can detect and jump out of the local area; hence, ANQTS has better search abilities than QTS." (p. 2) - "The core concept of QTS is that QTS moves the individuals toward the best solution and away from the worst solutions at the same time while enabling QTS to outperform other traditional optimization algorithms" (p. 3)		
[77] **Entanglement Local Search-Assisted Quantum-Inspired Optimization for Portfolio Optimization in G20 Markets (Kuo et al., 2023)**	In this paper, a quantum algorithm is proposed for PO problems, the ELSA-GNQTS. The ELSA-GNQTS is used to search for stable uptrend portfolios in the global g20 markets "This study discusses the expanded markets to demonstrate the superior ability of the proposed QIO method in a vast solution space." (p. 1) "This study aims to enhance the ability of QTS to solve a more complicated PO, and thus the quantum entanglement mechanism is simulated to propose a novel entanglement local search-assisted (ELSA) technique for PO" (p. 1) "This is the first study to apply trend ratio evaluation in an intermarket of G20 markets" (p. 2)	Objective(s) - Form a novel formulation of the QTS algorithm, employing ELSA to assist QTS in searching more accurately in the potential area with domain-dependent information where it is used. - Use the trend ratio-based improved ELSA-GNQTS formulation in an experiment where stable uptrends are to be identified from the global G20 markets from January 2013 to December 2022, selecting the top 30 companies from the G20. Specifications of the setting: initial funds of 1 billion local currency, 50 independent experiments, 10 populations, 10.000 iterations, equally weighted stocks. The results are benchmarked based by analyzing the financial performance of the found 'optimal portfolio' on different investment strategies, the robustness of results. - Use the trend ratio to evaluate a portfolio's utility and use it to further construct portfolios with stable uptrends. - Furthermore, use sliding window mechanism to find optimal training and testing periods, 13 sliding windows were used. Results: - Resulting portfolio had better results regarding risk than the single best stock performance for risk, the portfolio trend ratio was also higher	Quantum hardware: N/A Quantum algorithm: QIO inspired ELSA-GNQTS Methodology: Optimization Use case: Portfolio optimization	

		than that of the single best stock, indicating - The proposed QIO system demonstrates outstanding performance in managing risk and maximizing returns, significantly outperforming traditional strategies and market indexes in the G20 markets. - Considering the 13 chosen sliding windows, the ELSA-GNQTS outperformed the GNQTS every time based upon the given trend ratios. - The proposed QIO can effectively and efficiently find portfolios with stable trend ratios, and balance risk and return - Furthermore, ELSA-GNQTS outperformed other algorithms (GNQTS, GQTS, QTS, GA) based on the trend ratio - Adding more markets to them ix proved to incrementally improve performance of the ELSA-GNQTS in efficiency, and balancing risk-return. - “Through trend ratio evaluation, a global asset management system that integrates G20 markets can facilitate more robust investment.” (p. 5) - “The ELSA-GNQTS demonstrates its robustness by outperforming other QIO algorithms and GA in an integrated market analysis.” (p. 8) Important notes: - “The entanglement relationship can decrease the degree of freedom searched” (p. 1) - “QIO algorithms can serve as a bridge to realizing preliminary quantum advantages by exploiting classical computation abilities.” (p. 1) - NISQ computers still have many challenges considering error correction and fault tolerance. - QIO simulates quantum mechanics on a classical computer to exploit potential quantum benefits.		

[78] **Strategic Portfolio Optimization Using Simulated, Digital, and Quantum Annealing (Lang et al., 2022)**	In this paper, a new workflow is introduced for quantum annealing platforms to solve PO problems. A classical pre-processing step is combined with a modified QUBO model an evaluated using, simulated annealing (classical computer), digital annealing (Fujitsu's digital annealing unit), and quantum annealing on the D-wave advantage "In this paper, we focus on the applicability of annealing techniques to the NP-hard problem of portfolio optimization, a well-known topic for investment funds and individual investors" (p. 2)	Objectives: - Steps in the proposed workflow: 1. Markowitz's theory on PO is used in a classical pre-processing step where the most promising assets are found from an initial pool of assets. 2. The QUBO is modified to fit models for PO problems, it is modified such that there are no limitations on the number of stocks that be invested in. With optimization functions including Sharpe ratio maximization, diversification through covariance minimization, and budget constraints. 3. This QUBO formulation is then used on the identified set of assets from the New York Stock Exchange over a period of 5 years (31-12-2014, 31-12-2019) to find the percentage of capital that should be used on which asset. Specification of the experiment: 1000 random portfolios as benchmark, 10.000 samples for the annealing process, and the 10 best solutions each time are visualized in the paper. 4. As the QUBO formulation consists of three parts (a part for expected returns, a part for risk, and the third part being a budget constraint), tests are done using different weights for each part. - Perform the test for the QUBO formulation on real-world data from sets of stock in the New York Stock exchange as well as common ETFs. - Lastly, compare the results from the test against randomly generated portfolios using return, variance, and diversification measures. Results: - Looking at the given graphs for the results of the experiment, it can be seen that Digital and simulated annealing yield almost the same results. With quantum annealing performing not as good as simulated and digital annealing (probable cause is inherent noise missing error correction, and scaling of parameters) - Simulated annealing showed that the QUBO model approach worked as intended, meaning that portfolios	Quantum hardware: Classical computer (using simulated annealing), Fujitsu's digital annealing unit, and D-Wave advantage (~5000 qubits)as real quantum hardware. Quantum algorithm: QUBO model Methodology: Optimization Use case: Portfolio optimization (particularly how to spread funds over a portfolio)	

		were generated that respected the given preferences to either returns, risk, or budget constraint. - Changing the weights for either risk, return, and budget showed that results in the experiment gravitated accordingly and efficiently towards the objective weight distribution of the model (e.g. more weight relatively on expected returns yielded higher return portfolios) - Simulate and digital annealing both managed to use 100% of the budget every time, but for quantum annealing a bias of +- 9 percent was perceived in budget spending. - Sometimes over/underspending was needed for the optimal portfolio. - In part of the experiment, the differences between the different annealing approaches can be linked to better/worse diversification and different degrees of allocations of the budget to an asset. - "The results show that our QUBO formulation is capable of creating well diversified portfolios that respect certain criteria given by an investor, such as maximizing return, minimizing risk, or sticking to a certain budget." (p. 1) Important notes: - Heuristic methods such as simulated annealing, genetic algorithms, swarm intelligence have been found to not always fin the most optimal solution to a PO problem. - Current annealing solution for PO problems suffer from the following limitations: limited amount of assets to choose from, and use of naïve investment strategies for the calculation of future returns (meaning that the strategies rely mostly on basic assumptions and historical averages) - PO has been solved by two other quantum methods according to the paper: 1. Quantum linear systems algorithm, 2. Quantum annealing (afterwards the paper gives an example of an earlier study that		

		managed to use quantum annealing to construct a portfolio with a budget of 100 dollars and turn it into 121.176 dollars, showing how advantageous quantum annealing can be for PO)		
[80] **Portfolio Optimization Based on Quantum HHL Algorithm (Li et al., 2022)**	In this paper a quadratic HHL algorithm is proposed with equality constraints to solve combinatorial problems in finance. Results gathered from the proposed quadratic HHL algorithm design are measured analyzed, and compared with classical solutions “In this article, we proved the feasibility of the HHL algorithm to solve this type of portfolio problem (with constraints, NP-hard problem), and set up the actual problem to solve it” (p. 2)	Objectives: - Form a quadratic HHL algorithm with equality constraints and benchmark it using an example PO problem. (the exact origin of the values given to calculate the PO model have not been given) Results: - Compared to classical algorithms, the proposed HHL algorithm is able to solve combinatorial optimization problems, and the solution it gives is in good agreement with the exact optimal solution. - Proving the feasibility of the HHL algorithm on a PO problem showed solutions very close to the exact solution, and minimal error of each component. - Increasing then number of qubits (from 9) would likely increase the solution’s accuracy, but it will also increase the circuit complexity and quantum gates used. Important Notes: - The HHL algorithm was proposed by Harrow, Hassidim and Lloyd for solving linear systems with exponential acceleration compared to classical algorithms. - “The high computational complexity of financial problems sometimes makes them difficult to be solved on classical computers.” (p. 2) - “Some quantum algorithms applying in financial problems have been proved to be better than classical methods, which can provide considerable acceleration, such as quantum Monte Carlo algorithm,	Quantum hardware: N/A Quantum algorithm: Quantum HHL Methodology: Optimization Use case: Portfolio optimization	

		portfolio optimization algorithm" (p. 2)		
[81] **Hybrid Gate-Based and Annealing Quantum Computing for Large-Size Ising Problems (Liu, Chen-Yu. & Goan, his-Sheng. 2022)**	In this work the Large-System Sampling approximation (LSSA) algorithm is proposed to solve large size Ising problems with a hybrid quantum annealer / gate-based approach. "By dividing the full-system problem into smaller subsystem problems, the LSSA algorithm then solves the subsystem problems by either gate-based quantum computers or quantum annealers" (p. 1), and is then further optimized by VQE Both random Ising problems and PO problems are solved on simulators and real quantum hardware	Objective(s): - Form the LSSA algorithm for large size Ising problems, - solve different PO and random Ising problems (both on simulated and real hardware). Which are either: 1. fully connected random Ising problems with up to 160 variables on a 5-qubit quantum computer, or a PO problem with up to 4096 variables on 100 qubit quantum computer + a 7 qubit gate-based computer 2. A PO problem with up to 5120 variables. - Lastly, examine the effects that different sub-system sizes/numbers, and problem sizes have on the performance of LSSA on simulators and real hardware Results: For the simulated problems: For random Ising problems (using IBM Tabu for sub-system solving and IBM QASM for amplitude estimation): - For small size Ising problems with the QASM-simulator (simulated quantum computer), high approximation ratios are found, indication good performance of the LSSA algorithm - For larger size Ising problems with the Dwave-Tabu solver (classical solver), a decreasing trend in the approximation ratio as problem size increases, ultimately falling to 68%. For PO problems (IBM QASM simulator): - The LSSA achieved approximation ratio results close to 1, indicating similar performance to Dwave Tabu, the simulator showed robustness in results.	Quantum hardware: Simulators (IBM QASM Simulator), and real-quantum hardware (D-wave annealer advantage 4 with 5760 qubits, and IBM Auckland, IBM Cairo and IBM Guadeloupe gate-based computer) Quantum algorithm: LSSA algorithm (model) Methodology: Optimization Use case: Large-size Ising problem (portfolio optimization particularly)	Approximation ratio = Approximation ratios are different for each problem in this paper, the approximation ratio is a ratio that benchmarks solutions from experiments toward a given value obtained as an objective benchmark (so if approximation ratio is 1, it indicates performance alike to the given denominator (which changes each time to one of the two in this paper: e.g. results from the classical method Dwave Tabu, or the exact ground state energy (which is a measure of optimality))), so approximation ratios will look as follows: result obtained / result from dwave tabu, or result obtained / exact GSE (optimal solution). Overall if approximation

		- As problem size increased, approximation ratio stayed close to 1 Real-quantum hardware findings: For random Ising problems (with D-Wave advantage 4, and IBM gate-based computers): - "The trend of the average approximation ratio is similar to that obtained by the simulators, i.e., it decreases considerably to a low value when Np (problem size) > Ng (sub-system size), indicating a relatively poor performance." (p.8) For PO problems with simulated stock data (using D-Wave advantage 4 and IBM Auckland): - Approximation ratio for solving only the sub-systems using the D-Wave advantage 4 show good approximation ratios close to 1, indicating good performance. - Simulations with different PO problems on the IBM QASM Simulator showed similar results to a classical solver such as Dwave Tabu. - The impact sub-system size had was positive with greater sub-system sizes, and the fewer samples were performed, the better the results. For PO problems with real-world data over 47 months, and problem sizes (stock amounts) of 32 and 64 months from the US stock market to examine LSSA (using IBM Cairo): - Sharpe ratio of the LSSA was slightly lower than the classical solver for both problem sizes, indicating still good performance, but lower than the classical method - "Our proposed algorithm can solve fully-connected random Ising problems that are *O*(10^0) and portfolio optimization problems that are *O*(10^1) larger in size than the available quantum annealers and gate-based quantum computers" (p. 2), both with good performance from simulated and real-hardware - For Random Ising problems, performance declined with increasing problem size, which was not the case for PO problems		ratio is close to 1, it is good.

		- This paper shows promising results from a hybrid quantum annealing gate-based LSSA model. Important information - The given problem function is divided into sub-systems which are then solved first, after which an estimation of the full system is made. - "Even the largest gate-based quantum computer to date provided by IBM (IBM Washington) can only solve the problem with 127 variables if we use the original VQE and QAOA algorithms." (p. 1)		
[82] **QPSO algorithm based on Levy flight and its application in fuzzy portfolio (Lu, X, L. & He, G. 2021)**	In this paper, an improvised quantum-behaved particle swarm optimization algorithm (LQPSO) is proposed based upon the (Q)PSO The LQPSO is then used in an experimental setting with fuzzy portfolio models with transaction costs and background risk process to consider its practicality To enhance particle exploration (searching for potential solutions), Lévy flight strategy, premature prevention mechanism and contraction-expansion coefficient with non-linear structure are considered	Objectives: - Form an improved quantum-behaved particle swarm optimization algorithm (LQPSO), including Lévy strategy and contraction expansion coefficient with non-linear structure to enhance particle exploration - Evaluate the improvised algorithm via 12 basic benchmark functions and benchmark it against QPSO, PSO-w, RQPSO. With parameter setting being: population size of 100 (assets), search spaces of 10, 20, 30, with corresponding max iterations of 500, 1000, 1500. Results: - For the five uni-modal functions and seven multi-modal functions, LQPSO was superior to PSO-w, QPSO and RQPSO, showing higher accuracy and less standard deviation. - For the five uni-modal functions, LQPSO achieves theoretic optima each time - For the seven multi-modal functions, LQPSO shows that optimization results are better than the other three algorithms. - LQSPO overcame finding premature/sub-optimal solutions better than the other algorithms, jumping from local optima towards the global optimum (whilst the other algorithms often got stuck in local optima). - Under high-dimension and complex situations (30 dimensions, 1500	Quantum hardware: N/A Quantum algorithm: LQPSO Methodology: Optimization Use case: Portfolio optimization	Lévy flight strategy = a particular tool that enhances exploration capabilities of search algorithms to improve efficiency and effectiveness of the optimization process. Contraction-expansion coefficient with non-linear structure = a parameter used in optimization algorithms to control the movement of particles (possible solutions) in the search space, this helps balancing the exploration and exploitation phase of the algorithm. It is useful in complex search landscapes. Premature prevention mechanism = a mechanism that

		generations, convergence accuracy 10^-6), PSO-W successfully follows accuracy requirements in 2/30 functions, and QPSO and RQPSO accomplish error requitements in seven 7/30 functions with success rates of 100%, thus demonstrating strong robustness. - Wilcoxon rank sum test shows that LQPSO outperforms the rest of the algorithms. - "LQPSO demonstrates better convergence and robustness than PSO with inertia weight, QPSO and QPSO with a hybrid probability distribution in 12 benchmark functions." (p. 1) - "Experimental results indicate that LQPSO outperforms several metaheuristics when seeking optimal solution for the fuzzy portfolio model with constraints." (p. 1) Important notes: - The paper mentioned that QPSO has better converging speeds and global search ability than PSO - Investment proportions of each stock are constrained to a certain number.		ensures that the algorithm does not converge to a suboptimal solution by getting stuck in a local minima or maxima (which is often a problem for PSO algorithms) Uni-modal function = function with one local min/max (e.g. min risk) Multi-modal function = function with multiple local min/max (so it has multiple good solutions, but is prone to generating suboptimal solutions as there are more peaks, global best values are more complex to find)
[88] Diversifying Investments and Maximizing Sharpe Ratio: a novel QUBO formulation (Mattesi et al., 2023)	As classical optimization of the Sharpe ratio becomes more complex through additional needs such as new constraints or new objective function terms, the problem may become non-convex and thus not solvable via classical methods The proposed solution for this problem in this paper is a novel QUBO formulation of Sharpe ratio optimization with a diversification term	Objective(s): - Propose a novel QUBO formulation of a PO problem including maximization of Sharpe ratio with a diversification measure to spread risk. - Benchmark the novel QUBO formulation on two main aspects of the QUBO formulation: 1. Report the behavior of the complete model as parameters influencing the Sharpe ratio and diversification terms are employed, evaluate the performance of the formulation for the sole Sharpe ratio maximization compared to other techniques. - Benchmark performances of the QUBO model against classical solvers on a real-world dataset including	Quantum hardware: "existing QUBO solvers" (classical QBSOLV, and D-wave leap hybrid classical-quantum solver (which makes sub-systems that are then solved via tabu-search algorithm)) Quantum algorithm: QUBO Methodology: Optimization Use case: Portfolio optimization (maximizing Sharpe	Log returns = a different measure to assess assets in this case for the data pool, which employes assessment through the natural logarithmic of return of an asset, thereby aiming to increase efficiency of results.

		- Specifications of the experiment: 460 assets for simple returns, and 432 assets for log-returns. As the D-Wave system restricts high precision measures, the precision value of p = 12 bits. Results: - Results for Sharpe ratio maximization: more feasible optimal solutions are found as the diversification term is discarded, best Sharpe ratio values are observed when solving via the QBSOLV. - Results for Sharpe ratio including diversification measures: risk is lower, but the optimization is significantly impacted and the Sharpe ratio tends to decrease as more funds are allocated to spread the investments over more assets, thereby making it so that there is less impact on the expected returns or covariance of the assets. - For both formulations, the best performances are obtained by different solvers: D-Wave Hybrid and QBSOLV (which is mainly attributed to differing number of variables) - Furthermore, for the QUBo formulation, the QBSOLV performed best, being able to handle 5184 binary variables. - Constraints are fulfilled by several solvers, demonstrating a consistent viability as demonstrated by the proposed formulation. Competitive performance is shown by QUBO formulations when compared to PyPortfolioOpt, the classical solution. - The QUBO formulations offer a viable alternative to classical solvers, especially in handling complex optimization problems involving both Sharpe Ratio and diversification. Important notes: - “We do not emphasize the computational time required to obtain the solutions as it is not the primary focus of our study. Instead, we draw attention to the quality of the results	ratio with a diversification term)	

		in terms of objective function value" (p. 14) - "Portfolio optimization has been approached by different means, including linear programming, quadratic programming, semidefinite programming, meta-heuristics, deep learning, and reinforcement learning." (p. 2) - "It is widely believed, based on reasonable computational complexity assumptions [24], that neither classical nor Quantum Computers can efficiently solve NP-hard optimization problems." (p. 5), but significant speedup compared to classical algorithms is still proven.		
[89] **Applications of Quantum Machine Learning for Quantitative Finance (Mironowicz et al., 2024)**	This paper examines the connection between quantum computing and machine learning for applications in finance, in the summary of this paper, there will mostly ne looked at application for portfolio optimization. Further on in the paper, there is a specific section dedicated towards a review of current (2024) literature, which gives insight of Quantum Machine Learning from mother perspectives.	Uses of Quantum Machine Learning for portfolio optimization, a review: - Most commonly, the Sharpe ratio is taken as a measure of risk-adjusted return, this ratio is sought to be maximized in many of the quantum PO algorithms and use cases - The importance of taking crucial elements in the PO problem formulation is considered, as PO problems are not as black and white as max return and min risk, multiple measures come into play when achieving this (e.g. liquidity of assets, transactions costs, constraints set by the investor) - Two main types of PO problems are recognized: constrained and unconstrained, which respectively differ in the fact that one has certain set constraints (e.g. budget constraint or weights) an the other has a lack of constraints, but can still have weights assigned to certain parts of the function (e.g. giving higher allocation to expected return part of a formula). - When solving PO problems, you want to achieve portfolios that are on the line of the efficient frontier (see literature review for explanation) - There are also factor-based PO models that incorporate other factors influencing outcomes such as value, size, momentum, and quality. These are often measures used to estimate riskiness and relationship between	Quantum hardware: N/A Quantum algorithm: N/A Methodology: Optimization Use case: Portfolio optimization	

		securities in a portfolio, thereby being a good technique to form (un)correlated portfolios if needed. PO and Quantum Machine Learning (QML): - An example QML case is taken in the paper to explain the benefits of it. The example showed how QML was used for a multi-period PO problem on D-Wave systems ‘quantum annealer, showcasing high success rates in finding optimal portfolios with included transaction costs. - Furthermore, another study was taken where 63 securities listen on the Abu Dhabi Security Exchange were considered with certain budget and parameters to test whether the use of a D-Wave QPU could be beneficial for solving Markowitz portfolio. Results from this study showed that it could be sued to find optimal solutions - Next, the authors of the paper used an instance of another example paper where the importance of additional measures to optimize quantum models for efficiency is stressed. Even if a quantum model for a certain problem outperforms other benchmarked measures does not mean it cannot be significantly improved. In the case of the example paper, they discovered that certain measures such as seeding the algorithm with better data acquired from a quantum annealer and a reverse annealing protocol yielded 100 times faster time-to-solution as opposed to the corresponding forward quantum annealing process. - Furthermore, more examples are given to stress the notion that QML for PO problems are proven to be beneficial for efficiency and performances, - Lastly, In a comparison with the D-Wave 2000Q system and classical commercial solvers, results showed promising performances, coming close to the performance of existing classical solvers for same instance sizes.		

		- "Quantum technologies offer promising applications in portfolio optimization, leveraging quantum computing's potential to efficiently solve complex optimization problems." (p. 29) QML (Quantum Circuit Born Machines in this case) compared to classical ML methods (restricted Boltzmann machines predominantly): - "The quantum models demonstrated superior performance compared to RBMs when considering the same number of parameters" (p. 21) that was under data from the S&P500 - "The effectiveness of certain HHL enhancements is empirically demonstrated through the application to small portfolio optimization problems" (p. 21) - Another example was taken where the QML technique offered a quadratic speedup, along with statements of the great practical use of it. - Another instance where VQE is used on IBM 100 qubit simulators is analyzed, and it showed a strong relation between solution quality and quantum hardware size, VQE can generate solutions close to optimal/exact ones (even without error-mitigation) Important notes: - "As quantum computers continue to evolve and become more accessible, the integration of QML into finance applications is expected." (p. 1)		
[91] **Hybrid quantum investment optimization with minimal holding period (Mugel et al., 2021)**	A hybrid-quantum classical algorithm is proposed for dynamic PO problems with minimal holding periods. The hybrid quantum-classical algorithm is then experimented upon on a dataset consisting of 50 assets over a one-	Objective(s): - Form a hybrid-quantum classical algorithm for dynamic PO problems with minimal holding periods - Use clustering techniques to improve diversification and reduce risk, and at the same time reduce required resources from the quantum system. Do pre-processing of the assets on their historic volatility to measure and compare with a given risk threshold/category to form a pool of assets with require volatility.	Quantum hardware: Quantum annealing (D-Wave 2000Q) Quantum algorithm: A quantum-classical hybrid algorithm (exact name not specified) Methodology: Optimization Use case:	Integer bundles = the requirements that assets, in this case, must be sold in whole discrete units. (minimum) holding period = the amount of time elapsed between an investment's

	year period using the D-Wave 2000Q system.	- Experiment with the proposed hybrid algorithm on 50 international assets between May 31st 2019 and May 31st 2020 on a quantum annealer and compare to a random asset chosen portfolio (within risk requirements). Both portfolios are daily portfolios. Results: - During the given period of the experiment, the optimal investment trajectory was found for 50 assets on the D-Wave2000Q using five risk packages (5%, 10%, 15%, 20%). - Comparing with a randomly chosen portfolio of assets within the risk requirements, the quantum annealing method based upon dimensional reduction and post-selection showed solutions closers to the efficient frontier. - Computing time was "just a few minutes" on daily portfolios for 50 assets with the proposed method. Compared to classical (brute force), the algorithm performed way faster, and with comparison to other quantum methods (VQE), the proposed algorithm can compute greater problem sizes (as VQE could only perform this task with max 3 assets). - D-Wave2000Q showed to be faster than other solvers such as Gekko. - "Our study shows that the method is remarkably efficient and produces in few minutes results close to the optimal efficient frontier in portfolio space, much better than typical random portfolios." (p. 4) - Furthermore, this study showed that the proposed algorithm can perform well in giving out optimal investing trajectories for differing risk profiles. - "Our method is remarkably efficient, and produces results much closer to the efficient frontier than typical portfolios" (p. 1) - "Our results are a clear example of how the combination of quantum and classical techniques can offer novel valuable tools to deal with real-life	Portfolio optimization	purchase and its sale, and as investments are often taxed favorably in the long-term, a minimal holding period is imposed (minimal holding period in this paper is seven days, investing options that do not apply to the seven-day period are ruled out)

		problems, beyond simple toy models, in current NISQ quantum processors." (p. 1) Important notes: - The aim for the financial model is to maximize returns for a given level of risk considering the given constraints. - The metric used for comparing investments is the Sharpe ratio. - It is assumed that shares can only be sold in large bundles. - Number of objective variables is proportional to the number of assets. - NISQ devices are limited in their resources, therefore, dimensional reduction techniques are used to reduce required resources. - This work is a successor of a previous work entailing a hybrid algorithm alike, the differences proposed in this paper is an efficient post-selection protocol to impose a minimal holding period constraint, and a proposition that investors should invest in integer bundles - "There are many important optimization problems in finance which can be solved more efficiently using quantum computing." (p. 1)		
[92] **Dynamic portfolio optimization with real datasets using quantum processors and quantum-inspired tensor networks (Mugel et al., 2022)**	In this paper a PO problem involving transaction costs and other possible constraints is tackled using a number of quantum and quantum-inspired algorithms on different hardware platforms. The po problem data consists of daily prices from over 8 years of 52 assets Methods used are: Gekko exhaustive (classical), D-Wave hybrid quantum annealing, two VQE approaches on IBM-Q and a quantum-inspired	Objective(s): - Make use of D-Wave hybrid quantum annealing, IBM-Q with VQE and VQE-constrained and TN to solve a PO problem for a dataset of up to 52 assets over 8 years, with ultimate datasets varying in size. - Benchmark the solutions of the above algorithms with results obtained by classical methods (Gekko solver, and an exhaustive solver) via Sharpe ratio and computing times for different problem sizes (XS, S, M, L, XL, XXL - Results: Results from Gekko, Exhaustive, DWave Hybrid, VQE, VQE-Constrained, and TN solvers (results for problem sizes XS-XXL are only shown for XS, M, and XXL for a summarized overview, and N/A values for XS-	Quantum hardware: Gekko exhaustive (classical), D-Wave hybrid quantum annealing (D-Wave 2000Q), two VQE approaches on IBM-Q and a quantum-inspired optimizer based on tensor networks, Quantum algorithms: VQE, VQE-constrained, Quantum inspired tensor network (TN) Methodology: Optimization Use case: Portfolio optimization	

	optimizer based on tensor networks, To be able to fit the data on the platforms, pre-processing with clustering assets is performed.	XXL are taken out as there were no values obtained for that): - **Gekko: Sharpe ratio** Sharpe ratio (XS- 5.98, M- 8.39, XL-20.76), profits% (XS-5.8%, M-13.6%, XL-71.6%), time (XS-24s, M-21s, XL-261s) - **Exhaustive (brute-force search):** Sharpe ratio (XS-6.31) profits% (XS-5.1%) time (XS-0.005s) - **D-Wave Hybrid**: could solve problems up to 1272 fully connected qubits in 172 seconds, which is REALLY fast according to the authors. For the PO experiment, following results were obtained: Sharpe ratio (XS- 5.98, M-8.39, XL-12.16), profits% (XS-5.8%, M-13.6%, XL-67.6%), time (XS-8s, M-19s, XL-74s) - **VQE:** Sharpe ratio (XS-3.59) profits% (XS-2.4%) time (XS-278) - **VQE-constrained:** Sharpe ratio (XS-6.31, M-4.81) profits% (XS-5.1%, M-7.1%) time (XS-123s, M-490s) - **TN solver:** Sharpe ratio (XS-5.98, M-9.54, XL-15.83), profits% (XS-5.8%, M-15.4%, XL-39.7%), time (XS-0.838, M-120s, XL-82698s) Results showed that not all problem sizes could be computed for some methods, only D-Wave hybrid and TN could solve XXL problems, and VQE could not solve above XS problems. Computation times showed the increased. competition times that hybrid quantum-classical strategies can have over classical methods as these D-Wave hybrid was faster than the classical methods for increased problem sizes. Tn-solutions were quite high in computational times but did have better solution quality in finding minima than D-Wave hybrid, with different hyperparameters and fine-tuning the		

		authors propose that the solution quality and run-time of TN could be improved. the largest problem size XXL included 10^382 candidates, which is more than the number of observable atoms in the universe, 2 algorithms could find a solution to this problem, TN and D-Wave hybrid, showcasing the potential of quantum computing to tackle extreme problem sizes. Lastly, the authors propose to add more constraints and improved hardware to make solution quality better as a future work. Quotes on solution quality, speed, and overall results: - "From our results we also conclude that there seems to be no clear answer as to which is the "best" algorithm and hardware platform to solve the dynamic portfolio optimization problem for large systems. This is because there are several figures of merit at play: profits, Sharpe ratio, time cost, and also money cost. The performance of the algorithms is different depending on the figure of merit, leading us to conclude that, in practice, the more options we have, the better." (p. 11) - "We observed also that D-Wave Hybrid is remarkably fast, whereas Tensor Networks sometimes provide better portfolios at the expense of a longer calculation time" (p. 11) - "From our comparison, we conclude that D-Wave Hybrid and Tensor Networks are able to handle the largest systems, where we do calculations up to 1272 fully connected qubits for demonstrative purposes." (p. 1) - D-Wave Hybrid performed better than normal D-Wave, indicating classical-quantum to be better in this instance. - "We see that there is no clear answer as to which is the "best" algorithm and/or hardware platform to deal with large systems, as this depends strongly on different figures of merit." (p. 1)		

		- “In fact, the performance of Gekko is quite remarkable, sometimes even better than quantum and quantum-inspired solutions depending on the metric, but unfortunately the method hits a memory wall around 500 qubits” (p. 8)		
[93] **Use Cases of Quantum Optimization for Finance (Mugel et al., 2022)**	This paper gives an overview of some of the applications of quantum computing towards finance, however, in this summary there will only be looked at quantum computing use for PO.	For the summary considering PO, this paper mostly makes use of the paper above as an example to show performances of TN, VQE, classical methods, and D-Wave Hybrid, therefore only the following can be said on this paper for PO: - “Examples show that real business value can be derived from present day quantum computers. This is particularly true for the portfolio optimization case, where we found the best investment portfolio by optimizing over 52 assets and four years of data” (p. 224) - Tensor networks use by simulating quantum mechanics on classical computers	Quantum hardware: Gekko exhaustive (classical), D-Wave hybrid quantum annealing (D-Wave 2000Q), two VQE approaches on IBM-Q and a quantum-inspired optimizer based on tensor networks, Quantum algorithm: VQE, VQE-constrained, Quantum inspired tensor network (TN) Methodology: Optimization Use case: Portfolio optimization	
[94] **From Portfolio Optimization to Quantum Blockchain and Security: A Systematic Review of Quantum Computing in Finance (Naik et al., 2023)**	This paper gives a detailed and great overview of current (2023) quantum computing uses for PO, quantum blockchain and security. In this summary there will only be focused on the PO part, which gives great detail into recent contributions from other works in a neat table, use cases, previous survey works	Intro: - In PO problems, assets are chosen based upon factors like risk, return, liquidity, average return etcetera. PO problems can be categorized in two categories based on their formulation: 1. Convex and 2. Combinatorial optimization, where approaches have evolved from classical ways (e.g. mean-variance, variance with skewness, VaR, CVaR, mean absolute deviation, and minimax) to heuristic and meta-heuristic approach based methods. - Popular choices for these algorithms are: evolutionary algorithms, and swarm intelligence - Furthermore, some quantum approaches are also explored in the industry: as data increases exponentially (due to the curse of dimensionality), quantum computing methods become more of interest.	Quantum hardware: N/A Quantum algorithm: N/A Methodology: Optimization Use case: Portfolio optimization	

- The two major computation models used for quantum PO problems are quantum annealing and gate-based models. Where quantum annealing is more suitable for certain optimization problems, gate-based annealing is more suitable for universal problems but have less stable qubits on average than quantum annealing.

Table showing literature review results from this paper, the following table contains an overview of works that were cited in the literature review of the author that my paper has not covered, this gives a great overview of some literature evaluated in quantum computing application for PO:

Work Surveyed	**Contribution**
Financial portfolio management using d-wave's quantum optimizer: The case of Abu Dhabi securities exchange	"Portfolio Optimization problem for stocks from the Abu Dhabi Securities Exchange formulated as a QUBO, solved using DWave's simulator" (p. 16)
Improving variational quantum optimization using CVaR	"Proposed a method to improve the results by measurement system by using CVaR(Conditional Value at Risk)" (p. 17, where promising results were found
A variational approach for combinatorial optimization on noisy quantum computers	Layer-VQE was proposed in this paper, where it served the purpose of optimizing VQE that helps avoid local minima and improve chances of finding optimal solution Comapred to QAOA its gate count increased linearly, while that of QAOA increased quadratically, furthermore, layer-VQE had finite sampling errors, it was

			also simpler to implement than QAOA Quality of results improves with each additional layer in layer-VQE, unlike VQE.		
		Quantum metropolis solver: A quantum walks approach to optimization problems	“Developed an open software solution that used the Quantum Metropolis Hasting algorithm to provide a solution to optimization problems” (p. 17) It achieved a speedup over its classical counterpart, and as the problem scales, the quantum algorithm performed better than classical Metropolis Hasting algorithm, mostly with regard to time to solution.		
		Financial index tracking via quantum computing with cardinality constraints	“Tackled the problem of Financial Index Tracking by using discretized portfolio optimization to directly implement cardinality constraints in a single optimization procedure” (p. 17) The approach was successful in generating smaller portfoliosthat could track S&P 100 and S&P 500 indexes		
		Benchmarking the performance of portfolio optimization with QAOA	“Benchmarked the various versions of QAOA concerning its suitability to the current hardware” (p. 18) They imply that it is simpler to optimize examples with widely scattered correlations and returns as opposed to those with comparable correlations. This is because increased diversity in correlations and returns creates a more recognizable energy		

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<tr><td></td><td>landscape, which makes portfolios easier to identify and improve. Basically, it gives perspective into the different aspects of problems and how they affect solution quality, time etcetera for QAOA</td></tr>
<tr><td>Portfolio optimization with digitized counterdiabatic quantum algorithms</td><td>“Digitized counter adiabatic quantum computing (DCQC) and digitized counter adiabatic QAOA (DC-QAOA) were studied.” (p. 12)

Higher success rates of finding the optimal portfolio are achieved by optimizing the success rate in finding the ground state energy of the problem Hamiltonian (optimal solution)</td></tr>
<tr><td>Financial portfolio optimization: a QUBO formulation for Sharpe ratio maximization</td><td>“Proposed an improvement in the QUBO formulations of allowing the investor to decide the optimal fund allocation in each asset” (p. 18), which was achieved</td></tr>
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<td>[103] Experimental implementation of quantum-walk-based portfolio optimization (Qu et al., 2024)</td>
<td>In this paper, a quantum-walk based optimization algorithm experimented upon to show evidence for practical implementation of quantum-walk based algorithms.

“We realize the first experimental implementation of the QWOA mixing unitary and demonstrate its reliable convergence to high-quality solutions over a wide range of</td>
<td>Objective(s)
- Form a QWOA model for a combinatorial optimization problem for PO,
- For the experiment on a PO problem, there are three positions taken for the investor: 1. Short position, 2. Long position 3. No position. The PO problem is a discrete mean-variance Markowitz model for a cost function that considers historical behavior of the assets, it is expressed as a minimization problem.
- The experimental Po problem specifications are: 3 stocks (Google, IBM, and Microsoft), with zero constraints, in the period 1/1/2019-12/31/2020, on QuOp_MPI software,</td>
<td>Quantum hardware:
QuOp_MPI (simulator)

Quantum algorithm:
QWOA

Methodology:
Optimization

Use case:
Portfolio optimization</td>
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	quantum circuit depths" (p. 3)	test are done with 1 through 6 iterations of the algorithm.\ - "Our experimental approach is direct, flexible, and holds the potential for scalability" (p. 7) Results: - After comparing the results from the experiment with the known optimal solutions, it an be said that the experiment found the highest-quality portfolio with a probability of finding it to be 100% over 1 to 6 iterations. - Previous works on simulators compared QWOA with WAOA and it showed that QWAO was advantages over QAOA as it needed significantly less search space in achieving high-quality portfolios with fewer iterations. QWOA also showed great promise in solving heavily constrained formulations. - "Our work provides strong evidence for the potential of quantum-walk-based algorithms to solve complex optimization problems of practical significance" (p. 3) (complexity of setup is independent of number of iterations and only depends on number of dimensions, which is always 7) Important notes: - This experiment was performed under a noise-free system - "The exploration of quantum algorithms in practical applications is gaining momentum [53–55], even though they are currently in a preliminary stage. With the dedicated efforts of scientific researchers, we anticipate that quantum technology will soon be leveraged to tackle challenging real-life problems" (P. 7)		

[104] **A constrained multi-period portfolio optimization model based on quantum-inspired optimization (Ramaiah, K. Soundarabai, P, B. 2024)**	In this paper, a novel quantum-inspired whale optimization (QWOA) is proposed to tackle multi-period and multi-constrained portfolio optimization problems. Next to that, factors such as skewness, kurtosis, transaction costs, diversification, boundary and budget constraints are considered for assets The algorithm is then compared with Wale optimization (WOA), Gray Wolf Optimization (GWO), Fruit fly Optimization Algorithm (FOA), Particle Swarm Optimization (PSO), and Fruit fly Algorithm (FA), (MBO), (FSO), (CSO)	Objective(s): - Form a model based upon multi-constrained *(boundary constraint, budget constraint, diversification measure, high order constraints (kurtosis, skewness))* QWOA for multi-period portfolio optimization - Benchmark the model against QOA, GWO, FOA, PSO, and FA based upon excess mean return (EMR), net return, and transaction costs. - Dataset specifications: monthly return from 1963-2021 of the New York Stock Exchange, max iterations are 100, input size 32, initial population 100, performance indicators: Sharpe ratio, Sortino ratio, STARR ratio, information ratio, Shannon entropy, downside deviation. Experiment Results: - The proposed QWOA model showed to find the optimal results under different time periods - Compared to the other algorithms, QWOA achieved the highest mean Sharpe ratio (4.101048), indicating it to be the best algorithm under the other ones for this specific PO problem. - QWOA also achieved the best mean Sortino ratio and thus provides the best risk-adjusted returns. - QWOA also achieved the best mean STARR ratio - QWOA also achieved the best mean information ratio - QWOA also obtained the best mean Shannon entropy - The QWOA algorithm achieved better downward deviation than other classical models - Furthermore, the QWOA achieved higher net return rates, lower loss rates, and global optimal solutions were achieved more accurately and efficiently than traditional algorithms, - "QWOA provided an optimal portfolio with high return rates. The returns provided by the QWOA are high compared to the portfolios chosen by the other algorithms" (p. 21)	Quantum hardware: Classical computer. Quantum algorithm: QWOA Methodology: Optimization Use case: Portfolio optimization	Sortino ratio = a ratio that evaluates risk adjusted return of an investment STARR ratio = same as Sortino ratio but it also takes account of CVaR for tail risk, thereby making it more useful for portfolios with significant downside risk information ratio = helps to identify risk consistent returns Shannon entropy = a measure of uncertainty or randomness, in this case used to evaluate to what degree a portfolio is diversified. downside deviation = a measure that puts into perspective how well the formulated portfolios keep the volatility of returns below a specific threshold, often the minimum acceptable return line.

		- "Results suggested that the proposed model provided beneficial outcomes as compared with other algorithms" (p. 1) - Net return rate of the proposed model is always above 0.85%, Sharpe ratio is 5.016254 according to the experimental test. Statistical test results (to show strength of the proposed model): - QWOA had lowest standard deviation, lowest p-value (meaning high statistical significance of the results obtained in the test), and lowest t-statistic		
[111] **Quantum walk-based portfolio optimisation (Slate et al., 2021)**	In this paper, a quantum algorithm for PO on NISQ devices is proposed. A Quantum Walk Optimization algorithm (QWOA) is proposed for high-quality solutions to PO problems Furthermore, QWOA, Quantum Approximate Optimization Algorithm (QAOA), and Quantum Alternating Operator Ansatz (QAOAz) are compared against eachother	Objective(s): - Based on the mean-variance Markowitz model, form a PO problem that ought to be solved by QWOA, QAOA, QAOAz - Compare the results obtained from a PO experiment with two datasets (with long-position, short-position, and no-position) with the named algorithms to show which one performs better. - Dataset A specifications: 8 stocks with adjusted close price form the ASX20 index, period 01/01/2017 to 31/12/2018 - Dataset B specifications: 8 stocks with adjusted close price from ASX20 index, period 24/03/2020 to 06/09/2020 Results: Dataset A:	Quantum hardware: Classical computer (QUOP_MPI software) Quantum algorithm: QWOA, QAOA, QAOAz (all hybrid-quantum classical) Methodology: Optimization Use case: Portfolio Optimization (and periodic re-balancing)	

		- QAOA performs poorly compared to the other algorithms, has large standard deviation (with max 12.96), these results may be due to the classical solver part for the QAOA to have a higher likelihood of getting stuck in local minima than the other algorithms - QAOAz shows diminishing improvements after 8 iterations, - QWAO has superior performance at low iteration values, needing less search space for good results. Furthermore, QWAO performs significantly better considering annual return Dataset B: - Dataset B is consistent with the findings of dataset A - QWOA consistently finds the best expected solution quality, followed by QAOAz and QAOA. - QWAO had the best value for standard deviation, QAOAz in them idle, and then QAOA - QWOA shows significant advantage over QAOAz in optimizing portfolios - QWAO converges to the optimal solution efficiently - QWAo yields the best expected returns after iterations >2 (max 19) Overall results from the paper: - "Our earlier work indicated that QWOA offers significant advantages over pre-existing methods through a reduction in the search space and an unbiased encoding of optimization constraints" (P. 2) - QWOA outperforms QAOAz and QAOA in terms of amplifying optimal solutions and achieving higher expected returns with acceptable risk levels. The QWOA algorithm demonstrates robust performance in both convergence and optimization across different data sets. - QWAO also showed better performance in convergence, stability, and applicability to multiple combinatorial problems.		

		Important notes: - "QAOA and QAOAz are hindered by bias in the mixing operator over nontrivial feasible solution spaces." (p. 15) - For each dataset problem, the algorithms had different search space sizes (2^16 for QAOA, 1820 for QAOAz, and 266 for QWOA) - Each local optimal minimum for the algorithms is different (dataset A: -0.318 for QAOA and QWOA, and -0.305 for QAOAz), (dataset B: -1.25 for all three algorithms) - Highest returns did not mean lowest risk in the case of this paper as with the mean variance Markowitz model, a best combination of risk and return is to be found, therefore the highest return portfolio will not necessarily have the lowest risk.		
[118] Comparative Study between Quantum and Classical Methods: Few Observations from Portfolio Optimization Problem (Tripathy et al., 2022)	In this paper, the difference in overall efficiency and execution speed between classical and quantum computing for optimization problems is explored, where a Markowitz mean-variance PO problem is used to benchmark both methods. The data used for the benchmarking is historical data from 48 NSE stocks Quantum methods used are VQE and QAOA Classical method used is Monte Carlo,	Objective(s): - Compare classical and quantum methodologies in an example PO problem to show the advantages of quantum computing compared to classical - Overcome the qubit limitation (max 12) of the simulator by piling 48 stocks in 4 buckets. - Formulate the quadratic program as a QUBO formulation and optimize the parameters using optimizers. - Data specifications: 48 NSE stocks from the period 01/01/2011 to 01/11/2021, with 2011 till 2016 being used for training, and the rest for investing. A 16 asset portfolio ought to be made by the algorithms. Results: - Execution times were respectively: 11 minutes for VQE on Qiskit, 3.33 minutes for QAOA on Qiskit, 44 seconds for D-Wave CQM quantum annealing, and 16 hours for classical Monte Carlo. - Results achieved are comparable with classical approaches, however, calculation times were significantly less,	Quantum hardware (simulator): Gate model quantum computer (on Qiskit SDK), followed by D-Wave CQM (annealer, can handle up to 5000 variables and 100.000 constraints) Quantum algorithm: Quantum: VQE, QAOA on Qiskit and constrained quantum models (CQM) on D-Wave annealer Methodology: Optimization Use case: Portfolio optimization	

		- Gate-based quantum computers on average provide smaller numbers of qubits - "From the above CAGR plot, we observe that both classical and quantum approach are providing equally good and comparable results. From our experimentation performed on D-wave annealers and gate-model simulators, we observed that implementations using quantum methods were faster than the corresponding implementation of classical methods" (P. 5) - "We observed that implementations using quantum methods were faster than the corresponding implementation of classical methods" Important notes: - There was a qubit limitation in using the quantum simulator (max 12 qubits). - The paper stresses the importance of comparing classical and quantum computing methods through real-world tests to substantiate the difference. - An example is shown in the paper where a classical computer tires to solve a NP-hard PO problem, as can be seen, the total time to compute the ideal portfolio increases dramatically as assets increase along with required assets per portfolio. For a portfolio of 4 assets under 8 stocks to choose from, the computation time was 9 minutes, but for a portfolio of 10 stock with 50 stocks to choose from, the computation times is 11000 years.		
[119] Reverse quantum annealing approach to portfolio optimization problems (Venturelli, D. & Kondratyev, A., 2019)	In this paper, a hybrid quantum-classical solution method is proposed, where the mean-variance PO problem from Markowitz is taken as the objective problem. Several solvers for the QUBO formulation were used: Greedy	Objective(s): - Form a hybrid quantum annealing solver along with a specific setup to solve a mean-variance PO model casted into a QUBO formulation. - Benchmark the proposed model/algorithm along with the classical Genetic Algorithm (GA) on a dataset where the objective is to maximize risk-adjusted returns or Sharpe ratio on an unconstrained problem set.	Quantum hardware: D-Wave quantum annealer 2000Q Quantum algorithm: N/A Methodology: Optimization Use case: Portfolio optimization	

	search, genetic algorithm (GA), forward quantum annealing, and reverse quantum annealing.	- Benchmarking was done with different problem sizes, parameters, and solvers to evaluate the performance of the D-Wave 2000Q against classical heuristic methods (GA) - The test was performed on sets of assets: 24, 30, 36, 42, 48, 54, 60, and for reverse QA, pause times before resuming the process to mitigate errors Results: - Looking at the graphs depicting various information on time-to-solution (TTS), and effects of parameter settings, it can be said that as problem size increased: 1. Reverse QA with greedy search had best performances in TTS, 2. GA (from random starting point) performed worse in TTS than GA starting with Greedy Search but both increased in TTS quite stably, 3. Forward QA increased more in TTS as problem size increased, but was still faster than GA but not QA with Greedy Search. - Optimal results for Reverse QA were found using shorter annealing times. - Reverse QA with shorter pause times had less TTS - The performance of the greedy and classical approaches decreased as problem sizes increased, not taking away that the results obtained from the Greedy approaches were better, it still suggests that increased problem sizes may be difficult for them. - The best results in terms of time-to-solution for the hardest set instance were obtained by seeding the quantum annealer with better solution candidates found by greedy local search and then performing reverse annealing - “The optimized reverse annealing protocol is found to be more than 100 times faster than the corresponding forward quantum annealing on average.” (p. 1) Important notes:		

		- Greedy search was used as a benchmark, and to initialize the state for reverse quantum annealing, giving it a head start as it starts with a reasonably good approximation. - The D-Wave system has a maximum controllable energy, making it challenging to program accurately.		
[120] **Dynamic Asset Allocation with Expected Shortfall via Quantum Annealing (Xu et al., 2023)**	In this paper, a hybrid quantum-classical algorithm is proposed to solve dynamic asset allocation with target return and target risk metric (expected shortfall) The proposed algorithm is benchmarked using D-Wave 2000Q and D-Wave Advantage annealers against classical approaches. Contributions of this paper: 1: a demonstration of how NP constraints such as expected shortfall in an optimization problem can be solved using a hybrid quantum-classical approach 2: This paper serves as a first case employment in the industry of solving expected shortfall based dynamic asset allocation problems 3: this is one of the first papers to introduce the problem solving on a real quantum computer using real financial data 5 datasets are used and tested upon, the exact specifications of these	Objective(s): - Form a hybrid quantum-classical algorithm for a PO problem with dynamic asset allocation, target risk and target return - Compare the algorithms of classical and quantum kind against each other on D-Wave 2000Q and D-Wave Advantage with each other and simulated annealing on real-world financial data. - Form a modified Markowitz framework (to fit specifications of the objective problem) into a QUBO format - Objective problem = computing portfolios with minimum variance for a given target return - Data specifications overall: top-six ETFs by trading volumes, and six major Currencies exchange rates, respectively 12 and 23 assets in the experiments, expected shortfall of 5%, 30000 samples are taken on the QUBO formulation for more specific results. Ultimately, 5 datasets are made with different starting dates between 2010 and 2020 and each method has 100 days of data to work with. Results: - Simulated annealing followed the optimal solution in most tests - For test 4 of the currency tests, the real quantum annealers were able to find a portfolio with higher returns than simulated annealing (with a still acceptable but slightly increased risk) - It is observed that currency tests perform better on real quantum	Quantum hardware: D-Wave 2000Q (2048 qubits, up to 68 logical variables), D-Wave Advantage (5760 qubits, up to 180 logical variables) quantum annealers. Both simulated and physical quantum annealing are used. The simulator is not specified Quantum algorithm: N/A Methodology: Optimization Use case: Portfolio optimization	

	datasets are NOT mentioned	hardware than ETF tests on the same hardware. - "ealing. Both 2000Q and Advantage processors are able to compute returns that are consistently more than 80% of the optimal, except the two currency test cases where the algorithm fails to converge on the 2000Q" (p. 15) - "Both quantum annealers are able to generate portfolios with more than 80% of the return of the classical optimal solutions, while satisfying the expected shortfall" (P. 1) - "We observe that experiments on assets with higher correlations tend to perform better, which may help to design practical quantum applications in the near term." (p. 1) Remarks on the real quantum hardware: - 2000Q processor: can natively handle up to 12 assets - Advantage processor: can handle up to 23 assets, however due to defective qubits and connectors, only 119 qubits can be used currently (2023) - The Advantage processor fails to find the ground state effectively, with high chain lengths (up to 17) leading to poor performance. This indicates limitations in handling larger problems due to current hardware constraints. - The 2000Q processor struggles with embedding chain lengths of 16 and has difficulty finding the optimal solution. Important notes: - "Although we acknowledge there may be other factors contributing to our observations that currency tests do better than ETF tests on for quantum annealers, Figure 7 implies that more correlated assets tend to" (P. 15)		

Table 7, overview of articles used for literature synthetization