

BSc Thesis Civil Engineering

Comparison of two linear excess stress modelling methods of soil erodibility for JET and “Mini” JET devices

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Abstract

With increasing sea levels and climate change, soil erosion is becoming of greater concern in the Netherlands. With historical need and reliance on earthen structures such as dikes are becoming an emphasis with the changing climate. Securing these structures require adequate and reliable methods to pursue. One way of ensuring a reliable soil structure (such as clay coverage on dikes) can be done through erosion testing. However, the established method in the Netherlands is quickly becoming outdated. Newer methods of assessing cohesive soils to assess clays on dike structures are being asked of within the Netherlands.

Jet erodibility testing (JET) is a newer form of soil erosion testing that can directly procure an erosion coefficient (k_d) and critical shear stresses (τ_c) of soils. The Blaisdell method has been developed in methodology and written in literature as an established method to procure these parameters. Multiple newer methods have been developed to better derive these parameters and model erosion rates. A newer method, which will be referred to as the Linear regression method, has been developed with the objective to attempt the same derivations. It is necessary to assess the significance of the different methods to come to conclusions on what is appropriate for development.

The comparison between the linear regression method and Blaisdell is conducted with results of three different types of clays and their erosion results from a JET device. The erosion data was processed through both methods to have a direct comparison. By developing a spreadsheet to calculate both methods and deriving the parameters, other results were procured to develop an assessment of both methods. Erosion category plots and Bland-Altman plots were developed to assess their similarity. A scour-depth-time plot was derived, and an NOF analysis was conducted to assess the accuracy of the modelling method in comparison to the original data.

Both methods produced consistent results for the scour-depth-time plots which is in line with previously observed field research. However, the Linear regression method had produced significantly larger over-estimations. Despite both methods being statistically similar for parameter results, the Blaisdell method produced results closer to the original scour depth time data than with the linear regression method.

Keywords: *Erodibility, Critical shear stress, Jet erosion test, Scour depth.*

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1 Introduction

Dikes in the Netherlands are a significant structure both culturally and for safety. The reliance of the structural integrity of dikes is seen as a priority. With multiple laws and regulations in place, with the establishment of Water boards and their integration into Dutch society, it gives a brief explanation to the importance that dikes have in the country. Clays have been historically used in dike construction in the Netherlands as they are not only known as a material with good strength and stiffness as well as low permeability. The availability makes it still a popular and commonly used material in dike construction. The main purpose of a dike is the ability to prevent water bodies from entering a certain region or area. In the Netherlands, dikes are an essential aspect of flood protection and have been historically used since the 8th century. Dikes back then were mostly made up of mostly clay, peat and other naturally occurring materials. These structures were enhanced, repurposed, or modified over the coming centuries to have multiple layers and they varied based on the location and purposes of the dike. New and more modern dikes, such as the Afsluitdijk, use a modern method of dike construction. The simplified description is that they were created with a sand core and clay layers ovetop to retain the shape and stability of the structure. This, however, was primarily done in response to a lack of materials. Many dikes today are smaller than the Afsluitdijk. They are also modified dikes that were made mostly from clays.

To maintain the level of safety and standards of these dikes, determining features of the clays will allow for appropriate responses and conclusions for the future of the observed dike.

There are numerous reasons and definitions of what classifies a dike to be ‘safe’ such as factors like dike width, materials, height, inflow, and outflow rates of rivers, etc. A significant reason for dike stability is the materials themselves. Dikes are now traditionally built up of many layers to ensure the security and strength of the dike. Starting with a sand core, covered by an aggregate layer and the clay and dirt layers which sit on the surface. The idea of using clays is that it has a low permeability to prevent (or reduce) not only the erosion of the structure over time but also prevents seepage of water into the dike which is a cause for failure. The clay layers are crucial to the stability and security of dikes. Typically, these clays are erosion-resistant meaning that the force needed for erosion is high. Using them ensures the longevity and shape of the dike (TAW, 1996). Therefore, it is necessary to assess the clays to see to what extent failures will occur. This is mostly based on clay erodibility which can be tested for.

The current convention in the Netherlands evaluates the safety of the clays using methods such as the Atterberg limit tests¹. There are requirements which have been developed by TAW (1996) which are used to determine standards for clay as coverage layers. The standard procedure includes the Atterberg limits and are detailed in TAW (1996). It requires multiple different tests on these soils to get liquid limits and plasticity indices which can be used in multiple applications. The main two tests conducted to get these limits are the Casagrande cup method and the Cone penetration method. Both methods (in a laboratory) require that the soil sample is mixed to create a homogeneous soil sample. This is so that the results of the physical

¹ The convention isn’t an explicit nor concrete method. The statement that the current convention of the safety in clays is based on the classifications of types of erodible clays which have been derived from taking the Atterberg limits. TAW (1996) uses this convention to classify clays.

alterations that are conducted during the test presents consistent results given certain parameters such as water content. The results derived are used to gather information to determine the erodibility of the clays.

Although reliable, this method of testing is limiting. This test requires that the soil is disturbed and is no longer in its original state. The effect of state parameters such as density and water content are not considered during testing. This would mean that the results are only focusing on a potential state of the soil in a homogeneous matter which does not reflect the reality of the situation. The application of this test does not work for all soil types either. It cannot be applied to organic matter such as peat (O'Kelly, 2015). Given that, in the Netherlands, a lot of natural materials found are peat, it is limiting to determine erodibility parameters for soils that contain them. Finally, the final outputs only provide an erosion category and no parameters. This is limiting in the field for further calculations based on the results directly.

This is not to say that the Atterberg tests are not appropriate for soil testing compared to using the jet erosion devices. However, the jet tests can be used for a more diverse set of materials and can directly solve the problems that the Atterberg limits cannot. As of now, there is no one method to test for erodibility with a diverse group of materials for dike construction in the Netherlands.

A method that can be used, which solves the problems, is the jet erodibility test developed in the US by Dr Greg Hanson (2004) for direct testing of soil erodibility in situ (on the field). There is another smaller device produced based on the original, named the mini jet erodibility device which was developed by Al-Madhhachi et al. (2013), which can conduct these tests both in situ and in the laboratory. This is a method which can allow a direct testing of erodibility which is not based on deriving other features of the soil such as water content. It extrapolates the critical shear stress of the soil and a specific erosion coefficient by measuring the rate of scour of a soil. These devices will allow for newer methods of testing soils in the Netherlands which will be able to produce results more easily. Being widely used in the US with new standards being implemented for the devices (Wahl, 2016), this can provide significant use for the implementations of dike safety in the future.

Figure 1 is an image of the original jet erosion device. This device is used in situ and can take measurements on the field. This removes the need for the transportation of soil samples to a laboratory which is convenient if one is not accessible nearby. The mini jet erosion device can be used both in a laboratory and in situ. Due to the smaller size, it takes less effort to transport the device which makes it a cost-effective device. Moreover, it requires less resources to build and use (i.e., smaller sample sizes and smaller jets).

The way the jet erosion device works is using a jet stream that fires into the soil specimen chamber. A measuring gauge (a point gauge for the original device and depth gauge for the mini jet device) is used to measure the rate of erosion over time. This is the rate of the development of the scour hole over time in the soil specimen (see Figure 3 for reference).

The theory of the jet erodibility test is based on a common relationship which details the phenomena of erosion. It assumes that the erosion rate is proportional to the difference in effective and critical stresses (Hanson and Cook, 1991).

$$\varepsilon = k_d(\tau_e - \tau_c)^a \quad (1)$$

Where: ε is the erosion rate of the soil material; k_d (cm^3/Ns) is the erodibility coefficient; τ_e is the effective stress at the surface of the soil from the jet; τ_c (Pa) is the critical stress of the soil material; a is the exponential term but it is generally assumed to be equivalent to 1 (Al-Madhhachi, 2013).

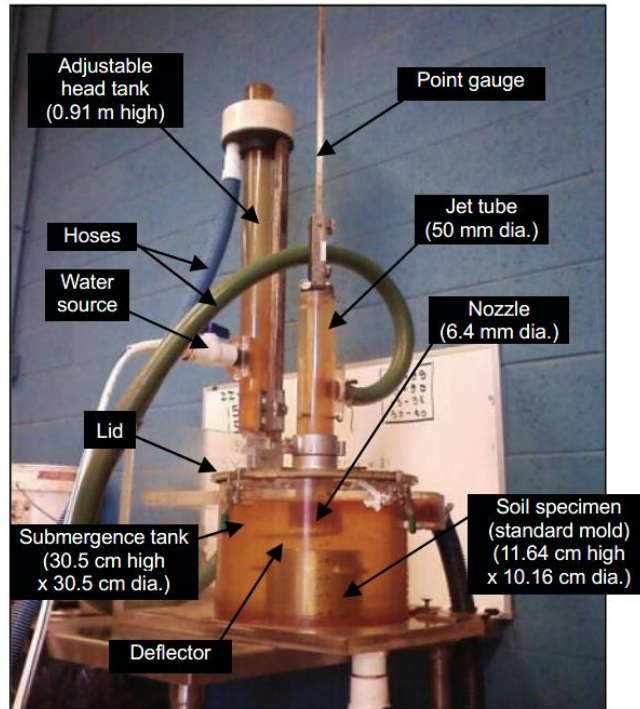


Figure 1 - Image with description of jet erosion device (Al-Madhhachi et al, 2013).

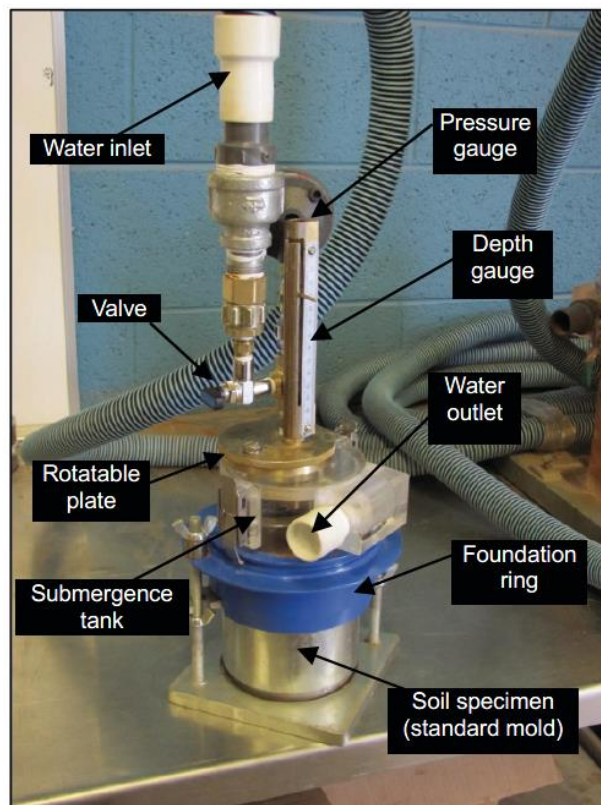


Figure 2 -Image with description of mini jet erosion device (Al-Madhhachi et al, 2013).



Figure 3 - Photos of scour hole formation in different soil physical conditions (a) wide and shallower, and (b) narrow and deeper (Mahalder et al. 2018).

The primary differences between the original JET device and the mini-JET are to do with sizes from the jet nozzles, water pressure, sample sizes, and other features. The differences are displayed below present the differences between the Hanson and Cook (2004) JET dimensions and the Al-Madhhachi (2013) Mini Jet in the table below:

Table 1- JET vs Mini-JET size components

	Jet Erosion Test	Mini-Jet Erosion Test
Diameter Nozzle	6.00 mm	3.18 mm
Sample height	180 mm	70 mm
Sample diameter	67-100 mm	67-100 mm
Location use	Field	Lab + Field

The overarching purpose of this study is to explore the jet erosion tests to, in the long run, aid the development of jet testing in the Netherlands. To do so, this thesis will aim to explore two modelling methods of the JET by comparing the two to present a clearer idea for the strengths and weaknesses of each.

As JET testing is still relatively new, having a tool to calculate the final outputs will prove to be more efficient in the overall process. As mentioned, the focus will be the comparative assessment of two different linear modelling methods: a popular method named the Blaisdell method, procured by Blaisdell et al. (1981) and popularly implemented by Hanson and Cook (1997) against; a simple linear regression model that directly extrapolates results from calculated shear stresses against rate of erosion.

The linear regression method is a much simpler method in comparison. It directly derives a linear regression from simpler calculation results in comparison to a model that requires fitting. This method will be assessed as it can be seen as a better alternative to the popular Blaisdell method.

This thesis will be answering a main research question with three sub-research questions:

Between the Blaisdell method and Linear regression method, which modelling method is the most appropriate to derive the Hanson erosion parameters for the jet erosion devices?

1. What are the differences between the JET and Mini-JET?

2. What are the differences between the linear regression model and the Blaisdell method(s)?
3. What defines the reliability of these methods?

These questions will be answered throughout the thesis with the final research question being discussed in the discussion. Starting with the methods section, it will go through the calculations behind the two modelling methods, the steps to determining erosion categories and the method for analysing the results. Followed by that is the results section which will present the results and briefly but simply describe the outcomes. The discussions page will answer the (sub)question(s) of the thesis using the outcomes in the results section including limitations. Finally, the conclusion will discuss the significance of the results in the overall context of dikes.

2 Methods

Both Blaisdell and the linear regression methods follow the idea of deriving a linear regression of some sort. They both use the linear excess shear stress equation (eq. 1) to derive their results. Therefore, the equations that are used specifically for those methods are derived from the same equations. Those equations were derived for the original JET device, proposed by Hanson and Cook (1997). The erosion rate of jet scour is written as (Hanson and Cook, 1997):

$$\frac{dJ}{dt} = k_d \left[\frac{\tau_0 J_p^2}{J^2} - \tau_c \right], \text{ for } J \geq J_p \quad (2)$$

Where: τ_0 is maximal shear stress; τ_c is critical shear stress; J is the scour depth (cm); J_p is the potential core length from the jet origin (cm); and k_d is the Hanson erosion coefficient.

Hanson and Cook (2004) have derived equations to solve the calculations for the critical stresses using the following equations (3-6). These equations follow the schematic which can be seen in Figure 4.

$$\tau_c = \tau_0 \left(\frac{J_p}{J_e} \right)^2 \quad (3)$$

$$J_p = C_d d_0 \quad (4)$$

$$\tau_0 = C_f \rho_w U_0^2 \quad (5)$$

$$U_0 = C \sqrt{2gh} \quad (6)$$

Where: d_0 is the nozzle diameter (cm); J_e is the equilibrium depth²; C_d (6.3) is the diffusion constant; C_f (0.000416) is the friction coefficient; ρ_w is water density (kg m^{-3}); U_0 is the velocity of the jet at the centre line; C is the discharge coefficient; g is the gravitational acceleration constant; and h is the pressure head (cm).

The proposed values for the discharge coefficient C for the original JET are between 0.95 and 1.00. According to Al-Madhhachi et al. (2013), the C value was proposed to 0.70 to 0.75 for

² Where the critical shear stress was assumed to occur when the rate of scour is zero (Hanson and Cook, 1997; Hanson et al., 2002b).

the mini-JET device. Figure 4 labels the specific lengths and measurements of the (generic) jet device³.

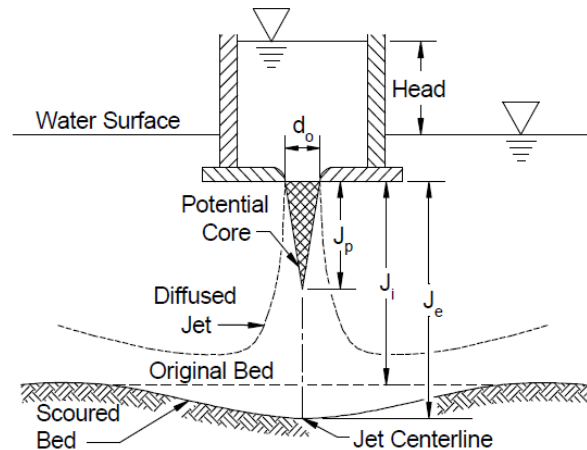


Figure 4 - Schematic of JET device with factor definitions (Hanson and Cook, 2004).

2.1 Blaisdell Method

The Blaisdell method is a common method basis for analysing jet erosion data (Wahl, 2016). It uses a hyperbolic logarithmic relation to model and determine the jet scour equilibrium J_e and critical shear stress τ_c .

Hanson and Cook (2001) use this model to then derive the erosion coefficients k_d . By using Excel solver to model dimensionless functions of time and depth, they fit it with the jet scour data (non-dimensionless) by adjusting k_d as a parameter. Al-Madhhachi et al. (2013) uses the same method except for adjusting two parameters (including k_d) instead of just the critical shear stress alone. This method will be used as Al-Madhhachi et al. (2013) uses this for the analysis of data for the mini-JET which can be applied to both.

The Blaisdell method itself has been criticised for the results producing a high jet equilibrium scour and a low critical shear stress as a result (Wahl, 2016). This is due to the assumption of the jet equilibrium scour occurring over an infinite time. As a result, the critical shear stress seems small to what is expected for the depth. These results were not to be compared to a 'practical equilibrium' as stated by Wahl (2016). However, a few justifications presented was that this method produces an overall consistency where, at the time, individual judgement often affected the results of the equilibrium scour. The reasoning for Hanson and Simon's (2001) use of this method was because of this consistency which they observed.

The first aspect of this method uses the hyperbolic logarithmic function of which as follows (Blaisdell et al., 1987):

$$(f - f_0)^2 - x^2 = A_1^2 \quad (7)$$

Where: A_1 is the value of the semi-transverse and semi-conjugate of the function; x is equal to $\log[(Uot)/d_o]$; f is $\log(J/d_o) - x$; and f_0 is $\log(J_e/d_o)$. The way it is solved is by using Excel solver to calculate the minimum squared error in the different x (the equation of x against the

³ Described as generic as this applies for the mini jet erodibility device as well as the normal device (developed by Hanson).

rearranged eq. 7) by adjusting the values of A_1 and f_0 . These results are plotted on the axes of f vs x for a visualisation of the fit with the adjusted parameters.

The equilibrium scour depth J_e can be determined with the final f_0 :

$$J_e = d_o 10^{f_0} \quad (8)$$

The critical shear stress is calculated by using equation 3 using the jet equilibrium depth from equation 8. These results will be used to determine the erosion coefficient kd by modelling it in a similar way. By comparing the fit to the observed results.

An integrated dimensionless form is used using equation 2 and 3 (Hanson and Cook, 2002a):

$$\frac{dJ^*}{dt^*} = \frac{(1 - J^{*2})}{J^{*2}} \quad (9)$$

Where $J^* = J/J_e$, and $J_p^* = J_p/J_e$. Reference time T_r is the indicated model time (from Blaisdell) of the occurrence of equilibrium depth:

$$T_r = \frac{J_e}{k_d \tau_c} \quad (10)$$

This can be formed into dimensionless time (T^*) with the collected data:

$$T^* = \frac{t}{T_r} \quad (11)$$

The integration of equation 4 results in the following (Hanson and Simon, 2001):

$$t_m = T_r \left[0.5 \ln \left(\frac{1 + J^*}{1 - J^*} \right) - J^* - 0.5 \ln \ln \left(\frac{1 + J_p^*}{1 - J_p^*} \right) + J_p^* \right] \quad (12)$$

Where t_m refers to measured time of equilibrium depth. In Al-Madhhachi et al. (2013), as mentioned before, equation 12 varies slightly. It uses two parameters (other than just kd) to fit the dimensionless time directly:

$$T^* - T_p^* = 0.5 \ln \left(\frac{1 + J^*}{1 - J^*} \right) - J^* - 0.5 \ln \ln \left(\frac{1 + J_p^*}{1 - J_p^*} \right) + J_p^* \quad (13)$$

With T_p^* being the second parameter to be adjusted. This is solved in Excel Solver (as with Blaisdell) for the minimum squared error in the fitted time (rearranged eq. 13) and the observed time (eq. 10). These are then plotted on the axes T^* against J^* . The k_d is directly calculated using excel solver using this method.

2.2 Linear Regression

The linear regression method uses a linear regression to directly derive the Hanson coefficient k_d and the critical stress of the soil τ_c . k_d is interpreted as the slope of the trendline and τ_c is interpreted as the intercept along the x axes.

The calculations are simpler in comparison to Blaisdell. Mostly requiring the calculation of the shear stresses and rate of scour directly from the data. This method is aimed to replicate the

linear results produced by the Blaisdell method for the rate of scour against the shear stresses without having the need to use the hyperbolic function fitting.

The linear regression method uses a variation of eq. 3 to calculate the shear stresses. The rate of scour is also calculated. The following equations present these:

$$\tau = \tau_0 \left(\frac{J_p}{J} \right)^2 \quad (14)$$

$$\frac{dJ}{dt} = \frac{\Delta J}{\Delta t} \quad (15)$$

The calculated rate of scour (eq. 15) is plotted against the shear stresses where a linear regression is calculated. Due to the nature of the test being non-regular, the range of stresses covered are discretized before applying a linear regression.

2.3 Mini JET

To answer the first sub-question, observations of the mini-JET will be presented. The primary differences between the JET and Mini-JET, with exception of the size, is also partially the calculations done to derive the critical shear stresses and Hanson erosion coefficients.

Al-Madhhachi et al. (2013) presents the variations in the calculations from the original JET device in comparison to the mini-JET device. The primary differences for calculations revolved around the values of coefficients. The first is the discharge coefficient values which was described in eq. 6.

The range of values were derived from the range of values provided the following discharge equation:

$$Q = CA\sqrt{2gh} \quad (16)$$

Where Q is the measured discharge and A is the area of the nozzle used (Al-Madhhachi et al., 2013).

It was observed that the mini-JET produced more consistent ranges with results for the k_d with the original device. However, the τ_c was always calculated to be smaller. To circumvent this issue, Al-Madhhachi used a coefficient to adjust the values for τ_c to produce comparable results. This was done by comparing the values of the equilibrium depths (J_e) over the jet height (J_0) for both devices (Al-Madhhachi et al., 2016):

$$C_{je} = \frac{(J_e/J_p)_o}{(J_e/J_p)_m} \quad (17)$$

Where o refers to the original jet and m refers to the mini jet.

The value proposed for C_{je} was 0.25. These values ranged between 0.1 and 0.8 (although, the majority were between 0.1 and 0.5). This, however, had not been validated for generic soils and was only produced to be used for the test their specific soils. These were evaluated on sandy to clayey soils.

These results of the mini-JET will be used for the calculations with the Blaisdell method and the Linear regression model.

2.4 Erodibility categorisation

Separating soils into categories of erodibility allow for easier identification of soils and their appropriateness for certain applications. In this context, the application is for dike composition and construction.

The categorisation of erodibility using the JET erosion tests uses the Hanson erosion coefficient. There are five categories for the Hanson erosion coefficient. The way that the erosion categories are defined are by plotting the Hanson erosion coefficients against the calculated critical stresses of the soils. This can be seen in Figure 5. The erosion classification is based on the criteria given by Hanson and Simon (2001). The erodibility categorisations can be defined by the critical stresses and Hanson erosion coefficient as seen in Table 2.

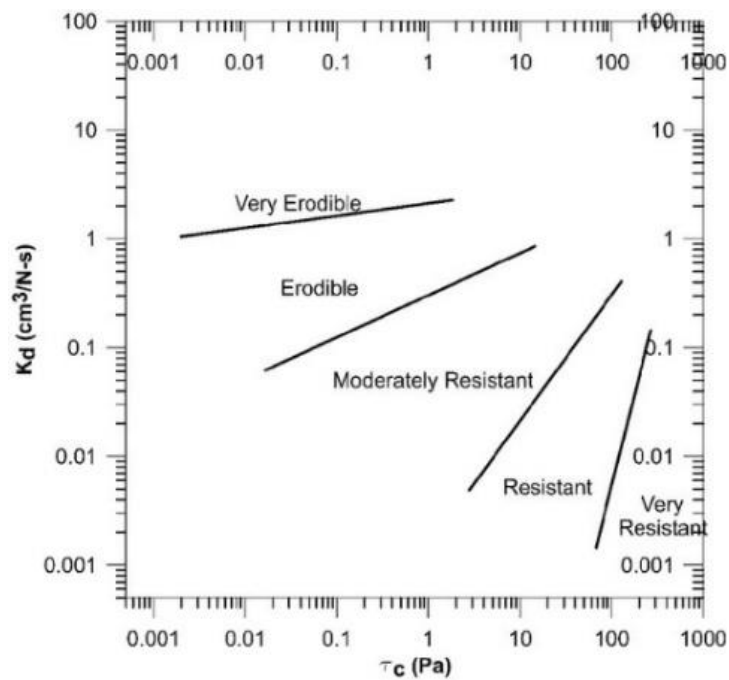


Figure 5 - Hanson & Simon (2001) erosion classification (Wibowo and Lopez-soto, 2021).

Table 2 - Hanson erosion coefficient categories.

Category	Hanson coefficient k_d (cm ³ /N-s)	Critical Stress τ_c (Pa)
Erodible	1.1	0.002
	2	2
Moderately Resistant	0.07	0.017
	0.95	18
Resistant	0.006	2
	0.8	150
Very Resistant	0.0015	45
	0.15	200

2.6 Data sets

Results from a French laboratory consultancy, geophyConsult, were produced in collaboration with Fugro NL. That report and results (k_d , τ_c and scour depth data) of their tests were used to develop and evaluate the excel tool. Initial scour depth inputs with recorded times, results of the k_d and the τ_c calculations, and outputs of these calculations were provided from these consult

report. The data used for testing was a consult comparing three types of clays sent from Fugro NL.

The types of clays that were assessed on were three samples of untreated clay, nine samples of Graustabiel treated clays, and nine samples of Komex Bio treated clays. The purpose of the research was to compare the treated clays at different intervals of the curing stages in comparison to untreated clays. In the end, two of the Komex Bio and Graustabiel clays results faired no erosion; one of the tests for both treated clays were not able to produce sufficient results and the data has not been provided.

The geophyConsult consult used the simple linear regression model and the data used will be analysed with the Blaisdell method as well as replicating the linear regression method.

2.7 Spreadsheet

The spreadsheet tool takes the inputs of jet scour and time measurements along with the diameter of the jet used, the height of the jet to the soil and the hydraulic head.

The spreadsheet outputs are the graphs for the test measurements (scour against time), the results for the Linear regression method and a graph presenting this, the results of the Blaisdell method, and the final plot of the erosion categories (see figures 6-8).

Due to the results for critical shear stresses being in the hundreds, the deviations should present minor change in the erosion category classifications compared to the original data. The k_d , on the other hand, will vary slightly more but is dependent on the order of magnitude of the result.

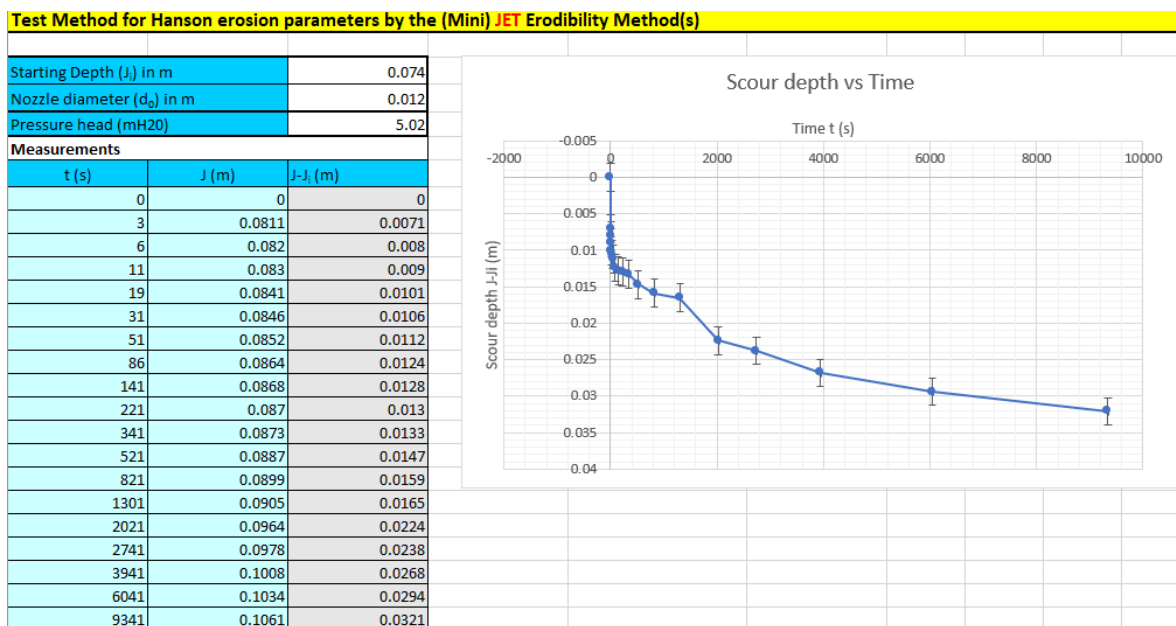


Figure 6 – Sample measurement input page from spreadsheet

<i>Method (1) Linear Regression</i>					
Results:	Symbols	Minimum	Calculated	Maximum	Units
Critical Stress	τ_c	-	260.35	-	Pa
Hanson Erosion coefficient	k_d	-	5.18	-	cm^3/Ns
Equilibrium scour depth	J_e	-	2.08	-	cm
Reference time	T_r	-	-	-	-
Modelling remarks					
Unable to replicate the discretization. The calculations for the confidence interval is not convergent. The overall shear stresses are smaller than what was modelled. The model predicted a higher critical stress of the soil than what the data provided. Same for the maximal scour depth (equilibrium depth). Both in the original report and this report, the Average trend was retained for modelling.					
<i>Method (2) Blaisdell</i>					
Results:	Symbols	Minimum	Calculated	Maximum	Units
Critical Stress	τ_c	-	290.58	-	Pa
Hanson Erosion Coefficient	k_d	-	0.50	-	cm^3/Ns
Equilibrium scour depth	J_e	-	1.58	-	cm
Reference time	T_r	-	620.02	-	s
Modelling remarks					
The model predicted a higher critical stress than what the data provided. Therefore, not all values for calculated T^* were valid and it affected the final Hanson coefficient value.					

Figure 7- Sample output results from spreadsheet. Outputs from Linear Regression method and Blaisdell.

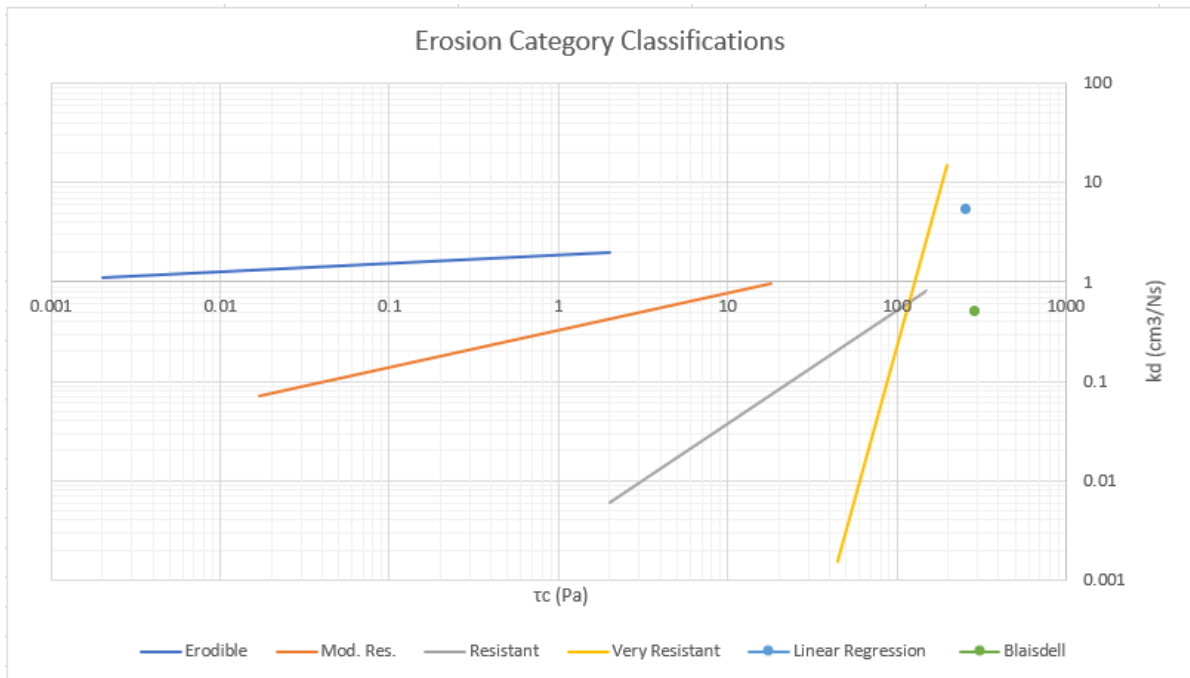


Figure 8 – Sample output results from spreadsheet. Erosion category plotting of results from Fig. 7.

2.8 Testing of Methods

To compare, after the development of the spreadsheet tool, there are multiple ways in which the data can be assessed comparatively. The first one method uses a ratio of the Standard deviations of the observed and predicted differences against the observed mean. In this assessment, the scour depth to time plots of each plot will be inferred from their rate of scour

against shear stress plots. This method is used in Daly et al. (2013). called the Normalised Objective Function (NOF) (Fox et al., 2006). to quantify the goodness of fit to the original data.

The NOF is given by the following equation. The NOF values are the deviation percentages from the observed, i.e., 1% = 0.01, 10% = 0.1, and 100% = 1.

$$NOF = \frac{STDD}{X_a} = \frac{\sqrt{\frac{\sum_{i=1}^N (x_i - y_i)^2}{N}}}{X_a} \quad (18)$$

The second method is the compare the two methods to each other and assess their agreeability. Bland-Altman plots will be used to assess the calculated critical shear stresses and the erosion coefficients. This is to compare the similarity and agreeability of the results produced but is not to compare the accuracy of these results. This analysis compares the average between the two method measurements and the difference between them; it will separately compare the k_d and τ_c results.

By extrapolating the data, the results of the diverse types were compared. As the Blaisdell method only calculates for one pair of results (k_d and τ_c), the initial results of the linear regression method will be compared. Although three data categories could be split, due to the limited data points and to compare the differences between the methods rather than the nature of what is being measured, it will be combined.

The third is the visual analysis of two different plots: the erosion category plots which is derived from the spreadsheet directly, and the scour depth time plots which is linked to the NOF calculations. This visual analysis will allow better understanding of the significance of the critical shear stresses and the erosion coefficients in the context of usable results affecting decisions in larger projects.

The final analysis is through the comparison of literature. There are other results and observations made that can be used to (in)validate the results and conclusions derived.

3 Results

The assessment was conducted for the JET device aspect of the spreadsheets. It was also conducted using the coefficients for the Mini-JET as well for comparison. However, because the device will use different nozzles, jet speeds, and sample sizes, the attempt to use the data for the mini-JET did not produce viable results to be able to compare the methods. Moreover, interpretation was not plausible with the Mini-JET coefficients.

3.1 Results of Spreadsheet

There was a total of fifteen soil samples that were processed through the spreadsheet; three untreated clay samples, six of the Graustabiel treated clay samples, and six of the Komex Bio clay samples. The main outputs are the k_d and the τ_c results (Table 6) and the erosion category plots. The results of the untreated clay samples are in Table 3. For Graustabiel, it's in Table 4, and for Komex Bio, in Table 5.

Looking at Graustabiel, the results are close together in results. However, the deviations in results vary a lot more for the Komex Bio clays. It's unclear as to why these deviations occur. Overall, the Blaisdell results lean towards the right which is a result of the higher estimations for critical shear stresses (also observed in Table 6).

Table 3 - Untreated clay samples, Erosion category classification spreadsheet output (1-3).

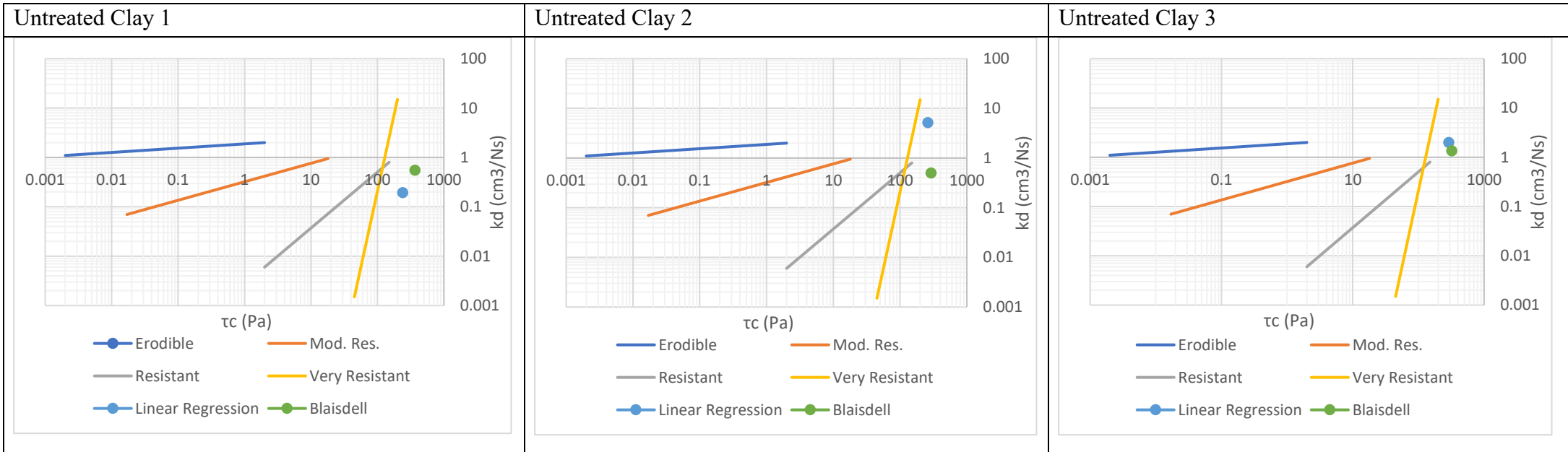


Table 4 – Graustabiel clay samples, Erosion category classification spreadsheet output (1-6)

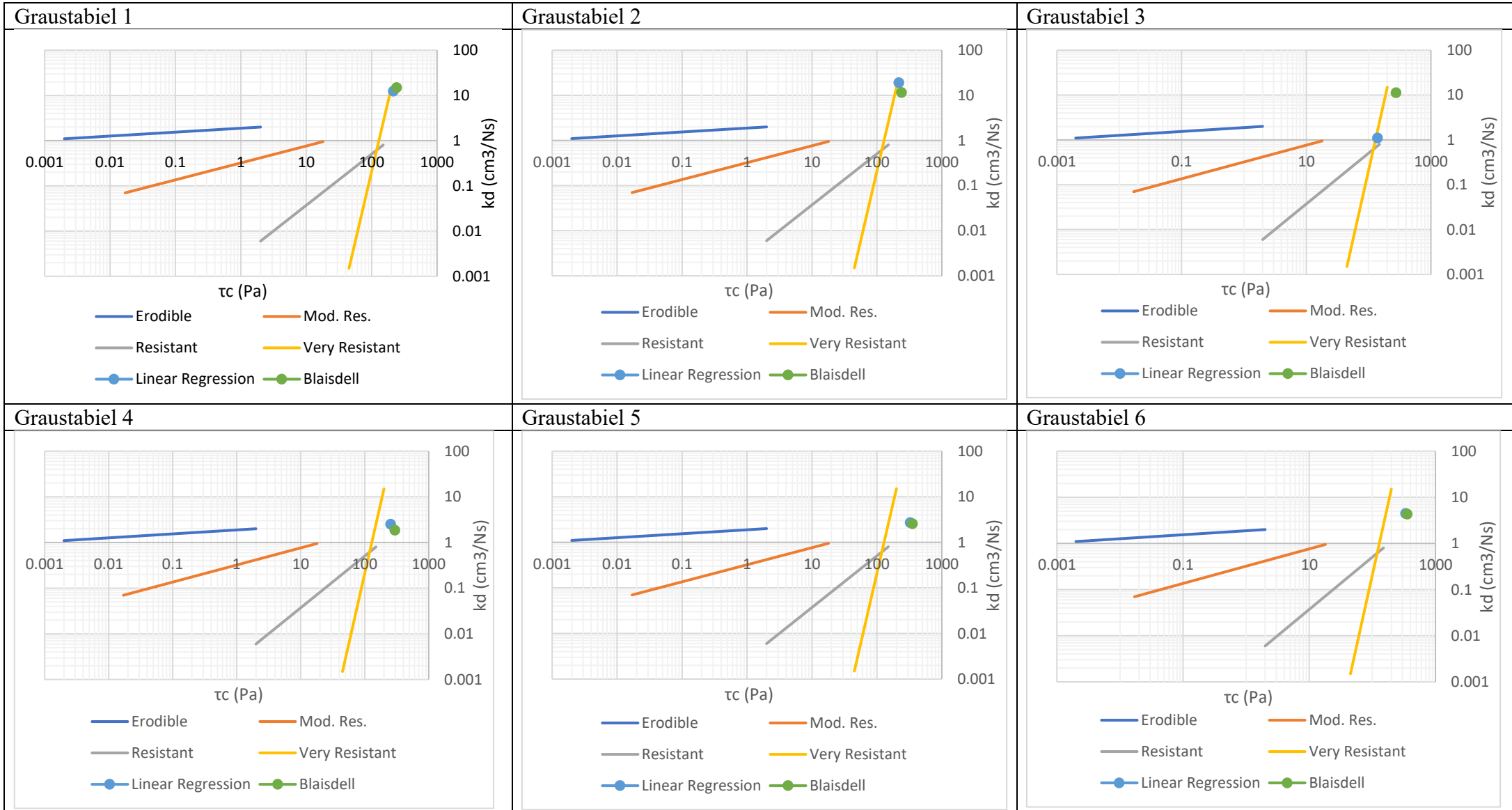
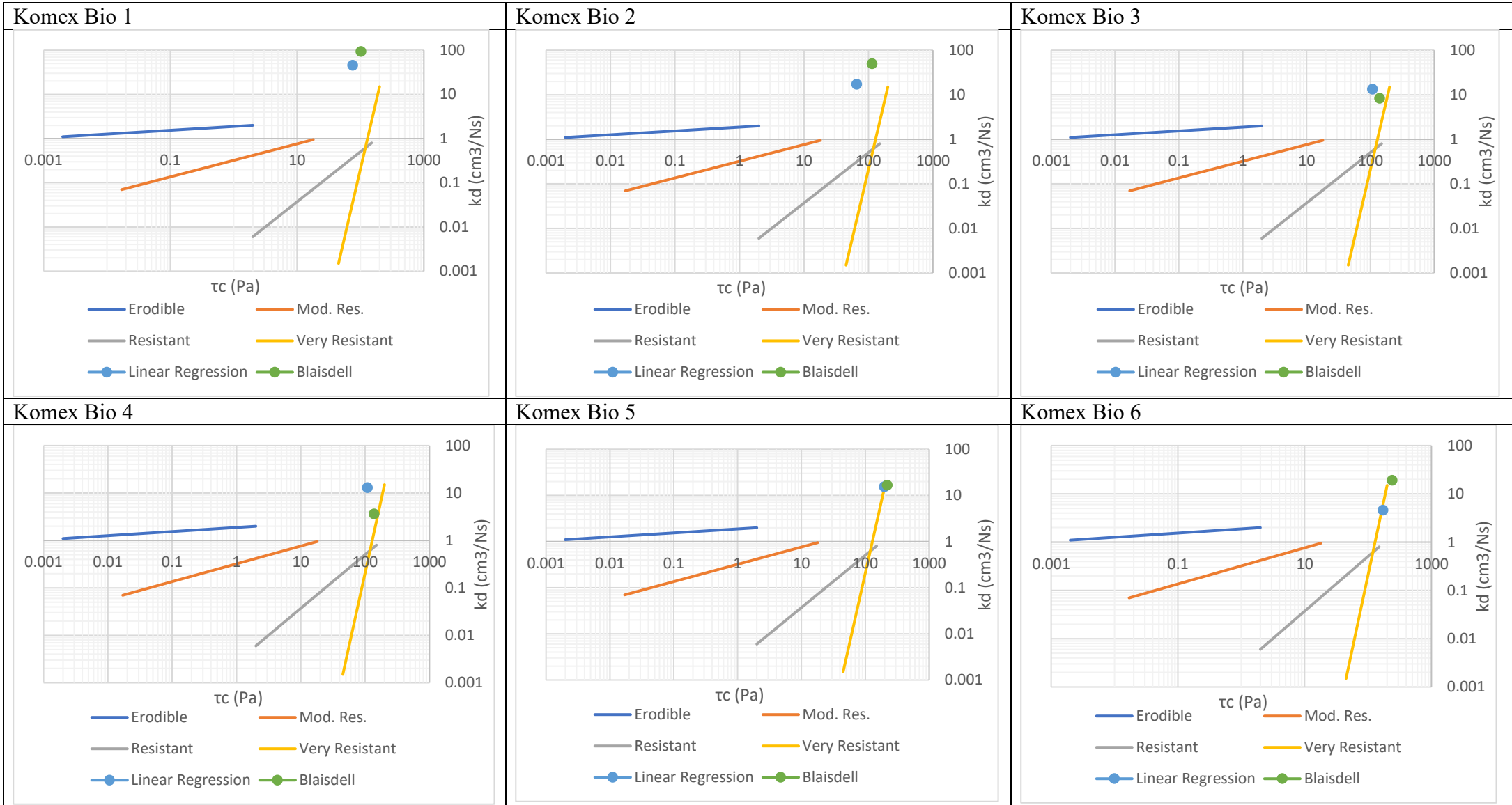


Table 5 – Komex Bio clay samples, Erosion category classification spreadsheet output (1-6)



3.2 Results of NOF and Scour depth time plots

The NOF values have also been calculated and are presented in Table 6. The magnitudes of the NOF values vary between 0.09 to 4.76 – 9% to 476% respectively. The Blaisdell method is closer to original data in comparison to the linear regression method. The average NOF values are 1.64 for Blaisdell and 2.08 for linear regression.

NOF values only present the deviations of the results but not the direction of deviations. Therefore, the analysis of the scour depth time plots is also important.

Table 6 - Results of Linear regression and Blaisdell method (Hanson and Simon, 2001) and the respective NOF values to the original data.

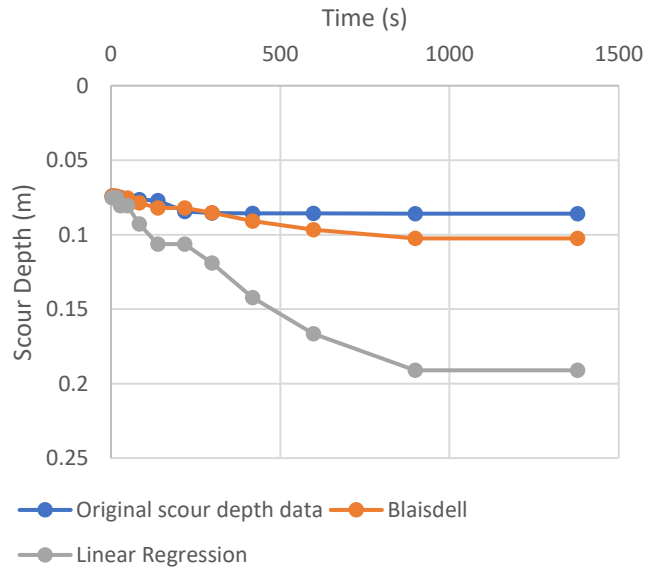
Sample	Linear Regression			Blaisdell			
	τ_c (Pa)	k_d (cm ³ /N-s)	NOF	τ_c (Pa)	k_d (cm ³ /N-s)	NOF	
Untreated Clay	1	239.29	0.19	0.64	366.91	0.55	0.09
	2	260.35	5.18	1.79	290.58	0.5	0.75
	3	288.5	1.99	2.23	321.38	1.35	0.64
Graustabiel	1	213.38	12.5	1.74	240.43	14.92	1.36
	2	217.62	19.17	1.60	238.6	11.58	1.18
	3	138.52	1.11	3.06	274.56	11.33	1.24
	4	254.49	2.54	1.41	295.49	1.87	0.82
	5	324.7	2.72	0.59	351.51	2.57	0.34
	6	334.49	4.49	0.58	353.44	4.36	0.39
Komex Bio	1	74.59	45.8	4.73	100.84	93.76	4.11
	2	65.73	17.41	4.76	113.66	50.21	3.82
	3	107.34	13.47	4.31	138.71	8.34	2.83
	4	108	13.08	4.43	137.69	3.63	1.76
	5	198.05	15.53	2.61	220.74	16.8	2.14
	6	234.28	5.24	1.11	238.59	19.11	1.53

Based on the visual analysis of the scour depth to time plots of all the samples, both methods overpredict the scour depth values. Based on the Blaisdell methods of Daly et al. (2013), this is to be expected and with large NOF values. The Blaisdell method consistently overpredicts and does so the least in comparison to the linear regression method.

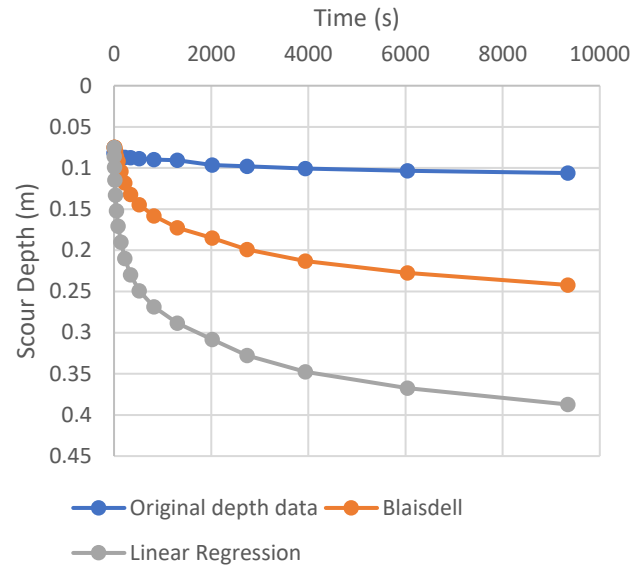
However, the only exception in this is with Komex Bio 6 where the linear regression method is closer to the results than with the Blaisdell method.

Table 7 – Untreated Clays, Scour depth time plots (1-3).

Untreated Clay 1



Untreated Clay 2



Untreated Clay 3

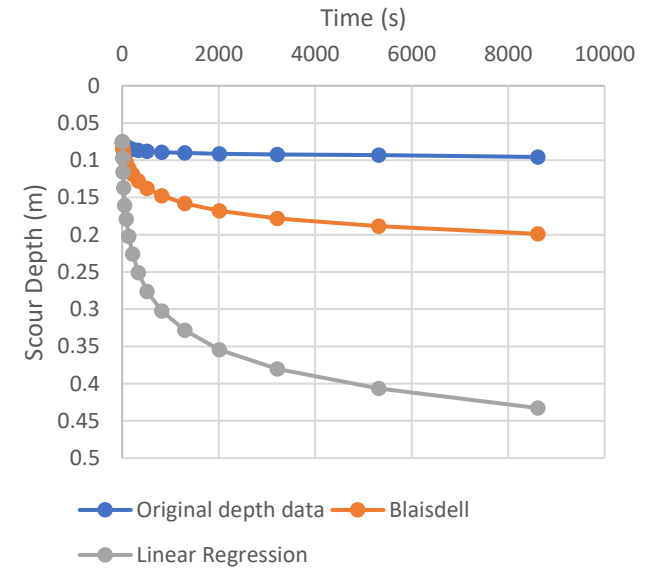


Table 8 – Graustabiel Clays, Scour depth time plots (1-6).

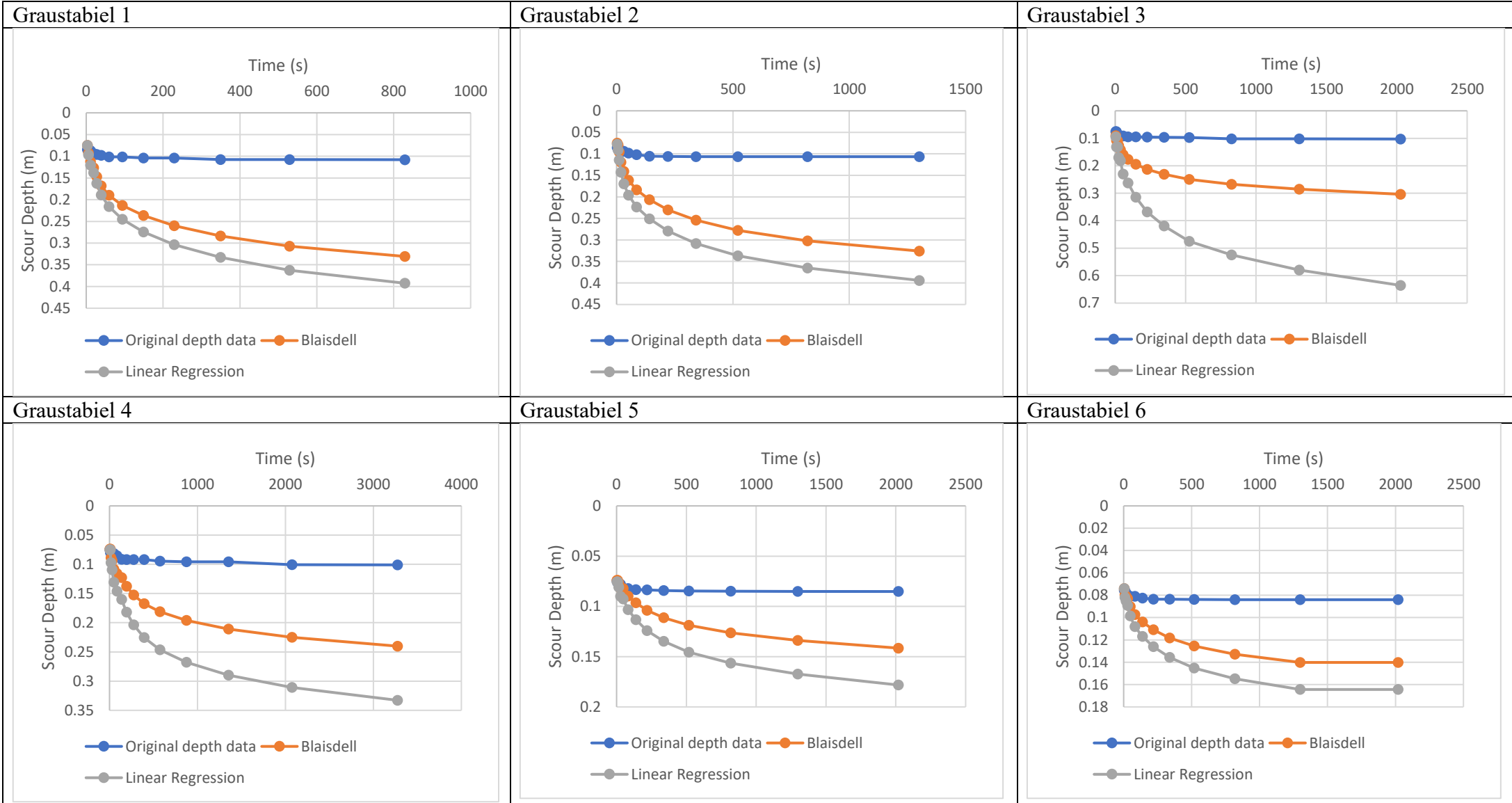
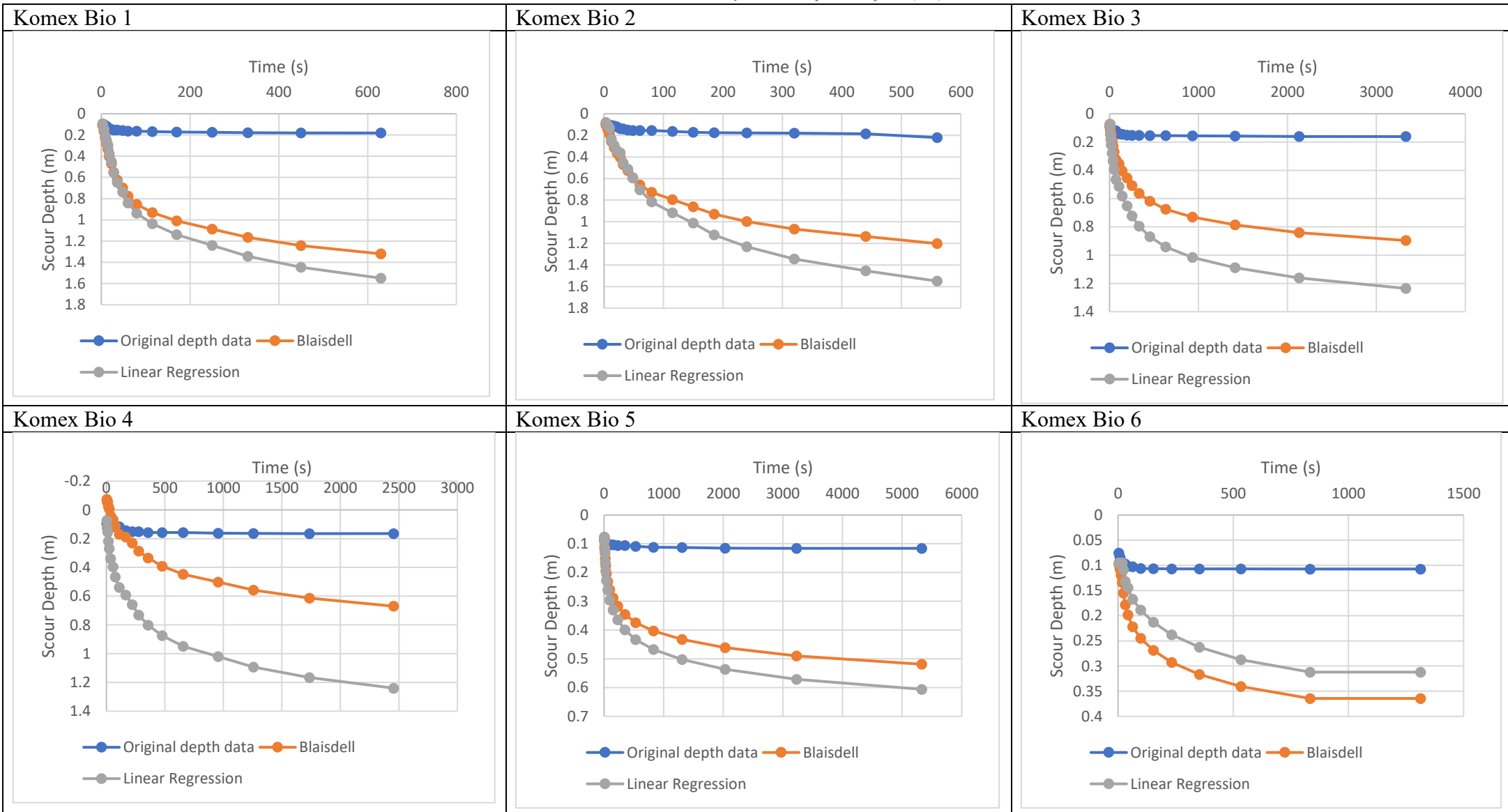


Table 9 – Komex Bio Clays, Scour depth time plots (1-6).



3.3 Results of Bland-Altman plots

For the Bland-Altman plot, the difference presents the difference between Blaisdell against the linear regression (Results of Blaisdell – linear regression).

Erosion Coefficient Bland-Altman plot

For erosion coefficient Bland-Altman plots (see Figure 10), the overall plots are more consistent towards the lower mean values of k_d . The difference increase towards the larger mean values of the results. This implies less consistency when higher values are presented. The mean level⁴ is at +5.36 as it includes the larger values.

The larger values in difference represent Komex Bio samples 1 and 2. Their NOF values 0.08 and 0.18 for the linear regression and 0.12 and 0.20 for Blaisdell. It is unclear whether those values are outliers and could be inferred as a trend due to lack of comparative data with similar qualities (of larger mean).

The all the points (except for the last) fall within the 95% limit of agreement (LOA) region or the ± 1.96 times the standard deviation⁵. Bland & Altman proposed this LOA region to be a region that 95% of data falls within two standard deviations of each other. This, however, is to present how statistically similar they are and does not necessarily present agreeability on its own.

Hanson Erosion Coefficient Bland-Altman plot

For the Bland-Altman critical shear stress plots (see Figure 9), most of the differences are more consistent with increasing means except for two points: Untreated clay 1 and Graustabiel 3. With the first untreated clay sample, the NOF values for both results are not majorly different for the average results of the rest. However, for Graustabiel 3, the NOF value for the linear regression is at 0.31 which can be seen as an outlier within the linear regression NOF values.

With exception of the two points, all the points fall closely within the mean line and are within the 95% LOA interval.

⁴ Referred to as the bias line in Figure 9 and Figure 10.

⁵ The 1.96 coming from the z-score of 95% (Bland & Altman, 1986)

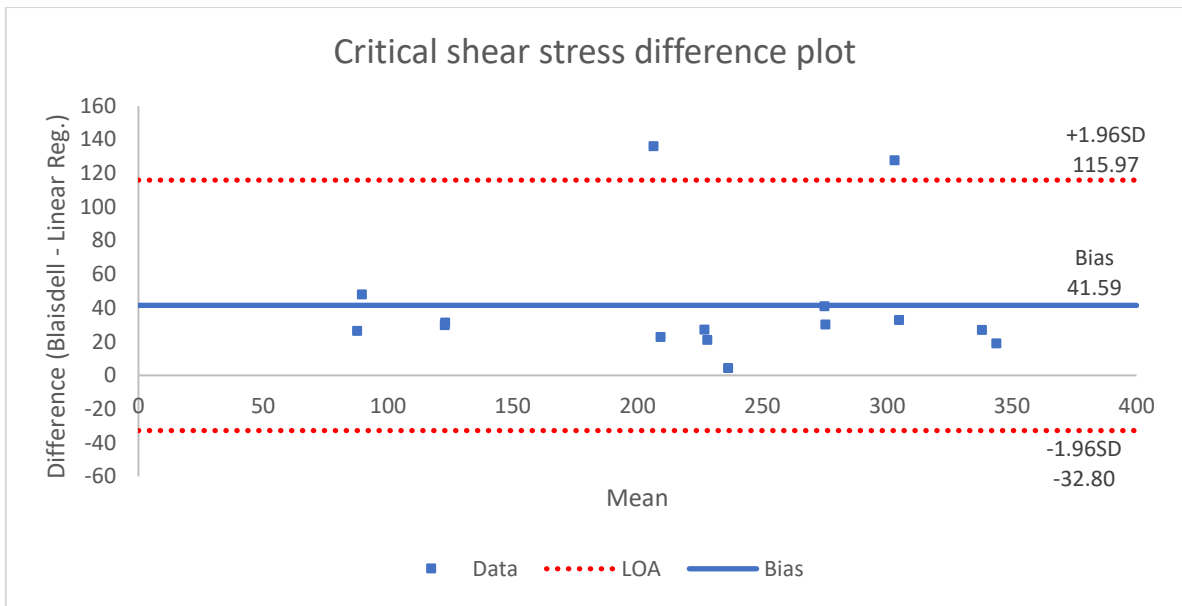


Figure 9 - Bland Altman plot of the critical shear stresses from both methods. The bias line represents the average difference, the upper and lower LOA (limit of agreeability) present the standard deviations of ± 1.96 Standard deviations.

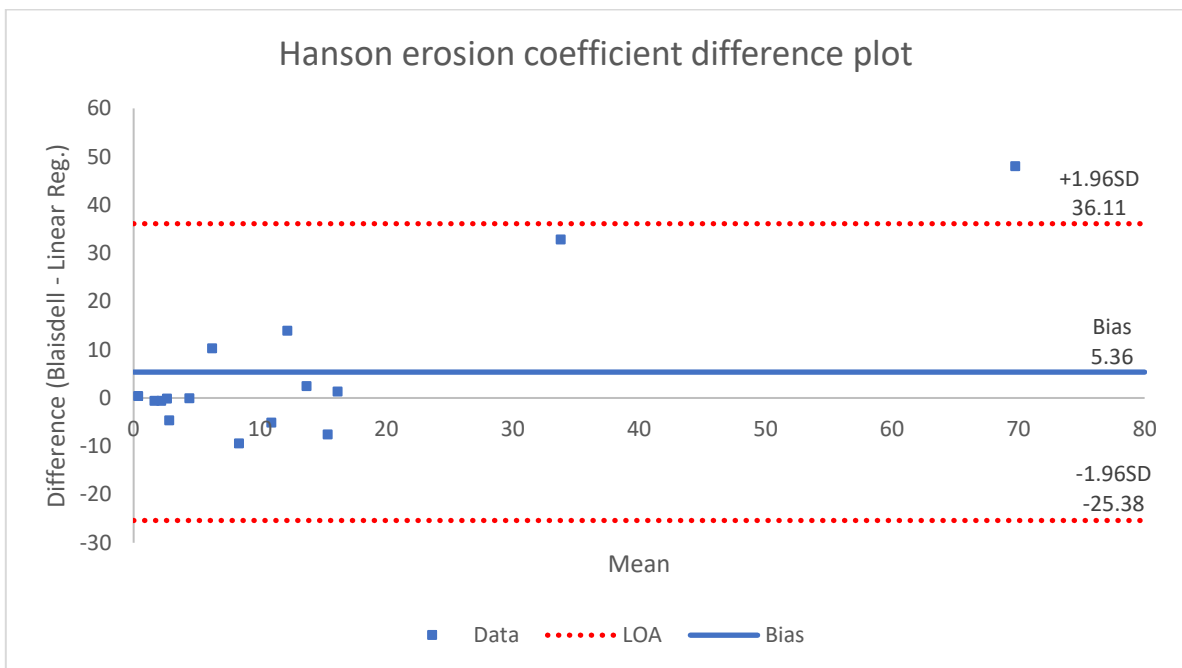


Figure 10 – Bland Altman plot of the Hanson erosion coefficients from both methods

4 Discussion

4.1 What defines the reliability of these methods?

To be able to discuss the results produced, the reliability of the methods must first be assessed.

Firstly, the reliability of the results is to do with the validity of the process and data. The origin of the data, as previously mentioned, comes from a GeophyConsult report that ran tests using a JET device. They utilised the linear regression method similarly to the one in this report. Despite the deviations mentioned prior, the results of this thesis are close to the original. This should help secure the reliability for the linear regression results.

As for the Blaisdell results, there is enough literature to compare the results to. Although most literature describe smaller τ_c and k_d , it can still be compared using the NOF values and Daly et al. (2013). In Daly et al (2013), there is a figure (see Figure 11) that describes the comparison of the Blaisdell method with the Scour depth method (another linear excess shear stress method). It presents an example of one of the samples' scour depth time plot which resembles the plots that are produced for both Blaisdell and the linear regression method.

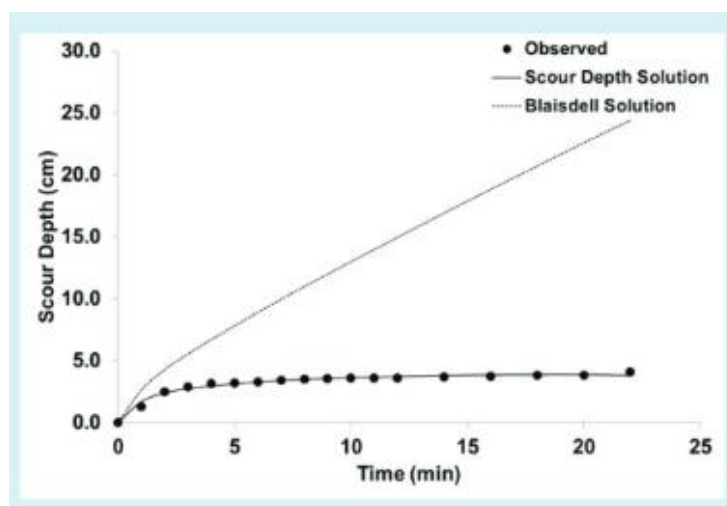


Figure 11- (Daly et al., 2013) Blaisdell against scour depth method for Scour depth time plot.

Table 1. Solutions based on varying initial guesses of τ_c and k_d for the Blaisdell solution and the scour depth solution. See figure 4 for an example solution for both approaches.

Site	Blaisdell Solution (Hanson and Cook, 2004)			Scour Depth Solution		
	k_d ($\text{cm}^3 \text{N}^{-1} \text{s}^{-1}$)	τ_c (Pa)	NOF	k_d ($\text{cm}^3 \text{N}^{-1} \text{s}^{-1}$)	τ_c (Pa)	NOF
1	23.3	0.1	3.4 ^[a]	121.2	1.4	0.1
2	13.2	<0.1	0.2	12.1	<0.1	0.2
3	4.4	0.4	4.6 ^[a]	27.5	2.8	0.1
4	3.6	0.0	0.8 ^[a]	6.6	3.1	0.1
5	1.1	1.3	3.3 ^[a]	6.6	14.3	0.1
6	4.1	0.1	2.5 ^[a]	17.6	3.1	0.1
7	28.6	0.3	5.3 ^[a]	210.8	1.2	0.1
8	1.4	0.0	0.5 ^[a]	2.0	2.8	0.2
9	7.7	0.4	4.6 ^[a]	50.8	2.5	0.1
10	22.1	1.3	5.8 ^[a]	194.1	2.3	0.1
11	11.0	0.5	4.8 ^[a]	74.7	2.2	0.1
12	0.3	1.3	1.9 ^[a]	0.9	16.4	0.1
13	3.1	0.9	5.4 ^[a]	21.9	5.6	0.1

^[a] Solution overpredicted the observed scour depth data, similar to the example in figure 4.

Figure 12 - Results with NOF of Daly et al. (2013) for Blaisdell and Scour depth method.

The results produced within Daly et al. (2013) also shows the NOF values which a range of results like the estimated deviations that was produced for this thesis. Moreover, the anticipated deviations are not only within the range but are also better than presented in Figure 12.

On the other hand, the reliability can be questioned based on the results of the critical shear stresses. In Wahl (2016), it is presented that the Blaisdell method produces significantly lower critical shear stresses. The paper that has used Blaisdell and have produced small results have been taken in situ along the inner parts of riverbanks. Examples in papers such as with Clark and Wynn (2007) and Daly et al. (2013). Why are the results produced higher?

The results are higher in this paper due to the nature of the soils being tested. The original report collected the data through laboratory tests. Moreover, the soils that have not been directly taken from riverbanks. This means that the conditions of the soils differ. Particularly with soil moisture and material composition. As the likelihood of the conditions of the soils are not the same, the comparison between the magnitudes of critical shear stress results is not reliable. The soils used in testing is likely to have different shear stresses due to differing conditions.

As the result produced can be considered similar to other results that have been published in literature (or in reports), the results are deemed reliable.

4.2 What are the differences between the Blaisdell and linear regression methods?

To summarise the results, the erosion category analysis presents that there is a similarity between the two methods with both producing the same erosion categories. However, some results are closely related compared to others. Moreover, the Blaisdell method predicts higher critical shear stresses. Critical shear stress describes the shear stress at which erosion stops as a result of (for example) compaction (at equilibrium depth). Having large shear stresses implies that the soil would need larger shear stresses ($\tau > \tau_c$) for erosion to occur. As the Blaisdell method estimates a larger value than with the linear regression, it would imply the soil is more resistant.

The results of the NOF result of the estimated scour depths indicate that the deviations for both methods are high. The Blaisdell method is closer to the original scour depth values in comparison to the linear regression method for all 15 samples except for Komex Bio 6. The deviations represent the similarities towards the original scour depth data. The larger the deviations mean the further away it is from representing the original data. In this case, Blaisdell resembles the data better than the linear regression method.

Finally, the results of the Bland-Altman plots demonstrate a correlation between the two methods for the mean values of the erosion coefficient and the difference. The higher the k_d value, the larger the difference. As k_d is the ratio between the erosion rate and the critical shear stress, it would entail that larger k_d would mean a higher rate of erosion for the critical shear stress. However, due to only being two points which describe significant k_d values, it is not clear whether this correlation is anomalous. Further research should be conducted.

It also indicates the consistently higher critical shear stress estimates for Blaisdell with the average difference being roughly 42 Pa higher than critical shear stresses from the linear regression model. This, again, relates to the shear stresses for erosion to occur.

4.3 What is the most appropriate method to model Jet erosion data?

It is important to firstly define what the most 'appropriate' model would look like. Looking through different literature on comparison of different modelling methods, the main strength and reasoning for using linear modelling methods such as the Blaisdell method is consistency. The consistency of results in terms of scour depths is the primary defining feature of the results.

Although these literatures are dependent on this consistency, another aspect should be further considered which is the application and purpose. With the assumption that there could be more than one modelling method that can be considered consistent, the purpose and use for the modelling method can allow to present what is 'appropriate'. In literatures, the soil samples are taken along riverbanks with expectations to properly model the situation. Therefore, it would be appropriate to model as closely as possible to the scour depths. However, with the assessment of dikes, the modelling process could be slightly lenient towards a negative expectation. It's safer to build a dike using very resistant soils with a model that estimates lower critical shear stresses (for example).

Looking at the Blaisdell method and the Linear regression method, the results present consistencies with the overestimations in scour depths and (more or less a) consistent differences between critical shear stresses (averaged 41.59 Pa difference). Blaisdell produces larger estimations for critical shear stresses but also presenting closer resembling scour depth predictions. This suggests that the Blaisdell method has slightly more positive expectations of critical shear stresses. Moreover, the scour depth predictions suggest that it is better for resembling the soil data. On the other hand, the linear regression method presents comparatively negative expectations of the critical shear stresses. Furthermore, the scour depth predictions would imply larger erosion as a result.

In the context of what defines the appropriateness in consistency, both Blaisdell and the Linear regression methods are consistent. However, for the applications of what methods should be preferred, Blaisdell is comparatively better at representing scour depths and the Linear regression is better to negatively assess states of soils.

4.4 Limitations

The calculations of the linear regression model were based on the calculations made in a GeophyConsult report given to Fugro. They also used the linear regression method except the report discretized the shear stress results. The process of discretizing the data was only mentioned to have occurred but no example of how it was conducted. Therefore, results vary slightly for the linear regression model compared to the original results that were produced in the report. This thesis did not assess those specific results produced in that report. This could be limiting as it could potentially produce better results than the ones presented in this thesis due to the discretisation which means that this aspect should be explored further.

In section 4.1 questioning the reliability of the results, it is mentioned that results from different research presented values that are significantly smaller than in this thesis. A limitation of this thesis is the exploration of the methods is the range of erodibilities of the soils. Having other data sets that allow a comparison of higher and lower values might prove to be more relevant. This thesis and the conclusions derived about the methods are limited to high shear stress values. As other literature produced lower shear stresses, it makes it difficult to compare whether the assessment is uniform throughout the orders of magnitude for the erosion parameters.

Another limitation is to do with the k_d analysis from the Bland-Altman plots. Lack of data points makes it difficult to point to a conclusion from it. This aspect should be explored further.

The final limitation regards the applicability to the Mini JET device. Due to the lack of data that could be used for this thesis, it was not covered. Although, it could be expected that the results would be similar based on the nature of the device and calculations, it still is unclear as to whether it is appropriate to come to the same conclusions.

5 Conclusion

The main research question of this thesis is “Between the Blaisdell method and Linear regression method, which modelling method is the most appropriate to derive the Hanson erosion parameters for the jet erosion devices?”. This question was partially answered in the discussion section, describing what defines ‘appropriateness’. It was determined that the Blaisdell method is more appropriate to represent scour depths with a comparatively more positive estimations on critical shear stresses. The Linear regression method is better suited for modelling (comparatively) negative estimations in critical shear stresses and scour depth estimates.

The significance of these two conclusions on both results are to do with the application. In the context of dike safety, using the Blaisdell method would prove to be a good way to measure already existing clays and soils on the surface of the dike. If the purpose of the test is to produce parameters to resemble the measured soil rate, then the Blaisdell method would be appropriate.

The Linear regression method has a negative modelling view and can be used to determine possible soil replacements of older, non-suitable soils. The negative estimations of critical shear stresses imply a stricter classification, possibly ensuring strong resistant soils to be used.

Having these different modelling methods and understanding the nature of these models can allow for a clear understanding of use if need be. Although the methods that were followed were aimed at presenting a ‘better’ outcome, it is important to understand what the models could potentially be used for as to present its individual strengths. Both methods are also fairly consistent in majority of the same ways based on the results of the test. Meaning that it is also difficult to outright select a ‘better’ method. Having the analysis look into more specific comparisons, such as the critical shear stresses, make it easier to pinpoint specific differences.

To conclude, in this assessment of the Blaisdell method and Linear regression method, it allowed for better insight to what the strength of each method is. In the broader context and use of the methods for jet erosion testing, this thesis may provide insight to a possible perspective on uses for the methods and how research can be conducted. In other aspects of modelling (for example) dikes, using modelling methods with negative outlooks (often stochastic models) present a sense of realism in relation to risk. The likelihood of risk is analysed. Therefore, knowing when to use the Blaisdell method or the linear regression method would prove to be useful.

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