Master Thesis

Concentrated Equity Portfolios for Pension Funds

Evaluating Concentrated Portfolio Approaches Based on Modeled Client Satisfaction and Expected Excess Yield

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Abstract

This research examines the feasibility and performance of concentrated equity portfolios for pension funds, focusing on client dissatisfaction risk and excess return. Concentrated portfolios, which consist of a limited number of securities, offer an alternative to widely diversified portfolios and may align with the investment philosophies of pension funds. This study aims to test its feasibility and to develop a comprehensive framework for constructing and managing such portfolios, using the S&P 500 index as a benchmark. The research evaluates various asset price path generating models, portfolio selection and capital allocation strategies.

Methodologies employed include computational simulations using models based on a Geometric Brownian Motion, historical data and the Capital Asset Pricing Model (CAPM). Next to that, a novel risk measure framework is developed that accounts for investor satisfaction, where the investment is assumed to be terminated after a memory-adjusted cumulative underperformance of five percent. The findings indicate that while well designed concentrated portfolios yield higher excess returns than passive benchmarks, they also exhibit substantially high risk of client dissatisfaction, reducing their overall appeal for pension funds enormously.

The analysis finds that if pursuing concentrated investing, pension funds should maintain at least forty assets, with both asset selection and capital distribution based on market capitalization. This approach generates an excess yield of 0.4% but with a probability of 50% that the investor will be unsatisfied over a ten-year period. Allocating only 35% of the capital to the concentrated portfolio, while investing the remaining 65% in the benchmark, reduces excess yield to 0.25% and lowers risk by 40%. However, this allocation significantly limits other potential benefits of adopting the concentrated portfolio strategy. Employing two managers to each manage half the capital would result in a 45% probability of client dissatisfaction over a ten-year period and an excess yield of 0.5% but would complicate the management process and potentially increase costs, reducing the attractiveness of the strategy.

Future research should incorporate more comprehensive datasets, investigate a broader set of time frames, explore alternative models for price path simulation and asset selection, and draw insights from historically successful portfolios to enhance the understanding of concentrated equity portfolios further.

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Contents

1 Introduction

1.1 Problem context

1.1.1 The history of the Dutch pension system

Until the late nineteenth century, no formal pension or retirement provisions existed in the Netherlands. Most people were unable to save enough money for their old age. Consequently, children were often the only means of support when individuals could no longer work. An extended family, where multiple generations lived together, was the norm until industrialization and urbanization began to change the social landscape in the Netherlands (IsGeschiedenis, n.d.).

The German Empire was the first nation to adopt a pension-like system, in 1889. The initial retirement age was set at 70 years, and the system was designed to help workers maintain a certain standard of living after they could no longer work. The amount received depended on the wage previously earned while working (Deutsche Rentenversicherung Bund, 2019).

The first pension scheme in the Netherlands was established right after the Second World War. Initially, the 'Noodwet Ouderdomsvoorziening' (emergency act age provision) provided financial aid to all people over 65 who had lived in the Netherlands long enough, had little or no income, and were considered useful members of society. After this initial emergency measure, a permanent act, the 'Algemene Ouderdomswet' (General Elderly Act), was introduced (Gerritse, 1954).

1.1.2 The current Dutch pension system

The contemporary Dutch pension system consists of three pillars. The AOW serves as basic income; everyone working or living in the Netherlands automatically gains a share in this system. The amount received depends on the living situation: singles receive 70% of the minimum wage, while couples receive 100% of the minimum wage per individual (Rijksoverheid, 2024). Working individuals contribute taxes to fund the income of the elderly. In 2021, the total AOW expenditure in the Netherlands amounted to 43.0 billion euros (Rijksoverheid, 2022).

The second pillar consists of a plan constructed through cooperation between workers and employers. About 90% of employees participate in such a pension plan, where employers usually pay two-thirds of the premium, and employees cover the remaining portion. Approximately 36% of retirees' income comes from this pillar (CBS, 2019).

The third pillar includes individual arrangements, such as annuities, life insurance, and private pension investments. These are tax-efficient ways to ensure a certain income level after retirement, especially for individuals who do not accrue a pension through an employer. Payments can be made voluntarily to an insurer or asset manager, who ensures a cash flow when the beneficiary reaches a specified age (Rijksoverheid, n.d.).

1.1.3 Managing a pillar two pension plan

Regular contributions are made to the pension fund throughout the participant's working life. Third parties invest the capital, manage risks, and administer pension rights and payments. Large pension funds, such as ABP and Pensioenfonds Zorg en Welzijn, have their own pension providers (APG and PGGM, respectively), while smaller pension funds hire third parties for the different tasks.

Pension asset managers invest capital across various instruments, including bonds, stocks, private equity, infrastructure, commodities, real estate, and derivatives. Equity asset management strategies vary across funds; some managers prefer passive investment in a broad range of stocks, while others adopt a more active investment approach (Hoekstra, 2023a)(Hoekstra, 2023b).

1.2 The concentrated equity portfolio

1.2.1 What is a Concentrated Portfolio?

One of the ways a pension fund can invest more actively is by focusing on a concentrated portfolio. Such a portfolio stands in contrast to widely diversified portfolios by concentrating investments on a smaller set of securities. The number of stocks can vary based on the context, typically ranging from ten to a hundred holdings selected from the portfolio manager's available options. This set of stocks is often also used to establish a benchmark for evaluating the concentrated portfolio's performance. The rationale behind concentrated portfolios is often rooted in the belief that careful selection and a deeper understanding of fewer investments can lead to superior returns.

The concept of concentration in investment can be traced back to some of the most successful investors in history. Notable figures such as Warren Buffett and Charlie Munger have championed the virtues of concentrated investing. Buffett's investment philosophy emphasizes the importance of understanding businesses deeply and investing heavily in those that exhibit strong fundamentals and competitive advantages. He famously remarked, "Diversification is protection against ignorance. It makes little sense if you know what you are doing," underscoring his preference for a concentrated approach.

1.2.2 What is needed to construct a concentrated portfolio?

Beyond selecting stocks for a benchmark, a concentrated portfolio approach requires methods to identify companies with strong fundamentals and competitive advantages, to decide which stocks to buy or avoid, and determine optimal selling times.

In addition to selecting appropriate stocks, a portfolio manager must develop a clear philosophy on how the capital entrusted to them should be distributed among the selected holdings. Effective allocation involves determining the appropriate weight for each holding based on factors such as the level of conviction in the investment, the associated risk, and the potential for return. For example, a manager might allocate more capital to stocks with higher confidence in their growth prospects and strong fundamentals, while limiting exposure to more speculative or volatile investments.

Effective capital allocation demands continuous assessment and rebalancing. As market conditions change and individual stock performances vary, the portfolio manager must adjust the weights of the holdings to maintain the desired risk-return profile. This dynamic process ensures that the portfolio remains aligned with its investment objectives and risk tolerance.

1.2.3 What considerations might pension funds take into account when investing in a concentrated portfolio?

A pension fund might consider investing in a concentrated portfolio because of the following reasons:

1. Possibility to outperform the market (where research shows that outperforming the market in the long run is unlikely [Bessembinder, 2021]).
2. Funds can better explain to their participants why they are investing in certain stocks.
3. Investment can be aligned with ethical or sustainability impact views of the fund or/and participants (like PMT, for example [PMT, n.d.]).
4. More possibilities for successful engagement between fund and company (which has some scientific bases (Kölbel et al., 2020)]).

Compared to passive investing, there are also some possible downsides to investing in a concentrated portfolio.

1. Higher probability on lower performance (Bessembinder, 2021).
2. More company-specific risk.
3. Higher management costs (Ellis, 2012).
4. Higher liquidity risk and transaction costs
5. Higher probability of missing the most profitable stocks (since only a small percentage of the stocks are responsible for the gains of the complete stock market (Bessembinder, 2018)).

Determining whether these considerations are valid is beyond the scope of this research; the above arguments are factors for pension fund directors to consider when evaluating a shift towards concentrated portfolios.

1.3 Research objective

Cardano aims to deepen its understanding of concentrated portfolios by studying how to manage these portfolios from a risk and return perspective. The objective research whether concentrated equity portfolios are viable solutions for pension funds, and to develop guidelines for constructing an robust concentrated portfolio, using the S&P 500 as a benchmark case. Specifically, we seek to answer the following questions: Which strategy should be employed, in what practical setting, and how does this strategy compare to the alternative of passive investment in the S&P 500? Therefore, our main research question is:

Is it feasible for a pension fund to invest in a concentrated equity portfolio from a risk and reward perspective? and if so, how should it be approached?

1.4 Research questions

The main research question leads to the following sub-questions.

1. How should the results of a concentrated portfolio be evaluated?
   - 1a. Which is the most appropriate risk measure for a concentrated portfolio?
   - 1b. How can the performance of a concentrated portfolio be evaluated in the most suited for this situation?
2. What are the desirable features of investing in concentrated portfolios?
   - 2a. How should stocks be selected in a concentrated portfolio?
   - 2b. How should capital be distributed among the selected stocks?
   - 2c. How many stocks should be selected in the concentrated portfolio to have an robust mix between risk and return?
   - 2d. What are the consequences of only allocating a part of the capital to a concentrated portfolio while investing the rest in a benchmark?
   - 2e. What are the effects of assigning the capital to multiple asset managers managing a concentrated portfolio?

2 Theory review

In this section, we discuss some theoretical aspects. First, we look briefly into the existing literature. After that, we delve into statistics, and discuss benchmark and portfolio evaluation methodologies.

2.1 Review of literature

The theoretical framework underpinning concentrated investment strategies can be traced back to the foundational principles of the modern portfolio theory (Markowitz, 1952). This theory posits that rational investors can construct the optimal portfolio by holding many assets to optimize or maximize expected return based on a given level of market risk (according to their risk appetite), emphasizing the benefits of diversification. due to that, debate have emerged around the efficiency of concentrated portfolios, with proponents arguing that they can outperform the market if carefully managed (Ivkovich et al., 2008), (Qin & Wang, 2021).

Empirical studies have focused on analyzing the risk-return trade-off in concentrated portfolios (Cremers & Petajisto, 2019). These studies often examine how concentrated investments fare against diversified portfolios, especially in volatile market conditions. The findings suggest that while concentrated portfolios can offer higher returns, they also come with increased volatility and risks, which can be critical considerations for pension funds with long-term liabilities .

It is debatable whether the success of concentrated portfolio strategies hinges on the effectiveness of active management. Active managers who employ these strategies might possess exceptional skills in identifying undervalued securities and must be disciplined to hold these investments over extended periods, often through market cycles. Research indicates that active management can significantly impact the performance outcomes of concentrated portfolios. The critical thought is that managers should not invest in their thirtieth best investment idea and, therefore, keep the number of holdings in the portfolio limited (Antón et al., 2021). However, it is known that enlarging a portfolio of assets gives the advantage of diversifying away risks.

To describe the characteristics of individual stocks, we use the Capital Asset Pricing Model (CAPM). The model is a foundational financial theory that seeks to explain and predict the relationship between the risk of an investment and its expected return. Developed in the 1960s by economists such as Jack Treynor, William Sharpe, John Lintner, and Jan Mossin independently, CAPM assumes that investors are risk-averse, meaning they require higher returns to compensate for greater risk. CAPM is built on the idea that investors need to be compensated in two ways: time value of money and risk. The time value of money is represented by the risk-free rate, compensating investors for placing their money in an investment over a period. The risk component requires additional compensation, as taking on more risk increases the potential variability of returns (Sharpe, 1964).

2.2 Statistics of a concentrated portfolio

In this subsection we aim to discover what we can learn from developed statistics theory in relation to concentrated portfolios. We discover how variance (a commonly used risk measure) of a portfolio works in relation to the capital distribution and portfolio size. Also, we examine the relation between the variance and the number of holdings in a concentrated portfolio and what the effect is of the way capital is distributed among different holdings.

The return of a (concentrated) portfolio in a certain timestep can be calculated:

$$R_{\mathrm{P}} = R_1 w_1 + R_2 w_2 + \cdots + R_{\mathrm{n}} w_{\mathrm{n}} = \vec{R}^T \vec{w}. \tag{1}$$

where R_p is the return of the portfolio, R_i is the return of an individual asset and w_i is the weight in a certain asset.

Risk is often expressed in the variance of the return. Calculating the variance gives:

$$\begin{aligned} \sigma_{\mathrm{P}}^2 &= \mathrm{Var}(R_P) = \mathrm{Var}(R_1 w_1 + R_2 w_2 + \cdots + R_{\mathrm{n}} w_{\mathrm{n}}) \\ &= \mathrm{Var}(R_1) w_1 + \mathrm{Var}(R_2) w_2 + \cdots + \mathrm{Var}(R_{\mathrm{n}}) w_{\mathrm{n}} + \sum_{\substack{i=1 \\ i \neq j}}^{n} \sum_{j=1}^{n} w_i w_j \mathrm{Cov}(R_i, R_j) \\ &= \vec{w}^T (\vec{\sigma}(\vec{\sigma}^T \boldsymbol{\rho})) \vec{w}. \end{aligned} \tag{2}$$

Next, we assume certain capital distributions to explore their implications. The simplest case is an equally weighted portfolio with N stocks with a single variance and a correlation for all stocks. When we want to calculate the risks of a portfolio.

Multiplying the variance vector with the correlation matrix and filling in the weight vector yields:

$$\sigma_{\mathrm{Pew}}^2 = \begin{bmatrix} \frac{1}{n} & \frac{1}{n} & \cdots & \frac{1}{n} \end{bmatrix} \cdot \begin{bmatrix} \sigma_a^2 & \rho_a \sigma_a^2 & \cdots & \rho_a \sigma_a^2 \\ \rho_a \sigma_a^2 & \sigma_a^2 & \cdots & \rho_a \sigma_a^2 \\ \vdots & \vdots & \ddots & \vdots \\ \rho_a \sigma_a^2 & \rho_a \sigma_a^2 & \cdots & \sigma_a^2 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{n} \\ \frac{1}{n} \\ \vdots \\ \frac{1}{n} \end{bmatrix}. \tag{3}$$

Solving this results in:

$$\sigma_{\mathrm{Pew}}^2 = \frac{\sigma_a^2 + (n-1)\rho_a \sigma_a^2}{n} = \rho_a \sigma_a^2 + (1 - \rho_a) \sigma_a^2 \frac{1}{n}. \tag{4}$$

The total variance consists of a structural component and a covariance component, which is the $(1 - \rho_a)\sigma_a^2 \frac{1}{n}$ term. This term can be reduced by enlarging the portfolio (i.e. diversification).

Next, we examine how the distribution of value influences the variance. in the case of a linear distribution following the distribution $w_i = \frac{2i}{n(n+1)}$, we end up with the following calculation:

$$\sigma_{\mathrm{Plin}}^2 = \begin{bmatrix} \frac{2}{n(n+1)} & \frac{4}{n(n+1)} & \cdots & \frac{2n}{n(n+1)} \end{bmatrix} \cdot \begin{bmatrix} \sigma_a^2 & \rho_a \sigma_a^2 & \cdots & \rho_a \sigma_a^2 \\ \rho_a \sigma_a^2 & \sigma_a^2 & \cdots & \rho_a \sigma_a^2 \\ \vdots & \vdots & \ddots & \vdots \\ \rho_a \sigma_a^2 & \rho_a \sigma_a^2 & \cdots & \sigma_a^2 \end{bmatrix} \cdot \begin{bmatrix} \frac{2}{n(n+1)} \\ \frac{4}{n(n+1)} \\ \vdots \\ \frac{2n}{n(n+1)} \end{bmatrix}. \tag{5}$$

Similar to Formula 4 , this simplifies to:

$$\sigma^2_{\text{Plin}} = \rho_a\sigma^2_a + (1-\rho_a)\sigma^2_a\frac{2(2n+1)}{3n(n+1)}. \tag{6}$$

This results is quite similar to the result of the equal weight portfolio, where the total variance also consisted of a structural and covariance part.
Next, we consider a portfolio with an exponential distribution, following $w_i = \frac{(2^b-1)2^{\text{bn}}}{(2^{\text{bn}}-1)}\frac{1}{2}^{\text{bi}}$, where b is a parameter determining the shape of the distribution. A higher value of b indicates a less equal distribution.

With similar calculations as the linear distribution case, this finally results in the following formula:

$$\sigma^2_{\text{Pexp}} = \rho_a\sigma^2_a + (1-\rho_a)\sigma^2_a\frac{(2^b-1)^2(4^{\text{bn}}-1)}{(4^b-1)(2^{\text{bn}}-1)^2}. \tag{7}$$

In Figure 1 we see the variance as function of the number of holdings for the equally-weight, linear-weight and exponentially-weight portfolio.

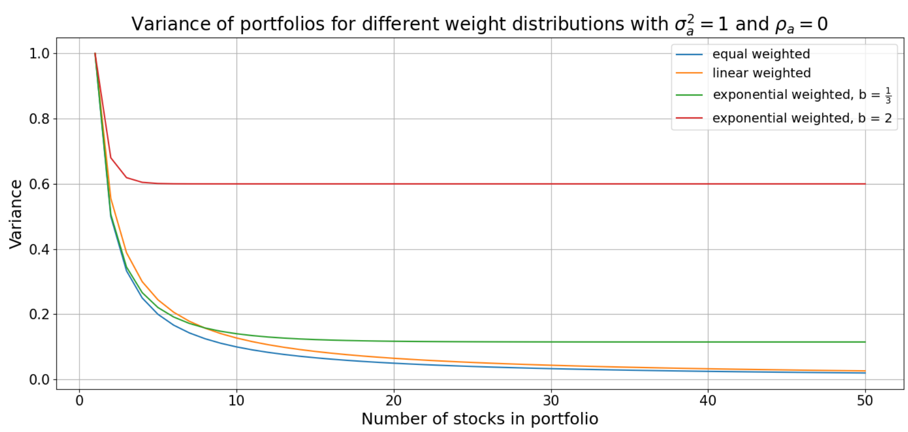

Figure 1: Variance of portfolios with different holding distributions for different numbers of holdings in portfolio according to Formulas 4, 48 and 61 .

With our concentrated portfolio in mind, we can see that shape of the capital distribution matters for how the variance evolves for different number of holdings in the concentrated portfolio. From Figure 1 we clearly see that adding more holdings to the portfolio lowers the variance, but there is an asymptote, for the equal and linear-weight portfolios, this asymptote is at zero, whereas the exponential-weight portfolios have an asymptote higher than zero (given that the corelation is zero).

Proof and further elaboration on the calculations can be found in the appendix A, Section 7.1.

2.3 Benchmark methodology

A benchmark serves to measure a fund's performance and align the manager's incentives with those of the investors (Alekseev & Sokolov, 2016). Which benchmark is the most appropriate to compare is not always obvious, and sometimes a custom benchmark is created. A framework of criteria mentioned by Jeffery V. Bailey (1992) can be used to evaluate how applicable a certain benchmark is for a certain portfolio.

Benchmarks come in various forms, with the most common being capitalization-weighted indices, such as the S&P 500 and AEX. Other types include price-weighted indices, like the Dow Jones Industrial Average and Nikkei 225, and equally weighted indices, such as the S&P 500 Equal Weighted and Nasdaq 100 Equal Weighted.

Research (Malladi & Fabozzi, 2016) has shown that, over 90 years of historical data, equal-weighted portfolios outperform value-weighted portfolios in both return and Sharpe ratio. Additional studies using 43 years of S&P data confirm these findings, showing that equal-weighted portfolios also perform better in terms of four-factor alpha, even when accounting for a 50 basis point trading cost. However, these portfolios exhibit higher volatility, kurtosis, and turnover (Plyakha et al., 2012).

From 2016 to 2021, equal-weight indices underperformed relative to capitalization-weighted indices, as documented by further research (Taljaard & Maré, 2021). This study concludes that while equal-weighted benchmarks may experience significant short-term underperformance, they tend to outperform capitalization-weighted benchmarks over the long term, as predicted by stochastic portfolio theory.

While equal-weight benchmarks perform better, most investors evaluate their investments using market-cap-weighted benchmarks. Therefore, we will use a market-cap index in our research.

2.4 Different possible performance measures

To evaluate the performance of an investment, such as a concentrated portfolio, an appropriate method is required. According to Cogneau and Hübner, 2009 there are at least 101 ways to measure portfolio performance a time period. A few commonly used methods are the simple return, Sharpe ratio, Jensen's alpha, Treynor ratio and Sortino ratio.

Simple Return

The Simple Return measures the percentage change in the value of an investment over a specific period of time. It is calculated as the difference between the final value and the initial value, divided by the initial value. The formula is:

$$R = \frac{V_f - V_b}{V_b}, \tag{8}$$

where R is the simple return, V_f is the final value of the investment after the time period, and V_b is the begin value of the investment at the start of the time period.

Sharpe Ratio

The Sharpe Ratio is a measure of the risk-adjusted return of an investment. It is defined as the ratio of the excess return of the investment over the risk-free rate to the standard deviation of the excess return. The formula is given by:

$$S = \frac{R_p - R_f}{\sigma_p}, \tag{9}$$

where S is the Sharpe Ratio, R_p is the average return of the portfolio, R_f is the risk-free rate, and σ_p is the standard deviation of the portfolio's excess return.

Jensen's Alpha

Jensen's Alpha measures the abnormal return of an investment relative to the expected return predicted by the Capital Asset Pricing Model (CAPM). The formula is:

$$\alpha = R_p - (R_f + \beta(R_m - R_f)), \tag{10}$$

where α is Jensen's Alpha, β is the portfolio's beta, and R_m is the return of the market.

Treynor Ratio

The Treynor Ratio is another measure of risk-adjusted return, similar to the Sharpe Ratio, but it uses beta as the risk measure instead of standard deviation. The formula is:

$$T = \frac{R_p - R_f}{\beta}, \tag{11}$$

where T is the Treynor Ratio.

Sortino Ratio

The Sortino Ratio is a modification of the Sharpe Ratio that differentiates harmful volatility from overall volatility by using the downside deviation. The formula is:

$$\text{Sortino Ratio} = \frac{R_p - R_f}{\sigma_d}, \tag{12}$$

where σ_d is the downside deviation of the portfolio.

2.5 Choosing the performance measure

The ultimate goal of a pension fund is to take care of a robust pension regardless of volatility beta, alpha and other metrics, that is why the simple return is most appropriate. To calculate the simple return over multiple periods, we use the CAGR (Compound Annual Growth Rate):

$$\text{CAGR} = \left(\frac{V_f}{V_b}\right)^{\frac{1}{n}} - 1 = \left(\prod_{i=1}^{n}(1+R_i)\right)^{\frac{1}{n}} - 1, \tag{13}$$

where R_i is the return for period i, and n is the number of periods.

2.6 The role of volatility

Because of the transition in the pension system in the Netherlands, volatility in the capital of pension funds will have a different role than before. In the old system, Volatility had to be minimized in the past because it impacted the fund's funding ratio, a key metric for its financial stability. When the funding ratio is above 110% (de Nederlandse Bank, n.d.), pension payments could be increased and should be decreased when the ratio is below 104.3 % (Koolmees, 2019). In the new pension system which is about to be adopted in the Netherlands, this funding ratio will be abolished and volatility will play a less significant role, which is the reason why this is not directly incorporated in the performance evaluation.

3 Constructing concentrated portfolio models and experiments

This chapter discusses the process leading to all the models and experiments in this research. First, we describe five models of how we generate asset prices, then we describe four variants for how holdings are selected, then our developed risk framework is explained, after that we describe the five policies for how value is distributed among the selected holdings. Finally, we explain what the different experiments look like and why we do them.

3.1 Asset price paths models

3.1.1 Used data

In our analysis, the time period from December 31, 2013, to January 1, 2024, is used as the reference. Return data were obtained via the Python yfinance library, while market capitalization data were collected through a macro designed to scrape annual data on the number of outstanding shares from Ycharts.com. These data was then multiplied by the stock price for the corresponding time span to calculate market capitalization. We considered all stocks included in the Wilshire 5000 index as of spring 2024, provided they existed at the start of our analysis period and had reliable share data available from Ycharts. In total, 1,898 stocks were included in our study, with their corresponding tickers listed in Appendix A.

The period from December 31, 2013, to January 1, 2024, was chosen for two main reasons. First, selecting a more recent period ensures that fewer stocks have disappeared due to mergers or bankruptcy, for which historical data may not be available. This allows for a more complete dataset and helps to reduce the impact of survivorship bias, thereby providing a more accurate representation of market dynamics. Second, the chosen time frame is sufficiently long to capture the various market dynamics, ensuring that our analysis covers a broad range of market conditions.

3.1.2 Survivorship bias in the data

Despite efforts to minimize it, survivorship bias is still present in our dataset. Stocks that existed ten years ago but no longer exist in 2024, due to mergers or bankruptcies for example, were investable during the early part of the analysis period and are part of the benchmark. Since both the portfolios under study and the benchmark are subject to this bias, its overall impact on our results is limited.

3.1.3 Global parameters used in all models

The following parameters are used throughout our analysis regardless of which of the five models is used.

- Simulation time: 10 years with 40 timesteps of 3 months.
- Trading frequency: once in the three months. Most companies report once every three months. This provides a good balance between simulation speed and precision.
- Transaction costs: 0.01%, this is a common value for institutional investors.
- Management fee: 0.16%, this is a common value for funds larger than 1 billion Euros.

- Only the largest 500 stocks make up the benchmark, and only those assets are investable for the fund manager.

3.1.4 Computational price path models

The most common way to simulate correlated price paths in academia today is using a multivariate geometric Brownian motion.
In a multivariate geometric Brownian motion, the random variable of the Wiener process can be designed to be correlated. This equation is represented in Formula 14 (Musiela & Rutkowski, 2004):

$$dS_t^i = \mu_i S_t^i \, dt + \sigma_i S_t^i \, dW_t^i, \tag{14}$$

$$\mathrm{E}(dW_t^i dW_t^j) = \rho_{i,j} \; dt, \tag{15}$$

where $\rho_{i,i} = 1$.

The computational model of the asset prices is mainly based on an article by van Heeswijk, 2021. A set of random standard normal variables is generated, which are matrix multiplied by a Cholesky decomposition of the correlation matrix (Cholesky, 1924), which creates a set of random variables which are correlated in line with the correlation matrix. The product of this matrix multiplication are the Wiener process variables.

In the model, the Wiener variables are calculated by the following formula:

$$d\vec{W}^t = \mathbf{R}\vec{X}^t, \tag{16}$$

where $\mathbf{R}$ is the Cholesky decomposition of the correlation matrix and $\vec{X}^t$ is a vector of independent random variables, which are normally distributed with a mean of zero and a standard deviation of one. From this basis, two computational models are constructed.

Plain model

This model is the most straightforward one. We assume all assets are the same except for their starting market capitalization.

The following assumptions are made for this model.

- Expected return μ_i: 3.365% on quarterly basis (14.15 % on yearly basis).
- Volatility σ_i: 21.668 % on quarterly basis (43.336 % on yearly basis).
- Correlation $\rho_{i,j}$: 0.288.
- The initial market capitations M are distributed according: $\mathrm{Log}_{10}(M) \sim \mathrm{Normal}(\mu_{\mathrm{cap}}, \sigma_{\mathrm{cap}})$, with $\mu_{\mathrm{cap}} = 9.064$ and $\sigma_{\mathrm{cap}} = 0.929$.
- No shares will be issued or bought back, keeping the number of outstanding shares constant. Therefore, the return of the asset price is equal to the return of the market capitalization.

The justification of the parameters above can be found in Appendix C, Section 7.3.

CAPM model

The plain model is beautifully simple but has a significant downside: the dynamics of all assets are the same, and therefore, a concentrated portfolio does not make much sense. In reality, some stocks have more risk than others, and with that they might have a higher expected return. This relation is described in the CAPM model. The two relevant formulas in this model are the following ones:

$$\mu_i = R_f + \beta_i(\mu_m - R_f), \tag{17}$$

$$\sigma_i^2 = (\beta_i \sigma_m)^2 + \sigma_{\text{id}}^2. \tag{18}$$

From these two formulas, our second model is created for which the variables are the imput for the Geometric Brownian Motion of Formula 14. The following assumptions are made for this model.

- Equity risk premium $\mu_m - R_f$: 2.63% on quarterly basis (10.94 % on yearly basis).
- Risk free rate R_f: 0.491 % on quarterly basis (1.98 % on yearly basis).
- Volatility of market σ_m : 7.692%.
- Correlation $\rho_{i,j}$: 0.288.
- The beta values of the assets are distributed according to: $\beta_i \sim \text{Normal}(\mu_{\text{beta}}, \sigma_{\text{beta}})$, with $\mu_{\text{beta}} = 1.251$ and $\sigma_{\text{beta}} = 0.756$.
- The values of the idiosyncratic volatilities on quarterly basis are distributed according: $\sigma_{\text{id}} \sim \text{LogNormal}(\mu_{\text{id}}, \hat{\sigma}_{\text{id}})$, with $\mu_{\text{id}} = 2.751$ and $\hat{\sigma}_{\text{id}} = 0.549$.
- The initial market capitations M are distributed according: $\text{Log}_{10}(M) \sim \text{Normal}(\mu_{\text{cap}}, \sigma_{\text{cap}})$, with $\mu_{\text{cap}} = 9.064$ and $\sigma_{\text{cap}} = 0.929$.
- No shares will be issued or bought back, keeping the number of outstanding shares constant. Therefore, the return of the asset price is equal to the return of the market capitalization.

The justification of the parameters above can be found in Appendix C, Section 7.3.

3.1.5 History based models

Next to creating our price paths of assets, we can also use our dataset by using the historically observed price paths. The advantage of this approach is that probabilities and correlations do not have to be guessed or determined, but the reality is used as it is. Two models are considered: backtesting and a historical simulation.

Backtesting simulation

In this model, we take the history of the asset prices and their market capitalizations and take these price paths as input price paths in our model. We calculate what would have happened when we would have started this concentrated portfolio ten years ago with these available stocks.

Historical simulation

In the historical simulation, forty times (ten years consisting of 4 quarters), a random quarter of the forty observed quarters is taken and used to calculate the next step of the asset prices and market capitalizations. In this model, some observed quarters can be used multiple times while other quarters are not used. This method is called bootstrapping.

3.1.6 Combined price path model

Our final model combines both computational and historical approaches. Market capitalizations are initialized using observed data from the dataset. For each time step, the model randomly selects between the historical method and a computational model, with an equal probability. When the historical model is selected, it uses data from a randomly chosen quarter, following the historical simulation approach.

Alternatively, when the computational model is selected, a time step is generated based on the Capital Asset Pricing Model (CAPM). In this case, instead of using randomly generated parameters, the model utilizes the observed betas and idiosyncratic volatilities for each stock. This modification ensures that the computational component closely reflects real-world data.

The rationale for this hybrid approach is that while historical data can provide valuable insights, it may not fully predict future outcomes. Therefore, the inclusion of a CAPM-based component that leverages observed data allows the model to better fit reality. This combination creates a balanced approach, blending past patterns with a theoretically grounded forecast model. As a result, we consider this to be our most advanced and reliable model.

The combined model is validated by constructing the same model and repeating all intermediary steps but then for 802 stocks, which were observed for 30 years.

3.2 Variants for selecting assets and updating selection

When asset price paths and market capitalizations are determined, the method (which we call variant) of selecting and trading assets should be constructed. For this, we come up with four variants, from now on called variants: the 'MC chance' variant, for which relative market capitalizations play a role and the 'MC random' variant, for which each stock has the same probability of being selected. Next to these first two variants, we construct two other variants based on the beta of the assets. The 'beta' variant selects assets with a beta close to one more likely than assets with a beta far from one and the 'beta cor' variant adds a maximum average correlation to the previous variant.

These variants only determine what holdings are selected, the way capital is distributed among the selected holdings is determined by the designed capital distribution policies, which is described in the next subsection.

3.2.1 Initial holding selection

A variant consists of two pieces, the initial selection algorithm and updating its selection of holdings by selling some holdings and buying others. First, the initial selection for all variants are discussed. First the probabilities of each stock being selected are established, and afterwards as many stocks as allowed in the concentrated portfolio are selected based on these probabilities.

MC chance variant

In the MC chance variant, several assets are selected equal to the size of the concentrated portfolio. The probability that an asset is selected is based on its relative market capitalization.

$$P_i = \frac{\text{marketCap}_i}{\sum_{i=0}^{N_{\text{inv}}} \text{marketCap}_i}, \tag{19}$$

where P_i is the probability a stock gets selected and N_{inv} is the number of stocks that is investable.

MC random variant

The MC random variant works the same as the MC chance variant; however, the selection procedure is entirely random and, thus, regardless of the stock's market capitalization. Mathematically this is expressed as:

$$P_i = \frac{1}{N_{\text{inv}}}, \tag{20}$$

Beta variant

The previous two selection variants select their holdings completely random or in a random process adjusted for their market capitalization. The beta variant and the beta cor variant try to select assets based on the specific dynamic of the stocks. A key metric for a stock is its beta value, empirical research (Brealey et al., 2024) suggest that stocks having a beta around one have higher expected yield than stocks with a beta far from one for the period after the introduction the Capital Asset Price Model. This finding questions the overall validity of this model, and makes it reasonable to select stocks with a beta value around one, which is why this variant is introduced.

The probability that an asset is chosen is computed with the following formula:

$$P_i = \frac{\frac{1}{|(\beta_i - 1)|}}{\sum_{i=0}^{N_{\text{inv}}} \frac{1}{|(\beta_i - 1)|}}, \tag{21}$$

where β_i is the individual beta value of a stock.

Beta cor variant

The beta cor variant is similar to the beta variant, except it has an extra step. When stocks are selected, the average correlation of all combinations may not be higher than 0.35. A new attempt is made when the random asset selection produces a holding combination for which the average correlation is higher than this value. The same holds for selecting new assets during the simulation.

Sometimes, it turns out this limit is not possible, and then this condition is dropped during the simulation. The reasoning behind this limit is that when one selects a combination of assets with relatively low correlation, there will be more diversification and with that less risk for the same number of holdings.

3.2.2 Updating holding selection

In reality, a manager sometimes sells some holdings to buy some new ones, with a dynamic that it not stable during time. Therefore a holding trading part is introduced. Every time step (which equals a quarter), a random X number of stocks are sold.

This number of X is determined the following way. Firstly, the expected value of X, X_μ is calculated:

$$X_\mu = \frac{N_{\text{inv}}}{\text{Average holding time}}. \tag{22}$$

The average holding time represents the average time a stock is expected to be in the portfolio, which is in our simulation is ten years. So, suppose we have a portfolio consisting of 40 stocks and on average, we hold a stocks for a period of ten years, which equals forty quarters, so that in this example, X_μ equals 1.

Then X itself itself follows a normal distribution:

$$X \sim Normal(X_\mu, \frac{X_\mu}{2}). \tag{23}$$

The standard deviation of the X variable is chosen to be half of X_μ, there is no research to justify this value, but it results in reasonable behavior of X. After X stocks are sold, new holdings should be selected, this happens based on the same procedure as in the initialization phase. The only difference is that now X holdings are selected to add to the remaining holding selection, where just sold assets and already owned assets have no probability of being selected. If the number of investable stocks is lower than the number of desired stocks in the portfolio, fewer holdings will be present in the concentrated portfolio than desired.

3.3 Capital distribution policies

Now that we have a way to select the holdings, we need to determine how we distribute our capital among the different holdings. For this, five policies are designed: market capitalization-weighted, equal-weighted, market capitalization-weighted with a cap, market capitalization-weighted with maximum normalized HHI index, and a linear combination between market capitalization and equal-weighted.

3.3.1 Market capitalisation-weighted policy

In this policy, the capital addressed to one stock is determined by the relative market capitalization of this particular stock compared to the market capitalizations of all selected stocks. To represent this in a formula:

$$C_i = \frac{\text{marketCap}_i}{\sum \text{marketCap}_i} \times \text{Capital total}, \tag{24}$$

where C_i is the capital assigned to stock i, marketCap_i is the market capitalization of selected stock i, and Capital total represents the total capital available to the manager.

3.3.2 Equal-weight policy

In the equal weight policy, the capital is equally distributed among all selected holdings. This is represented in Formula 25:

$$C_i = \frac{\text{Capital total}}{N}. \tag{25}$$

3.3.3 Market capitalization-weighted with a cap policy

In the market capitalization-weighted with a cap policy, the capital assigned to specific holdings is initially the same as the market capitalization policy. However, where if an individual holding has a higher concentration than a certain limit, it will be scaled down, and the freed capital will be re-distributed over the other holdings pro rata. A formula determining the cap based on the maximum number of stocks in the concentrated portfolio is needed because a maximum concentration of 0.1 is a serious concentration when one holds 100 assets, but not so much when one has 15 assets, and not even possible when one has eight assets.

Therefore, we construct the cap in the following way:

$$Cap = \frac{n^{\frac{2}{5}}}{n} = n^{-\frac{3}{5}}. \tag{26}$$

To illustrate the outcomes of this formula, some examples are calculated:

Number of holdings	equally weighted ($\frac{1}{n}$)	**cap** ($n^{-\frac{3}{5}}$)
1	1.00	1.0000
5	0.20	0.2783
10	0.10	0.1778
25	0.04	0.0891
50	0.02	0.0588
100	0.01	0.0376

Table 1: Table of concentration per holding for an equally weighted policy and the market capitalization cap policy for various holding sizes.

3.3.4 Market capitalization-weighted with maximum normalized HHI policy

In the market capitalization-weighted with maximum normalized HHI policy, the capital addressed to specific holdings is initially the same as the market capitalization policy, where the normalized HHI (Herfindahl–Hirschman Index) (Rhoades, 1993) may not exceed a specific limit. When the HHI of a distribution exceeds this limit, holding positions that contribute more than what, on average, is tolerable will be decreased. The freed capital will be distributed over the other holdings pro rata in small steps. This process is repeated until the portfolio's normalized HHI is lower than this chosen limit.

The normalized HHI is calculated in the following way:

$$\text{Normalized HHI} = \frac{\sum_{i=1}^{n}(\frac{marketCap_i}{\sum_{i=1}^{n} marketCap_i})^2 - \frac{1}{n}}{1 - \frac{1}{n}}. \tag{27}$$

A normalized HHI of one means that all value is in one asset, and an HHI of zero results in an equally weighted portfolio. To illustrate the dynamics of this number, suppose a portfolio of 50 assets; how high can the concentration of one asset be given that the other 49 assets are equally weighted? The results are shown in Table 2.

Normalised HHI	Concentration asset 1	Individual concentration other assets
1	1	0
10^-1	0.330	0.014
10^-2	0.118	0.018
10^-3	0.051	0.019
10^-4	0.030	0.020
0	0.02	0.02

Table 2: Normalized HHI values for portfolios of fifty shares when the maximum capital is put in the first asset and the rest of the capital equally weighted among the other forty-nine assets.

In this research, we chose a limit of 0.01 because initial experiments showed that this had the best results.

3.3.5 Linear combination of market capitalization and equal-weighted policy

This linear combination of market capitalization and equal-weighted policy, we create linear combination of the first two policies.

$$C_i = (f \times (\frac{\text{marketCap}_i}{\sum \text{marketCap}_i}) + (1-f)\frac{1}{N}) \times \text{Capital total}, \tag{28}$$

where f represents the fraction of value that should be in the market capitalization part of this policy, which we choose to be 0.5 throughout our research.

3.4 Portfolio evaluation

When we simulate a concentrated portfolio, we should also have a way to evaluate the performance and risk. The performance evaluation was already discussed in the theory and will be briefly mentioned. Next, we introduce the risk measure framework in this subsection.

3.4.1 Performance evaluation

As described in the theory review section, we use the simple return to calculate the performance over one time period, which we describe in Formula 8. To calculate the return over multiple periods, the cummulative annual growth rate (CAGR) is used, as described in Formula 13. To have an extensive image of the considerations behind the performance evaluation, have a look at Sections 2.4 and 2.5.

3.4.2 Risk measure framework

The risk measure framework is an essential and novel element of this research. In practice, funds that perform well relative to their benchmark gain positive coverage and receive more inflow. Funds that perform poorly relative to the benchmark will lose investors; investors will pull back, thus the fund will receive a lower fee (Cheraghali et al., 2022) (Sheng et al., 2021). A fund may have a long-term rewarding strategy, but when the performance in the short run is not satisfactory compared to an alternative investment strategy, a fund will not have the time to show this rewarding strategy. The alternatives for a specific concentrated stock portfolio are other actively managed equity funds or passive equity solutions based on a benchmark. These other funds will also be compared to other actively managed equity funds and passive equity solutions. The best, most objective comparison is the passive index.

Investors will pull back money when a fund is underperforming too much for too long; the quest here is to quantify the investor's limit for pulling back. The difficulty is that an investor may be dissatisfied after a one-year underperformance, which can be ten percent, but also three consecutive years with an underperformance of four percent. To adjust for time and memory, we come up with a way to calculate the risk in the following way:

$$RiskMeasure = -\sum_{i=0}^{T} \gamma^i (r_{\text{t,cp}} - r_{\text{t,b}} - r_{\text{tol}}), \tag{29}$$

where

γ is the temporal discount factor,
$r_{\text{t,cp}}$ is the return of the concentrated portfolio at year t,
$r_{\text{t,b}}$ represents the return of the benchmark at year t,
r_{tol} represents the long-term performance for which the investor is neutrally satisfied,
T is the number of years for which performance is available.

The risk measure formula is now defined. Now, we have to quantify the risk measure limit parameter, the return tolerance parameter, and the memory factor. The memory factor γ we set at 0.8, this value could not be found in the literature about investing. However, research on discounted utility suggests this value, indicating that it is a plausible and justifiable choice (Frederick et al., 2002)

We determine the value of r_{tol} to be -0.004. We choose this value because it shows that investors will tolerate a little underperformance, but not more than that.

The *RiskMeasure* limit is set to be 0.05. This value we determine in cooperation with field knowledge from experts at Cardano. If at some point this value is surpassed, it is a trigger that the investor is unsatisfied with the performances and might pull back its capital so that the investment is terminated, which needs to be prevented.

Summing up the log of the returns instead of the raw returns might address the task of quantifying this risk slightly better (when an investment goes up 50% one year and the other year it goes down by 50%, adding the two results, it looks like the investment did not move, where in reality it went down with 25% in this two years, which would not look like that when log returns would be added to each other). The reason why this is not done in this case is threefold:

1. Since the non-log method is more intuitive, it is easier to explain to executives and representatives of the investing pension fund.
2. The inaccuracy sketched in this example is only significant because the returns are far off zero (i.e. -50% and +50%). When the results are not far from zero, the difference between summing up returns and summing up log returns is not that large, and when the returns are large, the risk limit will be reached regardless of the method.
3. This simulation does not deal here with cumulative returns but with intermediate evaluations of possible underperformance of a portfolio compared to an index adjusted for risk tolerance.

Another consideration of this risk framework is the starting point. It might be arbitrary to start the evaluation from the moment of inception, but when the investment runs longer, this is not important. As shown in Table 3, the impact of the result of 5 years ago only contributes about 33% compared to the result of this year. Moreover, the result of something that happened ten years ago only counts for 10%.

To illustrate the calculation we show an example in Table 3, which presents a history of five years of performance of the portfolio relative to the benchmark (and adjusted for the underperformance tolerance of the investor) while applying the risk measure Formula 29.

Years ago	0	1	2	3	4	5
Relative performance: $(r_{\text{t,cp}} - r_{\text{t,b}} - r_{\text{tol}})$	-0.04	0	0.01	-0.04	-0.02	0.05
γ^i	0.8^0	0.8^1	0.8^2	0.8^3	0.8^4	0.8^5
Value of γ^i	1	0.8	0.64	0.512	0.410	0.328
$-\gamma^i(r_{\text{t,cp}} - r_{\text{t,b}} - r_{\text{tol}})$	0.04	0	-0.0064	0.0205	0.0082	-0.0164

Table 3: Risk framework example with five years of performance of the portfolio relative to the benchmark (and adjusted for the underperformance tolerance of the investor) while applying the risk measure Formula 29.

When all the values of the last row of Table 3 are summed up in line with Formula 29, it turns out that the value of the risk measure is 0.0463 for this case, which is just below 0.05, so within the risk boundary.

Now, suppose a year passes where the portfolio yields a relative return of -1.5%:

Years ago	(-1)	0	1	2	3	4	5
Relative performance: $(r_{\text{t,cp}} - r_{\text{t,b}} - r_{\text{tol}})$	-0.015	-0.04	0	0.01	-0.04	-0.02	0.05
γ^i	0.8^0	0.8^1	0.8^2	0.8^3	0.8^4	0.8^5	0.8^6
Value of γ^i	1	0.8	0.64	0.512	0.410	0.328	0.262
$-\gamma^i(r_{\text{t,cp}} - r_{\text{t,b}} - r_{\text{tol}})$	0.015	0.032	0	-0.00512	0.0164	0.00656	-0.0131

Table 4: Risk measure framework example for dummy returns.

Due to this new year, when we sum the last row of Table 4 as described in Formula 29, we get a risk measure value of 0.05184, just above 0.05. The investor's limit is now reached; a red flag is raised, and the investment is seriously at risk.

3.5 Different experiments

To answer the research questions, we set up a few experiments, all contributing to the research objective. There is a standard experiment which simulates the concentrated portfolio, we construct an experiment that examines the role of the investable universe. Next to that, we have two experiments with investments combining the benchmark and the concentrated portfolio. We have an experiment that looks at the consequences of hiring two asset managers with each their own concentrated portfolio. The last experiment will be about the results of selecting assets based on their beta values.

3.5.1 Standard experiment

This experiment is the most straightforward one. We regard all assets as investable and invest all of the available capital in different concentrated portfolios, varying the number of holdings in each portfolio. With this experiment, we aim to discover what kind of policy and variants yield good results in terms of risk and performance, how many holdings are needed and how the results change when the number of holdings changes.

3.5.2 Investable universe experiment

In reality, pension funds limit the investable universe to only the assets that align with their ethical beliefs. While the asset manager cannot invest in these stocks anymore, these are still used in the benchmark, affecting the performance and risk measure evaluation in our framework. This experiment examines the role of the size of the investable universe. In this experiment, we vary the size of the investable universe while keeping the number of holdings in our portfolio constant. The assets that are in the investable universe are selected entirely randomly, regardless of their nature. We chose the number of holdings in this experiment to be fifty because this is somewhere in the middle of the domain of the size of a portfolio that one could call a concentrated portfolio. This experiment explores the effect of limiting the investable universe and what percentage is needed to get satisfactory results.

3.5.3 Dynamical and statical fraction in benchmark experiment

A pension fund could invest in a concentrated portfolio strategy by investing partly in the concentrated portfolio and the remainder in the benchmark. This way, pension funds can heavily invest in the companies they want without massively underperforming when the concentrated portfolio is underperforming compared to the benchmark. If the concentrated portfolio gets liked by the fund, they might increase the share of the concentrated portfolio as a portion of their total portfolio; therefore, a dynamic fraction is built into this experiment.

This experiment explores the outcomes when a fraction of the capital is invested in concentrated portfolios, while the remaining capital is invested in the benchmark. The initial fraction invested in the benchmark is chosen to be 65 % because some pension funds are considering investing about one-third of their capital in concentrated portfolios. Every year, we reevaluate this fraction according to the proprietary Formula 30.

$$\Delta F_{\text{benchmark}} = \left\lfloor \frac{\text{RiskMeasure}}{\text{RiskMeasure Limit}} \right\rfloor \times \text{stepSize}. \tag{30}$$

A step size of 1 was chosen because that seemed a realistic parameter for modelling the relation between risk scores and the effect on the fraction a pension fund allocates to the benchmark.

To see a graphical outcome of Formula 30, have a look at Figure 2.

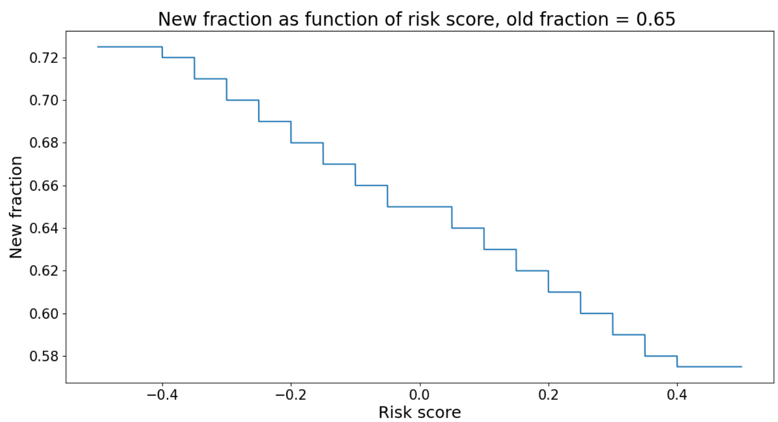

Figure 2: New fraction invested in the benchmark for different risk scores after one timestep when the old fraction was 0.65.

To compare the results of the experiment with a more straight forward way, we do the same experiment as stated above but then with a static fraction, which means that 35% of the initial capital is invested in the concentrated portfolio, and the rest in the benchmark, without any further capital transfers between these two categories. In this way we can separate the influences of dividing the capital into these two classes statically, and the influence of doing this in a dynamic way.

3.5.4 Two managers experiment

If a fund is willing to start a concentrated portfolio but is unsure which manager to employ, it could choose two managers instead of one, both having their own concentrated portfolio. In this experiment, we examine what happens when all assets are managed not by one manager but by two, without any transfer of capital among them.

4 Results

In this section, we evaluate the results of the experiments explained in the previous chapter. We look into the number of holdings, the different policies, the size of the investable universe, investing in the benchmark combination and allocating capital to two managers. We finalize with a scenario where assets are selected based on the beta of the stock instead of a random process.

4.1 Standard experiment

In this subsection, we examine the outcomes of the experiment for all different models and conclude which selection method and model works best, which we will continue to analyze.

4.1.1 Computational price path models

In Figure 3 and Figure 4, we can see the results of the standard experiment of the plain and CAPM models. The results for these two price path models are quite different. The only policy that results relatively well in both risk and yield for both models is the HHI MC chance policy. In general, the MC random variants of the policies result in better risk and yield outcomes, except for the yield of the plain model, which is easily explained by the fact that all stocks have similar behavior and, thus, regardless of the selection method, the yield will be the same. This explanation also applies to the observation that the equal weight strategy has better risk suitability in the plain model than in the CAPM model. In the plain model, all stocks exhibit the same characteristics.

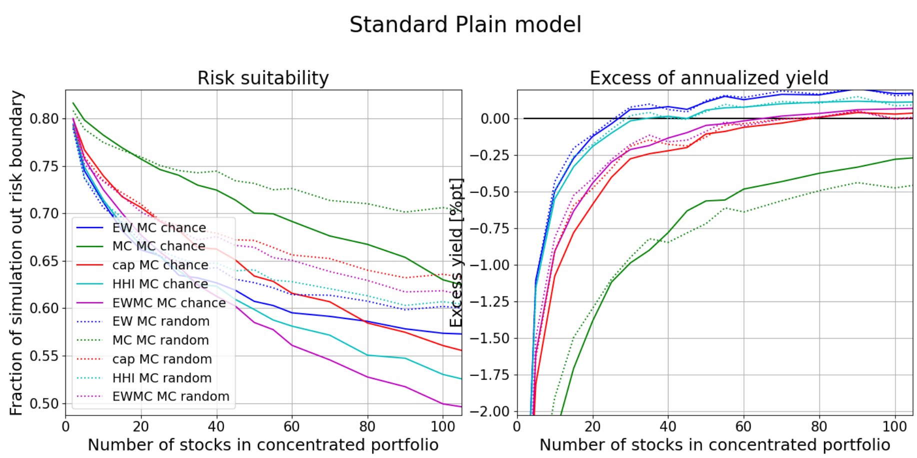

Figure 3: Risk suitability (left) and performance (right) for five to one hundred holdings in a concentrated portfolio using the plain model in the standard experiment.

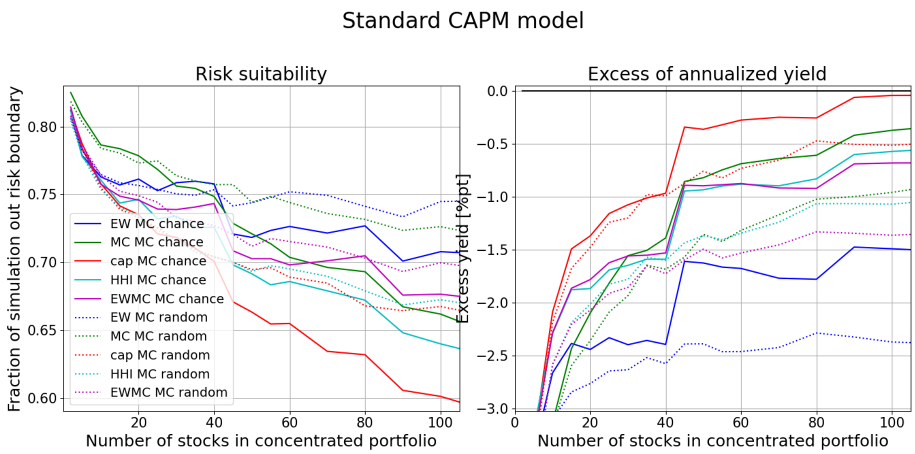

Figure 4: Risk suitability (left) and performance (right) for five to one hundred holdings in a concentrated portfolio using the CAPM model in the standard experiment.

To understand Figure 3, Figure 4 and all other similar figures that are still to come a bit better, below a brief explanation of how to understand these Risk suitability and Excess of annualized yield figures.

Risk suitability

In this figure, the percentage of simulations that do not remain within the limits of the risk boundary (as defined in Formula 29) throughout the simulation (ten years) is shown for different numbers of stocks in the portfolio and various trading strategies. This number should be as low as possible, indicating that the portfolio is well-designed to satisfy investors by adhering to the risk limit.

Excesses of Annualized yield

This plot shows the difference between the benchmark's performance and the concentrated portfolio, calculated using Formula 13. First, the CAGR is calculated for all portfolios in each simulation, then the average CAGR across all simulations is calculated for each portfolio. To convert this average performance to an excess yield, the benchmark's average performance is subtracted from the portfolio's average CAGR. A positive excess yield indicates superior performance of the concentrated portfolio over the benchmark.

The plots display jagged lines from time to time. This is because the simulations were done with 10,000 to 20,000 iterations, which cost around 8 hours of simulation time. Higher computational power would allow for smoother curves and a broader range of different portfolio sizes.

4.1.2 History based models

When comparing the backtesting model (Figure 5) with the historical model (Figure 6), we see quite overlapping results. The backtesting model shows us that if we had invested in the stocks available in the dataset according to the constructed variants and policies, an excess yield on top of the yield of the benchmark of 2 % point per year could have been made for a MC MC chance policy, with only a probability of 20% of getting out of the risk boundary. A similar image emerges from the historical simulation where the cap MC chance policy also gets out second best (both in terms of yield and risk). Also, here, the MC random variants of the policies result in way worse outcomes than their MC chance variants.

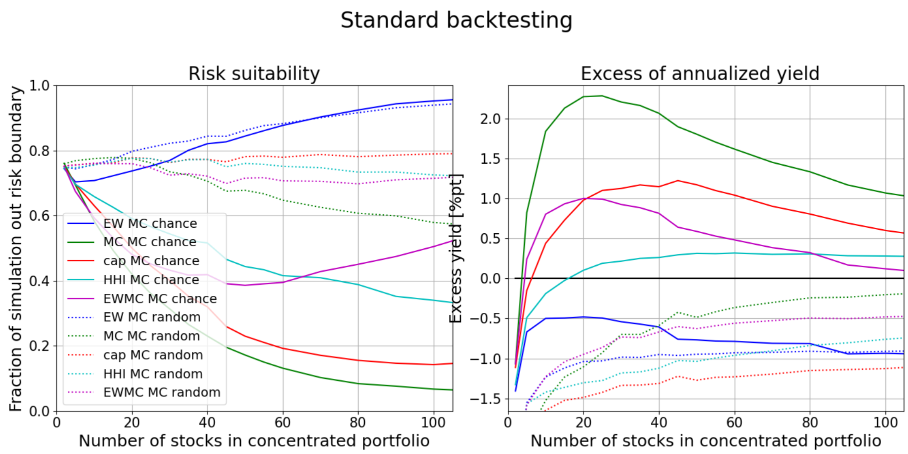

Figure 5: Risk suitability (left) and performance (right) for five to one hundred holdings in a concentrated portfolio using the backtest model in the standard experiment.

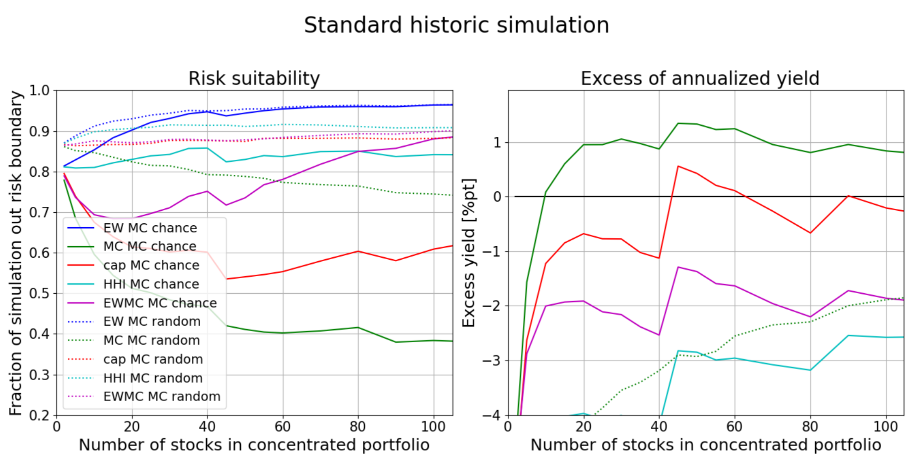

Figure 6: Risk suitability (left) and performance (right) for five to one hundred holdings in a concentrated portfolio using the historic model in the standard experiment.

4.1.3 Combined model

The final model analyzed here is the combined model, which integrates the historic model with the CAPM model using the observed betas and correlations for ten years and thirty years historical perod, both with a simulation time of ten years. In Figure 7 and 8, we see that both the ten-year and 30-year models have as best policy, both in terms of risk and performance, the MC MC chance policy, followed the cap MC chance and the EWMC MC chance policies, which are quite close to each other. Interestingly to note that while in the ten-year combined model, the MC chance variants are superior to their MC random variants, this is not necessarily the case for the thirty years model. This is true for nearly all policies in terms of performance, but not in terms of risk, where the only cap and MC policies have the MC chance variant superior to their MC random variant.

In Figure 7, we can see that the performance reaches an asymptote when around forty holdings are included in the concentrated portfolio, which is backed by the thirty years combined model, the historical model, somewhat by the backtest and plain model and in the CAPM model there is a plateau reached from that number of holdings on. There is a less clear image of the asymptote in the risk domain. In the ten-year combined model, the historical model and the backtest suggest a kind of asymptote that reached around fifty holdings, whereas the other models indicate that it is no asymptote at all and adding more holdings will keep being beneficial.

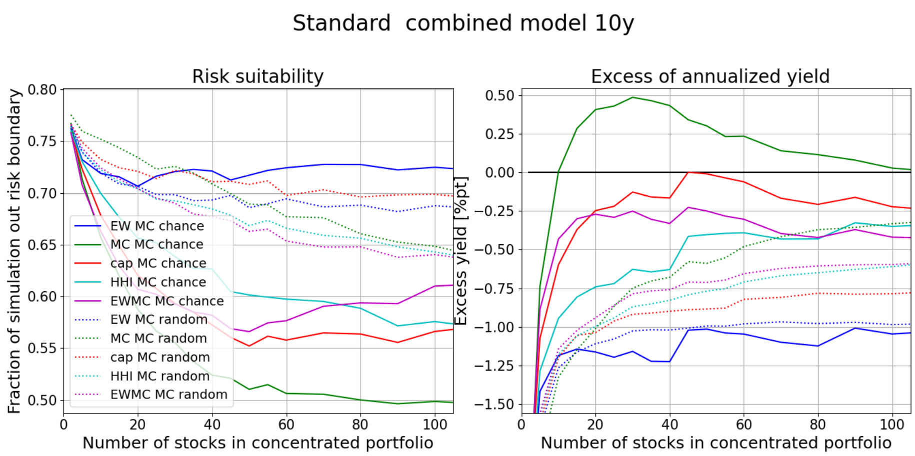

Figure 7: Risk suitability (left) and performance (right) for five to one hundred holdings in a concentrated portfolio using a combined model with 10-year data in the standard experiment.

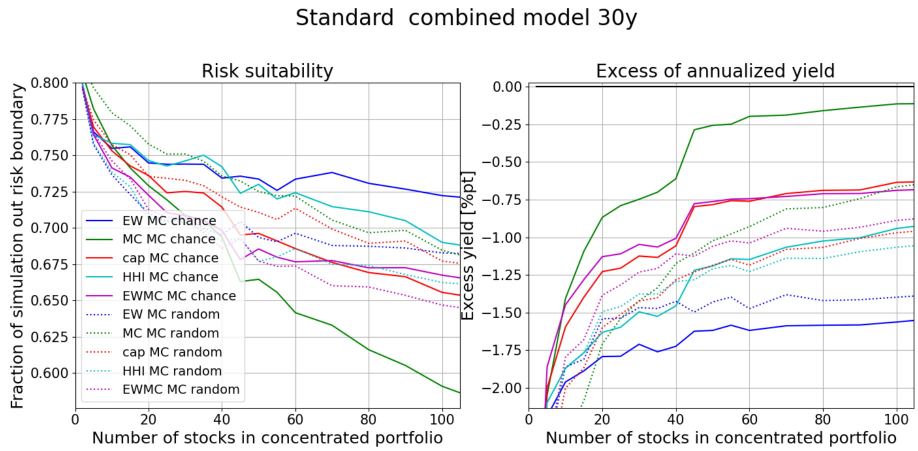

Figure 8: Risk suitability (left) and performance (right) for five to one hundred holdings in a concentrated portfolio using a combined model with 30-year data in the standard experiment.

In most of the models, we see that the MC chance variant has better outcomes than the MC random variant. For the risk, this can easily be explained: since the MC chance variant contains, on average, stocks with a higher market capitalization than the MC random variant, the portfolios of this variant will resemble more the benchmark, where larger stocks have a higher weight. For the performance, this is less evident, it might be because larger stocks perform better, because of the momentum effect.

When we have a close look at the yield of the MC MC chance portoflio for different number of holdings in Figure 7, we can see an interesting peak around thirty holdings. This might be the ideal mix between diversification and holding large big tech stocks which have outperformed the market massively.

To get a further image of the behaviour of the concentrated portfolio, in Figure 9, we can see the relative and absolute drawdown of the portfolios during ten years of simulation. We can see that more than twenty holdings are needed to prevent substantial drawdowns in an absolute and relative sense. We also see that MC chance variants are superior to MC chance variants of policies. We can also register that while the MC MC chance policy is mediocre in the absolute drawdown. Its drawdowns are primarily in line with the benchmark itself because, for the relative drawdown, it is one of the best policies (together with the EWMC MC chance policy).

Using Figure 7, 9 and all the analysis above, we can conclude that the best asset selection variant and best capital distribution policy is the MC MC chance method, which is nearly for all models superior to all the other methods. The right number of assets one should hold in its concentrated portfolio is forty to fifty holdings, for this number the risk nearly reached its assympote while is still an excess yield of around 0.3 % point.

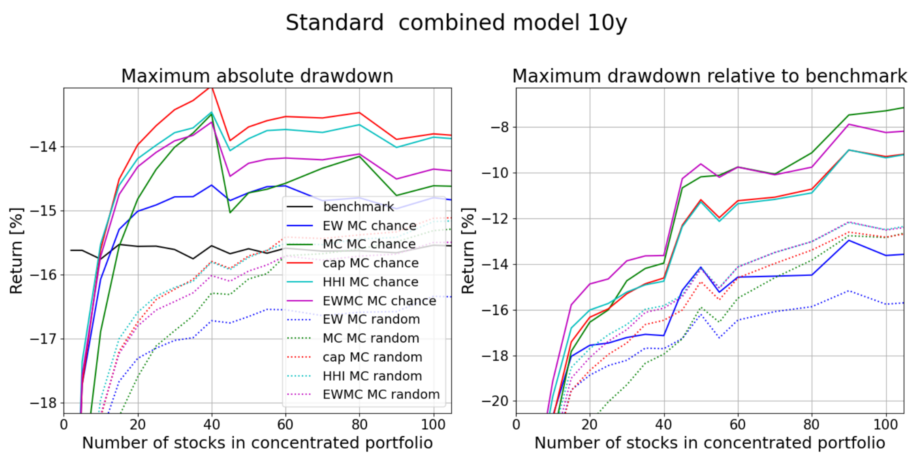

Figure 9: Maximum absoute drawdown (left) and maximum drawdown relative to the index (right) for five to one hundred holdings in a concentrated portfolio using the combined model with 10-year data in the standard experiment.

To understand Figure 9 and all other similar figures that are still to come, below a brief explanation of how to understand these Maximum absolute drawdown and Maximum drawdown relative to benchmark figures.

Maximum absolute drawdown

This figure shows an alternative to our used risk frame and is a commonly used risk measurement. Here, we calculate the portfolio's highest loss in one year (in terms of percentage) at the time of the simulation. We want this number to be as low as possible because a maximum drawdown indicates the downside risk in the worst-case scenario, which should be limited. Ideally, the maximum drawdown of a concentrated portfolio is lower than that of the benchmark, showing a lower downside risk than the benchmark.

Maximum drawdown relative to benchmark

This plot shows the highest underperformance of a concentrated portfolio relative to the benchmark. A high relative drawdown can be shown in newspapers from time to time. Because this is important for a pension fund manager, this can be a metric a fund manager chooses its concentrated portfolio for.

When a concentrated portfolio consists of all five hundred stocks, the policy with the best outcomes is the MC MC chance: It has 0 risk because it mimics the benchmark, but more interestingly, no other policy has a higher yield. On the contrary, this policy performs worst regarding the absolute drawdown.

Since the combined model of ten years is the most complete and advanced model, which is quite in line with the results of the thirty-year model, we take this model as the basis for our further analysis.

4.2 Influence of investable universe

To examine the influence of limiting the set of investable stocks, we have set up an experiment. In Figure 10, we see that the percentage of assets in the investable universe significantly impacts the results for MC chance variants but not for MC random variants. For the best-performing policies in the first category, there is a roughly linear relationship between the size of the investable universe and both the risk suitability and excess yield. For example, when we compare the risk suitability for the MC MC chance policy, the risk drops by around 0.3 (i.e., 30% points less probability of a simulation ending outside the risk boundary). The performance rises by one percentage point for an investable universe of 100% compared to one of 5%. The other models show a very similar outcome, which is shown in Section 7.4.1 of Appendix C. To have an optimal result, all stocks should be in the investable universe because the more stocks available for investing, the better the outcomes are.

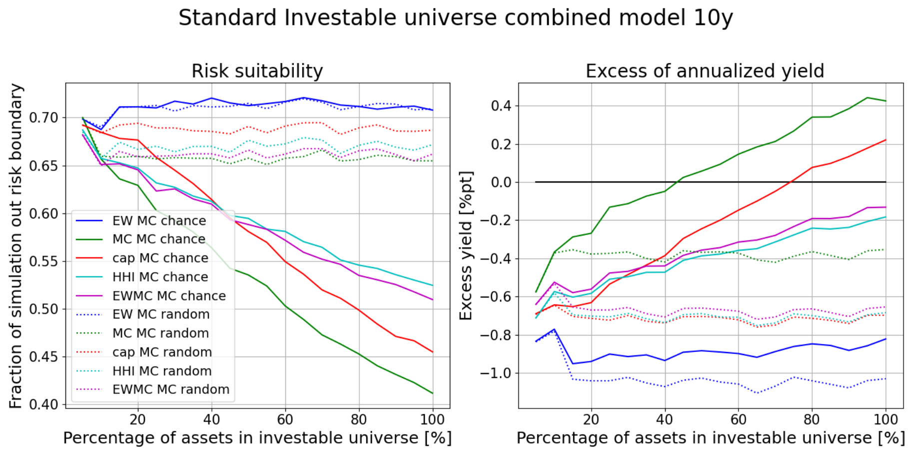

Figure 10: Risk suitability (left) and performance (right) for an investable universe of five to one hundred percent for a concentrated portfolio of fifty stocks using the combined model with ten years of data in the standard experiment.

4.3 Investing in benchmark and concentrated portfolio

An investor or pension fund may be willing to invest in a concentrated portfolio but may find it too bold to invest all the capital in such a portfolio. To address this concern, a fund could allocate only a fraction of the capital to the concentrated portfolio while investing the remaining part in the benchmark. Both the static and dynamic fraction strategies start with 65% in the benchmark, as explained in the method section. Since MC MC chance policies yield the best results, we will focus on this in further analysis.

In Figure 11, we can see that combining the concentrated portfolio with the benchmark decreases the risk suitability from 0.5 to 0.3 for the MC chance variants, whereas from 15 holdings on, the performance is slightly lower (up to 0.25% point). We can also see that investing in a combination with a static distribution is better than doing so on a dynamic basis because the static experiment has better results on all metrics for all holding sizes. These experiments for different models are shown in Section 7.4.2 in the appendix and show similar outcomes. Limiting the investments in the concentrated portfolio drastically in favor of capital allocated to the benchmark, and thus limiting the qualitative advantages (as stated in Section 1.2.3), causes the risk suitability to decrease significantly. However, there is still a 30% probability that the portfolio ends up outside the risk boundary. Conversely, the performance decreases slightly, making investing in a combination less attractive. Whether a pension fund wants to limit the qualitative advantages and accept a slight drop in yield, in exchange for better risk suitability by investing only partly in a concentrated portfolio, is a strategic choice.

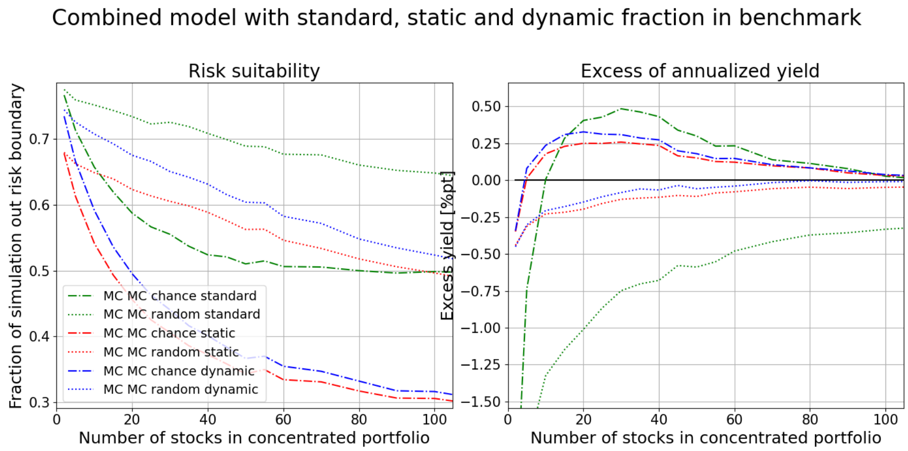

Figure 11: Risk suitability (left) and performance (right) for five to one hundred holdings in a concentrated portfolio using the combined model with ten-year data in the standard, static fraction in benchmark and dynamic fraction in benchmark experiment

4.4 Hiring two managers

When a pension fund wants to invest in a concentrated portfolio, it might want to choose two managers to limit manager-specific risk. In Figure 12, we see the combined results. Compared to the standard situation implementing two managers for the MC random variant, the risk drops to around 0.06, while the expected yield goes up by at least 0.3 % point. For the MC chance variant, the risk drops to around 0.04 while the expected yield goes up by around 0.1 % point. Other models show similar and more extensive outcomes, aligning with the combined model's outcomes, which are viewable in the appendix (Section 7.4.3). There is a clear gain in hiring two managers, especially when one wants around 20 holdings or fewer in the portfolio. The gains are clear but limited, and whether these limited gains justify hiring two managers is a question that requires a more extensive strategic analysis, which will not be done in this research. Part of this strategic consideration would be the extra governance required to control two managers instead of one. The effective concentration will be reduced; thus, the possible qualitative advantages (Section 1.2.3) will will decrease.

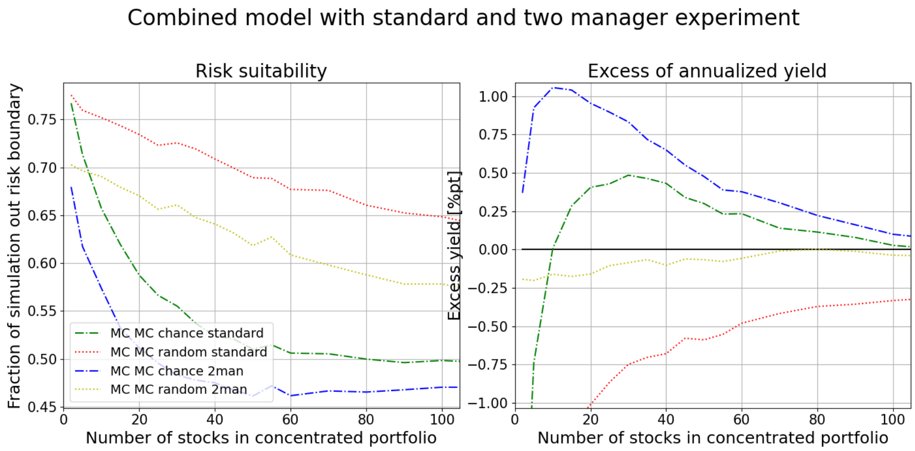

Figure 12: Risk suitability (left) and performance (right) for five to one hundred holdings in a concentrated portfolio using the combined model with ten-year data in the standard, static fraction in benchmark and dynamic fraction in benchmark experiment

4.5 Selecting assets based on betas

In this subsection, to examine the outcomes of the beta variants, we repeat the standard experiment, but select assets based on their betas (as described in the method chapter) to examine the vision of selection assets with a beta around one has as result.

In Figure 13 we see that the beta cor variant of the policies generally results in worse outcomes when compared to the beta variant of the policies. We also see that in this case, regarding the risk, the outcomes of the different policies are pretty close, only differing around 0.04 (4% point) from each other. Additionally, we see that the beta variant is slightly better than the beta cor variant, but they are close to each other in most policies.

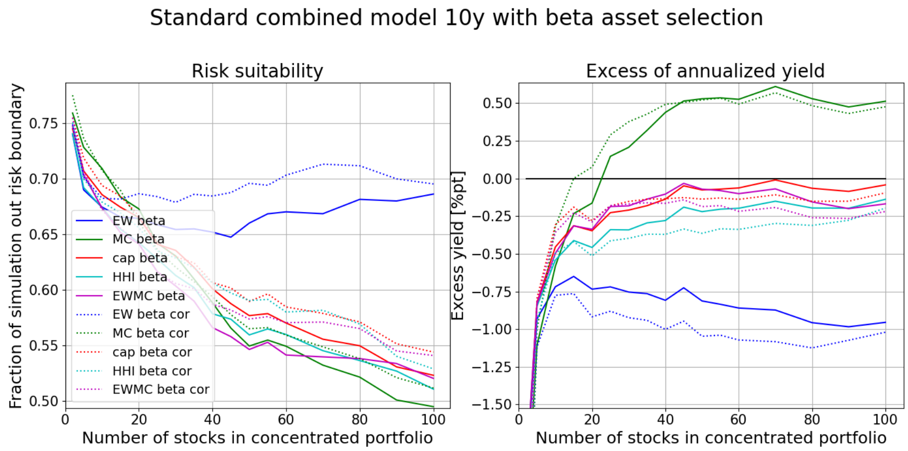

Figure 13: Risk suitability (left) and performance (right) for five to one hundred holdings in a concentrated portfolio using the combined model with ten-year data in the standard experiment with asset selection based on the betas of the stocks.

Comparing the most rewarding policy, the MC policy, across the four different variants (MC chance, MC random, beta, beta cor), we get the outcomes as shown in Figure 14. Regarding risk, these beta policies have worse results than the MC MC chance policy. Regarding performance, from forty holdings and more, the expected yield is higher for the beta variants, with a maximum difference of 0.5 % point when the number of stocks in the concentrated portfolio equals one hundred.

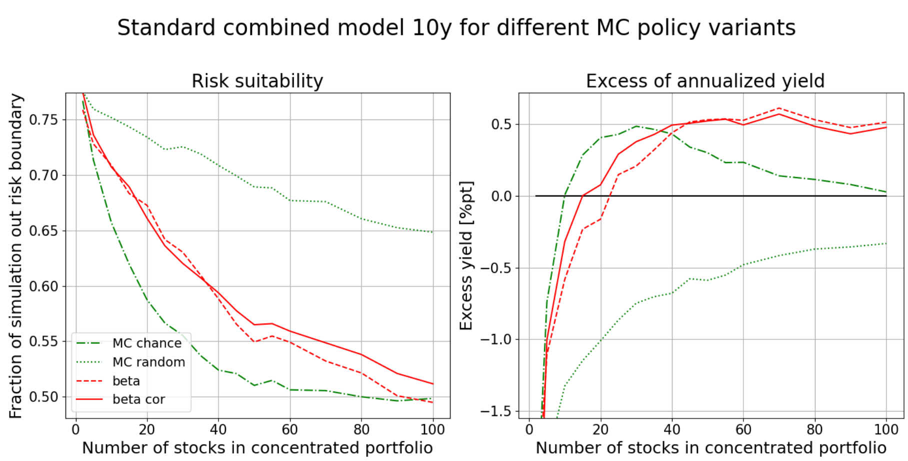

Figure 14: Risk suitability (left) and performance (right) for five to one hundred holdings in a concentrated portfolio using the combined model with ten-year data in the standard experiment with only all the variants of the MC policy.

5 Conclusion & Discussion

We simulated asset price paths using five different models and developed four methods for selecting holdings in concentrated portfolios, along with five strategies for allocating capital among these holdings. To evaluate the portfolios, we used the compound annual growth rate (CAGR) as a performance measure and introduced a custom risk measure: the memory-adjusted cumulative yield relative to the benchmark. We conducted five experiments to test various aspects of a concentrated portfolio approach, leading to the following conclusions. Finally, we discuss the limitations of this research and offer suggestions for future studies.

5.1 Conclusions

In the pension investment context, where asset performance is constantly evaluated against a benchmark, we developed a unique risk measure framework (Formula 29) that accumulates time-adjusted performance relative to the benchmark. This approach allows for underperformance up to a specific threshold, beyond which it is assumed that the investor would withdraw capital and terminate the investment. Through collaboration with pension experts at Cardano, the cumulative memory-adjusted underperformance threshold was set at five percent.

Performance was evaluated using the portfolio's CAGR, which was deemed the most suitable metric for comparing different portfolios in line with pension fund objectives.

We constructed five models to analyze the problem, these were based on:

1. Geometric Brownian Motion
2. Geometric Brownian Motion combined with the Capital Asset Pricing Model (CAPM)
3. Historical backtesting
4. Historical bootstrapping
5. A combined model of historical bootstrapping and correlated Brownian motion with CAPM

The fifth model was the most comprehensive, combining historical data bootstrapping with a computational asset price path model based on correlated Brownian motions and CAPM theory. We examined the effect of time by using stock data from both ten-year and thirty-year periods, which yielded similar results.

The main conclusion of this research is that, under the given assumptions and using the most successful method of constructing and maintaining a concentrated portfolio, it offers an expected excess yield of up to 0.5 percentage points. However, the risk of investors becoming unsatisfied due to underperformance—and consequently terminating the investment—is around 50%, making this portfolio approach unattractive for pension funds to employ.

When employing the concentrated portfolio, the optimal yield is achieved with portfolios consisting of forty to fifty holdings. Beyond that number, additional holdings contribute less to yield, while risk continues to decrease but at a diminishing rate.

We found that selecting assets based on market capitalization resulted in the most favorable in the risk-return trade-off. Allocating capital according to the market capitalization of the selected assets consistently led to superior performance in both risk and return across all history-based models, including the final combined model.

The impact of the investable universe on risk and performance was also significant, with a roughly linear relationship observed between the percentage of assets in the investable universe and both risk suitability and yield. Therefore, limiting the investable universe should be avoided as much as possible under the assumptions of this research.

When pension funds allocate a portion of their capital to a concentrated portfolio while keeping the rest in the benchmark, maintaining a fixed proportion of capital in both asset classes is more effective. Adjusting this proportion based on recent performance tends to increase risk and reduces return. Investing 35% of capital in the concentrated portfolio reduces risk by about 20 percentage points compared to fully investing in the concentrated portfolio. However, for portfolios with fifteen holdings or more, using a market capitalization policy causes the combined investment to underperform the pure concentrated portfolio. Thus, investing in a concentrated portfolio alongside the benchmark is more of a strategic decision than a result-driven choice based on this research.

Employing two managers to oversee investments improves risk suitability by a few percentage points, with a slight performance gain of approximately 0.1 percentage point compared to contracting all capital to one manager. Given this modest benefit, it is unclear if hiring two managers is justified based on this research, although it might still be a strategic choice for some pension funds.

Finally, the research indicates that using a beta-based selection method yields the best expected return for portfolios holding forty or more stocks, although risk suitability is better for portfolios selecting assets based on market capitalization.

5.2 Limitations of this research

Two primary concerns arise when applying this research to real-world practice: the limitations in modeling asset selection skills and the accuracy of the modeled asset prices.

Starting with the latter, while our modeled asset prices—especially in the combined model—may closely approximate future price dynamics, future stock prices remain inherently uncertain. The models rely on historical data to project future trends; therefore, the portfolios based on these projections may not precisely match future outcomes. Additionally, there is a possibility of survivorship bias, causing certain categories of stocks to perform better in the model than they would in reality. We also gave attention by looking to the thirty-year horizon, but this was done quite limited. This results in a limited applicability of this research, because for increasing that a broader set of time frames should be examined.

Regarding asset selection, we assumed four variants: random selection, random selection based on market capitalization, and two beta-based methods. In practice, asset managers claim that selection involves a detailed process considering various metrics, such as cash flow yield, return on equity, and expected earnings growth. Whether this process is genuinely effective or ultimately random is a topic of ongoing debate. However, what matters is that the outcomes from this selection process differ significantly from how we modeled it in this research.

5.3 Future research

The data used in this research included only currently existing stocks, so companies that went bankrupt or merged during the period were underrepresented. Moreover, only annual data on outstanding shares were available, making market capitalization data somewhat unreliable. Future research could obtain more accurate historical data to better understand actual market conditions.

Our analysis did not account for country and sector allocations. Incorporating these factors into the asset selection algorithm might improve outcomes of the experiments.

In practice, asset managers who aim to stay close to a benchmark often use an optimizer to find the combination of stocks that best mimics the benchmark's composition. Applying such optimization techniques could potentially improve the risk suitability metrics sufficiently to justify investing in the concentrated portfolio while staying within our risk limit.

An approach initially considered in this research was to model assets using the Heston model (Heston, 1993), but fitting its parameters proved challenging. The advantage of the Heston model is that it allows for stochastic volatility rather than constant volatility, better reflecting real-world behavior. Integrating the Heston model into the CAPM framework developed in this research could enhance the understanding of correlated asset dynamics.

Concentrated portfolios have a long history, and an analysis of best practices could reveal principles to help funds avoid significant outflows.

In this study, the risk approach was binary: a portfolio either stayed within the risk boundary or it did not and then the portfolio was closed. However, in reality, funds might partially withdraw capital instead of taking an all-or-nothing approach. Future research could analyze fund flow behaviors for concentrated portfolios to develop a more flexible risk measurement framework.

Additionally, a system could be designed to reduce risks when the risk limit is nearly reached, decreasing the likelihood of exceeding the limit and thus avoiding investor dissatisfaction.

Our two-manager experiment assumed no capital transfers between managers. New insights could be gained by developing a method for capital transfer between managers, aligning their incentives with their commercial objectives while diversifying managerial risk.

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7 Appendices

7.1 Appendix A: Elaboration of variance calculations

In this subsection, the various calculations done in Section 2.2 will be elaborated.

7.1.1 Proof equal weight distribution results in the calculated variance

Multiplying the variance vector with the correlation matrix and filling in the weight vector, we obtain the following equation for the portfolio variance:

$$\sigma_{\text{Pew}}^2 = \begin{bmatrix} \frac{1}{n} & \frac{1}{n} & \cdots & \frac{1}{n} \end{bmatrix} \cdot \begin{bmatrix} \sigma_a^2 & \rho_a\sigma_a^2 & \cdots & \rho_a\sigma_a^2 \\ \rho_a\sigma_a^2 & \sigma_a^2 & \cdots & \rho_a\sigma_a^2 \\ \vdots & \vdots & \ddots & \vdots \\ \rho_a\sigma_a^2 & \rho_a\sigma_a^2 & \cdots & \sigma_a^2 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{n} \\ \frac{1}{n} \\ \vdots \\ \frac{1}{n} \end{bmatrix}. \tag{31}$$

First, multiplying the correlation matrix by the weight vector on the right results in:

$$\begin{bmatrix} \sigma_a^2 & \rho_a\sigma_a^2 & \cdots & \rho_a\sigma_a^2 \\ \rho_a\sigma_a^2 & \sigma_a^2 & \cdots & \rho_a\sigma_a^2 \\ \vdots & \vdots & \ddots & \vdots \\ \rho_a\sigma_a^2 & \rho_a\sigma_a^2 & \cdots & \sigma_a^2 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{n} \\ \frac{1}{n} \\ \vdots \\ \frac{1}{n} \end{bmatrix} = \begin{bmatrix} \frac{\sigma_a^2+(n-1)\rho_a\sigma_a^2}{n} \\ \frac{\sigma_a^2+(n-1)\rho_a\sigma_a^2}{n} \\ \vdots \\ \frac{\sigma_a^2+(n-1)\rho_a\sigma_a^2}{n} \end{bmatrix}. \tag{32}$$

Next, multiplying this vector by the weight vector on the left yields:

$$\sigma_{\text{Pew}}^2 = \frac{1}{n} \cdot n \cdot \frac{\sigma_a^2 + (n-1)\rho_a\sigma_a^2}{n} = \frac{\sigma_a^2 + (n-1)\rho_a\sigma_a^2}{n}. \tag{33}$$

Finally, simplifying the expression results in:

$$\sigma_{\text{Pew}}^2 = \rho_a\sigma_a^2 + (1-\rho_a)\frac{\sigma_a^2}{n}. \tag{34}$$

7.1.2 Proof linear distribution used sums up to one

The summation to be solved is given by:

$$S = \sum_{i=1}^{n} \frac{2i}{n(n+1)}. \tag{35}$$

First, observe that the term $\frac{2}{n(n+1)}$ does not depend on the index i, so it can be factored out of the summation:

$$S = \frac{2}{n(n+1)} \sum_{i=1}^{n} i. \tag{36}$$

The summation $\sum_{i=1}^{n} i$ is the sum of the first n natural numbers, which is given by the formula:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}. \tag{37}$$

Substituting this result into the expression for S, we get:

$$S = \frac{2}{n(n+1)} \cdot \frac{n(n+1)}{2}. \tag{38}$$

Next, cancel out the terms $n(n+1)$ in the numerator and denominator:

$$S = 1. \tag{39}$$

Thus, the sum of the given series is equal to 1.

7.1.3 Proof linear weight distribution results in the calculated variance

We want to investigate how the way value is distributed influences the variance. Starting with a linear distribution of weights, $w_i = \frac{2i}{n(n+1)}$, we calculate the portfolio variance as follows:

$$\sigma^2_{\text{Plin}} = \begin{bmatrix} \frac{2}{n(n+1)} & \frac{4}{n(n+1)} & \cdots & \frac{2n}{n(n+1)} \end{bmatrix} \cdot \begin{bmatrix} \sigma_a^2 & \rho_a\sigma_a^2 & \cdots & \rho_a\sigma_a^2 \\ \rho_a\sigma_a^2 & \sigma_a^2 & \cdots & \rho_a\sigma_a^2 \\ \vdots & \vdots & \ddots & \vdots \\ \rho_a\sigma_a^2 & \rho_a\sigma_a^2 & \cdots & \sigma_a^2 \end{bmatrix} \cdot \begin{bmatrix} \frac{2}{n(n+1)} \\ \frac{4}{n(n+1)} \\ \vdots \\ \frac{2n}{n(n+1)} \end{bmatrix}. \tag{40}$$

We can make this equation a bit more abstract by transforming it to:

$$\sigma^2_{\text{Plin}} = \sum_{i=1}^{n}\sum_{j=1}^{n} w_i \Sigma_{ij} w_j. \tag{41}$$

In this case that can be split in two parts:

$$\sigma^2_{\text{Plin}} = \sum_{i=1}^{n}\sum_{j=i}^{n} w_i \Sigma_{ii} w_i + \sum_{i=1}^{n}\sum_{j\neq i}^{n} w_i \Sigma_{ij} w_j. \tag{42}$$

The first part of this equation simplifies to:

$$\sum_{i=1}^{n}\sum_{j=i}^{n} w_i \Sigma_{ii} w_i = \sigma_a^2 \sum_{i=1}^{n} w_i^2. \tag{43}$$

The second part of this equation simplifies to:

$$\sum_{i=1}^{n}\sum_{j\neq i}^{n} w_i \Sigma_{ij} w_j = \rho_a\sigma_a^2 \sum_{i=1}^{n}\sum_{j\neq i}^{n} w_i w_j. \tag{44}$$

Using the square of sum formula:

$$\sum_{i=1}^{n}\sum_{j\neq i}^{n} w_i \Sigma_{ij} w_j = \rho_a\sigma_a^2 \left(\left(\sum_{i=1}^{n} w_i \right)^2 - \sum_{i=1}^{n} w_i^2 \right). \tag{45}$$

Substituting that in Formula 42:

$$\sigma_{\text{Plin}}^2 = \sigma_a^2 \sum_{i=1}^{n} w_i^2 + \rho_a \sigma_a^2 \left(\left(\sum_{i=1}^{n} w_i \right)^2 - \sum_{i=1}^{n} w_i^2 \right). \tag{46}$$

Since $\sum_{i=1}^{n} w_i = 1$, this simplifies to:

$$\sigma_{\text{Plin}}^2 = \rho_a \sigma_a^2 + (1 - \rho_a)\, \sigma_a^2 \sum_{i=1}^{n} w_i^2. \tag{47}$$

Filling in the linear distribution formula:

$$\sigma_{\text{Plin}}^2 = \rho_a \sigma_a^2 + (1 - \rho_a) \sigma_a^2 \frac{2(2n+1)}{3n(n+1)}. \tag{48}$$

7.1.4 Proof exponential distribution used sums up to one

The summation to be solved is given by:

$$S = \sum_{i=1}^{n} \left(\frac{(2^b - 1)2^{bn}}{2^{bn} - 1} \cdot \frac{1}{2^{bi}} \right). \tag{49}$$

First, observe that the term $\frac{(2^b-1)2^{bn}}{2^{bn}-1}$ does not depend on the index i, so it can be factored out of the summation:

$$S = \frac{(2^b - 1)2^{bn}}{2^{bn} - 1} \sum_{i=1}^{n} \frac{1}{2^{bi}}. \tag{50}$$

Next, the remaining summation $\sum_{i=1}^{n} \frac{1}{2^{bi}}$ is a geometric series with the first term $\frac{1}{2^b}$ and the common ratio $\frac{1}{2^b}$. The sum of the first n terms of a geometric series is given by:

$$\sum_{i=1}^{n} r^i = \frac{r(1 - r^n)}{1 - r}. \tag{51}$$

where $r = \frac{1}{2^b}$. Applying this formula to the current series:

$$\sum_{i=1}^{n} \frac{1}{2^{bi}} = \frac{\frac{1}{2^b}\left(1 - \frac{1}{2^{bn}}\right)}{1 - \frac{1}{2^b}}. \tag{52}$$

This expression can be simplified as follows:

$$\sum_{i=1}^{n} \frac{1}{2^{bi}} = \frac{1 - \frac{1}{2^{bn}}}{2^b - 1}. \tag{53}$$

Substituting this result back into the original expression for S, we have:

$$S = \frac{(2^b - 1)2^{bn}}{2^{bn} - 1} \cdot \frac{1 - \frac{1}{2^{bn}}}{2^b - 1}. \tag{54}$$

Next, cancel the common factor $2^b - 1$ from the numerator and denominator:

$$S = \frac{2^{bn}}{2^{bn} - 1} \cdot \left(1 - \frac{1}{2^{bn}}\right). \tag{55}$$

Simplifying the remaining terms:

$$S = \frac{2^{bn}\left(1 - \frac{1}{2^{bn}}\right)}{2^{bn} - 1}. \tag{56}$$

The numerator simplifies as:

$$S = \frac{2^{bn} \cdot \frac{2^{bn}-1}{2^{bn}}}{2^{bn} - 1}. \tag{57}$$

Finally, the terms $2^{bn} - 1$ cancel, leaving:

$$S = 1. \tag{58}$$

Thus, the sum of the given series is equal to 1.

7.1.5 Proof exponential weight distribution results in the calculted variance

We start by applying the weight vector $w_i = \frac{(2^b-1)2^{bn}}{(2^{bn}-1)} \frac{1}{2^{bi}}$ to the portfolio variance expression:

$$\sigma^2_{\text{Pexp}} = \vec{w}^T(\vec{\sigma}(\vec{\sigma}^T\boldsymbol{\rho}))\vec{w} = \vec{w}^T\boldsymbol{\Sigma}\vec{w}. \tag{59}$$

Which resuls in:

$$\sigma^2_{\text{Pexp}} = \begin{bmatrix} \frac{(2^b-1)2^{bn}}{(2^{bn}-1)} \frac{1}{2^b} & \frac{(2^b-1)2^{bn}}{(2^{bn}-1)} \frac{1}{2^{2b}} & \cdots & \frac{(2^b-1)2^{bn}}{(2^{bn}-1)} \frac{1}{2^{nb}} \end{bmatrix} \cdot \begin{bmatrix} \sigma_a^2 & \rho_a\sigma_a^2 & \cdots & \rho_a\sigma_a^2 \\ \rho_a\sigma_a^2 & \sigma_a^2 & \cdots & \rho_a\sigma_a^2 \\ \vdots & \vdots & \ddots & \vdots \\ \rho_a\sigma_a^2 & \rho_a\sigma_a^2 & \cdots & \sigma_a^2 \end{bmatrix} \cdot \begin{bmatrix} \frac{(2^b-1)2^{bn}}{(2^{bn}-1)} \frac{1}{2^b} \\ \frac{(2^b-1)2^{bn}}{(2^{bn}-1)} \frac{1}{2^{2b}} \\ \vdots \\ \frac{(2^b-1)2^{bn}}{(2^{bn}-1)} \frac{1}{2^{nb}} \end{bmatrix}. \tag{60}$$

We can fill in Formula 47 in with the exponential distribution function, which results in:

$$\sigma^2_{\text{Pexp}} = \rho_a\sigma_a^2 + (1 - \rho_a)\sigma_a^2 \frac{(2^b - 1)^2(4^{\text{bn}} - 1)}{(4^b - 1)(2^{\text{bn}} - 1)^2}. \tag{61}$$

7.2 Appendix B: Tickers the 2013 to 2023 stocks

Below are all the symbols of stocks used in the analysis from 2013 to 2023.

1. A
2. AA
3. AAL
4. AAME
5. AAOI
6. AAON
7. AAP
8. AAPL
9. AAT
10. ABBV
11. ABCB
12. ABEO
13. ABG
14. ABIO
15. ABM
16. ABR
17. ABT
18. ACAD
19. ACCO
20. ACGL
21. ACHC
22. ACHV
23. ACIW
24. ACLS
25. ACM
26. ACN
27. ACNB
28. ACRE
29. ACTG
30. ACU
31. ADBE
32. ADC
33. ADI
34. ADM
35. ADMA
36. ADP
37. ADTN
38. ADUS
39. AE
40. AEE
41. AEHR
42. AEIS
43. AEMD
44. AEP
45. AES
46. AFG
47. AFL
48. AGCO
49. AGEN
50. AGIO
51. AGM
52. AGNC
53. AGO
54. AGYS
55. AHH
56. AHT
57. AIG
58. AIN
59. AIR
60. AIRI
61. AIRT
62. AIT
63. AIZ
64. AJG
65. AKAM
66. AKR
67. AL
68. ALB
69. ALCO
70. ALE
71. ALEX
72. ALG
73. ALGN
74. ALGT
75. ALK
76. ALKS
77. ALL
78. ALLE
79. ALNY
80. ALSN
81. ALV
82. ALX
83. AMAT
84. AMC
85. AMCX
86. AMD
87. AME
88. AMED
89. AMG
90. AMGN
91. AMH
92. AMKR
93. AMN
94. AMP
95. AMPE
96. AMRC
97. AMS
98. AMSC
99. AMSF
100. AMSWA
101. AMT
102. AMTX
103. AMWD
104. AMZN
105. AN
106. ANDE
107. ANGI
108. ANGO
109. ANIK
110. ANIP
111. ANIX
112. ANSS
113. AON
114. AOS
115. AOSL
116. AP
117. APA
118. APAM
119. APD
120. APDN
121. APEI
122. APH
123. APPS
124. APT
125. APTV
126. AR
127. ARAY
128. ARC
129. ARCB
130. ARE
131. ARI
132. ARKR
133. ARL
134. ARMK
135. AROC
136. AROW
137. ARR
138. ARTNA
139. ARTW
140. ARW
141. ARWR
142. ASB
143. ASGN
144. ASH
145. ASMB
146. ASPS
147. ASRT
148. ASRV
149. ASTC
150. ASTE
151. ASUR
152. ASYS
153. ATEC
154. ATGE
155. ATI
156. ATLC
157. ATLO
158. ATNI
159. ATNM
160. ATO
161. ATOS
162. ATR
163. ATRC
164. ATRI
165. ATRO
166. ATSG
167. AUBN
168. AUMN
169. AVA
170. AVAV
171. AVB
172. AVD
173. AVGO
174. AVNW
175. AVT
176. AVXL
177. AVY
178. AWI
179. AWK
180. AWR
181. AWRE
182. AX
183. AXDX
184. AXGN
185. AXL
186. AXP
187. AXR
188. AXS
189. AYI
190. B
191. BA
192. BAC

193. BAH
194. BANC
195. BANF
196. BANR
197. BAX
198. BBGI
199. BBSI
200. BC
201. BCBP
202. BCC
203. BCLI
204. BCO
205. BCOV
206. BCPC
207. BCRX
208. BDC
209. BDL
210. BDN
211. BDX
212. BECN
213. BELFA
214. BELFB
215. BEN
216. BERY
217. BFAM
218. BFIN
219. BFS
220. BG
221. BGFV
222. BGS
223. BH
224. BHB
225. BHE
226. BHLB
227. BHR
228. BIIB
229. BIO
230. BIOL
231. BJRI
232. BK
233. BKD
234. BKH
235. BKNG
236. BKTI
237. BKU
238. BLDR
239. BLFS
240. BLK
241. BLKB
242. BLMN
243. BMI
244. BMRC
245. BMRN
246. BMY
247. BOH
248. BOKF
249. BOOM
250. BOTJ
251. BPOP
252. BPTH
253. BR
254. BRC
255. BRID
256. BRKL
257. BRKR
258. BRN
259. BRO
260. BRT
261. BRX
262. BSET
263. BSRR
264. BSX
265. BUSE
266. BWA
267. BWEN
268. BWFG
269. BWXT
270. BXC
271. BXMT
272. BXP
273. BYD
274. BYFC
275. BZH
276. C
277. CAC
278. CACC
279. CACI
280. CADE
281. CAG
282. CAH
283. CAKE
284. CALM
285. CALX
286. CAMP
287. CAPR
288. CAR
289. CASH
290. CASI
291. CASS
292. CASY
293. CAT
294. CATC
295. CATY
296. CB
297. CBAN
298. CBFV
299. CBOE
300. CBRE
301. CBRL
302. CBSH
303. CBT
304. CBU
305. CBZ
306. CCBG
307. CCI
308. CCK
309. CCL
310. CCNE
311. CCO
312. CCOI
313. CCRN
314. CDE
315. CDMO
316. CDNS
317. CDW
318. CDXC
319. CDXS
320. CDZI
321. CE
322. CELH
323. CENT
324. CENTA
325. CENX
326. CERS
327. CEVA
328. CF
329. CFBK
330. CFFI
331. CFFN
332. CFR
333. CGNX
334. CHCI
335. CHCO
336. CHD
337. CHDN
338. CHE
339. CHEF
340. CHGG
341. CHH
342. CHMG
343. CHMI
344. CHRW
345. CHTR
346. CHUY
347. CI
348. CIA
349. CIEN
350. CIM
351. CINF
352. CIVB
353. CIX
354. CIZN
355. CKX
356. CL
357. CLAR
358. CLDT
359. CLDX
360. CLF
361. CLFD
362. CLH
363. CLIR
364. CLNE
365. CLRB
366. CLRO
367. CLW
368. CLX
369. CMA
370. CMC
371. CMCO
372. CMCSA
373. CMCT
374. CME
375. CMG
376. CMI
377. CMP
378. CMRX
379. CMS
380. CMT
381. CMTL
382. CNA
383. CNC
384. CNK
385. CNMD
386. CNO
387. CNOB
388. CNP
389. CNS
390. CNSL
391. CNTY
392. CNX
393. CNXN
394. CODA
395. COF
396. COHN
397. COHR
398. COHU
399. COKE
400. COLB
401. COLM
402. COMM

403. COO
404. COP
405. COR
406. CORT
407. COST
408. COTY
409. CPB
410. CPF
411. CPHC
412. CPIX
413. CPK
414. CPRI
415. CPRT
416. CPRX
417. CPS
418. CPSH
419. CPSS
420. CPT
421. CRAI
422. CRI
423. CRIS
424. CRMD
425. CRMT
426. CROX
427. CRS
428. CRUS
429. CRVL
430. CRWS
431. CSCO
432. CSGP
433. CSGS
434. CSL
435. CSPI
436. CSV
437. CSX
438. CTAS
439. CTBI
440. CTHR
441. CTO
442. CTS
443. CTSH
444. CTSO
445. CUBE
446. CUBI
447. CULP
448. CUTR
449. CUZ
450. CVBF
451. CVCO
452. CVGI
453. CVGW
454. CVI
455. CVLY
456. CVM
457. CVR
458. CVS
459. CVU
460. CVV
461. CVX
462. CW
463. CWBC
464. CWST
465. CWT
466. CXW
467. CYAN
468. CYCC
469. CYH
470. CYRX
471. CYTK
472. CZNC
473. CZWI
474. D
475. DAIO
476. DAKT
477. DAL
478. DAN
479. DAR
480. DCI
481. DCO
482. DCOM
483. DD
484. DDD
485. DE
486. DEI
487. DENN
488. DFS
489. DGICA
490. DGII
491. DGLY
492. DGX
493. DHI
494. DHIL
495. DHR
496. DHX
497. DIN
498. DIOD
499. DIS
500. DIT
501. DJCO
502. DK
503. DLA
504. DLB
505. DLHC
506. DLR
507. DLX
508. DMRC
509. DOC
510. DORM
511. DOV
512. DPZ
513. DRH
514. DRI
515. DRQ
516. DRRX
517. DSS
518. DTE
519. DUK
520. DVA
521. DVAX
522. DVN
523. DWSN
524. DX
525. DXC
526. DXCM
527. DXPE
528. DXYN
529. EA
530. EARN
531. EAT
532. EBAY
533. EBMT
534. EBS
535. EBTC
536. ECL
537. ECPG
538. ED
539. EDUC
540. EEFT
541. EFC
542. EFOI
543. EFSC
544. EFX
545. EGAN
546. EGBN
547. EGP
548. EGY
549. EHC
550. EHTH
551. EIG
552. EIX
553. EL
554. ELMD
555. ELS
556. ELSE
557. EME
558. EMKR
559. EML
560. EMN
561. EMR
562. ENG
563. ENPH
564. ENS
565. ENSG
566. ENSV
567. ENTA
568. ENTG
569. ENV
570. ENZ
571. EOG
572. EPAM
573. EPC
574. EPM
575. EPR
576. EQC
577. EQIX
578. EQR
579. EQT
580. ERIE
581. ERII
582. ES
583. ESCA
584. ESE
585. ESGR
586. ESNT
587. ESP
588. ESPR
589. ESRT
590. ESS
591. ESSA
592. ETN
593. ETR
594. EVBN
595. EVC
596. EVI
597. EVOK
598. EVR
599. EVRG
600. EVRI
601. EVTC
602. EW
603. EWBC
604. EXAS
605. EXC
606. EXEL
607. EXLS
608. EXP
609. EXPD
610. EXPE
611. EXPO
612. EXR

613. EXTR
614. EYPT
615. EZPW
616. F
617. FAF
618. FANG
619. FARM
620. FARO
621. FAST
622. FATE
623. FBIO
624. FBIZ
625. FBMS
626. FBNC
627. FBP
628. FC
629. FCAP
630. FCBC
631. FCCO
632. FCEL
633. FCF
634. FCFS
635. FCN
636. FCNCA
637. FCX
638. FDBC
639. FDP
640. FDS
641. FDX
642. FE
643. FEIM
644. FELE
645. FET
646. FF
647. FFBC
648. FFIC
649. FFIN
650. FFIV
651. FFNW
652. FGBI
653. FHN
654. FI
655. FIBK
656. FICO
657. FIS
658. FISI
659. FITB
660. FIX
661. FIZZ
662. FLIC
663. FLL
664. FLNT
665. FLO
666. FLR
667. FLS
668. FLWS
669. FLXS
670. FMAO
671. FMBH
672. FMC
673. FMNB
674. FN
675. FNB
676. FNCB
677. FNF
678. FNLC
679. FOLD
680. FONR
681. FOR
682. FORD
683. FORM
684. FORR
685. FOSL
686. FOXF
687. FR
688. FRD
689. FRME
690. FRPH
691. FRT
692. FSBW
693. FSFG
694. FSLR
695. FSP
696. FSS
697. FSTR
698. FTEK
699. FTI
700. FTK
701. FTNT
702. FUL
703. FULT
704. FUNC
705. FUSB
706. FWONA
707. FWRD
708. G
709. GABC
710. GAIA
711. GALT
712. GATX
713. GBCI
714. GBLI
715. GBX
716. GCBC
717. GD
718. GDEN
719. GDOT
720. GE
721. GEF
722. GEN
723. GENC
724. GEO
725. GEOS
726. GERN
727. GEVO
728. GFF
729. GGG
730. GHC
731. GHM
732. GIFI
733. GILD
734. GIS
735. GLBZ
736. GLDD
737. GLPI
738. GLRE
739. GLT
740. GLW
741. GM
742. GMED
743. GNE
744. GNRC
745. GNTX
746. GNW
747. GOGO
748. GOOD
749. GOOG
750. GOOGL
751. GORO
752. GPC
753. GPI
754. GPK
755. GPN
756. GPRE
757. GRBK
758. GRC
759. GRMN
760. GROW
761. GRPN
762. GS
763. GSAT
764. GSBC
765. GT
766. GTIM
767. GTLS
768. GTN
769. GTY
770. GVA
771. GVP
772. GWRE
773. GWW
774. H
775. HA
776. HAE
777. HAFC
778. HAIN
779. HAL
780. HALO
781. HAS
782. HASI
783. HAYN
784. HBAN
785. HBCP
786. HBI
787. HBIO
788. HCA
789. HCI
790. HCKT
791. HCSG
792. HDSN
793. HE
794. HEAR
795. HEES
796. HEI
797. HES
798. HFBL
799. HFWA
800. HHS
801. HI
802. HIFS
803. HIG
804. HII
805. HIW
806. HL
807. HLF
808. HLIT
809. HLT
810. HLX
811. HMN
812. HMNF
813. HMST
814. HNI
815. HNRG
816. HOG
817. HOLX
818. HOMB
819. HON
820. HOPE
821. HOV
822. HP

823. HPP
824. HPQ
825. HR
826. HRB
827. HRI
828. HRL
829. HROW
830. HRTX
831. HSIC
832. HSII
833. HSON
834. HST
835. HSTM
836. HSY
837. HTBI
838. HTBK
839. HTH
840. HTLD
841. HTLF
842. HUBB
843. HUBG
844. HUM
845. HUN
846. HURC
847. HURN
848. HUSA
849. HVT
850. HWBK
851. HWC
852. HWKN
853. HXL
854. HY
855. HZO
856. IART
857. IBIO
858. IBKR
859. IBM
860. IBOC
861. IBTX
862. ICAD
863. ICCC
864. ICE
865. ICFI
866. ICUI
867. IDA
868. IDCC
869. IDN
870. IDT
871. IDXG
872. IDXX
873. IESC
874. IEX
875. IFF
876. III
877. IIIN
878. ILMN
879. IMKTA
880. IMMR
881. INBK
882. INCY
883. INDB
884. INFN
885. INFU
886. INGR
887. INN
888. INO
889. INOD
890. INSG
891. INSM
892. INTC
893. INTG
894. INTT
895. INTU
896. INUV
897. INVA
898. INVE
899. IONS
900. IOR
901. IOSP
902. IOVA
903. IP
904. IPAR
905. IPG
906. IPGP
907. IPI
908. IPWR
909. IQV
910. IRBT
911. IRDM
912. IRIX
913. IRM
914. IROQ
915. IRT
916. IRWD
917. ISDR
918. ISRG
919. ISSC
920. IT
921. ITGR
922. ITI
923. ITIC
924. ITRI
925. ITT
926. ITW
927. IVAC
928. IVR
929. IVZ
930. JACK
931. JAKK
932. JAZZ
933. JBHT
934. JBL
935. JBLU
936. JBSS
937. JBT
938. JCI
939. JCTCF
940. JEF
941. JJSF
942. JKHY
943. JLL
944. JNJ
945. JNPR
946. JOB
947. JOE
948. JOUT
949. JPM
950. JVA
951. K
952. KAI
953. KALU
954. KAR
955. KBH
956. KDP
957. KELYA
958. KEQU
959. KEX
960. KEY
961. KFFB
962. KFRC
963. KFY
964. KIM
965. KINS
966. KKR
967. KLAC
968. KLIC
969. KMB
970. KMI
971. KMPR
972. KMT
973. KNX
974. KO
975. KOP
976. KOPN
977. KOS
978. KPTI
979. KRC
980. KRG
981. KRNY
982. KRO
983. KTCC
984. KTOS
985. KVHI
986. KW
987. KWR
988. L
989. LAD
990. LAMR
991. LANC
992. LAND
993. LARK
994. LAZ
995. LBTYA
996. LBTYK
997. LCII
998. LCNB
999. LCUT
1000. LDOS
1001. LECO
1002. LEE
1003. LEG
1004. LEN
1005. LEU
1006. LFUS
1007. LFVN
1008. LGIH
1009. LGL
1010. LGND
1011. LH
1012. LII
1013. LIN
1014. LINC
1015. LIND
1016. LIQT
1017. LIVE
1018. LKFN
1019. LKQ
1020. LL
1021. LLY
1022. LMAT
1023. LMNR
1024. LMT
1025. LNC
1026. LNG
1027. LNN
1028. LNT
1029. LOAN
1030. LODE
1031. LOPE
1032. LPCN

1033. LPLA
1034. LPSN
1035. LPTH
1036. LPX
1037. LQDT
1038. LRCX
1039. LRN
1040. LSBK
1041. LSCC
1042. LSTR
1043. LTBR
1044. LTC
1045. LTRX
1046. LUV
1047. LVS
1048. LWAY
1049. LXP
1050. LXRX
1051. LXU
1052. LYB
1053. LYTS
1054. LYV
1055. LZB
1056. MA
1057. MAA
1058. MAC
1059. MACK
1060. MAN
1061. MANH
1062. MAR
1063. MARA
1064. MARK
1065. MAS
1066. MASI
1067. MAT
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1069. MATX
1070. MAYS
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1072. MBI
1073. MBOT
1074. MBWM
1075. MCBC
1076. MCD
1077. MCHP
1078. MCHX
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1081. MCS
1082. MCY
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1084. MDGL
1085. MDLZ
1086. MDT
1087. MDU
1088. MED
1089. MEI
1090. MEIP
1091. MERC
1092. MET
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1094. MG
1095. MGEE
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1099. MGRC
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1101. MHH
1102. MHK
1103. MHLD
1104. MHO
1105. MIDD
1106. MITK
1107. MITT
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1109. MKL
1110. MKSI
1111. MKTX
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1113. MLI
1114. MLM
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1116. MLR
1117. MLSS
1118. MMC
1119. MMI
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1122. MMSI
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1143. MRIN
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1148. MS
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1151. MSEX
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1159. MTD
1160. MTDR
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1162. MTEX
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1165. MTN
1166. MTRN
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1168. MTSI
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1170. MTX
1171. MTZ
1172. MU
1173. MUR
1174. MUSA
1175. MUX
1176. MVBF
1177. MVIS
1178. MWA
1179. MXL
1180. MYE
1181. MYGN
1182. MYRG
1183. NAII
1184. NATH
1185. NATR
1186. NBHC
1187. NBIX
1188. NBN
1189. NBR
1190. NBTB
1191. NBY
1192. NC
1193. NCLH
1194. NCMI
1195. NDAQ
1196. NDLS
1197. NDSN
1198. NEE
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1201. NEOG
1202. NEON
1203. NEU
1204. NFBK
1205. NFG
1206. NFLX
1207. NGS
1208. NGVC
1209. NHC
1210. NHI
1211. NHTC
1212. NI
1213. NICK
1214. NJR
1215. NKE
1216. NKSH
1217. NKTR
1218. NL
1219. NLY
1220. NMIH
1221. NNBR
1222. NNI
1223. NNN
1224. NNVC
1225. NOC
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1227. NOV
1228. NOVT
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1230. NPK
1231. NPO
1232. NR
1233. NRC
1234. NRG
1235. NRIM
1236. NSC
1237. NSIT
1238. NSP
1239. NSSC
1240. NTAP
1241. NTGR
1242. NTIC

1243. NTIP
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1245. NTWK
1246. NUE
1247. NURO
1248. NUS
1249. NVAX
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1251. NVFY
1252. NVR
1253. NWBI
1254. NWE
1255. NWFL
1256. NWL
1257. NWLI
1258. NWN
1259. NWPX
1260. NWS
1261. NWSA
1262. NX
1263. NXST
1264. NYCB
1265. NYMT
1266. NYT
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1304. ORC
1305. ORCL
1306. ORI
1307. ORLY
1308. ORN
1309. ORRF
1310. OSBC
1311. OSIS
1312. OSK
1313. OSPN
1314. OSUR
1315. OTTR
1316. OVBC
1317. OVLY
1318. OXY
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1324. PAR
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1327. PAYX
1328. PB
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1330. PBHC
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1376. PLUS
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1384. PNM
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1390. POST
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1407. PRO
1408. PROV
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1413. PSA
1414. PSMT
1415. PSX
1416. PTC
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1421. PW
1422. PWOD
1423. PWR
1424. PXLW
1425. PZZA
1426. QCOM
1427. QCRH
1428. QDEL
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1433. QUAD
1434. QUIK
1435. R
1436. RAIL
1437. RAMP
1438. RBBN
1439. RBCAA
1440. RBCN
1441. RC
1442. RDI
1443. RDN
1444. RDNT
1445. RDUS
1446. REFR
1447. REG
1448. REGN
1449. REI
1450. RELL
1451. RES
1452. RF

1453. RFIL
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1470. RLGT
1471. RLI
1472. RLJ
1473. RM
1474. RMAX
1475. RMBS
1476. RMCF
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1478. RMTI
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1481. RNST
1482. ROCK
1483. ROG
1484. ROIC
1485. ROK
1486. ROL
1487. ROP
1488. RPM
1489. RRC
1490. RRGB
1491. RS
1492. RSG
1493. RUSHA
1494. RUSHB
1495. RVP
1496. RVSB
1497. RWT
1498. RYN
1499. SAFT
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1501. SAIA
1502. SAM
1503. SAMG
1504. SANM
1505. SANW
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1507. SATS
1508. SAVE
1509. SBAC
1510. SBCF
1511. SBFG
1512. SBGI
1513. SBH
1514. SBRA
1515. SBSI
1516. SBUX
1517. SCHL
1518. SCHW
1519. SCI
1520. SCL
1521. SCOR
1522. SCSC
1523. SCX
1524. SEAC
1525. SEB
1526. SEE
1527. SEIC
1528. SEM
1529. SENEA
1530. SENEB
1531. SF
1532. SFBC
1533. SFM
1534. SFNC
1535. SFST
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1539. SGMO
1540. SGRP
1541. SHBI
1542. SHEN
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1544. SHOO
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1548. SIGI
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1550. SITC
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1552. SJM
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1554. SKT
1555. SKX
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1562. SLGN
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1574. SNCR
1575. SNFCA
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1582. SON
1583. SPB
1584. SPG
1585. SPGI
1586. SPOK
1587. SPR
1588. SPSC
1589. SPTN
1590. SPWR
1591. SPXC
1592. SR
1593. SRCE
1594. SRCL
1595. SRDX
1596. SRE
1597. SRI
1598. SRPT
1599. SSB
1600. SSBI
1601. SSD
1602. SSKN
1603. SSNC
1604. SSP
1605. SSTK
1606. ST
1607. STAA
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1612. STE
1613. STLD
1614. STRA
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1617. STRT
1618. STT
1619. STWD
1620. STX
1621. SUI
1622. SUP
1623. SUPN
1624. SVT
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1626. SWKS
1627. SWN
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1631. SXT
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1633. SYK
1634. SYNA
1635. SYPR
1636. SYY
1637. T
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1639. TAIT
1640. TAP
1641. TAYD
1642. TBBK
1643. TBI
1644. TBNK
1645. TCBI
1646. TCBK
1647. TCI
1648. TDC
1649. TDG
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1652. TDY
1653. TECH
1654. TEL
1655. TELL
1656. TENX
1657. TER
1658. TEX
1659. TFSL
1660. TFX
1661. TG
1662. TGI

1663. TGLS
1664. TGNA
1665. TGTX
1666. THC
1667. THFF
1668. THG
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1670. THR
1671. THRM
1672. THS
1673. TILE
1674. TIPT
1675. TISI
1676. TKR
1677. TLF
1678. TMHC
1679. TMO
1680. TMP
1681. TMUS
1682. TNC
1683. TNDM
1684. TOL
1685. TOWN
1686. TPC
1687. TPH
1688. TPR
1689. TPX
1690. TR
1691. TRC
1692. TREE
1693. TREX
1694. TRGP
1695. TRIP
1696. TRMB
1697. TRMK
1698. TRN
1699. TRNO
1700. TRNS
1701. TROW
1702. TROX
1703. TRS
1704. TRST
1705. TRT
1706. TRV
1707. TSBK
1708. TSCO
1709. TSLA
1710. TSN
1711. TTC
1712. TTEC
1713. TTEK
1714. TTGT
1715. TTI
1716. TTMI
1717. TTNP
1718. TWI
1719. TWIN
1720. TWO
1721. TXMD
1722. TXN
1723. TXRH
1724. TXT
1725. TYL
1726. TZOO
1727. UAL
1728. UAMY
1729. UBCP
1730. UBFO
1731. UBOH
1732. UBSI
1733. UCBI
1734. UCTT
1735. UDR
1736. UEC
1737. UEIC
1738. UFCS
1739. UFI
1740. UFPI
1741. UFPT
1742. UG
1743. UGI
1744. UHAL
1745. UHS
1746. UHT
1747. UIS
1748. ULBI
1749. ULH
1750. UMBF
1751. UMH
1752. UNB
1753. UNF
1754. UNFI
1755. UNH
1756. UNM
1757. UNP
1758. UNTY
1759. UPS
1760. URI
1761. USAP
1762. USB
1763. USEG
1764. USIO
1765. USLM
1766. USM
1767. USNA
1768. USPH
1769. UTHR
1770. UTI
1771. UTL
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1773. UUU
1774. UVE
1775. UVSP
1776. UVV
1777. V
1778. VAC
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1781. VBIV
1782. VC
1783. VCEL
1784. VCYT
1785. VECO
1786. VERU
1787. VGR
1788. VHC
1789. VIAV
1790. VICR
1791. VLGEA
1792. VLO
1793. VLY
1794. VMC
1795. VMI
1796. VNDA
1797. VNO
1798. VNRX
1799. VOXX
1800. VOYA
1801. VPG
1802. VRSK
1803. VRSN
1804. VRTS
1805. VRTX
1806. VSAT
1807. VSEC
1808. VSH
1809. VSTM
1810. VTNR
1811. VTR
1812. VUZI
1813. VVI
1814. VXRT
1815. VZ
1816. WAB
1817. WABC
1818. WAFD
1819. WAL
1820. WASH
1821. WAT
1822. WBA
1823. WBS
1824. WCC
1825. WD
1826. WDC
1827. WDFC
1828. WEC
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1831. WERN
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1834. WFC
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1864. WTM
1865. WTS
1866. WU
1867. WVFC
1868. WVVI
1869. WW
1870. WWR
1871. WWW
1872. WY

1873. WYNN
1874. WYY
1875. X
1876. XEL
1877. XELB
1878. XNCR
1879. XOM
1880. XOMA
1881. XPL
1882. XPO
1883. XRAY
1884. XRX
1885. XTNT
1886. XXII
1887. XYL
1888. YELP
1889. YORW
1890. YTEN
1891. YUM
1892. ZBH
1893. ZBRA
1894. ZEUS
1895. ZG
1896. ZION
1897. ZTS
1898. ZYXI

7.3 Appendix C: Fitting parameters for asset price path model

To determine the essential parameters for our plain model and our CAPM model, we try to mimic the behaviour of the stocks of our dataset.

7.3.1 Parameters for both models

Distribution market capitalizations

In Figure 15, the market capitalizations at the end of 2013 are shown according to our collected data together with a fit. This fit fits the histogram very well.

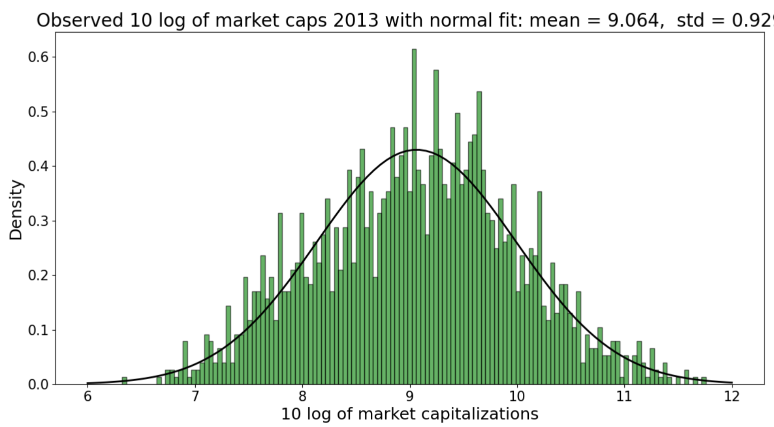

Figure 15: Histogram of the market capitalizations on the log scale of all assets on December 31, 2013, with fit used in the model.

Correlation

In Figure 16, a histogram of all pairs of stocks is shown; from this, we conclude that the correlation needed in our model is 0.288.

7.3.2 Parameters exclusively used in plain model

Return

Figure 17 shows a histogram of all quarterly returns of all stocks in the data set. From this, we conclude that the $\mu_i = 3.365\%$ on a quarterly basis.

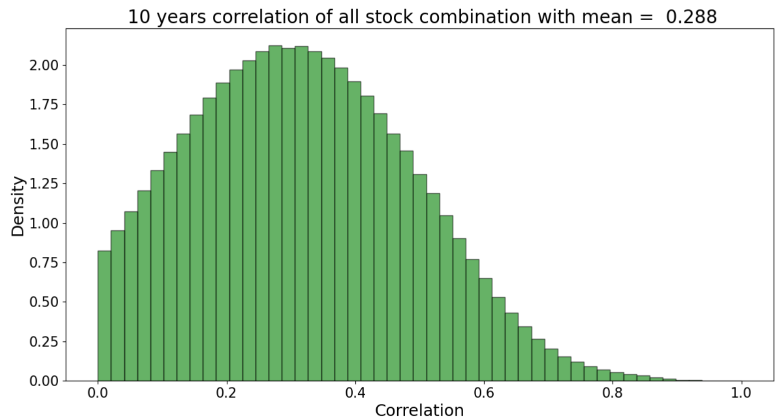

Figure 16: Histogram of all the correlation combinations of all assets

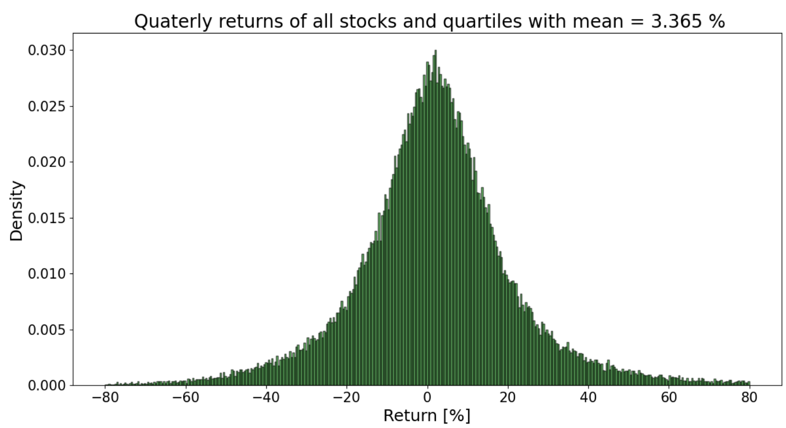

Figure 17: Histogram of all quarterly returns of all assets with fit used in the model.

Volatility

In Figure 18, a histogram of the volatilities of all stocks is shown. The volatility is calculated by calculating the standard deviation of the forty returns of a stock of our data set. We conclude that the volatility σ_i in our model should be 21.668 % on a yearly basis.

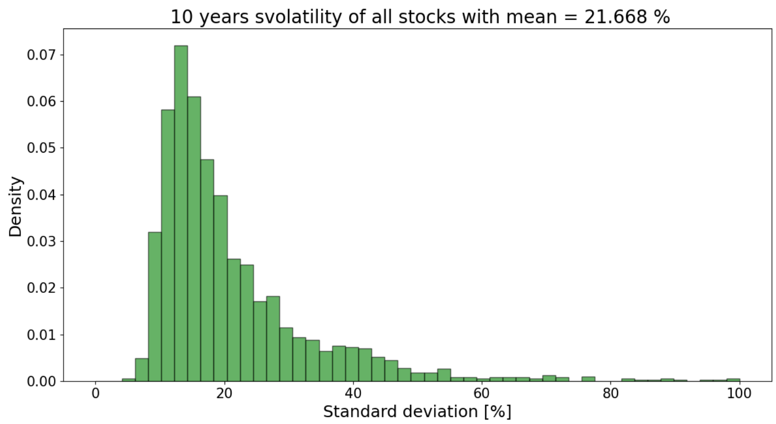

Figure 18: Histogram of the volatility of all assets with fit used in the model.

7.3.3 Parameters exclusively used in CAPM model

Risk free rate

Since we are dealing with American dollar stocks, we choose the 5-year Treasury yield for the risk-free rate. The behaviour of this yield can be seen in Figure 19. The average yield over this period is 1.98%, so we take that as our risk-free rate for the CAPM model.

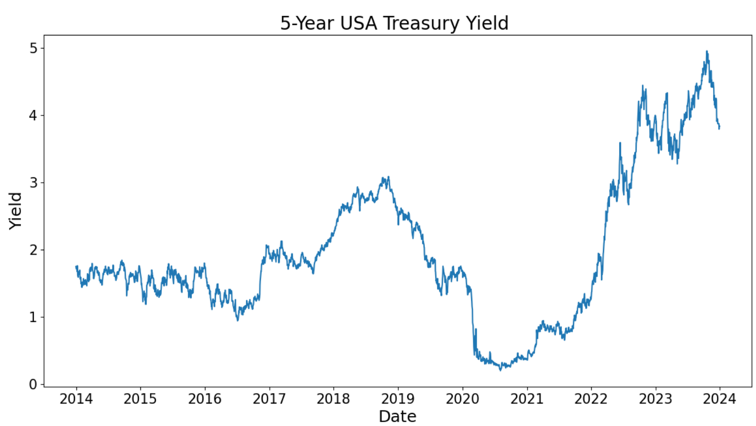

Figure 19: Risk free rate for CAPM model based on five-year USA Treasury yield.

Beta distribution

From our data, the betas are calculated in the following way: First, for every quarter, the quartelized risk-free rate is calculated. The market returns are calculated by taking the returns of all

stocks for a specific quarter and giving every return a weight corresponding to the fraction of its market capitalization compared to the market capitalizations of all stocks the market returns are calculated. Subtracting the risk-free rate from the market returns gives us the equity risk premium, and that is calculated for every quartile.
Finally, a linear fit is made by taking the observed returns of a stock subtracted by the risk-free rate for all quartiles and comparing that to the market equity premium of all quartiles. In Figure 20, one can see a histogram of all fitted betas with a fit through all fitted betas, which we can use in our model.

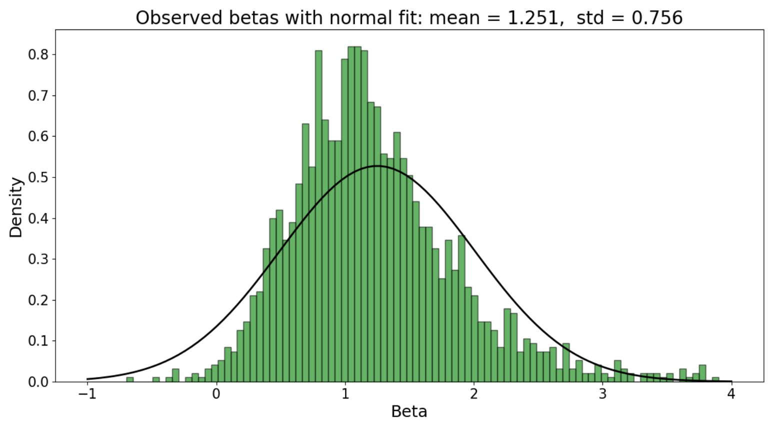

Figure 20: Histogram of the betas of all assets with fit used in the model.

Market risk premium

The previous section described how the market risk premium is calculated. We need a value to put this in our CAPM model. Since the average quarterly market risk premium observed was 2.63, we take that value for our model.

Ideosycratic volatility

As the last parameter, we need to calculate the idiosyncratic volatilities of all stocks and try to fit them.

We calculate the observed idiosyncratic volatilities by rearranging Formula 18:

$$\sigma_{\text{id}} = \sqrt{\sigma_i^2 - (\beta_i \sigma_m)^2}. \tag{62}$$

Volatilities σ_m and σ_i are calculated by calculating the standard deviation of the market returns and the individual stock returns, respectively.
In Figure 21, the idiosyncratic volatilities are plotted in a histogram, together with a fit for our CAPM model.

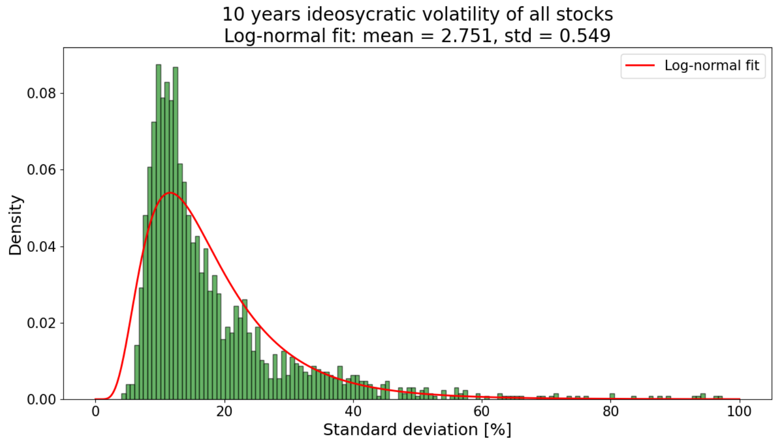

Figure 21: Histogram of the idiosyncratic volatility of all assets with fit used in the model.

Market volatility

Since we calculated the benchmark's quarterly returns minus the risk-free rate, we calculated the standard deviation of this value, which is the market's volatility, which is 7.692% on a quarterly basis.

7.4 Appendix D: More results

7.4.1 Investable universe

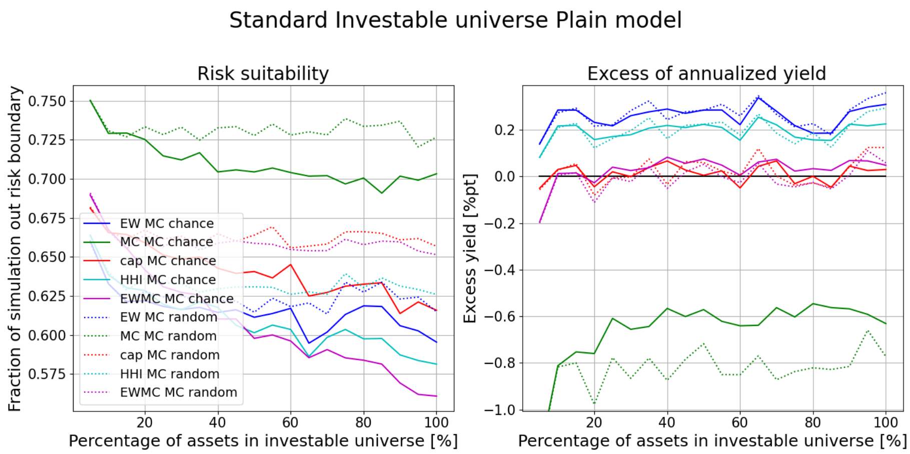

Figure 22: Risk suitability (left) and performance (right) for an investable universe of five to one hundred percent for a concentrated portfolio of fifty stocks using the plain model in the standard experiment.

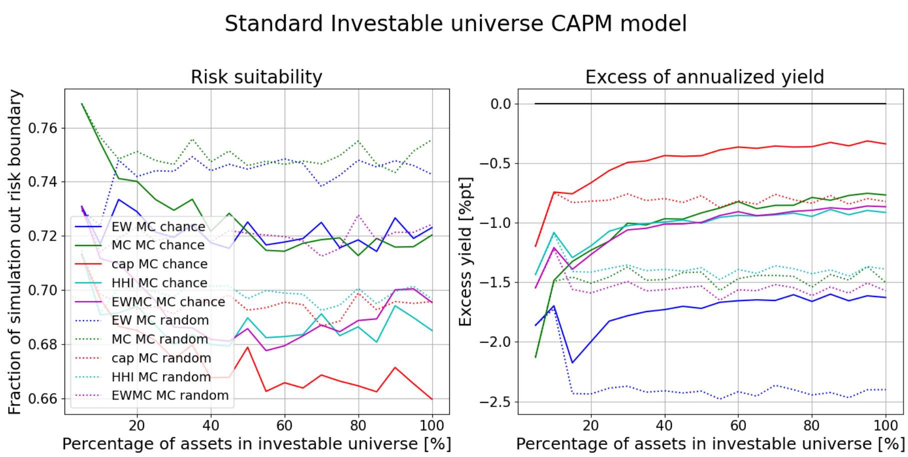

Figure 23: Risk suitability (left) and performance (right) for an investable universe of five to one hundred percent for a concentrated portfolio of fifty stocks using the CAPM model in the standard experiment.

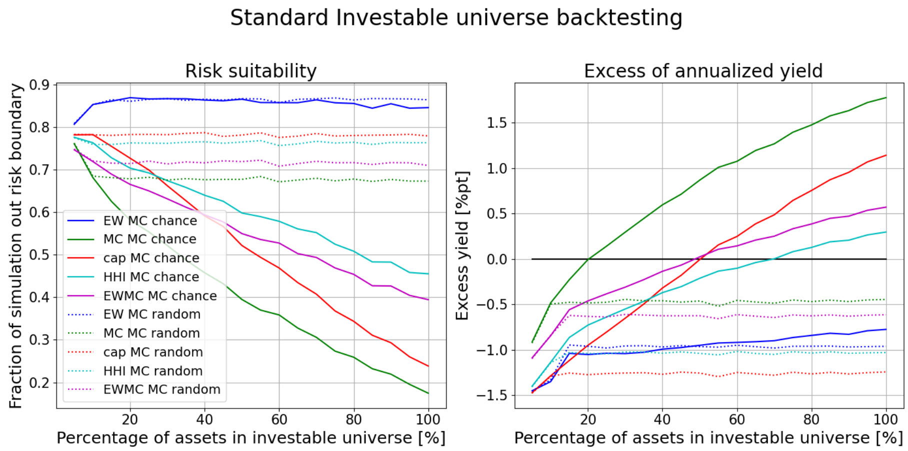

Figure 24: Risk suitability (left) and performance (right) for an investable universe of five to one hundred percent for a concentrated portfolio of fifty stocks using backtesting in the standard experiment.

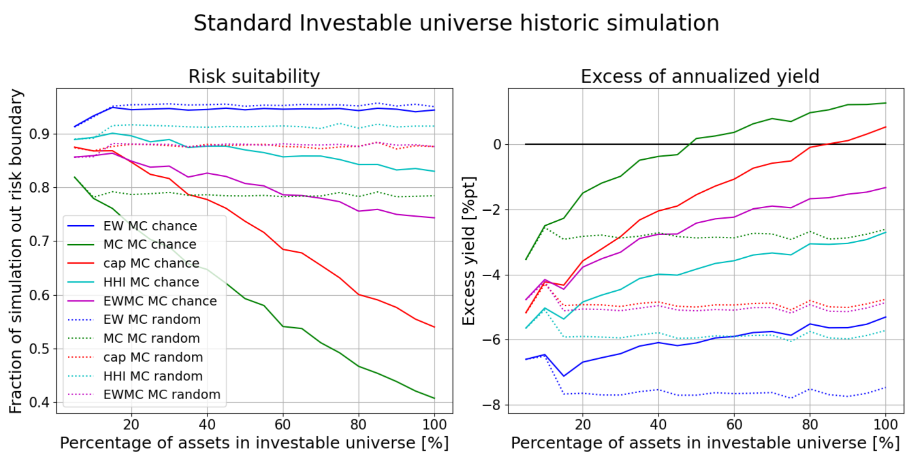

Figure 25: Risk suitability (left) and performance (right) for an investable universe of five to one hundred percent for a concentrated portfolio of fifty stocks using the historic model in the standard experiment.

7.4.2 Investing in benchmark and concentrated portfolio

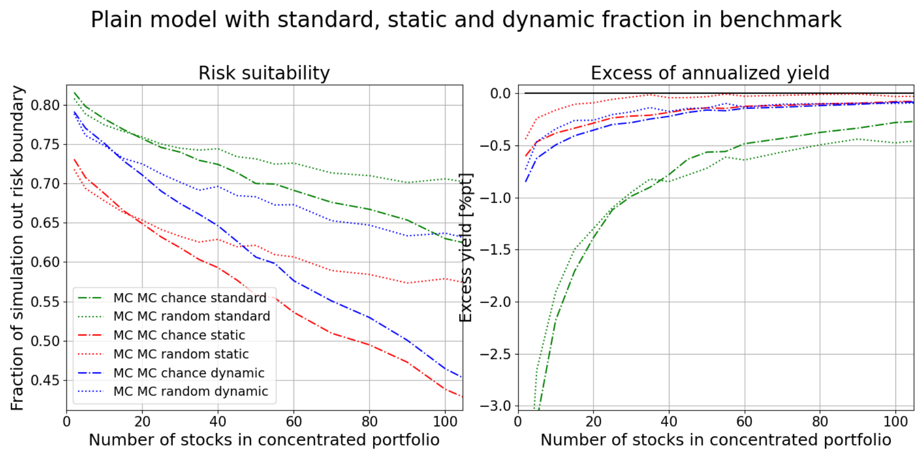

Figure 26: Risk suitability (left) and performance (right) for five to one hundred holdings in a concentrated portfolio using the plain model in the standard, static fraction in benchmark and dynamic fraction in benchmark experiment

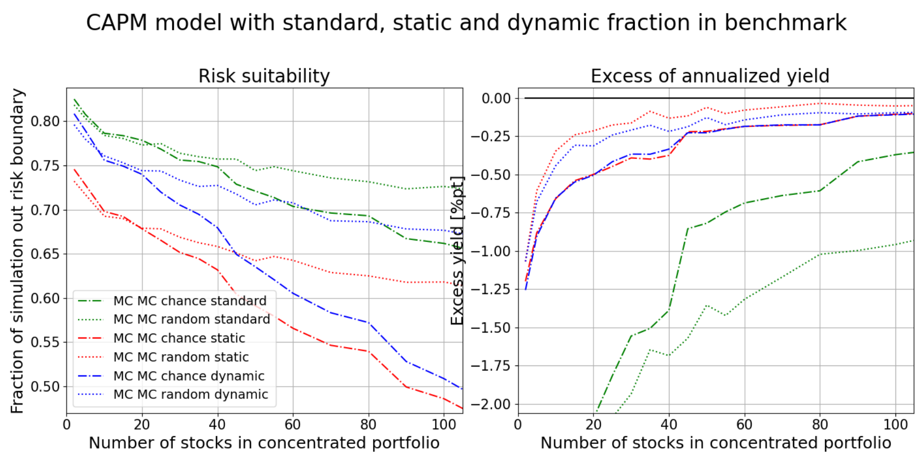

Figure 27: Risk suitability (left) and performance (right) for five to one hundred holdings in a concentrated portfolio using the CAPM model in the standard, static fraction in benchmark and dynamic fraction in benchmark experiment

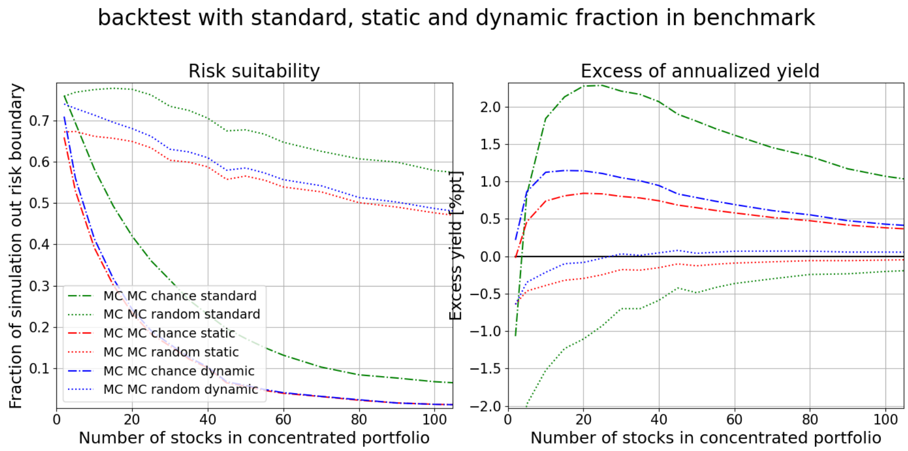

Figure 28: Risk suitability (left) and performance (right) for five to one hundred holdings in a concentrated portfolio using the backtesting model in the standard, static fraction in benchmark and dynamic fraction in benchmark experiment

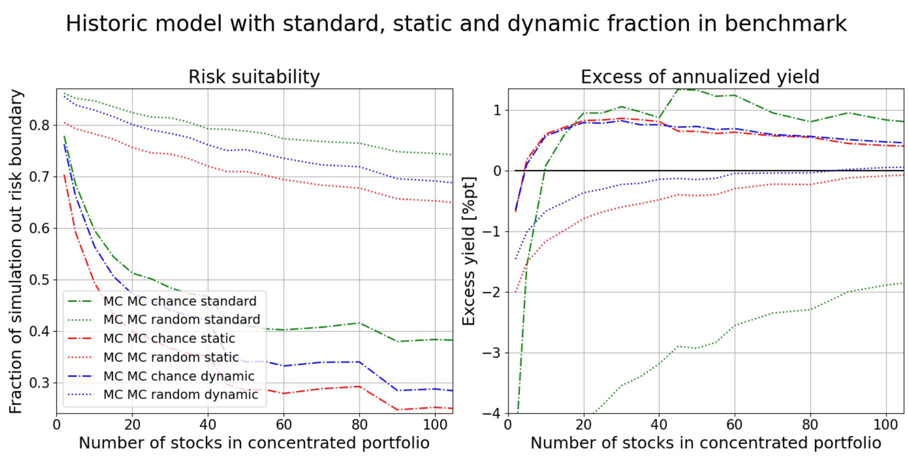

Figure 29: Risk suitability (left) and performance (right) for five to one hundred holdings in a concentrated portfolio using the backtesting model in the standard, static fraction in benchmark and dynamic fraction in benchmark experiment.

7.4.3 Two managers

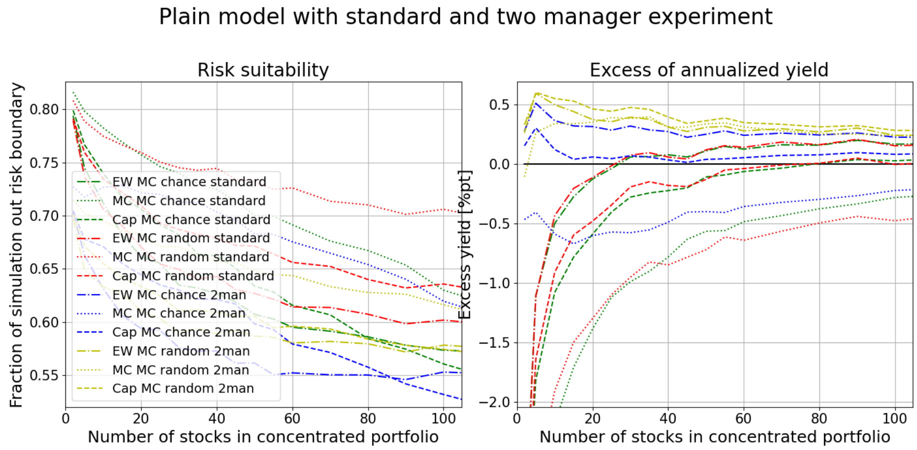

Figure 30: Risk suitability (left) and performance (right) for five to one hundred holdings in a concentrated portfolio using the plain model in the standard and two managers experiment.

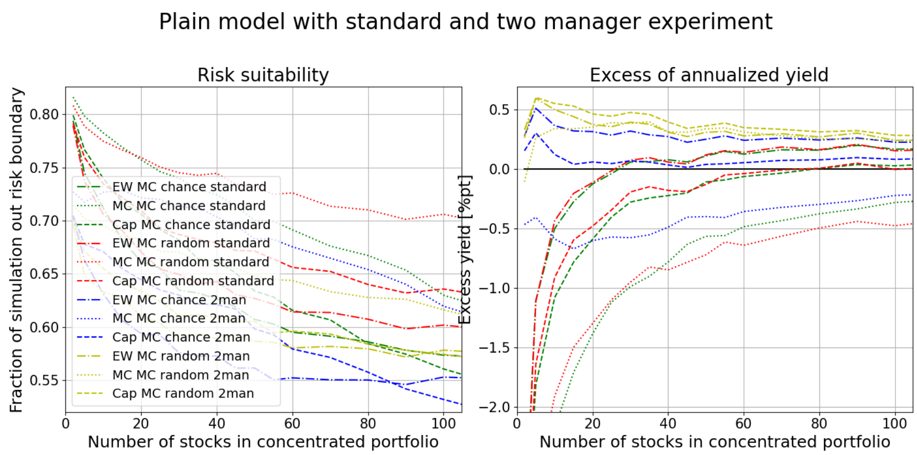

Figure 31: Risk suitability (left) and performance (right) for five to one hundred holdings in a concentrated portfolio using the CAPM model in the standard and two managers experiment.

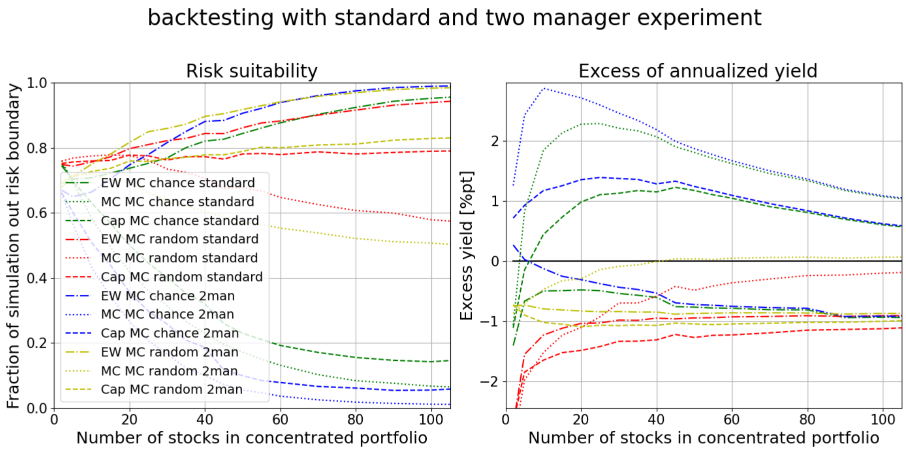

Figure 32: Risk suitability (left) and performance (right) for five to one hundred holdings in a concentrated portfolio using the backtesting in the standard and two managers experiment.

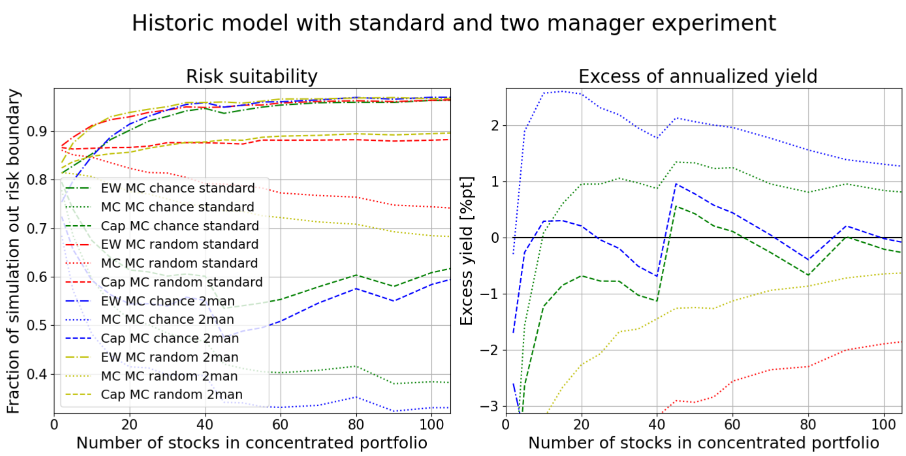

Figure 33: Risk suitability (left) and performance (right) for five to one hundred holdings in a concentrated portfolio using the historic model in the standard and two managers experiment.

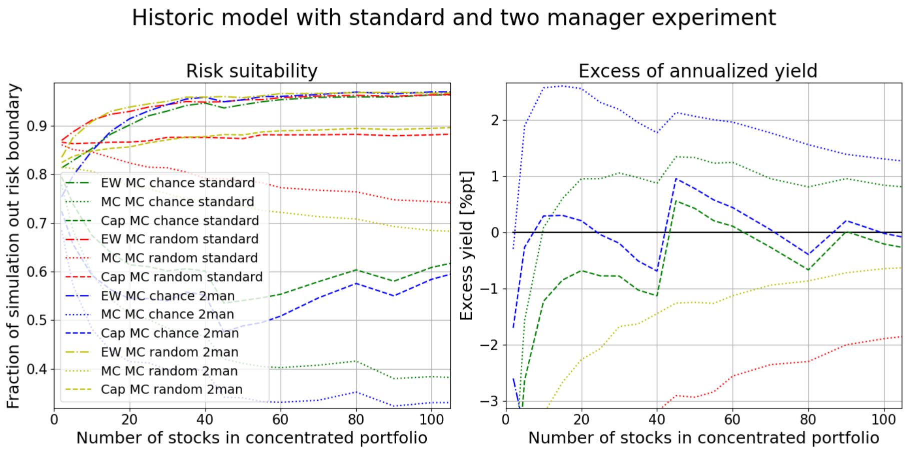

Figure 34: Risk suitability (left) and performance (right) for five to one hundred holdings in a concentrated portfolio using the combined model in the standard and two managers experiment.