Distortion mechanisms in 1-bit Sigma-Delta modulators

Huub Knijff (s2547929)

***Abstract*—One bit sigma-delta modulators (SDM) are used to convert analog signals to digital signals using a feedback loop. This feedback loop consists of a 1 bit quantizer and a frequency selective filter. The feedback loop however, does not attenuate memoryless distortion that occurs before the filter like a classical feedback loop would do. The reason for this is the binary nature of the feedback signal. Introducing a reconstruction filter in the feedback path can solve this problem. By lowering the cutoff frequency, better suppression of the memoryless distortion is obtained. Lowering the cutoff frequency too much can cause the system to not be able to suppress the distortion anymore, because the quantizer of the SDM will start to clip.**

I. Introduction

Analog to digital converters (ADC's) are used to convert analog signals to digital signals. An example of an ADC is the sigma-delta modulator (SDM), which typically consists out of a feedback loop with a loop filter and quantizer in it. An example of a first-order SDM can be seen in figure 1. The SDM is an oversampling ADC, which means that its sampling frequency is much higher than twice the maximum frequency of the input signal. An attractive feature of this SDM is that the resolution of its quantizer can be reduced to 1 bit, which causes the SDM to be inherently linear [1]. However, by reducing the resolution of the quantizer to 1 bit, the SDM becomes unable to suppress memoryless distortion inside its loop that occurs in between the subtraction and loop filter, unlike classical feedback loops. This memoryless distortion can come from a non-linearity in a mixer, like in [2], but also from non-idealities such as a finite DC gain of the loop filter [1] or a non-linearity in an amplifier that might be used. In [2] the authors claim that this is due to the binary nature of the feedback signal. To prevent the distortion from not being suppressed they suggest to add a reconstruction filter in the feedback path [3]. In this paper the distortion inside the SDM will be investigated by modeling the SDM and adding a distortion before the filter, and it will be verified that the distortion is indeed not being suppressed. Then the reconstruction filter will be introduced and the relation between the characteristics of the reconstruction filter and the suppression of distortion will be investigated, to get a better understanding of the design of such a filter. Furthermore, the effect of introducing a reconstruction filter on the system will be discussed.

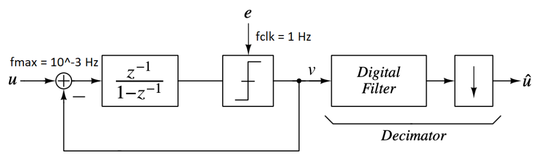


Fig. 1: System model of an ADC with a first-order modulator [1]

II. Modelling the SDM and distortion

In order to model the SDM, we used Matlab Simulink. This model consists out of a signal generator, a first order filter, quantizer and feedback loop, and can be seen in figure 2. The sampling frequency that we used is normalized to 1 Hz in all simulations. The loop filter has as transfer function:

$$H(z) = \frac{z^{-1}}{1 - z^{-1}} \tag{1}$$

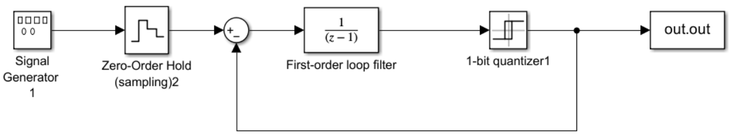


Fig. 2: Model of an ADC with a first-order modulator

To model the non-linearity (distortion) we used a third order Taylor series: $y = a_0 + a_1 * x + a_2 * x^2 + a_3 * x^3$ [2]. Now in order to add second and third order distortion to the model, the blocks shown in figure 3 are added. What happens here is that a signal comes in via the '1' and that signal gets multiplied by itself, which creates the second order distortion, and the resulting signal gets multiplied by the original signal again, which creates the third order distortion.

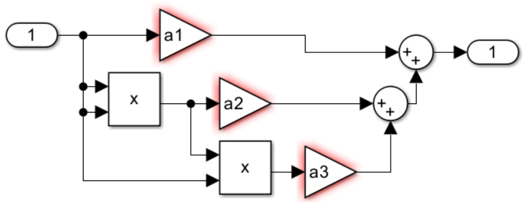


Fig. 3: Model of distortion

III. DISTORTION ANALYSIS

1) Distortion in classical feedback loops: The way conventional feedback loops suppress distortion can be explained by looking at figure 4.

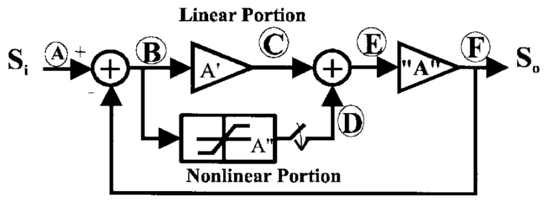


Fig. 4: A conventional feedback loop with a non-linear component [2]

In [2] is stated that a non-linear component of amplifier "A" in figure 4 can be described as

$$S_o = a_1 * S_i + a_2 * S_i^2 + a_3 * S_i^3 + ... \tag{2}$$

in which S_i is the input signal and S_o is the output signal. If this non-linear component is then placed in a feedback loop with feedback factor f the non-linear transfer can be described as

$$S_o = b_1 * S_i + b_2 * S_i^2 + b_3 * S_i^3 + ... \tag{3}$$

with

$$b1 = \frac{a_1}{1 + a_1 f} \tag{4a}$$

$$b2 = \frac{a_2}{(1 + a_1 f)^3} \tag{4b}$$

$$b3 = \frac{a_3(1 + a1f) - a_2^2 f}{(1 + a_1 f)^5} \tag{4c}$$

By using classical feedback theory the second harmonic distortion for the amplifier in figure 4 without feedback can be written as $HD_2 = \frac{1a_2}{2a_1^2} S_{OM}$ and with feedback can be written as $HD_2 = \frac{1a_2}{2a_1^2} \frac{S_{OM}}{1+a_1 f}$. The harmonic distortion with feedback is reduced by $1 + a_1 f$ [2].

2) Distortion in 1-bit quantized feedback loops: According to the authors in [2] the cause of distortion not being suppressed like in classical feedback loops is the binary feedback signal. To verify this, we have done three simulations, after which the frequency spectrum of the output of the ADC was analyzed. We created this frequency spectrum by first using a Hanning window, after which the absolute value of the fft of the output was taken, and converted to decibels. The three cases that were analyzed are:

1) The output of the system with no distortion (figure 2)
2) The output of the system with distortion outside of the feedback loop (figure 5)
3) The output of the system with the distortion placed inside of the loop before the filter (figure 6)

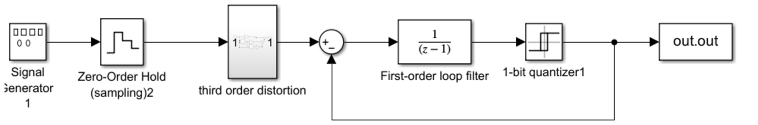


Fig. 5: SDM with distortion outside of the loop

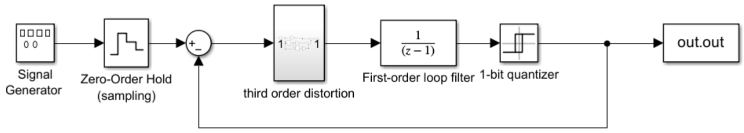
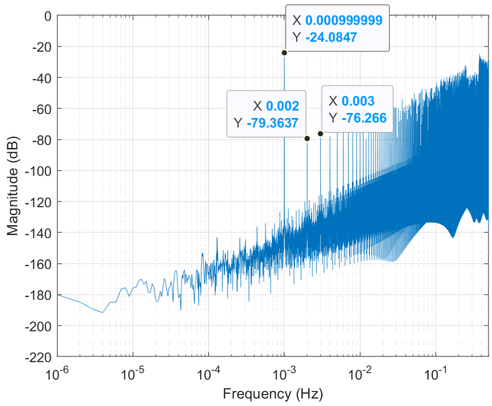


Fig. 6: SDM with distortion inside of the loop

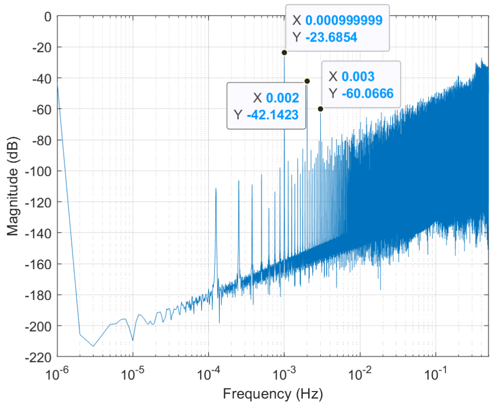
In these simulations a 10^{-3} Hz sinusoidal input with an amplitude of 0.25 and a sampling frequency of 1 Hz with a VDAC of $\pm 1V$was used . For easy comparison we chose the distortion coefficients to be $a1 = a2 = a3 = 1$. What we are interested in is the magnitude of the output spectrum at $2{*}10^{-3}Hz$ and $3{*}10^{-3}Hz$, because those magnitudes indicate whether second and third harmonic distortion is present in the output signal. If the ADC were to behave like a classical feedback loop, we would expect that, when the distortion is placed inside the feedback loop, the distortion would be attenuated and the spectrum would look more like the spectrum without distortion. However, this is not the case for the 1-bit SDM, as we can see in figure 7. We can see that in figure 7c there is still a big second order disortion term (-40 dB at $2 * 10^{-3}Hz$) compared to figure 7a (-80 dB). From these spectra we are able to conclude that the distortion is indeed not being suppressed.

A. *Why is the distortion not suppressed*

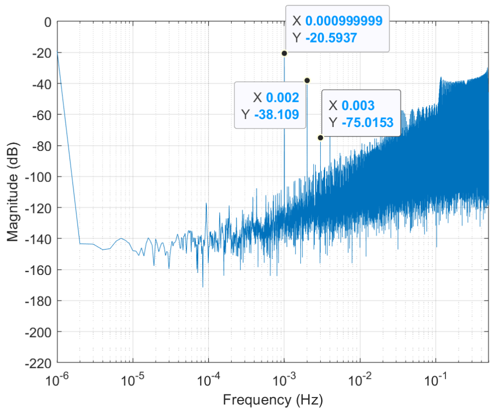
In [2] is stated that the distortion inside the loop is not suppressed due to the binary feedback signal. We can take a look at in what ways the feedback loop of a SDM acts different than a classical feedback loop does. In a classical feedback loop the output signal 'tracks' the input signal, but this is not the case for an SDM. The SDM produces a binary signal which does not look like the input at all. In a classical feedback loop the output is subtracted from the input, and as the output tracks the input, a very small error signal is left after the subtraction. This is not the case for the SDM as the binary output is subtracted from the input, which results in the signal from which a snippet can be seen in figure 8.



(a) Frequency spectrum with no distortion



(b) Frequency spectrum with the distortion placed outside of the loop



(c) Frequency spectrum with the distortion placed inside of the loop

Fig. 7: Frequency spectra of output of ADC

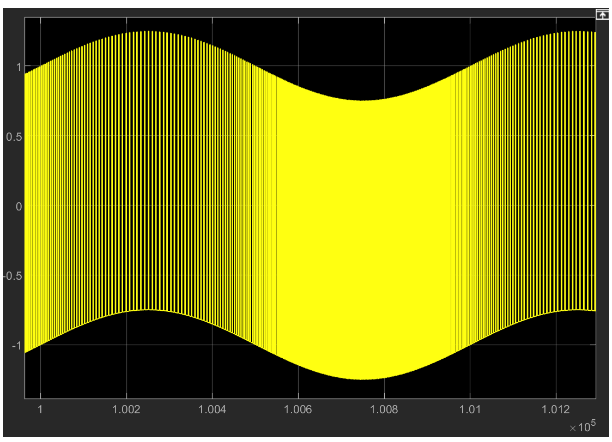


Fig. 8: Signal after subtraction

IV. The Proposed Solution

The authors in [2] propose to add a reconstruction filter in the feedback path. The model that we used for this system can be seen in figure 9.

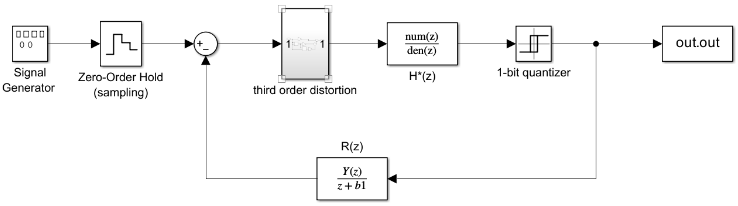


Fig. 9: Model of ADC with reconstruction filter

In order to dimension the loop and reconstruction filter we chose for the product of their transfer function to be equal to the transfer function of the original first order loop filter that can be seen in equation 1 and we define

$$H(z) = H^*(z) \cdot R(z) \tag{5}$$

This resulted in equation (6) for the reconstruction filter (R(z)) and equation (7) for the new loop filter ($H^*(z)$). Another constraint that we used, was that the magnitude of the reconstruction filter at low frequencies has to be equal to 1, which means c has to be equal to $1 + d$. In this way it was possible to sweep d an look at the resulting distortion in the output of the ADC.

$$R(z) = \frac{c \cdot z^{-1}}{1 + d \cdot z^{-1}} \tag{6}$$

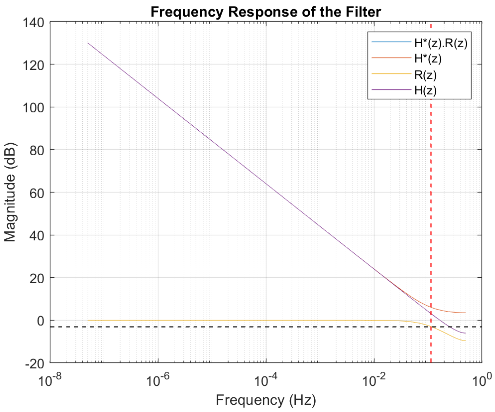
$$H^*(z) = \frac{\frac{1}{c} + (\frac{d}{c}) \cdot z^{-1}}{1 - z^{-1}} \tag{7}$$

By sweeping d it was possible to change the cut-off frequency of the loop and reconstruction filter. Two examples of this can be seen in figure 10. As we can see, as the cutoff frequency of the reconstruction filter becomes lower, the cutoff frequency of the loop filter moves with it, and the product of the two filters remains the same. We also plotted the original H(z) in this figure to show that the magnitude of $H^*(z) \cdot R(z)$, and H(z) is the same. The magnitude of $H^*(z) \cdot R(z)$ can't be seen because it exactly overlaps with H(z). We were also able to conclude that, when d is moved more towards -1, the cutoff frequency becomes smaller.

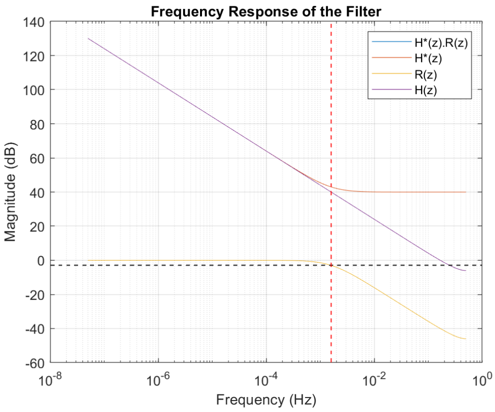
As the value of d isn't a very intuitive way of understanding the filter, we took another approach of doing a sweep. In this approach not d, but the cutoff frequency was swept. The corresponding value of d was calculated by using a bilinear transform, which creates the relation shown in equation (8), in which f_c is the cutoff frequency. From d we were then able to calculate the corresponding c value (9).

$$d = \frac{f_c * 2 * \pi - 2}{f_c * 2 * \pi + 2} \tag{8}$$

$$c = 1 + d \tag{9}$$



(a) Frequency response for d = -0.5



(b) Frequency response for d = -0.99

Fig. 10: Frequency reponses of filters for different values of d

Now in order for us to look at what the relation between filter characteristics and the suppression of distortion is, we did a sweep of the cutoff frequency, and plotted the resulting magnitude of the second and third harmonic in a graph in dBc (so with respect to the magnitude of the input frequency at $10^{-3}Hz$). This graph is shown in figure 11. For this sweep we took a set of 100 logarithmically spaced cutoff frequencies for the reconstruction filter. As we can see in figure 11, for a smaller cutoff frequency, the second and third order harmonics get attenuated better, until the cutoff frequency becomes lower than approximately $3 * 10^{-4}Hz$. Why the distortion is not suppressed below this frequency will be explained in section V.

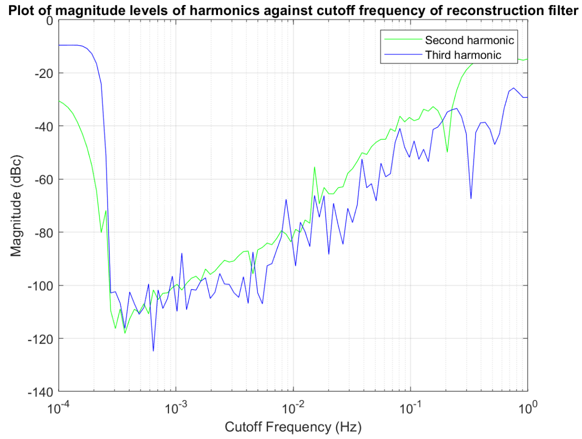


Fig. 11: Relative magnitude of second and third harmonic against cutoff frequency of reconstruction filter

From the graph in figure 11 we can be see that as the cut-off frequency goes up logarithmically, the magnitude of the second and third harmonic seem to go up with a linear trend. This linear relation can be seen more clearly if we put a trend line in the graph, which is done in figure 12.

So, why does filtering the feedback signal reduce the distortion in the output signal. We believe that this is because filtering restores some of the properties of a classical feedback loop. The lower the cutoff frequency, the more the feedback signal starts to look like the input signal, and the smaller the error signal after the subtraction becomes. To demonstrate this we again took a snippet of the signal after the subtraction, which is shown in figure 13. In this snippet we used reconstruction filter with a cutoff frequency of $10^{-3}Hz$. If we compare figure 13 to the signal shown in figure 8, we can see that the amplitude of the error signal has gone down with a factor 10^2.

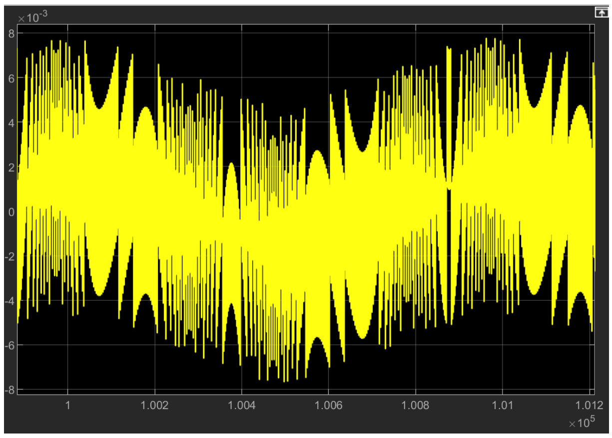
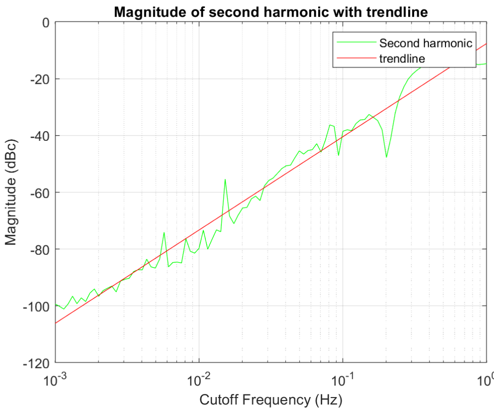
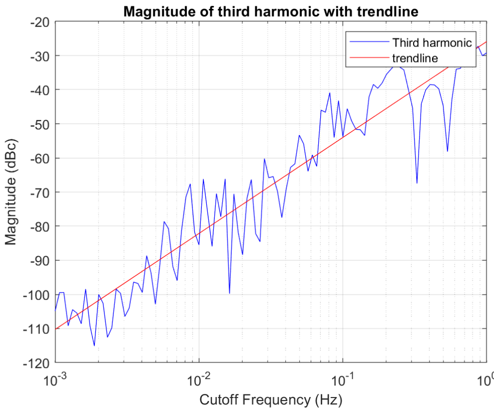


Fig. 13: Signal after subtraction with a cutoff frequency of $10^{-3}Hz$ and input frequency of $10^{-3}Hz$



(a) Relative magnitude of second harmonic with trend line



(b) Relative magnitude of third harmonic with trend line

Fig. 12: Relative magnitude of harmonics with trend line

V. Linearization

In order to investigate how the system acts for different cutoff frequencies of the reconstruction filter in the feedback path, the SDM is linearized. The quantization error is left out for now, because to investigate the stability we look at the STF (signal transfer function). The linearized model can be seen in figure 14.

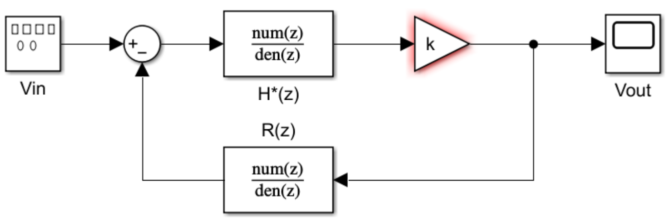


Fig. 14: Linear model of the SDM

Using this model it is possible to calculate the signal transfer function ($\frac{v_{out}(z)}{v_{in}(z)}$), which is

$$STF = \frac{H^*(z) \cdot k}{1 + R(z)H(z)^* \cdot k} \tag{10}$$

in which $H^*(z)$ is the loop filter, R(z) is the reconstruction filter and k is the quantizer gain. What is interesting about this STF is that if you make $H^*(z) \cdot R(z) \cdot k$ very big, the system will approach

$$STF \approx \frac{H(z)^* \cdot k}{R(z)H^*(z) \cdot k} \tag{11}$$

in which $H^*(z)$ and k cancel out such that you can rewrite 11 as

$$STF \approx \frac{1}{R(z)} \tag{12}$$

which is essentially a high-pass filter with the same cutoff frequency as the reconstruction filter. We can demonstrate this by plotting the STF of the SDM, which is done by sweeping the input frequency of the non-linear model and calculating the gain by dividing amplitude of the output over the amplitude of the input. The resulting system gain for three different cutoff frequencies of the reconstruction filter can be seen in figure 15.

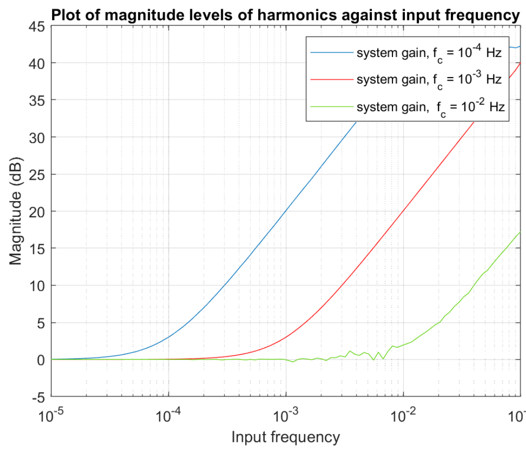


Fig. 15: STF of non-linear model for different cutoff frequencies of the reconstruction filter

How this causes distortion to not be suppressed anymore at low cut-off frequencies as can be seen in figure 11, can be explained by system having too much gain. As can be seen in figure 15 the system with $f_c = 10^{-4}Hz$ already has a gain of 20 dB at an input frequency of $10^{-3}Hz$. This is a problem because with such high gain the quantizer will start to clip. This can be shown by using a method to calculate the maximum input our quantizer can handle before it starts clipping, presented in [1]. The method is as following: we look at the quantizer input as we increase the amplitude of the input signal until the quantizer input blows up. The amplitude at which the quantizer input blows is known as the maximum stable amplitude. We determined the maximum stable amplitude for three scenarios: a cutoff frequency of 10^{-2} Hz, 10^{-3} Hz and 10^{-4} Hz, all with the input signal at 10^{-3} Hz. The result can be seen in figure 16. We can see that for a lower cutoff frequency the input explodes at a lower amplitude and for a cutoff of 10^{-4} Hz this explosion happens almost immediately. Using this result it is clear that when the cutoff frequency is too much lower than the input signal frequency, the system will have too much gain and the quantizer clips for a very low amplitude of the input signal.

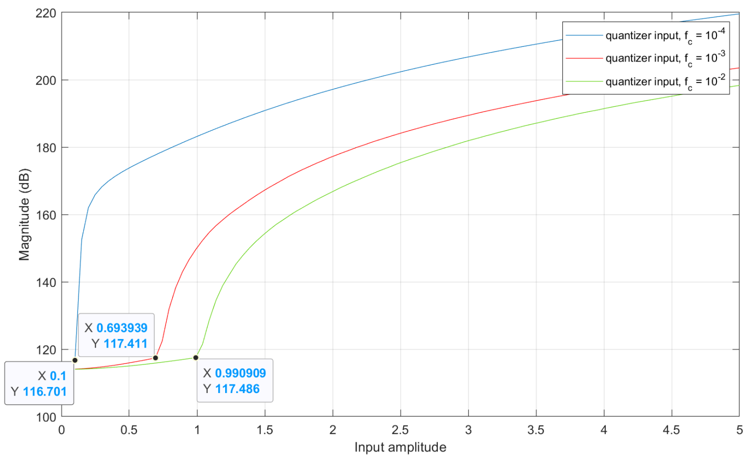


Fig. 16: MSA plot for three cut off frequencies of reconstruction filter

VI. Conclusion

In this paper a model of a sigma-delta converter was made, and was used to verify that memoryless distortion is not attenuated by the SDM if the quantizer resolution is one bit. A reconstruction filter was introduced into the feedback path, and a trend was found between the cutoff frequency of the reconstruction filter and suppression of distortion. Lowering the cutoff frequency resulted in better suppresion of distortion, until the quantizer started to clip. The quantizer starts clipping because the system has too much gain when the cutoff frequency of the reconstruction filter is too low compared to the input frequency. This was proven by first plotting the signal transfer function and then determining the maximum stable amplitude. By determining the maximum stable amplitude we were able to conclude that the quantizer starts to clip already for very low amplitudes of the input signal, when the cutoff frequency is too low compared to the frequency of the input signal. The reason why the reconstruction filter works is believed to be because it restores some of the properties of a classical feedback loop. Future research should find an intuitive explanation as to why distortion is not suppressed when the resolution of the quantizer is 1 bit and why introducing a reconstruction filter helps restoring that suppresion.

References

[1] G. C. T. Shanti Pavan, Richard Schreier, *Understanding Delta-Sigma Data Converters*. Wiley-IEEE Press, 2017.

[2] B. H. L. Ardeshir Namdar, "A 400-mhz, 12-bit, 18-mw, if digitizer with mixer inside a sigma–delta modulator loop," *IEEE JOURNAL OF SOLID-STATE CIRCUITS*, 1999.

[3] H. Knijff, "Survey paper," 2024.