MSc Thesis Applied Mathematics

Multimodal traffic assignment problem with drop-off pick-up and availability constraints

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February 19, 2025

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Abstract

In this thesis, a micro modelled tour-based multimodal traffic assignment model with availability and drop-off pick-up constraints is presented. This way, individual tours can be analysed for new innovations such as Mobility as a Service. The new method integrates non-line-based multimodal transport by using a new network structure with parking spaces and hubs on mode systems. The model is an adjusted version of the Household Activity Pattern Problem (Recker, 1995) with parking constraints from Chow and Djavadian (2015). However, the parking constraint does not comply with our drop-off pick-up constraint. An example is shown which is feasible with the parking constraint, while not feasible in our problem. A Binary Integer Linear Programming (ILP) is presented. The ILP model is evaluated and compared with numerical examples to an implementation of the Multi-state Super Network (MSN) (Arentze & Timmermans, 2004). The ILP model outperforms the MSN implementation when the number of parking spaces is increased. The new model generates plausible tour routes.

Preface

Ik wil veel mensen bedanken die mij hebben geholpen gesteund tijdens mijn afstudeertraject. Ik wil graag Thomas bedanken voor het grinden in de bachelor, master en ook tijdens het schrijven van de thesis. Zelfs toen Thomas afgestudeerd, was hij er vaak nog bij om samen aan de slag te gaan en de presentaties mee te oefenen of als persoonlijke assistent. Mijn begeleider van de universiteit, Alexander, voor de continue feedback en wekelijkse meetings. Jesse en Luuk bedank ik voor de begeleiding en wekelijkse spar sessies bij Goudappel. Verder wil ik Lilian bedanken voor de ondersteuning tijdens de gehele studie en het afstuderen, dat ze er op aandrong om toch iedere maand af te spreken. Jorn en Luuk bedankt voor het gezelschap tijdens het afstuderen, het doorlezen en feedback geven op het eindverslag. Ook Werner Sheinhardt en Marc Uetz bedankt dat jullie graag de commissie willen vullen en helpen beoordelen van mijn scriptie. Matthias Walter bedankt voor het inbrengen en controleren van mijn ILP versie van de HAPP, en het controleren dat de bestaande HAPP versies echt verkeerde tour routes toelaat. En de rest van mijn vrienden waaronder huisgenoten, studiegenoten en meer die mij op allerlei manieren hebben bijgestaan.

Summary

Imagine being able to predict and analyse how people adapt their travel behaviours to new modes of transportation. Such innovations include shared scooters and e-bikes. Traditional transportation prediction approaches mainly rely on gravity models, which predict the movements of entire groups. These models fail to capture individual travel behaviour, and can not detect the adaptation of new forms of shared mobility.

To address these limitations, we use micro-simulated tours. They determine which routes and transportation modes each individual will take to reach their destinations. However, designing these complex route selection models presents a challenge: How should they be structured?

The new modes of transportation introduce characteristics which have not been fully considered before in traffic modelling. For example, availability of a vehicle is a key factor. Will a rental bike still be there, or will another user have taken it? Route choices can also depend on previous decisions made throughout the day. For instance, if a user parks their car at a Park & Ride to use public transportation, they will later need to retrieve it. After all, nobody wants to have their car sit in a parking facility far away at the end of the day.

This research provides a network structure that explicitly accounts for these constraints. The network structure is formulated as a mathematical model. The solution method is an Integer Linear Programming (ILP) formulation. The ILP algorithm is tested and compared against existing algorithms.

With the newly developed method, transportation innovations, such as Mobility as a Service (MaaS), can be better studied. Furthermore, we will be able to design cities knowing the influence on personal mobility patterns.

Contents

Chapter 1

Introduction

Mobility as a Service (MaaS) is an upcoming development the Dutch government is funding to get cars off the road (Ministerie van Infrastructuur en Waterstaat, 2019). MaaS incorporates all possible modes of public and shared transport and combines them into one booking and payment system. Shared transport incorporates shared vehicles like scooters, bikes and cars. These can be of two types, one is that you need to bring back the vehicle that you share at the original pick-up location, the other one you can drop off at certain locations. Public transport includes line transport services like buses, trains or metros. According to the factsheet from Rijkswaterstaat (2023), road authorities need to know the origin and destinations of the intended target audience to measure whether MaaS is successful in offering an alternative transport mode for a location. One of the benefits is getting cars off the road, which reduces carbon emissions, improves air quality and reduces the demand for parking spaces, which can lead to more space for a liveable area. This all increases the social wellbeing ('Brede welvaart' in Dutch).

Next to MaaS, another measure to reduce cars on the roads in city centres is park and ride (P+R) facilities. The P+R facilities enable car travellers to park their cars outside city centres by replacing the last leg of their trip (i.e., the final part of their journey) with public transport. This reduces congestion and parking space in city centres. By reducing car kilometres, effective P+R locations reduce vehicle emissions significantly.

One way to measure the impacts of MaaS or P+R facilities is done with traffic simulation models. These models exist to measure mobility to and from regions. The simulation models combine demand and supply. Demand indicates where people want to travel to and from. We will call these people agents. Which we define as people who make choices on where and how they want to travel. Supply is the existing network on which agents can travel. Given demand and supply, traffic is assigned to networks, thereby modelling the route choices of travellers. This is called traffic assignment. The assignment determines the usage of the network on each link. The usage on each link is called the load or link flow and given in vehicles per hour. This output can help policymakers understand the effects of different transportation strategies.

In this thesis, we will propose a new mathematical modelling method. To understand why, we will describe some background on traffic modelling techniques in Section 1.1. After this, we will discuss the current modelling in which this project lies in the scope. In Chapter 2 we will discuss three existing methods. The three discussed are the notion of composite modes in Section 2.1, the Multi-state Super Network (MSN) (Arentze & Timmermans, 2004; Liao, 2013) in Section 2.2 and the Household Activity Pattern Problem (HAPP) (Recker, 1995) with additional parking constraints (Chow & Djavadian, 2015) and multimodal transport (Najmi, Rey, Waller, & Rashidi, 2020) in Section 2.3. Our method will be a modified version of the HAPP. In Section 2.3.1, we

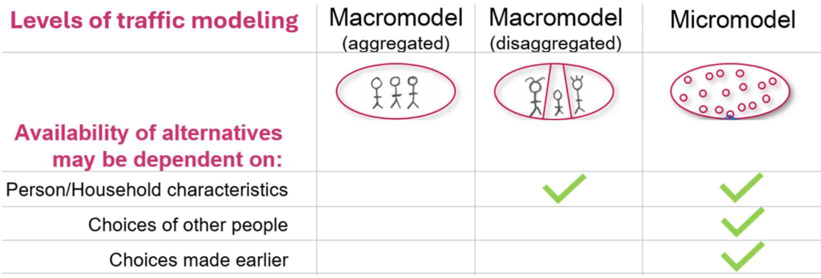


Figure 1.1: Different levels of traffic modelling (horizontal), which shows the different level of detail each level models. This research focusses on traffic assignment in a micromodel. Edited from (Brederode et al., 2020, p. 6).

demonstrate that the existing problem formulations in the literature do not comply with our availability and drop-off pick-up concepts. This is done by showing an example of incorrect, so called 'vehicle flow'. In Chapter 3 we will first set out a mathematical modelling introducing a new concept of 'mode systems'. These will make sure that we will have correct vehicle flow. After this, in Section 3.5, we will propose replacements constraints for the HAPP model. In Section 3.5.1, we will show a new Integer Linear Programming (ILP) formulation where we have removed the continuous variables and included the additional constraints to account for the vehicle flow problem. The new model is eventually evaluated in Chapter 4. These include model capabilities, scalability of the algorithm and stability. In section 4.1, the formulation is solved with a commercial MILP solver (Gurobi Optimization, LLC, 2024). The results are compared with an existing algorithm implemented by Goudappel based on MSN. Lastly, we conclude, discuss and provide future recommendations in Chapter 5.

1.1 Background

Traffic modelling can be done on three levels. These levels are outlined in Figure 1.1. In the first level, we know where groups of people want to go from A to B. This provides an Origin Destination (OD) matrix for all regions. This is a trip-based macro model with aggregated demand. A more accurate model considers different demographic groups, recognizing that each group has distinct mobility patterns. For instance, a student generally values costs over travel time, making them more likely to use public transport. Whereas, workers generally values time over costs, favouring car travel. This method, called disaggregated macro modelling, avoids aggregating across demographic groups and instead focuses on their specific travel behaviour. At an even more accurate level, even more dependencies on travel can be incorporated. This is micro modelling, where the travel route of each tour of each person is determined individually. This makes sure the choices within a tour are consistent with earlier made choices in the tour, for example, having to bring back a rented vehicle. Furthermore, the choices of other users can be incorporated as well, such as the dependency of availability on rental bikes.

In traffic modelling we can consider trip based our tour based models. Trip based demand models only find origin destination pairs. A tour based demand model, find trip chains. Such a chain of trips contains the all the trips needed to visit each activity location. A tour always starts with a trip that originates from a home location and ends in a trip that has the home as a destination. The advantage of tour based over trip based is the ability to add choices made earlier in the tour. For example, if a rented bike is used in the first trip, then in a later trip in

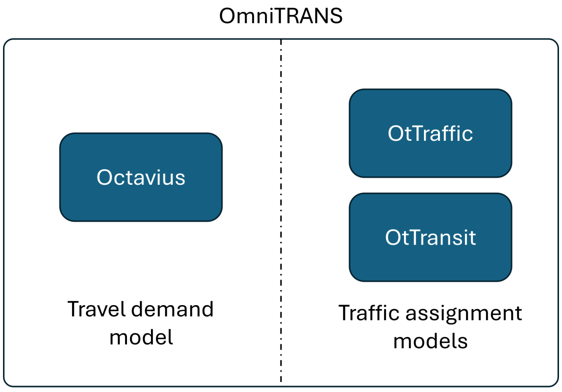


Figure 1.2: OmniTRANS software overview, showing the different relevant modules.

the tour that rented bike should be returned.

One traffic model software package is OmniTRANS, developed by Goudappel. In Figure 1.2 an overview of separate modules of OmniTRANS relevant to this paper is shown. The travel demand model Octavius is a tour based micro model. The traffic assignment models OtTraffic and OtTransit are trip based models. These modules are explained in this section.

The demand side of the traffic modelling framework is a tour based microsimulation model called Octavius. Octavius is recently developed by Goudappel and is a series of decision models that model certain choices an individual makes (van Essen & Voorhorst, 2022). The current demand models implemented in the Octavius framework has four decision model steps:

1. **Population Synthesizer:** Generates individuals with specific demographic characteristics and preferences. The agent preferences are preferences for mode, distance, costs, waiting time, transfer time, number of transfers or other. The next choice models base their choice on the properties and preferences of the agent.

2. **Tour generator:** For each individual, the tour generator determines the purpose of trips, the number of destinations and the order of destinations an individual will visit within a tour. There are six purposes (education, work, business, leisure, shopping and other).

3. **Destination choice:** Per transport mode, the destination choice determines the activity location per purpose within the tour. These destinations are conditional to the modes of transportation network. For each mode, there is a different destination.

4. **Mode choice:** The available modes of transportation will be chosen.

The outcome of the travel demand model consists of a travel diary for all persons in the synthetic population. For each synthetic person, the travel diary contains all tours conducted. Each tour includes information on the start location, the number of activity locations and their purposes, the order of the activities and the used mode for each trip within the tour.

The travel diaries are input for the traffic assignment model, which Goudappel implements using two methods tailored to different transport modes. For unimodal trips, the OtTraffic module is used to assign traffic on unimodal networks, achieving a user equilibrium with gravity models (Beckmann, McGuire, & Winsten, 1956). This method uses origin-destination (OD) matrices as input and produces link flows as output.

The second module assigns a certain type of multimodal transport, the line-based public transport. This includes transport, such as buses and trains, that only operate on a transit line. The line-based public transport is assigned together with access and egress mode using OtTransit. The access mode is the mode used to reach the first transit stop in the trip. The mode from the last transit stop to the destination is called the egress mode.

This leaves non-line-based multimodal transport, which is not assigned in OmniTRANS. Shared vehicles, such as shared cars, bikes and scooters are an example of this multimodal transport. This type of transport will be added in this research.

Furthermore, OmniTRANS lacks the ability to analyse individual travel patterns. As the assignment methods in OtTraffic only work with aggregated flows. To address these limitations, this research introduces a disaggregate traffic assignment model.

An overview of general context with transport demand and assignment modelling is given in Figure 1.3.

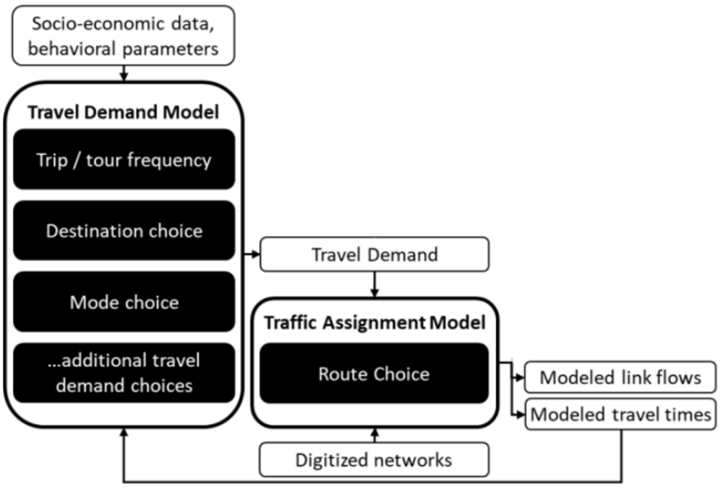


Figure 1.3: The role of a traffic assignment model in strategic transport model application, from (Brederode, 2023, p. 1)

1.1.1 A micro modelled multimodal traffic assignment module

Voorhorst (2021) defined the key properties of a micro-modelled tour. These properties allow for the availability of alternatives from Figure 1.1 and are therefore important to consider in this research. The three properties are outlined below:

- **Multimodal:** A multimodal assignment model should support being able to take multiple different modes, allowing travellers to switch modes during a trip. While this functionality already exists for line-based public transport within OtTransit, expanding it to include non-line-based modes enables the option to change from a car to a shared vehicle.

- **Availability constraint:** The availability constraint reflects that certain modes or mode transfers may not always be possible depending on certain circumstances. These circumstances can be influenced by choices made earlier by the agent or by actions of other agents in the model. Examples include, a car being unavailable if the tour is started without one, or a rental bike being unavailable because all rentable bikes are already used by other users. Such vehicles are only available at the places where a vehicle is located.

- **Drop-off and pick-up constraint:** This mechanism accounts for situations where a traveller needs to return to a certain spot because of a choice earlier in the tour. For instance, dropping off a car at a Park and Ride requires the agent to pick it up later. Or if a rental scooter is chosen, it may only be dropped off at certain locations.

These characteristics are implemented within the new traffic assignment method introduced in this research.

To emphasize the importance of these characteristics within a multimodal tour, an overview of various modes and events is provided in Table 1.1. For a more detailed explanation of the table, and to better understand the workings of these characteristics across different modes, we refer to Appendix A.

Using these characteristics, we arrive to the research question of this project: How can individual route choices be modelled to account for multimodal trips with availability and drop-off/pick-up constraints within a tour?

Table 1.1: Overview of modes and events that use the three characteristics. A 'v' indicates that the characteristic is needed. An 'o' indicates optional, in this case it is possible to have only one mode on a trip, or the requirement to pick-up an item after drop-off may be done by another agent. A more elaborated explanation is provided in Appendix A.

Mode or activity	**Type**	**Multimodal**	**Availability constraint**	**Drop-off and pick-up constraint**
Public transport	Line-based	v		
Car / scooter / bike / step sharing[a]	Free floating	o	v	
Car / scooter / bike / step sharing[a]	Station-based one-way	v	v	
Car / scooter / bike / step sharing[a]	Station-based round-trip	o	v	v
Carpool as passenger[a]	Ridesharing	o	v	
Carpool as driver[a]	Ridesharing			o
Taxi / autonomous vehicle[a]	Ridesourcing / Ridesplitting	o	v	
Personal vehicle sharing[a]			v	v
Park and Ride		v	v	v
Car parking		o	v	v

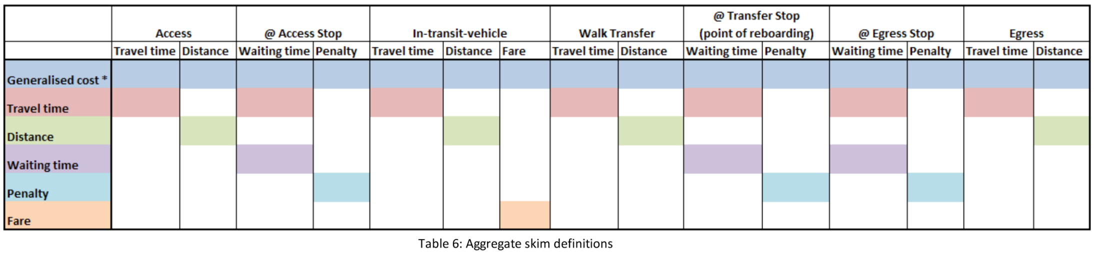
[a]Shared transport mode as described in Machado, De Salles Hue, Berssaneti, and Quintanilha (2018).

1.2 Scope

Our focus is on understanding travel behaviour once a stable equilibrium has been reached. While traffic conditions can change due to temporary disruptions, such as road closures or unexpected events, we are only interested in the long-term state. The stable equilibrium is the state where no agent wants to change their travel behaviour because every agent already found the best routes. A personal experience illustrates this concept well. When I first started commuting to a new company, like for my graduation, I initially took slightly longer routes. Over time, I discover the fastest and most efficient paths and no longer deviate from them. This mirrors with this research, which focusses on this final, stable phase of travel behaviour. In which, every individual refined their choices and the system is in equilibrium.

In this section, the scope of current Octavius modelling and the scope of this research are outlined, which defines the goal of this research.

To determine what the utility of every agent is, we have to define a utility function. The utility function calculates per user what the personal costs of travel is. The utility function may include multiple costs components for all the different modes. For unimodal modes this is usualy a component for the travel time multiplied by the value of time, and another component with the travel costs, like fuel or tolls. In the case of public transport, we can use utility function components from public transport economics (Hörcher & Tirachini, 2021). In this field of study, such a utility function sets monetary values to the different parts within a public transport trip. A value of time for each of the cost components can be set by estimates from empirical studies (e.g. (Wardman, 2004)) or with inverse optimization (Chow & Djavadian, 2015). Other cost components are parking costs, mode preferences, transfer penalties or waiting times at different stages in the trip. Currently, OtTransit is able to use all the cost components on line-based multimodal transport as outlined in Figure 1.4 (Cook, n.d.).



	Access		@ Access Stop		In-transit-vehicle			Walk Transfer		@ Transfer Stop (point of reboarding)		@ Egress Stop		Egress	
	Travel time	Distance	Waiting time	Penalty	Travel time	Distance	Fare	Travel time	Distance	Waiting time	Penalty	Waiting time	Penalty	Travel time	Distance
Generalised cost *															
Travel time															
Distance															
Waiting time															
Penalty															
Fare															

Table 6: Aggregate skim definitions

Figure 1.4: Utility function components possible within OtTransit (copy from Cook (n.d.))

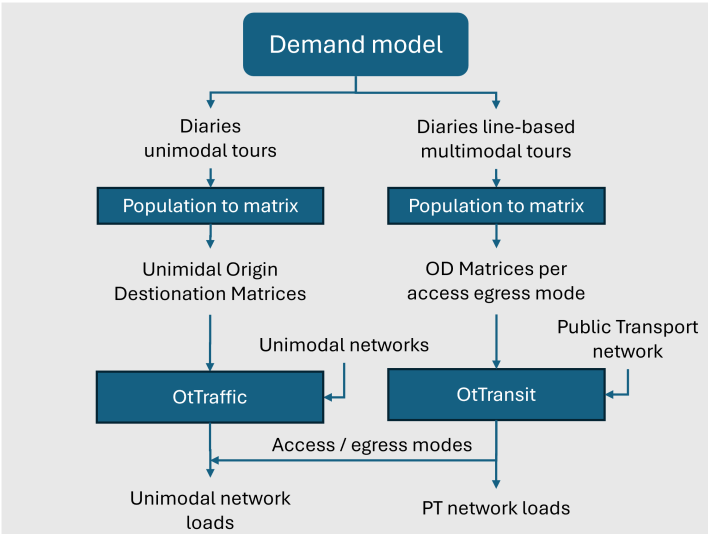
In this research, we will consider travel time, and discuss how fixed mode costs and mode transfer costs can be included. In the evaluation, in Chapter 4, we only consider the travel time of each mode and some weight factor to account for a mode preference.

1.2.1 Current modelling

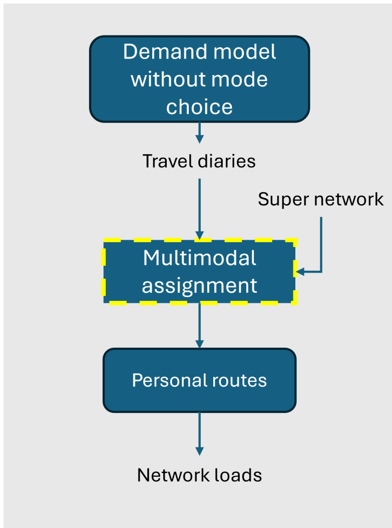
To give context to the scope, we will provide a summary of the current OmniTRANS multimodal transportation modelling. A schematic overview is outlined in Figure 1.5a, while the goal of this research is outlined in Figure 1.5b. The state of the art OmniTRANS traffic assignment model has as input the demand generated by Octavius. Octavius provides a travel diary per agent. This diary contains all the choices made by Octavius, which consists of the mode of transport, the amount of tours, the destinations, and the order of the destinations. To assign the demand to the supply, two steps are followed, the population to matrix step and the classical traffic assignment step. The population to matrix step translates the travel diaries into Origin Destination (OD) matrices, so that each trip in the tour is represented in these matrices. The classical assignment step is done with OtTraffic and is applied to unimodal modes, which are the cars, the bikes and walking. OtTraffic can do this in three ways, volume averaging, assignment until user equilibrium is reached (Beckmann et al., 1956) or according to shortest paths using Dijkstra's algorithm (Dijkstra, 1959). The public transport is assigned using OtTransit and executed per access and egress mode combination. OtTransit can assign demand onto line-based public transport. The output of the traffic assignment are the network link flows and properties per OD pair, such properties include car travel time or public transport transfer time per OD pair.

In theory, the model can use the output of OtTraffic to reevaluate the demand. This is a feedback loop, which adds the updated properties per OD pair back into the Octavius model. However, due to the running time of Octavius, this loop is not used. And will also not be used in our study. In other traffic models, such a feedback loop between the traffic assignment and demand generation is often present.

Currently, park and ride or non-line-based multimodal transport are not represented within OmniTRANS.



(a) Current traffic assignment: based on OD matrices.



(b) Our model: A micro simulated route finder including non-line based multimodal trips are added.

Figure 1.5: Overview of existing traffic assignment model within OmniTRANS and our proposed model.

1.2.2 The goal of this research

We will provide a traffic assignment model, that determines personal tour routes. The tour routes allow for non-line based modes, such as shared bikes or scooters. The input is given as travel diaries without pre-determined mode choices. Instead, the diaries provide which modes are available and provide a preference between these modes. The OtTraffic and OtTransit modules that work on OD matrices will be replaced. Instead, we will determine route patterns for each individual user, which can be seen in Figure 1.5b.

Our solution algorithm should be scalable. That can be in multiple dimensions, namely in model decision complexity by adding parking spaces and shared modes. Or in the network size with increasing the number of nodes and edges. Furthermore, our solution method should be stable, that is, there must be no stochastic in this. Each run should be reproducible. As we aim for optimality, this will not be an issue.

There are availability constraints on previous choices within the tour and the drop-off pick-up constraints. These two should be included. The availability constraint when there is a limit on a certain mode or parking facility will not be included. Since, we model a future equilibrium, we may assume there is always available. Examples are a limited number of parking spaces at a park and ride facility or a limit on the number of shared vehicles at a rental location.

Traffic assignment until a user equilibrium, is a positive trait of traffic assignment, but is not a requirement in this study.

Chapter 2

Related work

In this chapter, we present the related work introducing three concepts to model multimodal traffic assignment. In Section 2.1, we discuss the current trip based implementation of traffic assignment using OtTransit and OtTraffic. A demand model that expanded this to tour based modelling with rental bikes (Voorhorst, 2021) is also explained. We describe that the method to enumerate modes is not suitable for tour based non-line based multimodal traffic assignment. To make this more structured for tour based models, we introduce the concept of the Multi-state Super Network (MSN) in Section 2.2(Arentze & Timmermans, 2004; Liao, 2013). Solution methods use Dijkstra's algorithm on MSN. We will compare the MSN with a third method in the literature. This third method is the Household Activity Pattern Problem (HAPP) (Recker, 1995; Chow & Djavadian, 2015), which tries to find activity patterns. The activity patterns, state how activity visits are time scheduled by travellers. In our research, we do not consider time schedules, but only route assignments. In addition, existing HAPP solutions on multimodal networks allow for tours which are not allowed for our drop-off pick-up and availability constraints. A proof for this is provided in Section 2.3.1. To account for this, we introduce a new multimodal network model in Chapter 3. In that chapter, we also provide an ILP version of the HAPP problem and a way to resolve the issue of insufficient drop-off pick-up constraints within the HAPP framework.

2.1 Enumerating composite modes

One method of modelling multimodal tours is using composite modes. These are not existing modes, but combinations of modes which can be modelled as one for choosing a mode. The parts of the composite mode trip (legs) can be assigned separately as unimodal trips. The composite modes should inherently include all the options that a multimodal tour can have. We first describe current trip based composite modes, which are currently assigned using OtTransit and OtTraffic. We will expand this to tour based composite trips to also include non-line based multimodal tours and illustrate that this creates a problem.

Currently, composite trips can model line-based multimodal trips. The chosen modes for one trip will be combined and will be called a 'composite mode'. In Figure 2.1 the nine possible composite modes for line-based multimodal trips are given. For example, Option 2 has the composite mode 'BikePTWalk'. In the mode choice model of Octavius, one such composite mode will be selected.

The traffic assignment for a composite mode from a to b will be determined by OtTransit and goes as follows. First, the location of the origin transit stop and destination transit stop will be determined by a demand choice model. After which, OtTransit assigns the public transport

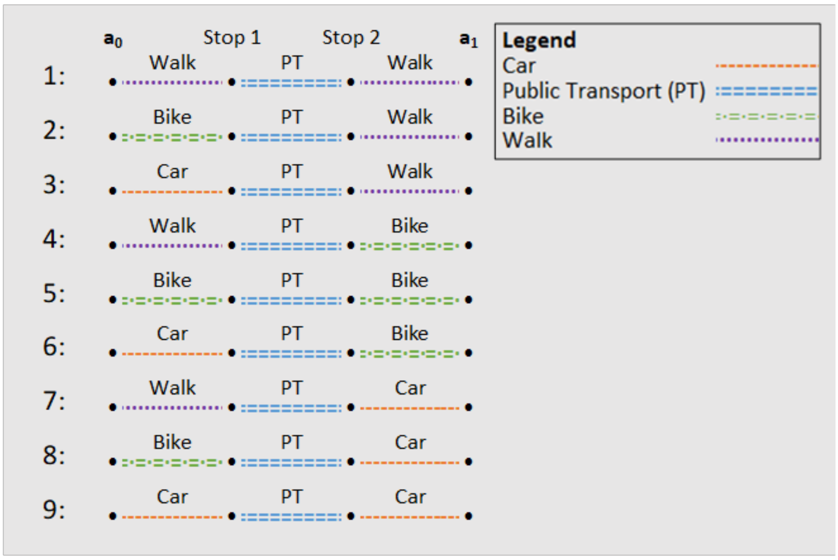


Figure 2.1: The nine possible composite modes for a line-based multimodal trip from location a_0 to location a_1. The line-based part of the trip uses public transportation and travels between 'Stop 1' and 'Stop 2'. The available access and egress modes are walk, bike and car.

lines used between these transit stops. The access trip from a to the first transit stop will be added to the OD matrices of the respective access mode. And similarly, the egress trip from the second transit stop to b will the added the OD matrix of the egress mode as well. All separate modes in the composite mode will thus be assigned independently. For one composite mode, a multiple of origin and destination transit stops are used. Probabilities are assigned to these different possibilities. Summarising, OtTransit enumerates routes with access mode, egress mode, all relevant transit stops, and the possible line choices. All the possible access and egress trips are added to the OD matrices, and will later be assigned using OtTraffic.

If we want to include non-line-based multimodal options, then we will have to enumerate more mode combinations, creating more composite modes. For a trip based model, this can be done relatively easy. For example, consider a trip from home to an activity location and the composite mode 'WalkPTBike'. Then after the public transport, the personal bike is not available and a rental bike will be used from the public transport stop. In this case, the composite mode 'WalkPTBike' includes a shared bike as egress mode, which will be assigned the same as a regular bike. However, there will be no guarantee that the trip backwards will again be with the appropriate composite mode 'BikePTWalk'. This can not be guaranteed, since we have a trip based assignment model. This conflicts with the drop-off pick-up concepts from Section 1.1.1. This example highlights again why we want tour based modelling.

To be able to add all the modes from Table 1.1 and to overcome the problem of the violated drop-off pick-up approach, we can use the enumeration strategy but then tour based, instead of trip based. For tour based, more mode combinations are possible.

We will lay out what this implies for the composite modes concept. For example, consider Figure 2.2, in this example the composite mode consists of the information of all the used modes and parking spaces. This makes the number of possible composite tours huge.

If you are not careful, there are many composite tours where the drop-off pick-up constraint will be violated. Such composite tours should not be considered.

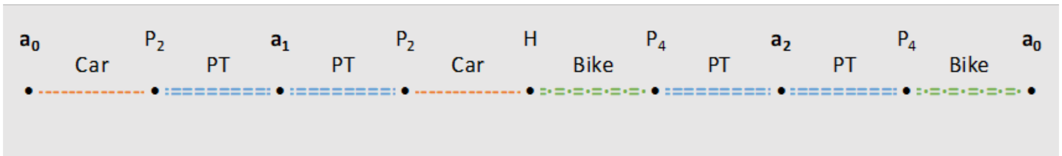


Figure 2.2: An example of a valid composite mode for a multimodal tour $T = \{a_1, a_2\}$. With drop-off pick-up constraints enforced on P_2 and P_4.

Voorhorst (2021) did a study on tour based approach adding the drop-off pick-up and availability including station-based round-trip bikes. The study of Voorhorst (2021) determines the destination choice. The goal was to determine how many people are using a rental bike system. While we must consider the traffic assignment, the route choice. For the route choice, we can still take the ideas from their approach on how the enumeration of different non-line based multimodal modes would work in a tour based setting.

Two composite modes can have similar elements. For example, if the car mode in the tour in Figure 2.2 is replaced with a shared bicycle, then the PT and bike modes remain the same on the rest of the tour. This property can be used to reduce the enumeration.

A smart way to list all the composite tours while satisfying the drop-off pick-up constraint is using a Multistate Super Network (MSN). In this, mode changes are encoded in a state transition graph. The next section goes in more detail on how a MSN works, and discusses the related work.

2.2 Multistate Super Network

The Multistate Super Network (MSN) can be used to determine a tour route to visit a set of activities while considering multiple modes of transport and also drop-off pick-up constraints. In Figure 2.3 we have an example of a MSN network. In this section we will discuss each element of a MSN like in the figure. We will discuss how the drop-off pick-up constraint of our research is incorporated in a MSN. Hereafter, an example tour route on the MSN network in Figure 2.3 is described. After this, we will talk about how the specific modelling need to be adjusted to our problem. At the end, we will discuss how Georgiou (2022) implemented availability on a certain mode.

In a MSN, each mode of transport has their own mode network, which indicate on which roads a user can use the mode for travel. For example, the car network contains all the roads where a person can drive. While the Public Transport Network (PTN) contains all the roads where a person can walk, and contains all the links between which places a person can use public transportation, like trains and buses. Each network is represented as a pentagon or hexagon on the left in Figure 2.3. In this case we have two Private Vehicle Networks (PVN), one with car and one with bike. And we have the Public Transport Network (PTN), which includes the public transportation links and the walk edges. Note that some roads may overlap between different mode networks, since you can use two different modes on the road.

To transfer to another mode, transfer points are added to each network. On these points, you can switch between two different modes of transportation. In our example, a transfer between the PV car and Public Transport (PT) can be made at parking spaces P_1 or P_2. In Figure 2.3 this can be seen, because P_1 and P_2 are both nodes in the PVN Car and the PTN. When such a transition is made, a link between the two mode networks is added. In Figure 2.3, such a link is represented as an arrow between two distinct mode networks. These links are only present on

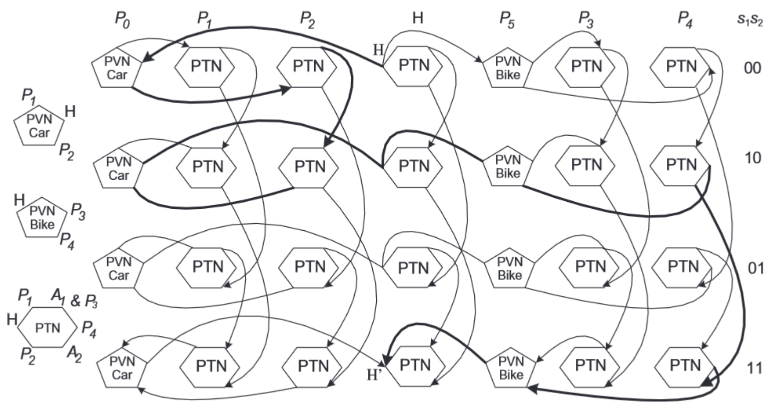


Figure 2.3: Multi-state Super Network transition diagram (copy from Liao et al. (2013, p.43))

the same row between two columns. We will explain why these links lay in one row, first we need to introduce activity visits.

To make a tour route, we will also keep track of which activities are visited, up to this point. A tour route must start at home H and will end at home H', whenever all activities are visited. When we start at home, no activities are visited, and we should end the tour route at home, where all activities are visited. To encode whether activities are visited, additional state information is introduced. Next to keeping track which mobility is currently in used, the state will also keep track which activities are visited. Whenever an activity a_i is visited on a node n with a mode m, a state transition is made. The new state indicates that a_i is visited (that is s_i goes from 0 to 1), while mode m is still in use. After this transition, if we look within the mode network, you are located on node n. Such an activity visit can be seen as switching to a different row in Figure 2.3.

To allow for drop-off pick-up we will also keep track where a Private Vehicle (PV) is. In our example, the car can be at home, at parking space $P1$ or at parking space $P2$, while the bike can be at home, at parking space $P3$ or at parking space $P4$. Whenever an activity is visited, we will keep the information where the PV is located. This ensures that the PVN can only be entered if the PV is picked up at the correct parking space, and ensures that the end location H' can not reached if any PV is not at home. In Figure 2.3 we can see that if we are in a state where a PV is at a parking space P_i (a column P_i), then there are only mode transition links from parking space P_i.

An example tour route, using the combination of modes like in Figure 2.2, is highlighted with thick edges in Figure 2.3. The tour route starts at H. The first state transitions towards column P_0 (the car is in use) indicates that we will start using the car. The car is driven to parking space P_2. In column P_2, the car is parked at parking space P_2 and we are travelling within the PTN. This enforces a drop-off pick-up constraint for the car at P_2. That is, before using any other mode, we first have to use the car starting at parking space P_2. We continue this tour by visiting the first activity using the PTN. This represented by the thick arrow from row 1 to row 2. Then, the PTN is used to pick up the PV car at P_2 again, lifting the drop-off pick-up constraint. Now the person wants to use the PV bike. To achieve this, the PV car is driven from P_2 back to home H. When the PV bike is used to visit the second activity, a similar drop-off pick-up constraint holds at parking space P_4 for the PV bike. At the end, the PV bike needs to be returned home H. This allows the tour route to be valid, because all activities are visited and all PV are returned to home H. Because the tour is valid, we represent home as H'. In the entire state transition graph, there is one valid ending state H', located in one mode network (In Figure 2.3 this is in column H, row $(s_1, s_2) = (1, 1)$, using the PTN network).

In our research, we have one activity location, per activity. But in general, a MSN can allow destination choice by allowing multiple activity locations where a transition can be made to visit (Liao et al., 2013). The destination choice is included by allowing a set of destination locations A_i per activity i, one can visit any node $n \in A_i$, to transition to the state where activity i is visited. To exclude the destionation choice, we will only have one activity location in the set A_i for all activities i.

Furthermore, in our research, we already know the order in which the activities are visited. While in general, a MSN can also determine this order. To fix the order of activity visit, we will only allow having transition links between activity states when such a activity can be visited. That is activity one first, after which activity two, etc... In our example in Figure 2.3 this can be seen as removing row three (the states where only activity two is visited).

We will now discuss two methods to resolve availability of shared modes of transportation (like a shared bike or scooter) (Georgiou, 2022). If there is a limited number of shared vehicles available at a transfer location, then such a transfer link can be removed after the shared vehicles are all used up by other users. A different approach to tackle the limited availability is by setting a probability on the transfer link. Then, before the route calculation, the probability determines whether the state transition link exists for this agent. In our research we have not used this principle of availability, but it may be implemented similarly as well.

The study of Georgiou (2022) only allows for one shared car, which can reach up to two parking spaces, a shared bike that can reach up to two parking spaces and the use of the PTN. Each agent can have different locations for their home, their available parking spaces and activities. The state transition graph is as big as in Figure 2.3.

In our evaluation framework, we test our proposed solution against the MSN implementation of Georgiou (2022). Which we have expanded to allow for an arbitrary number of parking spaces. But we have removed the ability to determine the order of activities.

2.3 Household Activity Pattern Problem

The Household Activity Pattern Problem (HAPP) tries to solve a similar problem as the MSN formulation stated in the previous section. That is, determine a tour route to visit a set of activities. But in addition to MSN, the HAPP tries to find activity patterns, these are tour routes including the arrival times at each activity location. The HAPP also may also add constraints or objective values to these activity arrival times. Such as, time window constraints, they state when an activity is open for visit. And, early or late arrival penalties, they penalize if the travel takes much longer or shorter than preferably planned. This way, they can model opening hours of shops. Or personal preferences, when to visit which activity, as a person might have a preference to do their leisure activities during the evening.

Formally, the HAPP is a mixed integer linear programming (MILP) formulation. Where the time variables are continuous time variables, and the routing variables are binary decision variables. For our research, we are only interested in finding the tour routing, and therefore we can omit the continuous time variables.

Activity travel patterns are similar to our problem. They try to find the routes for visiting a set of activities. However, the activity travel patterns also include the arrival time at each activity. Which in our research is not needed. So the HAPP is a more detailed version of our required tour route finder. We can use this implementation to find our more simplified tour routes.

In addition, most research on HAPP also try to incorporate household compositions, which make the vehicle availability between household members dependent. This is done more extensively when using the HAPP to solve the joint activity pattern problem (Vo, Lam, Chen, & Shao, 2020). The joint activity pattern problem is interested in finding activity patterns of a joint number of people, usually for a whole household. We will not consider household dependencies.

The first HAPP formulation allows ridesharing between a household (Recker, 1995). This formulation is based on the vehicle routing literature (Solomon & Desrosiers, 1988). Chow and Djavadian (2015) added multimodality with a parking constraint at parking facilities. This formulation is promising when comparing with our drop-off pick-up constraint, and the used network has overlapping parts with the MSN approach.

Most of these studies try to search a market equilibrium (Chow & Djavadian, 2015). However, this study will only focus on finding the feasible tour routes.

What is more interesting to is, is that the HAPP model also support multimodal transport options by using the network structure based on MSN approach (Najmi et al., 2020). This research allows non-line based multimodal modes, such as shared bikes. We will also introduce our own version of such multimodal networks.

Another improvement in HAPP is the addition of a parking constraint (Chow & Djavadian, 2015; Najmi et al., 2020), which makes sure a car is picked up later, after parking it at a parking facility. This relates to the drop-off pick-up concept.

Since we want the parking constraint, we will adjust the formulation of Chow and Djavadian (2015) and remove the time variables, and time based objectives. This will result in an Integer Linear Programing (ILP) formulation rather than a MILP. Even better, we only consider binary descion variables, making the solution space tractable. One important notion that we found is that the parking constraint presented in Chow and Djavadian (2015) and Najmi et al. (2020) still allows a tour that is not in line with our drop-off pick-up concept. In the next subsection, we will show why the formulations of Chow and Djavadian (2015) and Najmi et al. (2020) is insufficient for our study.

2.3.1 The insufficient parking constraint

To show that the parking constraint in existing research (Chow & Djavadian, 2015; Najmi et al., 2020) is not in line with our drop-off pick-up constraint, we will state the six rules of which a tour route of Najmi et al. (2020) must satisfy. We will present an undesirable example that satisfies these rules. Adding a rule on vehicle flow, would prevent such an unwanted tour. The additional rule, will require to make changes to the MILP formulations in Chow and Djavadian (2015) and Najmi, Rey, Rashidi, and Waller (2019). A newly proposed change to our ILP solution, incorporating vehicle flow, is presented in Section 3.5. In the remainder of this section, we will present the rules, and lay out an example that is allowed within these rules, but does not satisfy our drop-off pick-up constraint. In Appendix C, we will state the MILP with insufficient parking constraint from Chow and Djavadian (2015). The same example as presented here is shown to be feasible for this formulation.

The six rules are:

1. "A traveller can leave home with a private car, bicycle, public transport, or on foot to conduct out-of-home activities.

2. If a traveller leaves home by a private car/bicycle, he/she needs to return to his/her home with the same mode of private car/ bicycle at the end of their tour.

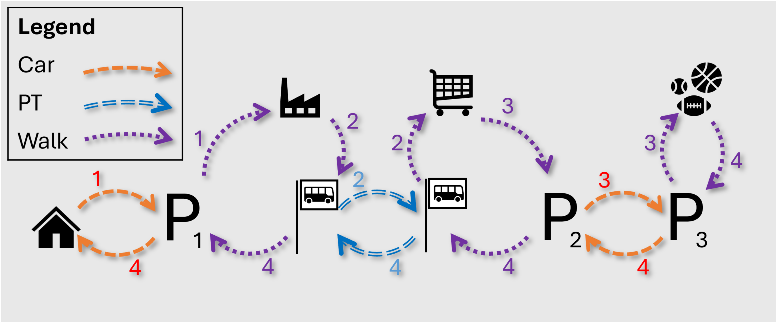


Figure 2.4: Unwanted mode combination for a tour with three activity locations. This combination of modes is feasible in existing HAPP research. But in this combination, a car is picked up at P_2, while this car should not be available (it is still at P_1). The labels on the arrows indicate in which trip the mode is used.

3. A traveller can not switch from the road network to the public transport network unless he/she parks his/her private vehicle in a parking lot at a parking node.
4. If a traveller parks his/her private vehicle in a parking space to use other travelling modes, he/she must return to the parking space to remove the car before the end of the day.
5. If a traveller uses a private vehicle to get to activity locations, he/she either goes to these locations directly or park the vehicle at parking nodes to change his/her modes to get there.
6. A traveller can not switch to the private vehicle network if he/she has left his/her home by public transport." (Najmi et al., 2020, p.8)

Statement The six rules from Najmi et al. (2020) allows for feasible routes that conflict with our requirements, the drop-off pick-up and availability constraints.

Proof Consider the mode combinations for the tour with three activity locations as in Figure 2.4. We will go over each of the rules stated above. The traveller uses all three available modes to travel to their activity locations, specially the private car is used to leave the home. Therefore, Rule 1 is satisfied. The tour starts at home with using the private car and also ends at home with using the private car. This is the same mode, so Rule 2 is satisfied. Rule 3 is also satisfied, as all transfers that include a car happen at parking spaces. Rule 4 happens, because the car is parked in Trip 1 at P_1 and later picked up in Trip 4. The same process happens at P_3 in Trip 3 and 4. Rule 5 is also satisfied, since the private vehicle, the car, is parked at the parking node P_1 when visiting the first two activity locations, and the car is parked at P_3 when visiting the third location. Lastly, Rule 6 is also satisfied. Since we have left the home with a car, we can transfer to the private vehicle network, the car network, at P_2. To show that the tour example hold for our research or not we go to our availability constraint. For this, we can see that the car is not available at P_2 in trip 3. At that moment, the car availability is only at P_1 (and the car can not be at two places at the same time, so the car is not available at P_2).

It is possible to add time based schedules to the example of Figure 2.4. Which will show that this example is also feasible in Chow and Djavadian (2015).

Chapter 3

New mathematical modelling of the traffic assignment with drop-off pick-up constraints

In this chapter, the mathematical problem formulation is stated. First, we define a newly developed multimodal network representation based on newly introduced ‘mode systems’ After this, we define the transport demand, and combine those to form the multimodal traffic assignment problem. In Section 3.4, we will compare the methods from the related works from Chapter 2 and compare how they will implement in the new ‘mode systems‘ framework. In Section 3.5, we will propose our ILP solution method. Which is a mathematical problem description to generate optimal tour routes. In Section 3.5.1, we present our vehicle flow to prevent the unwanted tours discussed in Section 2.3.1.

3.1 New network representation

A multimodal network is represented as a simple directed graph $G(V, E)$, where V is the set of nodes, and E the set of directed edges. Each node represents a place, these are intersections, parking facilities, shared mobility hubs or public transport stops. Edges denote the possibility of direct travel between nodes. An edge represents a road, a bike path, a footpath or a rail segment. In this section, we will go over each element in the multimodal network, of which an example is given in Figure 3.1.

A mode of transportation, hereafter simply ‘mode’, refers to the means by which a person travels. These are car, public transport, walk or bike. An edge ($e \in E$) is labelled with which modes can travel on that edge.

Transportation is organized into a set of mode systems M. Each mode system $m \in M$ consists of a mode of travel, a set of transfer nodes and a mode system network. The mode system network is a subgraph of the multimodal network with the edge set $E_m \subseteq E$ and the node set $V_m \subseteq V$. The subgraph is limited to the edges and nodes accessible by the mode and mode system.

For example, the car network from our multimodal network is given in Figure 3.2a. An example of a mode system network is a round-trip bike-sharing system. This system uses the bike network for its mode system network. Additionally, the mode system may be bounded by certain suburbs. In that case, the mode system network does not use the whole bike network.

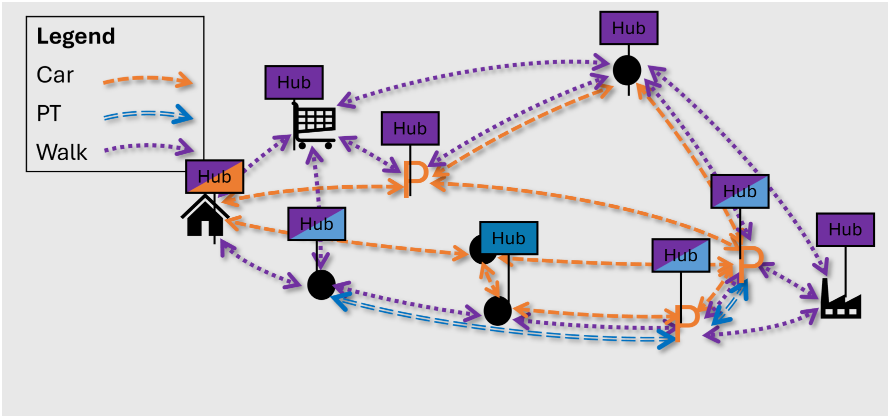
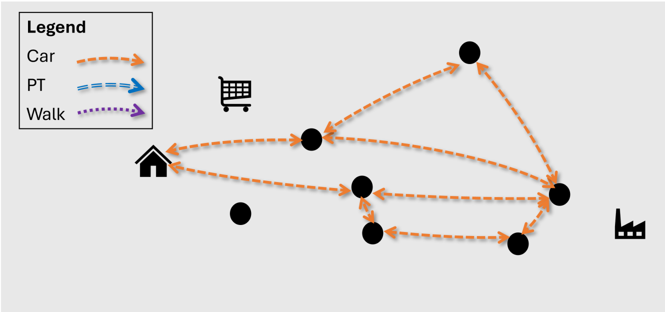


Figure 3.1: An example multimodal network. Each mode system $m \in M$ has their own colour. For each node $u \in V_m$ in the mode system network for mode system $m \in M$ the transfer nodes are indicated. The transfer nodes are either a 'Hub' sign when $u \in H_m$ or as parking node 'P' symbol when $u \in S_m$).

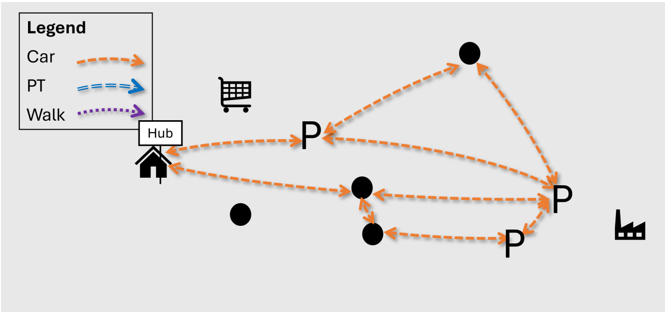
In the mode system network, a node $v \in V_m$, may be a transfer node. If a node is not a transfer node, it no connection to other mode systems, that is, no adjacent nodes with other mode system networks in the multimodal network. A transfer node is a node where a transfer to another mode system is possible. In the multimodal network, a transfer node $v \in V_m$ is adjacent with nodes from a system networks m_2 if v is also a transfer node in G_{m_2}. Each transfer node may either serve as a hub $h \in H_m \subseteq V_m$ or a parking node $s \in S_m \subseteq V_m$. The set of hubs combined with the set of parking nodes $V_m \cup S_m$ define all the transfer nodes. A hub $h \in H_m \subseteq V_m$ is a node within the mode system network where vehicles can originate or terminate their movement. A parking node $s \in S_m$ is a node where mode system m can be temporarily stopped to be used later from that node s. An important distinction from the hub is that the mode may not start on a parking node, and if the mode system is exited on s, the mode system must then later be entered on s again.

In Figure 3.2b, we have displayed the car mode system from our multimodal network in Figure 3.1. Note that the home node is the only hub. As that is where the car usage must originate and must end. The system has three distinct parking spaces where the car vehicle can temporarily be stored. The concept of walking is a special case of a mode system. To model walking as a mode system, every transfer point is a hub. Since walking allows individuals to initiate or terminate their movement at any accessible location within the network, rather than being constrained to a fixed start or end point of a vehicle. This principle can be seen in Figure 3.2d. A similar resoning applies to the public transport mode system, seen in Figure 3.2c.

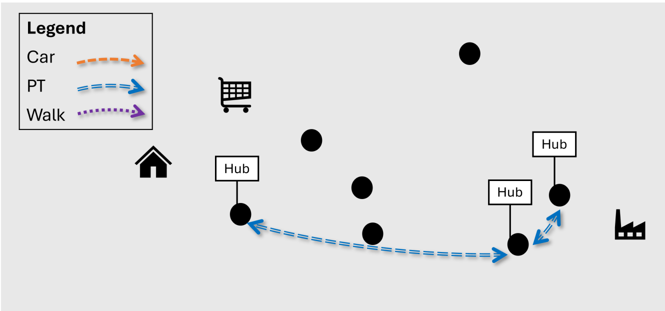
A round trip bike sharing system, like the public transport bikes in the Netherlands ("OV Fiets" in Dutch) can also be modelled as a mode system. In such a system, you can rent a bike from a specific rental place. After its use, it has to be returned to the same rental place. This can be done by setting only one hub, located at the rental place. Because there is only one hub, the shared bike may only start and end its usage at this location. This naturally enforces a drop-off pick-up constraint on such round-trip systems. However, the public transport bikes inherent an additional feature. A public transport bike can be returned at a different rental location for an additional fee. This mechanism can be implemented by introducing a second mode system, with a higher cost, in which hubs are placed at all rental locations. Since the start and end of the



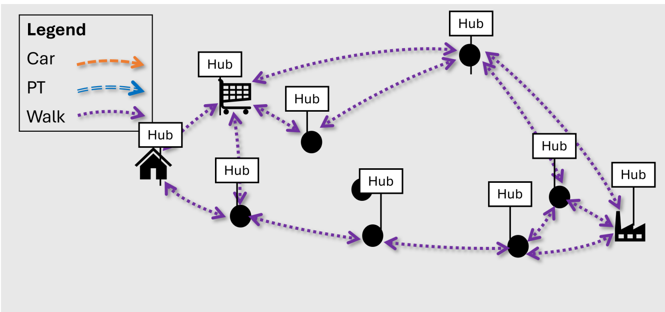
(a) The car network.



(b) The car system.



(c) The Public Transport (PT) system.



(d) The walk system.

Figure 3.2: Different mode and mode system networks from our example multimodal network in Figure 3.1

vehicles use must be at a hub, this mechanism makes sure to end of the public transport bike at a rental location.

In Table 3.1, examples are given of which mode can transfer on which of the example nodes. For example, one can not transfer from a car to another mode if there is no parking facility available. As there is no direct parking space available at Delft Station, it is not possible to transfer from car to another mode. Therefore, there is no transfer possibility at the Delft station for the mode car. A parking space is available at the car park and at home, so a transfer to another mode from the car mode is possible on these nodes.

Table 3.1: Transferability per mode system (rows) on some example nodes (the columns) in the Delft network. For each transfer possibility, it is indicated whether that transfer is a parking or hub.

Mode system	Delft station	Delft Zuid	Car parking	Home	Bus stop	Activity one
Car	✗	Parking	Parking	Hub	✗	✗
Walk	Hub	Hub	Hub	Hub	Hub	Hub
PT	Hub	Hub	✗	✗	Hub	✗
Bike	Parking	Parking	Parking	Hub	✗	Parking
$\text{OVfiets}_{\text{Delft station}}$	Hub	Parking	Parking	Parking	✗	Parking
$\text{OVfiets}_{\text{Delft Zuid}}$	Parking	Hub	Parking	Parking	✗	Parking
$\text{OVfiets}_{\text{any station}}$	Hub	Hub	Parking	Parking	✗	Parking

If we combine the mode systems with their hub and parking space information into one network, we will get the multimodal network. For example, when we combine the mode systems in Figure 3.2, we will get the multimodal network in Figure 3.1.

For each mode system $m \in M$, there is a weight component $w_m(e)$ on an edge $e \in E_m$. This weight component represents travel time, travel costs or a generalised cost function. The generalised cost function depends on the transport demand. The transport demand is explained in the next section.

3.2 Transport demand

In this model, there is a population P consisting of a finite number of people. Each person $p \in P$ may want to do activities. In order to do so, they visit the activity locations. The person will use the multimodal network to travel to the locations. Such activity locations are located on nodes in the multimodal network. The activities a person wants to do in a day are divided over tours. That is, a tour T is an ordered list of activity locations $T = (a_1, \ldots, a_{|T|})$, where $a_i \in V$ for $i \in \{1, \ldots, |T|\}$). If a person wants to travel to the activities in a tour T, they must originate from their home. The tour ends after the home is reached again, and all activities are visited in the given order. Therefore, the number of trips in a tour is $n = |T| + 1$. The travel between two activity nodes $a_1 \in V$ and $a_2 \in V$, within a tour, is called a trip. Therefore, a trip is a tuple of two activities (a_1, a_2). Each person $p \in P$ also has a home $a_0 \in V$, which is a node in the multimodal network. Additionally, a person can have multiple tours on a day. The list of tours is called the travel diary of the person. Furthermore, each person has a preference for each mode system that the person can use. This preference is reflected in the edge weights. Altogether, this population is called the transport demand.

3.3 Traffic assignment problem formulation

In traffic assignment, these two concepts are brought together. The goal is to assign transport demand to the multimodal network in a way that satisfies constraints and minimizes travel costs. To do this, we have trip paths, later referred to as paths. A path from $a \in V$ to $b \in V$ is a finite set of connected edges $(((a, v_1), (v_1, v_2), \ldots, (v_n, b)) \subset E)$, that is used to travel a trip from origin location $a \in V$ to destination location $b \in V$. Each edge in the path is labelled with the mode system used to traverse that edge. An edge can be used multiple times, if each time a different mode system is used. Note that the edges in each mode system subgraph are therefore unique, but not necessarily connected. When a node $v \in V$ in a path $((a, v_1), (v_1, v_2), \ldots, (v_n, b))$ is used to switch from one mode system to another, it called a transfer. Transfers can only occur if v is a transfer node in both mode systems, this is called transferability.

A tour route, later referred to as a route, is the combination of paths that a person $p \in P$ takes to visit all the activities in a specific tour T. The route starts and ends at the person's home ($a_0 \in V$) and visits each activity location in the order specified by the tour. The route paths consist of the first path, path zero, from home a_0 to the first activity location $a_1 \in T$, the paths between two consecutive activity locations (a_n to a_{n+1} for $n \in \{1, \ldots, |T| - 1\}$) and path $|T|^{\text{th}}$ from the final activity location $a_{|T|}$ back to a_0. In total, there are $|T| + 1$ paths for a tour T. Formally, the route is considered a walk, rather than one simple path, as two different trip paths may use the same edge with the same mode system, so not all edges are distinct. To overcome this, in the mathematical programming formulation, the network is expanded per trip. This is further elaborated in the next chapter.

Additionally, for a route to be valid, it must satisfy transferability and mode system requirements. The mode system requirements imply that, within a route, for all mode systems $m \in M$, the mode system may only start and end at a hub. This means whenever a mode system is exited at a parking node, it must re-enter the mode system at the same parking node later in the tour.

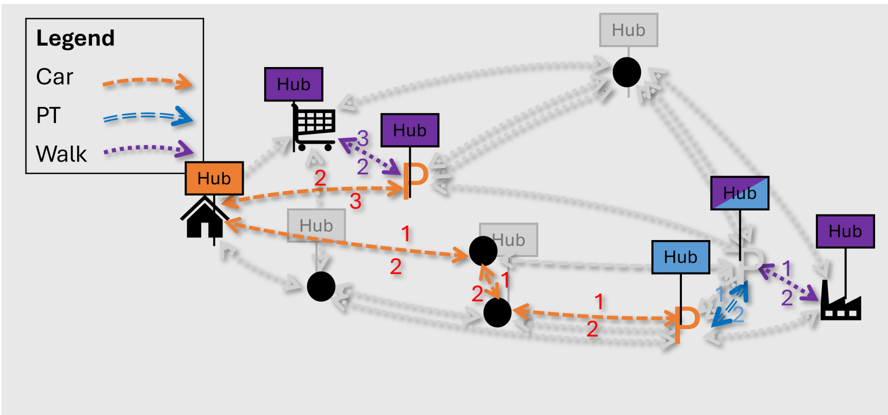


Figure 3.3: An example of a valid route assignment within the multimodal network from Figure 3.1. The edge labels indicate in which trip the edge is used. The hubs and parking spaces that are used, are colored with the mode system that is used.

An example of a valid route assignment is given in Figure 3.3. In this example, two activity locations are visited. In the first trip, the tour route uses a car to travel to a public transport stop, a drop-off pick-up constraint is activated on this transport stop (indicated with the orange P). So later in the tour the car must be picked up, which will be done in the second trip. The public transport and the walk mode systems have hubs on all nodes, so we do not have to worry about the drop-off pick-up constraint. In the second trip, the car is picked up and then driven to a second parking space near the second activity location. Similarly, the drop-off pick-up on this node is activated and lifted again in the last trip (trip 3). At the end of the tour the car will end up at the home location, which is a hub for the car, so the car use may end there as well. This makes a valid route assignment.

The optimal route for the tour T of a person $p \in P$ is the route that has minimizes the total edge weight. This naturally leads us to the multimodal traffic assignment problem, to find the most optimal route for every tour in the diaries of all individuals in the population P.

This problem formulation provides the foundation for modelling and solving traffic flows in the multimodal network. A new solution method is developed in the next chapter. This new algorithm will be evaluated in Chapter 4.

3.4 Properties of relevant models

In this section, we will contrast MSN (Liao, 2013; Georgiou, 2022) and the HAPP (Chow & Djavadian, 2015). We will select the desirable elements from each of these models, such they comply with the mode system formulation presented in Sections 3.1 to 3.3. Using these elements, we will state our proposed Integer Linear Programming (ILP) formulation. The newly developed method will be compared with MSN (Georgiou, 2022). The elements are outlined in Table 3.2. We will discuss each row of this table.

The most important capability is correct vehicle flow. What is meant with this concept, and why the HAPP from Chow and Djavadian (2015) does not comply, is explained in Section 2.3.1. A new version based of the HAPP will be presented in Section 3.5.1.

Second, in our mode system framework, we allow for two shared vehicles to be used after each other. In the MSN implementation of Georgiou (2022), this is not possible. However, by adding more states to the MSN, there exists ways to model this.

We consider fixed order of activities, since our input is a travel diary, where the order is already determined by Octavius (van Essen & Voorhorst, 2022). In both MSN and HAPP the order of visit of activities can still be determined. We do not consider this. Furthermore, our ILP does not allow for this, as we can not guarantee the prevention of subtours. This means that we do not have correct tour route, but a tour route not visiting all the activities and a separate cycle with some activities. Even if there is a method to prevent subtours, the new vehicle flow variables and constraints presented in Section 3.5.1 do also not allow for this feature. Luckily, Octavius already determines the order of activities in their tour generation model. The order of activities can easily be fixed in the existing MSN and HAPP works. For MSN, we have stated in Section 2.2, that this can be achieved by removing a row in the state transition graph. And in HAPP we can also easily update this by not allowing trips between activities that are not followed by each other.

All methods are individual route assignment methods, so individual networks can be considered by all algorithm.

An optional capability are the fixed mode system and mode system transfer costs are incorporated in HAPP, we can also implement this. This is stated in Section 3.5.2, however the complexity of adding these costs is not tested in the evaluation chapter.

Another optional capability is time scheduling using continuous time variables. The time scheduling is not needed in our research. Therefore, we will remove the continuous time variables, creating a ILP.

Our proposed solution and the MSN can incorporate the hubs and parking nodes from introduced in the previous sections.

Table 3.2: Comparison of requirements, input requirements and capabilities of our solution with other solution algorithms.

		(Georgiou, 2022)	(Liao, 2013)	(Chow & Djavadian, 2015)	Our solution
	Algorithm	MSN	MSN	HAPP	ILP
Requirements	Correct vehicle flow	✓	✓		✓
	A shared vehicle can be used while any other is still to be picked and dropped off		✓	✓	✓
	Fixed order of activities	✓[a]	✓[a]	✓[a]	✓
	Personalised networks	✓	✓	✓	✓
Optional capabilities	Fixed parking costs			✓	✓[b]
	Fixed mode system			✓	✓[b]
	Continuous time variables, including time window constraints, early or late penalties and variable parking costs			✓	
Input requirements	Hubs and parking nodes	✓			✓
Stability		✓	✓	✓	✓
Scalability	Number of parking nodes and number of mode systems	?	?	?	?

[a] Is also able to determine the order of activities, while this is not possible with our solution.
[b] Not considered in this chapter, but can be implemented as explained in subsection 3.5.2.

All the solution methods are stable, as they all compute an optimal tour. Whenever two different tours are found, they will have the same objective value.

In summary, MSN and ILP are both suitable for our problem. However, how well both algorithms scale, is unknown. This will be tested in Section 4.1. We will continue this chapter with stating our proposed ILP.

3.5 Proposed ILP

In order to visit an activity with a mode system, the activity node must be a transfer node. So in any case, an activity location can be vistited by the walk mode system, because each node $u \in V_m$ with $m =$ walk is a hub.

Below, a modified version of the HAPP in Chow and Djavadian (2015) is proposed. Time is removed, which removes all continuous decision variables. This only leaves the binary decision variables. So the HAPP turns into an Integer Linear Programming (ILP) problem instead of a mixed integer linear programming (MILP).

Now we will state the flow constraints, copied from Chow and Djavadian (2015). Their parking constraint is omitted, and our vehicle flow constraints in Section 3.5 must be added to account for correct vehicle flow and availability.

The ILP finds the optimal route for a person $p \in P$ with a tour $T = (a_1, \ldots, a_{|T|})$. Recall that the number of trips to complete tour T is $n = |T| + 1$. The first trip from home a_0 to the first activity a_1. Trip $i \in [n]$ goes from a_{i-1} to a_i, where the last trip $i = n$ goes from $a_{|T|}$ back to home $a_n = a_0$. For reference, see Figure 3.4. In this representation, the set of the first n integers is represented as $[n]$.

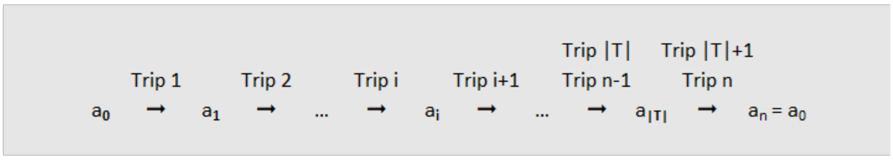


Figure 3.4: Trip numbering, for a tour $T = (a_1, \ldots, a_{|T|})$ with home location a_0. Trip $i \in [n-1]$ travels from activity a_{i-1} to a_i, and trip n travels from activity $a_{|T|}$ to a_0. The tour has $n = |T|+1$ trips.

Let $X^i_{uv,m}$ be the binary decision variable indicating whether the route for T uses the edge from node $u \in V_m$ to node $v \in V_m$ using mode $m \in M$ in trip $i \in [n]$. $X^i_{uv,m}$ is one if and only if the edge is used, otherwise $X^i_{uv,m}$ is zero. If there is no edge between node $u \in V_m$ to node $v \in V_m$, $X^i_{uv,m}$ does not exist and can be considered zero in the formulation. Later the vehicle flow constraints, presented in Section 3.5.1, will be added so that we valid have a route for tour T as described in Section 3.3.

$$\min \sum_{i\in[n]} \sum_{m\in M} \sum_{u\in V_m} \sum_{v\in V_m} c_{uw,m} X^i_{uw,m} \tag{3.1a}$$

subject to

$$\sum_{\substack{m\in M \\ \text{if } a_{i-1}\in H_m\cup S_m}} \sum_{v\in V_m} X^i_{a_{i-1}v,m} = 1, \qquad \forall i \in [n] \tag{3.1b}$$

$$\sum_{\substack{m\in M \\ \text{if } a_{i}\in H_m\cup S_m}} \sum_{u\in V_m} X^i_{ua_i,m} = 1, \qquad \forall i \in [n] \tag{3.1c}$$

$$\sum_{v\in V_m} X^i_{uv,m} - \sum_{v\in V_m} X^i_{vu,m} = 0, \qquad \forall m \in M, \forall i \in [n], \forall u \in V_m \text{ if } u \notin H_m \cup S_m \tag{3.1d}$$

$$\sum_{\substack{m\in M \\ \text{if } u\in H_m\cup S_m}} \sum_{v\in V_m} X^i_{uv,m} - \sum_{\substack{m\in M \\ \text{if } u\in H_m\cup S_m}} \sum_{v\in V_m} X^i_{vu,m} = 0, \qquad \forall i \in [n], u \in V \setminus \{a_{i-1}, a_i\} \tag{3.1e}$$

$$\sum_{i\in[n]} \sum_{m\in M} \sum_{u\in V_m} (X^i_{ua_{i+1},m} + X^i_{a_n u,m}) = 0, \tag{3.1f}$$

$$X^i_{uw,m} \in \{0, 1\}. \tag{3.1g}$$

In this formulation, Equations (3.1b) to (3.1f) are the flow conservation constraints. In Table 3.3, the transferability of some example node $u \in V$ is given and in Figure 3.5 the edges belonging to each of the flow situations on u are shown. This example highlights each of the possible flow cases for any node $u \in V$ and all mode systems $m \in M$. So for a general node $u \in V$, we have a constraint combining the flow of the modes where u is a transfer node for m (that is if $u \in H_m \cup S_m$). We have an additional flow conservation for each mode m where u is not a transfer node (if $u \notin H_m \cup S_m$).

Table 3.3: Transferability on example node u as seen in Figure 3.5.

Mode system	Edge colour	Transferability of u	u in
m_1		No	$\notin H_m \cup S_m$
m_2		Hub	$\in H_m$
m_3		Parking	$\in S_m$
m_4		No	$\notin H_m \cup S_m$
m_1		No	$\notin H_m \cup S_m$

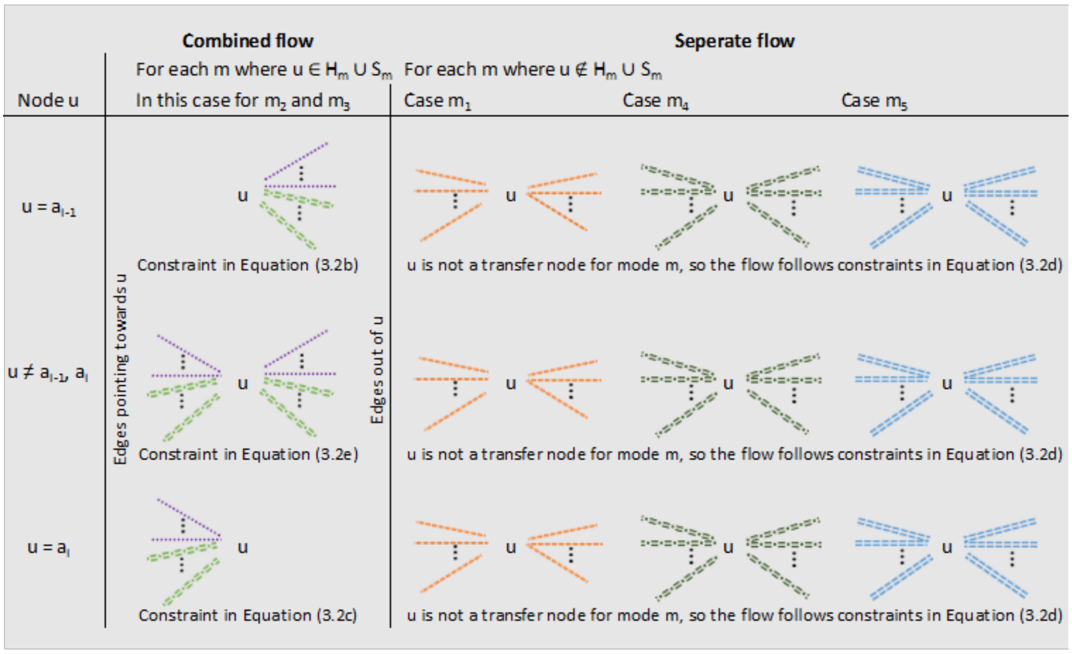


Figure 3.5: All possible flow constraints situations in Equations (3.1b) to (3.1e) for the example node u with transferability as in Table 3.3. All possible flow considerations are displayed. For the mode systems m_2 and m_3 we have transferability on u, and therefore a combined flow, while the other mode systems have no transferability ($u \notin H_m \cup S_m$), so their edges are not adjacent to other mode system edges at u. All edges point from left to right.

The meaning of each specific flow constraint is explained in this paragraph. It is required to have exactly one flow out and one flow in for each activity location (respectively Equations (3.1b) and (3.1c). In Equation (3.1b) for each trip $i \in [n]$ the trip origin node is a_{i-1}, so exactly one edge must originate from this node. In Equation (3.1c) for each trip $i \in [n]$, the trip destination node is a_i, so exactly one edge towards a_i must be used. Particularly in the last trip (when $i = n$), the destination is a_n which represents the home node a_0. The flow conservation constraints (Equations (3.1d) and (3.1e)) are split into two parts. On each none activity node $u \in V \setminus \{a_{i-1}, a_i\}$ in trip $i \in [n]$ we have a flow conservation for each mode system $m \in M$. If the node is a transfer node for mode $m \in M$ (that is if $u \in H_m \cup S_m$), then a transfer to another mode system is possible. In that case, the flow conservation is combined in Equation (3.1e). Otherwise, the flow can only go to and from edges of the same mode system m. This is resolved in Equation (3.1d).

Lastly, the binary decision variables are set with Equation (3.1g).

This formulation prevents subtours, as the tour must set consecutive paths. That is, the activities from tour T must be visited in order. In trip $i \in [n]$, the constraint in Equation (3.1b) makes sure that exactly one edge must be used to exit activity a_{i-1}. And the constraint in Equation (3.1c) makes sure that exactly one edge must be used to enter activity a_i (and a_0 whenever $i = n$). The only way the flow between a_{i-1} and a_i is satisfied is by having a path from a_{i-1} to a_i using the binary decision variables $X^i_{uv,m} \forall u, v \in V, m \in M$.

3.5.1 Proposed required vehicle flow

As discussed in the related work (Section 2.3.1) the formulation of Chow and Djavadian (2015) includes unwanted tours. In this section, an alternative constraint set is presented removing these unwanted tours.

This issue can be resolved in two ways

1. Add ordering (or non-zero time) variables, for every vehicle add that the drop-off must be before the pick-up at all nodes except station nodes of that vehicle.

2. Add flow variables for every vehicle usage and use flow conservation constraints on every vehicle. Flow conservation can be created when a parking variable is introduced on all non-station nodes of the vehicle network. On station nodes, you can drop off or pick up any vehicle, therefore no flow conservation is needed on these station nodes.

Option two is chosen, because it introduces less extra variables. A parking variable is introduced: $Y^i_{u,m} = 1$ if and only if mode system $m \in M$ is parked at parking node $u \in S_m$ at the end of the agent's trip $i \in [n]$, otherwise $Y^i_{u,m}$ is zero.

Add equations Equation (3.2) to Equation (3.1) to have flow conservation on the vehicle.

$$\sum_{v \in V_m} X^i_{uv,m} + Y^i_{u,m} - \sum_{v \in V_m} X^i_{vu,m} - Y^{i-1}_{u,m} = 0, \quad \forall i \in \{2, \dots, n-1\}, \forall m \in M, \forall u \in S_m \tag{3.2a}$$

$$\sum_{v \in V_m} X^1_{uv,m} + Y^1_{u,m} - \sum_{v \in V_m} X^1_{vu,m} = 0, \quad \forall m \in M, \forall u \in S_m \tag{3.2b}$$

$$\sum_{v \in V_m} X^n_{uv,m} - \sum_{v \in V_m} X^n_{vu,m} - Y^{n-1}_{u,m} = 0, \quad \forall m \in M, \forall u \in S_m \tag{3.2c}$$

$$Y^n_{u,m} = 0, \quad \forall m \in M, \forall u \in S_m \tag{3.2d}$$

$$X^i_{uw}, Y^i_{u\cdot} \in \{0, 1\} \qquad \forall i \in [n] \tag{3.2e}$$

Note that this only works when the ordering of activities is fixed. Otherwise, sub-tour elimination constraints must be added to avoid disconnected cycles.

This constraint ensures the following rule is satisfied:

1. A vehicle, has vehicle flow. The vehicle is only available at a hub, or at a place where the vehicle is last dropped off.

This rule may be added to the six rules of Najmi et al. (2020) presented in Section 2.3.1.

3.5.2 Incorporating additional costs

Additional costs to different parts in a journey can be added to the utility costs. In this section we discuss how fixed mode system costs and transfer costs may be added to the formulation in Equations (3.1) and (3.2).

We will start with fixed costs for using a mode system in a tour. This can be added by introducing the binary decision variables W_m for each mode system $m \in M$. Where W_m indicates whether the mode system is used in a route. The variable has a cost component of $w_m \geq 0$, which is the fixed costs of using the mode system m.

$$X^i_{uv,m} \leq W_m, \qquad \forall m \in M, u, v \in V_m, i \in [n] \tag{3.3a}$$

$$W_m \in \{0,1\} \qquad \forall m \in M. \tag{3.3b}$$

The constraint of Equation (3.3) essentially states that whenever any link of the mode system $m \in M$ is used. Then the decision variable W_m must be one, reflecting adding the fixed costs w_m for mode system m to the objective function. If no link of mode system m is used, then the decision variable W_m may be zero, resulting in no costs for mode system m.

Transfer costs can also be incorperated in the utility function. In addition to the formulation in Equations (3.1) and (3.2). This can be achieved by adding binary decision variables Z^i_{v,m_1,m_2} for mode systems $m_1, m_2 \in M$, a node $v \in V$ both in V_{m_1} and V_{m_2} (that is $v \in V_{m_1} \cap V_{m_2}$) and a trip $i \in \{1, \ldots |T|\}$. Which is one if and only if the transfer from m_1 to m_2 is made on node v in trip i. The associated transfer costs is represented as $w^i_{v,m_1,m_2} \geq 0$.

$$\sum_{u \in V_{m_1}} X^i_{uv,m_1} + \sum_{w \in V_{m_2}} X^i_{vw,m_2} - 1 \leq Z^i_{v,m_1,m_2}, \qquad \begin{array}{l}\forall m_1, m_2 \in M,\\ v \in V_{m_1} \cap V_{m_2}, i \in [n]\end{array} \tag{3.4a}$$

$$Z^i_{v,m_1,m_2} \in \{0,1\} \qquad \begin{array}{l}\forall m_1, m_2 \in M,\\ v \in V_{m_1} \cap V_{m_2}, i \in [n].\end{array} \tag{3.4b}$$

This is correct because if in trip i an edge from u to v with mode m_1 is used then the first sum is one, and if afterwards an edge from v to w is used with mode m_2 then the second sum is one. Making the left side of Equation (3.4a) one, making $Z^i_{v,m_1,m_2} = 1$, indicating that the transfer costs apply.

The constraints in Equations (3.3) and (3.4) are not tested for if they add decision complexity. It may be more complex, as adding fixed costs for a mode system $m \in M$ may result in preferring an alterative mode system over m. And after relaxing Equations (3.3b) and (3.4b), it is possible to have fractional solutions. Which requires more branch and bound or cutting planes.

Chapter 4

Model evaluation

This chapter compares the scalability performance of proposed binary Integer Linear Programming (ILP) and the Multi-state Super Network (MSN).

Requirements, optional capabilities and stability between the proposed ILP, the existing HAPP of Chow and Djavadian (2015) and the Liao (2013) are already discussed when selecting an appropriate method in Section 3.4 and Table 3.2.

The ILP model stated in Equations (3.1) and (3.2) is optimised with Gurobi Optimizer version 11 (Gurobi Optimization, LLC, 2024) and compared with the MSN implementation of Georgiou (2022) with a slight alteration. The order of activities is now fixed. The fixed order is achieved by removing the row which represents that only activity two is visited, eliminating the possibility to first complete activity two and then activity one.

4.1 Scalability

Scalability are tests on how well the algorithms scale in computation time and computation space as the number of decision possibilities increases. In the first experiment the number of parking spaces is increased, in the second experiment the number of shared vehicles is increased. The runtime to calculate the route for the tours of both algorithms is presented.

The instances of the experiments are the same for both algorithms. The output is equal, as both methods compute the optimal route on the same problem. Every new run of both algorithms a clean state is used, that is, no information of a previous iteration is used for the optimization.

The MSN and ILP models grow differently in size when the number of decision possibilities is increased. For example, in experiment one, an extra parking space for one mode system is added. In the ILP, this requires one extra constraint for every trip and one extra Y decision variable for every trip. While in MSN it reflects adding one column to the state transition matrix to account for the new state. For example, in Figure 2.3 a column is added for every new parking space u, which indicates whether the car is parked at parking space u.

4.1.1 Complexity analysis

Without the vehicle flow constraints in Equation (3.2), our ILP with only flow constraints in Equation (3.1) is an instance of the Shortest Path Tour Problem (SPTP) described in Gao et al. (2024):

Definition 4.1.1. "(Shortest Path Tour Problem (SPTP)). Given a directed graph $G = (O \cup N \cup D, E, W)$ with non-negative weights, the length $L(P)$ of a path P is defined as the sum of weights of the edges connecting consecutive nodes in P. Given an origin node $r \in O$, a destination node $s \in D$ and a disjoint node subsets $N_1, N_2, \ldots, N_l, (|V_i| \leq r, \forall V_i)$, where r represents the maximum size of node subsets, the aim is to find the shortest path P, which starts from r, sequentially passes through at least one node in each node subset, and ends at s." (Gao et al., 2024, p. 341)

In our case, the number of destinations in the subsets V_i for each i is one ($r = 1$). If destination choices would also be included, then $r \geq 1$. The SPTP problem without drop-off pick-up constraints can be solved in polynomial time in the number of nodes with time complexity $O(n^3)$ (Gao et al., 2024, p. 342), where n is the number of nodes in the multimodal network, here activity and transfer nodes. However, adding the vehicle flow constraints Equation (3.2) does make the problem more complex. To give insight on the complexity, we will take a more general approach.

Our formulation in Equations (3.1) and (3.2) is a binary Integer Linear Programming (0-1 ILP) formulation of the form,

$$\begin{aligned} &\min c^{\intercal} x \\ &\text{s.t. } Ax = b, \quad x \in \{0,1\}^{\mathbf{n}}. \end{aligned} \tag{4.1}$$

This is a minimization problem determining binary decision variables $x \in \{0,1\}^{\mathbf{n}}$, where $A \in \{-1,0,1\}^{\mathbf{m}\times\mathbf{n}}$ is a $\mathbf{m}$ by $\mathbf{n}$ matrix consisting of all entries from $\{-1,0,1\}$, $b \in \{0,1\}^{\mathbf{m}}$ is a column vector of length $\mathbf{m}$ with entries $\{0,1\}$, and $c \in \mathbb{R}^{\mathbf{n}}_{\geq 0}$ is a column vector of length $\mathbf{n}$ with non-negative real values.

Since the entries in A and b are all in $\{-1,0,1\}$ it is beneficial to have A be totally unimodular. In this case, the relaxation

$$\begin{aligned} &\min c^{\intercal} x \\ &\text{s.t. } Ax = b, \quad x \geq 0, \quad x \leq 1, \end{aligned} \tag{4.2}$$

yields only integer solutions as optimal value. Making the optimal value of eq. (4.1) easy to find by solving eq. (4.2), which can be solved in polynomial time.

In most cases in the experiments below, the root relaxation method finds integer solutions, suggesting that the matrix is totally unimodular. However, we encountered instances where the matrix is not totally unimodular in some experiments this does not happen. This implies that our formulation is not a perfect formulation. Considering the concept of drop-off pick-up constraint, it was also not expected for our formulation to be perfect.

In general, there exists a pseudo-polynomial algorithm for solving $\mathbf{m} \times \mathbf{n}$ integer programs, with fixed $\mathbf{m}$. In particular, consider the binary integer linear program with $A \in \{-1,0,1\}^{\mathbf{m}\times\mathbf{n}}$ and $b \in \{-1,0,1\}^{\mathbf{m}}$ (and $c \in \mathbb{R}^{\mathbf{n}}$) of the form,

$$\begin{aligned} &\min c^{\intercal} x \\ &\text{s.t. } Ax = b. \end{aligned} \tag{4.3}$$

Then Equation (4.3) has a pseudo-polynomial algorithm which can be carried out in time

$$O(\mathbf{n}^{2\mathbf{m}+2}\mathbf{m}^{(\mathbf{m}+1)(2\mathbf{m}+1)}),$$

which is a polynomial in $\mathbf{n}$ if $\mathbf{m}$ is fixed (Papadimitriou, 1981).

However, in our formulation in Equations (3.1) and (3.2) the number of columns $\mathbf{m}$ is not fixed. In particular, if the number of mode systems or parking spaces is increased, rows are added to Equations (3.1) and (3.2), increasing $\mathbf{m}$ in Equation (4.3).

The number of variables ($\mathbf{n}$ in Equation (4.3)) is

$$\mathbf{n} = |T||E||M| + |T||M| \sum_{m \in M} |S_m|,$$

where $|T|, |E|, |M|$ and $\sum_{m \in M} |S_m|$ are the number of trips, the number of edges, the number of mode systems and the number of parking nodes, respectively. The number of constraints ($\mathbf{m}$ in Equation (4.3)) is

$$\mathbf{m} = 2|T||M| + 1 + |T||M||V| + |T||M| \sum_{m \in M} |S_m|,$$

where $|V|$ is the number of nodes in the super network.

4.1.2 Runtime when the number of parking spaces is increased

In each iteration, one random parking space $u \in N$ is added to the list of available parking spaces for the car mode system.

In this experiment, the following elements are kept constant: The network used is the Delft network with 475 nodes and 1404 edges, with one agent completing a tour. The tour exists of two activity locations. The available mode systems are private car, public transport and walking.

The number of parking nodes in the private car mode system is increased every run. Each time the same list of parking nodes is used, in the next iteration, one randomly selected node that is reachable by car to and from the home node is added.

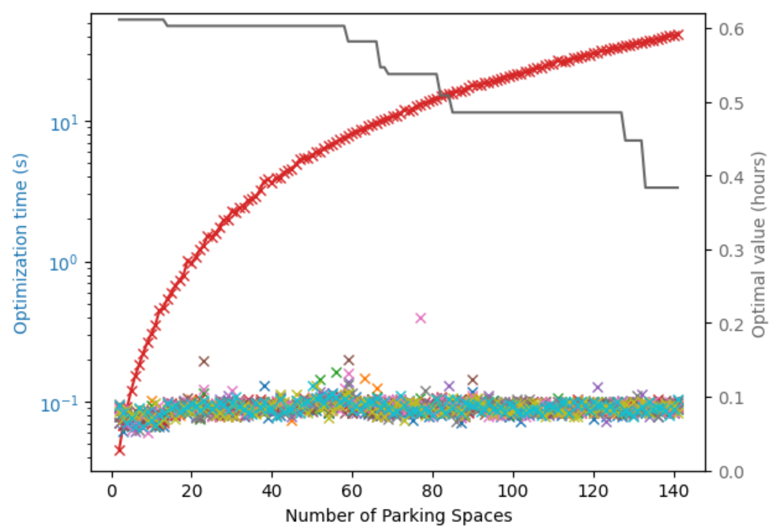


Figure 4.1: The case of adding one parking space to the set of parking spaces (x-axis). Difference in computation times between the ILP (10 runs per x value, seen in all the coloured dots at the bottom) and the MSN model (1 run per x value, seen in the red line) in logarithmic scale. And the optimal value of the computed route for each case (the gray line with the gray y-axis).

Figure 4.1 shows the runtime of the MSN and the ILP as parking spaces are added, up to 142 parking spaces. This is enough to encompass the complexity of the algorithm. The optimal value is included to demonstrate that there is no correlation between an increase in running time and a decrease in the optimal value. This figure further illustrates that when, a parking space is added, that will either result in a better route or stay the same. This is because adding a parking space may only decrease the travel time of the car mode.

The model build time of ILP is not included, this is takes about 0.4 seconds to 0.7 seconds and stays constant. This is not relevant as preprocessing can already account for a model build.

ILP runtime specifics

For each run, the Gurobi solver (Gurobi Optimization, LLC, 2024) performs the following steps:

- Presolving: This step removes redundant constraints and variables. In this case, 3,267 constraints and 10,824 variables are eliminated out of 6,973 constraints and 16,305 variables, respectively. This step takes 0.09 seconds.

- Root Relaxation: After presolving, the root relaxation finds the optimal solution in 0.02 seconds. Interestingly, even though the problem only involves binary decision variables, the optimal solution remains integer after relaxing the integer constraints in Equations (3.1g) and (3.2e). Thus no fractional routes are generated.

4.1.3 Runtime when the number of mode systems is increased

Two experiments were done using the ILP. The experiments try to find a route for the same tour as in the previous experiment. In the first experiment, the ILP allowed for 142 available shared public car systems. Where each car system has the same two parking spaces located close to the defined activity locations. And the hub node of each shared car system is drawn randomly from the node list. This experiment has an optimization time of 7.19s and a build time of about 46s.

A second experiments included 120 shared vehicles, each with randomly drawn hubs and all with the same 25 parking spaces. This experiment gives a build time of about 40s and an optimization time of 7.93s.

This increase in the number of mode systems is not tested on MSN, since the MSN implementation of Georgiou (2022) only supports up to two shared mode systems, with each two parking spaces. In this study we did not extend the given implementation of Georgiou (2022). Therefore, there are no results on the running time of MSN with increasing the number of mode systems.

Chapter 5

Discussion, conclusion and recommendations

The aim of this thesis was to develop a microsimulated tour based multimodal traffic assignment model with drop-off pick-up and availability constraints. The demand for more accurate traffic simulation models, requires us to develop a such a traffic model which takes in consideration new non-line based modes of transportation, that naturally evolve because of new developments as MaaS. This leads to traffic assignment on an individual person level, as opposed to traditional traffic assignment with origin destination matrices. This makes traffic simulation more accurate, generate consistent tours with drop-off pick-up and availability constraints and enables us to find tour routes of specific agents or agents within a specific demographic group. We proposed a new multimodal network representation with mode systems that require the information of hubs and parking spaces for each mode system. This new network representation inherently adopts drop-off pick-up and availability constraints that some of the existing research may be lacking (Chow & Djavadian, 2015). We have adjusted an existing MILP formulation for the HAPP to an ILP formulation (a mathematical description) that can describe the multimodal tours with drop-off pick-up and availability constraints. In a case study, we computed optimal tours that are realistic with the model input. After increasing the problem decision complexity by adding more parking spaces or mode systems, we compared the performance of our ILP formulation, solved with a commercial MILP solver (Gurobi Optimization, LLC, 2024), with an existing implementation of MSN (Liao, 2013; Georgiou, 2022), solved with Dijkstra. We observed that the new ILP is faster with computing these tours. Compared with the MSN implementation, the ILP moved the complexity of the problem into the solution algorithm of a ILP, explaining the better performance. Furthermore, we are able to generate routes with non-line-based multimodal transport modes, like shared vehicles. With this method, we will be able to determine the tour routes of all agents. With this, we will be able to see the influence of newly added Park and Ride systems and newly added MaaS providers.

5.1 Discussion

The solution algorithms is currently only tested on a small network (with 475 nodes and 1404 edges). However, this is also the case for the MSN. The compared MSN solution model is outperformed by the ILP solver. It is not likely that both models, the MSN and the ILP are optimally implemented. MSN model calculates shortest paths between two nodes that will not likely be in the optimal tour. For the ILP, we think that the formulation can be improved to allow more decision complexity. This improvement is needed to allow make larger networks more tractable. We have seen that adding more complexity in the number of parking spaces or the number of mode systems, does not result in a huge difference of running time.

To improve the ILP, we recommend investigating into decoupling the formulation. Such a decoupling can be done with methods like Benders' decomposition (Benders, 1962) or Danzig-Wolfe (Dantzig & Wolfe, 1960). This is promising, since the flows for different mode systems are independent of each other except at the transfer points. As discussed in the Section 4.1.1, adding the parking constraint to the ILP makes the problem NP-hard. So decoupling the complexity of determining which mode combinations and which parking spaces will be used from the route determination, between pairs of transfer nodes, is recommended.

The ILP may also be improved by adding cutting planes to the 0-1 ILP formulation. Examples are cover inequalities, for example when at most one out of a set of edges may be used. This is the case for two parallel paths of the same mode system, or similar parallel paths of two different mode systems. As a heuristic, you can exclude all parallel paths in a trip. A potential downside of this is that this eliminates the trip which uses the same edge in the same trip. This can happen with specific alignments of public transport lines, where the access trip uses edges in the same direction as the public transport line.

Current traffic assignment models can be replaced by our new method, as long as the network stays small. One can consider to only generate the routes of users who have a high probability of using multiple modes in their tour.

Our algorithm is not able to include the destination choice from the demand model in Section 1.1. This is because you need to carefully handle the route flow constraints and the newly added vehicle flow in Equation (3.1) after visiting an activity location. More investigation needs to be done if the destination choice needs to be included in the ILP. While still being consistent with the mode system network and the drop-off pick-up and availability constraints. Since we can not include the destination choice, it is even harder to model the tour generator from the demand model into the ILP formulation.

When thinking of the drop-off pick-up constraint, you might consider taking a child to the daycare facility or school location and later having to pick this up. This can not be modelled as a mode system. To resolve this, one can add additional activity locations for these visits.

Our ILP formulation is also flexible for determining alternative routes. In Appendix B, we show that after the ILP determines the optimal route, an additional constraint can be added to determine alternatives. The ILP solver, will remember most of the calculations, which benefits the flexibility of the ILP formulation.

5.2 Recommendations

For future research on micro simulated multimodal traffic assignment models, we recommend the following research directions.

- Use the newly developed mode system requirements and programming formulation for any future micro simulated tour generator. This creates a structured approach to deal with drop-off pick-up and availability constraints imposed by choices made earlier in the tour.

- To allow the ILP to work on networks with more nodes and edges, we recommend looking into decoupling methods such as Benders' decomposition or Danzig-Wolfe. Since the structure of the ILP looks promising for decoupling, by splitting the path calculation and the mode combination choices.

- The output of the microscopic demand model Octavius can already be used as input for the ILP and MSN. This will result in the ability to view tour routes of specific demographic groups. It is especially interesting for the agents who live in an environment that enables a high probability to use new shared transport such as shared cars, bikes or scooters.

- Vo et al. (2020) applied the HAPP to find joint activity patterns, which are time schedules to visit activities that need to be visited by any member of a household. To create dependencies between household members, this model can be further investigated and expanded upon.

5.3 Acknowledgement

This research commissioned by Goudappel in the form of a master thesis research at University of Twente. Some parts of this thesis are written with help of the writing centre of the University of Twente and ChatGPT. ChatGPT was only used to adjust the writing style and for feedback on some sections.

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Appendix A

The necessity of the three constraints in the introduction

Examples in which type of modes from Table 1.1 require the drop-off pick-up and availability characters will be explained in this appendix. There are a few types of modes, some are described by Machado et al. (2018) and some other this study also incorporates.

Starting with line-based public transport mode. This mode has defined transport lines with certain frequencies and stops. To access this transport, one needs access or egress modes, making the line-based public transport requiring multimodal tips.

There are numerous micromobility vehicle sharing options, which any user is able to use the vehicle, this means all of these options have the availability constraint. There are three levels of movement of these vehicles, free floating, station-based one-way or station-based round-trip. Free floating vehicles can be placed anywhere within a zone, making it an option that does not have to be a multimodal trip. In contrast, Station-based models have certain places where the shared vehicle can be picked up or dropped off. Because of the station constraint, you likely first need to get access or egress to a certain station, making the station-based modes require a multimodal trip. The two types of station-based shared transport are one-way or round-trip. One-way station-based do not require the vehicle to return to the original station, but a station near the destination needs to be available, hence another availability, example is a 'citi' bike system in New York. Round-trip shared vehicles requires the vehicle to be brought back to the original pick-up point, hence the drop-off pick-up constraint, examples include the Dutch 'OV fiets' or 'Greenwheels'.

Onto ride-sharing options, first in the context of carpooling, this is that a (part of) trip is common with another user, so they travel with the same car. We have two perspectives, one from the passenger which has to deal with the availability constraint, and might need an access or egress mode during their trip, the access can be a vehicle which needs to be picked up later in the tour, which then requires the drop-off pick-up, but this is a property of that vehicle and not with the carpool mode. From the perspective of the driver, the driver might need to drop off the passenger later in the tour at the pick-up point, hence the optional drop-off pick-up constraint. Other ride-share services, like Uber or autonomous vehicles, allow the passenger to call the vehicle, which can bring the passenger to their origin and destination, optionally there is an egress mode, these trips deal with the availability.

Another mode of shared transport is personal vehicle sharing, this can be in the form of Peer to Peer (P2P), where one owner makes their car available for others with their permission. Or fractional ownership, where a group of users owns a car. The availability constraint is included because the car might be used by other users. Furthermore, the car needs to get back like in a round-trip sharing system, therefore the drop-off pick-up constraint is included.

Then Park and ride trips inherently possess all these properties. First, you will have to drive to a park and ride, requiring the availability of a parking spot in the park and ride farcicality. Next, you will have to take another transit mode, requiring a multimodal trip. Lastly, you will have to pick up the car later in the tour, hence drop-off pick-up. A regular car parking also requires availability of the parking spot. A trip using a regular car parking is only multimodal if the parking is too far away from the origin or destination. Furthermore, a trip using a car parking requires the drop-off pick-up constraint as the car needs to be picked up later.

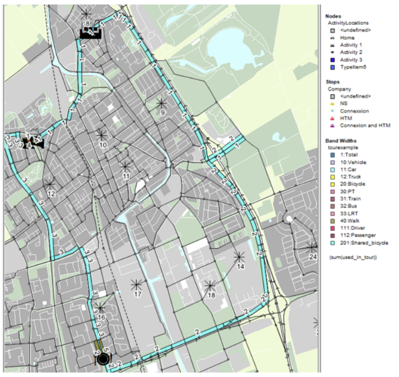
Then lastly, another use of the drop-off pick-up constraint is bringing children to a daycare or school facility. In these tours, the children need to be picked up again, bringing in a drop-off pick-up constraint. This however is not included in Table 1.1, as this project does not model them using the modelling structure of Chapter 3. The concept can be modelled in this project by adding the daycare or school facility as an activity location. As also discusses in the disussion.

Appendix B

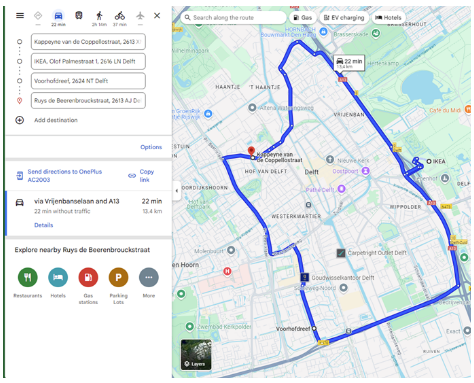
Proof of concept

B.1 Example tour route

The tour in Figure B.1 with a car to IKEA Delft and a restaurant in the south produces a tour route that takes 23.21 minutes. While Google Maps states it takes 22 minutes.



(a) Example tour using a private car (blue) and walking to the second activity (green)

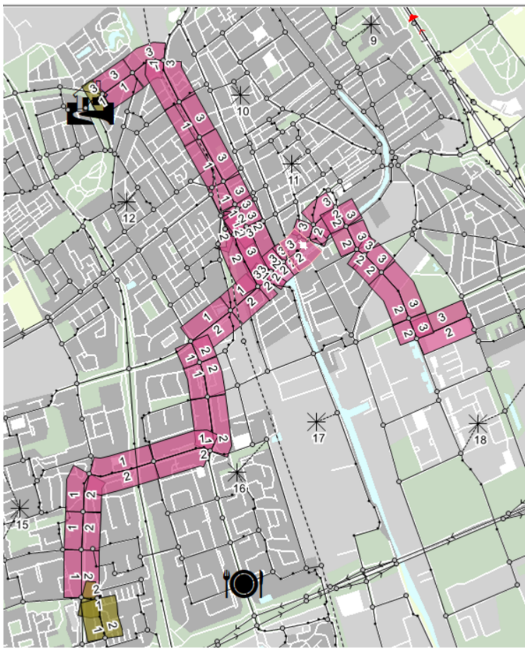


(b) Google Maps result of the example tour with a similar travel time as in Figure B.2

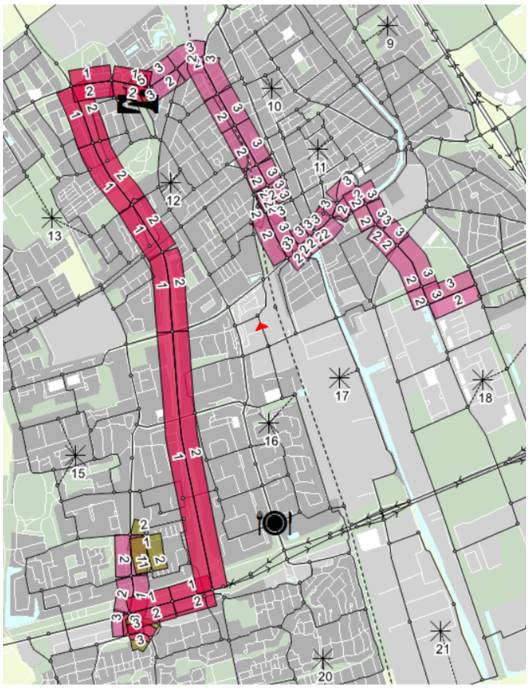
Figure B.1: Example tour route found with the ILP algorithm compared to the tour route found in maps.

B.2 Alternative routes

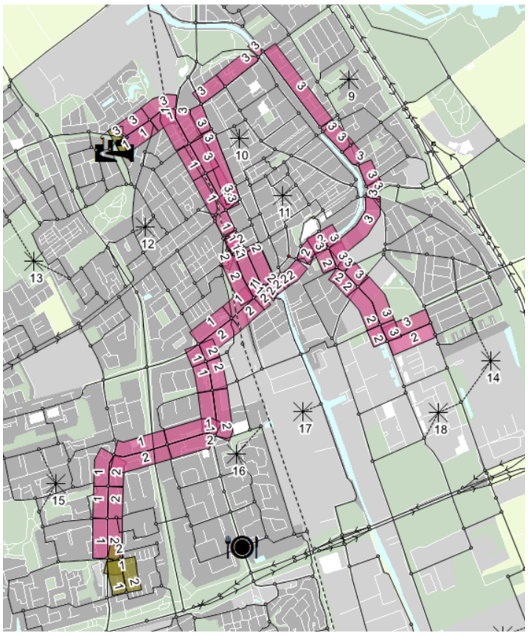
In this proof of concept, we will present a tour for which we will find alternative routes after removing a specific edges with mode, that were in the original optimal solution. Removing means that the specific edge may not be used by that mode in a newly calculated tour. The available modes were public transport, walking, private car, private bike and shared bike. The bike mode systems were never used in the optimal tour. The results can be seen in Figure B.2, on the next page.



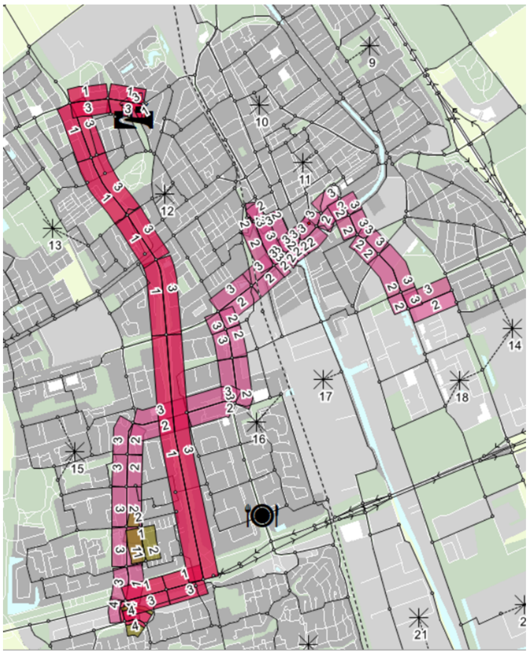
(a) Optimal tour (travel time 39.00 minutes).



(b) Optimal tour after removing the ability to walk to the first transit stop in the first trip (the trip with label 1) (travel time 39.62 minutes).



(c) Optimal tour after removing a specific transit link in the last trip (with label 3) (travel time 43.69 minutes).



(d) Optimal tour after removing the ability to walk from the last transit stop in the last trip (with label 3) (travel time 43.79 minutes).

Figure B.2: The optimal tour and alternatives after removing certain edges for certain modes. The numbers next to the edges indicate in which trip they are utilized. The colour of the edges represent the modes used (pink being public transport, green being walking and red being the private car).

Appendix C

Existing HAPP formulation

In this appendix, we will state the Mixed Integer Linear Programming (MILP) formulation of Household Activity Pattern Problem as stated in Chow and Djavadian (2015). This is only to be used as reference to Section 2.3.1. Where we have stated an example which does not comply with the drop-off pick-up and availability concepts in this project. First, we state the MILP formation. After which, we update the formulation to our research specifics. These are the removal of time variables and fixed order of activities. Note that this formulation still has the incorrect parking constraint. The correct ILP is presented in Section 3.5.

C.1 The new ILP version with incorrect parking constraint

In our research problem, the order of activity visits is already known. Therefore, we will modify the above formulation in which the order of visits is fixed. Furthermore, we do not need to know at what time each activity is visited, so we also removed the continuous time decision variables. What is left is an Integer Linear Programming (ILP) formulation. In this version we also have that the example from Section 2.3.1 works, and so the drop-off pick-up and availability constraints are also not satisfied in this formulation. In Section 3.5.1, we state new constraints that add vehicle flow. This way the drop-off pick-up and availability concepts are fully correct in our research. The parking constraint in Equation (C.2f) must be replaced by Equation (3.2) from Section 3.5.1 to achieve this.

In this appendix, different notation for of the variables is applied. This notation will only be used in this appendix.

The difference is the removal of time, the penalties for late or early delivery is omitted. The feature added is the fixed order of activities. A constraint, which includes the drop-off pick-up for more nodes other than parking, will be added later. Denote $V = \{1, 2, \ldots, |V|\}$ as the set of available modes, with 1 set default to automobile, 2+ for other modes for example shared vehicle or public transit. Zero refers to walking. The activity nodes are represented with N, each activity $n \in N$ must be visited after visit $n+1$. Other nodes in the network are represented by $\Lambda = \{\Lambda_1, \Lambda_2, \ldots, \Lambda_{|V|}\}$ these nodes represent parking facilities (Λ_1) or transit stops. Let O be the home node and D the return home node. The link costs variable t_{uw} is the travel time between u and w and c_{uw} is the monetary mode cost between u and w. The monetary mode costs include transit fares $v \geq 2$ and tolls $v = 1$.

X_{uw}^{rs} is a binary decision variable that indicates that a leg from node $u \in \{r, \Lambda\}$ to $w \in \{s, \Lambda\}$ for travel in a trip from node $r \in \{O, N\}$ to $s \in \{N, D\}$. The mode of a node is determined by the facilities u and w. If u and u belong to the same Λ_v, then the travel is by mode $v \in V$, otherwise by walking.

$$\min \sum_{k \in K} \beta_k Z_k \tag{C.1}$$

Where k is a combination of the objectives in (C.2a,C.2b)

$$\sum_{r \in \{N,O\}} \sum_{u\{r,\Lambda\}} \sum_{w\{r+1,\Lambda\}} c_{uw} X_{uw}^{r,r+1} \quad \text{Modal travel cost (includes fares } (v \geq 2) \text{ and tolls } (v = 1)) \tag{C.2a}$$

$$\sum_{r \in \{N,O\}} \sum_{u\{r,\Lambda\}} \sum_{w\{s,\Lambda\}} t_{uw} X_{uw}^{r,r+1} \quad \text{Travel time} \tag{C.2b}$$

subject to

$$\sum_{w \in \{r+1,\Lambda\}} X_{rw}^{r,r+1} = 1, \qquad \forall r \in \{O, N\} \tag{C.2c}$$

$$\sum_{u \in \{s-1,\Lambda\}} X_{us}^{s-1,s} = 1, \qquad \forall s \in \{D, N\} \tag{C.2d}$$

$$\sum_{w \in \{r+1,\Lambda\}} X_{uw}^{r,r+1} - \sum_{w \in \{r,\Lambda\}} X_{wu}^{r,r+1} = 0, \qquad \forall u \in \Lambda, r \in \{O, N\} \tag{C.2e}$$

$$\sum_{r \in \{N,O\}} \sum_{w \in \{\Lambda_1,N^-,D\}} X_{uw}^{r,r+1} - \sum_{r \in \{N,O\}} \sum_{w \in \{\Lambda_1,N^-,O\}} X_{wu}^{r,r+1} = 0, \qquad \forall u \in \Lambda_1 \tag{C.2f}$$

$$\sum_{r \in N} \sum_{u \in \{r,\Lambda\}} X_{uO}^{rO} + \sum_{s \in N} \sum_{w \in \{s,\Lambda\}} X_{Dw}^{Ds} = 0, \tag{C.2g}$$

$$X_{uw}^{r,r+1} \in \{0, 1\}. \tag{C.2h}$$

In this formulation, Equations (C.2c) to (C.2e) and (C.2g) are the flow conservation constraints, where it is required to have exactly one flow out and one flow in for each activity location (respectively Equations (C.2c) and (C.2d). In Equation (C.2c) the destination home node $s \in S$ is excluded to have no outflow from s. Similarly, the source home node $r \in O$ is excluded from Equation (C.2d). These two exclusions are also reflected in Equation (C.2g). Additionally, at each node, we have that the incoming flow equals the outgoing flow Equation (C.2e). Equation (C.2f) is the drop-off pick-up constraint for each parking node $u \in \Lambda_1$, once you enter a parking space u from the car network, you also have to exit the parking node into the car network at a different time. Lastly, the binary decision variables are set with Equation (C.2h).

This formulation prevents subtours, as there is no path into activity r after visiting activity r. There is no subtour within a trip from activity r to $r + 1$ because in this trip from constraint Equation (C.2c) exactly one edge must be used to exit activity r and from constraint Equation (C.2d) exactly one edge must be used to enter activity $r + 1$, the only way the flow at the other end of these edges is satisfied, is by having a path from activity r to $r + 1$.

C.2 Smaller counter example

The parking constraint in this formulation is not the drop-off pick-up constraint for our problem formulation. One can enter the car system from anywhere. It does not necessarily have to be where the car is currently located. A counter example would be network in Figure C.1 with a fixed order of visit (0 to 1 to 2 to 3) and the making the links 0-7, 0-6 very expensive. This can be the case if 0 is in a city centre not well accessible by car, but easily accessible by public transport. In that case, the optimal route will be 0-8-9-1-6-7-2-7-6-1-9-8-3. This is a feasible route in formulation in Appendix C.1. This is an issue because the car of the agent may only start and end at the home location, in this tour the car starts and ends at node 6. If parking costs are determined from this tour, there will be a negative parking costs at parking 6.

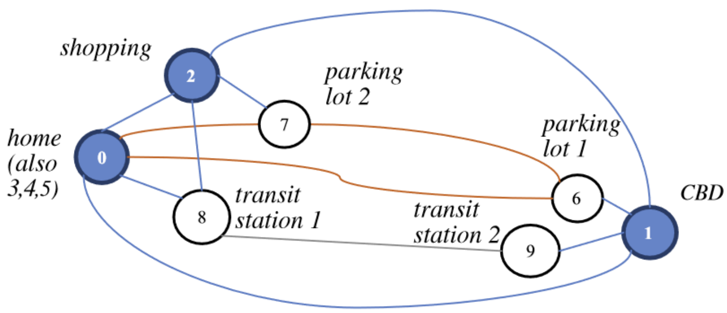


Figure C.1: Illustration network (blue lines – walking, red lines – driving, green line – transit). (Chow & Djavadian, 2015, p. 12, fig. 2)