Inflation Derivatives

Graduation Thesis Financial Engineering and Management

A research on the implementation of inflation derivatives within pension funds

Supervisors:
Dr. D.Y. Dupont
Dr. R. A. M. G. Joosten
Dr. B. Roorda
Drs. P. Bajema

Author: C.F.A.R. Wanningen
Company: Blue Sky Group
Date: July 2007

University of Twente
Enschede - The Netherlands

Preface

This thesis about inflation derivatives is the result of a six-month research which started in February and ended in July at pension fund provider Blue Sky Group. During this period I conducted a literature research on inflation linked products and developed a pricing model. Moreover I visited investment banks in London and the Euromoney Inflation Linked Products Conference 2007 in Frankfurt. The visits provided me with valuable information insights and business experience. Furthermore conversations with specialists working at Blue Sky Group and investment banks helped me to understand how inflation linked products are traded in the marketplace.

By writing this preface I would gladly grasp the opportunity to thank persons who made a contribution to this thesis. From the University of Twente I would like to thank Dr. D.Y. Dupont, Dr. R.A.M.G. Joosten and Dr. B. Roorda. Dr. Dupont supervised the research and gave information insights during the first months. Dr. Roorda supervised the research throughout the whole period and initiated useful discussions on inflation. Dr. Joosten supervised the research during the last months and gave supportive suggestions on writing a thesis.

From Blue Sky Group I would like to give thanks to Drs. P. Bajema, Drs. J.F. van Halewijn and A.A.M. Lute, RBA. Drs. Bajema was my mentor and traveled with me to Frankfurt and London. Furthermore Drs. Bajema actively monitored my proceeds and gave practical insights. Drs. van Halewijn, as head of the research department, provided the time to conduct the research. Mr. Lute, as chief investment officer, organized the internship at Blue Sky Group.

After reading this thesis I hope that the reader has knowledge about inflation and the inflation market. Moreover I would be glad if the reader has developed an intuition for inflation derivatives modeling and pricing. Finally, this thesis is especially written in order to contribute to the readers' opinion on the significance of inflation derivatives for pension funds.

Abstract

Nowadays the global inflation market enables pension funds to offer real pensions to their schemes' participants. Real pensions have a preference over their nominal counterparts because they are unaffected by unfortunate inflationary movements. Inflation linked bonds and inflation derivatives provide the required solutions that secure buying power of money over time horizons that are unrivaled in length.

By covering topics such as inflation measurement and inflation modeling this thesis is especially written for those who want to know more about inflation derivatives. Inflation derivatives greatly enhance investment portfolios of pension funds. However, due to no standardized pricing models, a pension fund must have an objective pricing model for inflation derivatives. This thesis provides a detailed methodology covering all the relevant factors on pricing inflation swaptions, which gives insight in the level of complexity of an appropriate pricing model for inflation derivatives.

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Introduction

Inflation is for most people a vague concept. This is unfortunate, because the inflation rate affects people's pension tremendously. In other words the inflation rate influences people's future income. For this reason I would like to see that inflation is a universally understood concept. Aiding this cause I shall introduce the definition of inflation as stated in Webster's new Universal Unabridged Dictionary:

"An increase in the amount of currency in circulation, resulting in a relatively sharp and sudden fall in its value and rise in prices: it may be caused by an increase in the volume of paper money issued or of gold mined, or a relative increase in expenditures as when the supply of goods fails to meet the demand."

This means that over time, as the cost of goods and services increase, the purchasing power of the currency is going to fall. Such an upward price movement of goods and services is usually measured by a consumer price index (CPI). Therefore indexing the money of participants in pension fund schemes to the CPI will offer a multiplicity of benefits. The indexation acts as a guarantee for buying power for the money paid to the retirees and mitigates social tension coming from inflation.

Currently pension funds invest most of their money in stocks, bonds and real estate. With these investments they cover their future liabilities. These liabilities comprise pensions, which are provisions of future income to the retirees. The (nominal) real coverage ratio is defined as assets divided by (nominal) real liabilities and denotes the capability of the pension fund to cover future liabilities.

Due to high volatility of the capital markets the value of investments is uncertain over time. Price movements in the market place affect coverage ratios of pension funds. Especially the inflation rate, interest rates and stock price indices affect the coverage ratios. Despite the fact that indexation against inflation of participants in Dutch pension schemes is not obligatory pension funds have the ambition to provide indexation against inflation for every participant. In general, high inflation in combination with underperforming investments affect pension fund's coverage ratios negatively.

Inflation linked products (ILPs), such as inflation linked bonds (ILBs) and inflation derivatives offer solutions that tackle inflation risk. The main benefit of ILPs is, as reflected in the name, the linkage to CPIs. This thesis conducts a study on ILPs and suggests a model to price them. Because this is perceived as a new and uneasy subject many pension funds lack sufficient knowledge about and experience with ILPs. Therefore pension funds tend to be hesitant in implementing ILPs in their investment portfolios.

Methodology

Research goal

If pension funds want to offer pensions with a fixed level of buying power, they should know what to take into consideration when conducting necessary investments. Consequently, the goal of the research is:

"Provide solutions to pension funds that enable a guaranteed level of buying power for pensions."

Problem description

In order to offer pensions with a guaranteed level of buying power a pension fund has to hedge against inflation risk. Therefore a pension fund must have the knowledge to buy products that fulfill their inflation hedging needs. Furthermore pension funds should develop or acquire pricing models for inflation linked products in order to conduct prudent investments. The main problem is denoted by:

"Which available inflation linked products are suitable to fulfill pension fund's inflation hedging needs and what is a correct methodology to price them."

Research questions

This thesis provides answers to research questions in a top-down fashion. Starting with inflation measurement in the market and ending with a pricing model for inflation derivatives.

- How does the market measure inflation?
- What types of inflation linked products are available?
- What comprises the market for inflation linked products?
- How are inflation linked products priced?

Approach

In the first phase of the research I will gather information about inflation by studying recently issued books and articles. Successively, when scrutinized a substantial amount of information I start working on a pricing model. In the second phase of the research I attend meetings with professionals at investment banks to obtain additional information insights. The next phase of the research deals with finalizing the pricing model. When the pricing model is finished I will be in the final phase of the research and visit the Euromoney Inflation Linked Products Conference 2007.

Chapter 1 Inflation Indices

Before implementing inflation linked products in an investment portfolio it is essential to have an understanding on how the market measures inflation. This chapter elaborates on market inflation measurement by introducing various concepts, such as CPI. At the end of this chapter special attention is given to Euro inflation indices, because these are interesting for Dutch pension funds.

1.1 Consumer price index

Deacon (2004) states that the consumer price index (CPI) is the most commonly used measure to link inflation linked bonds (ILBs) cash flow payments in order to compensate for inflation. Other used price indices are wholesale price indices, export price indices, earnings indices and the GDP deflator. For most inflation risk bearing investors a good choice of inflation index is the index that matches their assets and liabilities in the best possible way.

According to Deacon (2004) and Oldland (2006) pension funds should have a preference for Earnings-indexed bonds because these bonds compensate for wage inflation. Wage inflation has a good match with pension funds' future liabilities. Nevertheless most pension funds link to price inflation, such as consumer prices or retail prices.

Investors in ILBs have a certain basis risk, which is defined as the difference between the indexation from the referred price index of an ILB and the best possible indexation of their liabilities. If the price index has a tendency to overstate (understate) inflation, then the corresponding ILB compensates for a higher (lower) inflation than is necessary. This inflation mismatch is reflected in market prices of ILBs, because these will rise (drop) to reflect a positive (negative) inflation mismatch.

It is convenient for the investor in an ILB that the underlying price index of the ILB is not subject to regular revision. Moreover, rules describing the consequences of a revision on the price index have to be lucid. Generally, transparency will mitigate tension among investors in ILBs. In the ideal case of total transparency ILBs prospects provide all the relevant details about revisions, rebasing and other changes to the price index. Revision of the price index is not a problem if the liabilities of the investor are linked to the same price index of the ILB. On average the higher the correlation between in the price index of the ILB and the liabilities of the investor, the lower the effect of a revision.

1.2 Seasonality

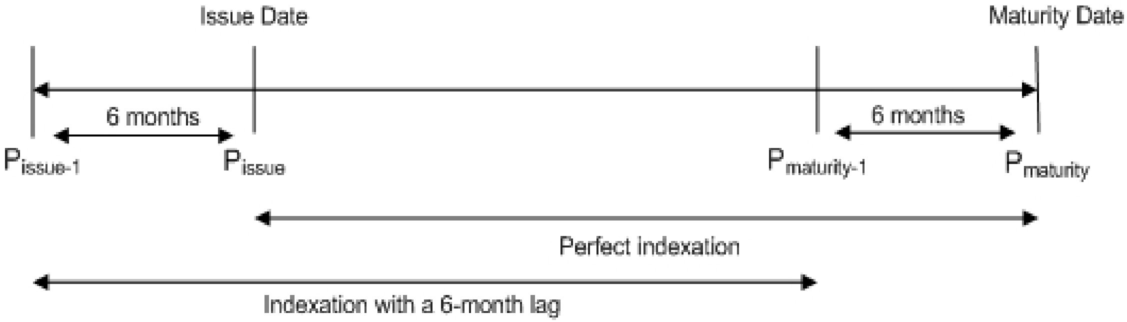
According to Benaben (2005), inflation is subject to a recurring pattern over the course of a year. This pattern comes from predictable price volatility that originates mainly from the demand for food and energy during the year. Consequently CPIs show seasonal movements. These seasonal movements intricate the analysis of ILB market prices. Subjective mathematical procedures are currently used in practice to estimate the seasonal movements on inflation and most of these procedures conduct back month revisions.

Almost all issuers choose to use a seasonal unadjusted price index as reference index for an ILB. A seasonal adjusted price index will cause the time of purchase during the year to affect the expected nominal size of future cash flows payments from the ILB and is therefore undesirable. Seasonal unadjusted price indices are for this reason better established, because they are most often applied. More clarification on seasonality shall be given in Chapter 7.

1.3 Indexation lag

According to Deacon (2004) unavoidable time-lags between actual movements in the CPI and their corresponding adjustments to cash flows payments from ILBs distort the hedging process. As a consequence of the time-lags there is a period at the end of the life of an ILB where there is no inflation protection at all. This is illustrated in Figure 1. On the other hand there exists a period of equal length before the issuance of an ILB for which inflation compensation is paid. As a general rule one can say that the longer the time-lag, the poorer the inflation hedge provided from an ILB. Furthermore, the shorter the residual maturity of an ILB existing after compensation for the period before the ILB issuance, the more a time-lag is relevant. As an illustration consider an ILB with two years to maturity and a six-month time-lag, which exposes investors to inflation risk over a six-month period at the final quarter of the life of the ILB.

Figure 1: The effect of a six-month time-lag



There are two main reasons why cash flows payments from ILBs are indexed to inflation with a time-lag:

- The time to compute and publish price indices.
  In most countries the CPI for a given month is typically published in the middle of the following month, which results in a lag of at least one month.
- Institutional arrangements for trading and settling bonds between coupon payment dates. Whenever a bond is traded on a day different from a coupon payment date, its valuation will reflect the propinquity of the cash flow payment on the next coupon payment date.

Extensive studies have been conducted trying to minimize the time-lag. The publication delay is generally minimized using a frequently published index. This is the main rationale of the predilection from issuers to incorporate CPIs in ILBs.

Most issuers have disputed with each other over the method for calibrating accrued interest. In order not to prolong the time-lag much beyond that what is necessary to allow for the publication of the index, the accrual method introduced by the Canadian authorities for Real Return Bonds (RRBs) is adopted by the majority of ILB issuers. The application of this method results in a total time-lag of three months. A disadvantage of application is that the accrual will vary from month to month within a given coupon period. A possibility to eliminate the accrued interest part of the time-lag is to issue indexed zero coupon bonds (IZCBs).

1.4 The Harmonized Indices of Consumer Prices

The purpose of CPIs is to be an economic indicator constructed to measure the changes over time in prices of consumer goods and services acquired, used or paid for by households. Eurostat[1] (2004) manages The Harmonized Indices of Consumer Prices (HICPs), which are a set of European CPIs. The application of a harmonized process and a unique set of definitions compute these indices. The aim from Eurostat is to have comparable results and not applying uniform methods for computing CPIs.
The three best known HICPs are:

- The Monetary Union Index of Consumer Prices (MUICP).
  This aggregate index covers the countries within the euro-zone.
- The European Index of Consumer Prices (EICP).
  This index covers the EU countries.
- The national HICPs.
  This index covers each corresponding country separately.

[1] Eurostat is the Statistical Office of the European Communities. Its task is to provide the European Union with statistics at European level. By harmonizing statistics from the European statistical system (ESS) to a single methodology, the statistics are made comparable.

HICPs are open to revision. Revision occurs when unprecedented or improved information becomes available. Furthermore Eurostat publishes so-called flash estimates. In the last week of each month these flash estimates are propagated. Moreover, flash estimates are based on the data from the first countries and energy prices. HICPs have several key characteristics:

- The geographical and population coverage is of all the purchases by resident and non-resident households within the territory of the corresponding country.
- The HICPs cover prices paid for goods and services in monetary transactions, hence bartering is excluded.
- The measured prices are those prices actually faced by consumers.
- The HICPs exclude interest and credit charges, regarding them as financing costs and not as consumption expenditure.

The differences between HICPs and national CPIs are caused by the countries' idiosyncratic treatment of subsidized healthcare and education, the aggregation formulas and geographical population coverage. Despite that these differences are diminishing, some structural differences will remain due to different methodologies. Moreover, in many countries national CPIs are set up to fulfill a different purpose and therefore some of the underlying concepts and methods of national CPIs are inappropriate.

HICPs strive to measure inflation and are published each month according to a pre-announced schedule. On average they are published eighteen days after the end of the month. This causes as described in section 1.3 a time-lag in the measuring of realized inflation resulting from the publication (1 month) and accrual method (3 month). The effect of the realized inflation from the Euro HICP affects ILBs with a three month lag according to Hurd (2006) and Deacon (2004). Time-lags for ILBs from several regions are depicted in Table 1.

Table 1: Time-lags of ILBs from several regions

Region	Index	Floor	Time-lag
France	Euro-area HICP ex Tobacco	Yes	3 months
Italy	Euro-area HICP ex Tobacco	Yes	3 months
Euro Zone	Euro-area HICP ex Tobacco	Yes	3 months
USA	US CPI	Yes	3 months
UK	UK RPI	No	8 months

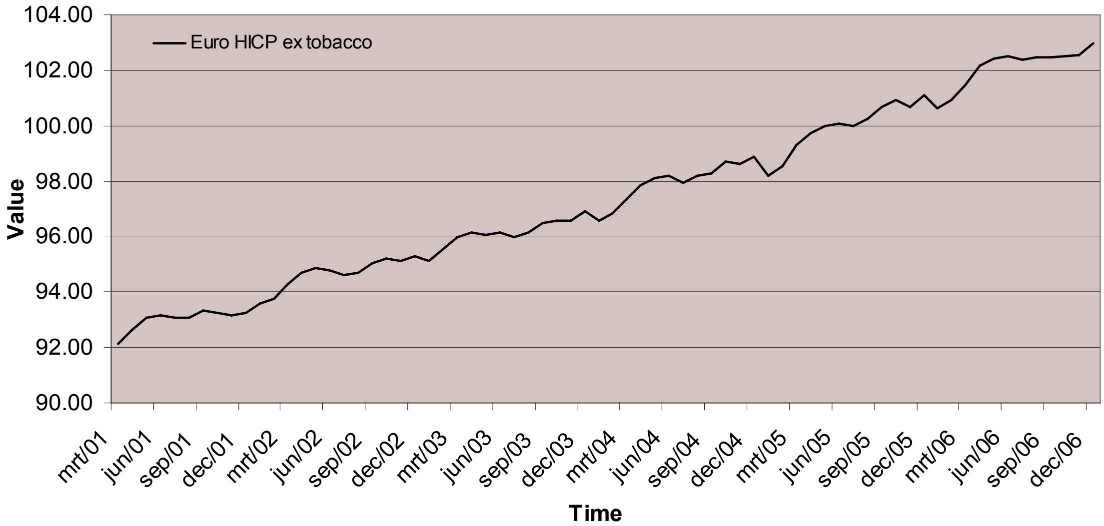
Source: Hurd (2006) and Deacon (2004)

The time-lags on ILBs cause hedging errors, which is undesirable when precise hedging is needed. However when maturity increases the negative effect originating from the time-lag on the inflation hedge decreases. The rationale is the juxtaposition of a fixed time-lag an longer maturity.
The HICP data, which is released monthly, covers the indices themselves, annual average price indices and rates of change, and monthly and annual rates of change. As provided in section 1.2 none of these are seasonally adjusted.

In addition to the regular HICPs a series of special aggregates are released. One of these aggregates, HICP excluding tobacco, is illustrated in Figure 2. The best known aggregates are:

- The HICP excluding energy.
- The HICP excluding energy, food, alcohol and tobacco.
- The HICP excluding unprocessed food.
- The HICP excluding energy and seasonal goods.
- The HICP excluding tobacco.

Figure 2: The HICP excluding tobacco



Source: DataStream, monthly seasonally unadjusted data

In order to produce comparable results, each country's HICPs rates are calculated using specific formulas. According to Eurostat (2004) the HICPs are in principal set up to provide the best measure for international comparisons of household inflation within the EU. Summarizing the above investors in indexed bonds linked to HICPs should note that:

- HICPs are subject to revision.
- HICPs are not seasonally adjusted.
- HICPs are published with an average of eighteen days after the month in question.
- HICPs have their influence on ILBs with a time-lag.

1.5 The Boskin Report and US CPI

Since 1996, when a committee headed by professor Boskin published a report on the fallacies of the US CPI, the Bureau of Labor Statistics (BLS) has made pivotal changes improving the US CPI. According to Johnson (2006) the key topics, that affect a CPI and are highlighted in this report, are:

- Calculating of CPIs.
- Consumer substitution.
- Quality change.
- New goods.

In the United States the CPI is extensively used by government and private entities in order to calculate cost-of-living adjustments. Therefore a cost of living index (COLI), such as the CPI, must be universally understood. The Boskin Report stated an upward bias in the US CPI of 1.1%, originating in biases from:

- Consumer substitution.
- New goods.
- Quality change.

In an update to the report, the Boskin committee estimated the bias to be 0.8% as of 1999. In another analysis in the Journal of Economic Literature, Lebow (2003) estimated the upward bias at 0.9%, with most of the bias coming from new goods and quality change. While conclusions from the report from the Boskin committee and Lebow (2003) caused a lot of turmoil around the US CPI, they are best to be known as single events in the history of the evaluation of the conceptual foundations and methodologies of CPIs.

Until 1999 the CPI was calculated using a formula that effectively assumed zero consumer substitution. The initial quantities used in the formula were to stay fixed after their introduction until the next expenditure weight update. In 1999, the BLS adopted a geometric means formula for item categories to calculate the CPI. Furthermore these item categories reflect 61% of the CPI. When using item categories, consumer substitution can be applied. Consumer substitution means a change in consumer purchasing behavior, which is caused by relative price changes of products.

In 2002 an additional index, the chained consumer price index for all urban consumers (C-CPI-U), was published accounting for higher substitution bias. To illustrate lower level consumer substitution a switch of consuming Swiss and cheddar cheese is a good example. On the other hand, a switch between beef and chicken is an example of upper level consumer substitution. The report form the Boskin committee predicts biases for both upper and lower level consumer substitution to be 0.15% per year and 0.25% per year, respectively. This results in a total upward bias from consumer substitution of 0.4%.

A geometric means formula (the n-th root of the sumproduct of n numbers) corrects for the lower level consumer substitution bias by diminishing the upward bias when the sample size increases. While the geometric means formula addresses lower level consumer substitution in the US CPI, upper level consumer substitution bias is not highlighted. This bias is addressed in the C-CPI-U. Thus if one believes that a COLI should take into account upper level consumer substitution, the C-CPI-U is a suitable price index to use.

Still, the most fundamental problem in constructing a CPI is the constantly changing market basket. An ideal price index must have methods in place to account for these changes. New goods can enter the CPI sample in one of three ways.

- During reprising.
- Sample rotation.
- During revision of the structure of the items.

It is crucial that the CPI receives new goods into its sample basket quickly, in order to have a market basket that accurately reflects current consumer purchases. The BLS has taken several steps to keep the market basket up to date. Since 2002, updated expenditure weights based on consumer expenditure surveys have been introduced every 2 years as opposed to every 10 years in the past. Moreover the time lag from survey to implementation is shorter and the survey is completed in a shorter time. The implementation of the survey results in weights for goods in the market basket.

Additionally the BLS changes its sample and commercial market rotation procedures. In 1998, the CPI went from rotating 20 percent of the commercial market sample each year to 25 percent, so that the entire sample is rotated every 4 years instead of every 5 years.

As stated before, the CPI tries to approximate a COLI. Therefore the goal for the CPI is to incorporate a level of quality which is constant over time. Meaning when the quality of goods and services in the market basket changes the CPI undergoes an adjustment based on the quantitative value of these changes. The Boskin Report suggests an upward bias of 0.6 percent for quality change. This is bigger than the substitution bias. However, Hulten (2001) argues that the quality change bias can be negative over time.

Operationally, the CPI deals with quality change in several different ways. For any given item being priced, the CPI economic assistant in the field must determine whether the item changed in such a way as if it has been replaced with a new version. Direct quality adjustment refers to the analyst making an estimate of the quantitative value of the quality change. The total effect of these estimates on the index is negligible.

Another recommendation from the Boskin report for the BLS is to improve its processes of bringing in outside information, research, and expertise. Therefore the Federal Economic Statistics Advisory Committee (FESAC) was established in 2000. FESAC allows the BLS to exchange ideas with the academic research community more efficiently, acting as a tool for the CPIs both by transmitting its latest research and methodology to the academic sector and to receive new relevant research for CPIs.

1.6 Summary

Pension funds have the ambition to offer pensions to retirees with a guaranteed buying power. One possible way to maintain the level of buying power is being compensated on the investments under management for inflationary movements. CPIs are economic indicators of inflation constructed to measure the changes over time in prices of consumer goods and services acquired, used or paid for by households. The main problem when constructing a CPI is the determination of a market basket containing the consumer goods. New goods can enter a CPI's market basket during reprising, sample rotation and during a revision of the structure of the items.

CPIs are used to link ILBs cash flows payments. However, investors in ILBs have a certain basis risk, which is the difference between the indexation from the reference CPI and the best possible indexation for their liabilities. Prices fluctuate in the form of seasonal movements and most issuers have chosen to use seasonally unadjusted CPIs. ILBs cash flows payments are indexed to inflation with a time lag, because of time to compute the accrual, announcement of CPIs rates, and institutional arrangements for trading and settling ILBs.

Eurostat manages the HICPs which are a set of comparable European CPIs striving to measure inflation. The monthly HICP data covers the price indices themselves, annual average price indices and rates of change, and monthly and annual rates of change.

Since the Boskin committee published a report on the fallacies of the US CPI the BLS has made pivotal changes to improve the US CPI with regards to:

- Calculating consumer price indices.
- Consumer substitution.
- Quality change.
- New goods.

With consumer substitution a change in consumer purchasing behavior is meant. In general CPIs try to incorporate a constant level of quality and when the quality of goods and services changes in the economy, the CPI should be adjusted by a quantitative equivalent of the quality change.

Chapter 2 Inflation Linked Bonds

A bond defines a certain amount of debt from the issuer of the bond to the buyer of the bond. At the end of the lifetime of the bond the buyer has been paid at least the amount he has paid to the issuer. Moreover the buyer often receives payments during the lifetime of the bond, so called coupon payments. This chapter provides explanations on issues related to bonds linked to inflation. Furthermore information insights on traditional inflation hedging within pension funds is presented.

2.1 Introduction

When speaking of ILBs the magnitude of the payments the buyer receives from the issuer depend on the CPI level. The international market for ILBs has increased in recent years. Especially government issuers are responsible for the increase. The importance of ILPs, such as an ILB, can be demonstrated by a juxtaposition of nominal bonds and ILBs. Cash flows of ILBs are linked to movements in a specific CPI. The goal of this linkage is to provide investors in ILBs with a means to maintain the buying power of their money. When invested in an ILB the buyer receives on top of the linkage a fixed interest rate, which results in a real return. Therefore, when assuming that the buying power of the debt remains constant, only the real rate influences the market price of an ILB. The nominal return on an ILB is dependent on the realized inflation during the lifetime of the ILB and the fixed real rate.

For a nominal bond the reverse holds, because the nominal return until maturity is fixed. As a consequence the real return depends on the realized inflation during the lifetime of the nominal bond. The dependency results in the effect that when inflation increases (decreases) the real return decreases (increases). Because a nominal bond has dependency towards realized inflation it has an inflation risk premium. The height of this premium depends on the volatility of inflation; the higher the volatility the higher the risk exposure. The market of ILBs is not as liquid as the nominal bond market and therefore ILBs have a liquidity premium. This liquidity premium depends on the difference in bid-ask spread. In the expected return of ILBs no inflation risk premium is taken into account, because the investor receives compensation for changes in inflation. Table 2 summarizes the components of nominal bonds and ILBs.

Table 2: Components of bonds

Nominal Bond	ILB/ILP
Real return Expected inflation Inflation risk premium	Real return Expected inflation Liquidity premium

Source: Blue Sky Group (2003)

2.2 Reasons for issuing

Blue Sky Group (2003) states that an issuer of ILB is incited for several reasons:

- For the issuing party, when looking at its cash flow position, cash outflows generated by ILBs are a better match for future obligations than cash outflows generated by nominal bonds.
- Cost reduction due to the fact that the issuer is not obliged to pay a premium on inflation risk.
- Issuing ILBs creates discipline to keep inflation low, because issuing governments have to pay inflation.
- Diversification for the issuing party by adding a funding instrument, which has relatively low correlation with other funding instruments.
- ILBs act as a valuable information source for both the issuer and buyer, because future inflation can be derived from their market prices.
- From a strategic point of view the issuer can enact the ILBs when having the opinion that expectations on inflation are high or when demand increases ILBs' market prices.

2.3 Inflation effects

Inflation increases and decreases affect the pension fund investment portfolio. Assuming real rates being constant, increasing inflation rates cause nominal interest rates to go up. Furthermore an inflation increase results in a decrease in stocks and bonds. The drop in stocks comes from the fact that money is more expensive, due to higher nominal interest rates. When money is more expensive financing costs increase. Higher financing cost result in lower profits and consequently lower returns on stocks. Lower returns on stocks causes stocks to be less valuable and are given a lower price. Increased inflation results in a decrease in bond value, because the higher nominal interest rate causes future payments to erode more quickly in terms of present value. Next to the drop in assets a higher nominal interest rate causes the nominal liabilities for a pension fund to decrease in value. This affects the nominal coverage ratio of the pension fund positively.

Inflation volatility increases and decreases affect the inflation risk premium on nominal bonds. When inflation becomes more (less) volatile, the inflation risk premium on nominal bonds will increase (decrease). ILBs are not sensitive to inflation volatility increases and decreases. The liquidity premium on ILB is sensitive to a comparison between the magnitude of the ILB market and the nominal bond market. It is assumed that the bigger (smaller) the ILB market compared to the nominal bond market the smaller (bigger) the liquidity premium on ILBs. According to Blue Sky Group (2003), the inflation risk premium of a nominal bond is unstable and not negligible. Its amount is between 0-100 basis points. The liquidity premium on ILBs is between 0 and 20 basis points.

Equation 1 depicts the expected difference between the nominal and real rate, which is defined as the break even inflation, BEI. This is defined by Elhedery (2004) and McDevitt (2006).

Equation 1: Definition of BEI

$$(1+\mathrm{BEI})=\frac{1+\mathrm{YTM}_{\mathrm{N}}}{1+\mathrm{YTM}_{\mathrm{R}}}$$

Where YTM stand for yield to maturity. The concept of yield to maturity is explained in the appendix.

2.4 Hedging instruments

According to Blue Sky Group (2003) instruments which are used for inflation hedging purposes are bonds, real estate, stocks, commodities (oil, gas, gold, etc) and cash.

Nominal bonds are not a suitable hedge against inflation increases. This excludes nominal floating rate bonds, because these compensate for inflation increases. The reasoning for nominal bonds not being a suitable hedge is that inflation increases lessen a nominal bond its value due to eroded real return. ILBs on the other hand are a suitable hedge due to incorporated price indices, which affect the coupon and principal payments. By having this characteristic ILBs compensate for future inflation.

Another possibility to hedge against inflation is to invest in real estate. By being invested in real estate one is linked to movements in real estate prices and rents. Intuitively this is a good hedge against inflation, because people spend a large portion of their income on rents or savings on houses. However there is a negative aspect to be taken into account that an inflation decrease causes rents to drop relatively less than inflation or not at all.

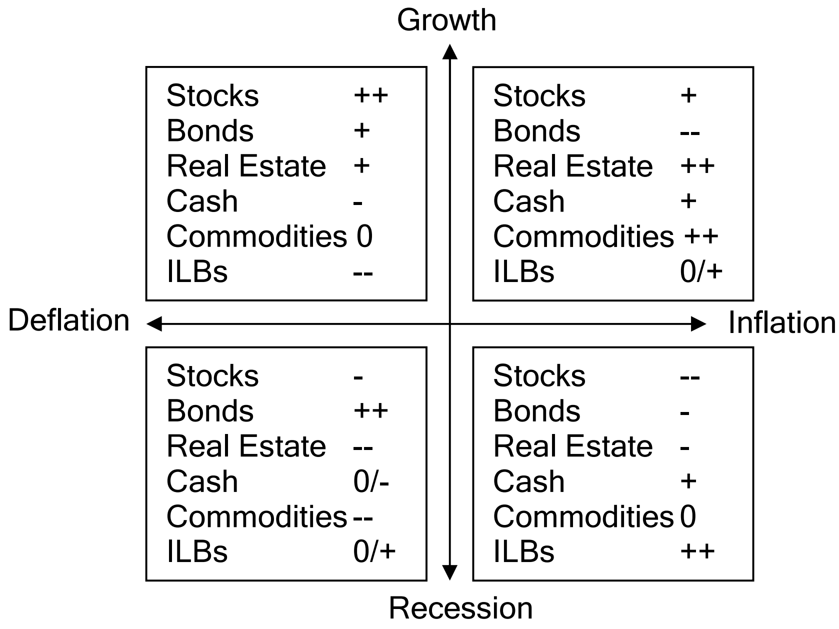
Hedging against inflation using stocks is in principle not a sound option, because in general stock indices are more capricious than price indices. This is the same comparison between prices of goods and expected profits made on goods, where the latter is intuitively far more volatile. Therefore stock, as a relatively high volatile hedging category on the short term, results in substantial hedging errors.

Commodities do not provide a full hedge against inflation because their prices are subject to political matters, subsidies, and international relationships. Moreover hedging this way is unattractive, because from a historical perspective commodities have small long term returns compared to other investment categories. This characteristic is for most pension funds a major drawback for investing in commodities as an inflation hedging category.

The last instrument available for hedging against inflation, cash, is intuitively correlated with inflation. Besides this very good characteristic, the long term return is even lower than for commodities. The rationale is that cash has very little exposure to the market. Figure 3 gives an overview of which inflation hedge is preferred in a given scenario.

Stocks are favorable in times of growth, because in this scenario they have relatively high returns due to high profits from companies. However when inflation increases, more money circulates in the economy, stock returns decrease and therefore stock becomes less preferable. Nominal bonds are favorable in times of deflation, especially when there is a recession. In this scenario they perform well due to their low risk exposure and relatively high real return. Real estate has good performance during economic growth, because in this situation the demand for offices, infrastructure and houses is relatively high. Furthermore, real estate returns have a better match with inflation movements in times of inflation than in times of deflation. Commodities only perform well in times of growth and high inflation, both factors causes their demand and prices to rise sharply. Cash on the other hand is only an interesting inflation hedging category in times of inflation due to the fact that the nominal interest rate is relatively high in this scenario.

Figure 3: Hedging instrument preferences



Unfortunately for Dutch pension funds, there are no ILBs linked to Dutch inflation. One possible reason for this phenomenon could be that inflation in the Netherlands is highly correlated with the ones abroad. Intuitively the correlation argument must also be true for the future. The rationale is that when Dutch inflation is at a high level compared to other Euro zone countries for a period of time, companies in the Netherlands will ask a higher price compared to foreign companies. This scenario is very unlikely in an open market economy such as the Euro zone. In Table 3 correlation figures are displayed

Table 3: Correlation of Dutch CPI with foreign CPIs

Correlation	1992-2006	2002-2006
France	0.9891	0.9859
Germany	0.9825	0.9577
United Kingdom	0.9891	0.9782
United States	0.9916	0.9730

Source: DataStream, monthly seasonally unadjusted CPI data

Since 1992 the correlation with foreign countries regarding inflation dropped due to a decreased inflation rate and relatively low inflation volatility. Table 4 shows that the monthly inflation rate has decreased significantly and Figure 4 depicts the 10 and 5 years moving average of the Dutch CPI. However, due to trading nature of the Dutch which is foreign oriented, an inflation shock in a foreign region will have an impact on the Dutch inflation rate. ILBs indexed on foreign inflation rates give for this reason a good performing hedge against Dutch inflation. The conducted monetary policy used to tackle inflation is therefore determined according to an average of the European countries. Furthermore, to consolidate this statement, inflation differences in the Euro zone are relatively low according to the Dutch Central Bank (DNB).

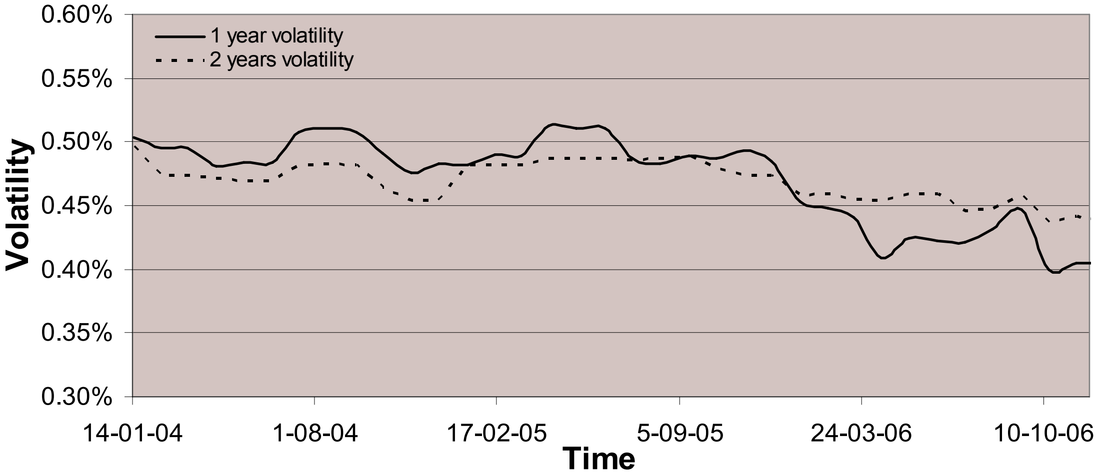
Table 4: Monthly inflation rate

Monthly Trend	1973 - 2006	2002-2006
France	0.41%	0.16%
Germany	0.22%	0.13%
The Netherlands	0.31%	0.15%
United Kingdom	0.48%	0.26%
United States	0.31%	0.22%

Source: DataStream, monthly seasonally unadjusted CPI data

The volatility of ILBs is less than the volatility of nominal obligations, because the risk of ILBs depends on volatility in the real rate. Over long time horizons, thus talking about high maturity bonds, the risk coming from nominal bonds compared to risk coming from ILBs decreases. The rationale is that for high maturities the difference between the real rate and nominal rate becomes less volatile. Until now ILBs proofed to be the best hedge against inflation over long time horizons compared to all other investment categories.

Figure 4: Volatility of Dutch CPI



Source: DataStream, monthly seasonally unadjusted CPI data

2.5 Purchasing power parity

According to Neary (2004) purchasing power parity (PPP) is the supposition that with a unit of purchasing power it should be possible to purchase the same basket of goods and services anywhere in the world. The PPP was put forward by the Swedish economist Gustav Cassel after the First World War.
The PPP is essential in international economics for three main reasons.

- Acting as a simple theory for estimation of exchange rates.
- Estimation of flexibility of the relative price of two currencies.
- The ratio of price levels provides a reference point for defining the exchange rate as under valued or overvalued.

Moreover, irrespective of whether PPP ever occurs in practice, it is wise to take deviations from PPP into account when conducting international comparisons of real income and return. PPP can be applied if the prices of all goods are equalized internationally by arbitrage. However, in reality, deviations are substantial. A possible rationale is that some kind of goods and services are not traded at all. Even traded goods face many barriers to fulfill the PPP such as tariffs, transport costs, product differentiation, and price discrimination by firms with market power. As a consequence PPP is better applicable to producer prices than to consumer prices.

There are many reasons explaining that PPP can not hold in the long run. A famous rationale is the "Balassa-Samuelson effect" introduced by Balassa (1964) and Samuelson (1964). The "Balassa-Samuelson effect" states that when high-income countries have an advantage in the production of traded goods compared to low-income countries, high-income countries produce traded goods cheaper. Applying the PPP to traded goods, the outcome is that the price of non-traded goods is lower in low-income countries compared to high-income countries, because traded goods are relatively more expensive. Another reason for not expecting the PPP to hold in the long run is in the case when productivity makes a sudden upward movement in an export sector. Then, as a consequence, the real exchange rate appreciates, which leads to a loss in competitiveness for other traded sectors.
Because deviations from the PPP are significant and persistent, international comparisons of gross domestic product (GDP) or of consumption per head give a misleading picture of international differences in productive potential or living standards when they are made using market exchange rates. To overcome this index number problem one needs comparable data across countries and an index number formula in order to produce a consistent set of "PPP adjusted" data on real incomes.

2.6 Types of inflation linked bonds

Deacon (2004) states that there are five types of ILBs: capital indexed, interest indexed, current pay, indexed annuity and indexed zero coupon. However, the ILBs often used in practice are capital indexed and interest indexed.

Capital indexed bonds (CIBs).
These bonds have a fixed real coupon rate and a nominal principal value that increases with inflation. Periodic coupon payments are calculated as the real coupon rate times the inflation adjusted principal, and the inflation-adjusted principal itself is repaid at maturity. Table 5 shows the cash flow structure of a 4% annual 5-year CIB. Here the coupon indexation component for year 2 is the fixed real coupon rate of 4% times the accumulated inflation rate of the past two years, $6\% + 5.5\% = 11.5\%$. The coupon payment totals in 4.47%.

Table 5: Example CIB

Year	Inflation Rate	Compounded Inflation	Coupon Indexation	Coupon Payment	Redemption Payment
1	6.00	1.06	0.24	4.24	
2	5.50	1.1183	0.47	4.47	
3	5.00	1.1742	0.70	4.70	
4	5.00	1.2329	0.93	4.93	
5	4.00	1.2822	1.13	5.13	128.22

Source: Deacon (2004)

Interest indexed bonds (IIBs).
IIBs pay a fixed coupon plus an indexation of the fixed principal every period. The principal repayment at maturity is not adjusted. All the inflation adjustment comes through the coupon rate of the bond. IIBs are often said to pay a margin over inflation and are best viewed as a form of inflation-protected floating rate bond. Table 6 presents the cash flow structure of an 4% annual coupon 5-year IIB. Here the coupon payment in year one is composed as the real coupon of 4% plus the inflation rate of 6% which totals in 10%.

Table 6: Example IIB

Year	Real Coupon	Inflation Rate	Principal Indexation	Coupon Payment	Redemption Payment
1	4.00	6.00	6.00	10.00	
2	4.00	5.50	5.50	9.50	
3	4.00	5.00	5.00	9.00	
4	4.00	5.00	5.00	9.00	
5	4.00	4.00	4.00	8.00	100

Source: Deacon (2004)

More exotic types of indexed bonds are current pay bonds, indexed annuity bonds and indexed zero-coupon bonds, which as said before are not often used.

Current pay bonds (CPBs).
These bonds are similar to IIBs. The principal repayment at maturity is not adjusted for inflation. However, CPBs pay both an inflation-adjusted coupon as well as an indexation of the fixed principal. In Table 7 one the mechanisms of a CPB are depicted. One notices that the “coupon indexation” is an addition to the inflation rate and the real rate. Here the coupon payment in year two is the fixed real coupon of 4% times the inflation rate of 5.5% plus the inflation and real coupon rate, which totals in 9.72%.

Table 7: Example CPB

Year	Real Coupon	Inflation Rate	Coupon Indexation	Principal Indexation	Coupon Payme nt	Redemption Payment
1	4.00	6.00	0.24	6.00	10.24	
2	4.00	5.50	0.22	5.50	9.72	
3	4.00	5.00	0.20	5.00	9.20	
4	4.00	5.00	0.20	5.00	9.20	
5	4.00	4.00	0.16	4.00	8.16	100

Source: Deacon (2004)

Indexed annuity bonds (IABs).
These bonds constitutes of a fixed annuity payment and a variable element to compensate for inflation. Table 8 shows an illustration of the cash flow structure of this type of bond. The calculation of the base payment is explained in the appendix. The varying element is calculated by multiplying the accumulated inflation rate with the base payment. The summation of the varying element and the base payment totals in the coupon payment. The varying element is calculated by subtracting one from the compounded inflation and multiplying it with the base payment.

Table 8: Example IAB

Year	Base Payment	Inflation Rate	Compounded Inflation	Varying Element	Coupon Payment
1	22.46	6.00	1.06	1.35	23.81
2	22.46	5.50	1.1183	2.66	25.12
3	22.46	5.00	1.1742	3.91	26.37
4	22.46	5.00	1.2329	5.23	27.69
5	22.46	4.00	1.2822	6.34	28.8

Source: Deacon (2004)

Indexed zero-coupon bonds (IZCBs)
These bonds consist of a single payment of inflation-adjusted principal on redemption. Table 9 illustrates the cash flow structure of a 5-year IZCB.

Table 9: Example IZCB

Year	Real Coupon	Inflation Rate	Compounded Inflation	Redemption Payment
1	0.00	6.00	1.06	
2	0.00	5.50	1.1183	
3	0.00	5.00	1.1742	
4	0.00	5.00	1.2329	
5	0.00	4.00	1.2822	128.22

Source: Deacon (2004)

An issuer is wise to take into consideration the potential investor demand for different structures as well as his own preferences, when he decides which cash flow structure to use for its indexed bonds. Factors that are important to both issuers and investors are duration, reinvestment risk and tax treatment of bonds. The duration of a bond is the average time to each of its cash flows weighted by present value. The duration is used to measure the interest rate risk of a bond. When the duration of a bond is high, the price will fluctuate more with moves in interest rates than when the duration is low. For any given maturity, of the main structures discussed earlier zero-coupon bonds will have the longest duration and annuities the shortest, with CIBs having a higher duration than IIBs. In general, indexed bonds will have a longer duration than conventional bonds, assuming they have a comparable cash flow structure. This causes indexed bonds to be popular with investors who have to hedge long-duration liabilities, such as pension funds.

2.7 Tax on inflation linked bonds

Unfortunately for investors, government issuers of ILBs have power to choose the tax regime that will apply to their securities. Therefore the tax regime is seen as a design feature of ILBs and tax regulations can influence ILBs in two ways:

- Cash flows taxed on a nominal basis, which results in uncertain post tax real yields.
- The moment during the life time of the ILB for paying tax on the inflation uplift.

As a consequence of tax regimes certain designs of ILBs are more attractive to investors than others. While perfect ILBs guarantee the certainty of pre tax real returns, any tax system that not makes a distinction between the real return and the inflation uplift, thus taxes on nominal return, reintroduces inflation risk. An increase in inflation will raise the nominal yield on ILBs, thereby increasing the tax liability and successively lowers the post tax real return. The nominal fluctuations due to inflation must be tax exempt to ensure that the real value of post tax cash flows is guaranteed.

In general tax is applied on the nominal income derived from nominal bonds. This phenomenon may cause preference for an ILB, because it provides an almost similar nominal yield, however one that is tax exempt. Although the tax system can reintroduce inflation risk for ILBs by taxing the nominal yield, this inflation risk is less compared to nominal bonds. The case of inflation uplift from 1.0 to 7.0 and the corresponding effect on a nominal bond and ILB is illustrated in Table 10. For a nominal bond the decline in the post tax real return caused by an increase in inflation is one for one, while for an ILB the decline is scaled by the tax rate.

Table 10: Effect of taxes on an ILB and a nominal bond

Case	Type of Bond	Inflation Rate	Pre-tax Real yield	Pre-tax nominal yield	Tax burden at 30%	Post tax nom yield	Post tax real yield	Change in post tax real yield
1	ILB	1.0	3.0	4.0	1.2	2.8	1.8	
2	ILB	7.0	3.0	10.0	3.0	7.0	0.0	-1.8
3	Nom	1.0	3.0	4.0	1.2	2.8	1.8	
4	Nom	7.0	-3.0	4.0	1.2	2.8	-4.2	-6.0

Source: Deacon (2004)

In several countries the inflation uplift is treated as current income for tax purposes. Holders of inflation CIBS or IZCBs are taxed on the inflation uplift component of the principal on an annual basis. With this mechanism unrealized income is taxed, because the uplift itself is paid at maturity. Furthermore this mechanism restricts the set of suitable investors. Suitable investors are exempt from taxes, such as pension funds. The alternative of deferring tax payments on the inflation adjustment until maturity would again favor ILBs over nominal bonds.

2.8 Summary

The issuance of ILBs by governments has increased over the past decade which makes ILBs more attractive than before. The nominal return on an ILB is dependent on realized inflation and a fixed real rate. On average ILBs are a more suitable hedge against inflation than real estate, stocks, commodities and cash.

The purchasing power parity, which enforces small inflation deviations between neighboring countries, is a rationale for the Dutch CPI to be highly correlated with foreign CPIs in the Euro zone. Moreover, the presence of a monetary union consolidates the high correlation of inflation in the Euro zone. Furthermore, the high correlation on inflation enables a Dutch pension fund to invest in an ILB linked to a foreign CPI.

There are five types of ILBs: capital indexed, interest indexed, current pay, indexed annuity and indexed zero-coupon. Capital indexed bonds and interest indexed bonds are most often put into practice. Tax regulations can affect ILBs by taxing cash flows on a nominal basis and on paying tax on the inflation uplift. Both cases result in a reintroduction of inflation risk with uncertain post tax real returns. In the Netherlands pension funds are tax exempt, which makes the reintroduction of inflation risk due to tax arbitrary for them.

Chapter 3 Inflation Derivatives

An inflation derivative is a contract between two parties, one buying party and one selling party. The payoff of the inflation derivative is derived from an agreement, often expressed in a mathematical formula, based on price movements in a CPI or inflation linked security. The following chapter expounds on the application of inflation derivatives in investment portfolios designed to hedge against inflation risk.

3.1 Motivation

According to Deacon (2004) when investor interest and issuer needs cannot be matched, inflation derivatives enable investors and issuers to overcome mismatches in maturity, timing, index, profile and size. According to Benaben (2005) and Deacon (2004) inflation derivatives are created for several reasons:

- Improve needs from issuers and investors.
- Allow issuers and investors to transfer risk.
- Increase liquidity of the inflation market.
- Optimize market timing.
- Enable enhanced investment portfolio hedging requirements.

These rationales support the execution of investment strategies that are cheaper and more efficient than strategies originating from the application of solely the ILB market. Furthermore investment strategies incorporating inflation derivatives help issuers and investors to meet needs that the ILB market can't satisfy.

3.2 Inflation swaps

Inflation swaps are the most widely used inflation derivatives. According to Dogra (2007) inflation swaps have several advantages over ILBs:

- More flexible risk structure than ILBs.
- Maturity is negotiable resulting in a better cash flow match than ILBs on average.
- Wider choice of CPIs.
- Avoidance of residual risk; not exposed to the mark-to-market risk of assets.

The last advantage is especially important for the parent company of the pension fund and not the pension fund itself. Since the introduction of IFRS 17 corporations are obligated to be less opaque in their annual report by reporting the market value of their pension funds assets.

The inflation swaps which are widely used are zero coupon (ZC) and year-on-year (YOY) swaps. The ZC inflation swap is easily constructed and constitutes an exchange taking place at maturity:

- Maturity T years.
- Counterparty A pays at time $T: \frac{CPI(T-Lag)}{CPI(0)} - 1$.
- Counterparty B pays at time T: a fixed percentage X.
  Where $X = (1+I)^T - 1$ and I is the annualized rate of inflation implied by the price of the ZC inflation swap price.

In short one can state that the cumulative inflation over the whole period of the swap contract is exchanged against a fixed payment at maturity. The main application of the ZC inflation swap is that it can be applied to match a bullet liability linked to inflation. Furthermore ZC inflation swaps are known as easy to be calibrated and provide protection for cumulative inflation. However ZC inflation swaps give the buyer and seller large credit exposure due the fact that the settlement takes place at maturity. Table 11 shows the behavior of a 10-year 2% ZC inflation swap during a low and high inflation scenario.

Table 11: ZC inflation swap behavior during low and high inflation

Scenario 1: Low Inflation Scenario	Scenario 2: High Inflation Scenario
Implied ZC Inflation 2% per Year	Implied ZC Inflation 2% per Year
Realized Inflation 1% per Year	Realized Inflation 3% per Year
In 10 years:	**In 10 years:**
Fixed CF: $1.02^{10} - 1 = 21.899\%$	Fixed CF: $1.02^{10} - 1 = 21.899\%$
Infl. Cash flow: $1.01^{10} - 1 = 10.4622\%$	Infl. Cash flow: $1.03^{10} - 1 = 34.392\%$

Source: Dogra (2007)

The YOY inflation swap compensates for realized inflation over one year at a time. The inflation payout, coupon payment, is the published inflation for the corresponding year. The structure is as follows:

- Maturity T years.
- Counterparty A pays at each period t $(t <= T): \frac{CPI(t-Lag)}{CPI(t-1\,year-Lag)} - 1$.
- Counterparty B pays at each period t $(t <= T)$: a fixed percentage I.
  Where I is the annualized implied rate of inflation between coupon start date and coupon end date.

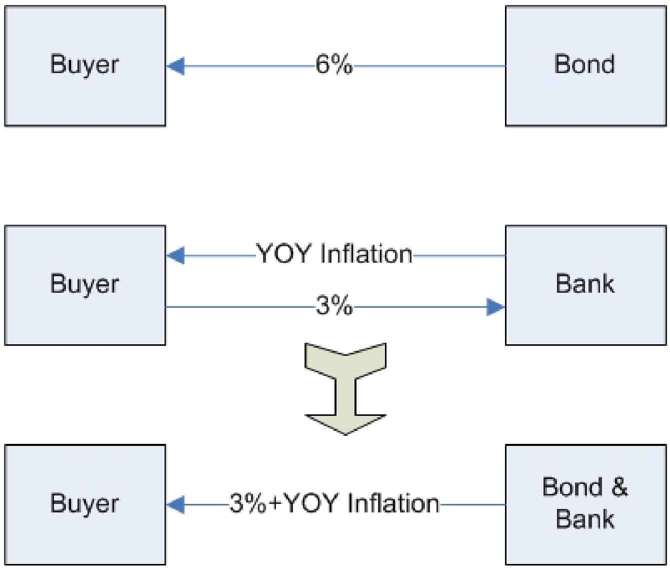
Application

Inflation swaps are used to swap out an issuer’s bullet inflation-linked issue and convert the debt profile to an annuity structure. This way the issuers expected cash flows are more closely matched.

An attractive break-even inflation (BEI) rate can be locked in by an issuer through the use of an inflation swap and a nominal swap. An issuer might take advantage of perceived market distortions and lock in wide BEI rates and subsequently issue an underlying security, such as an ILB, when it is profitable to do so.

Dogra (2007) states that an application of a YOY inflation swap as is illustrated in Figure 5 is the attachment of a YOY inflation swap to a nominal bond to create an exposure to the YOY real yield.

Figure 5: Application of YOY Inflation Swap



Source: Dogra (2007)

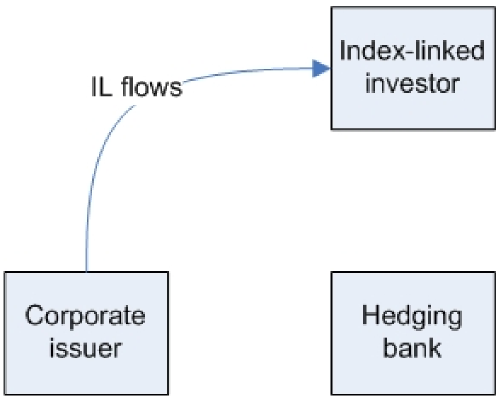
It is important to note that YOY inflation swaps can not be deduced directly form an inflation ZC curve. They have less credit exposure for the buyer and seller compared to ZC inflation swaps due to regular cash flows payments. YOY inflation swaps need a convexity adjustment for their forward rates. This subject is treated in detail in Chapter 8. The appendix displays more exotic inflation swap structures which are rarely used in practice.

3.4 Inflation payers

Issuers (inflation payers) of ILBs use inflation derivatives markets to modify or assist existing bond issuance. There are three classes of derivative solutions for issuers. The simplest option for issuers is to sell standard bullet profile bonds. Payers of inflation linked cash flows typically have a net exposure to an inflation linked income stream, which resembles a real rate annuity. It is appropriate to hedge this stream of future inflation-indexed cash flows with a structure to pay index-linked flows to the market. Nevertheless, in many cases these standard structures do not fully match the issuer's expected cash inflows.

Figure 6 illustrates a corporate issuer who wants to issue a 20-year bullet ILB to hedge their future inflation linked income stream. The commonly practiced approach as will be discussed in Chapter 4 is to bring a new issue targeted explicitly at a specified group of investors. An investment bank often acts as lead manager for the issue, contacting suitable investors.

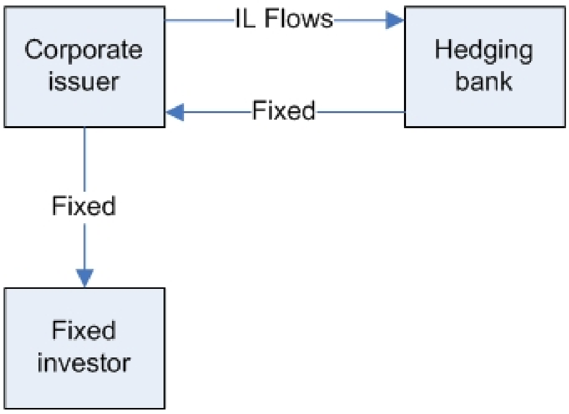
Figure 6: Standard ILB issuance



Source: Deacon (2004)

Another scenario is limited demand from investors for ILBs from a particular corporate issuer. Now the corporate issuer, subject to inflation-indexed future income, is forced to close deals with investors from the market for nominal bonds in order to satisfy its funding requirements. In this case it is plausible that it will not obtain its desired inflation-linked exposure. However, through the additional overlay of a real versus nominal swap the corporate issuer easily transforms the nominal bond into an ILB. Meaning that after receiving the real rate plus inflation from income it pays nominal and inflation and receives real. This totals in paying nominal and receiving real which is equivalent to paying inflation. Hence this way the corporate issuer has synthetically created an inflation-linked issuance, which meets its funding requirements. Figure 7 depicts the described scenario.

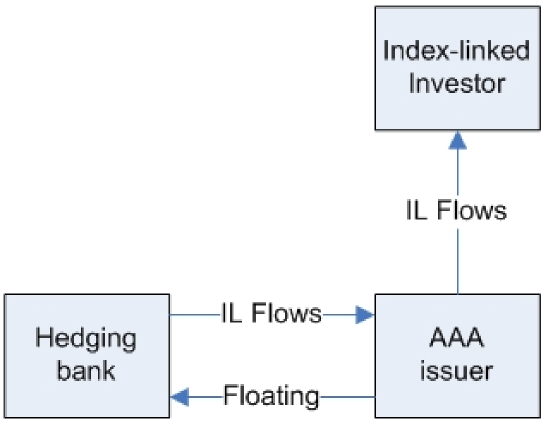
Figure 7: Synthetic ILB issuance



Source: Deacon (2004)

Figure 8 shows schematically that frequent issuers, such as AAA-rated sovereign issuers, try to exploit investor demand for AAA-rated ILPs They have to pay inflation plus real rate cash flows to their investors. However, since a hedging bank can bear the inflation risk or wants to obtain the position that the sovereign issuer can provide, the hedging bank can offer a swap to the AAA-raged issuer. This inflation swap comprises paying the real rate plus inflation by the hedging bank and receiving nominal floating from the AAA-rated issuer.

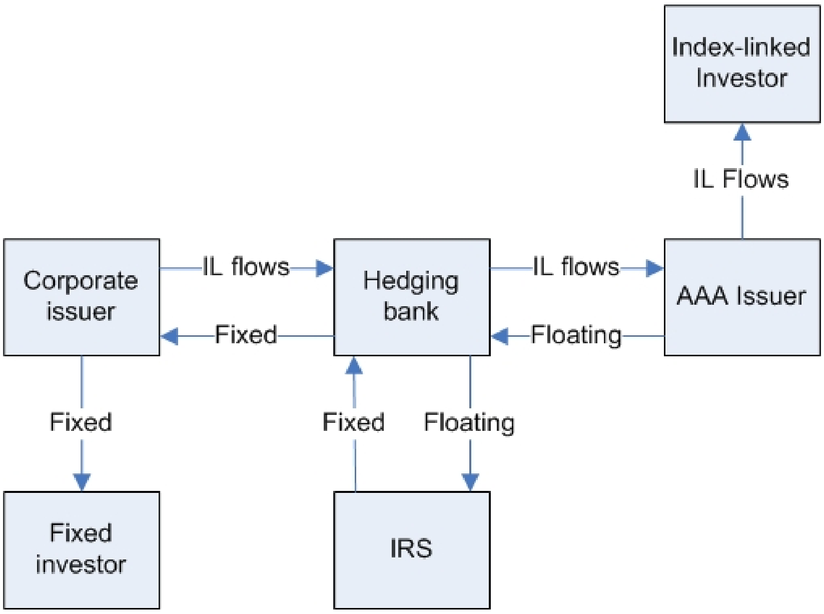
Figure 8: Swapped ILB issuance



Source: Deacon (2004)

The stream of cash flows paid and received by the hedging bank are similar. The combination of Figure 7 and Figure 8 illustrates how the hedging bank can neutralize its inflation-linked exposure by entering into both swaps. What is left is hedging the remaining fixed versus floating risk by entering an interest rate swap (IRS). In practice, the maturity, timing and notional size are rarely matched exactly, consequently the hedging bank retains part of the risks related to these mismatches. Figure 9 illustrates the situation

Figure 9: Combined matched hedging



Source: Deacon (2004)

To further explicate Figure 9, consider a corporate issuer that wants to lock-in a profitable level of BEI rates available in the market prior to the formal debt issuance of the sovereign issuer. The transaction of a real against nominal swap will lock in any break-even level for any issuer. Alternatively entering into nominal against real swaps by the corporate issuer, matching the maturities and coupons of already issued nominal bonds, will synthetically achieve the same result.

On the investor side several variations can take place. An investor can enter into a real versus nominal swap with the hedging bank directly, rather than purchase a swapped issue. The investor thereby gains exposure to inflation risk and is left free to invest in fixed rate nominal securities on its own. Such an approach is often used to synthetically create an ILB from a particular entity who themselves are hesitant of issuing this type of security.

Important to note, swaps with big notional amounts, for example between hedging banks and pension funds or insurance companies, are covered by collateralization agreements to reduce the credit exposure risk of both parties.

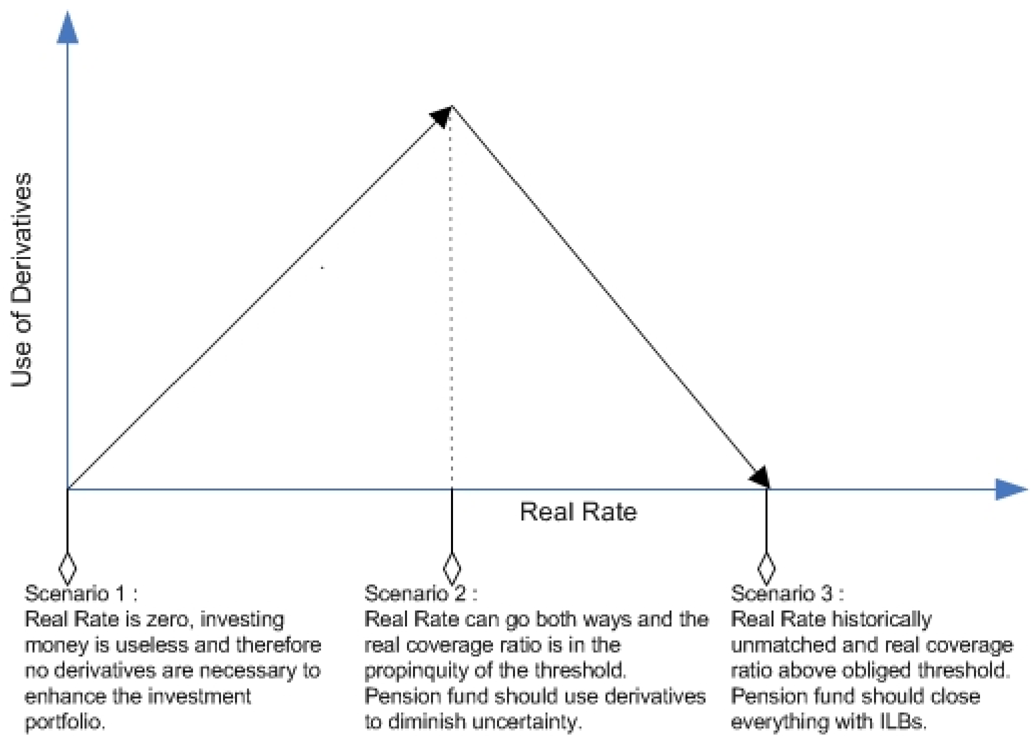
3.5 Inflation receivers

Inflation derivatives reflect the balance of demand from investors and the supply provided by issuers. There are two main sources of demand for inflation derivatives from investors:

- Pension fund managers, who seek to hedge their inflation-indexed liabilities.
- The retail sector, which is attracted by equity products with real principal guarantees and/or inflation plus fixed interest products.

Pension liabilities are typically linked to a COLI for their schemes participants. Not surprisingly many pension funds have investments in ILBs. Inflation derivatives provide alternative strategies for pension fund managers than the option of purchasing ILBs to match inflation linked liabilities. Mismatches in maturity, index, profile and size can be overcome with the use of inflation derivatives resulting in a more precise hedge. In general pension funds tend to apply inflation derivatives when real rates are neither high nor low. Figure 10 depicts three scenarios of low, mediocre and high real rates. In each of these scenarios there are obvious rationales about the application of inflation derivatives. The use of inflation derivatives increases from scenario one, when real rates are zero, to a zenith in scenario two, when a mediocre level of real rates is present. From that point the use of inflation derivatives drops towards no use in scenario three when real rates are historically unrivaled.

Figure 10: Derivatives and real rate relationship



3.6 Summary

Inflation derivatives are contracts which payoff is derived from an agreement, often expressed in a mathematical formula, based on price movements in a CPI or inflation linked security. Inflation derivatives enable investors and issuers to overcome mismatches in maturity, timing, index, profile and size.

Inflation swaps are the most widely applied inflation derivatives. They offer a more flexible risk structure than ILBs, negotiable maturity, wider choice of CPI and avoidance of residual risk. The latter is an advantage for the parent company originating from IFRS 17, which obligates all public companies listed on stock exchanges anywhere within the European Union to prepare accounts with a clear visibility of liabilities.

The types of inflation swaps that are widely used are ZC inflation swaps and YOY inflation swaps. ZC inflation swaps provide cumulative inflation until maturity with no intermediate payments. The YOY inflation swap compensates for inflation over one year at a time and need a convexity adjustment for estimating their forward rates.

Issuers of ILBs use inflation derivatives to modify or assist existing bond issuance. In practice, the maturity, timing and notional size are rarely matched exactly, consequently the hedging bank retains part of the risks related to these mismatches. Inflation swaps with big notional amounts are covered by collateralization agreements to reduce counterparty credit risk.

Demand for inflation derivatives comes especially from pension funds and the retail sector. In general pension funds tend to apply inflation derivatives when real rates are from a historical perspective neither high nor low.

Chapter 4 Inflation Markets

Being introduced to the concepts of CPI, ILB and inflation derivative an outline of the inflation markets is appropriate at this stage. According to Deacon (2004) the most developed international markets for inflation are Australia, Canada, France, Sweden, UK and the US. In order to get a feeling for the global inflation market each of these markets is highlighted by presenting its history, market players and current figures.

4.1 Australia

History

Australia's first indexed security was a CIB linked to the Australian CPI. The security was issued by the State Electricity Commission of Victoria in August 1983. Several years later, in July 1985 the Commonwealth Government department issued four treasury indexed bonds (TIBs), two CIBs and two interest indexed bonds (IIBs). The government had several motivations:

- Provision of cost effective financing.
- Prolonging the average maturity of government debt.
- Enhancement of the retirement incomes policy.

From 1987 quasi-government authorities entered the market and in 1988 annuity-style indexed securities were issued. The Australian Treasury resumed its issuing in 1993 and in 1997/1998 the Australian Treasury increased its auctions. The reason for this increase was to provide greater certainty of the timing of the auctions. As a consequence investors could enhance the planning of their acquisitions of TIBs. Another change introduced in 2001 was the index-linked auction calendar. This calendar would be announced at the start of each financial year. The auction dates were chosen to directly follow the TIB coupon payment dates to aid coupon reinvestment.

Market figures

The total indexed amount outstanding is according to Dogra (2007) AU$ 7.9 billion. This amount is represented by a total of three outstanding ILBs that are linked to the Australian consumer prices index. Furthermore these bonds mature between 2010 and 2020.

Market players
The top 15 investors in the market together hold 90% of the total amount outstanding and are domestic funds and insurance companies. The Commonwealth government department, representing half the securities, is the dominant issuer of Australian indexed bonds. Other issuers are state governments, quasi-government authorities, electricity companies and financiers of private projects.

ILB structure
In general the IIBs are linked to the Australian CPI. Both CIBs and IIBs have coupon payments on a quarterly basis, with cash flows indexed to the average percentage change in the CPI over the two quarters ending in the quarter that is two quarters prior to that in which the next interest payment falls.

4.2 Canada

History
Mark Deacon (2004) says that in December 1991 the Canadian government started issuing real return bonds (RRBs). RRBs are designed to stabilize the real cost of borrowing and fix the real rate of return for investors. Furthermore they are a new source of funding and produce cost savings for the issuer. The Canadian authorities encouraged participation in the market making process by reducing risk of submitting a fallacious bid by investors through fixed price subscriptions and single yield auctions.

Market figures
The indexed amount outstanding according to Dogra (2007) in RRBs is C$ 26.6 billion. This amount is represented by a total of 4 RRBs, all having a maturity of 30 years. These RRBs are linked to the Canadian CPI and mature between 2021 and 2036.

ILB structure
RRBs are CIBs that pay semi-annual coupons and are based on cumulative movements in the CPI since the last coupon date. This indexation methodology has currently been adopted by France, Italy, Sweden and the US.

Tax treatment
For RRBs the full nominal interest payments are taxable. Therefore RRBs are especially held by tax-exempt institutions as discussed in Chapter 1. These tax exempt institutions are pension funds and life insurance companies. Due to the buy and hold nature of these investors secondary market trading is limited.

4.3 France

History
After several small issuance of indexed debt, indexation increased in the 1950s in France, because of low confidence in the French Franc. Moreover due to low public finances the French government was incited to experiment with indexed bonds in order to raise revenues. Throughout the 1950s a number of public corporations and nationalized industries issued index-linked bonds, which in general were linked to their own product prices. In 1958 most forms of indexation were banned due to a devaluation of the French Franc and a total ban was enacted in 1968 due to improved government finances. In 1973 the issuing of indexed bonds resumed again, however in the 1977 the index bonds issues were put on hold again.

In 1997 the Trésor announced that it wanted to start with the issuance of ILBs. Reduced borrowing costs and becoming the first major sovereign issuer of ILBs in continental Europe were the main rationales. The choice of CPI was between Euro and French. The outcome was the French CPI due to vagueness of the Euro CPI at that time. The first French government Obligation Assimilables du Trésor (OAT) linked tot the French CPI was launched in September 1998.

The introduction by the French BLS of a new methodology for computing the CPI occurred in 1999. The improvements were aimed at reducing the Boskin bias, as discussed in Chapter 1, and included a better coverage of sales and rebates, the use of geometric mean formulas and better monitoring of price/quality effects for products. The French BLS estimated that these changes would shave 0.1% off CPI inflation per year and improve the index's correlation with the Eurostat's harmonized French CPI.

In 2001 Banks were asked to investigate the peculiarities of a new OAT with cash flows linked to the Euro MUICP. OATs, such as the one linked to the Euro MUICP, were issued by the French government in the upcoming years.

ILB structure
The Trésor decided to base the structure of its OATs on that of Canada's RRBs. The only difference is that interest is paid annually instead of semi-annually. Similar to the Canadian bonds, the French securities are CIBs with a three month time lag.

Market figures
Dogra (2007) states that the amount of government indexed debt outstanding is € 112.7 billion, €51.7 billion linked to Euro inflation and € 61 billion linked to French inflation. This amount is represented by a total of 10 (5 Euro and 5 French) OATs, which mature between 2009 and 2040.

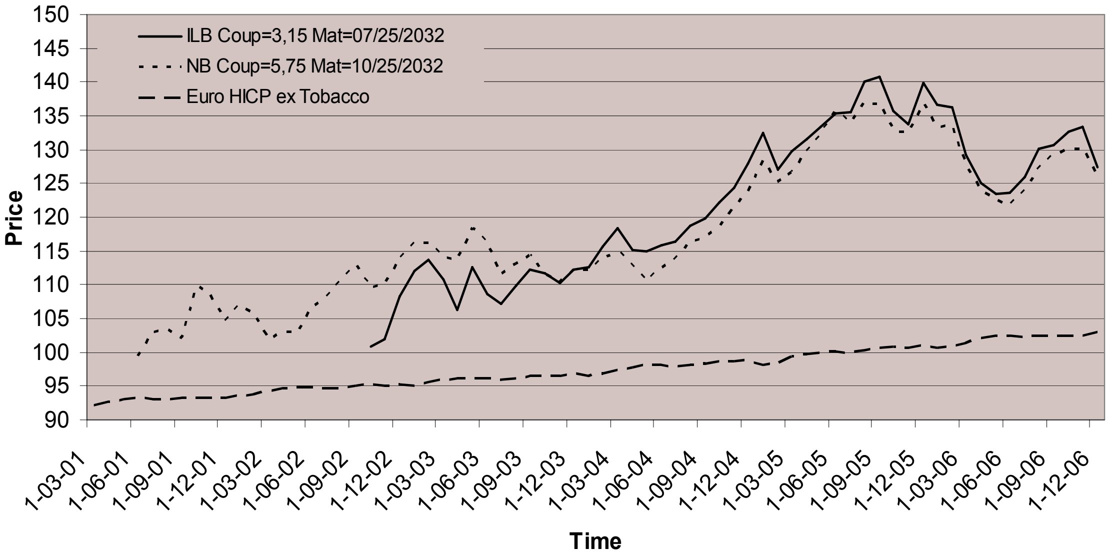
Market players

As the OAT market has grown other French institutions began issuing ILBs, such as Caisse Nationale de Autoroutes (CAN), and Reseau Ferre de France (RFF).

Tax treatment

The tax treatment of inflation-linked OATs is similar to that of nominal bonds. Domestic retail investors are taxed on the uplifted annual interest payments as well as on any capital gain realized either at redemption or when the bond is sold. Figure 11 shows the prices of al French nominal and French inflation linked OAT from March 2001 in conjunction with the Euro HICP ex Tobacco.

Figure 11: Prices of French nominal bond and ILB.



Source: Bloomberg

4.4 Sweden

History

Deacon (2004) states that the Swedish National Debt Office (SNDO) decided to put a ILBs issuance program into practice in April 1994. Main reasons for this program were:

- Reducing funding costs.
- Enhancing the credibility of monetary policy.
- Broadening the range of available investment options.
- Providing a means of estimating market expectations on inflation.

At end of 1994 the SNDO announced to issue shorter maturity ILBs in the near future. By selling small quantities of the shorter maturity ILBs to private investors at weekly auctions during 1995 they tried to encourage the development of the ILB market. In the beginning of January 1996 the SNDO announced that its ILBs would be issued by reopening rather than by regular price auctions. In October 1996, the SNDO announced a decrease in ILB issuances, which led to a slowdown in the growth of the ILB market. On 23 June 1998 the SNDO held the first ILB switch auction. In April 1999 the SNDO announced several reforms to the inflation market. In particular, issuance by reopening stopped and was replaced by yield auctions in order to improve the operation of the secondary market. Furthermore the SNDO announced its intention to introduce a deflation floor on the principal payment of any new ILB that it issued. As a logical result new issues from April 1999 lacked accrued inflation since 1994.

Four firms were appointed as authorized ILB dealers for 2000. The status of these firms is reviewed annually by the SNDO. In June 2002 the SNDO announced that it had decided to allow a degree of flexibility over the part of an ILB sold at an auction. Under this new system a fixed size need not be announced ahead of each auction, however a target interval is specified.

Deacon (2004) states that the size of the ILB market increased significantly during 2002, when the Swedish parliament decided to convert assets of three state entities: the Swedish Nuclear Waste Fund (KAF), the Premium Pension Authority (PPM) and the Deposit Guarantee Board (IGN). Assets held in the form of cash balances at the SNDO were converted to Treasury bonds, which resulted in the creation of SEK 41.5 billion of bonds on 1 July 2002, SEK 34.1 billion of which were inflation linked, with allocations spread across all the existing securities. This represented a 33% increase in the nominal size of the inflation-inked government debt outstanding at the time and a corresponding rise of 0.6 years in its duration. In its funding program for 2003, the SNDO announced a sharp rise in inflation-linked issuance, from SEK 9 billion in 2002 to SEK 15 billion in 2003.

Market figures

At the beginning of March 2007 the amount of government inflation indexed debt is SEK 237 billion according to Dogra (2007). This amount is represented by a total of 5 inflation linked government bonds. These bonds are linked to the Swedish CPI and mature between 2008 and 2028.

To date, only a tiny corporate ILB market has emerged in Sweden. At the end of March 2003 issuance stood at approximately 6 billion SEK and represented less than 4% of the total Swedish ILB market. The few bonds issued by these corporate issuers have all been linked to the Swedish CPI and the majority has been launched with a maturity between 8 and 20 years.

Market players

Besides the government, some corporate issuers are participating in the inflation markets:

- Electrolux (manufacturer of electrical appliances).
- Fysikhuset Stockholm KB (Construction Company).
- Investor AB (investment fund).
- Stanens Bostadsfinansiergsinstitut (government agency).
- Volvo (vehicle manufacturer).

4.5 United Kingdom

History

In the beginning of the 1970s rising inflation caused a decrease in real value of nominal debt. As a result the UK government issued savings contracts with returns linked to the retail prices index (RPI) in 1975. These contracts were not able to be marketed and investment opportunities were only limited to pensioners. A report in 1980 caused further anxiety with regard to the potential effects of inflation on the financial system. In the light of this report the UK government was advised that it should issue earnings-indexed gilts, only be purchased by pension funds. In March 1981 the first RPI-linked gilt was issued by the government. During March 1982 restrictions on ownership of the RPI-linked gilts were lifted and other buyers entered the inflation market.
Reasons for issuance were:

- Consolidation of the belief in the government's anti-inflation policy.
- Reduce cost of funding by saving the inflation risk premium.
- Improve monetary control by increasing the flexibility of funding.

The inflation market grew significantly during the early 1980s. Later that decade the UK government's finances moved into surplus and as a result a program was initiated to repurchase a portion of the outstanding nominal gilts, which made the RPI-linked gilts a higher portion of the government outstanding debt. The surplus of the government's finances prolonged and a freeze in issuance of gilts between October 1988 and January 1991 was initiated. Issuance of gilts resumed when the government's budget moved back into deficit during the early 1990s. Furthermore a policy of creating large issue sizes through the reopening of existing gilts improved liquidity in the gilts market significantly. The result can be illustrated by the fact that between September 1992 and July 2002 the size of the nominal and RPI-linked gilt market increased from £15 billion to £ 38 billion, with an average gilts' nominal size rising from 1.1 billion to 3.8 billion. Since March 1999 issuance was limited to RPI-linked gilts with maturities of 10 years and longer. To provide liquidity, the Debt Management Office (DMO) established repurchase facilities for nominal and RPI-linked gilts. By 1998 the authorities were convinced that the market was mature enough to support auctions. A pre-announced auction program would improve the predictability and transparency of issuance and consequently an increase of demand and liquidity.
Furthermore the DMO decided to specify a group of inflation market makers, in order to minimize the likelihood of "fair-weather" market-making behavior[2].
Mark Deacon (2004) says that in September 1998 an inflation market maker list was introduced, which comprises eight firms. The DMO announced that auctions would be on a single price basis. The first auction in this new program was held on 25 November 1998 and continued on a quarterly basis. In addition to improve auctions the DMO holds RPI-linked switch auctions, the first was held on 19 July 2001.

[2] It is meant here providing tremendous amounts of liquidity when markets are functioning well, but when markets don't function well tending to destabilize the market and driving liquidity away.

ILB structure
RPI-linked gilts are CIBs that pay semi-annual coupon payments and none have deflation floors. Furthermore an eight-month indexation time lag is incorporated. The eight months are comprised of two months to allow for the compilation and publication of the RPI and six months to ensure that the nominal size of the next coupon payment is known at the start of each coupon period. Although one agrees on that the Canadian design is an improvement over this one, many have expressed the concern that the launch of a new style of security will create a two-tier inflation market, with liquidity being concentrated in the new-style gilts at the expense of the existing issues. For this reason the DMO decided to keep the existing design.

Market figures
Dogra (2007) says that the current amount outstanding in inflation linked gilts is £123 billion, represented by a total of 12 RPI-linked gilts. These gilts mature between 2009 and 2055. It is nice to know that due to higher market inflation expectations compared to actual inflation since launch a tremendous reduction in cost of funding has been generated for the UK government.

Market players
Despite the fact that the inflation market has grown relatively quickly since it's inception in 1981, there has only been significant corporate inflation linked issuance since the late 1990s. Reasons for this phenomenon are:

- A Change in tax regime.
  By allowing the inflation uplift in the principal value of an inflation-linked gilt to be set against corporate tax liabilities they became more attractive.
- Privatization of many public sector corporations during the 1980s and 1990s.
  The revenue streams of these companies are inflation-linked due to the regulatory regime. Therefore these new privately owned companies seek liabilities having an inflation linked behavior.

From an historical point of view pension plans in the UK have a tendency to invest a relatively high proportion of their assets in equities. This is uncanny when one observers that their liabilities often resemble RPI-linked gilts more closely. Due to new accounting standards and the fall in equities in 2000 pension funds collectively shifted from equities to fixed income securities such as RPI-linked gilts. Other players in this market are life insurance funds, which are compelled by law to hold inflation linked gilts.

4.6 United States

According to Deacon (2004) the US Treasury started issuing inflation linked securities in 1997. The first inflation linked security was announced by President Bill Clinton. This security, linked to the CPI, had a maturity of 10 years and was issued by a single yield auction.

On 4 December 1996 the Boskin Committee reported to the US Senate on his study of the accuracy of the CPI as a measure for consumer price inflation. This report and its effects have been discussed thoroughly in section 2.5. The committee its main conclusion was that the US CPI overstated the true inflation rate by about 1.1 percentage points annually. This created concern that the CPI might be replaced.

Having this in mind, in January 1997 the size of the upcoming Treasury Inflation Indexed Note (TIIN) auction was announced and forward trading began. Expectations were diverse and analysts had forecasted that the TIIN would trade at a real rate of between 3,375% and 3,625%. The note was sold with a real rate of 3.449% at the auction on 29 January. In July 1997, the Treasury issued a 5 year TIIN, because of high demand for shorter maturity. This TIIN would be less vulnerable to revisions of the CPI. However in September 1998 the treasury announced to concentrate inflation indexed issuance on longer maturities. Due to federal budget surpluses during 1998 and 1999 the Treasury announced to reduce the frequency of its auctions. In the February 2000 financing announcement the Treasury outlined how it would implement a reduction of the size of public debt. Throughout 2001 there was much speculation surrounding the future of the TIIS program. Analysts started forecasting a small deficit for 2002. This was in contrast to the large surplus envisaged for the same year just months earlier. In the Quarterly Refunding Statement for February 2002, Assistant Secretary Brian Roseboro confirmed that the Treasury itself also envisaged small budgetary deficits for both 2002 and 2003. The Treasury used the May 2002 quarterly refunding announcement as an opportunity to publicize further changes to the TIIS program. An increase in supply reflected further deterioration of the fiscal situation. During the second half of 2002 the Treasury promoted inflation indexed securities by broadening its investor base and diversifying its funding sources. The rationale of the Treasury is the reduction of average funding costs over time.

ILB structure

The design of TIIN is based on that of Canada's RRBs. Like the Canadian bonds, TIIN are semi-annual CIBs with a three-month indexation time-lag. The only difference between the Canadian and American instruments is the deflation floor. The deflation floor means that the US Treasury guarantees that the nominal redemption payment of all its securities will not be less than the par value at issue.

Market figures

According to Dogra (2007) the total inflation indexed amount outstanding is around US$ 464 billion. This amount is represented by a total of 22 TIIN. These TIINs are linked to the US CPI and mature between 2008 and 2032.

Tax treatment

The increase due to inflation in the value of the principal of US indexed securities is treated as current income for tax purposes, resulting in a tax on the inflation uplift levied on an annual basis even though the inflation uplift payment itself is not made until maturity. This feature may limit demand for TIIN to that from tax-exempt investors and those taxpayers with tax-deferred accounts. For this reason the Treasury launched non-marketable inflation-indexed savings bonds in September 1998. These bonds are aimed at small investors and interest accrues monthly, compounds semi-annually and is payable at redemption. However the income tax is deferred until the securities are cashed in or until they stop earning interest after the full 30-years. Like TIIS, these bonds are also issued with deflation protection but in the form of a guarantee that the earnings rate never falls below zero.

Market players

A number of government agencies, corporations and municipalities have issued inflation-indexed securities since 1997. Swaps have played an important role in this process, since most issuers don’t want to bear the inflation risk incorporated in the instruments. Thus they have used the derivatives market to swap inflation risk for nominal rate risk. It is not likely to be a coincidence that the majority of non Treasury issues are IIBs, a structure that is better suited to swap inflation risk for nominal interest rate risk compared with the structure of TIIN. There have been a handful of corporate indexed issues since 1997. This indicates that there is only limited demand. An explanation is that it is plausible that risk-averse investors demand an instrument that removes credit as well as inflation risk.

Inflation derivatives

In July 1997 the Chicago Board of Trade (CBOT) introduced futures and options contracts on both medium and long term inflation indexed notes, and these were joined by comparable contracts on inflation indexed bonds from April 1998. These derivatives were designed to allow traders to take position or hedge movements either in real yields or in the CPI. Traders were expected to enhance overall liquidity. An interesting development in June 2003 was the launch by Deutsche Bank and Goldman Sachs of monthly auctions of options and forwards on the level of US CPI. This followed the successful launch of similar contracts on Euro-HICP ex-tobacco in May 2003.

Current TIPS market

In order to describe the current Treasury Inflation Protected Security (TIPS) market, the opinion from a London banker, Lupoli (2007), on current and future inflation is outlined below.

According to Lupoli as of March 2007, inflation risk will increase in the future. His rationale is a strong need for real rates and a widening move in breakevens. In order for breakevens to widen further from current level one needs a higher price for inflation linked bonds relative to nominal bonds. Another option would be a generalized sell-off in both nominal and real yields. Due to current US economy backdrop a sell-off in both nominal and real yields is not present because of the safety characteristics of instruments providing real and nominal yields. Hence, it is necessary for his opinion to become reality that the demand for real yields will increase.

The shape of the real yield curve is interesting at the moment, because from a historical perspective there is almost no curvature. Given the current backdrop, Lupoli (2007) sees the recent underperformance of the 10-year sector as continuing. To consolidate his view, historical data shows a seasonal underperformance of the 10-year sector into April. A 10-year TIPS re-opening and a new 5-year TIPS are scheduled. Therefore it is wise to go short the 10 years TIPS and go long the better performing maturities of the curve. Lupoli (2007) suggests that this helps in his view of positioning further out on the curve in TIPS and in breakevens.

Lupoli (2007) still thinks that inflation is a bigger threat than what is perceived by the market. Furthermore he has the opinion that re-rating of breakevens consistent with the level of nominal yields has begun. This phenomenon is better applicable for the front-end of the curve than the longer-end.

4.7 Inflation derivatives markets

Market development

Benaben (2005) describes that one can divide inflation linked securities (ILS) markets into four stages of development. These four stages of development are illustrated in Table 12. The stages of development and liquidity are ranked from stage 1, the least developed and liquid, to stage 4, the most developed and liquid.

In the first stage markets ILS are traded indirectly through an intermediate. It is plausible that swaps are traded through taking basis risk with instruments traded in higher stage markets, such as the interest rate swap (IRS) market. Stage 2 markets have one or more tradable ILS. However these markets lack a sufficient number of securities to cover all relevant maturities and are therefore characterized by low liquidity in inflation swaps. In the third stage markets have a full set of tradable securities at all relevant maturities. In these markets a full curve over maturities can be constructed. In the fourth stage, the market has reached a level of maturity and liquidity analogous to the interest rate market much that, independently of any models, implying prices based on the underlying real and nominal asset yields is possible.

Table 12: Four stages of development and liquidity of the ILS market

No Tradable underlying instrument	One or more tradable underlying instrument	Many tradable instruments possibility of constructing a forward CPI curve	Liquidity comparable to IRS market No model dependency, bid/offer tight
Belgian, German, Spanish indices	Italian inflation-indexed swap market	UK, FR, EU and US CPI swaps	some maturities in the EUR and UK ILS market from time to time
stage 1	**stage 2**	**stage 3**	**stage 4**
Increasing liquidity of inflation swaps from stage 1 to stage 4			

Source: Benaben (2005)

The development of the inflation derivatives market has been largely demand driven. The three main markets, Europe, UK and US represent over 95% of global inflation derivatives markets.

European inflation derivatives market
Benaben (2005) states that with the Italian Post Office's inflation protected equity product launched in 2001 and the issuance of the OATei 2012 by the French Trésor, the inflation swap market linked to the Euro HICP was given birth. In 2003 developments as the swapped issuance of approximately 8 billion of euro inflation linked medium term notes and an expanding curve of euro ILBs, helped maturing the Euro ILS market to a stage three market. This trend continued in 2004 and into 2005 where inflation swap trades are now a common in the market, especially the ZC inflation swaps trades.

UK inflation derivatives market
The earliest inflation swaps have been traded in the UK market in the early 1990s. Given the existence of a developed RPI-linked gilt market and interest from both payers and receivers, the UK inflation swap market developed the earliest among the European, UK and US.

The use of inflation swaps by corporations and project companies as well as provided the source of inflation streams, while pension liability hedging absorbed those flows directly through inflation swaps and through purchase of swapped new issues.

US Inflation derivatives market
The US CPI derivatives market remains the smallest amongst the major markets; however it has the potential to become the biggest. The US ILS market emerged in 2003, trading approximately $3 billion notional in the broker market and is dominated by a couple of investment banks. The market then expanded rapidly in 2004, trading approximately 12 billion $ notional in the broker market. A similar speed of growth occurred in 2005, with more investment banks entering the market. The rationale for the low liquidity in the US ILS market is the lack of issuers of inflation swaps, who want to retain inflation risk. The only issuer who retained inflation exposure is the Treasury. It is only with a development of a two-way flow market that we will see the US ILS market go through the next stage of development.

4.8 Future trends and market figures

Universally accepted standards from the International Swaps and Derivatives Association (ISDA) provide a great boost to the inflation derivatives market and assist in further development. Furthermore pension reforms in Europe are a significant factor responsible for the development of the inflation derivatives market. Although the current inflation derivatives market has been dominated by demand generated by the retail sector, pension reforms will switch the emphasis by demanding longer dated real assets and derivative solutions.

According to the International Accounting Standards Board (IASB) (2006) the European Union adopted International Financial Accounting Standard 17 “Retirement benefits” in January 2005. This IFRS obligates all public companies listed on stock exchanges anywhere within the European Union to prepare accounts with visibility of liabilities, which has widespread impact on the way companies report their financial statements. In particular companies with Defined Benefit (DB) pension schemes will be affected by the application of IFRS 17, which requires them to reflect the fully mark-to-market valuation in their financial statements. The key element of IFRS 17 is: that pension liabilities should be measured, incorporating actuarial assumptions such as life expectancy on cash flows basis.

Fixed income securities are the asset class that provides the closest liability match. Pension fund managers who tend to hold a relatively large amount of their assets in other asset classes are likely to introduce significant volatility into the financial statements of their parent companies. The rationale is that their asset valuations will move in line with their market prices while the present value of their liabilities will be affected by market prices of bonds. The potential role here of inflation derivatives is clear: if pension fund trustees want to minimize the mark-to-market volatility of the investments under their management, they need to purchase a mix of assets that match their liabilities as closely as possible.

Options, swaptions and other derivatives will be traded on a more standardized basis. Moreover in the future they will be developed in line with the requirements of the investors rather than simply replicating products that already exist in the interest rate market. As the underlying markets in swaps and forwards become more liquid, with tighter bid-offer spreads, confidence in the ability to efficiently and effectively hedge options will increase among market makers. The next step will be better price quotes, which results in increased interest from both buyers and sellers.

In principle the market shall evolve from historical volatility based pricing to implied volatility based pricing. This development would be similar to markets for interest rate volatility products during the course of the past 20 years. The existence of inflation derivatives in principle enables the emergence of more sophisticated pension hedging strategies, such as those incorporating inflation swaptions.

Market figures

The main investment banks have prices for ZC inflation swaps available on their Bloomberg websites. For the Euro, France, UK and US market ZC inflation swaps are traded in maturities varying from 2 until 30 years, for the UK up to 50 years. Because the inflation derivatives market contains a large number of OTC contracts its size is not transparent. Prices for swaptions on inflation are not published.

4.9 Summary

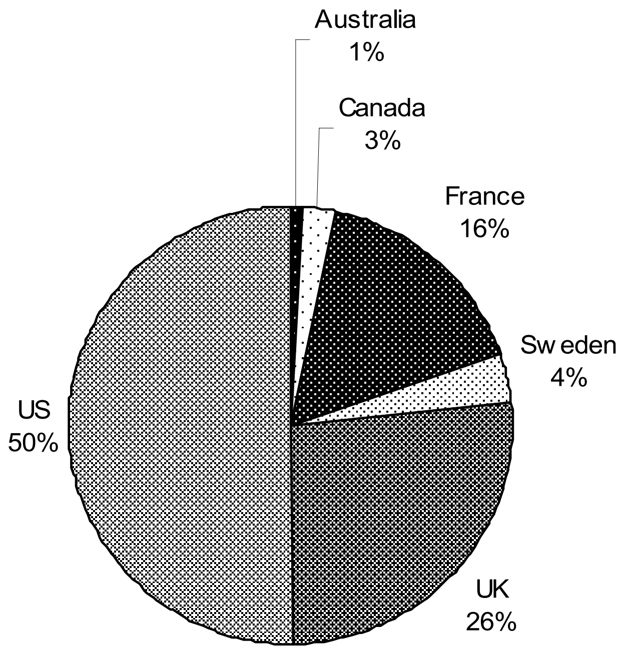
Currently the most developed international markets for inflation are Australia, Canada, France, Sweden, UK and the US. Canada, Australia and Sweden are small inflation markets compared to France, UK and US as comes clear from Table 13 and Figure 12. Furthermore France, UK and the US represent together a €640 billion large inflation market. Moreover the range of maturities form ILBs in these markets is more widespread compared to the small ones. In all developed inflation markets the sovereign issuers are dominating.

Table 13: Distribution of global inflation linked debt

Country	Debt	Currency	Euro Exchange Rate	Debt in Euros
Australia	7.9	AU$	0.629	4.97
Canada	26.6	C$	0.7035	18.71
France	112.7	Euro	1	112.70
Sweden	237	SEK	0.1061	25.15
UK	123	Pound	1.48	182.04
US	464	US$	0.7483	347.21
			Total Inflation Linked Debt	690.78

Source: Bloomberg & Dogra (2007)

Figure 12: Distribution of global inflation linked debt



Source: Bloomberg & Dogra (2007)

When inflation markets are in their first stage, securities are traded indirectly through an intermediate. At stage 2, inflation markets have one or more tradable inflation derivatives next to tradable securities. Third stage inflation markets are characterized by a full set of tradable securities at all relevant maturities. When reached the fourth stage, inflation markets enable investors to imply prices independently of any models. The three main markets, Europe, UK and US represent over 95% of global inflation derivatives markets.

Universally accepted standards form the International Swaps and Derivatives Association (ISDA) will provide a great boost to the inflation derivatives market in the future. Furthermore, inflation options and inflation swaptions will be traded on a more standardized basis. In principle the inflation market shall evolve from a market based on historical volatility pricing to implied volatility pricing.

The main investment banks have prices for ZC inflation swaps available on their Bloomberg websites. Prices for swaptions on inflation are not yet published.

Chapter 5 Inflation Modeling

Having described the global inflation market and inflation derivatives market this chapters will deal with modeling inflation. By providing insight in models that are applied in the market the reader should get a perception of the level of complexity an objective pricing model must have.

5.1 Introduction

Benaben (2005) says that the inflation derivatives market is a relatively new and lacks a standard model for estimating inflation. This is in contrast with the interest rate market in which, according to Hull (2003), the model of Black for swaption pricing is widely used.

Modeling future inflation is a difficult subject because of the economic nature of inflation. Inflation is not a rate quoted in the market, such as the interest rate, or a universal known traded liquid index. Inflation is defined from few monthly observations at a fixed schedule. Furthermore the inflation market exhibits seasonality of inflation, which significantly impinges inflation volatility.
To model derivatives in the inflation market, there are two main alternatives:

- Model the underlying spot value ($CPI(t)$) of the derivatives and calibrate parameters from prices of liquid products based on the same underlying.
  This modeling is flexible since one can choose any payoff function. Furthermore incorporation of non-observable or abstract parameters in the model is possible.
- Model the underlying forward values (CPI at T) with the current values as initial condition for the processes.
  Closer to the market because it assumes a dynamic for the prices of products through observable parameters (implied volatilities or implied correlations). This alternative is applied for swaption pricing described in Chapter 8 and Chapter 9.

The usual framework in financial mathematics modeling is in continuous time. Benaben (2005) defines, as for the ZC bonds that are defined via an instantaneous interest rate, the CPI via an instantaneous inflation rate and thinks of a dynamic such as: $\frac{dCPI}{CPI} = i_t dt$

In general the differentiated inflation $(\Delta i_t)_{t\geq 0}$ is modeled because differentiated inflation is stationary in an integrated multivariate framework. The models use in conjunction with differentiated inflation a set of exogenous variables and their past realizations. The resulting series is expected to be even more stationary.

In mathematical terms this can be explained by considering a stochastic vector $\vec{X}_t$ whose elements $\{X_t^i, i = 1,\ldots,n\}$ are integrated of order d, meaning they should be differentiated by taking the differences between two consecutive elements d times to become stationary. With an integrated multivariate framework is meant that there exists a vector $\vec{B}$ of order b such that the new process $\vec{B} \times \vec{X}_t$ is of order $d - b$.

5.2 Jarrow-Yildrim model

The Jarrow-Yildrim model as described in Malaveaz (2005) and Benaben (2005) is the most widely known inflation model. The model assumes that the CPI is an exchange rate between nominal and real yields. This naturally results in a three factor framework, where the factors are the nominal, real and inflation rate. The contribution of the Jarrow-Yildrim model is to provide non-arbitrage conditions among the three components, which leads to the dynamics shown in Equation 2.

Equation 2: Dynamics for the CPI

$$\frac{dCPI_t}{CPI_t} = \left(r_t^n - r_t^r\right)dt + \sigma_t^{CPI}\, dW_t^{CPI}$$
$$dr_t^k = a^k(b^k - r_t^k)\,dt + \sigma_t^k\, dW_t^k \quad k \in \{n, r\}$$

Where r^n (respectively r^r) is the nominal (respectively real) instantaneous short-term rate, factor correlations $d\left\langle W_t^k W_t^j \right\rangle = \rho_{k,j}\,dt \quad k, j \in \{n, r, CPI\}$, speeds of mean reversion a_n, a_r and means b_n, b_r. In the Jarrow-Yildrim model they use a standard Brownian motion, also called a Wiener process. This is a stochastic process $\{W_t\}_{t\geq 0}$, which means a family of random variables W_t, indexed by nonnegative real numbers t, defined on a common probability space (Ω, F, P) with the following properties:

- $W_0 = 0$.
- With probability 1, the function $t \to W_t$ is continuous in t.
- The process $\{W_t\}_{t\geq 0}$ has stationary, independent increments.
- The increment $W_{t+s} - W_s$ has the $N(0,t)$ distribution.

It is easy to draw the relationship that this model represents a combination of the Fisher equation ($i = r^n - r^r$) on the long term inflation and the co-integration approach ($\frac{dCPI}{CPI} = i_t dt$). However this only holds as long as both the nominal and real rates show close mean reversion: $a^n \approx a^r$. The advantages of this model are:

- Simple to understand due to analogy with exchange rate.
- Non arbitrage conditions cause framework easy to implement.

To calibrate the volatility coefficients and the correlations parameters one can use historical estimation using the series of nominal zero coupon and real zero coupon rates. The mean reversion coefficients are computed numerically using least-squares minimization (see appendix) between the theoretical values and the real market data. The model shows the problem of the instability of historically estimated parameters, because the outcome of all these parameters depends heavily on the sample choice and period looking back. Furthermore a large amount of market data is needed to validate the empirical estimates. The model is therefore better applicable in the UK and US than in Europe because the UK and the US have a more mature inflation market, which makes the estimation of the parameters more reliable.

In order to price inflation derivatives with the Jarrow Yildrim model one has to deal with a non-observable variable, the real rate. Therefore one computes inflation swaps on ILB prices. This brings up the following issues:

- One does not calibrate the model onto the most natural curve in the derivatives market, the swap curve, but use nominal bonds and ILBs.
- The BEI is higher in the swap than in the bond world, due to a lack of inflation payoffs. In the past investment banks have been forced to pay inflation via inflation swaps and recycling inflation from the linkers. As described in Chapter 3 this resulted in bearing a mismatch risk.
- The maturities of the products used for the calculation do often not closely match the maturities of the instruments one wants to price.

5.3 The Mercurio model

Based on an extension of the Jarrow-Yildrim model, Mercurio (2005) offers two additional approaches to improve the modeling of future inflation. Mercurio keeps the exchange rate analogy between nominal and real rate, however considers a Brace, Gatarek and Musiella (BGM) model on both the nominal and real London Inter Bank Offered Rates (LIBOR) forwards by linking LIBOR with the respective ZC bonds. The possibility of negative nominal interest rates and the difficulty of estimating historically the real rate parameters are treated by assuming log-normal dynamics on the nominal and the real LIBOR forwards.

Equation 3: Dynamics in Mercurio's model

$$\begin{cases} \dfrac{dF^k(t,T_{i-1},T_i)}{F^k(t,T_{i-1},T_i)} = \sigma_{k,i}\, dZ_i^k(t) \\[2ex] \dfrac{B^k(t,T_i)}{B^k(t,T_{i-1})} = \dfrac{1}{1+t_i F^k(t,T_{i-1},T_i)} \end{cases} \qquad k \in \{n, r\}$$

$$I_i(t) = CPI_t \frac{B^r(t,T_i)}{B^n(t,T_i)} \qquad\qquad \frac{I_i(t)}{I_{i-1}(t)} = \frac{1+t_i F^n(t;T_{i-1},T_i)}{1+t_i F^r(t;T_{i-1},T_i)}$$

Where $F^n(t,T_{i-1},T_i)$ (respectively $F^r(t,T_{i-1},T_i)$) is the value at time t of the nominal (respectively real) LIBOR forward between T_{i-1} and T_i, $Z_i^n(t)$ (respectively $Z_i^r(t)$) is a Brownian motion under the nominal (respectively real) forward payment probability, $t_i = T_i - T_{i-1}$ and $B^k(t,T)$ is the value at time t of a nominal (when $k = n$) or real (when $k = r$) ZC bond maturing at time T. Mercurio uses a result of the Jarrow -Yildrim model: the expected value of a forward inflation ratio can be expressed by only the real forward ZC bond and the nominal ZC bond, using ILBs and nominal bonds. The main advantages of the Mercurio model are:

- Use of LIBOR and bonds as input parameters, hence application of more market data than with the Jarrow-Yildrim model.
- More accurate calibration

5.4 Summary

The inflation derivatives market has no standard model to estimate inflation. Moreover, inflation is defined from few monthly observations and is not a rate quoted in the market, such as the nominal rate. Modeling inflation derivatives can be done on the basis of modeling the underlying spot rate or forward rate.

In models tailored at estimating future inflation rates, the differentiated inflation $(\Delta i_t)_{t\geq 0}$ is modeled. The rationale is that differentiated inflation is stationary in an integrated multivariate framework.

The Jarrow-Yildrim model is currently the most applied inflation model, because the model is simple to implement and easy to understand. The model contributes by providing non-arbitrage conditions among the nominal, real and inflation rate. A disadvantage of the Jarrow-Yildrim model is that it computes inflation swap rates based on bond prices.

The model introduced by Mercurio is an extension on the Jarrow-Yildrim model. Mercurio's model links LIBOR with the corresponding ZC bonds. The main advantages of the Mercurio model are more accurate calibration and use of more market input parameters.

Chapter 6 Interpolation

Enhanced interpolating between known data points is useful for estimating a ZC inflation rate curve covering all maturities or in order to compute inflation rates between known maturities which are not traded in the inflation marketplace. In the mathematical field of numerical analysis, spline interpolation is a form of interpolation where the function between two known points is a special type of polynomial. This polynomial is called a spline.

Generally interpolation methods are divided into two main categories:

- *Global interpolation.*
  This algorithm produces one equation. This equation is usually a higher degree polynomial fitting all known data points. These types of algorithms result in smooth curves but tend to overshoot at intermediate points.
- *Piecewise interpolation*.
  These algorithms produce a polynomial of low degree between pairs of known data points. If a first degree polynomial is used, it is called linear interpolation. For second and third degree polynomials, they are called quadratic and cubic spline interpolation respectively. In general, the higher the degree of the polynomials the smoother the curve will be. The smoothness of the curve is achieved by the fact that polynomials of degree m will have continuous derivatives up to degree $m-1$ at the known data points. The three algorithms, linear, quadratic and cubic spline interpolation are discussed in the appendix.

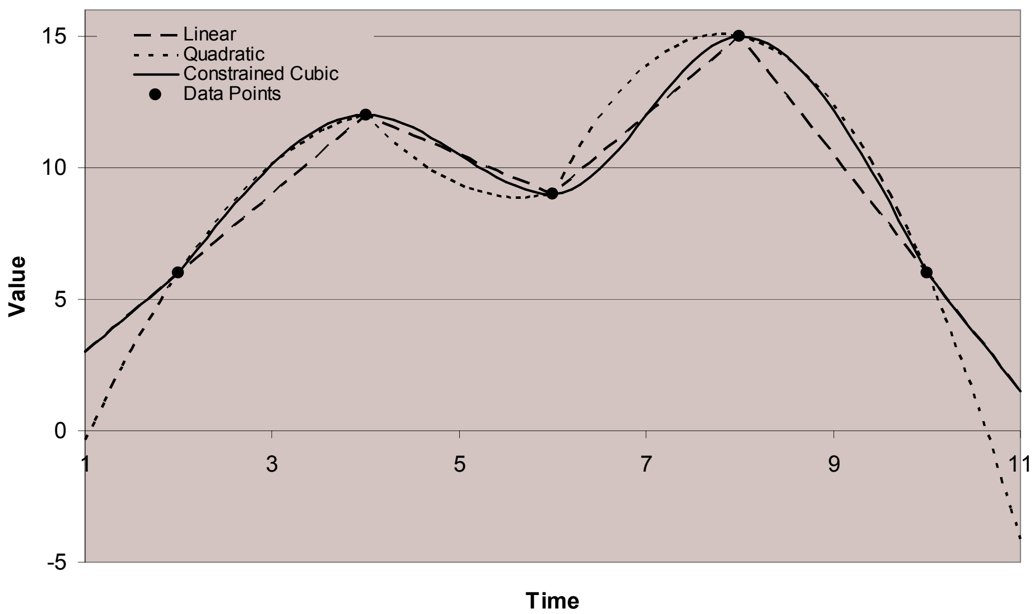
According to Kruger (2002) cubic spline interpolation is a good technique to interpolate between known data points. Main rationales presented by him are the stable and smooth characteristics that cubic spline interpolation offers. Kruger (2002) says that an even better interpolation method should combine the smooth curve characteristics of cubic spline interpolation with the non-overshooting behavior of linear interpolation.

In order to obtain a smooth curve, cubic splines are the minimum prerequisite. They are not very capricious and above all continuous up to the second order derivative at the data points, which is the constraint for smoothness. Even though cubic splines have lower tendency to overshoot than global polynomial equations, they do not prevent it. Therefore Kruger (2002) has come up with an extension on cubic spline interpolation, namely constrained cubic spline interpolation. The main benefits of this algorithm are:

- Generation of a relatively smooth curve.
- No overshooting at intermediate values.
- Interpolated values can be calculated directly without solving a system of equations.

For the inflation pricing model interpolation is applied to estimate inflation rates for maturities on the inflation curve which are not rated by the market. It is pivotal to use an algorithm which produces an intuitive correct shape of the inflation curve. In Figure 13 a comparison of the three interpolation algorithms is displayed.

Figure 13: Interpolation comparison



It is clear that linear interpolation is not suitable. Linear interpolation allows no smoothness on the curve. Smoothness is one of the characteristics of the inflation curve. Furthermore the algorithm for linear interpolation is simple and constitutes drawing a line between two consecutive points. The interpolation algorithm for quadratic interpolation comprises drawing a parabola between three consecutive points starting at the first and ending at the third point. Cubic interpolation is more difficult by having second order derivatives equal at every known point, meaning no sudden changes of the tangent between two consecutive splines. The bottom line using this algorithm is the construction of a smooth curve.

The inflation curve proves to be relatively flat over the maturities in practice. Therefore quadratic interpolation is sufficient because the parabolas drawn by the interpolation algorithm are relatively flat and therefore cause no fallacious interpolation outcomes. The main advantage of applying quadratic interpolation over cubic interpolation is that quadratic interpolation is quicker to calculate and easier to comprehend.

Chapter 7 Seasonality & Carry

Seasonality on inflation is essential to take into consideration, especially when it comes to estimate forward inflation rates. Seasonality on inflation influences the return on short term ILPs heavily and still significantly on long term ILPs. The following chapter discusses why seasonality on inflation is that crucial and introduces an approach on how to adjust seasonal unadjusted CPI data.

7.1 Introduction

According to Benaben (2005) CPIs, which cause for price movements in European and US ILBs, are seasonally unadjusted. This means that over the course of the year CPIs are subject to a recurring seasonal pattern. Furthermore once in a while the seasonal pattern is affected by unpredictable events, which impinges the CPI data tremendously from time to time. To make matters more complicated, seasonality on inflation is not even a constant pattern forever. A rationale for this could be a revision of the CPI for example.

Benaben (2005) says that the fact of seasonal development of CPIs has a direct effect on BEI and should therefore certainly not be neglected. Seasonal patterns play an important role in making tactical investment decisions on the short term. For example by being overweighed or underweighted in ILBs in an investment portfolio. You want to be overweighed when you think inflation is relatively high on the short term and underweighted when inflation is relatively low on the short term. The rationale is that because of the time lag incorporated in ILBs, as discussed in Chapter 1, compensation is paid for the relatively high or low inflation rate on the short term. Nevertheless, seasonal patterns can still be disturbed by unpredictable events. A strong increase in oil price is a good example.

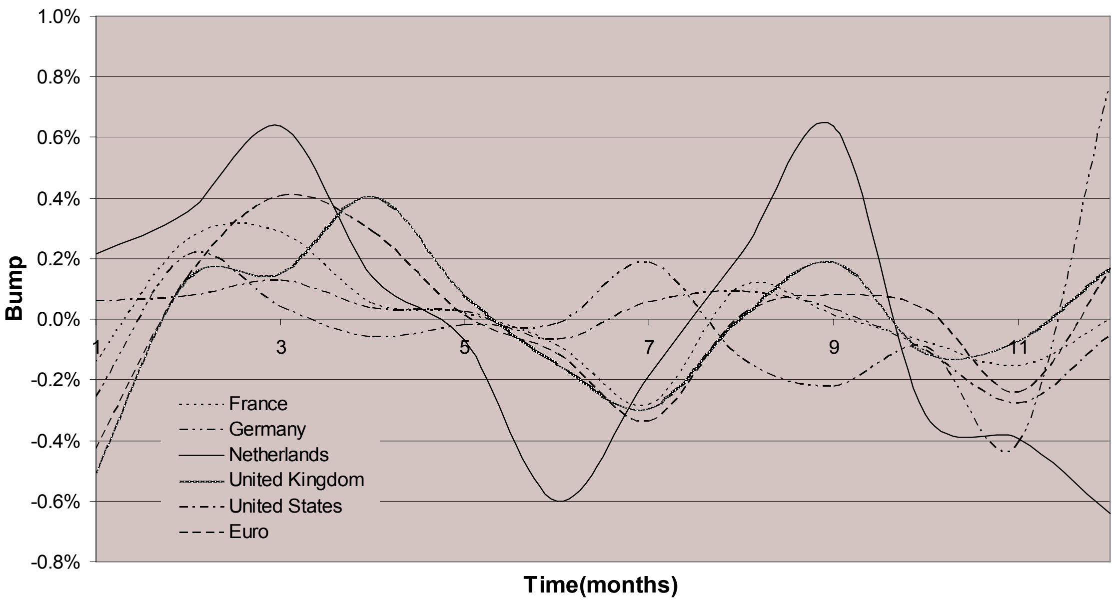
7.2 Seasonality

Belgrade (2004) says that a big difference between interest forward rates and inflation forward rates is that the latter is not really smooth and exhibits a recurrent seasonal pattern. This pattern is a result from various repeated economic phases, such as an increase in consumer spending during Christmas and cyclic variations in energy and food consumption. The seasonal pattern on inflation estimated over the period 2002-2006 for France, Germany, The Netherlands, United Kingdom, United States and Euro zone is shown in Figure 14.

As a logical result, the assumption that monthly inflation rate changes are constant is fallacious. The monthly inflation rate change may have a huge discrepancy from the monthly inflation rate change average over a year for a month with strong seasonal impact. In addition to this subject Dogra (2007) states that CPIs are affected by their idiosyncratic seasonal pattern during the year. This pattern appears on monthly inflation rates, but not on YOY inflation rates. YOY inflation rates eliminate the seasonality impact by making a comparison with the same month the year before. This uncanny phenomenon on monthly inflation data demands the incorporation of seasonality adjustments in an inflation pricing model which uses monthly inflation rate based data.

In addition Belgrade (2004) states that during the starting phase of the inflation derivatives market, when inflation derivatives were still in their infancy, seasonality was not a big issue. Nowadays, since the market has experienced tremendous growth modeling seasonality on inflation is a necessity.

Figure 14: Seasonality patterns over period 2002-2006



Source: DataStream, monthly seasonally unadjusted CPI data

Incorporating seasonality in a stochastic model is complex. The incorporation can be achieved by including seasonality in the diffusion equation. The diffusion equation causes the dynamics via the equation's drift and volatility term. The result is that the model is harder to calibrate. Belgrade (2004) therefore suggests an easier approach that is similar to the one used for end of year effect interest rate derivatives.

Instead of modeling seasonality dynamically, one should apply a static pattern to reshape the forward inflation rate curve. In other words modify the forward inflation rate curve and let the inflation dynamics unaltered. To simplify further, define yearly seasonality on a monthly basis. In order to do so one has to define a vector of monthly seasonal up and down bumps $\{B(i)\}_{i=1..12}$. The vector is indexed by months i with the supposition that January equals to 1. The mathematics for estimation of monthly seasonal up and down bumps is outlined in the appendix.

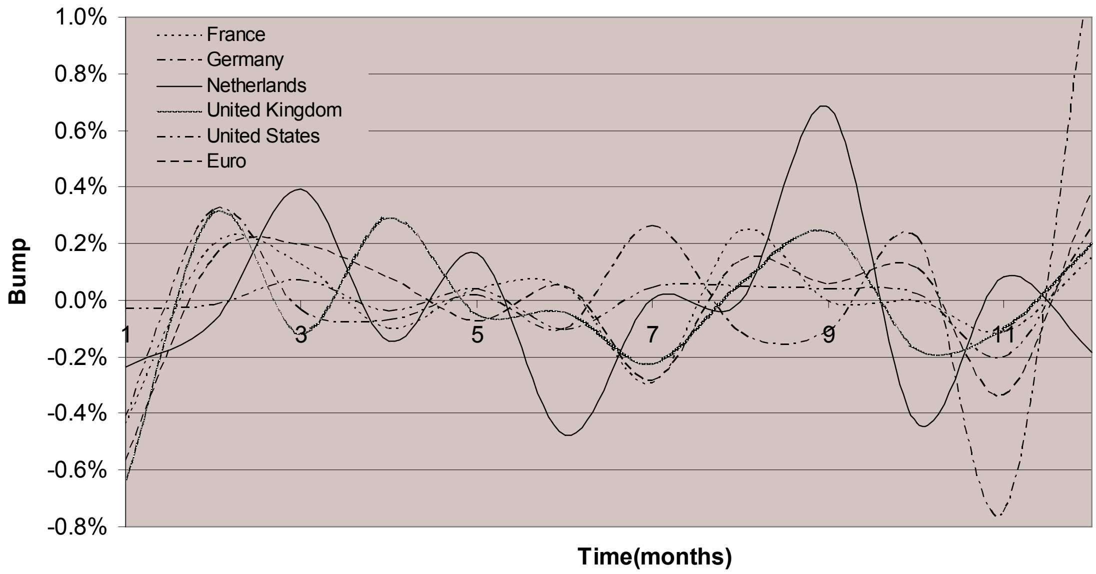
In addition to his approach, Belgrade (2004) uses Equation 4 to smooth the estimated seasonal up and down bumps.

Equation 4: Belgrade's smoothing formula

$$SB(T) = B(m) - \left((m - m_d) \frac{B(m_u) - B(m_d)}{m_u - m_d} + B(m_d) \right)$$

The smoothing formula is constructed in such a way that the seasonal bump for a specific month is defined by the seasonal bump for month m minus a three month seasonal bump trend and minus the impact of the seasonal bump of the previous month. The result is a smoothing effect on seasonality, which is depicted in Figure 15.

Figure 15: Seasonality patterns smoothed over period 2002-2006

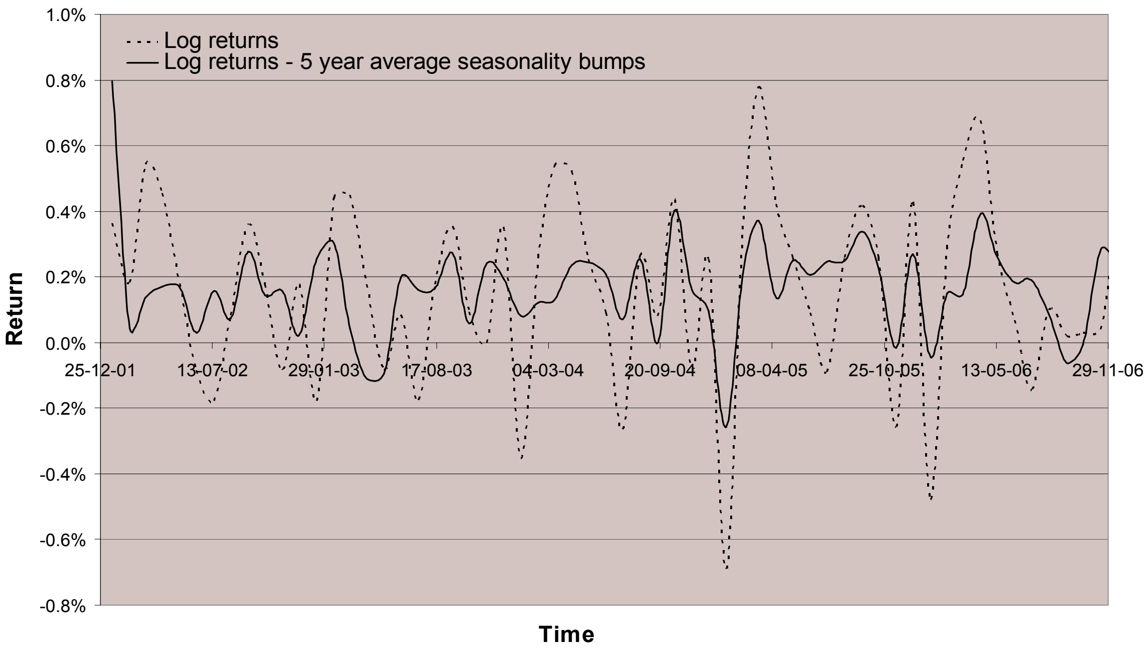


Source: DataStream, monthly seasonally unadjusted CPI data

In order to denote the estimated seasonal bumps as a significant factor causing volatility in monthly inflation rates they have to produce a series with reduced volatility when they are subtracted from realized monthly inflation rates. A serious error would be that the estimated seasonal bumps produce a series with increased volatility when being subtracted from realized inflation rates.

Figure 16 illustrates the monthly seasonal adjusted and unadjusted return series for the HICP Euro ex tobacco. The seasonal adjusted return series is clearly less volatile than the unadjusted one, which means that the estimated seasonal bumps are a significant factor causing volatility in inflation rates. Therefore seasonal bumps are suitable to adjust inflation forward rates.

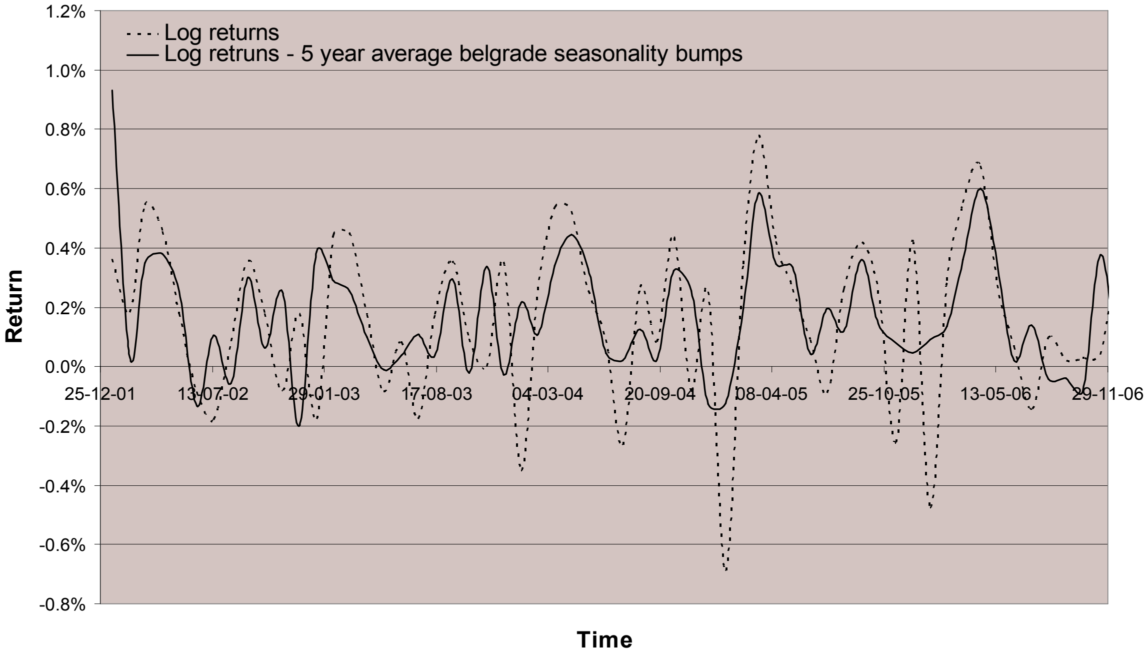
Figure 16: Seasonal bumps subtracted from realized inflation



Source: DataStream, monthly seasonally unadjusted HICP data

Since the seasonal bumps in the first place depend on the historical time frame where they are derived from they are also affected by the smoothing formula one applies on them. In general one can say that the estimation of seasonality on inflation is part of ones view on future inflation. The seasonal bumps smoothed with Belgrade's formula and subsequently used for adjusting the unadjusted monthly HICP ex Tobacco rates are depicted in Figure 16: Seasonal bumps subtracted from realized inflation. Again the adjusted series is less volatile than the original one. However the monthly return series is more volatile than the one retrieved from applying unsmoothed seasonal bumps.

Figure 17: Smoothed seasonal bumps subtracted from realized inflation



Source: DataStream, monthly seasonally unadjusted HICP data

7.3 Carry

Carry is particularly relevant in the short term valuation of ILBs. High monthly volatility on inflation causes the coupon and redemption payments to vary heavily from month to month. As a result there is considerable valuation volatility in short term ILBs compared to nominal bonds. The generally applied calculation for carry on bonds as shown in Equation 5 is derived from the difference between the bond's corresponding spot rate and the forward rate.

Equation 5: Standard carry estimation

$$\text{Carry}_{\text{nominal}} = \text{n} - \text{n}_{\text{T}} \quad \text{Carry}_{\text{real}} = \text{r} - \text{r}_{\text{T}}$$

Carry shows to what degree the bond interest rate must change in order for the investment return that results from the bond to be equal to an investment over the same horizon. Stating more difficult carry shows whether a long position in a bond, financed by a collateralized loan, produces a profit or a loss. As a consequence of carry, when interest rates do not change until the bond's maturity the difference between the forward and spot rate represents the advantage in holding bonds against deposits.

The advantage in holding ILBs against nominal bonds corresponds to the difference between carry on nominal interest and carry on real interest and is called breakeven carry.

A positive breakeven carry indicates that in the case of unchanged rates until maturity, one can earn a return equal to the break even carry by purchasing an ILB and selling a nominal bond. In other words, until maturity the breakeven inflation carry may decrease by its absolute value before the investors lose money. Therefore one can state that the longer the duration the higher the chance you will not profit from a carry trade. Consequently long term bonds are less sensitive for carry. In addition, BEI indicated by short term securities is not as representative as BEI indicated by long term securities. Furthermore, the demand for short term bonds will rise in times of high carry. This means in the case of ILBs, whose carry is impacted by a CPI, that they become more expensive than nominal bonds. Moreover the gap between nominal rates and real rates increases. This is similar to a rise in the inflation expectation that is displayed by BEI.

Carry on ILBs is affected by the development in their respective CPIs. As a result of a time lag as discussed in Chapter 1 the value of the reference CPI affecting the ILB is usually known a couple of months in advance. Therefore carry can be computed only for short term periods. However, carry can be projected for longer time horizons, if subjective assumptions on future CPI values are made.

In Europe and in the US, due to the 3-month lag embedded in most ILPs, the inflation accrual for the next month is known when the position is initiated. The inflation accrual depends on the difference between the monthly increase between the last two CPI values and the current BEI rate.

As an example, consider a month where the monthly increase is 0.6%, or 7.2% annually. Furthermore assume a BEI rate of 2%. Holding the ILB against current position for a month brings an “inflation carry” of 5.2% per year, or about 43 bp over the holding period.

In 10 years, carry translates into a BEI drop of $\frac{43\,\text{bp}*1.02^{10}}{10} = 5.24\,\text{bp}$

In 30 years, carry translates into a BEI drop of $\frac{43\,\text{bp}*1.02^{30}}{30} = 2.6\,\text{bp}$

7.4 Summary

The existence of seasonality on inflation has direct effect on ILPs and should certainly not be neglected when it comes to estimate forward inflation rates. Investors benefit from seasonality on inflation by being overweighed or underweighted in ILBs in times of high or low inflation.

Due to seasonality on inflation a monthly inflation rate change may have a huge discrepancy from the monthly inflation rate change average over a year. These days modeling seasonality is a necessity. Belgrade (2004) suggests that one can use a static pattern to reshape the forward inflation rate curve. This pattern is based on a vector of monthly seasonal up and down bumps. In order to define the estimated seasonal bumps as a significant factor causing volatility in monthly inflation rates they have to produce a series with reduced volatility when they are subtracted from realized monthly inflation rates. Ultimately, the seasonal bumps estimation is part of ones view on inflation.

Carry shows whether a long position in a bond, financed by a collateralized loan, produces a profit or a loss. Furthermore carry is especially essential in short term valuation of ILBs. This originates from high valuation volatility compared to nominal bonds. The difference between carry on nominal interest and carry on real interest is called breakeven carry.

Intuitively a longer duration cause less chance to profit from a carry trade, due to mean reversion characteristic of interest rates. Consequently the demand for short term bonds will rise in times of high carry.

Chapter 8 Inflation Swaption Pricing Theory

Inflation swaptions (ISWs) are interesting for pension funds due to vital roles the can play when implementing inflation hedging strategies. This chapter elaborates on pricing theory for ISWs and discusses an application of this type of inflation linked product.

8.1 Definition and application

Definition

According to Mia (2006) an ISW is an option to enter into an inflation swap at a pre determined date and rate in the future. This is similar to an interest rate swaption (IRSW). The definition of an IRSW according to Hull (2003):

"An IRSW is an option on an interest rate swap, which gives the holder the right to enter into a specified interest rate swap at a specified time in the future."

Application

Consider a pension fund which knows that in six months it shall invest in a twenty year nominal bond. Furthermore the pension fund wants to swap a portion of the fixed nominal coupon payments from the nominal bond for inflation linked coupon payments. This swap transforms the nominal bond into a more desired ILB. By paying a premium the pension fund can enter into an ISW. The ISW gives the pension fund the right to receive inflation linked coupon payments and pay fixed nominal coupon payments for a twenty year period starting in six months.

If a fixed nominal rate exchanged for inflation linked rate on a twenty year inflation swap available in the market in six months turns out to be lower than the specified fixed nominal rate in the ISW, the pension fund doesn't exercise the ISW and enters into the inflation swap without using the ISW. Alternatively, if the specified fixed nominal rate of the ISW turns out to be lower than the fixed nominal rate from the twenty year inflation swap available in the market in six months, the pension fund exercises the ISW.

One can conclude that ISWs provide pension funds with a guarantee that the fixed nominal coupon payments they will pay at a future date will not exceed a predefined level. Furthermore, when a pension fund has bought an ISW it is able to benefit from favorable inflation rate movements and simultaneously is protected against unfavorable inflation rate movements. There are two types of ISWs:

- A call ISW; providing the buyer the right to pay a fixed nominal rate and to receive inflation.
- A put ISW; providing the buyer the right to pay inflation and to receive a fixed nominal rate.

8.2 Swaption pricing model of Black

The inspiration for the pricing model comes from the model of Black, which is described in Hull (2003). Black considers in his model a call ISW in which the buyer has the right to pay a fixed nominal rate s_K and to receive inflation in an inflation swap which lasts n years starting in T years. Furthermore the inflation swap has m payments per year and a principal amount of L. Suppose that the inflation swap rate for this n year swap at maturity of the call ISW is s_T. By comparing cash flows on a inflation swap with fixed nominal rate s_T to cash flows on an inflation swap with fixed nominal rate s_K, the payoff from the call ISW is a series of cash flows $\frac{L}{m}\max(s_T - s_K, 0)$. These cash flows are received m times per year for n years, which represent the life of the inflation swap. Suppose that the payment dates are $T_1, T_2, \ldots T_{mn}$. Each cash flow at time T_i, as provided in Equation 6, is the payoff from a call option on s_T with strike price s_K.

Equation 6: Payoff at payment date

$$\frac{L}{m}P(0,T_i)\left[s_0 N(d_1) - s_K N(d_2)\right].$$

Where $d_1 = \frac{\ln(s_0/s_K) + \sigma^2 T/2}{\sigma\sqrt{T}}$, $d_2 = \frac{\ln(s_0/s_K) - \sigma^2 T/2}{\sigma\sqrt{T}}$ and s_0 the forward inflation swap rate.

The total value of the call ISW is $\sum_{i=1}^{mn}\frac{L}{m}P(0,T_i)\left[s_0 N(d_1) - s_K N(d_2)\right]$.

When one defines $A=\frac{1}{m}\sum_{i=1}^{mn}P(0,T_i)$, which is the value of a contract that pays $\frac{1}{m}$ at times $T_i (1 \le i \le mn)$, the value of a call ISW results in Equation 7.

Equation 7: Value of call inflation swaption

$$LA\left[s_0 N\left(d_1\right) - s_K N\left(d_2\right)\right]$$

A put ISW, which value is shown in Equation 8, gives the holder the right to receive fixed nominal rate s_K. The payoffs from the put ISW are $\frac{L}{m}\max(s_K - s_T, 0)$,

Equation 8: Value of put inflation swaption

$$LA\left[s_K N\left(-d_2\right) - s_0 N\left(-d_1\right)\right].$$

Before applying Blacks' model it necessary to build a ZC inflation curve to obtain appropriate inflation swap forward rates. According to Dogra (2006) the following has to be done in order to obtain ZC inflation forward rates and YOY inflation forward rates:

- Collect all currently tradable ZC inflation swaps.
- Interpolate non-quoted maturities.
- Apply a seasonal adjustment to obtain a monthly ZC inflation curve.

The derivation of ZC inflation swap forward rates is straightforward by dividing appropriate discount factors corresponding to the ZC inflation swaps' start and end date. For YOY inflation swap forward rates one has to do the following:

- Determine YOY inflation forward swap rates by considering a YOY inflation swap constituting multiple ZC inflation swaps.
- Apply convexity adjustments for the obtained YOY inflation forward swap rates.

8.3 Convexity adjustment for YOY inflation swap rates

Since an inflation swaption simply is as an option to exchange a floating rate bond with a fixed rate bond or vice versa depending on being call or put, it is pivotal to inspect forward rates on bonds. These rates are defined as the rate implied by the forward bond price. Suppose that B_T is the price of a bond at time T, y_T is its rate, and the relationship between B_T and y_T is $B_T = G(y_T)$. Furthermore assume that F_0 is the forward bond price of time zero and y_0 the corresponding forward bond rate for a contract maturing at time T. The definition of a forward bond price is defined by $F_0 = G(y_0)$ where F_0 and y_0 are related through the process $G(.)$. This means that, when the expected future bond price B_T equals the forward bond price F_0, the expected bond rate (y_T) does not equal the forward bond rate (y_0). The rationale is that, due to the risk neutrality assumption for estimating expected bond rate, $G(.)$ is defined as a linear process. However $G(.)$ is a non linear process when estimating the forward bond rate.

Figure 18 illustrates the problem, suppose that there are three possible bond prices, B_1, B_2, and B_3, and they are all equally likely in a world that is forward risk neutral with respect to probabilities $P(t,T)$. Furthermore assume that the bond prices are equally spaced, meaning that $B_2 - B_1 = B_3 - B_2$. The expected bond price is B_2 and this is also the forward bond price. The bond prices translate into three equally likely bond rates, Y_1, Y_2, Y_3, which are not equally spaced. The variable Y_2 is the forward bond rate, because it is the rate corresponding to the forward bond price. The expected bond rate is $\bar{Y} = \frac{Y_1 + Y_2 + Y_3}{3} > Y_2$.

Consider a ISW that provides a payoff dependent on an inflation swap at time T. It is known that the inflation derivative can be valued by:

- Calculating the expected payoff in a world that is forward risk neutral with respect to a ZC rate maturing at time T
- Discounting at the current risk-free rate for T

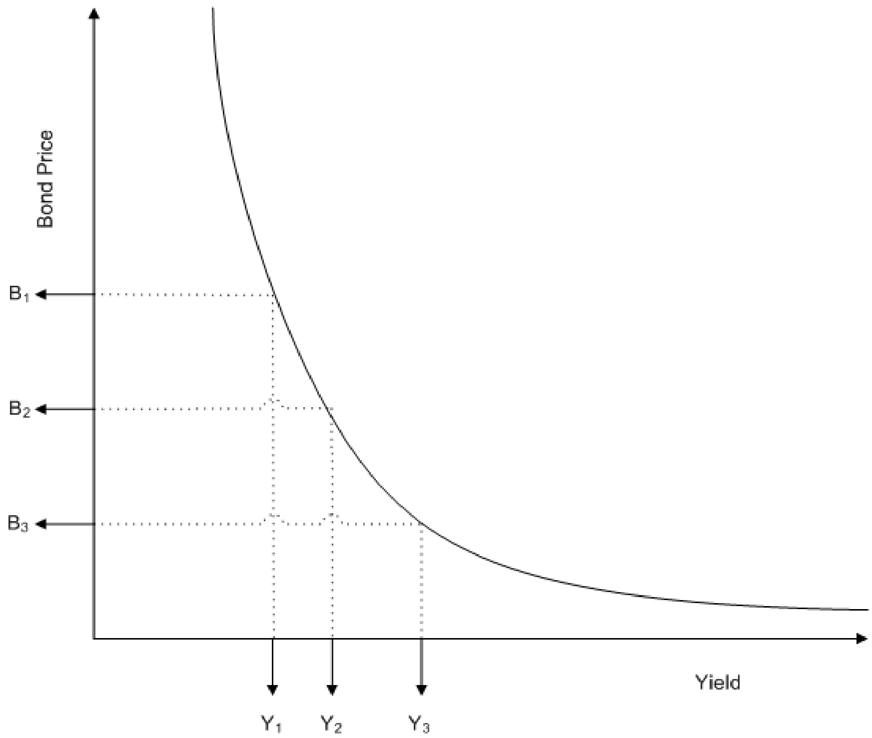
For the expected inflation swap price to equal the forward inflation swap price in a risk neutral world one needs to know the value of the expected inflation swap rate. Equation 9 expresses the expected inflation swap rate by Hull (2003):

Equation 9: Expected inflation swap rate

$$E_T(y_T) = y_0 - \frac{1}{2} y_0^2 \sigma_y^2 T \frac{G''(y_0)}{G'(y_0)}$$

Where G' and G'' denote the first and second partial derivatives of the process G, E_T denotes an expectation in a world that is forward risk neutral with respect to $P(t,T)$ and σ_y is the forward rate volatility. The difference between the expected bond rate and the forward bond rate $-\frac{1}{2} y_0^2 \sigma_y^2 T \frac{G''(y_0)}{G'(y_0)}$ is known as the convexity adjustment. Moreover the convexity adjustment is always positive because $G'(y_0) < 0$ and $G''(y_0) > 0$.

Figure 18: Reason for convexity adjustment on forward yields



8.4 Summary

An ISW is defined as an option to enter into an inflation swap at a pre determined date and rate in the future. These options provide pension funds with a guarantee that fixed nominal coupon payments they will pay at a future date will not exceed a predefined level. The two main types of ISWs are call ISWs and put ISWs, which respectively providers the buyer the right to receive or pay inflation for a given fixed nominal rate.

Black developed a model to value swaptions. He considers a call ISW where the buyer has the right to pay a fixed nominal rate s_K and to receive inflation rate on an inflation swap which lasts n years starting in T years. Furthermore the swap has m payments per year and a principal amount of L.
The total value of a call ISW according to the model of Black is:

$$\sum_{i=1}^{mn} \frac{L}{m} P(0,T_i)\left[s_0 N(d_1) - s_K N(d_2)\right]$$

Where $d_1 = \frac{\ln(s_0/s_K) + \sigma^2 T/2}{\sigma\sqrt{T}}$, $d_2 = \frac{\ln(s_0/s_K) - \sigma^2 T/2}{\sigma\sqrt{T}}$ and s_0 the forward inflation swap rate.

Before applying the model of Black one has to construct a ZC inflation curve to obtain appropriate forward inflation swap rates. The derivation of ZC inflation swap forward rates is easier than for YOY inflation swap forward rates. For YOY inflation swap forward rates a convexity correction has to be calculated and applied. A convexity adjustment constitutes the difference between a bond's expected rate and current forward rate.

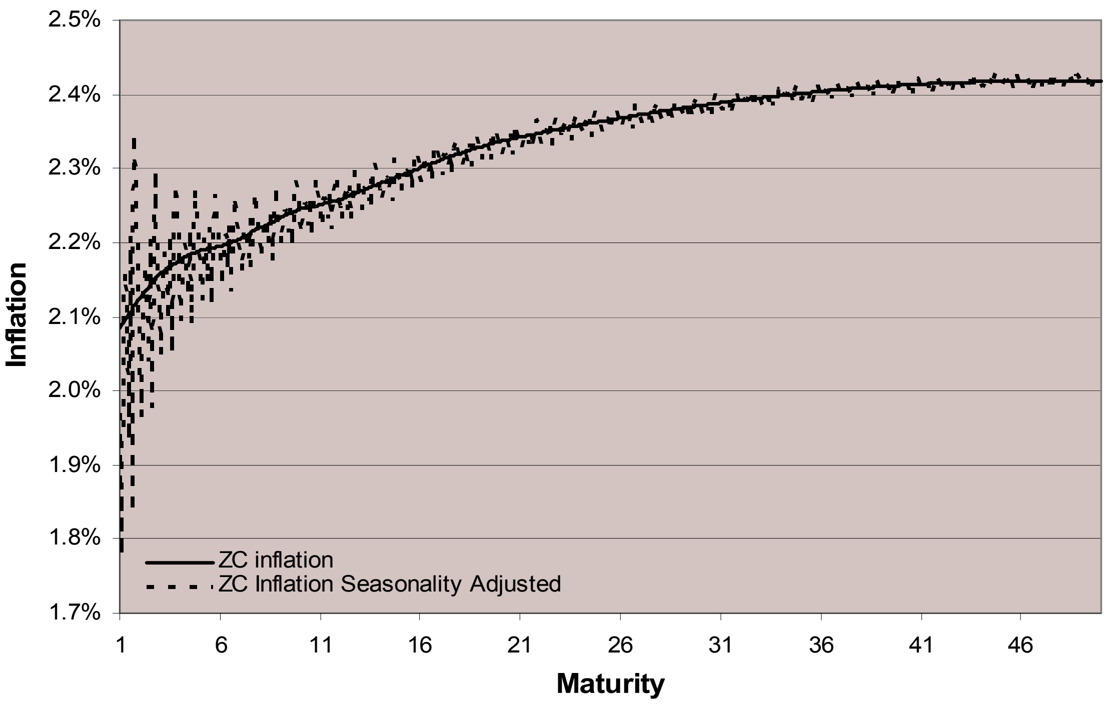
Chapter 9 Pricing Model Explanation

When a pension fund wants to invest in ISWs it should have a methodology to price these inflation linked products. This chapter combines the swaption pricing theory and the concept of seasonality on inflation by explaining a pricing model for YOY and ZC ISWs. Furthermore a suggestion to estimate a seasonality trend is introduced. The seasonality trend helps predicting future inflation more precisely.

9.1 Model

The model starts with a first step which is collecting ZC inflation swap data for Europe, France, United States and United Kingdom from investment banks. Successively the average of the mid prices over the investment bank for each maturity is considered as the ZC inflation rate for that maturity. With the use of quadratic interpolation, as described in Chapter 6, a ZC inflation curve is interpolated among maturities ranging from 1 year to 50 years in the future. This interpolated curve is the straight line in Figure 19. As mentioned earlier in this thesis, the seasonal impact on inflation is important and should be applied on the ZC inflation rate curve. Therefore the dotted line in Figure 19 represents the seasonal adjusted ZC inflation rates. The used seasonal factors are estimated as described in Chapter 7.

Figure 19: Seasonality on ZC Euro inflation curve



Remarkable from Figure 19 is that the seasonal impact fades away with increasing maturity on the ZC inflation curve. The proof for this phenomenon is illustrated in Figure 20. Here a ZC rate prediction algorithm, as depicted in Equation 10, is outlined by displaying four scenarios all having a negative seasonal impact in the last year. First an exact computation of the seasonal impact is done by estimating the ZC rate over the accumulated principal in the last year. Second the prediction algorithm is applied.

Equation 10: ZC prediction algorithm

$$ZC_{prediction}(T) = ZC_{unadjusted} - \frac{B(i)}{T}$$

Considering the small differences in Figure 20 between the exact ZC rates and predicted ones it can be concluded that this prediction algorithm suffices.

Figure 20: ZC prediction algorithm

Zero Coupon Rate	Maturity	year	Not seasonality adjusted	Seasonality adjusted
10%	2	1	1.10	1.10
Seasonalityimpact	Seasonalityimpact/maturity	2	1.21	1.166
4%	2.00%		**10.00%**	**7.98%**
ZC Prediction	Difference			
8.00%	0.02%			

Zero Coupon Rate	Maturity	year	Not seasonality adjusted	Seasonality adjusted
10%	3	1	1.10	1.10
Seasonalityimpact	Seasonalityimpact/maturity	2	1.21	1.21
4%	1.33%	3	1.331	1.2826
ZC Prediction	Difference			
8.67%	0.02%			
			10.00%	**8.65%**

Zero Coupon Rate	Maturity	year	Not seasonality adjusted	Seasonality adjusted
10%	4	1	1.10	1.10
Seasonalityimpact	Seasonalityimpact/maturity	2	1.21	1.21
4%	1.00%	3	1.331	1.331
ZC Prediction	Difference	4	1.4641	1.41086
9.00%	0.01%		**10.00%**	**8.99%**

Zero Coupon Rate	Maturity	year	Not seasonality adjusted	Seasonality adjusted
10%	5	1	1.10	1.10
Seasonalityimpact	Seasonalityimpact/maturity	2	1.21	1.21
4%	0.80%	3	1.331	1.331
ZC Prediction	Difference	4	1.4641	1.4641
9.20%	0.01%	5	1.61051	1.551946
			10.00%	**9.19%**

After having computed seasonal adjusted ZC inflation rates, the next step is to derive discount factors form the seasonal adjusted ZC inflation rates. The discount factors act as input for the algorithms that estimate the ZC inflation swap forward rates and YOY inflation swap forward rates. For ZC inflation swap forward rates the algorithm is displayed in Equation 11. The algorithm divides the discount factor at the start date with the discount factor at the end date. Successively the algorithm calibrates a yearly forward rate by taking the n^{th} root where n is equal to the length of the swap in years. The last step is subtracting one in order to obtain the ZC swap forward rate.

Equation 11: Forward rate ZC swap

$$F_{ZC}(t,n)=\sqrt[n]{\frac{\text{discountfactor(t)}}{\text{discountfactor(t+n)}}}-1$$

For YOY inflation swap forward rates the algorithm is depicted in Equation 12. This algorithm is a little more complicated than the algorithm for ZC inflation swap forward rates. Here the n^{th} root should be taken from the sum product of the divisions of discount factors that have a year time difference. Next, similarly as to ZC inflation swap forward rate calculation, one is subtracted to obtain a YOY inflation swap forward rate.

Equation 12: Forward rate YOY swap

$$F_{YOY}(t,n)=\sqrt[n]{\prod_{i=t}^{n-1}\frac{\text{discountfactor(i)}}{\text{discountfactor(i+1)}}}-1$$

Because YOY inflation swap forward rates demand a convexity adjustment, as stated in Chapter 8, the volatilities of the forward rates are needed before being able to compute the convexity adjustment. The forward rate volatilities for different strikes and maturities are provided by a number of investment banks. In the model the forward rate volatilities are averaged among investment banks and interpolated using quadratic interpolation in order to get forward rate volatilities defined by strike and maturity. Figure 21 shows volatility smiles. Important to note is that in general forward rate volatilities heavily depend on the characteristics of the underlying swap and reference index.

Using the YOY inflation swap forward rate, forward rate volatility, the length of the swap and the seasonal adjusted ZC inflation rates, convexity adjustments can be computed and added to the YOY inflation swap forward rates. Figure 22 displays the estimated forward rates for a YOY inflation swap and a ZC inflation swap. It is clear that the magnitude of the convexity adjustment increases with maturity and results after 5 year in an amount of 4 basis points Figure 22.

Figure 21: Volatility on different strikes on different forward rates

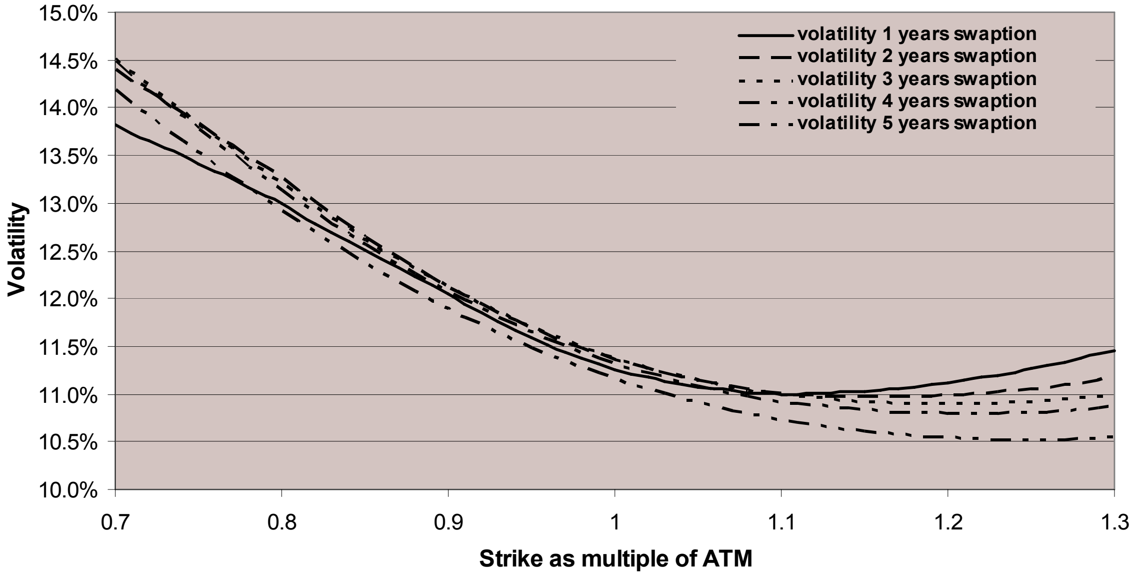
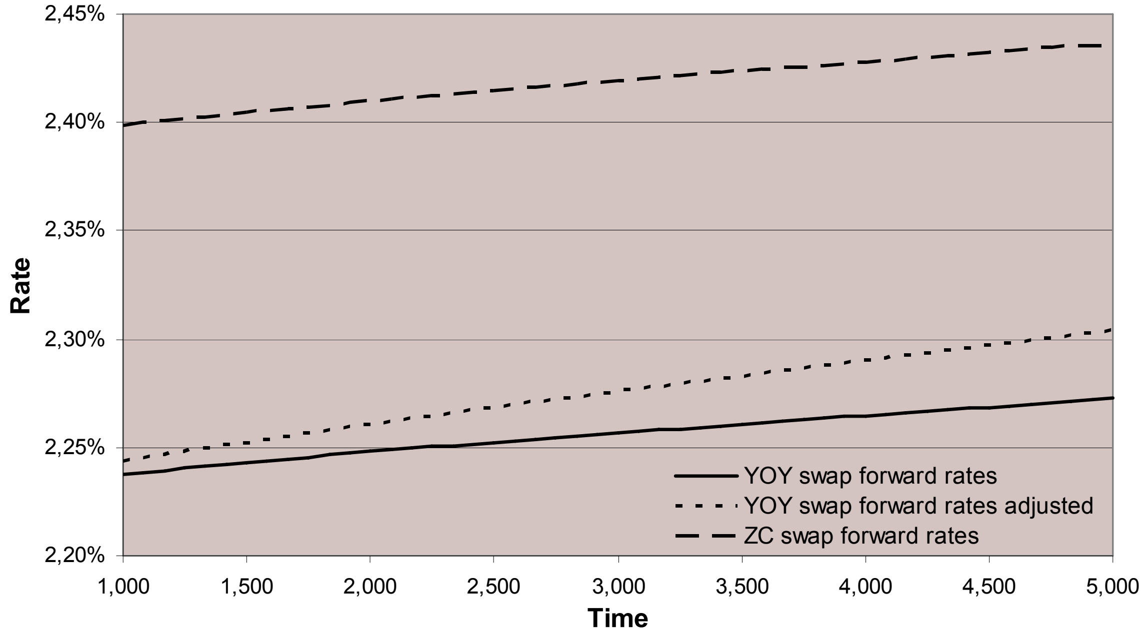


Figure 22: YOY and ZC Forward rates



Having the appropriate maturity, forward inflation swap rate, forward rate volatility, which is depicted in Figure 23, and strike level the calibration of the ISW price according to the model of Black is straightforward. In the model both call ISW prices and put ISW prices are calculated for given strike and maturity. Furthermore, nominal interest swap rates are used to discount payoffs at swap payment dates and therefore form the basis in the calculation of $A=\frac{1}{m}\sum_{i=1}^{mn}P(0,T_i)$.

Figure 24 and Figure 25 represent the prices for YOY put ISWs and ZC put ISWs. One notices that YOY ISWs are cheaper because of less credit risk compared to ZC inflation ISWs. The rationale is that the underlying ZC inflation swap pays only at maturity and the YOY inflation swap pays out yearly until maturity. Moreover, with increasing strike put ISWs are more expensive. This is logical because with a higher fixed nominal rate the protection against inflation risk is better. Furthermore when maturity increases the put ISW prices do also.

Figure 23: forward volatility rates

Inflation swap forward volatility rates

Maturity / Strike	70%	80%	90%	100%	110%	120%	130%
1	13.8%	13.0%	12.1%	11.3%	11.0%	11.1%	11.5%
2	14.4%	13.3%	12.1%	11.4%	11.0%	11.0%	11.2%
3	14.5%	13.2%	12.1%	11.4%	11.0%	10.9%	11.0%
4	14.5%	13.1%	12.1%	11.3%	10.9%	10.8%	10.9%
5	14.2%	12.9%	11.9%	11.2%	10.7%	10.5%	10.5%

Figure 24: ZC inflation swaption prices on Euro Inflation

Payer ZC inflation swaption prices

Maturity / Strike	70%	80%	90%	100%	110%	120%	130%
1	1	6	38	140	351	634	940
2	8	29	80	192	381	630	910
3	21	51	112	225	400	626	884
4	35	71	137	247	411	621	860
5	45	85	153	261	415	610	834

Figure 25: YOY inflation swaption prices on Euro Inflation

Payer YOY inflation swaption prices

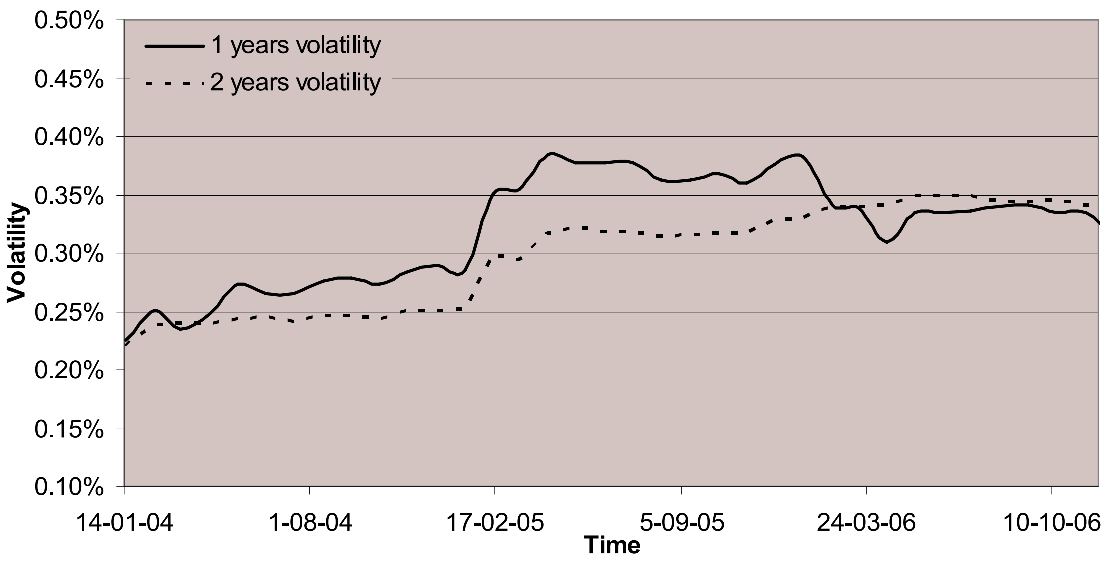
Maturity / Strike	70%	80%	90%	100%	110%	120%	130%
1	1	6	36	134	335	604	895
2	8	27	76	183	363	601	868
3	20	49	107	215	382	599	846
4	33	68	131	237	394	594	824
5	43	82	147	250	398	585	801

9.2 Seasonality trend

Until now the seasonal pattern, which causes most volatility in a CPI, is assumed to be constant in the internal model and in models used the financial world. My proposition to make a pricing model more precise is to introduce a seasonality trend. This trend is currently not reflected in market prices of ILPs. The calibration of this seasonality trend is done as follows:

- Consider the seasonal bumps B_t from Chapter 7 for a period n.
- Compute 1-year and 2-year moving averages for seasonal bumps B_t. Moving averages are depicted in Figure 26 for the HICP Euro ex Tobacco.
- Calculate the monthly change in the 1-years and 2-years moving averages, $C_1(t) = \frac{\sum_{i=t-12}^{t} B_i}{\sum_{i=t-13}^{t-1} B_i}$ and $C_2(t) = \frac{\sum_{i=t-24}^{t} B_i}{\sum_{i=t-25}^{t-1} B_i}$ respectively.
- Calculate the average monthly change in the 1-years and 2-years moving averages for period n.
- Take the average of the average 1-years and 2-years monthly change moving averages for period n.

Figure 26: 1 and 2 years volatility of Euro Inflation



The obtained monthly seasonality increase or seasonality decrease can be applied to modify inflation expectation. For example by powering the seasonality increase or seasonality decrease with the maturity in months and multiplying this with the seasonal impact at that maturity. Because this is a first order adjustment, it is not plausible that the seasonality trend is applicable for high maturities. In Figure 27 the values for the moving averages over the period 2004-2006 and the corresponding ratios of monthly volatility changes are shown.

Figure 27: Seasonality Growth Estimation over period 2004-2006

2 Years historical Euro Volatility	1 Years historical Euro volatility	Ratios 1and 2 years historical volatility	
0.2203%	0.2247%	Ratio of volatility changes 1 years	Ratio of volatility changes 2 years
0.2392%	0.2510%	1.117	1.086
0.2404%	0.2353%	0.937	1.005
0.2399%	0.2476%	1.052	0.998
0.2442%	0.2738%	1.106	1.018
0.2452%	0.2652%	0.969	1.004
0.2408%	0.2650%	0.999	0.982
0.2465%	0.2766%	1.044	1.023
0.2473%	0.2780%	1.005	1.004
0.2437%	0.2733%	0.983	0.985
0.2510%	0.2842%	1.040	1.030
0.2509%	0.2898%	1.020	1.000
0.2517%	0.2857%	0.986	1.003
0.2984%	0.3509%	1.228	1.185
0.2948%	0.3545%	1.010	0.988
0.3172%	0.3844%	1.084	1.076
0.3222%	0.3770%	0.981	1.016
0.3189%	0.3771%	1.000	0.990
0.3189%	0.3771%	1.000	1.000
0.3153%	0.3623%	0.961	0.989
0.3158%	0.3622%	1.000	1.002
0.3177%	0.3675%	1.015	1.006
0.3176%	0.3607%	0.981	1.000
0.3282%	0.3758%	1.042	1.034
0.3303%	0.3825%	1.018	1.006
0.3398%	0.3407%	0.891	1.029
0.3402%	0.3389%	0.995	1.001
0.3411%	0.3088%	0.911	1.002
0.3493%	0.3348%	1.084	1.024
0.3495%	0.3350%	1.001	1.001
0.3498%	0.3357%	1.002	1.001
0.3441%	0.3405%	1.014	0.984
0.3440%	0.3410%	1.001	1.000
0.3450%	0.3352%	0.983	1.003
0.3416%	0.3349%	0.999	0.990
0.3388%	0.3140%	0.938	0.992
		1.011	1.013
		1.0121706	Monthly Seasonality Growth

9.3 Summary

An internally developed model is discussed in this chapter. The model starts with collecting ZC inflation swap data from investment banks in order to estimate an interpolated ZC inflation curve. On this ZC inflation curve the seasonal impact fades away with increasing maturity. The proof for this phenomenon is provided. The next steps of the model are deriving discount rates and calculate ZC inflation swap forward rates and YOY inflation swap forward rates. YOY inflation swap forward rates need a convexity adjustment. The forward rate volatilities, necessary for the convexity adjustment, are assumed to be provided by investment banks.

The final step, calculating the put and call ISW prices with the model of Black, is done after having determined the appropriate maturity, forward swap rate, forward rate volatility and strike of the ISW contract. Furthermore I introduced a seasonality trend in order to make more precise inflation estimations. The seasonality trend is currently not reflected in market prices.

Chapter 10 Conclusions

Being a pension fund your ultimate ambition is to index all retirees and maintain the level of their buying power. Inflation linked bonds (ILBs) and inflation derivatives, which are linked to consumer price indices (CPIs), are investment solutions which value is affected by inflationary movements. Pension funds perceive these inflation linked products (ILPs) as a better hedge against inflation than nominal bonds, real estate, stocks, commodities and cash.

Despite the advantages of ILPs, pension funds retain a certain basis risk, which is the difference between the indexation from the underlying CPI and the best possible indexation for their liabilities. Furthermore CPIs exhibit seasonal up and down movements, and most ILB issuers apply a seasonal unadjusted CPI. In addition ILBs' cash flows compensate for inflation with a time-lag that originates from computing and publishing CPIs and the institutional arrangements necessary for trading ILBs. Eurostat manages HICPs, which are a set of comparable Euro zone CPIs. ILPs linked to Euro HICP are attractive for Dutch pension funds due to high correlation of Euro inflation with Dutch inflation.

Pension funds invested in ILBs might encounter five different ILB structure types, namely capital indexed, interest indexed, current pay, indexed annuity and indexed zero coupon. From these types, capital indexed and interest indexed is most often applied. Tax regulations can reintroduce inflation risk on ILBs. However in the Netherlands pension funds are tax exempt, which makes investing in ILBs for Dutch pension funds even more attractive by not having reintroduction of inflation risk.

Inflation derivates assist the inflation market by improving needs of investors and issuers and increase liquidity. Inflation swaps are used to insure for unfortunate future inflation movements by receiving floating inflation and paying a fixed nominal rate equal to the break even inflation during the lifetime of the swap. Moreover, inflation swaps are applied in conjunction with a nominal bond in order to create an ILB synthetically. This is especially done when a synthetic ILB turns out to be cheaper than a conventional ILB or has a better fit with a pension fund's future liabilities. Inflation swaps can provide exposure to inflation risk which does not exist in the ILB market. A pension fund first has to consider the level of the real rate before composing a portfolio of ILBs and inflation derivatives.

Developed international inflation markets are Australia, Canada, France, Sweden, UK and the US. In these markets sovereign issuers are dominating. Canada, Australia and Sweden are small markets compared to France, UK and US. The latter three represent together a market of about €640 billion, while the small ones represent not even 10% of this amount. Besides, the range of maturities covered by ILBs in the big markets is more widespread compared to the small ones.

As said on the Euromoney Inflation Linked Products Conference 2007, new inflation markets, such as Germany, Greece and Italy, are growing and new issues of ILBs from their governments are thought to happen in the future. Inflation derivatives are offered by major investment banks for the Euro, French, UK and US market with maturities ranging from 2 to 50 years. Many inflation derivatives contracts are OTC and therefore the magnitude of the inflation derivatives market is obscure.

Pricing inflation derivatives is not yet standardized in the inflation market. Moreover seasonality on inflation, originating form irregular customer spending over the year, makes price computation difficult. For pension funds it is essential to have a pricing model for inflation derivatives which is as objective as possible and takes into account the seasonality on inflation.

An inflation swaption is an option to enter into an inflation swap at a specified date and rate in the future. The two main types of inflation swaptions are call and put inflation swaptions, which respectively provides the buyer with the right to receive or pay inflation in exchange for a certain fixed nominal rate. Black has developed a model to value swaptions. A convexity adjustment, which is necessary for YOY inflation swap forward rates, encompasses the difference between a bond's expected spot rate and current forward rate.

The model suggested for pricing inflation swaptions constitutes the following:

- Retrieve ZC inflation swap rates for all possible maturities and regions.
- Interpolate a ZC inflation curve over a range of maturities for each region.
- Determine monthly seasonal bumps over the year for each region.
- Apply a seasonal correction on the ZC inflation curve.
- Calibrate discount factors.
- Calculate inflation swap forward rates.
- Collect forward volatility rates depending on strike and maturity of the inflation swaption.
- Apply convexity adjustments for YOY inflation swap forward rates.
- Use the specifications of the inflation swaption, the inflation swap forward rate and forward volatility rate in conjunction with the model of Black to estimate the inflation swaption price.

In the future generally accepted standards will provide a boost to the inflation derivatives market. This results in more frequent trading of inflation options and inflation swaptions. Furthermore the inflation market has the ability to evolve from historical volatility based pricing to implied volatility based pricing just as the interest rate market. This shall increase the liquidity and lower the prices of inflation derivatives.

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Appendix

A.1 Base payment calculation of IABs

Notional of the bond $F = 100$ and $w = \left(\frac{1}{1+r}\right) = \left(\frac{1}{1+0.04}\right) = 0.961538$

Annual real interest rate $r = 0.04$

$a_n = w + w^2 + w^3 + \ldots + w^n = \left(\frac{1 - w^n}{r}\right) = \left(\frac{1 - 0.961538^5}{0.04}\right) = 4.45187$

Base annuity payment: $B = \left(\frac{100}{4.45187}\right) = 22.46$

A.2 Seasonality estimation

- Download historical data to obtain monthly historical prices for a CPI of your choice.
- Select n periods A, each period start at January and end at December. Furthermore these periods do not overlap $A_i \cap A_{i+1} = \varnothing$ and $\bigcup_{i=1}^{n} A_i = [0, T]$.
- Determine log returns r_t for $[0, T]$ by $\ln(P_t) - \ln(P_{t-1}) = r_t$.
- Calibrate seasonal bumps B_t for January until December in each period A by taking into consideration that there must be a constant trend $\alpha_{[0,T]}$. The trend $\alpha_{[0,T]}$ must be subtracted from return r_t.
- When calibrating take into account that all seasonal bumps B_t must sum to zero over the period $[0, T]$: $\sum_{i=1}^{T} B_t = \sum_{i=1}^{T} (\ln(P_t) - \ln(P_{t-1}) - \alpha_{[0,T]}) = 0$.
- Compute the general seasonal bump for month i, $B(i) = \frac{\sum_{j=1}^{n} B_{i,j}}{n}$, which is the average of bumps B_t for a specific month i over n periods:
- Construct a seasonal bumps vector $\{B(i)\}_{i=1..12}$.

A.3 YTM and ZC rates

YTM

According to Investopedia (2007), when investors talk about yield they refer to YTM, which is the interest rate by which the sum of the net present values of all future cash flows, are equal to the bond's market price. An easier explication of the YTM of a bond is to think about it as the resulting interest rate the investor receives when investing all of the coupons payments from a bond at a constant interest rate until the bond matures. Moreover YTM can be seen as the return on the entire investment.

Investopedia (2007) says that YTM is an interest rate that must be calculated through trial and error. Because this type of approach is complicated investors typically have a computer program running the process. However if someone doesn't have such a program there is an appropriate approximation method available.

For this method one should have the following three concepts in mind. First, the relationship between a bond's market price and its yield:

- When a bonds' market price increases, yield decreases.
- When market interest rates increase, bond market prices decrease.

Second the basic price-yield properties of a bond:

- Premium bond, coupon rate is greater than market interest rates.
- Discount bond, coupon rate is less than market interest rates.

Thirdly, if you add up the present values of all future cash flows one calculates the market value and thus the market price of the bond. The formula used for a bond maturing at time n is the following:

$$\text{Bondprice} = \frac{\text{Cashflow}_1}{(1+\text{YTM})^1} + \frac{\text{Cashflow}_2}{(1+\text{YTM})^2} + \ldots + \frac{\text{Cashflow}_n}{(1+\text{YTM})^n}$$

This formula enables one to finds the YTM corresponding to a bond price observed in the market.

ZC rates

ZC bonds are bonds which do not pay intermediate coupons and can therefore be bought in the market at a discount from their value at maturity. Logically the discount is the time value of money scaled to the principal. The holder of a ZC bond has the privilege to receive a specified sum of money at a specified time in the future. ZC bonds can be, as described in chapter 1, inflation indexed. This means that the amount of money that will be paid has a fixed amount of purchasing power. Despite this interesting feature for pension funds, the majority of ZC bonds pay a fixed amount of money.

Bootstrapping

The procedure to calculate ZC rates from YTMs over a specific time horizon in the future is called bootstrapping. This procedure calculates the yield over a given period in the future starting from now and assuming only a cash flow at the end. When having estimated ZC rates, estimation of forward rates for given periods in the future is straightforward. Below is the mathematical and graphical representation in Figure 28 and Figure 29 of the bootstrapping procedure.

It is easy to comprehend that the YTM for the first year period is the same as the ZC rate. The ZC rate for the first two year period can be derived by substituting the YTM for the first year with the one year ZC rate. The ZC for the three year period can be found by using the previously found one and two year ZC rate. By applying this iterative process, the whole YTM curve can be bootstrapped into a ZC curve.

$$ZC_1 = YTM_1$$

$$\sum_{i=1}^{N-1} \frac{YTM_N}{(1+ZC_i)^i} + \frac{1+YTM_N}{(1+ZC_N)^N} = 1$$

$$\frac{1+YTM_N}{(1+ZC_N)^N} = 1 - \sum_{i=1}^{N-1} \frac{YTM_N}{(1+ZC_i)^i}$$

$$(1+ZC_N)^N = \frac{1+YTM_N}{1 - \sum_{i=1}^{N-1} \frac{YTM_N}{(1+ZC_i)^i}}$$

$$ZC_N = \sqrt[N]{\frac{1+YTM_N}{1 - \sum_{i=1}^{N-1} \frac{YTM_N}{(1+ZC_i)^i}}} - 1$$

$$\frac{YTM_2}{1+YTM_1} + \frac{1+YTM_2}{(1+ZC_2)^2} = 1$$

$$\frac{1+YTM_2}{(1+ZC_2)^2} = 1 - \frac{YTM_2}{1+YTM_1}$$

$$\frac{1+YTM_1}{1+YTM_1} - \frac{YTM_2}{1+YTM_1} = \frac{1+YTM_2}{(1+ZC_2)^2}$$

$$\frac{(1+YTM_1 - YTM_2)}{(1+YTM_1)} = \frac{(1+YTM_2)}{(1+ZC_2)^2}$$

$$(1+ZC_2)^2 = \frac{(1+YTM_1)(1+YTM_2)}{(1+YTM_1 - YTM_2)}$$

$$ZC_2 = \sqrt{\frac{(1+YTM_1)(1+YTM_2)}{(1+YTM_1 - YTM_2)}} - 1$$

Figure 28: Real Rates of Europe and France

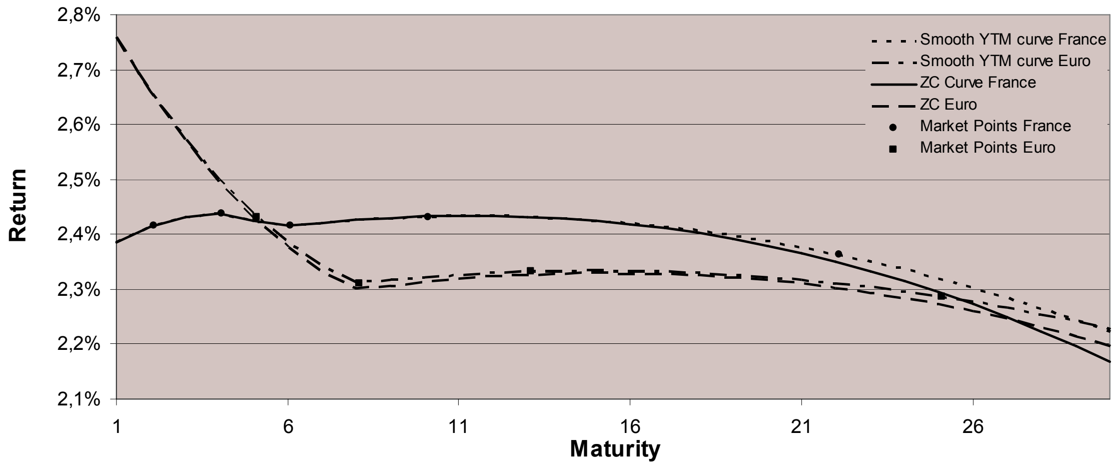
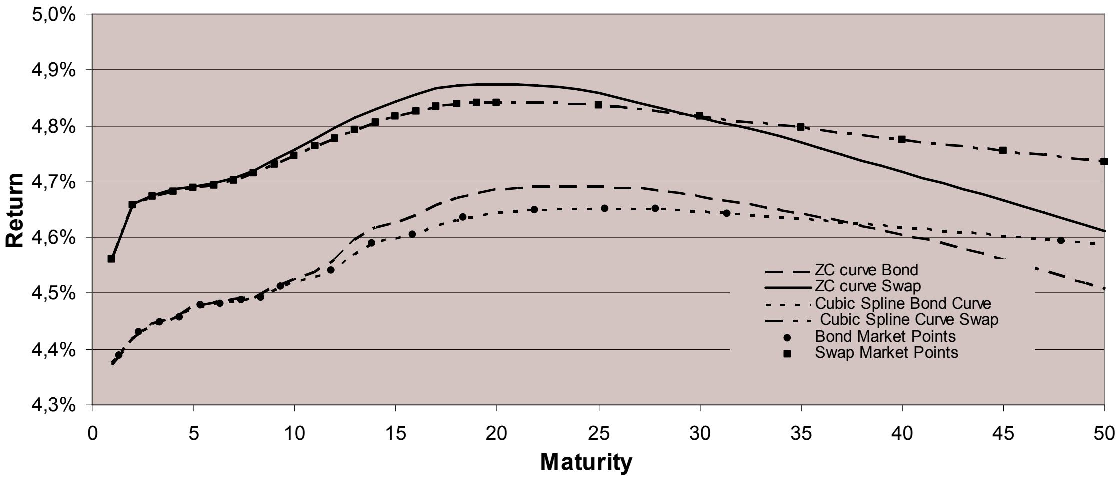


Figure 29: Swap and Bond Rates of France



A.4 Interpolation

Given $n+1$ distinct known points x_i such that $x_0 < x_1 < \ldots < x_{n-1} < x_n$ with $n+1$ known points, having values y_i the goal is to find a spline function of degree n.

$$S(x) := \begin{cases} S_0(x) & x \in [x_0, x_1] \\ S_1(x) & x \in [x_1, x_2] \\ \vdots & \vdots \\ S_{n-1}(x) & x \in [x_{n-1}, x_n] \end{cases}$$

Where each $S_i(x)$ is a polynomial of degree k.

Linear spline interpolation

Linear spline interpolation is the simplest form of spline interpolation and is also known as linear interpolation. The data points are connected by straight lines.

Algebraically, each S_i is a linear function constructed as $S_i(x) = y_i + \frac{y_{i+1} - y_i}{x_{i+1} - x_i}\left(x - x_i\right)$

The spline must be continuous at each data point, that is

$$S_i(x) = S_{i+1}(x_i) \quad i = 1, \ldots, n-1$$

This is the case when:

$$S_{i-1}(x_i) = y_{i-1} + \frac{y_i - y_{i-1}}{x_i - x_{i-1}}\left(x_i - x_{i-1}\right) = y_{i-1} + (y_i - y_{i-1}) = y_i$$

$$S_i(x_i) = y_i + \frac{y_{i+1} - y_i}{x_{i+1} - x_i}\left(x_i - x_i\right) = y_i + \frac{y_{i+1} - y_i}{x_{i+1} - x_i} * 0 = y_i$$

Quadratic spline interpolation

This is a more sophisticated form of spline interpolation. The procedure is as follows. Consider a vector of maturities $\vec{x}$ and corresponding yield vector $\vec{y}$, both of length n

When $x_1 > x$

$$S(x) = \frac{(x-x_2)}{(x_1-x_2)} * y_1 + \frac{(x-x_1)}{(x_2-x_1)} * y_2$$

When $x_1 < x$ and $x_{n-1} > x$:

$$S(x) = \frac{(x-x_{i+1})*(x-x_{i+2})}{(x_i-x_{i+1})*(x_i-x_{i+2})} * y_i + \frac{(x-x_i)*(x-x_{i+2})}{(x_{i+1}-x_i)*(x_{i+1}-x_{i+2})} * y_{i+1}$$
$$+ \frac{(x-x_i)*(x-x_{i+1})}{(x_{i+2}-x_i)*(x_{i+2}-x_{i+1})} * y_{i+2} \quad \text{where} \quad x_i \le x < x_{i+1}$$

When $x_{n-1} < x$:

$$S(x) = \frac{(x-x_{n-1})*(x-x_n)}{(x_{n-2}-x_{n-1})*(x_i-x_n)} * y_{n-2} + \frac{(x-x_{n-2})*(x-x_n)}{(x_{n-1}-x_{n-2})*(x_{n-1}-x_n)} * y_{n-1}$$
$$+ \frac{(x-x_{n-2})*(x-x_{n-1})}{(x_{n-1}-x_{n-2})*(x_n-x_{n-1})} * y_n$$

Cubic spline

Consider a set of data points $(x_0, y_0), (x_1, y_1), \ldots (x_{i-1}, y_{i-1}), (x_i, y_i)(x_{i+1}, y_{i+1}), \ldots (x_n y_n)$

With cubic spline the idea is to construct a third degree polynomial between each of these data points. For each data point there is a spline to the left indicated by f_i and a spline to the right indicated by f_{i+1}.

Each spline f_i is based on the following criteria

- Curves are third order polynomials:
  $f_i(x) = a_i + b_i x + c_i x^2 + d_i x^3$.
- Curves pass through all the known points:
  $f_i(x_i) = f_{i+1}(x_i) = y_i$.
- The first and second derivative is the same for both functions on either side of a known point: $f_i'(x_i) = f_{i+1}'(x_i)$ and $f_i''(x_i) = f_{i+1}''(x_i)$.

This results in $n-1$ equations and $n+1$ unknown y values. The two remaining equations are based on the border conditions for the starting point and end point.

Often one uses one of the following border conditions:

- Natural Splines: $f_i''(x_i) = f_{i+1}''(x_i) = 0$.
- Parabolic Runout Splines: $f_1''(x_0) = f_1''(x_1)$ $f_n''(x_n) = f_n''(x_{n-1})$.
- Cubic Runout Splines: $f_1''(x_0) = 2f_1''(x_1) - f_2''(x_2)$ $f_n''(x_n) = 2f_n''(x_{n-1}) - f_{n-1}''(x_{n-2})$.
- Clamped Splines: $f_1'(x_0) = f'(x_0)$ and $f_n'(x_n) = f'(x_n)$.

According to Kruger (2002) the main principle behind constrained cubic spline is to prevent overshooting by sacrificing smoothness. This is achieved by eliminating the requirement for equal second order derivatives at every point and replacing it with specified first order derivatives.
Hence: $f_i''(x_i) = f_{i+1}''(x_i) \rightarrow f_i'(x_i) = f_{i+1}'(x_i) = f'(x_i)$

The most important step becomes the calculation of the slope at each point. Intuitively the slope will be between the slopes of the adjacent straight lines, and should approach zero if the slope of either line approaches zero. An equation that satisfies these requirements is:

$$f_i'(x_i) = \frac{2}{\frac{\Delta x_{i+1}}{\Delta y_{i+1}} - \frac{\Delta x_i}{\Delta y_i}} = \frac{2}{\frac{x_{i+1} - x_i}{y_{i+1} - y_i} + \frac{x_i - x_{i-1}}{y_i - y_{i-1}}}$$ and $= 0$ if the slope changes in the point.

This equation is only valid for intermediate points. The slope at the end points:

$$f_1'(x_0) = \frac{3\Delta y_1}{2\Delta x_1} - \frac{f'(x_1)}{2} = \frac{3(y_1 - y_0)}{2(x_1 - x_0)} - \frac{f'(x_1)}{2} \qquad f_n'(x_n) = \frac{3\Delta y_n}{2\Delta x_n} - \frac{f'(x_{n-1})}{2} = \frac{3(y_n - y_{n-1})}{2(x_n - x_{n-1})} - \frac{f'(x_{n-1})}{2}$$

Now the first derivative is know at each point the spline function can be calculated based on two adjacent points on each side.
Calculation of actual parameters (a_i, b_i, c_i and d_i) for each spline (equation) is still possible.

$$f_i''(x_{i-1}) = \frac{-2\left[f_i'(x_i) + 2f_i'(x_{i-1})\right]}{\Delta x_i} + 6 * \frac{\Delta y_i}{\Delta x_i} * \frac{1}{\Delta x_i} = \frac{-2\left[f_i'(x_i) + 2f_i'(x_{i-1})\right]}{(x_i - x_{i-1})} + \frac{6(y_i - y_{i-1})}{(x_i - x_{i-1})^2}$$

$$f_i''(x_i) = \frac{-2\left[2f_i'(x_i) + f_i'(x_{i-1})\right]}{(x_i - x_{i-1})} - 6 * \frac{\Delta y_i}{\Delta x_i} * \frac{1}{\Delta x_i} = \frac{-2\left[2f_i'(x_i) + f_i'(x_{i-1})\right]}{(x_i - x_{i-1})} - \frac{6(y_i - y_{i-1})}{(x_i - x_{i-1})^2}$$

$$d_i = \frac{f_i''(x_i) - f_i''(x_{i-1})}{6(x_i - x_{i-1})} = \frac{f_i''(x_i) - f_i''(x_{i-1})}{6\Delta x_i}$$

$$c_i = \frac{x_i f_i''(x_{i-1}) - x_{i-1} f_i''(x_i)}{2(x_i - x_{i-1})}$$

$$b_i = \frac{(y_i - y_{i-1}) - c_i(x_i^2 - x_{i-1}^2) - d_i(x_i^3 - x_{i-1}^3)}{(x_i - x_{i-1})}$$

$$a_i = y_{i-1} - b_i x_{i-1} - c_i x_{i-1}^2 - d_i x_{i-1}^3$$

A.5 Inflation swaps and options

Inflation swaps

Define $\mathrm{CPI}(t)$ as the CPI value at time t

$T_i, i = 1,\ldots,n$ as the dates of flows of the product

T_0 as the initial time

N as the notional

$N' = (1+c)N$, where c is a coupon rate, as the notional at time T_i

- Zero Coupon swap.

$$N\left(\frac{\mathrm{CPI}(T)}{\mathrm{CPI}(T_0)} - 1\right)$$

T_0 $\qquad$ T

$$N\left[\left(1+Z_T\left(T_0\right)\right)^T - 1\right]$$

- An asset swap.

$$Nc\left(\frac{\mathrm{CPI}(T_i)}{\mathrm{CPI}(T_0)}\right) \qquad N'\left(\frac{\mathrm{CPI}(T_n)}{\mathrm{CPI}(T_0)} - 1\right)$$

T_0 $\qquad$ T_i $\qquad$ T_n

$$NS_n(T_0) \qquad N'S_n(T_0)$$

- An YOY swap.

$$N\left(\frac{\mathrm{CPI}(T_i)}{\mathrm{CPI}(T_{i-1})} - 1\right)$$

T_0 $\qquad$ T_i

$$NS_n(T_0)$$

Inflation options

One of the most commonly traded is a floor on YOY inflation, which guarantees a minimal forward inflation level and exists in the following forms.

- ZC option with a strike of $1+(1+k_T)^T$.

$$N\left(\frac{CPI(T)}{CPI(T_0)}-\left(1+(1+k_T)^T\right)\right)^+$$

T_0 $\qquad$ T

- An asset swap option, with a strike of K paying every year at time T_i.

$$Nc\left(\frac{CPI(T_i)}{CPI(T_0)}-K\right)^+ \qquad N'\left(\frac{CPI(T_n)}{CPI(T_0)}-(1+K)\right)^+$$

T_0 $\qquad$ T_i $\qquad$ T_n

- A YOY option, strike $1+K$ paying every year at time T_i.

$$N\left(\frac{CPI(T_i)}{CPI(T_{i-1})}-(1+K)\right)^+$$

T_0 $\qquad$ T_i

- Asian option on Inflation.

$$N\left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{CPI(T_i)}{CPI(T_{i-1})}-1\right)-K\right]^+$$

T_0 $\qquad$ T_n

A.6 Convexity adjustment example

Consider an inflation swap that provides a payoff in three years equal to the three-year inflation swap rate at that time multiplied by \$100. Suppose that payments are made annually on the inflation swap, the ZC inflation swap rates for all maturities are 12% per annum with annual compounding, the volatility for the three-year inflation swap rate in three years is 22%. The relevant function $G(y)$ is:

$$G(y) = \frac{0.12}{(1+y)} + \frac{0.12}{(1+y)^2} + \frac{0.12}{(1+y)^3}$$

$$G'(y) = -\frac{0.12}{(1+y)^2} - \frac{2*0.12}{(1+y)^3} - \frac{3*0.12}{(1+y)^4} = -\frac{0.12}{(1+y)^2} - \frac{0.24}{(1+y)^3} - \frac{0.36}{(1+y)^4}$$

$$G''(y) = \frac{1*2*0.12}{(1+y)^3} + \frac{2*3*0.12}{(1+y)^4} + \frac{3*4*0.12}{(1+y)^5} = \frac{0.24}{(1+y)^3} + \frac{0.72}{(1+y)^4} + \frac{1.44}{(1+y)^5}$$

The current YOY inflation swap forward rate, y_0, is 12%, so that

$G'(y_0) = -\frac{0.12}{(1+0.12)^2} - \frac{0.24}{(1+0.12)^3} - \frac{0.36}{(1+0.12)^4} = -0.495$ and

$$G''(y_0) = \frac{0.24}{(1+0.12)^3} + \frac{0.72}{(1+0.12)^4} + \frac{1.44}{(1+0.12)^5} = 1.4455$$

$$E_T(y_T) = y_0 - \frac{1}{2} y_0^2 \sigma_y^2 T \frac{G''(y_0)}{G'(y_0)} = 0.12 + \frac{1}{2} * 0.12^2 * 0.22^2 * 3 * \frac{1.4455}{-0.495} = 0.1231$$

Therefore the YOY inflation swap forward rate is 12,31% and not 12%. The value of the instrument is $\frac{\$100 * 0.1231}{1.12^3} = \$8.76$. The price without any convexity adjustment is $\frac{100 * 0.12}{1.12^3} = 8.54$.

A.7 Least squares algorithm

Suppose that a data set consists of points (x_i, y_i) with $i = 1, 2, \ldots, n$. If one wants to find a function f such that $f(x_i) \approx y_i$ you have to define the function f to be of a particular form and containing some parameters. When considering that f is quadratic, meaning that $f(x) = ax^2 + bx + c$, the solution is to find values for a, b and c which minimize the sum of the squares of the residuals $S = \sum_{i=1}^{n} (y_i - f(x_i))^2$.

A.8 Abbreviations

ATM	At the Money
BEI	Break Even Inflation
BLS	Bureau of Labor Statistics
CIB	Capital Indexed Bond
COLI	Cost of Living Index
CPI	Consumer Price Index
C-CPI-U	Price Index for All Urban Consumers
CPB	Current Pay Bond
DB	Defined Benefit
DMO	Debt Management Office
EICP	European Index of Consumer Prices
FESAC	Federal Economic Statistics Advisory Committee
GDP	Gross Domestic Product
HICP	Harmonized Index of Consumer Prices
IAB	Indexed Annuity Bond
IFRS	International Financial Reporting Standard
IIB	Interest Indexed Bond
ILB	Inflation Linked Bond
ILP	Inflation Linked Product
ILS	Inflation Linked Securities
IRSW	Interest Rate Swaptions
ISW	Inflation Swaption
IASB	International Accounting Standards Board
ISDA	International Swaps and Derivatives Association
IZCB	Indexed Zero Coupon Bond
LIBOR	London Inter Bank Offered Rates
MUICP	Monetary Union Index of Consumer Prices
OAT	Obligation Assimilables du Trésor
OTC	Over the Counter
PPP	Purchasing Power Parity
RPI	Retail Prices Index
RRB	Real Return Bond
SNDO	Swedish National Debt Office
TIB	Treasury Indexed Bond
TIIN	Treasury Inflation Indexed Note
TIIS	Treasury Inflation Indexed Securities
TIPS	Treasury Inflation Protected Securities
YTM	Yield to Maturity