

AGEING AND LARGE-CONSOLIDATION OF CEMENT-BASED GROUTS



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Forward

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Summary

This thesis describes a study of large and small consolidation of cement-based grouts using an analytical and a numerical (Finite Element) method. The study zooms in on the bleeding (drainage) behavior and efficiency (volume losses) resulting from reductions in the grouts water content under grouting (injection) pressures. The part of water content lost due to hydraulic drainage and that due to hydration are considered. Drainage can be studied (idealized) by Terzaghi's (small) or Biot's (large) consolidation theories. Hydration on the other hand results in errors in estimating drainage when applying the consolidation theories. Hydration losses are considered relevant if the consolidation test duration is long (aging). It remains to be seen from this study if aging is significant or not for a short experimental time of a thin 2 cm grout specimen tested, at different water contents and viscosity, to full consolidation.

From the above short description it is evident that a combination of some experimental and analytical work would be needed. In the first part (Part I) of this study a shortduration (experimental) aging appraisal was made to test to what extent are the simple index and strength properties are affected by hydration losses. In Part II the appropriate family of large and small deformations solutions were applied to assess drainage prediction comparing those to a series of previous laboratory bleeding tests. This work led to some further wisdom on the way this research should be conducted in the future. Suggestions are made for an experimental set-up modification to previous experiments to study bleeding efficiency due to grout-soil interaction and some encouraging preliminary results to be published soon are obtained although not reported in this thesis.

Parts I and II are summarized separately below to reflect the above described phases of the research and represent also the two publications accepted already by the reviewers in two separate international conferences. Part I will appear as a full paper in the International Civil Engineering Conference proceedings (ACE2008) in Famagusta, Cyprus, September 2008. Part II has will be presented at the 17th ICSMGE in September 2009 in Alexandria, Egypt and has been selected among several other papers by the Netherlands geotechnical engineering Society member of the international society of soil mechanics and geotechnical engineering ISSMGE. The author of this research is currently working with his supervisor Dr. U.F.A. Karim on a journal publication for the Geotechnical Engineering Journal of ICE to present the remaining further results on soil-grout interaction.

Part I

Changes with time (ageing) in shear strength and consistency of cement paste as grout material has been studied with vane and cone penetration tests. It is found that when hydrated and for a short duration afterwards the changes in the cement grouts remain insignificant. Cement paste may therefore be treated for the short pre-set period as a cohesive two- or three-phase soil. Therefore the odometer test may be used to study the consolidation behaviour of cement-based grouts. Moreover best fit relationships found between the shear strength of the grout, void ratio and time cone penetration are reported. The findings of these tests justify using the Oedometer method for soil by this author and others (Gustin et al. 2004¹¹) to study the (short) time-dependant bleeding behaviour of cement based grout.

Time-dependant deformations of cement and cement-bentonite grouts are then investigated using two consolidation models: the large strain consolidation theory and the traditional small strain consolidation theory of Terzaghi. A comparison between the deformation and the bleeding predicted by these two models is made. The following was found:

- Dissipation rate of pore water pressure of the grout predicted by the large strain consolidation theory is faster than that predicted by the small strain consolidation theory.

- Settlement of the grout predicted using the small strain consolidation theory is larger than that by the large strain consolidation theory.

- Settlement of the grout predicted by the small consolidation theory is slower.

Part II

In this part the consolidation behaviour of cement–based and cement-bentonite based mixture is investigated. These two types of mixtures are widely used as grout in geotechnical engineering. The consolidation behaviour is studied using two consolidation models. The first model is the one–dimensional large strain consolidation model proposed by Xie⁶. The second model is the traditional one-dimensional small strain consolidation theory of Terzaghi. It is shown from the result of the part I that:

The consolidation behaviour of cement paste with a range of void between 0,7 and 1,18 is similar to that of soft clays. This finding makes feasible to analyse bleeding of the grout using small and large strain consolidation theories. Bleeding is expressed in both theories as drainage of the water phase associated with pore pressure dissipation and volume reduction. The dissipation of the pore water pressures predicted by the large strain consolidation model was found to occur faster than those predicted using small strain assumption. The discrepancy between the excess pore water pressures of the two models varies with the boundary conditions, the load imposed and the permeability of the grout. These variables are significant for the effectiveness and control of grouting operations. The dissipation of the excess pore water pressure occurs faster in the cement-based than in the cement-bentonite based grout. Furthermore the settlement predicted by the large strain consolidation theory, indicating that geometric non-linearity from large deformation lead to a stiffer grout response predictions and less deformations. Development of the settlement from using small consolidation theory is slower.

Notation

Symbol	Quantity	Units
\mathcal{E}_{v}	Change in volume	[-]
ξ	Convective coordinate	[mm]
μ, μ _p	Plastic viscosity	[Pa s]
b	Bleeding	[%]
с	Cohesion of the soil	[Pa]
c_c	Compression index	$[m^2/kN]$
c_{cg}	Compression index of the grout	[-]
c_k	Permeability index	[-]
C_u	Undrained shear strength	[Pa]
C_{ug}	Undrained shear strength of the grout	[kN]
C_{us}	Undrained shear strength of the soil	$\{kN\}$
C_{v}	Coefficient of consolidation of the grout	$[m^2/s]$
Dz_{con}	Cone penetration	[mm]
е	Water ratio at t	[-]
e_g	Void ratio of the grout	[-]
e_o	Initial void ratio	[-]
e_{og}	Initial void ratio of the grout	[-]
Gs	specific gravity of the material particles	$[Kg/m^3]$
h	Thickness of the grout	[m]
h_f	Thickness of the grout after full consolidation	[m]
ks	Coefficient of permeability of the soil	[m/s]
k_v	Coefficient of permeability	[m/s]
k_v	Confined modulus	[s/m]
<i>k</i> _{vg}	Coefficient of permeability of the grout	[m/s]
k_{vo}	Initial coefficient of permeability	[m/s]
LL	Liquid limit	
М	Mass of the flowing fluid	[Kg]
m_{v}	Coefficient of volume compressibility	$[m^2/kN]$
m_{vg}	Coefficient of volume compressibility of the grout	$[m^2/kN]$
Р	Injection pressure	[Pa]
PL	Plastic limit	[-]
q	Pressure on the grout	[Pa]
q_{int}	Inflow	[-]
q_{out}	Outflow	[-]
S	Settlement	[mm]
t	Time	[s]
T_{v}	Time factor	[-]
и	Total water pressure	[Pa]

u_e	Excess water pressure	[Pa]
u_{eg}	Excess water pressure in the grout	$[kN/m^2]$
u_s	Static water pressure	$[kN/m^2]$
U_s	Excess water pressure in the soil	kN/m ²]
U_{v}	Degree of consolidation	[%]
V	Volume of the soil(particle)	[mm ³]
V_f	Volume of fluid	[mm ³]
$\check{V_t}$	Total volume	[mm ³]
Wcg	Water content of the grout	[-]
Wcs	Water content of the soil	[-]
γ	Shear rate	$[s^{-1}]$
ρ_f	Density of solid	$[kg/m^3]$
$\rho_{\rm s}$	Density of fluid	$[kg/m^3]$
σ	Total vertical stress	[Pa]
σ'	Effective stress	[Pa]
τ	Shear strength	[Pa]
το	Yield stress	[Pa]
Φ	Angle of internal friction	[°c]
h_d	Hydration degree	[cal/g.hour]
Mc	Stiffness matrix	[-]
Mbc	Boundary condition matrix	[-]
Мр	Points matrix,	[-]
Mmd	Mesh data matrix M _{md}	[-]
q_{tr}	Quality of triangle	[-]
M_w	The weight of the water	[kg]
M _s	The weight of the solids	[Kg]
$ ho_w$	Density of water	$[kg/m^3]$

PART I: EXPERIMENTAL AGEING BEHAVIOUR OF CEMENT-BASED GROUTS

1 Introduction

Cement and soils are the most used materials in construction. The interaction and behaviour of the materials when they are mixed with water are the subject of this study. Most of the time cement behaviour has been studied in relation to its chemical composition, bonding properties, water/cement interaction and its compressive strength development with time. In this research another aspect of cement is considered: namely the behaviour of a cement paste using a framework of soil mechanics based on consistency, phase relationship and drainage (consolidation) behaviour. Two types of mixtures widely used as grout materials in geotechnical engineering projects are studied: cement- and cement-bentonite based grouts.

In civil engineering, cement paste (or slurry) is frequently used as grout in compaction grouting. The initial consistency of cement- and cement-bentonite-based grouts prior to injection influences the process after grouting. Characteristics of cement paste, such as shear strength, viscosity, workability, pumpability and water to solid ratio will all change with time. The understanding of the way those characteristics change in short and long – term durations is essential in managing grouting operations during and after injection of the grout into soil. The behaviour of the grout material under injection and the soil reaction (stress and train) are a function of according to whether the ground is fractured permeated or compacted. In all these procedures a volume of water will be lost to the surrounding area reducing the effective volume of the grout. The consolidation of the grout due to the phenomenon of bleeding determines therefore the efficiency of the grouting process.

Consolidation of grout can be studied using the traditional small strain and Biot's¹ large strain consolidation theories if it is turns out that solids-water interaction for the duration of consolidation is predominantly governed by a mechanical rather than by a chemical processes.

Large strain deformation of soft soils has been studied for a long time in soil mechanics. Gibson² analyzed the consolidation problem when considering large strain deformation, nonlinear compressibility and soil permeability. Other improvements Xie et. Al.³, Kang-He Xie⁴, Nader Abbasi⁵, Zhuang Ying-chun⁶, Xueyu Geng⁷ have been made in consolidation predictions taking into account the non-linear behaviour of soils during consolidation. The nonlinear analysis of soil /grout consolidation is not new but consolidation analysis of cement-based grouts in relation to the phenomenon of bleeding is new.

Problem description

Tunnels boring or other ground works like massive excavations are most of the time accompanied with displacement or deformation of the ground around the site. A successful and effective grouting does not only depend on a good monitoring of the grouting process or soils behaviour but also depends of a good understanding of the behaviour of the grout before, during and after injection. The used of cement-based grout involves specialists from cement and soil technology. The problem one faces in practice is that when several specialists are involved in the same operation (for instance grouting), many efforts (time, money and procedures) have to be made to complete a simple engineering work. As result money may be lost and project completion may be delayed.

Objectives

The objective of this part of study is to provide geotechnical engineer with a quick, simple and inexpensive and efficient method to monitor and asses the consolidation behaviour of cement-based grouts using the framework of ground mechanics.

Question phrasing

The three main questions to be answered in this first part are:

- Whether or not ageing has significant impact for a short experimental time on cement grout, at different water contents to full consolidation?

- Does cement paste exhibit characteristics of a cohesive soil?

- May a simple standard oedometer test be used to investigate the consolidation behaviour of cement-based grouts?

In order to answer to those three questions, a grain size distribution analysis of cement is made; change with time (grout ageing) of void ratio and shear strength of cement-based grout is investigated and liquid limit is found. The use of the term ageing in the context of this research refers only to the part of the short-term behaviour of grout relevant to continuing bleeding during and shortly after injection. In that sense it can be viewed as experimental ageing.

A deformation analysis of cement grout due to the phenomenon of bleeding is also made.

The goal of this analysis is to make a preliminary assessment on the difference between bleeding predictions of small and large strain consolidation theories.

To the best knowledge of the author this is the first time that large strain consolidation theory has been used to investigate the deformation of cement- or bentonite based grouts together with a study of experimental ageing of the material.

2 Rheological and material characteristics

The rheological and material behaviour of fresh cement-based grouts is a topic of considerable interest. Fresh cement-based grout is a fluid material and its rheological behaviour affects or even limits the ways it can be processed. Therefore, the measurement and control of rheological parameters of the grout are very important in grouting. This section will describe some of the basic problem-related relationships, parameters and some simple tests to assess consistency and strength characteristics. Use of these tests will provide useful tools also to assess the effect of experimental ageing on these basic characteristics. There are many excellent books discussing the rheology of concrete and cement paste, among them Barnes et al⁸., Hunter⁹, Tattersall, and Banfill¹⁰. Much of the basic information used in this section is based on these references with the view of using basic concepts and parameters from cement and soil technology.

2.1 Basic relationships

The drainage and plasticity characteristics of the grouts used in this study are expected to be similar to those of silt or clayey-silt soils with high water content. The permeability range for this material is expected to result in full consolidation, of a thin (2 cm) consolidation sample, within a reasonably short time (less than an hour!). The initial low plasticity and high water content of the grouts are expected to result in large deformations at full consolidation and a measurable rate of viscosity change. During this short time to full consolidation the hydration (cementing) process will be less significant than the mechanical consolidation process (reduced water pressures due to bleeding combined with increased effective stresses). Basic models from soil mechanics could therefore be assumed to represent the grouts (2-phase material, effective stress concept, cohesive-frictional shear behavior, Mohr-Coulomb relationship, large-strain K_o -consolidation). Visco-elastic and visco-plastic stressstrain models are applicable also in sol mechanics and materials like cement paste for describing the viscous behaviour.

Viscosity – void ratio

The rheological behaviour of cement grout may be presented as a plot of viscosity, as a function of void ratio (see figure 3)

Viscosity (μ) is defined as $\mu = \tau / \gamma$. This is termed apparent viscosity. There are other ways to define viscosity (e.g., plastic viscosity: the slope of stress versus strain rate for a plastic material, and differential viscosity: the slope of the curve relating stress and strain rate). So it is important to identify the specific type of viscosity when reporting results. Plastic behaviour (also called Bingham), is when flow only commences above a certain level of stress called the yield stress. Another common behaviour is pseudo plastic (or shear thinning), in which viscosity decreases as strain rate increases.

Figure 1 illustrates the various phases of flow of a fresh mixed concrete under applied stresses. At low stresses, concrete behaves as a solid of extremely high viscosity. As stresses increase, concrete behaviour gradually changes to that of a liquid. From figure 1, it can be seen that frictional resistance which depends on the grain size has a great influence on the rheological behaviour of material. In the case study the frictional resistance depends on the grain size distribution of the cement used.

Figure 2 shows idealized types of curves that can be obtained when shear stress is plotted against shear rate. The line representing the Bingham model crosses the axe "Shear stress" at certain value which corresponds to yield stress in our case this value represents the undrained cohesion of the grout.



Applied stress/internal frictional stress

Applied stress/Cohesion Figure 1: Flow of concrete under various types of stress²²



Shear rate

Figure 2: Typical Shear stress/ shear rate curve²⁴

A study on the viscosity of cement-based grouts has been published by Gustin¹¹. A relationship between the void ratio and the plastic viscosity of cement-based grouts has been reported. This relationship is given by the equation (1) for cement-based and by the equation (2) for cement-bentonite based grout and is re-plotted in this paper in figure 3.



Figure 3: Plastic viscosity- void ratio curves¹¹

Mohr-Coulomb (M-C) Model

For the applicability of the M-C model on a cement-water mixture the following arguments are given:

- All the components of Portland cement are of mineralogical origin. Portland cement may be considered as a soil.

- The grain size distribution analysis of the cement shows that more than 81 % of the grains have a diameter of less than 62 μ m. (81 % of silt).

15 % of the grains have a diameter of less than $2\mu m$ (15% clay). The cement may be considered as a fine soil.

- Carefully prepared mixtures using de-aired water and careful mixing can be considered similar to two-phase soils under the water table (fully saturated).

- For short mixing and test time it is can be assumed that the water to solid ratio of the grout remains constant during the test.

The drainage and plasticity characteristics of the grouts used in this study are expected to be similar to those of Silt or clayey-Silt soils with high water content.

The M-C model is written in the following form:

$$\tau = c + \sigma \tan g\phi$$

(3)

where τ and c are the shear strength and the cohesion of the soil respectively and ϕ is the strength parameter described as the angle of internal friction or the angle of shearing resistance.

Usually the shear strength is obtained from the Mohr-Coulomb failure criterion. But for most saturated clays, tested in undrained conditions, the angle of shearing resistance is equal to zero. Under such conditions, the shear strength of a soil is equal to the undrained cohesion (or undrained shear strength C_u).

 $\tau = c_u \tag{4}$

It should be noted that this parameter is related to the void ratio of the soil or cement paste but is independent of the effective surrounding soil injection pressure (stress).

The Bingham model

The use of the Bingham model to determine the rheological characteristics of cement pastes has been reported by Tattersall and Banfill¹⁰. They used a two-point concrete rheometer to study the flow behaviour of concrete and cement paste. According to their experimental observation fresh cement paste can be described well using the Bingham model (with two intrinsic rheological constants, yield stress and plastic viscosity).

In the Bingham model, the shear stress of a material is expressed in terms of its cohesion, plastic viscosity, and the rate at which the shear load is applied.

During grout injection the movement of the grout is governed by the laws of fluid mechanics. A cement-water mixture as a non-Newtonian fluid possesses some thixotropy, requiring an initial force (shear stress) to become mobile. In grouting this initial force is the injection pressure needed to inject a grout into the soil. The magnitude of that initial force is referred to as the Bingham yield stress. With reference to soil (or grout) this initial force is equivalent to the cohesion.

The Bingham model is written as follows

$$\tau = \tau o + \mu \gamma \tag{5}$$

Where τ is the shear stress applied to the material, γ is the shear rate, τ o is the yield stress and μ is the plastic viscosity. The last two quantities characterize the flow-rate properties of a material, and both quantities depend for the case of grout on water to cement (w/c) ratio.

2.2 Basic tests

It is desirable for the purpose of this study, to ensure that bleeding (consolidation tests) are completed within a short time (few hours) after mixing. To ensure that one has to test the cement paste during its dormant period of hydration prior to maturing using slow-reacting type of cement. It remains however very necessary to detect if effects of ageing are significant.

The author has chosen to monitor this effect (the experimental ageing effect) on simple parameters using standard devices from soil mechanics. Shear strength (undrained

cohesion) parameter and a standard consistency (cone penetration) have chosen because both are strongly dependant parameters on the water-solid (w/c) ratio. If this ratio changes significantly with time it will be evident during these simple tests. These two measures indicate in a simple inexpensive manner the degree of mechanical (stressstrain) and plasticity changes occurring depending on the amount of water used by the cement particles during testing at different test intervals. As will be demonstrated later via test results, these simple tests proved very efficient in assessing these effects.

The type of cement used in all the tests is Portland composite cement. Many studies^{22, 23} on the rheology of concrete and cement pastes show that the chemical composition, the grain size distribution (indirectly the specific surface) of the cement have an influence on their rheological behaviour. The direct influence of the chemical composition and grain size distribution on viscosity and shear strength is out of the scope of this study. Nevertheless in order to inform the reader, the chemical composition of the cement given by the manufacturer and the laboratory test is given in table 1 and the grain size distribution is given in figure 4.

component	SiO ₂	Al ₂ O ₃	Na ₂ O	K ₂ O	MgO	CaO	Fe ₂ O ₃	P ₂ O ₅	SO ₃	TiO ₂	Mn ₂ O	Rest
Cement %	28.1	9.00	0.3	0.8	1.9	49.2	3.6	0.0	2.5	0.5	0.0	4.1
Bentonite	52.9	17.7	3.5	0.8	3.5	4.9	4.6	0.2	2.0	0.8	0.1	9.8
%												

Table 1: Chemical composition of cement and the bentonite

Table 2: Material characteristics CEM II/B-M	(V-L)) sources:ENCI, Cebogel
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	Specific	Specific	Setting	Compressive
	surface[m ² /kg]	weigth[Kg/m ³]	time[min]	strength at 28
				days [Nmm ²]
Cement	410	2950	165	48
Bentonite	130	2750	-	-



Figure 4: Grain size distribution of used cement and betonite¹¹

Vane shear test

Two series of tests are carried out. The first is a series of vane shear test used to relate the shear strength of cement-based grout to its void ratio. The second series is done in order to monitor how the shear strength changes with time (experimental ageing of the grout). De-aired water is used to prepare mixtures. The de-aired water and the cement were previously placed in the same environment (laboratory) 24 hours before the test to avoid a temperature difference between the two materials when preparing samples.

The test is carried out with a standard vane shear apparatus with a motorised constant rate of 90 degrees per minute according to the British Standard BS1377:Part19906 with the exception that a vane of 24 by 24 mm is used. A cement-based grout is prepared by mixing a portion of Portland cement with de-aired water at different void ratios of 0.74, 0.83, 0.89, 1.03, and 1.18. The mixing time is around three minutes. Test is repeated three times for each value of void ratio.

The void ratio *e* of the cement paste sample is calculated as follow $e = \frac{V_f}{V}$ where V_f

represents the volume of fluid and V is the volume of solids.

The samples are considered to be fully saturated therefore the volume of fluid in sample is equal to the volume of water used to prepared this sample. The void ratio can be written as follow

$$e = \frac{M_w}{\rho_w} \times \frac{Gs\rho_w}{M_s} = \frac{M_w}{M_s}Gs$$

Where:

e is the void ratio, M_w is the weight of the water, G_s - the specific weight of solids : for cement type CEM II/B-M/V-L, $Gs= 2.950 \text{ kg/m}^3$ W- w/c ratio.

Test	Cement[kg]	Water[kg]	w/c ratio	void ratio
1	1.2	0.300	0.25	0.74
2	1.2	0.336	0.28	0.83
3	1.2	0.360	0.3	0.89
4	1.2	0.420	0.35	1.03
5	1.2	0.480	0.4	1.18

Table 3: composition of samples

Fall Cone test

By analogy with the shear strength test two series of cone penetration tests are carried out using the British Standard BS1377: Part 2:19904.3. The cement used is the same as in the vane shear tests series. The first series of tests is performed on cement mixtures at different void ratios varying from 0.35 up to 1.18. The second series of tests was carried out half-hourly on different specimens from the same sample with an initial void ratio of 1.18. The goal is to monitor how the cone penetration of the sample changes with the ageing of the sample. A Liquid Limit test of the grout sample is also made.

3 Experimental ageing test results

3.1 Van shear test

The measurement of the shear strength of sample as a function of void ratio is shown in figure 5. It can be seen from these results that the shear strength is very sensitive to the change in the void ratio. An increase of 5 % of the void ratio of a sample results in a decrease of 17 % of the shear strength. The results are found to be consistent showing that void ratio, if changed for whatever reason will reflect on the measured shear strength. In other words ageing could be correlated to changes of shear strength via this parameter which is also related directly to w/c ratio and indirectly to viscosity.

The M-C and the Bingham models are used to analyze the results of the shear test for that propose. As mentioned above the test is carried out using the standard laboratory vane test apparatus, at a shear rate of 9 degrees per minute. This represents 40 minutes per revolution or 0.0004 s^{-1} . In this case equation (3) can be written as:

$$\tau = \tau o + 0.0004 \mu \tag{6}$$

The type of sample used is the same as that used by Gustin¹¹ in his viscosity and consolidation tests. According to Gustin¹¹, the viscosity (μ) of cement-based grouts with a void ratio between 0.5 and 1 have an average value equal to (0.7) ^{-8.85} = 23.9 mPa.s by using the equation (1). Note that the number 0.7 (= (0.5+1)/2) is the average value of void ration of the sample used by Gustin. Thus the value 0.0004 μ = 0.0004 × 0.0000239 = 9.2 × 10⁻⁹ (in kPa) is negligible compared to τ_0 . The Bingham model becomes:

 $\tau = \tau_{o.}$

(7)

Equations (7) and (4) are similar. It can therefore be concluded that the shear strength of the grout samples may be computed using the M-C or Bingham model.

Two curve fitting methods are used to interpolate a relationship between the shear strength and the void ratio data. The first is the least squares method (fit 1, trend/regression type power function using MS Excel 2007) and the second is the polynomial method (fit 2 using Matlab. V6.1). Fit 1 is preferred to fit 2 because the former gives the relationship between the void ratio and the shear strength in a power form. Figure 5 gives the shape of the relationship between the void ratio and the shear strength, which is calculated as follows:

$$\tau = 1.81 e_g^{-5.36} \tag{8}$$



void ration Figure 5: Shear strength versus void ratio

With regards to the ageing of the sample, during the first 90 minutes after mixing the cement with water, the shear strength of the grout sample remains practically unchanged. After this period of time the shear strength increases progressively to reach a value of 4.25 kPa after 4 hours. That is an increase of 425 % at the rate of 1 kPa/h. This gives a standard error of 0.22 kPa that may be acceptable or one that can be allowable in any further testing. These results show that the hydration of cement (under the testing conditions) remains in a dormant period during the first hour after the mixing. Therefore if during this short period of time a cement paste of grout is put under pressure the consolidation is mainly due to drainage rather than hydration. This strengthens the argument for using the oedometer test to investigate consolidation (bleeding) of the same cement in a short- term (1 hour) testing time after mixing with water.

Two more curve fitting methods are also used to interpolate for a relationship between shear strength and experimental ageing (time). The best fit relationship is found to be in exponential form, which is

$$\tau = 0.9e^{0.35t} \tag{9}$$

where t is the time (ageing), e is the Euler's number.

Figure 6 gives the shape of a relationship between the shear strength of the cement paste and the ageing. This relationship is only valid for a period of time between 1 and 4 hours from the mixing and is only for the type of cement paste used in this study.



Figure 6: Shear strength versus ageing

3.2 Fall cone penetration test

An initial liquid limit of 30% that corresponds to a void ratio of 0.9 was determined using the cone penetration test. From the experimental ageing tests (measuring penetration at different time intervals), it was observed that penetration did not change during the first 60 minutes after the mixing with water. After this time, cone penetration begins to change rapidly. It was also observed that during the cone penetration test, when the cone falls into the cement slurry, a quantity of water appears on the surface of the slurry. This phenomenon can be regarded as instantaneous bleeding. That is to say that at the moment the cone falls into the grout, the void ratio of the grout is instantaneously modified. The results of the evolution cone penetration tests are shown in figures 7 and 8.

It can be seen from figure 7 that the consistency of the cement paste expressed in term of cone penetration is sensitive to any change of void ratio (thus indirect viscosity). In this test the void ratio varies from 0.74 up to 1.18 which represents a variation of 60 %. As result of this, the consistency (cone penetration) has changed from 15 to more than 36 mm this represents a variation of 73%.

The results of the "ageing" fall cone test show that the consistency of the used cement paste begun significantly to change only after more than one hour from the mixing. This result confirmed that of the shear strength. With the results obtained on the ageing tests (shear and fall cone) of the cement paste, it seems to be increasingly certain that for a short period of time (1,5 hours maximum) that in this study case is approximately one hour, the odometer test seems to be applicable to investigate the consolidation behaviour of cement paste.

$$\Delta z_{con} = 36.5 - 0.21t - 1.29t^2 \tag{10}$$

where, Δz_{con} is the cone penetration, t ($1 < t \le 4$ hours) is the time. The curve representing this relationship is shown in figure 8.



Figure 7: Cone penetration versus void ratio



Figure 8: Cone penetration versus ageing

3.3 Hydration Error

The hydration of cement begins immediately after mixing with water. It is a complex process which consists of a succession of chemical reactions of individual cement compounds. Several factors influence the rate at which cement hydrates, amount them: chemical composition of cement, its particles sizes, w/c ration, the ambient temperature etc whether or not additives are added into the cement paste.

A number of techniques are available to estimate the degree of hydration of cement paste in real time. Amount them, the Differential Thermal Analysis (DTA), Thermogravimetric Analysis (TG) reported by Ramachandran²³, the computer-based 3-D computer model HYMOSTRUC (HYdration, MOrphology and STRUCture formation) developed at the Technical University of Delft in The Netherlands, reported by Chen²⁵ and Brouwers²⁶. According to all those references, the hydration of cement can be divided in several (5) stages and the rate of hydration of cements is determined through the heat development as shown in figure 9. In the first (I) stage, as soon as cement comes into contact with water, a quantity of heat is released. This first stage also called pre-induction period and takes approximately 10 minutes to complete. During this first stage cement is mixed with water to form cement paste. The second stage known as "dormant" or induction period is characterized by a low release of heat. This period may take several hours to cease .Tests carried out in this study correspond to the second stage of hydration. A low quantity of heat released during the second stage means also a small quantity of water reacting with cement. Therefore it can be concluded that the error due to the hydration on the results of the consolidation tests is negligible.



Figure 9: Conduction calorimetric curves of hydrating tricalcium silicate and cement²³

4 Large Deformation Analysis

This section reports some initial results from large deformation analysis of cement paste.. The initial result investigates of the permeability to simplify modelling the drainage (bleeding) to a better accuracy with large deformation instead of using small- strain assumptions. The section will provide therefore some insight into the potential for further investigation of the problem both numerically and experimentally using large deformation.

The mechanical model involves a soil with an impervious top layer and a pervious bottom layer (ITPB) as illustrated in figure 10. A vertical load q is applied suddenly to the top of the grout and maintained at a constant pressure for a short period of time. It is assumed that the consolidation of the grout is only due to the change of the void ratio i.e. there is no creep or deformation of the solid particles and the pore water is considered to be incompressible. Thus permeability, effective stress and settlement in the soil are only related to the void ratio.



Figure 10: Model of the case study

4.1 Main large deformation equations

In Xie³, the coefficient of the volume compressibility m_v is considered to be constant and has the following form:

$$m_v = -\frac{1}{1+e}\frac{de}{d\sigma'} = \text{constant}.$$

In reality, the coefficient of compressibility changes during the consolidation of the soil. Because the sample is laterally confined, the lateral strain is zero. With regards to consolidation settlement, the reduction in the volume of the grout sample is only due to the reduction of the void ratio. Therefore, the reduction in volume per unit volume is equal to the reduction in thickness per unit thickness. m_v may be computed as follows:

$$m_{\nu} = -\frac{1}{1+e_0} \frac{de}{d\sigma'} = \frac{1}{h(\frac{h_0 - h}{\sigma - \sigma_0})} = -\frac{1}{h_{(n-1)}(\frac{h_{ni} - h_{(n-1)}}{\sigma_n - \sigma_{(n-1)}})}$$
(11)

where h_0 , h_{n-1} h_n is the initial at time t₀= 0, t_{n-1} and t_n (n = 1, 2, 3...) height of the soil layer respectively, and σ' the effective stress.

According to Xie³ the equation of the excess pore water pressure describing a large strain problem has the following form :

$$\frac{1}{\gamma_{w}}\frac{\partial}{\partial z}\left[\frac{k_{vg}(1+e_{0g})}{1+e_{g}}\frac{\partial u_{eg}}{\partial z}\right] = m_{vg}\frac{1+e_{g}}{1+e_{0g}}\left[\frac{\partial u_{eg}}{\partial t}\right]$$
(12)

where k_{vg} is the coefficient of permeability of the grout, and u_{eg} the excess pore water pressure in the grout.

Equation (12) is a diffusion equation written in terms of the state variables excess pore water pressure u_{eg} , time (t), and distance (z) with a variable coefficient of consolidation.

Another model, which will be referred to as Model 2, describes Tezaghis' traditional onedimensional small strain consolidation theory.

The excess pore water pressure during the consolidation process is given by the wellknown equation of consolidation:

$$\frac{\partial u_{eg}}{\partial t} = \frac{k_{vg}(1+e_{0g})}{\gamma_{v.cc}} \frac{(\sigma_g - \sigma_{og})}{\log(\sigma_g / \sigma_{0g})} \frac{\partial^2 u_{eg}}{\partial z^2}$$
(13)

Equation (13) is referred to as Model 2.

Both Models describe the evolution of the consolidation process in terms of dissipation of the excess pore water pressure. Model1 describes large strain consolidation of a cohesive soil while model 2 describes conventional small strain consolidation.

4.2 Numerical solution of the problem

For a numerical solution of (12) and (13), the coefficients of permeability and the coefficient of volume compressibility of the grout are needed. These coefficients are calculated using the results of Gustin' s'¹¹ bleeding (consolidation) tests.

- Cement-based grout: $c_c=0.17, k_{vg}=1.2 \times 10^{-7} m/s$.

- Cement-bentonite grout: $c_c = 0.22$, $k_{vg} = 8.81 \times 10^{-8} m/s$.

Initial and Boundary conditions:

The initial condition used: at t = 0, $u_{eg}(z,0)=q$.

- Boundary conditions: $u_{eg}(h_{1},t) = 0$

At any time other than zero, the hydraulic gradient is zero on the impermeable surface, that is: $\frac{\partial u_{eg}}{\partial u_{eg}} = 0$

that is: $\frac{\partial u_{eg}}{\partial z} = 0$.

The Partial Differential equations Tools (PDE tools) of the standard software Matlab version 7.3.0.267 is used to solve numerically the equations (12) and (13).

The numerical solution shows some difference on the consolidation. Specifically the two models show different rates of consolidation. The difference between the rates of consolidation of two models varies with time. The consolidation (bleeding) predicted by the large deformation algorithm occurs at higher rate. From the beginning of the consolidation up to a time factor t_v of 0.0025 the difference between the degrees of consolidation of the two models is around 10%. After this value of time factor the difference decreases progressively and becomes less than 1 % at the factor time of 0.006. In Gustin et. al.¹¹ a comparison between the two models is also reported. According to Gustin, the consolidation (displacement) predicted by the model 1 is greater than that predicted by the model 2 and that the discrepancy between the two predictions grows with the load imposed on the top of the sample(figure 12). The second part of Gustin's

statement is contrary to our findings and seems to be very debatable. In the view of the author the discrepancy between the models is mainly due to the assumption wherever the coefficient of permeability can be considered constant or not during large consolidation. In large one-deformation of cohesive soils (clays) the coefficient of permeability is considered to vary during the consolidation process. As the effect of this change, permeability will diminish progressively and consequently the rate of consolidation (bleeding) will also decrease and tends for some time to become equal or less than that assumed in the small-train theory. Therefore the discrepancy between consolidation (displacement or excess water pressure dissipation) predict by both models will decrease or at most remain unchanged. this is well illustrated in the figure 11 below.



Figure11: Comparison Small and large deformation



Figure12: Comparison Small and large deformation: Gustin¹¹

5 Conclusions

- The shear strength of a cement-based grout (for the studied type of cement) with a range of initial void ratio between 0.7 and 1.18, remains constant during the first hour commencing at the point at which the cement is mixed with water.

- Both the results of Fall Cone and Vane Shear tests show that cone penetration (consistency) and shear strength of grout samples remain practically unchanged during the first hour after the mixing with water. In addition, the result of Vane Shear test shows that shear strength of cement-grout is very sensitive to any change in the void ratio. For the type of the cement used, an increase of 5% of the void ratio results in a decrease of 17% of the shear strength. It is also well-known from literature that in the "dormant" period hydration of cement is very low.

Therefore, to the question whether or not ageing has significant impact for a short experimental time on cement grout, at different water contents to full consolidation? It can be answered: No, it does not. The hydration of the cement has a marginal effect on the experimental ageing tests provided the time of the bleeding test is less than 30 minutes for the range of values of the void ratio used.

-The grain size distribution analysis of the cement shows that more than 81 % of the grains has a diameter, less than 62 μ m (81 % of silt) and 15 % of the grains has a diameter of less than 2 μ m (16% clay). The cement that was used may be considered as equivalent to and may exhibit mechanical and physical characteristics of very clayey silt prior to maturing and hardening of the cement paste.

-The undrained shear strength of the tested samples is found to be less than 20 kPa which is the range of shear strength of very soft soils²¹. Therefore cement paste used in this research may be considered as a very soft soil.

- The Liquid limit test has given a liquid limit of 30%, which is a characteristic of soils of low plasticity.

To answer to the second question one can conclude that: Yes, cement paste with above mentioned range of w/c ratio may be considered as a cohesive soil (clayey silt) for a short period after mixing with water.

From the above named arguments, it follows that cement-based grout used in this way may be considered as fully saturated silt.

Affirmative answer to the two first questions leads to the conclusion that oedometer test is applicable to cement-based grout with the ranges given: range of void ratio: from 0.74 up to 1.18; viscosity range from 20 up to 100 mPa.s and shear strength range: up to 20 kPa.

The shear strength of cement-based grout may be related to its void ratio (equation 8).

All the derived equations (8, 9 and 10) represent a limited range of materials characteristics and test conditions but nevertheless one encouraging for further investigation using the short-term consolidation tests.

A preliminary analysis of results of consolidation of cement-based grout leads to the following conclusions.

- The excess pore water pressure predicted by large strain consolidation model differs from that predicted by the traditional one-dimensional small consolidation theory.

The dissipation of the excess pore water pressure predicted by the large strain consolidation model is faster than that predicted by the small strain consolidation theory.

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PART II: LARGE CONSOLIDATION OF CEMENT-BASED GROUTS.

1 Introduction

In civil engineering, cement slurry or paste are frequently used as grout in compaction grouting applications. It was the desire of the author to investigate the behaviour of two types of mixtures widely used as grout material in geotechnical engineering namely: a cement-water and a water-cement-bentonite mixture which are referred to as the cement-and the cement-bentonite based grout respectively. The consolidation behaviour of these grouts is studied using two consolidation models: The traditional one-dimensional small strain consolidation theory of Tergazhi¹ and the one-dimensional large strain consolidation theory.

The fundamental basis of Trezaghi theory is the assumptions that during the consolidation process the coefficient of permeability and compressibility remain constant and that the strains are small. Other several simplifying assumptions in the Trezaghi's theory are made to resolve practical problems in geotechnical engineering. But a strict application of the Terzaghi's consolidation theory may lead to uncertainty, specially arising from the two above mentioned assumption as in case of soft materials subject to large strains.

It has been shown that the compressibility for a saturated soil is a non-linear function of the effective stress state of the soil. This must be accounted for in the theory if large deformations are anticipated. When large strains take place, the soil skeleton pores are significantly deformed resulting decrease in the void ratio of the soil that in geometrically non-linearity related to the effective stress increase. A smaller void ratio means that water has less room to flow and there is a resulting decrease in the permeability.

Mikasa² was one of the first to suggest that the permeability and the compressibility must not be considered constant during the consolidation of soft clays. Davis and Ray³ developed an analytical solution for a case of constant loading based on the assumptions that permeability decreases proportionally to the decrease in compressibility and the effective stress remains constant with the depth. For the first Gibson⁴ analyzed the consolidation problem considering large strain deformation, nonlinear compressibility and permeability of soil. Gibson's work removes the limitation of small strain consolidation and provides a comprehensive model which takes into account the more realistic behaviour of the soil during the consolidation process. Since then many efforts have been made to improve consolidation predictions. Wissa *et al*⁵ applied the Mariska-Davis-Raymond model to the constant rate of strain (CRS). Xie et al.⁶ proposed an analytical solutions of one-dimensional large strain consolidation of saturated and homogeneous clays based on Gibson's works. Yuan and Tadanobu⁷ analysed the liquefaction of saturated soil taking into account variation in porosity and permeability with large deformation. Many others works are published on the nonlinear consolidation like Kang-He Xie⁸, Nader Abbasi⁹, Zhuang Ying-chun¹⁰, Xueyu Geng¹¹.

In this paper the author has chosen to use Xie et al. (2004) to study the bleeding from large consolidation of grout. Exact solution of the large consolidation governing equation is provided. But it is restricted to one-dimensional consolidation assuming constant

permeability and compression parameters. When these parameters are not constant the author resort to numerical solution of the same governing equation of Xie et al. the validity of using the consolidation models in this work has been justified for the not-soil grouts in part I

Objectives

The objective of the second part of this research is to investigate the consolidation behaviour of cement based grouts in order to assess drainage predictions. Therefore the author aims is to lay a basic for future research in this direction using small en large consolidation theories.

Question phrasing

The questions the author wants to answer are:

What is the discrepancy between drainage predictions of the two consolidation theories in the case of cement grout?

Which of the two consolidation theories shows drainage prediction closer to the result of the short term consolidation test for the case study?

This paper is organised as follow: firstly a short review of the rehological behaviour of cement-based grout in relation to experimental ageing¹² is presented. Secondly the theory of small and large consolidation is presented leading to the main governing equation. Further a numerical solution of a case study is presented. Finally a comparison between the numerical solutions of the two models is made. The numerical solutions of the small en large consolidation are also compared with the result of oedometer test.

2 Ageing of cement- base grout

In part I of this research, an experimental ageing¹² test was carried out. The results of this experimental ageing have laid to the following conclusions:

Ordinary Portland cement-based grout when mixed with water exhibits geotechnical characteristics including grain size distribution, permeability, plasticity, liquid Limit similar to those of a two/ three phase cohesive soil prior to setting and hardening. Furthermore the error due to hydration in the drainage prediction is marginal. Therefore consolidation of the grout during the experimental ageing is mainly due to the bleeding phenomena rather than to chemical reaction of the cement with water

From above mentioned arguments, it follows that standard oedometer test may be used to investigate the consolidation of cement paste. The results obtain in the Part I of this research form an experimental basic for further research on the consolidation behaviour of cement- based grouts using the framework of ground mechanics.

3 Background of consolidation Theory

Before deriving the consolidation equations coordinate system and the dependant variable(s) need to be defined.

3.1 Coordinate system and independent variables

The most widely used coordinate system in geotechnical engineering is the Eularian system. In the Eulerian system material deformations are related to planes fixed in space. This fixed plane is commonly referred to as a datum. Distances are then measured relative to this datum. Properties of a Representative Elementary Volume (REV) are referenced to a specific distance from the datum. Terzaghi's consolidation theory is based on this type of system and assumes that both the size and position of the element remain the same over time. Any deformation that takes place in the soil element is assumed to be small in comparison to the size of the element.

The consolidation problem is a time-dependant and is non-linear in the load –deformation domain (material properties). In large consolidation, the deformations are large compared to the thickness of the compressible layer. Both the position and the size of the REV change over the time. In such case, properties referenced to a certain coordinate may suddenly be outside the element they are referring to. A system must be found that deforms with the material particles. Such a coordinate system can be either a convective system or a lagrangean. When the soil element deforms, the location and size of the soil element change and this is reflected by the changing coordinates. Changes with time can

be related to the either the convective system (ξ, t) or to the lagrangian system (a,t) as shown in figure 13. This type of problem is also transient, geometrical and materially non-linear



Figure 13: Lagrangian and convective coordinates: (a) initial configuration at t = 0; (b) configuration at time t (Xie⁶)

The choice of the independent variable to obtain a governing equation of consolidation affects the way to proceed in order solves it. The most used dependent variables include pore-water total head, pore-water pressure, effective stress, and void ratio. In the traditional Terzaghi's one dimensional consolidation equation, pore-water pressure is used as the dependent variable. If pore-water pressure is used as the dependent variable, boundary conditions can be easily specified. Boundary conditions in this case are generally no flow and constant pressure. The primary disadvantage of using pressure as a dominant variable seems to be that in large deformation formulation the resulting equation is highly non-linear. The drawback will finally results in the difficulties to find a solution method to solve a non-linear nature of the equation.

For the first, Davis & Raymond1⁴ developed a consolidation equation taking into account variations of permeability and compressibility of the soil during the consolidation process. The equation was based on effective stress. The finding was that when deformation of the soil was taken into account, pore-water pressure dissipation occurred at a slower rate than Terzaghi's equation predicted. Later E. Gibson⁴ developed another consolidation equation in which void ratio was used as dependent variable. The use of void ratio seemed to result in a consolidation equation that was easier to solve. This seems to be the raison-d'être and thus the popularity of Gibson's equation.

In The following section the equations describing one-dimentional small- stain and largestrain consolidations respectively will be presented; the difference between them is illustrated in detail.

3.2 Small strain consolidation theory

To develop the governing consolidation equation, let us consider a representative elementary volume element (RVE) in the soil as shown in figure 14. If the pressure is applied on the top surface the water (or fluid) contained in the soil will flow. The flow of the water through the soil is governing by the Darcy's law.



Figure 14: Representative elementary volume element

The Darcy's law is written: $q_v = -k \frac{dh}{dy}$, the law of continuity stipulates that $\frac{\partial M}{\partial t} = q_{in} - q_{out}$ (1) where q_{in} q_{out} are the in- and outflow respectively, M is the mass of the flowing fluid (water).

Per definition $M = \rho_w \cdot V_w$ and $V_w = q_y dx dz$, where: ρ_w is the density of water , V_w is the volume of water and q_y is the flow of water per unit area. It can also be written that

$$q_{\text{out}} = q_{\text{in}} + \Delta q = \rho_w . q_y . dx. dz + \frac{\partial}{\partial y} (\rho_w . q_y . dx. dz). dy$$
⁽²⁾

If we put the equation (2) into the equation (1) we obtain:

$$\frac{\partial(\rho_{w}V_{w})}{\partial t} = \rho_{w}.q_{y}.dx.dz - \left[\rho_{w}.q_{y}.dx.dz + \frac{\partial}{\partial y}(\rho_{w}.q_{y}.dx.dz)dy\right] = \frac{\partial(\rho_{w}V_{w})}{\partial t} = -\frac{\partial}{\partial y}(\rho_{w}.q_{y}.dx.dz)dy.$$
(3)

The total volume of the REV is $Vt = dy \cdot dz \cdot dx$. Putting this V_t into the equation (3) and cancelling densities we obtain:

$$\frac{\partial \left(\frac{V_w}{V_t}\right)}{\partial t} = -\frac{\partial q_y}{\partial y} \tag{4}$$

Let us transform the Darcy's Law and write it using the definition of the head.

 $q_y = -k_y \frac{dh}{dy}$, $h = head = \frac{u}{\rho_w g} + z$ (m), where u is the pore-water pressure, g represents the acceleration of gravity.

Let us substitute the above expression of q_y into the equation (4) we obtain:

 $\frac{\partial \left(\frac{V_w}{V_t}\right)}{\partial t} = \frac{\partial}{\partial y} \left(k_y \frac{\partial h}{\partial y}\right).$ If the coefficient of permeability varies with the depth, i.e. k is a

function of y or k = k(y) than the equation (4) becomes:

$$\frac{\partial \left(\frac{V_w}{V_t}\right)}{\partial t} = \frac{\partial}{\partial y} \left(k(y)\frac{\partial h}{\partial y}\right)$$
(5)

The equation (5) is a composite of two functions. We must use the chain rule to derivate it. In order to save time not all the derivation steps are shown in this paper. As the result of the chain rule we obtain:

$$\frac{\partial \left(\frac{V_w}{V_t}\right)}{\partial t} = \frac{\partial k(y)}{\partial y} \frac{\partial h}{\partial y} + k(y) \frac{\partial^2 h}{\partial y^2}$$
(6)

The changes in the REV may be expressed in term of its volume change ε_v . The volume change of the REV may be written as follow: $\varepsilon_v = \frac{\Delta V}{V_t}$. The relationship between the coefficient of volume compressibility m_v and the change in volume is give by

$$m_{\nu} = -\frac{\Delta \varepsilon_{\nu}}{\Delta \sigma'}.$$
(7)

Let us assume now that all the volume change of the REV is only due to due the drainage or the loss of water this is the case of fully saturated soils (the term loss of water or drainage is equivalent to the term "bleeding" used in this research). Therefore $\Delta V_w = \Delta V$, and that the total stress does not change or $\Delta \sigma' = \Delta \sigma_t - \Delta u$, so $\Delta \sigma' = -\Delta u$.

$$m_{v} = \frac{\partial \left(\frac{V}{V_{t}}\right)}{\partial u} \Longrightarrow \partial \left(\frac{V}{V_{t}}\right) = m_{v} \partial u$$
(8)

If we differentiate the equation (8) with respect to time we obtain:

$$\frac{\partial \left(\frac{V}{Vt}\right)}{\partial t} = m_v \frac{\partial u}{\partial t}$$
(9)

Since

$$u = (h - y)\rho_w g = h\rho_w g - y\rho_w g \tag{10}$$

Since the rest of the equation is in total head, the pore-water pressure term will be converted:

$$\frac{\partial h}{\partial t} = \frac{\partial u}{\partial t} \frac{1}{\gamma_w} + \frac{\partial y}{\partial t}$$
(11)

Let assume that the strain is infinitesimal strain then the term $\partial y/\partial t = 0$,

$$\frac{\partial \left(\frac{V}{V_{t}}\right)}{\partial t} = \gamma_{w} m_{v}(y) \cdot \frac{\partial h}{\partial t}$$
(12)

Substituting the equation (12) into the equation (6) we obtain:

$$\gamma_{w}m_{v}(y).\frac{\partial h}{\partial t} = \frac{\partial k(y)}{\partial y}\frac{\partial h}{\partial y} + k(y)\frac{\partial^{2}h}{\partial y^{2}} \quad \text{or} \quad \frac{\partial h}{\partial t} = \frac{1}{\gamma_{w}m_{v}(y)}\left[\frac{\partial k(y)}{\partial y}\frac{\partial h}{\partial y} + k(y)\frac{\partial^{2}h}{\partial y^{2}}\right]$$
(13)

In order to follow the traditional way of presenting consolidation equations, let us rewrite equation the equation (13) in terms of pore-water pressure by differentiating the equation (10)

We obtain:
$$\frac{\partial h}{\partial t} = \frac{\partial u}{\partial t} \frac{1}{\gamma_w} + \frac{\partial y}{\partial t}$$
 (14)

and assuming that $\partial y/\partial t=0$ we have:

$$\frac{\partial h}{\partial t} = \frac{\partial u}{\partial t} \frac{1}{\gamma_{w}}$$
(15)

Substituting the equations (15) into equation (13) we obtain

$$\frac{\partial u}{\partial t} = \frac{1}{\gamma_w m_v(y)} \left[\frac{\partial k(y)}{\partial y} \frac{\partial u}{\partial y} + \gamma_w \frac{\partial k}{\partial y} + k(y) \frac{\partial^2 u}{\partial y^2} \right]$$
(16)

If permeability remains constant during the consolidation process, e.g. $\partial k/\partial y = 0$, then we have:

$$\frac{\partial u}{\partial t} = \frac{k(y)}{\gamma_w m_v(y)} \frac{\partial^2 u}{\partial y^2}$$
(17)

Finally, if k and m_v do not vary with depth and by setting

$$c_{v} = \frac{k}{m_{v}\gamma_{w}} \tag{18}$$

then we have:

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial y^2} \tag{19}$$

The equation (19) is the Terzaghi's traditional one-dimensional consolidation equation. This fundamental equation is used to predict the settlement or the dissipation of the pore water pressure.

To derivate the equation (19), a number of assumptions have been made. Clearly assumptions may not always reflect the large consolidation problems that one have to solve in geotechnical engineering particularly on the large deformations. Permeability typically changes with void ratio. The coefficient of compressibility m_v varies with to effective stress which changes with pore-water pressure.

3.3 Davis & Raymond Consolidation Equation

Davis & Raymond³ used in their consolidation equation the effective stress as the main independent variable. They found that the dissipation of pore-water pressures was slower using the Davis & Raymond equation than that Terzaghi's at smaller deformation, the degree of settlement was found identical with the Terzaghi one-dimensional theory but the rate of pore-water pressure dissipation depends on the loading increment ratio σ_i/σ_f , σ_i is the initial load and σ_f is the final load as illustrated in figure 15.



Figure 15: Solution for percentage settlement and dissipation, respectively of maximum pore pressure (After Davis & Raymond) Lee¹³

In order to understand how the soil deforms, let us use the three below figures: they represent the most common way of describing how a soil deforms.



Figure 16: models of soil deformation

This leads to the following equations describing a saturated volume change:

$$-a_{v} = \frac{\Delta e}{\Delta \sigma'}$$
(20)

$$C_c = \frac{-\Delta e}{\log \frac{\sigma'_2}{\sigma'_1}}$$
(21)

$$-m_{\nu} = \frac{\Delta \nu}{\Delta \sigma'}$$
(22)

Using the assumption that decrease in permeability is proportional to the decrease in compressibility¹⁴ it can been demonstrate that,

$$m_v = \frac{C_c}{1 + e_o} \frac{1}{\sigma' \log 10}$$
(23)

Let
$$A = \frac{C_c}{1+e} \frac{1}{\log 10}$$
 (24)

we obtain

$$m_v = \frac{A}{\sigma'}$$
(25)

It is well known that

$$Cv = \frac{k}{m_v \gamma_w} \Longrightarrow k = C_v m_v \gamma_w \tag{26}$$

Therefore,

$$\frac{\partial v}{\partial y}dy = \frac{\partial}{\partial y}\left(\frac{-k}{\gamma_w}\frac{\partial u}{\partial y}\right)dy = -C_v A \frac{\partial}{\partial y}\left(\frac{1}{\sigma'}\frac{\partial u}{\partial y}\right)dy$$
(27)

Let us differentiate the expression (27) using the product rule

$$\frac{\partial v}{\partial y}dy = -C_v A \left[\frac{1}{\sigma'} \frac{\partial^2 u}{\partial y^2} - \left(\frac{1}{\sigma'}\right)^2 \frac{\partial u}{\partial y} \frac{\partial \sigma'}{\partial y} \right] dy$$
(28)

Let considerer now the strain in the element

$$\frac{\partial \varepsilon}{\partial t} = \frac{A}{\sigma'} \frac{\partial \sigma'}{\partial t}$$
(29)

and substituting that into (28) we obtain:

$$\frac{\partial v}{\partial y} dy = \frac{\partial \varepsilon}{\partial y} dy$$
(30a)

or

$$\frac{\partial \sigma'}{\partial t} = -C_{v}\sigma' \left[\frac{1}{\sigma'} \frac{\partial^{2} u}{\partial y^{2}} - \left(\frac{1}{\sigma'} \right)^{2} \frac{\partial u}{\partial y} \frac{\partial \sigma'}{\partial y} \right]$$
(30b)

As can been seen the equation (30) is non-linear, describes the consolidation process (the governing equation). The results predicted from this equation are close to the Terzaghi formulation when the coefficient of consolidation remains constant. As effective stress increases, an increase in the confined modulus K_v was balanced by a decrease in the coefficient of permeability. The result of this is a relatively constant coefficient of consolidation while including the nonlinear properties of permeability and compressibility, this formulation still assumes infinitesimal strains. This is shown in Figure 15.



Figure 17: Typical oedometer test on normally consolidated soil c_v with changing k and m_v (unpublished) Lee¹³

3.4 Large strain consolidation: Gibson's Consolidation Equation

The following derivation is taken from Gibson⁴. If we assume the density of pore fluid ρ_f and solids ρ_s are both constant than the vertical equilibrium requires that:

$$\frac{\partial \sigma}{\partial z} \pm (e\rho_f + \rho_s) = 0 \tag{36}$$

Also the equilibrium of the pore-fluid requires that:

$$\frac{\partial u}{\partial z} - \frac{\partial u_e}{\partial z} \pm \rho_f \frac{\partial \xi}{\partial z} = 0$$
(32)

where u is the pore water pressure, and u_e is the excess pore water pressure. ξ is a Lagrangean coordinate describing the height of the soil element.

$$\frac{\partial}{\partial z} \left[\frac{e(v_f - v_s)}{1 + e} \right] + \frac{\partial e}{\partial t} = 0$$
(33)

where v_f and v_s are the velocities of the fluid and solid phases relative to the datum plane then continuity of pore fluid flow is ensured. Finally, Darcy's law requires that

$$\frac{e(v_f - v_s)}{k} \pm (1 + e) + \frac{1}{\rho_f} \frac{\partial u}{\partial z} = 0$$
(34)

If there is no creep of the soil skeleton and the consolidation is monotonic, then the permeability k may be expected to depend only on the void ratio e.g.

$$k = k(e) \tag{35}$$

The principle of the effective stress stipules $\sigma' = \sigma - u$ The void ratio will control the effective stress e.g.. $\sigma' = \sigma'(e)$ (36)

The combination of the equations (32),(33), (34), (35), (36) gives the following equation for the void ratio:

$$\left(\frac{\rho_s}{\rho_f} - 1\right)\frac{\partial}{\partial e}\left[\frac{k(e)}{1+e}\right]\frac{\partial e}{\partial z} + \frac{\partial}{\partial z}\left[\frac{k(e)}{\rho_f(1+e)}\frac{\partial\sigma'}{\partial e}\frac{\partial e}{\partial z}\right] + \frac{\partial e}{\partial t} = 0$$
(37)

The equation (37) is the Gibson's equation describing the consolidation of soil taking into account the variability of the permeability and the compressibility of the soil. As it can be seen it is highly nonlinear. If the permeability and the compressibility of the soil are considered constant, this equation can be simplified back to the Terzaghi's or David's.

4 Large Deformation of cement-based grout

In the previous section a review was made of the consolidation theory. In this section the consolidation of cement-based grouts is investigated using large strain consolidation model proposed by Xie⁶ based on the governing equation (37). The proposed large deformation equation (37) is solved numerically and the result is compared to that of oedometer test and with the Terzaghi's small strain consolidation. Xie provided an exact solution of the simplified problem assuming constant consolidation parameters with depth (z) and time (t).



Figure 18: Model of the grout sample

One seeks a numerical solution for impervious top pervious bottom boundaries (ITPB) as illustrated in figure 18. A vertical load q is applied suddenly on the top of the grout and maintained constant. This load is equivalent to grouting (pumping) pressure minus energy losses. It is assumed that the consolidation of the grout is only due to the change of the void ratio i.e there is no creep or deformation of the solids particles and the pore water is considered to be incompressible. Thus the permeability, the effective stress and the settlement in the soil are only related to the void ratio.

Let, $c_c u$, σ' , k_v , c_v be respectively the coefficient of consolidation, the excess water pressure, the effective stress, the coefficient of permeability and the coefficient of compression(compression index) of the of the grout. The coordinate z of any particle in the grout is measured downwards from the top surface of the material and *t* is the time.

4.1 Equations for large consolidation

A general case of a soil subjected to large deformation is considered. This case is similar to those proposed by Xie⁶ using a convective coordinate system.

The consolidation of the soil begins at the moment that a load q is applied on its top surface. The total vertical σ stress the pore water pressure u, the effective stress σ ' and the settlement S are expressed by the following relationships.

The total vertical stress at any point in the sol is determined by the sum of the unity weight of all materials including free water above that point and the load imposed on the top surface of the soil.

$$\sigma = \sigma(z,t) = \sigma(0,t) + \int_{0}^{z} \frac{(G_s + e)\gamma_w}{1 + e_0} dz$$
(38)

where $\sigma(0,t)$ is the vertical stress at the top of the surface of the soil i.e. at z = 0.

$$\sigma(0,t) = q \tag{39}$$

where e_o , e are initial and instantaneous void ratios of the soil. γ_w is the specific weight of water.

ne specific weight of water.

 G_s is the specific gravity of the material particles.

The total water pressure *u* is given by the following equation:

$$u = u(z,t) = u_e + u_s \tag{40}$$

where u_e is the excess pore water pressure and u_s is the static pore water pressure.

$$u_{s} = \int_{0}^{z} \frac{(1+e)\gamma_{m}}{1+e_{0}} dz$$
(41)

In this research the soil is considered to have a thin layer therefore the self-weight is negligible

From the principle of effective stress

$$\sigma' = \sigma'(z,t) = \sigma - u = q = u_e = [\sigma(o,t) + \int_0^z \frac{(G_s + e)\gamma_m}{1 + e} dz] - [u_e + \int_0^z \frac{(1 + e)\gamma_m}{1 + e_0} dz]$$
(42)

In Xie⁶, the coefficient of the volume compressibility m_v is considered to be constant and has the following form:

$$m_{\nu} = -\frac{1}{1+e_o} \frac{de}{d\sigma} \tag{43}$$

Because the sample is laterally confined, the lateral strain is zero. For a consolidation settlement, the reduction in the volume of the grout sample is only due to the reduction of the void ratio. Therefore, the reduction in volume per unit volume is equal to the reduction in thickness per unit thickness.

$$m_{\nu} = -\frac{1}{1+e_0} \frac{de}{d\sigma'} = \frac{1+}{(1+e_0)} \frac{e_i - e_0}{\sigma_i - \sigma_0} = -\frac{1}{h_i(\frac{h_0 - h}{\sigma - \sigma_0})}$$
(44)

Where h_0 is the initial height of the soil layer.

In reality, the coefficient of compressibility changes during the consolidation of the soil. Experimentally the coefficient of the volume compressibility can be found as follows: Let the void ratio at time t = 0 of a layer equal, to e_0 (initial void ratio), at time t_n (n = 1,2,3,...) the void ratio becomes e_n (n = 1,2,3,...). At any time during the consolidation process, the coefficient of the volume compressibility m_v may be computed as follows

$$m_{\nu} = -\frac{1}{1+e_{n-1}} \left(\frac{e_n - e_{n-1}}{\sigma_n - \sigma_{n-1}} \right) = -\frac{1}{h_{n-1} \left(\frac{h_n - h_{n-1}}{\sigma_n - \sigma_{n-1}} \right)}$$
(45)

To obtain a good estimation of the compressibility, several steps have to be considered in the consolidation. That means the vertical load has to be applied step by step (n steps) in small increment on the top surface of the sample.

According to Xie⁶ the equation of the excess pore water pressure describing a large strain problem has the following form:

$$\frac{1}{\gamma_{w}}\frac{\partial}{\partial z}\left[\frac{k_{v}(1+e_{0})}{1+e}\frac{\partial u_{e}}{\partial z}\right] = m_{v}\frac{1+e}{1+e_{0}}\left[\frac{\partial u_{e}}{\partial t}-\frac{\partial q}{\partial t}\right]$$
(46)

Where m_v is the coefficient of the volume compressibility of the soil.

Equations (45) and (46) describe a one-dimensional large strain consolidation (defined by stress) of a soil with variable compressibility. Because of a variable coefficient of compressibility an analytical solution of the equation (46) seems to be very difficult even impossible to find. Therefore a numerical approach will be the more realistic. Now let us implement the equation (46) to our case of cement-based grout.

The load q remains constant during the consolidation, $\frac{\partial q}{\partial t} = 0$, (46) becomes:

$$\frac{1}{\gamma_{w}}\frac{\partial}{\partial z}\left[\frac{k_{vg}(1+e_{0g})}{1+e_{g}}\frac{\partial u_{eg}}{\partial z}\right] = m_{vg}\frac{1+e_{g}}{1+e_{0g}}\left[\frac{\partial u_{eg}}{\partial t}\right]$$
(47)

Where k_{vg} the coefficient of permeability of the grout, e_{0g} , e_0 the initial void ratio and the void ratio at time *t* of the grout respectively and u_{eg} is the excess pore water pressure in the grout.

The equation (47) may be written in the following form:

$$\frac{\partial u_{eg}}{\partial t} = c_{v \times} \frac{\partial^2 u_{eg}}{\partial z^2} \tag{48}$$

where

$$c_v = \frac{k_{vg}}{m_{vg}\gamma_w} \left(\frac{1+e_{0g}}{1+e_g}\right)^2$$

The equation (48) is the diffusion equation written in terms of the state variables excess pore water pressure u_{eg} , time (*t*), and distance (*z*) with a variable coefficient of consolidation c_v .

$$m_{vg} = \frac{c_c}{1 + e_{0g}} \frac{\log(\sigma_g - \sigma_{0g})}{\sigma_g - \sigma_{0g}}$$
(49)

The combination of (47) (48) and (49) gives:

$$\frac{\partial u_{eg}}{\partial t} = \left[\frac{k_{vg}}{\gamma_{w.cc}} \frac{(1+e_{0g})^3}{(1+e_g)^2} \frac{(\sigma_g - \sigma_{0g})}{\log_{10}(\sigma_g / \sigma_{0g})}\right] \frac{\partial^2 u_{eg}}{\partial z^2}$$
(Model1: large strain consolidation) (50)

The equation (50) represents a mathematical model of a large strain consolidation of a grout. In this paper, this model is referred as Model 1

Another model which will be referred as Model 2 is those describing the traditional onedimensional small strain consolidation theory of Terzaghi.

Excess pore water pressure during the consolidation process is given by the well known Terzaghi's one-dimensional small strain consolidation equation.

$$\frac{\partial u}{\partial t} = cv \frac{\partial^2 u}{\partial z^2}$$
(51)

Where

$$c_{\nu} = \frac{1}{\gamma_{\nu}} \frac{k_{\nu}}{m_{\nu}} \tag{52}$$

The combination of (49), (51) and (52) gives:

$$\frac{\partial u_{eg}}{\partial t} = \frac{k_{vg}(1+e_{0g})}{\gamma_{v.cc}} \frac{(\sigma_{g}-\sigma_{0g})}{\log(\sigma_{g}/\sigma_{0g})} \frac{\partial^{2}u_{eg}}{\partial z^{2}} \quad (\text{Model 2: Small-strain consolidation})$$
(53)

The equation (53) is referred as Model 2.

Theoretically the Model 1 and 2 describe both the evolution of the consolidation process in term of dissipation of the excess pore water pressure.

For the solution of these governing equations the coefficients of permeability and that of volume compressibility of the used grout mixtures are needed. These coefficients are determined experimentally.

4.2 Determination of the coefficients of permeability k_{vg} and compression index c_c

In this research the model proposed by Mckinley¹⁴ is adopted to calculate the coefficient of permeability of the grout.

$$k_{vg} = \frac{1}{2} \frac{h_f^2}{t\sigma} \left(\frac{1 + e_{0g}}{1 + e_g} \right)$$
(54)

Where h_f is the thickness of sample after full consolidation, *t* is the consolidation time, σ is the load increment.

The results of the filtration test in Gustin¹⁵ are used to calculate k_{vg} The compression index C_c is given by

$$C_c = \frac{e_{0g} - e_g}{\log(\sigma_g / \sigma_{og})}$$
(55)

Grout materials

Water-cement and water-cement-bentonite mixtures are used in this research. The cement used is a composite Portland cement (Dutch mark CEM II/B-M/V-L; ENCI) and bentonite (Dutch type CEBOGEL CSR; CEBO). The chemical Characteristics of both materials are given in Part I (see table 1, figure 4: part I). The mixtures are prepared at different void ratios using de-aired water as the experimental ageing tests.

The coefficients of permeability and compressibility are calculated using the results of the filtration test of Gustin¹⁵. For every test, k_{vg} and C_c are calculated using (54) and (55) respectively. These values of k_{vg} and C_c are than substituted into (50) and (53) to obtain the governing diffusion equations for the two consolidation models.

	Pressure	$\sigma = 11kPa$	$\sigma = 22kPa$	$\sigma = 33kPa$
Coefficient of	Filter 1	$1.5 \times 10^{-7} m/s$	$1.38 \times 10^{-7} m/s$	$1.03 \times 10^{-7} m/s$
permeability k_g	Filter 2	$2 \times 10^{-7} m/s$	$7 \times 10^{-8} m/s$	$1.2 \times 10^{-7} m/s$

Table 2 : Coefficient of permeability of grout sample: Cement-based with $e_0 = 1.745$

The average coefficient of permeability is $1 \ 1.22 \times 10^{-7} m/s$ for the filter and $0.8 \times 10^{-7} m/s$ for the filter 2. The compression index is $c_c=0.17$ for the filter 1 and 0.114 for the filter 2.

For the cement-bentonite grout with the initial void ratio of 1.475 the coefficient of permeability is given in the table 3.

Table 3: Coefficient of r	permeability of grout	sample: Cement-be	ntonite with $e_0 = 1.745$

	Pressure	$\sigma = 11kPa$	$\sigma = 22kPa$	$\sigma = 33kPa$
Coefficient of permeability k_g	Filter 1	$1.17 \times 10^{-7} m/s$	$8.44 \times 10^{-8} m/s$	$4.4 \times 10^{-8} m/s$

The average value of k_g is than 8.18 ×10⁻⁸ m/s and C_c is 0.22 for the filter 1.

The following geotechnical properties representing a cement- and cement-bentonite grout have been adopted for a further analysis.

- Cement-based grout: $c_c = 0.17, k_{vg} = 1.2 \times 10^{-7} m/s$.

- Cement-bentonite grout: $c_c = 0.22$, $k_{vg} = 8.81 \times 10^{-8} m/s$.

The coefficient kv_g for a cement-based grout is similar to those obtained by Mckinley¹⁴. For a cement with the same specific surface Mickley¹⁴ found the coefficient of permeability of a cement grout to be between $4.6 \times 10^{-7} m/s$ and $1.47 \times 10^{-6} m/s$.

5 Numerical solution of the problem

5.1 Initial and Boundary conditions

- Initial condition:

The initial condition is: at t = 0 and at any distance z from the impermeable surface which is in this case the top surface of the grout sample, the pore water pressure is equal to the total vertical pressure imposed on the top surface of the sample. ueg(z,0)=q.

- Boundary conditions:

Boundary conditions are defined by the particular physical conditions imposed to the sample during the consolidation process.

At any time other than zero at the drainage surface which is in this case the bottom of the sample the excess pore water pressure is zero; that is

ueg(h1,t)=0

At any time other than zero at the impermeable surface, the hydraulic gradient is zero; that is

$$\frac{\partial u_{eg}}{\partial z} = 0$$

The Partial Differential equations Tools (PDE tools) of the standard software Matlab version 7.3.0.267 is used to solve the equations (50) and (53) numerically. The numerical solution of the equation is based on the finite elements method (FEM) using the following model.

5.2 Model Description and data structure

The numerical solution starts by defining computational domain which is represented by a rectangle the length and the width of which are equal to the thickness (20 mm) and the width (75mm) of the sample respectively. This rectangle represents the Constructive solid geometry model. This computational domain is divided into simple geometric objects, in this case triangles.



Figure I: Discretization (Triangulation) of the computational domain

After the triangulation, the quality of each triangle is checked using the formula ²⁴:

 $q_{tr} = \frac{4a\sqrt{3}}{h_1^2 + h_2^2 + h_3^2}$

where *a* is the area and h_1, h_2, h_3 the side length of the triangle.

If $q_{tr} > 0.6$, the triangle is of acceptable quality. $q_{tr} = 1$ when triangle is equilateral. The result of calculation gives an average value 0f $q_{tr} = 0.976$.



Figure II: Triangle quality It can be seen that the quality of all mesh elements is sufficiently good ($q_{tr} > 0.7$)

The output of the triangulation is Mesh data matrix M_{md}.

Mmd = [Mp Me Mt], where Mp is the Points matrix, Me the Edge matrix and Mt is the triangle Matrix.

Note that $Mp = [p_1 p_2 p_n]$, where $p_1 , p_2 ... p_n$ are the point describing the centre of the circumcircles of the triangles 1, 2...n respectively.

Boundary condition matrix $Mbc = [u_k \ u_{k+1} \ u_{k+2}....]$ where u_k , u_{k+1}, u_{k+2} ... represents a solution value at the corresponding point from the boundary.

Coefficients matrix Mc represents a stiffness matrix. In the case study Mc is a constant and equal to Cv.



Figure 19: Data structure model

5.3 Analysis of the results

For a good interpretation of the numerical solutions, let us analyse the result according to the composition of the grout and to the consolidation model.

Cement- versus cement-bentonite gout

Drainages of the two types of grout are different. The numerical solution of the governing equations show that when bentonite is added to a cement grout the rate of excess water pressure changes (decrease). This tendency is the same for small en large deformation. Indeed for values of time factor up to 0.0023 the large deformation model predicts a discrepancy of 15% between the degrees of consolidation of the two types of studied grouts. This discrepancy decreases progressively to less than 1% at a time factor of 0.006 as shown in figure 11

The same tendency is observed for the traditional small deformation model of Terzaghi. For this model, the discrepancy between the degrees of consolidation of the two grouts is around 20% up to a time factor of 0.003 and then decrease to 10 % at a time factor of 0.006 as shown in figure 12. Therefore it can be suggested that a grout with bentonite has a small coefficient of permeability compare to the cement-grout. This can be seen from the figure 25 and 26.



Figure 20: Excess pore water pressure (large deformation)

Small versus large deformation

Small and the large consolidation models show different drainage predictions of the same type of grout as shown in the 3-D figure 20 and 21. For both cement- and cement-bentonite grout large deformation model predicts a more rapid dissipation of excess water pressure than the traditional small consolidation model of Terzaghi.

The difference between the degrees of consolidation predict by the two models varies with time (time factor). From the beginning of the consolidation up to a time factor of 0.0025, the discrepancy between the degrees of consolidation of the two models is around 10%. After this value of time factor the discrepancy between the rates of consolidation of the two models diminishes progressively. At a time factor of 0.006 the difference decreases and becoming less than 1 %. The same tendency is observed for both cement- and cement-bentonite grouts samples as shown in figures 22 and 23

In the small stain consolidation theory, the coefficient of permeability is assumed to be constant while in the large strain consolidation the coefficient of permeability changes. According to the opinion the author the behaviour of discrepancy between the drainage evolution (dissipation of excess water pressure) of the two consolidation illustrates the way the coefficient of permeably in large consolidation change with time.



Figure 21: Excess pore water pressure (Small strain deformation)



Figure 22: Comparison Large versus Small strain of cement grout (with 5% bentonite)



Figure23: Comparison Large versus Small deformation of cement grout (without bentonite)



Figure24: Comparison Large and Small average degree of consolidation (Xie⁶)

Comparing the Large and small strain consolidations, our findings in the rate of the dissipation of excess seems to mach those of Xie⁶. According to Xie, in one-dimensional consolidation of saturated and homogenous clays, the dissipation of excess pore water pressure is found to be faster than in small strain consolidation. Another finding that is in accordance with Xie is that the discrepancy between large and small strain theories diminishes with reducing compressibility. At this point the findings of Gustin¹⁵ are totally opposed. An interesting observation is that Xie used an analytical solution and geotechnical properties representing soft clays to compare the two solutions while the author used a numerical solution and geotechnical properties representing a cement-based grout.

Further research is needed to investigate the underlying reasons for the discrepancy between our finding and that of Xie with those of Gustin.









5.4 Comparison with the result of the oedometer test

The comparison between the numerical solutions and the result of the oedometer test is made. In general it is found that the trend of results of the oedometer test is different from

result of numerical solution as shown in figure 27. the difference increases from a time factor of 0.0015. This could indicate either not full consolidation at short experimental time and/or experimental ageing of the cement pate due to bonding from that time factor onwards. However it is found that the consolidation predict by the small strain consolidation seems give a result that is closer to the result of the oedometer. The Large strain consolidation predicts a faster rate of consolidation both the Large and small stain consolidation predict a rate of consolidation greater that the oedometer test has shown. When the degree of consolition is greater than 50%, the rate of consolidation of the oedometer becomes greater.

To interpret these findings, the author remembers the reader that the result of the numerical solution comes from solving a diffusion equation with water pressure as the dependant variable. But the oedometer test in fact gives consolidation settlement. Therefore when comparing the two solutions, in fact one is comparing the degree of consolidation defined by stress (excess pore water) and that defined by strain (displacement-settlement). It is well know that in small strain consolidation both the consolidation defined by the stress and by the strain are the same what is no the case in large consolidation. That may justify our finding that the result of the oedometer is closer to the numerical solution of the small consolidation problem.



Figure 27: Comparison Large, Small strain with the result of oedometer test

6 Discussion

The numerical solutions found in the previous section represent the evolution of the excess water pressure in the sample during the primary consolidation. Therefore, not all the mechanism of volume change including the deformation, the displacement the dispersion of particles is taken into account. However, the dissipation of the excess pore water pressure is the essential component of the consolidation and it is thus representative the all consolidation.

To evaluate the prediction of the consolidation settlement of a cement based grout, let us consider exact solution of two other models.

The first model is that proposed by Xie in⁶. This model describes the evolution of the settlement of the homogenous clays according to the large strain consolidation theory. The model is expressed by the following equation

$$\rho_{l} = H(1 - \exp(-m_{vx}\sigma_{x})) \left(1 - \sum_{m=1}^{\infty} \frac{2}{M2} \exp(-MT_{v})\right)$$
(56a)

The equation (56a) can be written in the following form:

$$\rho_l = A(1 - \exp(-m_{vx}\sigma_x)) \tag{56b}$$

Where
$$A = H\left(1 - \sum_{m=1}^{\infty} \frac{2}{M2} \exp(-MT_v)\right)$$
 (56c)

The second model is the traditional small strain consolidation theory of Terzaghi whereby the settlement is expressed as follow

$$\rho_{sd} = Hm_{vx}\sigma_{x}\left(1 - \sum_{m=1}^{\infty} \frac{2}{M2}\exp(-MT_{v})\right) = m_{v}\sigma_{x}A$$
(57)

Let us now introduce two new functions Y1 and Y2.

 $Y_1 = 1 - \exp(-m_{vx}\sigma_x)$ And $Y_2 = m_{vx}\sigma_x$.

To compare the behaviours of the functions defined by equations (56a) and (57) with each other it will be sufficient to compare the behaviours of the functions Y1 and Y2.

(1) Conditions where m_{vx} is constant and σ is variable:

Let's consider that the only variable in Y1 and Y2 is the load imposed on surface of the grout. The volume compressibility m_{yx} of the ground remains constant during the consolidation process.

It is very easy to see that for any value of the total vertical pressure α ($\alpha > 0$) acting on the top surface of the ground the curve of Y2 remains above that of the Y1 The discrepancy between the two curves increases with the increase of the value of the total vertical pressure.

(2) The conditions where the compressibility m_{vx} is variable:

Let us now consider the two functions Y1 and Y2 as parametric functions. The volume compressibility of the ground is considered as a parameter. It s already showed that that for any value of the total vertical stress imposed on the top surface, the curve of Y1 is above that of Y2.

When the value of the parameter compressibility is very small (nearly zero), the discrepancy between the curves of Y1 and Y2 is very small or almost non-existent. When the compressibility of the ground increases, the discrepancy between the two curves become important.

There are three possibilities to combine the coefficient of volume compressibility with the total vertical pressure in settlement analysis of the grout presented here

(a)- Both m_{vx} and σ are constant

(b)- m_{vx} is variable and σ constant

(c) -Both m_{vx} and σ are variable

For all the three above named combinations of $m_{\nu x}$ and σ the curve of Y2 remains above those of Y1 as it is shown in figure 28. So the settlement predicted by the traditional small consolidation theory is at any time larger than that predicted by the large strain consolidation theory. But the development of the settlement in the small consolidation theory is slower compare to the large deformation theory proposed by Xie.



Figure 28: Comparison of exact solutions small versus large consolidation

7 Grout-soil interaction

When a grout is injected into a soil, the way it consolidates depends on the interaction between the grout and soil. In an interaction soil-grout the rate of consolidation of the grout mainly depends on the geotechnical characteristics of the soil.

When grout is placed in soil under pressure, it behaves in one of three ways. The first way is as a penetrant filling pore space, if the pumping rate is equal to or less than the rate at which the pore structure will accept the grout. The second way is as a fluid causing hydraulic fracture of the soil if the pumping rate exceeds the permeation rate or if the grout consistency does not allow permeation. The third is as an expanding mass and will compact the soil. The transition and mechanism associated with threes different procedures could be controlled by primary and secondary variables. Of the primary variables are the grout design water/solid ration and plasticity (including viscosity, and mineral composition. etc.); the soil classification (cohesion, grading and relative density, compressibility, water limits etc.). Secondary parameters refer to the boundary conditions (pressure and displacements) and boundary interphase zone characteristics that change during grouting. Of all those parameters one considerer the most important in grout-soil interaction are relative value of the stiffness (and relative compressibility or deformation parameters) and relative parameters and the consolidation parameters. These, together with secondary boundary conditions parameters dictate which grouting presses will be dominant: permeation, fracturing, compaction combination of one or more of these mechanisms.

The problem of grout-soil interaction may also be viewed as those involving discontinuities. Discontinuity problems include the existence of an interface or a transition layer wherein the properties of material rapidly change. Let us consider two types of soil under which grouting operations take place:

- (a) Condition where the soil is granular

In a case of a granular soil for instance sand, when a load is imposed on the top surface the sand will immediately deforms (no time-dependant consolidation). At a time t = 0, the pore water pressure in the sand will be the same as at the contact boundary. After that, the pore water pressure in the sand immediately decays to zero at any time t > 0. Because the permeability of the sand is much greater than that of cement- grout, the sand acts as a filter and does not have a particular influence on the consolidation of the grout. - (b) In the case of a cohesive soil for instance clays, the rate of the consolidation depends

on geotechnical properties of both grout and soil mainly on the permeability.

To study the interaction soil-grout one has to commence by examining what may be formed as material at the 'interface' soil-grout. If the properties (for instance permeability and compressibility) of the 'new' formed material are determined, it will be possible to model the interaction grout-soil. This may be made by considering the groutsoil system as a three layered soil: layer 1 grout, layer 2 the 'new' interphase material (mixture of grout solids with soil) and layer 3 the soil. The properties of the new martial depends on the amount of the grout that penetrates the soil. In order to monitor how the grout interacts with the soil, the author has design the following experimental setup.

8 Experimental setup

The following experimental setup shown in figure 15 has been designed to study groutsoil interaction







A modified oedometer consist of a transparent perpex cylinder of 250 mm height. The cylinder has an internal diameter of 75 mm. A transparent cylinder allows to observe what happens at the grout-soil interface and the parameters controlling the various

grouting processes. The cylinder is reinforced with fiber rings to avoid it deformation. The top of the cylinder is made impermeable with a metallic disk, around which a rubber ring is wrapped. Frictions between the disk and the cylinder are minimized by lubricating the rubber ring. The metallic ring is fixed to the loading cap to maintain it horizontal during the consolidation. A transducer is mounted on the top of the loading cap and connected to a log apparatus that is itself connected to a computer. A special software installed on the computer is used to monitor and record the settlement of the grout-soil. More details of the modified oedometer is showed in appendix 2.

The test set-up is recommended for further testing to verify the results of this study.

9 Conclusions en recommendations

In the second part of this study the following is found:

Excess pore water pressure predicted by large strain consolidation model proposed by Xie⁶ differs from that predict by the traditional one-dimensional small consolidation theory of Terzaghi. The dissipation of the excess pore water pressure predicted by the large strain consolidation model is faster than that predicted by the small strain consolidation theory of Terzaghi. The discrepancy depends on the composition of the grout: The dissipation of the excess pore water pressure in cement-based grout happens faster than in cement-bentonite based grout.

The settlement predicted by the traditional small consolidation theory is larger than that predicted by the large strain consolidation theory. But the development of the settlement in one-dimensional small strain consolidation theory of Terzaghi is slower. The compressibility of the soil plays an important role. It is has been showed that the more the soil is compressible the more the discrepancy between the two models is great. The above arguments constitute an answer the first question asked at the beginning of the part II.

The comparison of the numerical solution with the result of the oedometer test made in this research has show that the small strain consolidation seems to gives a closer result to oedemeter test. This is only up to the time factor corresponding to 50% consolidation. The comparison of numerical solution to the result of the oedometer test has not given a clear response the second question. To assess which drainage prediction is closer to experimental the author proposes to use a more sophisticate equipment (hydraulic oedometer for instance) that allows the measurement of excess water pressure and displacement at the same time.

Recommendation for future work:

One expects that the grouting process i.e., the transition from permeation to fracturing (compensation) to compaction is controlled by relationship between the major parameters from the soil and grout (k_g/k_s ; C_{ug}/C_{us} ; U_{eg}/U_s ; W_{cg}/W_{cs}). An experimental setup like the one the author here suggested in section 8 (part II) is of limited value in investigating the controlling mechanism and parameters of the problem. The author suggest investigating this problem further using a more complete setup in which the grouting process and is parameters can be monitored.

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Appendix

Details of the Modified Consolidation Cell



Cell for the bleeding test



Cell base (material: steel)



The cell body will be mounted on the base. See the next page











This disc will be mounted on the loading cap



Rods for oedemeter(material: steel)

