# A Holonic Architecture for the Global Automated Transportation System (GATS)

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## Abstract

*High costs of congestion and collisions have triggered the research into completely automated transportation systems. The GAT system differentiates itself by its holonic architecture. In the simplified situation where there is just one vehicle and a deterministic system, we propose an algorithm for this architecture that can create a point-to-point shortest path. Because of the special features of the holonic architecture we use the proven label-correcting algorithm of Pallottino and decrease the number of areas searched by an A* search like approach. The result is a flexible algorithm that is scalable to as many hierarchical levels as needed, thus usable for small as well as extended areas.*

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# 1. Introduction

The annual costs resulting of errors and accidents in traffic are estimated to be over $1000 per person (Cambridge Systematics, Inc., 2008). Combined with the high costs of congestion, estimated at $430 per person per year, this has lead to the development of automated transport systems (Zelinkovsky, 1994). The Global Automated Transportation System (GATS) is such a system, which uses advanced algorithms to guide vehicles to their destination and reschedule routes in case of unforeseen events. This study contributes to the development of these algorithms.

## 1.1. The Global Automated Transportation System

The Global Automated Transport System aims to provide an automated, driver-less, road-vehicle transport system, which optimizes travel, in terms of speed, safety and economy (Zelinkovsky, 1994). The GAT system can do this better than when current systems would be expanded, because of the new technology and architectural design. In this section we will describe these new factors that give the GATS its competitive advantage.

The basis lies in the communication between the vehicle and the system. The vehicles are controlled by objects called Road-Units (RUs), that lie in an intelligent cable about 10-15 centimetres below the road. The vehicle sends short radio transmissions down towards the RUs at regular time intervals. The RU receives a transmission, processes it and responds with a radio transmission back to the vehicle (Versteegh, Salido, & Giret, 2007).


**Figure 1: Communication among RUs**

The communication within the system is done by two types of networks. First the RUs are connected to each other by direct serial lines (the upper line in Figure 1). The communication that passes through these lines is about the control of arriving vehicles. A group of RUs however, is controlled by a level controller. This level controller communicates with the RUs through the lower parallel line. Communication through these lines is for the purpose of planning routes and is therefore most important to this research.

The GAT system is efficient due to its modular nature (Figure 2). Only small groups of RUs are controlled by the level controller of the first level. Multiple controllers however, are by itself controlled by another level controller of a higher level. This creates a clear hierarchy of controller levels. Different from the labelling in Figure 2, we will refer to the first level of controllers as level *1* controllers and the highest level controller will be called the level *m* controller. Due to this decentralized, modular architecture the GAT system can be implemented with the same ease and simplicity in small contained areas as in larger areas (Zelinkovsky, 1994) The only thing that needs adjusting in this case is the number of levels.

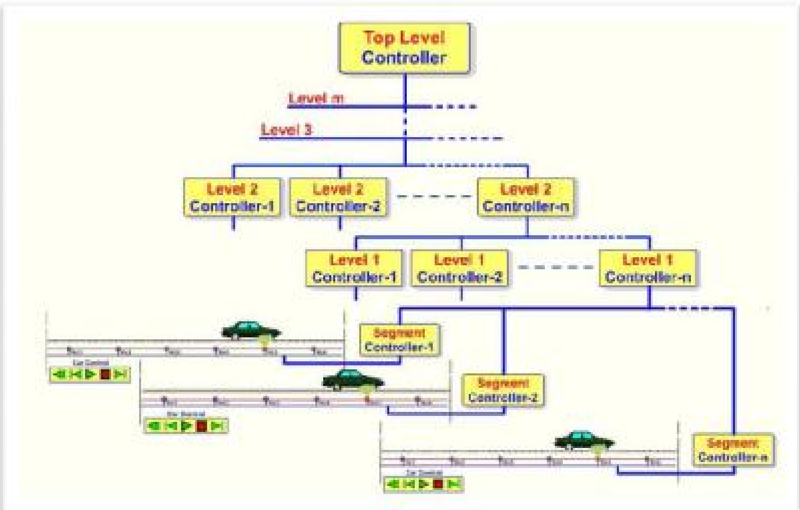



Figure 2: modular architecture of GATS

## 1.2. Results of Previous Studies

Since this work is a part of an ongoing process, it is necessary to briefly describe the current state of the research. The research focuses on multiple fields at the same time, although for the purpose of this study, the work on the algorithmic part is most important.

In a previous study (Versteegh, Salido, & Giret, 2007) the theoretical foundation was created for a static distributed algorithm for the shortest path problem. For the shortest path calculations, Dijkstras algorithm was proposed, with a general recursive algorithm capable of calculating shortest paths within as many levels as necessary. Practical implications of such an algorithm were not yet considered at that time.

Another study (Salido & Giret, 2008) aimed to resolve one of the main problems of the hierarchical architecture. Although this so called 'holonic' architecture is supposed to be more efficient by storing information only locally, it creates inefficiency as well, because local information has to be gathered at all times. Not only the problem is in division though, the solution is too. By dividing the problem into several sub problems, ordering and solving them concurrently instead of consecutively, a lot of time can be saved. This is possible because all controllers work as independent agents in the system and can thus work as parallel machines.

## 1.3. Research Questions

Although the architecture of the GAT system is the very thing that should give the system its competitive advantage, too little attention has been paid to these characteristics in the creation of the algorithm so far. Especially in the choice of the basic shortest path algorithm, very little thought was given to the system requirements. An optimal connection between architecture, system goals and algorithm is a necessity when creating a successful system, though. The goal of this research is *to create an algorithm for the static shortest path problem that fits the requirements and* *architecture of the Global Automated Transportation System.* This research is meant to find this connection by answering the following questions:

1) What is the state of the art in the area of shortest path algorithms?
2) How can the algorithm be created, considering system architecture, requirements and shortest path algorithm possibilities?
3) How well does the algorithm perform?
4) In what direction could the research continue?

We limit ourselves to the situation where there is only one car in the system. Moreover we keep to the design of a static algorithm as in Versteegh, Salido & Giret (2007). We do however, look at the theory on dynamic shortest path algorithms. This knowledge must keep us from creating an algorithm that is only valid for the static case, but can be extended to the dynamic case during future research.

## 2. Theoretical Background

The GAT system efficiently guides vehicles from starting point to destination. In order to do this it calculates the optimal route between the two points. In the literature this problem is known as the shortest path problem. This chapter gives a short overview of some of the important aspects of the problem to the GAT system. Since sorting is an important issue as well for the GAT system, we added a short introduction to sorting problems too.

### 2.1. Shortest Path Problems

First we define the different terms often used in shortest path calculations and the different types of shortest path requests, for not in every problem the same information is needed. Then we describe some of the solutions that are known to the problem.

The formulation of a shortest path problem consists of a network, a pair (source, destination) of points in the network and possibly a starting time (Barrett, Bisset, Holzer, Konjevod, Marathe, & Wagner, 2006). The network is represented by a directed graph $G = (V,E)$, where $V$ is a finite set of nodes and $E$ is a set of edges, where an edge is an ordered pair $(u, v)$ of nodes $u, v \in V$ (Wagner & Willhalm, 2006). Each edge $(u, v)$ has a weight $w\ (u, v)$ and we term a path with minimum weights as a shortest path. The solution is an algorithm that finds the most efficient route from the source $s \in V$ to the destination $d \in V$ leaving the source at the given time $t$. Additional constraints may be imposed, restricting the set of feasible routes (Barrett, Bisset, Holzer, Konjevod, Marathe, & Wagner, 2006). The problem is only well defined, if $G$ does not contain negative cycles. This problem is also sometimes called the single-pair -or *one-to-one* shortest path problem, meaning that there is one source and one destination. It thereby distinguishes itself from other forms, like the *one-to-some* shortest path problem, with one source and multiple destinations, the *one-to-all* shortest path problem, with one source and an interest in the distance to all other locations, and the *all pairs* shortest path problem, where every possible pair of source and destination is requested (Wikipedia, Shortest_Path_Problem, 2008).

The static shortest path problem is one of the most studied problems in algorithmic graph theory. In the static situation arc travel times are deterministic and there are many algorithms that solve the problem to optimality. In reality, however, many networks tend to have dynamic characteristics, like stochastic or time-dependent travel times and changes in the availability of roads, which require more sophisticated approaches for computing shortest paths (Dean, 2004). The dynamic shortest path problem arises when it is required to model a transportation network in which travel times change significantly as a function of time (Gao, 2005). There are two common types of dynamic shortest path problems, that can differentiate themselves by their proactive versus reactive nature. The first type is the time-dependent and/or stochastic shortest path problem. Here network characteristics change, sometimes with time, in a, more or less, predictable fashion. This type of problem has a proactive nature, since it deals with the dynamic network characteristics during the creation of the shortest path. The second type of dynamic problem is reactive. It assumes frequent, instantaneous, and unpredictable changes in network data and recomputes the shortest path based on these changes. This is essentially the reoptimization of the original problem (Dean, 2004).

We discuss the dynamic problem later on in this chapter, but first turn to solutions to the static shortest path problem.

### 2.1.1. Static Shortest Path Approaches

Algorithms for solving the shortest path problem are typically classified into two groups: label-setting and label-correcting algorithms. Both approaches iteratively assign distance labels to nodes at each step. These labels are estimates of the shortest path distance from the source node to these nodes. However, while label-setting algorithms designate one label as permanent and thus optimal in each step, in label-correcting algorithms all labels are temporary up to the last iteration when all labels become permanent (Hasselberg, 2000).

We first describe label-setting algorithms, especially the classical label-setting algorithm for computing shortest paths of Dijkstra (Wagner & Willhalm, 2006). Then we turn to the label-correcting algorithms. We start the description with some basic theory about the algorithms. Because we aim to find a proper connection between system architecture and shortest path algorithm qualities, we discuss the features of the algorithm with respect to the GAT system in the last part of each sub section.

### *2.1.1.1. Label-Setting Algorithms: The Dijkstra Algorithm*

The Dijkstra algorithm is a uniform cost search, which means that, starting from the root node, the node with the least total cost from the root node is visited in each step (Wikipedia, Uniform-Cost_Search, 2008). This rule ensures that the shortest path tree is constructed by permanently labeling one node at a time. The biggest feature of the label-setting algorithms is that once a node is permanently labeled, its optimal shortest path distance from the source node is known. Hence, for the *one-to-one* shortest path problem the Dijkstra algorithm can be terminated as soon as the destination node is labeled (Zhan & Noon, 1998). Especially when the search area is large, this can mean huge savings.

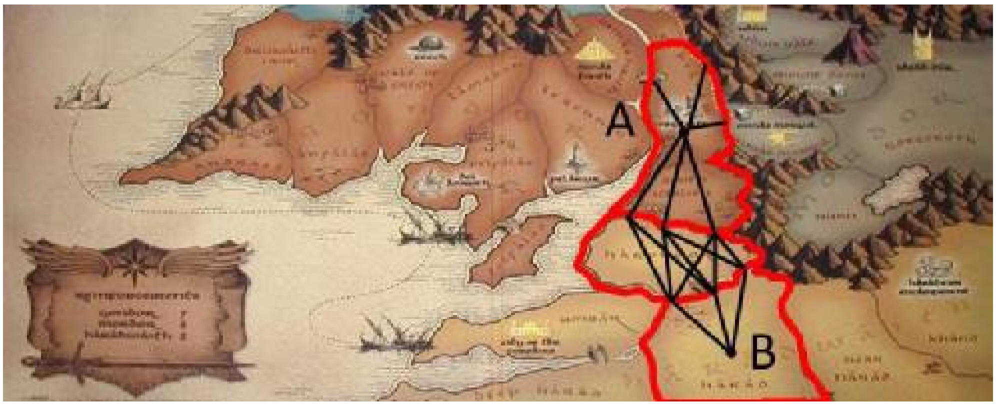



**Figure 3: Example different type of shortest path calculations**

For the static shortest path problem in the GAT system the Dijkstra algorithm was suggested by Versteegh, Salido & Giret (2007). However, because of the distributed nature of the GAT system, only if the starting point and destination fall within the same region there will be a *one-to-one* shortest path problem. For all other situations there is a problem similar to the *one-to-all* shortest path problem (for example in Figure 3, between the starting point A and the border points) , or even the *some-to-all* problem (the calculations in the middle region). Therefore there is not much need for the special feature of this algorithm.

#### *2.1.1.2. Label-Correcting Algorithms*

Based on empirical evidence, a label-correcting algorithm is often used, instead of a uniform cost search algorithm, in transportation planning applications. Especially when multiple routes have to be identified. Because the label-correcting algorithm updates the labels of the nodes with the scan of every arc, it cannot provide the shortest path between two nodes before the shortest path to every node in the network is identified. The necessity of this *one-to-all* search mode makes the label-correcting algorithms more suitable in situations when many shortest paths from a root node need to be found (Fu, Sun, & Rilett, 2006).

Inherent to the distributed architecture of the GAT system is that in most cases there is more than one starting point in a region. Moreover, not all of these starting points are available at the same time, since the path leading to some of the source points of this region might not have been calculated yet. Therefore multiple shortest paths need to be found within one region. This creates the need to either combine the information of separated shortest path calculations, or to use an algorithm that is capable of updating the labels of the nodes at every calculation.

#### *2.1.1.3. Results of Real Road Network Studies*

We discussed the difference between label-setting and label-correcting algorithms in theory. However, only few studies take into account the fact that real road networks are different from generated networks. One of the differences is the arc-to-node ratio, which lies considerably lower on real road networks. Ratios for generated networks are reported up to 10, while for real road networks these ratios lie between 2 and 3 (Zhan & Noon, 1998). For example, the road network of the Netherlands has a ratio of 2.136 (Klunder & Post, 2006). The fact that arc lengths are often drawn randomly in generated networks also contributes to the existence of differences. Consequently, algorithms that are considered to be fastest in studies on generated networks might not be the best for real road networks.

Studies on real road networks show that for the case of the *one-to-all* shortest path problem, the label-correcting algorithms of Pape (Pape, 1974) and Pallottino (Pallottino, 1984) outperform the others. However, the polynomial worst case complexity of the latter, compared to the exponential worst case complexity of the Pape algorithm, makes the graph growth algorithm with two queues of Pallottino algorithm preferable (Zhan & Noon, 1998).

The study of Hasselberg (Hasselberg, 2000) on algorithm speed on real road networks indicates that the Pallottino algorithm loses some of its efficiency on huge road networks. The test cases used contain 500000 to 1500000 nodes, with the intention of creating more realistic road networks, i.e. of larger scale than in the study of Zhan and Noon (1998). However, since the distributed nature of the GAT system will prevent the networks to have this size, the study has less significance for the GATS problem.

#### *2.1.1.4. Label-Correcting Algorithms: The Pallottino Algorithm*

Since the Pallottino algorithm is the algorithm that performs best on real road networks for the *one-to-all* shortest path problem, we want to give special detail to the working of this specific algorithm.

The algorithm starts by labeling all nodes with infinity, except for the root node, which receives the label zero (Zhan, 1997). Of course this value is higher when the root node is not the current location of the vehicle, but a random border point in the system. Moreover, the number of starting points can be larger than one. The labels for the nodes can also be lower than infinity if it is not the first time that the region controller is calculating shortest paths. In the implementation of this algorithm nodes are partitioned into two sets: the first set of nodes, the queue *Q* (Figure 4),contains those nodes that have not yet been used to find a shortest path and the second set contains the remaining nodes. Nodes in the second set are further split into two categories, based on whether they have been examined already or not. After initialization all nodes are marked unexamined and only the starting points will belong to the queue.

The queue $Q$ can be split up again into two parts, $Q'$ and $Q''$, where $Q'$ has priority over $Q''$ and both parts of the queue will have a 'First In First Out' (FIFO) strategy. $Q'$ will contain already examined nodes, while $Q''$ will contain the unexamined nodes. The starting points will be added to $Q'$ and nodes are removed from this queue one at a time. It is fairly easy to see from the name of the FIFO search strategy that the oldest node in the set of temporarily labeled nodes is selected first. All the nodes connected to the removed node are examined and if the label has to be updated, the successor will be added to the queue. This will thus be either in $Q'$ or $Q''$ based on its status. This process will continue until the queue $Q$ is empty. (Zhan, 1997)

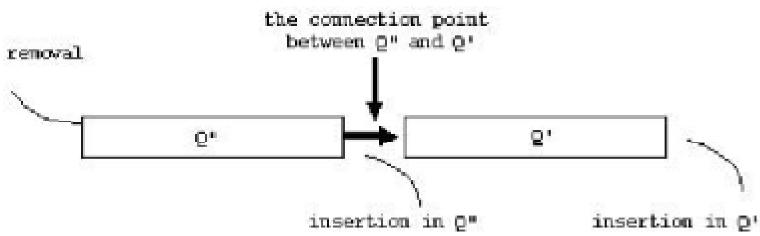



Figure 4: Queue system in the Pallottino Algorithm

### 2.1.1.5. Improvements on Shortest Path Algorithms

The algorithms described in the previous sections will give solutions to the shortest path problems. When these solutions are required very often, though, or when the response is required immediately, these algorithms sometimes fall short. The inefficiency of many algorithms stems primarily from the fact that the algorithms employ uninformative outward search techniques without making use of a priori knowledge on the location of the origin and destination nodes, path composition and network structure. However some important techniques can be used to improve the algorithm. The next subsections describe three main types of shortest path algorithm improvements. These are the limiting of the search area, the decomposition of the search problem and the limiting of the searched arcs (Fu, Sun, & Rilett, 2006).

### 2.1.1.5.1. A* Search

An improvement of the first type is A* (Russel & Norvig, 2003). The A* search algorithm evaluates the cost *f* (*n*) of the cheapest path through node *n* by combining the actual cost *g*(*n*) from the starting point to node *n* and the *estimated* cost *h*(*n*) from node *n* to the destination. The addition of the distance to the destination will prohibit the algorithm to search in the direction opposite to the destination too much, thus dramatically decreasing the search area. In order to obtain an optimal solution with A* search algorithm, the heuristic function *h*(*n*) must exhibit monotonicity. If there are two nodes *n* and *n*', where *n* is previous to *n*' , the *h*(*n*) can never be larger than the sum of *h*(*n*') and the cost from *n* to *n*'. The estimation *h(n)* thus has to be a lower bound between the two points *n* and the destination (Yue & Shao, 2007). Figures 5 and 6 show the improvement that can be realized when applying A* search techniques.

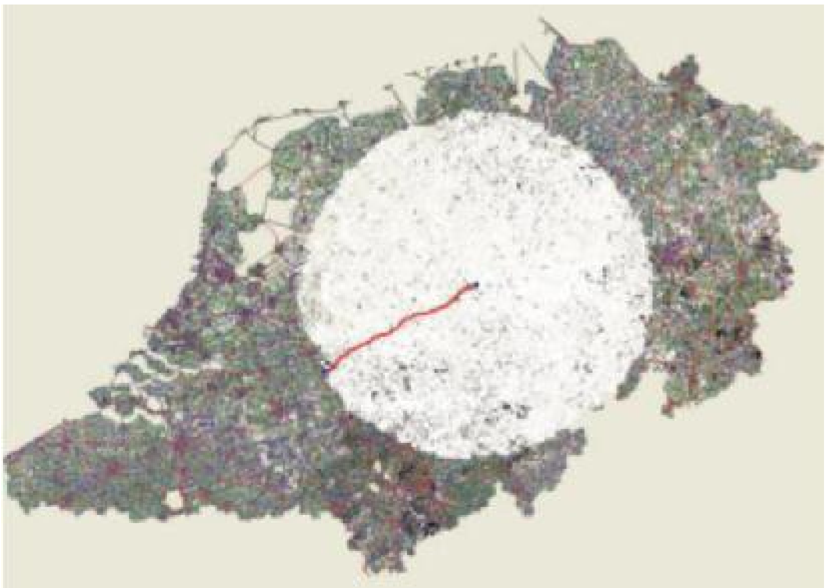


**Figure 5: Normal Algorithm Search Area**

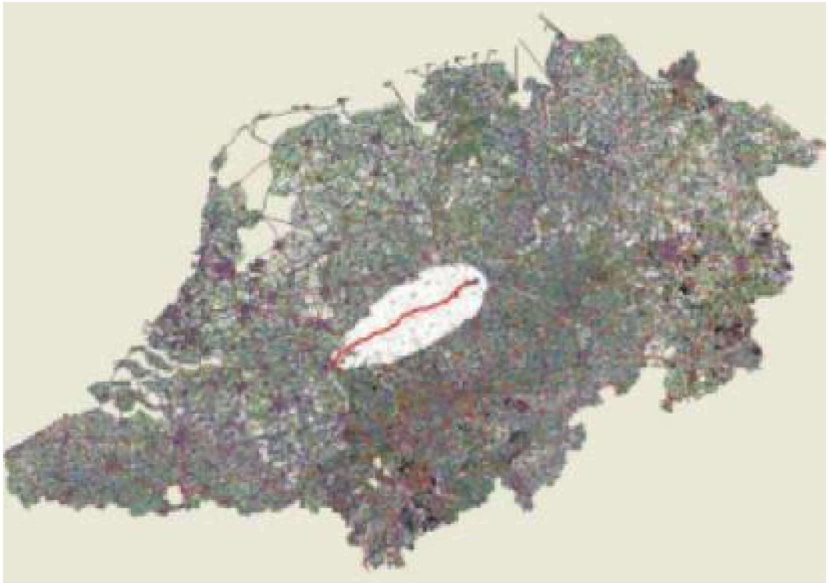


Figure 6: A* Algorithm Search Area

### *2.1.1.5.1.1. Haversine Formula*

Because every RU is given a unique name based on its location in latitudes and longitudes, the distance between two points can easily be calculated. The form of the earth approximates the form of a sphere (Wikipedia, Earth Radius, 2008), therefore creating the need to calculate great-circle distances (Weisstein, 2008). A good formula to calculate this great-circle distance is the Haversine formula (Wikipedia, Haversine_Formula, 2008) because of its ability to calculate accurately on small distances (Sinnott, 1984). For this purpose it uses earth's radius and the differences in latitudes and longitudes between two points. A slight deterioration occurs, though, because the earth is not a perfect sphere. The accuracy can be improved by estimating earth´s radius for every calculation. For this purpose a formula is used that calculates the radius of the earth based on latitude. This is done by taking the average latitude between the two points in the calculation (Wikipedia, Earth Radius, 2008).

### 2.1.1.5.2. Decomposing the Search Area

It is commonly recognized that the computational effort required to solve a problem to optimality usually grows faster than the size of the problem. As a result, if the original problem can be decomposed into smaller sub-problems, substantial computational savings can be realized (Fu, Sun, & Rilett, 2006). Although the distributed nature of the GATS decomposes the problem into sub-problems already, this will not simply make the algorithm more efficiently. More calculation time is needed to gather, transfer and combine the information, while not explicitly cutting the search area.

A method that does cut down the search area is bi-directional search (Fu, Sun, & Rilett, 2006), since it is more efficient to search utilizing both the origin and the destination uniformly by searching alternatively from the origin side and from the destination side (Klunder & Post, 2006). Unfortunately this technique is not applicable to the GAT system. The reason will be discussed in section 2.1.2.1.1. on time-dependent networks.

#### 2.1.1.5.3. Hierarchical Search

An improvement of the third type is the hierarchical search method. The hierarchical search strategy is well known in the artificial intelligence field and is also known as an abstraction problem solving strategy. The basic idea behind the hierarchical search is that in order to effectively find a solution to a complex problem, the search procedure should at first concentrate on the essential features of the problem without considering the lower level details, and then complete the details later (Fu, Sun, & Rilett, 2006). In order to make this differentiation the system needs multiple layers of information. Furthermore, the improvement is very complex and many studies have been dedicated to the implementation. One issue inherited with a hierarchical search algorithm is that it usually does not allow any shortcuts such as moving from one arterial road to another by using a residential road. In a traffic network there usually exist many types of shortcuts and some of them may even be unavoidable. To overcome these problems, difficult pre-processing steps will have to be taken (Fu, Sun, & Rilett, 2006) and this makes this improvement less desirable for non-centralized systems.

### 2.1.2. Dynamic Shortest Path Approaches

Compared to the static shortest path problem, few works have been done on the dynamic shortest path problem (Gao, 2005). Two different aspects, i.e. stochastic and time-dependent networks and the deviation in realization of the route from the plan, will be discussed here.

#### *2.1.2.1. Stochastic and Time-dependent Networks*

There are two ways in which travel times can be changing or even unavailability might arise. Obviously, not at every point in time there is an equal number of travelers on a road and accordingly the travel time varies with time. When the number of travelers is large, this can even lead to recurrent congestion. Recurrent congestion is due to the mismatch between demand and supply under normal conditions. However, usually traffic infrastructure is updated in a fairly long cycle. Recurrent congestion is usually seen in peak hours, but if the capacity is significantly low compared to average demand, congestion is likely to spread outside peak hours (Gao, 2005).

The other reason for variability in travel times is due to disturbances to the traffic network. Disturbances, such as incidents, vehicle breakdown, bad weather, work zones, special events, and so on, occur with various types of predictability. For instance incidents and vehicle breakdown cannot be predicted and are therefore unavoidable. Others are predictable to some extent, such as bad weather, work zones and special events, but usually there are prediction errors. A weather forecast is usually in a probabilistic format, e.g. a precipitation probability of 90% (Gao & Chabini, 2006). Completely unpredictable disturbances will be ignored for now, but will be dealt with in the following part of this chapter on rerouting. The incidences that are to some extent predictable can be used to create better routes.

##### 2.1.2.1.1. Time-dependent Networks

If there is only deviation in travel times due to time, then this can easily be solved. Although at this time travel times in transportation networks are time-varying quantities that are at best known a priori with uncertainty (Miller-Hooks & Mahmassani, 2003), in a totally automated transportation system all future travel times are approximately deterministic. Figure 7 shows a time-dependent network, where arriving at different times at node 4 influences the travel time over arc *e.* The path *abe* has a length of 10. This is a lot less then the length of path *cde* (14), although the difference between *ab* and *cd* is only 1. However, due to the monotonistic character of roads it is never possible that arriving later at a node will result in arriving earlier at its successor. The information about incoming cars can be used to predict the travel time over an arc. In order to do this, the approximate time of arrival of the vehicle at the node is needed, creating the necessity to search unidirectional. This is the reason that bi-directional search is not an option for the GAT system.

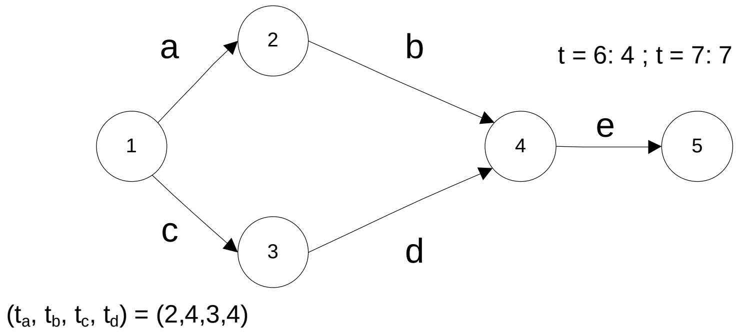
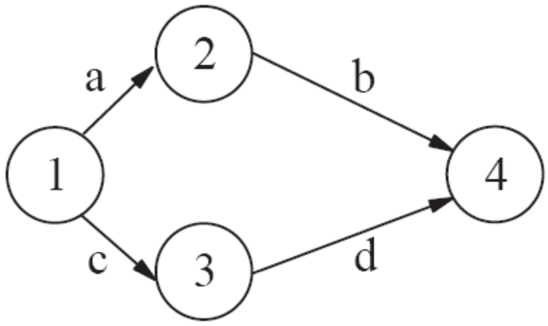



Figure 7: Time-dependent Network

### 2.1.2.1.2. Stochastic Networks

A stochastic network is a network where the link travel times are random variables with some a priori distributions. If the underlying network is assumed to be static (non-time-dependent), the link travel times remain unchanged after they are revealed to the travelers (Gao, 2005). Figure 8 is an example of such a network. The expected travel time for path *ab* is 6+*m*/2 while for *cd* this is 7+*m*/2. It is obvious that it would be best to travel path *ab.*




$$(t_a, t_b, t_c, t_d) = \begin{cases} (5, M, 1, 9) & w.p.\ 0.5 \\ (1, 6, 4, M) & w.p.\ 0.5 \end{cases}, \text{ where } M \text{ is a very large positive number}$$

**Figure 8: Stochastic Network**

### 2.1.2.1.3. Stochastic Time-Dependent Networks

Of course the travel times can change as well based on both the previous factors. In a time-dependent network the travel time of every link at every time period is an individual random variable, so travel times revealed at different time periods could be different (Gao, 2005). In Figure 9 the travel times are both dependent on the time at which you cross the arc, but as well on the probability of the occurrence of disturbances. Here traveling path *ab* would cost 2,5+7,5=10, while traveling *ac* would cost 5+6,5=10,5. Here we would again choose to travel path *ab.*

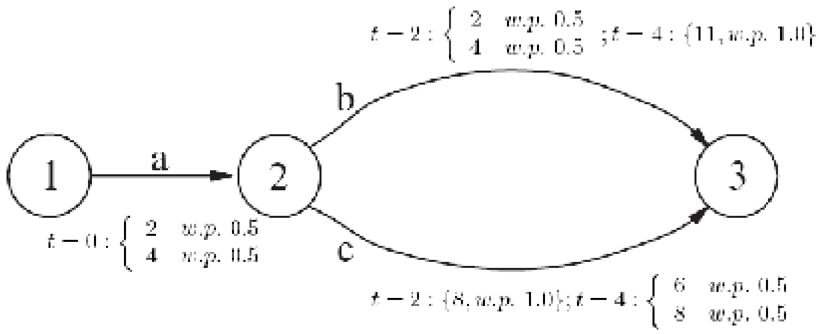



**Figure 9: Stochastic Time-dependent Network**

### 2.1.2.1.4. Addaptiveness to Time-dependent Information

So far, we assumed that the choice of the optimal path had to be made before starting the trip. However, if the decision which path to take could be postponed until information about arcs *a* and *c* in Figure 8 or arc *a* in Figure 9 would be available, the choice would be easier, because now the system would be deterministic in the first case and only stochastic in the second case. The closer you get to the arc you want to travel, the more accurate the information will be and the better the choice. The decision rule which specifies what node to take next out of the current node based on the current time and online information is called a routing policy. Users are assumed to choose routing policies rather than paths (Gao, 2005). Although a routing policy will give a better route, it also requires the system to continuously update information and recalculate all options and will therefore be much more demanding of the system than a simple model.

### 2.1.2.1.5. Equilibrium Assignment Models in Stochastic Time-Dependent Networks

Gao (2005) recognizes four different models for networks, the so called equilibrium assignment models, that distinguish themselves by three features: the knowledge of the incident probability function, adaptiveness to online information, and optimal adaptive decision. The policy model is the most advanced and has all the features. It uses dynamic programming like methods to take the next best step anticipating what lies ahead. A little less involving is the online path model. It calculates a path with minimum expected travel time from the current node to the destination based on the current information and follows the first link along this path. When the user arrives at the next node, a new minimum expected travel time path is computed and the first link followed, and so on. This is also an adaptive model. However, it assumes no future change in network conditions and is therefore a little more short-sighted than the policy model. The path model is not adaptive anymore. It does consider the random incident, but users follow simple paths instead of being adaptive. It thus focuses on stochastic networks. The base model is even simpler. It does not have knowledge of the probability function of the incident and is model for the simple static situation.

#### *2.1.2.2. Rerouting decisions*

Until now, the dynamic models discussed were merely dealing with proactive decision making. However, when such models are chosen that there exists the possibility of changing the path along the way, rerouting problems arise. These are reactive models. There are different models in literature that describe rerouting decisions. The rational-boundary model (Mahmassani & and Jayakrishnan, 1991), assumes that one reoptimizes the current route either when the relative difference travel time between two paths is larger than a predefined relative threshold parameter or when the absolute difference between these two paths is higher than a pre-defined absolute threshold. In the binary-logit model (Ben-Akiva & Lerman, 1985) each driver makes its rerouting decision according to two values; a probability value to change its current following path to newly found shortest path calculated from binary-logit model and a random value, representing the modeling error. The probability is based not only on the improvement of the route, but is representing each driver's familiarity with the traffic network and the resistance to change the route during the trip too (Yang & Recker, 2006).

Both models, as well as the most involving equilibrium assignment models, assume knowledge of changes throughout the system. This knowledge makes it possible to review the current path at every node. Usually, in a distributed system, information is not shared throughout the entire system, though and since there is no time to recalculate the entire route at every node, other ways must be found to initiate rerouting.

## 2.2. Sorting Problems

The previous section shows that 2 kinds of algorithms can provide solutions to the shortest path problem; the label-setting algorithms and the label-correcting algorithms. The first kind produces neatly sorted lists of distance labels. The label-correcting algorithms do not guaranty these sorted lists after the termination of the algorithm. Though, for simplicity in back-tracking the optimal path, this is desirable. Several sorting algorithms are known that can solve this problem. This section discusses some of the possibilities.

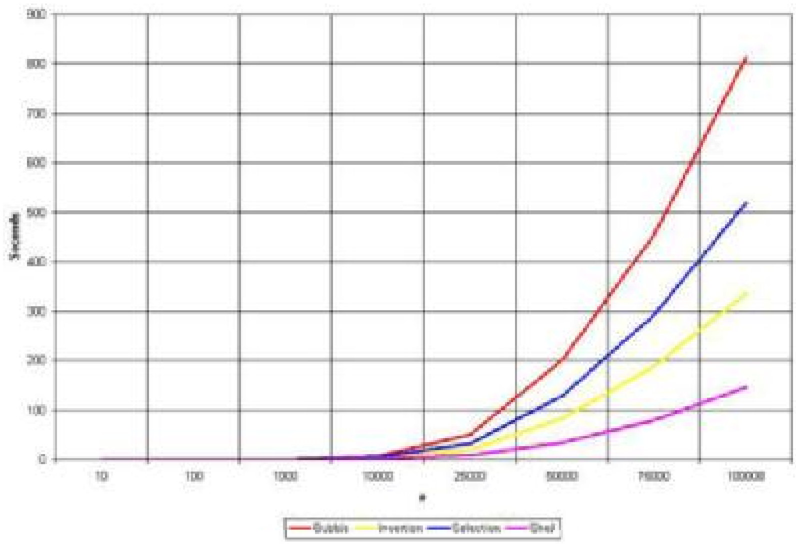



**Figure 10: Sorting Methods O(n2)**

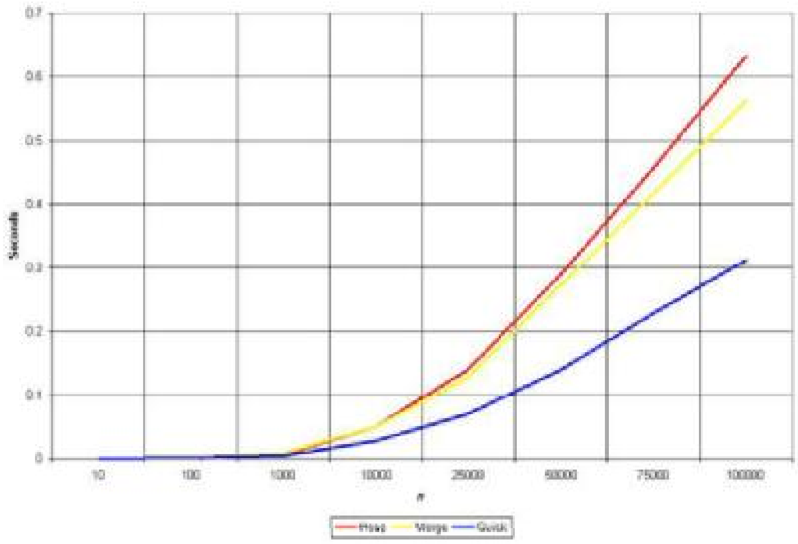



Figure 11: Sorting Methods O(n log n)

Sorting processes can be very time consuming. The most simple algorithms have a complexity of $O(n^2)$. Figures 10 and 11 show an implementation of some of the different algorithms. From these figures the huge differences between the various algorithms become visible. The difference does not only lie within the complexity of the algorithm, since algorithms with the same complexity can produce very different results. Dependent on the data series and the problem characteristics, a specific algorithms will outperform all others. Therefore it is important to assess some of the different algorithms.

One of the first sorting algorithms to be invented and the most popular $O(n^2)$ algorithm is bubblesort (Astrachan, 2003). Bubblesort is a straightforward and simplistic method of sorting data, comparing every two items and swapping them if the first is bigger than the second. While simple, this algorithm is highly inefficient and is rarely used except in education (Wikipedia, Sorting_Algorithms, 2008).

A better sorting algorithm is insertion sort. This algorithm is often called card sort, because it works the way most people sort a hand of playing cards. It starts by sorting the first 2 items. Then it adds the third into its proper place and continues until all items are sorted. The algorithm is very easy to implement and is amongst the most efficient when dealing with small or nearly sorted lists. Still, the average and the worst-case complexity are *O*($n^2$).

Both sorting algorithms described above work by comparing different items to each other and are therefore called comparison sort algorithms. In theory, these algorithms have a lower bound complexity of *O(n log n)*. An example of an algorithm with average complexity of *O(n log n)* is quicksort.

Quicksort works by partitioning all the items around a pivot and recursively sort the sub lists of items larger and smaller than the pivot (Wikipedia, Quicksort, 2008). Ideally, quicksort partitions sequences of size N into sequences of size approximately N/2, those sequences in sequences of size N/4, and so on, implicitly producing a tree of sub problems whose depth is approximately log2 N. Sequences that cause many unequal partitions result in the growth of the sub problem tree in a linear rather than a logarithmical way. This is for instance the case in partially sorted lists (Musser, 1997). Another problem encountered in quicksort is that it does not work very efficiently for small sub problems (Sedgewick, 1987). It is very strong, though, for large lists of randomly sorted items and in practice it is in fact faster than most other sorting algorithms (Musser, 1997). This makes quicksort one of the most popular sorting algorithms, available in many standard libraries (Wikipedia, Sorting_Algorithms, 2008).

The literature provides some solutions to solve the weaknesses of quicksort. To avoid a linear growth of the problem tree, Sedgewick (1987) suggests that a median of 3 items be used as a pivoting item. However, sequences can be found that will still have a complexity of *O*($n^2$) under the median of 3 strategy. Rather than a median-of-3 items Musser (Musser, 1997) proposes a limit on the depth of the search, to limit the complexity to *O(n log n)*, although recognizing the fact that the probability that these sequences occur is very small.

The inefficiency and storage space requirements can be solved by choosing a different method for small sequences. Leaving small sub problems to insertion sort is one of the usual optimizations of quicksort (Musser, 1997). Because the number of recursive calculations will increase exponentially with the number of divisions, aborting the recursive process for small sequences will save a lot of memory. The result thus will be a hybrid method capable of choosing the most efficient sorting scheme based on the length of the sequence.

# 3. Algorithm Formulation

The algorithm for the GAT system is described in this chapter. The basis for the creation of this algorithm is the shortest path algorithm. An overview of the literature suggests that, given the specific distributed architecture of the GAT system and given that the algorithm will deal with real road networks, the Pallottino algorithm is the best choice. This algorithm gives the possibility to use multiple starting points at the same time and easily combines information from multiple calculations. However, the algorithm does not produce entirely sorted lists. Given that the results of the algorithm will already be partially sorted, the choice of the insertion sort algorithm is logical for small lists. For longer lists the quicksort is better suited.

The Pallottino algorithm, combined with the 2 sorting algorithms will be the main part of the algorithm. We guide the creation of the rest of the algorithm by the use of an engineering framework. The logical steps of the framework can be put almost one-on-one with the sections of this chapter.

## 3.1. A Multi-Agent Engineering Framework

The framework of Wood & DeLoach (2000) (Appendix A. MaSE Framework)is particularly useful because it takes an initial system specification, and produces a set of formal design documents in a graphically based style. The graphical, universally accepted style makes sure that both the steps in the process of creation and the algorithm itself are understandable for others. Moreover, since the methodology is independent of particular multi-agent system architecture, agent architecture, programming language, or message-passing system, it will not influence the implementation of our results. The framework is a little bit too extended though for this purpose, so we will not follow every step as extensively and focused mainly on the analysis part of the framework.

We follow the steps of the framework in order to answer the following questions:

a. What are the requirements of the system?
b. What are the goals of the system?
c. What do the processes of the algorithm look like?

d. What information passes through the system?
e. What actors can be identified in the system?
f. What are the tasks and procedures that these actors have to perform?

## 3.2. Transportation System Requirements

The first step of the framework determines the system requirements. As explained, we focus on a static system with just one car. This means that the requirements at this time are different from the ultimate system requirements. In this phase the system should be able to:

1. Calculate a shortest path from source to destination.
2. Limit the search area by excluding regions that are unlikely to contain the shortest path.
3. Use any number of region levels without changing the algorithm (scalability).
4. Store the future arrival of a vehicle in a RU when this RU is on the shortest path in combination with the arrival time.
5. Exclude particular roads for particular vehicles, either because of temporary unavailability or because of structural inaccessibility to that kind of vehicle.
6. Calculate the shortest path based on real time information.
7. Calculate the shortest path based on time or absolute distance.
8. Adjust the travel time based on car characteristics.
9. Sorting out useful information and deleting useless information from system memory.

## 3.3. Transportation System Goals

In the next step we transform the initial requirements into a structured set of goals. This means both identifying and structuring the goals. The result of this structuring process is a hierarchical goal-tree where all sub-goals relate functionally to their parent (Wood & DeLoach, 2000). One of the requirements of the system is that it is perfectly scalable. For this purpose we add a recursive element to the system. This recursive element is not easily made visible in a tree structure. So for clarity we sketch a situation with just 1 level.

Appendix B presents the goal-tree. This figure visualizes that the goal of creating a shortest path is supported by the goals of choosing the search area, sending and receiving information of the down lying regions, storing this information and using it to create the shortest path. In the recursive algorithm, all these goals will be full-filled multiple times.

## 3.4. A Detailed Transportation System Description

The third step of the framework consists of the creation of use cases. Use cases are descriptions of system behaviour as it responds to a request that originates from outside of that system (Wikipedia, Use_Case, 2008). They describe in a narrative way what we want the system to do in what situation. One of the benefits of use cases is that they put requirements in context, describing them in a clear relationship to tasks. For now, we do not make a clear distinction between the different use cases. Rather we will provide a detailed system description, explaining all actions and functions. In this description we sometimes make some side trips, that, strictly speaking, might not be part of the use cases, but that do give a broader understanding of the systems working.

Consider a vehicle at a random starting point A that wants to travel to destination B. After entering this destination the vehicle seeks contact with the nearest RU. This request contains the request for an itinerary to the destination, but also characteristics about the vehicle, the kind of vehicle and whether you want to follow the shortest route or the fastest route. We will call this information (reflected in Appendix D) 'car information.' This information is vital to every procedure and will therefore be sent to every level controller during the process of creating the shortest path.

The vehicle characteristics are important when calculating the fastest route because the engine power, acceleration speed, maximum speed and vehicle weight, among others, will influence the travel time. Of course, these characteristics do not have the same effect on every road. On a highway the maximum speed matters more than the acceleration speed. In this phase of the project we keep the information pretty basic though, but it can easily be made more realistic later. The kind of vehicle is important because not every road is accessible to every kind of vehicle. Trucks are often banned from city centres or mountain passes, while small motorized vehicles might not be allowed on highways. If the road is temporarily unavailable due to weather conditions or other circumstances, the accessibility can be turned off for every type of vehicle.

A RU does not calculate shortest paths. This is done by the controllers, so the RU will immediately pass this message on to the level *1* controller. The controller picks the starting point (A) and starts calculating the shortest path using Pallottino´s algorithm. At this point, every arc is evaluated on accessibility and current travel time based on both vehicle and road characteristics. This evaluation is done by the RU, because this agent has knowledge of the status in real-time, so the controller will receive a travel time and a travel distance.

| «enumeration»<br>**GeneralInformationStorageFile** |
|---|
| +LocationOfOrigin = 0<br>+LocationOfDestination = 1<br>+TravelTime/DistanceToDestination = 2<br>+LvLControllerOfInvestigation = 3<br>+LvlControllerOfOrigin = 4<br>+Optional: TravelTimeToDestination = 5 |
| |

Figure 12: General Information Storage File

For every point (1, in Figure 12) and vehicle not only the travel time is stored, but also the previous point visited (0) to get to that point and, if the RU is a border point between two regions, the name of the region that is not being investigated now (3). This last information has two purposes. First we immediately know what RUs are border points and second we know what the next region is that we want to explore without consulting the particular RU. The region we are exploring now is also stored (4) because we need this information when we want to back-trace our path after finding the destination. Both travel time and distance can be passed because, when calculating the shortest distance path, the time of arrival needs to be stored and thus known in the end. Therefore a place is reserved for travel time (5) in case (2) is travel distance. All information about the passing RUs is thus stored in arrays of the type 'General Information Storage Files:'

If all points are evaluated and the destination is not reached yet, then a selection of nodes has to be made. All the information about nodes that are on the border of two regions will be send to the appropriate higher level controller. This information is always accompanied by the information about the vehicle. Here, the information about the border points (3) is needed, because we can easily separate the border from the non-border points.

The level *2* controller now has information about the travel times to all the borders. It will ask all the regions that have RUs on the border of the already calculated area to start calculating from these known points. The information about the arrival time of the car is passed. The RU requires this information to give a good estimation about the travel time at the moment in time at which the car arrives. Once the RU is selected as the next RU to explore, the LvLControllerOfInvestigation information is altered to indicate that the points are already investigated and thus to prevent calculating from the same points over and over again. When all regions have been calculated and the destination is not found, again the information will be send upwards.

Unlike in the lower levels, where the computational time needed to explore all RUs is still limited, the search in the higher level controllers will continue one region at a time. The next region to be explored will be based on a heuristic approximation of the total distance or time to the destination. So the way the A* method chooses the next node, in the GAT the next region will be selected. Because while it might be profitable on lower levels to drive away from the destination, for instance to prevent crossing a large and busy city centre, on higher levels, province or country sized, the direct approach is most often the best.

The process described above will continue until the highest necessary level (the level *m* controller) that involves both the starting point and destination. As explained above, before sending the information to higher level controllers a sorting procedure is executed.

Although the GATS is a driverless system, this does not mean that there is no human agent inside the vehicle. So we still identify the driver, as the person that wants to drive from starting point to destination. This driver is an important part of the system and influences the requirements. A driver would for instance feel very uneasy driving towards another vehicle, without a clue if his vehicle would slow down (Labiale, 1997). So there is a need to let the driver know what is going to happen. That also means that although normally all information is stored distributed over all level controllers, there has to be a central knowledge of the route as well, at least in the vehicle. Therefore, after creating the shortest path, the entire path will be transferred to the starting RU, which will communicate it back to the vehicle.

Both to visualize this information and to be able to enter a destination into the system, the vehicle should be aware of the entire static system, i.e. the location of the RUs and the present arcs between these RUs. For this reason a system is needed that allows us to quickly find the locations. Since the internet is a distributed system as well with a system that has already proven itself, we propose a similar system for the level controllers in GATS. The Internet uses the so called Domain Name System (DNS) (Wikipedia, Domain_name_system, 2008). In DNS, different domains exist at different levels. Every domain carries, besides its own name, the name of its parents as well. For example '.org' might be the top level domain, having a sub domain 'wikipedia.org,' having another sub domain 'en.wikipedia.org.' In the same manner every level controller name will also contain the names of all its parents.

Although this system makes it very easy to see what is the first level controller shared by both starting point and destination, this is not necessarily the level *m* controller. Only in the case of convex regions can this be guaranteed. A region is said to be convex if every point on the line segment between two points (*x,y*) in this region lies within the region (Wikipedia, Convex_set, 2008). In case of non-convex regions the optimal path might lead through a different level controller of the same level and thus need a higher level *m* controller. Of course a lower level *m* controller is never possible. The example of Figure 13 shows a planned route (Google, 2008) between 2 points in Spain leading through Portugal and thus needing a higher level controller than the one controlling only Spain. Even though a region might look geographically convex, the fact that most often travel times are used for the determination of the used path can make the region lose its convexity.


Figure 13: Concave region

The level *m* controller will calculate the shortest path between the points A and B. After finding the shortest path, all RUs need to be informed of the path that will be taken. Therefore the controllers will pass this message on down until it reaches the RUs, at the same time, the information about the entire path will be sent up, so that it can be sent to the car. Furthermore all stored information about the vehicles possible routes will be deleted from the memory.

## 3.5. Information Flows in the Transportation System

The narrative description of the system is visualized in this phase into the sequence diagrams of Appendix C. These make the information flows visible among the different agents. Any communication between different levels is visible here in the form of an arc between the two regions. From Figure 21 it becomes clear that the calculations start in a small region around the starting point and spread in the direction of the destination with the increase of the number of the level of the controllers. As well it can be seen that the request for information at the level *1* controller triggers the execution of the '*Pallottino algorithm*' and the request for information at the RU triggers the procedure '*return arcs.*' These procedures are only visualised once, but are of course executed every time. The procedures are thoroughly explained in chapter 3.7.

Figure 22 shows the process of informing the RUs of the arrival of a vehicle after the shortest path has been found. After this information has passed through the system, the path will be sent to the vehicle and all stored information will be deleted. This last process has a sequence diagram very similar to that of informing the RUs of the arrival of the vehicle. The main differences are that it´s only a downwards process and it affects all down lying level controllers instead of only the ones on the shortest path. Because of these only minor differences the process isn´t shown in a separate diagram.

## 3.6. Actors in the Transportation System

Both in the system description and in the sequence diagrams from the section above, a lot of actors are already introduced. This section gives a full explanation of all actors, though, and more importantly, it gives an overview of the different roles that the agents can take.

Although there are only few agents that seem to interact in this system, we can define multiple different roles within these seemingly simple agents. The simplest role is that of the RU. This role is responsible for the realization of the goals that have to do with the time or distance measurement and are defined by area A (Appendix B). Normally goals and roles exist one-to-one, however similar or related goals might be combined into one role (Wood & DeLoach, 2000). The RU interacts with the car and the level 1 controller, which we will both define as roles too. Although the car can be used as an agent, in the algorithmic part of the research it hardly plays an important part and is mainly used as an initiator for the processes.

Not every level of controllers needs to be identified as a different role, since most controllers react in the same way. Of course this is inherent to the requirement that the system should be extendable to as many levels as required. The controllers realize all the goals in area B of Appendix B. However there are some differences. The lower level controllers determine the shortest paths by requesting the information about all possible points (current borders of the investigated area) at the same time, while the higher level controllers only calculate the most promising region. The controllers all have to receive and send information to higher and lower level controllers or RUs, except for the level *m* controller, which only communicates with lower level controllers. Most level controllers coordinate with other level controllers, while the level *1* controller also coordinates with RUs. Depending on the route and preferences, a level controller could take any of these roles or multiple at the same time. Therefore we define the above four controller roles, but have them all carried out by the same agent.

When we look at our class structure (Appendix D), obviously we define the different agents as classes, but we also add another: the arcs. These contain the static information about the road network in the system.

## 3.7. Specific Algorithm Tasks and Procedures

At this point the general way of working of the algorithm is clear, as are the agents participating in the system and the system goals. Therefore we can now specify the specific tasks and procedures of which the algorithm is made up. With the tasks and procedure diagrams presented in this section, the code behind the tool of chapter 4. and our algorithm becomes clear. The other way around, we use the tool to clarify the procedures and tasks.

The different tasks that have to be performed are visualized in task diagrams (Appendix E). In the diagrams, the nodes represent the processes, while the arcs represent information flows, information changes or decisions. The tasks are carried out by one single role and thus by one agent. However, multiple tasks can be performed consecutively. When a task has a '*waiting*' process, it indicates that another task is started and that the return of that task is input for the continuation of the task.

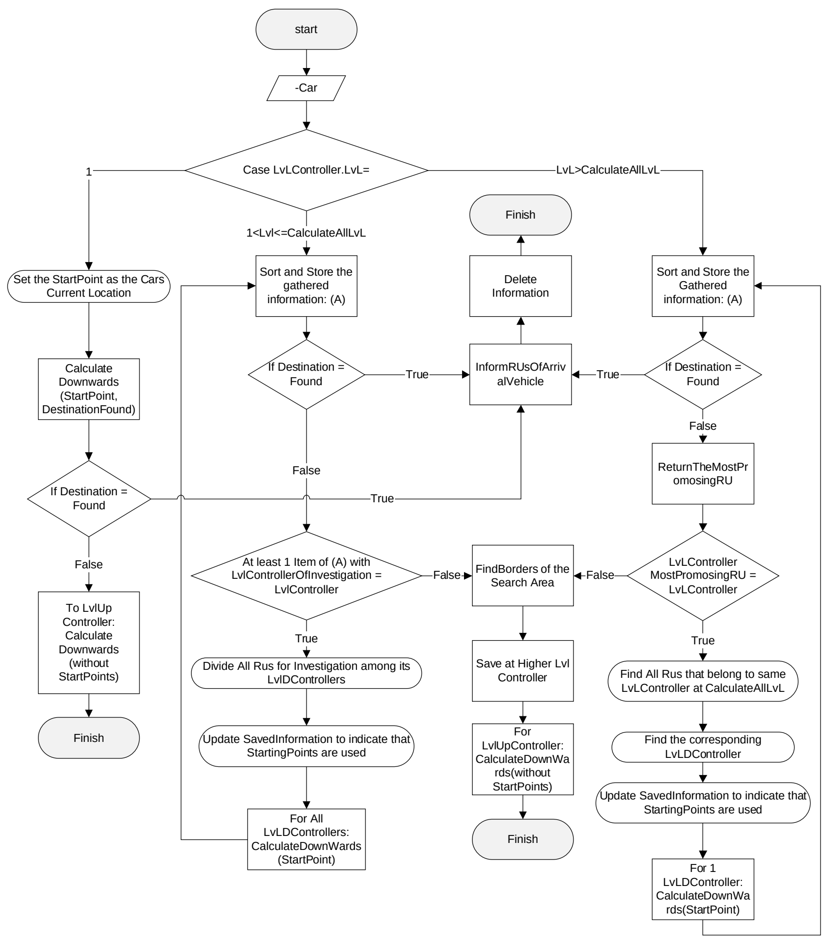
Within the tasks, there are subtle differences between the situations where the tasks are requested by a higher level controller (Figure 26, Figure 28 and Figure 30) or by a lower level controller or RU (Figure 25, Figure 27 and Figure 29). The search area is always expanded by a request to a higher level controller. After that the search is continued to the lower level controllers that return the information. Therefore, the fact that a task of a level controller is requested by a lower level controller means that the executing level controller is the highest level controller at that time and thus possibly the level *m* controller. If it is the level *m* controller the task will end and no other task will be executed, otherwise the task will end by expanding the search area even more. It holds as well that a level controller task that is requested by a higher level controller can neither contain the creation of the ultimate shortest path nor the informing of the RUs of that path. Therefore these tasks always end by returning the border-to-border information to the higher level controllers. Another difference between the two situations is that in case of a request of a lower level controller, there a no starting points yet, while a request from a higher level controller is always accompanied by a set of starting points.

Also it can be noticed that the task will continue until either no more border points of the search area can be found that are within this level controllers region or the destination is found. This last event will only initiate a new procedure (*inform RU of arriving vehicle)* if the task is requested by a lower level controller. The procedures of informing the RU of the arriving vehicle and deleting the information are not represented in task graphs. Although these procedures will trigger tasks at the different level controllers, these tasks are very simple and will be only displayed in the form of a procedure.

As stated, every '*waiting*' process indicates the initiation of another task and tasks can serve as input for other tasks. Therefore we indicate the relationships between the different tasks. In Figure 23 the request for the shortest path (indicated by '*newDestination*') is passed to a level *1* controller. This is the initialization of the task in Figure 25. In Figure 25 and Figure 26 the '*waiting*' process is the start of the task of Figure 24. For Figure 27 to Figure 30 it is not directly certain which task is initiated since it is dependent on the level of the level controller and the place of the border between lower and higher level controllers.

Next we will provide some explanation to the diagrams that describe the different processes or procedures of the algorithm. The procedures go from start to finish and use the input from the parallelogram directly following the start node. The squares are other procedures that are called, while the rounded rectangles represent actions in the procedure itself. The diamonds are conditional nodes, where the options are on the arcs leaving the node.

## Calculate Downwards (without StartingPoints)




The first procedure is called *'calculate downwards without starting points.'* This procedure is the beginning of the main iterative procedure and is always called by the highest level controller involved in the calculations at that moment. The procedure will be different for the different roles of the level controller and we will separate three separate possibilities; level *1,* below the level where we calculate everything and above this level. The last two possibilities might never occur, depending on the height of the level below which we calculate everything and the distance between the starting point and destination.

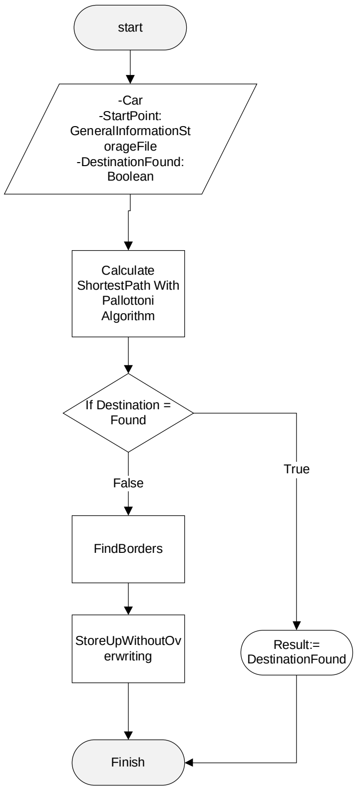
If the level is level *1,* then we set the current location of the vehicle as the starting point and we call the next procedure *'calculate downwards with destination check.'* After the procedure returns and the destination wasn´t in the same region, the search area will be expanded by making the level *2* controller the highest level controller. Otherwise the RUs will be informed of the arrival of the car and the car will be informed of the journey.

If the controller is of a higher level, then the procedure starts by finding the initial starting points. In case of a 'lower-level' controller, these are all the unexplored points within this region. In the case of a 'higher-level' controller, this is the set of points belonging to the region of the most promising RU. However, it is possible that multiple calculations after each other are executed in the same region. Therefore, after the first calculations based on the initial found starting points have finished, a check is added to see whether the next RU that has to be explored belongs to this area. If this is no longer the case, then again the search area is expanded.

Every time calculations are carried out on level controllers higher than level *1,* the saved information will have to be updated to indicate that the border points are used as starting points. Otherwise, the procedure will be appointing the same border points all the time as starting points and will get stuck in a loop.

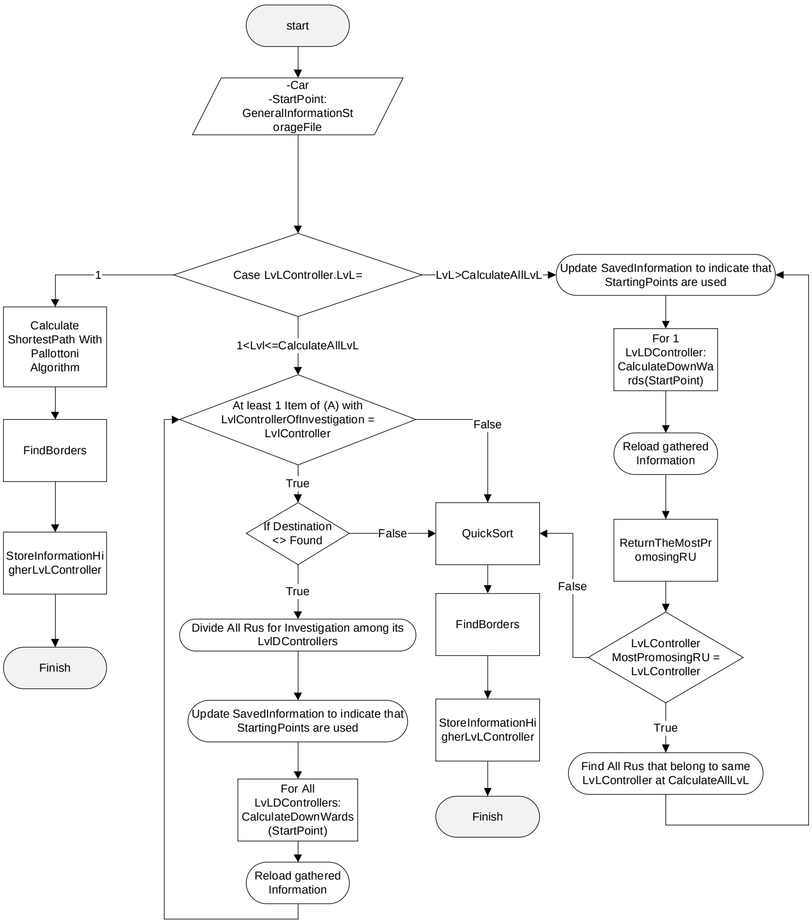
Because this procedure is always executed by the highest level controller in the search area, this controller also has the possibility to calculate the ultimate shortest path after the destination has been found.

## Calculate Downwards (with Destination Check)




The procedure *'calculate downwards with destination check'* is the next procedure. This procedure exists in contrast to the other *'calculate downwards'*-procedures only for the level *1* controller. After calculating the shortest paths through the level *1* region, this procedure decides either to store the starting point to border information at the level *2* controller or to return a message that the destination has been found.

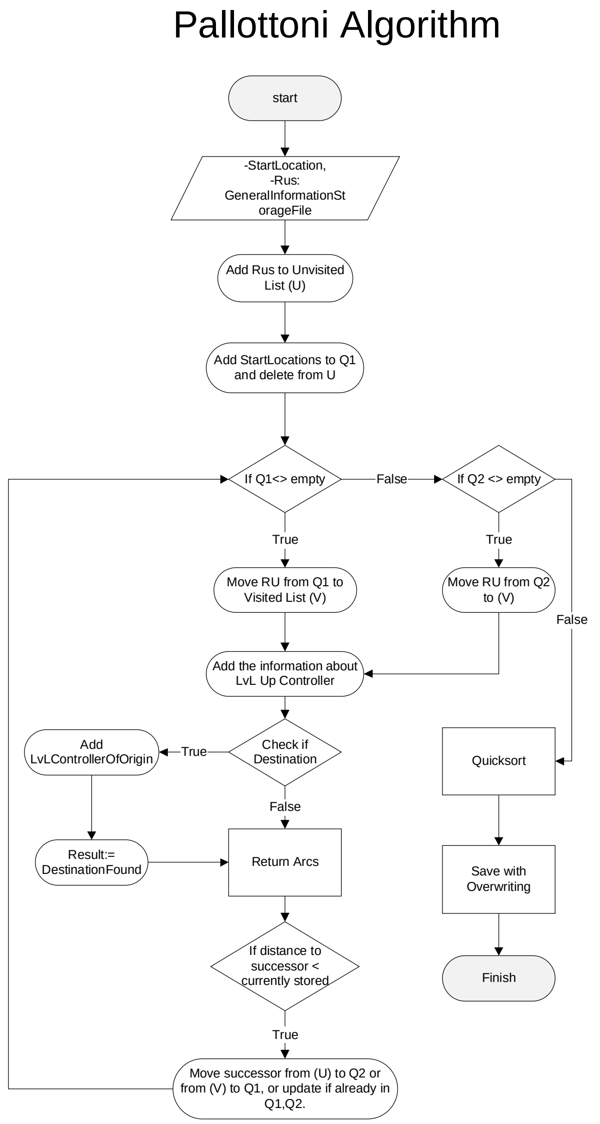
## Calculate Downwards (StartingPoints)



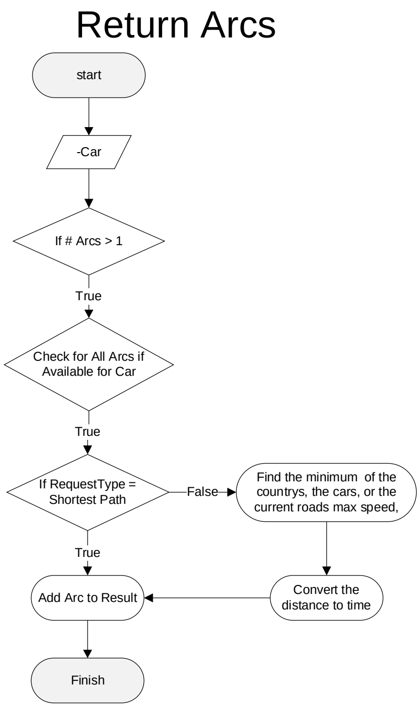

The procedure *'calculate downwards with starting points'* is very similar to the other two. It does not have a destination check at the first level, because it is already known that level *1* is not the highest level if this procedure is called at all. For the higher levels there are three main differences. First, there is no need to do the initial calculations, because the starting points are already provided. Second, since this is never the highest level controller, no ultimate shortest path will be calculated and last, after the discovery of the destination, the current calculations are finished, but no new ones are started.

The real calculation of the shortest path in the level *1* regions is done by the *'Pallottino algorithm'*. This algorithm is a label updating algorithm and uses 2 queues and 2 lists.

First these lists and queues are loaded. The starting points go to the priority queue (Q1), the other RUs go to the list of unvisited items. The distance to these RUs is infinite if this is the first time the region is explored or equal to the found minimum distance in the previous exploration. The other list contains the already checked RUs. When RUs are moved from the queues to the visited items list, the distance to the RU combined with the distance to its successors is compared to the existing distance label of the successors. If the distance is shorter, the label is updated and the RU is moved into a queue, if it is not yet in one. If the RU was in the visited list, it will be added to the priority queue, RUs from the unvisited items list enter Q2. After the procedure is finished, the list is sorted and saved.

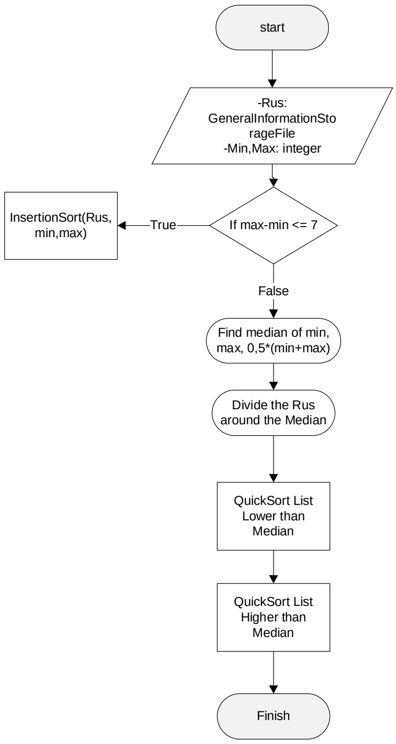



The distance and time to all successors are calculated by the procedure *'return arcs'*, that finds all links and travel distances and converts these into a travel time based on the maximum speed on the arc at that particular time, the maximum speed allowable for that type of vehicle in that region and the maximum speed of the car. It eliminates arcs that are unavailable to the vehicle, either because of congestion or because of structural unavailability to this type of vehicle.

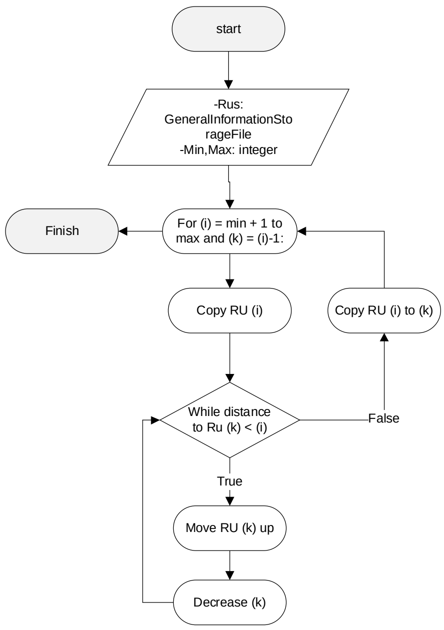



The *'quicksort'* procedure sorts the RUs on distance or travel time. The method is calling itself recursively for smaller sub lists. Therefore the minimum and maximum number of the list are added to indicate the position and size of the sub list. If the length is smaller than 9, the list is sorted by the *'insertion sort'* procedure. If not, the method finds the median of the first, middle and last RU of the list. The items are moved around this number, the higher numbers on the right, the lower on the left.

# Quicksort

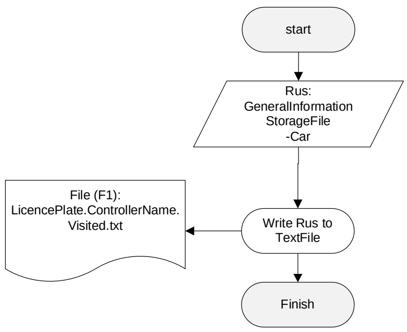

start
-Rus:
GeneralInformationSto
rageFile
-Min,Max: integer
If max-min <= 7
True
InsertionSort(Rus,
min,max)
False
Find median of min,
max, 0,5*(min+max)
Divide the Rus
around the Median
QuickSort List
Lower than
Median
QuickSort List
Higher than
Median
Finish

## Insertionsort



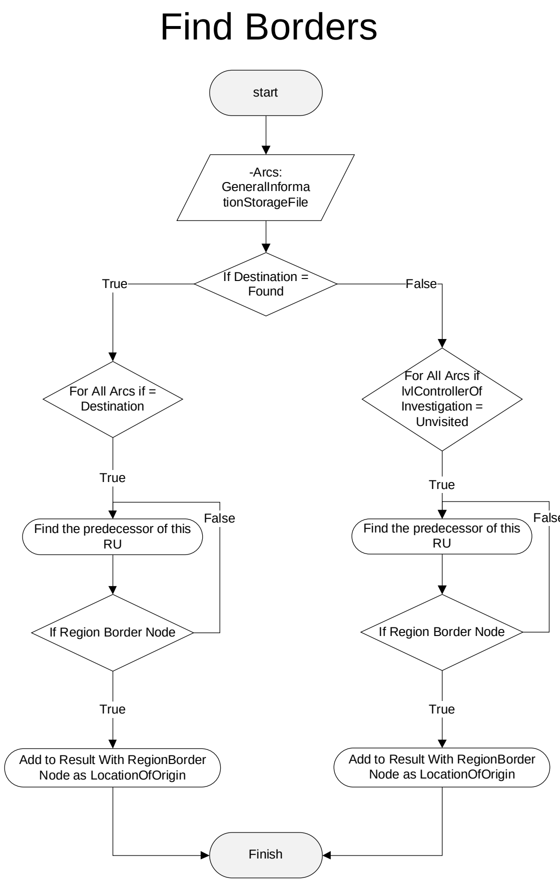

After sorting the RUs, the sorted list is saved. The procedure *'save with overwriting'* is used, because after the *´Pallottino algorithm'* all the RUs are in the list and have the lowest possible value.

## Save with Overwriting



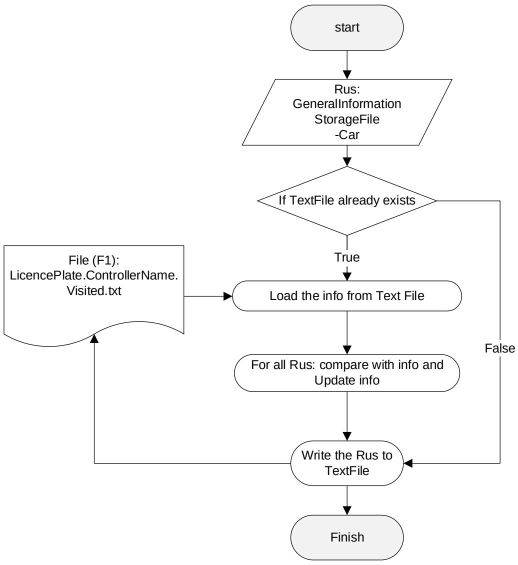

Not all information will be sent to the higher level controllers. Therefore the procedure *'find borders'* selects the proper information. There are two cases that can be separated;

either the destination is found already, or it is not. If the destination is found, only the distance to the destination is valuable information. So this is saved and the ultimate predecessor in this region is back tracked. Otherwise, all RUs that haven´t been investigated yet are selected and combined with their ultimate predecessors.



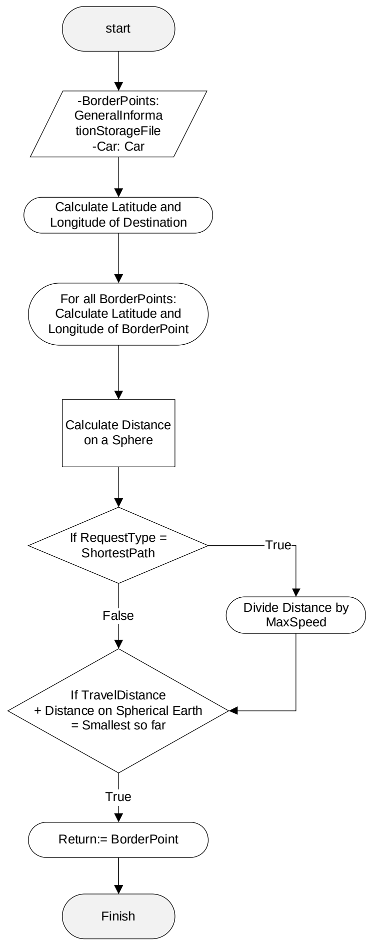

Multiple level down controllers have to save their information at the level up controller. If they would save their information and overwrite old information, most information would get lost. Moreover, because a region might be explored multiple times from different starting points, information might be passed up multiple times. Therefore the procedure *'save without overwriting'* will check whether the RUs are already in the list and adds or updates the RUs.

Save without Overwriting

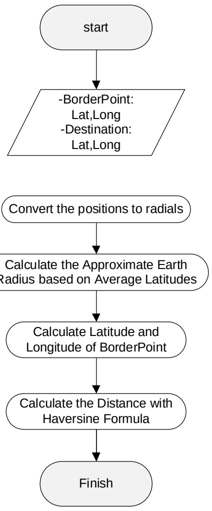



The procedure *'return most promising region'* finds the location of all the unexplored points and the destination in terms of latitude and longitude. Then the procedure *'calculate distance on a sphere'* uses the haversine formula to calculate the distance between the two points. This way it enables *'return most promising region'* to find the most promising region based on the total expected distance.

# ReturnMostPromosing RU

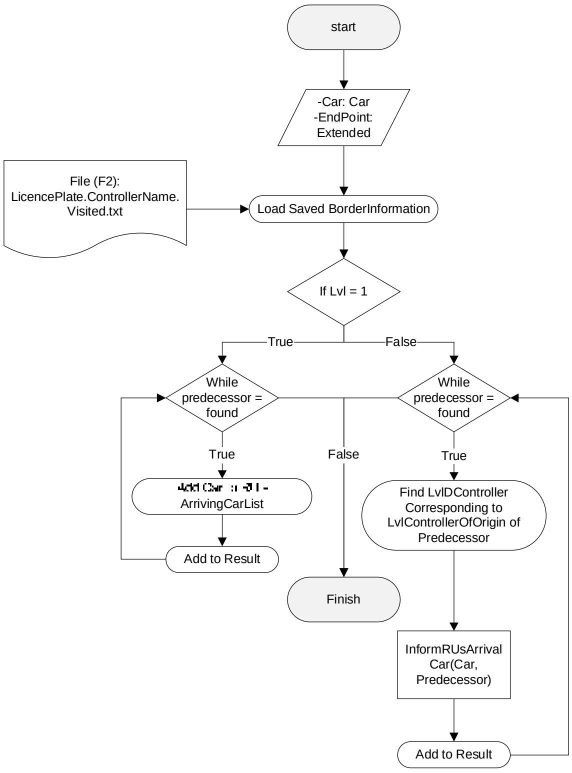


Calculate Distance on a Sphere

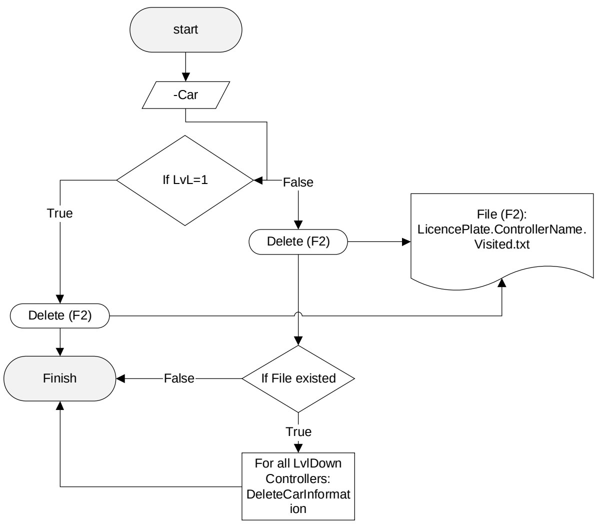



The procedure *'inform RU of arriving vehicle'* is a recursive formula that starts at the destination at the level *m* controller and back tracks the route over every level. It stops at the starting point of a region, because that is the end point for another. The level controller knows what region to use because this is stored in the 'general information storage file.' Then it returns the information so that in the end the level *m* controller is aware of the entire path and can send it to the vehicle. After that there is no more use for the stored information and the procedure *'delete information'* will, surprisingly, delete all information stored by the level controllers.

# Inform RUs of Arrival Car




# Delete Information



# 4. Evaluation of the Algorithm and Tool-design

To test the algorithm we have developed a tool, using Borland Delphi. This tool uses a simple test case based on a small part of Valencia. The first part of this chapter discusses the features of this test case to get some basic knowledge of the tools working. Then we explain the working of the algorithm with the help of an example. In the third part of this chapter we compare our algorithm to the optimal solution calculated by Dijkstra's algorithm. With this comparison we can evaluate our algorithm and make some conclusions about the quality of our algorithm.

## 4.1. Tool for the GAT system

In this part we explain what the major features of the tool and the test case data are. The test case is based on real locations, because the algorithm uses latitudes and longitudes of locations. Therefore we picked an area of points *abcd* (Figure 14). These points mark the boundaries of the highest level controller (a level *3* controller). The highest level controller has four lower level controllers (the controlling area of the first level *2* controller is marked in the left upper corner) and all of these level controllers again have four lower level controllers. This creates the 16 regions in Figure 14. In the tool we represent the regions by numbers counting from left to right, from top to bottom. Since we use an internet-like naming system, the regions inherit the names of their parents. So the region in the top left corner is called 111, where the first 1 stands for the top left position of the level *1* controllers, the second for the top left position of the level *2* controllers and the third because there is just 1 level *3* controller. This means that the name of the bottom right region is called 441. In every region 5 points are randomly picked, creating 80 RUs. However, due to the fact that every border point belongs to two different regions, the number of RUs in the regions will be larger than 5. The small size of the test case creates a ratio of border-to-non-border-points that is much higher than in real situations. Therefore RUs are likely to connect more than 2 regions, which is impossible for this algorithm. To solve this problem we created virtual RUs close to the RUs that had this problem. These virtual RUs will be used to create paths, but can never be a starting point or destination. The lengths of the arcs are calculated based on a small factor combined with the lower bound distance between the two real points, while the travel speed is set equal to 45 for simplicity.

Figure 14: Map Area in Tool

The tool allows us to select different options based on the requirements of the GAT system specified in chapter 3. These requirements can be separated into two categories, routing characteristics and car characteristics. Under routing characteristics one can choose the *CalcAllLevel*. This is the level up to which one considers all down lying points. Above this level, the algorithm will switch to a choice of the next region based on the A* method. There are three possibilities: even the level *1* controllers will be chosen by the A* like method, only the level *2* controllers will be chosen based on this method or none of the controllers will be chosen this way. Furthermore the current location and the destination can be chosen. After entering the locations, the tool will show the two points and a route between the points based on the Google maps functionality. This route doesn´t represent the route chosen by the algorithm because the test case doesn´t follow the real roads, but it allows us to get some feeling about the route. Last, there is a possibility to choose either a fastest routing approach, based on time, or a shortest routing approach, based on distance.

The car characteristics contain the vehicle type, the speed and the acceleration. This is not a very exhaustive representation of a vehicle and definitely not realistic, but it requires the system to account for differences between vehicles and this makes the system easily adaptable to different inputs. After entering the car characteristics, the tool can be initialized, which enters the current preferences into the system and the search can be started. The information that would normally pass back to the vehicle now will be shown in the tool; the name of the passing nodes and the time of arrival at these nodes.

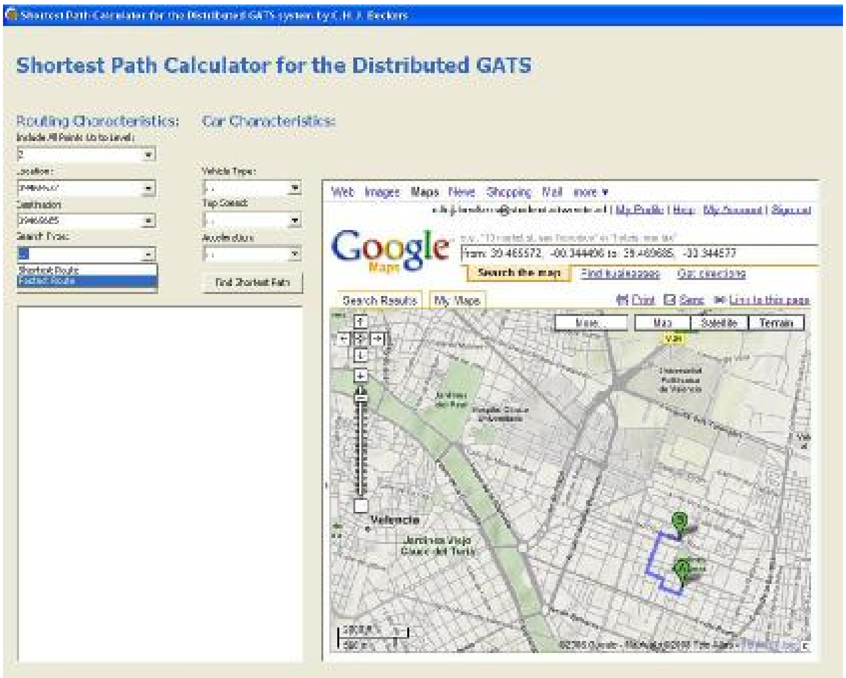



Figure 15: Tool Interface

## 4.2. Example of the Algorithm in the Developed Tool

In this part we follow the algorithm in its calculations of the shortest path between two randomly chosen points of the test case. More than the theoretical description in the previous chapter, this will give insight into the workings of the algorithm. We show at what times the different procedures start working and why certain choices are made. To be able to explain this properly, we present the inputs and outputs to different procedures in table format. From the enormous amounts of information we indicate what is important and notable.

We initiate the system. Let´s assume we´re at starting point *A,* with coordinates (39.465572, -0.344496), which lies in region 431. We want to travel to destination *B* at location (39.473347, -0.339594). The *CalcAllLevel* is set at 2.

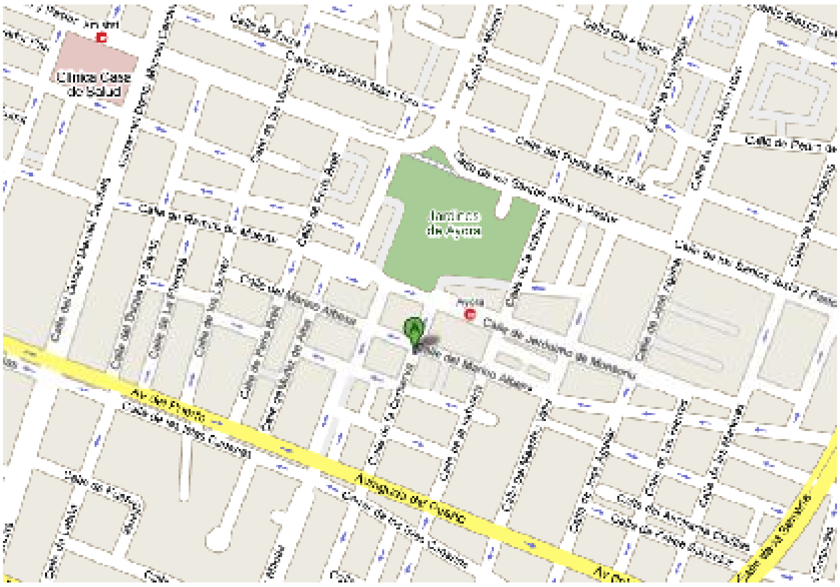


**Figure 16: Starting Point Example**

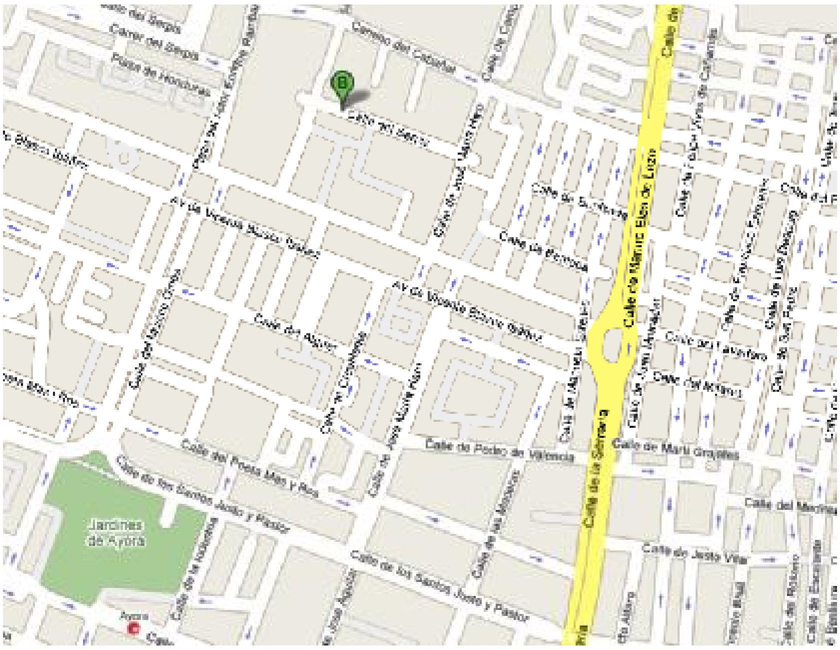



Figure 17: Destination Example

The level controller starts the *'pallottino algorithm'* in the first region. In the initialization of this procedure all RUs from the level controller will be divided amongst the lists. The starting point(s) will go to Q1. In this case only one RU will go to that list (Table 1), our current location *A*. The rest of the RUs are added to the list of unvisited RUs. 7 RUs are now in this list, the distance to each of these is marked as infinite.

| From: | To: | Distance (in minutes): | Region of Investigation: | Region of Origin: |
|---|---|---|---|---|
| (39.465572, -0.344496) | (39.465572, -0.344496) | 0.0000 | -1 | - |

Table 1: Starting Point 431

The algorithm picks the first starting point from Q1, moves it to the visited RU list and checks if this is a border point of the region. In this case, the starting node is not a border point, so we do not have to add any region information. Therefore we immediately start looking for the arcs emanating from this point by calling the procedure *'return arcs'*. We only select the ones that lie within this region. We find that there are four RUs we can add to Q2. We update the distance to these points and now have the following queue:

| From: | To: | Distance (in | Region of | Region |
|---|---|---|---|---|

| | | minutes): | Investigation: | of Origin: |
|---|---|---|---|---|
| (39.465572, -0.344496) | (39.465205, -0.347125) | 0.3149 | - | - |
| (39.465572, -0.344496) | (39.465397, -0.346526) | 0.2408 | - | - |
| (39.465572, -0.344496) | (39.465907, -0.344935) | 0.0728 | - | - |
| (39.465572, -0.344496) | (39.467125, -0.343259) | 0.2783 | - | - |

**Table 2: Q2**

The Q1 is empty, so we have to pick from Q2 now. We pick the first RU from the queue and check if it is a border point, which again it is not. Therefore we move it to the visited list and proceed to find all the emanating arcs:

| From: | To: | Distance (in minutes): | Region of Investigation: | Region of Origin: |
|---|---|---|---|---|
| (39.465205, -0.347125) | (39.465655, -0.349067) | 0.2390 | - | - |
| (39.465205, -0.347125) | (39.466674, -0.347921) | 0.2431 | - | - |
| (39.465205, -0.347125) | (39.465572, -0.344496) | 0.3149 | - | - |
| (39.465205, -0.347125) | (39.465397, -0.346526) | 0.0764 | - | - |
| (39.465205, -0.347125) | (39.465907, -0.344935) | 0.2795 | | |

**Table 3: Arcs from RU**

Note that the third arc is going back to the origin and that the last two arcs are pointing to points that are already in Q2. We add the first two RUs to Q2 and compare the last three on the total distance. Obviously this will lead to the omitting of the path to the origin, but the other paths are less desirable than the already known labels as well. For instance, the distance to (39.465397, -0.346526) through (39.465205, -0.347125) will take 0.3149+0.0764=0.3913, which is larger than the already known distance of 0.2408. So this will give a new queue Q2, with updated distances and locations of origin:

| From: | To: | Distance (in minutes): | Region of Investigation: | Region of Origin: |
|---|---|---|---|---|
| (39.465572, -0.344496) | (39.465397, -0.346526) | 0.2408 | - | - |
| (39.465572, -0.344496) | (39.465907, -0.344935) | 0.0728 | - | - |
| (39.465572, -0.344496) | (39.467125, -0.343259) | 0.2783 | - | - |
| (39.465205, -0.347125) | (39.465655, -0.349067) | 0.5539 | - | - |
| (39.465205, -0.347125) | (39.466674, -0.347921) | 0.5580 | - | - |

**Table 4: Q2**

This process will continue until all RUs in the region have the lowest possible label. Then the information will be stored by the procedure *'store with overwriting.'* The final information will be stored like this:

| From: | To: | Distance (in minutes): | Region of Investigation: | Region of Origin: |
|---|---|---|---|---|
| (39.465572, -0.344496) | (39.465572, -0.344496) | 0.0000 | -1 | - |
| (39.465572, -0.344496) | (39.465907, -0.344935) | 0.0728 | 231 | 431 |
| (39.465572, -0.344496) | (39.465397, -0.346526) | 0.2408 | - | - |
| (39.465572, -0.344496) | (39.467125, -0.343259) | 0.2783 | 231 | 431 |
| (39.465572, -0.344496) | (39.465205, -0.347125) | 0.3149 | - | - |
| (39.465397, -0.346526) | (39.466674, -0.347921) | 0.4958 | 331 | 431 |
| (39.465397, -0.346526) | (39.465655, -0.349067) | 0.5428 | 331 | 431 |
| (39.467125, -0.343259) | (39.468527, -0.341495) | 0.5768 | 141 | 431 |

**Table 5: Saved Information Level 1 Controller 431**

We can see that there was only one starting point in this region (indicated by -1), that there are 2 RUs that have no connection to other regions (indicated by the blanks in the region fields), four RU´s that have a connection to a region that has the same level *2* controller (31) and one with a connection to a region that only has the same level *3* controller (1). The procedure *'find borders'* will filter out all the useful information for the level *2* controller and replace the current origins of the arcs with the ultimate predecessors. In this case, all paths originally start at the starting point *A.* Therefore the following information will be stored at level *2*:

| From: | To: | Distance (in minutes): | Region of Investigation: | Region of Origin: |
|---|---|---|---|---|
| (39.465572, -0.344496) | (39.465572, -0.344496) | 0.0000 | -1 | - |
| (39.465572, -0.344496) | (39.465907, -0.344935) | 0.0728 | 231 | 431 |
| (39.465572, -0.344496) | (39.467125, -0.343259) | 0.2783 | 231 | 431 |
| (39.465572, -0.344496) | (39.466674, -0.347921) | 0.4958 | 331 | 431 |
| (39.465572, -0.344496) | (39.465655, -0.349067) | 0.5428 | 331 | 431 |
| (39.465572, -0.344496) | (39.468527, -0.341495) | 0.5768 | 141 | 431 |

**Table 6: Saved Information Level 2 Controller 31**

The search area is expanded and the level *2* controller (31) is now the highest level controller. Because the level below which we calculate everything is level *2* we select all the border points of this region for the new search. This means we get two sets of starting points. These two sets will normally be calculated at the same time by two different level controllers. However, in our tool they have to be calculated consecutively.

| From: | To: | Distance (in minutes): | Region of Investigation: | Region of Origin: |
|---|---|---|---|---|
| (39.465907, -0.344935) | (39.465907, -0.344935) | 0.0728 | -1 | 431 |
| (39.467125, -0.343259) | (39.467125, -0.343259) | 0.2783 | -1 | 431 |

**Table 7: Starting Points 231**

| From: | To: | Distance (in minutes): | Region of Investigation: | Region of Origin: |
|---|---|---|---|---|
| (39.466674, -0.347921) | (39.466674, -0.347921) | 0.4958 | -1 | 431 |
| (39.465655, -0.349067) | (39.465655, -0.349067) | 0.5428 | -1 | 431 |

**Table 8: Starting Points 331**

The procedure *'calculate downwards with starting points'* will be executed for both regions. When the regions *231* and *331* will execute the *'Pallottino algorithm,'* we will know a distance to all the reachable RUs in those regions. Here it becomes clear why it is useful to have a label-correcting algorithm. Labels to RUs in the regions will differ based on which of the 2 RUs we take as starting point. It might even be faster to go through one starting point to the other. The label-correcting algorithm can update the labels every time a better solution is found and therefore we only have to do the calculations once, instead of once for every starting point. The results are saved and the border to border information is sent up to the level *2* controller:

| From: | To: | Distance (in minutes): | Region of Investigation: | Region of Origin: |
|---|---|---|---|---|
| (39.465572, -0.344496) | (39.465572, -0.344496) | 0.0000 | -1 | - |
| (39.465572, -0.344496) | (39.465907, -0.344935) | 0.0728 | -1 | 431 |
| (39.465572, -0.344496) | (39.467125, -0.343259) | 0.2783 | -1 | 431 |
| (39.465907, -0.344935) | (39.467887, -0.346222) | 0.4110 | 331 | 231 |
| (39.467125, -0.343259) | (39.468006, -0.343266) | 0.4128 | 411 | 231 |
| (39.467125, -0.343259) | (39.468006, -0.343267) | 0.4128 | 141 | 231 |
| (39.465572, -0.344496) | (39.466674, -0.347921) | 0.4958 | -1 | 431 |
| (39.465907, -0.344935) | (39.468206, -0.347218) | 0.5381 | 331 | 231 |
| (39.465572, -0.344496) | (39.465655, -0.349067) | 0.5428 | -1 | 431 |
| (39.465572, -0.344496) | (39.468527, -0.341495) | 0.5768 | 141 | 431 |
| (39.467125, -0.343259) | (39.469055, -0.344503) | 0.6075 | 411 | 231 |
| (39.467125, -0.343259) | (39.469685, -0.344577) | 0.7040 | 411 | 231 |
| (39.466674, -0.347921) | (39.468556, -0.350578) | 0.9534 | 131 | 331 |
| (39.466674, -0.347921) | (39.468885, -0.350398) | 0.9926 | 131 | 331 |
| (39.466674, -0.347921) | (39.468726, -0.351005) | 1.0006 | 131 | 331 |
| (39.466674, -0.347921) | (39.468557, -0.351875) | 1.0503 | 131 | 331 |

**Table 9: Saved Information Level 2 Controller 31**

There are a few notable things in Table 9. There are for instance still RUs that have the just explored region 331 as RegionOfInvestigation (rows 4 and 8). This indicates that there was a faster way to the border point of region *331* through region *231* instead of directly from region *431*. This result is another argument for the label-correcting algorithm, because region *331* will now be investigated again. Instead of creating a path from scratch, all labels already have a reasonable upper bound. Also, we can see the 'virtual' border points we created for this test case. Rows 5 and 6 have an almost similar location, similar distance, but a different RegionOfInvestigation. If we would not have used the 'virtual' points, the real RU would need multiple RegionOfInvestigations.

The iterative procedure continues in the same region since there are still RUs within region 31 that have not been explored. We use the starting points in Table 10 and Table 11. Once these are all explored, the procedure 'find borders' again filters the individual results of the level *1* controllers. The information will be sent to the level 2 controller and another time, this controller will check if there are RUs left to investigate within its region of control.

| From: | To: | Distance (in minutes): | Region of Investigation: | Region of Origin: |
|---|---|---|---|---|
| (39.465907, -0.344935) | (39.467887, -0.346222) | 0.4110 | -1 | 231 |
| (39.465907, -0.344935) | (39.468206, -0.347218) | 0.5381 | -1 | 231 |

**Table 10: Starting Points 331**

| From: | To: | Distance (in minutes): | Region of Investigation: | Region of Origin: |
|---|---|---|---|---|
| (39.466674, -0.347921) | (39.468556, -0.350578) | 0.9534 | -1 | 331 |
| (39.466674, -0.347921) | (39.468885, -0.350398) | 0.9926 | -1 | 331 |
| (39.466674, -0.347921) | (39.468726, -0.351005) | 1.0006 | -1 | 331 |
| (39.466674, -0.347921) | (39.468557, -0.351875) | 1.0503 | -1 | 331 |

**Table 11: Starting Points 131**

As it turns out, no more RUs are left for investigation and the search area must be expanded to the entire level *3* controller region. This means that the border is crossed between the two levels of searching and we will start using A* for the choice of the next level *2* controller to explore. The information stored at the moment at the level *3* controller is the distance from the starting point to all of the border points of the last search area (Table 12).

| From: | To: | Distance (in minutes): | Region of Investigation: | Region of Origin: |
|---|---|---|---|---|
| (39.465572, -0.344496) | (39.468006, -0.343266) | 0.4128 | 411 | 231 |
| (39.465572, -0.344496) | (39.468006, -0.343267) | 0.4128 | 141 | 231 |
| (39.465572, -0.344496) | (39.468527, -0.341495) | 0.5768 | 141 | 431 |
| (39.465572, -0.344496) | (39.469055, -0.344503) | 0.6075 | 411 | 231 |
| (39.465572, -0.344496) | (39.469685, -0.344577) | 0.7040 | 411 | 231 |
| (39.465572, -0.344496) | (39.469825, -0.351300) | 1.1650 | 311 | 131 |

**Table 12: Saved Information Level 3 Controller 1**

Now the procedure *'return most promising region'* is executed to find out which of the two possible level *2* regions (11 and 41) will be explored. From every RU the distance to the destination is calculated and converted to a lower bound on the time left to travel. This value combined with the known value of the distance to this RU will determine the best next region to investigate. Table 13 shows that the RU through which the path with the lowest expected total distance goes, (39.468006, -0.343266) is. Therefore the next level controller that will execute the procedure, will be *11*.

| RU: | Known Distance (in minutes): | Region of Invest: | Estimated Distance (in km) | Maximum Speed | Estimated Distance (in minutes) | Estimated Total Distance (in minutes) |
|---|---|---|---|---|---|---|
| (39.468006, -0.343266) | 0.4128 | 411 | 0.6722 | 130 | 0.3102 | 0.7230 |
| (39.468006, -0.343267) | 0.4128 | 141 | 0.6722 | 130 | 0.3102 | 0.7230 |
| (39.468527, -0.341495) | 0.5768 | 141 | 0.5601 | 130 | 0.2585 | 0.8353 |
| (39.469055, -0.344503) | 0.6075 | 411 | 0.6365 | 130 | 0.2938 | 0.9013 |
| (39.469685, -0.344577) | 0.7040 | 411 | 0.5904 | 130 | 0.2725 | 0.9765 |
| (39.469825, -0.351300) | 1.1650 | 311 | 1.0782 | 130 | 0.4976 | 1.6626 |

**Table 13: A*-type Calculations**

The procedures explained above will continue until the destination has been found. At this point all the necessary information is gathered at the level *m* controller to be able to back track the shortest path. Table 14 shows the information present at the level *3* controller after the destination has been found.

| From: | To: | Distance (in minutes): | Region of Investigation: | Region of Origin: |
|---|---|---|---|---|
| (39.465572, -0.344496) | (39.468006, -0.343266) | 0.4128 | -1 | 231 |
| (39.465572, -0.344496) | (39.468006, -0.343267) | 0.4128 | -1 | 231 |
| (39.465572, -0.344496) | (39.468527, -0.341495) | 0.5768 | -1 | 431 |
| (39.465572, -0.344496) | (39.469055, -0.344503) | 0.6075 | -1 | 231 |
| (39.465572, -0.344496) | (39.469685, -0.344577) | 0.7040 | -1 | 231 |
| (39.468006, -0.343266) | (39.470017, -0.343148) | 0.7202 | -1 | 411 |
| (39.468527, -0.341495) | (39.468892, -0.339008) | 0.8751 | -1 | 141 |
| (39.468527, -0.341495) | (39.470691, -0.340895) | 0.9147 | -1 | 141 |
| (39.469055, -0.344503) | (39.468527, -0.341494) | 1.0198 | -1 | 411 |
| (39.468527, -0.341495) | (39.469882, -0.338541) | 1.0359 | -1 | 141 |
| (39.468527, -0.341495) | (39.469366, -0.337773) | 1.0376 | -1 | 241 |
| (39.465572, -0.344496) | (39.469825, -0.351300) | 1.1650 | -1 | 131 |
| (39.468527, -0.341495) | (39.469344, -0.336387) | 1.1916 | -1 | 241 |
| (39.468527, -0.341495) | (39.471682, -0.337485) | 1.3377 | -1 | 141 |
| (39.470691, -0.340895) | (39.473347, -0.339594) | 1.3654 | -2 | 121 |
| (39.468527, -0.341495) | (39.471329, -0.336455) | 1.3665 | -1 | 141 |
| (39.468527, -0.341495) | (39.471329, -0.336456) | 1.3752 | -1 | 241 |
| (39.469055, -0.344503) | (39.474073, -0.342759) | 1.6847 | -1 | 211 |
| (39.469055, -0.344503) | (39.472406, -0.342112) | 1.9504 | -1 | 211 |

**Table 14: Saved Information Level 3 Controller 1**

The destination is the RU indicated with the *-2* as the RegionOfInvestigation. Since a destination doesn't need to be a border point, the RegionOfOrigin is added to support the back tracking. The back tracking in itself is simple and done by the procedure '*inform RU of arriving vehicle.'* It starts at the destination and looks for the RegionOfOrigin. This is the region of the destination. The level *m* controller contacts the level controller of the destination region and, starting from the destination, it keeps looking for predecessors of the destination on the shortest path. When there are no predecessors left, the procedure returns to a higher level controller. The lower the level of the level controller, the more detailed the stored information is. This is one of the characteristics of the hierarchical system architecture. In the end, the entire path should be known by the vehicle, though. Therefore, when the procedure returns to a higher level controller, the path known until that point, will be returned as well. Once the level *m* controller is reached again, it finds the predecessor of the destination. In this case, this one can be found in row 8 of Table 14. Then the same steps are taken, but now for the RegionOfOrigin *141.* In the end, the entire path will be known by the level *m* controller and this one will send it to the vehicle. The following path is found to be shortest:

| From: | To: | Minutes: | Seconds: |
|---|---|---|---|
| | (39.465572, -0.344496) | 00 | 00 |
| (39.465572, -0.344496) | (39.467125, -0.343259) | 00 | 17 |
| (39.467125, -0.343259) | (39.468527, -0.341495) | 00 | 35 |
| (39.468527, -0.341495) | (39.468527, -0.341494) | 00 | 35 |
| (39.468527, -0.341494) | (39.470691, -0.340895) | 00 | 55 |
| (39.470691, -0.340895) | (39.471835, -0.339776) | 01 | 08 |
| (39.471835, -0.339776) | (39.473347, -0.339594) | 01 | 22 |

**Table 15: Shortest Path**

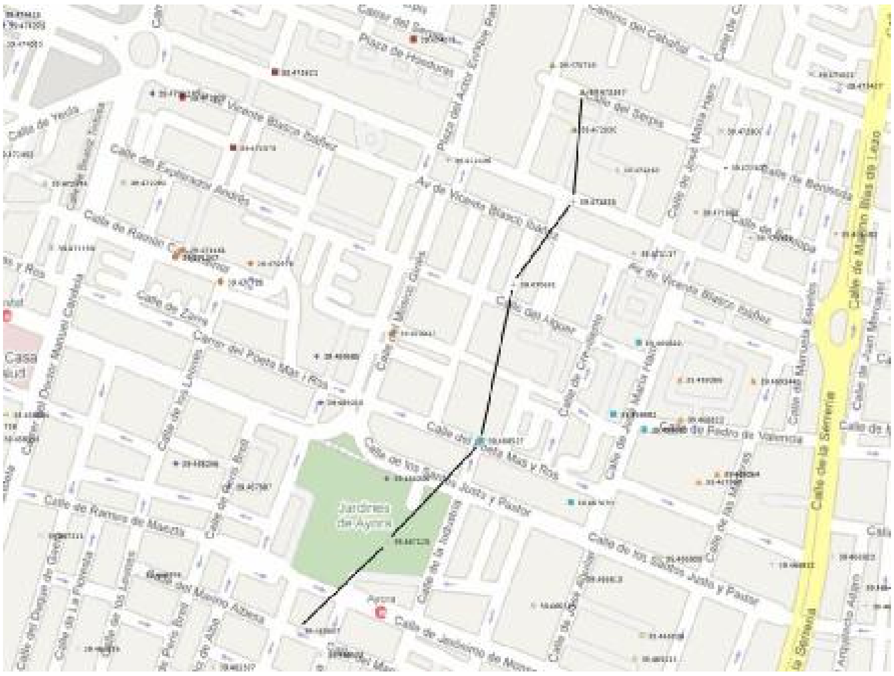



**Figure 18: Shortest Path**

## 4.3. Comparing the Algorithm to the Optimal Solution

Now that we have both some theoretical and some practical feeling for the working of the algorithm, we take a look at the quality of the algorithm. Although we cannot say anything about the calculation time or the complexity of the algorithm, we can compare the results to the optimal solution found by the simple Dijkstra algorithm. Therefore we set-up an experiment in which we calculate the shortest paths from all points to all other points. For simplicity, we only look at these combinations in one direction, thus reviewing half of all the possible paths. This gives a total of 80*79*½=3160 paths. Because the CalcAllLevel determines what part of the area is searched, we calculate these 3160 paths for all three CalcAllLevels. When discussing the results, we first focus on the general quality of the algorithm. Later, we discuss specific paths with reasons for the sub optimality of the algorithm and worst case solutions.

| | **CalcAll={1 }** | **CalcAll={2 }** | **CalcAll={3 }** |
|---|---|---|---|
| **Percentage optimal** | 53,61% | 70,54% | 69,56% |
| **Average algorithm-to-optimality ratio** | 110,26% | 105,86% | 106,45% |
| **Conditional average algorithm-to-optimality ratio** | 122,12% | 119,88% | 121,20% |

**Table 16: comparison Algorithm to Optimal**

First we look at how often our algorithm gives the optimal solution. In table 16 we see the results of the experiment. In the best case (CalcAllLevel *2*) about 70% of the calculations results in the optimal solution. The next question is, what on average the quality of the solution in comparison to the optimal solution is. In order to give an answer to this question we calculate 2 ratios. First the average algorithm-to-optimality ratio. This ratio is simply the average of the result of our algorithms calculation divided by the result of Dijkstras algorithms calculations. We see that on average (again for a CalcAllLevel of 2) the results of our algorithm are almost 6% worse than the results of Dijkstras algorithm. Since more than 70% of the results is actually exactly the same as the result of Dijkstras algorithm, it is perhaps more useful to ask what the value of this ratio is, given the fact that the result is not optimal. This is the conditional average algorithm-to-optimality ratio. This ratio is for all CalcAllLevels pretty much the same. The algorithm performs about 20% worse than Dijkstras algorithm in case it is not optimal. Figure 19 gives a more detailed view of the distribution. The first column represents the percentage of paths that are calculated to optimality. Every next column represents the percentage of results that was less than 5% worse than the previous column. So approximately 11% of the results is less than 5% worse, but not equal to the optimal solution.

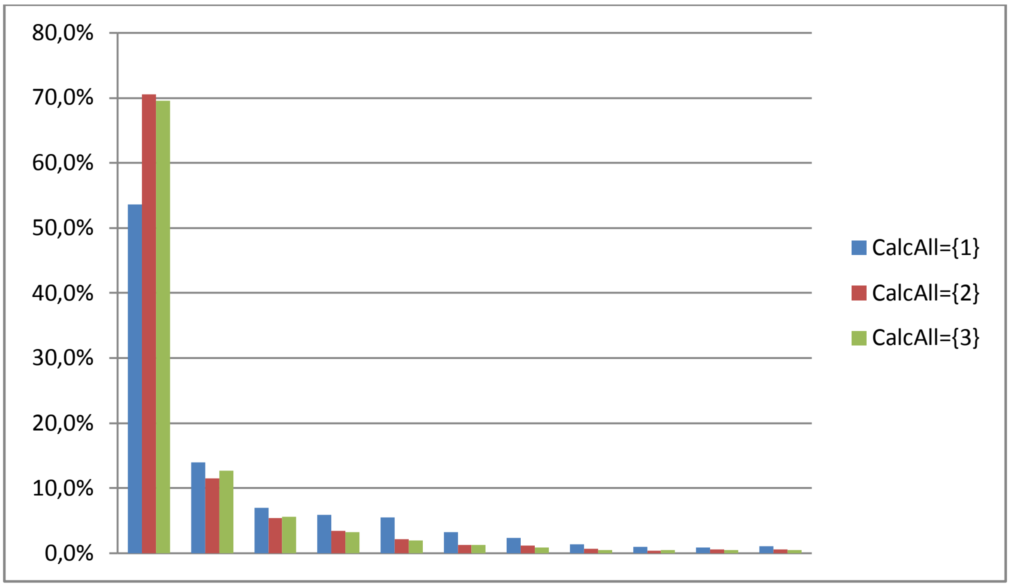



Figure 19: Distribution of Algorithm Performance

The lower the CalcAllLevel, the faster the algorithm turns to the A*-like algorithm and the more the algorithm cuts of the solution space. However, as opposed to the A* algorithm, our heuristic doesn't guaranty optimality, only the limiting of the search area. Therefore we can expect the algorithm to perform better for higher CalcAllLevels. The CalcAllLevel thus is a trade-off between solution space and quality. And indeed we see that going from level *1* to level *2* increases the number of optimal solutions with as much as 20%, while simultaneously decreasing the ratios. From Table 17 it can be seen that, although for CalcAllLevel *1* the result is still sub optimal, for the higher CalcAllLevels the results improve.

| Starting Point | Destination | Optimal | CalcAll={1} | CalcAll={2} | CalcAll={3} |
|---|---|---|---|---|---|
| (39.466245, -0.340522) | (39.467957, -0.337415) | 0,451184 | 0,566487 | 0,451184 | 0,451184 |

**Table 17: Example of Quality Increasing with CalcAllLevel**


Figure 20: Optimal (Left) versus Suboptimal Solution

The sub optimal solution is actually the result of multiple algorithm characteristics. After the first regions calculations (in case of CalcAllLevel *1*) immediately the A*-like algorithm is used. Furthermore, the estimation of the distance is quite a lot smaller than the real distance. Therefore, the choice will always be made for RUs that are further away from the destination. In this specific example, this leads to the investigation of a region that is not used in the shortest path. However, the fact that a RU can be part of two different regions when it is a border point, creates a path from starting point to destination anyway. At the level *m* controller, the algorithm is programmed to finish the calculations as soon as there is a path from starting point to destination. These characteristics combined give the sub optimal solution in the previous example. Therefore we decide to alter the algorithm slightly. The new algorithm will only finish at the level *m* controller once there are no more border points within its region that it wants to investigate. This should give improved results at the cost of calculation time.

| | **CalcAll={1 }** | **CalcAll={2 }** | **CalcAll={3 }** |
|---|---|---|---|
| **Percentage optimal** | 65,92% | 73,73% | 72,66% |
| **Average algorithm-to-optimality ratio** | 105,67% | 105,06% | 105,11% |
| **Conditional average algorithm-to-optimality ratio** | 116,63% | 119,28% | 118,71% |

Table 18: Comparison Algorithm to Optimal 2

Indeed we see that this change gives an improvement both in the percentage of optimal results and in the deviation of our algorithm results in comparison to the optimal solution. Still, even in the improved version of the algorithm, the result for the CalcAllLevel *2* is more often optimal than the result for the CalcAllLevel *3.* We check how often a lower CalcAllLevel produces better results than a higher CalcAllLevel. From Table 19 we see that this actually occurs quite a lot. Especially the CalcAllLevel *3* performs very bad. This is not what we expected, since we only anticipated improvements at higher levels.

| | **1>2** | **2>3** | **1&2>3** |
|---|---|---|---|
| **Algorithm** | 2,82% | 4,87% | 2,85% |
| **Improved Algorithm** | 0,54% | 1,33% | 0,51% |

**Table 19: CalcAllLevels compared**

The tool and the test case presented in this chapter gave us the opportunity to review the algorithm. As it turned out, the characteristics of the algorithm made sub optimal solutions quite common. A short evaluation provided opportunities for improvement, although even the improved algorithm didn't always perform as expected. Moreover, still almost 30% of all paths are sub optimal and more extensive research has to be conducted to find the nature of these non-anticipated results.

# 5. Towards a Dynamic Algorithm

The algorithm we propose is for the relatively simple static situation. More challenges arise when creating proper rules for the dynamic situation. Especially because there is the need to control many vehicles at the same time, the question whether to reroute or not cannot be answered too often. In this part we give some topics on which future research could focus. A sort of brainstorm into the possibilities of the dynamic algorithm, based on the experiences with the static algorithm. The ideas in this chapter should thus not be viewed as perfect solutions to the problem, but rather as possibilities for future researchers to explore these areas.

The time dependency of the system can be solved relatively easy. There are different types of methods that can be used to predict the travel time over an arc, but the so called BPR (bureau of public roads, creator of the functions) functions are probably the most used and have the following form:

$$T_f = T_o * \left(1 + \alpha * \left(\frac{V}{C}\right)^{\beta}\right)$$

The final travel time ($T_f$) is composed of the free flow time ($T_o$) multiplied by a factor based on the number of cars ($V$) and the capacity of the road ($C$). The coefficients $\alpha$ and $\beta$ often have values of around 0.15 and 4 respectively and control the shape of the function (Spiess, 1990). Since the number of cars arriving at any moment is stored at the RU and the distributed system requires an information exchange at every calculation, the variables are easily filled in.

Difficulties arise however because these formulas assume one steady state travel time, while in reality you would calculate different times for every new request. The sequence of cars arriving at the RU is not necessarily the same as the sequence in which the cars were planned. Therefore, if a vehicle arrives earlier than expected, the travel time over the arc will always be shorter than expected too, while a vehicle that arrives later than expected will always take a longer time. This thus will lead to an increase of the difference between planning and realization. Too minimize this effect the coefficients $\alpha$ and $\beta$ should be chosen such that the differences in travel times between situations with a different number of vehicles on the arc are smallest. Luckily, time penalties on busy roads will be much smaller in automated systems than in non-automated systems, which will justify this decision.

The second problem in the dynamic situation is stochasticity. If the travel time would just be stochastic and not time-varying, then the calculation of the shortest path based on the expected values of the travel times is usually considered the best approach (Miller-Hooks & Mahmassani, 2000). However, imagine the case of an accident that has blocked the road. Emergency services are working to clear the area but no one knows for sure when they will finish. At the time of an arriving car, there is only a 10% chance that the road is cleared again and the traffic is moving at its normal speed of 130 km/h. This will create an expected travel speed of 13 km/h.

The main reason we cannot search bi-directional is that we need the travel time to a RU to know the speed at which to travel an RU, because the system is time-varying. There is no possibility however, that you will arrive at the next RU with a speed of 13 km/h. Therefore it doesn't make any sense to use this speed. Assuming we are arriving at the next RU, we arrive there with a speed of 130 km/h. This is the conditional expected value under the condition that you can drive and is therefore the most useful. Most likely in this case, the road will not be available at all, though. The next question should thus be whether to use the conditional expected value in the first place, or not. A possibility might be to use a personal risk acceptance level that every driver can enter. If the probability that the vehicle cannot enter the road is larger than the risk acceptance level, then the road will be regarded as completely unavailable to that vehicle.

In the two situations described above, we have two major and obvious risks, i.e. that the difference between the planning and realization become too large and that the road will be unavailable anyway. In these cases we need rerouting. We reviewed the topic of rerouting in chapter two and concluded that most systems found in literature reconsider the path at every node. Since we expect that to be too time consuming for the GAT system, we propose four other events to initiate the rerouting in the GATS algorithm.

- Incident Initiated Rerouting

If disturbances arise in the system it can be possible that arcs are (partially) inaccessible. After a sudden car breakdown the road might be blocked for 30 minutes or with heavy weather the maximum speed might be 10 km/h lower, reducing the capacity of the road. The system, knowing what cars are arriving in that time period, might inform the cars that are not able to pass the arc anymore of the situation and initiate rerouting.

- Vehicle Initiated Rerouting

The route cannot be planned to the second. Therefore some approximation about arrival times will take place, both when requesting the travel time over an arc and when storing the arrival information. The realization of the route will thus slightly differ from the schedule. Accordingly there is a possibility that vehicles arrive in batches instead of evenly spread. This will lower the handling speed of the system and might lead to vehicles being behind of schedule. On some roads the system might be able to compensate this, but on others the delay might only grow. At some point the old schedule will differ so much from the current situation that the vehicle is interfering with the schedules of other vehicles. Therefore at some point, when behind schedule, the vehicle should initiate rerouting.

- Location Initiated Rerouting

The equilibrium assignment models in the previous section give one possible reason to reroute based on location. The most involving methods review the system at every node for updated information and optimal decisions. Other reasons can be thought of, though. In the case of a sudden car-breakdown the road might be blocked for approximately 30 minutes. A vehicle will arrive there in 32 minutes, but investigation shows that rerouting will add 15 minutes to the total travel time. The new route would deviate from the old at the last intersection before the roadblock. Since there is a probability that the road might be cleared in time and postponing decisions will lead to better choices, the arrival at the intersection might initiate reevaluating. This will either lead to a continuation of the old road or the choice of deviating.

- Time Initiated Rerouting.

In the least adaptive equilibrium assignment models no rerouting is considered. And although the system might initiate rerouting if things get worse, no action is taken if things get better. Imagine the situation that a path is selected based on the assumption that a road would be inaccessible all day due to a crash. Later it turns out that the road can be used after all. This information will only become available to the vehicles after a new routing request. To overcome this problem, a timer might initiate a recalculation after a fixed period since the last calculation.

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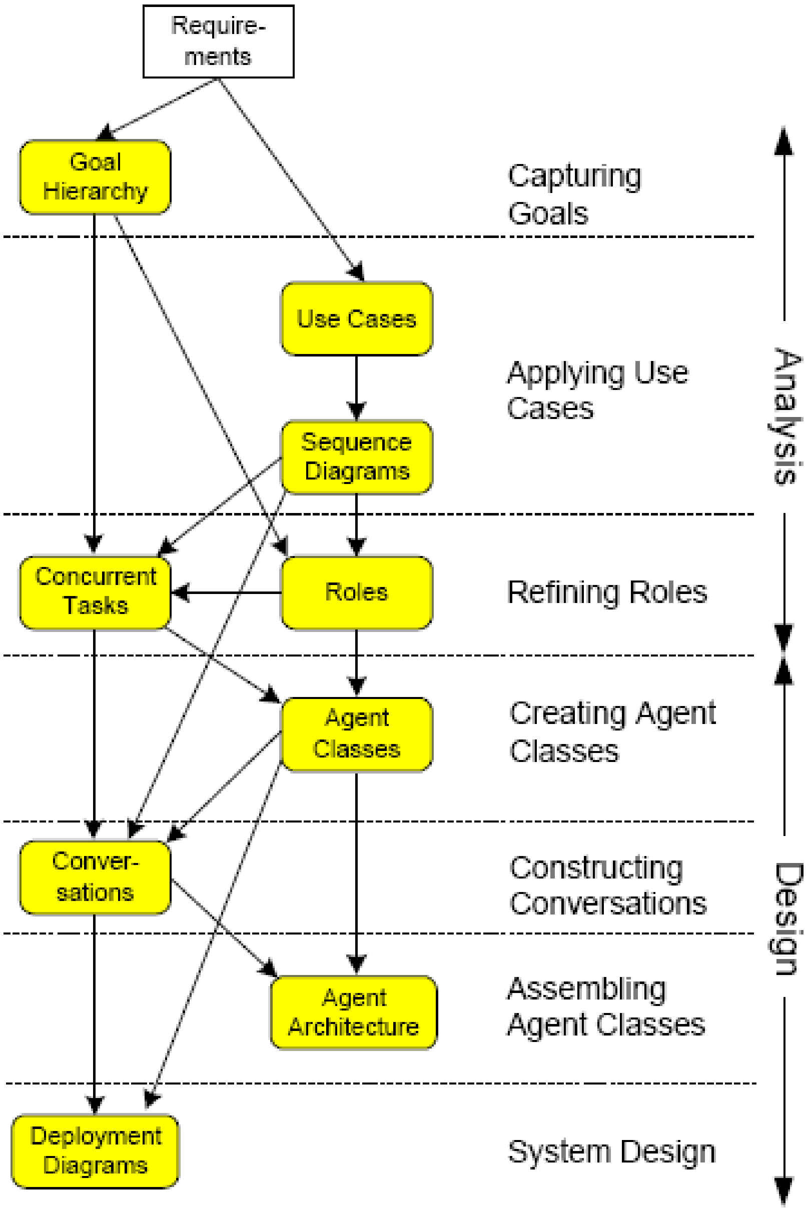
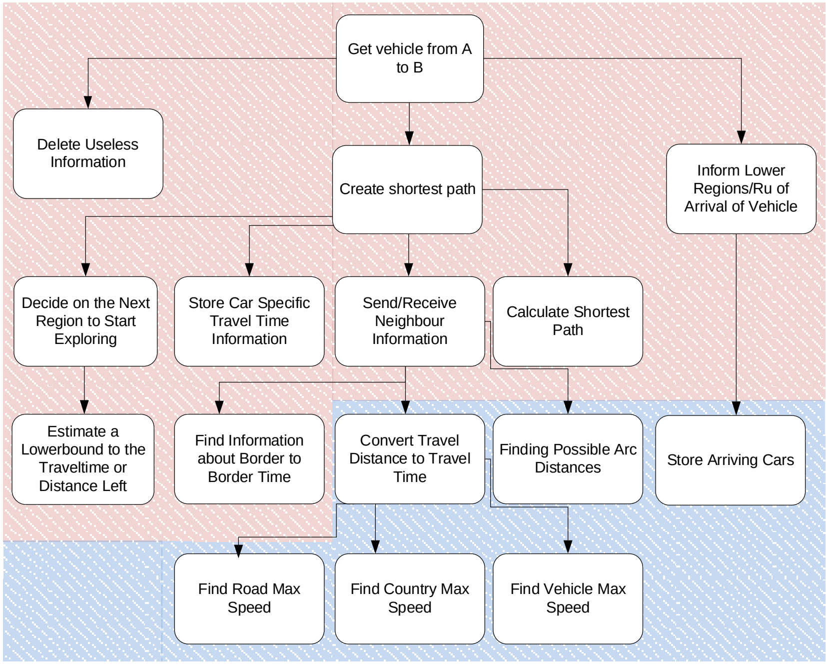
# Appendices

## A. MaSE Framework



## B. Static Distributed Algorithm Goals



## C. Static Distributed Algorithm Sequence Diagrams

Find Nearest Neighbour with A and B in same level *m* region until the destination is found

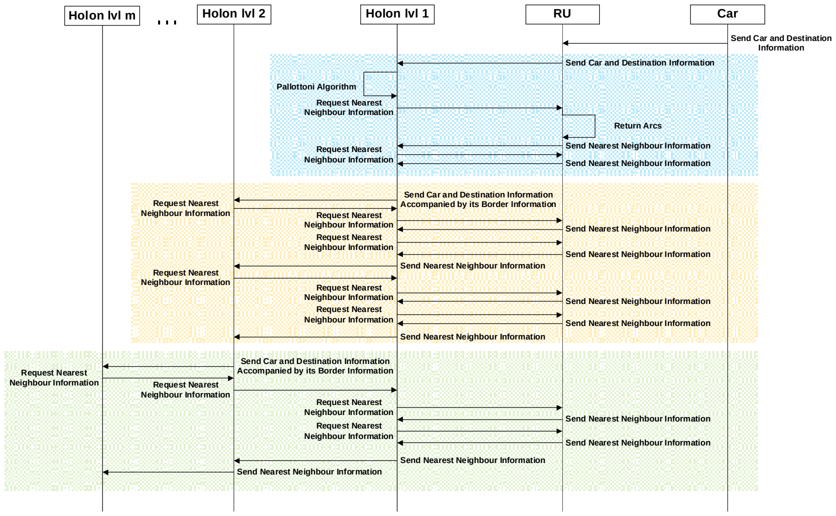



Figure 21: Sequence diagram 1

Inform the Rus of the Arrival of a Vehicle

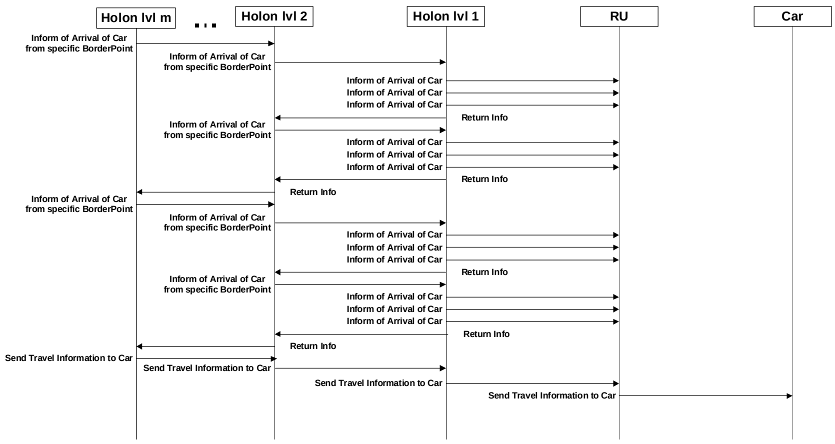
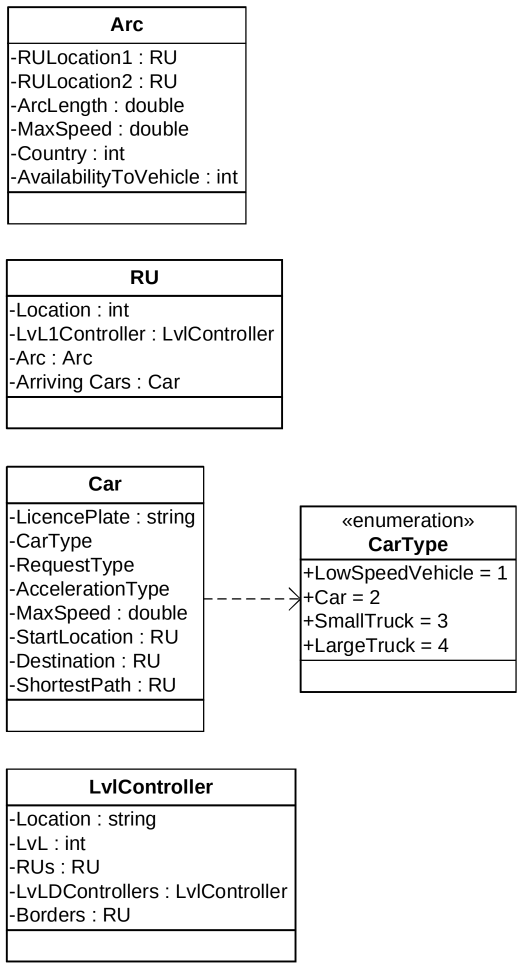



Figure 22: Sequence Diagram 2

## D. Static Distributed Algorithm Classes



## E. Static Distributed Algorithm Tasks

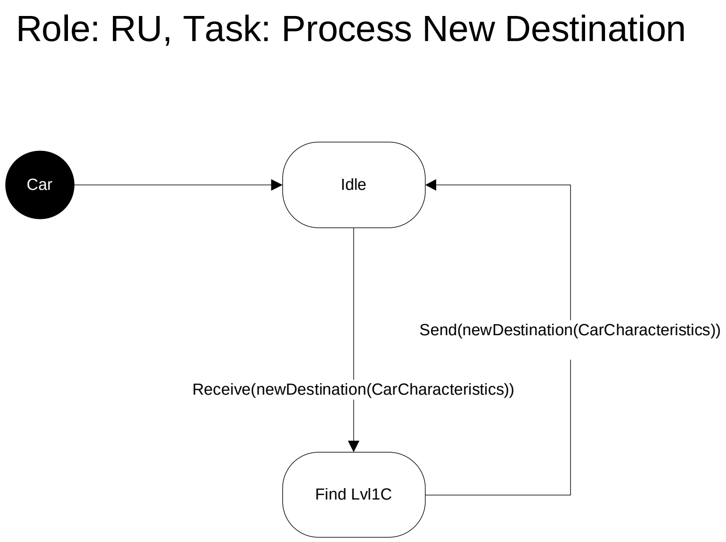



Figure 23: Task Diagram 1

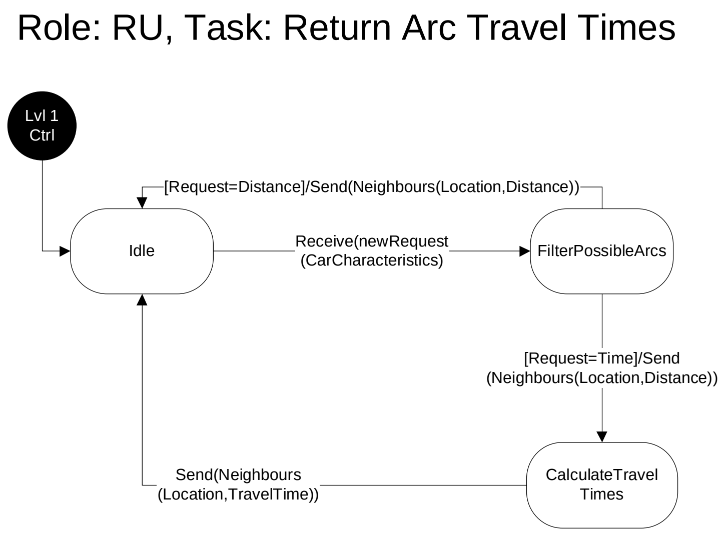



Figure 24: Task Diagram 2

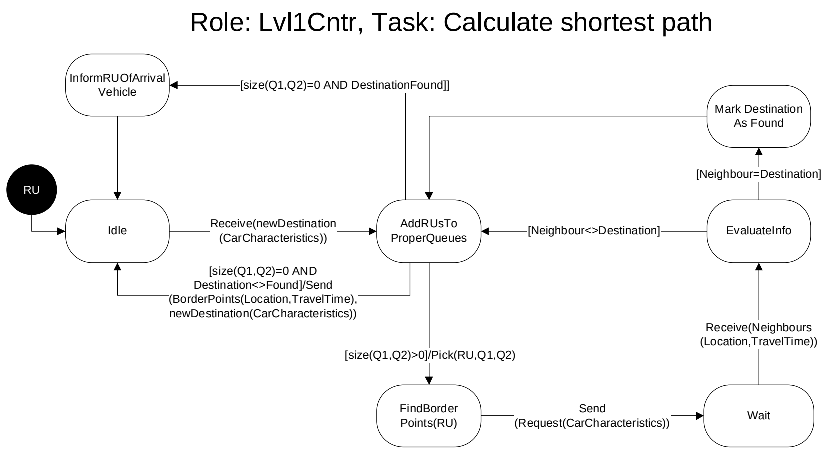



Figure 25: Task Diagram 3

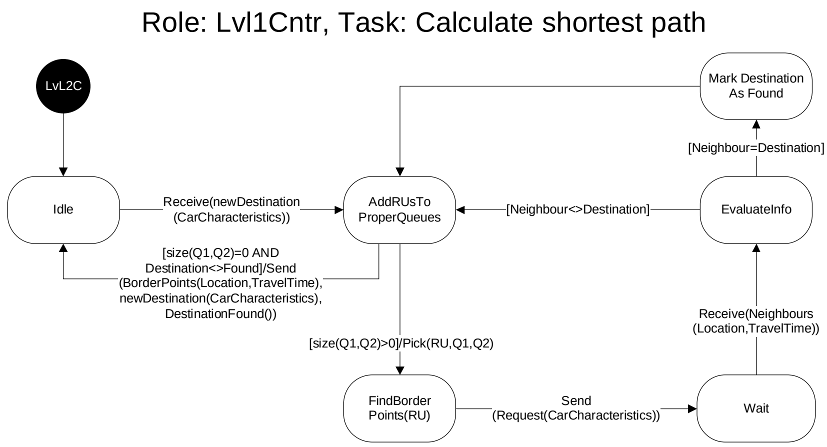



Figure 26: Task Diagram 4

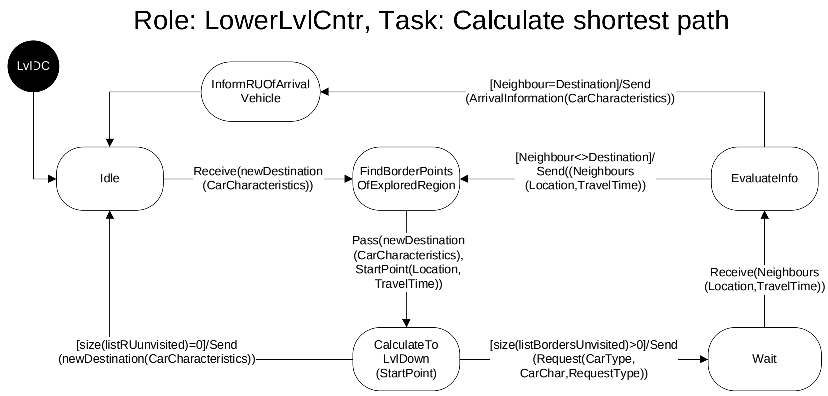



Figure 27: Task Diagram 5

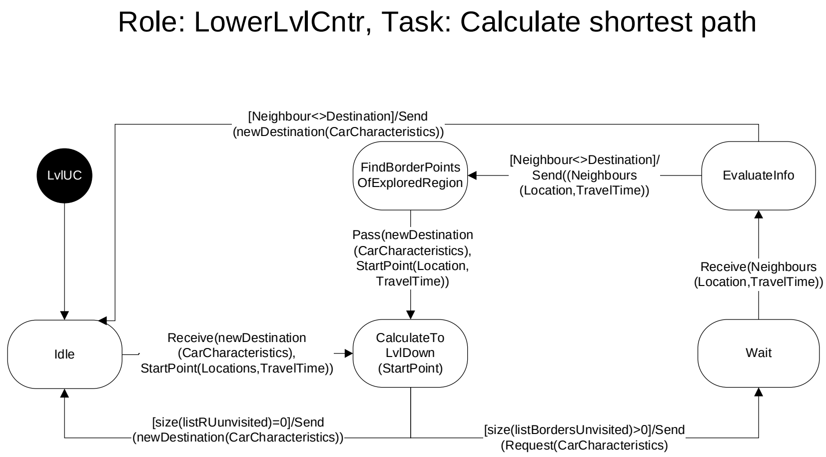



Figure 28: Task Diagram 6

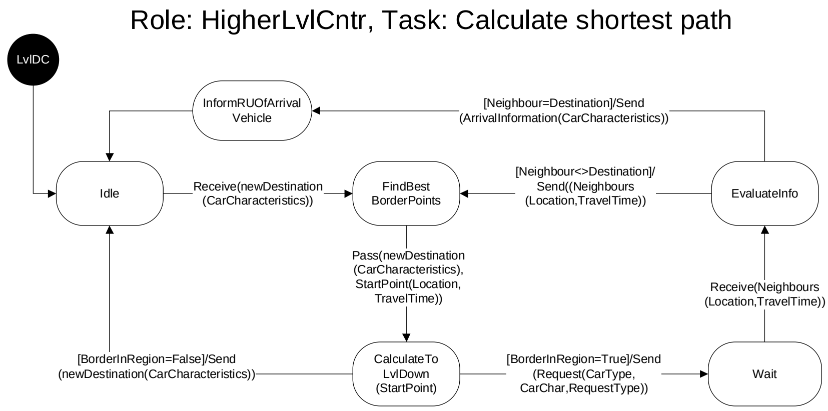



Figure 29: Task Diagram 7

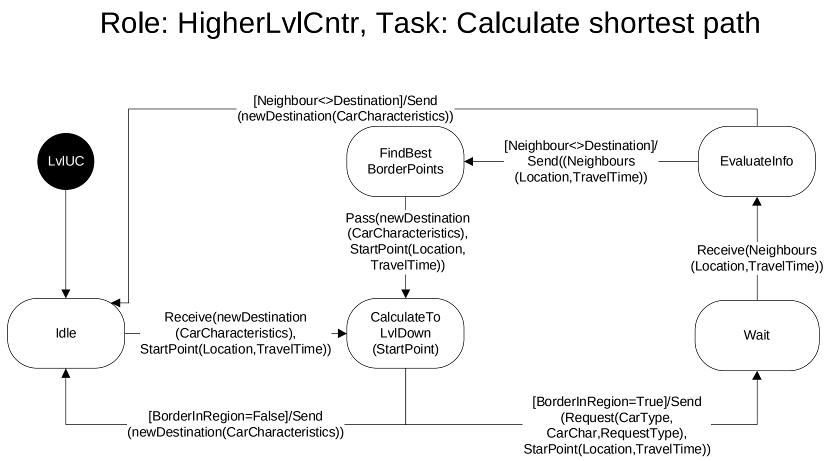



Figure 30: Task Diagram 8