



Looking inside
the Endoscopy department

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12 July 2008

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Optimization of the master schedule of the Hepato-Gastroenterology part of the Endoscopy department at the Academic Medical Centre, Amsterdam

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Date: 9 June 2008

Master thesis

Master: Industrial Engineering and Management

School of Management and Governance

University of Twente

Organization: Academic Medical Centre

Quality Assurance and Process Innovation

Patient Oriented Logistics

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Photo on the front page: ERCP room in a Venezuelan hospital,

source: <http://www.flickr.com/photos/21974686@N03/2500907422/>



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Summary

The Hepato-Gastroenterology part of the endoscopy department of the Academic Medical Centre faces big problems. The access times, the time patients have to wait for a consultation, are far too long, but the physicians and nurses felt they already needed to work too hard. The planning was not optimal, as it did not spread the usage of scopes and the recovery room and did not always provide residents to learn. The planning was also a cause of the long access time, as the skipping of shifts and the bad division of capacity under the consultation types are main causes for not meeting the access time standards. This research tries to improve the planning approach.

We recommend implementing a planning approach based on an iterative usage of simulation and Mixed Integer Linear Programming (MILP). This approach delivers master schedules, which the department can easily incorporate in the planning. One can use heuristic 1 to solve the MILP, but the solution quality varies for different data sets. We also recommend buying no OGD-suction for room 120 or new x-ray system for room 213.

The proposed approach delivers a master schedule, which will bring the access time into the standards. The approach also decreases the number of double bookings, which normally result in overwork. The developed Mixed Integer Linear Programming (MILP) does also take the maximum usage of the recovery room and the OGD-scopes into account. Furthermore, this new planning approach is more robust than the old, manually one. Direct solving the MILP gives better solutions, so is preferable above the proposed heuristic.

Implementing a new planning approach does bring many changes. The planning for the physicians will change and they need to adapt to it. Another consequence is that the department needs to do the planning process in a new way, partly also by other people.

Preface

This report is my master thesis for Industrial Engineering and Management. It deals with the application of Linear Programming for developing a planning for the endoscopy department of the AMC. Linear Programming is a popular technique from the field of Operations Research (OR), a branch of applied mathematics. Some people are reluctant to bring OR-techniques to hospitals. These methods are more widely applied in the industry than in service-oriented organizations like hospitals indeed. Furthermore, people feel quality is more important in a hospital than efficiency. The budgets for hospitals are tight however, and many patients need treatment. Hospitals need an optimized process to give every patient the attention he or she deserves. Improving the efficiency does not automatically bring down the quality; it is often needed to improve it. This report is a further step in optimizing the processes and can help to give all patients the attention they need.

This research finishes my master study at the University of Twente, so it needs to bring a contribution to the scientific community as well. It certainly does, as the iterative approach of simulation and Linear Programming is quite new. Paul Joustra, my AMC supervisor, is currently writing a scientific article about this approach. This research also adds some insights about the working of heuristics to find faster solutions for optimization problems.

I could not do this research on my one of course. I thank my AMC colleagues Veerle Struben, Paul Joustra and Henk Greuter. Veerle and Henk have helped the endoscopy department improve in other aspects than the planning, but also did help this research. Paul Joustra was my supervisor from the AMC, who has sent me in the right direction. Bob Overbeek did his master thesis in the same period and on the same department as me. This research was not possible without his data gathering and his simulation model, so a big thank to him. I thank all my AMC colleagues as they have helped me to feel at home in Amsterdam quickly. Furthermore, Bruce Jamison help me with my English, which is clearly not my strongest point. Last but not least, I thank my supervisors from the University – Erwin Hans and Peter Vanberkel. Their comments were of invaluable help and this would definitely be a clearly different and worse report without them.

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1. Introduction

This chapter describes the situation and problem. We describe the context in Section 1.1, with attention for the hospital, the two relevant departments and the project itself. Section 1.2 describes the problems, Section 1.3 state the objective of this problem and Section 1.4 define the research questions.

1.1 Context description

Academic Medical Centre (AMC)

The AMC is one of two academic hospitals in the city of Amsterdam in the Netherlands. It cooperates with the University of Amsterdam for research and educational purposes. It is one of the largest hospitals in the Netherlands with more than 6,000 employees and 1,000 beds. (AMC, 2008)

Quality Assurance and Process Innovation

The department Quality Assurance and Process Innovation (KPI, from the Dutch name Kwaliteit en Proces Innovatie) is a department within the AMC consisting of about forty people with different backgrounds, such as logistics, medical, evidence based practice and quality management. KPI delivers improvements for the departments of the AMC and advises these departments and the Board of Directors. The main objective of KPI is to support the departments within the AMC by helping them to improve the logistics and service in health-care processes. The activities are project-based and KPI carries them out in close cooperation with the department (most) concerned.

Endoscopy

Endoscopy is a minimally invasive medical procedure used to look inside gastrointestinal organs by inserting a camera into the body. The word endoscopy literally means “looking inside”. A physician executes this procedure with an endoscope, which consists of a camera, a light and a lens at the end of a tube. The tube also has an extra channel to allow insertion of medical instruments, to take a sample of the organ tissue for example. The part of the endoscopy department that is the object of this case study is the specialism Hepato-Gastroenterology (HGE, in Dutch Maag-Darm-Lever). This means all endoscopic consultations directed at the stomach, colon and liver procedures. The lung- and surgery-specialists also do consultations in the department, but we do not consider these as they use their own room, schedulers and physicians.

The studied part of the endoscopy department in the AMC handles about 9,300 procedures each year. The department consists of six different scope rooms, two waiting rooms, a pre-assessment room and a recovery room. The patients need to spend about two hours in the recovery room to allow the effects of the narcotics to wear off after most of the consultations. There are presently ten attending physicians and six resident physicians (in Dutch: arts-assistenten); both groups also serve at other departments. The attending physicians are experienced doctors, who do difficult consultations. The resident physicians are still learning “on the job”. At the beginning of their four year cycle they only observe attending physicians at work. They quickly learn to perform the simpler tasks independently with the possibility to call a supervisor, an attendant-physician, when required. A resident and an attendant preferably are scheduled together for the more complicated consultations, so the resident can

do more of the work as he gets more experience. There are also 19.4 FTE of nurses, although not all of them received the same medical training.

Project

This study is part of a bigger logistics-project of KPI and endoscopy departments. The main reason for initiating this project was the long access times, which prompted the endoscopy department to ask for support from the KPI process specialists. One of the problems that became clear during the analysis is the planning. Other problems include incorrect use of the recovery rooms, after care and availability of information.

1.2 Problem analysis

KPI and Endoscopy started the project to reduce the *access times*. Access time is the time between the day patients makes an appointment and the actual date of this appointment. We distinguish access time from waiting time, as the last one is the period a patient has to wait in the waiting rooms at the day of his appointment; access time is measured in days or weeks, waiting times in minutes. The current access times are unacceptable from both a medical and a patient point of view. Furthermore, the long access times lead to more patients that are urgent. Some general practitioners send their patients to other hospitals with shorter access times.

The project-team decided to create *standards* for the access times. There are different consultation types, all directed at different (parts of) gastrointestinal organs. We give a short explanation of all consultation types in the list of used terms at the end of the report. The standards for the access times vary over the consultation types for medical reasons. The standards are that the department must handle 95% of the patients within three weeks. The department wants to handle the patients within one week for three more urgent consultation types. We skip patients that want to or have to wait longer than five days for these standards. Table 1 shows that the department does not meet the standards for the most types of consultation.

Type consultation	Frequency (patients in 2006)	Average access time in 2006 in days (Standard Deviation)	Standard for the access time
OGD	3027	31 (34)	95% within 21 days
Colonoscopy	1961	37 (33)	95% within 21 days
Sigmoscopy	989	30 (19)	95% within 21 days
ERCP	918	5 (4)	100% within 7 days
EUS	747	6 (4)	100% within 7 days
Oesdil	518	7 (4)	100% within 7 days
Proctoscopy	89	11 (7)	95% within 21 days
Other	242	8 (2)	Different

Table 1: access time and standard (Source: Struben and Greuter, 2007)

The lack of *capacity* is one reason for the long access times for OGDs, Colonoscopy and Sigmoscopy. The department performed 9,113 consultations in 2006, while there was a demand for 9,334 (Struben and Greuter, 2007). The availability of physicians is a main limitation for the number of consultations performed. Some of the physician's time is unused due to no-shows (on average 2.4% of the patients), bad planning and unused emergency spots.

This lack of capacity has less influence on other types of consultation as the department gives implicit priority to these. A capacity analysis per type is impossible to provide, as the types share most of the resources. There is a nationwide shortage of HGE physicians, which is expected to grow in the future, as the number of endoscopic treatments is likely to grow rapidly (Van der Velden et al., 2003). Improving the efficiency of HGE physicians is one way to solve (part of) this problem.

Different numbers of patients come in for different types of consultations, and they have a varying urgency. The consultation types *require* different rooms and different physicians. Based on these differences in urgency and requirements the department has created different blocks, in which the desk employees schedule the patients. The planning of these blocks has many requirements. As stated, every block has its own possible rooms and physicians. These physicians each have their own maximal working time and their own shifts in which they are available. Furthermore, the physicians are sometimes unavailable due to holidays and congresses for example. Training days leads to the unavailability of both physicians and nurses. However, patients will arrive at these days. Physicians are now working 8% too much (Struben and Greuter, 2007) and want to minimize their working hours.



Figure 1: a physician performs a colonoscopy,
source: <http://www.flickr.com/photos/awong37/493065731/>

Some of the consultations are also for research purposes. Four different research programs group these specific consultations. They need complete shifts as researchers come for specific researches to the department. Other constraints to the planning are the number of scopes and the capacity of the recovery room. The average usage of these facilities varies among different types. The exact demand for a scope of the recovery room is impossible to predict, but the expected demand in one shift can be set to a maximum.

1.3 Objective

This study focuses on the planning approach and so we state the objective as:

To develop a planning approach that minimizes the access time of the different types of endoscopy with the current resources.

A promising approach seems to be to optimize the current master schedule. A *master schedule* is a cyclic scheduling approach of elective consultations. The endoscopy department is currently working with such a schedule. The cyclic length of this one is one week, although a couple of shifts are once every two or four weeks. Next to the capacity offered other methods to reduce access time with the planning are spreading appointment shifts for one type of consultations more equally over the week and making the planning more flexible.

The access times are not the only concern for the planning. Other important concerns are the availability and skills of the physicians, the capacity of the recovery room and the available equipment.

1.4 Research questions

1. *What is the current process, planning approach and performance?*
2. *What has already be done in similar projects, and which other theories are available?*
3. *What are the goals and restrictions of a suitable planning approach?*
4. *What is the performance of different planning approaches?*

The research questions contain and refer to different subjects and we discuss them in separate chapters. Chapter 2 answers the first question, and so deals with the current situation. Chapter 3 surveys the theory, which shows that similar projects are scarce. Chapter 4 handles the requirements of a planning approach. We discuss the construction of a heuristic in Chapter 5. Chapter 6 discusses the results of the heuristics and lists other results as well. Furthermore, this chapter discusses the implementation of this model. Chapter 7 lists the conclusions and recommendations. A list of the terms used is included at the end of this report.

2. Process description

This chapter describes the current situation in more detail. Section 2.1 sketches the processing of a patient. We discuss the current planning of these patients in Section 2.2. Section 2.3 discusses the most important characteristics of patients.

2.1 Patient process

Most of the patients visit the endoscopy department only once. This means that a contact starts by making an appointment with one of the desk employees. The desk employee also informs nearly all patients to do some kind of preparation, usually fasting and/or drinking a laxative.

When an outpatient arrives at the desk for his appointment, he or she has to wait in a waiting room. The department strives to schedule two nurses in each scope room. One of them receives the patient and often has to do some kind of preparation: mostly filling in a questionnaire and insert a needle for the narcosis or painkiller. The patient then has to wait in a second waiting room. If the previous patient has left the scope room, a nurse escorts the patient to and installs him or her into the room. Normally the physician has read the patient file at this point, so the actually endoscopy can start. When the physician has finished the consultation, he or she updates the patient file, while the two nurses clean the room and prepare it for the next patient. The patient can go directly home if the patient did not get narcosis; otherwise, one of the nurses brings the patient to the recovery room, where the patient stays for a couple of hours. Sometimes the physician wants to inform the patient the outcome of the consultation as soon as possible, so the patient waits after his recovery in the recovery room until the physician finds time.

This process is slightly different for the inpatients, a minority at this department. The transport department will bring them to the recovery room to wait until the consultation can start. The preparation, if necessary, takes place there. After the consultation, the patients will wait in the recovery room for the transport department to bring them back to their own ward. Inpatients from other hospitals face a similar routine: the only difference being the waiting time for the ambulance.

This process is shown in a flowchart is Figure 1. The process will vary for different types of consultation; the depicted process is the median process.

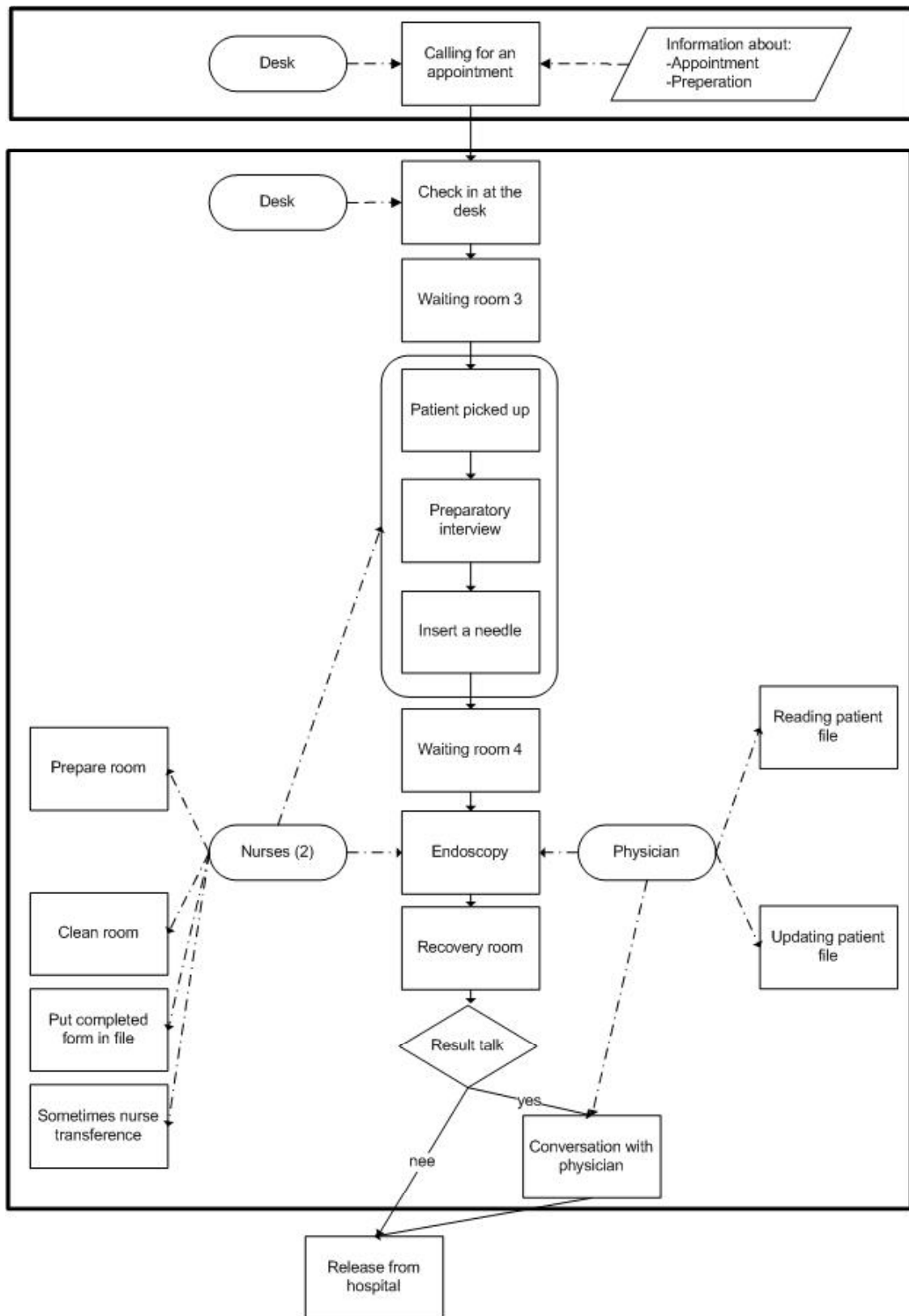


Figure 2: flowchart of the process

2.2 Planning process

The current planning process works in three steps, which we discuss in this paragraph.

First, one of the administrative employees makes a weekly *master schedule*. This master schedule plans the attendant and resident physicians in different rooms on different shifts. Normally the morning shift lasts 3.5 hours and the afternoon shift 2.5 hours, but sometimes both are 3 hours to plan complete consultations. Another part of the master schedule is devoted to dividing the available time over the different planning categories, currently nine. For four of the categories some of their time is reserved for urgent patients. The department revises the master schedule every three months, but the changes are small. In particular, the manned shifts and the division among the different categories hardly change.

The second stage of planning is using the master schedule to make the *plan*. One of the desk employees does this. Shifts are skipped for national holidays or training days. Some shifts are skipped due to unavailability of physicians, which happens for example as a lot of them choose the same week for their holidays. In total about 14% shifts are skipped in this step.

The third and last stage is to *schedule patients* in the available shifts. The patient communicates with one of the desk employees and chooses one of the provided appointment slots. These are normally the non-urgent times available for his category of consultation. A week in advance, the department releases the blocks, so schedules patients not only in the times of the specific category, but also in unused times of other categories. The only category left open is day urgency, as emergency patients use that time. Urgent and emergency patients for whom no suitable slot is available are double booked. This means that the department schedules them on an already filled appointment slot and handles them after the normal planned patients. On the day itself, the supervising nurse can move appointments from one room to another, to prevent too much overtime in any of the rooms.

This process is made more complicated by the *research blocks*. The desk employees reserve certain shifts for evaluating new diagnosis or treatment methods. There are four of these programs, namely fluorescence, CRC, FAP and IBD. Specific research nurses schedule the patients for the fluorescence and CRC programs.

2.3 Patient types

The endoscopy department performs annually 9,300 consultations. We can divide these consultations on several ways: new patient or not, urgency, referrer and type of consultation. This section discusses the two relevant ones: the urgency and the consultation type.

The *urgency* of a patient is an important factor and the department reserves time slots for more or less urgent patients. However, the definition of urgency is not clear at the department: as the access times for some of them are about three months, physicians call every patient who needs a consultation in a shorter period urgent and the department helps most of them within one day. More generally: the number of urgent patients depends on the access time. The department does not register urgency, so we take all patients that the department served on the same day as the registration of their appointment as urgent. Figure 2 shows that there were more urgent patients on Friday in 2006 due to the patients who need to be handled before the weekend.

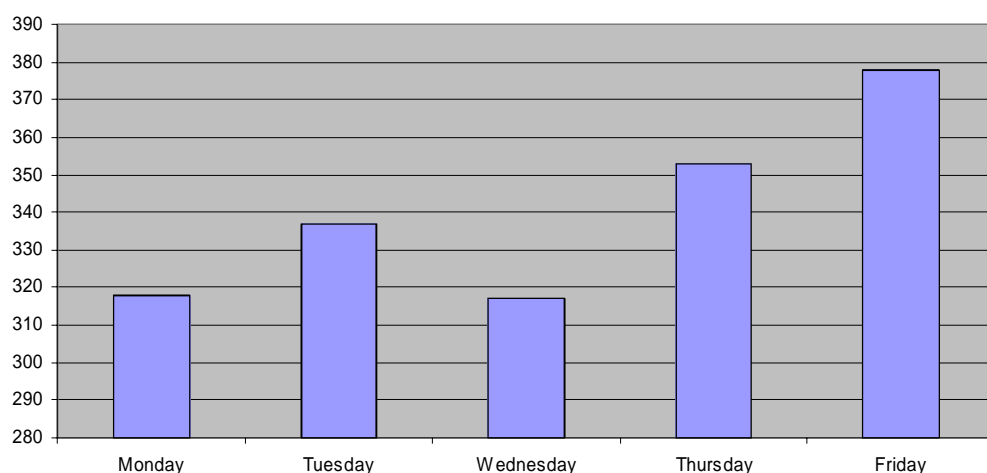


Figure 3: Emergency arrivals per weekday in 2006 (source: Overbeek, 2008)

There are different kinds of consultations. Nearly all consultations are ERCP, EUS, oesdil, OGD, colonoscopy, sigmoidoscopy, feeding tubes or proctoscopy. However, the four research programmes contain some of the consultations of these types. The department schedules sigmoidoscopies and OGDs together with some less common types, but schedules the other types mentioned in separate blocks. We will group more patients together, as it improves the efficiency (Overbeek, 2006).

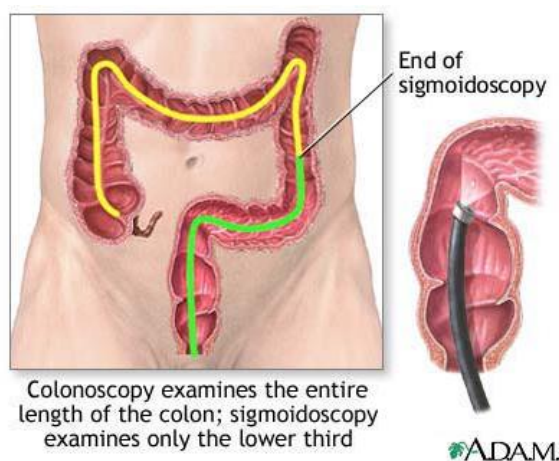


Figure 4: difference between colonoscopy and sigmoidoscopy, both consultations are for diagnosing the colon (source: <http://adam.about.com/care/Bloodless-medicine-glossary.htm>)

2.4 Current performance

As we stated in Section 1.2 the access time is far too long, especially for the consultations regarded as simple. This report tries to reduce the access time by the planning approach and more specific the master schedule. In this section, we face the question what is wrong with the current master schedule.

The department offers enough capacity to prevent long access times. The department has distributed capacity unequally over the different consultation types. During the project, there are changes in the demand for two different consultation types. First, the department will ask patients for simple consultations from outside the AMC-region to visit their local hospital instead. Second, one of the research programs, CRC, will attract a lot more patients as the department set-up a new screening program. These changes improve the difficulty of the making of the master schedule as a limited number of physicians can handle this research program. A manual made master schedule for the new situation does not exist, so it is hard to judge if enough capacity could be planned with this method.

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The skipping of shifts is a major aspect in the long access times in this case (Overbeek, 2008). Skipping of shifts means that the desk employees do not incorporate a shift from the master schedule in a certain week. This happens mainly due unavailability of physicians. This causes longer access time due a decreased capacity and variation in the offered capacity. We can minimize this by plan for every shift a reserve in the master schedule. We cannot judge the current performance on this aspect as the department plans no reserves.

Another goal of the department is to teach the resident-physicians the difficult procedures ERCP and EUS as good as possible. Therefore, we wish to plan always a resident alongside an attendant-physician for these consultation types. The current master schedule plans in eleven of the fourteen cases a resident to learn.

The performance could be better as discussed in this section. The planning is complicated due changes in the case mix of patients. First, the desk employees will advice the patients for simple consultations outside the neighbourhood of the AMC to go to their nearby hospital. Second, the department will start shortly with a new screening program, which will bring in more patients that are complicated. Therefore, the CRC blocks will contain these new patients. Only four physicians can handle these blocks. Concluding the amount of needed HGE-blocks will decrease, where the amount of CRC-blocks will increase. This makes the construction of the master schedule more difficult. It is not known what the performance of a master schedule constructed with the old method in the new situation is, as this is never tried.

We conclude that the current performance can be significantly improved. Furthermore, the current method is hard to continue due changes in the case mix. Therefore, we need a new method to construct the master schedule. This report will develop one.

3. Theory

This chapter deals with three different theory fields. Section 3.1 deals with the technique we use in this research: Linear Programming. Section 3.2 discusses the application of OR techniques, like linear programming, in the field of health care. Section 3.3 deals with the iterative combination of Simulation and Linear Programming, which is done in this and Overbeek (2008)'s report.

3.1 MILP

A popular tool for scheduling is Mixed Integer Linear Programming (MILP). Linear Programming was developed during the Second World War and has become a popular tool during the past fifty year. It is an optimization modelling technique, where linear equations express the constraints and optimization criteria. MILP is a generalization of Linear Programming, as in a MILP a part of the decision variables is integer. Winston (2004) discusses this more extensively.

Different methods can solve a MILP. We will use the most common one, Branch-and-Bound, with the barrier method to solve the resulting relaxations. The simplex method is more often used, but this research uses the barrier method as it turns out to be faster in this specific case. This method is built-in in specialized solvers. For this research the solvers CPLEX and XA are used, as part of the modelling environment AIMMS.

The barrier method finds the optimal solution through the inside of the feasible region, different from the simplex-method. The largest improvement in objective and the smallest increase to the boundary given by the constraints gives the direction to travel. One needs a mathematical transformation to make this path work (Winston, 2004).

The simplex and barrier methods however, cannot solve problems that contain integers, like the one in this research. The AIMMS solver uses the Branch-and-Bound method to solve problem with integers. This method uses the splitting of the solution space in nodes, the branching process. This is done by picking a variable v that must be integer and require in one node that $v \leq x$ and in the other that $v \geq x+1$. An attempt is made at eliminate those nodes, which means that the optimal solution cannot be there. This is the bounding process. If the problem is a minimization problem like this one, the lower bounds will be node-specific and the upper bound general. The solver can eliminate that node if the lower bound of a node is equal to or higher than the upper bound. The upper bound is the best feasible solution found so far, where the lower bound is the LP relaxation (the MIP without the integrality constants) with the extra requirements from the branching. The exact sequence of branching matters a lot for the speed of finding a solution, but is an AIMMS company secret (Tijms, 2002; Bisschop, 2007).

3.2 Planning in healthcare

We research the possibility of similar work by looking at other usages of operational research techniques in healthcare. We search on combinations of operations research, linear programming, integer programming with healthcare, hospital and endoscopy. We consider only articles up to a maximum age of ten years old and available on the UVA/AMC licence. We use the search machines Google Scholar, Scirus and PubMed. We scan these hundreds of

hits on their title. We read the abstract of about a hundred of them. We read twenty articles completely, but we found only four that solve comparable problems. We survey the citations and the articles that cite them of these four articles, but we find no additional articles. In the following sections we give an overview of the articles found, focussing on the articles that solve similar problems.

The nurse-scheduling problem has attracted a lot of attention, as it is a standard problem in employee shift planning. Cheang et al. (2003) provide an overview of nurse scheduling articles. Sherali et al. (2002) and Day et al. (2006) do something similar for residents; Beliën and Demeulemeester (2005) look at the problem for trainees. However, in these articles assume a known demand over time, which makes them different from this research. Centeno et al. (2003) first calculate the demand for staff over time in an ER department with simulation and use that as input for an ILP to optimize the shifts. Yeh and Lin (2007) do something similar, but use a simulation model and a genetic algorithm (GA) in a cyclic approach, which means that they use simulation to test each generation of the GA-generated schedules.

There are four differences between this research and the articles last paragraph describes. First, the goal of an ER department is to minimize waiting times in minutes rather than access time in days. The ER department can hardly schedule the demand and normally treats the patients in order of urgency and arrival. Second, the nurses and physicians of an ER department have more generic skills than in the endoscopy department. Third, an ER department has seldom problems with the rooms, but the rooms and equipment are a constraint in the endoscopy department.



Figure 5: nurse prepares a patient for an ERCP,
source: <http://www.flickr.com/photos/21974686@N03/2447726019/>

Scholars have done a lot of research on deciding the ideal appointment times. The goal is to choose those appointment times in a way that minimizes patient-waiting time, server (physician) idle and over time. The most of those articles test different rules with a simulation model against a varying variation, no show percentage and number of emergency patients (among others Klassen and Rohleder, 2003; Ho and Lau, 1999; Liu and Liu, 1998).

A popular topic, which has more similarities with the problem of this research, is

Operation Room scheduling. Many articles try to maximize the number of operations planned given e.g. a maximum risk on overtime, like Marcon et al. (2003) which use negotiation combined with an ILP to minimize risk of no realization. Jabali et al. (2005) and Guinet and Chaabane (2002) do something similar, but schedule first operations over operating rooms and optimize individual room schedules afterwards. Dexter et al. (among others 2002) looked to optimizing the OR planning with respect to the financial rewards for the operations.

A subtopic comparable to problem of this report that received some attention is the operation room scheduling with restrictions on the Intensive Care (IC) beds. Van Oostrum et al. (2006) and Van Houdenhoven et al. (2007) solve this problem with column generation: a repetitive LP-approach in which they generate standard days for an OR in the first phase and they divide those over the available ORs in the second phase. Santibáñez et al. (2007) take IC and ward beds into account and use a greedy branch and bound algorithm. Vissers et al. (2005) schedule cardiothoracic surgery and take IC beds, ward beds and nurses into account. They use the standard MIP-solver, but limit the number of nodes to reach an acceptable solution within limited time.

We have found no articles that plan both doctors and consultations in the same phase. The most scholars focus on nurse scheduling and appointment times, from which we cannot copy the solutions or techniques. There are three researches with comparable problems: operation room scheduling with respect to other resources.

3.3 Simulation and Linear Programming

We survey the literature also on iterative combinations of simulation and linear programming. We search on combinations of “linear program”, simulation and either iterative or repetitive in the search machines Google scholar and Scirus. We enforce no limitations on the age or the number of citations. We survey also the references of and the articles citing the found interesting articles. We find only a few interesting articles with this method.

The combination of simulation and linear programming is seldom, although both tools are widely used today. We found only one combination in health care as mentioned before (Centeno et al. 2003). More researches use the combination of these tools outside health care, but we found only one iterative approach: Hung and Leachman (1996). They use simulation to predict the flow times and use that to determine a production schedule for a semiconductor company. They later on replace their simulation model by a model based on queuing theory to speed up the algorithm (Hung and Hou, 2001). We can use their approach, but we cannot use their MILP and the simulation model as the problem is too different.

Henderson and Mason (1998) lead out the framework for such an approach ten years ago, calling it Rostering by Iterating Integer Programming and Simulation (RIIPS). Atlason et al. (2004) give their framework a mathematical foundation. Both articles focus on minimizing staff costs while reaching minimal service standards, but we can use their framework. Their approach is to make a start schedule and simulate it to discover where it fails on the customer service level. They add an extra constraint to the MILP to enforce an extra employee in a certain period. The simulation model tests the schedule made by the MILP with the extra constraint. They repeat these steps until they reach a satisfying solution. Nobody brought their framework into practice as far as we known.

4. Model

This chapter focuses on methods to solve the problem. Section 4.1 explains the solution approach, which consists of three different steps. The first step is defining the planning categories and we discuss this in Section 4.2. The second step is the making of a master schedule. Section 4.3 outlines a MILP to do this. Section 4.4 explains the method of data case generation for the experiments, which will start from here. We use the first experiment, Section 4.5, to determine the weighting factors. Section 4.6 proves that this program is computationally hard to solve. We will develop a heuristic to solve this MILP in Chapter 5. The third and last step of the solution approach is a simulation model: this part is outside the scope of this research as Overbeek (2008) already dealt with this.

4.1 Solution approach

This project focuses on the *master schedule*. In the current situation, the department works with a weekly master schedule with some small variations. The department needs a repetitive schedule, as it is otherwise hard to schedule employees and patients every week anew. A shorter period is not possible due to the effects of the weekend and research programs. There is no need for a longer period, as there is no need for shifts that rotate every two weeks. Another reason for a weekly schedule is that the consultations need to have an access time within three or one week. A longer schedule would make clever division of the shifts inside the master schedule inevitable, as the department needs to help many of the patients in a shorter time. Optimizing a schedule of a week is easier than of a longer period, due to the smaller number of variables. Therefore, a weekly master schedule is ideal for the endoscopy department.

We would ideally solve the problem in one-step. However, we cannot calculate the access time with analytical methods, which are required for direct optimization. Consequently, we use an iterative process, with a stepwise use of Linear Programming and simulation; we change the input parameters of the MILP on the hand of the results of the simulation model. The most important input parameter that we adjust is the quantity of time planned for different types of consultation. Figure 4 depicts this process, which starts with determining how we group the consultations together in planning categories: we do this in Section 4.2. The next paragraphs explain the choice for MILP and simulation in the described steps.

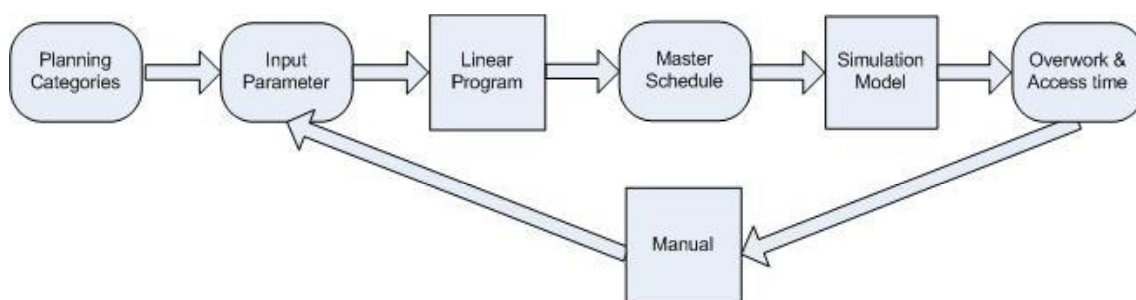


Figure 6: iterative process used in this research

Ideally, we make the master schedule with a *Mixed Integer Linear Program*. The problem is too difficult to solve manually, due to the interrelations between different consultation types and rooms. The new master schedule should provide a reserve physicians if the first one is unavailable as varying offered capacity is a major factor in the long access times (Overbeek, 2008). However, this is impossible to guarantee when planning manually. Facilities as scopes

and recovery room are also hard to take into account manually. The choice for an automatic process leaves two possibilities: optimization modelling or a heuristic. Optimization modelling can give an optimal solution to the problem and is easier to develop than a heuristic. The quality of a solution constructed by a heuristic is harder to judge, but cannot be better than an optimization modelling with a right objective. Optimization modelling means in practice Mixed Integer Linear Programming (MILP) as the problem is impossible to model without integers and solve without linearity. However, solving the MILP within reasonable time turned out to be hard, so we made a heuristic that had to do the same. The rest of this report explains this in more detail.

We test the developed schedules with *simulation* for a simple reason. Those other methods are testing with the real system, with a physical model or with mathematics, e.g. the earlier mentioned queuing theory. The first two options are too expensive and the last one cannot deal with the system's complexity. (Overbeek, 2008)

We choose for the approach is an iterative usage of MILP and simulation. This is comparable to the approach Henderson and Mason (1998) describe, which they call Rostering by Iterating Integer Programming and Simulation (RIIPS). They describe an approach in which a MILP makes crew schedules and simulation tests those; if the schedule does not meet the standards the MILP makes a new schedule with an extra constraint. This is comparable to what we are going to do. A difference is that we will add or adjust the constraints manually. This decreases

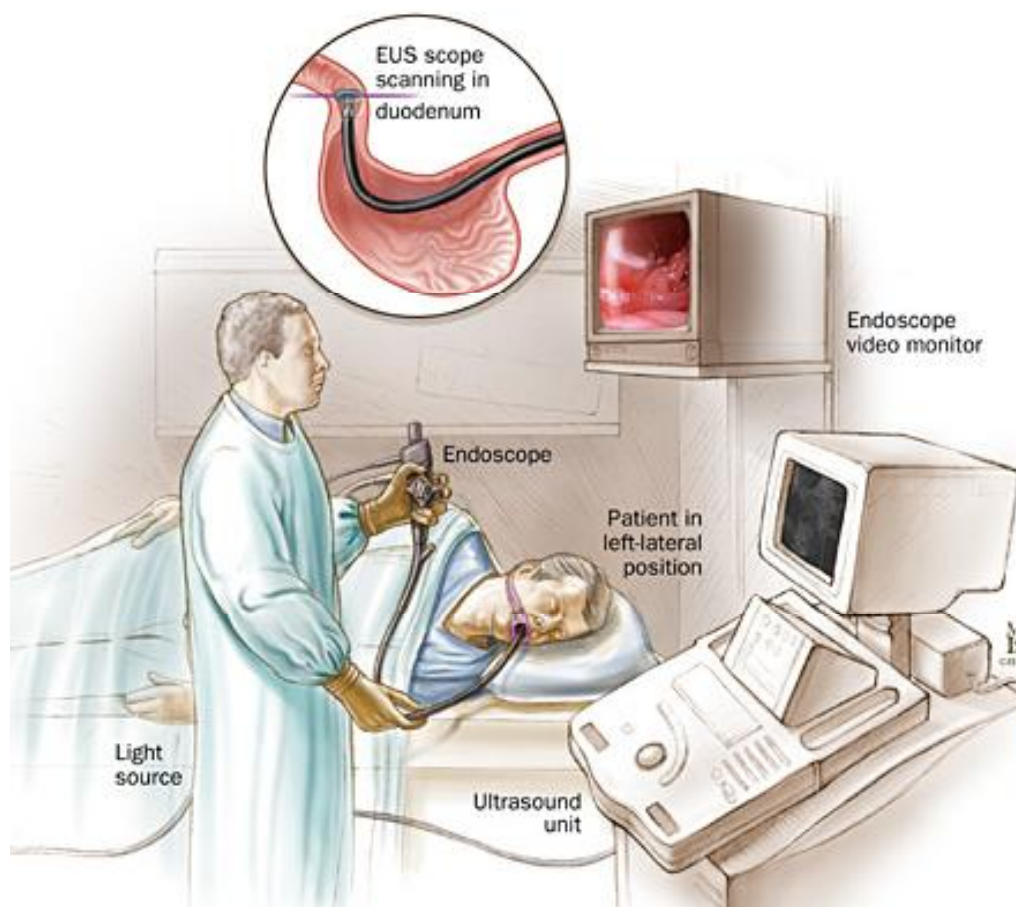


Figure 7: schematic drawing of an EUS consultation, source: <http://www.thaimed.us/what-is-endoscopic-ultrasound-eus/2008/04/24/>

the computational time, but we cannot guarantee optimality, as the tested input parameters are subjective. Another difference is the subject: Henderson and Mason work on call centres with customer grade of service, where we use a medical department with access times. Third and main difference is that they describe a theoretical model, which we bring into practice. Hung and Leachman (1996) are the only article found which uses an iterative combination of MILP and simulation, but their models are different.

Overbeek (2008) already built the simulation model, so this research focuses on the MILP. However, the next section explains the first step in the approach, the choice of consultation types.

4.2 Consultation types

To develop a suitable planning approach we have to determine which consultation types we will use. Fewer categories leads to a shorter access time (Overbeek, 2008), but the differences within a category becomes larger. There are a couple of research programs, which need own shifts and physicians and are therefore distinct categories. Now these categories are fluores (3 shifts per week), fluorecho (1), CRC (2), IBD (1) and FAP (1).

We divide also the normal consultations, mainly by planning requirements. ERCP and EUS needs to be distinct categories, as a limited number of physicians can do them, both in one specific room. We separate Oesdil as it needs shorter access time and can only be handled in one of the scope rooms. The remaining, mainly colonoscopies, OGDs and sigmoidoscopies, are divided into normal and urgency part, as the characteristics differ. Table 2 lists each consultation in the rows and the consultation types, the planning categories, in the columns. Table 3 lists the results for the planning.

	Duration (minutes)	Week urgency	Day urgency	Olver	FUS	OesDil	ERCP	CRC	FAP	Fluores	IBD
Colonoscopy	60	48	136	1159	0	0	0	701	4	8	117
OGD without anesthesia	15	123	349	959	0	0	0	17	35	0	21
OGD with anesthesia	30	22	61	0	0	0	0	4	23	690	13
Sigmoidoscopy without anesthesia	15	42	121	439	0	0	0	19	89	0	43
Sigmoidoscopy with anesthesia	30	1	4	81	0	0	0	4	1	0	6
Oes	30	10	27	95	0	0	0	0	0	0	0
VS and PEG	30	0	228	0	0	0	0	0	0	0	0
EUS	45	0	0	0	757	0	0	0	0	0	0
OesDil	30	0	0	0	0	528	0	0	0	0	0
ERCP	60	0	0	0	0	0	920	0	0	0	0

Table 2: 2006 demand of consultations per consultation type (own data)

Category	Research program	Possible rooms	Possible physicians	Standard for access time (days)
Fluores	yes	5	3	-
Fluorecho	yes	1	3	-
CRC	yes	5	4	14
IBD	yes	4	2	14
FAP	yes	4	2	-
ERCP	no	1	5	7
EUS	no	1	4	7
Oesdil	no	2	16	7
Week Urgency	no	4	16	7
Day Urgency	no	4	16	1
Other	no	4	16	21

Table 3: types of consultation in new planning

4.3 Model

This section discusses the mixed integer linear program (MILP). We start with a formal problem definition of the generation of master schedules. We discuss the different parts of a linear model after that: indices, variables, parameters, constraints and the objective.

Formal problem description

The goal is to make a master schedule that the department will repeat weekly. For every shift s it has to be decided which of the rooms r are open, which attendant-physicians a and/or resident-physicians d serve, and how much consultations of type t will be done. The goal is to schedule reserve physicians for all filled shifts, so that the department can also execute the master schedule when physicians are unavailable. A further aim of the model is that resident-physician also spends time in learning the more difficult consultations, ERCP and EUS.

Indices

- a Attendant Physician, experienced physicians.
- d Resident Physician, physicians learning on the job.
- r Rooms, there are available rooms with various equipment. Furthermore, we create one dummy room: supervisor. See constraint (c).
- s Shifts. There are different shifts: mornings and afternoons of the five working days.
- t, τ Types of consultation.

Parameters

- AttAvail_{as} =1 if attendant-physician a is available during shift s
=0 otherwise
- AttShifts_a Number of shifts attendant-physician a can serve in the endoscopy department
- AttPos_{at} =1 if consultation type t can be done by attendant-physician a
=0 otherwise
- $c_1, c_2, \text{etc.}$ Weight factors in the objective for the different unwanted effects.
- CostDem_t The penalty per unit for not meeting the demand of consultation type t .
- DayLength The maximal length of a day.
- Demand_t Demand for consultations of type t .

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Dur_t	Duration of one unit of consultation type t .
Influence	The importance of being a shift reserve for one room compared to serve normal on one shift.
M	Sufficient large number
$MaxLength_s$	Maximal length of shift s . Normally 3.5 hours for morning shifts and 3 hours for afternoon shifts.
MinMorn	The minimal amount of time from one specific type of consultation that needs to be in morning shifts.
$MinShift_{ts}$	Minimal number of consultations planned in shift s of consultation type t . This is zero for most cases, only some consultation types with a lot of urgent patients a number larger than zero is noted.
RecCap	The maximum expected number of patients that we can send to the recovery room in one shift.
$RecSent_t$	The expected number of patients sent to the recovery room per unit of consultation type t .
$ResAvail_{ds}$	=1 if resident-physician d is available during shift s =0 otherwise
$ResShifts_d$	Number of shifts resident-physician d can serve in the endoscopy department
$ResPos_{dt}$	=1 if consultation type t can be done by resident- physician d = 0 otherwise
$RoomPos_{rt}$	=1 if consultation type t can be done in room r =0 otherwise
$ScopesAv_s$	The maximum expected number of OGD-scopes that can be used in shift s .
$ScopesNeed_t$	Expected number of OGD-scopes needed during one unit of consultation type t .

Decision Variables

X_{str}	Number of patients of type t in shift s in room r
Att_{asr}	=1 if attendant a planned in room r in shift s =0 otherwise
$Resid_{dsr}$	=1 if resident d planned in room r in shift s =0 otherwise
$RAtt_{asr}$	=1 if attendant a planned as reserve in room r in shift s =0 otherwise
$RResid_{dsr}$	=1 if resident d planned as reserve in room r in shift s =0 otherwise

Auxiliary Variables (derived from decision variables):

$DifDem_t$	Number of patients of type t not handled
$NoResid_{sr}$	=1 if no resident planned on shift s in room r if certain categories are performed =0 otherwise
$NoReserve_{sr}$	=1 if no reserve planned on shift s in room r if consultations are planned =0 otherwise

Physician related Constraints

(a) Either a skilled attendant- or a resident-physician can perform a consultation. We want a reserve physician for every filled shift, who can serve the patients when the first one is unavailable.

$$X_{str} \leq \left[\sum_a (Att_{asr} * AttPos_{at}) + \sum_d (Resid_{dsr} * ResPos_{dt}) \right] * M \quad \forall s, t, r$$

$$X_{str} \leq \left[\sum_a (RAtt_{asr} * AttPos_{at}) + \sum_d (RResid_{dsr} * ResPos_{dt}) + NoReserve_{sr} \right] * M \quad \forall s, t, r$$

(b) We want a resident-physician to learn for the consultation types ERCP and EUS. Therefore, we record the absence of one in the NoResid_{sr}-variable and penalized later on.

$$X_{str} \leq \sum_d (Resid_{dsr}) + NoResid_{sr} * M \quad \forall s, r, v, t \in \{ERCP, Eus\}$$

(c) There must be an attendant as supervisor.

$$\sum_a Att_{asr} = 1 \quad \forall s, r \text{ supervisor}$$

$$\sum_a RAtt_{asr} + NoReser_{sr} = 1 \quad \forall s, r \text{ supervisor}$$

(d) Both attendant- and resident-physicians can only be planned on shifts they are available.

$$Att_{asr} + Influence * RAtt_{asr} \leq AttAvail_{as} \quad \forall a, s, r$$

$$Resid_{dsr} + Influence * RResid_{dsr} \leq ResAvail_{ds} \quad \forall d, s, r$$

(e) We bound the time spent by different attendant- and resident-physicians to a maximum.

$$\sum_{srv} (Att_{asr} + RAtt_{asr} * Influence) \leq AttShifts_a \quad \forall a$$

$$\sum_{sra} (Resid_{dsr} + RResid_{dsr} * Influence) \leq ResShifts_d \quad \forall d$$

Consultation related constraints

(f) We define DifDem_t as the difference between the demand and the planned units.

$$Demand_t - \sum_{rs} X_{srt} = DifDem_t \quad \forall t$$

(g) We must meet the minimum number of units of consultation type to be planned in shift s.

$$\sum_r X_{srt} \geq MinShift_{ts} \quad \forall t, s$$

(h) The length of a shift cannot exceed its maximal duration.

$$\sum_t (X_{srt} * Dur_t) \leq MaxLength_s \quad \forall r, s$$

(i) No day can last longer than the given maximum length. Monday is shift 1 and 2, Tuesday shift 3 and 4, et cetera.

$$\sum_t [(X_{srt} + X_{s-1,rt}) * Dur_t] \leq LengthDay \quad \forall r, s \in \{1,3,5,7,9\}$$

(j) All consultation types can only be done in some of the rooms.

$$X_{srt} \leq RoomPos_{rt} * M \quad \forall s, t, r$$

(k) The department performs some consultations with narcosis and those patients need to recover in the recovery room afterwards. We maximize the number of patients sent to the recovery room per shift. This minimizes the chance that patients have to recover on the corridor.

$$\sum_{r,t} X_{srt} * RecSent_t \leq RecCap \quad \forall s$$

(l) We bound the expected number of OGD-scopes needed to a maximum, as their number is limited. Other types of scopes do not give problems, so we do not consider them.

$$\sum_{rt} X_{str} * ScopesNeed_t \leq ScopesAv_s \quad \forall s$$

(m) Some consultation types, the research programs, need whole shifts. Note that $X_{srt} \in \{0,1\}$

$$(1 - X_{srt}) * M \geq X_{srt} \quad \forall s, r, t \in \{fluores, fluorecho, IBD, CRC, FAP\}, t \neq \tau$$

(n) We cannot concentrate the shifts of a research program of two different consultation types, due capacity restrictions in preceding and succeeding steps. The number of shifts in four subsequent shifts, two days, is bound to a maximum of two, where the total number of shifts is four.

$$\sum_{t \in \{fluores, fluorecho\}} \sum_r (X_{srt} + X_{s-1,rt} - X_{s-2,rt} - X_{s-3,rt}) \leq 2 \quad \forall s \in \{shifts\} \quad 3$$

(o) One consultation type needs a part of its time in mornings.

$$\sum_r (X_{1rt} + X_{3rt} + X_{5rt} + X_{7rt} + X_{9rt}) \geq MinMorn \quad \forall t \in \{fluores\}$$

(p) Other departments than HGE use rooms of the endoscopy department as well. One department needs a room every Tuesday morning, and room 120 is the least useful room that they can use. The other department needs room 219 on Thursday three out of four weeks, so this room is not used the fourth week as we use a master schedule of one week.

$$X_{str} = 0 \quad \forall s \in \{3,5,7,9\}, t, r \in \{room120\}$$

$$X_{str} = 0 \quad \forall t, s \in \{7,8\}, r \in \{room219\}$$

(q) Twice a week another department uses the first part of a shift in one specific room. The HGE-part of the endoscopy department can fill the remaining. We therefore model this as a different consultation type.

$$X_{str} = 1 \quad \forall s \in \{1,5\}, t \in \{selecton\}, r \in \{room213\}$$

(r) One consultation type, CRC, consists of different consultations. Most of them can be done in five different rooms, but some only in four of them. Therefore, this constraint must make sure at least one of them is in another room.

$$\sum_{s,r=room120} X_{str} \geq 1 \quad \forall_t = CRC$$

Objective

The objective is a weighted sum of the different criteria. This are the number of shifts attendants and residents have to work, the difference between the demand and the planned consultations and the number of shifts that contain consultation types ERCP or EUS without a resident to learn.

min

$$z = c_1 * \sum_{asr} (Att_{asr} + Influence * RAtt_{asr}) + c_1 * \sum_{dsr} (Resid_{dsr} - Influence * RResid_{dsr}) \\ + c_2 * \sum_t (DifDem_t * Weigth_t) + c_3 * \sum_{srv} NoResid_{sr} + c_4 * \sum_{srv} NoReser_{sr}$$

Alternative model descriptions

There are four other possible set ups of the model:

- We could add a new variable Y_{str} that is 1 if $X_{str} > 0$ and 0 otherwise. Y_{str} would replace the X_{str} * M-part that occurs in different constraints. This could make the LP-relaxation better and thereby decreasing the needed computation time.
- We could made reserves only dependent on the shift and not on the room. This means that the variables RAtt and RResid are only depending on shift and physician and not on the room. NoReserve depends still on the room due physician's abilities.
- We could model the attendant-physician, resident-physicians and rooms by one integer variable, rather than all by distinct binary variables. This could replace the numerous binary variables with less integer variables. With attendant-physicians and rooms this turn out to be impossible, as from a planning point of view, there are no two attendants similar and only two out of six rooms. This could be possible with residents, as five out of six are identical from a planning point of view. We try to model the residents on this way, but it results in master schedules that are impossible for the residents. We can conclude that this is no option.
- We could model this problem with scenarios: in each scenario, one physician is unavailable and the program needs to staff all shifts. This model has more options and so normally leads to better solutions. However, this problem has far too many binary variables to solve within acceptable time.

We also research the first two set-ups; we will see in Section 4.6 that these descriptions are slower.

4.4 Data case generation

We perform several experiments for this report. We do not only experiments with the AMC case, but also with randomly generated data sets. This increases the chance that the chosen solution technique still delivers a good solution if changes in the department occur. Another goal of the different data sets is testing the applicability of the model in other hospitals. This section describes the methods we use for generation of these data case and some other general settings; we show the results of the different experiments in different sections. We generate new data for each experiment, as this is easier than storing all the data cases. We use the same data cases within one experiment and table to make fair comparisons possible. We check the feasibility for all generated data cases by finding a feasible solution for a simpler version of the problem, without reserves or residents to learn, with the solver CPLEX.

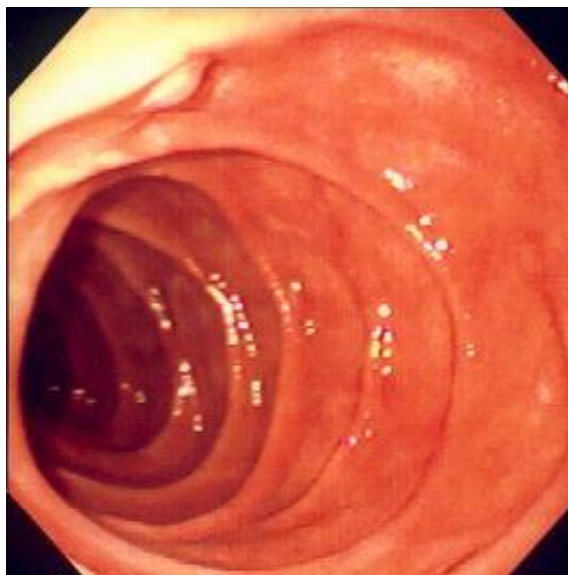


Figure 8: photo of duodenum made during endoscopy (source: <http://flickr.com/photos/testastretta/327749867/in/set-72157594429210279/>)

The values are averages of three different data case-generation-methods, except when mentioned otherwise. Those three methods are:

- 1) The AMC data case, we refer to it as *data case 1: AMC*
- 2) The average of different data cases with a varying demand. We generate the demand with a rounded exponential distribution. The mean of this distribution depends on the duration of one consultation, as there is typically less demand for consultations with a longer duration. Consultation types included in the MinShift_{is} -parameter do get a minimal value of the total amount in this parameter afterwards. The other parameters are equal to the AMC case. We mention this average as *data case 2: varying demand*.
- 3) The average of different data cases with seven parameters varying. We randomly generate the parameters Demand_t , RoomPos_{rt} , PhysPos_{at} , ResidPos_{dt} , PhysAvail_{as} and ResidAvail_{ds} . The second marker describes the generation of Demand_t ; we generate the other listed parameters with a binary-distribution, with the chance in line with the real values. We refer to this average as *data case 3: varying parameters*.

All experiments were done with a Pentium 4 with a CPU of 2.80 GHz and 504mb RAM. We cut off all searches after 30 seconds and perform them with CPLEX 11.0, except when mentioned otherwise. We base the number of experiments on the 95% confidence interval and the duration of one experiment.

4.5 Weight factors

Choosing correct weight factors can be of major influence in the final solution. One needs expert opinion to obtain these. The different parts of the objective are comparable in this case,

as all count one shift at a time. However, it is good to show the minimum and maximum values that are possible for the different optimization criteria for different objectives. This makes a better judgement of the weight factors possible. Table 4 lists the objectives that we use for obtaining the minima and maxima. Meeting the demand is the main priority in all objectives. The demand is a hard constraint in reality, which we weaken to speed up the solver and allow nearly always a feasible solution. The first three objectives all focus on one optimization criteria. The last two consists of two steps: in the first step Objective 1 is used, afterwards we fix the consultations and physicians and a second objective is used. For Objective 4 the second objective is Objective 2; for Objective 5 the second objective is Objective 3. We add these two as the number of shifts without reserve and resident to learn can be changed after the first step without changing anything else. The values for shifts without reserve and resident to learn under Objectives 4 and 5 do say more than the values under Objective 1.

Objective 1	Minimizing/maximizing the number of working shifts of the physicians	$c_1=1, c_2=100 c_3= c_4=0$
Objective 2	Minimizing/maximizing the number of shifts without a reserve	$c_1=c_4=0 c_2=100 c_3= 1$
Objective 3	Minimizing/maximizing the number of shifts without resident to learn	$c_1=c_3=0 c_2=100 c_4= 1$
Objective 4	First step: Objective 1. Second step: Objective 2.	
Objective 5	First step: Objective 1. Second step: Objective 3.	

Table 4: different objectives

Table 5 shows the results: these numbers fractional numbers as it are averages over the data case methods. These three different data case-methods are the AMC case, the average over 23 data cases in which we vary the demand en the average over 64 cases for which we vary six parameters. This means that the numbers of Table 5 are weighted averages over the 88 cases.

For all objectives we define, we solve the MILP and give the resulting working shifts, shifts without reserve and shifts without resident to learn. The bottom rows show the minima and maxima of both the minimizations and the maximizations. The differences in shifts without resident to learn are the smallest as we expect: we need only one for two consultation types. The differences in working shifts are not too big either, as the department needs many of the possible shifts to fulfil the demand. The differences in shifts without reserve are big: it is possible to schedule a reserve for every filled shift, but also to maximize the number of shifts and schedule no reserves on it. The maximum number of working shifts is not equal to the maximum number of shifts without reserve as in the first cases residents to learn are included. The number of working shifts with objective 5 is larger than with objectives 1 and 4 as we record the resident to learn-shifts also as working shifts.

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Objective	Explanation	Working shifts	Shifts without reserve	Shifts without resident to learn
Objective 1a	Minimize working shifts	48.4	35.6	8.8
Objective 1b	Maximize working shifts	67.6	42.1	7.4
Objective 2a	Minimize shifts without reserve	52.9	2.1	8.2
Objective 2b	Maximize shifts without reserve	62.1	51.4	8.4
Objective 3a	Minimize shifts without resident to learn	60.8	35.4	0.1
Objective 3b	Maximize shifts without resident to learn	54.5	35.6	10.3
Objective 4a	1. minimize working shifts; 2. minimize shifts without reserve	48.4	10.2	8.6
Objective 4b	2. maximize working shifts; 2. maximize shifts without reserve	67.6	42.4	3.8
Objective 5a	1. minimize working shifts; 2. minimize shifts without resident to learn	57.2	38.4	0.7
Objective 5b	2. maximize working shifts; 2. maximize shifts without resident to learn	67.6	42.4	9.4
Minimum	Objective's a (minimizations)	48.4	2.1	0.1
	Objective's b (maximizations)	60.8	38.4	8.8
Maximum	Objective's a (minimizations)	54.5	35.6	3.8
	Objective's b (maximizations)	67.6	51.4	10.3

Table 5: averages of the optimization criteria-experiments (n=1, 23 and 64)

We made no difference between attendant- and resident-physicians: both are currently working too much according to themselves and the management. A further reason for this equal weight is that in both cases the department needs two nurses as well. The management team judges the scheduling of the residents to learn with the consultation types as more important than working more shifts. A further argument for its importance is that the variation of the objective in this aspect is much smaller than in the others. We set this value 1.5 times as high as the weight for working an extra shift. The program record scheduling a resident to learn also as a working shift, so the net benefit in the objective is 0.5 working shift. Finally, we have to determine the weight of NoReserve. This is the hardest to judge as its effect is indirect: through the number of shifts that the department skips on the access time. The variance in offered capacity is a main factor in the access time (Overbeek, 2008). Furthermore, the system in which the master schedule is leading rather than the physicians is new to the department. To make a good start it is important that all shifts have a reserve. We judge this aspect as important for these two reasons and the providing weight factor set higher than the weight factor for the residents to learn. This gives the complete weighting factors we use: $c_1=1$, $c_2=100$, $c_3=1.5$ and $c_4=2$.

4.6 Calculation time

This section shows some results of simplified versions of this model to illustrate the computational complexity.

We survey four different simpler versions of the problem. Problem 1 is the relaxed linear problem, meaning that all integer variables can have any positive value and all binary variables can have any value between 0 and 1. Afterwards, we survey three problems with only a part of the objective, removing thereby also a part of the relevant variables. Problem 2 ignores the reserves and residents to learn and minimizes the number of working shifts. Problem 3 focuses on minimizing the number of shifts without reserve. Problem 4 is minimizing the number of shifts with ERCP- and EUS-consultations without resident to learn. We include the complete problem as Problem 5, although we never prove optimality.

Table 6 shows for these five problems and both solvers the percentage of cases in which we find a solution, the found objective and the time it costs. These three numbers are an average of the three different data case-methods: the AMC case, the average over 54 data cases in which we vary the demand in 76 cases for which we vary six parameters; these three numbers are weighted averages over the 132 cases. If we could find no solution, we do not take the data case for that problem and both solvers into account in the shown averages.

Table 6 shows that CPLEX performs much better than XA for the MILPs: CPLEX finds for more cases a solution within the given time and if both solvers do find the optimal solution the calculation time is much smaller. When we cut both solvers off after 30 seconds, the solution found by XA is always equal or worse than the solution found by CPLEX. However, XA is faster than CPLEX for the LP-relaxation. A possible reason for this is that the CPLEX-solver has to connect to a distant license server. Considering the differences between the problems, remarkable is that problem 4 is quite easy: we need to find a resident only for a limited number of shifts. For CPLEX Problem 3 and especially Problem 4 is a lot easier than problem 2, as we found often a solution which equals the lower bound, which is in all used cases 0.

Problem	Description	Weight factors		CPLEX	XA
Problem 1	LP-relaxation	$c_1=1$ $c_2=100$ $c_3=1.5$ $c_4=2$ (LP)	Solution found	100%	100%
			Objective	21	21
			Time (s)	2.0	0.6
Problem 2	Working shifts without reserves and residents	$c_1=1$ $c_2=100$ $c_3=c_4=0$	Solution found	100%	3%
			Objective	36	40
			Time (s)	16.4	30.7
Problem 3	Shifts without reserves	$c_1=c_4=0$ $c_2=100$ $c_3=1$	Solution found	100%	0%
			Objective	-	-
			Time (s)	-	-
Problem 4	Residents to learn	$c_1=c_3=0$ $c_2=100$ $c_4=1$	Solution found	100%	2%
			Objective	0	0
			Time (s)	0.4	6.8
Problem 5	Complete problem	$c_1=1$ $c_2=100$ $c_3=1.5$ $c_4=2$ (MILP)	Solution found	54%	0%
			Objective	-	-
			Time (s)	-	-

Table 6: Results experiments different problems (n=1, 57 and 74)

We conclude that we can seldom solve even the simpler version, Problem 3, by XA within small time. However, in reality the department only will solve this problem once every three months. Therefore, the acceptable calculation time is a couple of hours. We investigate the AMC case more careful with this in mind. XA could find no solution at all in nine hours for Problem 2, so we can conclude that direct solving with XA is still not an option.

The problem consists of nine attendant-physicians, six resident-physicians, six rooms, ten shifts and twelve consultation types. This gives a total of 2,940 decision variables, 152 auxiliary variables, 4,241 constraints and 41,509 non-zeros. We have shown that this problem is computationally hard, although it is not that large. A possible reason for this is the dispersed solution space.

We cannot solve the described model without CPLEX. To solve the problem within a reasonable time a different set-up of the model would be the easiest. We test two other descriptions, as promised in Section 4.3. Description 1 is the one given in Section 4.3 and used for the previous experiments. In Description 2, we do not specify the reserves to the room. This means that the RAtt and RResid-variables depends only on the shift and the physician, and not on the room as in the model depicted in Section 4.3. As the number of variables decreases, the new model could be easier to solve than the old one. Description 3 is the addition of a new variable, Y_{str} , which would be 1 if $X_{str} > 0$ and 0 if $X_{str} = 0$. This variable could replace X_{str} in all constraints where the parameter M shows up and thereby improve the lower bound of the relaxation.



Figure 9: photo of the stomach made during an endoscopy, with endoscope visible (source: <http://flickr.com/photos/testastretta/327749879/>)

The alternative definitions did increase for both alternatives as can be seen in Table 7 contrary to our expectations. The table shows the objective found after 60 seconds of calculation time. We did no research into the cause of the worse performance of the alternative descriptions.

Number	Explanation	Objective
Description 1	Depicted description	60.4
Description 2	Without specifying the rooms for reserves	63.3
Description 3	Addition of variable Y_{str}	61.9

Table 7: results experiments different descriptions (n=1, 37 and 20)

5. Heuristic

The model we describe in Section 4.3 is complete, but hard to solve. The solver CPLEX is able to find good solutions in acceptable time as we have seen in Section 4.6, but takes long to prove optimality. We define solutions as good when they meet all demand and plan for most of the shifts a reserve and, if needed, a resident to learn. We want another method to solve the problem, as CPLEX is not available in the AMC and expensive to buy. We could first solve the model focussing on the number of shifts, without reserves and residents to learn, and add them later. However, as we have seen in the last section also that model is too hard to solve with XA. Therefore, we built a heuristic that solves the problem of scheduling consultations and physicians in steps. Section 5.1 describes this heuristic and Section 5.2 tests it. This gives feasible solutions to the problem, but this construction heuristic does not find reserves and residents to learn. We deal with that problem afterwards: Section 5.3 describes a heuristic on this subject; Section 5.4 shows the performance of different configurations of this heuristic on this part. Although we call the parts a construction and improvement heuristic, we made the division not on that aspect. The last step we propose for the heuristic of Section 5.1 is an improvement step, and one can see the first step of Section 5.2 as a construction step. We make the division at planning the consultations with a physician and planning the extras – the reserves and residents to learn. We call this heuristic 1; we will introduce heuristic 2 in section 6.2. Appendix 1 shows the complete MILPs used for all steps of the heuristic.

5.1 Construction heuristic

This section describe a heuristic that schedules consultations and physicians. The results are comparable to problem 3 of Section 4.5, which is hard to solve without CPLEX. To get solutions quicker and with a less sophisticate solvers, this section describes a construction heuristic based on three steps. This heuristic does give feasible solutions to the problem, although the quality is low, as it schedules no reserves or residents to learn– even if they are available without any other change to the master schedule. We describe also an improvement step for this problem, but the next section proves it is unsuccessful.

The first step is to plan the consultations without planning physicians. We add some constraints to increase the chance at a good total solution. The second step is to find physicians for the filled shifts. The heuristic skips shifts for which it finds no physician and the consultations of these shifts become unplanned. We replan these consultations in the third step. Figure 8 depicts these steps. We repeat these steps multiple times with different random numbers in the first step to increase the chance of a better solution. The next section surveys the influence of this number of repetitions.

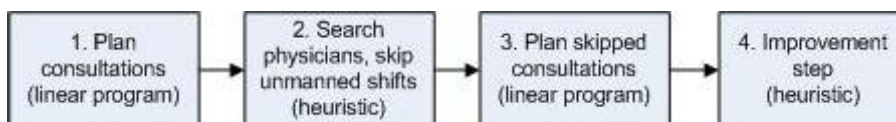


Figure 10: steps of heuristic

We choose to plan the consultations first and then continue with the physicians. We will discuss four other stepwise heuristics below, and give reasons why we choose this one.

An alternative is to plan the physicians first and then continue with the consultations. We did not choose this second option, because the MILP has more consultation related constraints than physician related ones. Especially the needed spread of certain consultation types over the week makes this sequence hard. A further difficulty with this kind of heuristic is that certain consultations can only be performed in one room and by a small group of physicians. This means that the heuristic needs to spread the shifts of a couple of physicians correctly over the rooms.

A third possibility for building a heuristic is to start with linking physicians and consultations and then plan them on shifts and rooms. This is hard to do, as we would have to define a new type of variable. This new type would consist of many new variables, as we would have to link every physician with all consultation types. It is hard to find clever ways to make the good links between consultations and physicians. Also planning the consultation-physician combinations into the shifts and rooms would be very hard, as we would have to deal with most of the constraints in this phase.

A fourth option for this problem is to build the master schedule shift by shift. However, the problem is too complex for this kind of heuristic. Making the master schedule at this method would certainly give problems later on, as the heuristic could end up with the hard-to-plan consultations and the least useful physicians for the last shifts. Even if we could prevent this, some physicians need to be planned only for one or two consultation types to plan all demand.

As we have seen, other methods to make a master schedule are probably worse than the chosen one. Therefore, we use the consultations-first method.

Step 1 - consultations

The first step is to plan the consultations in rooms and shifts, without dealing directly with the physicians. We do this with a MILP, with the consultations-related constraints of the complete program. We add some extra constraints to increase the chance that we can schedule the physicians on the planned shifts. Another reason for these extra constraints is to make the outcome of this step dependent of random variables, thereby make it possible to generate different possible schedules and look which one gives the best total solution. The reason for this multiple repetitions is that the difference between a good and a bad solution is not clear at the beginning of the construction process. We describe eight extra consultations and test them in Section 5.2. We decide after these tests which of these extra constraints to include.

Extra Parameters

Maxshiftdur Minimum shift time without penalty. We generate them randomly with a uniform division between 0 and 2.5.
 r_1, r_2, r_3 Weight factors in the objective for relaxing the extra constraints. We generate these randomly with a uniform division between 0 and 50.

Extra auxiliary variables

except_{str} =1 if no two physicians are available for this consultation type and it is planned anyway.
 =0 otherwise
 echos_s =1 if consultation type EUS are planned in a shift
 =0 otherwise
 open_{sr} =1 if total duration of shift s in room r is longer than maxshiftdur
 =0 otherwise

Extra constraints

(s) The first extra constraint makes sure that the heuristic only plans consultations if there is at least one physician available to handle them. We built an exception possibility to make sure the heuristic can always plan all consultations.

$$X_{str} \leq 7 * \sum_p (AvailPhys_{ps} * PosPhys_{pt}) + ResidPos_{dt} * ResidAv_{sd} - 1 + excep_{str} \quad \forall s, t, r$$

(t) The second extra constraint tries to prevent too many consultations that are difficult to plan in one shift.

$$\sum_{r,t} X_{str} * \left(30 - \sum_p PosPhys_{pt} - \sum_d ResidPos_{dt} \right) \leq r_1 * 100 + 1000 \quad \forall s, r$$

(u) This constraint penalizes the number of shifts opened, as there need to be physicians for them.

$$\sum_t X_{str} \geq 7 * openshift_{sr} \quad \forall s$$

(v) There are a limited number of physicians who can do consultation type EUS, so it is wise to concentrate those in a limited number of shifts. This constraint will reward doing so. Note that we can only plan EUS in one of the available rooms.

$$X_{str} \leq M * echos_s \quad \forall s, t \quad EUS, r \quad Room127$$

(w) Only one consultation type is possible in one of the rooms. The heuristic plans all except one shifts of this type in that room to give some extra space to shuffle with consultations. The shift in one of the rooms is necessary to perform certain consultations within this type.

$$\sum_s X_{str} \leq Demand_t - 1 \quad \forall t - crc, r - Room120$$

(x) This constraint provides an incentive to plan consultations in more shifts. This is useful as it increases the chance that the next steps can still plan all consultations if it needs to skip some shifts if it could find no physician for it.

$$\sum_t (X_{str} * Dur_t) \geq Maxshiftdur * open_{sr} \quad \forall s, r$$

(y) This constraint makes sure the heuristic plans all demands. This is a condition for a good solution and is normally the case.

$$DifDem_t = 0 \quad \forall t$$

(z) We add an extra constraint that should have a slightly negative influence to quantify the need for such constraints better. This constraint forbids planning one type of consultation in one room, but there are enough rooms left for that consultation type and enough consultations for that room.

$$\sum_s X_{str} = 0 \quad \forall t - fluores, r - Room127$$

Objective

The objective is the weighted sum of the difference between the demand and the planned consultations, the number of shifts with consultation type EUS, the number of exceptions made to constraint s and the number of shifts with lasts longer than maxshiftdur.

$$z = c_3 * \sum_t (DifDem_t * Weigth_t) + r_1 * \sum_s echos_s + r_2 * \sum_{str} excep_{str} + r_3 * \sum_{sr} open_{tr}$$

Other variables, parameters and constraints come from section 4.3.

Step 2 - physicians

The second step plans the physicians. The heuristic starts with scheduling for every shift an attendant as supervisor. The heuristic prefers physicians that cannot do many different types of consultations and spread the supervisor shifts among the physicians. Afterwards the heuristic tries to find a physician for every different shift that contains consultations. The heuristics first looks if residents can do this shift, and otherwise prefers the physician who can handle the least number of different consultation types. The heuristic deletes these consultations from the proposed master schedule if no resident or physician can handle this shift. A heuristic rather than a MILP performs this step, as a MILP costs too much time due the large amount of comparable solutions.

Step 3 – removed consultations

The third step tries to schedule the consultations that the previous step deleted on a shift a physician is available. To make this possible all consultations are scheduled again, but normally most have to pick up their old place due all constraints and the fixed physicians. Therefore, we made a MILP with the physicians fixed, all consultation-related constraints and physician related constraint a1, which deals with the qualifications of physicians.

Extra parameters

- XFixed_{str} Values of variable X receiving from last step.
- PhysFixed_{srp} Values of variable Phys receiving from last step.
- ResidFixed_{srd} Values of variable Resid receiving from last step.

Extra constraints

(aa) This constraint fixes the planning of the physicians, in this case to replan the consultations.

$$Phys_{srp} = PhysFixed_{srp} \quad \forall s, r, p$$

$$Resid_{srd} = ResidFixed_{srd} \quad \forall s, r, d$$

Objective

$$z = \sum_t (DifDem_t * Weigth_t)$$

Other constraints, other parameters and all variables come from section 4.3.

Improvement step

We experiment with a fourth step. This step should reduce the number of shifts, as it can occur that there are half filled shifts. The program tries to remove a shift and uses the program of step 3 to divide the consultations among the remaining shifts. We keep the new solution if it is possible to plan all consultations; otherwise, we restore the old situation. We repeat this whole process multiple times. This heuristic is a time consuming one due the repetitive calling of earlier steps. The next section discusses the results of this step.

5.2 Experiments construction heuristic

We have to optimize different settings for an optimal working of the heuristic. The most important ones are which extra constraints are useful in the first step and the number of repetitions of the heuristic. We experiment to determine those in this section. We do the

experiments to these two factors separately as the needed number of experiments would be too large otherwise.



Figure 11: physician and nurse performing an ERCP, source: <http://www.flickr.com/photos/21974686@N03/2448545470/>

We need to test 256 different set-ups for the constraints, as all combinations of the described eight constraints were tested. The constraints do influence each other, so only experiments with one constraint each are not sufficient. We try all possible configurations on ten different data cases with nine repetitions. We use the best performing configurations to determine the ideal number of repetitions. This second test offers also the possibility to test these configurations better. This test repeats the heuristic hundred times for the chosen configurations and the best solution found until then is stored.

The selection of the best configurations required time-consuming experiments for all 256 possible configurations. We

use the results of these experiments to calculate three performances: one for the AMC case and two averages over the varying demand and varying parameters-data cases. The configurations that we use for the next test are the ones with the lowest average over the three types of data cases. We extend this set with the configurations that score the lowest in one of the three performance indicators. Table 8 shows these configurations and indicates of each constraint of it is included in the configuration. We see that the strange Constraint z is normally not included, but turns out to be essential for generating the best solutions with

varying demand. The configurations with the lowest average include always the Constraints s, w and y.

		Constraint s	Constraint t	Constraint u	Constraint v	Constraint w	Constraint x	Constraint y	Constraint z	Average	AMC case	Varying demand	Varying Parameters
Configurations	1	In	In	In	In	In		In		18%	0%	31%	21%
	2	In	In		In	In		In		21%	0%	42%	21%
	3	In		In		In		In		21%	0%	42%	21%
	4	In				In		In		21%	0%	42%	21%
	5	In		In	In	In		In		21%	0%	42%	21%
	6	In			In	In		In		21%	0%	42%	21%
	7	In		In	In	In				21%	0%	42%	21%
	8	In	In							32%	44%	50%	1%
	9					In	In		In	36%	29%	17%	61%

Table 8: configurations we analyze further with performance as average percentage between minimum and maximum found objective by a heuristic (n=3, 5, 5)

Figures 9 and 10 show the results of the experiments to the needed number of repetitions. Not only for the varying demand and parameters, but also for the AMC case we did multiple experiments, as the heuristic depends on random. The chance of finding a better solution is larger in the beginning than in the end, but stays significant. Therefore, the number of repetitions one needs to choose as large as possible: a better solution is still possible, also after eighty earlier repetitions. The chance of improving a solution is much higher with the AMC case than with others. We can see that Configuration 9 performs badly on average, just as it did in our last test. It is remarkable that Configuration 8 is among the best, as it did not perform that good in the first test. The other configurations are close to each other. We can conclude that there is little room between the configurations 1 until 8 and that many repetitions are useful. It would be even better to perform the heuristic multiple times with different constraints, so further enlarging the chance at a good solution.

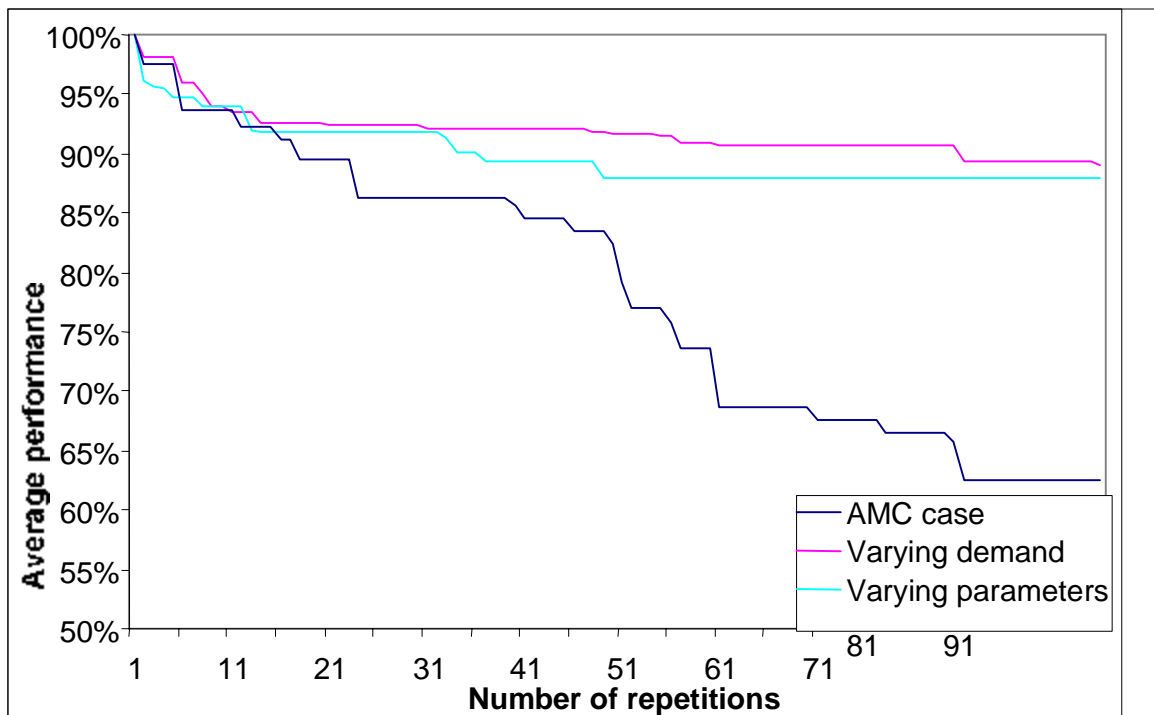


Figure 12: performance of the heuristic for the types of different data sets against the number of repetitions (n=6, 5, 7)

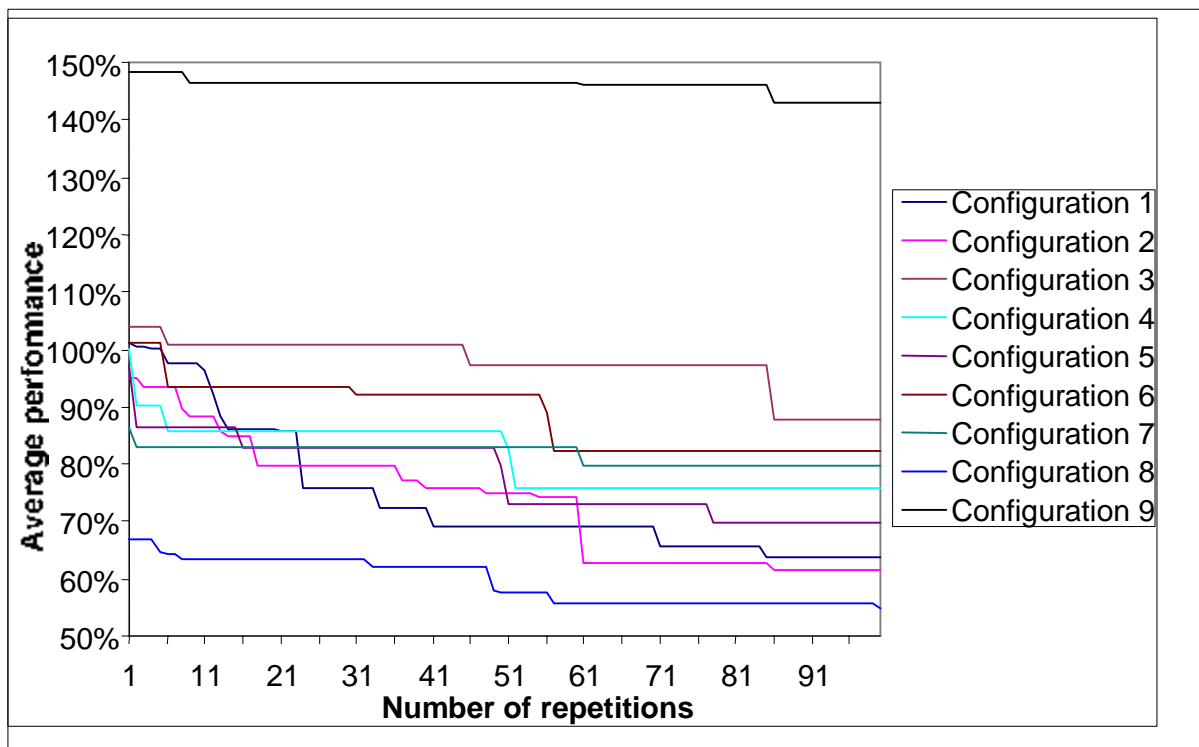


Figure 13: performance of different configurations against the number of repetitions (n=6, 5, 7)

After determining the ideal settings of a construction heuristic, we look to the proposed improving step. We choose to generate a solution by fifty repetitions from configuration 1. The improvement step tries to improve the solution and the optimization criteria before and afterwards are stored. We do this ten times for the AMC case, twenty times for cases with varying demand and twenty cases with varying parameters. In none of these fifty cases, the proposed step makes any improvement, so we abandon this improvement step.

5.3 Improvement heuristic

We built a second heuristic to find reserves and resident heuristics, as the construction heuristic gives a schedule with consultations and physicians. We search not only for reserves and for residents to learn when they are possible, but also try to reschedule physicians and consultations. We propose different heuristics and we will test them in Section 5.4.

Adding

The first step of this heuristic is to search reserve physicians for the planned shifts and resident-physicians to learn with ERCP and EUS-consultations. We perform this step by fixing the consultations and the scheduled physicians and run a MILP with all physician-related constraints.

Extra parameters

$X_{Fixed_{str}}$ Values of variable X receiving from the construction heuristic
 $Phys_{Fixed_{srp}}$ Values of variable Phys receiving from the construction heuristic
 $Resid_{Fixed_{srd}}$ Values of variable Resid receiving from the construction heuristic

Extra constraints

(v) This constraints are similar to those of step 3 of the construction heuristic, but now also the consultations are fixed. In this case this focuses the problem on the reserves and residents to learn the consultation types ERCP en EUS.

$$X_{srt} = X_{Fixed}_{str} \quad \forall s, t, r$$

$$Phys_{srp} = Phys_{Fixed}_{srp} \quad \forall s, r, p$$

$$Resid_{srd} \geq Resid_{Fixed}_{srd} \quad \forall s, r, d$$

Objective

$$z = c_3 * \sum_{srv} NoResid_{sr} + c_4 * \sum_{srv} NoReser_{sr}$$

Other parameters, other constraints and all variables come from section 4.3.

Within shifts swap

This step consists of a heuristic that optimizes the physicians in one shift among the rooms. Aim is to reduce the shifts without reserve-physician or without resident to learn. The heuristic tries to optimize all shifts with at least one shift without reserve or extra resident. We fix all consultations and include all possible physicians in such an optimization step. We use these as input for the MILP. We cut this MILP off after 30 seconds, as the program sometimes does not find a feasible solution within acceptable time. The heuristic keeps the old solution in those cases. We made a complete new MILP for this step, but this one is similar to the described complete MILP. However, this MILP is limited to one shift with all

physician-related constraints. Optimizing within shifts is a clearly separated sub problem, which we can usually solve quickly to optimality.

Physician swap

A possibility to expand the neighbourhood in which the heuristic searches is to swap two physicians from unrelated shifts.

Consultation swap

Another procedure to improve the solutions is to swap all consultations from two different shifts.

Consultation move

A third possible procedure for improvement is to move all consultations from one shift to another. The heuristic searches a new physician, as just taking the old one will decrease the chance that a move is feasible.

These last three steps share some characteristics. All three do not improve the solution directly, but can give a new basis to solutions for the within shift optimizer. Another common characteristic is the small number of feasible changes due the large number of restrictions. A third characteristics is we do not accept changes if they decrease the quality of the solution after the within shifts-optimizer has been used.

We will describe why we choose these steps in this paragraph. The adding is a simple search for the easy ones and therefore logical to include. The optimizing within shifts is the best-defined sub-problem. The other three steps are the three methods to search a neighbouring solution.



Figure 14: an EUS is performed, source: <http://www.flickr.com/photos/21974686@N03/2447812991/>

5.4 Experiments improvement heuristic

We test the improvement steps described in last section. The adding of reserves and residents to learn is always the first step of this heuristic, as it is a simple search for the easy ones. We compare four different configurations to the result of the construction heuristics expanded with the adding step. The first configuration is to improve it only with the only within shifts swap. The other three configurations also use that step, as the other tested steps change rather than improve the schedule. These changes could enable more improvements. The second configuration uses the physician swap-step, the third swaps consultations between two shifts and the last moves consultations from one shift to an empty one. Table 9 shows the results of these experiments for the different types of data cases. The improvement is the relative decrease in the value of the optimization criteria.

	Average improvement	AMC case	Varying demand	Varying parameters
Only within shifts swap	2.18%	0.00%	0.04%	4.32%
Physician swap	2.28%	0.00%	0.05%	4.52%
Consultation swap	2.28%	0.00%	0.05%	4.52%
Consultation move	2.31%	0.00%	0.05%	4.57%

Table 9: results experiment improvement heuristic

Table 9 shows that the only within shift swap does largely improve the solution. In that respect the other three steps do less, but can still improve the solution. These steps cost a lot of time as they use the within shifts-swap repeatedly. We conclude that the within shift-swap is always useful to do, but that the other steps are disputably. Remarkable is that the more the case differs from the AMC case, the larger the improvements are; the heuristic does not find any improvements for the AMC case, some improvements for the varying demand case and large improvements for the varying parameters case. A possible explanation for this is that the input parameters in the AMC case are optimized: the items with the largest demand also have the most physicians and rooms available to do them. If we randomly generate those parameters the total planned demand will on average be lower. Therefore, there is more space to swap or move with consultations and physicians.

6. Results

This chapter reports the results of the concluding experiments of this research. We start with results of more simple calculations that we did next to or in support of building the model in Section 6.1. Section 6.2 compares the results of the described heuristic with a CPLEX-based one. This is followed by a sensitivity analysis to these conclusions in Section 6.3. We investigate the influence of the different constraints using CPLEX in Section 6.4. We use these results to determine if some changes to soften some constraints are worthwhile. Section 6.5 compares the current master schedules with master schedules made with the iterative approach described in this report.

6.1 General results

Skipping of shifts

Overbeek (2008) concludes that the skipping of shifts is a main factor in the increasing access time. In the current situation the endoscopy rooms skips 231 shifts a year, excluding national holidays. A major cause is the unavailability of physicians. HGE physicians are unavailable about 16 week a year for patient care (XCare, 2008). The physicians are hardly tuning their unavailability to each other, so their unavailability leads often to the skipping of shifts.

Demand policy

A method to deal with the long access time and business of physicians is to reduce the demand. This paragraph looks at the number of patients that request simple consultations from outside the service area of the AMC. The department can ask them to go to other hospitals, as the AMC does not benefit from them. HGE specialists performed in 2006 9,338 consultations at the department. Of those 6,716 were patients who lived outside the service area of the AMC. Reasons for this large number are the third line healthcare function of the AMC and the popularity of the department due its medical expertise. We investigated how much of this group of 6,716 consultations are simple ones and which can be carried out in the hospital closest to the patient. OGDs, colonoscopies and sigmoidoscopies are simple consultations, if they do not belong to any research program. Patients referred to the AMC by other hospitals for these consultations are mostly part of a research program. General practitioners refer in total 228 of those consultations for patients outside the service area to the AMC. The HGE and internal medicine-outpatients department refer another 896 simple consultations for patients from outside the service region to the endoscopy department. These two groups of patients are a low priority. Other departments of the AMC also refer patients to the endoscopy department, but an endoscopy is often a small step in a complete diagnosis and treatment-process in the AMC for these patients, so the endoscopy department has to handle them.

Division of consultations

The endoscopy department made 9,338 appointments for an endoscopy by an HGE specialist at the AMC in 2006. These numbers will change in the nearby future as a new screening program will start in July and will require around 530 extra colonoscopies per year. Furthermore, the department will advise patients mentioned in last paragraph to go to another hospital. Incorporation of these changes will achieve the demand data as shown in Table 3. The department can then expect annually 7,905 consultations, of which 3,905 will be simple ones. Of those 3,905 simple consultations, around 926 are day urgency and another 245 week

urgency. Other important consultation types are ERCP (920 consultations), EUS (757), CRC (744) and Fluores (698).

Backlog

The access time is now 16 weeks, which means that there are 16 weeks of backlog. If the department is able to offer enough capacity to meet the access time-standards in the stable situation, this backlog will be still there. The capacity offered is larger than the demand in this situation, to be able to absorb variation in this demand. The access time will therefore decrease slowly. We predict the rate with a simple calculation based on eight assumptions listed in Appendix 2. We predict that it will last until 11 April 2010 when the department has removed the backlog and will meet the standards. If the department adds three extra shifts to the master schedule from 1 August 2008 the backlog will be removed on 17 February 2009. Appendix 2 lists the calculation and the complete results, next to the assumptions.

6.2 Results heuristics

The ideal settings of heuristic 1 are determined in the previous chapter; now it is time to test the results from the developed heuristic against another heuristic and the optimum. Figure 11 depicts the used steps of heuristic 1, which shows that we use all steps of the improvement heuristics. The construction heuristic works with hundred repetitions and with Configuration 1.

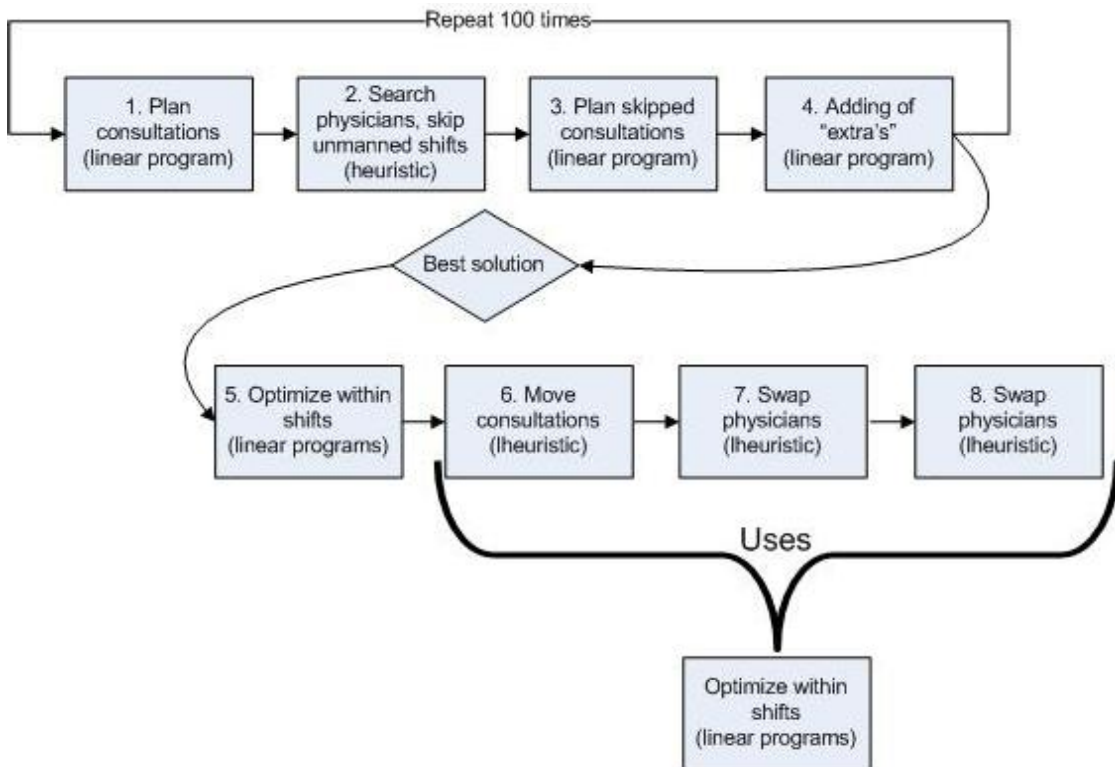


Figure 15: steps of the complete heuristic

We compare two heuristic with each other and a lower bound. We developed heuristic 1 in Chapter 5. Heuristic 2 is the best-found integer result of a five-minute run of CPLEX. This

five-minute CPLEX run gives also the lower bound; we compare both heuristics to that lower bound. We see this in Figure 12, where the runtime of CPLEX is on the horizontal axis and the value of the optimization criterion is on the vertical one. The horizontal line is the results found by heuristic 1, the decreasing line is the best-found integer solution by CPLEX and the increasing line is the lower bound of the MILP. We stop CPLEX after a runtime of five minutes, this is the vertical line, and we obtain the performances of heuristic 1 and 2 as shown with the arrows. These are upper bounds of the performance, as the optimal solution lies between the lower bound and the best solution found by one of the heuristics.

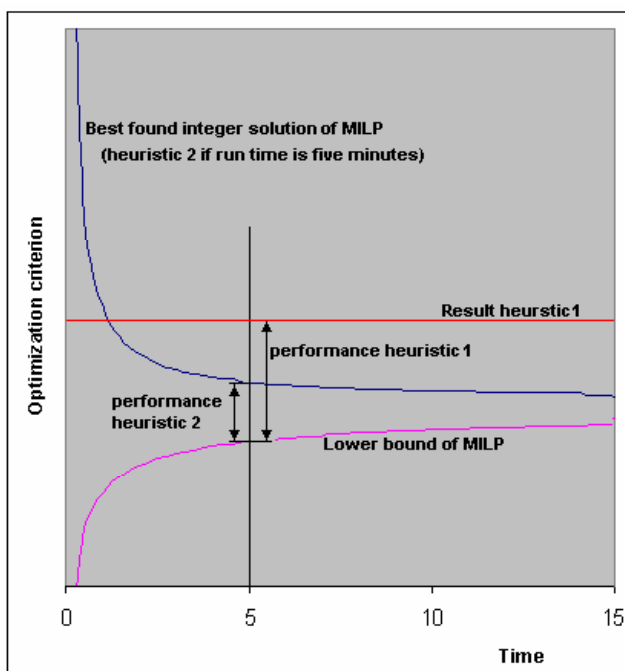


Figure 16: meaning upper and lower bound performance

Table 10 shows that the results of heuristic 1 vary largely per data case; sometimes the quality of the solution found by the heuristic is better than the found solution by heuristic 2, but often the heuristic leaves too much demand unplanned. Heuristic 1 sometimes even finds no feasible solution for a data set from the varying parameters-case. We record this as all demand unplanned, but also no filled shifts or missing reserves and residents to learn. Heuristic 1 performs on average the best on the AMC case. A possible cause for this is that we optimized the heuristic with the AMC case as the most important data case. The performance varies a lot when we randomly generate data cases, especially if they vary more from the AMC case. This means one cannot use this heuristic in general, but one can use it for this AMC case.

	Average	Data set 1: AMC	Data set 2: varying demand	Data set 3: varying parameters
Heuristic 1	29079%	17%	11241%	75980%
Heuristic 2	13%	10%	13%	18%
Computation time heuristic 1	0:07:58	0:11:00	0:07:16	0:05:38

Table 10: the results of the heuristics in per cent above the lower bound found in five minutes by CPLEX. The computation time of heuristic 2 is always five minutes. (n=12, 27 and 30; more AMC cases as heuristic 1 depends on random)

Table 11 shows the results of the same experiment, but splits the results to different optimization criteria. It shows that heuristic 1 uses less shifts, but fails on providing reserves and residents to learn. The earlier mentioned finding of no feasible solution at all explains the low number for working shifts for data set 3 and heuristic 1.

Data case	Heuristic	Total	Demand	Working shifts	Reserves	Residents to learn
Average	Heuristic 1	12109.4	120.5	45.3	2.3	4.6
	Heuristic 2	55.7	0.0	54.4	0.0	0.6
Data set 1: AMC case	Heuristic 1	66.0	0.0	58.0	4.0	1.0
	Heuristic 2	62.0	0.0	62.0	0.0	0.0
Data set 2: varying demand	Heuristic 1	5837.4	57.8	52.1	1.3	0.9
	Heuristic 2	58.0	0.0	54.5	0.0	1.7
Data set 3: varying parameters	Heuristic 1	30424.8	303.7	25.7	1.5	11.8
	Heuristic 2	47.0	0.0	46.8	0.0	0.1

Table 11: results of heuristics splitted up to optimization criteria. (n=12, 27 and 30)

6.3 Sensitivity analysis

We did two different sensitivity analyses. First, we analyzed the performances for variations in the demand. Afterwards, we did some tests about some probable scenarios in real life. Experimenting with all possible changes in physicians' and rooms' possibilities and unavailability's is nearly impossible due to the large number of possible changes.

The first experiment is about variations in the demand for one consultation type at a time. We experiment with the demand for one consultation type 20% up or down. This makes no difference for five of the thirteen consultation types, as the demand for those types is one or two. The case with only 80% of the demand for the consultation type 'day urgency' is infeasible as the minimal number of consultations per day cannot be met then (Constraint g). All cases that are infeasible or equal to the AMC case are not considered; the average performance of the other cases is shown in table 12. It shows that heuristic 2 is more reliable than heuristic 1. Heuristic 1 sometimes fails to plan all demand for cases with lower demand than the AMC case.

Data cases	Heuristic	Performance	Objective	Demand	Working shifts	Reserves	Residents to learn
Total	Heuristic 1	1944%	1154.5	10.9	60.9	2.4	1.7
	Heuristic 2	12%	63.3	0.0	63.3	0.0	0.0
Cases with lower demand	Heuristic 1	1476%	879.5	8.1	61.0	2.0	1.5
	Heuristic 2	12%	62.3	0.0	62.3	0.0	0.0
Cases with higher demand	Heuristic 1	2466%	1468.9	14.0	60.9	2.9	1.9
	Heuristic 2	13%	64.4	0.0	64.4	0.0	0.0

Table 12: performance and different optimization criteria for variations in demand

The second experiment is about scenarios that are likely in the AMC endoscopy department. We distinguished four of those.

- Scenario 1 is that one of the residents is becoming an attendant, as this will happen in reality. This change will bring in some differences in time spending and possibilities of this physician.
- Scenario 2 is the hiring of a new attendant to decrease the workload for the physicians, so the maximum number of shifts spent in the department for other physicians will decrease.
- Scenario 3 is the planning of nine hours of extra simple consultations to bring the backlog within short time within the standards as explained in Section 6.1 and Appendix 2.
- Scenario 4 is a combination of those two: we include a new physician and extra demand. This is a logical combination, as reducing the backlog would be the most important reason to hire an extra physician now. Another department will probably require a new attendant somewhere in 2009.

Table 13 shows the results. The results of the first two scenarios are comparable to the results of the AMC case. Heuristic 1 performs badly if we try to plan the extra hours, with or without the extra physician. It is unexpected that heuristic 2 performs less on the easier case of scenario 4 than that of scenario 2. This is just based on just one case in which heuristic 1 could not find an adequate solution.

Data case	Heuristic	Performance	Objective	Demand	Working shifts	Reserves	Residents to learn
Scenario 1	Heuristic 1	24%	70.0	0.0	62.0	4.0	1.0
	Heuristic 2	12%	63.0	0.0	63.0	0.0	0.0
Scenario 2	Heuristic 1	21%	67.0	0.0	61.0	4.0	0.0
	Heuristic 2	13%	63.0	0.0	63.0	0.0	0.0
Scenario 3	Heuristic 1	2615%	1568.5	15.0	62.0	3.0	1.0
	Heuristic 2	14%	66.0	0.0	66.0	0.0	0.0
Scenario 4	Heuristic 1	1680%	1069.9	10.0	62.7	3.3	1.1
	Heuristic 2	10%	66.0	0.0	66.0	0.0	0.0

Table 13: performance and different optimization criteria for different scenarios

We have seen in this and the last section that the results of heuristic 1 are very sensitive to the given data case. This strongly varying performance of the developed heuristic can have two possible causes. The first possible cause is the heuristic itself. This reason is unlikely in this case, as the repetitive calling of the first part has to prevent a varying performance. The second possible cause for the varying performance is the solution space. Two neighboring solutions could have a vastly different quality, for example as one of both could not plan all demand. Furthermore, a highly restricted, dispersed solution space makes good improvement heuristics hard to construct, as feasible neighboring solutions are hard to find. These two reasons make it very hard to make a good heuristic for the endoscopy department. Making a general applicable heuristic for problems like this is even harder and seems to be nearly impossible.

6.4 Expanding the equipment

There are many constraints as Section 4.3 shows. We survey the influence of those constraints on the quality of the solution in this section. We can use the results to evaluate whether it is profitable to relax some of the constraints. Appendix 3 shows that removing most of the constraints does not give better solutions. The constraints that prevent a better solution are impossible to relax. Consultations cannot be done without physician, days cannot be made longer and the department does not want to change the current physicians and their capabilities. The only constraint that prevents a better solution and can be relaxed is expanding the available equipment in the rooms. More specific there are two interesting cases: purchasing a new x-ray system or an extra OGD-suction. We survey these two possibilities as we expect them to be (close to) worthwhile. A new x-ray system can be useful as the only room with a good working one, room 124, is heavily used. We could merge two consultation types in the planning with a new x-ray system. We choose to investigate an OGD-suction as it is relatively cheap and OGDs are a common consultation, which appear in six consultation types. We look at the influence of adding this equipment in the following two paragraphs.

The first option that we look at is purchasing a new x-ray system for room 213. There is currently a very old one, which the department uses only in case of emergency. A new one would make it possible to do consultation type oesdil in that room. This would make it possible to merge it with the week urgency category. This merging of categories improves the efficiency. However, moving the consultation type oesdil to the more pressured room 213 lowers the possibilities and thereby the efficiency. We discover with the iterative approach that the master schedule does not improve. Another advantage would be that the department can do consultation type ERCP in room 213 in emergency cases or if the x-ray system in room 124 is broken.

Another possible relaxation of the room-constraint would be to install suction-equipment in room 120 to make it possible to do OGDs there. As OGDs are part of different consultation types, the number of consultation types that could be done in room 120 rises from one to seven. However, this extra planning flexibility does not improve the master schedule.

We also try to add both pieces of equipment. This can improve the situation, as due the new suction-equipment we could move some consultation types from room 213 to room 120. This gives the possibility to move consultation type oesdil to room 213. However, also adding both types of equipment does not improve the master schedule.

6.5 Results compared to current

This section compares the results of the iterative approach of MILP and simulation with the current master schedule. We used Heuristic 2 for the MILP-step, so a five minutes run of CPLEX. The simulation model used is the one made by Overbeek (2008).

We compare these on the optimization criteria regarding the number of working shifts as well as the number of shifts with a missing resident to learn. Furthermore, we look at the percentage of patients within the standards for different consultation types, the number of double bookings and the amount of offered capacity. Double bookings are patients that need a consultation within one day and for whom no regular place can be found. In the comparison

we do not include the optimization criteria unmet demand and number of shifts without reserves. The demand is always met, so comparing it is not useful. The number of shifts without reserves cannot be compared, as the current master schedule does not contain them. The number of skipped shifts will decrease due the introduction of reserves; therefore, we lower the number of skipped shifts also in the simulation model. The new number of skipped shifts – as included in the simulation model - is chosen on basis of the length of important congresses which a large number of physicians will attend.

We will compare with both the 2006 demand and with the demand including the upcoming changes. The changes in demand require some changes to the current schedule, as there is far too less capacity for a consultation type that increases in the number of patients. The chosen changes are the smallest possible to make the master schedule feasible in the new situation. The reason for including both cases is that neither can be missed: we compare for the old demand data as the current master schedule is made for that and the changes we made to use it for the new case are arbitrary; however, we are more interested in the performance in the realistic scenario, so we also include the expected demand data.

Demand Master schedule	2006 demand		Expected demand	
	Current	New	Adapted current	New
ERCP	99%	98%	98%	97%
EUS	94%	97%	94%	97%
Colonoscopy	19%	98%	21%	95%
Oesdil	81%	96%	61%	97%
HGE with duration of 15 minutes	84%	98%	93%	96%
HGE with duration of 30 minutes	77%	97%	89%	95%
CRC	No standard set		88%	100%
Urgency	95%	97%	95%	97%
Double bookings	254	227	265	176
Hours of offered capacity per week	135.75	128.5	135.75	134.5
Number of shifts offered	48.25	45	48.25	48
Number of shifts without resident to learn	5	0	5	0

Table 14: results proposed and old master schedule for 2006 and expected demand data (source: author with simulation model of Overbeek (2008) for 2006 demand; Overbeek (2008) for expected demand)

We can see that for both the 2006 as the expected demand the performance on all aspects increases. Not only do 95% or more of the patients for all consultation types meet the standards, the performance on double bookings improves too. We obtain this while lowering the offered capacity per week, and thereby requiring less shifts each week. The optimization of the master schedule also leads to no shifts without resident to learn when that is needed. The expected demand case requires more offered capacity than the 2006 demand case for the new master schedule. This is mainly caused by the change in the CRC program, which is hard to plan as it is a research program.

7. Conclusions and discussion

We end this report with three concluding aspects. We summarize the outcomes and answer the research question in Section 7.1. We discuss the limitations of these conclusions and our approach in Section 7.2. The last section of this report lists the recommendations to the department and topics that need further research.

7.1 Conclusions

The planning process of the endoscopy department did not function well. The access time kept increasing while both physicians and nurses felt that they work too hard. The planning was not optimal because the old planning did not spread the usage of scopes and the recovery room evenly and did not always provide residents to learn. The planning was also a cause for the long access times, just as the skipping of shifts and the poor division of capacity under the consultation types were a main factor in the access times.

Our objective was to optimize the planning approach by using optimization programming. The only kind of optimization programming which is feasible for this type of problem is Mixed Integer Linear Programming (MILP). We cannot calculate the access time and amount of overwork analytically, so Overbeek (2008) built a simulation model to predict these. We used an iterative approach, in which we made master schedules by a MILP, tested them with the simulation model and adjusted them manually according to the results. Henderson and Mason (1998) describe such an approach, but we found no article bringing this theory into practice.

The MILP makes a master schedule of one week for the Hepato-Gastroenterology physicians on the endoscopy department. It plans consultations, attendant- and resident-physician at shifts in the rooms. Both the rooms and the physicians can only support or do part of the consultation types. It plans a fixed number of consultations of all consultation types. The MILP can only plan physicians if they are available and for a limited number of shifts per week. Further restrictions of the MILP are a maximum usage of OGD-scopes and recovery beds in one shift. The optimization criterion is to minimize the number of working shifts, shifts without reserve and shifts without residents to learn. Nurses are not taken into account, as their planning is more flexible and can be adjusted according to the consultation- and physician-based schedule.

The developed MILP is difficult to solve. Experiments show that we need the solver CPLEX to solve the program for problems like the AMC case. We developed a step-wise heuristic for this MILP, called heuristic 1. This heuristic plans

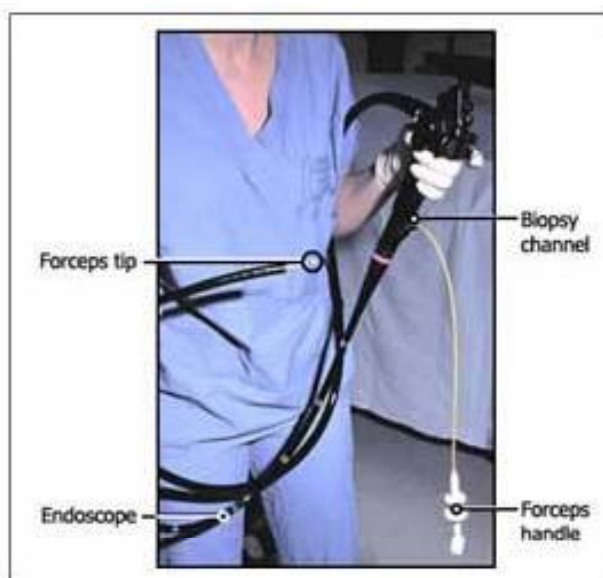


Figure 17: an OGD-scope with equipment to take a sample, source: http://www.barrettsinfo.com/content/3a_what_is_egd_with_biopsy.htm

consultation without physicians in the first step. The second step searches physicians with the filled shifts, and skips the unmanned shifts. The third step replans the consultations of those skipped shifts. The fourth step adds reserves and residents to learn without changing the master schedule so far. Afterwards we can use three improvement steps, which all uses a MILP that optimizes the physicians within one shift. The other three improvement steps involve swapping physicians and consultations from two different steps, and moving the consultations from one shift to another and finding new physicians for it. We tested heuristic 1 and optimized some settings of it. The performance of heuristic 1 varies a lot: it is reasonable for the AMC case, but on average bad for the randomly generated cases; heuristic 1 is very sensitive to small changes in the data case and the restrictions. This makes it impossible to rely on it in general. However, the performance is acceptable for the AMC case and therefore we can use it.

We used the described iterative approach of simulation and MILP to make the master schedule of the department. This master schedule would ensure that the department reaches the access time standards if there is no backlog of patients. The implementation takes a long time due to objections by the attendant-physicians, which has lead to a number of changes to the MILP. This research shows that an iterative process of simulation and Mixed Integer Linear Programming can help to reduce access times in theory. We will have to wait for the results of the implementation for the conformation in practice.

We return to the objective as stated in Section 2.2:

To develop a planning approach that minimizes the access time of the different types of endoscopy with the current resources.

We developed a planning approach based on a MILP to make a master schedule. The department needed to incorporate this in its planning. The only problem with this step is that we need to minimize the skipping of shifts due unavailability of physicians. Some rules have to change in the scheduling of patients, but this follows directly from the master schedule. These changes are the introduction of week urgency, the combination of colonoscopy with OGDs and sigmoidoscopies in one planning category and reserving spots for urgent ERCPs. We advise implementation of this approach.

7.2 Limitations

The MILP/simulation approach works well, but is not perfect. We could not model all details of the department in the simulation and MILP model. This section focuses on the important limitations of the MILP model and also mentions some general limitations.

The whole approach described in this report has focused exclusively on the planning. We found other measures that reduce the access time and implemented them. Examples are planning OGDs always in 15 minutes, instead of sometimes in 30 minutes, and using other methods of narcoses. When implementing an iterative approach of a MILP and a simulation model, one has to remember to look for other improvements. Other measures are often easier to develop as well.

The data used has some uncertainty. We used historic demand data, adjusted with some changes that took place. These changes were the introduction of a region-based rejection policy, the start of a new screening program and the introduction of an extra research block. We predicted the impact of these changes; however, the impact can be smaller or larger than

expected. It is always also possible that the demand changes due external factors. We could of course be optimizing our system for the wrong situation. We can of course adjust the planning as soon as new demand data is available: the planning approach can stay the same.

Some patients have to be seen by a specific physician. The physician will claim a research block for these patients if this number is large. However, there are also patients that the department plans with a specific physician outside a research block. We partly model this in the simulation model (Overbeek, 2008). In short: the strict division between research and standard patients is less definite in reality than modeled.

The iterative approach of simulation and MILP is time-consuming. We adjusted the MILP-constraints according to the outcomes of the simulation manually to save time. The constraints are changed in an intelligent way rather than trying everything to bring the outcomes of the simulation in line with the standards. This decreases the number of iterations needed and thereby speeds up the process.

7.3 Recommendations

We recommend the endoscopy department of the AMC the following:

1. Implement the new planning approach, based on an iterative usage of MILP and simulation. Replacing the old, manual one, the proposed master schedule will lead to shorter access times, less double bookings and fewer overloads of the recovery room and OGD scopes.
2. Decrease the number of planning categories and extra scheduling restrictions as much as possible. Blocks and extra scheduling restrictions influence the access time negatively. An easy way to decrease the number of planning categories with one is to merge two existing planning categories, the HGE general and colonoscopies. This improves the efficiency, as the pooling of more consultations will reduce the access time with the same demand and available capacity.
3. Introduce week urgency. Currently the department only differentiates between day urgency and elective patients. A physician sees the first group of patients on the same day; the department serves some patients of the last group after sixteen weeks. However, many of this first group of patients needs to be served within one week rather than one day. The introduction of week urgency offers physicians the possibility to indicate that, so the desk employees can actually schedule these patients on likely quiet days.
4. Plan every OGD or sigmoidoscopy without anesthesia in fifteen minutes. Currently the desk employees schedule them for fifteen minutes in room 213, but for thirty minutes in other rooms. Reasons for this are unclear, but probably had to do with the former situation. A couple of years ago an attendant and two nurses were planned in room 213 and a resident and one nurse in the other rooms where simple consultations were performed. However, this situation changed some years ago and nowadays there are no clear difference between room 213 and the other rooms. Therefore, we can conclude there is no reason to make a difference and the physicians and nurses of the department agrees that those consultations must be possible within fifteen minutes.

5. Strive to minimize the number of skipped shifts. Skipped shifts are shifts of the master schedule that are not incorporated in the plan and are a main cause for the long access time. (Overbeek, 2008) The most important reason for skipping them is the unavailability of physicians. HGE physicians are 16 weeks a year unavailable for patient care (Xcare, 2008), where the standard is 10. The HGE physicians should reduce their unavailability for the endoscopy department and tune their unavailability to each other.

6. Introduce an application-policy based on region. AMC HGE physicians spent too much of their time in patient care in 2006 (Struben and Greuter, 2007) and an important reason for this is the large amount of patients. As the endoscopy department of the AMC is well known for its medical expertise there are more patients than the department can handle. The AMC does not 'profit' from these 'extra' patients. Refusing patients is legally impossible, but the department can advise patients from outside the service area of the AMC with rather simple problems to go to a local hospital.

7. Offer temporary extra capacity to bring the access time within the standards more quickly. The access time would be within the standards within roughly half a year should the department offer three extra shifts a week. It would take until 2010 to bring the access times within the standards when the department offers no extra capacity. One can find the results of the calculations to the backlog in Appendix 2.

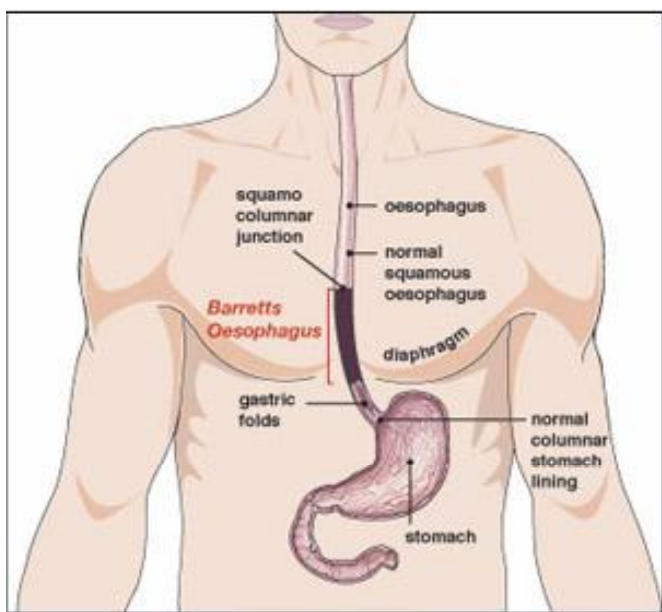


Figure 18: the fluores program surveys the Barrett oesophagus, source: <http://www.kennislink.nl/web/show?id=123222>

8. Implement methods to monitor the current performance. This will prevent further problems becoming as big as the current ones. A possible method to monitor the performance is the introduction of an operational scorecard. Relevant indicators are the access time, the number of double-bookings and the number of skipped shifts.

9. Improve the administration. Nobody registers whether consultations are urgent or part of a research program. This is valuable management information and essential for logistical projects like this.

10. Try to standardize the patient processes in the department as much as possible. Although the consultations do vary a lot from a medical point of view, there is little difference from a process point of view. A more standardized approach will reduce the time used for one patient, thereby reducing waiting times for patients and overtime for nurses.

11. Do not expand the available equipment with a new x-ray system in room 213 or an OGD-suction in room 120. This would relax some of the constraints, but they do not result in better master schedules. Extra suction-equipment brings no additional benefits besides a possible better master schedule. A new x-ray system has more benefits: it is a back up for the current

one, which is old and fails sometimes. However, last year there were only two days in which the x-ray system was broken and a new one is expensive.

We suggest the following subjects for further research:

1. The iterative approach of simulation and MILP worked well for this case and can probably be used in more cases. The model provided could be adjusted and used. Another possibility is to use this model standalone, so without an iterative approach with simulation. The generalization of both the approach and the model are likely, but needs to be verified with further research.
2. Another practical research would be to enlarge the level of detail. The capacity of the recovery room and the scopes are only taken roughly into account, but their usage could be better predicted when consultations are planned on time and not only in shifts. More information is required to do so however, especially on the length of stay in the recovery room. This length of stay is in practice not constant, but is different for outpatients, inpatients from the AMC and inpatients from other hospitals. The best way to deal with this is probably to make a simulation model that models the operational process, taking patient schedules as input. This could also help to minimize the chance on overtime with parameters as the planned duration of certain consultations and the planned duration of a shift.

List of used terms

AMC	Academic Medical Centre. The hospital in which this research has taken place.
Attendant	Attendant-physician, an experienced physician.
Colo	Colonoscopy, a type of consultation in which the colon is inspected through the anus.
CRC	Colon Rectal Carcinoma, but the name is outdated. One of the existing research programs. A gathering of different colon-fixated research programs.
ERCP	Endoscopic Retrograde CholangioPancreatography, a type of consultation in which the physician inspects the pancreas through the mouth. It needs x-ray equipment and is one of the more complicated consultations.
EUS	Endoscopic UltraSound, a type of consultation that needs ultrasound equipment.
FAP	Familial Adenomatous Polyposis. A research program to this kind of diseases, which mainly consists of sigmoidoscopies.
Fluores	Fluorescence, but the name is outdated. One of the existing research programs for a special kind of oesophagus-patients, who get a special OGD, often an Endoscopic Mucosal Resection, or EUS.
Fluorecho	A special consultation type with the EUS consultations of fluores.
HGE	Hepato-Gastroenterology, medical specialism directed at the digestive tract and liver. In Dutch: maag-darm-lever (mdl).
HGE general	Category in the planning that consist of all remaining treatments, mainly OGDs and sigmoidoscopies. We propose to include also colonoscopies, PEGs and VS in this category in this report.
IBD	Inflammatory Bowel Disease. A research program that will start shortly, consisting of special kinds of colonoscopy, sigmoidoscopy and OGDs, directed at this disease.
Master schedule	A planning that is repeated, in this research every week.
OGD	Oesophago-Gastro-Duodenoscopy, a type of consultation in which mainly the stomach is inspected through the mouth. Also called gastro or EGD, the second after the American spelling Esophago.
PEG	Percutaneous Endoscopic Gastrostomy. A type of consultation that consist of the placement of a feeding tube to the stomach through the abdominal wall.
Procto	Proctoscopy, a type in consultation in which the anal cavity, rectum and/or sigmoid colon are examined. Also called recto.
Resident	Resident-physician, a doctor in training who needs supervision from an Attendant-Physician.
Sigmo	Sigmoidoscopy, a type of consultation in which the lower part of the colon is inspected through the anus.
VS	Feeding tube, after the Dutch word VoedingSonde. A type of consultation in which a feeding tube is placed through the mouth.

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Appendix 1: Complete MILPs for each step

Construction: Step 1

Indices

- a Attendant Physician, nine experienced physicians.
- d Resident Physician, six physicians who are learning on the job.
- r Rooms, there are seven available rooms with varying equipment.
- s Shifts. There are ten different shifts: mornings and afternoons of the five working days.
- t, τ Types of consultation.

Parameters

- c_3 . Weight factors in the objective for the unplanned demand.
- CostDem_t The penalty per unit for not meeting the demand of consultation type t.
- DayLength Maximal length of a day.
- Demand_t Demand for consultations of type t.
- Dur_t Duration of one unit of consultation type t.
- M Sufficient large number
- MaxLength_s Maximal length of shift s. Normally 3.5 hours for morning shifts and 3 hours for afternoon shifts.
- Maxshiftdur Minimal shift time to be not penalized. Randomly generated with an uniform division between 0 and 2.5.
- MinMorn The minimal amount of time from one specific type of consultation that needs to be in morning shifts.
- MinShift_{ts} Minimal number of consultations planned in shift s of consultation type t. For most cases this is zero, only for a limited number of urgent consultation types for some days a number larger than zero is noted.
- r_1, r_2, r_3 Weight factors in the objective for the different unwanted effects. These are randomly generated with an uniform division between 0 and 5.
- RecCap The maximum expected number of patients that can be sent to the recovery room in one shift.
- RecSent_t The expected number of patients sent to the recovery room per unit of consultation type t.
- RoomPos_{rt} =1 if consultation type t can be done in room r
=0 otherwise
- ScopesAv_s The maximum expected number of OGD-scopes that can be used in shift s.
- ScopesNeed_t Expected number of OGD-scopes needed during one unit of consultation type t.

Decision variables

- X_{str} Number of patients of type t on shift s in room r

Auxiliary variables

- DifDem_t Number of patients of type t not handled
- excep_{str} =1 if no two physicians are available for this consultation type and it is planned anyway.
=0 otherwise
- echos_s =1 if EUS are planned in a shift
=0 otherwise
Note that EUS can only be done in one room.

$open_{sr}$ =1 if total duration of shift s in room r is longer than $maxshiftdur$.
 =0 otherwise

Constraints

$$Demand_t \quad \sum_{rs} X_{srt} = DifDem_t \quad \forall t$$

$$\sum_r X_{srt} \geq MinShift_{ts} \quad \forall t, s$$

$$\sum_t (X_{srt} * Dur_t) \leq MaxLength_s \quad \forall r, s$$

$$\sum_t [(X_{srt} + X_{s-1,rt}) * Dur_t] \leq LengthDay \quad \forall s \in \{1,3,5,7,9\}$$

$$X_{srt} \leq RoomPos_{rt} * M \quad \forall s, t, r$$

$$\sum_{r,t} X_{srt} * RecSent_t \leq RecCap \quad \forall s$$

$$\sum_{rt} X_{str} * ScopesNeed_t \leq ScopesAv_s \quad \forall s$$

$$(1 - x_{stru}) * M \geq \sum_{t \neq u} x_{srt} \quad \forall s, r, u \in \{fluores, fluorecho, IBD, CRC, FAP\}$$

$$\sum_{t \in \{fluores, fluorecho\}} \sum_r (X_{srt} + X_{s-1,rt} + X_{s-2,rt} + X_{s-3,rt}) \leq 2 \quad \forall s \leq 7$$

$$\sum_r (X_{1rt} + X_{3rt} - X_{5rt} + X_{7rt} + X_{9rt}) \geq MinMorn \quad \forall t \text{ fluores}$$

$$X_{str} = 0 \quad \forall s \in \{3,5,7,9\}, t, r \in \{120\}$$

$$X_{str} = 0 \quad \forall t, s \in \{7,8\}, r \in \{219\}$$

$$X_{str} = 1 \quad \forall s \in \{1,5\}, t \in \{selectron\}, r \in \{213\}$$

$$\sum_{s,r \neq 120} X_{str} \geq 1 \quad \forall t = CRC$$

$$X_{str} \leq \max \left\{ 0, 7 * \sum_p (AvailPhys_{ps} * PosPhys_{pt}) + ResidPos_{dt} * ResidAv_{sd} - 1 + excep_{str} \right\} \quad \forall s, t, r$$

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$$X_{str} \leq M * echos_s \quad \forall s, t = eus, r = 124$$

$$\sum_s X_{str} \leq Demand_t - 1 \quad \forall t = crc, r = 120$$

$$\sum_t (X_{str} * Dur_t) \geq Maxshiftdur * open_{sr} \quad \forall s, r$$

Objective

$$z = c_3 * \sum_t (DifDem_t * Weigth_t) + r_1 * \sum_s echos_s + r_2 * \sum_{str} excep_{str} + r_3 * \sum_{sr} open_{sr}$$

Construction: Step 3

Indices

- a Attendant Physician, nine experienced physicians.
- d Resident Physician, six physicians who are learning on the job.
- r Rooms, there are seven available rooms with varying equipment.
- S Shifts. There are ten different shifts: mornings and afternoons of the five working days.
- t, τ Types of consultation.

Parameters

- AttFixed_{srp} Values of variable Att receiving from last step.
- DayLength Maximal length of a day.
- Demand_t Demand for consultations of type t.
- Dur_t Duration of one unit of consultation type t.
- M Sufficient large number
- MaxLength_s Maximal length of shift s. Normally 3.5 hours for morning shifts and 3 hours for afternoon shifts.
- Maxshiftdur Minimal shift time to be not penalized. Randomly generated with an uniform division between 0 and 2.5.
- MinMorn The minimal amount of time from one specific type of consultation that needs to be in morning shifts.
- MinShift_{ts} Minimal number of consultations planned in shift s of consultation type t. For most cases this is zero, only for a limited number of urgent consultation types for some days a number larger than zero is noted.
- RecCap The maximum expected number of patients that can be sent to the recovery room in one shift.
- RecSent_t The expected number of patients sent to the recovery room per unit of consultation type t.
- ResidFixed_{srd} Values of variable Resid receiving from last step.
- RoomPos_{rt} =1 if consultation type t can be done in room r
=0 otherwise
- ScopesAv_s The maximum expected number of OGD-scopes that can be used in shift s.
- ScopesNeed_t Expected number of OGD-scopes needed during one unit of consultation type t.
- Weight_t The penalty per unit for not meeting the demand of consultation type t.

Decision variables

- X_{str} Number of patients of type t on shift s in room r
- Att_{asr} =1 if attendant a planned on room r on shift s
=0 otherwise
- Resid_{dsr} =1 if resident d planned on room r on shift s
=0 otherwise

Auxiliary variables

- DifDem_t Number of patients of type t not handled

Constraints

$$X_{str} \leq \left[\sum_a (Att_{asr} * AttPos_{at}) + \sum_d (Resid_{dsr} * ResPos_{dt}) \right] * M \quad \forall s, t, r, v$$

$$Demand_t - \sum_{rs} X_{srt} = DifDem_t \quad \forall t$$

$$\sum_r X_{srt} \geq MinShift_{ts} \quad \forall t, s$$

$$\sum_t (X_{srt} * Dur_t) \leq MaxLength_s \quad \forall r, s$$

$$\sum_t [(X_{srt} + X_{s-1,rt}) * Dur_t] \leq LengthDay \quad \forall s \in \{1,3,5,7,9\}$$

$$X_{srt} \leq RoomPos_{rt} * M \quad \forall s, t, r$$

$$\sum_{r,t} X_{srt} * RecSent_t \leq RecCap \quad \forall s$$

$$\sum_{rt} X_{str} * ScopesNeed_t \leq ScopesAv_s \quad \forall s$$

$$(1 - x_{stru}) * M \geq \sum_{t \neq u} x_{strt} \quad \forall s, r, u \in \{fluores, fluorecho, IBD, CRC, FAP\}$$

$$\sum_{t \in \{fluores, fluorecho\}} \sum_r (X_{srt} + X_{s-1rt} + X_{s-2rt} + X_{s-3rt}) \leq 2 \quad \forall s \leq 7$$

$$\sum_r (X_{1rt} + X_{3rt} + X_{5rt} + X_{7rt} + X_{9rt}) \geq MinMorn \quad \forall t \in \{fluores\}$$

$$X_{str} = 0 \quad \forall s \in \{3,5,7,9\}, t, r \in \{1,2\}$$

$$X_{str} = 0 \quad \forall t, s \in \{7,8\}, r \in \{1,2\}$$

$$X_{str} = 1 \quad \forall s \in \{1,5\}, t \in \{selecton\}, r \in \{1,2\}$$

$$\sum_{s, r \neq 120} X_{str} \geq 1 \quad \forall t = CRC$$

$$Att_{srp} = AttFixed_{srp} \quad \forall s, r, p$$

$$Resid_{srd} = ResidFixed_{srd} \quad \forall s, r, d$$

Objective

$$\min z = \sum_t (DifDem_t * Weigth_t)$$

Improvement: Adding

Indices

- a Attendant Physician, nine experienced physicians.
- d Resident Physician, six physicians who are learning on the job.
- r Rooms, there are seven available rooms with varying equipment. Furthermore, one dummy room is created: supervisor.
- s Shifts. There are ten different shifts: mornings and afternoons of the five working days.
- t Types of consultation.

Parameters

- AttAvail_{as} =1 if attendant-physician a is available during shift s
=0 otherwise
- AttFixed_{srp} Values of variable Att receiving from last step.
- AttShifts_a Number of shifts attendant-physician a can serve in the endoscopy department
- AttPos_{at} =1 if consultation type t can be done by attendant-physician a
=0 otherwise
- c₄, c₅ Weight factors in the objective for the two unwanted effects.
- Influence The importance of being a shift reserve compared to serve.
- M Sufficient large number
- ResAvail_{ds} =1 if resident-physician d is available during shift s
=0 otherwise
- ResShifts_d Number of shifts resident-physician d can serve in the endoscopy department
- ResidFixed_{srd} Values of variable Resid receiving from last step.
- ResPos_{dt} =1 if consultation type t can be done by resident-physician d
= 0 otherwise
- XFixed_{str} Values of variable X receiving from last step.

Decision Variables

- X_{str} Number of patients of type t on shift s in room r
- Att_{asr} =1 if attendant a planned on room r on shift s
=0 otherwise
- Resid_{d sr} =1 if resident d planned on room r on shift s
=0 otherwise

Summary Variables (derived from decision variables):

- DifDem_t Number of patients of type t not handled
- NoResid_{sr} =1 if no resident planned on shift s in room r if certain categories are performed than
=0 otherwise
- NoReserve_{sr} =1 if no reserve planned on shift s in room r if consultations are planned than
=0 otherwise

Constraints

$$X_{str} \leq \sum_a (Att_{asr} * AttPos_{at}) + \sum_d (Resid_{d sr} * ResPos_{dt}) * M \quad \forall s, t, r$$

$$X_{str} \leq \left[\sum_a (Reser_{asr} * AttPos_{at}) + \sum_d (ResResid_{d sr} * ResPos_{dt}) + NoReserve_{sr} \right] * M \quad \forall s, t, r$$

$$X_{str} \leq \left[\sum_d (Resid_{dsr}) + NoResid_{sr} \right] * M \quad \forall s, r, t \in \{ERCP, Eus\}$$

$$\sum_a Att_{asr} = 1 \quad \forall s, r = supervisor$$

$$\sum_a Reser_{asr} + NoReser_{sr} = 1 \quad \forall s, r = supervisor$$

$$Att_{asr} + Influence * Reser_{asr} \geq AttAvail_{as} \quad \forall a, s, r$$

$$Resid_{dsr} \geq ResAvail_{ds} \quad \forall d, s, r$$

$$\sum_{sr} (Att_{asr} + Reser_{asr} * Influence) \leq AttShifts_a \quad \forall a$$

$$\sum_{sr} (Resid_{dsr} + ResResid_{dsr} * Influence) \leq ResShifts_d \quad \forall d$$

$$X_{srt} = XFixed_{srt} \quad \forall s, t, r$$

$$Att_{srp} = AttFixed_{srp} \quad \forall s, r, p$$

$$Resid_{srd} = ResidFixed_{srd} \quad \forall s, r, d$$

Objective

$$\text{Min } z = c_4 * \sum_{srv} NoResid_{sr} + c_5 * \sum_{srv} NoReser_{sr}$$

Improvement: Within shift swaps

Indices

- a Attendant Physician, nine experienced physicians.
- d Resident Physician, six physicians who are learning on the job.
- r Rooms, there are seven available rooms with varying equipment. Furthermore, one dummy room is created: supervisor.
- t Types of consultation.

Parameters

- ShiftAttAvail_a Number of (reserve) shifts an attendant-physician a can do during the specific shift, with a maximum of 1.
- AttPos_{sat} =1 if consultation type t can be done by attendant-physician a
=0 otherwise
- c₄, c₅ Weight factors in the objective for the two unwanted effects.
- Influence The importance of being a shift reserve compared to serve on a shift.
- M Sufficient large number
- ShiftResAvail_d Number of (reserve) shifts a resident-physician a can do during the specific shift, with a maximum of 1.
- ResPos_{dt} =1 if consultation type t can be done by resident- physician d
= 0 otherwise
- ShiftX_{tr} Values of variable X receiving from last step for this specific shift.

Decision Variables

- ShiftAtt_{ar} =1 if attendant a planned on room r on the specific shift
=0 otherwise
- ShiftResid_{dr} =1 if resident d planned on room r on the specific shift
=0 otherwise
- ShiftRAtt_{ar} =1 if attendant a planned as reserve on room r on the specific shift
=0 otherwise
- ShiftRResid_{dr} =1 if resident d planned as reserve on room r on the specific shift
=0 otherwise

Auxiliary Variables (derived from decision variables):

- ShiftNoResid_r =1 if no resident planned on the specific shift in room r if certain categories are performed than
=0 otherwise
- ShiftNoReserve_r =1 if no reserve planned on the specific shift in room r if consultations are planned than
=0 otherwise

Constraints

$$SX_{str} \leq \sum_a (\text{ShiftAtt}_{ar} * \text{AttPos}_{at}) + \sum_d (\text{ShiftResid}_{dr} * \text{ResPos}_{dt}) \quad \forall t, r$$

$$\text{ShiftX}_{str} \leq \sum_a (\text{ShiftRAtt}_{ar} * \text{AttPos}_{at}) + \sum_d (\text{ShiftRResid}_{dr} * \text{ResPos}_{dt}) + \text{ShiftNoReserve}_r * M \quad \forall t, r$$

$$ShiftX_{str} \leq \left[\sum_d (ShiftResid_{dr}) + ShiftNoResid_{sr} \right] * M \quad \forall r, t \in \{ERCP, Eus\}$$

$$\sum_a ShiftAtt_{ar} = 1 \quad \forall s, r \text{ supervisor}$$

$$\sum_a ShiftReser_{ar} + ShiftNoReser_r = 1 \quad \forall s, r \text{ supervisor}$$

$$ShiftAtt_{ar} + Influence * ShiftRAtt_{ar} \geq ShiftAttAvail_a \quad \forall a, r$$

$$ShiftResid_{dr} - Influence * ShiftRResid_{dr} \geq ShiftResAvail_d \quad \forall d, r$$

Objective

$$\min z = c_4 * \sum_{srv} ShiftNoResid_{sr} + c_5 * \sum_{srv} ShiftNoReser_{sr}$$

Appendix 2: Calculations backlog

This appendix shows the results of the calculations to the backlog. We calculated when the department would finish the backlog and meet the standards for the access time. The input variables used are the amount of extra capacity offered, and the time this starts. This calculation is a simple one: the reader has to see the results as general impression rather than the exact truth. We base our calculations on six assumptions:

- It handles only about the consultation types HGE general (OGDs and sigmoidoscopies) and colonoscopy, as these are the types with a significant backlog
- We use a utilization rate of 100% as long as there is backlog.
- The department schedules no patients in other blocks. The desk employees plan some patients in other blocks when time is available a week in advance in reality.
- The demand stays stable, except that patients from outside the service area of the AMC will take the advice and go to a nearby hospital.
- The current access time is three months.
- The situation with enough capacity to handle the variations in the demand starts at 1 April, so the access time will also decrease from 1 April until the time the extra capacity will start.

We will now show our calculations for the end date. The start date and the extra capacity are the input variables that are varied.

$$Backlog_{1July} = 399 \text{ hours}$$

$$Rate_{beforestart} = 4.3 \text{ hours/week}$$

$$Time_{beforestart} \text{ (weeks)} = \frac{startdate_{1July}}{7}$$

$$Backlog_{startdate} = Backlog_{1July} - Rate_{beforestart} * Time_{beforestart}$$

$$Rate_{afterstart} = Rate_{beforestart} + Extracapacity$$

$$Time_{afterstart} = \frac{Backlog_{startdate}}{Rate_{afterstart}}$$

$$Enddate = startdate + \frac{Backlog_{startdate}}{Rate_{afterstart}}$$

These assumptions and formulas result in the calculation as shown in Table 12. We calculated the extra capacity for every three hours per week, as this is the size of a normal shift. We choose the different start dates used after consultation of the department, as those can be possible.

Table 15: date when department would finish backlog

Extra capacity	Start date	Predicted end data
Hours per week	Extra capacity starts	Backlog is gone
0	-	11 April 2010
3	1 July 2008	18 July 2009
6	1 July 2008	29 March 2009
9	1 July 2008	27 January 2009
12	1 July 2008	19 December 2008
15	1 July 2008	22 November 2008
3	1 Augustus 2008	31 July 2009
6	1 Augustus 2008	16 April 2009
9	1 Augustus 2008	17 February 2009
12	1 Augustus 2008	11 January 2009
15	1 Augustus 2008	16 December 2008
3	1 September 2008	13 Augustus 2009
6	1 September 2008	4 May 2009
9	1 September 2008	10 March 2009
12	1 September 2008	3 February 2009
15	1 September 2008	9 January 2009
3	1 October 2008	25 Augustus 2009
6	1 October 2008	21 May 2009
9	1 October 2008	30 March 2009
12	1 October 2008	25 February 2009
15	1 October 2008	2 February 2009

Appendix 3: Influence constraints

Constraint		MILP solution found after 300 seconds	
Name	Short description	LB	UB
-	With all constraints	62.31	67
a1	Physician planned if consultations planned	26*	26
a2	Reserve physicians	60.04	65
b	Residents to learn	51.52	56
c1	Supervisor	52.27	57
c2	Reserve Supervisor	62.04	67
d	Physicians available if planned	61.69	67
e1	Time spent by attendants	62.14	67
e2	Time spent by residents	62.52	67
f	Demand planned	37*	37
g	Minimal demand per shift	58.36	65
h	Length of a shift	45.73	51
i	Length of both shifts on a day	62.42	66
j	Equipment in rooms	44.17	57
k	Recovery room	61.95	67
l	OGD scopes	62.15	67
m	Research programs need whole shifts	61.29	67
n	Spread of one research program	62.62	67
o	Minimal part of consultations in morning for one type	62.25	67
p	Other departments uses rooms for complete shifts	61.99	67
q	Other departments uses rooms for a part of two shifts	62.43	67
r	CRC ones outside room 120	61.73	67
	OGDs in room 120	62.38	67
	X-ray system in room 213	62.73	67

Table 16: The quality of the solution found after five minutes with one constraint less. The two bottom rows are by changing some constraints to reflect easy possible changes. *=optimal solution found.