The Reconstruction of Vehicle Trajectories with Dynamic Macroscopic Data

Master's Thesis

Final Report

11th August 2009

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University of Twente
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Summary

In recent years the calculation of emissions is becoming more important. The amount of emissions of traffic can be calculated using simulation models, to investigate ex-ante the effects of certain measures. Most of the appropriate models are microscopic simulation models. This type of models is vehicle based and is not able to consider large areas. Models which can consider large areas are not vehicle based. The outcomes of these models are averaged values for speeds and flow rates. Literature shows that emission calculation with these averaged values is not accurate enough. Next to this, visualization of vehicle movements helps to understand the performance of a traffic system. Microscopic models are able to visualize the cars, while macroscopic models are unable to do that. On the other hand, microscopic models are stochastic models. Macroscopic models are, on the contrary, deterministic and able to calculate the traffic flows in larger areas.

Therefore, in this research a method is developed which is able to combine the best of the two different models by reconstructing vehicle trajectories based on dynamic macroscopic data. This method makes it possible to visualize vehicle movements and to calculate emissions, while it is still macroscopic and deterministic.

Method

The method uses an interpolation technique to describe density in the considered time-space frame. Based on the density speed can be calculated using a fundamental diagram. In this research the fundamental diagrams of Newell and Smulders are used. Interpolation of density over space and time is done using a second order Taylor-series expansion. With this relation two different methods are worked out in detail and are compared with simple trajectory reconstruction methods and linear interpolation methods.

The first method is a four-corner interpolation. Every point in a time-space frame lies between four known points: between two detectors and two time intervals. With the Taylor series expansion density is calculated from every point towards a certain point in the middle. The four generated values are averaged based on the distance to the four corner points.

The second method is a two-point interpolation. First density between two time intervals is calculated based on a linear interpolation. This interpolation is performed at both detectors. Between the two new points the Taylor series expansion is used to calculate density between the two detectors. The two resulting values are averaged based on the distance to the detectors.

The methods for the reconstruction of the trajectories need a fundamental diagram. The fundamental diagram can either be given or estimated based on the dynamic macroscopic data.

Conclusions
The developed methods have been validated and compared with other methods. This validation illustrates that the methods perform reasonable well, compared to the results of simple reconstruction methods and a linear interpolation method. On the level of trajectories the developed methods are not the best way to estimate the arrival times at different locations, but the calculated speed is better than with all other methods. At the level of macroscopic data the developed methods score better compared to the linear interpolation method, measured in density. The difference between the four-corner and the two-point interpolation is very small. Also the differences between the fundamental diagram of Newell and Smulders are very small.

The reconstructed trajectories can be used for the visualization of car movements. The method is able to calculate the location for each car at any moment. Using this ability the method can update the locations of the cars every time step.

Further research
In this research trajectories are reconstructed with macroscopic data. These trajectories contain the essential information for the calculation of external effects. Before the external effects can be calculated some work has to be done. It is, for instance, not known in what way that effectively can be done with this method.

The methods are only applied on very simple networks. Before the method can be used on a larger network further research is needed. One difficulty will be intersections and other merging and diverging areas. In these areas cars go to several directions. It needs research to develop a method to draw trajectories along these areas.

Samenvatting

In de laatste jaren is het berekenen van emissies steeds belangrijker geworden. De hoeveelheid uitstoot, veroorzaakt door verkeer, kan worden uitgerekend met simulatiemodellen. Daarmee kan ook van te voren worden bepaald wat de effecten zijn van maatregelen. De meeste van deze modellen zijn microscopische simulatiemodellen. Dit type modellen is gebaseerd op voertuigniveau, waardoor het niet in staat is grote gebieden te berekenen. Modellen die dat wel kunnen zijn niet gebaseerd op voertuigniveau, maar op wegniveau. De resultaten van een dergelijk model zijn gemiddelde waarden voor snelheid en aantallen voertuigen per tijdseenheid. Uit de literatuur blijkt dat de berekening van emissies, gebaseerd op deze getallen, niet nauwkeurig genoeg is. Daarnaast helpt de visualisatie van voertuigbewegingen om het functioneren van het verkeerssysteem te begrijpen. Microscopische modellen zijn in staat om die bewegingen weer te geven, terwijl macroscopische modellen dat niet zijn. Aan de andere kant zijn microscopische modellen stochastisch. Macroscopische modellen daarentegen zijn deterministisch en in staat om in grotere gebieden de verkeersstromen te berekenen.

Daarom is in dit onderzoek een methode ontwikkeld die in staat is het beste van beide modellen te combineren door het reconstrueren van voertuig trajectoriën aan de hand van dynamische macroscopische data. Deze methode maakt het mogelijk om voertuigbewegingen te visualiseren en om emissies uit te rekenen, terwijl het nog steeds een macroscopisch en deterministisch model is.

Methode

De ontwikkelde methode maakt gebruik van een interpolatietechniek voor het beschrijven van de dichtheid in zowel de ruimtelijke als de tijdsdimensie. Gebaseerd op de dichtheid kan dan de snelheid worden uitgerekend aan de hand van een fundamenteel diagram. In dit onderzoek zijn voornamelijk de diagrammen van Newell en Smulders gebruikt. De interpolatie van de dichtheid wordt beschreven met behulp van een Taylor-reeks. Met deze functie zijn twee verschillende methoden in detail uitgewerkt en vergeleken met simpele reconstructiemethoden en met een lineaire interpolatiemethode.

De eerste methode is een 'vier-hoeken'-interpolatie. Ieder punt in een tijd-ruimte gebied ligt tussen vier bekende punten: tussen twee detectoren en twee tijdsintervallen. Met de Taylor-reeks is de dichtheid uitgerekend vanuit elke hoek. De vier resulterende waarden zijn vervolgens gemiddeld, gebaseerd op de afstand tot de hoek.

De tweede methode is een 'twee-punts'-interpolatie. Eerst is de dichtheid tussen twee tijdsintervallen uitgerekend middels een lineaire interpolatie. Dit is gedaan voor beide detectoren. Tussen de twee nieuwe punten is vervolgens een Taylor-reeks gebruikt voor het berekenen van de dichtheid tussen de twee detectoren. De twee resulterende waarden zijn vervolgens weer gemiddeld op basis van de afstand naar de detector.

Voor de reconstructie is een fundamenteel diagram vereist. Dit diagram kan gegeven zijn of geschat worden aan de hand van de macroscopische data.

Conclusies
De ontwikkelde methoden zijn gevalideerd en vergeleken met andere methoden. De validatie laat zien dat de methode tamelijk goed scoort vergeleken met het resultaat van de simpele reconstructiemethoden en de lineaire interpolatie. Op het niveau van trajectoriën zijn de nieuwe methoden niet de beste manier voor het schatten van de aankomsttijd bij de verschillende detectoren, maar de uitgerekende snelheid is wel beter dan bij alle andere methoden. Op het niveau van macroscopische data scoren de nieuwe methoden beter dan de lineaire interpolatie, gemeten in dichtheid. De verschillen tussen de 'vier-hoeken'-interpolatie en de 'twee-punts'-interpolatie zijn erg klein. Ook zijn de verschillen tussen de fundamentele diagrammen van Newell en Smulders erg klein.

De gereconstrueerde trajectoriën kunnen gebruikt worden voor de visualisatie van voertuigbewegingen. De methode is in staat om op ieder moment de locatie van een voertuig uit te rekenen. Gebruik makend van deze functie kan dus elke tijdstap de nieuwe locatie van elk voertuig worden bepaald.

Verder onderzoek
In dit onderzoek zijn trajectoriën gereconstrueerd aan de hand van macroscopische data. Deze trajectoriën bevatten de benodigde informatie voor het berekenen van de externe effecten. Voordat deze externe effecten uitgerekend kunnen worden moet echter nog wat werk gedaan worden. Het is bijvoorbeeld nog niet bekend hoe dat effectief met deze methode gedaan zou kunnen worden.

Verder zijn de methoden alleen toegepast op eenvoudige netwerken. Voor de toepassing op grotere netwerken moet ook nog werk verzet worden. Een moeilijk punt zijn de kruispunten en op- en afritten. Op deze plekken kunnen auto's verschillende kanten op. Het vereist verder onderzoek om een methode te ontwikkelen die in staat is trajectoriën te tekenen over zulke gebieden.

Preface

If there would be a list of frequently asked questions about my research the first question would be: what are you doing? Then I explained that I am developing a method to reconstruct vehicle trajectories (*What?*). But why? That was always the second question. After explaining why I am doing this there was a third question: do you really like this? The subject of this research is quite technical and most people would not like to perform this research, but I think this research fits my interests and skills. Most of the time I have enjoyed working on it and I think the result is satisfying.

During the process a lot of people helped me to finish it. First of all I want to thank Michiel and Luuk for their help with the development of the methods. We had a lot of useful discussions about new ideas, problems with ideas and interpreting the results. These discussions have contributed to the final result as it is now. I am also indebted to Eric and Jing for their feedback on my work.

Next to that, I also want to thank Bastiaan for his reactions on all the nice pictures and for all the coffee talks about politics, Michael Jackson, the Tour de France, movies, football, and so on. You made the life of a graduate student much more easier. At last I want to thank Margreet for her everlasting support. Thank you for all you helped me with!

I hope you will enjoy reading this report.

Lieuwe Krol

Deventer, July 2009

Contents

List of Tables and Figures

List of Tables

List of Figures

Frequently used Symbols

V_f	Free flow speed
V_{cap}	Speed at capacity
Q_{cap}	Capacity flow
k_{jam}	Jam density
k_{cap}	Density at capacity
k, K	Density
v, V	Speed
q, Q	Flow rate
t	Time
x	Location
i	Integer, mostly used for different detector locations
j	Integer, mostly used for different time steps
n	Integer

Chapter 1

Introduction

In this research vehicle trajectories are reconstructed based on dynamic macroscopic data. In this chapter background information is given on the general idea of this research. First more information is given on the concept of vehicle trajectories. After that, macroscopic data is described in more detail. Beside this introduction the relevance of this research is described. At the end, a preview of this research is given.

1.1 Vehicle Trajectories

According to The Free Dictionary a trajectory is 'the path of a projectile or other moving body through space'. Applied on a vehicle this is the path of a car driving on a road. With a known trajectory the location of the car is known at every moment. With a microsimulation model it is easy to record the location of all vehicles on each time step. With these data, a trajectory plot can be made for all vehicles. Furthermore, by taking the first and second derivative of the distance with respect to time, speed and acceleration are known as well.

In figure 1.1 a trajectory plot is given. The bold line corresponds with a trajectory of an individual car. The data for this plot is extracted from Vissim, in which a piece of road was simulated, with a speed limitation at about 1100 meters from the beginning. In figure 1.2 a detailed view of the trajectories is given. The changes in speed are clearly visible in the figure.

1.2 Macroscopic Data

Vehicle trajectories are not used in macroscopic models. In such models aggregated or averaged values of speeds and flow rates are used. In real traffic averaged values are also used often. These averaged values are the result of a number of cars passing a detector loop during a certain interval. In this interval the speed and the number of cars are measured. With that data several other things can be calculated, like density and headways. When each vehicle is detected separately, headway distributions, vehicle length distribution, and occupancy rates can be calculated. In this research, only average speeds and flow rates are used, which also could be obtained from detector loops. It is also assumed that the detector loop locations are known, as well as the length of a time interval.

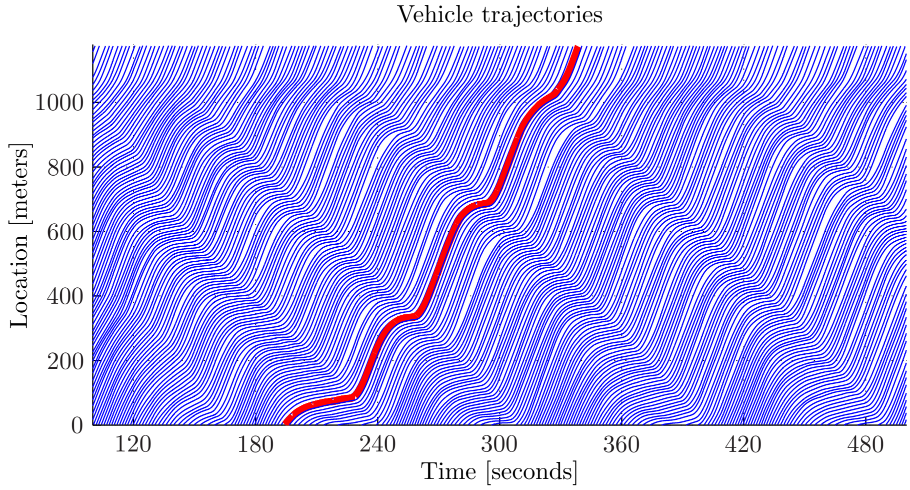


Figure 1.1: Trajectories of cars in a time-space region

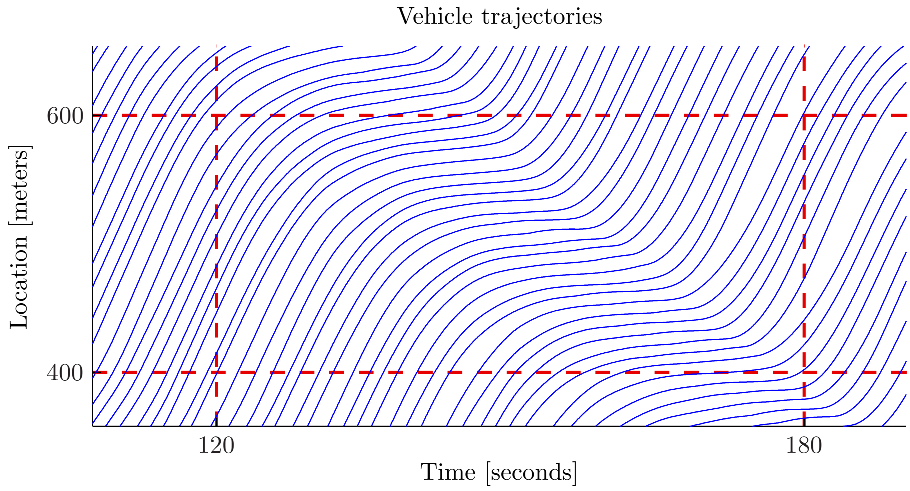


Figure 1.2: Detailed view of trajectories

The fundamental relation between speed, flow rate and density is important, since only the speed and the flow rate are measured. The proof of this relation is based on Yperman (2007).

The flow q is measured as the number of vehicles m passing a point during a certain interval:

$$q = \frac{m}{\Delta t}$$

When multiplied by a small differential of space, dx, the formula for flow becomes:

$$q = \frac{mdx}{\Delta tdx} = \frac{\text{Total distance in time-space region}}{\text{Area of time-space region}} \tag{1.1}$$

In this formula, the numerator is the total distance travelled and the denominator represents a certain time-space region. The density k is defined as the number of cars m' on a certain stretch of road:

$$k = \frac{m'}{\Delta L}$$

When multiplied by a small differential of time dt this results in:

$$k = \frac{m'dt}{\Delta Ldt} = \frac{\text{Total time in time-space region}}{\text{Area of time-space region}} \tag{1.2}$$

In this formula the numerator represents the total time spend within the time-space region and the denominator represents the time-space region. The average speed in a time-space region is the total distance travelled, divided by the total time needed for this travelling. Thus by combining equation 1.1 and equation 1.2 the speed becomes:

$$v = \frac{\text{Total distance in time-space region}}{\text{Total time in time-space region}} = \frac{q}{k}$$

So, with only the average speed and the average flow rate of a certain time-space region known, the density can be calculated.

With only the average speed and flow rate known, the passings of the cars can be graphically depicted as displayed in figure 1.3. As shown, the flow rate is defined by the number of red lines on a horizontal line between t_x and t_{x+1}, and the speed is defined by the steepness of the slope of the (red) lines. With this information the density can be calculated. In this time-space region, vehicle trajectories can be drawn, with the continuous line shown as an example. However, there is an infinite number of possible trajectories, which fulfil the macroscopic data requirements.

In reality all horizontal grid lines between t_x and t_{x+1} are filled with these red lines.

1.3 Relevance

Vehicle trajectories contain a lot of useful information about accelerations and decelerations. With microsimulation models it is possible to visualize the vehicle movements, because the trajectories are known. This visualization improves the understanding of the traffic system. When trajectories can be drawn from macroscopic data then macroscopic model are able to visualize the movements too. Furthermore, for the calculation of external effects, like air pollution and emissions[1] and traffic safety accelerations are important.

[1] e.g. CO, CO_2, NO_2, NO_x and PM_{10}

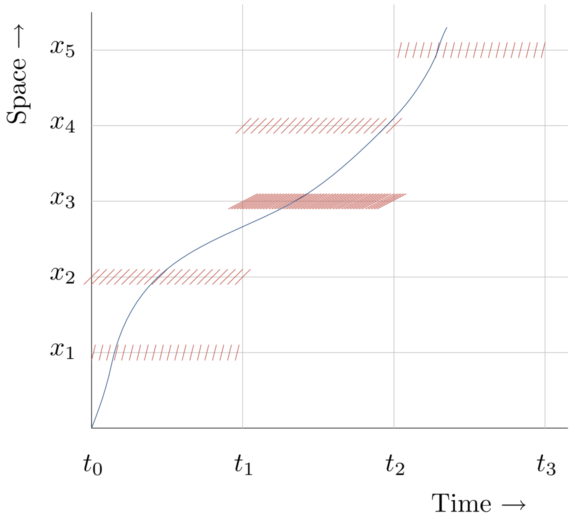


Figure 1.3: Representation of macroscopic data, with a possible trajectory

In chapter 3 more information is provided on this topic. When it is possible to reconstruct the trajectories from dynamic macroscopic data, it will be possible to visualize vehicle movements and to calculate external effects with a dynamic macroscopic traffic model. With a macroscopic model larger networks can be supported, so the emissions for a larger area can be determined with the new model.

In this research only the first step is done: the reconstruction of the trajectories. Further research is needed before it can be used for the calculation of external effects.

1.4 Preview

With known trajectories of cars it is easy to aggregate the data to average speeds and flow rates. Doing this the other way around is much more difficult. The aim of this research is to redraw the vehicle trajectories based on the macroscopic data, as explained above.

In chapter 2 a description of this research is given. Also, the aim of the research is worked out in more detail and the research questions are posed. When the aim of the research is clear a literature review is given in chapter 3. After the literature review the real work starts: in chapter 4 possible methods on vehicle trajectory reconstruction are described. The developed methods are validated in chapter 5. In a case study in chapter 6 the methods are applied on real data of the A12. At last, in chapter 7, the conclusions of this research are presented, as well as some discussion and ideas for further research. In figure 1.4 an overview is given of the structure of this report, as well as the structure of the research.

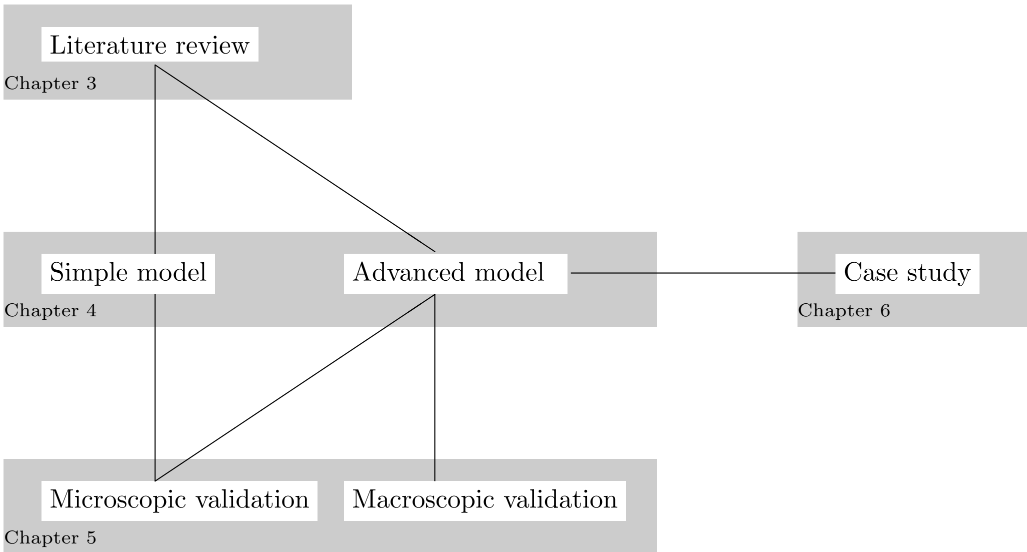


Figure 1.4: Structure of the report

Chapter 2

Description of the research

Traffic simulation models are used for a lot of studies in traffic engineering. Depending on the purpose of the study, an appropriate model is selected which fits the targets of the research the best. Macroscopic models have a different purpose than microscopic or mesoscopic models, as can be seen in the literature review (chapter 3). In general, for traffic and transport planning on a larger spatial scale mostly macroscopic models are used, while microscopic simulations are used for intersections or strings of roads.

Recently, environmental damage caused by traffic is becoming more important. For a good calculation of the effects, models are needed. But here a problem emerges: microscopic models can calculate emissions quite well, but are stochastic and not able to take a large network into account, while macroscopic models are deterministic and able to predict future travel demand, but are not able to calculate emissions quite well.

In this research this paradox is tried to be tackled. This is done by reconstructing vehicle trajectories, based on the outcomes of a dynamic macroscopic simulation. Reconstructed trajectories provide the necessary information for the calculation of emissions of cars. So, in the end, it is possible to calculate the effects on the environment caused by traffic for a larger area and a longer period of time. With the reconstructed trajectories it is also possible to visualize individual vehicle movements. The visualization of vehicle movements helps to understand the performance of the traffic system.

2.1 Objective

Comparing macroscopic and microscopic models the objective can be formulated as:

Reconstruct vehicle trajectories based on the outcomes of a macroscopic model

This objective is worked out in more detail, by posing the questions why, what, and how.

Why is it necessary to reconstruct trajectories?
Vehicle trajectories contain a lot of useful information, for several purposes. Vehicle trajectories are the time-space diagram of a car so the speed and acceleration of a car can be calculated easily. Knowing the trajectories, it is also possible to visualize the vehicle movement and to calculate the emissions or the time-to-collision. For a good calculation of externalities like air quality, air pollution and traffic safety, this data is needed.

It is certainly possible to do this with a microscopic simulation model, but the spatial scale of this type of models is too small. Furthermore, these models are stochastic, which implies that an outcome is just a certain random combination of values drawn out of distributions.

In this research dynamic outcomes of a macroscopic model are used to reconstruct the vehicle trajectories. With that a part of the drawbacks of a microscopic model can be overcome. In that case, using a macroscopic model to calculate vehicle emissions and traffic safety indicators is possible.

What data is needed/available for this reconstruction?
For this study only limited data is used. This data contains the aggregated time-varying time mean speed and flow rates. The virtual network and the locations of the detectors are known. In fact, this data can be compared with the data obtained by loop detectors. Next to this data, calculations will be done on the propagation of traffic streams. This combination leads to the reconstruction of vehicles trajectories.

How can this be achieved?
To reconstruct the trajectories first some simple methods are developed, followed by more advanced models. The development process is further described in section 2.4.

2.2 Research question

To reach the objective the main question is formulated as follows:

How can vehicle trajectories be reconstructed with only dynamic macroscopic data?

The question is divided into a number of sub-questions:

1. *What data is needed for the reconstruction of trajectories?*
   First, an investigation is made on the data needed. In general there is an optimum where the more data would decrease the quality of the results. On the other hand, if too many assumptions have to be made, the error can be very large. The most simple methods need less data compared to the more complex methods.

2. *What would a conceptual model look like?*
   Second, a conceptual method is developed. This method is the basis for the deployment of a method which is made in Matlab. The conceptual method will evolve during the process: the first method is simple, and the successive methods try to improve the results.

3. *How can a more complicated model be supported?*
   In section 2.4 more information is given on the way the model is developed. For the extensions of the model it is necessary to think about feasibility.

4. *What is the quality of the reconstructed trajectories?*
   At last, the results have to be compared to the input data. How can that be done and what are indicators for a good model? Those questions should be addressed before the quality question can be answered.

2.3 Scope

Reconstructed trajectories provide information which can be used for the visualization of vehicle movements and for the calculation of external effects, like air pollution and traffic safety. However, the calculation of external effect is beyond the scope of this project, but it is a topic for further research.

There might be other methods to improve the quality of the calculation of external effects with a macroscopic model, but in this research the focus will be on vehicle trajectories. The first argument to use this method is the fact that it, if possible, ensures that emissions can be calculated, although additional research might be needed. A second argument is that the reconstructed of trajectories can be used for the visualization of the car movements, so a gap between microscopic models and macroscopic models is filled. Next to that, trajectory reconstruction came as a request from Goudappel Coffeng.

The data for this research is captured with both microscopic and macroscopic simulation models. The microscopic data is used to compare the vehicle trajectories of the 'real' situation in the simulation model with the reconstructed trajectories where only the aggregated data is used. The data of the macroscopic simulation model is used to compare the results of different interpolation techniques. The data which is measured at the detector is compared to the results of the interpolation between the previous and the next detector.

2.4 Approach

The reconstruction of the trajectories will be tried with several methods, with an increasing level of the use of traffic data and traffic flow theory. The methods can be divided into two different models: the first one is a very naive and simple model. This model has only simple mathematical relations and does not use any traffic flow theory. The second model is an interpolation model which uses a fundamental diagram for the interpolation between detector locations and time intervals.

For this research two different data sets are used. The first data set is an aggregation of microscopic data obtained with a microscopic simulation model. The other data set contains macroscopic data of a simple network which is constructed in MaDAM, a dynamic macroscopic simulation model.

Since the two different models use very different methods for the reconstruction of vehicle trajectories the validation is performed in two different ways. The first way compares the outcomes the trajectories of the microscopic simulation with the reconstructed trajectories. The second validation methods is only performed on the advanced methods, since these methods are based on an interpolation algorithm.

2.4.1 Two models

For this research two models are developed. The first is a very simple one and the second is more advanced, in order to improve the results of the reconstruction. Both models are described in detail in chapter 4.

Simple model
The first and simple model is a model which estimates the moments that vehicles pass the detector locations. After this distribution of the headways the passings at different detector locations have to be linked. This is only done for the first car in the network and all other cars are linked according to the first-in-first-out principle. On this first method some improvements are made in order to increase the quality of the model. The most important improvement takes speed at the detector locations into account which is used to draw a spline[1] between the detector locations.

On this simple model only the data set with the aggregated microscopic data is used. The reason for this is that the simple model reconstructs the trajectories directly, so the results of the reconstruction can be compared directly with the microscopic data. The validation is, for the same reason, only done at the trajectories level. It is possible to use the method for larger data sets.

Advanced model
The advanced model is a model which reconstructs trajectories in two steps. In the first step density is interpolated between the known points. After that, speed is calculated based on the fundamental diagram. This speed is used to calculate the distance which a car travels in a certain time step. The flow rate is used to calculate when a car enters the network. One of the differences with the first model is that the algorithm only considers the current time step, while the simple model at least needs to know the entire time interval.

With the advanced model it is possible to compare the results of the reconstructed trajectories with the microscopic data. In this way it is possible to validate the results at the trajectories level. Next to that, it is also possible to compare the results of the interpolation when detector data is leaved out with the macroscopic data.

2.4.2 Two data sets

Both data sets are obtained with simulation models. The first data set is created in Vissim, a microscopic simulation model. In Vissim all trajectories are recorded. This data is aggregated with Matlab to virtual loop detector data. The other data set is made with MaDAM, a dynamic macroscopic model.

Aggregated microscopic data set
The first data set is generated with Vissim. The data contains information of the location, time and speed of the cars in the network, for every tenth of a second. With this data it is possible to visualize real vehicle trajectories of all cars. These trajectories are the basis for the comparison with the reconstructed trajectories. The Vissim data is aggregated to 'loop detector data'. This means that the simulation period is divided in equal time intervals and that a number of detectors are placed on the road. At all detector locations and for all time intervals an algorithm counts the passing vehicles and records their harmonic mean speed. At the end of this procedure, the macroscopic data for all time intervals and detector locations are known. This is the input for the development of the reconstruction algorithm.

[1] A spline is, in this case, a cubic polynomial function

The advantage of the Vissim data is that the real trajectories are available for comparison with the reconstructed trajectories. The use of a microscopic simulation model has also some disadvantages. The first disadvantage is that the outcome is the result of a stochastic process. This implies that the outcomes can be different when another seed number would have been used. A second disadvantage is that the fundamental diagram is not known for a certain link. This diagram has to be estimated based on the results of the aggregation of the data. Therefore also a data set is generated with a deterministic and macroscopic model, MaDAM.

Macroscopic data set
For the methods which use interpolation methods, also a data set is created with MaDAM, a dynamic macroscopic model. This data contains only the speed, density and flow rates for every link. The simulated time, as well as the link length, is much longer in this data set. The advantage of a macroscopic model is its deterministic character. Therefore the outcome is unique.

2.4.3 Two validations

Since both models are build very differently, it is not possible to do the validation in only one way. The first validation method compares the trajectories and the second method investigates the quality of the interpolation methods. The validation is performed in chapter 5.

Trajectories level
At the trajectories level of the validation the arrival time and the speed of the 'real' trajectories are compared with those of the reconstructed trajectories. Since all methods are able to reconstruct trajectories it is possible to validate the results of all methods.

Interpolation level
The interpolation method is of great importance for the advanced model. Therefore the quality of the interpolation is validated. In this validation the speed and the density at different detector locations are interpolated and compared with the real values for the speed and the density.

Chapter 3

Literature Review

In this chapter the background of the topic is discussed. In this review a description on existing models on all spatial scales is given. After that, information is given on topics which are in the near field of the subject of this research.

3.1 Traffic models

In general, traffic models are used for forecasting travel demand and traffic behaviour. Each type of model has its own spatial and temporal scale on which it performs best. In this section the main types of models are described. The macroscopic models are discussed first, followed by microscopic models and mesoscopic models.

3.1.1 Macroscopic models

The aim of a macroscopic model is to investigate travel and traffic demand between a certain set of origins and destinations. The dominating principle used on this level is the classical four-step model (McNally 2000; Ortuzar and Willumsen 2001; Transportation Association of Canada 2008). The outcome of the model is an amount of traffic per link during a certain time period. The main task for this type of models is to forecast future travel demand. Most of the macroscopic models follow the four steps: generation, distribution, modal split, and assignment, although they use different principles within the modelling of the sub-steps. Macroscopic models originally were used for the planning of car transport, based on a given land-use plan (SRTC 2006). Macroscopic models are not able to visualize vehicle movements, to calculate speeding patterns or to do a detailed study on a small area like an intersection or a bottleneck. Furthermore, most macroscopic simulation models are deterministic, i.e. they do not use random seeds or distributions for the calculations and give only one answer. The result of the model will be the same every model run. Most of the macroscopic models are based on the first order traffic flow theory, using fluid dynamics, as developed by Lighthill, Whitham and Richards (Lighthill and Whitham 1955). Other models are based on gas kinetics theories, starting with the Prigogine model (Klar et al. 1996; Moet 2003).

Traditionally, macroscopic models used all-or-nothing or equilibrium assignment methods. Nowadays also dynamic assignment methods are used, which do not necessarily produce an equilibrium solution (Yperman 2007).

Examples of dynamic macroscopic assignment models are INDY, METANET, Marple and MaDAM (Verkeersmodellering.nl 2009; Hoogendoorn and Hoogendoorn-Lanser 2008).

3.1.2 Microscopic models

A second type of model is the microscopic simulation model. Microscopic simulation models are designed for road facility system analysis. Microsimulation models are not designed for optimization of control strategies. These models show time-dependent operations results (Ca DoT 2002).

Microscopic simulation models are based on the interaction between vehicles. The simulation is based on the vehicle movements, i.e. the position of the vehicle is calculated each time step. With these models, it is possible to visualize the vehicle movements and to record the speeding patterns of the cars. With this data it is possible to calculate emissions. Microscopic models are usually used to study a smaller area than what is common for macroscopic models. The spatial scale can be one or more possibly signalized intersections, a corridor or a weaving section on a highway.

A drawback of a microscopic model is its stochastic character. Therefore, multiple simulations with different seed numbers are needed to get a reliable average outcome.

Examples of microscopic models are Vissim, AIMSUN, Paramics, and CORSIM (Verkeersmodellering.nl 2009; Lieberman and Rathi 2001).

3.1.3 Mesoscopic models

The mesoscopic models aim to fill the gap between microscopic and macroscopic models. On one hand, it provides the modelling of choices of individual drivers, while on the other hand, it limits the level of detail of the driving behaviour (Burghout 2005).

Mesoscopic models are in the field between microscopic and macroscopic model, in both spatial and temporal scale. Vehicles are represented as groups. These groups are simulated each time step. This type of models mostly uses a dynamic assignment method. Most of these models are based on the gas kinetics theory (Klar et al. 1996).

Examples of mesoscopic models are DYNASMART, DYNAMIT, and CONTRAM (Verkeersmodellering.nl 2009; Lieberman and Rathi 2001).

3.2 Related issues

No evidence is found in literature on research that tries to reconstruct vehicle trajectories, based on macroscopic data. Without direct literature the focus is turned to indirect literature or literature that describes research in the near field.

The first issue related to this research concerns hybrid models. Secondly, research describing a methodology of reconstructing trajectories was found in literature. After that, further investigation is done on the quality of the calculation of externalities with macroscopic models. The fourth topic is about interpolation techniques, which can be used to describe the whole area between time intervals and detectors. At last some fundamental diagrams are described.

3.2.1 Hybrid models

Hybrid models combine two types of models, in order to use the opportunities of both models. The link with the proposed research lies in combining two models. The difference is found in the way the calculations are done: in this research, no microsimulation is used, but all calculations are based on the outcomes of the dynamic macroscopic assignment. The outcomes will look like microsimulation results.

Burghout (2004) assessed in his doctoral dissertation a number of models which integrate aspects of different models. The hybrid models he assessed are models with a static assignment with simulation, mesoscopic models with microscopic simulation and macroscopic models with microscopic simulation.

On the first type, SATURN combines a static assignment over a larger network with traffic simulation in a smaller part of the network. Another hybrid model is the combination of EMME/2 (macroscopic) and AIMSUN/2 (microscopic). In this model the output of EMME/2 can be used in the microscopic assignment. A final combination described by Burghout is the combination VISUM - Vissim. This combination works about the same as the EMME/2 - AIMSUN/2 model.

The second type of hybrid models are mesoscopic models with the microscopic simulation. A first combination is made by Paramics and Dynasmart. In this model a larger network can be simulated, and it takes the route choices from Dynasmart. Another combination is Metropolis and MITSIMLab. The mesoscopic model Metropolis is run on a larger network. The output flows of Metropolis are used to define a time dependent OD-matrix, which is used for MITSIMLab. A last combination in this type is Transmodeler. In this GIS-based simulation the user defines for each link whether it is microscopic, mesoscopic or macroscopic. This model was still in development in 2004. The first release of Transmodeler was in December 2005.

The last type of hybrid models are the macroscopic models with microscopic simulation. The combination of Pelops and SIMONE, MICMAC, and Hystra, all face the problem of using data from macroscopic models into microscopic models. Pelops/SIMONE and MICMAC define the transition of the macroscopic part of the network to the microscopic part of it in different ways. Pelops/SIMONE uses an aggregation algorithm to reduce the length of the time steps. Furthermore, data about speeds and headways have to be calculated. MICMAC uses the fundamental diagram constraints to ensure that the models communicate in a good way. At last, Hystra uses two models for the modelling of the different parts of the network. One part is modelled macroscopically, the other part microscopically. Both sub-models are consistent with the kinematic flow theory, as developed by Lighthill, Whitham and Richards (Lighthill and Whitham 1955).

3.2.2 Trajectory studies

Coifman (2000) describes a methodology to calculate link travel times based on the estimation of vehicle trajectories. In this method he uses the speed and the headway of individual vehicles. With one additional parameter he is able to create a trajectory, which he uses to define the link travel time. The results are reasonable considering the simplicity of the method.

Some problems, both technical and procedural, exist with this method. The technical problems are the way of measuring. Most operators do not measure individual car speeds and headways, but aggregate the measurements over a certain time period (30 seconds or longer). On the procedural side, there are problems with traffic queuing. With a traffic jam partly on the link, but not on the detector, the calculated travel time will be shorter than the real travel time. This mechanism also works the other way around: with the jam on the detector, but not on the entire link, the calculated travel time will be overestimated.

3.2.3 Environmental studies

For most of microscopic simulation models, it is already possible to calculate exhaust emissions (European Commission, Directorate General VII - Transport 1999), containing at that time at least Vissim, CORSIM, FLEXSYT II, Paramics, and AIMSUN2. In the field of macroscopic and mesoscopic models, the list is much shorter and the results are not overwhelming: Cappiello (2002) did a literature review on emission models, concluding that speed based models[1] are too simple. There are just few dynamic non-microscopic emission models.

On the mesoscopic level, Yue (2008) developed the VT-meso model, which uses three variables: average travel speed, number of vehicle stops per unit distance, and average stop duration. Emissions are calculated per link. The results at this level are reasonable, compared to VT-micro and field tests.

3.2.4 Macroscopic interpolation

With only limited data available, interpolation techniques are useful to make a prediction about the traffic states between the measurements. A very simple one-dimensional model is to interpolate linearly between the measurement points. In this model, the inverted distance to the measurement points is the weighing factor for the value in between. For a two-dimensional situation the linear interpolation will be based on four points. The weighing factor for the point in between remains the inverted distance.

Treiber and Helbing (2002) developed a model which uses an non-linear interpolation technique. In this *adaptive smoothing method* a non-linear filter transforms the discrete detector data into a smooth spatio-temporal function for the data. With this method it is possible to calculate the value of, for instance, the speed or flow rate on any point. Although the method is not a linear interpolation, it still is a mathematical one. A better way of interpolating would use more traffic science. For this reason no further attention is paid on this interpolation method.

3.2.5 Fundamental diagrams

Fundamental diagrams could be of great use for the reconstruction of trajectories using traffic flow theory. Fundamental diagrams describe the fundamental relation between the flow, speed and density. The basic relation between flow, speed and density is given by $Q = k * v$. With the fundamental diagram known, only one of these three values has to be known to calculate the other two. Most of the macroscopic models rely on this diagram, which can have several shapes. Below three fundamental diagram are described, which are important for this research, as well as the first fundamental diagram known in traffic flow theory.

[1] A speed based model is a model that only considers average speed and flow rates

Greenshields
The first one to derive a relation between flow, speed and density was Bruce D. Greenshields (Hoogendoorn and Hoogendoorn-Lanser 2008). In 1933, he used photographic measurements methods to study traffic and travel behaviour. Greenshields came up with a parabolic flow-density relation (Greenshields 1933, 1935). According to Hall (2001), Greenshields model dominated the field of traffic engineering for over 50 years, until 1994. Meanwhile, others developed relations between traffic flow, speed and density as well. One of these others was Newell.

Newell
In his simplified theory of kinematic waves Newell (1993) developed a much more simple fundamental diagram: a triangular shaped flow-density diagram.

$$v(k) = \begin{cases} V_f & \text{if } k < k^* \\ \dfrac{Q_{cap}}{k^* - k_j}\left(1 - \dfrac{k_j}{k}\right) & \text{if } k \geq k^* \end{cases}$$

Where k^* is the optimal density.

With this triangular shaped diagram it is possible to solve the conservation law as posed by Lighthill and Whitham (1955).

$$\frac{\partial k(x,t)}{\partial t} + \frac{\partial q(x,t)}{\partial x} = \frac{\partial k(x,t)}{\partial t} + \frac{\partial Q(k(x,t))}{\partial k}\frac{\partial k(x,t)}{\partial x} = 0 \tag{3.1}$$

According to Yperman (2007), this partial differential equation can be solved, given the initial and boundary conditions. In this equation $\dfrac{\partial Q(k(x,t))}{\partial k}$ is the slope of the fundamental diagram, which is a constant when a triangular shaped form is assumed.

Van Aerde en Rakha
Later, Van Aerde and Rakha (1995) described a continuous relation between flow, speed and density. In this method four parameters have to be estimated, by which this relation can be described. These parameters are free speed, V_{free}, speed at capacity, V_{cap}, jam density, k_{jam} and capacity flow, Q_{cap}. The relation between flow and density is given by the following equation:

$$Q(k) = k(v) * v = v(k) * v \tag{3.2}$$

$$k(v) = \frac{1}{c_1 + \frac{c_2}{V_f - V} + c_3 V} \tag{3.3}$$

Where

$$\begin{aligned} p =& \frac{2 * V_{cap} - V_f}{(V_f - V_{cap})^2} \\ c_1 =& p * c_2 \\ c_2 =& \frac{1}{k_j * \left(p + \frac{1}{V_f}\right)} \\ c_3 =& \frac{-c_1 + \frac{V_{cap}}{Q_{cap}} - \frac{c_2}{V_f - V_{cap}}}{V_{cap}} \end{aligned}$$

An advantage of this fundamental diagram is the continuity of the function. This continuity enables formulating a derivative. The derivative $\frac{\partial q}{\partial k}$ can be calculated when $q(k)$ is known. Therefore, $k(v)$ must be rewritten to $v(k)$:

$$k(v) = \frac{1}{c_1 + \frac{c_2}{V_f - V} + c_3 V} \tag{3.4}$$

$$\begin{aligned} v(k) =& \frac{-\left(V_f c_3 - c_1 + \frac{1}{k}\right) + \sqrt{\left(V_f c_3 - c_1 + \frac{1}{k}\right)^2 + 4c_3\left(c_1 V_f + c_2 - \frac{V_f}{k}\right)}}{-2c_3} \\ q(k) =& k * \frac{-\left(V_f c_3 - c_1 + \frac{1}{k}\right) + \sqrt{\left(V_f c_3 - c_1 + \frac{1}{k}\right)^2 + 4c_3\left(c_1 V_f + c_2 - \frac{V_f}{k}\right)}}{-2c_3} \end{aligned} \tag{3.5}$$

Then $\frac{\partial q}{\partial k}$ becomes:

$$\begin{aligned} \frac{\partial q}{\partial k} =& \frac{-\left(V_f c_3 - c_1 + \frac{1}{k}\right) + \sqrt{\left(V_f c_3 - c_1 + \frac{1}{k}\right)^2 + 4c_3\left(c_1 V_f + c_2 - \frac{V_f}{k}\right)}}{-2c_3} + \\ & \frac{\frac{1}{k} * \left[1 + \frac{V_f c_3 + c_1 - \frac{1}{k}}{\sqrt{\left(V_f c_3 - c_1 + \frac{1}{k}\right)^2 + 4c_3\left(c_1 V_f + c_2 - \frac{V_f}{k}\right)}}\right]}{-2c_3} \end{aligned} \tag{3.6}$$

Smulders
Smulders (1990) developed a fundamental diagram which is parabolic in the free flow part of the fundamental diagram and linear in the congestion part of it.

$$v(k) = \begin{cases} V_f\left(1 - \dfrac{k}{k_j}\right) & \text{if } k < k^* \\ V_f k^* \left(\dfrac{1}{k} - \dfrac{1}{k_j}\right) & \text{if } k \geq k^* \end{cases}$$

Where k^* is the optimal density. This function looks like the fundamental diagram of Newell, but is not linear in the first part of the diagram. This function is more consistent with real traffic situation, where is it is also not likely that the speed at capacity is equal to the free flow speed.

The four described fundamental diagrams developed by Greenshields (1933), Newell (1993), Van Aerde and Rakha (1995) and Smulders (1990), are graphically depicted in figure 3.1.

Use of the fundamental diagram
With the fundamental diagram, the speed of lines with a constant density, characteristic lines, can be found. The speed of this line is the tangent of the fundamental diagram, $\frac{\partial q}{\partial k}$. When a characteristic line departs at a certain location, the arrival at the next location can be estimated. At that location, the density can be checked, and when this is almost

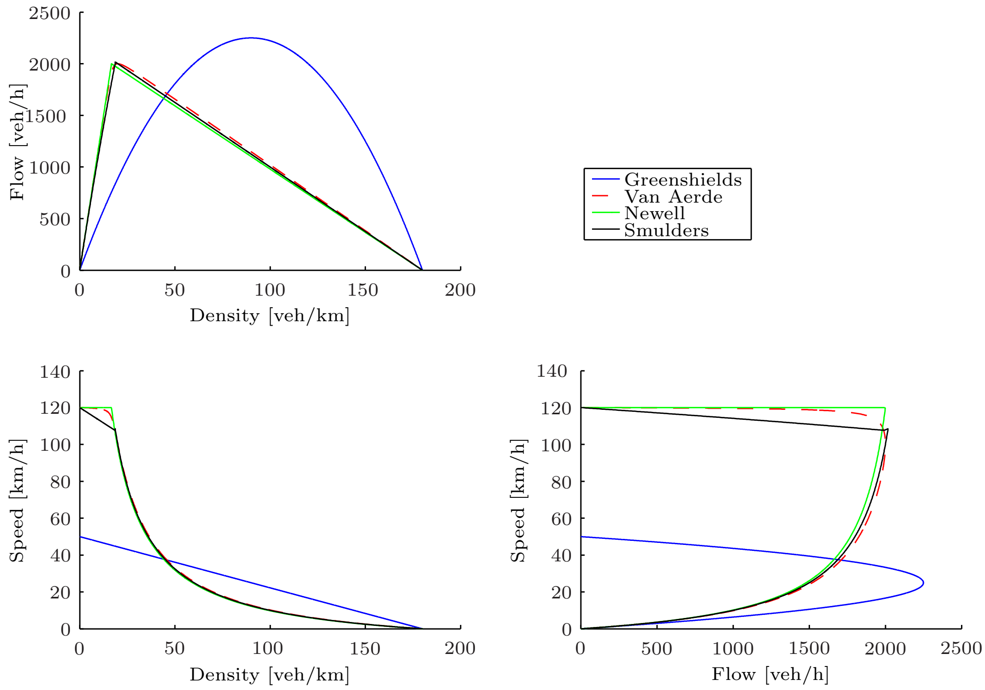


Figure 3.1: Four fundamental diagrams

the same as at its departure, it is likely that this line is not intersected by a shock wave. In this way the area between two locations can be estimated using the characteristic line.

The independent variable could, in theory, be either the flow rate, the density or the speed. But there are two values for the speed and the density for every flow rate and, when Newell's fundamental diagram is used, the speed in a free flow traffic situation is independent of the density. Therefore, only when the density is used as independent variable it gives unique answers for the flow rate and the speed.

Chapter 4

Trajectory reconstruction methods

In this chapter, the development of the reconstruction algorithm is described. In the first stage of the development a simple method is constructed, in which no traffic flow theory is used. The second method uses interpolation techniques to calculate the speed for the whole time-space frame. When the speed is known everywhere the trajectories can easily be reconstructed.

4.1 Simple model

In the first model only simple methods are used for the reconstruction of the trajectories. These methods only use mathematical relations and no traffic flow theory.

The development of the first model is done according to the following steps:

1. Distribution of the passings
2. Estimation of arrival times at all locations
3. Drawing of the trajectories

These steps are worked out in more detail below. Additional information about the used formulas can be found in appendix A.

Step 1: Distribution of the passings
The macroscopic, or aggregated microscopic, data can be represented by a number of cars passing a certain point during a certain time interval with a certain average speed. The cars passing the detector within the time interval have an average headway distribution equal to the interval length divided by the number of cars. The most simple distribution of the cars is a homogeneous distribution over the time interval. With the assumption that all cars pass the detector with the average speed, the macroscopic constraints of the number of cars and their average speed, are fulfilled. This distribution is very naive, but a good starting point. In equation 4.1 the headway and speed of the cars is given.

The passing time and speed of car c_{ij}^{n} are:

$$\begin{aligned} c_{ij}^{tn} &= t_0 + (j-1)\Delta t + (n-0.5)\frac{\Delta t}{N_{ij}} \qquad n = 1, 2, ..., N_{ij}-1, N_{ij}, \qquad \forall i, j \\ c_{ij}^{vn} &= \bar{V}_{ij} \qquad \forall i, j \end{aligned} \tag{4.1}$$

From this point on, a new distribution is calculated, which takes into account the macroscopic data of the previous and next time intervals. This new distribution still fulfils the macroscopic constraints, but decreases the homogeneous nature of the first model. The basic assumption is that the speed and headway at the end and the beginning of a time interval is the mean of the two intervals. Half way down the interval the speed and the headway is equal to twice the mean speed of the interval minus the mean of the first and last car within the interval. All other cars are distributed linearly between the first car, the car in the midst and the last car in the interval. In this way the speed and the headways are not homogeneous distributed, but still fulfil the macroscopic constraints for the average speed and headway.

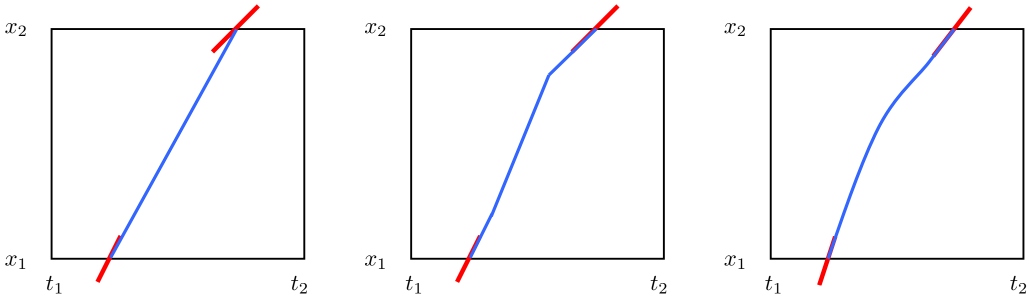
Step 2: Estimation of arrival times at all locations
After the distribution, the passings of the cars are known at the detector locations, but it is unknown which passings of the different detectors have to be connected with each other for the right trajectories. Therefore an estimation of the travel time has to be made. In the estimation only the travel time of the first car is estimated; all other cars are assigned according to the first-in-first-out principle.

A simplest way to estimate the travel time between two detector points takes the only speed at one detector location into account. Using the speed at the current detector the travel time can be estimated by dividing the distance between the points by the speed of the car. The advantage of this estimation is its simplicity. Only the speed of one detector is needed, combined with the distance between the detectors. A big disadvantage is the fact that the travel time will be overestimated or underestimated. When the speed at the current detector is low, compared to the next detector, the travel time will be overestimated. The other way around, the travel time will be underestimated. Therefore, an algorithm which considers the speeds at both detectors would reduce this problem, although its simplicity decreases. In this more advanced method the time-space region for which the speed is considered to be constant is the same for both detectors, i.e. high speeds have relative large space for a relative small time period, and low speeds have a smaller space for a larger time. In this way the product of the time period and the space is the same for both detectors. Of course, the sum of the time periods is the travel time of the car between both points. From a database with the data of the passings of the vehicles, calculated in step 1, the passing which is closest to the arrival time is selected. When the space between two detectors is split up in x_1 and x_2 and the travel time between those detectors is split up in t_1 and t_2, then:

$$\begin{aligned} x_1 t_1 &= x_2 t_2 \\ x_1 &= v_1 t_1 \\ x_2 &= v_2 t_2 \\ x_1 + x_2 &= \text{Distance between two detectors} \end{aligned} \tag{4.2}$$

v_1 and v_2 are the speeds at the detector locations. By solving these equations the travel time can be calculated.

Since the arrival time of the first car is known for all locations, all other cars car be put behind this car according to the first-in-first-out principle. The data for the arrival time and the passing speed is extracted from the passing data, as made in step 1.



(a) Simple linear trajectory reconstruction

(b) Advanced linear trajectory reconstruction

(c) Spline trajectory reconstruction

Figure 4.1: Different trajectory reconstruction techniques

Step 3: Drawing of the trajectories
After step 2, for all cars the arrivals at the detectors are known, so the lines between those points can be drawn. For the drawing of the lines three methods are developed, with an increasing complexity. The first method is to connect the lines without taking the speed at the current location into account. An advantage of this method is its simplicity and the fact that cars can not drive backwards. A disadvantage is the fact that the connector line does not consider the speed at any location. Therefore, a more advanced, linear method is developed, which takes into account the speed of the car. The speed of the car at the detector is considered to be constant over a certain adjustable space, being a constant fraction of the space between two detectors. In this way a certain space is already covered and the ends of both lines are simply stitched together with a connector line. In this way, the trajectory between two detectors is created with three linear lines, of which two have the speed of the car at the detector.

A last method is a mathematical method for drawing a continuous differentiable line between two points with known tangents, which is the speed of the car. Therefore, a spline is used, based on the arrival time, location and speed at the two detectors. Along the road, all splines are linked, in order to get a trajectory. A big advantage of this method is that it takes the speed of the car at the detectors into account, as well as the fact that the trajectory is continuous differentiable, like it is in reality. A drawback of this method is the fact that the spline can be such that the car can drive backwards. In figure 4.1 a graphical representation of the different trajectory reconstruction methods is given.

4.2 Reconstruction based on interpolation techniques

To improve the quality of the reconstructed trajectories, a more complex method is developed. In this method, the speed is interpolated between the known locations to get data for all time steps and locations. With this dense grid of data the reconstruction of the trajectories can be done by extrapolating the speed over the small time interval to calculate the next location of the car. The reconstruction is done in three steps:

1. Estimation of the fundamental diagram

2. Interpolation of the density

3. Drawing of the trajectories

In the first step the fundamental diagram has to be estimated, if it is not known. A simple algorithm is developed to estimate a fundamental diagram per section of the road. The algorithm is described in section 4.2.1.

In the second step the density is interpolated. The macroscopic data can be represented by a grid of data, where the results are allocated to the intersections of the detector location and the time intervals. For each time-space block of 4 data point, between two location and between two time intervals, an interpolation method is used to describe the whole time-space frame. After the interpolation, the density and speed are known at any location and at any time.

To achieve the best result with an interpolation method, different methods are used to calculate the density within the time-space frame. In the next sections the development of the methods is described. The most simple method uses a linear interpolation technique in both directions. Then a method is developed in which a Taylor series expansion is used for the interpolation of the density. This Taylor series expansion is based on the fundamental diagram and the conservation law. These methods are described in more detail in the sections 4.2.2 and 4.2.3.

In the last step the trajectories are drawn. The method to do this is described in section 4.2.4.

4.2.1 Estimation of the fundamental diagram

An algorithm is developed to estimate the fundamental diagram for each section of the road. The fundamental diagram is used for the interpolation of the density and for the calculation of the speed. The algorithm searches good values for the free flow speed, V_f, the speed at capacity, V_{cap}, the jam density, k_{jam}, and the capacity flow Q_{cap}. From these parameters the density at capacity, k_{cap} can be calculated.

The algorithm is a one parameter optimization, which optimizes k_{cap}. For this optimization V_f, k_{jam}, Q_{cap} are estimated based on the given data, where:

$$\begin{aligned} V_f^i &= \min(130, \max(V_j^i)) && \forall j \\ Q_{cap}^i &= \max(Q_j^i) && \forall j \\ k_{jam} &= \max(\max(K_{ij}), 150) && \forall i, j \end{aligned} \tag{4.3}$$

The free flow speed and the capacity are based on the data of the current section, but the jam density is the maximum density found in the entire network, with a minimum value of 150. The free flow speed is bounded to a maximum of 130 km/h. With these parameters the best value for the density at capacity can be found. The difference between the measured density and the calculated density is measured with the root mean squared error. The best value for the density at capacity is stored and replaced when a better value is found.

In figure 4.2 an example is given of an estimation of a fundamental diagram. The data is retrieved from a loop detector on the A12 in the Netherlands, near Gouda. The data is used in the case study in chapter 6.

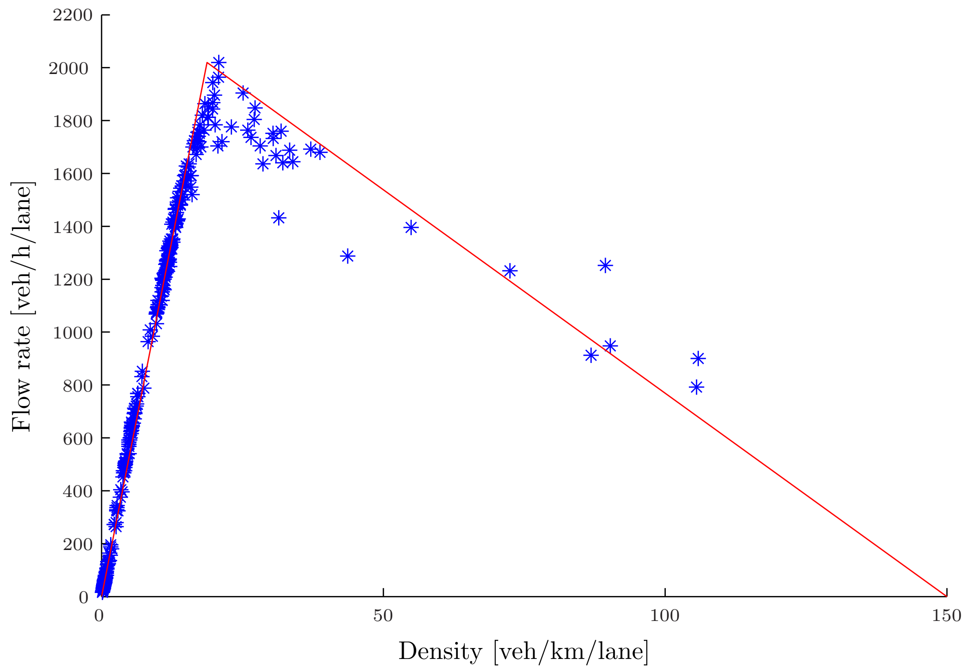


Figure 4.2: Example of estimated fundamental diagram

4.2.2 Linear interpolation

The most simple way of interpolation is a two dimensional linear interpolation. This method is very quick and pre-defined in Matlab. The linear interpolation only needs the four corner points of the two-dimensional area. This is the area between two detectors and two time intervals. The function can then be described as follows:

$$k = \sum_{i=1}^{2}\sum_{j=1}^{2} k_{ij} \frac{(\Delta t - |t^* - t_j|)(\Delta x - |x^* - x_i|)}{\Delta t \Delta x} \tag{4.4}$$

Where

k_{ij} =Density at detector i and time interval j
Δt =Time between two time intervals
Δx =Space between two detector locations
t_j =Time at the corner point
x_i =Location of the corner point
t^* =Time at an intermediate point
x^* =Location of an intermediate point

The results are not accurate (see figure 4.3): the contour lines, where the density is constant, is not a straight line, but are curved along the left-hand side of the queue. In reality this would be more or less a straight line, since the inflow on the link is constant.

Since the outcomes of the model are not considered to be satisfying, new methods are developed to describe the density over the whole time-space frame.

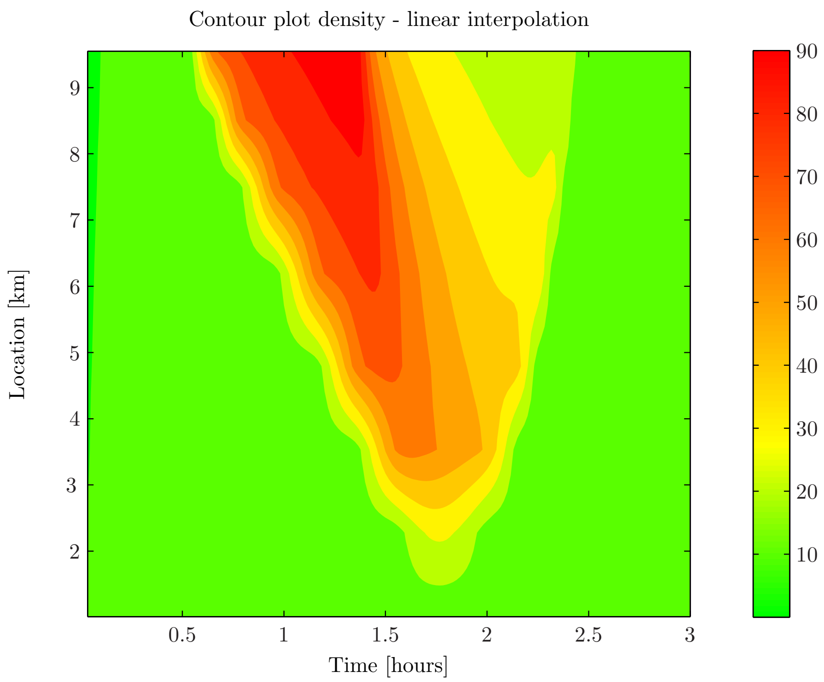


Figure 4.3: Linear interpolation of density [veh/km]

4.2.3 Taylor series expansions

To describe the whole time-space frame, a Taylor series expansion is formulated which is based on relations given by the conservation law and the fundamental diagram. The fundamental diagram describes the relation between flow, speed and density, which is given by a function. A lot of research is done to describe a good relation between these traffic flow quantities. Examples of these diagrams can be found in Greenshields (1935); Smulders (1990); Newell (1993); Van Aerde and Rakha (1995).

The general idea is that, when the density and speed are known everywhere, the trajectories can be drawn very easily. Every time step a new density and a new speed is calculated, which is used to calculate the space step. So if one is able to calculate the density very accurate, the resulting trajectories will have a good quality.

A general two-dimensional second order Taylor series expansion is given in equation 4.5.

$$k(x,t) \approx k(x_0,t_0) + \begin{bmatrix} x-x_0 \\ t-t_0 \end{bmatrix}^T \nabla k(x_0,t_0) + \frac{1}{2}\begin{bmatrix} x-x_0 \\ t-t_0 \end{bmatrix}^T \nabla^2 k(x_0,t_0) \begin{bmatrix} x-x_0 \\ t-t_0 \end{bmatrix} \tag{4.5}$$

Since the fundamental diagram is known, $\frac{\partial q}{\partial k}$ is known. The conservation law states that:

$$\frac{\partial k}{\partial t} + \frac{\partial q}{\partial k}\frac{\partial k}{\partial x} = 0 \tag{4.6}$$

So, if the derivative of k with respect to either time or space is known or assumed the other one can be calculated. The second derivative can also be calculated, in order to fill in the Taylor series expansion. The result of this is given in equation 4.7. In this equation it is assumed that $\frac{\partial k}{\partial t}$ is known or can be calculated.

$$\begin{aligned} k(x,t) \approx & k(x_0,t_0) - \frac{\partial k}{\partial t}\left((x-x_0)\left(\frac{\partial q}{\partial k}\right)^{-1} + (t-t_0)\right) + \\ & \left(\frac{\partial q}{\partial k}\right)^{-2}\frac{\partial^2 q}{\partial k^2}\left(\frac{\partial k}{\partial t}\right)^2\left(-(x-x_0)^2\left(\frac{\partial q}{\partial k}\right)^{-1} + (x-x_0)(t-t_0)\right) \end{aligned} \tag{4.7}$$

In appendix B the Taylor series expansion is worked out in more detail.

The outcome of the equation is dependent of the chosen fundamental diagram. If $\frac{\partial k}{\partial t}$ is assumed to be constant only the first and second derivative of $q(k)$ have to be defined. This is done for three different fundamental diagrams: Newell, Smulders and Van Aerde.

Newell

$$\frac{\partial q}{\partial k} = \begin{cases} V_f & \text{if } k < k_{cap} \\ \dfrac{Q_{cap}}{(k^* - k_j)} & \text{if } k \geq k_{cap} \end{cases} \tag{4.8}$$

$$\frac{\partial^2 q}{\partial k^2} = 0 \tag{4.9}$$

Van Aerde

$$\frac{\partial q}{\partial k} = \frac{f_1 - \sqrt{f_2} - \dfrac{1}{k} + \dfrac{f_1 - 2V_f c_3}{k\sqrt{f_2}}}{2c_3} \tag{4.10}$$

$$\frac{\partial^2 q}{\partial k^2} = \frac{(f_1 - 2c_3 V_f)^2 - f_2}{2c_3 k^3 f_2^{3/2}} \tag{4.11}$$

Smulders

$$\frac{\partial q}{\partial k} = \begin{cases} V_f - 2k\dfrac{V_f}{k_j} & \text{if } k < k_{cap} \\ -\dfrac{\phi}{k_j} & \text{if } k \geq k_{cap} \end{cases} \tag{4.12}$$

$$\frac{\partial^2 q}{\partial k^2} \begin{cases} -2\dfrac{V_f}{k_j} & \text{if } k < k_{cap} \\ 0 & \text{if } k \geq k_{cap} \end{cases} \tag{4.13}$$

Where

$$
\begin{aligned}
f_1 =& V_f c_3 - c_1 + \frac{1}{k} \\
f_2 =& \left(V_f c_3 - c_1 + \frac{1}{k}\right)^2 + 4c_3\left(c_1 V_f + c_2 - \frac{V_f}{k}\right) \\
c_1 =& p \cdot c_2 \\
c_2 =& \frac{1}{k_j\left(p + \frac{1}{V_f}\right)} \\
c_3 =& \frac{-c_1 + \frac{V_{cap}}{Q_{cap}} - \frac{c_2}{V_f - V_{cap}}}{V_{cap}} \\
p =& \frac{2V_{cap} - V_f}{\left(V_f - V_{cap}\right)^2} \\
\phi =& V_f \cdot k_{cap} \\
V_f =& \text{Free-flow speed} \\
V_{cap} =& \text{Speed at capacity} \\
Q_{cap} =& \text{Maximum flow rate, capacity} \\
k_j =& \text{Maximum density} \\
k_{cap} =& \frac{Q_{cap}}{V_{cap}} = \text{Optimal density}
\end{aligned}
\tag{4.14}
$$

Four-corner interpolation
Using the Taylor series expansion different interpolation methods are tried to see what the results would be. The first methods used the values of four corner points of a 'time-space'-block. From each corner the density is calculated for several points to interpolate the density in the space within the block. In figure 4.5(a) a schematic overview is given of this interpolation method. The values, obtained with the Taylor series expansion, are averaged based on the distance to the corner. The density of the interpolated point is calculated using equation 4.15.

$$
k = \sum_{i=1}^{2}\sum_{j=1}^{2} k^*_{ij} \frac{\left(\Delta t - |t^* - t_j|\right)\left(\Delta x - |x^* - x_i|\right)}{\Delta t \Delta x} \tag{4.15}
$$

Where

$$
\begin{aligned}
k^*_{ij} =& \text{Interpolated density, based on the density of location } i \text{ and time interval } j \\
\Delta t =& \text{Time between two time intervals} \\
\Delta x =& \text{Space between two detector locations} \\
t_j =& \text{Time at the corner point} \\
x_i =& \text{Location of the corner point} \\
t^* =& \text{Time at an intermediate point} \\
x^* =& \text{Location of an intermediate point}
\end{aligned}
$$

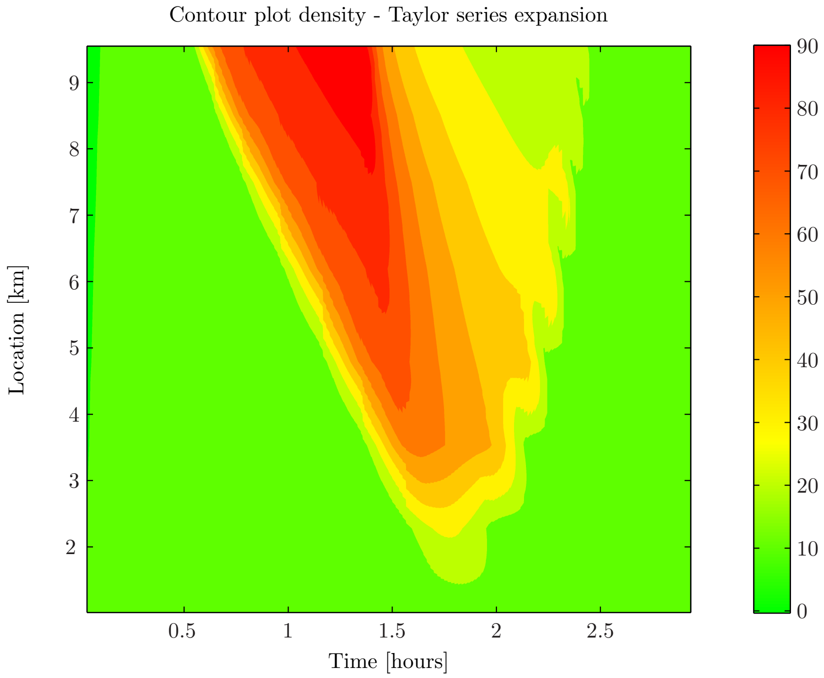


Figure 4.4: Four-corner interpolation of density with the fundamental diagram of Newell

When the density is known, the speed can be calculated from the fundamental diagram. The result of the interpolation of the density is shown in figure 4.4.

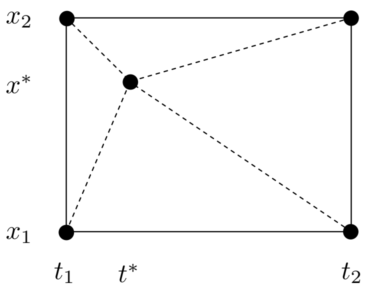
Two-point interpolation

With a two-point interpolation the density is calculated in two steps. In figure 4.5(b) it is graphically depicted. In the first step the density between two time intervals is calculated. The density is in this step only a function of time, which can be calculated when the change of the density is assumed. Between the two new points a Taylor series expansion is used to interpolate the density in the space. In this way $k(x,t)$ can be reduced to $k(x)$, and some terms go to zero, since $\Delta t = t - t_0 = 0$:

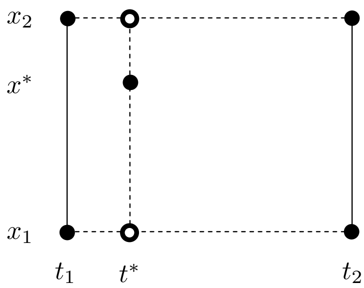
$$k(x) \approx k(x_0) - (x - x_0)\left(\frac{\partial q}{\partial k}\right)^{-1} \frac{\partial k}{\partial t}\left(1 + (x - x_0)\left(\frac{\partial q}{\partial k}\right)^{-2} \frac{\partial^2 q}{\partial k^2}\frac{\partial k}{\partial t}\right) \tag{4.16}$$

Now, for each point between the two detector locations the density can be calculated at any time step. Again, when the density is known, also the speed can be calculated. The calculation of the density is done from both sides of the link and the outcomes are averaged with a weighing factor based on the distance to the detector.

The result of the interpolation is almost the same as for the four-corner interpolation (see figure 4.4). Therefore no graphical result is given for the two-point interpolation.



(a) Four-corner interpolation, between two detector locations and two time intervals



(b) Two-point interpolation, between two detector locations and two time intervals

Figure 4.5: Different interpolation techniques

4.2.4 Trajectory reconstruction

With the interpolation methods it is possible to calculate speed and density at any time and at any location. So, if a car enters the network its speed can be calculated at that time and place. From that point on a stepwise procedure can be used to calculate the new location, based on the current speed. At that new location the density can be interpolated and the speed can be calculated with the fundamental diagram. Then the procedure can restart.

Vehicles entering the network
The only thing left is to calculate when a new car enters the network. Since the flow rate is known, the fraction which enters the network in a time step can be calculated. The loaded fraction is the flow rate times the length of time step. Since the number of cars which are loaded on the network will be an integer number, the fraction is added to a stored non-integer number, which is zero at the first time step. If, by adding the fraction, the non-integer number becomes larger than one, the floored value of the non-integer number is added to the network. The non-integer number is than lowered with its own floored value, which is then again lower than one.

4.3 Other possible reconstruction methods

Next to the above described methods others can be used to reconstruct vehicle trajectories. These methods are not worked out in full detail and therefore not used in the validation (chapter 5).

4.3.1 Characteristic lines

In the macroscopic traffic model of Yperman (2007) characteristic lines are used to estimate traffic states. A characteristic line is a line where the density is constant. This line is a straight line trough the time-space frame, so its speed is constant. The speed of this line is equal to the tangent of the fundamental diagram. At any moment in time and space the density defines the speed of the characteristic line. When the fundamental diagram

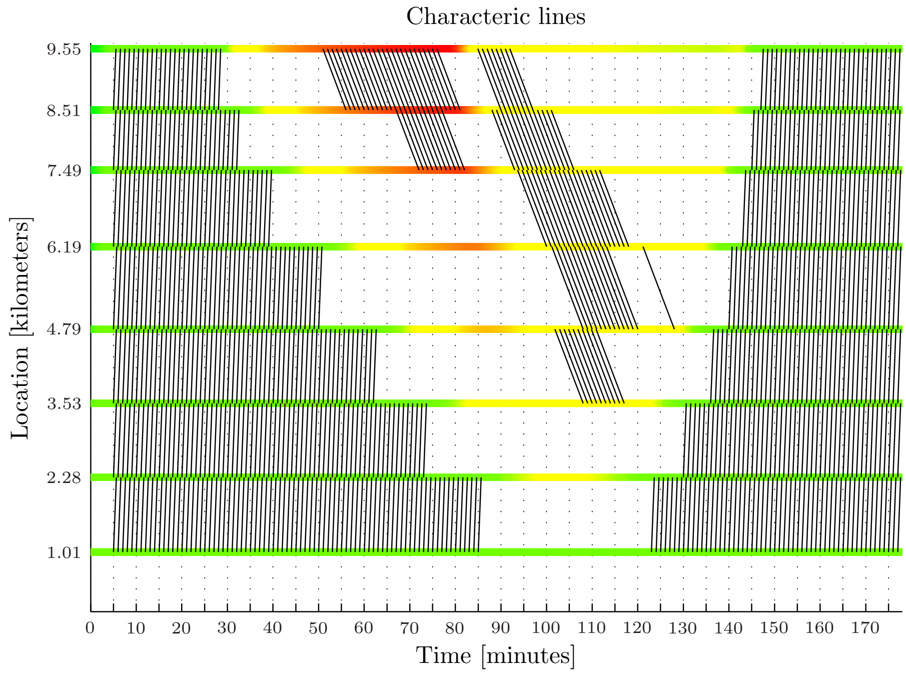


Figure 4.6: Plot of characteristic lines, with a tolerance of 1% for free flow traffic and 5% for congested traffic

of Newell is used, as in the model of Yperman, the speed of this line can only take two values: one for the free-flow stage and one for the congestion stage. When two different characteristic lines intersect, a shock wave occurs.

The general idea is to estimate the density between two detector locations using characteristic lines. The density along these lines is constant. For this procedure the density over time is linearly interpolated, so the density for each time step at the detector location is known. For each moment in time an estimation can be done when the characteristic line would arrive at the next detector location, when this line would not be intersected by another characteristic line. At the arrival time at the next location the density can be looked up, to see whether this is equal (with a certain tolerance) or not. If the density is equal between those two detector locations, the density along the line can be filled in. In this way the whole field can be interpolated. In figure 4.6 an example is given of the result of this method.

The main problem of this method is that is does not describe the whole field, and especially the transition between the two different traffic states. For a description of the transition period another method needs to be developed.

4.3.2 Stepwise interpolation

Another interpolation method uses a stepwise interpolation. With the same procedure as the two-point interpolation, the density can be calculated for a location nearby at $x + \Delta x$. In the next step, this location becomes the starting location, from where the new density is calculated. In this way the density along a line between two detectors can be calculated. This can be done from both detectors and then be averaged. The density is not known everywhere, but only at the calculated location. All other locations can be interpolated linearly between the known points.

An advantage of this method is that it can be more accurate, if Δx is very small. The calculated density differs only very little from the true density, so in the next step the error will be small. A disadvantage is that it can not be used directly for the reconstruction of the trajectories, since the density can not be calculated at a specific point.

4.3.3 Trajectories solved with a differential equation

The density can be written as a function of time and space by a Taylor series expansion. Furthermore, when the fundamental diagram is known speed can be written as a function of density. Also the time step and space step can be linked together by the actual speed at the current point. These three statements can be used to calculate the travelled distance:

$$\begin{aligned} k =& f(x,t) \\ v =& g(k) = g(f(x,t)) \\ \frac{\partial x}{\partial t} =& v \\ \frac{\partial x}{\partial t} =& g(f(x,t)) \end{aligned} \tag{4.17}$$

The last one is the differential equation which has to be solved to get the travelled distance. The density function can be rewritten as follows:

$$\begin{aligned} k(x,t) \approx& k(x_0,t_0) - \frac{\partial k}{\partial t}\left((x-x_0)\left(\frac{\partial q}{\partial k}\right)^{-1} + (t-t_0)\right) + \\ &\left(\frac{\partial q}{\partial k}\right)^{-2}\frac{\partial^2 q}{\partial k^2}\left(\frac{\partial k}{\partial t}\right)^2\left(-(x-x_0)^2\left(\frac{\partial q}{\partial k}\right)^{-1} + (x-x_0)(t-t_0)\right) \\ \approx& A + B(x-x_0) + C(t-t_0) + D(x-x_0)^2 + E(x-x_0)(t-t_0) \end{aligned} \tag{4.18}$$

Where A, B, C, D, E are constants.

In the most simple case, $v(x,t)$ is a linear function of k, which it is in the fundamental diagram of Greenshields. Equation 4.18 can then be reformulated as:

$$\begin{aligned} v(x,t) =& f\left(A + B(x-x_0) + C(t-t_0) + D(x-x_0)^2 + E(x-x_0)(t-t_0)\right) \\ =& A' + B'(x-x_0) + C'(t-t_0) + D'(x-x_0)^2 + E'(x-x_0)(t-t_0) \end{aligned} \tag{4.19}$$

Where A', B', C', D', E' are constants.

It is not easy to solve this differential equation analytically, although it would give the exact answer. Therefore, a numerical solution to reconstruct the trajectories can be found with an iterative procedure. In the first step the density is calculated for a certain point. From this density the speed is found with the fundamental diagram. For the next time step the new location can be found, since $x_{new} = x_{old} + v * \Delta t$. Then the procedure starts again with calculating the density at that location.

A problem with this method is that the density for a certain point is calculated from only one point. In the other methods the values of two points are averaged to get a better answer. When there are no changes in the road lay-out, the error will be small. Notice that this method is not a interpolation method, but a direct way to draw the trajectories.

Chapter 5

Model validation

The methods described in chapter 4 are validated to investigate the quality of the model. This chapter contains the methods and results of this validation.

5.1 Validation methods

Validation of the developed methods is done in two different ways. The first method compares the reconstructed trajectories with the real trajectories. For this method the data of individual cars is necessary, so a microscopic simulation is used, where the data of the individual cars is recorded. Almost all methods[1] are able to reconstruct the trajectories, so all results can be compared with the real outcomes. The second method of validating uses macroscopic data. The reason to validate this type of model and data is that macroscopic outcomes are deterministic, instead of the stochastic results of the microscopic simulation. Furthermore, the macroscopic model supports larger networks with longer simulation times. The validation with macroscopic data compares the dynamic macroscopic results with the results of the interpolation methods, to see what is the best method for interpolation.

Table 5.1 gives an overview of what is validated. Not all combinations of methods, fundamental diagrams and data sets are possible, for different reasons. This is worked out in more detail in the sections below.

Validation at microscopic data level

For the validation of the methods at the microscopic level every 20^{th} trajectory of the real data is used. At the passing time of the trajectories at the first detector location, the reconstruction of the trajectory also starts. At about 200 locations the travel time and speed of the real trajectory is compared with the travel time and speed of the reconstructed trajectory. If the difference between these two is very small, the reconstruction method performs reasonable well. A total of about 25 trajectories is used.

The difference in travel time and speed at all locations is measured in two different ways: the Root Mean Squared Error (RMSE) and the Mean Adjusted Absolute Percentage Error (MAAPE). The formulas for these measures are given in equation 5.1.

[1] Only the combination of the fundamental diagram of Van Aerde and four-corner or two-point interpolation method does not work.

Model	Method	Fundamental diagram	Micro-scopic	Macro-scopic
Simple model	Linear	-	x	-
	Spline	-	x	-
Advanced model	Linear interpolation	Newell	x	x
		Van Aerde	x	-
		Smulders	x	x
	Four-corner interpolation	Newell	x	x
		Van Aerde	-	-
		Smulders	x	x
	Two-point interpolation	Newell	x	x
		Van Aerde	-	-
		Smulders	x	x

Table 5.1: Overview of the validated methods

$$\begin{aligned} RMSE &= \sqrt{\frac{1}{n}\sum_{i=1}^{n} e_i^2} \\ MAAPE &= \sum_{i=1}^{n} \frac{|e_i|}{\frac{1}{2}(y_i + \hat{y}_i)} * 100\% \end{aligned} \tag{5.1}$$

Where

$$\begin{aligned} y_i &= \text{Value of real trajectory} \\ \hat{y}_i &= \text{Value of reconstructed trajectory} \\ e_i &= y_i - \hat{y}_i = \text{Error between } y_i \text{ and } \hat{y}_i \end{aligned}$$

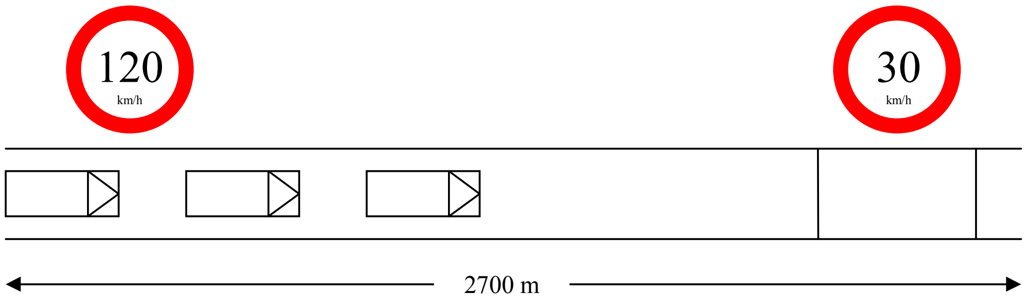
For the validation at the microscopic level it is not possible to use advanced interpolation methods in combination with the fundamental diagram of Van Aerde. This method requires the first and second derivative of the fundamental diagram. In some situation this leads to a combination in which the calculated density takes enormous values. This overreaction makes the method unusable for the reconstruction of the trajectories.

For the simple methods it is not possible to start the reconstructed trajectories at the exact same time as the real trajectory. The simple methods calculate the passings at the detector locations, so the trajectory which starts closest to the real trajectory is used.

Validation at macroscopic data level

At the macroscopic level the validation is done by leaving out detector data. The macroscopic data at the detector location is then estimated by the interpolation algorithm and compared to the real data. This is done every 20 seconds. The real and the estimated data are compared by calculating the same two error measures as for the microscopic validation (see previous section and equation 5.1).

Detectors 2 to 7 are deleted, and at these locations the density and the speed are estimated with the previous and the next detector. So, if the data of detector n is deleted, the data of detector $n-1$ and detector $n+1$ are used.



Time	Flow rate
0 - 10 min	800 veh/h
10 - 20 min	2000 veh/h
20 - 30 min	800 veh/h

Figure 5.1: Network for microscopic validation

At the macroscopic validation level it is only possible to use interpolation algorithms. Therefore it is not possible to use the simple method for validation. Note that the combinations of the interpolation methods with the fundamental diagram of Van Aerde is problematic, so such combinations can not be used for validation.

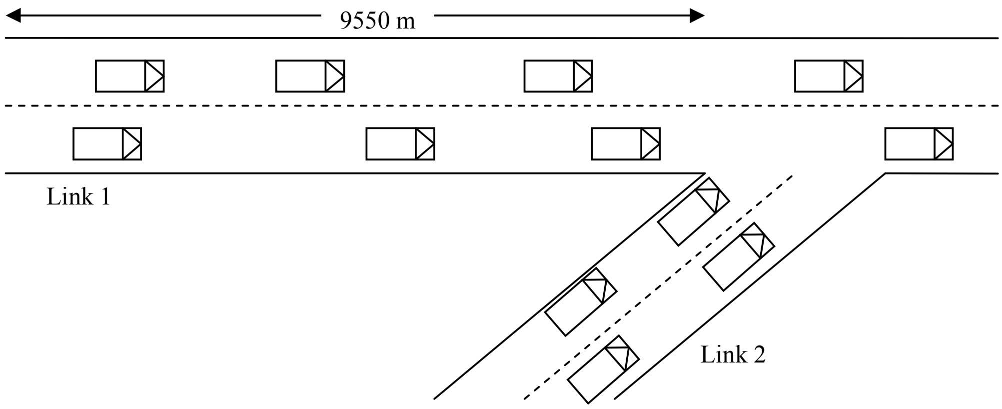
5.2 Network description

Two different networks are used for validation, of which one is set up in microscopic modelling environment (Vissim) and the other in a macroscopic modelling environment (MaDAM). Both modelling environments are used for their own advantages. The advantage of the microscopic environment is the ability to see the real trajectories, while in the macroscopic environment more data can be used in more complex networks. Moreover, the macroscopic model is based on the fundamental diagram as described by Van Aerde, so the parameter describing the fundamental diagram are known. Since the parameters don't need to be estimated the quality of the reconstruction will improve.

Before the interpolation methods can be used in the microscopic environment, the parameters of the fundamental diagrams have to be estimated at each detector location.

Microscopic environment
In the microscopic simulation data of all vehicles is recorded, so the simulated time and network size can not be too large. A single lane in one direction is simulated (see figure 5.1). The traffic flow on this road is varied in time, starting with a low flow rate, later increasing it to its capacity and finally returning to the low flow rate. The free flow speed on the road is about 120 km/h, but at the end of the link is a speed reduction area, in which car drivers are obliged to drive about 30 km/h. As a matter of consequence a static bottleneck occurs at the start of the speed reduction area, which begins to grow upstream when the flow rate increases. When the flow rate goes back to its original level, the queue decreases at the tail.

The road has a length of about 2700 meters, at which 10 detectors are located. The



Time	Flow rate link 1	Flow rate link 2
0 - 60 min	3040 veh/h	0 veh/h
60 - 120 min	3040 veh/h	2280 veh/h
120 - 180 min	3040 veh/h	0 veh/h

Figure 5.2: Network for macroscopic validation

simulation time is 30 minutes and the level of aggregation is 1 minute.

Macroscopic environment
The macroscopic validation is performed on a network with a merging link. The network can be represented by a two lane, one direction road with a merging link of also two lanes in one direction. The link downstream of the merging area also has two lanes in one direction. On the first link the flow rate is constant during the entire simulation, at a flow rate of 1500 veh/h/lane. On the entering link, link 2, the flow rate is zero in the beginning, then increasing to about 1100 veh/h/lane and at last again zero. Before the merging area, a queue will arise which will grow as long as the flow on the entering link is non-zero. When the flow on the entering link becomes zero again, the bottleneck will disappear, but the queue will grow from its tail, while it dissolves from the front. This is a non-static bottleneck. Only the link upstream from the merging area will be considered. The capacity of the link is 2000 veh/h/lane.

The upstream link has a length of about 10 kilometres, at which 8 detectors are located. The simulation time is 180 minutes and the level of aggregation is 1 minute. In total about 540 data points are used for the calculation of the error. A graphical representation of the network is given in figure 5.2.

5.3 Validation results

Microscopic validation
During the microscopic validation the errors are calculated at all locations. The errors are plotted as a function of the location. In figure 5.3 an example is given of the result, in this case for the two-point interpolation. Since the real trajectories and the reconstructed trajectories start at the same time the error of the arrival time is zero at the beginning of the link. During the link this error increases. For the estimation of the speed the

fundamental diagram is used. In appendix C more information is given on the results of the microscopic validation.

Macroscopic validation
The error of the interpolation is measured at different detector location for a lot of time periods. The errors are again plotted as a function of the location. In figure 5.4 the results of the validation of the interpolation with the fundamental diagram of Newell are given. The differences between the four-corner and the two-point interpolation are quite small. In appendix C more results of the macroscopic validation are given.

The distance between two detectors, when the intermediate detector is leaved out, is about 2 kilometres. In reality this distance is most of the time smaller, which implies that the error is overestimated.

5.4 Analysis of the results

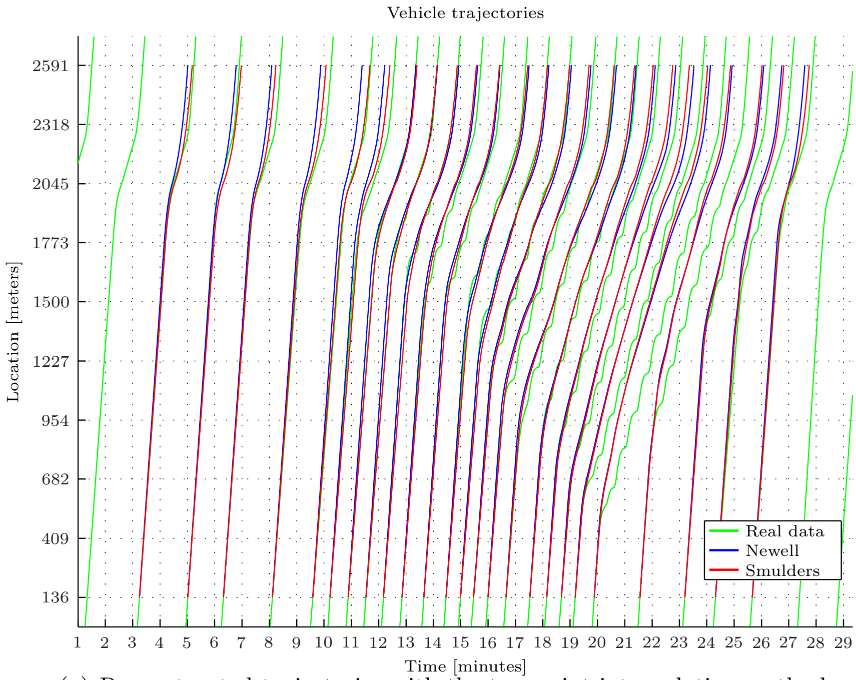
The results of the validation are shown in table 5.2 and 5.3. Below an analysis is made of the results of the validation, starting with the microscopic validation and followed by the macroscopic validation.

5.4.1 Microscopic validation

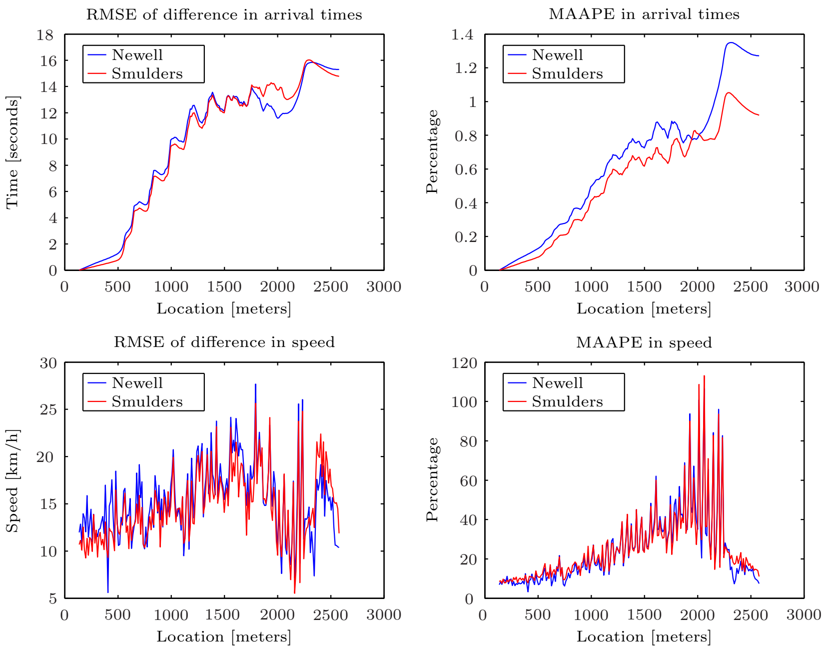
In the validation process on the microscopic level real trajectories are compared to the reconstructed trajectories. The difference between the trajectories is measured using the RMSE and the MAAPE of the difference between the real and the reconstructed travel time and speed at about 200 locations.

If the results are compared considering the RMSE of the travel time, the most simple methods score best. The maximum RMSE is 10.7 seconds for the linear reconstruction and 11 seconds for the spline reconstruction. The other methods have, with one exception, an error of about 15 to 16 seconds. On the other hand, the advanced methods techniques perform better when using the measure of the percentage error. The maximum percentage error is lowest when two-point interpolation with the fundamental diagram of Smulders is used. The maximum RMSE of the speed is lowest when the fundamental diagram of Smulders is used, almost independent of the interpolation technique. The difference with the simple techniques is large: 26.8 versus 56.1 km/h. The mean MAAPE of the speed is lowest for Van Aerde in the linear interpolation. Newell performs better for all interpolation techniques than Van Aerde, measured in terms of MAAPE of speed.

Based on these results it can be concluded that the simple models are able to predict the right time of arrival of a trajectory, during trip over the link. For travel time estimation over a link this method can be applied. On the other hand, the simple methods are not able to predict the right speed. The speed of the real trajectories fluctuates strongly, so it is quite difficult to predict the correct speed. The percentage error shows this too. The maximum MAAPE of the speed is large, more than 100%, but the mean MAAPE is much bigger for the simple methods than for the interpolation methods. This can be explained by the fact that trajectories in the simple methods can drive backwards, eihter in space or time. As a matter of consequence the difference between the real and the reconstructed trajectory will be large. So, for an application where the speed is important (i.e. emission calculation, visualization) the simple methods are not good enough. For these applications

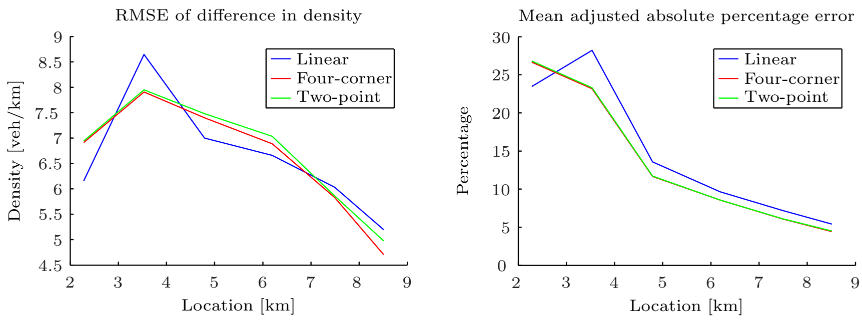


(a) Reconstructed trajectories with the two-point interpolation method

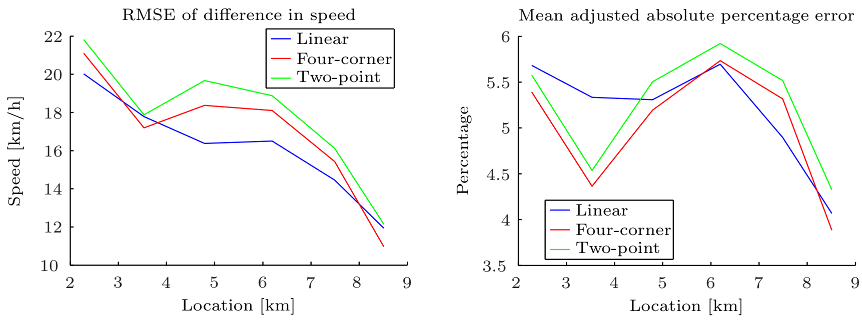


(b) Difference in travel times with the two-point interpolation method

Figure 5.3: Results of the reconstruction with the two-point interpolation method



(a) Difference in density with Newell



(b) Difference in speed with Newell

Figure 5.4: Results of the error with the fundamental diagram of Newell

		Arrival time RMSE [sec]			Arrival time MAAPE [%]			Speed RMSE [km/h]			Speed MAAPE [%]		
		min	mean	max	min	mean	max	min	mean	max	min	mean	max
Simple model	Linear	1.2	6.4	10.7	0.2	0.8	1.44	7.6	29.9	45.5	13.9	40.5	116
	Spline	1.1	6.3	11	0.2	0.81	1.3	6.4	28.9	56.1	7.9	40.3	111
Linear int	Newell	0	9.2	14.7	0	0.63	1.25	5.9	15.3	28.8	3.5	25.7	104
	Van Aerde	0	13	26.1	0	0.9	2.21	3.8	14.7	29.1	3	23.9	106
	Smulders	0	9.6	16	0	0.57	1.06	5.1	14.9	26.8	7.4	26.3	110
Four-corner	Newell	0	9.7	15.2	0	0.63	1.22	5.9	15.2	29	3.5	25.5	103
	Smulders	0	10.1	16.8	0	0.58	1.10	5.4	14.8	26.8	7.4	26	109
Two-point	Newell	0	9.8	15.3	0	0.64	1.22	5.9	15.2	28.9	3.6	25.5	103
	Smulders	0	10	16.4	0	0.56	1.04	5.5	14.8	27	7.4	26.1	110

Table 5.2: Summary of the results of the microscopic validation

		Density RMSE [veh/km]			Density MAAPE [%]			Speed RMSE [km/h]			Speed MAAPE [%]		
		min	mean	max	min	mean	max	min	mean	max	min	mean	max
Newell	Linear	5.2	6.6	8.6	5.4	14.6	28.2	11.9	16.2	20	4.1	5.2	5.7
	Four-corner	4.7	6.6	7.9	4.4	13.4	26.7	11	16.9	21.1	3.9	5	5.7
	Two-point	5	6.7	7.9	4.5	13.5	26.8	12.1	17.7	21.8	4.3	5.2	5.9
Smulders	Linear	5.2	6.6	8.6	5.4	14.6	28.2	10.6	14.3	17.4	4	5	5.5
	Four-corner	4.8	6.4	7.8	4.5	13.3	26.1	9.7	14.5	18.5	4.1	4.9	5.6
	Two-point	4.8	6.5	7.8	4.5	13.4	26.3	9.4	14.7	18.7	3.9	4.9	5.6

Table 5.3: Summary of the results of the macroscopic validation

an interpolation technique has to be used. The best one is the four-corner or the two-point interpolation. The differences between the fundamental diagrams of Newell and Smulders are very small. Newell scores best in the RMSE of the arrival time and the MAAPE of speed, while Smulders scores best at the MAAPE of the arrival times and the RMSE of the speed.

5.4.2 Macroscopic validation

At the macroscopic level the differences between density and (calculated) speed are compared to the interpolation results of density and speed. For this method only the fundamental diagram of Newell and Smulders are used, with three different interpolation techniques.

The interpolation of the density is performed best by the four-corner interpolation. The minimum and maximum of both the RMSE and the MAAPE is lower than for the other interpolation techniques, for Newell's fundamental diagram as well as for Smulders's diagram. The errors of the interpolation of the speed are almost the same for all interpolation techniques. Smulders performs better than Newell, measured in the RMSE of the speed. The MAAPE is about 5% for both fundamental diagrams and for all interpolation techniques.

It is hard to point out which combination of interpolation technique and fundamental diagram is best. Only the linear interpolation is clearly not good enough. If the two-point interpolation is used, the fundamental diagram of Smulders performs best. The RMSE of the speed is lower when the fundamental diagram of Smulders is used. On the other hand, the fundamental diagram of Newell fits better to the real data. That makes it difficult to say which fundamental diagram performs best.

5.4.3 Conclusions

It is hard to define the best method for the reconstruction of trajectories. What it is *not* is a simple, non traffic science based reconstruction. The simple linear or the spline reconstruction give trajectories, but the outcomes are not good enough, since some of the cars are at more than one location at a time. The linear interpolation is also not the best method to reconstruct the trajectories. Furthermore, the fundamental diagram of Van Aerde can not be used for this purpose, because the outcomes of the Taylor-series can take enormous values for the density.

What is left are two interpolation techniques and two fundamental diagrams. The differences in the outcomes are very small. As can be seen in the macroscopic validation, Smulders performs slightly better than Newell, but the error which is made by the calculation of the speed from the density is bigger than for Newell. The technical differences between the four-corner interpolation and the two-point interpolation are quite small.

If other criteria are taken into account, like calculation time and the estimation of parameters of the fundamental diagram, the best method would be the two-point interpolation with the fundamental diagram of Newell. The two-point interpolation only needs to calculate the density from two point, instead of four points needed for the four-corner interpolation. The number of parameters needed for the fundamental diagram of Newell is three (V_f, Q_{cap} and k_j), while the fundamental diagrams of Smulders and Van Aerde also need the speed at capacity, V_{cap}.

Remarks
The validation is only performed on two simple networks with a very distinct traffic flow. The quality of the method is in these cases quite good, but it is unknown what it would be when complex networks with very varying flows would have been used. The quality of the estimated fundamental diagrams is not validated, so, although it seems to be good, it is not known what the quality is.

Chapter 6

Case studies

The developed method is applied to real traffic data from the highway A12. The data is captured in April 2006. In this chapter the results are described of what is happening when the reconstruction method is applied to real traffic data.

6.1 Network description

In this case study the detector loop data of a part of the A12 is used. The A12 is a highway between The Hague and the German border near Zevenaar. The data set only contains data from the section between Gouda and The Hague. The network is depicted in figure 6.1.

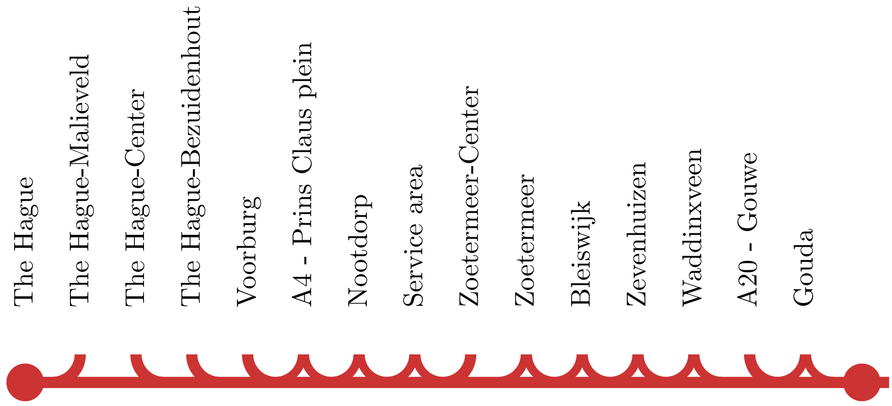
The data is recorded on April, 4$^{\text{th}}$. On this day there was serious congestion especially around the intersections with the A20 and the A4. The data is recorded with detector loops, with a total of 67 detector locations. The data is averaged over all lanes and aggregated to 5-minute data. The length of the section is about 28 kilometres. In 2006 the junction near Nootdorp was not in service and the effect of the service station near Zoetermeer is neglected. The last detector is just before the on-ramp The Hague-Malieveld. So in total there are eleven junctions in this section.

6.2 Assumptions

1. The flow which enters the highway is defined by difference between the downstream and the upstream flow. The same procedure as described in section 4.2.4 is used to calculate the amount of vehicles which enters the highway during a simulation step.

   Vehicles leave the highway according to a chance defined by the difference between the upstream and the downstream flow of the off-ramp. If Q_1 is the upstream flow of an off-ramp and Q_2 the downstream flow of this off-ramp, the fraction which leaves the highway is $Q_1 - Q_2$; The chance that a vehicle will leave the highway can be calculated:
   $$p_{\text{off}} = \frac{Q_1 - Q_2}{Q_1} \tag{6.1}$$

2. The lane of a car is constant, unless there is a junction or a change in the number of lanes. The chosen lane depends on the cumulative number of cars which already



(a) Schematic overview of the road

(b) Map of the area

Figure 6.1: Overview of a section of the A12 between Gouda and The Hague

have entered the section. If car n enters a certain section the cumulative flow is n. When the section has k lanes, the chosen lane for car n is $(n \bmod k)+1$. This means that each next car takes the next lane.

3. Two parameters can be set in the reconstruction and the visualization, being the length of a simulation step and the number of cars. When this last parameter is set to n, every n^{th} car is reconstructed. In this reconstruction the simulation step is 2 seconds where every 5^{th} car is reconstructed. This implies that the maximum distance which a car can travel within a simulation step is about 70 meters. For the plot of the trajectories every 25^{th} is represented, for a better visibility.

4. Although the data is available for the whole day, only the morning rush hour is taken into account. This is defined as the period between 6:00 and 10:00 am.

5. The used method for the reconstruction is a two-point interpolation based on the fundamental diagram of Newell. The difference between the contour plots of the speed is very small, which means that the result is not very different when Smulders would have been used.

6. The data of the A12 contains one clear error, on a detector before the off-ramp to the A4 between 0:30 and 6:00 am. This error has no effect on the reconstruction of the rush hour traffic.

6.3 Results

With the developed methods it is possible to reconstruct the trajectories and to make interpolations for speed and density on the whole time space frame. In the section on the A12 are a lot of junctions and changes in the number of lanes. Some extensions are made on the method to be able to reconstruct the trajectories and to visualize the movements of the cars.

The results of the reconstruction are quite good. The congestion on the road is clearly visible and the heads of the queues are at expected locations (e.g. before lane drops or on-ramps). The first problem on the road occurs around the junction near Zevenhuizen. The tail of this queue goes back until Gouda. Meanwhile another problem occurs at the on-ramp from the A4. A lot of traffic enters the A12 and these weaving movements result in congestion. At the end of the A12 the traffic light is unable to process the traffic, so a queue emerges. In the reconstructed trajectories also stop-and-go waves can be seen (see figure 6.3, especially the lower figure).

A screen shot of the animation of the reconstruction is given in figure 6.4. In the animation the on-ramps and off-ramps are marked with the green and red lines, as well as the number of lanes and the time. The road is at 6:00 am almost empty, because the network has still to be loaded, which is completed at about 6:15 am.

When the trajectories are known it is very easy to calculate the actual travel time, as a function of the departure time. In figure 6.2 an overview is given of the change in travel time during the morning peak. The lowest travel time is 14 minutes and 36 seconds, but the longest travel time is 58 minutes and 8 seconds. So the ratio between the maximum and the minimum travel time is 3.98. The maximum average speed is 115.7 km/h and the minimum average speed is 29.0 km/h.

This case study shows that the method can be applied for the visualization of macroscopic traffic data. The results can be recorded and shown as an animation. For further application the method should be developed in order to support complex networks.

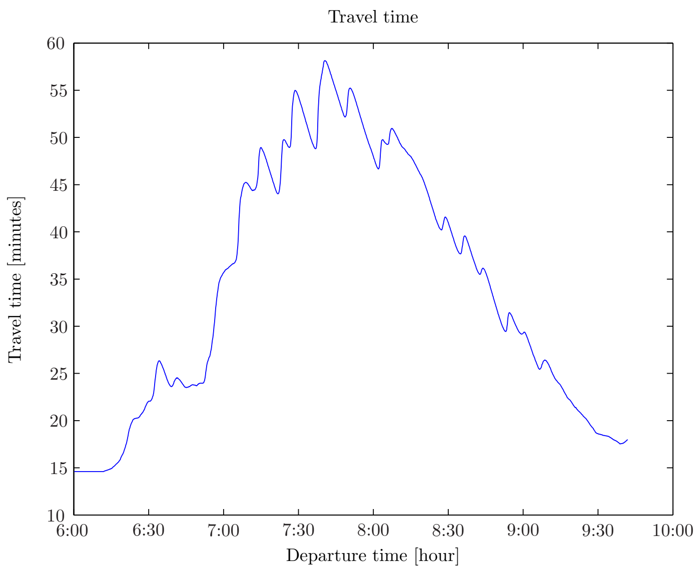
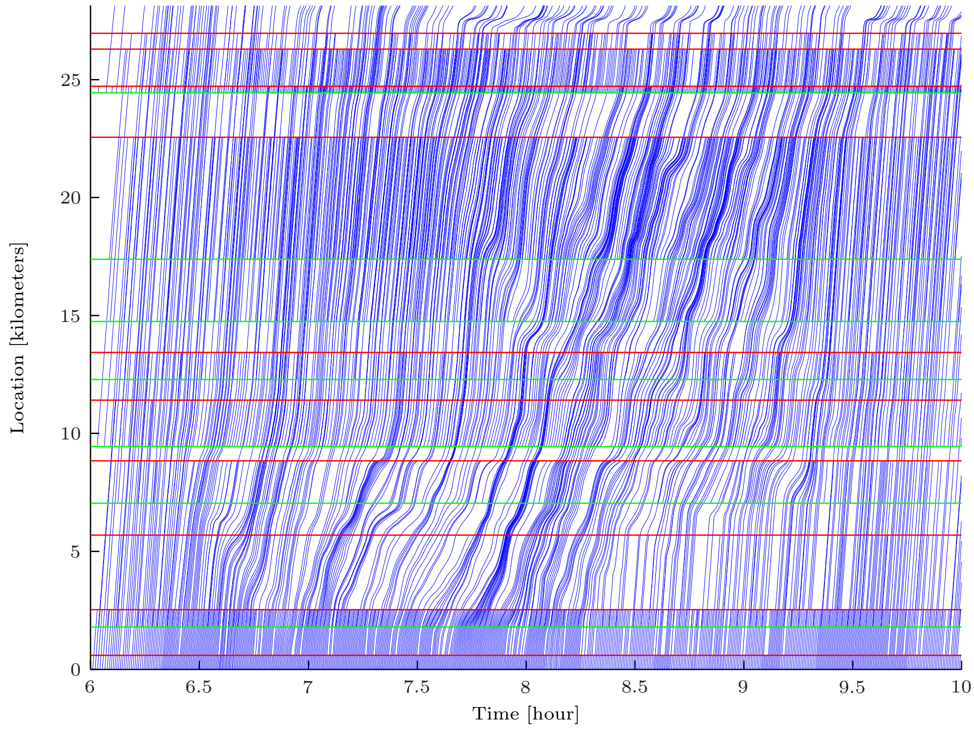
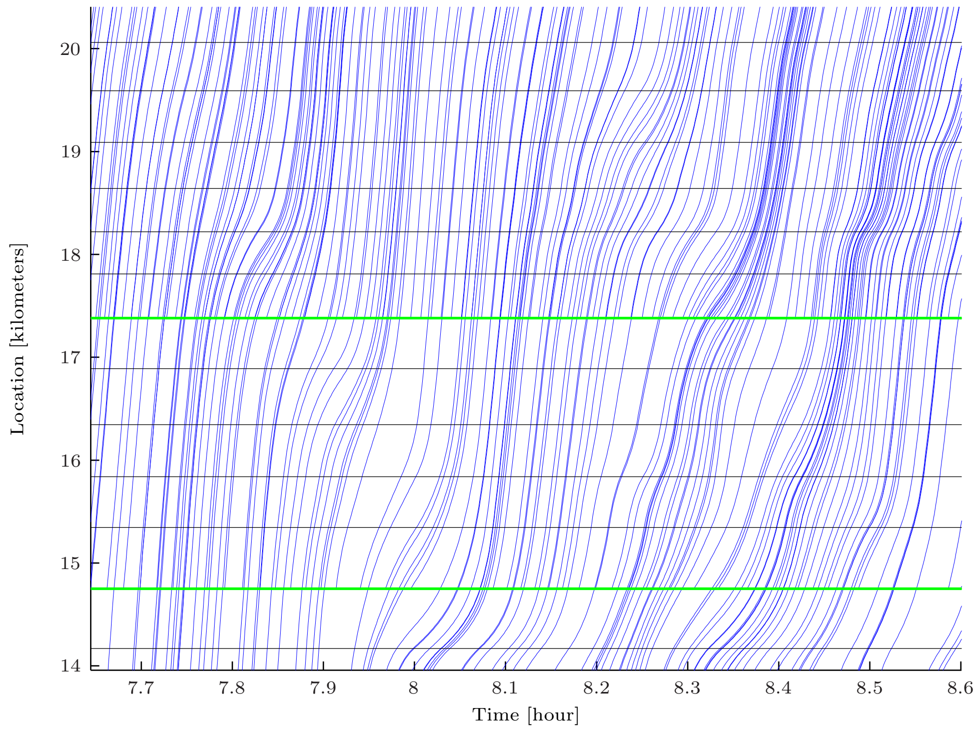


Figure 6.2: Actual travel time as a function of the departure time



(a) Reconstructed trajectories; every 25th car



(b) Reconstructed trajectories; zoom with stop-and-go waves ares

Figure 6.3: Reconstructed trajectories on the A12; the green lines represent the on-ramps, the red lines the off-ramps and the black lines the detector locations.

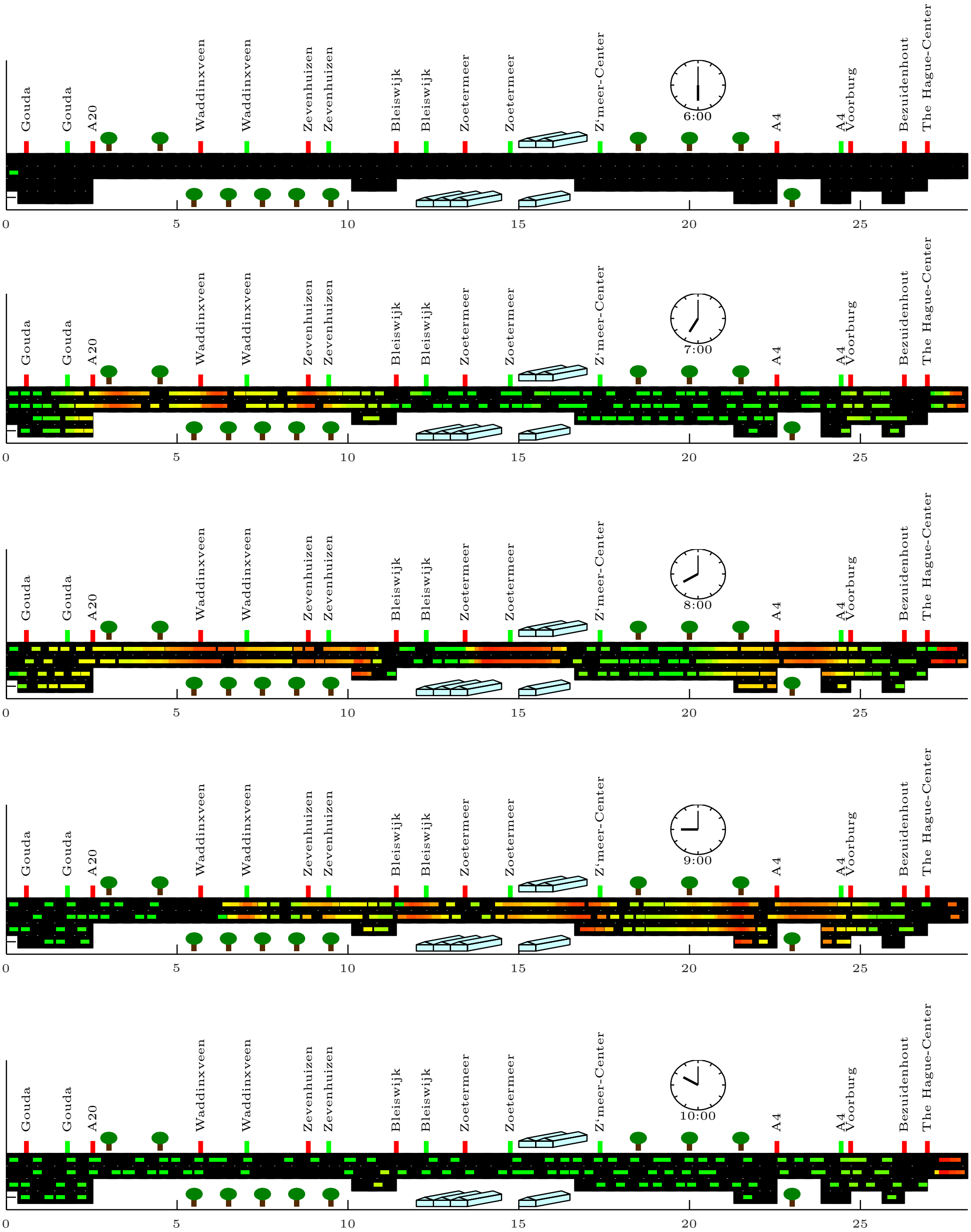


Figure 6.4: Screen shot of the animation

Chapter 7

Conclusions

In this research several methods are developed to reconstruct vehicle trajectories with dynamic macroscopic data. The reconstructed trajectories can be used for the visualization of vehicle movements. The developed methods are validated to measure their performance under different circumstances.

7.1 General conclusions

In chapter 2 the aim of this research is formulated as follows: *Reconstruct vehicle trajectories based on the outcomes of a macroscopic model.* All developed methods are able to reconstruct vehicle trajectories. The methods which use interpolation techniques are better than the simple methods. The simple methods use only mathematical relations and do not use any traffic flow theory. The linear interpolation method uses the fundamental diagram only to calculate the speed from the density. The four-corner and the two-point interpolation methods also use the fundamental diagram to estimate the course of the density pattern within a time interval and between detector locations.

It can be concluded that it is possible to reconstruct trajectories, even with a good quality. For this reconstruction several methods are tried of which the four-corner and the two-point interpolation methods perform best.

The research questions, as posed in section 2.2, are answered below:

1. *What data is needed for the reconstruction of trajectories?*
   The minimum data needed for reconstructing trajectories is the macroscopic data of the density, combined which either a fundamental diagram or the macroscopic data of speed and flow rate. From the macroscopic data of speed, density and flow rate the parameters forming the fundamental diagram can be estimated. If the fundamental diagram is given speed and flow rate can be calculated. In fact any fundamental diagram can be used, but the results will improve when the fundamental diagram fits better to the data.

2. *What would a conceptual model look like?*
   The first developed methods are very naive and do not use any kind of traffic flow theory. Although the simple methods are able to reconstruct trajectories, the results are not considered to be satisfying. For this reason better methods are developed.

3. *How can a more complicated model be supported?*
   The best developed method is a interpolation method which uses traffic flow theory to describe the density. From the density, speed can be calculated according to a fundamental diagram. When speed is known at any time and place a trajectory can be drawn. This is done by calculating the movement of a vehicle during a small time step.

4. *What is the quality of the reconstruction?*
   The validation study shows that the best methods are the four-corner and the two-point interpolation. In the macroscopic validation the errors of the interpolated speed are about 5%. In the microscopic validation it is about 28%. An error of 28% seems large, but if the course of the real trajectories is taken into account it becomes clear that it would be very hard to estimate the stop-and-go waves. If the reconstructed trajectories are used for visualization this is not a problem.

The reconstructed trajectories can be used for visualization of car movements. In order to do that the locations of the cars need to be updated every time step. The results can then be shown as an animation.

The calculation time of the reconstruction is not investigated. In general, the interpolation methods do not need to store much data. Only the current position is needed to calculate a new speed. With this new speed the movement of the car is defined. The linear interpolation is assumed to be the fastest algorithm, since the density can be calculated very fast, without calculating derivatives of the density. For the Taylor series interpolation methods, two and four Taylor series expansions have to be calculated each time step, for respectively the two-point and the four-corner interpolation. The calculation of the Taylor series expansion takes some time, although it is not much. The visualization of the results is by far the most time consuming process.

7.2 Discussion

In the research some choices are made which have an influence on the results. Below some of these choices are discussed.

Fundamental diagram

If the fundamental diagram is not known it has to be estimated based on dynamic macroscopic data. The quality of the fundamental diagram has a great influence on the outcomes of the model, because of the importance of the fundamental diagram for the interpolation methods. The estimation of the free flow part of the fundamental diagram is easier to estimate than its congestion part. Although it is easier to estimate the free flow part, the application of trajectory reconstruction is mainly meant for non-stable traffic situations. The data used for the validation study contained congestion situations and the resulting errors are not very large.

The algorithm to estimate the parameters of the fundamental diagram is only visually examined.

Network design
The developed methods are validated on very simple networks with distinct flow rates. Two remarks have to be made. The first one is that the methods are not tested on a more complicated network (with more than one link). Implementation problems which come along with enlarging the network are not known at this moment. The other remark has to do with the change in flow rates. In case of distinct time-varying flow rates it is quite safe to assume that the change of density in time is constant. In real traffic the flow is not that distinct, but varies smoothly and continuously. When the time interval is small this will not be a very decisive problem. However in case of large flow variations and a time interval of 5 minutes, it might be that there occur problems with the method. For the visualization of the vehicle movements this is not a problem, since the accuracy of the speed is less important for this application. If external effects are calculated it is likely that the emission results are underestimated.

Data collection
In this research the methods are developed and tested with data from simulation models. The advantage is that the data is complete and correct, but the disadvantage is that the model development is not based on real traffic data. The size of the error is unknown, but is depends on the quality of the simulation model and the quality of the real data. Since this is the first research on this topic, the scope of the research had been restricted to the development of the algorithm.

The use of Vissim
Vissim is a stochastic microscopic simulation model. In Vissim, by default, the car following model of Wiedemann is used to calculate the speed and following distance of the cars. In this research it is assumed that the result of the simulation is an average outcome. Nevertheless, it might be that the chosen random seed and the car following model have a certain effect. It could be that the strongly curved trajectories would be more smooth with other parameters for the car following model or with another car following model. On the other hand, the validation error is overestimated is this case. If the real trajectories would have been smoother, the difference with the reconstructed trajectories would have been smaller.

Applicability
For the use of the interpolation method the most important parameters are the fundamental diagram and the change of the density in time. When the quality of these parameters is good the method is applicable in a lot of situation, at least for visualization. It is not known what the quality of calculating external effects would be if the method is used or what the quality of the main parameters would be.

7.3 Further research

Upscaling
In theory the developed method is applicable for large networks. In this research only a single link network is used for the development. The advantage of a single link network is its simplicity and the speed of calculation. For further implementation of the method

further research is needed on how larger networks can be supported and how the speed of the algorithm can be maintained.

Application
If the trajectories are known, it is possible to calculate the external effects. In this research no attention is paid to the calculation of the external effects. For this topic too, further research is needed in order to use this method for the calculation of external effects with a macroscopic modelling environment.

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Appendix A

Calculations simple model

In this appendix the way the calculations are made are described. The first section, section A.1, described the most simple model, in which only simple mathematics are applied.

A.1 Simple model

For the reconstruction of the trajectories in the simple model a three-step model is followed. In this model the first step is the distribution of the headways over the different detectors. After this step, the arrivals of the first car at the different detectors are calculated. In the last step the connection between the different points is made, in three different ways. It is assumed that the first car which enters the first detector will also be the first car which enters the next detector. Furthermore, the n^{th} car will arrive at all detectors at the n^{th} position.

A.1.1 Step 1: Headway distribution

Homogeneous headway distribution

With the real trajectory data an algorithm is run which gathered the aggregated data for several sections and time intervals. Now, for all locations and time intervals the average speed and the number of cars passing that point are known. The headways are distributed equally within the time interval.

Let x_i denote the location of detector i and t_j the time interval j, beginning at $t_0 + (j-1)\Delta t$. N_{ij} is the number of cars passing location x_i at time interval t_j with the average speed $\bar{V}_{ij}$.

The passing time of car c_{ij}^{tn} is:

$$c_{ij}^{tn} = t_0 + (j-1)\Delta t + (n-0.5)\frac{\Delta t}{N_{ij}} \qquad n = 1, 2, ..., N_{ij}-1, N_{ij}, \qquad \forall i, j \tag{A.1}$$

The speed of car c_{ij}^{vn} is:

$$c_{ij}^{vn} = \bar{V}_{ij} \qquad \forall i, j \tag{A.2}$$

As a result, all headways at all locations and time intervals are distributed equally. This is also visually depicted in figure A.1, for a certain time interval and location.

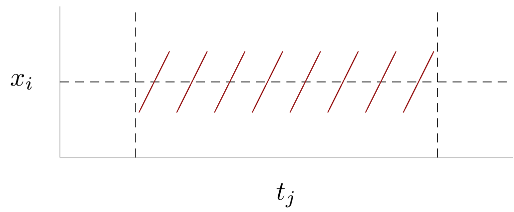


Figure A.1: Distribution of headways in a simple model

Headway distribution with inhomogeneous speed distribution
To get a more accurate distribution of the speeds over the time a algorithm is developed which smooths the speeds as a more continuous function in time. This new distribution still fulfils the macroscopic average speeds on the different time intervals and locations.

At any location the average speeds of the time intervals are known. Let there be j intervals at location i. The (average) speed at this interval is $\bar{V}_{i,j}$, with individual speeds $v_{i,j}^n$; the number of passing vehicles is $C_{i,j}$, described by $c_{i,j}^n$ with $n = 1, 2, ..., C_{i,j} - 1, C_{i,j}$. In 5 steps the individual speeds are assigned to the cars.

Step 1 Between all intervals, the speed of the last car at j and the first car at $j+1$ is the average of $\bar{V}_{i,j}$ and $\bar{V}_{i,j+1}$.

Step 2 The speed of the first car at $j = 1$ is $2 * \bar{V}_{i,1} - v_{i,1}^{C_{i,j}}$.

Step 3 The speed of the last car in the last time interval $j = j_{max}$ is $2 * \bar{V}_{i,j} - v_{i,j}^1$.

Step 4 In the next step the speed of the cars halfway the interval is assigned. If $C_{i,j}$ is even, the speed of the 2 cars halfway is $2 * \bar{V}_{i,j} - mean\left(v_{i,j}^1, v_{i,j}^{C_{i,j}}\right)$. If $C_{i,j}$ is odd, the speed is assigned to 1 car.

Step 5 At last the other speeds are linearly interpolated between the known points.

Headway distribution with inhomogeneous speed and headway distribution
Next to a more smooth distribution of the speeds it is also possible to do this for the headways. In the same procedure as described above, the average headways are smoothed over the interval, while it still fulfils the macroscopic average. In this way, the speeds and the headways are both inhomogeneous within the time interval.

A.1.2 Step 2: Arrival time estimation

Simple estimation
When the arrivals at the detectors are distributed over the time, the trajectory of the first car can be estimated. For this estimation the starting location is known, as well as the speed and the passing time. With this information an estimation is done on the arrival time at the next location. Let c_i^t denote the passing time of the first car at location x_i. The speed, as calculated in equation A.2, of this car is $\bar{V}_{i1} = c_i^v$. Starting at $i = 1$, a straight line through the time-space plane is made to the next measure location x_2, where it arrives at c_2^t. In general terms it becomes:

$$c_{i+1}^t = c_i^t + \frac{(x_{i+1} - x_i)}{c_i^v} \tag{A.3}$$

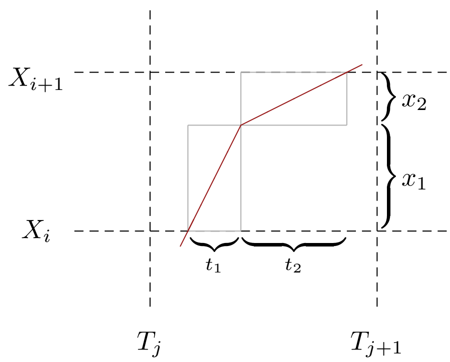


Figure A.2: Advanced estimation of the arrival time at the next section

Since it is very likely that a number of cars have passed before, the algorithm searches for the next car that passes after c^t_{i+1}. Now the speed can be updated and the actual arrival time can be filled in. This procedure is repeated for all locations.

All other cars will use the next passing time at all locations. Now, for all cars is known when they passed the locations and with what speed. From this point, it is also possible to calculate the average travel time for all cars. The measured average travel time can be compared with the real average travel time. By adding a certain 'offset time' to the arrival time of the first car at the different locations the average travel time can be influenced. Redoing the whole arrival time distribution, with the offset-parameter, gives a new average travel time. When the real travel time and the estimated travel time are (almost) equal, the next phase can start: the construction of the trajectories.

Advanced estimation

By extrapolating the speed a first guess is made for the arrival time at the next detector. This quite rough estimation is improved by taken into account the speed of the next section as well.

The general idea is to cut the distance between two detectors in two pieces, and use the speed of the first detector at the first part and the speed of the second detector at the second part. The length of the pieces is such that $x_1 t_1 = x_2 t_2$. Furthermore, the speeds are known, being v_1 at detector 1 and v_2 at detector 2. The total travel time T is the sum of t_1 and t_2; the distance X between two successive detectors is the sum of x_1 and x_2 and is known. t_1 and t_2 can be rewritten as $\frac{x_1}{v_1}$ and $\frac{x_2}{v_2}$. All is graphically depicted in figure A.2.

With this data three (linear) equations can be formulated, which can be solved easily with Matlab.

$$t_1 + t_2 = T \Rightarrow t_1 + t_2 - T = 0 \Rightarrow \frac{x_1}{v_1} + \frac{x_2}{v_2} - T = 0 \tag{A.4}$$

$$x_1 t_1 = x_2 t_2 \Rightarrow \frac{x_1^2}{v_1} = \frac{x_2^2}{v_2} \Rightarrow \frac{x_1}{\sqrt{v_1}} = \frac{x_2}{\sqrt{v_2}} \Rightarrow \frac{x_1}{\sqrt{v_1}} - \frac{x_2}{\sqrt{v_2}} = 0 \tag{A.5}$$

$$x_1 + x_2 = X \tag{A.6}$$

This can be summarized as a structure of three equations with three unknowns:

$$\begin{bmatrix} \frac{1}{v_1} & \frac{1}{v_2} & -1 \\ \frac{1}{\sqrt{v_1}} & -\frac{1}{\sqrt{v_2}} & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ T \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ X \end{bmatrix} \tag{A.7}$$

Before equation A.7 can be solved it is necessary to get the speed at the second detector. This speed depends on the arrival time, so an iterative procedure is necessary to solve this. In the first iteration it is assumed that the arrival at the next section is in the same time interval as the departure at the first detector. When it is possible to solve the procedure stops. If it is not possible, the speed of the next time interval is used to calculate the arrival time.

In equation (A.5) the square root is token over $\frac{x^2}{v}$. To get a non-imaginary outcome x^2 and v should be both positive (or negative), which implies that x can have any value to satisfy this condition, but v should be greater than zero. Since v is always greater or equal to zero, only a problem occurs when v is equal to zero. In that case, v is the average of the the previous period, the current period and the next period.

Since the arrival time at the second detector is (mostly) not equal to the distributed headways, the closest one is chosen. By doing this for all detectors it is possible to draw the trajectory through all these points.

A.1.3 Step 3: Trajectory reconstruction

Linear lines
In the most simple model a line is made between two points, without taking the measured speeds into account. Since all locations and arrival times are known when this is calculated for the first car, it is possible to draw a straight line between two successive points.

Advanced linear lines
In a more advanced model the speed is taken into account at the measurement locations. By assuming that this speed is constant over a certain time and period a line can be drawn around the measurement locations. In this algorithm a parameter `leng` is used to draw the trajectory over `1/leng` of the distance between the current detector and the previous and next detector. In this way the trajectory between two point is known for $2 * \frac{1}{\texttt{leng}}$. Both ends of the lines are connected by a straight line, so a trajectory between two detectors consists of three connected lines. Note that if `leng` is equal to 2, the connecting line is horizontal.

Splines
To calculate a spline between two point a number of values should be known. In this case the values of the location and time are known for both point, as well as the speeds in that points. In total six values are known: two locations, two times, and two speeds. With these six values it is possible to solve a cubic function, with four variables, of the form $y = ax^3 + bx^2 + cx + d$.

To solve the problem four equations are known:

$$y_0 = ax_0^3 + bx_0^2 + cx_0 + d \tag{A.8}$$

$$y_1 = ax_1^3 + bx_1^2 + cx_1 + d \tag{A.9}$$

$$y_0' = 3ax_0^2 + 2bx_0 + c \tag{A.10}$$

$$y_1' = 3ax_1^2 + 2bx_1 + c \tag{A.11}$$

In other words, the following matrix should be solved:

$$\begin{bmatrix} x_0^3 & x_0^2 & x_0 & 1 \\ x_1^3 & x_1^2 & x_1 & 1 \\ 3x_0^2 & 2x_0 & 1 & 0 \\ 3x_1^2 & 2x_1 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} y_0 \\ y_1 \\ y_0' \\ y_1' \end{bmatrix} \tag{A.12}$$

With Matlab it is easy to solve these equations with the `linsolve` command. Matlab contains a function which is able to draw a spline between a number of points, but this function only takes the values in that points into account. It is not able to handle derivatives of the points.

Appendix B

Calculations interpolation models

B.1 Derivation of the Taylor series expansion

To derive the Taylor series, one starts with the conservation law:

$$\frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} = 0 \tag{B.1}$$

Since $q(k)$, the fundamental diagram, is known the conservation law can be rewritten as:

$$\frac{\partial k}{\partial t} + \frac{\partial q}{\partial k}\frac{\partial k}{\partial x} = 0 \tag{B.2}$$

Which again can be rewritten as:

$$\frac{\partial k}{\partial x} = -\frac{\frac{\partial k}{\partial t}}{\frac{\partial q}{\partial k}} \tag{B.3}$$

With the fundamental diagram $\frac{\partial q}{\partial k}$ can be calculated. Only $\frac{\partial k}{\partial t}$ has to be assumed or estimated.

To estimate the density at a location nearby a location with a known density, a general Taylor series expansion can be formulated:

$$k(x,t) \approx k(x_0,t_0) + \begin{bmatrix} x - x_0 \\ t - t_0 \end{bmatrix}^T \nabla k(x_0,t_0) + \frac{1}{2}\begin{bmatrix} x - x_0 \\ t - t_0 \end{bmatrix}^T \nabla^2 k(x_0,t_0) \begin{bmatrix} x - x_0 \\ t - t_0 \end{bmatrix} \tag{B.4}$$

$$\begin{aligned} \nabla k(x,t) &= \begin{bmatrix} \frac{\partial k}{\partial x} \\ \frac{\partial k}{\partial t} \end{bmatrix} = \begin{bmatrix} -\left(\frac{\partial q}{\partial k}\right)^{-1} \\ 1 \end{bmatrix} \frac{\partial k}{\partial t} \\ \nabla^2 k(x,t) &= \begin{bmatrix} \frac{\partial^2 k}{\partial x^2} & \frac{\partial^2 k}{\partial x \partial t} \\ \frac{\partial^2 k}{\partial x \partial t} & \frac{\partial^2 k}{\partial t^2} \end{bmatrix} \end{aligned} \tag{B.5}$$

$$
\begin{aligned}
\frac{\partial^2 k}{\partial x \partial t} =& \frac{\partial}{\partial t}\left(\frac{\partial k}{\partial x}\right) = \frac{\partial}{\partial t}\left(-\left(\frac{\partial q}{\partial k}\right)^{-1}\frac{\partial k}{\partial t}\right)\\
=& \left(\frac{\partial q}{\partial k}\right)^{-2}\frac{\partial^2 q}{\partial k^2}\left(\frac{\partial k}{\partial t}\right)^2 - \left(\frac{\partial q}{\partial k}\right)^{-1}\frac{\partial^2 k}{\partial t^2}\\
\frac{\partial^2 k}{\partial x^2} =& \frac{\partial}{\partial x}\left(\frac{\partial k}{\partial x}\right) = \frac{\partial}{\partial x}\left(-\left(\frac{\partial q}{\partial k}\right)^{-1}\frac{\partial k}{\partial t}\right)\\
=& -2\left(\frac{\partial q}{\partial k}\right)^{-3}\frac{\partial^2 q}{\partial k^2}\left(\frac{\partial k}{\partial t}\right)^2 + \left(\frac{\partial q}{\partial k}\right)^{-2}\frac{\partial^2 k}{\partial t^2}\\
\frac{\partial^2 k}{\partial t^2} =& \frac{\partial}{\partial t}\left(\frac{\partial k}{\partial t}\right) = \frac{\partial^2 k}{\partial t^2}
\end{aligned}
\tag{B.6}
$$

The partial derivatives can be filled in into the Hessian, which then becomes:

$$
\begin{aligned}
\nabla^2 k(x,t) =& \begin{bmatrix} \frac{\partial^2 k}{\partial x^2} & \frac{\partial^2 k}{\partial x \partial t} \\ \frac{\partial^2 k}{\partial x \partial t} & \frac{\partial^2 k}{\partial t^2} \end{bmatrix}\\
=& \begin{bmatrix} -2\left(\frac{\partial q}{\partial k}\right)^{-3}\frac{\partial^2 q}{\partial k^2}\left(\frac{\partial k}{\partial t}\right)^2 + \left(\frac{\partial q}{\partial k}\right)^{-2}\frac{\partial^2 k}{\partial t^2} & \left(\frac{\partial q}{\partial k}\right)^{-2}\frac{\partial^2 q}{\partial k^2}\left(\frac{\partial k}{\partial t}\right)^2 - \left(\frac{\partial q}{\partial k}\right)^{-1}\frac{\partial^2 k}{\partial t^2} \\ \left(\frac{\partial q}{\partial k}\right)^{-2}\frac{\partial^2 q}{\partial k^2}\left(\frac{\partial k}{\partial t}\right)^2 - \left(\frac{\partial q}{\partial k}\right)^{-1}\frac{\partial^2 k}{\partial t^2} & \frac{\partial^2 k}{\partial t^2} \end{bmatrix}
\end{aligned}
\tag{B.7}
$$

Writing this out, $k(x,t)$ becomes:

$$
\begin{aligned}
k(x,t) \approx& k(x_0,t_0) - \frac{\partial k}{\partial t}\left((x-x_0)\left(\frac{\partial q}{\partial k}\right)^{-1} + (t-t_0)\right) + \\
&\left(\frac{\partial q}{\partial k}\right)^{-2}\frac{\partial^2 q}{\partial k^2}\left(\frac{\partial k}{\partial t}\right)^2\left(-(x-x_0)^2\left(\frac{\partial q}{\partial k}\right)^{-1} + (x-x_0)(t-t_0)\right)
\end{aligned}
\tag{B.8}
$$

Appendix C

Results of the validation

The validation is performed on two different data sets. The microscopic data set contains individual vehicle trajectory data and the macroscopic data set contains data for speed, density and flow rate. In this section the quality of the trajectory reconstruction methods is tested.

C.1 Validation at microscopic data level

The results of the validation are described in the next sections. Every section describes a different method. First the results with the simple model are described, followed by the results of the linear interpolation, the four-corner interpolation and the two-point interpolation. The results of all methods are then summarized in table C.1.

Simple model results
The simple model uses only very basic techniques for the reconstruction of the trajectories. Nevertheless the results of this method are quite well, compared to the data generated by Vissim. The only thing that has to be estimated by the model is the distribution of the individual headways and speeds of the cars at the detector locations.

In figure C.1 the results are given. In one sub-plot both the real and the reconstructed trajectories are drawn. In the other sub-plot the errors of the difference in arrival times and speed are given.

As can be seen in the trajectory plot some of the reconstructed trajectories are sometimes at more than one location, which is a result of a non-intelligent method. The percentage error (MAAPE) of the arrival times is lower than 1.5%. The RMSE in arrival times is for both the linear and the spline reconstruction smaller than 11 seconds. The error is speed is much higher: the maximum MAAPE is almost 120%. Also the RMSE of the speed is high, with maxima of about 55 km/h. The size of the MAAPE can be explained taking the curve of the real trajectories into account. These trajectories contain very a lot of stop-and-go waves, which are almost undetectable. Consequence is that the speed will be overestimated.

To calculate the error in speed, the speed of the reconstructed trajectory is bounded to 0 km/h and 150 km/h. If that is not done, the RMSE explodes to about 1400 km/h, because the cars in the reconstruction at some moment drive more than 4000 km/h!

When the number of the detector locations is decreased to 5 (instead of 10), the

		Arrival time RMSE [sec]			Arrival time MAAPE [%]			Speed RMSE [km/h]			Speed MAAPE [%]		
		min	mean	max	min	mean	max	min	mean	max	min	mean	max
Simple model	Linear	1.2	6.4	10.7	0.2	0.8	1.44	7.6	29.9	45.5	13.9	40.5	116
	Spline	1.1	6.3	11	0.2	0.81	1.3	6.4	28.9	56.1	7.9	40.3	111
Linear int	Newell	0	9.2	14.7	0	0.63	1.25	5.9	15.3	28.8	3.5	25.7	104
	Van Aerde	0	13	26.1	0	0.9	2.21	3.8	14.7	29.1	3	23.9	106
	Smulders	0	9.6	16	0	0.57	1.06	5.1	14.9	26.8	7.4	26.3	110
Four-corner	Newell	0	9.7	15.2	0	0.63	1.22	5.9	15.2	29	3.5	25.5	103
	Smulders	0	10.1	16.8	0	0.58	1.10	5.4	14.8	26.8	7.4	26	109
Two-point	Newell	0	9.8	15.3	0	0.64	1.22	5.9	15.2	28.9	3.6	25.5	103
	Smulders	0	10	16.4	0	0.56	1.04	5.5	14.8	27	7.4	26.1	110

Table C.1: Summary of the results of the microscopic validation maximum percentage error in arrival times is still smaller than 2.5%, and the RMSE in arrival times is smaller than 19 seconds for the linear reconstruction and smaller than 14 seconds for the spline reconstruction. The RMSE of the speed is still fluctuating between 20 and 60 km/h. By increasing the length of a time interval from 1 to 5 minute, the MAAPE of the arrival times is about 2.5% and the RMSE is smaller than 26 seconds. The RMSE of the speed of the spline reconstruction is strongly fluctuating between 5 and 60 km/h. The maximum MAAPE is still almost 120%. When the time step is increased from 1 minute to 5 minutes and the number of detector location is 5, the MAAPE of the arrival time is about 2.2% and the maximum RMSE in arrival times is about 20 seconds. The RMSE of the speed is almost 50km/h, while the MAAPE is still very high at 130%. Note that no graphical results are given of the outcomes of the changed network.

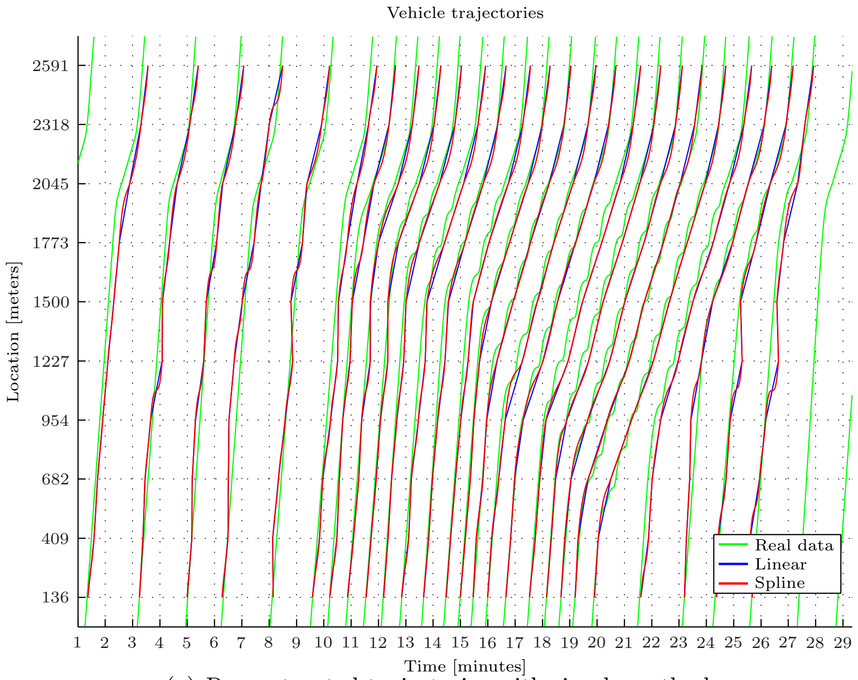
No explanation is found why the results of the increased time intervals, combined with reduced detector locations, perform better than the situation in which only one of them is changed. It is most likely that this is due to a sort of randomness.

Linear interpolation results

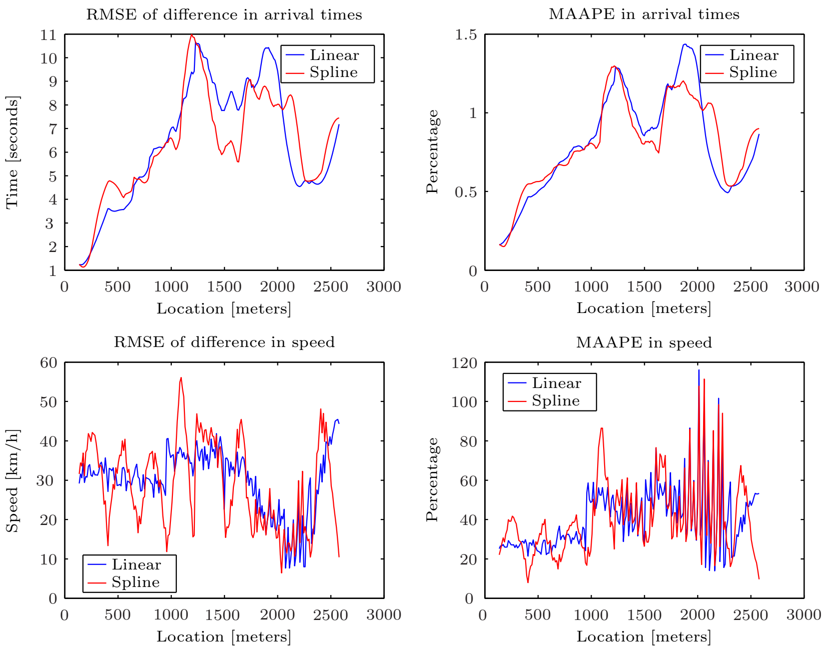
To reconstruct trajectories based on interpolation methods two steps have to be taken. First, the density has to be interpolated between the known points. After that, the speed can be calculated based on the density and a chosen fundamental diagram. When the data from Vissim is used the parameters of the fundamental diagram have to be estimated. For the linear interpolation model the fundamental diagrams of Newell, Van Aerde and Smulders are used.

As can be seen from the results (figure C.2), the reconstructions in which the fundamental diagrams of Newell and Smulders are used perform much better than the reconstruction using Van Aerde's fundamental diagram. The maximum percentage error with Newell and Smulders is lower than 1.5%, while with Van Aerde the error is larger than 2%. Also the RMSE in arrival times is with Van Aerde about 10 seconds more: 14.7 for Newell, 16 for Smulders and 26 for Van Aerde. The maximum RMSE for the speed is for Smulders 27 km/h and for Newell and Van Aerde 29 km/h. The MAAPE is still very large, about 110%.

When the length of a time interval is enlarged from 1 to 5 minutes, the reconstruction with Smulders' fundamental diagram performs best, where the RMSE of the arrival time takes values of about 17 seconds, compared to 20 seconds for Newell and 60 seconds for Van Aerde. The percentage error is about 1.2% for Smulders, 1.6% for Newell and 5% for Van Aerde. The differences between the fundamental diagrams in the RMSE for the speed is small. The RMSE is about 30 km/h. The MAAPE is 140% for Van Aerde and about 105% for Newell and Smulders. When the number of detectors is decreased to 5, with 1 minute time intervals, the performance with Newell is best, with a percentage error in arrival time smaller than 2.4%, compared to 2.9% for Smulders and 3.4% for Van Aerde. The maximum RMSE is varying between 22 seconds (Newell) and 38 seconds (Van Aerde). The maximum RMSE of the speed is about 35 km/h, for all three fundamental diagrams. The MAAPE is again very big, at about 140% for all diagrams. With the time interval set to 5 minutes and the number of detector locations of 5, the differences between the different methods are small: the maximum percentage error in arrival times is between 1.8% and 2.5%. The RMSE is between 20 and 29 seconds. For the speed the differences are also small: 30 to 35 km/h for the RMSE and 130% to 140% for the MAAPE.

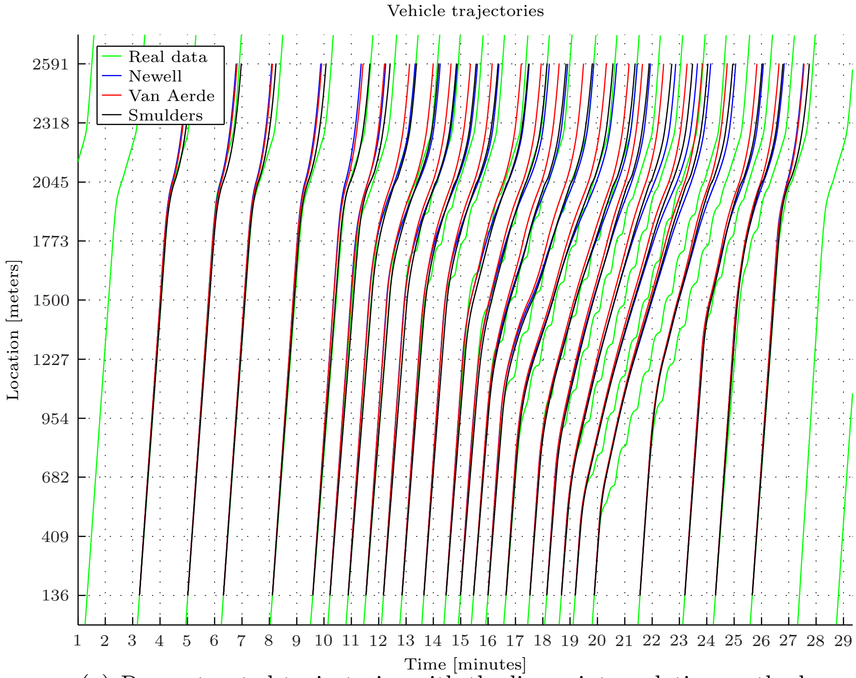


(a) Reconstructed trajectories with simple methods

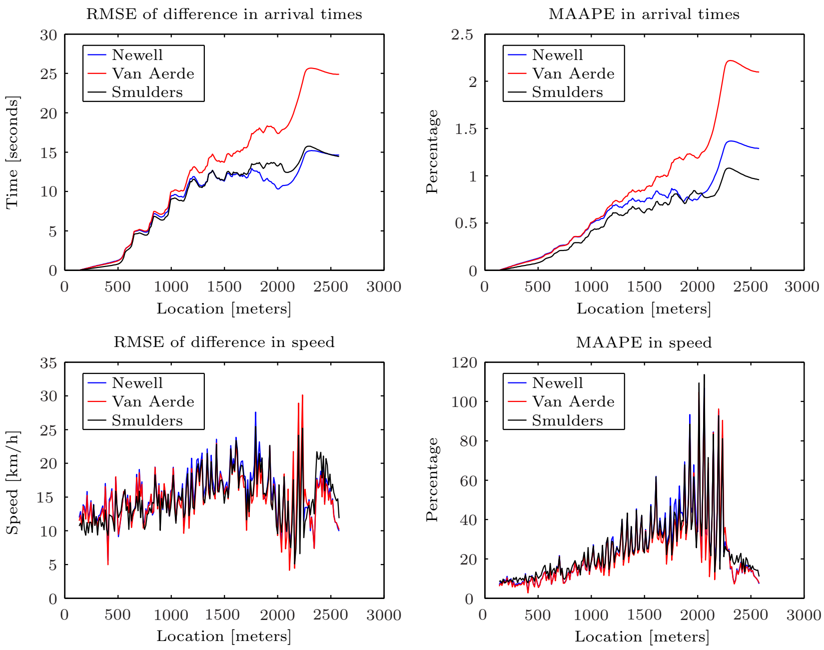


(b) Difference in travel times with simple methods

Figure C.1: Results of the reconstruction with simple methods



(a) Reconstructed trajectories with the linear interpolation method



(b) Difference in travel times with the linear interpolation method

Figure C.2: Results of the reconstruction with the linear interpolation method

Four-corner interpolation results
With a more advanced interpolation method the density between two detector locations is not interpolated linearly. Still, the fundamental diagram has to be estimated to use this method. With this method it is not possible to use the fundamental diagram of Van Aerde, because the combination of the first and second derivative of this fundamental diagram is such that the density can take impossible values (extremely negative or positive). Therefore, only Newell's and Smulders's fundamental diagrams are used.

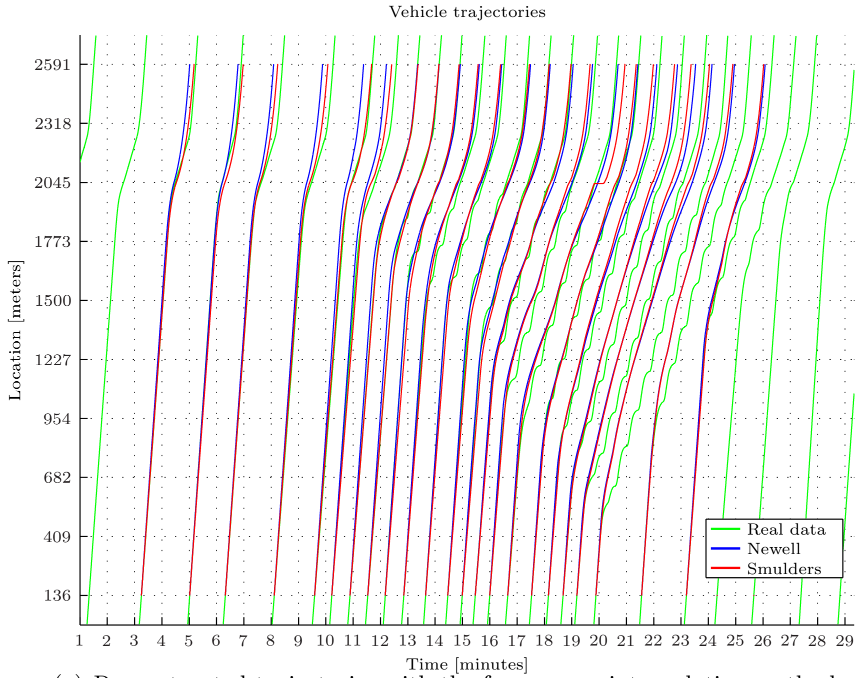
The results of this interpolation technique are given in figure C.3. The maximum percentage error in arrival times of both fundamental diagrams is about 1.2% for Newell. The maximum RMSE in arrival times is about 16 seconds. For the speed the RMSE is 27 km/h for Smulders and 29 km/h for Newell. The MAAPE is about 105%.

When the time interval is set to 5 minutes the differences between Newell and Smulders are very small. The maximum percentage error for the arrival times becomes about 1.2% and the maximum RMSE in arrival time becomes 20 seconds. The RMSE of the speed is about 30 km/h, with a MAAPE of about 100%. With 5 detectors and a 1 minute time interval, the percentage error of the arrival times is 2.5% for Newell and 3% for Smulders. Also the RMSE in arrival time has a slight advantage for Newell: 26 seconds, versus 32 seconds for Smulders. With 5 detectors and a 5 minute time interval the percentage error in arrival times is 1.9% for Newell and 2.3% for Smulders. The maximum RMSE in arrival times is about 25 seconds. The RMSE of the speed is 34 km/h, with a MAAPE of 140%.

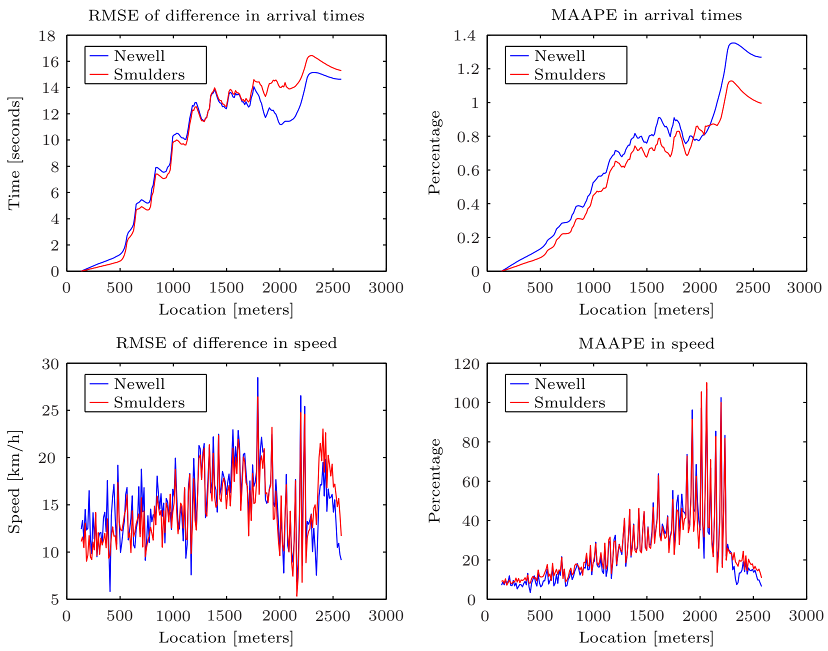
Two-point interpolation results
The difference between the four-corner interpolation method and the two-point interpolation method is very small. The way the interpolation of the density is performed differs, but the other steps are the same. The results are graphically shown in figure C.4.

The percentage error (of the arrival times) of the reconstruction is about 1% for Smulders and 1.2% for Newell. The maximum RMSE in arrival time is about 16 seconds. The error is speed is about 27 km/h and 100%.

With 5 detectors the percentage error of the arrival time is about 2.5% and the maximum RMSE is about 30 seconds. The RSME for the speed is 35 km/h and the MAAPE is 145%.With a 5 minute time interval and 10 detectors the percentage error in arrival times is 1.5% for Newell and 1.2% for Smulders. The maximum RMSE is 20 seconds for Newell and 19 seconds for Smulders. The speed error is 30 km/h and 100% for both fundamental diagrams. With a 5 minute time interval and 5 detectors the percentage error in arrival times is 2%, and the RMSE is about 23 seconds. The error for the speed is 33 km/h and 135%.

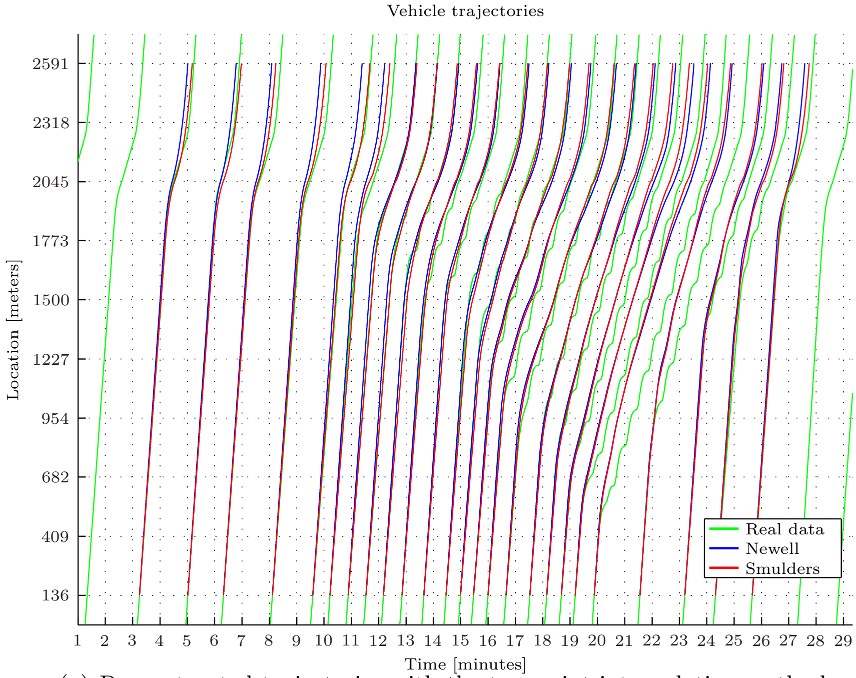


(a) Reconstructed trajectories with the four-corner interpolation method

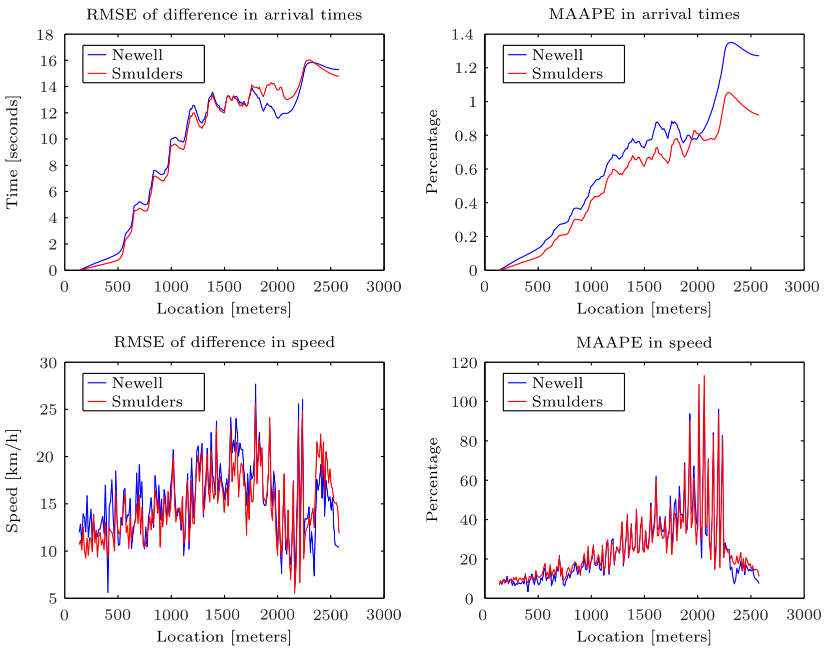


(b) Difference in travel times with the four-corner interpolation method

Figure C.3: Results of the reconstruction with the four-corner interpolation method



(a) Reconstructed trajectories with the two-point interpolation method



(b) Difference in travel times with the two-point interpolation method

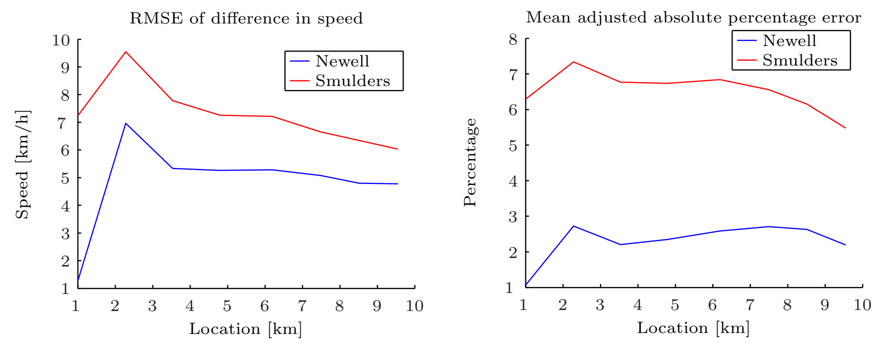
Figure C.4: Results of the reconstruction with the two-point interpolation method

C.2 Validation at macroscopic data level

At the macroscopic level the data of a detector is compared with the interpolated values at that location. In this procedure the density is interpolated between two locations and from this density the speed is calculated, according to the fundamental diagram. In this part of the validation, the results of different methods are compared for the fundamental diagram of Newell and Smulders. The results are summarized in table 5.3.

C.2.1 Speed calculation

In the model the speed is calculated based on the density, so in this validation the data at the detector location is also the speed which is calculated with the fundamental diagram and therefore not necessarily the exact value of the measured speed. The error which comes along with this method is calculated and visualized in figure C.2.1. As can be seen, the relative error in speed is lower for Newell compared to Smulders, about 2.5% versus 6.5%.



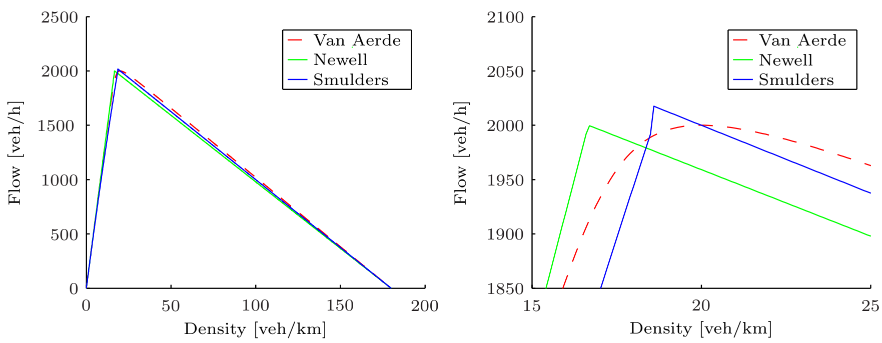


Figure C.5: Fundamental diagrams used for the validation of the macroscopic data

The error made by using the fundamental diagram's speed arises from the fact that MaDAM uses the fundamental diagram of Van Aerde. For the interpolation methods it is not possible to use this diagram, as explained in section 5.1. The fundamental diagrams used for the validation of the data are visualized in figure C.5. In the left part of the figure the complete diagrams are shown, and in the right part a detailed area of the top of the diagrams, since at that location the diagrams differs most.

C.2.2 Results

The results of the macroscopic validation are given in table C.2. The results are described in the next sections.

Newell
In figure C.6, the results of the interpolation with Newell's fundamental diagram are given. At some locations the linear interpolation performs better, and at other locations the four-corner or two-point interpolation give better results. In general, the linear interpolation method performs worse than the other two, as can be seen in the mean absolute percentage error of especially the density. At the beginning of the road, between kilometre 2 and kilometre 4, the error of the density is large; more than 25%. At these locations the error in speed is much smaller, which can be explained with the fundamental diagram: with the traffic state in free flow, the speed is, with Newell's fundamental diagram, constant for all density values. Therefore, a large error in density will not automatically result in a large error in the speed. For the reconstruction, it is necessary that the error in the speed is very small, since the trajectories are built up with this data. The error of the speed fluctuates between 4% and 6%.

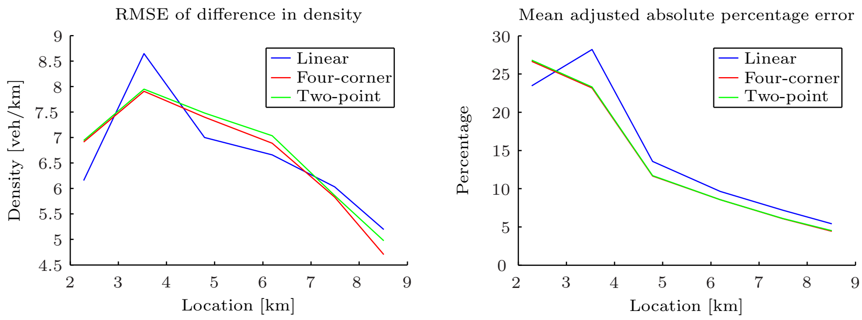
If the time interval is enlarged to 5 minutes, the relative error in the density is always smaller than 30%, where the four-corner and the two-point interpolation techniques perform best. At the end of the link, the relative error is about 5%. The RMSE lies between 3.5 and 8.5 veh/km. The relative error of the speed fluctuates somewhat between 3% and 6%. The RMSE decreases from 20 km/h at the beginning of the section to 8 km/h at the end of the section. There are no big differences between the different interpolation techniques.

Smulders
The results of the interpolation with Smulders's fundamental diagram are presented in figure C.7. The difference with the results of Newell are not so big. The results of the linear interpolation of the density are exactly the same. The MAAPE is lower for the four-corner and the two-point interpolation compared to the linear interpolation. The differences are very small for the speed. The maximum of the MAAPE is about 5.5%, and the RMSE decrease from 19 km/h at the beginning of the link to 9 km/h at the end

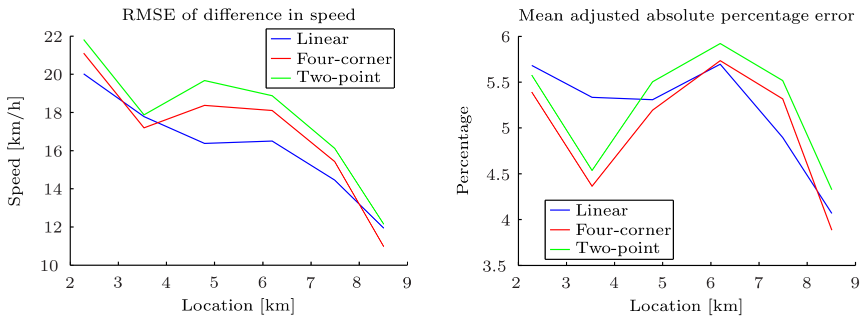
If the time interval is set to 5 minutes, the maximum MAAPE in density is about 27% for all interpolation techniques. The minimum of this relative error is 5% for the linear interpolation and 3.5% for the four-corner and the two-point interpolation. The RMSE varies between 3 and 8 veh/km. The percentage error for the speed varies between 2.7% and 5.8%. The RMSE lies between 6 and 19 km/h. The four-corner and the two-point interpolation score in general best, but the differences when compared to the linear interpolation technique are quite small.

		Density [veh/km] RMSE			Density [%] MAAPE			Speed [km/h] RMSE			Speed [%] MAAPE		
		min	mean	max	min	mean	max	min	mean	max	min	mean	max
Newell	Linear	5.2	6.6	8.6	5.4	14.6	28.2	11.9	16.2	20	4.1	5.2	5.7
	Four-corner	4.7	6.6	7.9	4.4	13.4	26.7	11	16.9	21.1	3.9	5	5.7
	Two-point	5	6.7	7.9	4.5	13.5	26.8	12.1	17.7	21.8	4.3	5.2	5.9
Smulders	Linear	5.2	6.6	8.6	5.4	14.6	28.2	10.6	14.3	17.4	4	5	5.5
	Four-corner	4.8	6.4	7.8	4.5	13.3	26.1	9.7	14.5	18.5	4.1	4.9	5.6
	Two-point	4.8	6.5	7.8	4.5	13.4	26.3	9.4	14.7	18.7	3.9	4.9	5.6

Table C.2: Summary of the results of the macroscopic validation

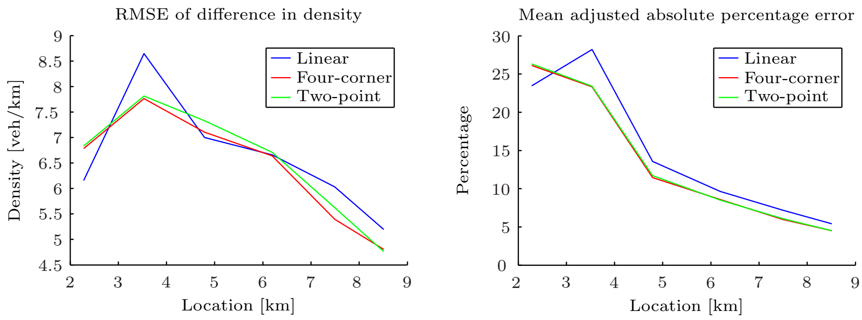


(a) Difference in density with Newell

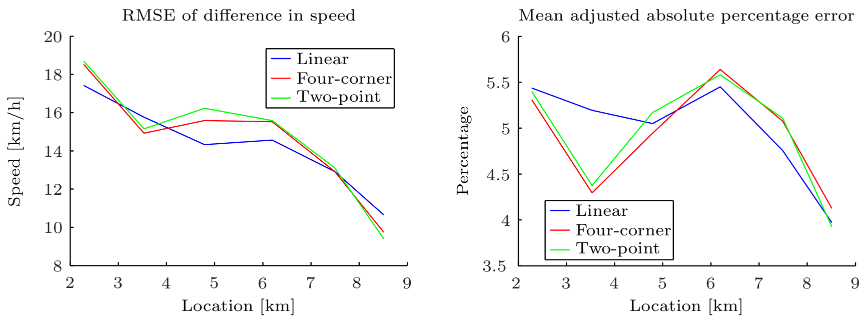


(b) Difference in speed with Newell

Figure C.6: Results of the error with the fundamental diagram of Newell



(a) Difference in density with Smulders



(b) Difference in speed with Smulders

Figure C.7: Results of the error with the fundamental diagram of Smulders