The Influence of Lateral Depth Variations on Tidal Dynamics in Semi-enclosed Basins

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The Influence of Lateral Depth Variations on Tidal Dynamics in Semi-enclosed Basins

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Summary

Understanding tidal dynamics is important for coastal safety, navigation and marine ecology. Many tidal basins around the world can be classified as semi-enclosed, i.e. bounded at three sides by coasts. Therefore, it is relevant to study tidal wave propagation in this type of basins. Taylor (1921) performed an idealized study on tidal wave propagation in a rectangular, rotating (due to the Earth's rotation) semi-enclosed basin of uniform depth and derived fundamental wave solutions, i.e. Kelvin and Poincaré waves. His model has proven to be useful to obtain insight in the physical mechanisms underlying tidal wave propagation in semi-enclosed basins. In this study Taylor's classical problem is extended to account for a basin geometry with basin-scale, lateral depth variations.

In a straight infinite channel wave solutions (i.e. modified Kelvin and Poincaré modes) are found by means of a semi-analytical, hydrodynamic model allowing for depth variations in lateral direction. For small depth variations (compared to uniform depth) the properties of the modified wave modes remain close to those of the uniform depth solutions. However, for large depth variations the modified wave modes show considerably different behavior. It is found that the wave lengths of the modified Kelvin waves depend on the water depth near the coastal boundaries. The wave length increases with increasing depth near the coastal boundary along which the Kelvin wave propagates, whereas the wave length decreases with decreasing depth. Furthermore, it is found that the Kelvin waves obtain a cross-channel velocity component, which is radically different from the classical Kelvin wave solution. Depending on the type of lateral depth profile the (evanescent) Poincaré modes obtain a propagative and/or more evanescent character compared to the uniform depth solutions. Also the lateral amplitude structures of the free surface elevation (with respect to still water) and velocity components change considerably as a result of lateral depth variations, especially at locations where the water depth is relatively shallow.

The solution to the Taylor problem allowing for lateral depth variations (i.e. a semi-enclosed basin of non-uniform depth) is written as a superposition of the modified wave modes in an infinite channel: an incoming Kelvin wave, a reflected Kelvin wave and a truncated sum of (reflected) Poincaré modes. A collocation technique is applied to satisfy the no-normal flow boundary condition at the basin's closed end. In general, we find that for symmetrical depth profiles the elevation amphidromic points (EAPs) remain on the centre line of the basin (as for uniform depth), whereas for asymmetrical depth profiles they shift in lateral direction towards the deeper side of the basin on a straight line parallel to the longitudinal coast. In addition, the EAPs shift in longitudinal direction, due to altered Kelvin wave lengths. The displacements in longitudinal direction are generally much larger than the displacements in lateral direction. The current amphidromic points (CAPs) show similar shifts as the EAPs and remain located between two EAPs. As a result of the cross-channel velocity component of the modified Kelvin waves, the cross-channel velocity is not only present close to the basin's closed end, but also farther away from this boundary.

Finally, a practical case is studied by means of our hydrodynamic model: the Southern North Sea with and without large-scale sand extraction on the Netherlands Continental Shelf (NCS). Based on bathymetrical data a longitudinally averaged, lateral depth profile is determined for the Southern North Sea. It is found that adopting this realistic lateral depth profile for the Southern North Sea instead of assuming uniform depth leads to considerable changes in the tidal amplitudes and currents. Consequently, large-scale sand extraction is modeled by dividing the basin into two parts: one part with a sand extraction trench and one part without extraction trench

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in the lateral depth profile. It is concluded that large-scale sand extraction may considerably impact on the tidal system of the Southern North Sea. The tidal amplitudes and currents are not only locally affected, but also far away from the extraction area. These changes may have severe impacts on the morphodynamics and, consequently, coastal safety, ecology and cable and pipeline infrastructure.

Based on the potential impacts of large-scale sand extraction on the tidal system of the Southern North Sea, it is recommended to account for tidal changes in present and future studies on this issue. For further research it is recommended to include bottom friction and (simple) longitudinal depth variations in our hydrodynamic model and study the consequent effects on the tidal system. Furthermore, it is recommended to compare these model results (i.e. with bottom friction included) with tide observations in actual (semi-enclosed) seas.

Samenvatting

Begrip van de getijdendynamiek is belangrijk voor kustveiligheid, scheepvaart en ecologie. Veel getijdenbassins op aarde kunnen worden geclassificeerd als half-ingesloten bassins (aan drie zijden begrensd door kusten). Om deze reden is het relevant om de voorplanting van getijdengolven in dit soort bassins te bestuderen. In een geïdealiseerde studie heeft Taylor (1921) onderzoek gedaan naar de voortplanting van getijdengolven in een rechthoekig, roterend (als gevolg van de draaiing van de aarde) half-ingesloten bassin van uniforme diepte. Hierin leidt hij fundamentele golfoplossingen af, te weten Kelvin en Poincaré golven. Taylor's model is bruikbaar gebleken om inzicht te verkrijgen in de fysische mechanismen die ten grondslag liggen aan de voortplanting van getijdengolven in half-ingesloten bassins. In deze studie wordt het klassieke Taylor probleem uitgebreid door een bassingeometrie met grootschalige dieptevariaties (op bassin-schaal) in laterale richting mee te nemen in het model.

In een recht kanaal van oneindige lengte worden de golfoplossingen (gemodificeerde Kelvin en Poincaré modi) gevonden door middel van een semi-analytisch, hydrodynamisch model dat diepteveranderingen in laterale richting toestaat. Voor kleine dieptevariaties (ten opzichte van uniforme diepte) blijven de golfeigenschappen van de gemodificeerde modi min of meer gelijk aan die van de oplossingen voor uniforme diepte. Voor grotere diepteveranderingen vertonen de golfmodi echter aanzienlijk ander gedrag. We zien dat de golflengtes van de gemodificeerde Kelvin golven afhangen van de waterdiepte dichtbij de kustlijn. De golflengte neemt toe naarmate het dieper wordt in de buurt van de kust waarlangs de Kelvin golf zich voorplant, terwijl de golflengte afneemt naarmate het daar ondieper wordt. Verder vinden we dat de Kelvin golven een snelheidscomponent verkrijgen in de dwarsrichting van het kanaal, wat radicaal verschilt van de klassieke Kelvin golfoplossing. Afhankelijk van het type laterale diepteprofiel verkrijgen de (gebonden) Poincaré modi een propagerend en/of meer afvallend karakter vergeleken met de oplossingen voor uniforme diepte. Ook de laterale amplitudestructuren voor de veranderingen in het vrije wateroppervlak (ten opzichte van stilstaand water) en de snelheidscomponenten veranderen aanzienlijk als gevolg van laterale dieptevariaties, vooral op locaties waar het relatief ondiep is.

De oplossing van het Taylor probleem met inbegrip van laterale diepteveranderingen (ofwel een half-ingesloten bassin van niet-uniforme diepte) is een superpositie van de gemodificeerde golfmodi in een oneindig recht kanaal: een inkomende Kelvin golf, een gereflecteerde Kelvin golf en een getrunceerde som van (gereflecteerde) Poincaré modi. Een collocatietechniek is toegepast om aan de randvoorwaarde te kunnen voldoen die geen normale stroming toestaat door de afgesloten rand aan het eind van het bassin. In het algemeen vinden we dat de amfidromische punten voor veranderingen in het vrije water oppervlak (EAPs) voor symmetrische diepteprofielen op de middenlijn van het bassin blijven liggen (net als voor uniforme diepte), terwijl deze voor asymmetrische diepteprofielen lateraal verschuiven richting het diepere gedeelte van het bassin op een rechte lijn, parallel aan de longitudinale kust. Tevens verschuiven de EAPs in longitudinale richting als gevolg van de veranderde golflengtes van de Kelvin golven. In het algemeen zijn de longitudinale verschuivingen aanzienlijk groter dan de laterale verschuivingen. De amfidromische punten voor de getijdenstroming (CAPs) vertonen verschuivingen vergelijkbaar met die van de EAPs en blijven zich bevinden tussen twee EAPs. Als gevolg van de snelheidscomponent in dwarsrichting van de gemodificeerde Kelvin golven is de dwars-snelheid niet alleen aanwezig dichtbij de afgesloten rand van het bassin, maar ook verder weg van deze rand.

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Tot slot is een praktische casus bestudeerd met behulp van ons model: de zuidelijke Noordzee met en zonder grootschalige zandwinning op het Nederlands Continentaal Plat (NCP). Met behulp van bathymetrische data is een longitudinaal gemiddeld, lateraal diepteprofiel bepaald voor de zuidelijke Noordzee. Door dit realistischer diepteprofiel te modelleren in plaats van uniforme diepte, treden aanzienlijke veranderingen op in getijdenamplitudes en –stromingen. Vervolgens is grootschalige zandwinning gemodelleerd door het bassin op te splitsen in twee delen: een deel met een zandwinninggeul en een deel zonder zandwinninggeul in het laterale diepteprofiel. We concluderen dat grootschalige zandwinning het getij in de zuidelijke Noordzee aanzienlijk kan beïnvloeden. De getijdenamplitudes en -stromingen worden niet alleen lokaal beïnvloed, maar ook ver weg van het winningsgebied. Deze veranderingen kunnen aanzienlijke gevolgen hebben voor de morfodynamiek en daardoor ook voor de kustveiligheid, ecologie en kabel- en pijplijninfrastructuur.

Door de potentiële impact van grootschalige zandwinning op het getijdensysteem van de zuidelijke Noordzee, wordt aanbevolen om veranderingen in het getij mee te nemen in huidig en toekomstig onderzoek op dit gebied. Voor nader onderzoek wordt tevens aanbevolen om bodemwrijving en (eenvoudige) longitudinale dieptevariaties mee te nemen in ons hydrodynamische model en de daaraan gerelateerde effecten op de getijdendynamiek te bestuderen. Verder wordt aanbevolen om deze modelresultaten (waarin bodemwrijving is meegenomen) te vergelijken met getijdenobservaties in bestaande (half-ingesloten) zeeën.

Preface

After extending my student life with another five months in Norway, the time had come to finish my study Civil Engineering & Management at the University of Twente. Since there are so many things that have my interest, finding a suitable Master's Thesis topic was not easy. Finally, I decided to choose for a challenging topic, something that pushed my boundaries and forced me to gain new knowledge. Well, challenge is what I got with this study on tidal dynamics! More than once I was lost in the forests of mathematics or frustrated by the bugs in MATLAB[®]. Probably Johan Cruijff would have said something like: "mathematics is simple, but nothing is harder than applying simple mathematics". I think this is very applicable to my research. Almost every problem I faced turned out to have a relatively simple solution, but finding that solution proved to be not always an easy task. Nevertheless, I enjoyed gaining a lot of knowledge in the past months!

Many people helped me finishing this thesis. First of all, I would like to thank my daily supervisor Pieter Roos. Pieter, thank you for your patience, enthusiasm and motivation while guiding me through the world of mathematics! I do not think I could have managed this without you. I am also grateful to Suzanne Hulscher and Ad Stolk for reading my reports and their useful comments. Furthermore, thanks to Blanca Perez-Lapeña and Henriët van der Veen for helping me out with the GIS-profiles of the Southern North Sea.

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Frequently Used Symbols

Symbol ¹	Definition	Non-dimensional (scaled) notation
A^*	Amplitude of lateral depth variations (m)	À
B^*	Width of semi-enclosed basin (m)	В
$\mathbf{B}^{*}_{\text{trench}}$	Width of sand extraction trench (m)	B _{trench}
c*	Wave speed (m s^{-1})	С
C _n	(Complex) Collocation coefficient of the n-th reflected wave	
	solution (-)	
f	Coriolis parameter (s^{-1})	f
g	Gravitational acceleration (m s^{-2})	
$h^*(y^*)$	Lateral depth profile (m)	h(y)
$h^*_{trench}(y^*)$	Lateral depth profile of sand extraction trench (m)	$h_{trench}(y)$
h [*] _{trench;max}	Maximum trench depth (m)	h _{trench;max}
H [*]	Uniform water depth (m)	
\mathbf{k}^*	(Complex) Wave number (m^{-1})	k
K^*	Reference wave number, associated with a classical Kelvin wave (m^{-1})	
τ*.	Wave length Kelvin waves (m)	Ι.
	E-folding length scale Poincaré waves (m)	L ₀ I
L _n I [*] ,	Length of sand extraction trench (m)	L _n
M ₂	Semi-diurnal (principal lupar) tidal constituent (-)	Ltrench
\mathbf{R}^*	Rossby deformation radius (m)	
s	Rottom slope of linear lateral denth profiles (-)	
t [*]	Time (s)	t
	Depth-averaged velocity component in longitudinal	11
u .	direction (m s^{-1})	u
<u>^*</u>	(Complex) Amplitude of the depth-averaged velocity in	û
и	longitudinal direction (m s^{-1})	и
1 1*	Typical velocity scale, associated with a classical Kelvin	
U	wave (m s^{-1})	
\mathbf{v}^*	Depth-averaged velocity component in lateral direction	V
	$(\mathbf{m} \mathbf{s}^{-})$	
\hat{v}^*	(Complex) Amplitude of the depth-averaged velocity in	\hat{v}
*	lateral direction (m s ⁻¹)	
X *	Coordinate in longitudinal direction (m)	Х
У _*	Coordinate in lateral direction (m)	У
y trench	Lateral coordinate of seaward boundary of sand extraction	Ytrench
*	trench (m)	
Y 20m	Lateral location of established -20 m depth contour –	y _{20m}
7*	(Arbitrary) Maximum alouation amplitude at the acost (m)	
∠ β*	(A) of a single state of the second	ß
β	Latitude, positive in the Northern Hemisphere and resetive	h
U	Lanuac, positive in the Normern Hemisphere ($_{-}$)	
۲*	Free surface elevation with respect to still water level (m)	٢
5	The surface crevation, with respect to suff water rever (III)	5

¹ Symbols denoted with an asterisk are dimensional quantities

Symbol	Definition	Non-dimensional (scaled) notation
\hat{z}^*	(Complex) Amplitude of free surface elevation (m)	^ Z
\hat{Z}_0^*	Mean maximum elevation amplitude at the coast for actual basin (m)	\hat{Z}_0
σ^{*}	Tidal angular frequency (rad s ⁻¹)	σ
$\stackrel{\phi}{\Omega^{*}}$	Phase shift for asymmetrical sinusoidal depth profile (-) Angular velocity of the Earth's rotation (rad s ⁻¹)	

1. Introduction

In this Chapter the problem definition of the study is discussed (section 1.1), resulting in the research objective (section 1.2) and research questions (section 1.3). Finally, the outline of the report is presented (section 1.4).

1.1 Problem Definition

Understanding tidal dynamics is important with respect to coastal safety, navigation and marine ecology. Since many tidal basins around the globe are bounded by coasts at three sides, such as the Adriatic Sea, the Gulf of California and the Southern North Sea (when the Dover Strait is assumed to be closed), it is of practical relevance to study tidal dynamics in semi-enclosed basins. Although (complex) numerical models are necessary to study the local effects of the tide, idealized studies are useful to obtain insight in the physical mechanisms underlying tidal wave propagation in these basins. Taylor (1921) performed such an idealized study and derived analytical wave solutions in rectangular, rotating (due to the Earth's rotation) semi-enclosed basins of uniform depth, i.e. Kelvin and Poincaré waves. The resulting free surface elevation and velocity fields are presented in amphidromic systems. Elevation amphidromic systems describe the spatial structure of tidal amplitudes and phases in a marine basin and are usually presented in terms of co-amplitude and co-phase lines (i.e. lines of equal tidal amplitude and equal tidal phase). Current amphidromic systems describe the spatial structure of the velocity components which can be presented in terms of tidal ellipses. Taylor's study has proven to be useful to gain a better understanding of the tidal dynamics in semi-enclosed basins. Consequently, many studies have been dedicated to further analyze the Taylor problem, also for more complex cases than Taylor's idealized study. Although considerable progress has been achieved in these studies, there remains a need for a comprehensive study of the effects of basin-scale, lateral depth variations on the tidal dynamics in semi-enclosed basins. Here basin-scale, lateral depth variations are defined as depth variations with length scales extending over the whole width of the basin (i.e. tens to hundreds of kilometers).

The Taylor approach accounting for lateral depth variations can be useful for studying the impacts of large-scale sand extraction on the Netherlands Continental Shelf (NCS) on the tidal system of the Southern North Sea. Over the past years the demand of sand from the North Sea has increased. In the future a further increase in sand demand is expected due to the policy of maintaining the sand budget of the near shore zone (coastal defense), increasing scarcity of sand extraction locations on land and plans for land reclamation such as the second Maasvlakte (Boers, 2005). In the coming 200-300 years these activities may lead to the development of a large sand extraction trench in front of the Dutch coast of tens to hundreds of kilometers in length, tens of kilometers in width and tens of meters in depth (Stolk, 2009). Several concerns are related to sand extraction on this scale, such as the (local) impacts on flow conditions and morphodynamics, but also impacts on the environment and the coastal defense system. However, potential impacts of large-scale sand extraction on the tidal system as a whole (i.e. the amphidromic system), also affecting the flow conditions and morphodynamics, are hardly considered so far. Therefore, this study investigates the potential impacts of large-scale sand extraction on the tidal system of the Southern North Sea.

1.2 Research Objectives

The main objective of the study is to analyze the influences of basin-scale, lateral depth-profiles on the wave properties and the amphidromic systems in semi-enclosed basins, using Taylor's approach. Moreover, the study aims to investigate the tidal effects of adopting a realistic lateral depth profile of the Southern North Sea with and without large-scale sand extraction on the NCS.

1.3 Research Questions

Based on the problem definition and research objectives the following research questions are formulated:

- 1. How do the wave properties of the Kelvin and Poincaré modes change for idealized lateral depth profiles compared to uniform depth?
- 2. What are the influences of the (idealized) lateral depth profiles on the resulting elevation and current amphidromic systems (i.e. the solution to the Taylor problem)?
- 3. What are the potential effects of large-scale sand extraction on the Netherlands Continental Shelf (NCS) on the tidal system of the Southern North Sea?

1.4 Outline of the Report

Chapter 2 discusses the theoretical background of the study, describing the Taylor problem for uniform depth and the findings of previous studies on extensions of the Taylor problem. Chapter 3 contains a description of the hydrodynamic model that is used to find wave solutions for a basin geometry with basin-scale, lateral depth variations. In Chapter 4 the properties of the wave solutions corresponding to idealized lateral depth profiles are presented. Chapter 5 discusses the resulting amphidromic systems for these profiles. Consequently, in Chapter 6 the case of the Southern North Sea with and without large-scale sand extraction on the NCS is analyzed. This Chapter examines the effects of adopting a realistic lateral depth profile for the Southern North Sea instead of assuming uniform depth and, in addition, studies the impact of large-scale sand extraction on its tidal system. Chapter 7 contains a discussion of the study results. Finally, the conclusions and recommendations of the study are presented in Chapter 8.

2. Theoretical Background: A Review of the Taylor Problem

In this chapter the literature underlying the Taylor problem is reviewed. This includes a brief (qualitative) description of the Taylor problem (section 2.1), extensions of the original Taylor problem (section 2.2) and applications of the Taylor approach to existing semi-enclosed basins around the world (section 2.3).

2.1 The Taylor Problem: Tidal Wave Propagation in Semienclosed Basins

Taylor (1921) considered a rectangular, rotating (due to the Earth's rotation) semi-enclosed basin of width² B^{*} as indicated in Figure 1, forced by a Kelvin wave entering the basin from infinity, while allowing reflected Kelvin and Poincaré waves to radiate outward. This has become known as the Taylor problem. The solution to Taylor's problem can be written as a superposition of wave solutions in an infinite open-channel (see Figure 2): an incoming and reflected Kelvin wave and an infinite set of Poincaré waves. In this section the properties of the individual Kelvin and Poincaré wave solutions in an infinite channel as well as the solutions to the Taylor problem (the superposition of the individual wave modes) are discussed in qualitative terms.



Figure 1. Top-view of a rectangular semi-enclosed basin of width B* in the Northern Hemisphere.

2.1.1 Kelvin and Poincaré Waves in an Infinite Open-channel

A Kelvin wave is a gravity wave in the ocean or atmosphere that balances the Earth's Coriolis force against a topographic boundary like a coastline. In uniform depth Kelvin waves are nondispersive, i.e. the group velocity (the speed of the wave group) and, hence, the speed of the wave energy, is equal to the phase velocity (the speed of individual waves), so that the Kelvin waves are not deformed by linear processes. Another typical feature of a classical Kelvin wave is the absence of flow velocity in cross-channel-direction. The dispersion relationship for Kelvin waves in uniform depth can be written as follows:

$$k_0^* = \pm \frac{S^*}{\sqrt{g^* H^*}}$$
. (Eq. 1)

² Dimensional quantities are denoted with an asterisk.

Where:

- \mathbf{k}_0 Wave number in m⁻¹
- σ^{*} Tidal angular frequency in rad s⁻¹
- g H Gravitational acceleration in m s⁻²
- Uniform (undisturbed) water depth in m



Figure 2: Top-view of an infinite open-channel of width B* in the Northern Hemisphere.

Poincaré waves are another type of gravity waves which are, in contrast to Kelvin waves, dispersive, i.e. the group velocity is not equal to the phase velocity. In addition, Poincaré waves have a sinusoidally varying cross-channel velocity structure (v^{*}), whereas the Kelvin waves do not have a v^{*}-component at all. In uniform depth a Poincaré wave has either a real or a purely imaginary wave number (k_n^*) . Poincaré waves are called *free* (unbound) when k_n^* is purely real and *trapped* (evanescent, bound) when k_n^* is purely imaginary. However, the introduction of horizontally viscous effects (Roos and Schuttelaars, 2009) or, as we will see later, lateral depth variations, complicates the classification into free and trapped Poincaré waves, because k_n^{*} is not purely real or imaginary anymore. Since it is useful to classify the Poincaré waves and the modifications of k_n^* are relatively small, we will refer to free modes when k_n^* is dominantly real and trapped modes when k_n^* is dominantly imaginary. A critical channel width (B_{crit}^*) exists for the existence of *free* Poincaré waves, which is defined in equation (2).

$$B_{Crit}^{*} = \frac{p\sqrt{g^{*}H^{*}}}{\sqrt{s^{*^{2}} - f^{*^{2}}}} \qquad \text{with} \qquad f^{*} = 2\Omega^{*}\sin q \,. \tag{Eq. 2}$$

Where, in addition to the symbols explained below equation (1):

- f^* Coriolis parameter in s⁻¹
- Ω^* Angular velocity of the Earth $(7.292*10^{-5} \text{ rad s}^{-1})$

θ Latitude, positive in the Northern Hemisphere and negative in the Southern Hemisphere

Every channel admits a finite number of free Poincaré modes (possibly zero when $B^* < B_{crit}^*$) and an infinite number of trapped Poincaré modes. The dispersion relationship for Poincaré waves in an infinite open-channel can be written as follows:

$$k_n^* = \pm \left\{ \frac{s^{*2} - f^{*2}}{s^* H^*} - \frac{n^2 p^2}{B^{*2}} \right\}^{1/2}.$$
 (Eq. 3)

Where, in addition to the symbols explained below equations (1) and (2):

- \mathbf{B}^* Channel width in m
- Integer value (n = 1, 2, 3, ...), indicating the different Poincaré modes n

2.1.2 Solutions to the Taylor Problem

To find analytical solutions for tidal wave propagation in a rectangular, rotating semi-enclosed basin, Taylor (1921) made several assumptions (also see Pedlosky (1982) and De Swart (2008)):

- I. <u>Homogeneous fluid:</u> constant density of water (ρ=constant), no stratification.
- II. <u>f-plane</u>: Coriolis force is included, but it is assumed that the dynamics are hardly influenced by the curvature of the earth.
- III. Ideal, inviscid fluid: effects of bottom friction and viscosity are neglected.
- IV. <u>Linear problem:</u> after scaling against typical scales it turns out that the Froude number is small, so that the non-linear terms in the continuity and momentum equations can be neglected.
- V. Depth-averaged motion: velocity variations over depth are not considered.
- VI. <u>Uniform depth:</u> flat bottom, resulting in a uniform (undisturbed) water depth.

The solution to the Taylor problem is a superposition of the Kelvin and Poincaré modes derived in an infinite open-channel, resulting in an elevation and current amphidromic system. In Figure 3 and Figure 4 the elevation and current amphidromic system are presented for a basin comparable to the Southern Bight of the North Sea with $B^* = 200$ km, $H^* = 30$ m, $\theta = 53^{\circ}N$, $\sigma^* = 1.41 \times 10^{-4}$ rad s⁻¹ (corresponding to the semi-diurnal, principal lunar tidal constituent M₂) and maximum elevation amplitude at the coast $\hat{z}_0^* = 1.5$ m (see Brown, 1987; Sinha and Pingree, 1997; Roos and Schuttelaars, 2009). For motivation for the selected parameter values is referred to section 4.1. In Figure 3 one can see that elevation amphidromic points (EAPs) occur where all co-phase lines come together and, by definition, the tidal amplitude is zero. The tidal amplitude is at its maximum at the coastal boundaries. The current amphidromic system consists of tidal ellipses, which reduce to lines where one of the velocity components is zero (i.e. at the closed boundaries) and to circles where both velocity components are equal (Figure 4). Current amphidromic points (CAPs) occur where the tidal ellipse reduces to a point (i.e. both velocity components are zero) and, in this case, are located between two EAPs. The maximum tidal currents occur close to the coastal boundaries. The cross-channel flow and the cross-channel wave propagation near the closed boundary at $x^* = 0$ of the basin are the result of the Poincaré modes.

2.1.3 Properties of Solutions to the Taylor Problem

The solution of the Taylor problem has the following properties:

- 1) The solution is a superposition of an incoming Kelvin wave, a (partially) reflected Kelvin wave and an infinite set of Poincaré waves.
- 2) In uniform depth a Poincaré wave either has a real or purely imaginary wave number (k_n^*) . Poincaré waves are called free when k_n^* is real and trapped (evanescent) when k_n^* is imaginary. Every channel admits a finite number of free Poincaré modes (possibly zero) and an infinite number of trapped Poincaré modes. A critical channel width (B^*_{crit}) exists for the existence of *free* Poincaré waves $(B^* > B^*_{crit})$.
- 3) The spatial scale on which trapped Poincaré modes emerge (e-folding length-scale $1/\text{Im}(k_1^*)$) is for most basins much smaller than the length of the basin, so that trapped Poincaré modes are important only close to the boundary $x^* = 0$ and decay with increasing distance x^* from the closed end. In basins where free Poincaré modes cannot exist (i.e. narrow basins in which $B^* < B^*_{crit}$), the velocity field at distances from $x^* = 0$ larger than the e-folding length-scale is only determined by the two Kelvin waves.

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- 4) The tidal wave rotates cyclonically (clockwise in the Southern Hemisphere and anticlockwise in the Northern Hemisphere) around amphidromic points. In case of full reflection (i.e. the incoming and reflected Kelvin waves have the same amplitude) in a narrow basin ($B < B_{crit}$), sufficiently far away from the boundary at $x^* = 0$ not to be affected by the trapped Poincaré modes, both the elevation and current amphidromic points are located on the centre-axis of the basin. In this case the distance between successive amphidromic points is half the wave length of the Kelvin waves.
- 5) Between successive elevation amphidromic points (EAPs) there are current amphidromic points (CAPs). The current amphidromic system consists of tidal ellipses which reduce to lines when one of the velocity components (u^{*} or v^{*}) is zero and to points when both velocity components are zero.



Figure 3. Elevation amphidromic system for uniform depth. The solid lines indicated co-amplitude lines with intervals of 20 cm and the dashed lines indicate co-phase lines dividing the tidal period into 12 intervals. Note that for each EAP one of the co-phase lines is presented as a set of multiple lines. This is because of a numerical limitation: it is not recognized that -180° and 180° are the same phases.



Figure 4. Current amphidromic system for uniform depth. Black squares indicate cyclonic rotation (i.e. counter-clock-wise in the Northern Hemisphere) and open circles indicate anti-cyclonic rotation (i.e. clockwise in the Northern Hemisphere).

2.2 Extensions of the Taylor Problem

To gain a better understanding of the physics underlying tidal wave propagation in a semienclosed basin and, moreover, to be able to explain observations that differed significantly from Taylor's original solution, several authors have extended the Taylor problem to account for energy dissipation or depth variations. This section gives a brief overview of the research that has been dedicated to these topics and provides motivation for the need of the present study.

2.2.1 Energy Dissipation Mechanisms

Many authors focused on the effects of energy dissipation mechanisms in the Taylor problem, such as energy dissipation at the basin's closed end (Proudman, 1941; Hendershott and Speranza, 1971), bottom friction (Rienecker and Teubner, 1980; Rizal, 2002), an oscillating boundary (Brown 1987; Brown 1989) and vertical and horizontal viscosity (Davies and Jones, 1995; Roos and Schuttelaars, 2009). Hendershott and Speranza (1971) allow power-flux absorption at the

closed end of the basin by means of a dissipative boundary condition ($u^* = \alpha^* \zeta^*$ with ζ^* indicating the free surface elevation in m and α^* the partial absorption coefficient in s⁻¹). Their results indicate that the reflected Kelvin wave decreases in amplitude as α^* increases. This implies that the elevation amphidromic points (EAPs) move towards the wall along which the reflected Kelvin wave propagates, on a straight line parallel to the longitudinal coastal boundary. Rienecker and Teubner (1980) and Rizal (2002) show that the EAPs are located on a straight line that tends to move towards the wall along which the reflected Kelvin wave travels (i.e. has a small angle to the longitudinal direction), become virtual and eventually disappear completely as bottom friction increases. Brown (1987, 1989) has shown that shifts of the EAPs could also be attributed to tidal oscillations at the basin's closed end (representing an open connection to other tidal waters), and not solely to friction. Roos and Schuttelaars (2009) find an additional type of (viscous) Poincaré modes as a result of horizontally viscous effects. Similar to bottom friction the EAPs are located on a straight line making a small angle to the longitudinal direction due to horizontal viscous effects. Although these studies have considerably contributed to our understanding of the effects of energy dissipation mechanisms on the tidal dynamics in semienclosed basins, most studies maintained the uniform depth assumption, so that the effects of depth variations remain unrevealed.

2.2.2 The Role of Depth Variations

There have been studies that did account for depth variations in their analysis, although often not as the main topic of investigation. Godin and Martinez (1994), Davies and Jones (1995), Van der Molen et al. (2004) and Winant (2007) included *longitudinal* depth variations in their analyses. Hunt and Hamzah (1967) derived an analytical solution for the case of linear lateral bottom slope. It turns out that the wave speed depends on the slope s. The wave speed c is increased in case of positive slope s (defined as decreasing water depth with y^{*}, where the water depth increases at the boundary along which the wave propagates) and decreased in case of negative s compared to uniform depth. Furthermore, Hunt and Hamzah (1967) conclude that the minimum basin-width (B_{crit}^{*}) for the existence of Poincaré waves is increased due to cross-channel depth variations. Hendershott and Speranza (1971) allowed for cross-channel bottom relief by letting the water depth vary in steps in y⁻-direction. They observe displacement effects on the amphidromic system, but argue that these effects are minor compared to the effects of energy dissipation at the basin's closed end. Staniforth et al. (1993) obtain exact wave solutions for a channel with linear lateral depth variations, only for positive s. For mild slopes they find the same trends in the wave speed as Hunt and Hamzah (1967). Furthermore, the positive Kelvin-wave solution shows radically different behavior for higher values of s than the pure Kelvin-wave type solution. The u^{*}- and v^{*}-components are of the same order for high s, whereas the v^{*}-component is zero for the classical Kelvin wave. Since Staniforth et al. (1993) look for wave solutions in terms of the tidal frequency (σ_n^*) for a fixed, real-valued wave number k^* , they do not find trapped Poincaré modes and, hence, do not solve the Taylor problem, which typically is a forced problem. Despite the work that has been done on the topic of lateral depth variations in the Taylor problem so far, most studies are limited by the focus on one type of lateral depth profile (linear), the focus on Kelvin waves rather than Poincaré modes or the assumption of a non-rotating basin (no Coriolis force). Consequently, a comprehensive study on the influences of lateral depth variations on the tidal dynamics in the Taylor problem, not subject to the above stated limitations, is still lacking.

2.3 Applications of the Taylor Approach

The tidal dynamics in several existing semi-enclosed basins have been explained by the Taylor approach. Several authors (a.o. Taylor, 1921; Hendershott and Speranza, 1971; Brown, 1987) have reflected upon their results by comparing their findings to tide observations in basins around the world. These comparisons can give an indication of the relevance and relative magnitude of the study results. Therefore, the semi-enclosed basins that have been studied in previous studies are also interesting for studying the impact of lateral depth variations on the tidal dynamics. In Table 1 an overview of these basins is presented as well as their main characteristics in terms of dimensions, latitude and dominating tide.

Basin	Author	Width	Depth	Latitude	Tide
Adriatic Sea	Hendershott and	180 km	35-700 m	43°N	M ₂
	Speranza (1971)				
	Mosetti (1986) –	150 km	40 m	45°N	M ₂
	Northern Pit				
	Winant (2007)	170 km	100-1000 m	?	M_2, K_1
Arctic	Kowalik (1979)	500 km	Deep ocean:	75°N	M ₂
Ocean		(narrow	2000 m		
		part)	Shelf:		
		_	20-200 m		
Bungo	Yanagi (1987)	50 km	80 m	33°N	M ₂
Channel					
Chesapeake	Winant (2007)	?	20 m	?	M ₂
Bay					
Gulf of	Hendershott and	190 km	690-960 m	26°N	M ₂
California	Speranza (1971) ³				
	Winant (2007)	170 km	200-2000 m	?	M ₂ , K ₁
North Sea	Taylor (1921)	463 km	73.5 m	53°N	M ₂
	Rienecker and Teubner	500.5 km	74 m	54.46°N	M ₂
	(1980)				
	Brown (1987) –	200 km	30 m	?	M_2
	Southern Bight				
	Roos and Schuttelaars	150 km	25 m	52°N	M ₂
	(2009) - Southern				
	Bight				

Table 1. Overview of semi-enclosed basins, used in application of the Taylor problem, and their main characteristics.

³ Hendershott and Speranza (1971) studied a standing Kelvin wave.

3. Hydrodynamic Model

In this Chapter the semi-analytical, hydrodynamic model that is used for finding the individual wave solutions and, consequently, solving the Taylor problem is discussed. The model is qualified as semi-analytical, because the equations of motion are partly solved analytically (as far as mathematically possible) and partly numerically. Firstly, the model equations and boundary conditions are examined (section 3.1). Secondly, a scaling procedure is applied in order to obtain a non-dimensional differential problem (section 3.2). Consequently, the method for finding the wave solutions in an infinite channel is discussed (section 3.3). Finally, the results of the model verification and sensitivity analysis are examined (section 3.4).

3.1 Equations of Motion and Boundary Conditions

Taking into account the assumptions of Taylor (1921) (see section 2.1.2), but now allowing for a non-uniform depth (for the moment both in x^* - and y^* -direction), the conservation of momentum and mass reduce to the linear depth-averaged shallow water equations for a homogeneous fluid on the *f*-plane (Pedlosky, 1982). Note that for a uniform depth h^* is independent of x^* and y^* and, hence, can be placed outside the differentials in equation (6).

$$\frac{\partial u^*}{\partial t^*} - f^* v^* = -g^* \frac{\partial z^*}{\partial x^*}, \qquad (Eq. 4)$$

$$\frac{\partial v^*}{\partial t^*} + f^* u^* = -g^* \frac{\partial z^*}{\partial y^*}, \qquad (Eq. 5)$$

$$\frac{\partial z^*}{\partial t^*} + \frac{\partial (h^* u^*)}{\partial x^*} + \frac{\partial (h^* v^*)}{\partial y^*} = 0.$$
 (Eq. 6)

Where:

- u*.v* Depth-averaged velocity components in longitudinal (x^*) and lateral (y^*) direction in m/s t Time in s
- Coriolis parameter in s⁻¹
- Gravitational acceleration in m/s²
- $\int_{\xi^*}^{s}$ Free surface elevation in m
- (Undisturbed) Water depth in m, now allowed to vary with x^* and y^* h

Because no water can be transported through the walls of the semi-enclosed basin, the following boundary conditions apply:

$$v^* = 0$$
 at $y^* = -B^*/2, B^*/2$ and $u^* = 0$ at $x^* = 0.$ (Eq. 7)

Furthermore, the problem is forced by an incoming Kelvin wave from infinity, while allowing reflected Kelvin and Poincaré waves to radiate outward.

3.2 Scaling

Analogously to Roos and Schuttelaars (2009) we introduce the following non-dimensional quantities in order to scale the equations of motion and the boundary conditions:

$$Z = \frac{Z^*}{Z^*}, \quad (u,v) = \frac{(u^*,v^*)}{\hat{U}^*}, \qquad (x,y) = K^*(x^*,y^*), \qquad t = S^*t^*, \quad f = f^*/S^*. \text{ (Eq. 8)}$$

With (arbitrary) maximum elevation amplitude at the coast Z^* , typical velocity scale $\hat{U}^* = Z^* \sqrt{g^*/H^*}$ and reference wave number $K^* = s^* / \sqrt{g^*H^*}$, associated with a classical Kelvin wave (i.e. for uniform depth, without dissipation). H^* denotes the lateral reference depth i.e. the mean depth in lateral direction.

In addition, we scale the lateral depth profile with the reference depth (equal to the average basin depth H^*):

$$h = h^* / H^*$$
. (Eq. 9)

Consequently, the dimensionless equations of motion can be rewritten as:

$$\frac{\partial u}{\partial t} - fv = -\frac{\partial z}{\partial x},$$
 (Eq. 10)

$$\frac{\partial v}{\partial t} + fu = -\frac{\partial z}{\partial y}, \qquad (Eq. 11)$$

$$\frac{\partial z}{\partial t} + \frac{\partial (hu)}{\partial x} + \frac{\partial (hv)}{\partial y} = 0.$$
 (Eq. 12)

With corresponding dimensionless boundary conditions:

$$v = 0$$
 at $y = -B/2, B/2$ and $u = 0$ at $x = 0$. (Eq. 13)

With dimensionless basin width $B = B^*K^*$.

3.2.1 Klein-Gordon Equation

Pedlosky (1982) shows that the equations (10), (11) and (12) can be rewritten into a single equation for the free surface elevation ζ , which extends the Klein-Gordon equation for uniform depth:

$$\frac{\partial}{\partial t} \left[\left(\frac{\partial^2}{\partial t^2} + f^2 \right) \mathbf{z} - \frac{\partial}{\partial x} \left(h \frac{\partial z}{\partial x} \right) - \frac{\partial}{\partial y} \left(h \frac{\partial z}{\partial y} \right) \right] - f \left(\frac{\partial h}{\partial x} \frac{\partial z}{\partial y} - \frac{\partial h}{\partial y} \frac{\partial z}{\partial x} \right) = 0.$$
 (Eq. 14)

Note that the Klein-Gordon equation for uniform depth can easily be obtained from equation (14) by omitting all the terms containing derivatives of h to x and y. When we take only into account depth variations in y-direction (i.e. $\partial h/\partial x = 0$), equation (14) reduces to:

$$\frac{\partial}{\partial t} \left[\left(\frac{\partial^2}{\partial t^2} + f^2 \right) \mathbf{z} - h \frac{\partial^2 \mathbf{z}}{\partial x^2} - \frac{\partial h}{\partial y} \frac{\partial \mathbf{z}}{\partial y} - h \frac{\partial^2 \mathbf{z}}{\partial y^2} \right] + f \frac{\partial h}{\partial y} \frac{\partial \mathbf{z}}{\partial x} = 0.$$
 (Eq. 15)

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3.2.2 Polarization Equations

The scaled polarization equations, which relate the velocity components u and v to ζ , can be derived by manipulating the momentum equations, i.e. equations (10) and (11):

$$\begin{bmatrix} \frac{\partial^2}{\partial t^2} + f^2 \end{bmatrix} u = -\frac{\partial^2 z}{\partial x \partial t} - f \frac{\partial z}{\partial y},$$

$$\begin{bmatrix} \frac{\partial^2}{\partial t^2} + f^2 \end{bmatrix} v = -\frac{\partial^2 z}{\partial y \partial t} + f \frac{\partial z}{\partial x}.$$
 (Eq. 16)

By means of the polarization equations the boundary conditions for v can be rewritten in terms of ζ :

$$\frac{\partial^2 z}{\partial y \partial t} - f \frac{\partial z}{\partial x} = 0 \qquad \text{at} \qquad y = -B/2, B/2.$$
 (Eq. 17)

3.3 Method for deriving Wave Solutions in an Infinite Channel

In order to find the wave solutions in an infinite channel (i.e. without the boundary at x = 0), we look for solutions of the general form:

$$z = \Re \left\{ \hat{z}(y) e^{i(kx-t)} \right\},$$

$$u = \Re \left\{ \hat{u}(y) e^{i(kx-t)} \right\},$$

$$v = \Re \left\{ \hat{v}(y) e^{i(kx-t)} \right\}.$$

(Eq. 18)

Where $\hat{z}(y)$, $\hat{u}(y)$ and $\hat{v}(y)$ denote the (complex) lateral amplitudes; k denotes the (complex) wave number defined as $k = k^*/K^*$; and \Re indicates that only the real part of the argument is physically relevant.

When the trial solutions (equations (18)) are substituted into the extended Klein-Gordon equation (equation (15)) and its boundary conditions (equation (17)), the following equation is obtained:

$$\frac{\partial^2 \hat{z}(y)}{\partial y^2} + B(k, y) \frac{\partial \hat{z}(y)}{\partial y} + C(k, y) \hat{z}(y) = 0,$$
 (Eq. 19)

With:

$$B(k, y) = \frac{1}{h} \frac{\partial h}{\partial y}, \qquad (Eq. 20)$$

$$C(k, y) = \frac{1 - f^2}{h} - k^2 + \frac{fk}{h} \frac{\partial h}{\partial y}.$$
 (Eq. 21)

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And Boundary Conditions:

$$\frac{\partial \hat{z}}{\partial y} + fk\hat{z} = 0$$
 at $y = -B/2, B/2$. (Eq. 22)

This is a second order linear Ordinary Differential Equation (ODE) with variable coefficients, since h is not a constant anymore. This set of equations can generally not be solved analytically. Therefore, a numerical method is applied to find the solutions $\hat{z}(y)$ to this equation.

3.3.1 Fourth Order Runge-Kutta Method

The differential problem in equations (19)-(22) is a Boundary Value Problem (BVP) with one boundary condition on either side of the domain and, in addition, an unknown wave number k. By fixing k to a trial value, we now proceed by treating this as an Initial Value Problem (IVP) for ζ while ignoring the second boundary condition (i.e. at y = B/2) for the moment. This IVP can be solved by means of a 4th order Runge-Kutta (RK4-) method and, in addition, an iterative search routine for k (section 3.3.2), i.e. to find the k-value for which the second boundary condition is satisfied as well. In order to be able to apply the RK4-method the second order ODE must be rewritten as a set of first order ODEs. Let us define:

$$y = \frac{\partial \hat{z}}{\partial y}.$$
 (Eq. 23)

Then the second order ODE in (19) can be written as:

$$\frac{\partial}{\partial y} \begin{bmatrix} \hat{z} \\ y \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -C(y,k) & -B(y,k) \end{bmatrix} \begin{bmatrix} \hat{z} \\ y \end{bmatrix} = \begin{pmatrix} f_1(y, \hat{z}, y) \\ f_2(y, \hat{z}, y) \end{pmatrix}.$$
(Eq. 24)

Or:

v

Vг

v - 1

$$\frac{\partial f}{\partial y} = I f = f(y, f) \quad \text{with} \quad f = \begin{bmatrix} \hat{z} \\ y \end{bmatrix} \quad \text{and} \quad I = \begin{bmatrix} 0 & 1 \\ -C(y, k) & -B(y, k) \end{bmatrix}. \quad (Eq. 25)$$

The procedure of the RK4-method is then as follows for $j = 0...N_v$, where N_v indicates the number of numerical intervals in y-direction:

$$\begin{aligned} \mathbf{\dot{a}}_{1} &= \Delta y * \hat{f} \Big[y(j), \hat{f}(j) \Big], \\ \mathbf{\ddot{y}}_{2} &= \Delta y * \hat{f} \Big[y(j) + \frac{\Delta y}{2}, \hat{f}(j) + \frac{\mathbf{\ddot{a}}_{1}}{2} \Big], \\ \mathbf{\ddot{y}}_{3} &= \Delta y * \hat{f} \Big[y(j) + \frac{\Delta y}{2}, \hat{f}(j) + \frac{\mathbf{\ddot{a}}_{2}}{2} \Big], \\ \mathbf{\ddot{y}}_{4} &= \Delta y * \hat{f} \Big[y(j) + \Delta y, \hat{f}(j) + \mathbf{\ddot{a}}_{3} \Big], \\ y(j+1) &= y(j) + \Delta y, \\ \mathbf{\ddot{f}}(j+1) &= \hat{f}(j) + \frac{1}{6} (\mathbf{\ddot{a}}_{1} + 2\mathbf{\ddot{a}}_{2} + 2\mathbf{\ddot{a}}_{3} + \mathbf{\ddot{a}}_{4}). \end{aligned}$$
(Eq. 26)

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With boundary conditions:

$$\mathbf{Y}_{f(0)} = \begin{bmatrix} \hat{z}_{0} \\ -fk\hat{z}_{0} \end{bmatrix}.$$
(Eq. 27)

The RK4-method gives approximations \mathbf{f}_{j+1} to the solutions $\mathbf{f}(y_{j+1})$ at $y_{j+1} = y_0 + (j+1)^* \Delta y$, where $j = 0, 1, ..., N_y$ and Δy denotes the step size.

By means of the RK4-method as outlined above, the wave solutions can be approximated by "walking" in small steps from one side to the other side of the basin as indicated in Figure 5. Since this is an approximation of the analytical solution, a so-called numerical error is introduced. The numerical error can be minimized by taking a small value for the step size Δy (i.e. a large number of intervals in y-direction N_y).



Figure 5: Indication of the procedure of "walking" from one side of the basin to the other side.

3.3.2 Search Routine for Finding Wave Numbers (k_n)

Although the wave numbers (k_n) corresponding to a given lateral depth profile are the variables that we would like to determine (i.e. the wave solutions), we need an initial guess for k_n in order to be able to run the RK4-method. Therefore, we use the k_n -values that are known from the uniform depth case as trial k_n 's by using the dispersion relationships for Kelvin and Poincaré modes as presented in section 2.1.1. Since these trial k_n 's are just an approximation of the k_n 's we are looking for, an error will occur (on top of the numerical error). Therefore, it is most likely that the boundary condition at y = B/2 is not satisfied.

Let us define the error ε as the absolute deviation from the boundary condition at y = B/2, i.e. the absolute lateral velocity amplitude $|\hat{v}|$ at the other side of the basin. Since $|\hat{v}|$ at y = B/2 must be zero, ε is defined as follows (see also equations (16) and (22)):

$$\boldsymbol{e} = \left| \hat{\boldsymbol{v}} \right| = \left| \frac{-i}{1 - f^2} \left(\frac{\partial \hat{\boldsymbol{z}}}{\partial y} + f k \hat{\boldsymbol{z}} \right) \right| \quad \text{at} \quad \boldsymbol{y} = B / 2 \,. \tag{Eq. 28}$$

The wave numbers (k_n) corresponding to an arbitrary lateral depth profile can be found by an iterative procedure which minimizes the error until below a certain error criterion (order of magnitude 10^{-10}). In Figure 6 an indication is given of the ε -contours around different k_n plotted in the complex wave number plane for (a) uniform depth and (b) a linear lateral depth profile. The

error ε decreases as we come closer to the solution k_n . In uniform depth (Figure 6 (a)) the Kelvin waves are found on the real k-axis whereas the Poincaré waves are found on the imaginary k-axis. Figure 6 (b) indicates that for linear lateral depth variations the Kelvin and Poincaré modes are still found, but they have shifted in the complex wave number plane. The search routine, which is used to find the values of k_n , is described below. The search routine for Kelvin and Poincare modes is similar, only with a different trial k.



Figure 6. k_r,k_i -plot with 200 logarithmically spaced error (ϵ)-contours from 10⁻¹⁰ to 10 for (a) uniform depth and (b) a linear depth profile defined as $h(y) = H^*(1-s^*y/B)$ with a slope (s) of 1.10. The following (scaled) parameter settings have been used B = 1.64, $\zeta_0 = 1, f = 0.83$ and $N_y = 100$.

Search Routine

- 1. Define a search radius Δk (real number) around the trial k_n . The search radius is defined as a small deviation both in the positive and negative real and imaginary direction (as indicated in Figure 7a).
- 2. Determine the errors ε for the trial k_n and the neighboring k's on the search radius. The k for which ε is the smallest, determines the next step in the search process.

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- 3. (1) When the smallest ε corresponds to our trial k_n, again k_n is taken as the starting point for the search procedure, but now with half the search radius (as indicated in Figure 7b).
 (2) When the smallest ε corresponds to one of the neighboring k's on the search radius, the same procedure as described in step 1 and 2 is repeated, but now around the best value for k (as indicated in Figure 7c), retaining the search radius Δk.
- 4. Repeat steps 1-3 until ε is smaller than the error criterion (order of magnitude 10^{-10}). Then we have numerically found the wave solution corresponding to the new lateral depth profile.
- 5. Increase the slope (s) or amplitude (A) of the depth profile and use the k_n that is found for the previous s or A (by applying steps 1-4) as the initial guess for the next increment of s or A, in order to prevent that another wave mode is found than the one that we are looking for. For example, when the given depth profile is radically different from the uniform depth case, the second Poincaré mode corresponding to the given depth profile could be located close to the third initial Poincaré mode. In this case it is possible that we find the second Poincaré mode although we were looking for the third. This possibility is shown in Figure 8a. Figure 8b shows how slope/amplitude increments can assist in preventing this "undesired" result.



Figure 7: Overview of (a) the initial search radius, (b) decreasing search radius and (c) new search radius around new k_n .



Figure 8: (a) The danger of finding the 2nd Poincaré mode when looking for the 3rd. (b) Finding the correct Poincaré mode by applying slope/amplitude increments.

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The whole search routine for finding the wave number k_n is summarized in Figure 9. This routine can be applied for every k_n , i.e. every Kelvin and Poincaré mode. So we are able to find the wave solutions for any lateral depth profile as long as the magnitude (slope/amplitude) of the depth variations is gradually increased.



Figure 9: Overview of search routine for finding k_n

3.3.3 Solutions for u- and v-Amplitudes

When the trial solutions for ζ , u and v (equations (18)) are substituted into the polarization equations (equations (16)) the equations for the complex amplitudes \hat{u} and \hat{v} become:

$$\hat{u}(y) = \frac{1}{1 - f^2} \left(k\hat{z}(y) + f \frac{\partial \hat{z}}{\partial y} \right)$$

$$\hat{v}(y) = -\frac{i}{1 - f^2} \left(\frac{\partial \hat{z}}{\partial y} + fk\hat{z}(y) \right)$$
(Eq. 29)

The structures of $\hat{u}(y)$ and $\hat{v}(y)$ depend completely on $\hat{z}(y)$. This means that once the solutions for $\hat{z}(y)$ and $\partial \hat{z}(y)/\partial y$ are found at each location numerical point (by means of the RK4-method), the solutions for $\hat{u}(y)$ and $\hat{v}(y)$ are also known.

3.4 Model Verification and Sensitivity Analysis

After the implementation of the hydrodynamic model in The MathworksTM MATLAB[®], its performance has been verified for the uniform depth case (Pedlosky, 1982; we should recover equations (1) and (3)) and for a linear depth profile (Staniforth et al., 1993). The results of the model verification are presented in Appendix A and show that our model performs well in both cases. In addition to the model verification, the model sensitivity has been tested. Appendix B shows the results of the sensitivity analysis with respect to the number of numerical steps N_y in the RK4-method, the search radius Δk in the search routine and the slope increments Δs for a linear lateral depth profile and a specific basin width B. The sensitivity analysis shows that for non-dimensional (scaled) quantities and specific B our model is hardly sensitive to changes in Δk and Δs and only slightly to changes in N_y.

4. Results: Wave Solutions in an Infinite Channel of Nonuniform Depth

This Chapter examines the wave solutions in an infinite open-channel that are found by means of the hydrodynamic model for different types of idealized, lateral depth profiles. Firstly, the selected idealized depth profiles are presented (section 4.1). Consequently, the properties of the modified Kelvin and Poincaré waves are discussed in terms of wave numbers and typical length scales (section 4.2) as well as lateral amplitude structures for ζ , u and v (section 4.3).

4.1 Selection of Idealized, Lateral Depth Profiles to be studied

With our model the impact of various idealized lateral depth profiles on the tidal system can be investigated. The motivation for studying several lateral depth profiles is to study the general influences lateral depth variations on the tidal dynamics, i.e. to obtain a better insight in the physics of the tidal system. To select different lateral depth profiles, the following criteria have been formulated:

- Since this is an idealized study, it is sufficient to use idealized depth profiles.
- In order to remain as close to the analytical solutions as possible, we require the depth profiles to be described by continuous and differentiable (smooth) mathematical functions.
- The selection should include at least some of the lateral depth profiles that have been subject to previous studies on the Taylor problem (i.e. especially linear profiles), in order to be able to compare the results.
- One of our hypotheses is that adopting symmetrical and asymmetrical depth profiles leads to significantly different wave solutions and amphidromic systems (as was already indicated by Hendershott and Speranza, 1971). Therefore, the selection should include both symmetrical and asymmetrical lateral depth profiles.

Based on these criteria the following idealized profiles are proposed (in non-dimensional notation), see also Figure 10:

- Linear profile:

$$h(y) = 1 - \frac{sy}{B}, \qquad (Eq. 30)$$

- S-curved profile:

$$h(y) = 1 - \frac{2A}{p} \arctan(by), \qquad (Eq. 31)$$

- Symmetrical sinusoidal profiles:

$$h(y) = 1 + A\cos\left(\frac{2py}{B}\right),\tag{Eq. 32}$$

- Asymmetrical sinusoidal profiles:

$$h(y) = 1 + A\cos\left(\frac{2py}{B} - j\right).$$
 (Eq. 33)

Where:

- h(y) scaled basin depth in lateral direction with $h(y) = h^*(y^*)/H^*$
- s slope (linear profile), can be both positive and negative
- B scaled basin width with $B = K^*B^*$
- A scaled amplitude with $A = A^*/H^*$, can be both positive and negative
- β steepness factor for S-curve (dimensionless)
- φ phase shift relative to pure cosine, in this study we fix φ to the value $\pi/4$

It is important to note that the defined lateral depth profiles are uniform in the longitudinal direction, i.e. there are no depth variations in longitudinal direction. Furthermore, the depth profiles are defined such that the mean depth of each profile is equal to uniform depth (H).

In order to study the effects of the lateral depth profiles on the wave properties in the next sections, basin characteristics typical for the Southern Bight of the North Sea have been applied (see Table 2). The basin width B^{*} and mean basin depth H^{*}_{mean} are roughly average values for this part of the North Sea (see Brown, 1987 and Roos and Schuttelaars, 2009). The width of the basin B^{*} is smaller than the critical width B^{*}_{crit} for the existence of free Poincaré modes. The maximum elevation amplitude at the coast \hat{z}_0^* , which can be chosen arbitrarily as long as \hat{z}_0^* is small compared to H^{*}_{mean} (i.e. the gravitational Froude number Fr is small), is based on observations used in Sinha and Pingree (1997). The latitude θ is assumed to be constant over the basin and taken at Den Helder, which is roughly the middle between the Dover Strait and the second elevation amphidromic point of the North Sea. The tidal angular frequency σ^* corresponds to the M₂-tide, which is the dominating tidal component in the Southern Bight of the North Sea. A graphical overview of the depth profiles with the given parameter settings is presented in Figure 10. Note that the depth h^{*}(y^{*}) is plotted in the negative z^{*}-direction, i.e. z^{*} = -h^{*}(y^{*}).

Parameter	Symbol	Value	Dimension
Basin width	B^*	200	km
Mean basin depth	H^{*}	30	m
Maximum elevation amplitude at the coast	$\hat{z_0}^*$	1.5	m
Latitude	θ	53	°N
Angular frequency of the M ₂ -tide	σ^*	$1.41*10^{-4}$	rad s ⁻¹
Basin width (dimensionless)	В	1.64	-
Coriolis parameter (dimensionless)	f	0.83	-
Gravitational Froude number (dimensionless)	$\operatorname{Fr}\left(=\hat{\boldsymbol{z}}_{0}^{*}/\mathrm{H}^{*}\right)$	0.05	-

Table 2. Overview of parameter values, typical for the Southern Bight of the North Sea



Figure 10: Overview of the selected idealized profiles for different slopes/amplitudes with $H^* = 30$ m and $B^* = 200$ km

4.2 Typical Length Scales of Modified Wave Modes

In this section the properties of the modified Kelvin and Poincaré Modes are discussed in terms of wave numbers and typical length scales. Analogously to Roos and Schuttelaars (2009) the Kelvin wavelength L_0^* is used as a typical length scale for the Kelvin modes and the e-folding decay length L_n^* for the evanescent Poincaré modes. These typical length scales are defined as follows:

$$L_{0}^{*} = \frac{2p}{K^{*}|k_{0,re}|},$$
(Eq. 34)
$$L_{n}^{*} = \frac{1}{K^{*}|k_{n,im}|}.$$
(Eq. 35)

The analysis of Staniforth et al. (1993), used to verify our semi-analytical model (see Appendix A), focused on linear depth profiles with positive s (i.e. deep at -B/2 and shallow at B/2). However, by fixing k to a real value, Staniforth et al. (1993) only find free wave modes, whereas in our case the trapped modes are just as important in solving the Taylor problem for non-uniform depth. Therefore, it is still useful to analyze the wave solutions corresponding to the linear depth profile by means of our hydrodynamic model as well. Although we focus here on linear depth profiles, the trends in the wave numbers and typical length scales of the modified Kelvin and Poincaré modes are also discussed for the other idealized profiles. For figures and tables showing these trends is referred to Appendix C. Without loss of generality, we restrict our attention to the positive Poincaré modes are relevant to solve the Taylor problem for non-uniform depth in Chapter 5. Note that the positive Kelvin wave (i.e. $k_{re} \geq 0$) represents the reflected Kelvin wave in the Taylor problem.

The numerical parameters that are used in the model are presented in Table 3.

Parameter	Symbol	Value
Number of steps in RK4-method	Ny	100
Initial search radius around trial wave number	Δk	0.05
Error criterion for solving RK4-method	€ _{crit}	10-10

Table 3. Numerical parameters used to find wave numbers and typical length scales

4.2.1 Kelvin Waves

Figure 11 shows the wave numbers k_n of the positive (reflected) Kelvin wave and the first four positive Poincaré waves plotted in the complex wave number plane for linear depth profiles with (a) positive s and (b) negative s. The trends for the Kelvin waves are exactly the opposite for positive and negative s. Therefore, only the results for positive s (i.e. deep at -B/2 and shallow at B/2) are discussed here. The values of L_0^* corresponding to positive s are shown in Table 4. It is observed that $|k_0|$ of the negative (incoming) Kelvin wave is increasing with increasing s, which implies a smaller wavelength L_0^* (see Table 4). This can be explained by the decreasing water depth h(y) at the coast along which the incoming Kelvin wave propagates with increasing s. Because waves travel slower in shallow water than in deep water, the wave speed ($c = \sigma/k$) decreases and, hence, $|k_{re}|$ increases. The positive (reflected) Kelvin wave shows the opposite

behavior: $|k_{re}|$ is decreasing with increasing s (see Figure 11(a) and Table 4), which implies a larger L_0^* , because of increasing h(y) at the side of the channel along which the reflected Kelvin wave propagates.



Figure 11. k_r,k_i-plot for the positive Kelvin mode and the first four positive Poincaré modes for linear depth profiles with (a) positive and (b) negative slopes.

	Negative (incoming) Kelvin		Negative (incoming) Kelvin Positive (reflected) Kelvin	
s [-]	k_0^{inc} [-]	L_0^* [km]	k ₀ [-]	L_0^* [km]
0	-1.000	767	1.000	767
0.5	-1.058	725	0.955	803
1.1	-1.159	662	0.912	841
1.5	-1.266	606	0.889	863
1.95	-1.503	510	0.866	886

Table 4. Wave numbers and wave lengths of incoming (k_0^{inc}) and (k_0) reflected Kelvin waves for linear depth profile with different positive slopes (s)

The S-curved depth profile shows more or less the same trends as the linear profile, as could be expected based on its similar shape, only the effects are generally much stronger. The symmetric sinusoidal depth profiles with increasing positive A^* (i.e. deep in the centre, shallow at the sides) show an equal increase in $|k_0|$ for the incoming and reflected Kelvin waves. The increase in $|k_0|$ for both Kelvin waves (and corresponding decrease in L_0^*) could be linked to decreasing h(y) at the sides of the channel. This indicates that the water depth along the coasts is more important for

determining L_0^* than the depth farther away from the coasts. The equal wave length for both Kelvin waves is explained by the symmetric shape of the depth profile.

In case of the opposite profile, i.e. symmetric sinusoidal depth profiles with negative A^{*} (shallow in the centre, deep at the sides), the Kelvin waves do not show exactly the opposite behavior. The $|k_0|$ -values of both Kelvin waves decrease until A^{*} \approx -27 m, which is expected due to increasing h(y) at the sides of the channel. However, for A^{*} \leq -27 m $|k_0|$ is increasing, which is hard to link to increasing h(y). Possibly, the presence of the very shallow area in the centre of the channel might cause wave trapping or the existence of a double Kelvin mode of the type described by Longuet-Higgins (1968). Longuet-Higgins (1968) argued that "the existence of a sloping bottom, particularly a discontinuity in depth, will tend to restrict the horizontal component of the particle motion on the deeper side". However, it is not clear whether this could explain the increase of $|k_0|$. These kind of effects are not found for symmetric profiles with positive A^{*}, i.e. increasing h(y) in the centre of the channel. This apparent contradiction might be explained by the fact that Kelvin waves propagate along coasts, so that h(y) is especially important near the coasts. In case of positive A^{*}, the water depth is decreasing near the coast and in shallow water no trapping effects occur (Longuet-Higgins, 1968).

For the asymmetric sinusoidal depth profiles $|k_0|$ is increasing (i.e. decreasing L_0^*) for the Kelvin wave that is travelling in the relatively shallower side of the channel and decreasing (i.e. increasing L_0^*) for the Kelvin wave that is traveling in the relatively deeper side of the channel. Both Kelvin waves do not show exactly the opposite behavior for positive and negative A^{*}, since both profiles are not each others exact opposite.

4.2.2 Poincaré Waves

As for the Kelvin waves, the trends for the Poincaré waves are exactly the opposite for linear depth profiles with positive and negative s (see Figure 11(a) and (b)). Again we discuss only the results for positive s here. The positive Poincaré wave numbers $k_{n>0}$ obtain, besides a slightly larger imaginary part $k_{n,im}$, also a considerable (positive) real part $k_{n,re}$ compared to uniform depth (see Figure 11(a) and Table 5). In terms of their absolute values both $k_{n,re}$ and $k_{n,im}$ increase with increasing s, which implies that the Poincaré modes obtain a propagative character, but their e-folding length-scale L_n^* is slightly decreasing. Therefore, the Poincaré waves influence the wave field over a shorter distance than in uniform depth.

For the S-curved profile roughly the same trends are observed as for the linear profile, only the effects are generally much stronger. In contrast to the linear and S-curved depth profiles, the Poincaré modes hardly obtain a real part $k_{n,re}$ for symmetric sinusoidal depth profiles with positive A^{*}. The imaginary part $k_{n,im}$ increases considerably in magnitude (i.e. decreasing L_n^*) with increasing A^{*}. Similarly, for symmetric profiles with negative A^{*}, $k_{n>0}$ keep mainly an imaginary character. An interesting feature is that for very large negative A^{*} the second and third Poincaré modes have nearly the same $k_{n,im}$ and nearly equal $|k_{re}|$, but with opposite sign (see Figure 36 in Appendix C).
	s = 0		s = 0.5		s = -0.5	
n	k _n [-]	L_n^* [km]	k _n [-]	L_n^* [km]	k _n [-]	L_n^* [km]
1	0.0+1.8i	66.6	-0.1+1.8i	66.4	0.1+1.8i	66.4
2	0.0+3.8i	32.2	-0.1+3.8i	32.1	0.1+3.8i	32.1
3	0.0+5.7i	21.3	-0.1+5.7i	21.3	0.1+5.7i	21.3
4	0.0+7.6i	16.0	-0.1+7.7i	16.0	0.1+7.7i	16.0
5	0.0+9.6i	12.8	-0.1+9.6i	12.8	0.1+9.6i	12.8
6	0.0+11.5i	10.6	-0.1+11.5i	10.6	0.1+11.5i	10.6
7	0.0+13.4i	9.1	-0.1+13.4i	9.1	0.1+13.4i	9.1
8	0.0+15.3i	8.0	-0.1+15.3i	8.0	0.1+15.3i	8.0
9	0.0+17.3i	7.1	-0.1+17.3i	7.1	0.1+17.3i	7.1
10	0.0+19.2i	6.4	-0.1+19.2i	6.4	0.1+19.2i	6.4

Table 5. Wave numbers (k_n) and e-folding decay lengths (L_n^*) for the first ten Poincaré modes for linear depth profile with different slopes (s)

The Poincaré modes for asymmetric sinusoidal profiles with positive A^* keep mainly an evanescent character, except for the first mode which obtains a propagative character. The value of $k_{n,im}$ increases with A^* (i.e. L_n^* decreases) for the first Poincaré mode as well, but hardly changes for the other Poincaré modes. For asymmetric profiles with negative A^* the Poincaré modes show somewhat different behavior. The wave number of the first Poincaré mode hardly has a real part and $k_{1,im}$ is strongly decreasing with $|A^*|$ (i.e. L_n^* increases). For the second and third Poincaré modes $k_{n,im}$ slightly increases with increasing $|A^*|$, and $k_{n,re}$ increases (in terms of its absolute values) especially for large $|A^*|$.

4.2.3 Wave Properties compared to Effects of Energy Dissipation

Previous studies on the Taylor problem focusing on energy dissipation (Hendershott and Speranza, 1971; Rienecker and Teubner, 1980) seldom discussed the properties of the wave solutions (i.e. Kelvin and Poincaré modes). Roos and Schuttelaars (2009), studying bottom friction and horizontally viscous effects, are an exception to this. They find that bottom friction causes the wave numbers of the Kelvin and the Poincaré modes to shift slightly in the complex plane. We also observe this shift in the complex plane for the Poincaré waves, but not for the Kelvin waves. In contrast to bottom friction effects we find that most lateral depth profiles cause the wave numbers of the Kelvin and Poincaré modes to shift considerably in the real plane, i.e. the wave lengths of the Kelvin waves change and the Poincaré modes obtain a propagative character. When also horizontally viscous effects are added, Roos and Schuttelaars (2009) observe that the wave numbers of the Kelvin and Poincaré modes shift further in the complex plane and, additionally, that a new type of mode emerges, which they call the viscous modes. Such a type of mode is not found in case of lateral depth variations. Roos and Schuttelaars (2009) find that viscous effects reduce L_n^* of the Poincaré modes, what is also found for most lateral depth profiles that we studied.

4.3 Lateral Amplitude Structures of Modified Wave Modes

In this section the properties of the modified Kelvin and Poincaré Modes for the selected lateral depth profiles are discussed in terms of lateral $\hat{z} -, \hat{u} -$ and \hat{v} -structures. These structures have already been studied by Staniforth et al. (1993) for linear lateral depth profiles. However, Staniforth et al (1993) only find real lateral structures for \hat{z} , \hat{u} and \hat{v} , whereas we find imaginary structures for \hat{v} as well. In order to be able to relate our results to Staniforth et al. (1993) the focus is again on the linear depth profiles are also discussed here, but for figures is referred to Appendix D. We restrict our attention to the incoming (negative) and reflected (positive) Kelvin waves and the first positive Poincaré mode. The reason for only discussing the first Poincaré mode is that this mode has the largest e-folding length scale and, hence, the largest impact on the amphidromic system (discussed in Chapter 5). In order to be able to evaluate the wave modes in a fair manner, the amplitude structures are corrected for differences in wave energy, so that all wave modes represent the same amount of energy (see Appendix E).

4.3.1 Kelvin Waves

The lateral structures for both Kelvin waves and the first positive Poincaré mode for linear depth profiles with positive s are presented in Figure 12, Figure 13 and Figure 14 respectively. In accordance with Staniforth et al. (1993) the \hat{z} - and \hat{u} -fields of the positive and negative Kelvin waves remain in phase and decay exponentially away from the boundary for small s (see Figure 12 and Figure 13). For bigger s this exponential character is less profoundly visible, which was also found by Staniforth et al. (1993). In contrast to Staniforth et al. (1993) the maximum \hat{z} and \hat{u} of the positive Kelvin wave for large s do not occur near the boundary at y = B/2, but remain at the boundary at y = -B/2. Nevertheless, \hat{z} and \hat{u} of both Kelvin waves become larger in terms of their absolute values compared to uniform depth near the boundary at y = B/2 (i.e. the shallowest part of the channel). The local increase of \hat{u} on the shallow side of the channel might be linked to continuity, i.e. the mass flux must be maintained all over the channel. The consequent increase of \hat{z} is hard to explain. In case of uniform depth the increase of \hat{z} could be linked to geostrophy, i.e. the balance between the pressure gradient $\partial \zeta / \partial y$ acting perpendicular to the direction of wave propagation and the Coriolis force also acting perpendicular to the wave propagation but in opposite direction. However, with depth variations the Kelvin waves obtain a v-component (which is discussed below), so that the relation between \hat{z} and $\hat{\mu}$ is not purely dictated by geostrophy anymore. With increasing s, \hat{z} of the positive Kelvin wave (propagating in relatively deep water) decays less rapidly away from its boundary, whereas \hat{z} of the negative Kelvin (propagating in relatively shallow water) decays more rapidly away from its boundary. This differences in decay speed can be linked to a increasing Rossby radius ($R^* = \sqrt{(g^*H^*)/f^*}$) in deeper water and a decreasing R^{*} in shallower water.

In accordance with Staniforth et al. (1993) we also observe that both Kelvin waves obtain a \hat{v} -structure, which becomes more profound for increasing s (see Figure 14). The existence of the cross-channel current component is probably the result of slope effects that have been introduced in the channel by allowing lateral depth variations.



Figure 12. ζ-amplitudes for the positive and negative Kelvin waves and the first positive Poincaré mode corresponding to a linear depth profile with positive slopes.



Figure 13. u-amplitudes for the positive and negative Kelvin waves and the first positive Poincaré mode corresponding to a linear depth profile with positive slopes.



Figure 14. v-amplitudes for the positive and negative Kelvin waves and the first positive Poincaré mode corresponding to a linear depth profile with positive slopes.

For the other lateral depth profiles we note that the \hat{z} -structures of both Kelvin waves generally maintain their exponential character and show an increase when it is relatively shallow near the boundary along which they propagate and a decrease when it is relatively deep. For the symmetric profiles with a large negative A^{*} the \hat{z} -structures of the Kelvin waves do not maintain their exponential character, but show a considerable peak in the centre of the basin where it is very shallow. This feature might be a result of wave trapping or the existence of a double Kelvin mode (see also section 4.2.1).

The \hat{u} -structures of both Kelvin waves remain in phase with the \hat{z} -structures and generally show big deviations from the exponential structure in uniform depth, especially at the locations in the channel where it is relatively shallow. The deviation increases for increasing $|A^*|$. In the deeper parts of the channel the deviations in the \hat{u} -structures in the uniform depth case are generally relatively small compared to the deviations in the shallower parts of the channel. In general, $|\hat{u}|$ increases in the shallower parts of the channel and decreases in the deeper parts, like \hat{z} .

For all lateral depth profiles the Kelvin waves obtain a \hat{v} -structure, as opposed to the uniform depth case. The \hat{v} -structures become more profound for increasing $|A^*|$. For all depth profiles both Kelvin waves obtain their maximum \hat{v} where the water depth is the shallowest.

4.3.2 First Positive Poincaré Wave

Since we fixed σ and allowed the wave number to be complex, we obtain two different lateral wave structures for the Poincaré modes: real and imaginary amplitudes. These two different wave structures indicate a phase difference, i.e. the structure is different when one looks at the channel at different moments. The $\hat{z} -, \hat{u} -$ and \hat{v} -structures for linear depth profiles with positive s are presented in Figure 12, Figure 13 and Figure 14 respectively. Looking at the \hat{z} -structures for small s (see Figure 12), the real \hat{z} -structure of the first positive Poincaré mode is sinusoidal with large free surface elevation at the boundary at y = -B/2 and low free surface elevation at the opposite boundary, similar to Staniforth et al. (1993). The imaginary structure is also sinusoidal, but shows zero free surface elevation at the boundaries and low free surface elevation in the centre. With increasing s both \hat{z} -structures maintain their shapes, but the real structure shifts downwards and the maximum of the imaginary structure shifts towards the boundary at y = B/2 (i.e. the shallower side of the channel).

For linear depth profiles with small positive s the \hat{u} -structures of the first positive Poincaré mode have similar shapes to the \hat{z} -structures (see Figure 13). With increasing s the imaginary structure maintains its shape and shifts downward, as was the case for the real \hat{z} -structure as well. However, the real \hat{u} -structure becomes distorted for large s.

The \hat{v} -structures are two sinusoidal shaped structures with zero velocities at the coasts, as required by the boundary conditions (see Figure 14). The real structure is increasing in magnitude with increasing s, the imaginary one is moving with its maximum towards the shallower part of the channel, i.e. towards the boundary at y = B/2. Note that the magnitude of the real \hat{v} -structure is comparable to the positive Kelvin mode, whereas that of the imaginary one is comparable to the negative Kelvin mode.

The $\hat{z} \cdot \hat{u}$ - and \hat{v} -structures of the first positive Poincaré mode for the S-curved profiles show similar behavior to the linear profiles, but the trends are much stronger. However, for the sinusoidal depth profiles, both symmetric and asymmetric, the structures show radically different behavior. For the symmetric sinusoidal depth profiles with increasing positive A^{*} a phase shift seems to occur for the \hat{z} -, \hat{u} -, and \hat{v} -structures. Therefore, for large positive A^{*} the imaginary structures look more like the real structures for small A^{*} and the real structures more like the initial imaginary structures. For symmetric sinusoidal profiles with negative A^{*} such a phase shift is not observed. In this case the \hat{z} -, \hat{u} - and \hat{v} -structures, except for the real \hat{v} -structure, remain close to the uniform-depth solutions for small $|A^*|$, but show a big peak around the centre of the channel (i.e. where it is shallow) for large $|A^*|$. Also the structures for the asymmetric sinusoidal profiles, both with positive and negative A^{*}, show a big peak where it is very shallow.

5. Results: Solutions to the Taylor Problem with Lateral Depth Variations

This Chapter discusses the solutions to the Taylor problem (i.e. in a semi-enclosed basin) allowing for lateral depth variations. Section 5.1 examines the collocation method that is used to satisfy the no-normal flow boundary condition at the basin's closed end. In section 5.2 the resulting elevation and current amphidromic systems are presented.

5.1 Collocation Method

The solutions to the Taylor problem with lateral depth variations, i.e. the ζ -, u- and v-fields in a semi-enclosed basin, are a superposition of the individual Kelvin and Poincaré modes in an infinite channel. Due to the geometry of a semi-enclosed basin an additional boundary is introduced at x = 0 with corresponding no-normal flow boundary condition u = 0. The value of u at the boundary can be approximated by taking a truncated sum of the incoming and reflected Kelvin waves and M reflected Poincaré modes. Only the reflected Poincaré modes are relevant to solve the Taylor problem, since these modes are trapped in the positive x-direction at the basin's closed end. By means of a collocation method the boundary condition at x = 0 can be satisfied at a finite number of M+1 collocation points (one for the reflected Kelvin wave denoted by n=0 and M for the M reflected Poincaré modes). This results in a set of M+1 equations and M+1 unknown complex coefficients (C_n), from which the complex coefficients can be solved (see equation (36)).

$$\hat{u}_{0}^{inc}(y) + \sum_{n=0}^{M} C_{n} \hat{u}_{n}(y) = 0$$
 at $x = 0.$ (Eq. 36)

A schematic overview of the collocation method is given in Figure 15. Once the values of C_n are determined, the corresponding ζ -, u- and v-fields can be obtained as follows:

$$z = \hat{z}_{0}^{inc}(y) \exp(i[k_{0}^{inc}x-t]) + \sum_{n=0}^{M} C_{n}\hat{z}_{n}(y) \exp(i[k_{n}x-t]),$$

$$u = \hat{u}_{0}^{inc}(y) \exp(i[k_{0}^{inc}x-t]) + \sum_{n=0}^{M} C_{n}\hat{u}_{n}(y) \exp(i[k_{n}x-t]),$$

$$v = \hat{v}_{0}^{inc}(y) \exp(i[k_{0}^{inc}x-t]) + \sum_{n=0}^{M} C_{n}\hat{v}_{n}(y) \exp(i[k_{n}x-t]).$$
(Eq. 37)



Figure 15. Overview of the basin, in which F represents the forcing by the incoming Kelvin wave and N_1 the finite set of reflected Kelvin and Poincaré waves. The boundary condition $u^*=0$ at $x^*=0$ is satisfied at a finite number (M+1) of collocation points.

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5.2 Amphidromic Systems for Idealized Depth Profiles

Table 6 presents the numerical parameters that are used to generate the amphidromic systems for the different lateral depth profiles. A limited number of wave modes (i.e. collocation points) is sufficient to approximate the amphidromic systems, since the amplitudes of the highest Poincaré modes are so small that they hardly contribute to the overall solution. However, for the determination of the displacements of the amphidromic points in x^* - and y^* -direction, more numerical steps and collocation points have been taken into account (i.e. the grid is finer) in order to be able to observe the trends more accurately (presented between brackets in Table 6).

Parameter	Symbol	Value
Number of steps in y-direction for RK4-method	Ny	100 (400)
Number of steps per wave length L [*] in x-direction	N _x	100 (200)
Number of collocation points	М	11 (41)
Initial search radius around trial wave number	Δk	0.05
Error criterion for solving RK4-method	٤ _{crit}	10-10

Table 6. Numerical parameters used for generating amphidromic systems

5.2.1 Elevation Amphidromic Systems

In Figure 16 (a) to (f) the elevation amphidromic systems are presented for the different lateral depth profiles (for a specific slope or amplitude). The S-curved depth profile is omitted from the analysis, because it shows more or less the same trends as the linear depth profile, though a bit stronger. In Table 7, Table 8 and Table 9 the shifts of the elevation amphidromic points (EAPs) in x^* - and y^* -direction are presented for the linear and symmetric and asymmetric sinusoidal profiles respectively. Since we used a numerical approach, the shifts of the EAPs cannot be determined very accurately. However, in order to describe the general direction and order of magnitude of the displacements the accuracy is sufficient.

Figure 16 (a) to (f) and Table 7, Table 8 and Table 9 indicate that, generally, for the symmetric profiles the EAPs remain at the centre-line of the basin (i.e. at $y^* = 0$) as in the uniform depth case, but for the asymmetric profiles the EAPs are shifted towards the deeper side of the basin with equal Δy^* for all EAPs. Extreme amplitudes A^* are an exception to this general rule. For example, for a symmetric profile with $A^* = -29$ m the second, third and fourth amphidromic point show a major shift in y^* -direction (see Table 8 and Figure 17). Figure 17 shows that the second, third and fourth EAP seem to consist of more than one point. This feature can be explained by the distortion in the \hat{z} -structures of both Kelvin waves in the centre of the channel (see section 4.3 and Appendix D). Due to this distortion the Kelvin waves can interfere at multiple locations in lateral direction, leading to multiple EAPs. The same feature is found for an asymmetric profile with $A^* = -29$ m. This behavior may be related to the very shallow water depth in or near the centre of the basin and might be the result of wave trapping in the shallow region or the existence of double Kelvin modes, as described by Longuet-Higgins (1968).

Furthermore, we can conclude from Figure 16 (a) to (f) and Table 7, Table 8 and Table 9 that the EAPs also show a displacement in x^* -direction as a result of lateral depth variations. The displacement in x^* -direction can be related to changes in the length scales (L_0^*) of the incoming and reflected Kelvin waves. L_0^* of both Kelvin waves decreases for the symmetrical sinusoidal depth profile with positive A^* . This implies that the intersection (interference) points of both Kelvin waves shift in the negative x^* -direction. In case of symmetrical sinusoidal profiles with negative A^* this is exactly opposite. For linear depth profiles with positive s, the decrease in L_0^* of the reflected Kelvin wave is much stronger than the increase in L_0^* of the incoming Kelvin.

Therefore, the EAPs move into the negative x^* -direction. For the asymmetrical sinusoidal depth profiles it is a bit harder to observe general trends in the shifts of the EAPs in x^* -direction. For positive A^* the EAPs roughly shift in the negative x^* -direction, whereas for negative A^* the EAPs roughly shift in the other direction. Again these trends can be related to the length scales of the Kelvin waves.

The order of magnitude of the displacements of the EAPs is generally increasing with increasing s or A^* and much larger in x^* -direction than in y^* -direction. For most lateral depth profiles that have been investigated the ratio $\Delta y^*/B^*$ is of the order of magnitude 0-0.075. The displacements in x^* -direction are negligible for small depth variations, but become more significant for increasing s or A^* and for EAPs located farther away from the closed end of the basin. For the first two EAPs the ratio $\Delta x^*/B^*$ is of the order of magnitude 0-0.5.

In previous studies on the Taylor problem focusing on energy dissipation (Hendershott and Speranza, 1971; Rienecker and Teubner, 1980; Roos and Schuttelaars, 2009) the EAPs move towards the wall along which the reflected Kelvin wave travels with increasing energy dissipation. In case of bottom friction and horizontal viscous effects the EAPs are on a line making a small angle to the longitudinal direction (Rienecker and Teubner, 1980 and Roos and Schuttelaars, 2009), whereas the EAPs are on a line with constant distance y^* to the boundary for an energy-absorbing barrier (Hendershott and Speranza, 1971). The effects of lateral depth variations are comparable to the latter, although the direction of the displacement in y^* -direction is not dependent on the boundary along which the reflected Kelvin wave travels but on the direction of the asymmetry in the lateral depth profile $h^*(y^*)$. Roos and Schuttelaars (2009) also reflect on the order of magnitude of the displacements in y^* -direction and find ratios $\Delta y^*/B^*$ of the order of magnitude 0-1 for ratios of the friction coefficient and horizontal eddy viscosity and their reference values ranging from 0 to 2. Therefore, generally energy dissipation has a larger impact on the displacements of the EAPs in y^* -direction than lateral depth variations. Nevertheless, for large s* or A* the effects of lateral depth variations can be considerable.

The studies on energy dissipation in the Taylor problem hardly reflect on the shifts of the EAPs in x^* -direction. Roos and Schuttelaars (2009) find that L_0^* is decreasing equally for both Kelvin waves (because of the uniform depth assumption) with increasing bottom friction or horizontal eddy viscosity. This indicates that the EAPs show a displacement in the negative x^* -direction, which we also observe for most lateral depth profiles. However, the order of magnitude of the displacement in x^* -direction (relative to the width of the basin B^*) is for most lateral depth profiles larger than for bottom friction and horizontally viscous effects (for realistic values of the friction coefficient and the horizontal eddy viscosity).



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Figure 16. Elevation amphidromic systems for (a) uniform depth, (b) linear depth profile with s = 1.50, symmetrical sinusoidal profiles with (c) $A^*=25$ m (d) and $A^*=-25$ m, and asymmetrical sinusoidal profiles with (e) $A^*=25$ and (f) $A^*=-25$. The solid lines indicate co-amplitude lines with intervals of 20 cm and the dashed lines indicate co-phase lines dividing the tidal period into 12 intervals. Note that for each EAP one of the co-phase lines is presented as a set of multiple lines. This is because of a numerical limitation: it is not recognized that -180° and 180° are the same phases.

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S	Δx_1^* [km]	Δx_2 [km]	Δx_3 [km]	Δx_4 [km]	∆y₁ [*] [km]	Δy_2 [km]	Δy_3 [km]	Δy_4 [km]
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.5	0.0	0.0	-3.8	-7.7	-5.0	-5.0	-5.0	-5.0
1.1	-3.8	-15.3	-30.7	-42.2	-10.0	-10.0	-10.0	-10.0
1.5	-11.5	-38.4	-65.2	-92.0	-10.0	-10.0	-10.0	-10.0
1.95	-26.8	-84.4	-141.9	-203.3	-15.0	-15.0	-15.0	-15.0
-0.5	0.0	0.0	-3.8	-7.7	5.0	5.0	5.0	5.0
-1.1	-3.8	-15.3	-30.7	-42.2	10.0	10.0	10.0	10.0
-1.5	-11.5	-38.4	-65.2	-92.0	10.0	10.0	10.0	10.0
-1.95	-26.8	-84.4	-141.9	-203.3	15.0	15.0	15.0	15.0

Table 7. Displacement of the first four EAPs in x^* - and y^* -direction for linear lateral depth profiles with different slopes.

A [*] [m]	Δx_1^{*} [km]	Δx_2 [km]	Δx_3 [km]	Δx_4 [km]	Δy ₁ [km]	Δy_2 [km]	Δy_3 [km]	Δy_4 [km]
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	-3.8	-7.7	-15.3	0.0	0.0	0.0	0.0
15	-3.8	-19.2	-34.5	-53.7	0.0	0.0	0.0	0.0
25	-11.5	-38.4	-72.9	-107.4	0.0	0.0	0.0	0.0
29	-11.5	-49.9	-92.0	-134.2	0.0	0.0	0.0	0.0
-5	0.0	7.7	11.5	15.3	0.0	0.0	0.0	0.0
-15	0.0	15.3	26.8	38.4	0.0	0.0	0.0	0.0
-25	0.0	15.3	34.5	53.7	0.0	0.0	0.0	0.0
-29	-7.7	3.8	23.0	42.2	-10.0	-45.0	-40.0	-40.0

Table 8. Displacement of the first four EAPs in x^* - and y^* -direction for symmetric sinusoidal lateral depth profiles with different amplitudes.

A [m]	Δx_1 [km]	Δx_2 [km]	Δx_3 [km]	Δx_4 [km]	Δy_1 [km]	Δy_2 [km]	Δy_3 [km]	Δy_4 [km]
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	-3.8	-7.7	-11.5	0.0	0.0	0.0	0.0
15	-3.8	-19.2	-38.4	-53.7	5.0	5.0	5.0	5.0
25	-19.2	-61.4	-107.4	-153.4	10.0	10.0	10.0	10.0
29	-38.4	-122.7	-210.9	-295.3	10.0	10.0	10.0	10.0
-5	0.0	3.8	7.7	11.5	0.0	0.0	0.0	0.0
-15	0.0	3.8	11.5	19.2	-5.0	-5.0	-5.0	-5.0
-25	-7.7	-3.8	3.8	11.5	-5.0	-5.0	-5.0	-5.0
-29	-19.2	-23.0	-19.2	-19.2	10.0	-15.0	-10.0	-15.0

Table 9. Displacement of the first four EAPs in x^* - and y^* -direction for asymmetric sinusoidal lateral depth profiles with different amplitudes.



Figure 17. Elevation amphidromic system for a symmetrical sinusoidal profile with A^* =-29 m, displaying that EAPs may consist of more than one point. The solid lines indicate co-amplitude lines with intervals of 20 cm and the dashed lines indicate co-phase lines dividing the tidal period into 12 intervals. Note that for each EAP one of the co-phase lines is presented as a set of multiple lines. This is because of a numerical limitation: it is not recognized that -180° and 180° are the same phases.

5.2.2 Current Amphidromic Systems

In Figure 18 (a) to (f) the current amphidromic systems are presented for the different lateral depth profiles in terms of tidal ellipses, in which the black squares indicate cyclonic rotation of the ellipses (i.e. counter-clockwise in the Northern Hemisphere) and the open circles anticyclonic rotation (i.e. clockwise in the Northern Hemisphere). Though elevation amphidromic systems have been well studied in literature, current amphidromic systems are not so well understood (Xia et al., 1995). Current amphidromic points (CAPs) are the points where both velocity components are zero, i.e. the ellipse reduces to a point. We can distinguish between two groups of CAPs: (1) middle CAPs, existing in pairs located between two EAPs and (2) end CAPs, existing singularly located between the closed end of the basin and the first EAP (Xia et al., 1995).

In case of uniform depth (Figure 18 (a)) tidal ellipses only occur in the region close to the closed end of the basin, due to the influence of the cross-channel velocity component v^* of the Poincaré waves in this region. With increasing distance from the closed end, the Poincaré waves decay and their influence eventually reduces to zero. Therefore, at distances farther away than the e-folding length of the first Poincaré mode, the tidal ellipses reduce to lines (currents parallel to the shore), since Kelvin waves do not have a cross-channel velocity component (v^*). Furthermore, in the region close to the closed end, the direction of rotation of the ellipses is the same as the rotation of the earth (i.e. cyclonically), whereas at distances further away from the closed end (around the first middle CAP) the direction of rotation reverses.

Due to lateral depth variations the current amphidromic systems are altered significantly. Tidal ellipses do not only occur near the closed end, but also farther away from this boundary, because the modified Kelvin waves obtain a cross-channel velocity component v^* . Generally, v^* is the largest in the shallowest part of the basin (Figure 18 (b) to (f)), as was already indicated by the \hat{v} -structures of the Kelvin waves in section 4.3.1. In addition, we find that the direction of rotation changes. For the linear depth profile (Figure 18 (b)) and the symmetric and asymmetric sinusoidal depth profiles with positive A* (Figure 18 (c) and (e)) the direction in which the particles move round the ellipses is for most ellipses cyclonical (i.e. the same as the rotation of the Earth). However, most of the tidal ellipses do reverse direction of rotation at some distance from the closed end for the symmetric and asymmetric sinusoidal profiles with negative A* (Figure 18 (d) and (f)). This opposite behavior might be explained by the very shallow water depth in or close to the centre of the basin for the sinusoidal depth profiles with negative A*, which may cause wave trapping (as was also indicated in section 4.2.1 and 4.3.1). Only near the closed end the tidal ellipses keep a cyclonical character, which is in agreement with the direction of propagation of the tidal waves through the basin.

We find that, looking in the x^{*}-direction, also with lateral depth variations the CAPs remain located between two EAPs. In y^{*}-direction we find that the CAPs remain on the centerline of the basin for symmetric profiles and are displaced towards the deeper side of the basin for asymmetric profiles, as was the case for the EAPs as well. The shift towards the deeper side of the basin occurs because the u^{*}-and v^{*}-components are the smallest in the deeper part of the basin, as was already found in the analysis of \hat{u} - and \hat{v} -structures of the Kelvin waves (see section 4.3 and Appendix D).



Figure 18. Current amphidromic systems for uniform depth (a), linear depth profile with s = 1.50 (b), symmetrical sinusoidal profiles with $A^*=25$ m (c) and $A^*=-25$ m (d), and asymmetrical sinusoidal profiles with $A^*=25$ m (e) and $A^*=-25$ m (f). Black squares indicate cyclonic rotation (i.e. counter-clock-wise in the Northern Hemisphere) and open circles indicate anti-cyclonic rotation (i.e. clockwise in the Northern Hemisphere).

6. Case: Large-scale Sand Extraction on the Netherlands Continental Shelf (NCS)

In this chapter a practical case is studied: the Southern North Sea with and without large-scale sand extraction. This practical case is meant to study (1) whether a realistic lateral depth profile of an actual sea (instead of a highly idealized one) leads to significant changes in the tidal dynamics compared to uniform depth and (2) whether large-scale sand extraction on the Netherlands Continental Shelf (NCS), as indicated in section 1.1, affects the tidal dynamics of the Southern North Sea. Firstly, the study area and the dimensions of large-scale sand extraction are described (section 6.1). Consequently, the method for modeling the Southern North Sea with and without sand extraction is discussed (section 6.2). Section 6.3 presents the model results for adopting a realistic lateral depth profile for the Southern North Sea instead of assuming uniform depth. Finally, section 6.4 discusses the effects of large-scale sand extraction on the tidal system of the Southern North Sea.

6.1 Study Area and Sandpit Dimensions

The study area is the Southern North Sea, especially the NCS. Figure 19 gives an overview of the bathymetry of the Southern North Sea and its rectangular approximation upon visual inspection. The bathymetrical data is taken from Van der Veen (2008) and originated from Boon and Gerritsen (1997) en Ten Brummelhuis et al. (1997).

In the coming 200-300 years sand extraction activities may lead to the development of a large sand extraction trench in front of the Dutch coast (Stolk, 2009). Rijkswaterstaat has given instruction to Deltares to investigate the effects of increasing sand extraction on the hydrodynamics and morphodynamics in the near-shore zone. The most extreme scenarios under investigation take into account a sand extraction trench with depths of 6 to 20 m between the established -20 m depth contour and the 12 miles-line (ca. 10-25 km) along the whole Dutch coastline (Stolk, 2009).

This sand extraction trench is indicated in Figure 20 (adapted from Ruiter and Gerrits, 2006). The area between the established -20 m depth contour and the 12 miles-line (indicating the territorial sea) is regarded as the most suitable for sand extraction, since, on the one hand, it is located seaward of the established -20 m depth contour (which is



Figure 19. Bathymetry of the Southern North Sea. The black lines indicate the rectangular approximation of the Southern North Sea and the red lines indicate the lateral cross-sections (with an interval of ca. 22.5 km) for which bathymetrical data is obtained.

prescribed by legislation for coastal safety) and, on the other hand, close enough to the coast to keep sand extraction economically attractive. At the moment it is unknown whether and to what extent these extreme extraction scenarios will affect the tidal system of the Southern North Sea.

6.2 Modeling the Southern North Sea and Sand Extraction

This section discusses the implementation of the Southern North Sea with and without sand extraction in our hydrodynamic model. Therefore, firstly a representative lateral depth profile is determined for the Southern North Sea. Consequently, the implementation of sand extraction trenches in this depth profile is examined. Finally, the collocation procedure for modeling sand extraction in a semi-enclosed basin is discussed, which is slightly more complicated than the collocation procedure that was used for a basin without sand extraction (see section 5.1).

6.2.1 Lateral Depth Profile and Parameter Settings for Modeling Southern Bight of the North Sea

Lateral Depth Profile of Southern Bight of the North Sea

To model the Southern North Sea we first have to determine a representative lateral depth profile for this area. Therefore, the following procedure is followed:

- 1. We have obtained 6 lateral cross-sections with an interval of approximately 22.5 km from a GIS map with bathymetrical data of the Southern Bight of the North Sea (from Van der Veen, 2008), indicated by the 2nd to 7th red line in Figure 19 looking from South to North. These cross-sections are presented in Appendix F. The remaining cross-sections have been omitted from the analysis, because they either cross land (1st cross-section) or are too wide compared to the width of our semi-enclosed basin (8th to 10th cross-section).
- 2. The bathymetrical data of these 6 crosssections has been squeezed or stretched to the chosen width of our basin by multiplying the coordinates by a factor $B_{\text{basin}}/B_{\text{profile}}$.
- 3. Because the intervals between the data points of the GIS data are not equal for the six cross-sections (due to squeezing or stretching), the water depths have been linearly interpolated to the coordinates used in the RK4-method for each cross-section.
- 4. Based on the modified cross-sections an average lateral depth profile for the Southern North Sea is determined.



Figure 20. The Netherlands Continental Shelf (NCS) with the location of the sand extraction trench between the established -20m depth contour and the 12-miles line, marked by the black shaded area (adapted from Ruiter and Gerrits, 2006).

The resulting lateral depth profile is presented in Figure 21 (red line). However, in order to remain as close as possible to the analytical solutions, our model requires the depth profile to be described by a continuous and differentiable mathematical function (see section 4.1). Therefore, we fit a polynomial to the representative lateral depth profile by means of a least squares method (i.e. the best fit is found by minimizing the squared differences between the representative profile and polynomial approximation). Figure 21 (blue line) indicates that a 5th order polynomial (with a

mean square error of 1.92) gives a satisfactory fit to the bathymetrical data for the purpose of our study. In the remaining of this Chapter this fit is used in order to study the Southern North Sea with and without large-scale sand extraction.



Figure 21. Longitudinally averaged lateral depth profile of the Southern North Sea (red line) and approximation by the 5th order polynomial $h(y) = \alpha_0 + \alpha_1 * (y/B) + \alpha_2 * (y/B)^2 + \alpha_3 * (y/B)^3 + \alpha_4 * (y/B)^4 + \alpha_5 * (y/B)^5$ with $\alpha_0 = 37.3$, $\alpha_1 = 43.3$, $\alpha_2 = -69.0$, $\alpha_3 = -355.3$, $\alpha_4 = -175.8$, $\alpha_5 = 857.9$ (blue line).

Parameter Settings for Modeling Southern Bight of the North Sea

The basin characteristics for modeling the Southern North Sea are listed in Table 10. The basin width B^* and mean basin depth H^*_{mean} are now derived more precisely than in section 4.1, based on the dataset of Boon and Gerritsen (1997) en Ten Brummelhuis et al. (1997). The other parameters are the same as in Table 2.

Parameter	Symbol	Value	Dimension
Basin width	B^*	157	km
Mean basin depth	$\mathrm{H}^{*}_{\mathrm{mean}}$	29	m
Maximum elevation amplitude at the coast	\hat{z}_{0}^{*}	1.5	m
Latitude	θ	53	°N
Angular frequency of the M ₂ -tide	σ	$1.41*10^{-4}$	rad s ⁻¹
Basin width (dimensionless)	В	1.31	-
Coriolis parameter (dimensionless)	f	0.83	-
Gravitational Froude number (dimensionless)	$\operatorname{Fr}\left(=\hat{z_{0}^{*}}/\operatorname{H}^{*}\right)$	0.05	-

Table 10. Parameter values for modeling the Southern North Sea.

6.2.2 Modeling Sand Extraction Trenches

Assumptions for Modeling Sand Extraction Trenches in an Infinite Channel

To model the sand extraction trenches on the NCS in an infinite channel the following assumptions have been made:

- 1. The -20 m depth contour is assumed to be a fixed, straight line along the Dutch coast and assumed to be located where the water depth is ca. 20 m in the polynomial approximation.
- 2. In lateral direction the sand extraction trench is assumed to have the shape of a cosine with the following function:

$$h_{pit}(y) = \frac{h_{trench;max}^{*}}{2} \left[1 - \cos\left(2p \, \frac{y^{*} - y_{20m}^{*}}{B_{trench}^{*}}\right) \right] \quad \text{with} \quad \begin{array}{c} y_{20m}^{*} \leq y^{*} \leq y_{trench}^{*} \\ B_{trench}^{*} = y_{trench}^{*} - y_{20m}^{*} \end{array} \right.$$
(Eq. 38)

Where:

 $h_{trench;max}^*$ Maximum trench depth in m

- y_{20m}^* Straight -20m depth contour in polynomial approximation in m (coastward lateral boundary of the extraction trench)
- y_{trench}^{*} Seaward lateral boundary of extraction trench in m

 B_{trench}^{*} Maximum width of the extraction trench in m



Figure 22. Approximation of lateral depth profile of the Southern North Sea with sand extraction trenches of dimensions $B^*_{trench} = 25$ km and $h^*_{trench;max} = 6$, 12 and 20 m.

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In Figure 22 an example is given of sand extraction trenches in the Southern North Sea with dimensions $B^*_{trench} = 25$ km and $h^*_{trench;max} = 6$, 12 and 20 m. In the remaining of this study we focus on the most and least extreme sand extraction scenarios, i.e. extraction trenches of $B^*_{trench} = 25$ km and $h^*_{trench;max} = 20$ m (corresponding to $8.0*10^4$ Mm³ of sand) and $B^*_{trench} = 10$ km and $h^*_{trench;max} = 6$ m (corresponding to $9.6*10^3$ Mm³ of sand) respectively. As a reference, for the creation of the islands of the mega-project *The World* in Dubai, 320 Mm³ of sand has been dredged (Van Oord, 2009).

Collocation Method for Modeling Sand Extraction

Since only sand extraction on the NCS is investigated, the extraction trench is of limited length and, hence, present in only part of the semi-enclosed basin. This makes the collocation procedure described in section 5.1 slightly more complicated, since we have to introduce an additional collocation line at the transition of the basin with and without sand extraction. In order to solve the Taylor problem, we require the free surface elevation (ζ) and longitudinal mass flux (uh) at this transition line to be equal for both parts of the basin, in addition to the no-normal flow requirement (u^{*} = 0) at the basin's closed end. Furthermore, the following assumptions are made:

- 1. The sand extraction trench is assumed to have the same lateral shape over its whole length.
- 2. The Southern boundary of the extraction trench is assumed to coincide with the basin's closed end, which is useful for the collocation procedure. Therefore, the length of the extraction trench, roughly from the closed end to Den Helder, is assumed to be 320 km (i.e. ca. 225 km in front of the Dutch coast and ca. 95 km in front of the Belgian/French coast)

The collocation procedure for modeling large-scale sand extraction is described in Appendix G. Table 11 presents the numerical parameters that are used to generate the $|z^*|$ -, $|u^*|$ - and $|v^*|$ - fields for the Southern North Sea with and without sand extraction. Note that more numerical steps and collocation points have been taken into account than for analyzing the amphidromic systems of the idealized profiles (section 5.2) in order to incorporate the shape of the sand extraction trench sufficiently accurate (because of its relatively small scale).

Parameter	Symbol	Value
Number of steps in y [*] -direction for RK4-method	Ny	400
Number of steps per extraction trench length L^*_{trench} in x [*] -direction	N _x	200
Number of collocation points	М	41
Initial search radius around trial wave number	Δk	0.05
Error criterion for solving RK4-method	ε _{crit}	10-10

 Table 11. Numerical parameters used for modeling the Southern North Sea with and without large-scale sand extraction.

6.3 Results: Realistic Lateral Depth Profile for the Southern North Sea instead of Uniform Depth

The modified wave solutions corresponding to the realistic lateral depth profile of the Southern North Sea, found by our hydrodynamic model, are discussed in Appendix H. This section discusses the impacts of adopting a realistic lateral depth profile instead of uniform depth on the tidal amplitudes and tidal currents.

6.3.1 Effects on Elevation Amphidromic System

In Figure 23 the elevation amphidromic systems for the Southern North Sea are presented with (a) uniform depth and (b) a realistic lateral depth profile as well as (c) the differences between

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them. In Table 12 the corresponding shifts of the first two EAPs in x^* - and y^* -direction are shown. When adopting a realistic depth profile we find that the EAPs shift slightly in the negative x^* -direction and the positive y^* -direction. The shift in x^* -direction is related to altered Kelvin wave lengths, whereas the shift in y^* -direction is related to the relatively larger water depths on the "English side" of the basin than on the "Dutch side" of the basin. Consequently, the tidal amplitudes at the Dutch coast increase with up to 0.20 m compared to uniform depth, whereas the tidal amplitudes at the English coast decrease with up to 0.08 m (related to the increased and decreased distances between the EAPs and the coast respectively).



Figure 23. Elevation amphidromic system for the Southern North Sea with (a) uniform depth $(H^*_{uniform} = 29 \text{ m})$ and (b) a realistic lateral depth profile and (c) the differences between them. In (a) and (b) the solid lines indicate co-amplitude lines with intervals of 20 cm and the dashed lines indicate co-phase lines dividing the tidal period into 12 intervals. In (c) the differences are plotted with intervals of 5 cm.

	Δx_1^{*} [km]	Δx_2 [km]	Δy_1 [km]	Δy_2 [km]
Realistic profile vs. Uniform depth	-1.6	-11.2	3.9	3.9

Table 12. Displacements of the first two EAPs in x^* - and y^* -direction for the realistic lateral depth profile of the Southern North Sea compared to uniform depth.

6.3.2 Effects on Tidal Currents

In Figure 24 and Figure 25 respectively the long-channel (u^*) and cross-channel (v^*) tidal currents for the Southern North Sea are presented (a) with uniform depth and (b) with a realistic lateral depth profile as well as (c) the differences between them. When adopting a realistic depth profile we find that u^* considerably increases (with up to 0.21 m/s) near the Dutch coast, whereas u^* decreases (with up to 0.10 m/s) near the English coast (see Figure 24 (c)). These changes in currents can be linked to the water depths near the coasts. It is relatively shallower compared to uniform depth near the Dutch coast leading to higher velocities, whereas it is relatively deeper near the English coast leading to lower velocities. In Figure 25 (a) it is shown that in uniform depth the cross-channel velocity is only present near the closed end of the basin (as a result of the Poincaré waves). By adopting a realistic lateral depth profile this is changed considerably, because the modified Kelvin waves obtain a v^{*}-component (see Appendix H). The changes in v^{*} can be up to 0.17 m/s.



Figure 24. Long-channel currents u^* for the Southern North Sea with (a) uniform depth ($H^*_{uniform} = 29 \text{ m}$) and (b) a realistic lateral depth profile and (c) the differences between them. In (a) and (b) the u^* -contours are plotted with intervals of 0.20 m/s. In (c) the differences are plotted with intervals of 0.05 m/s.



Figure 25. Cross-channel currents v^* for the Southern North Sea with (a) uniform depth ($H^*_{uniform} = 29 \text{ m}$) and (b) a realistic lateral depth profile and (c) the differences between them. The v^* -contours in (a) and (b) as well as the differences in (c) are plotted with intervals of 0.05 m/s.

6.4 Results: Large-scale Sand Extraction on the NCS

The modified wave solutions corresponding to a lateral depth profile accounting for large-scale sand extraction on the NCS are discussed in Appendix H. This section discusses the impacts of large-scale sand extraction on the tidal amplitudes and tidal currents in the Southern North Sea. We focus on the most ($B^*_{trench} = 25$ km and $h^*_{trench;max} = 20$ m) and least ($B^*_{trench} = 10$ km and $h^*_{trench;max} = 6$ m) extreme sand extraction scenarios.

6.4.1 Effects on Elevation Amphidromic System

In Figure 26 the elevation amphidromic systems for the Southern North Sea are presented (a) without sand extraction and (b) for the most extreme sand extraction scenario as well as (c) the differences between them. Note that Figure 26 (a) is equal to Figure 23 (b). In Table 13 the corresponding displacements of the first two EAPs in x^* - and y^* -direction are shown for both the most and least extreme sand extraction scenarios. For extreme sand extraction the EAPs show a slight shift in the positive x^* -direction, which can be linked to altered Kelvin wave lengths. The shift in y^* -direction is negligible. As a result the tidal amplitudes decrease with up to 0.15 m at the Dutch coast near the extraction trench. Also far away from the extraction trench the tidal amplitudes at the coast are affected by the sand extraction (up to 0.10 m). For the least extreme

extraction scenario the EAPs hardly shift. In this case the maximum changes in coastal amplitudes are about 0.02 m. Both increasing trench depth and increasing trench width lead to changes of similar order of magnitude in the tidal amplitudes.



Figure 26. Elevation amphidromic system for the Southern North Sea (a) without sand extraction and (b) with a sand extraction trench of $B^*_{trench} = 25$ km and $h^*_{trench;max} = 20$ m and (c) the differences between them. In (a) and (b) the solid lines indicate co-amplitude lines with intervals of 20 cm and the dashed lines indicate co-phase lines dividing the tidal period into 12 intervals. In (c) the differences are plotted with intervals of 5 cm.

	Δx_1^{*} [km]	Δx_2 [km]	∆y₁ [km]	Δy_2^{*} [km]
B [*] _{trench} = 10 km				
$h_{\text{trench;max}} = 6 \text{ m}$	0.0	1.6	0.0	0.0
B _{trench} = 25 km				
h [*] _{trench;max} = 20 m	6.4	11.2	0.0	0.0

Table 13. Displacements of the first two EAPs in x^* - and y^* -direction for sand extraction trenches of $B^*_{trench} = 10$ km and $h^*_{trench;max} = 6$ m and $B^*_{trench} = 25$ km and $h^*_{trench;max} = 20$ m compared to a Southern North Sea profile without sand extraction.

6.4.2 Effects on Tidal Currents

In Figure 27 and Figure 28 respectively the long-channel (u^*) and cross-channel (v^*) tidal currents for the Southern North Sea are presented (a) without sand extraction and (b) for the most extreme extraction scenario as well as (c) the differences between them. Note that Figure 27 (a) and Figure 28 (a) are equal to Figure 24 (b) and Figure 25 (b) respectively. For extreme sand extraction we find that u^* considerably decreases (with up to 0.10 m/s) near the Dutch coast (Figure 27 (c)), which can be related to the decreased water depth as a result of sand extraction. Also the u^* -field far away from the trench is affected by the sand extraction. Note that the sudden increase in u^* at the transition line of the basin is caused by the sudden rise in sea floor from the part of the basin with extraction trench to the part of the basin without extraction trench (Figure 27 (c)).



Figure 27. Long-channel currents u^{*} for the Southern North Sea (a) without sand extraction and (b) with a sand extraction trench of $B^*_{trench} = 25$ km and $h^*_{trench;max} = 20$ m and (c) the differences between them. In (a) and (b) the u^{*}-contours are plotted with intervals of 0.20 m/s. In (c) the differences are plotted with intervals of 0.05 m/s.

The Influence of Lateral Depth Variations on Tidal Dynamics in Semi-enclosed Basins

In Figure 28 (c) it is shown that v^* is considerably decreased (with up to 0.18 m/s) as a result of sand extraction, especially within the extraction trench. The decrease in v^* is especially apparent near the longitudinal boundaries of the trench, where the v^* -component was relatively strong without sand extraction. The v^* -field outside the trench is hardly affected by the sand extraction. For the least extreme scenario the maximum changes in tidal currents are of the order of magnitude 0.02 m/s for u^* and 0.06 m/s for v^* . Generally, an increase in trench depth has a stronger effect on the tidal currents than an increase in trench width.



Figure 28. Cross-channel currents v^* for the Southern North Sea (a) without sand extraction and (b) with a sand extraction trench of $B^*_{trench} = 25$ km and $h^*_{trench;max} = 20$ m and (c) the differences between them. The v^* -contours in (a) and (b) as well as the differences in (c) are plotted with intervals of 0.05 m/s.

7. Discussion

This Chapter examines the possibilities and limitations of studying tidal wave propagation in semi-enclosed basins by means of our hydrodynamic model. We also reflect on the interaction between lateral depth variations and bottom friction. Finally, the implications of our findings on large-scale sand extraction in the Southern North Sea are discussed.

7.1 Possibilities and Limitations of Hydrodynamic Model

In principal, our hydrodynamic model is capable of finding wave solutions for every arbitrary lateral depth profile. However, we chose to restrict our attention mainly to idealized lateral depth profiles that are described by differentiable and continuous (smooth) mathematical functions in order to remain as close to the analytical solution as possible. It is possible to find solutions for every arbitrary depth profile, but then the local derivatives need to be determined by means of inter- or extrapolation, which introduces additional errors into the model.

It is stressed that studying tidal dynamics in semi-enclosed basins by means of Taylor's approach is subject to severe simplifications, such as straight coastlines, depth-averaged motion and omitting bottom friction and viscosity. Therefore, our hydrodynamic model is especially useful to obtain insight in the global effects of lateral depth variations on the tidal system as well as the order of magnitude of these effects. For studying the small-scale, quantitative effects of lateral depth variations, (complex) numerical models are necessary.

Care must be taken when using the collocation technique for the superposition of the individual wave modes. The collocation technique satisfies the boundary condition(s) at a finite number of pre-determined collocation points. In between these points the boundary condition(s) is (are) not necessarily satisfied. Possible errors due to this characteristic of the collocation technique can be minimized by increasing the number of collocation points (i.e. decreasing the distance between sub-sequent collocation points). Note that the number of numerical intervals (N_y) should be increased accordingly in order to include the amplitude structures of the highest Poincaré modes sufficiently accurately.

Our model is capable of modeling sand extraction trenches of large dimensions. Because of the relatively small scale of the sand extraction trenches compared to the basin width, a sufficient number of collocation points (usually more than for modeling the idealized, basin-scale depth profiles) should be taken into account to incorporate the shape of the sand extraction trenches sufficiently accurately. It is found that our hydrodynamic model has difficulties with the collocation procedure for sand extraction trenches of relatively small width ($B^*_{trench} = 10$ km) and relatively large depths ($h^*_{trench;max} = 12, 20 \text{ m}$), see also Appendix G. For these dimensions the model has problems with satisfying the boundary conditions for the velocity components at the walls of the semi-enclosed basin. These problems might be related to the relative large depth variations on a relatively small scale and may be overcome by considerably increasing the number of collocation points. However, even increasing the number of collocation points by a factor 3 did not solve the problem. In addition, increasing the number of collocation points implies increasing the number of wave modes and numerical intervals, which severely increases the computation time of our model and may lead to too many elements to be handled by the program The MathworksTM MATLAB[®]. Therefore, our model is particularly capable of modeling extraction trenches of relatively larger width (i.e. $B_{trench}^* = 25$ km or larger), which can be modeled accurately with a smaller number of collocation points.

7.2 Lateral Depth Variations and Bottom Friction

This study focuses on the impacts of lateral depth variations on tidal dynamics in semi-enclosed basins while omitting energy dissipation. Other studies (Hendershott and Speranza, 1971; Rienecker and Teubner, 1980; Roos and Schuttelaars, 2009) focused on the effects of energy dissipation on the tidal system, but generally assumed uniform depth. In reality, depth variations and energy dissipation (as a result of bottom friction) are interrelated, i.e. in shallow water the water movement can be highly affected by the bed, whereas in deep water these effects can be negligible. Based on the findings in previous studies and this study, in which both aspects are investigated separately, it is hard to reflect on the *combined* impacts on the tidal system in quantitative terms.

In qualitative terms we note the following. The *separate* effects of lateral depth variations and bottom friction on the tidal system may enhance as well as oppose each other, depending on the type of lateral depth profile. For example, when it is deep where the Kelvin wave enters the basin and shallow where it is reflected, our analysis indicates that the elevation amphidromic points (EAPs) shift to the deeper part of the basin (i.e. towards the wall along which the incoming Kelvin wave propagates). However, previous studies focusing on energy dissipation (Hendershott and Speranza, 1971; Rienecker and Teubner, 1980; Roos and Schuttelaars, 2009) indicate that the EAPs shift towards the wall along which the reflected Kelvin wave propagates (i.e. the opposite side), which may be enhanced by increased energy dissipation for the reflected Kelvin as a result of the shallow water. Thus, in this case the separate effects of lateral depth variations and energy dissipation oppose each other. In the opposite situation, i.e. shallow where the Kelvin wave enters the basin and deep where it is reflected, the separate effects of lateral depth variations and bottom friction enhance each other. However, lateral depth variations and bottom friction may also have combined effects on the tidal system. Based on this study, we cannot reflect on the properties and magnitude of these combined effects. For this purpose, both lateral depth variations and bottom friction should be included in the hydrodynamic model.

7.3 Implications of Large-scale Sand Extraction

In this study we found that large-scale sand extraction can have considerable impacts on the tidal system of the Southern North Sea, both in terms of tidal amplitudes at the coasts as well as tidal currents. These impacts are not only local, but can be present far away from the extraction area as well. The changes in coastal amplitudes may be important with respect to coastal safety. The changes in tidal currents play an important role with respect to the morphodynamics in the Southern North Sea. Even small changes in the tidal currents can severely affect the morphodynamics, because they are non-linearly related to each other (by power-laws). The changes in morphodynamics in turn can affect coastal safety, ecology and cable and pipeline infrastructure. Based on this study it is hard to reflect on the quantitative impacts on the morphodynamics, since we deal with depth-averaged velocities whereas the velocities near the bed are the most important for sediment transport.

It is stressed that our study only focuses on the tidal impacts of sand extraction as a result of changes in the lateral depth profile of the Southern North Sea. However, also bottom friction and longitudinal depth variations may be important in this respect. These effects are not taken into account in this study. Therefore, the quantitative changes in tidal amplitudes and currents as a result of sand extraction may differ in practice.

8. Conclusions and Recommendations

In this Chapter the conclusions (section 8.1) and recommendations (section 8.2) of the study are presented. In the conclusions the answers on the research questions, formulated in section 1.3, are examined. The recommendations are divided into recommendations on large-scale sand extraction in the Southern North Sea and recommendations on further research.

8.1 Conclusions

In this study the influence of lateral depth variations on the wave properties and amphidromic systems in rectangular, rotating semi-enclosed basins is investigated. Therefore, we extended the Taylor problem (1921) of Kelvin wave reflection in a semi-enclosed basin by allowing depth variations in lateral direction. In order to find the modified wave solutions, i.e. Kelvin and Poincaré modes, we fixed the tidal frequency and used a semi-analytical, hydrodynamic model with a search routine to find the complex wave numbers. The model is applied to a variety of idealized lateral depth profiles. In addition, our model is used to investigate the impacts of large-scale sand extraction on the tidal system of the Southern North Sea. In this section the research questions are answered.

Research question 1: How do the wave properties of the Kelvin and Poincaré modes change for idealized lateral depth profiles compared to uniform depth?

As a result of lateral depth variations the wave lengths of the Kelvin wave are considerably altered compared to uniform depth. These changes are related to (local) changes in water depth: Kelvin waves propagate faster in deep water, implying larger wave lengths and the other way around in shallow water. Therefore, generally, the wave lengths become larger when the water depth increases (relative to uniform depth) near the boundary along which the Kelvin wave propagates and smaller when the water depth decreases there. For symmetrical depth profiles the incoming and reflected Kelvin waves have equal wave lengths, whereas for asymmetrical profiles both Kelvin waves have different wave lengths. The lateral amplitude structures of the modified Kelvin modes remain close to the uniform depth variations. Furthermore, we find that the Kelvin waves obtain a cross-channel velocity component due to the lateral depth variations, as opposed to the classical Kelvin wave solution in uniform depth.

Also the (evanescent) Poincaré modes alter considerably due to lateral depth variations, but it is hard to discover general trends. Depending on the type of lateral depth profile the Poincaré modes obtain a propagative and/or more evanescent character compared to the uniform depth solutions. The e-folding length scales are generally smaller than for the uniform depth solutions. By fixing the tidal frequency and allowing the wave number to be complex, we find real as well as imaginary lateral structures for the cross-channel velocity-component of the Poincaré waves. These two different structures indicate the wave structures at two different moments. As for the kelvin waves the lateral amplitude structures of the modified Poincaré waves remain close to the uniform depth solutions for small depth variations and become more distorted for larger depth variations.

Research question 2: What are the influences of the lateral depth profiles on the resulting elevation and current amphidromic systems (i.e. the solution to the Taylor problem)?

The solution of the Taylor problem allowing for lateral depth variations is written as a superposition of the individual wave modes: an incoming Kelvin wave, a reflected Kelvin wave and a truncated sum of (reflected) Poincaré modes. We applied a collocation technique to satisfy the no-normal flow boundary condition at the basin's closed end. In general, we find that for symmetrical depth profiles the elevation amphidromic points (EAPs) remain on the centre line of the basin, whereas for asymmetrical depth profiles they are laterally displaced towards the deeper side of the basin on a straight line parallel to the longitudinal coast. Lateral depth variations also lead to displacements of the EAPs in longitudinal direction due to altered Kelvin wave lengths. In general, the displacements in longitudinal direction are larger than the displacements in lateral direction.

We find that the CAPs experience more or less the same lateral and longitudinal displacements as the EAPs and, as a result, remain located between two EAPs in longitudinal direction (as in uniform depth). In contrast to the uniform depth case the tidal ellipses do not only occur in the region near the closed end of the basin, but also far away from this boundary, because the Kelvin waves obtain a cross-channel velocity-component. The cross-channel velocity-component is the strongest in the shallower parts of the basin. Furthermore, the direction of rotation is changed due to lateral depth variations. For most of the depth profiles the major part of tidal ellipses rotates cyclonically (i.e. in the same direction as the rotation of the earth).

Research question 3: What are the potential effects of large-scale sand extraction on the Netherlands Continental Shelf (NCS) on the tidal system of the Southern North Sea?

We studied a practical case by means of our hydrodynamic model: the Southern North Sea with and without large-scale sand extraction on the Netherlands Continental Shelf (NCS). Based on bathymetrical data a longitudinally averaged lateral depth profile is determined for the Southern North Sea. It is found that adopting this realistic lateral depth profile instead of assuming uniform depth leads to considerable changes in the tidal amplitudes and currents. To model large-scale sand extraction, the basin is divided into a part with and a part without sand extraction trench in the lateral depth profile. It is found that sand extraction may considerably impact on the tidal amplitudes and currents, not only locally, but also farther away from the extraction area. These changes may have severe impacts on the morphodynamics and, consequently, coastal safety, ecology and cable and pipeline infrastructure, also for the least extreme sand extraction scenario.

8.2 Recommendations

On Large-scale Sand Extraction in the Southern North Sea

The results of this study indicate that large-scale sand extraction in the Southern North Sea can have considerable impacts on the tidal system, both in terms of tidal amplitudes at the coasts and tidal currents. These tidal changes can affect coastal safety, ecology and cable and pipeline infrastructure, not only locally, but also far way from the extraction area. Therefore, it is recommended to take into account these changes in the tidal forcing in present and future studies on large-scale sand extraction.

This study only investigated the tidal impacts of large-scale sand extraction by adapting the lateral depth profile of the Southern North Sea. Taking into account bottom friction and longitudinal depth variations as well, as suggested in the next sub-section, may provide more accurate information on the tidal impacts. Therefore, it is recommended to study the impacts of large-scale sand extraction also with bottom friction and longitudinal depth variations included in our hydrodynamic model

On Further Research

As pointed out in section 7.2 lateral depth variations and bottom are interrelated. In this study bottom friction is omitted and, hence, the interaction between these aspects is neglected. Therefore, it is recommended to include bottom friction in our hydrodynamic model in order to study the interaction between lateral depth variations and bottom friction and the consequent impacts on the tidal system.

The effects of longitudinal depth variations on the tidal dynamics in semi-enclosed basins have been investigated in previous studies, in contrast to lateral depth variations often also in relation to frictional effects (see Hendershott and Speranza, 1971; Godin and Martinez, 1994; Davies and Jones, 1995; Winant, 2007). However, to the author's knowledge, no comprehensive studies on the combined effects of various types of lateral and longitudinal depth variations exist. It could be interesting to study these effects both with and without bottom friction. For the longitudinal depth variations it is recommended to start with a relatively simple division into two basins with different lateral depth profiles, like we did in this study for sand extraction. Later also more complex longitudinal depth variations could be investigated. Be aware that this severely complicates the solution procedure, since for more complex longitudinal depth variations not only the complex wave amplitudes (transverse wave structures) but also the longitudinal wave structures change.

Although the primary objective of our idealized model is to gain insight in the physical mechanisms underlying tidal wave propagation in semi-enclosed basins, it could be useful to compare the model results on qualitative properties with tide observations (amplitudes and phases) in existing semi-enclosed basins. For this purpose, it is recommended to include bottom friction in our hydrodynamic model, because of the potential combined effects of lateral depth variations and bottom friction on the tidal system (see section 7.2). Another possibility is to compare the results of the current model (i.e. without bottom friction) with an actual (semi-enclosed) sea where bottom friction is relatively unimportant, such as the Adriatic Sea (see Hendershott and Speranza, 1971).

References

Boers, M. (2005). *Effects of a deep sand extraction pit*. Ministerie van Verkeer en Waterstaat, Rijkswaterstaat North Sea Directorate, RIKZ/2005.001 (ISBN 90-369-3498-2).

Boon, J.G. and Gerritsen, H. (1997). *Modelling of suspended particulate matter (SPM) in the North Sea: A dedicated orthogonal boundary-fitted modeling approach (PROMISE).* Res. Rep. Z2025, Delft Hydraulics, Delft, The Netherlands.

Brown, T. (1987). *Kelvin wave reflection at an oscillating boundary with applications to the North Sea.* Continental Shelf Research, Vol. 7, No. 4, pp. 351-365.

Brown, T. (1989). *On the general problem of Kelvin wave reflection at an oscillating boundary*. Continental Shelf Research, Vol. **9**, No. 10, pp. 931-937.

Davies, A.M. and Jones, J.E. (1995). *The influence of bottom and internal friction upon tidal currents: Taylor's problem in three dimensions*. Continental Shelf Research, Vol.**15**, No. 10, pp. 1251-1285.

De Swart, H.E. (2008). *Ocean Waves – Lecture Notes*. Utrecht University, Utrecht, The Netherlands, accessed through <u>http://www.phys.uu.nl/~deswart/owaves_docs</u>

Hendershott, M.C. and Speranza, A. (1971). *Co-oscillating tides in long, narrow bays; the Taylor problem revisited.* Deep-Sea Research, Vol. **18**, pp. 959-980.

Hunt, J.N. and Hamzah, A.M.O. (1967). *Tidal Waves in a Canal of Non-Uniform Depth*. Pure and Applied Geophysics, Vol. **67**, No. 1, pp. 133-142.

Godin, G. and Martinez, A. (1994). *On the possibility of Kelvin-type motion in actual seas*. Continental Shelf Research, Vol. **14**, No. 7/8, pp. 707-721.

Kowalik, Z. (1979). A Note on the Co-oscillating M_2 -Tide in the Arctic Ocean. Dt. Hydrogr. Z., Vol. **32**, pp.100-112.

Kreyszig, E. (1999). Advanced Engineering Mathematics. δ^{th} Edition. Wiley & Sons, Inc., Singapore, ISBN 0-471-33328-X (paperback).

Longuet-Higgins, M.S. (1968). On the trapping of waves along a discontinuity of depth in a rotating ocean. J. Fluid Mech., Vol. **31**, Part 3, pp. 417-434.

Mosetti, R. (1986). Determination of the current structure of the M2 tidal component in the Northern Adriatic by applying rotary analysis to the Taylor problem. Boll. Oceanol. Teor. Appl., Vol. 4, No. 3, pp. 165-172.

Pedlosky, J. (1982). *Geophysical Fluid Dynamics*. Springer-Verlag, New York, U.S.A., ISBN 0-387-90745-9/ISBN 3-540-90745-9

Proudman, J. (1941). *The effect of friction on the tides*. Mon. Not. R. astr. Soc. geophys. Suppl., Vol. 1, pp. 23-26.

Rienecker, M.M. and Teubner, M.D. (1980). A note on frictional effects in Taylor's problem. Journal of Marine Research, Vol. 38, No. 2, pp. 183-191.

Rizal, S. (2002). *Taylor's problem – influences on the spatial distribution of real and virtual amphidromes*. Continental Shelf Research, Vol. **22**, pp. 2147-2158.

Roos, P.C. and Schuttelaars, H.M. (2009, accepted). *Solving the viscous Taylor Problem*. Accepted by Journal of Fluid Mechanics.

Ruiter, E.R. and Gerrits, S. (2006). Zandwinning op de Noordzee. Startnotitie milieueffectrapportage. Stichting La Mer, File: X3276-01.001, Version 4, Registration number: MD-WR20060126

Sinha, B. and Pingree, R.D. (1997). *The principal lunar semidiurnal tide and its harmonics: baseline solutions for M2 and M4 constituents in the North-West European Continental Shelf.* Cont. Shelf Res. Vol. **7**, No. 11, pp. 1321-1365

Staniforth, A.N., Williams, R.T. and Neta, B. (1993). *Influence of Linear Depth Variation on Poincaré, Kelvin and Rossby Waves*. Journal of the Atmospheric Sciences, Vol. **50**, No. 7, pp. 929-940.

Stolk, A. (2009). Rijkswaterstaat North Sea Directorate. June 16th 2009 (Interview).

Taylor, G.I. (1921). *Tidal oscillations in gulfs and rectangular basins*. Proceedings of London Mathematical Society, Vol. **20**, pp. 148-181.

Ten Brummelhuis, P.G.J., Gerritsen, H. and Van der Kraay, T. (1997). *Sensitivity analysis and calibration of the orthogonal boundary-fitted coordinate model PROMISE for tidal flow: The use of adjoint modelling techniques.* Res. Rep. Z2024, Delft Hydraulics, Delft, The Netherlands.

Van der Molen, J., Gerrits, J. and De Swart, H.E. (2004). *Modelling the morphodynamics of a tidal shelf sea*. Continental Shelf Research, Vol. **24**, pp. 483-507.

Van der Veen, H.H. (2008). *Natural and Human Induced Seabed Evolution*. PhD Thesis, University of Twente, Enschede, The Netherlands, ISBN: 978-90-365-2613-5.

Van Oord (2009). *Dubai: The World*. Van Oord, Rotterdam, The Netherlands, accessed through: <u>http://www.vanoord.com/gb-en/our projects/project_selector/the_world/index.php</u>.

Winant, C.D. (2007). *Three-Dimensional Tidal Flow in an Elongated, Rotating Basin.* Journal of Physical Oceanography, Vol. **37**, pp. 2345-2362.

Xia, Z., Carbajal, N. and Südermann, J. (1995). *Tidal current amphidromic system in semi*enclosed basins. Continental Shelf Research, Vol. 15, No. 2/3, pp. 219-240.

Yanagi, T. (1987). *Kelvin Wave Reflection with Phase Lag in the Bungo Channel*. Journal of the Oceanographical Society of Japan, Vol. **43**, pp. 377-382.

Appendix A: Model Verification

To test the RK4-method and the search routine we compare the model outcomes with two cases: (1) the uniform depth case (Pedlosky, 1982) and (2) linear lateral depth profiles (Staniforth et al., 1993).

A.1 Verification with Uniform Depth

In the uniform depth case the wave numbers of the Kelvin and Poincaré modes can be determined by means of the dispersion relationships as derived in Pedlosky (1982). The dimensional dispersion relationships for Kelvin and Poincare waves are presented in the equations (1) and (3) respectively. The non-dimensional wave numbers (k_n) can be obtained by dividing k_n^* by K^{*}. For a given latitude ($\theta = 53^\circ$ N) and tidal angular frequency ($\sigma^* = 1.405*10^4 \text{ s}^{-1}$ or T^{*} = 12.42 hr) and known basin characteristics B^{*} (= 200 km) and H^{*}_{uniform} (= 30 m) the values of k_n can be obtained analytically. The wave numbers are also determined by means of our hydrodynamic model (using N_y = 100 and $\Delta k = 0.05$), but now using the analytical k_n -values as trial values. When the modeled k_n 's converge to the k_n 's that have been obtained analytically, the model is able to reproduce the uniform depth wave solutions. In Table 14 is shown that the model results resemble the analytical solutions very well.

n	k _{n;analytical}	k _{n;modeled}	n	k _{n;analytical}	k _{n;modeled}
0	1.000	1.000	0	-1.000	-1.000
1	0.000+1.687i	0.000+1.687i	-1	-0.000-1.687i	0.000-1.687i
2	0.000+3.725i	0.000+3.725i	-2	-0.000-3.725i	0.000-3.725i
3	0.000+5.680i	0.000+5.680i	-3	-0.000-5.680i	0.000-5.680i
4	0.000+7.616i	0.000+7.616i	-4	-0.000-7.616i	0.000-7.616i
5	0.000+9.545i	0.000+9.545i	-5	-0.000-9.545i	0.000-9.545i
6	0.000+11.469i	0.000+11.470i	-6	-0.000-11.469i	0.000-11.470i
7	0.000+13.392i	0.000+13.392i	-7	-0.000-13.392i	0.000-13.392i
8	0.000+15.314i	0.000+15.314i	-8	-0.000-15.314i	0.000-15.314i
9	0.000+17.234i	0.000+17.235i	-9	-0.000-17.234i	0.000-17.235i
10	0.000+19.154i	0.000+19.156i	-10	-0.000-19.154i	0.000-19.156i

Table 14. Analytical and modeled wave numbers for the positive and negative Kelvin wave (n = 0) and first 10 Poincaré modes $(n \neq 0)$ for uniform depth

A.2 Verification with Results of Staniforth et al. (1993)

The model performance is also verified with the results of Staniforth et al. (1993), who found wave solutions for linear lateral depth variations of the form (which is equal to the linear profile that has been defined in section 4.1, but here presented in dimensional notation):

$$h^{*}(y^{*}) = H^{*}\left(1 - \frac{sy^{*}}{B^{*}}\right).$$
 (Eq. 39)

However, a fundamental complexity arises here. We fix σ^* to a real value and look for wave numbers k_n^* , which can be real or complex. Staniforth et al. (1993) assume a fixed, real-valued wave number k^* and look for wave solutions in terms of tidal frequencies (σ_n^*). In addition, they deal with dimensional quantities whereas our model is based on non-dimensional quantities. Therefore, in order to be able to reproduce the results of Staniforth et al. (1993), the model must be rewritten into a search routine for σ_n^* for a fixed k^* . In their paper Staniforth et al. present a

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figure (see Figure 29) in which the dimensional phase speeds ($c^* = \sigma_n^*/k^*$) of the wave modes are given as a function of the (lateral) bottom slope (s).



Figure 29: Phase speeds (c^{*}) for Poincaré and Kelvin modes as a function of s from s = 0.05 to 1.95, for B^{*} = 5*10⁶ m, H^{*}_{uniform} = 4*10⁴/g^{*} = 4077 m, $f^* = 10^4$ s⁻¹ and k^{*} = 2 π /B^{*} = 1.257*10⁻⁶ m⁻¹ (Figure 1 from Staniforth et al., 1993)

In order to generate model results we used $N_y = 100$ and $\Delta k = 0.05$. Care must be taken with the choice of slope increments Δs . For too large Δs , especially in case of large s, another wave mode is found than the one we were looking for. After some fine-tuning it turns out that the results of Staniforth et al. are reproduced very well for slope increments of 0.10, 0.30, 0.50, 0.70, 0.90, 1.10, 1.30, 1.50, 1.70, 1.75, 1.77, 1.80, 1.83, 1.85, 1.87, 1.90, 1.93 and 1.95 see Figure 30).



Model verification with Staniforth et al. (1993) based on phase speed (c*)

Figure 30: Model results in terms of phase speeds for lateral bottom slope increments of 0.10-0.30-0.50-0.70-0.90-1.10-1.30-1.50-1.70-1.75-1.77-1.80-1.83-1.85-1.87-1.90-1.93-1.95.

In addition to the results in terms of phase speeds (c), Staniforth et al. (1993) also present the wave solutions in terms of ζ^* -, u^{*}- and v^{*}-amplitude structures for the positive and negative Kelvin modes as well as the first and fourth positive Poincaré modes. The results of Staniforth et al. for the positive Kelvin mode and the first positive Poincaré mode are presented in Figure 31. The same initial parameters are used as indicated in Figure 29.

For the same model settings as used before, the model results in terms of the eigenstructures of the positive Kelvin wave and the first positive Poincaré are identical to the structures presented in Staniforth et al. (1993) (see Figure 32 and Figure 33). Although only the results for the positive Kelvin wave and the first positive Poincaré mode are presented here, also the structures of the negative Kelvin wave and the fourth positive Poincaré mode match the results of Staniforth et al. (1993).

Thus, the model is able to produce the correct results for uniform depth as well as for linear depth variations. Therefore, we can have reasonable faith in the model accuracy for investigating the influences of other lateral depth profiles on the tidal system.



Figure 31. Eigenstructures of positive Kelvin wave (left) and first positive Poincaré mode (right) as a function of y for s = 0.1 (upper curve) and s = 1.95 (lower curve). The curves are labeled as u^* : solid, v^* : dotted, ζ^* : chain dot. The velocity scale is on the left and the height scale on the right (Figures 8 and 11 from Staniforth et al., 1993).



Figure 32: Same as Figure 31 (left), but now produced by our model.



Figure 33: Same as Figure 31 (right), but now produced by our model.

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Appendix B: Sensitivity Analysis

Since our model is partly numerical, the model outcomes can be sensitive to numerical parameters, such as the number of numerical steps N_y in the RK4-method, the search radius Δk in the search routine and the slope increments Δs for linear profiles (or the increments in amplitude ΔA^* for other lateral depth profiles as defined in section 4.1). In order to obtain insight in the sensitivity, a sensitivity analysis is performed with respect to these numerical parameters. The outcomes of this analysis assist in choosing the right parameter settings for the model calculations in the remaining of the study. The choice for these settings is mainly a balance between accuracy and computation speed, i.e. the higher the accuracy, the lower the computation speed and the other way around.

The model sensitivity is investigated only for a linear profile, since we expect the sensitivity of the other profiles to be more or less of equal order of magnitude. The analysis focuses on the sensitivity with respect to the wave numbers (k_n) , since the ζ -, u- and v-profiles are directly dependent on the values of k_n . Since the largest slope s is likely to be the most sensitive to changes in the model parameters, only the results for s = 1.95 are presented. Although the sensitivity analysis is performed for a wider range of numerical parameter settings, only selected parameter settings are presented here. In Table 15 and Table 16 respectively the basin characteristics and numerical parameters are presented with which the sensitivity analysis is performed.

Parameter	Symbol	Value	Dimension
Basin width	B^*	200	km
Average basin depth	H^{*}	30	m
Maximum elevation amplitude at	ź.	1.5	m
the coast	20		
Latitude	θ	53	°N
Angular frequency of the M ₂ -tide	σ	$1.41*10^{-4}$	rad s ⁻¹
Basin width (dimensionless)	В	1.64	-
Coriolis parameter (dimensionless)	f	0.83	-

Table 15. Overview of basin characteristics used to perform sensitivity analysis

Parameter	Symbol	Value(s)
Number of steps in RK4-method	N _v	50
_		100
		200
Number of modes taken into account	N _{modes}	11
(1 positive Kelvin and 10 positive		
Poincaré modes)		
Initial search radius around trial wave	$\Delta \mathbf{k}$	0.025
number		0.05
		0.10
Slope increments	Δs	0-0.50-1.10-1.50-1.75-1.90-1.95
		0-0.10-0.30-0.50-0.70-0.90-1.10-1.30-1.50-
		1.70-1.75-1.80-1.85-1.90-1.95
		0-1.95
Error criterion for solving RK4-	Ecrit	10 ⁻¹⁰
method		

Table 16. Numerical parameters used to perform sensitivity analysis
B.1 Model Sensitivity with respect to N_y

In Table 17 the resulting values of k_n for the first 11 modes (1 Kelvin and 10 Poincaré modes) are presented for different N_y (25; 50; 100) and fixed Δk (0.05) and Δs (0-0.50-1.10-1.50-1.75-1.90-1.95). From the table can be concluded that the differences in k for different N_y are negligible for the first wave modes, but become bigger for the higher Poincaré modes, especially for N_y = 50. N_y = 100 is sufficiently accurate for analyzing the wave modes. However, a higher N_y may be necessary when more Poincaré modes need to be taken into account in the collocation procedure.

	N _v					
n	100	200	50			
0	0.866	0.866	0.866			
1	-0.487+1.976i	-0.487+1.976i	-0.488+1.976i			
2	-0.645+4.039i	-0.645+4.039i	-0.645+4.040i			
3	-0.732+6.001i	-0.732+6.001i	-0.732+6.001i			
4	-0.793+7.934i	-0.793+7.934i	-0.793+7.934i			
5	-0.840+9.855i	-0.840+9.855i	-0.840+9.855i			
6	-0.876+11.770i	-0.876+11.770i	-0.877+11.772i			
7	-0.906+13.682i	-0.906+13.682i	-0.907+13.685i			
8	-0.931+15.593i	-0.931+15.593i	-0.932+15.599i			
9	-0.951+17.503i	-0.951+17.503i	-0.953+17.513i			
10	-0.969+19.413i	-0.968+19.412i	-0.972+19.429i			

Table 17: Resulting wave numbers (k_n) for the first 11 wave modes (n) for different values of N_y and fixed Δk (0.05) and Δs (0-0.50-1.10-1.50-1.75-1.90-1.95).

B.2 Model Sensitivity with respect Δk

In Table 18 the resulting k_n -values are presented for different Δk (0.025; 0.05; 0.10) and fixed N_y (50) and Δs (0-0.50-1.10-1.50-1.75-1.90-1.95). From the table can be concluded that the differences in k_n are negligible for different Δk , even for the higher Poincaré modes. A Δk of 0.5 is sufficiently accurate.

	Δk					
n	0.05	0.025	0.10			
0	0.866	0.866	0.866			
1	-0.487+1.976i	-0.487+1.976i	-0.487+1.976i			
2	-0.645+4.039i	-0.645+4.039i	-0.645+4.039i			
3	-0.732+6.001i	-0.732+6.001i	-0.732+6.001i			
4	-0.793+7.934i	-0.793+7.934i	-0.793+7.934i			
5	-0.840+9.855i	-0.840+9.855i	-0.840+9.855i			
6	-0.876+11.770i	-0.876+11.770i	-0.876+11.770i			
7	-0.906+13.682i	-0.906+13.682i	-0.906+13.682i			
8	-0.931+15.593i	-0.931+15.593i	-0.931+15.593i			
9	-0.951+17.503i	-0.951+17.503i	-0.951+17.503i			
10	-0.969+19.413i	-0.969+19.413i	-0.969+19.413i			

Table 18: Resulting wave numbers (k_n) for the first 11 wave modes (n) for different values of Δk and fixed N_v (100) and Δs (0-0.50-1.10-1.50-1.75-1.90-1.95).

B.3 Model Sensitivity with respect to Δs

In Table 19 the resulting k_n -values are presented for different Δs (0-0.50-1.10-1.50-1.75-1.90-1.95; 0-0.10-0.30-0.50-0.70-0.90-1.10-1.30-1.50-1.70-1.75-1.80-1.85-1.90-1.95; 0-1.95) and fixed N_y (50) and Δk (0.05). From the table can be concluded that the differences for different slope increments are negligible, also for the higher Poincaré modes. This seems to be in contrast with our earlier observation in the model verification (Appendix A) where the model results showed considerable sensitivity to Δs . This apparent contradiction can be explained by the fact that for the model verification dimensional quantities have been used whereas our model uses non-dimensional (scaled) quantities. Using non-dimensional quantities Table 19 indicates that slope increments are not really necessary to obtain accurate results. Nevertheless, it might be safer to apply some slope increments.

	Δs					
n	0-0.50-1.10-1.50- 1.75-1.90-1.95	0-0.10-0.30-0.50- 0.70-0.90-1.10-1.30- 1.50-1.70-1.75-1.80- 1.85-1.90-1.95	0-1.95			
0	0.866	0.866	0.866			
1	-0.487+1.976i	-0.487+1.976i	-0.487+1.976i			
2	-0.645+4.039i	-0.645+4.039i	-0.645+4.039i			
3	-0.732+6.001i	-0.732+6.001i	-0.732+6.001i			
4	-0.793+7.934i	-0.793+7.934i	-0.793+7.934i			
5	-0.840+9.855i	-0.840+9.855i	-0.840+9.855i			
6	-0.876+11.770i	-0.876+11.770i	-0.876+11.770i			
7	-0.906+13.682i	-0.906+13.682i	-0.906+13.682i			
8	-0.931+15.593i	-0.931+15.593i	-0.931+15.593i			
9	-0.951+17.503i	-0.951+17.503i	-0.951+17.503i			
10	-0.969+19.413i	-0.969+19.413i	-0.969+19.413i			

Table 19: Resulting wave numbers (k_n) for the first 11 wave modes (n) for different values of Δs and fixed N_y (100) and Δk (0.05).

B.4 Concluding Remarks

Although only the sensitivity of the linear lateral depth profiles is investigated in this section, it is believed that the sensitivity of the other depth profiles is more or less of the same order of magnitude, since the same modeling principles apply. The sensitivity analysis shows that for non-dimensional (scaled) quantities our model is hardly sensitive to changes in Δk and Δs and only slightly to changes in N_y.

Appendix C: Wave Numbers and Length Scales for Idealized Depth Profiles

C.1 Linear Depth Profiles



Figure 34. k_r,k_i-plot for the positive Kelvin mode and the first four positive Poincaré modes for linear depth profiles with (a) positive and (b) negative slopes.

	Negative (incoming) Kelvin		Positive (reflected) Kelvin	
s [-]	k_0^{inc} [-]	L^{*} [km]	k ₀ [-]	L^{*} [km]
0	-1.000	767	1.000	767
0.5	-1.058	725	0.955	803
1.1	-1.159	662	0.912	841
1.5	-1.266	606	0.889	863
1.95	-1.503	510	0.866	886
-0.5	-0.955	803	1.058	725
-1.1	-0.912	841	1.159	662
-1.5	-0.889	863	1.266	606
-1.95	-0.866	886	1.503	510

Table 20. Wave numbers and wave lengths of incoming (k_0^{inc}) and (k_0) reflected Kelvin waves for linear depth profiles with different slopes (s)

	s = 0 $s = 0.5$		s = 0.5		s = -	0.5
n	k _n [-]	L_n^* [km]	k _n [-]	L_n^* [km]	k _n [-]	L_n^* [km]
1	0.0+1.8i	66.6	-0.1+1.8i	66.4	0.1+1.8i	66.4
2	0.0+3.8i	32.2	-0.1+3.8i	32.1	0.1+3.8i	32.1
3	0.0+5.7i	21.3	-0.1+5.7i	21.3	0.1+5.7i	21.3
4	0.0+7.6i	16.0	-0.1+7.7i	16.0	0.1+7.7i	16.0
5	0.0+9.6i	12.8	-0.1+9.6i	12.8	0.1+9.6i	12.8
6	0.0+11.5i	10.6	-0.1+11.5i	10.6	0.1+11.5i	10.6
7	0.0+13.4i	9.1	-0.1+13.4i	9.1	0.1+13.4i	9.1
8	0.0+15.3i	8.0	-0.1+15.3i	8.0	0.1+15.3i	8.0
9	0.0+17.3i	7.1	-0.1+17.3i	7.1	0.1+17.3i	7.1
10	0.0+19.2i	6.4	-0.1+19.2i	6.4	0.1+19.2i	6.4

Table 21. Wave numbers (k_n) and e-folding decay lengths (L_n^*) for the first ten Poincaré modes for linear depth profiles with different slopes (s)



C.2 S-curved Depth Profiles

Figure 35. k_r,k_i-plot for the positive Kelvin mode and the first four positive Poincaré modes for Scurved depth profiles with (a) positive and (b) negative amplitudes.

	Negative (incoming) Kelvin		Positive (reflected) Kelvin	
A^*	k_0^{inc} [-]	L^{*} [km]	k ₀ [-]	L^{*} [km]
[m]				
0	-1.000	767	1.000	767
5	-1.052	729	0.958	800
15	-1.184	648	0.905	848
30	-1.565	490	0.861	890
38	-2.501	307	0.849	904
-5	-0.958	800	1.052	729
-15	-0.905	848	1.184	648
-30	-0.861	891	1.565	490
-38	-0.849	904	2.501	307

Table 22. Wave numbers and wave lengths of incoming (k_0^{inc}) and (k_0) reflected Kelvin waves for S-curved depth profiles with different amplitudes (A^*)

	$A^* =$	$A^* = 0 m$		$A^* = 5 m$		-5 m
n	k _n [-]	L_n^* [km]	k _n [-]	L_n^* [km]	k _n [-]	L_n^* [km]
1	0.0+1.8i	66.6	-0.1+1.8i	66.8	0.1+1.8i	66.8
2	0.0+3.8i	32.2	-0.0+3.8i	32.2	0.0+3.8i	32.2
3	0.0+5.7i	21.3	-0.1+5.7i	21.3	0.1+5.7i	21.3
4	0.0+7.6i	16.0	-0.0+7.6i	16.0	0.0+7.6i	16.0
5	0.0+9.6i	12.8	-0.1+9.6i	12.8	0.1+9.6i	12.8
6	0.0+11.5i	10.6	-0.0+11.5i	10.6	0.0+11.5i	10.6
7	0.0+13.4i	9.1	-0.1+13.4i	9.1	0.1+13.4i	9.1
8	0.0+15.3i	8.0	-0.1+15.3i	8.0	0.1+15.3i	8.0
9	0.0+17.3i	7.1	-0.1+17.3i	7.1	0.1+17.3i	7.1
10	0.0+19.2i	6.4	-0.1+19.2i	6.4	0.1+19.2i	6.4

Table 23. Wave numbers (k_n) and e-folding decay lengths $(L_n^{\,*})$ for the first ten Poincaré modes for S-curved depth profiles with different amplitudes (A^*)



C.3 Symmetric Sinusoidal Depth Profiles

Figure 36. k_r,k_i-plot for the positive Kelvin mode and the first four positive Poincaré modes for symmetric sinusoidal depth profiles with (a) positive and (b) negative amplitudes.

	Negative (incom	Negative (incoming) Kelvin		ted) Kelvin
$A^*[m]$	$\mathbf{k}_{0}^{\text{inc}}$ [-]	L^{*} [km]	k ₀ [-]	L^{*} [km]
0	-1.000	767	1.000	767
5	-1.014	757	1.014	757
15	-1.047	732	1.047	732
25	-1.095	701	1.095	701
29	-1.123	683	1.123	683
-5	-0.988	777	0.988	777
-15	-0.967	793	0.967	793
-25	-0.951	806	0.951	806
-29	-0.953	805	0.953	805

Table 24. Wave numbers and wave lengths of incoming (k_0^{inc}) and (k_0) reflected Kelvin waves for symmetric sinusoidal depth profiles with different amplitudes (A^*)

	$A^* =$	0 m	$A^* = 5$	m	$A^* = -1$	5 m
n	k _n [-]	L_n^* [km]	k _n [-]	L_n^* [km]	k _n [-]	L_n^* [km]
1	0.0+1.8i	66.6	-0.0+2.0i	61.6	-0.0+1.7i	71.6
2	0.0+3.8i	32.2	-0.0+3.8i	32.1	-0.0+3.8i	32.1
3	0.0+5.7i	21.3	-0.0+5.7i	21.3	-0.0+5.7i	21.3
4	0.0+7.6i	16.0	-0.0+7.7i	16.0	-0.0+7.7i	16.0
5	0.0+9.6i	12.8	-0.0+9.6i	12.8	-0.0+9.6i	12.8
6	0.0+11.5i	10.6	-0.0+11.5i	10.6	-0.0+11.5i	10.6
7	0.0+13.4i	9.1	-0.0+13.4i	9.1	-0.0+13.4i	9.1
8	0.0+15.3i	8.0	-0.0+15.3i	8.0	-0.0+15.3i	8.0
9	0.0+17.3i	7.1	-0.0+17.3i	7.1	-0.0+17.3i	7.1
10	0.0+19.2i	6.4	-0.0+19.2i	6.4	-0.0+19.2i	6.4

Table 25. Wave numbers (k_n) and e-folding decay lengths (L_n^*) for the first ten Poincaré modes for symmetric sinusoidal depth profiles with different amplitudes (A^*)



C.4 Asymmetric Sinusoidal Depth Profiles

Figure 37. k_r,k_i-plot for the positive Kelvin mode and the first four positive Poincaré modes for asymmetric sinusoidal depth profiles with (a) positive and (b) negative amplitudes.

	Negative (incoming) Kelvin		Positive (reflected) Kelvin	
$A^*[m]$	k_0^{inc} [-]	L^{*} [km]	k ₀ [-]	L^{*} [km]
0	-1.000	767	1.000	767
5	-0.988	776	1.034	742
15	-0.966	794	1.128	680
25	-0.947	810	1.326	578
29	-0.944	813	1.637	468
-5	-1.013	758	0.972	789
-15	-1.040	738	0.927	828
-25	-1.071	716	0.895	857
-29	-1.086	706	0.896	856

Table 26. Wave numbers and wave lengths of incoming (k_0^{inc}) and (k_0) reflected Kelvin waves for asymmetric sinusoidal depth profiles with different amplitudes (A^*)

	$A^* = 0 m$		$A^* = 5 m$		A [*] = -5 m	
n	kn	Ln*	kn	Ln*	k _n [-]	L_n^* [km]
1	0.0+1.8i	66.6	0.1+1.9i	63.2	-0.1+1.7i	70.3
2	0.0+3.8i	32.2	0.0+3.8i	32.2	-0.0+3.8i	32.2
3	0.0+5.7i	21.3	-0.0+5.7i	21.3	0.0+5.7i	21.3
4	0.0+7.6i	16.0	0.0+7.7i	16.0	-0.0+7.7i	16.0
5	0.0+9.6i	12.8	0.0+9.6i	12.8	-0.0+9.6i	12.8
6	0.0+11.5i	10.6	0.0+11.5i	10.6	0.0+11.5i	10.6
7	0.0+13.4i	9.1	-0.0+13.4i	9.1	-0.0+13.4i	9.1
8	0.0+15.3i	8.0	-0.0+15.3i	8.0	-0.0+15.3i	8.0
9	0.0+17.3i	7.1	-0.0+17.3i	7.1	-0.0+17.3i	7.1
10	0.0+19.2i	6.4	-0.0+19.2i	6.4	-0.0+19.2i	6.4

Table 27. Wave numbers (k_n) and e-folding decay lengths (L_n^*) for the first ten Poincaré modes for asymmetric sinusoidal depth profiles with different amplitudes (A^*)

Appendix D: Lateral Amplitude Structures for Idealized Depth Profiles

D.1 Linear Depth Profiles

Positive s



Figure 38. ζ-amplitudes for the positive and negative Kelvin waves and the first positive Poincaré mode corresponding to a linear depth profile with positive slopes.



Figure 39. u-amplitudes for the positive and negative Kelvin waves and the first positive Poincaré mode corresponding to a linear depth profile with positive slopes.



Figure 40. v-amplitudes for the positive and negative Kelvin waves and the first positive Poincaré mode corresponding to a linear depth profile with positive slopes.

D.2 S-curved Depth Profiles

Positive A^{*}



Figure 41. ζ-amplitudes for the positive and negative Kelvin waves and the first positive Poincaré mode corresponding to S-curved profile with positive A^{*}.



Figure 42. u-amplitudes for the positive and negative Kelvin waves and the first positive Poincaré mode corresponding to S-curved profile with positive A^{*}.

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Figure 43. v-amplitudes for the positive and negative Kelvin waves and the first positive Poincaré mode corresponding to S-curved profile with positive A^{*}.

D.3 Symmetric Sinusoidal Depth Profiles

Positive A^{*}



Figure 44. ζ-amplitudes for the positive and negative Kelvin waves and the first positive Poincaré mode corresponding to symmetric sinusoidal profile with positive A^{*}.

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Figure 45. u-amplitudes for the positive and negative Kelvin waves and the first positive Poincaré mode corresponding to symmetric sinusoidal profile with positive A^{*}.



Figure 46. v-amplitudes for the positive and negative Kelvin waves and the first positive Poincaré mode corresponding to symmetric sinusoidal profile with positive A^{*}.



Negative A^{*}

Figure 47. ζ-amplitudes for the positive and negative Kelvin waves and the first positive Poincaré mode corresponding to symmetric sinusoidal profile with negative A^{*}.



Figure 48. u-amplitudes for the positive and negative Kelvin waves and the first positive Poincaré mode corresponding to symmetric sinusoidal profile with negative A^{*}.



Figure 49. v-amplitudes for the positive and negative Kelvin waves and the first positive Poincaré mode corresponding to symmetric sinusoidal profile with negative A*.

D.4 Asymmetric Sinusoidal Depth Profiles

Positive A^{*}



Figure 50. ζ-amplitudes for the positive and negative Kelvin waves and the first positive Poincaré mode corresponding to asymmetric sinusoidal profile with positive A^{*}.

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Figure 51. u-amplitudes for the positive and negative Kelvin waves and the first positive Poincaré mode corresponding to symmetric sinusoidal profile with positive A^{*}.



Figure 52. v-amplitudes for the positive and negative Kelvin waves and the first positive Poincaré mode corresponding to asymmetric sinusoidal profile with positive A^{*}.



0.5

0

У

shallow

Negative A^{*}

-0.5

deep

Figure 53. ζ-amplitudes for the positive and negative Kelvin waves and the first positive Poincaré mode corresponding to asymmetric sinusoidal profile with negative A^{*}.

-0.5

deep

0

У

shallow

0.5



Figure 54. u-amplitudes for the positive and negative Kelvin waves and the first positive Poincaré mode corresponding to symmetric sinusoidal profile with negative A^{*}.



Figure 55. v-amplitudes for the positive and negative Kelvin waves and the first positive Poincaré mode corresponding to asymmetric sinusoidal profile with negative A^{*}.

Appendix E: Wave Energy Correction

In uniform depth the positive and negative wave modes (both Kelvin and Poincaré) had the same structures for \hat{z} , \hat{u} and \hat{v} , only in opposite directions. Therefore, the energy of the positive and negative wave modes was the same as long as the wave structures were forced by the same coastal wave amplitude \hat{z}_0 . However, by allowing lateral depth variations, the \hat{z} -, \hat{u} - and \hat{v} -structures and, hence, the potential and kinetic energy of the wave modes are altered. Therefore, the positive and negative modes do not necessarily represent the same amount of wave energy anymore. In order to be able to evaluate the \hat{z} -, \hat{u} - and \hat{v} -structures in a fair manner, the wave modes are analyzed representing the same amount of energy. For this purpose, the wave energy will be considered in this section.

As shown in De Swart (2008), the total wave energy (E) is the sum of the potential (E_p) and kinetic energy (E_k) of the wave (given in non-dimensional notation):

$$E = E_p + E_k, \qquad (Eq. 40)$$

With:

$$E_p = \frac{1}{2}z^2,$$
 (Eq. 41)

$$E_k = \frac{1}{2}H(v^2 + u^2).$$
 (Eq. 42)

In our case H (indicating uniform depth) in (42) is replaced by h(y), since we deal with depth variations in y-direction. In order to be able to evaluate the structures of the wave modes we would like to know the amount of energy of the waves averaged over a single wavelength (L) and integrated over the width of the channel (B). Then the energy equations become (with <...> indicating the average operator for spatially averaging over L):

$$\int_{-B/2}^{B/2} \langle E \rangle dy = \int_{-B/2}^{B/2} \langle E_p \rangle dy + \int_{-B/2}^{B/2} \langle E_k \rangle dy, \qquad (Eq. 43)$$

With:

$$\int_{-B/2}^{B/2} \langle E_p \rangle dy = \int_{-B/2}^{B/2} \langle \frac{1}{2} z^2 \rangle dy = \int_{-B/2}^{B/2} \left| \hat{z} \right|^2 dy, \qquad (Eq. 44)$$

$$\int_{-B/2}^{B/2} \langle E_k \rangle dy = \int_{-B/2}^{B/2} h(y) \left\langle \frac{1}{2} (u^2 + v^2) \right\rangle dy = \int_{-B/2}^{B/2} h(y) \left(|\hat{u}|^2 + |\hat{v}|^2 \right) dy.$$
 (Eq. 45)

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Since we deal with numerical expressions instead of continuous functions for \hat{Z} , \hat{u} and \hat{v} , the energy function has to be integrated over the channel width B by means of a numerical integration method. Therefore, Simpson's Rule of Integration is applied (Kreyszig, 1999). To apply Simpson's Rule of Integration, the interval of integration ($-B/2 \le y \le B/2$) needs to be divided into an even number of equal subintervals of length a = B/N (with N is an even number). For each two subintervals the function f(y) is approximated by the Lagrange polynomial p_2 (y), i.e. a parabola. By summing all these approximations of every set of two subintervals, Simpson's rule is obtained:

$$\int_{-B/2}^{B/2} f(y) dy \approx \frac{a}{3} \left(f_0 + 4f_1 + 2f_2 + 4f_3 + \dots + 2f_{N-2} + 4f_{N-1} + f_N \right).$$
 (Eq. 46)

This rule can be applied to both the integrals for potential and kinetic energy, so that the total energy can be calculated for each wave mode. The $\hat{z} \cdot , \hat{u} \cdot$ and \hat{v} -fields can be corrected for differences in wave energy by division with the square root of the wavelength-averaged, width integrated wave energy:

$$E_{corr} = \sqrt{\int_{-B/2}^{B/2} \langle E \rangle dy} \quad .$$
 (Eq. 47)

This way the wave modes can be evaluated, representing an equal amount of energy.

Appendix F: Lateral Cross-sections of the Southern North Sea



Figure 56. Overview of the six lateral cross-sections that are used to determine a realistic lateral depth profile of the Southern North Sea. The bathymetrical data is taken from Van der Veen (2008) and originated from Boon and Gerritsen (1997) and Ten Brummelhuis et al. (1997).

Appendix G: Collocation Technique for Modeling Sand Extraction

In Figure 57 a schematic overview of the basin, the wave modes and the boundary conditions is presented. In this figure the semi-enclosed basin is divided in a part with (basin 1) and a part without (basin 2) sand extraction trench (marked by the black shaded area). The characters indicate the forcing by the negative (incoming) Kelvin wave in basin 2 (F), the positive (reflected) Kelvin and a finite set of positive Poincaré waves in basin 1 $\binom{V}{N_2}$ and the positive (reflected) Kelvin and a finite set of positive Poincaré waves in basin 1 $\binom{V}{N_2}$ and the positive (reflected) Kelvin and a finite set of positive Poincaré waves in basin 1 $\binom{V}{N_3}$ respectively. The arrows indicate the direction in which the waves are propagating.



Figure 57. Overview of the wave modes and boundary conditions in the semi-enclosed basin, which is divided in a part with (basin 1) and a part without (basin 2) sand extraction trench.

The ζ -, u- and v-fields in each part of the basin (i.e. (1) with and (2) without sand extraction) can be obtained by means of the following equations:

$$Z_{1} = \left[\sum_{n=0}^{M+1} C_{2;n} \hat{Z}_{n}(y) \exp(i[k_{n}x-t])\right]_{N2} + \left[\sum_{n=0}^{M+1} C_{3;n} \hat{Z}_{n}(y) \exp(i[k_{n}x-t])\right]_{N3},$$

$$Z_{2} = \hat{Z}_{0}^{inc}(y) \exp(i[k_{0}^{inc}x-t]) + \left[\sum_{n=0}^{M+1} C_{1;n} \hat{Z}_{n}(y) \exp(i[k_{n}x-t])\right]_{N1},$$

$$u_{1} = \left[\sum_{n=0}^{M+1} C_{2;n} \hat{u}_{n}(y) \exp(i[k_{n}x-t])\right]_{N2} + \left[\sum_{n=0}^{M+1} C_{3;n} \hat{u}_{n}(y) \exp(i[k_{n}x-t])\right]_{N3},$$

$$u_{2} = \hat{u}_{0}^{inc}(y) \exp(i[k_{0}^{inc}x-t]) + \left[\sum_{n=0}^{M+1} C_{1;n} \hat{u}_{n}(y) \exp(i[k_{n}x-t])\right]_{N1},$$

$$v_{1} = \left[\sum_{n=0}^{M+1} C_{2;n} \hat{v}_{n}(y) \exp(i[k_{n}x-t])\right]_{N2} + \left[\sum_{n=0}^{M+1} C_{3;n} \hat{v}_{n}(y) \exp(i[k_{n}x-t])\right]_{N3},$$

$$v_{2} = \hat{v}_{0}^{inc}(y) \exp(i[k_{0}^{inc}x-t]) + \left[\sum_{n=0}^{M+1} C_{1;n} \hat{v}_{n}(y) \exp(i[k_{n}x-t])\right]_{N1}.$$
(Eq. 48)

We use a collocation technique with M+1 collocation points to find the unknown complex coefficient vectors C_1 , C_2 and C_3 (each of length M+1). The collocation procedure is expressed as:

$$Ac = b, (Eq. 49)$$

Where:

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ 0 & A_{32} & A_{33} \end{bmatrix}, \qquad c = \begin{bmatrix} \mathbf{v} \\ \mathbf{v}_1 \\ \mathbf{v}_2 \\ \mathbf{v}_2 \\ \mathbf{v}_3 \end{bmatrix}, \qquad b = \begin{bmatrix} \mathbf{v} \\ \mathbf{b}_1 \\ \mathbf{v}_2 \\ \mathbf{b}_2 \\ \mathbf{0} \end{bmatrix}.$$
(Eq. 50)

A is a 3(M+1)*3(M+1) matrix, i.e. (M+1) wave modes and (M+1) collocation points for each boundary condition. For example:

$$A_{11} = \begin{bmatrix} \boldsymbol{z}_0 \left(\boldsymbol{x} = \boldsymbol{L}_{trench}, \boldsymbol{y} = \boldsymbol{y}_0 \right) & \mathbf{L} & \boldsymbol{z}_M \left(\boldsymbol{x} = \boldsymbol{L}_{trench}, \boldsymbol{y} = \boldsymbol{y}_0 \right) \\ \mathbf{M} & \mathbf{M} & \mathbf{M} \\ \boldsymbol{z}_0 \left(\boldsymbol{x} = \boldsymbol{L}_{trench}, \boldsymbol{y} = \boldsymbol{y}_M \right) & \mathbf{L} & \boldsymbol{z}_M \left(\boldsymbol{x} = \boldsymbol{L}_{trench}, \boldsymbol{y} = \boldsymbol{y}_M \right) \end{bmatrix}_{N1}$$
(Eq. 51)

Likewise, the other terms in the first row of A are the matrices with the ζ -values at $x^* = L^*_{trench}$ corresponding to N_2 and N_3 . The second and third row of A respectively represent the values of uh at $x^* = L^*_{trench}$ and u at $x^* = 0$ corresponding to N_1, N_2 and N_3 . The vectors c and b are 3(M+1) column vectors, where c represents the collocation coefficients for the wave modes for each boundary condition and b the forcing at the collocation points for each boundary condition.

A sufficient number of collocation points needs to be taken into account in order to incorporate the shape of the sand extraction trench sufficiently accurate (because of its relatively small scale). It is found that 41 collocation points is sufficient to model most of the sand extraction trenches of interest (as indicated in Figure 58 for an extraction trench of $B^*_{trench} = 25$ km and $h^*_{trench;max} = 20$ m). However, for extraction trenches of relatively small width and relatively large depth, 41 collocation points is not sufficient (see Figure 59 for an extraction trench of $B^*_{trench} = 10$ km and $h^*_{trench;max} = 10$ m). In this particular case, only two collocation points are located within the extraction trench. It is found that, in this case, our model has difficulties with solving the boundary conditions. This problem may be solved by considerably increasing the number of collocation points by a factor 3 does not solve the problem. Increasing the number of collocation even further, leads to limitations with respect to computation time and the number of elements that the program The MathworksTM MATLAB[®] is able to handle. Therefore, our model is particularly capable of modeling extraction trenches of larger width (i.e. $B^*_{trench} = 25$ km or larger), which can be modeled accurately with a smaller number of collocation points.



Figure 58. Lateral depth profile of the Southern North Sea with a sand extraction trench of $B^*_{trench} = 25$ km and $h_{trench;max} = 20$ m with the location of the 41 collocation points.



Figure 59. Lateral depth profile of the Southern North Sea with a sand extraction trench of $B^*_{trench} = 10$ km and $h_{trench;max} = 20$ m with the location of the 41 collocation points.

After solving the complex coefficients c, the ζ -, u- and v-fields are found using equations (48). As required by the boundary conditions the ζ - and hu-fields are continuous across the transition line. However, the u-field has a discontinuity at this location, as expected by the abrupt difference in water depth.

Appendix H: Wave Solutions for the Southern North Sea with and without Sand Extraction

In this Appendix the properties of the (modified) Kelvin and Poincaré modes are discussed for the Southern North Sea with uniform depth, a realistic lateral depth profile and sand extraction trenches with dimensions $B^*_{trench} = 10$ and 25 km and $h^*_{trench;max} = 6$, 12 and 20 m. These trench dimensions correspond to the most ($B^*_{trench} = 25$ km) and least ($B^*_{trench} = 10$ km) extreme extraction scenarios currently under investigation. In section H.1 the properties of the (modified) wave modes are discussed in terms of their wave numbers and typical length scales. Section H.2 examines the wave energy corrected lateral amplitude structures of the wave modes. The numerical parameters that are used in the model are the same as for the other lateral depth profiles and are presented in Table 3 (see section 4.2).

H.1 Typical Length Scales of Modified Wave Modes

Kelvin Waves

In Figure 60 the wave numbers of the positive (reflected) Kelvin wave and the first four positive Poincaré waves are plotted in the complex wave number plane for all the Southern North Sea profiles under consideration. In Table 28 the corresponding values of the wave lengths L_0^* of the Kelvin waves are presented. We find that $|k_0|$ is decreased (i.e. L_0^* increased) for the negative (incoming) Kelvin wave and increased (i.e. L_0^* decreased) for the positive (reflected) Kelvin wave by adopting the realistic lateral depth profile instead of assuming uniform depth. This behavior can be linked to the water depth h(y), which is relatively deeper than uniform depth where the negative Kelvin wave propagates (corresponding to a larger c, hence smaller k) and relatively shallower where the positive Kelvin wave propagates (corresponding to a smaller c, hence larger k). As a result of sand extraction the wave numbers decrease both for the negative and positive Kelvin wave number can be explained by the locally increasing water depth due to sand extraction. The slight decrease of the negative Kelvin wave number cannot be explained by the locally increase in the average (overall) water depth of the basin as a result of sand extraction.



Figure 60. k_r , k_i -plot for the positive Kelvin mode and the first four positive Poincaré modes for the Southern North Sea with $B_{trench} = 25$ km (a) and $B_{trench} = 10$ km (b).

	Negative (inc	oming) Kelvin	Positive (reflected) Kelvi	
	k_0^{inc} [-]	L_0^* [km]	k ₀ [-]	L_0^* [km]
H _{uniform}	-1.000	756	1.000	756
h(y) _{North Sea}	-0.982	769	1.067	709
$h(y)_{North Sea}, h_{trench} = 6m,$				
$\mathbf{B}^*_{\text{trench}} = 25 \text{ km}$	-0.979	772	1.050	720
$h(y)_{North Sea}, h_{trench}^{*} = 12m,$				
$\mathbf{B}^*_{\text{trench}} = 25 \text{ km}$	-0.975	775	1.033	731
$h(y)_{North Sea}, h_{trench} = 20m,$				
$\mathbf{B}^*_{\text{trench}} = 25 \text{ km}$	-0.971	779	1.013	746
$h(y)_{North Sea}, h_{trench} = 6m,$				
$\mathbf{B}^*_{\text{trench}} = 10 \text{ km}$	-0.981	770	1.059	714
$h(y)_{North Sea}, h_{trench} = 12m,$				
$\mathbf{B}^*_{\text{trench}} = 10 \text{ km}$	-0.980	771	1.052	719
$h(y)_{North Sea}, h_{trench} = 20m,$				
$B^*_{trench} = 10 \text{ km}$	-0.978	773	1.043	725

Table 28. Wave numbers and wave lengths of incoming reflected Kelvin waves for uniform depth, a realistic lateral depth profile of the Southern North Sea and different sand extraction scenarios.

Poincaré Waves

Most positive Poincaré wave numbers $k_{n>0}$ obtain, besides a larger imaginary part $k_{n,im}$ (i.e. decreasing L_n^*), also a small real part $k_{n,re}$ for the realistic lateral depth profile compared to uniform depth (see Figure 60 and Table 29). This implies that the Poincaré modes obtain a propagative character, but their influence on the wave field is present over a shorter distance than for uniform depth. The presence of a sand extraction trench slightly alters the real and imaginary components of the wave numbers of the first Poincaré modes.

	H _{uniform}		h(y) _{North Sea}		$h(y)_{North Sea}, B_{trench}^* = 25 \text{ km}$		$h(y)_{North Sea}, B_{trench}^* = 10 \text{ km}$	
					$h^*_{trench} = 20 \text{ m}$		$h^*_{trench} = 6 m$	
n	k _n [-]	L_n^* [km]	k _n [-]	L_n^* [km]	k _n [-]	L_n^* [km]	k _n [-]	L_n^* [km]
1	0.0+2.3i	51.6	0.2+2.8i	43.0	0.1+2.7i	44.4	0.2+2.8i	43.3
2	0.0+4.8i	25.2	0.0+5.3i	22.9	-0.2+5.4i	22.3	-0.0+5.3i	22.9
3	0.0+7.2i	16.8	0.0+7.6i	15.8	-0.1+8.2i	14.6	-0.0+7.7i	15.6
4	0.0+9.6i	12.5	0.1+10.0i	12.0	1.5+9.9i	12.2	0.1+10.1i	11.9
5	0.0+12.0i	10.0	0.1+12.4i	9.7	-0.2+11.6i	10.4	0.3+12.4i	9.7
6	0.0+14.4i	8.4	0.2+14.7i	8.2	-0.4+14.5i	8.3	0.3+14.5i	8.3
7	0.0+16.8i	7.2	0.2+17.1i	7.0	-0.3+17.4i	6.9	0.1+16.9i	7.1
8	0.0+19.2i	6.3	0.2+19.5i	6.2	0.0+20.3i	5.9	-0.0+19.4i	6.2
9	0.0+21.6i	5.6	0.2+21.9i	5.5	1.2+21.9i	5.5	-0.0+22.0i	5.5
10	0.0+24.0i	5.0	0.2+24.3i	5.0	0.1+23.9i	5.0	0.1+24.6i	4.9

Table 29. Wave numbers (k_n) and e-folding decay lengths (L_n^*) for the first ten Poincaré modes for different lateral depth profiles of the Southern North Sea.

H.2 Lateral Amplitude Structures of Modified Wave Modes

Kelvin Waves

The $\hat{z} -, \hat{u}$ - and \hat{v} -structures of the incoming and reflected Kelvin waves and the first positive Poincaré mode are presented in Figure 61, Figure 62, Figure 63 respectively (for $B^*_{trench} = 25$ km, when sand extraction is applicable). It is found that the \hat{z} -structure of the positive Kelvin wave slightly increases near the boundary at y = -B/2 for the realistic North Sea profile, but hardly alters for the negative Kelvin wave. The \hat{u} -structures of both Kelvin waves are in phase with their \hat{z} -structures. The \hat{u} -structure of the positive Kelvin wave shows a slight increase at the side of the channel along which it propagates (i.e. y = -B/2). This might be linked to the shallower water depth at this side of the channel compared to uniform depth. The \hat{u} -structure of the negative Kelvin wave shows a slight decrease (in absolute terms) at the side of the channel along which it propagates (i.e. y = B/2), probably because it is relatively deeper at the English side the channel. In addition, both Kelvin waves obtain a small \hat{v} -structure. Sand extraction causes \hat{z} of the positive Kelvin to decrease slightly near y = -B/2. The \hat{u} - and \hat{v} -structures become distorted at the location of the sand extraction trench, resulting in lower velocities. These trends are more apparent for larger extraction dimensions.



Figure 61. ζ -amplitudes for the positive and negative Kelvin waves and the first positive Poincaré mode corresponding to different lateral depth profiles of the Southern North Sea (with $B^*_{trench} = 25$ km when sand extraction is applicable).



Figure 62. u-amplitudes for the positive and negative Kelvin waves and the first positive Poincaré mode corresponding to different lateral depth profiles of the Southern North Sea (with $B^*_{trench} = 25$ km when sand extraction is applicable).

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Figure 63. v-amplitudes for the positive and negative Kelvin waves and the first positive Poincaré mode corresponding to different lateral depth profiles of the Southern North Sea (with $B^*_{trench} = 25$ km when sand extraction is applicable).

First Positive Poincaré Wave

The \hat{z} -, \hat{u} - and \hat{v} - structures of the first positive Poincaré mode are considerably altered for a realistic depth profile compared to uniform depth. The presence of sand extraction causes a considerable distortion in the \hat{u} - and \hat{v} - structures at the location of the extraction trench.