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Link performance of the time offset transmitted reference system

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Summary

There has been an interest in recent years in exploiting noise modulation for the transmission of analog and digital information in radio communications. Because of their broad bandwidth noise signals can be used to spread the information signal letting us to transmit an ultra wideband (UWB) signal. Noise communication can use a delay line both at the transmitter and at the receiver. To distinguish between the modulated signal and the reference signal the time offset is much larger then the coherence time of the noise carrier.

In this thesis the performance of the time offset transmitted reference is determined in the presence of the AWGN in terms of signal-to-beat noise ratio (SNR) and bit error rate (BER). The broadband noise carrier is modeled as a Gaussian distribution. The result is a simple expression where the beat-noise and the number of users are involved.

By the study of a band limited broadband noise carrier, the bit error rate is determined as a function of E_b/N_0. It is proved that the number of users and the processing gain G limits the BER. Moreover, the BER degrades as the number of users increases. Also it is shown that the BER develops an error floor and this increases rapidly with the number of users. Theoretically we found that the BER at fixed E_b/N_0 attains its minimum at a certain processing gain G.

It was shown that narrowband interferences could jam completely the information signal. Significant improvement in performance can be obtained if Manchester line coding is used. In a multipath environment, the difference between each pair of multipath delays should be larger than the coherence time of the noise carrier.

Simulations are also carried out in Simulink and the results match very closely with the analytical BER.

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List of symbols

$A(t)$ The amplitude of the narrowband signal

B Bandwidth of the noise carrier

$c(t)$ Pseudo noise signal

E_b Received bit energy

f: Frequency variable used as an argument of power spectral density functions

f_0 The carrier frequency for the narrowband signal

G processing gain

$J(t)$ Narrowband interference signal

$H(f)$: transfer function of the broadband noise filter.

$h(t)$ The impulse function of the broadband filter

$h_2(t)$ The time-reversed version of $h(t)$

m_i: Data bit sent by user i

$\hat{m}_i$: Detected data bit

M Number of transmitters

$n(t)$: Additive white Gaussian noise

N Number of multipath

N_0 Power spectral density of the AWGN.

P_e Bit error rate

$R_{nn}(\tau)$ Autocorrelation function of $n(t)$

$R_{xx}(\tau)$ Autocorrelation function of $x(t)$

$s(t)$: transmitted signal

SNR signal-to-beat noise ratio

$S_{nn}(f)$ Power spectral density function of the random process $n(t)$

$S_{xx}(f)$ Power spectral density function of the random process $x(t)$

t: Time variable used as an argument of time signal

T_b: Bite time

v_i: The integrator output

v_J The integrator output resulting only from the narrowband filter $J(t)$

$w(t)$ Broadband filter output

$w_J(t)$ The broadband filter output resulting from the narrowband signal $J(t)$

$x(t)$: Noise carrier

$y(t)$: Received signal

$z(t)$ Mixer output

$z_J(t)$ The mixer output component resulting from the narrowband signal $J(t)$

α_i Time variable used in integrals

$\gamma_i(t)$ Attenuation factor for the *i*-th path

τ : Time difference variable used as an argument of correlation functions

τ_c Coherence time

τ_i : Time offset for user *i*

τ_r The time offset at the receiver.

ν Frequency variable used in integrators

κ_i Propagation delay for the *i*-th path

$\phi(t)$ The phase of the narrowband signal

List of abbreviations

AWGN	Additive white Gaussian noise
BER	Bit error rate
CDSK	Correlation delay shift keying
DCSK	Differential chaos shift keying
FCC	Federal communication commission
NRZ	Non return to zero
TR	Transmitted reference
UWB	Ultra wideband

List of operators and special functions

$\otimes$:	Convolution operator
$\mathrm{E}\{x\}$	Mean value of the random variable x
$\lvert x \rvert$	Absolute value of variable x
$\delta(t)$	Continuous dirac delta function

1 Introduction

The basics of radio transceivers have hardly changed since the first radio communication experiments carried out by Marconi around 1900. In general radio transceivers consist of tuned circuits with limited bandwidth. Sinusoidal waveforms are used as carriers to transport the user information. Hence the transmitter implementation contains expensive filters and oscillators.

Moreover the radio spectrum is limited and there are a lot of users. Therefore multiple access techniques are used to allow many mobile users to share the finite amount of the radio spectrum simultaneously. Examples of radio multiple access techniques are Time Division Multiple Access (TDMA), Frequency Division Multiple Access (FDMA), and Code Division Multiple Access CDMA. These techniques are based on absolute frequency, time, or code sequence, which must be known both at the receiver and the transmitter. All these are extensively discussed in [1]. The conventional radio transmission has some disadvantages. It requires a lot of co-ordination between the transmitter and the receiver. Moreover the narrow bandwidth limits the applications used in the mobile telecommunications. This situation pushed a lot of research groups to look for new communications techniques to increase for instance the radio channel bandwidth and therefore the bit rate of the information signal, or to make the radio transceiver cheaper, or to implement a transceiver to fulfill the two previous requirements.

Recently a new modulation technique has been investigated which breaks with the conventional tuned transceiver and which is not based on a sine waveform carrier. This technique is called Ultra Wideband transmission (UWB). The next section will give a brief description of the advantages of UWB transmission.

1.1 Ultra wideband technology

UWB is defined by the Federal Communication Commission (FCC) as: the bandwidth of the UWB signal is at least 20% of the center frequency or more than 500 MHz [2].

UWB technology holds great promise for a vast array of new applications that have the potential to provide significant benefits in a variety of applications such as tracking and positioning, radars, and telecommunications. Potential applications in telecommunication include wireless local area networks (WLAN), and multiple-access communication systems for short-range or indoor applications with high-speed data transmissions. It is expected that the UWB technology can deliver wireless connections as fast as 100Mbps.

UWB does have many advantages such as:

- By using UWB technique high data rate can be reached.
- Simple RF architecture [3].
- UWB doesn't use sine waves as carriers. Therefore the transceiver doest not contain expensive filters and oscillators. Not using sine waves can also results in no multipath cancellation effects. Non-sinusoidal nature of waves prevents cancellation of waves that occurs when waves are reflected and meet partially or totally out of phase with one another.
- The UWB radio transceiver can result in a cheap implementation. Transceivers will be low cost because the electronics can be completely integrated onto a CMOS.
- Low power spectral density: the low power spectral density enables the UWB system to co-exist with other communication systems [3] (Figure 1.1).

To be able to co-exist with other communication systems some rules were made by the FCC. Figure 1.2 shows the maximum power spectral density of UWB signals for indoor and outdoor communication in the frequency band between one gigahertz and ten gigahertz. The frequency band proposed for UWB systems used in telecommunication systems should be between 3.1 GHz and 10.6GHz [4] for indoor and outdoor communication. In this frequency band the maximum power spectral density should be less then –41dBm/MHz [4]. Therefore the interference from UWB transmission has a minimal impact to narrowband receivers, i.e. power spectral density is below the thermal noise floor [5].

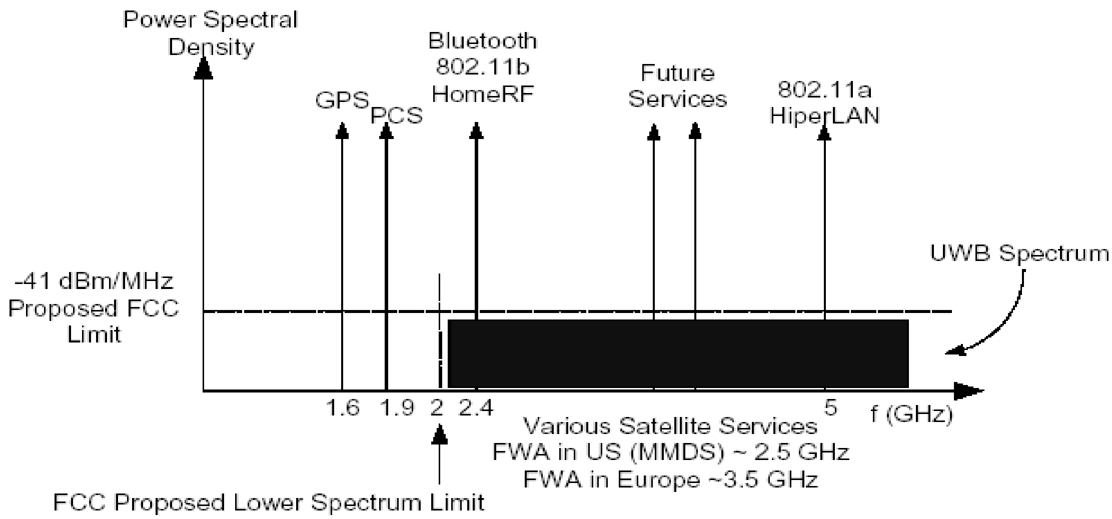

Figure 1.1:UWB power spectral density showing the co-existence of the UWB systems with other communication systems.

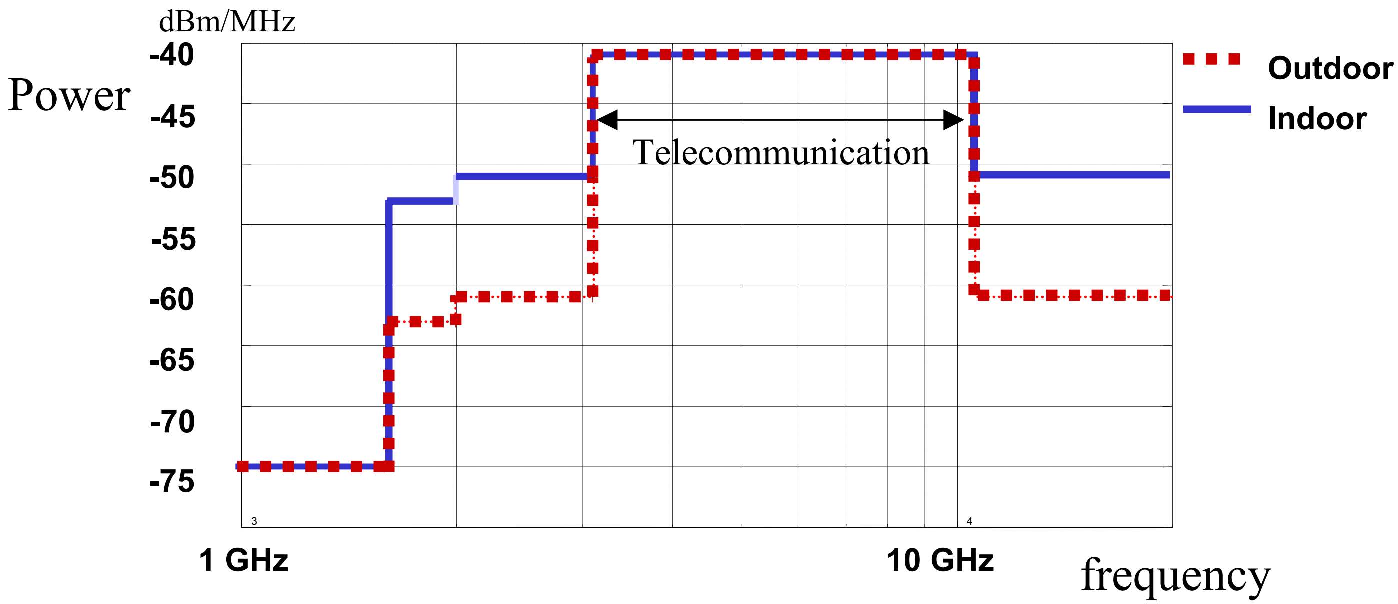

Figure 1.2: Allowed UWB emission for indoor and outdoor telecommunication

The modulation technique proposed for UWB by most research groups is based on ultra-short pulses. It uses pulse position modulation of very short duration pulses. The pulse width is between 0.1 nanosecond and 1.5 nanosecond. To enable multiple access in radio impulse modulation time hopping is used. However, a modulation alternative is to use broadband noise signals. This thesis is about using noise signals for spread spectrum communication. To put this work into context, we will first briefly discuss noise signals.

1.2 Noise signals

Noise signals are bounded, and have aperiodic behavior that exhibits sensitivity to initial conditions. This sensitivity can be demonstrated by giving two very close initial states to a chaotic map; After a few iterations, the two resulting sequences will look completely decorrelated [6]. That means that the time evolution of a noise system, while governed by strictly deterministic equations, is impossible to predict beyond extremely small time scales. Noise is extremely common in nature. In fact, many common non-linear dynamical systems are often found to exhibit chaotic behavior for a certain parameter ranges. In electrical and electronic systems, there are many circuit components that are inherently non-linear, such as diodes, transistors, switches and oscillators. Circuits containing these elements often display chaotic behavior. The inherently broadband nature of noise signals has made them promising for spread spectrum communication systems. Moreover an advantage of a noise-based communication system is a less complicated circuitry in comparison with conventional spread spectrum approaches. Consequently, the weight and volume requirements of the devices are reduced and efficiency is increased. It may be possible to put a complete transmitter or receiver on one small chip [6].

There have been a number of modulation schemes proposed for noise communication. Most of them use a chaotic signal to modulate the information-carrying signal. Chaos Shift Keying (CSK) [7] [8], which uses a chaotic signal to directly modulate digital data, is the most primitive scheme, corresponding to Amplitude Shift Keying (ASK) in conventional systems. Various improved CSK schemes have also been proposed. The most popular is Differential Chaos Shift Keying (DCSK) and its variants. DCSK has shown promising results in simulation, especially in extremely noisy conditions. A brief description of the DCSK scheme will be given in the fourth section in this chapter.

1.3 Conventional Transmitted reference (TR) systems.

This section will give a brief description of the conventional transmitted reference schemes and some disadvantages using these schemes.

The conventional transmitted reference system was studied for the first time in the beginning of the 1950s. Simultaneously at M.I.T and at the USA army some experimental works were done. The conventional transmitted reference systems accomplish spread spectrum operation by transmitting two versions of a wideband signal, unpredictable carrier, one modulated by data and the other unmodulated (Figure 1.3) [9]. These two signals are transmitted separately over two different channels (one may be displaced in frequency from the other). The two signals, being separately recovered by the receiver, are the input for a correlator detector, which recovers the data [9]. The wideband carrier in a conventional transmitted reference system may be random, wideband noise source, unknown by the transmitter and the receiver until the instant it is generated for use in communication.

The conventional transmitted reference does have fundamental weaknesses:

- The available bandwidth is divided between the two signals, the reference and the modulated signal. This system is not bandwidth efficient because half of the bandwidth is not used to transport data.
- The transmitted reference system's two channels may be difficult to match [9].
- Any listener who has access to both transmitted signals easily determines the data.
- The conventional transmitted reference uses two transmitters and two receivers and that makes the cost for the radio implementation expensive.
- But the major problem resulting from the use of two different channels is the interferences that corrupt the two channels. By using two channels the two signals can be affected differently.

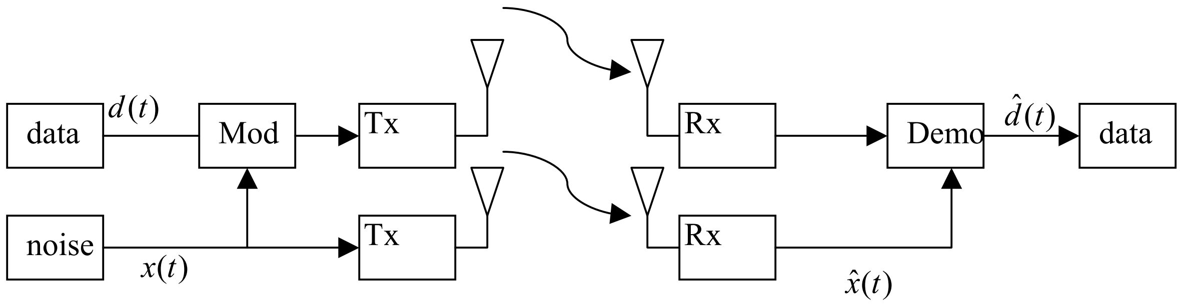

Figure 1.3: Conventional Transmitted Reference system [9]

1.4 New system using transmitted reference principle

The spread-spectrum strategy used in the conventional transmitted reference employing two channels can be more efficiently used if the two signals are sent over one channel. One of the most popular solutions proposed is the differential chaos shift keying DCSK [10]. The structure of a typical DCSK transmitter and receiver is shown in Figure 1.4 [10]. A reference chaotic waveform $x(t)$ is transmitted during the first half of each data bit. If the bit is a ‘1’, $x(t)$ is transmitted again during the second half. If the bit is a ‘0’, $-x(t)$ is transmitted. At the receiver, the signal is delayed by half a bit period and correlated with the undelayed signal to get the decision variable for producing the output data stream. In DCSK, part of the information associated with the chaotic carrier remains unexploited. In fact, the total channel capacity is shared between the reference signal that is transmitted during the first half of the bit period and the modulated signal that is transmitted during the second half of the bit period. A multiple-access technique for the use with DCSK is proposed and analyzed in [12].

To be able to use the whole channel capacity another modulation technique is proposed. This technique is called *correlation delay shift keying* CDSK Figure 1.5 [10]. In CDSK, instead of the sequence transmission of the modulated chaotic signal and the reference, as it is done in DCSK, the two are added together with a certain time delay. This permits a continuous operation of the transmitter and makes the transmitted signal more homogenous.

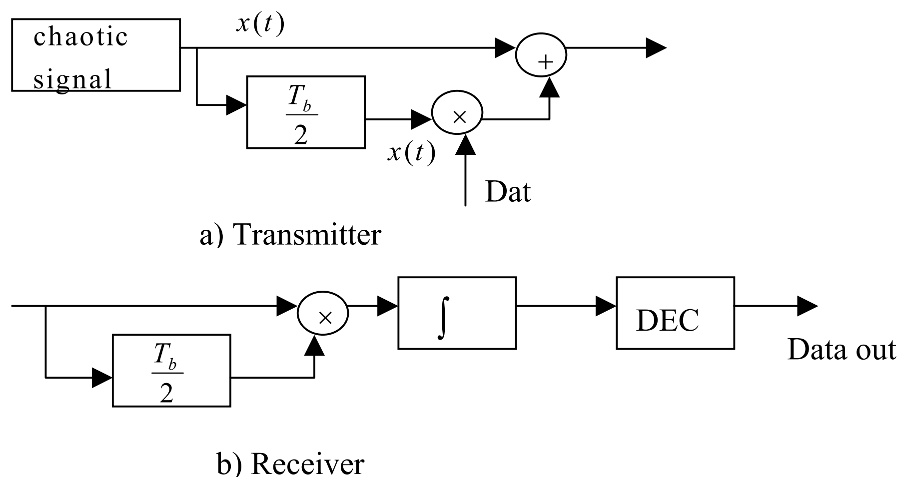

Figure 1.4: DCSK scheme

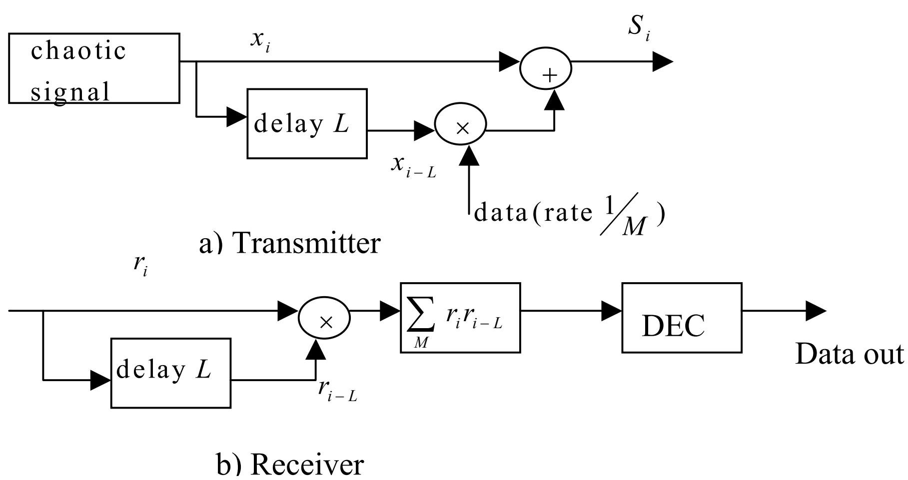

Figure 1.5: CDSK scheme [10]

This thesis will investigate a similar scheme as CDSK. This technique could be called *time offset transmitted reference*. The transmitted reference and the modulated signal are combined at the transmitter and transmitted over one channel. The differences between the two schemes can be summarized in two points: the noise signal in our work has a Gaussian distribution and the chaotic signal scheme discussed in [10] used a symmetric tent map signals and has a uniform distribution. The time delay used in the CDSK scheme is longer than the bit time to be sure that the cross-correlation between the modulated signal and the reference is minimum. But in our work the time delay is much smaller than the bit time.

At the telecommunication Engineering group at the University of Twente another UWB transmission technique is studied. The system is called frequency offset division multiple access FODMA [11]. This technique is based on a frequency offset rather than the time offset and uses also noise signals as a carrier.

1.5 Research goal

The goal of this research is to study the architecture and the principle of the time offset transmitted reference system and to determine the link performance of the time offset transmitted reference scheme taking into account the auto-correlation terms in the case of one user. The link performance for multiple users will also be determined, where the cross-correlation and the number of users are taken into account. The work contains an analytical part and a simulation part and a comparison between the achieved results.

1.6 Thesis organization

This thesis has been organized as follows. Chapter two gives more detailed description of the time offset transmitted reference scheme. Chapter three deals with the theoretical calculation of the link performance in AWGN of the time offset transmitted reference scheme for one user. Chapter four deals with the theoretical calculation of the link performance in AWGN for multiple users. Chapter five deals with the effect of multipath and narrowband interferences on the communication system. Chapter sex presents the simulation results carried out in Simulink. It also presents a comparison between the analytical and simulation results. Conclusions and recommendations on this research are given in chapter 7.

2 Time offset transmitted reference principle

This thesis is about using noise signals for spread spectrum communication. To put this work into context, we will first briefly discuss spread spectrum communication. Then we will discuss a detailed description of the time offset transmitted reference scheme and the principle of this technique.

2.1 Spread spectrum communication

Spread spectrum communication aims at using a transmitted signal designed to have a bandwidth much larger than the minimum required to transmit data at a given rate. The process of increasing the bandwidth of the data signal before transmitting it is called spreading. Conventional spreading techniques fall into two basic classes:

1. Direct Sequence Spread Spectrum (DS-SS): a high-speed pseudo-random binary sequence known as the spreading code is multiplied with the data signal to increase its bandwidth.
2. Frequency Hopping Spread Spectrum (FH-SS): the data signal is modulated by a carrier whose frequency is varied over time by a frequency synthesizer under the control of a high-speed pseudo-random binary sequence.

At the receiver, the signal is subjected to despreading in order to reduce the bandwidth to the original value. This is typically performed by a correlation. Based on the correlation method, the main types of spread spectrum systems are:

1. Stored Reference (SR) systems: in such systems, the transmitter and the receiver are synchronized, the spreading pseudo-random binary sequence by the transmitter is regenerated at the receiver and the received signal is correlated with it to recover the transmitted data. CDMA is stored reference system.
2. Transmitted reference (TR) system: in such systems, the receiver and transmitter are not synchronized. The original spreading sequence is transmitted to the receiver along with the data stream after spreading. The receiver correlates the two signals to despread the data. Such systems are needed when the transmitter and receiver codes cannot be synchronized (or synchronization is too difficult). This is the case when chaotic codes are used for spreading.

In general, transmitted reference communication is advantageous when transmitting through an unknown channel that severely distorts the transmitted waveforms making the use of a stored, phase synchronous filter problematic. The reference, because it passes through the same channel as the modulated signal, is distorted in the same way as the carrier of the modulated signal. That means that no synchronization is needed between the reference signal and the modulated signal.
The next section will investigate the time offset transmitted reference system using a broadband noise signal and based on transmitted reference.

2.2 Time offset transmitted reference architecture

2.2.1 Transmitter

Figure 2.1 shows the architecture of the time offset transmitted reference scheme. The transmitter generates a broadband noise signal with a mean value equal to zero (Figure 2.2). To make difference between the AWGN and the generated signal at the receiver, we will call the generated signal *noise carrier*. The signal is split into two branches. The signal in the upper branch is used as a reference for the modulated signal. The signal in the second branch is delayed by a time offset equal to τ_1 (we will see in the next chapter that the time offset τ_1 should be greater then the coherence time of the noise carrier). The delayed signal is modulated by the information signal. The modulation involves a multiplication of the noise carrier by the information-signal. The modulator is assumed to be an ideal multiplier. That means that the output signal is the product of the incoming noise carrier and the information signal. The information-signal is assumed to be a rectangular polar NRZ signal *m*(t) (Figure 2.3), which takes the values +1 and –1. The two signals coming from the two branches are combined and then transmitted. To be able to detect the information signal at the receiver the time offset τ_1 should be much larger than the coherence time of the noise carrier. In this case the cross correlation between the noise carrier and its shifted version is almost equal to zero [16].
By applying different time offsets τ_k, and different noise carriers, different channels can be defined. This technique can provide a multiple access technique. It can be called *Delay Division Multiple Access* (DDMA). As condition to achieve this kind of multiple access is that the cross correlation between the different noise carriers equal to zero or at least is minimum.

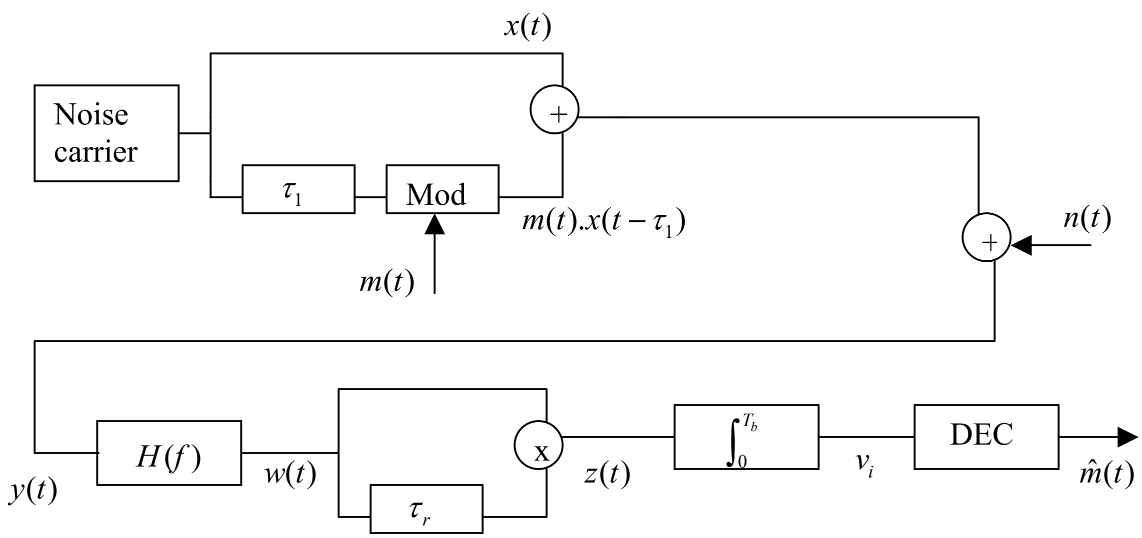

Figure 2.1: The time offset transmitted reference scheme

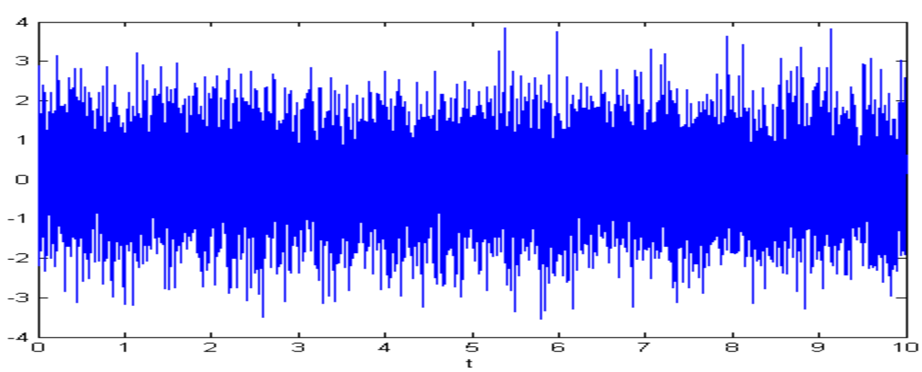

Figure 2.2: Noise carrier with a mean value equal to zero

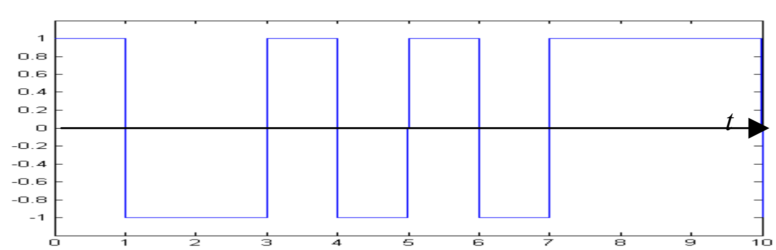

Figure 2.3: Information signal

2.2.2 Receiver

The transmitted signal will be corrupted by additive white Gaussian noise (White noise cannot exist in real life. However, white noise approximates random processes encountered in nature). At the receiver, the received signal passes through a wideband filter to limit the effect of the additive white Gaussian noise. The bandwidth of the broadband filter should be at least equal to the bandwidth of the noise carrier. The signal coming from the broadband filter is split into two signals; the signal in the lower branch is delayed by a time offset equal to τ_r and then multiplied by the original version of the received signal. We will see in the next chapter that if the absolute value of the difference between the time offsets at the receiver τ_r and the at the transmitter τ_1 is much smaller than the coherence time of the noise carrier then the information signal is de-spread directly to base band at the mixer output (Figure 2.4). After low-pass filtering the information signal remains. Figure 2.5 shows the integrator output. Otherwise, if the absolute value of the difference between the time offsets at the receiver τ_r and the at the transmitter τ_1 is much larger than the coherence time of the noise then the signal at the mixer output is a noise with a mean value equal to zero (Figure 2.6). Figure 2.7 shows the integrator output. It is quite impossible for the receiver to take the right decision about the transmitted bit. The entire de-spreading operation includes only a delay line and a correlator to demodulate the signal.

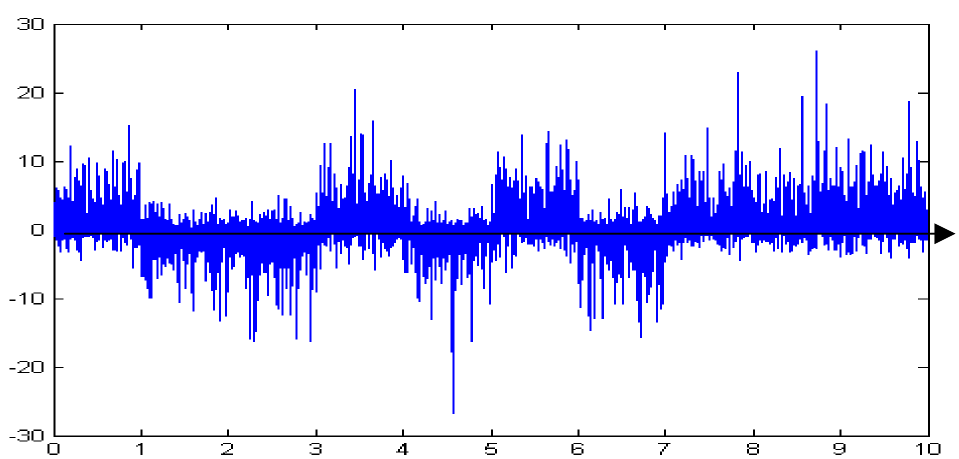

Figure 2.4: The mixer output if the delays at the both receiver and the transmitter are equal

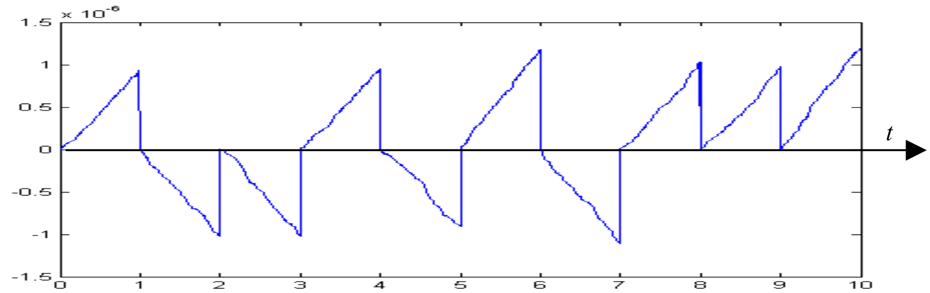

Figure 2.5: Integrator output if the delays at both the receiver and the transmitter are equal

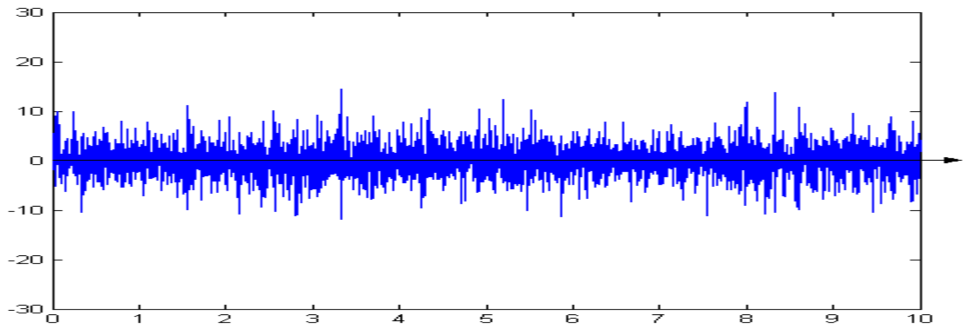

Figure 2.6: The mixer output if the delays at both the receiver and the transmitter are different

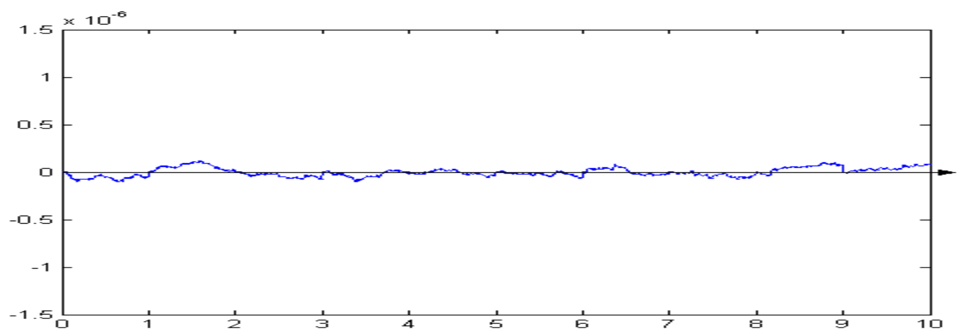

Figure 2.7: integrator output if the delays at bothe the receiver and the transmitter are different

3 Link performance in AWGN for one user

In chapter 2 we described the architecture of the time offset transmitted reference system and its principles. This chapter will investigate the link performance of the time offset transmitted reference system, when it is corrupted by AWGN. In this chapter we will study the case of one transmitter and one receiver. The last section will investigate an example in detail.

3.1 Transmitter

In this section we will determine the signal output of the transmitter.

Figure 3.1 illustrates the transmitter architecture. The transmitter generates a noise carrier. This noise carrier is unknown by the transmitter and the receiver until the instant it is generated for use in communication. We assume that the noise carrier has a Gaussian distribution and its mean value is equal to zero (Figure 3.2).

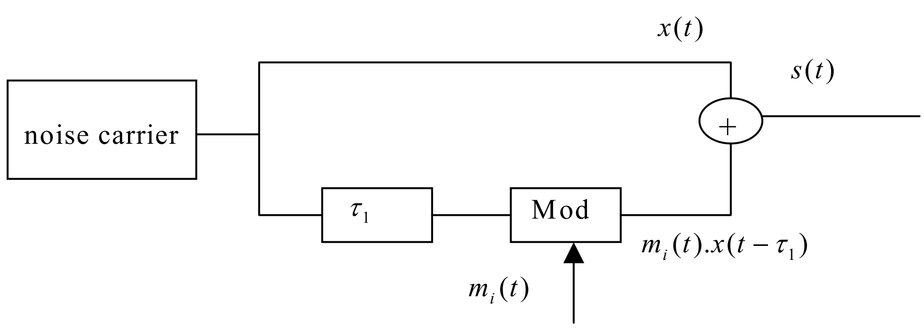

Figure 3.1: The time offset transmitted reference transmitter

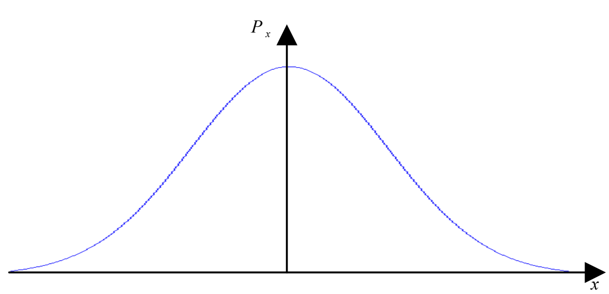

Figure 3.2: PDF of the Gaussian distribution with a mean value equal to zero

The signal in the lower branch in the interval $0 \le t \le T_b$ can be written as:

$$m_i(t).x(t-\tau_1) \qquad 0 \le t \le T_b \qquad (3\text{-}1)$$

$m_i(t)$ Data signal sent by the user

$x(t)$ Noise carrier

τ_1 Time offset used by the transmitter

In our calculation we will assume that the bit time T_b is large enough to assume that all signals in the system are quasi stationary during one bit time. Thus the bit signal $m_i(t)$ is assumed to be constant. Equation (3-1) can be written as

$$m_i.x(t-\tau_1) \qquad 0 \le t \le T_b \qquad (3\text{-}2)$$

We assume also that the time offset τ_1 is much smaller than the bit time T_b.

Combining the two signals coming from the two branches, the output of the transmitter can be written as:

$$s(t) = x(t) + m_i.x(t-\tau_1) \qquad 0 \le t \le T_b \qquad (3\text{-}3)$$

3.2 Receiver input

The channel is assumed to corrupt the signal by the addition of white Gaussian noise (AWGN) as illustrated in Figure 3.3. We assume that the channel causes no attenuation, no delay, and no distortion. The received signal may be expressed as:

$$y(t) = s(t) + n(t) \qquad (3\text{-}4)$$

Where $n(t)$ denotes the Additive White Gaussian Noise (AWGN) process having a mean value equal to zero and having a double sided spectral density $\frac{N_0}{2}$ W / Hz (Figure 3.4). $s(t)$ denotes the transmitted signal.

Substituting equation (3-3) in equation (3-4) we get:

$$y(t) = x(t) + m_i.x(t-\tau) + n(t) \qquad (3\text{-}5)$$

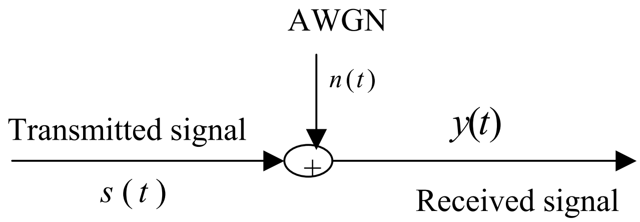

Figure 3.3: Received signal passed through AWGN channel

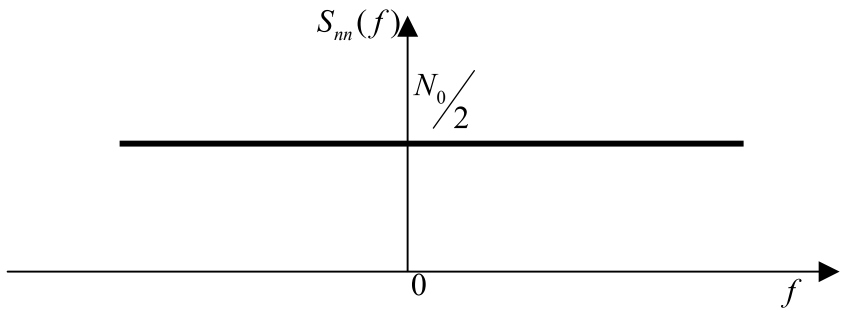

Figure 3.4: Power spectral density of the AWGN

3.3 Mixer output

It is convenient to divide the receiver into two parts, the signal demodulator and the signal detector, as shown in Figure 3.5. The function of the demodulator is to convert the received waveform to a base band signal. The function of the detector is to decide which bit was transmitted based on the value v_i.

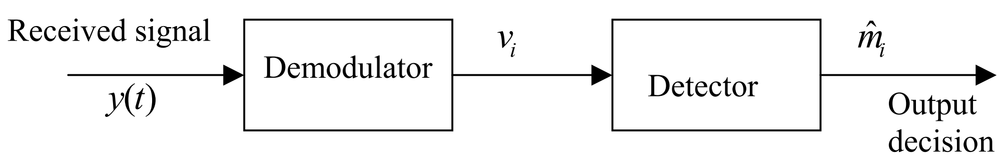

Figure 3.5: The receiver configuration

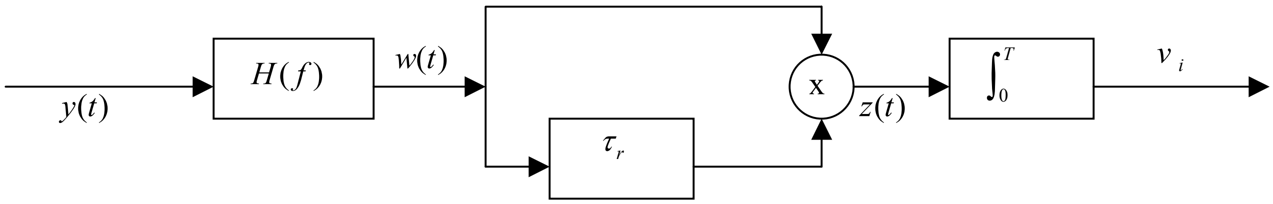

Figure 3.6: The time offset transmitted reference demodulator

Figure 3.6 shows the architecture of the demodulator. It contains a broadband filter, a delay line, a mixer, and an integrate and dump filter. The integration time is equal to the information bit time T_b. Based on the observation of v_i, we will determine the link performance of the time offset transmitted reference system. To reach this goal different steps should be followed. First, we will determine the mixer instantaneous output $z(t)$ and its mean value. Then we will determine the covariance of the mixer output $z(t)$. Second, we will determine the power spectral density of the mixer output by Fourier transforming the covariance. The third step is to determine the noise in the information band. The latter step in the calculation of the link performance is to find out the distribution of the integrate and dump filter output v_i.

3.3.1 Instantaneous mixer output

In this section we will determine the instantaneous mixer output $z(t)$. The input signal $y(t)$ to the receiver is given by (3-5). The signal passes through the broadband filter to remove all high frequencies. The output of the broadband filter is given by:

$$w(t) = y(t) \otimes h(t) \qquad (3\text{-}6)$$

$\otimes$: Convolution operator

$h(t)$ the impulse function of the broadband filter

We write out the convolution

$$w(t) = \int_{-\infty}^{\infty} y(t-\alpha).h(\alpha)\mathrm{d}\alpha \qquad (3\text{-}7)$$

The signal coming from the filter will be delayed by a time offset equal to τ_r and then mixed with the original received signal. The mixer output $z(t)$ can be written as:

$$z(t) = w(t).w(t-\tau_r) \qquad (3\text{-}8)$$

$z(t)$ is the instantaneous mixer output.

τ_r is the time offset at the receiver.

We substitute $w(t)$ and $w(t-\tau_r)$ by their expression given by (3-7), so equation (3-8) becomes:

$$z(t)=\int_{-\infty}^{\infty} y(t-\alpha_1).h(\alpha_1)\mathrm{d}\alpha_1 \int_{-\infty}^{\infty} y(t-\tau_r-\alpha_2).h(\alpha_2)\mathrm{d}\alpha_2 \tag{3-9}$$

The product of the two integrals in equation (3-9) can be written as a double integral, which gives:

$$z(t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y(t-\alpha_1).y(t-\tau_r-\alpha_2).h(\alpha_1).h(\alpha_2)\mathrm{d}\alpha_2\mathrm{d}\alpha_1 \tag{3-10}$$

If we substitute $y(t-\alpha_1)$ and $y(t-\tau_r-\alpha_2)$ by their expressions given by equation (3-5), equation (3-10) can be written as:

$$\begin{aligned} z(t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} & h(\alpha_1).h(\alpha_2).\{x(t-\alpha_1)+m_i.x(t-\alpha_1-\tau_1)+n(t-\alpha_1)\} \\ & .\{x(t-\alpha_2-\tau_r)+m_i.x(t-\alpha_2-\tau_1-\tau_r)+n(t-\alpha_2-\tau_r)\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \end{aligned} \tag{3-11}$$

Writing out the product, equation (3-11) can be written as:

$$z(t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)\left\{\begin{aligned} & x(t-\alpha_1).x(t-\alpha_2-\tau_r) \\ & +m_i.x(t-\alpha_1).x(t-\alpha_2-\tau_1-\tau_r) \\ & +x(t-\alpha_1).n(t-\alpha_2-\tau_r) \\ & +m_i.x(t-\alpha_1-\tau_1).x(t-\alpha_2-\tau_r) \\ & +m_i.m_i.x(t-\alpha_1-\tau_1).x(t-\alpha_2-\tau_1-\tau_r) \\ & +m_i.x(t-\alpha_1-\tau_1).n(t-\alpha_2-\tau_r) \\ & +n(t-\alpha_1).x(t-\alpha_2-\tau_r) \\ & +m_i.n(t-\alpha_1).x(t-\alpha_2-\tau_1-\tau_r) \\ & +n(t-\alpha_1).n(t-\alpha_2-\tau_r) \end{aligned}\right\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \tag{3-12}$$

The integration is a linear operation, thus equation (3-12) can be written as:

$$
\begin{aligned}
z(t) = &\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)x(t-\alpha_1).x(t-\alpha_2-\tau_r)\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\
&+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} m_i.h(\alpha_1).h(\alpha_2)x(t-\alpha_1).x(t-\alpha_2-\tau_1-\tau_r)\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\
&+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)x(t-\alpha_1).n(t-\alpha_2-\tau_r)\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\
&+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} m_i.h(\alpha_1).h(\alpha_2)x(t-\alpha_1-\tau_1).x(t-\alpha_2-\tau_r)\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\
&+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} m_i.m_i h(\alpha_1).h(\alpha_2)x(t-\alpha_1-\tau_1).x(t-\alpha_2-\tau_1-\tau_r)\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\
&+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} m_i.h(\alpha_1).h(\alpha_2).x(t-\alpha_1-\tau_1).n(t-\alpha_2-\tau_r)\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\
&+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)n(t-\alpha_1).x(t-\alpha_2-\tau_r)\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\
&+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} m_i.h(\alpha_1).h(\alpha_2)n(t-\alpha_1).x(t-\alpha_2-\tau_1-\tau_r)\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\
&+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)n(t-\alpha_1).n(t-\alpha_2-\tau_r)\mathrm{d}\alpha_1\mathrm{d}\alpha_2
\end{aligned}
\qquad (3\text{-}13)
$$

3.3.2 Average mixer output

The average mixer output $z(t)$ can be found by taking the expected value of $z(t)$ given in equation (3-13).

$$
\mathrm{E}\{z(t)\} = \mathrm{E}\left\{
\begin{aligned}
&\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)x(t-\alpha_1).x(t-\alpha_2-\tau_1)\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\
&+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} m_i.h(\alpha_1).h(\alpha_2)x(t-\alpha_1).x(t-\alpha_2-2\tau_1)\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\
&+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)x(t-\alpha_1).n(t-\alpha_2-\tau_1)\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\
&+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} m_i.h(\alpha_1).h(\alpha_2)x(t-\alpha_1-\tau_1).x(t-\alpha_2-\tau_1)\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\
&+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} m_i.m_i h(\alpha_1).h(\alpha_2)x(t-\alpha_1-\tau_1).x(t-\alpha_2-2\tau_1)\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\
&+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} m_i.h(\alpha_1).h(\alpha_2).x(t-\alpha_1-\tau_1).n(t-\alpha_2-\tau_1)\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\
&+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)n(t-\alpha_1).x(t-\alpha_2-\tau_1)\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\
&+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} m_i.h(\alpha_1).h(\alpha_2)n(t-\alpha_1).x(t-\alpha_2-2\tau_1)\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\
&+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)n(t-\alpha_1).n(t-\alpha_2-\tau_1)\mathrm{d}\alpha_1\mathrm{d}\alpha_2
\end{aligned}
\right\}
\qquad (3\text{-}14)
$$

Since taking the expected value of an expression is a linear operation, we can interchange the order of integration and taking the expected value. The information bit time T_b is large in comparison to the broadband filter impulse response and to the autocorrelation function $R_{xx}(\tau)$ of the noise carrier. Therefore we can assume that the information bit is constant during one bit time and we can take it out of the integral, which gives:

$$
\begin{aligned}
\mathrm{E}\{z(t)\} = &\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)\mathrm{E}\{x(t-\alpha_1).x(t-\alpha_2-\tau_r)\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\
&+ m_i.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)\mathrm{E}\{x(t-\alpha_1).x(t-\alpha_2-\tau_1-\tau_r)\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\
&+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)\mathrm{E}\{x(t-\alpha_1).n(t-\alpha_2-\tau_r)\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\
&+ m_i.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)\mathrm{E}\{x(t-\alpha_1-\tau_1).x(t-\alpha_2-\tau_r)\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\
&+ m_i m_i.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)\mathrm{E}\{x(t-\alpha_1-\tau_1).x(t-\alpha_2-\tau_1-\tau_r)\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\
&+ m_i.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2).\mathrm{E}\{x(t-\alpha_1-\tau_1).n(t-\alpha_2-\tau_r)\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\
&+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)\mathrm{E}\{n(t-\alpha_1).x(t-\alpha_2-\tau_r)\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\
&+ m_i.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)\mathrm{E}\{n(t-\alpha_1).x(t-\alpha_2-\tau_1-\tau_r)\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\
&+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)\mathrm{E}\{n(t-\alpha_1).n(t-\alpha_2-\tau_r)\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2
\end{aligned} \tag{3-15}
$$

$m_i(t)$ is polar non-return to zero. Therefore $m_i.m_i = 1$

$x(t)$ and $n(t)$ are two stationary signals, therefore

$\mathrm{E}\{x(t_1).x(t_2)\} = R_{xx}(t_1 - t_2)$

$\mathrm{E}\{n(t_1).n(t_2)\} = R_{nn}(t_1 - t_2)$

$x(t)$ and $n(t)$ are mutually independent, therefore the cross correlation between them is equal to zero. Taking into account the previous properties expression (3-15) can be reduced to:

$$
\begin{aligned}
\mathrm{E}\{z(t)\} = &\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)R_{xx}(-\alpha_1+\alpha_2+\tau_r)\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\
&+ m_i.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)R_{xx}(-\alpha_1+\alpha_2+\tau_1+\tau_r)\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\
&+ m_i.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)R_{xx}(-\alpha_1+\alpha_2-\tau_1+\tau_r)\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\
&+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)R_{xx}(-\alpha_1+\alpha_2+\tau_r)\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\
&+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)R_{nn}(-\alpha_1+\alpha_2+\tau_r)\mathrm{d}\alpha_1\mathrm{d}\alpha_2
\end{aligned} \tag{3-16}
$$

We write the double integral as a double convolution. Equation (3-16) can be written as:

$$
\begin{aligned}
\mathrm{E}\{z(t)\} = &(h \otimes h_2 \otimes R_x)(\tau_r) + m_i.(h \otimes h_2 \otimes R_{xx})(\tau_1+\tau_r) + m_i.(h \otimes h_2 \otimes R_{xx})(-\tau_1+\tau_r) \\
&+ (h \otimes h_2 \otimes R_{xx})(\tau_r) + (h \otimes h_2 \otimes R_{nn})(\tau_r)
\end{aligned} \tag{3-17}
$$

Where h_2 is the time-reversed version of h, i.e. $h_2(t) = h(-t)$.

Equation (3-17) is a general expression of the mean value of the mixer output $z(t)$ at the receiver. If the absolute value of the difference between the two time offsets is much smaller than the coherence time of the noise carrier, the information signal will remain. To simplify the calculation we will assume that the time offset τ_r at the receiver is equal to the time offset τ_1 at the transmitter, so expression (3-17) can be written as:

$$\mathrm{E}\{z(t)\} = (h \otimes h_2 \otimes R_x)(\tau_r) + m_i.(h \otimes h_2 \otimes R_{xx})(2\tau_r) + m_i.(h \otimes h_2 \otimes R_{xx})(0) \\ + (h \otimes h_2 \otimes R_{xx})(\tau_r) + (h \otimes h_2 \otimes R_{nn})(\tau_r) \quad (3\text{-}18)$$

We prove in appendix A1 that the expected value of the mixer output $z(t)$ can be reduced to:

$$\mathrm{E}\{z(t)\} = m_i.\int_{-\infty}^{\infty} |H(f)|^2 S_{xx}(f)\mathrm{d}f \quad (3\text{-}19)$$

$S_{xx}(f)$ is the power spectral density function of the noise carrier $x(t)$.

We can write it also as the double convolution

$$\mathrm{E}\{z(t)\} = m_i.(h \otimes h_2 \otimes R_{xx})(0) \quad (3\text{-}20)$$

Expression (3-19) illustrates that the expected value of the mixer output $z(t)$ is proportional to the transmitted bit m_i. The broadband filter transfer function and the power spectral density of the noise carrier are assumed to be time invariant; therefore the integral of their product is also time invariant.

If the time offsets at the receiver and at the transmitter are not equal, and following the same steps as we did in appendix A1, the mean value of the mixer output $z(t)$ is equal to zero. Therefore no information bit can de detected.

Conclusion

If the absolute value of the difference between the time offsets at the transmitter and at the receiver is much smaller than the coherence time of the noise carrier, then the expected value of the mixer output $z(t)$ is proportional to the transmitted bit m_i, otherwise if the difference between the time offsets at the transmitter and at the receiver is larger than the coherence time of the noise carrier, the mixer output is a noise signal with a mean value equal to zero.

3.4 The time offset transmitted reference signal-to-beat noise ratio

The performance of the time offset transmitted reference is limited mainly by the beat noise. The beat noise arises as the effect of the mixing between different signals (broadband noise signal, additive white Gaussion noise, and their shifted versions) at the receiver. The effect of the beat noise on the link performance can be found by computing the power spectral density of the beat noise in the information band. This can be done by: first calculating the covariance of the mixer output and then the power spectral density of the beat noise in the information band by taking the Fourier transform of the covariance [16].

3.4.1 Mixer beat noise power spectral density

The bandwidth of the broadband noise signal and the broadband filter at the receiver are much broader than the information bandwidth. Therefore we can assume that the beat noise spectrum is flat in the information band so we can treat it as white noise [16]. Hence the power spectral density of the beat noise in the information band can be approximated by the power spectral density $S_{zz}(f)$ of the beat noise at the mixer output $z(t)$ for $f = 0$.

We define $S_{zz}(f)$ as the Fourier transform of the covariance function $C_{zz}(\tau_1, \tau)$, which is equal to the autocorrelation function without the DC-term. The calculations are done in appendix A3, the result is:

$$S_{zz}(0) = 7.\int_{-\infty}^{\infty} |H(\nu)|^4 .S_{xx}^2(\nu)\mathrm{d}\nu + 4.\int_{-\infty}^{\infty} |H(\nu)|^4 .S_{xx}(\nu).S_{nn}(\nu)\mathrm{d}\nu + \int_{-\infty}^{\infty} |H(\nu)|^4 .S_{nn}^2(\nu)\mathrm{d}\nu \tag{3-21}$$

The beat noise power spectral density contains three terms. The first term is the noise power resulting from the mixing of the of the random noise carrier $x(t)$ and its shifted signals. The second term is the noise power resulting from the mixing between the different shifted signals of the noise carrier $x(t)$ and the additive white Gaussian noise $n(t)$. The third term is the noise power resulting only from the additive white Gaussian noise $n(t)$ mixed with itself.

The power spectral density of the additive white Gaussian noise $n(t)$ is equal to $\frac{N_0}{2}$. If we substitute it in equation (3-21) we get:

$$S_{zz}(0) = 7.\int_{-\infty}^{\infty} |H(\nu)|^4 .S_{xx}^2(\nu).\mathrm{d}\nu + 2.N_0.\int_{-\infty}^{\infty} |H(\nu)|^4 .S_{xx}(\nu).\mathrm{d}\nu + \frac{1}{4}N_0^2.\int_{-\infty}^{\infty} |H(\nu)|^4 .\mathrm{d}\nu \tag{3-22}$$

3.4.2 Integrate and dump filter output

As we mentioned before we are only interested in the information signal and the noise that corrupts the information band. We prove in section appendix A2 that the signal at the mixer output contains a desired part and a noise part, which is assumed to be white noise.

$$z(t) = \mathrm{E}\{z(t)\} + n'(t) \tag{3-23}$$

$n'(t)$ White noise with a power spectral density equal to $S_{zz}(0)$.

The signal coming from the mixer passes through the integrate and dump filter. The integration time is equal to the information bit time T_b.

$$v_i = \int_0^{T_b} z(t)\mathrm{d}t \tag{3-24}$$

v_i The output of the integrate and dump filter.

Substituting (3-23) in equation (3-24) gives:

$$\begin{aligned} v_i &= \int_0^{T_b} \left[\mathrm{E}\{z(t)\} + n'(t)\right]\mathrm{d}t \\ &= \int_0^{T_b} \mathrm{E}\{z(t)\} + \int_0^{T_b} n'(t)\mathrm{d}t \\ &= \mathrm{E}\{z(t)\}.T_b + \int_0^{T_b} n'(t)\mathrm{d}t \end{aligned} \tag{3-25}$$

We define

$$n_i'' = \int_0^{T_b} n'(t)\mathrm{d}t \tag{3-26}$$

$$v_{s,i} = \mathrm{E}\{z(t)\}.T_b \tag{3-27}$$

$v_{s,i}$ represents the signal component at the integrate and dump output.

n_i'' represents the noise component at the integrate and dump output.

Substituting (3-26) and (3-27) in equation (3-25) gives:

$$v_i = v_{s,i} + n_i'' \tag{3-28}$$

We conclude that the output of the integrator is composed of two parts, the signal part and the noise part, which has a Gaussian distribution. The noise component $\{n_i''\}$ is Gaussian, since it can be viewed as the output of the integrate and dump filter with a large integration time in comparison to the coherence time of the beat noise. Its mean value is

$$\begin{aligned} \mathrm{E}\{n_i''\} &= \mathrm{E}\left\{\int_0^{T_b} n'(t)\mathrm{d}t\right\} \\ &= \int_0^{T_b} \mathrm{E}\{n'(t)\}\mathrm{d}t \\ &= 0 \end{aligned} \tag{3-29}$$

and its variance is

$$\mathrm{E}\left\{\left[n_i\,''\right]^2\right\} = \int_0^{T_b}\int_0^{T_b} \mathrm{E}\{n'(t_1).n'(t_2)\}.\mathrm{d}t_1\mathrm{d}t_2$$

$$\simeq S_{zz}(0).\int_0^{T_b}\int_0^{T_b} \delta(t_2 - t_1).\mathrm{d}t_1\mathrm{d}t_2$$

$$= S_{zz}(0).\int_0^{T_b} \mathrm{d}t_2 \tag{3-30}$$

$$= S_{zz}(0).T_b$$

From the above development, it follows that the integrate and dump filter output is a Gaussian random variable with mean value equal to:

$$\mathrm{E}\{v_i\} = \mathrm{E}\{z(t)\}.T_b + \mathrm{E}\{n_i\,''(t)\}$$

$$= \mathrm{E}\{z(t)\}.T_b \tag{3-31}$$

and variance equal to:

$$\mathrm{E}\left\{\left[v_i\right]^2\right\} = \mathrm{E}\left\{\left[n_i\,''\right]^2\right\} = S_{zz}(0).T_b \tag{3-32}$$

3.4.3 Signal-to-beat noise ratio

In the previous section we determined demodulator output: the signal component and its variance. In this section we will determine the signal-to-beat noise ratio at the detector input. It is given by:

$$SNR = \frac{\{v_{s,i}\}^2}{\mathrm{E}\left\{\left[n_i\,''\right]^2\right\}} \tag{3-33}$$

Where $\{v_{s,i}\}^2$ denotes the average signal component power and $\mathrm{E}\left\{\left[n_i\,''\right]^2\right\}$ the variance of the beat noise. Substituting (3-27) and (3-30) in equation (3-33) we get:

$$SNR = \frac{\{\mathrm{E}\{z(t)\}.T_b\}^2}{S_{zz}(0).T_b}$$

$$= \frac{\left[\mathrm{E}\{z(t)\}\right]^2.T_b}{S_{zz}(0)} \tag{3-34}$$

Substituting (3-19) and (3-22) in equation (3-34) we get:

$$SNR = \frac{\left\{\int_{-\infty}^{\infty} \left|H(f)\right|^2 S_{xx}(f)df\right\}^2.T_b}{7.\int_{-\infty}^{\infty}\left|H(\nu)\right|^4.S_{xx}^2(\nu)d\nu + 2.N_0.\int_{-\infty}^{\infty}\left|H(\nu)\right|^4.S_{xx}(\nu).d\nu + \frac{1}{4}N_0^2\int_{-\infty}^{\infty}\left|H(\nu)\right|^4 d\nu} \tag{3-35}$$

Expression (3-35) illustrates a general formula of the signal-to-beat noise ratio of the time offset transmitted reference system. The only assumption made in the calculation is that the distribution of the noise carrier $x(t)$ is Gaussian.

In the next section we will determine the link performance of the time offset transmitted reference system.

3.5 Detection probability

We have demonstrated that, for a transmitted signal over the AWGN channel, the demodulator produces the values $\{v_i\}$, which contains the relevant information in the received waveform. In this section we will determine the bit error rate of the time offset transmitted reference based on the values $\{v_i\}$. For this development, we assume that the decoder doesn't have any memory.

We also assume that the channel causes no attenuation, and no distortion and that synchronous clocks are available at the transmitter and the receiver for determining the time interval T_b. The information bits are polar, antipodal signal. We assumed that 1 and –1 are equally likely. We proved in the previous section that the demodulator outputs $\{v_i\}$ have a Gaussian distribution and can be written as:

$$v_i = \mathrm{E}\{v_i\} + n'' \qquad (3\text{-}36)$$

$\mathrm{E}\{v_i\}$ is assumed to be constant and depends on the signal being transmitted.

For a polar antipodal signal in Guassian noise and with a threshold equal to 0, the bit error rate is given by the Gaussian tail function, which has an argument equal to the square root of the signal-to-beat noise ratio [14]. Thus:

$$P_e = \mathrm{Q}\left(\sqrt{SNR}\right) \qquad (3\text{-}37)$$

3.6 Evaluation for band limited spectrally flat noise carrier

In the previous section we determined a general expressions for the most important properties of the time offset transmitted reference system such as the signal to noise ratio and the link performance. In this section we will study a simple example. We assume that the noise carrier is band limited spectrally flat (perfect square in frequency domain) and its bandwidth is equal to B.

We also assume that the transfer function of the broadband filter has a perfect rectangular shape. It has the following power spectral density and transfer function:

$$S_{xx}(f) = \begin{cases} S_{xx} & -B/2 < f < B/2 \\ 0 & \text{elsewhere} \end{cases}$$
$$H(f) = \begin{cases} 1 & -B/2 < f < B/2 \\ 0 & \text{elsewhere} \end{cases} \tag{3-38}$$

Other power spectrum density distribution for the noise carrier and other transfer functions for the broadband noise filter are also possible. In this thesis we will only investigate one case.

3.6.1 Signal-to-beat noise ratio

Combining equation (3-35) and equation (3-38), the SNR can be written as:

$$SNR = \frac{\left\{ \mathrm{S}_{xx} \int_{-B/2}^{B/2} \mathrm{d}\nu \right\}^2 .T_b}{7.S_{xx}^2 \int_{-B/2}^{B/2} d\nu + 2.N_0 \int_{-B/2}^{B/2} .S_{xx}.d\nu + \frac{1}{4} N_0^{\ 2} \int_{-B/2}^{B/2} d\nu} \tag{3-39}$$

Working out the integrals, equation (3-39) is reduced to:

$$SNR = \frac{\left\{ \mathrm{S}_{xx}.B \right\}^2 .T_b}{7.S_{xx}^2 B + 2.N_0.S_{xx}.B + \frac{1}{4} N_0^{\ 2}.B} \tag{3-40}$$

This is equal to:

$$SNR = \frac{\mathrm{S}_{xx}^2 B.T_b}{7.S_{xx}^2 + 2.N_0.S_{xx} + \frac{1}{4} N_0^{\ 2}} \tag{3-41}$$

We assumed before that the transmitted energy per bit is constant, and that the channel causes no attenuation and no distortion. Therefore the received energy per bit will be equal to:

$$E_b = 2.T_b. \int_{-\infty}^{\infty} S_{xx}(f) df \tag{3-42}$$

The factor 2 is the result of transmission of two versions of the same signal. Substituting equation (3-38) in equation (3-42) we get:

$$\begin{aligned} E_b &= 2.T_b.S_{xx} \int_{-B/2}^{B/2} df \\ &= 2.B.T_b.S_{xx} \end{aligned} \tag{3-43}$$

Using equation (3-43) we write S_{xx} as:

$$S_{xx} = \frac{E_b}{2.B.T_b} \tag{3-44}$$

We combine equation (3-44) and equation (3-41), the result is:

$$SNR = \frac{\left\{\frac{E_b}{2.B.T}\right\}^2 .B.T_b}{7.\left\{\frac{E_b}{2.B.T}\right\}^2 + 2.N_0.\frac{E_b}{2.B.T_b}.+\frac{1}{4}N_0{}^2} \tag{3-45}$$

This is equal to:

$$SNR = \frac{1}{4.B.T}.\frac{\left\{\frac{E_b}{N_0}\right\}^2}{\frac{7}{4}.\left\{\frac{E_b}{N_0}\right\}^2.\frac{1}{\{B.T_b\}^2}+\frac{1}{B.T_b}.\frac{E_b}{N_0}.+\frac{1}{4}}$$

$$= \frac{\left\{\frac{E_b}{N_0}\right\}^2}{7.\left\{\frac{E_b}{N_0}\right\}^2.\frac{1}{B.T_b}+4.\frac{E_b}{N_0}+B.T_b} \tag{3-46}$$

Where $\frac{E_b}{N_0}$ is the received SNR per bit.

Let G denote the processing gain of the time offset transmitted reference system:

$$G = B.T_b \tag{3-47}$$

B	The bandwidth of noise carrier
T_b	Information bit time

Substituting equation (3-47) in equation (3-45) we get:

$$SNR = \frac{\left\{\frac{E_b}{N_0}\right\}^2}{7.\left\{\frac{E_b}{N_0}\right\}^2.\frac{1}{G}+4.\frac{E_b}{N_0}+G} \tag{3-48}$$

The signal-to-beat noise ratio depends on the processing gain G. We have proven that the first term in the denominator is the noise power resulting of the mixing of the different versions of the noise carrier $x(t)$. Equation (3-48) illustrates that this beat noise power term is inversely proportional to the processing gain. That means: if we increase the processing gain (noise carrier bandwidth to information signal bandwidth ratio) this kind of beat noise will decrease. On the other hand the third term in the denominator is resulting from the power noise of the additive white Gaussian noise $n(t)$. This term is proportional to the processing gain. If the processing gain increases, the additive white Gaussian noise power in the information band will also increase. Thus by increasing the processing gain we decrease the beat noise power resulting of the mixing of the different versions of the noise carrier $x(t)$, and at the same time we increase the noise power resulting from the AWGN $n(t)$. We have a tradeoff between the two-sort beat noise powers. We can maximize the SNR by looking for the optimum processing gain. This can be done by minimizing the denominator. Differentiating the denominator and taking G as the parameter we find that the SNR is maximal if

$$G = \sqrt{7}.\frac{E_b}{N_0} \tag{3-49}$$

This relation states that for each value of the SNR per bit $\frac{E_b}{N_0}$, the processing gain G should satisfy equation (3-49) to maximize the signal-to-beat noise ratio at the input of the detector. Equation (3-49) also illustrates that the optimal processing gain G is a linear function of $\frac{E_b}{N_0}$.

Substituting equation (3-49) in equation (3-48) we get:

$$SNR_{\text{optimal}} = \frac{\left\{\frac{E_b}{N_0}\right\}^2}{7.\left\{\frac{E_b}{N_0}\right\}^2.\frac{1}{\frac{E_b}{N_0}}.\frac{1}{\sqrt{7}} + 4.\frac{E_b}{N_0} + \sqrt{7}.\frac{E_b}{N_0}} = \frac{1}{4+2\sqrt{7}}.\frac{E_b}{N_0} \tag{3-50}$$

Expression (3-50) illustrates the optimum signal-to-beat noise ratio. This can be reached only if equation (3-49) is fulfilled.

At large values of $\frac{E_b}{N_0}$, equation (3-48) can be approximated by:

$$SNR_{\text{floor}} \approx \frac{1}{7.\left\{\frac{E_b}{N_0}\right\}^2 . \frac{1}{G}} . \left\{\frac{E_b}{N_0}\right\}^2 \qquad (3\text{-}51)$$

$$= \frac{G}{7}$$

Expression (3-51) illustrates that the signal-to-beat noise ratio, given a processing gain G, is limited by the maximum value of $\frac{G}{7}$. The event causing this maximum is the cross correlation between the noise carrier $x(t)$ and its different shifted signals. Equation (3-51) also illustrates that the maximum signal-to-beat noise ratio is proportional to the processing gain G.

3.6.2 Numerical evaluation

We proved in section 2.5 that the bit error rate is equal to the Gaussian tail function with an argument equal to the square root of the signal-to-beat noise ratio. Combining equation (3-37) and equation (3-48) leads to:

$$P_e = \mathrm{Q}\left(\sqrt{\frac{\left\{\frac{E_b}{N_0}\right\}^2}{7.\left\{\frac{E_b}{N_0}\right\}^2 . \frac{1}{G} + 4.\frac{E_b}{N_0} + G}} \right) \qquad (3\text{-}52)$$

A plot of the BER for the time offset transmitted reference system is shown in Figure 3.7. The BER is plotted as a function of the SNR per bit $\frac{E_b}{N_0}$ for different processing gains (G = 20dB and G = 23dB), and also for the optimal case (G is optimal for each value of $\frac{E_b}{N_0}$). It is apparent that for a processing gain equal to 20 dB the bit error rate is minimized by $7.8*10^{-5}$, which is equal to the error-floor. The error floor is mainly due the contribution of the noise carrier-noise carrier cross talk. Since the noise carrier-noise carrier cross talk are present even when the AWGN amplitude is zero, bit error rate saturates at large $\frac{E_b}{N_0}$ at the value:

$$P_{\text{error-floor}} = \mathrm{Q}\left(\sqrt{\frac{G}{7}}\right) = \mathrm{Q}\left(\sqrt{\frac{100}{7}}\right) = \mathrm{Q}\left(\sqrt{14.3}\right) \tag{3-53}$$

Figure 3.7 also illustrates that by increasing the processing gain; we can reach lower bit error for high values of $\frac{E_b}{N_0}$. This is conforming to expression (3-52). At high values of $\frac{E_b}{N_0}$ the first term in equation (3-52) dominates the other terms and because this term is inversely proportional the processing gain G the BER will be improved.

We can see from Figure 3.7 that for a fixed processing gain G, the BER curve is always above the curve of the optimal BER except for one point where G is optimum. The optimal processing gain G is mainly due the trade of between the noise carrier-noise carrier interferences and the contribution of the AWGN in the information signal. Figure 3.8 shows the BER as a function of the processing gain G for a fixed value of $\frac{E_b}{N_0}$. We can see that the BER decrease for a while and then reached its minimum at unique value of the processing gain G and then starts increasing as the G increases. Hence for a given value of $\frac{E_b}{N_0}$ the processing gain G should be as close to the to the optimal processing gain as possible to get the low bit error rate.

Figure 3.9 compares between our analytical results and those of Sushchik [10]. We can see that Sushchik model has a better link performance than our model. This is due to the kind of the noise carrier used in the two schemes. We used a noise carrier having a Gaussian distribution. But Sushchik used a chaotic carrier having a uniform distribution. A disadvantage of Chaotic signals with uniform distribution is that are difficult to generate.

We proved that for a given SNR per bit $\frac{E_b}{N_0}$, the signal-to-beat noise ratio is optimal only for one value of the processing gain. Thus if equation (3-49) is fulfilled, the optimal bit error rate will be equal to:

$$P_{\text{optimal}} = \mathrm{Q}\left(\sqrt{\frac{1}{4+2\sqrt{7}}.\frac{E_b}{N_0}}\right) \tag{3-54}$$

Figure 3.10 shows the BER for the optimal bit error rate (left). The BER is optimal for each value of $\frac{E_b}{N_0}$. This is due mainly to the trade off between the different contributions of the noise carrier interferences and the contributions of the AWGN interferences in the information band for different processing gains. Figure 3.10 also shows the optimal processing gain *G* as a function of $\frac{E_b}{N_0}$ (right). The optimal processing gain is a linear function of $\frac{E_b}{N_0}$. It implies that the processing gain can be adaptively optimised to different values of $\frac{E_b}{N_0}$ to get the lower BER. This is undesirable because it will increase the complexity of the system.

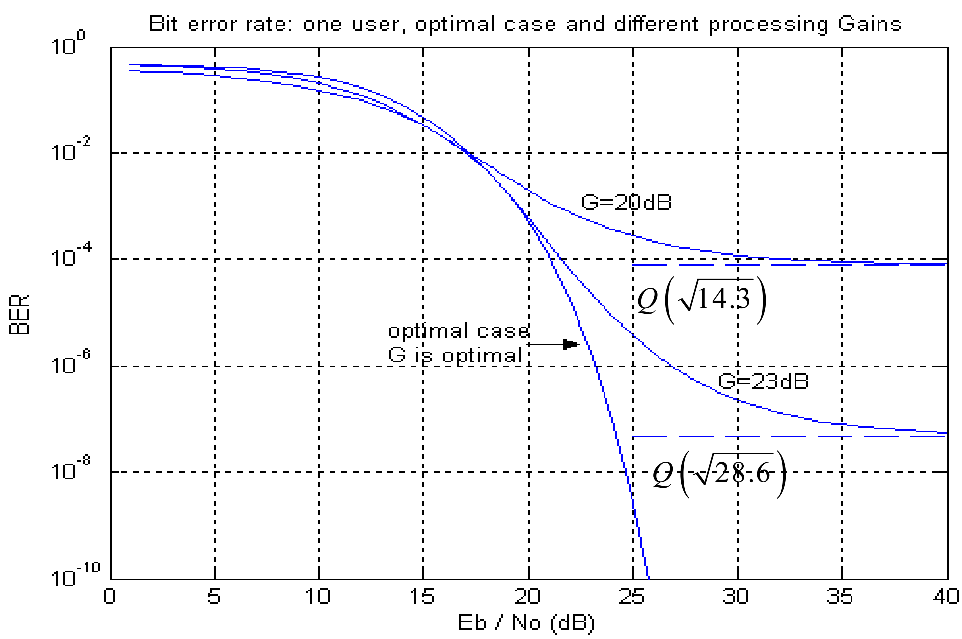

Figure 3.7:The BER as a function of E_b/N_o and for different processing gains (G=20dB, G=23 dB), and for the optimal bit error rate (G is optimal for each value of the SNR per bit E_b/N_o)

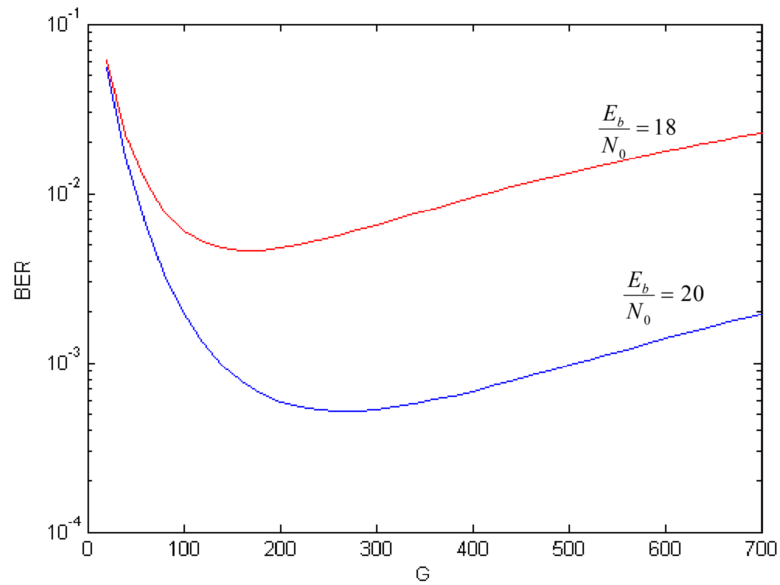

Figure 3.8: The bit error rate as a function of the processing gain *G* at a fixed value of E_b/N_o

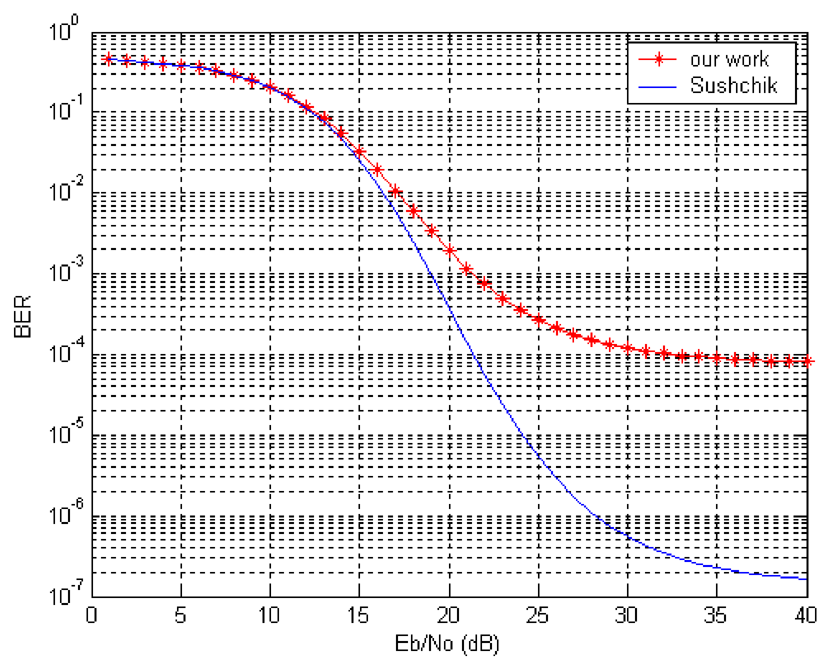

Figure 3.9: The BER, our model and Sushchik's model

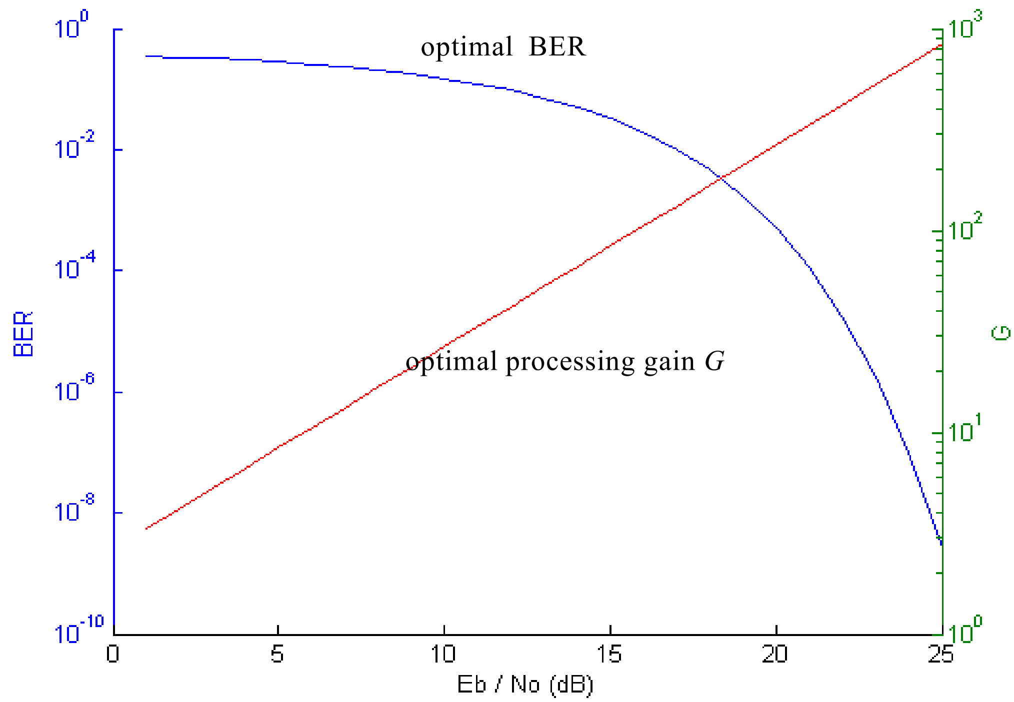

Figure 3.10: The optimum BER as a function of E_b/N_o (left) and the corresponding optimum processing gain *G* (right)

4 Link performance for multiple users

In chapter 3 we investigated the link performance for one transmitter and one receiver. In practice multiple users transmit and receive at the same time. In this chapter we will determine the link performance of the time offset transmitted reference system if there are multiple transmitters and multiple receivers. The last section will study an example in detail.

4.1 Mixer output

4.1.1 Instantaneous mixer output

Figure 4.1 illustrates the time offset transmitted reference system when multiple users transmit at the same time. We will assume that the number of users is equal to *M*.

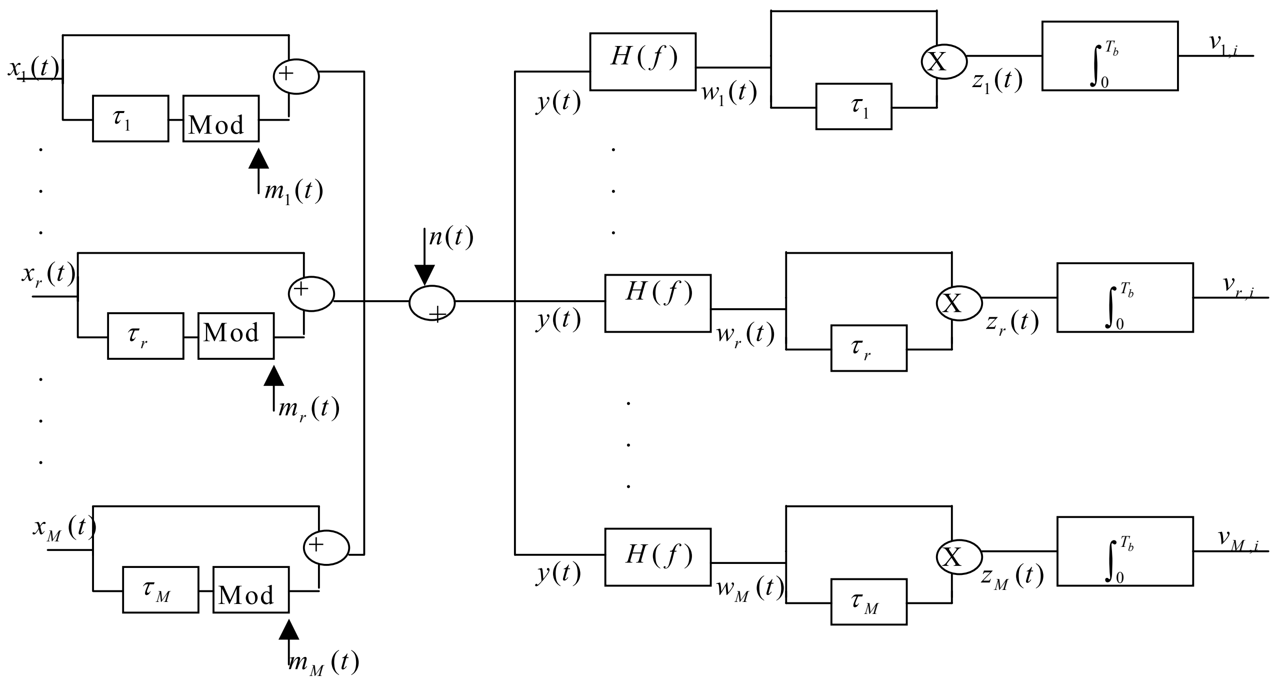

Figure 4.1: The time offset transmitted reference system with multiple transmitter and receivers

We denote the signal transmitted by transmitter *j* by:

$$y_j(t) = x_j(t) + m_j . x_j(t - \tau_j) \tag{4-1}$$

$y_j(t)$ The transmitted signal by transmitter *j*.

m_j Information signal transmitted by transmitter *j*.

τ_j The time offset used by transmitter *j*, which is unique for each transmitter.

$x_j(t)$ The noise carrier used by transmitter *j*.

We assume that the noise carriers are mutually independent, and all of them have a Gaussian distribution with a mean value equal to zero. Each receiver receives all transmitted signals corrupted by AWGN. Therefore the input of each receiver can be written as:

$$y(t) = \sum_{j=1}^{M} y_j(t) + n(t) \tag{4-2}$$

$n(t)$ The Additive White Gaussian Noise process having a mean value equal to zero and having a double sided spectral density $\frac{N_0}{2}$ W / Hz.

M The number of users transmitting at the same time

By writing out $y_j(t)$ we get:

$$y(t) = \sum_{j=1}^{M} \left[x_j(t) + m_j . x_j(t - \tau_j) \right] + n(t) \tag{4-3}$$

N.B: strictly speaking, it is incorrect to assume that the AWGN is the same for all receivers because each receiver generates a different AWGN. But for the calculations we can assume that the AWGN is the same for all users because this assumption will lead to the same result.

The received signal $y(t)$ passes through a broadband filter, which removes all high frequencies present in the signal. We assume that all filters are identical with an impulse response equal to *h(t)* and a transfer function equal to $H(f)$. Therefore the output of the broadband filter will be the same for all receivers and can be written as:

$$w(t) = y(t) \otimes h(t) \tag{4-4}$$

We write out the convolution, so equation (4-4) can be written as:

$$w(t) = \int_{-\infty}^{\infty} y(t - \alpha).h(\alpha)\mathrm{d}\alpha \tag{4-5}$$

Although each receiver receives the same signal $y(t)$ as expressed in equation (4-3), the output signal of each receiver is unique since it has a unique time offset. Next we will determine the instantaneous mixer output for the user *r*. This is equal to:

$$z_r(t) = w(t).w(t-\tau_r) \tag{4-6}$$

$z_r(t)$ The instantaneous mixer output for receiver r.

We substitute equation (4-5) in equation (4-6), we get:

$$z_r(t) = \int_{-\infty}^{\infty} y(t-\alpha_1).h(\alpha_1)\mathrm{d}\alpha_1 \int_{-\infty}^{\infty} y(t-\tau_r-\alpha_2).h(\alpha_2)\mathrm{d}\alpha_2 \tag{4-7}$$

The product of two integrals can be written as a double integral, which gives:

$$z_r(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y(t-\alpha_1).y(t-\tau_r-\alpha_2).h(\alpha_1).h(\alpha_2)\mathrm{d}\alpha_2\mathrm{d}\alpha_1 \tag{4-8}$$

We substitute $y(t-\alpha_1)$ and $y(t-\tau_r-\alpha_2)$ by their expressions given in equation (4-3), equation (4-8) can be written as:

$$\begin{aligned} z_r(t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2).&\left\{\sum_{i=1}^{M}(x_i(t-\alpha_1)+m_i.x_i(t-\tau_i-\alpha_1))+n(t-\alpha_1)\right\} \\ &.\left\{\sum_{j=1}^{M}(x_j(t-\tau_r-\alpha_2)+m_j.x_j(t-\tau_j-\tau_r-\alpha_2))+n(t-\tau_r-\alpha_2)\right\}.\mathrm{d}\alpha_2\mathrm{d}\alpha_1 \end{aligned} \tag{4-9}$$

We work out the product in equation (4-9), we get:

$$\begin{aligned} z_r(t) = &\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)\sum_{i=1}^{M}\sum_{j=1}^{M}\left\{\begin{array}{l} x_i(t-\alpha_1).x_j(t-\alpha_2-\tau_r) \\ +m_j.x_i(t-\alpha_1).x_j(t-\alpha_2-\tau_j-\tau_r) \\ +m_i.x_i(t-\alpha_1-\tau_i).x_j(t-\alpha_2-\tau_r) \\ +m_i.m_j.x_i(t-\alpha_1-\tau_i).x_j(t-\alpha_2-\tau_j-\tau_r) \end{array}\right\}.\mathrm{d}\alpha_2\mathrm{d}\alpha_1 \\ &+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2).n(t-\alpha_2-\tau_r)\sum_{i=1}^{M}\left\{x_i(t-\alpha_1)+m_i.x_i(t-\alpha_1-\tau_i)\right\}.\mathrm{d}\alpha_2\mathrm{d}\alpha_1 \\ &+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2).n(t-\alpha_1)\sum_{j=1}^{M}\left\{x_j(t-\tau_r-\alpha_2)+m_j.x_j(t-\tau_j-\tau_r-\alpha_2)\right\}.\mathrm{d}\alpha_2\mathrm{d}\alpha_1 \\ &+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2).n(t-\alpha_1).n(t-\alpha_2-\tau_r).\mathrm{d}\alpha_2\mathrm{d}\alpha_1 \end{aligned} \tag{4-10}$$

Since the integration is a linear operation, we interchange the sum and the double integral. Therefore equation (4-10) can be written as:

$$z_r(t)=\sum_{i=1}^{M}\sum_{j=1}^{M}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_1).h(\alpha_2)\begin{Bmatrix}x_i(t-\alpha_1).x_j(t-\alpha_2-\tau_r)+\\ m_j.x_i(t-\alpha_1).x_j(t-\alpha_2-\tau_j-\tau_r)+\\ m_i.x_i(t-\alpha_1-\tau_i).x_j(t-\alpha_2-\tau_r)+\\ m_i.m_j.x_i(t-\alpha_1-\tau_i).x_j(t-\alpha_2-\tau_j-\tau_r)+\end{Bmatrix}\mathrm{d}\alpha_1\mathrm{d}\alpha_2$$
$$+\sum_{j=1}^{M}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_1).h(\alpha_2).n(t-\alpha_1).x_j(t-\tau_r-\alpha_2).\mathrm{d}\alpha_1\mathrm{d}\alpha_2$$
$$+\sum_{j=1}^{M}m_j.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_1).h(\alpha_2).n(t-\alpha_1).x_j(t-\tau_j-\tau_r-\alpha_2).\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \qquad (4\text{-}11)$$
$$+\sum_{i=1}^{M}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_1).h(\alpha_2).n(t-\alpha_2-\tau_r).x_i(t-\alpha_1).\mathrm{d}\alpha_1\mathrm{d}\alpha_2$$
$$+\sum_{i=1}^{M}m_i.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_1).h(\alpha_2).n(t-\alpha_2-\tau_r).x_i(t-\alpha_1-\tau_i).\mathrm{d}\alpha_1\mathrm{d}\alpha_2$$
$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_1).h(\alpha_2).n(t-\alpha_1).n(t-\alpha_2-\tau_r).\mathrm{d}\alpha_1\mathrm{d}\alpha_2$$

4.1.2 The average of the mixer output

The average of the mixer output at receiver r can be found by calculating the expected value of the instantaneous mixer output $z_r(t)$. Since taking the expected value of an expression is a linear operation, we can interchange the order of integration and taking the expected value, thus equation (4-11) can be written as:

$$\mathrm{E}\{z_r(t)\}=\sum_{i=1}^{M}\sum_{j=1}^{M}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_1).h(\alpha_2)\begin{Bmatrix}\mathrm{E}\{x_i(t-\alpha_1).x_j(t-\alpha_2-\tau_r)\}\\ +m_j.\mathrm{E}\{x_i(t-\alpha_1).x_j(t-\alpha_2-\tau_j-\tau_r)\}\\ +m_i.\mathrm{E}\{x_i(t-\alpha_1-\tau_i).x_j(t-\alpha_2-\tau_r)\}\\ +m_i.m_j.\mathrm{E}\{x_i(t-\alpha_1-\tau_i).x_j(t-\alpha_2-\tau_j-\tau_r)\}\end{Bmatrix}\mathrm{d}\alpha_1\mathrm{d}\alpha_2$$
$$+\sum_{i=1}^{M}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_1).h(\alpha_2).\mathrm{E}\{n(t-\alpha_2-\tau_r).x_i(t-\alpha_1)\}.\mathrm{d}\alpha_1\mathrm{d}\alpha_2$$
$$+\sum_{i=1}^{M}m_i.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_1).h(\alpha_2).\mathrm{E}\{n(t-\alpha_2-\tau_r).x_i(t-\alpha_1-\tau_i)\}.\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \qquad (4\text{-}12)$$
$$+\sum_{j=1}^{M}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_1).h(\alpha_2).\mathrm{E}\{n(t-\alpha_1).x_j(t-\tau_r-\alpha_2)\}.\mathrm{d}\alpha_1\mathrm{d}\alpha_2$$
$$+\sum_{j=1}^{M}m_j.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_1).h(\alpha_2).\mathrm{E}\{n(t-\alpha_1).x_j(t-\tau_j-\tau_r-\alpha_2)\}.\mathrm{d}\alpha_1\mathrm{d}\alpha_2$$
$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_1).h(\alpha_2).\mathrm{E}\{n(t-\alpha_1).n(t-\alpha_2-\tau_r)\}.d\alpha_1 d\alpha_2$$

The noise carriers $x_i(t)$ and the AWGN $n(t)$ are mutually independent. Therefore the cross correlation between them is equal to zero. The cross correlation between the additive white Gaussian noise and its shifted version is equal to zero as we saw in the previous chapter. Expression (4-12) can be written as:

$$\mathrm{E}\left\{z_r(t)\right\}=\sum_{i=1}^{M}\sum_{j=1}^{M}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_1).h(\alpha_2).\begin{Bmatrix}\mathrm{E}\left\{x_i(t-\alpha_1).x_j(t-\alpha_2-\tau_r)\right\}\\+m_j.\mathrm{E}\left\{x_i(t-\alpha_1).x_j(t-\alpha_2-\tau_j-\tau_r)\right\}\\+m_i.\mathrm{E}\left\{x_i(t-\alpha_1-\tau_i).x_j(t-\alpha_2-\tau_r)\right\}\\+m_i.m_j.\mathrm{E}\left\{x_i(t-\alpha_1-\tau_i).x_j(t-\alpha_2-\tau_j-\tau_r)\right\}\end{Bmatrix}.\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \qquad (4\text{-}13)$$

$\left\{x_i(t)\right\}$ are stationary signals and are mutually uncorrelated, therefore

$$\mathrm{E}\{x_i(t_1).x_j(t_2)\}=R_{x_ix_j}(t_1-t_2)=0 \qquad \forall\ \ i\neq j \qquad (4\text{-}14)$$

We substitute the expected value by its autocorrelation function; equation (4-13) can be written as:

$$\mathrm{E}\left\{z_r(t)\right\}=\sum_{i=1}^{M}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_1).h(\alpha_2)\left\{R_{x_ix_j}(\tau_r)+m_j.R_{x_ix_i}(\tau_i+\tau_r)+m_i.R_{x_ix_i}(-\tau_i+\tau_r)+m_i.m_j.R_{x_ix_i}(\tau_r)\right\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \qquad (4\text{-}15)$$

We write the double integral as a double convolution, thus expression (4-15) can be written as:

$$\mathrm{E}\left\{z_r(t)\right\}=\sum_{i=1}^{M}\begin{Bmatrix}\left(h\otimes h_2\otimes R_{x_ix_i}\right)(\tau_r)+m_i.\left(h\otimes h_2\otimes R_{x_ix_i}\right)(\tau_i+\tau_r)\\+m_i.\left(h\otimes h_2\otimes R_{x_ix_i}\right)(-\tau_i+\tau_r)+m_i.m_i\left(h\otimes h_2\otimes R_{x_ix_i}\right)(\tau_r)\end{Bmatrix} \qquad (4\text{-}16)$$

We assume that the broadband signals $x_i(t)$ are identically distributed and have the same cross-correlation function, which means that:

$$R_{x_ix_i}(\tau)=R_{xx}(\tau) \qquad \forall i \qquad (4\text{-}17)$$

m_i is polar, thus: $m_i.m_i=1$

Using equation (4-17), equation (4-16) can be written as:

$$\mathrm{E}\left\{z_r(t)\right\}=\sum_{i=1}^{M}\begin{bmatrix}\left(h\otimes h_2\otimes R_{xx}\right)(\tau_r)\\+m_i.\left(h\otimes h_2\otimes R_{xx}\right)(\tau_i+\tau_r)\\+m_i.\left(h\otimes h_2\otimes R_{xx}\right)(-\tau_i+\tau_r)\\+\left(h\otimes h_2\otimes R_{xx}\right)(\tau_r)\end{bmatrix} \qquad (4\text{-}18)$$

We proved in appendix A1 that all terms where the time offset or the sum of two time offsets appear can be omitted. Hence equation (4-18) can be written as:

$$\mathrm{E}\{z_r(t)\} = \sum_{i=1}^{M} m_i.\left(h \otimes h_2 \otimes R_{xx}\right)(-\tau_i + \tau_r) \tag{4-19}$$

The average mixer output consists of M information terms, of which only one (the r-th term) is to be detected. Thus at receiver r, $\tau_i = \tau_r$, and equation (4-19) can be written as:

$$\mathrm{E}\{z_r(t)\} = m_r.h \otimes h_2 \otimes R_{xx}(0) + \sum_{\substack{i=1 \\ i \neq r}}^{M} m_i.h \otimes h_2 \otimes R_{xx}(-\tau_i + \tau_r) \tag{4-20}$$

The first term in equation (4-20) is the desired signal. The other M-1 information terms are cross talk. The cross talk terms are removed if the next condition is fulfilled:

$$|\tau_r - \tau_i| >> \tau_c \qquad \forall \;\; i \neq r \tag{4-21}$$

With τ_c the coherence time of the noise carriers.

We conclude that if expression (4-21) is fulfilled, equation (4-20) can be written as:

$$\mathrm{E}\{z_r(t)\} = m_r.h \otimes h_2 \otimes R_{xx}(0) \tag{4-22}$$

We can also write is as:

$$\mathrm{E}\{z_r(t)\} = m_r.\int_{-\infty}^{\infty} |H(f)|^2 \, S_{xx}(f)\mathrm{d}f \tag{4-23}$$

Conclusion

Each transmitter should have a different time offset. Moreover, the absolute value of the difference between each pair of time offsets should be greater than the coherence time of the of the noise carriers.

4.2 Power spectral density of the beat noise in the information band

As we did in chapter 3, but now with multiple transmitters and multiple receivers, we will determine the power spectral density of the beat noise in the information band. We will determine first the autocorrelation function at the mixer output and then the power spectral density of the beat noise in the information band. For the calculation of the power spectral density of the beat noise in the information band we follow the same steps as we did in appendix A2 and A3. The following equation results:

$$S_{z_r z_r}(0) = 6.\sum_{i=1}^{M}\int_{-\infty}^{\infty}\left|H(\nu)\right|^4 .S_{xx}^2(\nu)d\nu + 4.\sum_{i=1}^{M}\sum_{\substack{j=1 \\ j\neq i}}^{M}\int_{-\infty}^{\infty}\left|H(\nu)\right|^4 .S_{xx}^2(\nu)d\nu + \int_{-\infty}^{\infty}\left|H(\nu)\right|^4 .S_{xx}^2(\nu)d\nu$$
$$+ 4.\sum_{i=1}^{M}\int_{-\infty}^{\infty}\left|H(\nu)\right|^4 .S_{xx}(\nu).S_{nn}(\nu)d\nu + \int_{-\infty}^{\infty}\left|H(\nu)\right|^4 .S_{nn}^2(\nu)d\nu \qquad (4\text{-}24)$$

We assumed that the noise carriers x_i's are identically distributed, and that all received signals have the same power (no near-far effect). If we also assume that the broadband filters are identical, expression (4-24) can be written as:

$$S_{z_r z_r}(0) = 6.M.\int_{-\infty}^{\infty}\left|H(\nu)\right|^4 .S_{xx}^2(\nu)d\nu + 4.M.\left(M-1\right).\int_{-\infty}^{\infty}\left|H(\nu)\right|^4 .S_{xx}^2(\nu)d\nu + \int_{-\infty}^{\infty}\left|H(\nu)\right|^4 .S_{xx}^2(\nu)d\nu$$
$$+ 4.M.\int_{-\infty}^{\infty}\left|H(\nu)\right|^4 .S_{xx}(\nu)..S_{nn}(\nu)d\nu + \int_{-\infty}^{\infty}\left|H(\nu)\right|^4 .S_{nn}^2(\nu)d\nu \qquad (4\text{-}25)$$

If we substitute the AWGN power spectral density by its expression $\frac{N_0}{2}$, equation (4-25) can be written as:

$$S_{z_r z_r}(0) = \left(4M^2 + 2.M + 1\right).\int_{-\infty}^{\infty}\left|H(\nu)\right|^4 .S_{xx}^2(\nu)d\nu + 2.M.N_0\int_{-\infty}^{\infty}\left|H(\nu)\right|^4 .S_{xx}(\nu).d\nu + \frac{1}{4}N_0^{\,2}\int_{-\infty}^{\infty}\left|H(\nu)\right|^4 .d\nu \qquad (4\text{-}26)$$

The power spectral density of the noise in the information band contains three terms. The first term is the noise power resulting from the mixing of the different random noise carriers x_i' s. This noise power is proportional to the factor ($4M^2 + 2M + 1$), where M is the number of users. Consequently the number of users will be a limiting factor if we want to achieve a low bit error rate. The second term is the noise power resulting from the cross correlation between the different noise carriers x_i' s and the additive white Gaussian noise $n(t)$. This power is proportional to the factor *2M*. The third term is the noise power resulting only from the AWGN $n(t)$ mixed with itself.

4.3 Signal-to-beat noise ratio at the integrate and dump filter output

Following the same steps as we did in chapter 3, we can prove that the integrate and dump filter output is a Gaussian random variable with a mean value equal to:

$$\mathrm{E}\left\{v_{r,i}\right\} = \mathrm{E}\{z_r(t)\}.T_b + \mathrm{E}\left\{n_i''(t)\right\} \qquad (4\text{-}27)$$
$$= \mathrm{E}\{z_r(t)\}.T_b$$

and a variance equal to:

$$\mathrm{E}\left\{\left[v_{r,i}\right]^2\right\} = \mathrm{E}\left\{\left[n_i''\right]^2\right\} = S_{z_r z_r}(0).T_b \tag{4-28}$$

The signal-to-beat noise ratio is

$$SNR = \frac{\left[\mathrm{E}\left\{v_{r,i}\right\}\right]^2}{\mathrm{E}\left\{\left[n_i''\right]^2\right\}} \tag{4-29}$$

Where $\left[\mathrm{E}\left\{v_{r,i}\right\}\right]^2$ denotes the average signal component power for the *i*-th bit at receiver *r*. $\mathrm{E}\left\{\left[n_i''\right]^2\right\}$ denotes the variance of the noise.

Substituting equation (4-27) and equation (4-28) in equation (4-29) we get:

$$SNR = \frac{\left\{\int_{-\infty}^{\infty}\left|H(f)\right|^2.S_{xx}(f).df\right\}^2.T_b}{\left(4.M^2+2.M+1\right).\int_{-\infty}^{\infty}\left|H(\nu)\right|^4.S_{xx}^2(\nu)d\nu + 2.M.N_0.\int_{-\infty}^{\infty}\left|H(\nu)\right|^4.S_{xx}(\nu).d\nu + \left\{\frac{N_0}{2}\right\}^2.\int_{-\infty}^{\infty}\left|H(\nu)\right|^4.d\nu} \tag{4-30}$$

(Note: for M=1, equation (4-30) is reduced to the expression of the SNR for one user given by expression (3-48)).

We will see in the next section via a study of an example that the number of users is a limiting factor if we want to achieve a specific bit error rate.

4.4 Evaluation for band limited spectrally flat noise carriers

In the previous section we determined the general expressions for the signal-to-beat noise ratio of the time offset transmitted reference system. As we did in the previous chapter we will end this chapter with the study of a simple example. We assume that the noise carriers are band limited spectrally flat (perfect square in frequency domain) and their bandwidth is equal to B. We assume also that the transfer function of the broadband filter has a perfect square shape. They have the following power spectral density and transfer function:

$$S_{xx}(f) = \begin{cases} S_{xx} & -B/2 < f < B/2 \\ 0 & \text{elsewhere} \end{cases}$$

$$H(f) = \begin{cases} 1 & -B/2 < f < B/2 \\ 0 & \text{elsewhere} \end{cases} \tag{4-31}$$

Following the same steps as we did in chapter 3 the signal-to-beat noise ratio will be equal to:

$$SNR = \frac{\left\{\frac{E_b}{N_0}\right\}^2}{\left(4M^2 + 2.M + 1\right).\left\{\frac{E_b}{N_0}\right\}^2 .\frac{1}{G} + 4.M.\frac{E_b}{N_0} + G} \quad (4\text{-}32)$$

G: The processing gain

$\frac{E_b}{N_0}$: Received SNR per bit.

As it was the case for one user, the beat noise power resulting from the mixing between the different noise carriers and their shifted versions is inversely proportional to the processing gain G, and the noise power resulting from the AWGN is proportional to the processing gain G. Expression (4-32) illustrates that the number of users is a limiting factor if we want to achieve a specific link performance.

We can maximize the SNR by looking for the optimum processing gain. It can be done by minimizing the denominator. Doing this, the SNR is maximum if

$$G = \sqrt{4.M^2 + 2.M + 1}.\frac{E_b}{N_0} \quad (4\text{-}33)$$

This relation states that for a specified value of $\frac{E_b}{N_0}$, the processing gain G should satisfy equation (4-33) to maximize the signal-to-beat noise ratio. The optimal processing gain G is a linear function of $\frac{E_b}{N_0}$ and depends also on the number of users active at the same time.

Combining equation (4-33) and equation (4-32), we get:

$$SNR_{\text{optimal}} = \frac{1}{2\sqrt{4.M^2 + 2.M + 1} + 4.M}.\frac{E_b}{N_0} \quad (4\text{-}34)$$

Expression (4-34) illustrates the optimum signal-to-beat noise ratio for the time offset transmitted reference if equation (4-33) is fulfilled. Expression (4-34) also illustrates that if the number of transmitters increases, the optimal SNR will decreases. Expression (4-34) also illustrates that the optimal signal to noise ration is a linear function of $\frac{E_b}{N_0}$.

Given a processing gain G, we can determine the maximum signal-to-beat noise ratio that can be achieved by the time offset transmitted reference system. This can be done by letting $\frac{E_b}{N_0}$ approaching the infinity. Expression (4-32) can be reduced to:

$$SNR_{\max imal} = \frac{G}{\left(4M^2 + 2.M + 1\right)} \tag{4-35}$$

Expression (4-35) illustrates that, given a processing gain, the maximum signal-to-beat noise ratio is approximately inversely proportional to factor $4M^2 + 2.M + 1$. Where M is the number of users. Hence the performance of the system will degrade quite rapidly with the number of users. Note: Equation (4-36) is similar to the SNR equation for coherence multiplexing in optical domain [16].

4.4.1 Numerical evaluation

We proved in chapter 3 that the link performance for the time offset transmitted reference system is equal to the Gaussian tail function with an argument equal to the square root of the signal-to-beat noise ratio. Therefore the bit error rate for a system with multiple users can be written as:

$$P_e = \mathrm{Q}\left(\sqrt{\frac{\left\{\frac{E_b}{N_0}\right\}^2}{\left(4M^2 + 2.M + 1\right).\left\{\frac{E_b}{N_0}\right\}^2.\frac{1}{G} + 4.M.\frac{E_b}{N_0} + G}}\right) \tag{4-36}$$

Figure 4.2 illustrates the bit error rate as a function of $\frac{E_b}{N_0}$ for different number of users. The processing gain is equal to 30 dB for all cases. It is clear that the number of users is a limiting factor for the link performance of the time offset transmitted reference. At a given value of $\frac{E_b}{N_0}$ the BER degrade as the number of users increases. This is due to the increasing contribution of the cross talk from the other users. Figure 4.2 also illustrates the presence of an error-floor as the number of users increases. This is mainly due to increasing contribution of the interferences from the other users. Since these interferences are present even when the amplitude of the AWGN is zero, the BER saturates at large values of $\frac{E_b}{N_0}$ at:

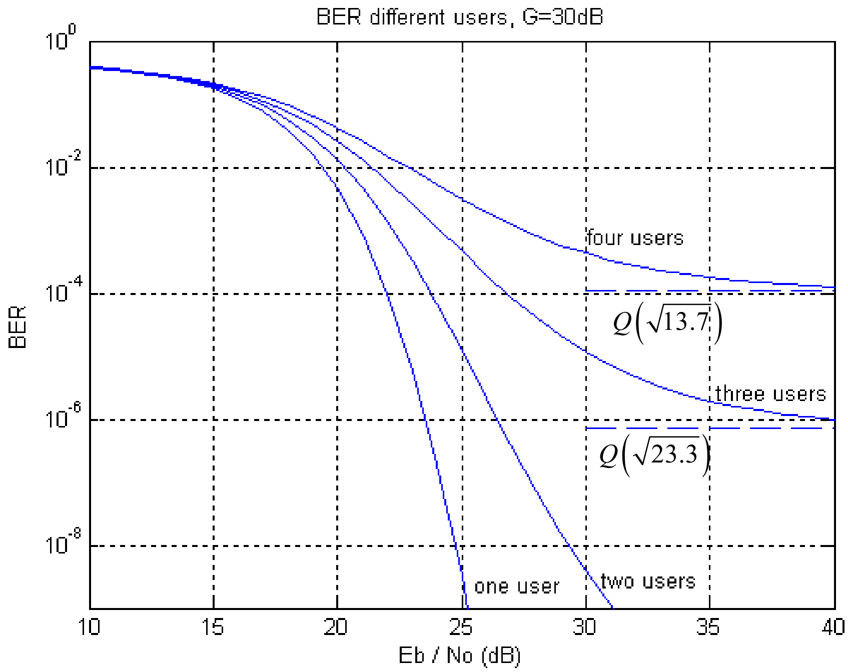

Figure 4.2:The BER as a function of $\frac{E_b}{N_0}$ for different users, G=30dB

$$P_{\text{error-floor}} = \mathrm{Q}\left(\sqrt{\frac{G}{\left(4M^2+2.M+1\right)}}\right) \tag{4-37}$$

For instance, if the number of users is equal to four, and the processing gain *G* is equal to 30dB the error-floor is approximately equal to 10^{-4}.

We also proved that for a given value of $\frac{E_b}{N_0}$, the signal-to-beat noise ratio is optimal only for one value of the processing gain *G*. Thus if equation (4-33) is fulfilled, the optimal bit error rate will be equal to:

$$P_{\text{optimal}} = \mathrm{Q}\left(\sqrt{\frac{1}{2\sqrt{4.M^2+2.M+1}+4.M}\cdot\frac{E_b}{N_0}}\right) \tag{4-38}$$

Therefore for a given $\frac{E_b}{N_0}$ the processing gain should be as close as possible to the optimal processing gain to get the lower bit error.

Figure 4.3 illustrates two different curves. The optimal bit error rate (left) for one user, two users, and three users, given the optimum processing gain *G* for each value of $\frac{E_b}{N_0}$. This figure shows also that when the number users increases the optimal BER increases. This is due as we said before to the increasing interferences from the different users. The second curve in Figure 4.3 is the optimal processing gain *G* (right) as a function of $\frac{E_b}{N_0}$ for one user, two users, and three users. The optimal processing gain *G* is a linear function of the $\frac{E_b}{N_0}$.

Figure 4.4 shows the error-floor achieved by the time offset transmitted reference as a function of the number of users, for a processing gain *G*=20dB, and *G*=23dB. It is apparent in this figure that the error-floor increase rapidly as the number of users increases. This is due to the increasing contribution of the interferences from the other users. Figure 4.4 also shows that by increasing the processing gain from 20 dB to 23dB the link performance is improved. This is consistent to expression (4-36). By increasing the processing gain the effect of the noise carrier-noise carrier interferences decreases.

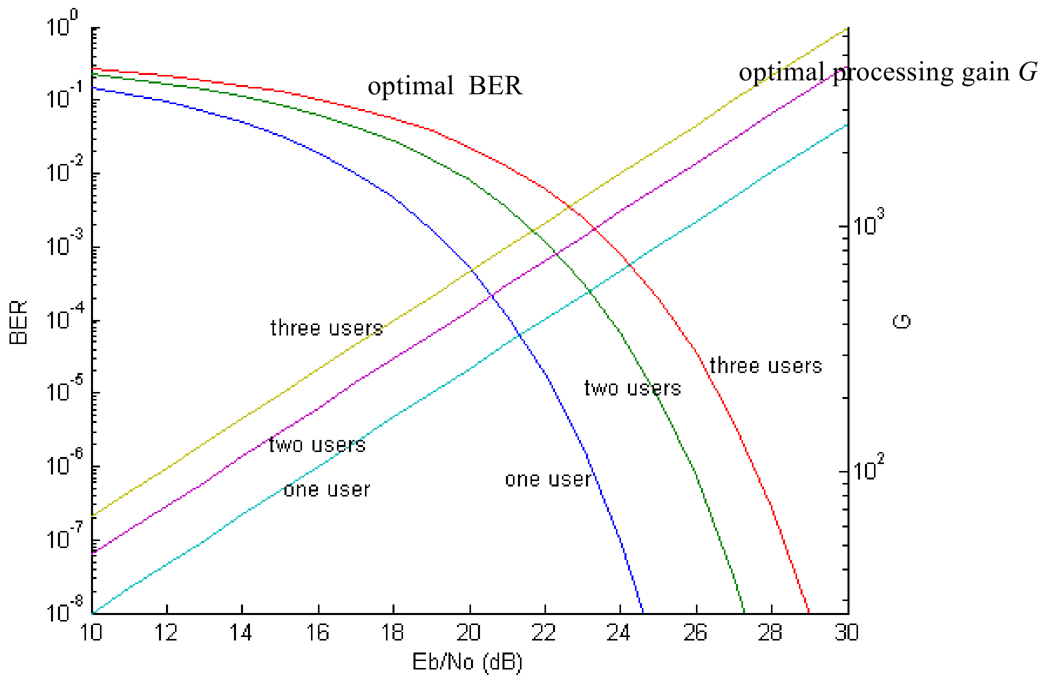

Figure 4.3: Optimal BER (left) and optimal processing gain (right) as a function of E_b/N_0

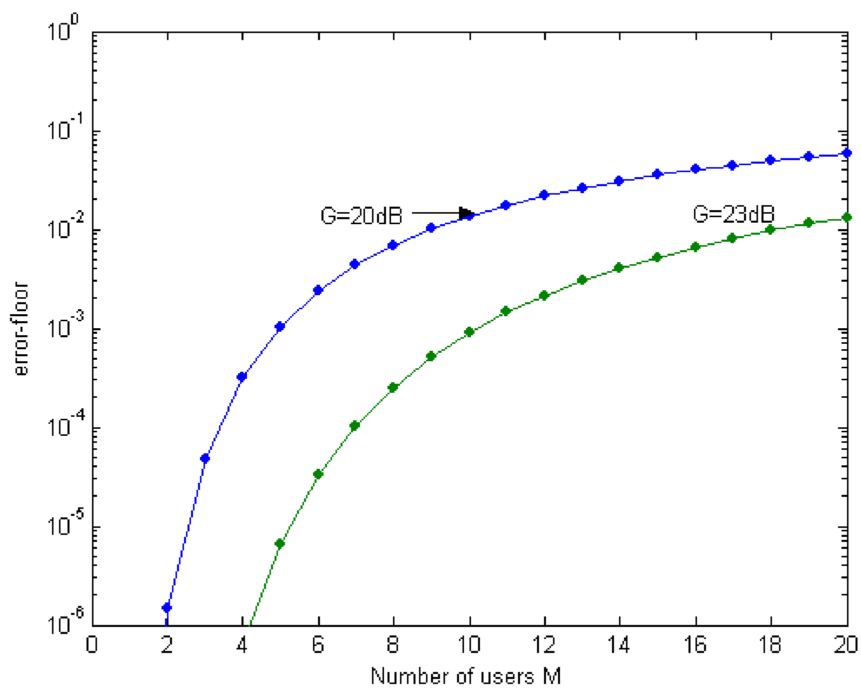

Figure 4.4:The error floor as a function of the number of users for *G*=20dB and *G*=23dB

5 Multipath effect and narrowband interference

We investigated in the previous chapter the link performance of an ideal system. In this chapter we will look at the multipath effect and the narrowband interference.

5.1 Multipath effect

Multipath occurs due to reflection from the ground and surrounding structures. The incoming radio waves arrive from different directions with different propagation delays. In this section we will investigate the multipath effect on the transmitted signal for one transmitter and one receiver. The transmitted signal is given by:

$$s(t) = x(t) + m(t).x(t - \tau_1) \tag{5-1}$$

Associated with each path is a propagation delay and a attenuation factor. Both propagation delays and attenuations factors are time-variant as a result of changes in the structure of the medium. The received signal may be expressed by [18]:

$$y(t) = \sum_{i=1}^{N} \gamma_i(t) s(t - \kappa_i(t)) + n(t) \tag{5-2}$$

Where $\gamma_i(t)$ is the attenuation factor for the signal received on the *i*-th path and $\kappa_i(t)$ is the propagation delay for the *i*-th path. $n(t)$ is the additive white Gaussian noise corrupting the signal at the receiver. *N* is the number of multipaths. Depending on the channel, the number of the paths varies in time. In our calculation, we will assume that the channel impulse response is time invariant, therefore:

$$\gamma_i(t) = \gamma_i \qquad \kappa_i(t) = \kappa_i \tag{5-3}$$

Expression (5-2) can be written as:

$$y(t) = \sum_{i=1}^{N} \gamma_i s(t - \kappa_i) + n(t) \tag{5-4}$$

We assume that

$\{\gamma_i\}$ are mutually independent and identically distributed.

$\{\kappa_i\}$ are mutually independent and identically distributed.

We substitute equation (5-1) in equation (5-4) we get:

$$y(t)=\sum_{i=1}^{N}\gamma_i\left\{x(t-\kappa_i)+m(t-\kappa_i).x(t-\tau_1-\kappa_i)\right\}+n(t) \tag{5-5}$$

The time offset at the receiver is equal to τ_r. Following the same steps as we did in chapter 3 the instantaneous mixer output $z(t)$ can be written as:

$$\begin{aligned} z(t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} &\left\{\sum_{i=1}^{N}\gamma_i.\{x(t-\kappa_i-\alpha_1)+m(t-\kappa_i-\alpha_1).x(t-\tau_1-\kappa_i-\alpha_1)\}+n(t-\alpha_1)\right\} \\ &.\left\{\sum_{j=1}^{N}\gamma_j.\{x(t-\tau_r-\kappa_j-\alpha_2)+m(t-\tau_r-\kappa_j-\alpha_2).x(t-\tau_1-\tau_r-\kappa_j-\alpha_2)\}+n(t-\alpha_2-\tau_r)\right\}.h(\alpha_1).h(\alpha_2)\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \end{aligned} \tag{5-6}$$

This can be written as:

$$\begin{aligned} z(t)=&\sum_{i=1}^{N}\sum_{j=1}^{N}\gamma_i.\gamma_j.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}x(t-\kappa_i-\alpha_1).x(t-\kappa_j-\alpha_2-\tau_r)h(\alpha_1).h(\alpha_2).\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\ &+\sum_{i=1}^{N}\sum_{j=1}^{N}\gamma_i.\gamma_j.m(t-\tau_r-\kappa_j).\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}x(t-\kappa_i-\alpha_1).x(t-\kappa_j-\alpha_2-\tau_1-\tau_r)h(\alpha_1).h(\alpha_2).\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\ &+\sum_{i=1}^{N}\sum_{j=1}^{N}\gamma_i.\gamma_j.m(t-\kappa_i).\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}.x(t-\kappa_i-\alpha_1-\tau_1).x(t-\kappa_j-\alpha_2-\tau_r)h(\alpha_1).h(\alpha_2).\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\ &+\sum_{i=1}^{N}\sum_{j=1}^{N}\gamma_i.\gamma_j.m(t-\kappa_i).m(t-\tau_r-\kappa_j).\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}x(t-\kappa_i-\alpha_1-\tau_1).x(t-\kappa_j-\alpha_2-\tau_1-\tau_r)h(\alpha_1).h(\alpha_1).\mathrm{d}\alpha_2\mathrm{d}\alpha_2 \\ &+\sum_{j=1}^{N}\gamma_j.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}x(t-\kappa_j-\alpha_2-\tau_r).n(t-\alpha_1)h(\alpha_1).h(\alpha_2).\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\ &+\sum_{j=1}^{N}\gamma_j.m(t-\tau_r-\kappa_j).\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}n(t-\alpha_1).x(t-\kappa_j-\alpha_2-\tau_1-\tau_r)h(\alpha_1).h(\alpha_2).\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\ &+\sum_{i=1}^{N}\gamma_i.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\{x(t-\kappa_i-\alpha_1-\tau_1).n(t-\alpha_2-\tau_r)\}h(\alpha_1).h(\alpha_2).\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\ &+\sum_{i=1}^{N}\gamma_i.m(t-\kappa_i).\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}x(t-\kappa_i-\alpha_1-\tau_1).n(t-\alpha_2-\tau_r)h(\alpha_1).h(\alpha_2).\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\ &+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}n(t-\alpha_1).n(t-\alpha_2-\tau_r)h(\alpha_1).h(\alpha_2).\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \end{aligned} \tag{5-7}$$

5.1.1 The mean value

The average of the mixer output $z(t)$ can be found by taking the expected value of $z(t)$. $\{\gamma_i\}$ and $x(t)$ are statistically independent. Therefore:

$\mathrm{E}\{\gamma_i.x(t)\}=\mathrm{E}\{\gamma_i\}.\mathrm{E}\{x(t)\}$

We follow the same steps as we did in chapter three. We get the next result:

$$\begin{aligned}
\mathrm{E}\{z(t)\} &= \sum_{i=1}^{N}\sum_{j=1}^{N}\mathrm{E}\{\gamma_i.\gamma_j\}.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}R_{xx}(-\alpha_1+\alpha_2-\kappa_i+\kappa_j+\tau_r)h(\alpha_1).h(\alpha_2).\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\
&+\sum_{i=1}^{N}\sum_{j=1}^{N}\mathrm{E}\{\gamma_i.\gamma_j\}.m(t-\tau_r-\kappa_j).\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}R_{xx}(-\alpha_1+\alpha_2-\kappa_i+\kappa_j+\tau_1+\tau_r)h(\alpha_1).h(\alpha_2).\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\
&+\sum_{i=1}^{N}\sum_{j=1}^{N}\mathrm{E}\{\gamma_i.\gamma_j\}.m(t-\kappa_i).\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}R_{xx}(-\alpha_1+\alpha_2-\kappa_i+\kappa_j-\tau_1+\tau_r)h(\alpha_1).h(\alpha_2).\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\
&+\sum_{i=1}^{N}\sum_{j=1}^{N}\mathrm{E}\{\gamma_i.\gamma_j\}.m(t-\kappa_i).m(t-\tau_r-\kappa_j).\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}R_{xx}(-\alpha_1+\alpha_2-\kappa_i+\kappa_j+\tau_r)h(\alpha_1).h(\alpha_1).\mathrm{d}\alpha_1\mathrm{d}\alpha_2
\end{aligned} \tag{5-8}$$

We write the double integral as a double convolution. Equation (5-8) can be written as:

$$\begin{aligned}
\mathrm{E}\{z(t)\} &= \sum_{i=1}^{N}\mathrm{E}\{\gamma_i^2\}.m(t-\kappa_i).(h\otimes h_2\otimes R_{xx})(-\tau_1+\tau_r) \\
&+\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\mathrm{E}\{\gamma_i.\gamma_j\}.m(t-\kappa_i).(h\otimes h_2\otimes R_{xx})(-\kappa_i+\kappa_j-\tau_1+\tau_r) \\
&+\sum_{i=1}^{N}\sum_{j=1}^{N}\mathrm{E}\{\gamma_i.\gamma_j\}.(h\otimes h_2\otimes R_{xx})(-\kappa_i+\kappa_j+\tau_r) \\
&+\sum_{i=1}^{N}\sum_{j=1}^{N}\mathrm{E}\{\gamma_i.\gamma_j\}.m(t-\tau_r-\kappa_j).(h\otimes h_2\otimes R_{xx})(-\kappa_i+\kappa_j+\tau_1+\tau_r) \\
&+\sum_{i=1}^{N}\sum_{j=1}^{N}\mathrm{E}\{\gamma_i.\gamma_j\}.m(t-\kappa_i).m(t-\tau_r-\kappa_j).(h\otimes h_2\otimes R_{xx})(-\kappa_i+\kappa_j+\tau_r)
\end{aligned} \tag{5-9}$$

From expression (5-9) we can distinguish between two cases:

- The time offset at the transmitter and at the receiver are equal
- The time offset at the transmitter and at the receiver are different.

If the time offsets are equal, equation (5-9) can be written as:

$$
\begin{aligned}
\mathrm{E}\{z(t)\} &= \sum_{i=1}^{N} \mathrm{E}\{\gamma_i^2\}.m(t-\kappa_i).(h \otimes h_2 \otimes R_{xx})(0) \\
&+ \sum_{i=1}^{N} \mathrm{E}\{\gamma_i^2\}.m(t-\tau-\kappa_j).(h \otimes h_2 \otimes R_{xx})(2\tau_r) \\
&+ \sum_{i=1}^{N} \mathrm{E}\{\gamma_i^2\}.(h \otimes h_2 \otimes R_{xx})(\tau_r) \\
&+ \sum_{i=1}^{N} \mathrm{E}\{\gamma_i^2\}.m(t-\kappa_i).m(t-\tau-\kappa_j).(h \otimes h_2 \otimes R_{xx})(\tau_r) \\
&+ \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \mathrm{E}\{\gamma_i.\gamma_j\}.(h \otimes h_2 \otimes R_{xx})(-\kappa_i+\kappa_j+\tau_r) \\
&+ \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \mathrm{E}\{\gamma_i.\gamma_j\}.m(t-\tau-\kappa_j).(h \otimes h_2 \otimes R_{xx})(-\kappa_i+\kappa_j+2\tau_r) \\
&+ \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \mathrm{E}\{\gamma_i.\gamma_j\}.m(t-\kappa_i).(h \otimes h_2 \otimes R_{xx})(-\kappa_i+\kappa_j) \\
&+ \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \mathrm{E}\{\gamma_i.\gamma_j\}.m(t-\kappa_i).m(t-\tau-\kappa_j).(h \otimes h_2 \otimes R_{xx})(-\kappa_i+\kappa_j+\tau_r)
\end{aligned}
\tag{5-10}
$$

We proved in chapter 3 and appendix A1 that all terms that contain only the time offset or the sum of two time offsets are assumed to be equal to zero. Thus equation (5-10) can be reduced to:

$$
\begin{aligned}
\mathrm{E}\{z(t)\} &= \sum_{i=1}^{N} \mathrm{E}\{\gamma_i^2\}.m(t-\kappa_i).(h \otimes h_2 \otimes R_{xx})(0) \\
&+ \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \mathrm{E}\{\gamma_i.\gamma_j\}.(h \otimes h_2 \otimes R_{xx})(-\kappa_i+\kappa_j+\tau_r) \\
&+ \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \mathrm{E}\{\gamma_i.\gamma_j\}.m(t-\tau-\kappa_j).(h \otimes h_2 \otimes R_{xx})(-\kappa_i+\kappa_j+2\tau_r) \\
&+ \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \mathrm{E}\{\gamma_i.\gamma_j\}.m(t-\kappa_i).(h \otimes h_2 \otimes R_{xx})(-\kappa_i+\kappa_j) \\
&+ \sum_{i=1}^{N} \sum_{\substack{j=1 \\ j \neq i}}^{N} \mathrm{E}\{\gamma_i.\gamma_j\}.m(t-\kappa_i).m(t-\tau-\kappa_j).(h \otimes h_2 \otimes R_{xx})(-\kappa_i+\kappa_j+\tau_r)
\end{aligned}
\tag{5-11}
$$

The mean value of the mixer output $z(t)$ is the sum of DC values coming from the different paths and the cross-terms as a result of the mixed signal between the different multi-path signals. Since only the DC values coming from the different paths are desired, the cross-terms should be equal to zero. This can be achieved if the next condition is fulfilled.

$$\left.\begin{array}{l} \left|-\kappa_i + \kappa_j + \tau_r\right| > \tau_c \\ \left|-\kappa_i + \kappa_j + 2\tau_r\right| > \tau_c \\ \left|-\kappa_i + \kappa_j\right| > \tau_c \end{array}\right\} \forall\ i \neq j \tag{5-12}$$

The third condition in (5-12) illustrates that the absolute value of the difference between each pair of multipath delays should be larger than the coherence time of the noise carrier. Expression (5-12) illustrate also that the time offset τ_r should be chosen on a way that also the first and the second conditions.

If the time offset at the receiver and at transmitter are different, the mean value of the mixer output $z(t)$ should be zero. That means all terms in expression (5-9) should equal zero. This can be achieved only if the next conditions are fulfilled:

$$\left.\begin{array}{l} \left|-\tau_1 + \tau_r\right| > \tau_c \\ \left|-\kappa_i + \kappa_j + \tau_r\right| > \tau_c \\ \left|-\kappa_i + \kappa_j + \tau_1 + \tau_r\right| > \tau_c \\ \left|-\kappa_i + \kappa_j - \tau_1 + \tau_r\right| > \tau_c \end{array}\right\} \tau_r \neq \tau_1, \quad \forall\ i, j \tag{5-13}$$

The first condition in (5-13) is always fulfilled (see chapter four). Expression (5-13) illustrates that the choice of the time offsets should take into account the path delays.

5.2 Narrowband interference

The UWB systems use very low power spectral density. This enables UWB radio systems to co-exist with other narrowband systems over the same frequency band without interfering the narrow band systems. Nevertheless, these narrowband systems may cause interferences, which may jam the UWB receiver completely. This strong interference occurs mainly because the power spectral density of the UWB signals is very low in comparison to the narrow band signals. These high levels of narrowband interferences are expected from a nearby narrow band radio systems, like cellular phones or other such devices. Note, that a typical cellular system uses a bandwidth of up to a few megahertz, while a typical UWB system uses a bandwidth of a few gigahertz.

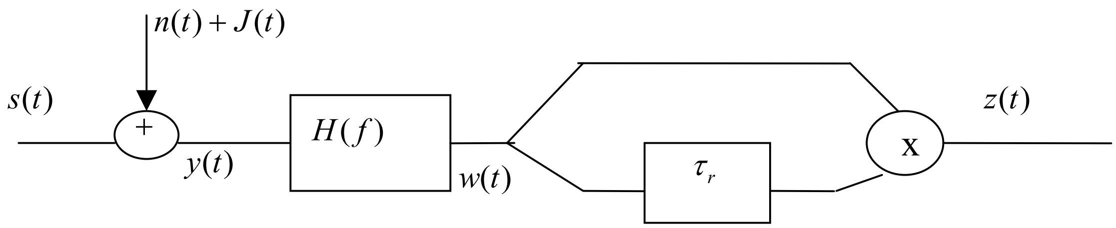

Figure 5.1: The received signal corrupted by a narrowband signal *J(t)*, and a broadband signal *n*(*t*)

In the previous chapters we assumed that the transmitted signal is corrupted only by a broadband noise signal, namely the additive white Gaussian noise. In this section we will also investigate the interference between the transmitted signal and a narrowband signal. Figure 5.1 shows the received signal corrupted by a broadband noise signal $n(t)$, and narrowband signal $J(t)$ at the receiver side.

The signal at the input of the receiver can be written as:

$$y(t) = s(t) + n(t) + J(t) \tag{5-14}$$

With $J(t)$ the narrowband signal. We define $J(t)$ by:

$$J(t) = A(t)\cos(2\pi f_0 t + \phi(t)) \tag{5-15}$$

$A(t)$ The amplitude of the narrowband signal

$\phi(t)$ The phase of the narrowband signal

f_0 The carrier frequency.

Since $J(t)$, $s(t)$ and $n(t)$ are mutually independent, the cross correlation between them is equal to zero. We proved in the previous chapter that the received signal without the narrowband signal is composed of the desired signal that is proportional to the information bit and white noise in the information signal band. Next we will investigate only the signal component resulting from the narrowband signal $J(t)$.

The narrowband signal $J(t)$ passes through the broadband filter. The filter output is

$$w_i(t) = J(t) \otimes h(t) \tag{5-16}$$

Since $J(t)$ is a narrowband signal and the filter is broadband, the transfer function $H(f)$ of the broadband filter can be regarded as constant over the frequency range of the narrowband interference [17]. Therefore the output of the filter is approximated by:

$$w_J(t) \approx |H(f_0)| A(t)\cos(2\pi f_0 t + \phi(t) + \arg(H(f_0))) \tag{5-17}$$

w_J is the component at the filter output resulting only from the narrowband signal.

The signal is split in two branches; the signal in the lower branch is delayed. Mixing the two signals coming from the two branches gives the next result:

$$\begin{aligned} z_J(t) &= w_J(t).w_J(t-\tau_r) \\ &= |H(f_0)|^2 A(t).A(t-\tau_r)\cos\left(2\pi f_0 t + \phi(t) + \arg(H(f_0))\right).\cos\left(2\pi f_0 (t-\tau_r) + \phi(t-\tau_r) + \arg(H(f_0))\right) \end{aligned} \tag{5-18}$$

$z_J(t)$ is the mixer output component resulting only from the narrowband signal.

When we work out the product, expression (5-18) can be written as:

$$\begin{aligned} z_J(t) &= \frac{1}{2}.|H(f_0)|^2 A(t).A(t-\tau_r).\cos\left(4\pi f_0 t + 2\arg(H(f_0)) - 2\pi f_0 \tau_r + \phi(t) + \phi(t-\tau_r)\right) \\ &\quad + \frac{1}{2}.|H(f_0)|^2 A(t).A(t-\tau_r).\cos\left(2\pi f_0 \tau_r + \phi(t) - \phi(t-\tau_r)\right) \end{aligned} \tag{5-19}$$

A(t) and $\phi(t)$ change slowly since $n_i(t)$ is narrowband. Therefore we assume that *A(t)* and $\phi(t)$ won't change during one bit time T_b (this can be seen as the worst case). The time offset τ_r is so small that we can assume that $A(t)$ and $\phi(t)$ are equal to their shifted version $A(t-\tau_r)$ and $\phi(t-\tau_r)$ during one bit time T_b.

$$\left.\begin{aligned} A(t) &= A(t-\tau_r) = A \\ \phi(t) &= \phi(t-\tau_r) \end{aligned}\right\} \qquad 0 \le t < T_b \tag{5-20}$$

Expression (5-19) can be reduce to:

$$\begin{aligned} z_J(t) &= \frac{1}{2}.|H(f_0)|^2 .A^2.\cos\left(4\pi f_0 t + 2\arg(H(f_0)) - 2\pi f_0 \tau_r + 2\phi(t)\right) \\ &\quad + \frac{1}{2}.|H(f_0)|^2 .A^2.\cos(2\pi f_0 \tau_r) \end{aligned} \tag{5-21}$$

The output of the mixer passes through the integrate and dump filter. The integration time is equal the bit time T_b:

$$v_J = \frac{1}{2}.\left|H(f_0)\right|^2.A^2.\int_0^{T_b}\cos\left(4\pi f_0 t + 2\arg(H(f_0)) - 2\pi f_0\tau_r + 2\phi(t)\right)\mathrm{d}t$$
$$+\frac{1}{2}.\left|H(f_0)\right|^2.A^2.\cos(2\pi f_0\tau_r).\int_0^{T_b}\mathrm{d}t \qquad (5\text{-}22)$$

v_J is the integrate and dump filter output resulting only from the narrowband filter.

The integration time T_b is large enough that we can assume that the first term in expression (5-22) is equal to zero, and thus expression (5-22) can be reduced as:

$$v_J = \frac{1}{2}.\left|H(f_0)\right|^2.A^2.\cos(2\pi f_0\tau_r)\int_0^{T_b}\mathrm{d}t$$
$$= \frac{1}{2}.\left|H(f_0)\right|^2.A^2.\cos(2\pi f_0\tau_r).T_b \qquad (5\text{-}23)$$

Expression (5-23) illustrates that the integrator output has a DC value resulting from the narrowband signal $J(t)$ that depends on the term $\cos(2\pi f_0\tau_r)$. If $f_0\tau_r = \frac{n}{2}$, the narrowband interference is maximal and is equal to:

$$v_J = \left|H(f_0)\right|^2.A^2.T_b \qquad (5\text{-}24)$$

We can conclude that the narrow band interference can jam the information signal completely and cause a severe damage on the desired signal. The next section will deal with an approach to suppress the narrow band interference.

5.2.1 Narrow band suppression using Manchester line code

To improve the system with the presence of a narrowband signal an adaptive notch filter can be used to suppress all narrowband interferences. Since the narrow band interference center frequency can range from very low frequency to few gigahertz, implementing an analog notch filter requires a bank of analog notch filters, each with different range of possible center frequencies. This approach increases the receiver complexity considerably. This thesis will present another possible approach to suppress narrow band interference. It is based on the use of Manchester line codes at the transmitter to encode the information signal, Figure 5.2, rather than using polar line code. Et the receiver, Figure 5.3, the signal coming from the mixer is multiplied by a block signal that takes values ± 1 alternatively, Figure 5.4. The bit rate of the block signal is equal to the bit rate of the Manchester encoded signal.

The mixer output resulting from the narrow band interference component is given by equation (5-21). Multiplying the narrow band interference component by $c(t)$ will lead to:

$$z_J(t).c(t) = \frac{1}{2}.\left|H(f_0)\right|^2.A^2.\cos\left(4\pi f_0 t + 2\arg(H(f_0)) - 2\pi f_0\tau_r + 2\phi(t)\right).c(t)$$
$$+\frac{1}{2}.\left|H(f_0)\right|^2.A^2.\cos(2\pi f_0\tau_r).c(t) \qquad (5\text{-}25)$$

Integrating between 0 and T_b lead to the next equation

$$\int_0^{T_b} z_J(t).c(t).\mathrm{d}t = \int_0^{T_b}\frac{1}{2}.\left|H(f_0)\right|^2.A^2.\cos\left(4\pi f_0 t + 2\arg(H(f_0)) - 2\pi f_0\tau_r + 2\phi(t)\right).c(t).\mathrm{d}t$$
$$+\int_0^{T_b}\frac{1}{2}.\left|H(f_0)\right|^2.A^2.\cos(2\pi f_0\tau_r).c(t).\mathrm{d}t \qquad (5\text{-}26)$$

$c(t)$ is equal to 1 in the first half of the information bit time and is equal to –1 in the second half. Equation (5-26) can be written as:

$$\int_0^{T_b} z_J(t).c(t).\mathrm{d}t = \int_0^{\frac{T_b}{2}}\frac{1}{2}.\left|H(f_0)\right|^2.A^2.\cos\left(4\pi f_0 t + 2\arg(H(f_0)) - 2\pi f_0\tau_r + 2\phi(t)\right).\mathrm{d}t$$
$$-\int_{\frac{T_b}{2}}^{T_b}\frac{1}{2}.\left|H(f_0)\right|^2.A^2.\cos\left(4\pi f_0 t + 2\arg(H(f_0)) - 2\pi f_0\tau_r + 2\phi(t)\right).\mathrm{d}t$$
$$+\int_0^{\frac{T_b}{2}}\frac{1}{2}.\left|H(f_0)\right|^2.A^2.\cos(2\pi f_0\tau_r).\mathrm{d}t \qquad (5\text{-}27)$$
$$-\int_{\frac{T_b}{2}}^{T_b}\frac{1}{2}.\left|H(f_0)\right|^2.A^2.\cos(2\pi f_0\tau_r).\mathrm{d}t$$
$$=0$$

Equation (5-27) illustrates that the narrowband interference can be suppressed completely by using Manchester line code at the transmitter and multiplying the mixer output by the block signal $c(t)$.

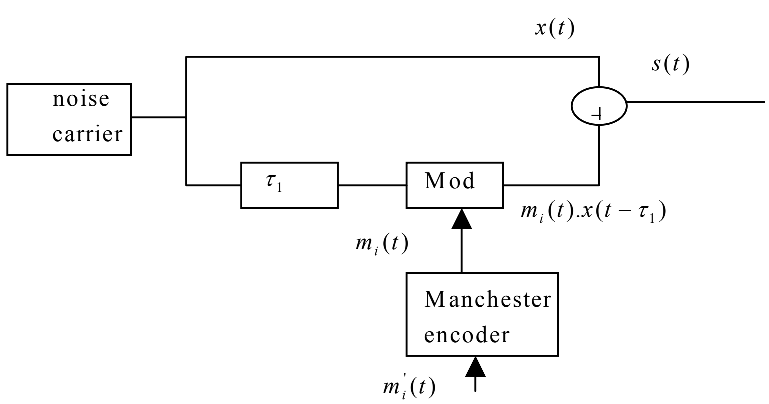

Figure 5.2: Modified transmitter scheme

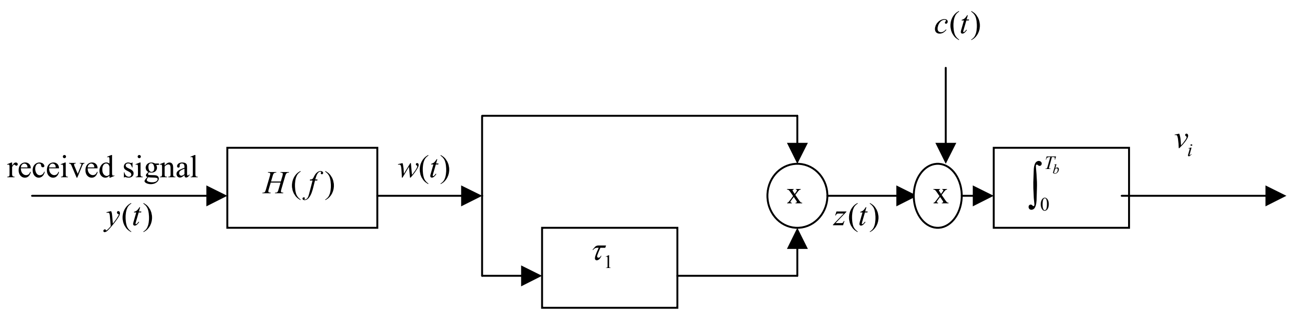

Figure 5.3: Modified receiver scheme

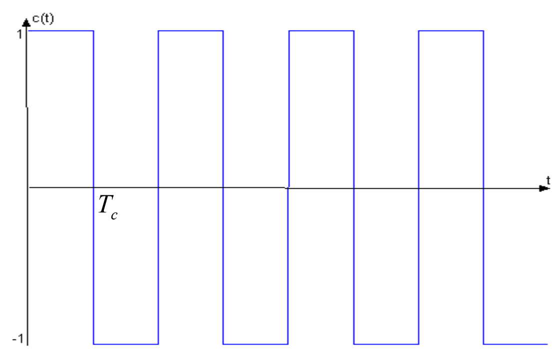

Figure 5.4: Bloc signal *c*(*t*)

6 Simulation results

We determined the link performance for the time offset transmitted reference in the previous chapters by analysis. In this chapter we will verify these results. A scheme is build in Simulink under Matlab. Simulink is MATLAB Graphical User Interface (GUI) to build dynamic systems. The MATLAB interpreter makes all calculations. The simulator of Simulink only works in the time-domain. Therefore the behavior of most dynamic systems is described in the time domain.

6.1 Simulation model

Figure 6.1 shows the implementation in Simulink of the time offset transmitted reference system having one transmitter and one receiver. In this section we will describe the behavior of the most important components.

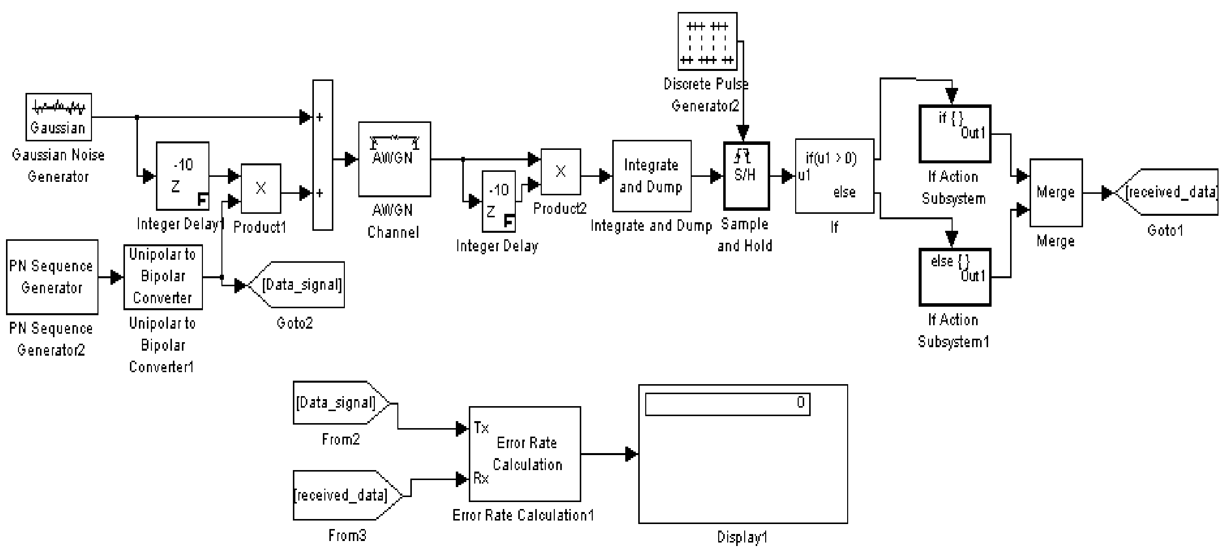

Figure 6.1: The implementation of the time offset transmitted reference in Simulink

6.1.1 The Gaussian noise generator

The Gaussian Noise Generator block generates discrete-time white Gaussian noise. There are several ways to generate random sequences having the properties of Gaussian noise. These functions use a uniform random generator since it is very easy to generate these numbers. These functions are available in programming languages. Possible solutions are 1) central limit theory

2) box-Muller method. Both models has a uniform distribution and transform it to a normal distribution with mean value equal to zero and a variance equal to one [19]. From here one can transform the function into every mean value and every variance. To have realistic simulations, different Gaussian noises should be generated for each simulation. Also, if there is more than one Block, each block should generate a different Gaussian noise. Otherwise the Noise carriers will be correlated and the system will not work. To achieve these conditions we choose different initial seeds for all blocks and for each simulation. It can be realized by using the randseed function.

6.1.2 Data signal generator

The data signal generator can be any random signal generator. In our simulation we used a pseudo noise sequence generator. It generates a sequence of pseudorandom binary numbers. The pseudo noise sequence generator block uses a shift register to generate sequences. Since the generator values are ones and zeros, we used a unipolar to bipolar converter to transform data to a polar signal. To have realistic results, a different data signal should be generated for each experiment. Also if there is more than one user, each block should generate a different data signal. This can be achieved by giving random values for the initial state or by giving a random generator polynomial, or both of them, as it was the case in our simulations.

6.1.3 Modulator

The modulator shifts the phase of the waveform by $\{0, \pi\}$, resulting in a multiplication by ± 1. In the simulations the modulator just multiplies the applied sequence at the inputs by each other.

6.1.4 The additive white Gaussian noise block

The additive white Gaussian noise block adds a white Gaussian noise to the signal transmitted over the channel. The white Gaussian noise is generated on the same way, as it was explained in section 6.1.1. We can specify the variance of the noise generated by the AWGN channel block using four modes. In our simulations we only used the signal to noise ratio per bit $\frac{E_b}{N_0}$ mode. The block calculates the variance from these quantities that we specify in the block mask:

- $\frac{E_b}{N_0}$ Signal to noise ratio per bit.

- s : The input signal power
- T_b : Symbol period.

The sample period T_{samp} is inherent in the signal at the input of the block. The AWGN block detects it automatically. The relation between all this parameters is given by:

$$\frac{E_b}{N_0} = \frac{s.T_b}{\frac{N}{B_n}} = \frac{s.T_b}{\frac{N}{F_s}} = \frac{s.T_b}{N.T_{samp}}$$

where

N: Noise power

B_n: Noise bandwidth

F_s: Sampling frequency

$$B_n = F_s = {}^{1}\!/\!{}_{T_{samp}}$$

The bandwidth of the white Gaussian noise added to the signal over the channel is limited. Its bandwidth is equal to these of the noise carrier. Therefore the broadband filter is omitted in the simulations.

Note:

The additive white Gaussion noise is a stochastic process with a constant power spectral density for all frequencies. The autocorrelation function of the white noise is the inverse Fourier transform of the power spectral density $R_x(\tau) = \frac{N_0}{2}\delta(\tau)$. The average power is $R_x(0)$. In this case it is infinity. This means that white noise cannot exist in the nature. However, white noise approximates noise processes in the nature.

6.1.5 Integer delay

The Integer Delay block delays its input by N sample periods. The block accepts one input and generates one output, both of which can be scalar or vector.

6.1.6 Integrate and dump filter

The Integrate and Dump block integrates the input signal in discrete time, and resets to zero according to a fixed schedule. The reset times are the positive integral multiples of the Integration period parameter. At each reset time, the block performs its final integration step, sends the result to the output port, and then clears its internal state for the next time step. This block uses the Forward Euler integration method.

6.1.7 Decoder

Figure 6.2 shows the decoder used in the simulations. It consists of a comparator. It compares the output of the correlator to the threshold zero. Via if action, it takes the decision whether the detected bit is 1 or –1.

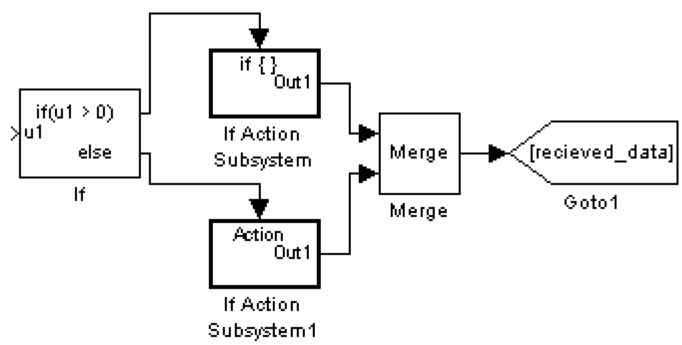

Figure 6.2:Decoder

6.1.8 Error rate calculation block set.

The Error Rate Calculation block compares the input data from the transmitter with the input data from the receiver. It calculates the error rate as a running statistic, by dividing the total number of unequal pairs of data elements by the total number of input data elements from one source.

6.2 Simulation setup and configurations

All simulations use the same model; that is why we will mention the setup here once. If nothing is mentioned about the setup we assume these settings. Otherwise the proper values of the components are mentioned in the appropriate section.

We assumed in our theoretical work that the bit time should be large enough so that all signals in the system are quasi stationary during one bit time. Therefore the processing gain G should be large enough to fulfill this condition. In our simulations we chose a processing gain equal to 30dB. All blocks generating a Gaussian noise do have a sampling time equal to one nanosecond. That is equivalent to one gigahertz bandwidth. For each simulation a different random noise signal is generated. The data signal has a bit time equal to one microsecond. That is equivalent to one megahertz bandwidth. The data signal is also random for each simulation.

6.2.1 System with one transmitter and one receiver

In all simulations done for one transmitter and one receiver and with a processing gain equal to 30 dB, the time offset is equal to 10 nanoseconds. Figure 6.3 shows a comparison between the theoretical link performance curve and the curve resulting from the simulations. It is clear that the two curves do match. There are some differences between the two curves for values of $\frac{E_b}{N_0}$ greater than 20. The difference can probably be explained by the fact that for values of $\frac{E_b}{N_0}$ greater than 20, the bit error becomes low and the number of samples wasn't enough to have good statistical results. To have good statistical results, the simulations should last longer periods of time and the two curves would probably much. Another possibility to justify the differences for high values of $\frac{E_b}{N_0}$ is that the integrator output distribution is probably not Gaussion for high values of $\frac{E_b}{N_0}$.

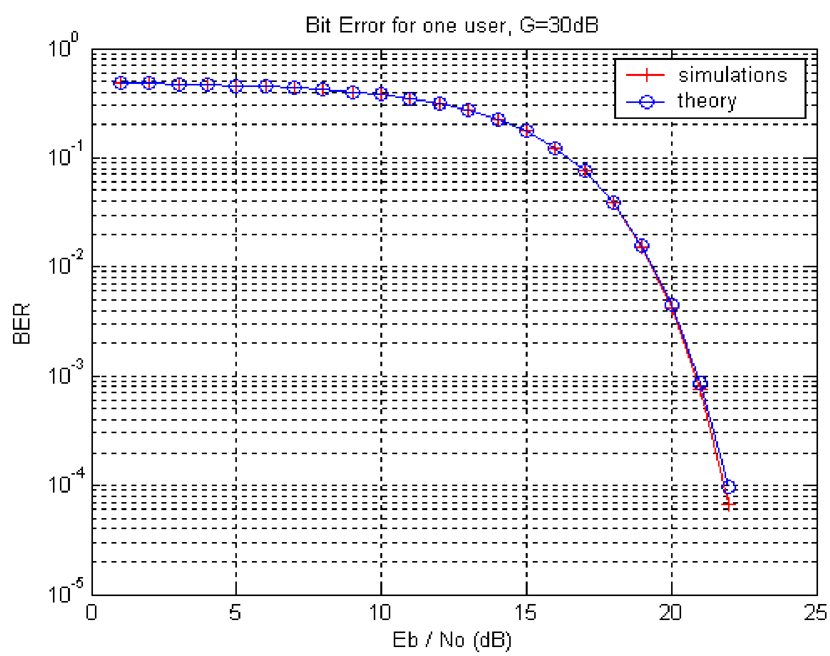

Figure 6.3: Bit error rate for one transmitter and one receiver. G=30dB

6.2.2 System with two transmitters and two receivers

Figure 6.4 shows the implementation of the time offset transmitted reference systems with two transmitters and two receivers. The time offset used in these simulations is 4 nanoseconds for the first user and 10 nanosecond for the second user. During the simulations the bit error rate is determined for the two users. The difference between the two simulated values is very small. We took the average of the two simulated bit error rate as the total bit error rate. Figure 6.5 shows the theoretical and the simulation curves. The two curves do mach except for values of $\frac{E_b}{N_0}$ greater than 21 dB, where we can see some differences. As we mentioned in the previous section, the differences is probably due to the fact that for high value of $\frac{E_b}{N_0}$ the integrator output does not have a Gaussion distribution.

6.2.3 System with four users

The time offset used in these simulations is 2 nanoseconds for the first user, 7 nanoseconds for the second user, 11 nanoseconds for the third user, and 23 nanoseconds for the fourth user. The choice of the time offsets was random. Other values can be chosen. During the simulations the bit error rate is determined for the four users. It can be observed that all users achieve quite similar number of error with small differences between them. The reason is that all users are suffering from similar amount of interference from all other users. The total bit error rate is the average of the four simulated bit error rates. Figure 6.6 shows the theoretical and simulation curves. It can be observed that simulation results match very closely the theoretical results except for values of $\frac{E_b}{N_0}$ above 25 dB. As we said before, this is probably due to the fact that for high values of $\frac{E_b}{N_0}$ is that the integrator output distribution is not Gaussion. The simulations also illustrate the presence of the error floor witch is almost equal to the theoretical value.

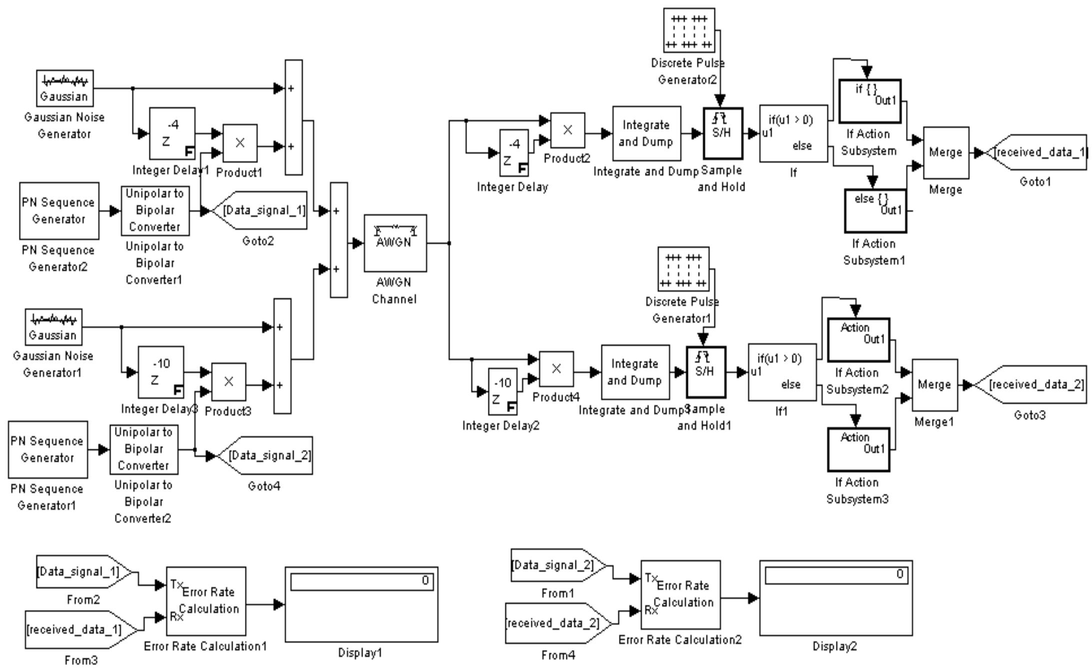

Figure 6.4: The time offset transmitted reference with two transmitters and two receivers

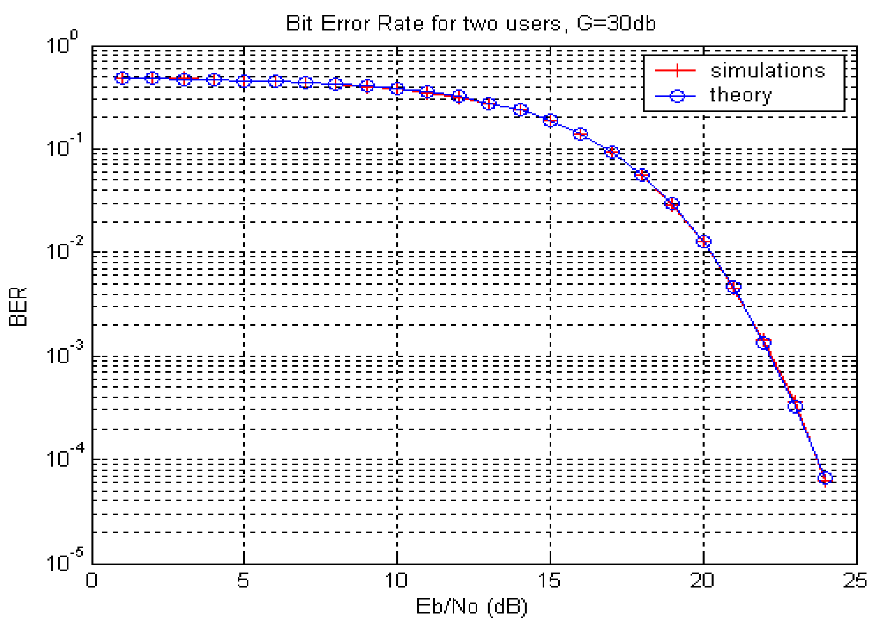

Figure 6.5:Bit Error Rate for two transmitters and two receivers, G=30dB

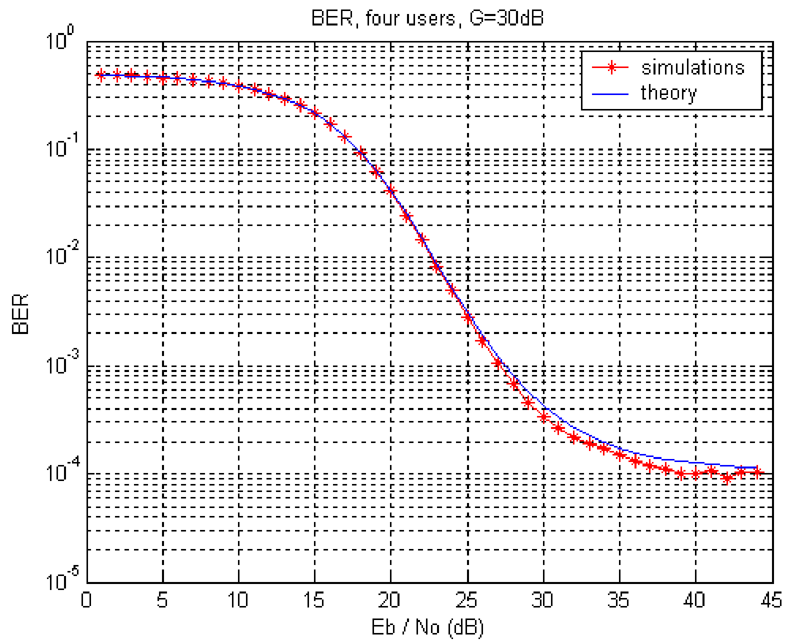

Figure 6.6: Bit Error Rate, simulations and theoretical results for four users

6.2.4 Error floor

We saw in chapter four that the error floor is a function of two parameters, namely the processing gain and the number of users. As we mentioned before, the processing gain is equal to 30 dB. To be able to check the error-floor, we increase the number of users. Two simulations were done. The first one is done with three users, and the second one is done for four users. The SNR per bit $\frac{E_b}{N_0}$ is equal to 200dB. The theoretical and simulation results are given in table 1. We can see that the simulation results are almost equal to the theoretical values.

Table 1:Error-floor for three and four users, G=30dB

Number of users	Processing gain G (dB)	Theoretical value	Simulation value
Three	30	$7.1*10^{-7}$	$9.3*10^{-7}$
Four	30	$1.1*10^{-4}$	$0.9 * 10^{-4}$

7 Conclusions and recommendations

7.1 Conclusions

This thesis has concentrated on the application of chaotic signals as carrier in radio communications. Delay lines are used at the transmitter to distinguish the modulated signal and the reference signal. The study of this system led to the next conclusions

- The time offset at the transmitter and the receiver should be equal to demodulate the information signal to base band signal, or the difference between the time offsets is less than the coherence time of the noise carrier.
- To minimize the effect of the cross-talks between the different users, each transmitter should have a different time offset. Moreover the difference between each pair of time offsets should be larger than the coherence time of the noise carrier.
- The general expression of the BER for the time offset transmitted reference system has shown that the BER is dependent of the total cross-talks and the number of users
- It was shown that narrowband interference can add a significant DC value at the integrator output and therefore can affect severely the performance of our system. Significant improvement in performance can be obtained when Manchester line coding is used.
- To suppress the cross-talk in a multipath environment, the absolute value of the difference between each pair of multipath delays should be larger than the coherence time of the noise carrier

The study of the band limited Gaussian noise carrier shows the next results:

- The BER as a function of E_b/N_0 degrades as the number of users increases.
- We found that the BER at fixed E_b/N_0 reaches its minimum at a certain processing gain G.
- The BER develops an error-floor and that the error floor increases rapidly with the number of users or if the processing Gain G decreases.

7.2 Recommendations

- In our work we used a noise carrier with a Gaussion distribution. Further work can investigate other kind of noises that have different distribution.
- In our work we gave a brief study of the behavior of the time offset transmitted reference in the presence of multipath. Further work can include the calculation of the BER in various multipath fading environments.
- In our work we proposed a general idea on how to suppress narrow band interferences. Further work can include the calculation of the BER taking into account the presence of narrowband interferences.
- We assumed in our work that all receivers receive equal amount of power. Further work can include the calculation of the BER taking into account the near-far effect.
- During all calculation we assumed that the transmitted bit energy is time invariant. Further work can derive the link performance taking into account that the transmitted or the received power is time variant.

Appendix A Calculation on the time offset transmitted reference system with one transmitter and one receiver

Appendix A1 Calculation of the average mixer output

We write all terms in equation (3-17) as the inverse Fourier transform of the power spectral density. The Fourier transform of a convolution of functions is equal to the product of the Fourier transform of those functions. The Fourier transform of the time-reversed version of a real function is equal to the complex conjugate of the Fourier transform of that function [1].

$$\begin{aligned} \mathrm{E}\{z(t)\} = & \int_{-\infty}^{\infty} H(f).H^{*}(f) S_{xx}(f) \exp(j2\pi f \tau_1)\mathrm{d}f + m_i.\int_{-\infty}^{\infty} H(f).H^{*}(f) S_{xx}(f) \exp(j2\pi f 2\tau_1)\mathrm{d}f \\ & + m_i.\int_{-\infty}^{\infty} H(f).H^{*}(f) S_{xx}(f)\mathrm{d}f + \int_{-\infty}^{\infty} H(f).H^{*}(f) S_{xx}(f) \exp(j2\pi f \tau_1)\mathrm{d}f \\ & + \int_{-\infty}^{\infty} H(f).H^{*}(f) S_{nn}(f) \exp(j2\pi f \tau_1)\mathrm{d}f \end{aligned} \qquad \text{(A.1)}$$

$S_{nn}(f)$ is the power spectrum density of $n(t)$

$S_{xx}(f)$ is the power spectrum density of $x(t)$

We can prove that all terms where τ_1 appears can be omitted. For example:

$$\int_{-\infty}^{\infty} |H(f)|^2 .S_{xx}(f) \exp(j2\pi f \tau_1)\mathrm{d}f = \int_{-\infty}^{\infty} |H(f)|^2 .S_{xx}(f).\cos(2\pi f \tau_1)\mathrm{d}f + j\int_{-\infty}^{\infty} |H(f)|^2 .S_{xx}(f).\sin(2\pi f \tau_1)\mathrm{d}f \qquad \text{(A.2)}$$

The integrals in (A.2) are assumed to be zero since $H(f)$ and $S_{xx}(f)$ are broadband

$$\int_{-\infty}^{\infty} |H(f)|^2 .S_{xx}(f) \exp(j2\pi f \tau_1)\mathrm{d}f \approx 0$$

By omitting all terms where τ_1 appears the expected value of $z(t)$ will be reduced to:

$$\mathrm{E}\{z(t)\} = m_i.\int_{-\infty}^{\infty} |H(f)|^2 S_{xx}(f)\mathrm{d}f \qquad \text{(A.3)}$$

Appendix A2 Calculation of the mixer output autocorrelation function

The autocorrelation of $z(t)$ can be determined by calculating the expected value of the product $z(t)$ and its shifted version $z(t+\tau)$.

$$R_{zz}(t,t+\beta) = \mathrm{E}\{z(t)z(t+\tau)\} \tag{A.4}$$

We replace $z(t)$ and $z(t+\tau)$ by their expressions given in equation (3-13), (A.4) can be written as:

$$\mathrm{E}\left\{z(t)z(t+\tau)\right\} = \mathrm{E}\left\{\begin{array}{l} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)\left\{\begin{array}{l} x(t-\alpha_1).x(t-\alpha_2-\tau_1) \\ +m_i.x(t-\alpha_1).x(t-\alpha_2-2\tau_1) \\ +x(t-\alpha_1).n(t-\alpha_2-\tau_1) \\ +m_i.x(t-\alpha_1-\tau_1).x(t-\alpha_2-\tau_1) \\ +x(t-\alpha_1-\tau_1).x(t-\alpha_2-2\tau_1) \\ +m_i.x(t-\alpha_1-\tau_1).n(t-\alpha_2-\tau_1) \\ +n(t-\alpha_1).x(t-\alpha_2-\tau_1) \\ +m_i.n(t-\alpha_1).x(t-\alpha_2-2\tau_1) \\ +n(t-\alpha_1).n(t-\alpha_2-\tau_1) \end{array}\right\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2 \\ .\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_3).h(\alpha_4)\left\{\begin{array}{l} x(t+\tau-\alpha_3).x(t+\tau-\alpha_4-\tau_1) \\ +m_i.x(t+\tau-\alpha_3).x(t+\tau-\alpha_4-2\tau_1) \\ +x(t+\tau-\alpha_3).n(t+\tau-\alpha_4-\tau_1) \\ +m_i.x(t+\tau-\alpha_3-\tau_1).x(t+\tau-\alpha_4-\tau_1) \\ +x(t+\tau-\alpha_3-\tau_1).x(t+\tau-\alpha_4-2\tau_1) \\ +m_i.x(t+\tau-\alpha_3-\tau_1).n(t+\tau-\alpha_4-\tau_1) \\ +n(t+\tau-\alpha_3).x(t+\tau-\alpha_4-\tau_1) \\ +m_i.n(t+\tau-\alpha_3).x(t+\tau-\alpha_4-2\tau_1) \\ +n(t+\tau-\alpha_3).n(t+\tau-\alpha_4-\tau_1) \end{array}\right\}\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \end{array}\right\} \tag{A.5}$$

Eliminating the brackets, writing products of double integrals as fourfold integrals, interchanging the order of integration, taking the expected value, and taking in account the next points

- The terms in which $n(t)$ and $x(t)$ occurs only once are omitted, since $n(t)$ and $x(t)$ are mutually independent and their mean value is equal to zero. For instance:
  $\mathrm{E}\{x(t_1).x(t_2).x(t_3).n(t_4)\} = \mathrm{E}\{x(t_1).x(t_2).x(t_3)\}.\mathrm{E}\{n(t_4)\} \approx 0$

- The terms in which $n(t)$ and $x(t)$ occurs twice, and since $n(t)$ and $x(t)$ are mutually independent, the expected value can be written as the product of the autocorrelation of $x(t)$ and $n(t)$ for example:

  $\mathrm{E}\{x(t_1).x(t_2).n(t_3).n(t_4)\} = \mathrm{E}\{x(t_1).x(t_2)\}.\mathrm{E}\{n(t_3).n(t_4)\} = R_{xx}(t_1 - t_2).R_{nn}(t_3 - t_4)$

Hence (A.5) can be written as:

$$
\begin{aligned}
&R_{zz}(t,t+\tau)= \\
&\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4). \mathrm{E}\{x(t-\alpha_1).x(t-\alpha_2-\tau_1)x(t+\tau-\alpha_3).x(t+\tau-\alpha_4-\tau_1)\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \\
&+m_i\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4). \mathrm{E}\{x(t-\alpha_1).x(t-\alpha_2-\tau_1)x(t+\tau-\alpha_3).x(t+\tau-\alpha_4-2\tau_1)\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \\
&+m_i\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4). \mathrm{E}\{x(t-\alpha_1).x(t-\alpha_2-\tau_1)x(t+\tau-\alpha_3-\tau_1).x(t+\tau-\alpha_4-\tau_1)\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \\
&+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4). \mathrm{E}\{x(t-\alpha_1).x(t-\alpha_2-\tau_1).x(t+\tau-\alpha_3-\tau_1).x(t+\tau-\alpha_4-2\tau_1)\}d\alpha_1 d\alpha_2 d\alpha_3 d\alpha_4 \\
&+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).R_{xx}(-\alpha_1+\alpha_2+\tau_1).R_{nn}(-\alpha_3+\alpha_4+\tau_1)\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \\
&+m_i\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4). \mathrm{E}\{x(t-\alpha_1).x(t-\alpha_2-2\tau_1)x(t+\tau-\alpha_3).x(t+\tau-\alpha_4-\tau_1)\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \\
&+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4). \mathrm{E}\{x(t-\alpha_1).x(t-\alpha_2-2\tau_1)x(t+\tau-\alpha_3).x(t+\tau-\alpha_4-2\tau_1)\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \\
&+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4). \mathrm{E}\{x(t-\alpha_1).x(t-\alpha_2-2\tau_1)x(t+\tau-\alpha_3-\tau_1).x(t+\tau-\alpha_4-\tau_1)\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \\
&+m_i\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4). \mathrm{E}\{x(t-\alpha_1).x(t-\alpha_2-2\tau_1).x(t+\tau-\alpha_3-\tau_1).x(t+\tau-\alpha_4-2\tau_1)\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \\
&+m_i\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).R_{xx}(-\alpha_1+\alpha_2+2\tau_1).R_{nn}(-\alpha_3+\alpha_4+\tau_1)\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \\
&+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).R_{xx}(-\alpha_1+\alpha_3-\tau).R_{nn}(-\alpha_2+\alpha_4-\tau)\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \qquad \text{(A.6)} \\
&+m_i\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).R_{xx}(-\alpha_1+\alpha_3+\tau_1-\tau).R_{nn}(-\alpha_2+\alpha_4-\tau)\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \\
&+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).R_{xx}(-\alpha_1+\alpha_4+\tau_1-\tau).R_{nn}(-\alpha_2+\alpha_3-\tau_1-\tau)\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \\
&+m_i\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).R_{xx}(-\alpha_1+\alpha_4+2\tau_1-\tau).R_{nn}(-\alpha_2+\alpha_3-\tau_1-\tau)\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \\
&+m_i\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4). \mathrm{E}\{x(t-\alpha_1-\tau_1).x(t-\alpha_2-\tau_1)x(t+\tau-\alpha_3).x(t+\tau-\alpha_4-\tau_1)\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \\
&+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4). \mathrm{E}\{x(t-\alpha_1-\tau_1).x(t-\alpha_2-\tau_1)x(t+\tau-\alpha_3).x(t+\tau-\alpha_4-2\tau_1)\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \\
&+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4). \mathrm{E}\{x(t-\alpha_1-\tau_1).x(t-\alpha_2-\tau_1)x(t+\tau-\alpha_3-\tau_1).x(t+\tau-\alpha_4-\tau_1)\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \\
&+m_i\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4). \mathrm{E}\{x(t-\alpha_1-\tau_1).x(t-\alpha_2-\tau_1).x(t+\tau-\alpha_3-\tau_1).x(t+\tau-\alpha_4-2\tau_1)\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \\
&+m_i.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).R_{xx}(-\alpha_1+\alpha_2).R_{nn}(-\alpha_3+\alpha_4+\tau_1)\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4
\end{aligned}
$$

$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \begin{array}{l} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4). \\ \mathrm{E}\{x(t-\alpha_1-\tau_1).x(t-\alpha_2-2\tau_1)x(t+\tau-\alpha_3).x(t+\tau-\alpha_4-\tau_1)\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \end{array}$$

$$+m_i.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \begin{array}{l} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4). \\ \mathrm{E}\{x(t-\alpha_1-\tau_1).x(t-\alpha_2-2\tau_1)x(t+\tau-\alpha_3).x(t+\tau-\alpha_4-2\tau_1)\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \end{array}$$

$$+m_i.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \begin{array}{l} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4). \\ \mathrm{E}\{x(t-\alpha_1-\tau_1).x(t-\alpha_2-2\tau_1)x(t+\tau-\alpha_3-\tau_1).x(t+\tau-\alpha_4-\tau_1)\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \end{array}$$

$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \begin{array}{l} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4). \\ \mathrm{E}\{x(t-\alpha_1-\tau_1).x(t-\alpha_2-2\tau_1).x(t+\tau-\alpha_3-\tau_1).x(t+\tau-\alpha_4-2\tau_1)\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \end{array}$$

$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).R_{xx}(-\alpha_1+\alpha_2+\tau_1).R_{nn}(-\alpha_3+\alpha_4+\tau_1)\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 +$$

$$+m_i\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).R_{xx}(-\alpha_1+\alpha_3-\tau_1-\tau).R_{nn}(-\alpha_2+\alpha_4-\tau).\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4$$

$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).R_{xx}(-\alpha_1+\alpha_3-v).R_{nn}(-\alpha_2+\alpha_4-\tau).\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4$$

$$+m_i\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).R_{xx}(-\alpha_1+\alpha_4-\tau).R_{nn}(-\alpha_2+\alpha_3-\tau_1-\tau).\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4$$

$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).R_{xx}(-\alpha_1+\alpha_4+\tau_1-\tau).R_{nn}(-\alpha_2+\alpha_3-\tau_1-\tau).\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4$$

$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).R_{xx}(-\alpha_2+\alpha_3-\tau_1-\tau).R_{nn}(-\alpha_1+\alpha_4+\tau_1-\tau).\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4$$

$$+m_i\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).R_{xx}(-\alpha_2+\alpha_3-\tau).R_{nn}(-\alpha_1+\alpha_4+\tau_1-\tau).\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4$$

$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).R_{xx}(-\alpha_2+\alpha_4-\tau).R_{nn}(-\alpha_1+\alpha_3-\tau).\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4$$

$$+m_i\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).R_{xx}(-\alpha_2+\alpha_4+\tau_1-\tau).R_{nn}(-\alpha_1+\alpha_3-\tau).\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4$$

$$+m_i\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).R_{xx}(-\alpha_2+\alpha_3-2\tau_1-\tau).R_{nn}(-\alpha_1+\alpha_4+\tau_1-\tau).\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4$$

$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).R_{xx}(-\alpha_2+\alpha_3-\tau_1-\tau).R_{nn}(-\alpha_1+\alpha_4+\tau_1-\tau)\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4$$

$$+m_i\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).R_{xx}(-\alpha_2+\alpha_4-\tau_1-\tau).R_{nn}(-\alpha_1+\alpha_3-\tau)\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4$$

$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).R_{xx}(-\alpha_2+\alpha_4-\tau).R_{nn}(-\alpha_1+\alpha_3-\tau)\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4$$

$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).R_{xx}(-\alpha_3+\alpha_4+\tau_1).R_{nn}(-\alpha_1+\alpha_2+\tau_1)\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4$$

$$+m_i\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).R_{xx}(-\alpha_3+\alpha_4+2\tau_1).R_{nn}(-\alpha_1+\alpha_2+\tau_1)\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4$$

$$+m_i\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).R_{xx}(-\alpha_3+\alpha_4).R_{nn}(-\alpha_1+\alpha_2+\tau_1)\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4$$

$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).R_{xx}(-\alpha_3+\alpha_4+\tau_1).R_{nn}(-\alpha_1+\alpha_2+\tau_1)\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4$$

$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \begin{array}{l} h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4). \\ \mathrm{E}\{n(t-\alpha_1).n(t-\alpha_2-\tau_1)n(t+\tau-\alpha_3).n(t+\tau-\alpha_4-\tau_1)\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \end{array}$$

In (A.6) appears terms in the form of $\mathrm{E}\{x(t_1).x(t_2)x(t_3).x(t_4)\}$. This is the fourth moment of $x(t)$. Since $x(t)$ is a Gaussian noise signal with a mean value equal to zero, the fourth moment of $x(t)$ can be written as [15]:

$$\begin{aligned}\mathrm{E}\{x(t_1).x(t_2).x(t_3).x(t_4)\} = {} & \mathrm{E}\{x(t_1).x(t_2)\}.\mathrm{E}\{x(t_3).x(t_4)\} + \mathrm{E}\{x(t_1).x(t_3)\}.\mathrm{E}\{x(t_2).x(t_4)\} \\ & + \mathrm{E}\{x(t_1).x(t_4)\}.\mathrm{E}\{x(t_2).x(t_3)\}\end{aligned} \tag{A.7}$$

Writing the expected value as the autocorrelation function, (A.7) becomes:

$$\begin{aligned}\mathrm{E}\{x(t_1).x(t_2).x(t_3).x(t_4)\} = {} & R_{xx}(t_1 - t_2).R_{xx}(t_3 - t_4) + R_{xx}(t_1 - t_3).R_{xx}(t_2 - t_4) \\ & + R_{xx}(t_1 - t_4).R_{xx}(t_2 - t_3)\end{aligned} \tag{A.8}$$

Substituting equation (A.8) in equation (A.6) and writing the fourfold integral as the product of two double integrals, this will reduce equation (A.6) to:

$$
\begin{aligned}
R_{zz}(t,t+\tau) = &(h\otimes h_2\otimes R_{xx})(0).(h\otimes h_2\otimes R_{xx})(0)\\
&+5.(h\otimes h_2\otimes R_{xx})(-\tau).(h\otimes h_2\otimes R_{xx})(-\tau)\\
&+6.(h\otimes h_2\otimes R_{xx})(\tau_1-\tau).(h\otimes h_2\otimes R_{xx})(-\tau_1-\tau)\\
&+(h\otimes h_2\otimes R_{xx})(\tau_1-\tau).(h\otimes h_2\otimes R_{xx})(-\tau)\\
&+(h\otimes h_2\otimes R_{xx})(2\tau_1-\tau).(h\otimes h_2\otimes R_{xx})(-\tau)\\
&+(h\otimes h_2\otimes R_{xx})(2\tau_1-\tau).(h\otimes h_2\otimes R_{xx})(-2\tau_1-\tau)\\
&+4.(h\otimes h_2\otimes R_{xx})(-\tau).(h\otimes h_2\otimes R_{nn})(-\tau)\\
&+2.(h\otimes h_2\otimes R_{xx})(-\tau+\tau_1).(h\otimes h_2\otimes R_{nn})(-\tau-\tau_1)\\
&+(h\otimes h_2\otimes R_{xx})(-\tau-\tau_1).(h\otimes h_2\otimes R_{xx})(-\tau-\tau_1)\\
&+(h\otimes h_2\otimes R_{xx})(-\tau).(h\otimes h_2\otimes R_{xx})(-2\tau_1-\tau)\\
&+2.(h\otimes h_2\otimes R_{xx})(-\tau-\tau_1).(h\otimes h_2\otimes R_{nn})(-\tau+\tau_1)\\
&+(h\otimes h_2\otimes R_{nn})(-\tau)(h\otimes h_2\otimes R_{nn})(-\tau)\\
&+(h\otimes h_2\otimes R_{nn})(\tau_1-\tau).(h\otimes h_2\otimes R_{nn})(-\tau_1-\tau)\\
&+m_i.\left\{\begin{array}{l}
6.(h\otimes h_2\otimes R_{xx})(-\tau).(h\otimes h_2\otimes R_{xx})(\tau_1-\tau)\\
+2.(h\otimes h_2\otimes R_{xx})(2\tau_1-\tau).(h\otimes h_2\otimes R_{xx})(-\tau_1-\tau)\\
+6.(h\otimes h_2\otimes R_{xx})(-\tau).(h\otimes h_2\otimes R_{xx})(-\tau-\tau_1)\\
+2.(h\otimes h_2\otimes R_{xx})(\tau_1-\tau).(h\otimes h_2\otimes R_{xx})(-2\tau_1-\tau)\\
+(h\otimes h_2\otimes R_{xx})(-\tau+\tau_1).(h\otimes h_2\otimes R_{nn})(-\tau)\\
+(h\otimes h_2\otimes R_{xx})(-\tau+2\tau_1).(h\otimes h_2\otimes R_{nn})(-\tau-\tau_1)\\
+3.(h\otimes h_2\otimes R_{xx})(-\tau-\tau_1).(h\otimes h_2\otimes R_{nn})(-\tau)\\
+(h\otimes h_2\otimes R_{xx})(-\tau).(h\otimes h_2\otimes R_{nn})(-\tau-\tau_1)\\
+(h\otimes h_2\otimes R_{xx})(-\tau).(h\otimes h_2\otimes R_{nn})(-\tau+\tau_1)\\
+(h\otimes h_2\otimes R_{xx})(-\tau-2\tau_1).(h\otimes h_2\otimes R_{nn})(-\tau+\tau_1)
\end{array}\right\}
\end{aligned}
\tag{A.9}
$$

We determined equation (3-19) that the expected value of $z(t)$ is equal to $m_i.(h\otimes h_2\otimes R_{xx})(0)$.

Taking the square of the expected value gives:

$$\{\mathrm{E}\{z(t)\}\}^2 = \{m_i.(h\otimes h_2\otimes R_{xx})(0)\}^2 \tag{A.10}$$

$$m_i^2 = 1$$

Equation (A.10) can be written as:

$$\{\mathrm{E}\{z(t)\}\}^2 = (h\otimes h_2\otimes R_{xx})(0).(h\otimes h_2\otimes R_{xx})(0) \tag{A.11}$$

Substituting equation (A.11) in equation (A.9) we get:

$$R_{zz}(t,t+\tau) = \{\mathrm{E}\{z(t)\}\}^2 + C_{zz}(\tau_1,\tau) \tag{A.12}$$

Where

$$
\begin{aligned}
C_{zz}(\tau_1,\tau) = {} & 5.(h\otimes h_2\otimes R_{xx})(-\tau).(h\otimes h_2\otimes R_{xx})(-\tau) \\
& +6.(h\otimes h_2\otimes R_{xx})(\tau_1-\tau).(h\otimes h_2\otimes R_{xx})(-\tau_1-\tau) \\
& +(h\otimes h_2\otimes R_{xx})(\tau_1-\tau).(h\otimes h_2\otimes R_{xx})(-\tau) \\
& +(h\otimes h_2\otimes R_{xx})(2\tau_1-\tau).(h\otimes h_2\otimes R_{xx})(-\tau) \\
& +(h\otimes h_2\otimes R_{xx})(2\tau_1-\tau).(h\otimes h_2\otimes R_{xx})(-2\tau_1-\tau) \\
& +4.(h\otimes h_2\otimes R_{xx})(-\tau).(h\otimes h_2\otimes R_{nn})(-\tau) \\
& +2.(h\otimes h_2\otimes R_{xx})(-\tau+\tau_1).(h\otimes h_2\otimes R_{nn})(-\tau-\tau_1) \\
& +(h\otimes h_2\otimes R_{xx})(-\tau-\tau_1).(h\otimes h_2\otimes R_{xx})(-\tau-\tau_1) \\
& +(h\otimes h_2\otimes R_{xx})(-\tau).(h\otimes h_2\otimes R_{xx})(-2\tau_1-\tau) \\
& +2.(h\otimes h_2\otimes R_{xx})(-\tau-\tau_1).(h\otimes h_2\otimes R_{nn})(-\tau+\tau_1) \\
& +(h\otimes h_2\otimes R_{nn})(-\tau)(h\otimes h_2\otimes R_{nn})(-\tau) \\
& +(h\otimes h_2\otimes R_{nn})(\tau_1-\tau).(h\otimes h_2\otimes R_{nn})(-\tau_1-\tau) \\
& +m_i.\left\{\begin{aligned}
& 6.(h\otimes h_2\otimes R_{xx})(-\tau).(h\otimes h_2\otimes R_{xx})(\tau_1-\tau) \\
& +2.(h\otimes h_2\otimes R_{xx})(2\tau_1-\tau).(h\otimes h_2\otimes R_{xx})(-\tau_1-\tau) \\
& +6.(h\otimes h_2\otimes R_{xx})(-\tau).(h\otimes h_2\otimes R_{xx})(-\tau-\tau_1) \\
& +2.(h\otimes h_2\otimes R_{xx})(\tau_1-\tau).(h\otimes h_2\otimes R_{xx})(-2\tau_1-\tau) \\
& +(h\otimes h_2\otimes R_{xx})(-\tau+\tau_1).(h\otimes h_2\otimes R_{nn})(-\tau) \\
& +(h\otimes h_2\otimes R_{xx})(-\tau+2\tau_1).(h\otimes h_2\otimes R_{nn})(-\tau-\tau_1) \\
& +3.(h\otimes h_2\otimes R_{xx})(-\tau-\tau_1).(h\otimes h_2\otimes R_{nn})(-\tau) \\
& +(h\otimes h_2\otimes R_{xx})(-\tau).(h\otimes h_2\otimes R_{nn})(-\tau-\tau_1) \\
& +(h\otimes h_2\otimes R_{xx})(-\tau).(h\otimes h_2\otimes R_{nn})(-\tau+\tau_1) \\
& +(h\otimes h_2\otimes R_{xx})(-\tau-2\tau_1).(h\otimes h_2\otimes R_{nn})(-\tau+\tau_1)
\end{aligned}\right\}
\end{aligned}
\tag{A.13}
$$

The autocorrelation function $R_{zz}(t,t+\tau)$ is the sum of the square of the expected value of the mixer output *z*(*t*), and the covariance function $C_{zz}(\tau_1,\tau)$ of *z*(*t*). This implies that that *z*(*t*) is the sum of a DC value and a noise term. The covariance $C_{zz}(\tau_1,\tau)$ depends on two parameters, τ_1, the time offset, and τ, the time difference variable of the autocorrelation function.

Appendix A3 Calculation of the mixer output power spectral density

We define the power spectral density function of the mixer output $S_{zz}(f)$ as the Fourier transform of the covariance function $C_{zz}(\tau_1,\tau)$, which is equal to the autocorrelation function without the DC-term. $C_{zz}(\tau_1,\tau)$ is given in expression (A.13). Using the next proprieties:

- The Fourier transform of a time-reverse real function is equal to the complex conjugate of the Fourier transform of that function
- Multiplication in the time domain corresponds to the convolution in the frequency domain and vice versa.
- And using the next Fourier transformation

  $$x(at-\tau) \Leftrightarrow \frac{1}{|a|}X(\frac{f}{a}).\exp(-j2\pi\frac{f}{a}\tau)$$

  If a = -1

  $$x(-t-\tau) \Leftrightarrow X(-f).\exp(j2\pi f\tau)$$

By writing out the convolution operation, we get the next formula for the power spectral density function $S_{zz}(f)$

$$\begin{aligned}
S_{zz}(f) = {} & 5.\int_{-\infty}^{\infty}\left|H(\nu)\right|^2.S_{xx}(\nu).\left|H(f-\nu)\right|^2.S_{xx}(f-\nu)\mathrm{d}\nu \\
& + 6.\int_{-\infty}^{\infty}\left|H(\nu)\right|^2.S_{xx}(\nu).\exp(-j2\pi\nu\tau_1).\left|H(f-\nu)\right|^2.S_{xx}(f-\nu).\exp(j2\pi(f-\nu)\tau_1).\mathrm{d}\nu \\
& + \int_{-\infty}^{\infty}\left|H(\nu)\right|^2.S_{xx}(\nu).\exp(-j2\pi\nu\tau_1).\left|H(f-\nu)\right|^2.S_{xx}(f-\nu).\mathrm{d}\nu \\
& + \int_{-\infty}^{\infty}\left|H(\nu)\right|^2.S_{xx}(\nu).\exp(-j2\pi\nu\tau_1).\left|H(f-\nu)\right|^2.S_{xx}(f-\nu).\mathrm{d}\nu \\
& + \int_{-\infty}^{\infty}\left|H(\nu)\right|^2.S_{xx}(\nu).\exp(-j4\pi\nu\tau_1).\left|H(f-\nu)\right|^2.S_{xx}(f-\nu).\exp(j4\pi(f-\nu)\tau_1).\mathrm{d}\nu \\
& + 4.\int_{-\infty}^{\infty}\left|H(\nu)\right|^2.S_{xx}(\nu).\left|H(f-\nu)\right|^2.S_{nn}(f-\nu)\mathrm{d}\nu \\
& + 2.\int_{-\infty}^{\infty}\left|H(\nu)\right|^2.S_{xx}(\nu).\exp(-j2\pi\nu\tau_1).\left|H(f-\nu)\right|^2.S_{nn}(f-\nu).\exp(j2\pi(f-\nu)\tau_1).\mathrm{d}\nu \\
& + \int_{-\infty}^{\infty}\left|H(\nu)\right|^2.S_{xx}(\nu).\exp(j2\pi\nu\tau_1).\left|H(f-\nu)\right|^2.S_{xx}(f-\nu).\exp(j2\pi(f-\nu)\tau_1).\mathrm{d}\nu \\
& + \int_{-\infty}^{\infty}\left|H(\nu)\right|^2.S_{xx}(\nu).\left|H(f-\nu)\right|^2.S_{xx}(f-\nu).\exp(j4\pi(f-\nu)\tau_1).d\nu \\
& + 2.\int_{-\infty}^{\infty}\left|H(\nu)\right|^2.S_{xx}(\nu).\exp(j2\pi\nu\tau_1).\left|H(f-\nu)\right|^2.S_{nn}(f-\nu).\exp(-j2\pi(f-\nu)\tau_1).d\nu \\
& + \int_{-\infty}^{\infty}\left|H(\nu)\right|^2.S_{nn}(\nu).\left|H(f-\nu)\right|^2.S_{nn}(f-\nu)\mathrm{d}\nu \qquad \text{(A.14)} \\
& + \int_{-\infty}^{\infty}\left|H(\nu)\right|^2.S_{nn}(\nu).\exp(-j2\pi\nu\tau_1).\left|H(f-\nu)\right|^2.S_{nn}(f-\nu).\exp(j2\pi(f-\nu)\tau_1).\mathrm{d}\nu
\end{aligned}$$

$$+ m_i.\left\{\begin{aligned}
& 6.\int_{-\infty}^{\infty}\left|H(\nu)\right|^2.S_{xx}(\nu).\exp(-j2\pi\nu\tau_1).\left|H(f-\nu)\right|^2.S_{xx}(f-\nu).\mathrm{d}\nu \\
& +2.\int_{-\infty}^{\infty}\left|H(\nu)\right|^2.S_{xx}(\nu).\exp(-j2\pi\nu\tau_1).\left|H(f-\nu)\right|^2.S_{xx}(f-\nu).\mathrm{d}\nu \\
& +6.\int_{-\infty}^{\infty}\left|H(\nu)\right|^2.S_{xx}(\nu).\left|H(f-\nu)\right|^2.S_{xx}(f-\nu).\exp(j2\pi(f-\nu)\tau_1)\mathrm{d}\nu \\
& +2.\int_{-\infty}^{\infty}\left|H(\nu)\right|^2.S_{xx}(\nu).\exp(j2\pi\nu\tau_1).\left|H(f-\nu)\right|^2.S_{xx}(f-\nu).\mathrm{d}\nu \\
& +\int_{-\infty}^{\infty}\left|H(\nu)\right|^2.S_{xx}(\nu).\exp(-j2\pi\nu\tau_1).\left|H(f-\nu)\right|^2.S_{nn}(f-\nu).\mathrm{d}\nu \\
& +\int_{-\infty}^{\infty}\left|H(\nu)\right|^2.S_{xx}(\nu).\exp(-j4\pi\nu\tau_1).\left|H(f-\nu)\right|^2.S_{xx}(f-\nu).\exp(j2\pi(f-\nu)\tau_1).\mathrm{d}\nu \\
& +3.\int_{-\infty}^{\infty}\left|H(\nu)\right|^2.S_{xx}(\nu).\exp(j2\pi\nu\tau_1).\left|H(f-\nu)\right|^2.S_{nn}(f-\nu).\mathrm{d}\nu \\
& +\int_{-\infty}^{\infty}\left|H(\nu)\right|^2.S_{xx}(\nu).\left|H(f-\nu)\right|^2.S_{nn}(f-\nu).\exp(j2\pi(f-\nu)\tau_1).\mathrm{d}\nu \\
& +\int_{-\infty}^{\infty}\left|H(\nu)\right|^2.S_{xx}(\nu).\left|H(f-\nu)\right|^2.S_{nn}(f-\nu).\exp(-j2\pi(f-\nu)\tau_1).\mathrm{d}\nu \\
& +\int_{-\infty}^{\infty}\left|H(\nu)\right|^2.S_{xx}(\nu).\exp(j4\pi\nu\tau_1).\left|H(f-\nu)\right|^2.S_{xx}(f-\nu).\exp(-j2\pi(f-\nu)\tau_1).\mathrm{d}\nu
\end{aligned}\right\}$$

The noise carrier bandwidth and the filter bandwidth at the receiver are much broader than the information bandwidth. Therefore we can assume that the noise spectrum is flat in the information band [16], so we can treat it as white noise. Hence the power spectral density of the noise in the information bit can be approximated by the power spectral density $S_{zz}(f)$ for f = 0. By doing this we get:

$$
\begin{aligned}
S_{zz}(0) = {} & 5.\int_{-\infty}^{\infty}|H(\nu)|^2.S_{xx}(\nu).|H(-\nu)|^2.S_{xx}(-\nu)\mathrm{d}\nu \\
& + 6.\int_{-\infty}^{\infty}|H(\nu)|^2.S_{xx}(\nu).\exp(-j2\pi\nu\tau_1).|H(-\nu)|^2.S_{xx}(-\nu).\exp(j2\pi(-\nu)\tau_1).\mathrm{d}\nu \\
& + \int_{-\infty}^{\infty}|H(\nu)|^2.S_{xx}(\nu).\exp(-j2\pi\nu\tau_1).|H(-\nu)|^2.S_{xx}(-\nu).\mathrm{d}\nu \\
& + \int_{-\infty}^{\infty}|H(\nu)|^2.S_{xx}(\nu).\exp(-j2\pi\nu\tau_1).|H(-\nu)|^2.S_{xx}(-\nu).\mathrm{d}\nu \\
& + \int_{-\infty}^{\infty}|H(\nu)|^2.S_{xx}(\nu).\exp(-j4\pi\nu\tau_1).|H(-\nu)|^2.S_{xx}(-\nu).\exp(j4\pi(-\nu)\tau_1).\mathrm{d}\nu \\
& + 4.\int_{-\infty}^{\infty}|H(\nu)|^2.S_{xx}(\nu).|H(-\nu)|^2.S_{nn}(-\nu)\mathrm{d}\nu \\
& + 2.\int_{-\infty}^{\infty}|H(\nu)|^2.S_{xx}(\nu).\exp(-j2\pi\nu\tau_1).|H(-\nu)|^2.S_{nn}(-\nu).\exp(j2\pi(-\nu)\tau_1).\mathrm{d}\nu \\
& + \int_{-\infty}^{\infty}|H(\nu)|^2.S_{xx}(\nu).\exp(j2\pi\nu\tau_1).|H(-\nu)|^2.S_{xx}(-\nu).\exp(j2\pi(-\nu)\tau_1).\mathrm{d}\nu \\
& + \int_{-\infty}^{\infty}|H(\nu)|^2.S_{xx}(\nu).|H(-\nu)|^2.S_{xx}(-\nu).\exp(j4\pi(-\nu)\tau_1).\mathrm{d}\nu \\
& + 2.\int_{-\infty}^{\infty}|H(\nu)|^2.S_{xx}(\nu).\exp(j2\pi\nu\tau_1).|H(-\nu)|^2.S_{nn}(-\nu).\exp(-j2\pi(-\nu)\tau_1).\mathrm{d}\nu \\
& + \int_{-\infty}^{\infty}|H(\nu)|^2.S_{nn}(\nu).|H(-\nu)|^2.S_{nn}(-\nu)\mathrm{d}\nu \\
& + \int_{-\infty}^{\infty}|H(\nu)|^2.S_{nn}(\nu).\exp(-j2\pi\nu\tau_1).|H(-\nu)|^2.S_{nn}(-\nu).\exp(j2\pi(-\nu)\tau_1).\mathrm{d}\nu \\
& + m_i.\left\{\begin{aligned}
& 6.\int_{-\infty}^{\infty}|H(\nu)|^2.S_{xx}(\nu).\exp(-j2\pi\nu\tau_1).|H(-\nu)|^2.S_{xx}(-\nu).\mathrm{d}\nu \\
& +2.\int_{-\infty}^{\infty}|H(\nu)|^2.S_{xx}(\nu).\exp(-j2\pi\nu\tau_1).|H(-\nu)|^2.S_{xx}(-\nu).\mathrm{d}\nu \\
& +6.\int_{-\infty}^{\infty}|H(\nu)|^2.S_{xx}(\nu).|H(-\nu)|^2.S_{xx}(-\nu).\exp(j2\pi(-\nu)\tau_1)\mathrm{d}\nu \\
& +2.\int_{-\infty}^{\infty}|H(\nu)|^2.S_{xx}(\nu).\exp(j2\pi\nu\tau_1).|H(-\nu)|^2.S_{xx}(-\nu).\mathrm{d}\nu \\
& +\int_{-\infty}^{\infty}|H(\nu)|^2.S_{xx}(\nu).\exp(-j2\pi\nu\tau_1).|H(-\nu)|^2.S_{nn}(-\nu).\mathrm{d}\nu \\
& +\int_{-\infty}^{\infty}|H(\nu)|^2.S_{xx}(\nu).\exp(-j4\pi\nu\tau_1).|H(-\nu)|^2.S_{xx}(-\nu).\exp(j2\pi(-\nu)\tau_1).\mathrm{d}\nu \\
& +3.\int_{-\infty}^{\infty}|H(\nu)|^2.S_{xx}(\nu).\exp(j2\pi\nu\tau_1).|H(-\nu)|^2.S_{nn}(-\nu).\mathrm{d}\nu \\
& +\int_{-\infty}^{\infty}|H(\nu)|^2.S_{xx}(\nu).|H(-\nu)|^2.S_{nn}(-\nu).\exp(j2\pi(-\nu)\tau_1).\mathrm{d}\nu \\
& +\int_{-\infty}^{\infty}|H(\nu)|^2.S_{xx}(\nu).|H(-\nu)|^2.S_{nn}(-\nu).\exp(-j2\pi(-\nu)\tau_1).\mathrm{d}\nu \\
& +\int_{-\infty}^{\infty}|H(\nu)|^2.S_{xx}(\nu).\exp(j4\pi\nu\tau_1).|H(-\nu)|^2.S_{xx}(-\nu).\exp(-j2\pi(-\nu)\tau_1).\mathrm{d}\nu
\end{aligned}\right\}
\end{aligned}
\tag{A.15}
$$

In expression (A.15) are some terms with an exponential factor. All this terms will be omitted, for example

$$\int_{-\infty}^{\infty} |H(\nu)|^2 .S_{xx}(\nu).\exp(-j2\pi\nu\tau_1).|H(-\nu)|^2 .S_{xx}(-\nu).\exp(-j2\pi\nu\tau_1).d\nu \qquad \text{(A.16)}$$

The Fourier transform of the time-reversed version of a real function is equal to the complex conjugate of the Fourier transform of that function [14].

$$|H(-\nu)|^2 = \left|\{H(\nu)\}^*\right|^2 = |H(\nu)|^2$$

$x(t)$ is a real signal; therefore: $S_{xx}(-\nu) = S_{xx}(\nu)$

Equation (A.16) will be written as:

$$\int_{-\infty}^{\infty} |H(\nu)|^4 .S_{xx}^2(\nu)\exp(-j4\pi\nu\tau_1)d\nu \qquad \text{(A.17)}$$

We proved in appendix A1 that such integral are approximately equal to zero, thus

$$\int_{-\infty}^{\infty} |H(\nu)|^4 .S_{xx}^2(\nu)\exp(-j4\pi\nu\tau_1)d\nu \approx 0 \qquad \text{(A.18)}$$

By omitting all terms in (A.15) having an exponential term, equation (A.15) will be reduced to

$$S_{zz}(0) = 7.\int_{-\infty}^{\infty} |H(\nu)|^4 .S_{xx}^2(\nu)d\nu + 4.\int_{-\infty}^{\infty} |H(\nu)|^4 .S_{xx}(\nu)..S_{nn}(\nu)d\nu + \int_{-\infty}^{\infty} |H(\nu)|^4 .S_{nn}^2(\nu)d\nu \qquad \text{(A.19)}$$

Appendix B Calculation on the time offset transmitted reference with multiple transmitters and receivers

Appendix B1 Calculation of the mixer output autocorrelation function

The autocorrelation of the mixer output at receiver r can be determined by calculating the expected value of the product between $z_r(t)$ and its shifted version $z_r(t+\tau_r)$.

$$R_{z_r z_r}(t, t+\tau) = \mathrm{E}\{z_r(t) z_r(t+\tau)\} \tag{B.1}$$

We replace $z_r(t)$ and $z_r(t+\tau_r)$ by their expressions given in (4-11), (B.1) can be written as:

$$
R_{z_r z_r}(t,t+\tau)=\mathrm{E}\left\{
\begin{array}{l}
\sum_{i=1}^{M}\sum_{j=1}^{M}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2)\left\{\begin{array}{l}
x_i(t-\alpha_1).x_j(t-\alpha_2-\tau_r)\\
+m_j.x_i(t-\alpha_1).x_j(t-\alpha_2-\tau_j-\tau_r)\\
+m_i.x_i(t-\alpha_1-\tau_i).x_j(t-\alpha_2-\tau_r)\\
+m_i.m_j.x_i(t-\alpha_1-\tau_i).x_j(t-\alpha_2-\tau_j-\tau_r)
\end{array}\right\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2\\
+\sum_{j=1}^{M}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2).n(t-\alpha_1).x_j(t-\tau_r-\alpha_2).\mathrm{d}\alpha_1\mathrm{d}\alpha_2\\
+\sum_{j=1}^{M} m_j.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2).n(t-\alpha_1).x_j(t-\tau_j-\tau_r-\alpha_2).\mathrm{d}\alpha_1\mathrm{d}\alpha_2\\
+\sum_{i=1}^{M}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2).n(t-\alpha_2-\tau_r).x_i(t-\alpha_1).\mathrm{d}\alpha_1\mathrm{d}\alpha_2\\
+\sum_{i=1}^{M} m_i.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2).n(t-\alpha_2-\tau_r).x_i(t-\alpha_1-\tau_i).\mathrm{d}\alpha_1\mathrm{d}\alpha_2\\
+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2).n(t-\alpha_1).n(t-\alpha_2-\tau_r).\mathrm{d}\alpha_1\mathrm{d}\alpha_2.\\
+\sum_{i'=1}^{M}\sum_{j'=1}^{M}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_3).h(\alpha_4)\left\{\begin{array}{l}
x_{i'}(t-\alpha_3+\tau).x_{j'}(t-\alpha_4-\tau_r+\tau)\\
+m_{j'}.x_{i'}(t-\alpha_3+\tau).x_{j'}(t-\alpha_4-\tau_{j'}-\tau_r+\tau)\\
+m_{i'}.x_{i'}(t-\alpha_3-\tau_i+\tau).x_{j'}(t-\alpha_4-\tau_r+\tau)\\
+m_{i'}.m_{j'}.x_{i'}(t-\alpha_3-\tau_{i'}+\tau).x_{j'}(t-\alpha_4-\tau_{j'}-\tau_r+\tau)
\end{array}\right\}\mathrm{d}\alpha_3\mathrm{d}\alpha_4\\
+\sum_{j'=1}^{M}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_3).h(\alpha_4).n(t-\alpha_3+\tau).x_{j'}(t-\tau_r-\alpha_4+\tau).\mathrm{d}\alpha_3\mathrm{d}\alpha_4\\
+\sum_{j'=1}^{M} m_{j'}.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_3).h(\alpha_4).n(t-\alpha_3+\tau).x_{j'}(t-\tau_{j'}-\tau_r-\alpha_4+\tau).\mathrm{d}\alpha_3\mathrm{d}\alpha_4\\
+\sum_{i'=1}^{M}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_3).h(\alpha_4).n(t-\alpha_3-\tau_r+\tau).x_{i'}(t-\alpha_3+\tau).\mathrm{d}\alpha_3\mathrm{d}\alpha_4\\
+\sum_{i'=1}^{M} m_{i'}.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_3).h(\alpha_4).n(t-\alpha_4-\tau_r+\tau).x_{i'}(t-\alpha_3-\tau_{i'}+\tau).\mathrm{d}\alpha_3\mathrm{d}\alpha_4\\
+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_3).h(\alpha_4).n(t-\alpha_3+\tau).n(t-\alpha_4-\tau_r+\tau).\mathrm{d}\alpha_3\mathrm{d}\alpha_4
\end{array}\right\} \quad \text{(B.2)}
$$

Eliminating the brackets, writing products of double integrals as fourfold integrals, interchanging the order of integration, taking the expected value, and taking in account the next points

- The terms in which $n(t)$ and $x_k(t)$ (where $k=i,i',j,j'$) occurs only once are omitted since $n(t)$ and $x_k(t)$ are mutually independent, $n(t)$ and $x_k(t)$ have the mean value equal to zero. For instance:

  $\mathrm{E}\{x_k(t_1).x_k(t_2).x_k(t_3).n(t_4)\}=\mathrm{E}\{x_k(t_1).x_k(t_2).x_k(t_3)\}.\mathrm{E}\{n(t_4)\}\approx 0$

Hence equation (B.2) can be written as:

$$
\begin{aligned}
&R_{z_r z_r}(t,t+\tau)= \\
&\sum_{i=1}^{M}\sum_{j=1}^{M}\sum_{i'=1}^{M}\sum_{j'=1}^{M}\left\{
\begin{array}{l}
\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{E}\left\{x_i(t-\alpha_1).x_j(t-\alpha_2-\tau_r).x_{i'}(t-\alpha_3+\tau).x_{j'}(t-\alpha_4-\tau_r+\tau)\right\}. \\
\quad h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \\
+m_{j'}.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{E}\left\{x_i(t-\alpha_1).x_j(t-\alpha_2-\tau_r).x_{i'}(t-\alpha_3+\tau).x_{j'}(t-\alpha_4-\tau_{j'}-\tau_r+\tau)\right\}. \\
\quad h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \\
+m_{i'}.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{E}\left\{x_i(t-\alpha_1).x_j(t-\alpha_2-\tau_r).x_{i'}(t-\alpha_3-\tau_{i'}+\tau).x_{j'}(t-\alpha_4-\tau_r+\tau)\right\}. \\
\quad h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \\
+m_{i'}.m_{j'}.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{E}\left\{x_i(t-\alpha_1).x_j(t-\alpha_2-\tau_r).x_{i'}(t-\alpha_3-\tau_{i'}+\tau).x_{j'}(t-\alpha_4-\tau_{j'}-\tau_r+\tau)\right\}. \\
\quad h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \\
+m_{j}.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{E}\left\{x_i(t-\alpha_1).x_j(t-\alpha_2-\tau_j-\tau_r).x_{i'}(t-\alpha_3+\tau).x_{j'}(t-\alpha_4-\tau_r+\tau)\right\}. \\
\quad h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \\
+m_{j'}.m_{j}.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{E}\left\{x_i(t-\alpha_1).x_j(t-\alpha_2-\tau_j-\tau_r).x_{i'}(t-\alpha_3+\tau).x_{j'}(t-\alpha_4-\tau_{j'}-\tau_r+\tau)\right\}. \\
\quad h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \\
+m_{i'}.m_{j}.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{E}\left\{x_i(t-\alpha_1).x_j(t-\alpha_2-\tau_j-\tau_r).x_{i'}(t-\alpha_3-\tau_{i'}+\tau).x_{j'}(t-\alpha_4-\tau_r+\tau)\right\}. \\
\quad h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \\
+m_{i'}.m_{j'}.m_{j}.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{E}\left\{x_i(t-\alpha_1).x_j(t-\alpha_2-\tau_j-\tau_r).x_{i'}(t-\alpha_3-\tau_{i'}+\tau).x_{j'}(t-\alpha_4-\tau_{j'}-\tau_r+\tau)\right\}. \\
\quad h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \\
+m_{i}.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{E}\left\{x_i(t-\alpha_1-\tau_i).x_j(t-\alpha_2-\tau_r).x_{i'}(t-\alpha_3+\tau).x_{j'}(t-\alpha_4-\tau_r+\tau)\right\}. \\
\quad h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \\
+m_{j'}.m_{i}.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{E}\left\{x_i(t-\alpha_1-\tau_i).x_j(t-\alpha_2-\tau_r).x_{i'}(t-\alpha_3+\tau).x_{j'}(t-\alpha_4-\tau_{j'}-\tau_r+\tau)\right\}. \\
\quad h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \\
+m_{i'}.m_{i}.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{E}\left\{x_i(t-\alpha_1-\tau_i).x_j(t-\alpha_2-\tau_r).x_{i'}(t-\alpha_3-\tau_{i'}+\tau).x_{j'}(t-\alpha_4-\tau_r+\tau)\right\}. \\
\quad h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \\
+m_{i'}.m_{j'}.m_{i}.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{E}\left\{x_i(t-\alpha_1-\tau_i).x_j(t-\alpha_2-\tau_r).x_{i'}(t-\alpha_3-\tau_{i'}+\tau).x_{j'}(t-\alpha_4-\tau_{j'}-\tau_r+\tau)\right\}. \\
\quad h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \\
+m_{i}.m_{j}.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{E}\left\{x_i(t-\alpha_1-\tau_i).x_j(t-\alpha_2-\tau_j-\tau_r).x_{i'}(t-\alpha_3+\tau).x_{j'}(t-\alpha_4-\tau_r+\tau)\right\}. \\
\quad h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \\
+m_{j'}.m_{i}.m_{j}.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{E}\left\{x_i(t-\alpha_1-\tau_i).x_j(t-\alpha_2-\tau_j-\tau_r).x_{i'}(t-\alpha_3+\tau).x_{j'}(t-\alpha_4-\tau_{j'}-\tau_r+\tau)\right\}. \\
\quad h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \\
+m_{i'}.m_{i}.m_{j}.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{E}\left\{x_i(t-\alpha_1-\tau_i).x_j(t-\alpha_2-\tau_j-\tau_r).x_{i'}(t-\alpha_3-\tau_{i'}+\tau).x_{j'}(t-\alpha_4-\tau_r+\tau)\right\}. \\
\quad h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4 \\
+m_{i'}.m_{j'}.m_{i}.m_{j}.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \mathrm{E}\left\{x_i(t-\alpha_1-\tau_i).x_j(t-\alpha_2-\tau_j-\tau_r).x_{i'}(t-\alpha_3-\tau_{i'}+\tau).x_{j'}(t-\alpha_4-\tau_{j'}-\tau_r+\tau)\right\}. \\
\quad h(\alpha_1).h(\alpha_2)h(\alpha_4).h(\alpha_4).\mathrm{d}\alpha_1\mathrm{d}\alpha_2\mathrm{d}\alpha_3\mathrm{d}\alpha_4
\end{array}
\right\}
\end{aligned}
\tag{B.3}
$$

$$+\sum_{i=1}^{M}\sum_{j'=1}^{M}\left[\begin{array}{l}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_3).h(\alpha_2).\mathrm{E}\left\{n(t-\alpha_2-\tau_r).n(t-\alpha_3+\tau)\right\}\mathrm{d}\alpha_2\mathrm{d}\alpha_3\\ .\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_1).h(\alpha_4).\mathrm{E}\left\{x_i(t-\alpha_1).x_{j'}(t-\tau_r-\alpha_4+\tau)\right\}\mathrm{d}\alpha_1\mathrm{d}\alpha_4\end{array}\right]$$

$$+\sum_{i=1}^{M}\sum_{j'=1}^{M}m_{j'}.\left[\begin{array}{l}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_3).h(\alpha_2).\mathrm{E}\left\{n(t-\alpha_2-\tau_r).n(t-\alpha_3+\tau)\right\}\mathrm{d}\alpha_2\mathrm{d}\alpha_3\\ .\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_1).h(\alpha_4).\mathrm{E}\left\{x_i(t-\alpha_1).x_{j'}(t-\tau_{j'}-\tau_r-\alpha_4+\tau)\right\}\mathrm{d}\alpha_1\mathrm{d}\alpha_4\end{array}\right]$$

$$+\sum_{i=1}^{M}\sum_{i'=1}^{M}\left[\begin{array}{l}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_3).h(\alpha_2).\mathrm{E}\left\{n(t-\alpha_2-\tau_r).n(t-\alpha_3-\tau_r+\tau)\right\}\mathrm{d}\alpha_2\mathrm{d}\alpha_3\\ .\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_1).h(\alpha_4).\mathrm{E}\left\{x_i(t-\alpha_1).x_{i'}(t-\alpha_3+\tau)\right\}\mathrm{d}\alpha_1\mathrm{d}\alpha_4\end{array}\right]$$

$$+\sum_{i=1}^{M}\sum_{i'=1}^{M}m_{i'}.\left[\begin{array}{l}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_3).h(\alpha_2).\mathrm{E}\left\{n(t-\alpha_2-\tau_r).n(t-\alpha_3-\tau_r+\tau)\right\}\mathrm{d}\alpha_2\mathrm{d}\alpha_3\\ .\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_1).h(\alpha_4).\mathrm{E}\left\{x_i(t-\alpha_1).x_{i'}(t-\alpha_3-\tau_{i'}+\tau)\right\}\mathrm{d}\alpha_1\mathrm{d}\alpha_4\end{array}\right]$$

$$+\sum_{i=1}^{M}\sum_{j'=1}^{M}m_{i}.\left[\begin{array}{l}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_3).h(\alpha_2).\mathrm{E}\left\{n(t-\alpha_2-\tau_r).n(t-\alpha_3+\tau)\right\}\mathrm{d}\alpha_2\mathrm{d}\alpha_3\\ .\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_1).h(\alpha_4).\mathrm{E}\left\{x_i(t-\alpha_1-\tau_i).x_{j'}(t-\tau_r-\alpha_4+\tau)\right\}\mathrm{d}\alpha_1\mathrm{d}\alpha_4\end{array}\right]$$

$$+\sum_{i=1}^{M}\sum_{j'=1}^{M}m_{i}.m_{j'}.\left[\begin{array}{l}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_3).h(\alpha_2).\mathrm{E}\left\{n(t-\alpha_2-\tau_r).n(t-\alpha_3+\tau)\right\}\mathrm{d}\alpha_2\mathrm{d}\alpha_3\\ .\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_1).h(\alpha_4).\mathrm{E}\left\{x_i(t-\alpha_1-\tau_i).x_{j'}(t-\tau_{j'}-\tau_r-\alpha_4+\tau)\right\}\mathrm{d}\alpha_1\mathrm{d}\alpha_4\end{array}\right]$$

$$+\sum_{i=1}^{M}\sum_{i'=1}^{M}m_{i}.\left[\begin{array}{l}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_3).h(\alpha_2).\mathrm{E}\left\{n(t-\alpha_2-\tau_r).n(t-\tau_r-\alpha_3+\tau)\right\}\mathrm{d}\alpha_2\mathrm{d}\alpha_3\\ .\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_1).h(\alpha_4).\mathrm{E}\left\{x_i(t-\alpha_1-\tau_i).x_{i'}(t-\alpha_4+\tau)\right\}\mathrm{d}\alpha_1\mathrm{d}\alpha_4\end{array}\right]$$

$$+\sum_{i=1}^{M}\sum_{i'=1}^{M}m_{i}.m_{i'}.\left[\begin{array}{l}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_3).h(\alpha_2).\mathrm{E}\left\{n(t-\alpha_2-\tau_r).n(t-\tau_r-\alpha_3+\tau)\right\}\mathrm{d}\alpha_2\mathrm{d}\alpha_3\\ .\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_1).h(\alpha_4).\mathrm{E}\left\{x_i(t-\alpha_1-\tau_i).x_{i'}(t-\alpha_4-\tau_{i'}+\tau)\right\}\mathrm{d}\alpha_1\mathrm{d}\alpha_4\end{array}\right]$$

$$+\sum_{i'=1}^{M}\sum_{j'=1}^{M}\left[\begin{array}{l}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_1).h(\alpha_2).\mathrm{E}\left\{n(t-\alpha_1).n(t-\alpha_2-\tau_r)\right\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2\\ .\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_3).h(\alpha_4).\mathrm{E}\left\{x_{i'}(t-\alpha_3+\beta).x_{j'}(t-\alpha_4-\tau_r+\tau)\right\}\mathrm{d}\alpha_3\mathrm{d}\alpha_4\end{array}\right]$$

$$+\sum_{i'=1}^{M}\sum_{j'=1}^{M}m_{j'}.\left[\begin{array}{l}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_1).h(\alpha_2).\mathrm{E}\left\{n(t-\alpha_1).n(t-\alpha_2-\tau_r)\right\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2\\ .\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_3).h(\alpha_4).\mathrm{E}\left\{x_{i'}(t-\alpha_3+\beta).x_{j'}(t-\alpha_4-\tau_{j'}-\tau_r+\tau)\right\}\mathrm{d}\alpha_3\mathrm{d}\alpha_4\end{array}\right]$$

$$+\sum_{i'=1}^{M}\sum_{j'=1}^{M}m_{i'}.\left[\begin{array}{l}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_1).h(\alpha_2).\mathrm{E}\left\{n(t-\alpha_1).n(t-\alpha_2-\tau_r)\right\}\mathrm{d}\alpha_1\mathrm{d}\alpha_2\\ .\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h(\alpha_3).h(\alpha_4).\mathrm{E}\left\{m_{i'}.x_{i'}(t-\alpha_3-\tau_i+\beta).x_{j'}(t-\alpha_4-\tau_r+\tau)\right\}\mathrm{d}\alpha_3\mathrm{d}\alpha_4\end{array}\right]$$

$$+\sum_{i'=1}^{M}\sum_{j'=1}^{M} m_{i'}.m_{j'}.\left[\begin{aligned}&\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2).\mathrm{E}\left\{n(t-\alpha_1).n(t-\alpha_2-\tau_r)\right\}.\mathrm{d}\alpha_1\mathrm{d}\alpha_2\\ &.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_3).h(\alpha_4).\mathrm{E}\left\{x_{i'}(t-\alpha_3-\tau_{i'}+\tau).x_{j'}(t-\alpha_4-\tau_{j'}-\tau_r+\tau)\right\}.\mathrm{d}\alpha_3\mathrm{d}\alpha_4\end{aligned}\right]$$

$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_2).\mathrm{E}\left\{n(t-\alpha_1).n(t-\alpha_2-\tau_r)\right\}.\mathrm{d}\alpha_1.\mathrm{d}\alpha_2.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_3).h(\alpha_4).\mathrm{E}\left\{n(t-\alpha_3+\tau).n(t-\alpha_4-\tau_r+\tau)\right\}.\mathrm{d}\alpha_3\mathrm{d}\alpha_4$$

$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_3).\mathrm{E}\left\{n(t-\alpha_1).n(t-\alpha_3+\tau)\right\}.\mathrm{d}\alpha_1.\mathrm{d}\alpha_3.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_2).h(\alpha_4).\mathrm{E}\left\{n(t-\alpha_2-\tau_r)n(t-\alpha_4-\tau_r+\tau)\right\}.\mathrm{d}\alpha_2\mathrm{d}\alpha_4$$

$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_1).h(\alpha_4).\mathrm{E}\left\{n(t-\alpha_1).n(t-\alpha_4-\tau_r+\tau)\right\}.\mathrm{d}\alpha_1.\mathrm{d}\alpha_4.\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\alpha_2).h(\alpha_3)\mathrm{E}\left\{n(t-\alpha_2-\tau_r).n(t-\alpha_3+\tau)\right\}.\mathrm{d}\alpha_2\mathrm{d}\alpha_3$$

Since $\left\{x_k(t)\right\}$ are mutually independent and their mean value is equal to zero:

$$\mathrm{E}\left\{x_i(t_1).x_j(t_2)\right\} = R_{x_i x_j}(t_1-t_2) \tag{B.4}$$

Using equation (A.8) and equation (B.4), and writing the double integral as a double convolution, equation (B.3) can be written as:

$$R_{z_r z_r}(t,t+\tau)=$$

$$\sum_{i=1}^{M}\sum_{i'=1}^{M}\sum_{j=1}^{M}\sum_{j'=1}^{M}\left[\begin{array}{l}
\left(h\otimes h_2\otimes R_{x_i x_j}\right)(\tau_r).\left(h\otimes h_2\otimes R_{x_{i'} x_{j'}}\right)(\tau_r) \\
+\left(h\otimes h_2\otimes R_{x_i x_{i'}}\right)(-\tau).\left(h\otimes h_2\otimes R_{x_j x_{j'}}\right)(-\tau) \\
+\left(h\otimes h_2\otimes R_{x_i x_{j'}}\right)(\tau_r-\tau).\left(h\otimes h_2\otimes R_{x_j x_{i'}}\right)(-\tau_r-\tau) \\
+m_{j'}.\left[\begin{array}{l}
\left(h\otimes h_2\otimes R_{x_i x_j}\right)(\tau_r).\left(h\otimes h_2\otimes R_{x_{i'} x_{j'}}\right)(\tau_r+\tau_{j'}) \\
+\left(h\otimes h_2\otimes R_{x_i x_{i'}}\right)(-\tau).\left(h\otimes h_2\otimes R_{x_j x_{j'}}\right)(-\tau+\tau_{j'}) \\
+\left(h\otimes h_2\otimes R_{x_i x_{j'}}\right)(\tau_r+\tau_{j'}-\tau).\left(h\otimes h_2\otimes R_{x_j x_{i'}}\right)(-\tau_r-\tau)
\end{array}\right] \\
+m_{i'}.\left[\begin{array}{l}
\left(h\otimes h_2\otimes R_{x_i x_j}\right)(\tau_r).\left(h\otimes h_2\otimes R_{x_{i'} x_{j'}}\right)(\tau_r-\tau_{i'}) \\
+\left(h\otimes h_2\otimes R_{x_i x_{i'}}\right)(\tau_{i'}-\tau).\left(h\otimes h_2\otimes R_{x_j x_{j'}}\right)(-\tau) \\
+\left(h\otimes h_2\otimes R_{x_i x_{j'}}\right)(\tau_r-\tau).\left(h\otimes h_2\otimes R_{x_j x_{i'}}\right)(-\tau_r+\tau_{i'}-\tau)
\end{array}\right] \\
+m_{i'}.m_{j'}.\left[\begin{array}{l}
\left(h\otimes h_2\otimes R_{x_i x_j}\right)(\tau_r).\left(h\otimes h_2\otimes R_{x_{i'} x_{j'}}\right)(\tau_r-\tau_{i'}+\tau_{j'}) \\
+\left(h\otimes h_2\otimes R_{x_i x_{i'}}\right)(\tau_{i'}-\tau).\left(h\otimes h_2\otimes R_{x_j x_{j'}}\right)(\tau_{j'}-\tau) \\
+\left(h\otimes h_2\otimes R_{x_i x_{j'}}\right)(\tau_r+\tau_{j'}-\tau).\left(h\otimes h_2\otimes R_{x_j x_{i'}}\right)(-\tau_r+\tau_{i'}-\tau)
\end{array}\right] \\
+m_{j}.\left[\begin{array}{l}
\left(h\otimes h_2\otimes R_{x_i x_j}\right)(\tau_j+\tau_r).\left(h\otimes h_2\otimes R_{x_{i'} x_{j'}}\right)(\tau_r) \\
+\left(h\otimes h_2\otimes R_{x_i x_{i'}}\right)(-\tau).\left(h\otimes h_2\otimes R_{x_j x_{j'}}\right)(-\tau_j-\tau) \\
+\left(h\otimes h_2\otimes R_{x_i x_{j'}}\right)(\tau_r-\tau).\left(h\otimes h_2\otimes R_{x_j x_{i'}}\right)(-\tau_j-\tau_r-\tau)
\end{array}\right] \\
+m_{j}.m_{j'}.\left[\begin{array}{l}
\left(h\otimes h_2\otimes R_{x_i x_j}\right)(\tau_j+\tau_r).\left(h\otimes h_2\otimes R_{x_{i'} x_{j'}}\right)(\tau_r+\tau_{j'}) \\
+\left(h\otimes h_2\otimes R_{x_i x_{i'}}\right)(-\tau).\left(h\otimes h_2\otimes R_{x_j x_{j'}}\right)(-\tau_j+\tau_{j'}-\tau) \\
+\left(h\otimes h_2\otimes R_{x_i x_{j'}}\right)(\tau_r+\tau_{j'}-\tau).\left(h\otimes h_2\otimes R_{x_j x_{i'}}\right)(-\tau_j-\tau_r-\tau)
\end{array}\right] \\
+m_{j}.m_{i'}.\left[\begin{array}{l}
\left(h\otimes h_2\otimes R_{x_i x_j}\right)(\tau_r+\tau_j).\left(h\otimes h_2\otimes R_{x_{i'} x_{j'}}\right)(\tau_r-\tau_{i'}) \\
+\left(h\otimes h_2\otimes R_{x_i x_{i'}}\right)(\tau_{i'}-\tau).\left(h\otimes h_2\otimes R_{x_j x_{j'}}\right)(-\tau_j-\tau) \\
+\left(h\otimes h_2\otimes R_{x_i x_{j'}}\right)(\tau_r-\tau).\left(h\otimes h_2\otimes R_{x_j x_{i'}}\right)(-\tau_r+\tau_{i'}-\tau_j-\tau)
\end{array}\right] \\
+m_{j}.m_{i'}.m_{j'}.\left[\begin{array}{l}
\left(h\otimes h_2\otimes R_{x_i x_j}\right)(\tau_r+\tau_j).\left(h\otimes h_2\otimes R_{x_{i'} x_{j'}}\right)(\tau_r-\tau_{i'}+\tau_{j'}) \\
+\left(h\otimes h_2\otimes R_{x_i x_{i'}}\right)(\tau_{i'}-\tau).\left(h\otimes h_2\otimes R_{x_j x_{j'}}\right)(\tau_{j'}-\tau_j-\tau) \\
+\left(h\otimes h_2\otimes R_{x_i x_{j'}}\right)(\tau_r+\tau_{j'}-\tau).\left(h\otimes h_2\otimes R_{x_j x_{i'}}\right)(-\tau_r+\tau_{i'}-\tau_j-\tau)
\end{array}\right]
\end{array}\right] \qquad \text{(B.5)}$$

$$
+\sum_{i=1}^{M}\sum_{i'=1}^{M}\sum_{j=1}^{M}\sum_{j'=1}^{M}
\begin{bmatrix}
m_i.\begin{bmatrix}
\left(h\otimes h_2\otimes R_{x_i x_j}\right)(\tau_r-\tau_i).\left(h\otimes h_2\otimes R_{x_{i'} x_{j'}}\right)(\tau_r)\\
+\left(h\otimes h_2\otimes R_{x_i x_{i'}}\right)(-\tau-\tau_i).\left(h\otimes h_2\otimes R_{x_j x_{j'}}\right)(-\tau)\\
+\left(h\otimes h_2\otimes R_{x_i x_{j'}}\right)(\tau_r-\tau_i-\tau).\left(h\otimes h_2\otimes R_{x_j x_{i'}}\right)(-\tau_r-\tau)
\end{bmatrix}\\
+m_i.m_{j'}.\begin{bmatrix}
\left(h\otimes h_2\otimes R_{x_i x_j}\right)(\tau_r-\tau_i).\left(h\otimes h_2\otimes R_{x_{i'} x_{j'}}\right)(\tau_r+\tau_{j'})\\
+\left(h\otimes h_2\otimes R_{x_i x_{i'}}\right)(-\tau_i-\tau).\left(h\otimes h_2\otimes R_{x_j x_{j'}}\right)(-\tau+\tau_{j'})\\
+\left(h\otimes h_2\otimes R_{x_i x_{j'}}\right)(\tau_r-\tau_i+\tau_{j'}-\tau).\left(h\otimes h_2\otimes R_{x_j x_{i'}}\right)(-\tau_r-\tau)
\end{bmatrix}\\
+m_i.m_{i'}.\begin{bmatrix}
\left(h\otimes h_2\otimes R_{x_i x_j}\right)(\tau_r-\tau_i).\left(h\otimes h_2\otimes R_{x_{i'} x_{j'}}\right)(\tau_r-\tau_{i'})\\
+\left(h\otimes h_2\otimes R_{x_i x_{i'}}\right)(\tau_{i'}-\tau_i-\tau).\left(h\otimes h_2\otimes R_{x_j x_{j'}}\right)(-\tau)\\
+\left(h\otimes h_2\otimes R_{x_i x_{j'}}\right)(\tau_r-\tau_i-\tau).\left(h\otimes h_2\otimes R_{x_j x_{i'}}\right)(-\tau_r+\tau_{i'}-\tau)
\end{bmatrix}\\
+m_i.m_{i'}.m_{j'}.\begin{bmatrix}
\left(h\otimes h_2\otimes R_{x_i x_j}\right)(\tau_r-\tau_i).\left(h\otimes h_2\otimes R_{x_{i'} x_{j'}}\right)(\tau_r-\tau_{i'}+\tau_{j'})\\
+\left(h\otimes h_2\otimes R_{x_i x_{i'}}\right)(\tau_{i'}-\tau_i-\tau).\left(h\otimes h_2\otimes R_{x_j x_{j'}}\right)(\tau_{j'}-\tau)\\
+\left(h\otimes h_2\otimes R_{x_i x_{j'}}\right)(\tau_r+\tau_{j'}-\tau_i-\tau).h\otimes h_2\otimes R_{x_j x_{i'}}(-\tau_r+\tau_{i'}-\tau)
\end{bmatrix}\\
+m_i.m_j.\begin{bmatrix}
\left(h\otimes h_2\otimes R_{x_i x_j}\right)(\tau_r-\tau_i+\tau_j).\left(h\otimes h_2\otimes R_{x_{i'} x_{j'}}\right)(\tau_r)\\
+\left(h\otimes h_2\otimes R_{x_i x_{i'}}\right)(-\tau_i-\tau).\left(h\otimes h_2\otimes R_{x_j x_{j'}}\right)(-\tau_j-\tau)\\
+\left(h\otimes h_2\otimes R_{x_i x_{j'}}\right)(\tau_r-\tau_i-\tau).\left(h\otimes h_2\otimes R_{x_j x_{i'}}\right)(-\tau_r-\tau_j-\tau)
\end{bmatrix}\\
+m_i.m_j.m_{j'}.\begin{bmatrix}
\left(h\otimes h_2\otimes R_{x_i x_j}\right)(\tau_r-\tau_i+\tau_j).\left(h\otimes h_2\otimes R_{x_{i'} x_{j'}}\right)(\tau_r+\tau_{j'})\\
+\left(h\otimes h_2\otimes R_{x_i x_{i'}}\right)(-\tau_i-\tau).\left(h\otimes h_2\otimes R_{x_j x_{j'}}\right)(\tau_{j'}-\tau_j-\tau)\\
+\left(h\otimes h_2\otimes R_{x_i x_{j'}}\right)(\tau_r-\tau_i+\tau_{j'}-\tau).\left(h\otimes h_2\otimes R_{x_j x_{i'}}\right)(-\tau_r-\tau_j-\tau)
\end{bmatrix}\\
+m_i.m_j.m_{i'}.\begin{bmatrix}
\left(h\otimes h_2\otimes R_{x_i x_j}\right)(\tau_r-\tau_i+\tau_j).\left(h\otimes h_2\otimes R_{x_{i'} x_{j'}}\right)(\tau_r-\tau_{i'})\\
+\left(h\otimes h_2\otimes R_{x_i x_{i'}}\right)(\tau_{i'}-\tau_i-\tau).\left(h\otimes h_2\otimes R_{x_j x_{j'}}\right)(-\tau_j-\tau)\\
+\left(h\otimes h_2\otimes R_{x_i x_{j'}}\right)(\tau_r-\tau_i-\tau).\left(h\otimes h_2\otimes R_{x_j x_{i'}}\right)(-\tau_r-\tau_j+\tau_{i'}-\tau)
\end{bmatrix}\\
+m_i.m_j.m_{i'}.m_{j'}.\begin{bmatrix}
\left(h\otimes h_2\otimes R_{x_i x_j}\right)(\tau_r-\tau_i+\tau_j).\left(h\otimes h_2\otimes R_{x_{i'} x_{j'}}\right)(\tau_r-\tau_{i'}+\tau_{j'})\\
+\left(h\otimes h_2\otimes R_{x_i x_{i'}}\right)(\tau_{i'}-\tau_i-\tau).\left(h\otimes h_2\otimes R_{x_j x_{j'}}\right)(-\tau_j+\tau_{j'}-\tau)\\
+h\otimes h_2\otimes R_{x_i x_{j'}}(\tau_r+\tau_{j'}-\tau_i-\tau).h\otimes h_2\otimes R_{x_j x_{i'}}(-\tau_r-\tau_j+\tau_{i'}-\tau)
\end{bmatrix}
\end{bmatrix}
$$

$$+\sum_{j=1}^{M}\sum_{j'=1}^{M}\left(h\otimes h_2\otimes R_{x_j x_{j'}}\right)(-\tau).\left(h\otimes h_2\otimes R_{nn}\right)(-\tau)$$
$$+\sum_{j=1}^{M}\sum_{j'=1}^{M}m_{j'}.\left(h\otimes h_2\otimes R_{x_j x_{j'}}\right)(\tau_{j'}-\tau).\left(h\otimes h_2\otimes R_{nn}\right)(-\tau)$$
$$+\sum_{j=1}^{M}\sum_{i'=1}^{M}\left(h\otimes h_2\otimes R_{x_j x_{i'}}\right)(-\tau_r-\tau).\left(h\otimes h_2\otimes R_{nn}\right)(\tau_r-\tau)$$
$$+\sum_{j=1}^{M}\sum_{i'=1}^{M}m_{i'}.\left(h\otimes h_2\otimes R_{x_j x_{i'}}\right)(-\tau_r+\tau_{i'}-\tau).\left(h\otimes h_2\otimes R_{nn}\right)(\tau_r-\tau)$$
$$+\sum_{j=1}^{M}\sum_{j'=1}^{M}m_j.\left(h\otimes h_2\otimes R_{x_j x_{j'}}\right)(-\tau_j-\tau).\left(h\otimes h_2\otimes R_{nn}\right)(-\tau)$$
$$+\sum_{j=1}^{M}\sum_{j'=1}^{M}m_j.m_{j'}.\left(h\otimes h_2\otimes R_{x_j x_{j'}}\right)(-\tau_j+\tau_{j'}-\tau).\left(h\otimes h_2\otimes R_{nn}\right)(-\tau)$$
$$+\sum_{j=1}^{M}\sum_{i'=1}^{M}m_j.\left(h\otimes h_2\otimes R_{x_j x_{i'}}\right)(-\tau_j-\tau_r-\tau).\left(h\otimes h_2\otimes R_{nn}\right)(\tau_r-\tau)$$
$$+\sum_{j=1}^{M}\sum_{i'=1}^{M}m_j.m_{i'}.\left(h\otimes h_2\otimes R_{x_j x_{i'}}\right)(-\tau_j+\tau_{i'}-\tau_r-\tau).\left(h\otimes h_2\otimes R_{nn}\right)(\tau_r-\tau)$$
$$+\sum_{i=1}^{M}\sum_{j'=1}^{M}\left(h\otimes h_2\otimes R_{x_i x_{j'}}\right)(\tau_r-\tau).\left(h\otimes h_2\otimes R_{nn}\right)(-\tau_r-\tau)$$
$$+\sum_{i=1}^{M}\sum_{j'=1}^{M}m_{j'}.\left(h\otimes h_2\otimes R_{x_i x_{j'}}\right)(\tau_{j'}+\tau_r-\tau).\left(h\otimes h_2\otimes R_{nn}\right)(-\tau_r-\tau)$$
$$+\sum_{i=1}^{M}\sum_{i'=1}^{M}\left(h\otimes h_2\otimes R_{x_i x_{i'}}\right)(-\tau).\left(h\otimes h_2\otimes R_{nn}\right)(-\tau)$$
$$+\sum_{i=1}^{M}\sum_{i'=1}^{M}m_{i'}.\left(h\otimes h_2\otimes R_{x_i x_{i'}}\right)(\tau_{i'}-\tau).\left(h\otimes h_2\otimes R_{nn}\right)(-\tau)$$
$$+\sum_{i=1}^{M}\sum_{j'=1}^{M}m_i.h\otimes h_2\otimes R_{x_i x_{j'}}(-\tau_i+\tau_r-\tau).h\otimes h_2\otimes R_{nn}(-\tau_r-\tau)$$
$$+\sum_{i=1}^{M}\sum_{j'=1}^{M}m_i.m_{j'}.h\otimes h_2\otimes R_{x_i x_{j'}}(-\tau_i+\tau_{j'}+\tau_r-\tau).h\otimes h_2\otimes R_{nn}(-\tau_r-\tau)$$
$$+\sum_{i=1}^{M}\sum_{i'=1}^{M}m_i.h\otimes h_2\otimes R_{x_i x_{i'}}(-\tau_i-\tau).h\otimes h_2\otimes R_{nn}(-\tau)$$
$$+\sum_{i=1}^{M}\sum_{i'=1}^{M}m_i.m_{i'}.h\otimes h_2\otimes R_{x_i x_{i'}}(-\tau_i+\tau_{i'}-\tau).h\otimes h_2\otimes R_{nn}(-\tau)$$
$$+\left(h\otimes h_2\otimes R_{nn}\right)(-\tau).\left(h\otimes h_2\otimes R_{nn}\right)(-\tau)$$
$$+\left(h\otimes h_2\otimes R_{nn}\right)(\tau_r-\tau).\left(h\otimes h_2\otimes R_{nn}\right)(-\tau_r-\tau)$$

To calculate expression (B.5) four cases should be investigated:
First case $i = j = i' = j'$

Second case $i = j \neq i' = j'$

Third case $i = i' \neq j = j'$

Fourth case $i = j' \neq j = i'$

The terms for all other cases are equal to zero.

$$
\begin{aligned}
R_{z_r z_r}(t,t+\tau) &= \left(h\otimes h_2\otimes R_{xx}\right)(0).\left(h\otimes h_2\otimes R_{xx}\right)(0)\\
&+\sum_{\substack{i=1\\ i\neq r}}^{M}\left(h\otimes h_2\otimes R_{xx}\right)(\tau_r-\tau_i).\left(h\otimes h_2\otimes R_{xx}\right)(-\tau_r+\tau_i)\\
&+\sum_{i=1}^{M}\left(h\otimes h_2\otimes R_{xx}\right)(\tau_r-\tau_i-\tau).\left(h\otimes h_2\otimes R_{xx}\right)(-\tau_r+\tau_i-\tau)\\
&+\sum_{i=1}^{M}\left[\begin{array}{l}
4.\left(h\otimes h_2\otimes R_{xx}\right)(-\tau).\left(h\otimes h_2\otimes R_{xx}\right)(-\tau)\\
+4.\left(h\otimes h_2\otimes R_{xx}\right)(\tau_r-\tau).\left(h\otimes h_2\otimes R_{xx}\right)(-\tau_r-\tau)\\
+\left(h\otimes h_2\otimes R_{xx}\right)(\tau_i-\tau).\left(h\otimes h_2\otimes R_{xx}\right)(\tau_i-\tau)\\
+\left(h\otimes h_2\otimes R_{xx}\right)(\tau_r+\tau_i-\tau).\left(h\otimes h_2\otimes R_{xx}\right)(-\tau_r+\tau_i-\tau)\\
+\left(h\otimes h_2\otimes R_{xx}\right)(\tau_r+\tau_i-\tau).\left(h\otimes h_2\otimes R_{xx}\right)(-\tau_i-\tau_r-\tau)\\
+\left(h\otimes h_2\otimes R_{xx}\right)(\tau_i-\tau).\left(h\otimes h_2\otimes R_{xx}\right)(-\tau_i-\tau)\\
+\left(h\otimes h_2\otimes R_{xx}\right)(-\tau_i-\tau).h\otimes h_2\otimes R_{xx}(-\tau_i-\tau)\\
+\left(h\otimes h_2\otimes R_{xx}\right)(\tau_r-\tau_i-\tau).\left(h\otimes h_2\otimes R_{xx}\right)(-\tau_r-\tau_i-\tau)\\
+\left(4.h\otimes h_2\otimes R_{xx}\right)(-\tau).\left(h\otimes h_2\otimes R_{nn}\right)(-\tau)\\
+\left(2.h\otimes h_2\otimes R_{xx}\right)(-\tau_r-\tau).\left(h\otimes h_2\otimes R_{nn}\right)(\tau_r-\tau)\\
+2.\left(h\otimes h_2\otimes R_{xx}\right)(\tau_r-\tau).\left(h\otimes h_2\otimes R_{nn}\right)(-\tau_r-\tau)\\
+\left(h\otimes h_2\otimes R_{nn}\right)(\tau_r-\tau).\left(h\otimes h_2\otimes R_{nn}\right)(-\tau_r-\tau)\\
\\
m_i.\left[\begin{array}{l}
4.\left(h\otimes h_2\otimes R_{xx}\right)(-\tau).\left(h\otimes h_2\otimes R_{xx}\right)(-\tau+\tau_i)\\
+\left(2.h\otimes h_2\otimes R_{xx}\right)(\tau_r+\tau_i-\tau).\left(h\otimes h_2\otimes R_{xx}\right)(-\tau_r-\tau)\\
+2.\left(h\otimes h_2\otimes R_{xx}\right)(\tau_r-\tau).\left(h\otimes h_2\otimes R_{xx}\right)(-\tau_r+\tau_i-\tau)\\
+4.\left(h\otimes h_2\otimes R_{xx}\right)(-\tau).\left(h\otimes h_2\otimes R_{xx}\right)(-\tau_i-\tau)\\
+2.\left(h\otimes h_2\otimes R_{xx}\right)(\tau_r-\tau).\left(h\otimes h_2\otimes R_{xx}\right)(-\tau_i-\tau_r-\tau)\\
+2.\left(h\otimes h_2\otimes R_{xx}\right)(\tau_r-\tau_i-\tau).\left(h\otimes h_2\otimes R_{xx}\right)(-\tau_r-\tau)\\
+2.\left(h\otimes h_2\otimes R_{xx}\right)(\tau_i-\tau).\left(h\otimes h_2\otimes R_{nn}\right)(-\tau)\\
+\left(h\otimes h_2\otimes R_{xx}\right)(-\tau_r+\tau_i-\tau).\left(h\otimes h_2\otimes R_{nn}\right)(\tau_r-\tau)\\
+2.\left(h\otimes h_2\otimes R_{xx}\right)(-\tau_i-\tau).\left(h\otimes h_2\otimes R_{nn}\right)(-\tau)\\
+\left(h\otimes h_2\otimes R_{xx}\right)(-\tau_i-\tau_r-\tau).\left(h\otimes h_2\otimes R_{nn}\right)(\tau_r-\tau)\\
+\left(h\otimes h_2\otimes R_{xx}\right)(\tau_i+\tau_r-\tau).\left(h\otimes h_2\otimes R_{nn}\right)(-\tau_r-\tau)\\
+\left(h\otimes h_2\otimes R_{xx}\right)(-\tau_i+\tau_r-\tau).\left(h\otimes h_2\otimes R_{nn}\right)(-\tau_r-\tau)
\end{array}\right]
\end{array}\right]\text{when i} = i' = j = j'
\end{aligned}
\qquad \text{(B.6)}
$$

$$\left[\begin{array}{l}
+\sum_{i=1}^{M}\sum_{\substack{j=1\\ j\neq i}}^{M} 4.\left(h\otimes h_2\otimes R_{xx}\right)(-\tau).\left(h\otimes h_2\otimes R_{xx}\right)(-\tau)\\
+\sum_{i=1}^{M}\sum_{\substack{j=1\\ j\neq i}}^{M} m_i.\left[\begin{array}{l} 2.\left(h\otimes h_2\otimes R_{xx}\right)(\tau_i-\tau).\left(h\otimes h_2\otimes R_{xx}\right)(-\tau)\\ +2.\left(h\otimes h_2\otimes R_{xx}\right)(-\tau_i-\tau).\left(h\otimes h_2\otimes R_{xx}\right)(-\tau)\end{array}\right]\\
+\sum_{i=1}^{M}\sum_{\substack{j=1\\ j\neq i}}^{M} m_j.\left[\begin{array}{l} 2.\left(h\otimes h_2\otimes R_{xx}\right)(-\tau).\left(h\otimes h_2\otimes R_{xx}\right)(\tau_j-\tau)\\ +2.\left(h\otimes h_2\otimes R_{xx}\right)(-\tau).\left(h\otimes h_2\otimes R_{xx}\right)(-\tau_j-\tau)\end{array}\right]\\
+\sum_{i=1}^{M}\sum_{\substack{j=1\\ j\neq i}}^{M} m_i.m_j.\left[\begin{array}{l} \left(h\otimes h_2\otimes R_{xx}\right)(\tau_i-\tau).\left(h\otimes h_2\otimes R_{xx}\right)(\tau_j-\tau)\\ +\left(h\otimes h_2\otimes R_{xx}\right)(\tau_i-\tau).\left(h\otimes h_2\otimes R_{xx}\right)(-\tau_j-\tau)\\ +\left(h\otimes h_2\otimes R_{xx}\right)(-\tau_i-\tau).\left(h\otimes h_2\otimes R_{xx}\right)(\tau_j-\tau)\\ +\left(h\otimes h_2\otimes R_{xx}\right)(-\tau_i-\tau).\left(h\otimes h_2\otimes R_{xx}\right)(-\tau_j-\tau)\end{array}\right]
\end{array}\right] when..i=i'\neq j=j'$$

$$\left[\begin{array}{l}
\sum_{i=1}^{M}\sum_{\substack{j=1\\ j\neq i}}^{M}\left[4.\left(h\otimes h_2\otimes R_{x_ix_i}\right)(\tau_r-\tau).\left(h\otimes h_2\otimes R_{x_jx_j}\right)(-\tau_r-\tau)\right]\\
+\sum_{i=1}^{M}\sum_{\substack{j=1\\ j\neq i}}^{M} m_i.\left[\begin{array}{l} 2.\left(h\otimes h_2\otimes R_{x_ix_i}\right)(\tau_r+\tau_i-\tau).\left(h\otimes h_2\otimes R_{x_jx_j}\right)(-\tau_r-\tau)\\ +2.\left(h\otimes h_2\otimes R_{x_ix_i}\right)(\tau_r-\tau_i-\tau).\left(h\otimes h_2\otimes R_{x_jx_j}\right)(-\tau_r-\tau)\end{array}\right]\\
+\sum_{i=1}^{M}\sum_{\substack{j=1\\ j\neq i}}^{M} m_i.\left[\begin{array}{l} 2.\left(h\otimes h_2\otimes R_{x_ix_i}\right)(\tau_r-\tau).\left(h\otimes h_2\otimes R_{x_jx_j}\right)(-\tau_r+\tau_j-\tau)\\ +2.\left(h\otimes h_2\otimes R_{x_ix_i}\right)(\tau_r-\tau_i-\tau).\left(h\otimes h_2\otimes R_{x_jx_j}\right)(-\tau_r-\tau)\end{array}\right]\\
+\sum_{i=1}^{M}\sum_{\substack{j=1\\ j\neq i}}^{M} m_i.m_j.\left[\begin{array}{l} \left(h\otimes h_2\otimes R_{x_ix_i}\right)(\tau_r+\tau_i-\tau).\left(h\otimes h_2\otimes R_{x_jx_j}\right)(-\tau_r+\tau_j-\tau)\\ +\left(h\otimes h_2\otimes R_{x_ix_i}\right)(\tau_r+\tau_i-\tau).\left(h\otimes h_2\otimes R_{x_jx_j}\right)(-\tau_j-\tau_r-\tau)\\ +\left(h\otimes h_2\otimes R_{x_ix_i}\right)(\tau_r-\tau_i-\tau).\left(h\otimes h_2\otimes R_{x_jx_j}\right)(-\tau_r+\tau_j-\tau)\\ +\left(h\otimes h_2\otimes R_{x_ix_{j'}}\right)(\tau_r-\tau_i-\tau).\left(h\otimes h_2\otimes R_{x_jx_{i'}}\right)(-\tau_r-\tau_j-\tau)\end{array}\right]
\end{array}\right] \text{when } i=j'\neq j=i'$$

$$+\left(h\otimes h_2\otimes R_{nn}\right)(-\tau).\left(h\otimes h_2\otimes R_{nn}\right)(-\tau)$$
$$+\left(h\otimes h_2\otimes R_{nn}\right)(\tau_r-\tau).\left(h\otimes h_2\otimes R_{nn}\right)(-\tau_r-\tau)$$

The expected value of the mixer output $z(t)$ is given by expression (1.24) and is equal to $m_r.(h\otimes h_2\otimes R_{xx})(0)$. We take the square of the expected value of the mixer output and we get

$$\left\{\mathrm{E}\{z(t)\}\right\}^2=\left\{m_r.(h\otimes h_2\otimes R_{xx})(0)\right\}^2 \qquad \text{(B.7)}$$

$m_r(t)$ is a polar antipodal signal, therefore $m_r^2=1$
Equation (B.7) can be written as:

$$\left\{\mathrm{E}\{z(t)\}\right\}^2=(h\otimes h_2\otimes R_{xx})(0).(h\otimes h_2\otimes R_{xx})(0) \qquad \text{(B.8)}$$

Substituting equation (B.8) in equation (B.6) we get:

$$R_{z_r z_r}(t,t+\tau) = \left\{\mathrm{E}\{z_r(t)\}\right\}^2 + C_{z_r z_r}(\tau_k,\tau) \qquad k = i, i', j, j' \tag{B.9}$$

$C_{z_r z_r}(\tau_k,\tau)$ Denotes the covariance of the mixer output $z(t)$.

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