Master thesis

# An improved LGD model for the hire purchase and financial lease portfolio of Volkswagen Bank GmbH Branch NL

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# Management summary

## *Problem background*

For its financial lease and hire purchase portfolio Volkswagen Bank aspires to use the Internal Ratings Based approach for the determination of the credit risk capital. Under this approach the bank needs to find its own estimate for loss given default using a LGD model. The risk management department of Volkswagen Bank had identified some problems in the current LGD model. Therefore a student Industrial Engineering and Management from the University of Twente was given the task to back test the current model and to find improvements to increase the predictive power of the LGD model and to make it more intuitively explainable within the applicable regulatory constraints.

## *Recommendations and motivation*

It is recommended to implement the following improved LGD model. Like the current model this model consists of two sub models. This is based on the idea that a contract can go in default and then cure to become a normal working contract again without a loss being incurred. The probability of this happening is modeled by the cure model. The loss given non-cure model predicts the loss in case a contract does not cure. The model prediction for LGD can be found by multiplying the outcomes of both sub models.

Loss Given Default = (1 - Probability cure ) x Loss Given Non-cure

### Cure model

<<Figure confidential>>

**Figure 1 - Improved cure model**

The probability of cure model uses contract vintage to distinguish between different groups of contracts. The newest contracts cure most frequently as was shown by univariate analysis and confirmed by the debt control department. Contracts that are past their original end date cannot cure by definition and this is captured too in the improved model.

For hire purchase contracts client age is important because this variable is used to model the mortality risk. In case of death of a client the contract cannot cure so as client age increases, the probability of cure decreases. For financial lease contract the whether the car is new or used is the variable that distinguishes best between groups of contracts in terms of their probability of cure.

### Loss given non-cure model

<<Figure confidential>>

**Figure 2 - Improved loss given non-cure model**

The loss given non-cure model adds the hire purchase contracts of self employed clients to the group of financial lease contracts, because both client groups have many similarities in terms of income source and as the client age is lower for self employed clients mortality risk plays only a minor role. A distinction is made between commercial vehicles and regular cars as in case of a commercial vehicle the contract is treated more conservatively at acceptation. Loan to value is an important variable to predict loss which is shown by univariate analysis and literature, therefore an equation with loan to value is used.

In the group of non-self employed clients mortality risk plays an important role which is modeled using the variable client age. For contracts that have a start date before July 1st 2008 a large loss is encountered in case of death of the client, because in that case **.*** euro of the exposure does not need to be repaid. This can be modeled with an equation that uses client age. Starting July 1st 2008 the full exposure of the contract needs to be repaid even in case of death of the client. It is expected that loss can be predicted using loan to value in a similar way as for the financial lease contracts and hire purchase contracts for self employed clients.

### *Consequences*

The performance of the recommended new LGD model has been tested and it shows improvements in calibration and precision. At the level of the sub models the performance is also better in comparison to the current LGD model. There is a small difference in performance of the model on a dataset on which it has been developed and data for which that was not the case, but still the model is expected to work well on new data. However for individual contracts the difference between predicted LGD and the realized LGD can be large.

The new LGD model is easier to explain intuitively because the hard to explain cut-offs between categories of contracts in the current model have been removed. On average the LGD predicted by the new model will be lower compared to the current model. This is because the dataset contained an error which was resolved before the development of the new LGD model.

# Contents

## Abbreviations

| | |
|---|---|
| **AUROC** | Area Under Receiver Operating Characteristic curve |
| **BaFin** | Bundesanstalt für Finanzdienstleistungsaufsicht |
| **BELGD** | Best Estimate Loss Given Default |
| **CAP curve** | Cumulative Accuracy Profiles curve |
| **CCF** | Credit Conversion Factor |
| **CoC** | Coefficient of Concurrence |
| **EAD** | Exposure At Default |
| **FL** | Financial Lease |
| **HP** | Hire Purchase |
| **IRB approach** | Internal Ratings Based approach |
| **LGD** | Loss Given Default |
| **LGN** | Loss Given Non-cure |
| **LtV** | Loan to Value |
| **PD** | Probability of Default |
| **ROC curve** | Receiver Operating Characteristic curve |
| **TKV** | Terug Koop Verklaring (buy back agreement) |
| **VW** | Volkswagen |
| **VWPFS** | Volkswagen Pon Financial Services |

# Preface

The combination of risk management and a company in the automotive sector seemed very interesting when I started my master thesis in May 2009, and it certainly was. Looking for improvements for the LGD model of Volkswagen Bank provided me with the opportunity to learn more and experience firsthand what is involved in developing a risk model. I got some more insight in the regulation and in the particularities of loss given default modeling.

In this preface I would like to thank Mr. Roorda and Mr. De Bakker for their work as supervisors during my research. Looking back at our discussions and their comments I feel that these were very helpful for my master assignment.

Mr. Stienstra as Basel II coordinator for VWPFS was the commissioner of this research. I thank him for the frequent feedback and providing me with company information whenever I needed it.

I hope my suggested improvements for the LGD model will help Volkswagen Bank to get a more accurate view on the credit risk in their portfolio and that the number of defaults will remain problematically low in the future.

Pieter Heeres

# 1 Introduction

*This chapter aims to give an introduction to the research that has been carried out. It starts with a problem description, research questions and the research approach. After that the organizations VWPFS and Volkswagen Bank GmbH Branch NL will be introduced. So this chapter aims to give an overview of the research that has been done and the organization that commissioned it.*

## 1.1 Problem statement

### 1.1.1 Problem background

With the introduction of the new Basel II Accord became the possibility for banks to make more extensive use of internal models for credit risk. This came as a response to the criticisms on the old Accord which did not differentiate between more and less risky credit exposures within exposures to banks or companies. Formerly all loans by a bank or corporation were given the same risk weight. (Hull, 2007)

The Basel II Accord meant new possibilities for more sophisticated way of determining the capital requirements for credit risk. Banks are under certain conditions allowed to use their own internal models. Under this Internal Ratings Based approach risk parameters like PD, LGD, CCF and EAD have to be determined using internal models.

Because these risk parameters play an important role in determining the minimal required capital for banks, their estimation needs to be done with care. Regulation sets out requirements for the models used and the process of estimating these risk parameters. This includes annual validation of models to determine their predictive ability and to demonstrate this to the supervisor.

The way in which this validation should be carried out is still very much under discussion by supervisors and banks. The topic is still changing as there is not yet a prescriptive approach of how this should be carried out. (Basel Committee on Banking Supervision, 2005)

In Europe the Basel II Accord became mandatory for banks after a European Directive was translated into national laws. In Germany this resulted in the new Solvency Ordinance. (Deutsche Bundesbank, 2006)

Volkswagen Bank Branch The Netherland, in short Volkswagen Bank, as part of Volkswagen Bank GmbH falls under the supervision of the *Bundesanstalt für Finanzdienstleistungsaufsicht* and it needs to comply with the rules of the German Solvency Ordinance. For its financial lease and hire purchase portfolio Volkswagen Bank has chosen to implement the Internal Ratings Based approach.

To fulfill the requirements of this IRB approach the first model for the risk parameter LGD was developed in 2007. A back test showed that the performance of this model was not satisfactory. Therefore redevelopment was recommended which in 2008 resulted in the current model. (Linker & Van Baal, Backtest report LGD model 2007 VW Bank, 2008)

### 1.1.2 Problem overview

The current LGD model consists of two sub models. This is based on the idea that a contract can go in default but if the required payments are made again it becomes a normal contract again. This process is called the cure of a contract. In case of cure the result is no direct loss on a contract but costs for debt collection may be incurred. The current LGD model has a sub model to determine the probability that a contract will cure once it is in default, called the cure model. There is also a sub model that models the loss, given that the contract does not cure. This is called the loss given non-cure model.

The LGD model uses a decision tree structure for both the cure and loss given non-cure model. Although the model has not yet been back tested there have been some conceptual problems identified. The decision tree creates hard cut-off points in the risk drivers. So a small change in the value of a risk driver could result in a large change in risk parameter model output. This is not intuitively plausible and it is not possible to explain the economic logic behind this.

Therefore research into possible model improvements was commissioned by the Risk Management department of Volkswagen Pon Financial Services. This department is responsible for risk management of Volkswagen Bank in the Netherlands. Finding improvements for the LGD model was done as a master thesis project under supervision of the University of Twente.

### 1.1.3 Problem scope

The research will be limited to the financial lease and hire purchase portfolios of Volkswagen Bank. These portfolios include retail exposures to companies and private clients respectively in The Netherlands. The model that will be studied in this research is the LGD model for those two portfolios, the main focus is on improving this model. Related issues such as the Best Estimate LGD model, determining the conservative margin, allocation of direct and indirect costs and LGD downturn factor will be looked at but emphasis is on finding improvements for the LGD model.

### 1.1.4 Research objective

The objective of the research is to find improvements for the existing LGD model of Volkswagen Bank NL for the hire purchase and financial lease portfolios such that the model:

- Has increased predictive power
- Complies with the requirements of the German Solvency Ordinance
- Is intuitively explainable to the business

### 1.1.5 Research questions

1. What LGD model structures can be found in literature?
2. What regulatory requirements for LGD models are specified by the German Solvency Ordinance?
3. How does the current Volkswagen Bank LGD model perform in terms of predictive power?
    a. How can the Volkswagen Bank LGD model be back tested as part of validation?
    b. What criteria and accompanying statistics can be found to test the performance of a LGD model according to literature?
    c. What regulatory requirements for back testing are specified by the German Solvency Ordinance?

4. What problems can be identified by the business and from a back test of the current Volkswagen Bank LGD model?
5. How can the current Volkswagen Bank LGD model be improved?
    a. What risk drivers should be included in the improved LGD model according to univariate analysis?
    b. How can loan to value be used as a risk driver in the improved LGD model?
    c. What LGD model structure should be used for the improved LGD model?
    d. How does the improved LGD model perform in terms of predictive power?
    e. How does the improved LGD model work on different data than on which it was developed?

### 1.1.6 Conceptual model

The conceptual model describes the research graphically in terms of information needed and the order in which activities need to be performed. It also gives the outline of this report by showing in which chapter each activity will be reported.

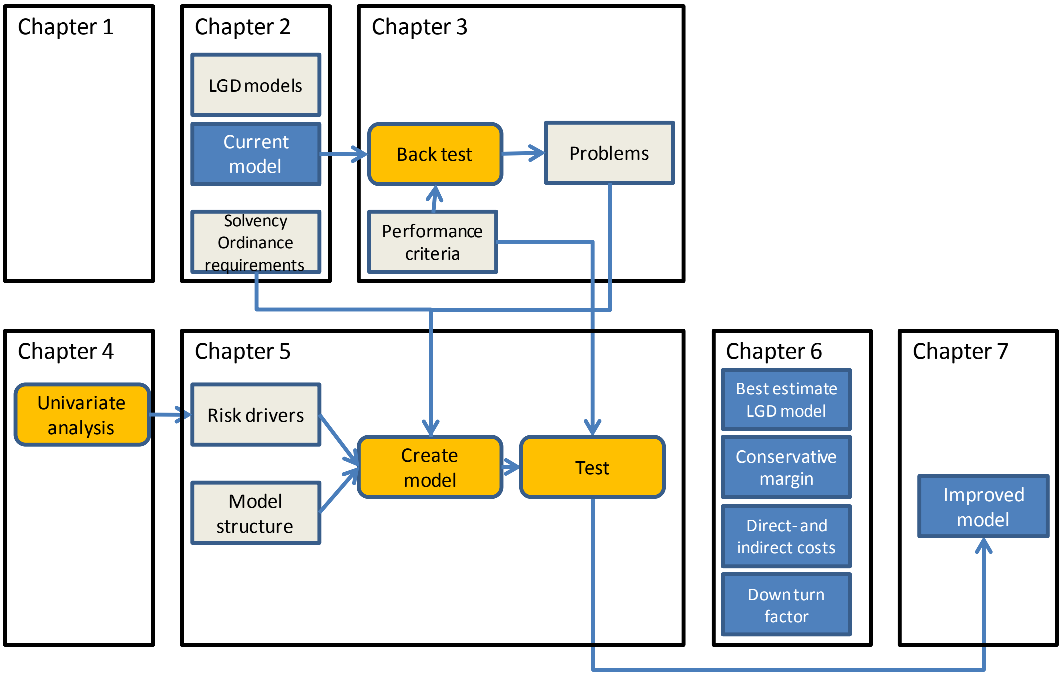


Figure 3 - Conceptual model

## 1.2 Research approach

The research is aimed at developing an improved model, so it can be classified as 'designing research'. (Verschuren & Doorewaard, 2003, p. 54) Designing cannot be done immediately as first the problems of the current model need to be mapped out more extensively. This step can be categorized as evaluative research.

Now per research question the research approach will be explained. The research approach describes how the answer to the question will be found in terms of information sources and activities to be carried out.

### 1.2.1 What LGD model structures can be found in literature?

This question aims to map out the different approaches that can be identified towards modeling LGD in terms of model structure. Together with the different possibilities their strengths and weaknesses will be examined so that in a later stage the best suitable model can be chosen. This will be done by studying literature, both scientific and articles published by regulators.

### 1.2.2 What regulatory requirements for LGD models are specified by the German Solvency Ordinance?

In order to find acceptable improvements for the LGD model it is vital that these are compliant with the regulation to which Volkswagen Bank is subject. Therefore the English translation of the German Solvency Ordinance will be studied to find requirements that apply to LGD models for the type of exposures that Volkswagen Bank has.

### 1.2.3 How does the current Volkswagen Bank LGD model perform in terms of predictive power?

Determining predictive power is done when a model is back tested as part of validation. A back test will be performed to find how the current model performs in terms of predictive power. Before this can be done research sub questions have to be answered that show how to do this.

- How can the Volkswagen Bank LGD model be back tested as part of validation?
  This question will be answered by studying documents from regulatory institutions such as central banks.
- What criteria and accompanying statistics can be found to test the performance of a LGD model according to literature?
  To find an answer to this question documents by regulators as well as scientific literature will be used.
- What regulatory requirements for back testing are specified by the German Solvency Ordinance?
  The obvious source of information to answer this question is the English translation of the Solvency Ordinance.

The result of this research question is not only that a back test can be carried out but also the criteria and statistics are found for measuring performance. These are also useful for testing improvements in a later stage of the research.

### 1.2.4 What problems can be identified by the business and from a back test of the current Volkswagen Bank LGD model?

The aim of this question is to map out all problems that need to be addressed to find improvements for the current LGD model. A source of information for this question is discussing the model with the Risk Management department. A report of an external review that was carried out in 2008 could also be a useful source of information.

### 1.2.5 How can the current Volkswagen Bank LGD model be improved?

This question is central in this thesis and is too large to be answered without sub questions. It can only be answered when the previous research questions have been answered.

- What risk drivers should be included in the LGD model according to univariate analysis?
  A univariate analysis will be carried out to identify the relations that variables have with the probability of cure or loss. Also key employees will be interviewed to find explanations for the relations found.
- How can loan to value be used as a risk driver in the LGD model?
  A more detailed analysis of the variable loan to value will be carried out.
- What LGD model structure should be used for the model?
  Based on the theoretical strengths and weaknesses of the different model structures a choice is made of a structure that will most likely work well.
- How does the improved model perform in terms of predictive power?
  Various possible improved models will be tested based on the criteria and statistics found and used earlier to back test the model.
- How does the improved model work on different data than on which it was developed?
  Analysis of performance will be done on both a dataset on which the improvements were made and a so far not used set. The performance on both sets will be compared to have an indication of the capability of the model to work on new data.

## 1.3 Overview organization

### 1.3.1 Volkswagen Pon Financial Services

Volkswagen Pon Financial Services aims to provide financial mobility to automotive clients according to its mission statement. (VWPFS, 2009) The company does so by providing financial-, insurance and lease products specially tailored for the automotive clients. These products are often provided as a package to add customer value.

At the moment over 300 people work for VWPFS. It is located in six different Dutch cities with Amersfoort being the central location. The company is a joint venture between Volkswagen Financial Services AG and Pon Holdings B.V. VWPFS itself operates four different business lines in separate companies. These are DFM N.V., Lease+ BV, Dutchlease and Volkswagen Leasing. Besides these there are Volkswagen Bank GmbH Branch The Netherlands and Volkswagen Verzekeringen Serivce N.V. which are both managed by VWPFS. An overview of these entities and their relations can be found below in Figure 4. (SSC Risk Management, 2008, p. 5)

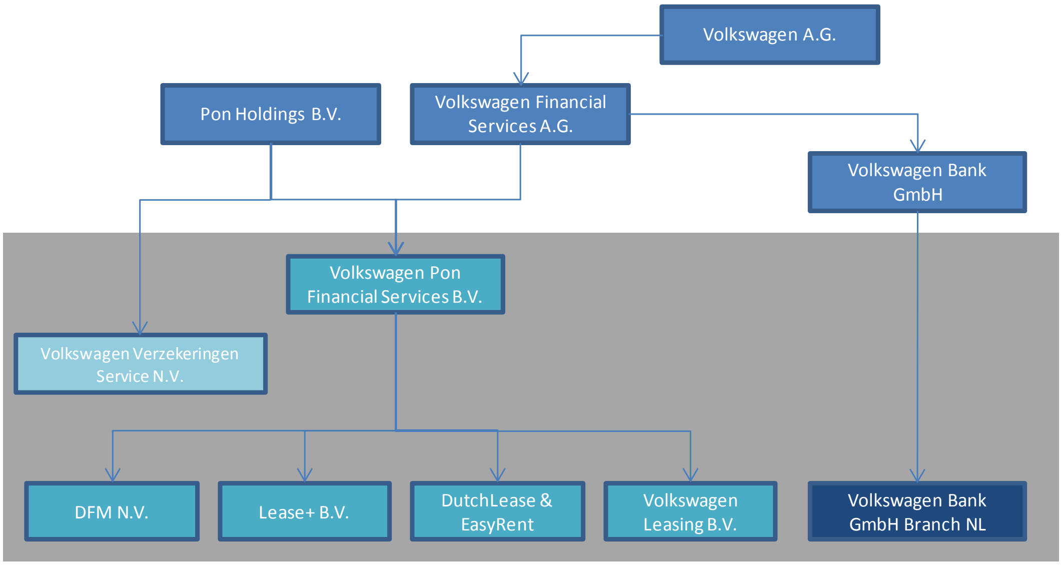


**Figure 4 – Organization entities**

This research was done for the Risk Management department of VWPFS. The department is also responsible for the risk management of Volkswagen Bank.

The different business of VWPFS each serve a different customer target group and offer different products. An overview of these can be found in Table 1.

Table 1 – Organizational entities and their target customer groups

| Entity | Customer group | Financing & Financial Lease | Insurance | Operational lease | Rent |
|---|---|---|---|---|---|
| **Volkswagen Verzekeringen Service N.V.** | Corporate & Consumers | | ✓ | | |
| **DFM N.V.** | Car dealers, lease and rental companies | ✓ | | | |
| **Lease+ B.V.** | Small and medium size enterprises | | | ✓ | |
| **DutchLease & EasyRent** | Small and medium size enterprises | | | ✓ | ✓ |
| **Volkswagen Leasing B.V.** | Small and medium size enterprises | | | ✓ | |
| **Volkswagen Bank** | Consumers & Small and medium size enterprises | ✓ | | | |

### 1.3.2 Volkswagen Bank GmbH Branch NL

Volkswagen Bank GmbH Branch NL will be described in more detail because the research was done for this bank. It is full subsidiary of Volkswagen Bank GmbH. For convenience Volkswagen Bank will be used in this report to refer to Volkswagen GmbH Branch NL. Volkswagen Bank is managed by VWPFS and reports to Volkswagen Financial Services A.G.

The clients of Volkswagen Bank can be divided in consumers and businesses. The consumers' category does not only include natural persons but self employed clients as well. Products are mainly sold through car dealerships of the brands imported by Pon Holdings. These are Volkswagen, Audi, SEAT, Skoda and Porsche. The dealers act as intermediaries between the client and Volkswagen Bank selling the product under the brand name of the car. In total four different types of products offered. (SSC Risk Management, 2008)

#### *Financial Lease*

Financial lease is the only available credit facility for business clients at Volkswagen Bank. This type of contract is primarily for financing a car. A contract between bank and client is made based on the value of the car, its expected trade-in value at the end of the contract and duration of the contract. Usually at the end of the contract a final installment has to be paid because the contract does not fully pay off the car. The resale value of the car should cover this final installment. During the life of the contract the car acts as collateral. In case the customer stops making the required payments the car can be impounded and sold.

### *Hire purchase*

Hire purchase is only for consumers and is technically similar to financial lease in terms of the product offered. There is one important legal difference for Volkswagen Bank. Hire purchase contracts with a credit amount up to 40.000 euro are subject to the *Wet op het consumentenkrediet*. This law aims to protect consumers and restricts the possibility of the lender to seize the collateral in case the client stops paying. In case 75% of the credit amount has been repaid it is not allowed to impound the car. (Article 41, Wet op het consumentenkrediet) The lender still has a claim on the car but to seize it requires costly additional legal procedures.

### *Personal Loan*

Personal loan is a credit facility for consumers in the form of a regular loan where the financed amount will be fully paid off by monthly installments. Possibly a car is used as collateral.

### *Revolving Credit*

Revolving credit is also for consumers and mostly a car is used as collateral. When the client uses the credit facility, the client has to pay at least one monthly payment to pay off the borrowed amount before making use of the credit facility again.

To give an indication of the portfolio of Volkswagen Bank Table 2 gives the number of contracts and exposure for each type of product at September 30th 2008. (SSC Risk Management, 2008, p. 13)

**Table 2 – Key figures for the products of Volkswagen Bank**

<<Figure confidential>>

### 1.3.3 Solvency Ordinance implementation at Volkswagen Bank

Volkswagen Bank GmbH as a bank needs to adhere to the requirements of the Solvency Ordinance. It was decided to adopt the internal ratings based approach for credit risk. Volkswagen Bank Branch NL therefore needs to do the same. To use the IRB method for retail exposures the bank needs to provide its own estimates for the probability of default, exposure at default and loss given default.

In the portfolio of VW bank there are four types of products. For the Personal Loan and Revolving Credit the standard approach will be used instead of the IRB approach. This is because the number of contracts is very small so it is not possible to develop models for these two types of contracts. (SSC Risk Management, 2008, p. 13)

The Financial Lease and Hire Purchase exposures are classified as retail claims. These claims satisfy the criteria for retail exposures because they are small individual exposures that are treated all the same.  (SSC Risk Management, 2008, p. 17)

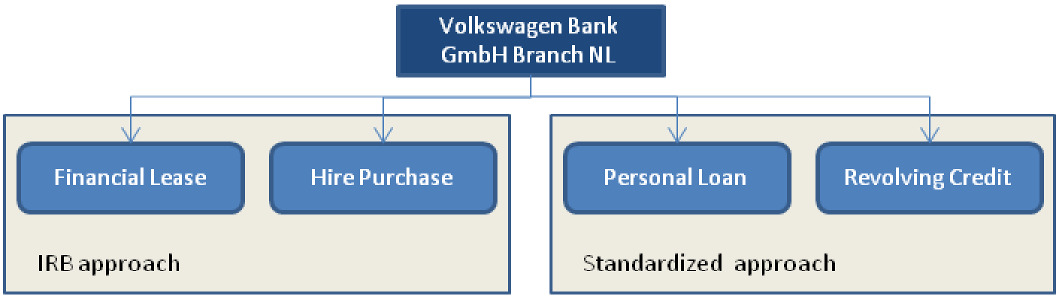


Figure 5 – Volkswagen Bank products and approach under the Solvency Ordinance

### 1.3.4 Modeling credit risk at Volkswagen Bank

With its portfolio of financial lease, hire purchase and loan contracts Volkswagen Bank is exposed to credit risk from its counterparties. To manage this risk and to comply with regulation several models are used by the bank. Some are used prospectively to assess the client creditworthiness at the time of application. For this purpose there are credit score cards. Other models are used to monitor the period after the contract has been signed. These are the models for PD and LGD.

An application is first assessed using an application model. If the client is accepted a contract can start directly or some time later. This could be because the car has been ordered but not yet delivered.

Now a short overview of the models used will be given. (SSC Risk Management, 2008)

Table 3 – Volkswagen Bank risk models and their use

| Model | Portfolio | Application | Behavior |
|---|---|---|---|
| **Private clients scoring model** | Hire purchase | ✓ | |
| **SME  scoring model** | Financial lease | ✓ | |
| **PD model** | Financial lease | | ✓ |
| **PD model** | Hire purchase | | ✓ |
| **LGD model** | Hire purchase and Financial lease | | ✓ |

# 2 LGD models

*This chapter will introduce the key concepts related to LGD models. Then an overview of the LGD model structures is given to answer the first research question. After that the current LGD model will be described. Finally the second research question will be answered by discussing the regulatory framework that applies for the LGD model of Volkswagen Bank.*

## 2.1 LGD models

This research is focused on LGD models. Therefore it is important to pay attention to what LGD exactly is and what is being modeled.

According to section 132 of the Solvency Ordinance LGD is the expected ratio of the loss on an exposure due to the default of a counterparty to the amount outstanding at default. That means LGD is the expected loss as a fraction of the exposure on counterparty in case of a default of that counterparty. LGD is like PD, EAD and CCF called a risk parameter.

### 2.1.1 Measuring LGD

There are a number of ways in which the LGD can be measured. There are subjective methods that use expert opinion and objective methods that are based on numerical data. In this case objective methods are the most relevant, an overview of those is given below based on working paper 14 of the Basel Committee on Banking supervision (2005, p. 62)

Table 4 – Overview of methods to determine LGD

| Source | Measure | Defaulted facilities | Non-defaulted facilities | Most applicable to |
|---|---|---|---|---|
| **Market values** | Price differences | Market LGD | | Large corporate, sovereigns, banks |
| | Credit spreads | | Implied market LGD | Large corporate, sovereigns, banks |
| **Recovery and cost experience** | Discounted cash flows | Workout LGD | | Retail, SMEs, large corporates |
| | Historical total losses and estimated PD | Implied historical LGD | | Retail |

Implied historical LGD means using the total historical losses as a prediction for future losses. From the historical losses and the estimated PD it is possible to imply what the LGD will be. Workout LGD uses actual cash flows.

Workout LGD was chosen to be used in this research. This is because it was used in developing the current model. Methods that use market values cannot be used because the exposures are not traded.

### *Workout LGD*
Ratio of discounted recoveries minus discounted costs due to the default of a counterparty to the amount outstanding at default. (Basel Committee on Banking Supervision, 2005, p. 66) These cash flows can occur at different points in time after default, to make them comparable they have to be discounted. If there are *i* cash flows, at different times t and a discount rate *r* is used the formula below gives the LGD.

$$LGD = 1 - \frac{\sum_i NPV(Cash\,flow_i, t, r)}{EAD}$$

## 2.1.2 Loss
Before LGD can be calculated it is important to know what exactly qualifies as a loss. According to section 126 of the Solvency Ordinance loss means economic loss. This definition includes material direct and indirect costs associated with collecting.

## 2.1.3 Default
Default is a certain event that can happen to a counter party to which one has a credit exposure. According to the Solvency Ordinance, Section 125, article 1 default has occurred for a particular obligor if any of the following two events has occurred:

- The institution has material reason to consider that the obligor is unlikely to pay its credit obligations in full to the institution or any group enterprise belonging to the group of institutions or financial holding group to which the institution belongs without recourse by the institution to actions such as realizing the security (if held)
- The obligor is past due more than 90 successive calendar days on any material part of its overall credit obligation to the institution or to a group enterprise belonging to the group of institutions or financial holding group to which the institution belongs.

Events that should be taken as indications of unlikeliness to pay are the following. These are set out against the default reasons identified by Volkswagen Bank. (Linker & Van Baal, 2008, p. 11) The abbreviations used by Volkswagen Bank for the various default reasons can be found in appendix I.

**Table 5 – Solvency Ordinance default reasons set out against the default resaons used by Volkswagen Bank**

| Solvency Ordinance | Volkswagen Bank Category |
|---|---|
| **Value adjustment** | |
| **Sell at material loss** | |
| **Distressed restructure** | Individual payment deadline granted, obligor in debt restructuring |
| **Filed obligor bankruptcy** | Bankruptcy obligor |
| **Obligor in bankruptcy or similar** | Confiscation by tax authority, bankruptcy obligor |
| **Other** | Untraceable, repossession , cancelled by VW Bank, decease of contractor, loss of an object, actual loss, migration obligor to foreign country, false information, embezzlement |

### 2.1.4 The nature of LGD modeling

To explain the nature of LGD modeling it is clarifying to compare them with PD models. LGD models are so called regression models while PD models can be described as binary classification problems. The difference is in what is being modeled.

#### *Binary classification problems*

The objective of a binary classification problem is to discriminate between two classes. For PD models this mean discriminating between defaults and non-defaults. (Van Gestel & Baesens, 2009, p. 188) What is modeled is the occurrence or non-occurrence of an event.

#### *Regression problems*

The name regression problem should not be confused with the regression model. This may be confusing but it will be used because that is how this type of problem is referred to by Van Gestel and Baesens (2009). The goal for a regression problem is to model in such a way that the difference between the estimated variable and the realized value is minimized. (Van Gestel & Baesens, 2009, p. 186) This is not the same and does not imply a regression model structure to be used such as described in paragraph 2.2. It simply means looking for a model that models a continuous variable in such a way that it has the smallest difference between model estimations and realizations.

### 2.1.5 LGD model classification

LGD models can be distinguished by their model structure and the risk drivers used. This division will be used throughout this research.

Models can have different basic ideas on which the modeling is based. Three types have been identified. (Van Gestel & Baesens, 2009, p. 174)

#### *Financial models*

Financial models use financial theory and they do not rely on data to explain their functioning. The risk drivers used in the model are based on financial theory. Problem with this type of model is that the theoretical back ground may not match completely with empirical observations. All assumptions made for the theoretical model may not hold in practice.

#### *Expert models*

Expert models use human knowledge to find estimates for risk parameters. The process which financial experts use can be more or less formalized. Experts can analyze and interpret information in detail and can do further research if necessary. In this sense expert models are flexible and can adapt to changes. On the downside the expert judgment itself may not be transparent.

#### *Empirical data based models*

Empirical data based models use past observations to estimate and identify risk drivers. These rely purely on data from past observations. Data is used to determine an empirically valid relation between risk drivers and risk parameters.

It is also possible to combine these approaches. Because of regulatory constraints it is not possible to base a model purely on financial models or expert opinions. This is because estimates should be based on historical experience and empirical evidence. (Section 128, Solvency Ordinance) In practice therefore a combination of empirical data based and expert approaches are used and will be used in this research.

## 2.2 LGD model structures

*This paragraph will give attempt to give some taxonomy of the different types of LGD model structures that exist. The idea is that a LGD model can be described in terms of its structure and risk drivers included.*

### 2.2.1 Direct and indirect models

The variable being modeled provides the first possibility for distinction between models. Direct models predict the risk parameter without any step in between. This means that the model output is directly the risk parameter that is modeled.

Indirect models take on ore more steps before the risk parameter is modeled. They predict an intermediate result that serves to find the final prediction of the risk parameter. (Van Gestel & Baesens, 2009, p. 194)

Indirect models allow the intermediary results to be used as input for different models. It may however cause problems if the intermediate variables are correlated. Also during model construction this may cause the optimization of the intermediate results instead of the final risk parameter.

In practice indirect models are still the most useful. This is because in practice it is often too complicated to create a global model for the risk parameter. Also data considerations play a role as there is often not enough data available from one source. (Van Gestel & Baesens, 2009, p. 194)

### 2.2.2 LGD model structures

Schuermann (2004) gives an overview of LGD modeling structures classified along different levels of sophistication. Within this overview the model architectures described by Van Gestel and Baesens (2009, p. 183) can be fitted. It should be noted that Schuermann made this overview particularly for LGD models, Van Gestel and Baesens do not limit to LGD but include all models. Gupton (2005, p. 62) describes look-up tables as the traditionally most used model and they identify some weaknesses of this approach.

Based on the work of these authors the overview in table 6 was created which gives the different model structures and the accompanying strengths and weaknesses.

Table 6 – LGD model structures and their strengths and weaknesses

| Sophistication level | Type | Details | Strengths | Weaknesses |
|---|---|---|---|---|
| Low | Look-up table | Each cell in the look-up table gives a LGD for a particular exposure type. | Easy to construct and use | Data intensive in case of large table |
| Medium | Linear regression | LGD found by linear regression. $LGD = w_1 x_1 + \ldots + w_n x_n + b$ Risk drivers *x* can be dummies for buckets. | Relatively easy to construct, flexible on data quantity | If used, bucketing must be done with care |
| Medium-high | Advanced regression | LGD found by regression where risk drivers are functions with parameters $\lambda$. Intrinsically linear. $LGD = w_1 f_1(x_1, \lambda) + \ldots + w_n f_n(x_n, \lambda) + b$ | Increased fit to data | Somewhat prone to over fitting and data mining |
| High | Neural nets, tree methods, machine learning | Variety of models. Often deal better with ordinal variables than regression. | Even better fit to data | Prone to over fitting and data mining |

This gives an overview with a variety of model structures that could be used and some of their strengths and weaknesses as identified in literature. Generally speaking the more sophisticated ones provide the best fit to the data but are the most prone to problems like data mining or over fitting. These problems are more profound if limited data is available.

## 2.3 LGD risk drivers from literature

Risk drivers are the factors that influence risk parameters. These risk drivers are presumed to have a causal effect on the risk parameter value.

There is a wide variety of literature on risk drivers for LGD models. The difficulty is in identifying the ones that are relevant for Volkswagen Bank. Therefore the search was limited to factors that are mentioned for the car leasing industry.

First of all the exposures of Volkswagen Bank have collateral in the form of the car that has been financed. This is the case generally in the leasing industry. ABN AMRO Lease Holding (2001) and Schmitt (2002) identify the value of the collateral as important risk driver for LGD on car leasing exposures. The existence of collateral is important in mitigating risk and therefore car leasing is seen as less risky as unsecured loans. It should be noted that the value of the collateral is not fixed but fluctuates with prices for second hand cars.

Laurent and Schmit (2005, p. 318) also mention collateral. Besides that they identify the age of the contract as a risk driver for LGD.

## 2.4 Current Volkswagen Bank LGD model

There are separate LGD models for contracts that are not in default and for contracts that have already a default status. The model for contracts that are in default is called the best estimate LGD model, for contracts that are not in default the model will simply be referred to as LGD model.

The current LGD model should be classified as an indirect model because it consists of two separate sub models, a cure model and a loss given non-cure model. This is based on the idea that a defaulted contract can cure if the client makes the required payments. In case of cure the defaulted contract becomes a normal contract again and as a result there is no loss. The cure model gives the expected probability of that happening.

If a contract does not cure a loss can occur. This is modeled by the loss given non-cure model.

With the outcomes of the two sub models the final LGD estimate can be calculated.

Loss given default (LGD) = (1 - Probability cure ) x Loss given non-cure (LGN)

**Figure 6 – The relation between sub models and the LGD estimate**

### 2.4.1 Cure model

The cure model makes use of a decision tree structure. However it is also possible to convert this structure in to a simple lookup tables so it is not a highly sophisticated model. All end nodes give the different probabilities of cure, in the decision tree cure this is denoted as recovery.

<<Figure confidential>>

**Figure 7 - The current cure model, the percentages show the fraction of contracts that cure and the number behind the percentage give the number of contracts used to determine the percentage.**

### 2.4.2 Loss given non-cure model

Again a decision tree model is used to model the loss given non-cure. The same remark that this tree can be converted to lookup table also holds for this model.

<<Figure confidential>>

**Figure 8 – The current loss given non-cure model, the percentages give the expected loss as a fraction of the exposure.**

### 2.4.3 LGD conservative margin

The LGD conservative margin was found using the bootstrap method. New data sets were created by drawing random samples from the cure modeling set. This was done 1000 times and the 95$^{th}$ percentile of these was compared with the average LGD estimate. The average LGD estimate is *****% and the 95$^{th}$ percentile of 1000 bootstraps is equal to *****%. This is *.**** times larger than the average so the to find the conservative margin LGD for each contract the LGD is multiplied by *.****.

### 2.4.4 Best Estimate LGD model

The best estimate LGD model is used to find the LGD for contracts that are already in default.

Again this model consists of two separate sub models, a cure model and a loss given non-cure model.

#### *Best Estimate Cure model*

The idea is to look at the first reason for default. There is a difference if it is because the contract is more than 90 days past due or if there is another reason. Therefore using regression analysis there are two equations found that give the probability that the contract is cured.

- <<Confidential>>

### *Best Estimate LGN model*

For the best estimate LGN model three different default reasons are identified:

1. Loss of the car and embezzlement
2. 90 days default (90 days over payment date)
3. Non 90 days, non loss or embezzlement

This gives the following equations for the loss percentage:

<<Confidential>>

### 2.4.5 Downturn LGD

To consider worse economic situations an additional downturn margin needs to be added to the estimated LGD. This margin shows what the LGD would be if the LGD was modeled during an economic downturn.

The downturn LGD analysis was done by looking at past provision levels. The Downturn LGD factor was determined as the average provision rate of the two highest consecutive rates divided by the average provision rate of the modeling period. This showed a downturn factor of *****. So the current LGD estimates should be multiplied by that factor to find an estimate for an economic downturn period.

## 2.5 Regulation for LGD models

*Regulation plays an important role in developing LGD models. After all any model that is developed needs to be approved by regulators and that can only happen if it is compliant with all relevant regulation. This paragraph will explore the legal background first to identify the applicable sources of regulation. Subsequently specific regulatory requirements for LGD models for retail portfolios will be identified.*

### 2.5.1 Legal background

The Bank of International Settlements has no direct regulatory or supervisory power. This means that the Basel II Accord has no direct legal consequences for German banks. In this sense the Accord has no legal force but is a best practice for banks.

The Basel II Accord is however the basis for future regulation. In Europe it has been implemented through Directive 2006/48/EC on the business of credit institutions and Directive 2006/49/EC on capital adequacy of investment firms and credit institutions. These are legislative acts by the European Union that requires each member state to adopt regulation to achieve the results described in the directive. This directive is only legally binding for the member states of the European Union and has no direct effect for banks.

There is a lot of freedom given to the member states to determine how to implement the directive into national regulation. In Germany this was done by adapting the existing Banking Act and the creation of a Solvency Regulation. (Deutsche Bundesbank, 2006, p. 67) This *Solvabilätsverordnung* will be referred to as Solvency Ordinance in this report. It came into force on January 1st 2007. (Regulation governing the capital adequacy of institutions,groups of institutions and financial holding groups (Solvency Regulation (Solvabilitätsverordnung)) of 14 December 2006)

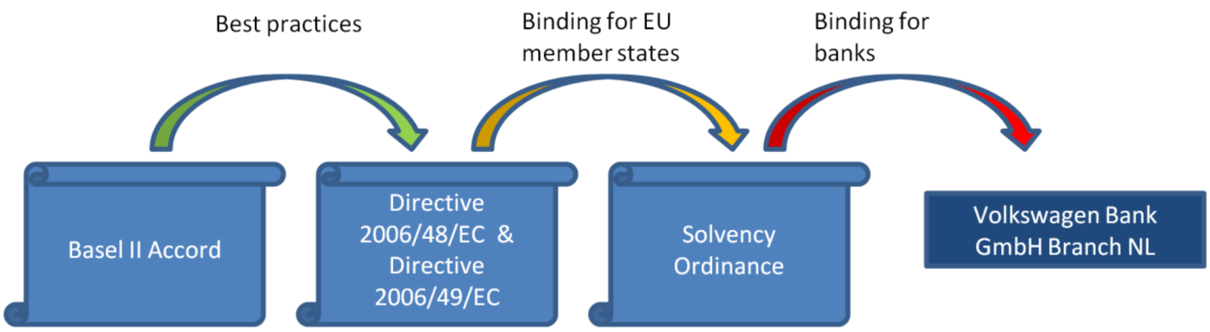


Figure 9 – Legal documents and their effect on Volkswagen Bank

#### *Basel II Accord*

The Basel II Accord was published by the Bank of International Settlements in June 2004. It replaces the original Basel accord that was introduced in 1988. It responded to some to the criticisms on the first accord by treating credit risk in a more sophisticated way.

The Basel II Accord is based on three pillars.

1. Minimal capital requirements
2. Supervisory review
3. Market discipline

The minimal capital requirements determine the minimal amount of capital that banks have to hold for credit, market and operational risk. Supervisory review is aimed achieving consistent application of the supervisory review process. Supervisors' role should include early intervention and the spread of better risk management techniques. The third pillar requires banks to disclose more about their allocation of capital and the risks they take. (Hull, 2007, p. 179)

The Accord gives three choices for modeling credit risk, however for retail exposures banks can only chose the standardized approach or internal ratings based approach. Under the IRB approach the bank can provide its own estimates for PD, EAD and LGD. These approaches enable the banks to calculate the capital that they are required to hold for credit risk. (Hull, 2007, p. 186)

### *Solvency Ordinance*

As the Solvency Ordinance is based on the Basel II Accord its contents are very similar as it is the German implementation of the Basel II Accord. In case of difference these are usually not material but could simply be due to translations. The original Solvency Ordinance is in German however for practical reasons the English translation by the Bundesbank has been used in this research.

Like the Basel Accord the Solvency Ordinance aims to ensure that financial institutions have an appropriate minimal capital. It details minimal amounts of capital that must be held for credit risks, market risks and operational risks. It also contains disclosure requirements for financial institutions similar to the Pillar 3 section of the Basel Accord. (Deutsche Bundesbank, 2006)

The Solvency Ordinance has different ways in which capital required for credit risk can be determined.

- Credit Risk Standardized Approach (CRSA or SA)
- Internal Ratings-Based Approach (IRBA)

The Internal Ratings-Based Approach is new for German banks because previous regulation did not allow the use of this. Bank willing to use this method must gain prior approval by the BaFin, the German Financial Supervisory Authority.

### 2.5.2 Solvency Ordinance requirements for LGD models

LGD models are part of the IRB approach to modeling credit risk. Therefore requirements on LGD models can be found in the sections of the Solvency Ordinance that describe minimum requirements for using Internal Ratings-Based Approach. This is Division 5 of the Solvency Ordinance; the contents of this Division are summarized in the figure below.

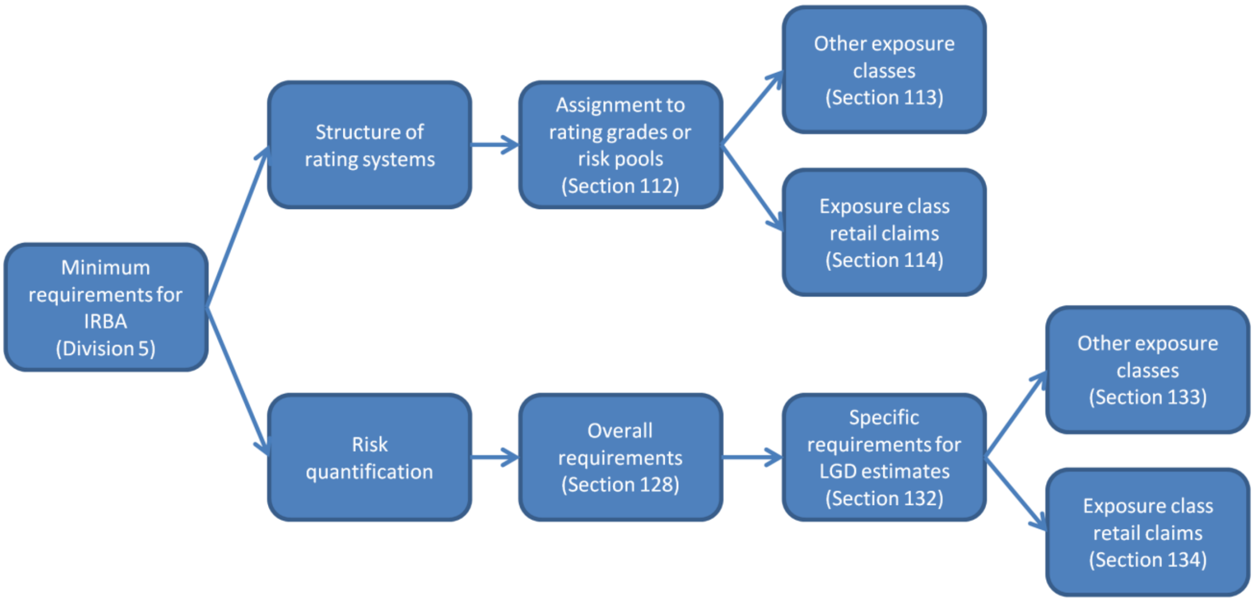


**Figure 10 – Overview of the requirements on LGD model that can be found in division 5 of the Solvency Ordinance**

#### *Structure of rating systems*

The risk quantification can be done for two different levels, for pools of contracts or for individual contracts. (Section 109) In case of individual contract the Solvency Ordinance treats these estimates as if they were outputs of grades on a continuous rating scale.

In case of pools of contracts the rating systems should make sure that obligors with similar risk are in the same category. Sections 112 until 115 describe the requirements on this process as well as how to deal with manual overrides of such a system. This is not relevant for the research because at Volkswagen Bank risk parameters are estimated at individual contract level.

#### *Risk quantification for retail portfolios*

Risk quantification refers to the estimation of the risk parameters such as LGD. Attention is only paid to the relevant parts that apply to LGD estimation for retail portfolios. The contents are summarized in table 7, sorted by subject.

Table 7 – Overview of requirements on the estimation of LGD from the Solvency Ordinance

| Estimation | Section |
|---|---|
| Risk parameter estimation<br>• on all relevant data, information and methods.<br>• use historical experience and empirical evidence not just judgmental conditions.<br>• plausible, intuitive and shall be based on material drivers of the respective risk parameters.<br>• less data means need to be more conservative | 128 (1) |
| The institution must be able to decompose its risk parameter estimates into factors it sees as drivers of the respective risk parameters | 128 (2) |
| Take into account any changes in practice, process. Reflect implications of technical advances and new data as soon as it becomes available. | 128 (3) |
| Add to estimation a margin of conservatism related to the expected range of estimation errors. | 128 (6) |
| Per grade or pool find LGD by average of realized LGDs by pool using all observed defaults within data sources (default-weighted average) | 132 (2) |
| LGD estimates for IRBA exposure class Retail claims may be derived from realized losses and appropriate estimates of PDs. | 134 (1) |

| Data | Section |
|---|---|
| Compare current exposures with data used to develop estimates. Number of credit exposures in sample and data period used for quantification shall be sufficient to provide the institution with confidence in accuracy and robustness. | 128 (4) |
| LGD estimates for IRBA exposure class Retail claims should be based on data over a minimum of two years. Each year this period is increased by one year until it covers a period of five years. Historical data does not need to be given the same weight if it can be demonstrated that more current data has more predictive power. | 134 (4) |

| Collateral | Section |
|---|---|
| Consider dependence between obligor and collateral or collateral provider. In case of dependence address conservative. | 132 (4) |
| Treat currency mismatch between underlying obligation and collateral conservatively in LGD assessment. | 132 (5) |
| In case collateral is used in LGD, estimate not solely based on collateral estimated market value. Should include effect of potential inability to efficiently gain control of their collateral and liquidate it. | 132 (6) |

| | |
|---|---|
| **In case collateral is used in LGD the institution must establish internal requirements for collateral management, legal certainty and risk management must meet minimum requirement for recognizing collateral.** | 132 (7) |

| **Downturn** | **Section** |
|---|---|
| **Use estimates that are appropriate for an economic downturn if these are more conservative than the long-run average. If need to deliver constant LGD over time, adjust estimates to limit capital impact of economic downturn.** | 132 (3) |

| **Best-estimate** | **Section** |
|---|---|
| **Defaulted exposures (ex section 125), institution shall use the sum of its best estimate EL for each exposure, given current economic circumstances and IRBA exposure status, and its estimate for the potential rise in EL owing to additional unexpected losses during the recovery period.** | 132 (9) |

## 2.6 Conclusion

Besides introducing important concepts in the research and the current LGD model, this chapter dealt with the first two research questions.

1. *What LGD model structures can be found in literature?*

LGD modeling means finding a model that minimizes differences between model predictions and realizations, in literature called a regression problem. The cure sub model is a binary classification problem meaning that it needs to discriminate between two events, in this case either cure or non-cure.

LGD modeling can be done on three basic ideas, based on financial models, expert models and empirical data. In this research a combination of expert and empirical data based modeling is used as this is in line with the requirements of the Solvency Ordinance.

Model structures can be classified as direct and indirect models based on the presence of sub models. Model structures found in literature are in order of increasing sophistication: look-up tables, linear regression models, advanced regression models and a variety of advanced models. Fit with data will increase with model sophistication so will the problem of over fitting.

2. *What regulatory requirements for LGD models are specified by the German Solvency Ordinance?*

The LGD model of Volkswagen Bank must comply with the German Solvency Ordinance. This ordinance gives broad requirements for LGD models. A model should be created using all relevant information and data, based on empirical evidence, plausible, intuitive and based on material drivers of LGD. It also poses requirements for data and for the inclusion of collateral in the model and demands that a downturn factor should be added to the model. A best estimate model should be created for contracts that already are in default.

# 3 LGD model back testing

*As part of the research a back test has to be carried out to assess the performance of the currently used LGD model. This will give an answer to the research question of how the current LGD model performs in terms of predictive power. Before this can be done attention will be paid to validation in relation to back testing. Performance criteria for predictive power will be set out as well as the accompanying statistics to measure them. The regulatory requirements for back testing are examined and when that all has been done it is possible to carry out a back test of the current LGD model.*

*The back test results will be used to answer the research question of how the current LGD model performs in terms of predictive power. This question forms part of the input for discussing the problems that can be identified in the current LGD model. This will be done in the final section of this chapter and this answers the next research question.*

## 3.1 LGD model validation

In a newsletter the Basel Committee states that validation is fundamentally about assessing the predictive ability of a bank's risk estimates and the use of ratings in credit processes. (Newsletter 4: Update on work of the Accord Implementation Group related to validation under the Basel II Framework, 2005) It is necessary that a banks' internal estimates are correct because under the internal ratings based approach these estimates form an important input for determining the required capital the bank needs to keep.

Validation is a topic that is still very much under development. Therefore formal regulation is not very prescriptive and it is important to look at other documentation. Especially publications by the Bank of International Settlements are relevant because these could form the basis of new regulation and set the standards banking supervisors will expect banks to use in the future. The Deutsche Bundesbank is also a place to look for information on validation.

As expected the Solvency Ordinance does not explicitly specify what constitutes validation and how validation has to be done. Below there is an overview of what the Ordinance does state about validation of estimates of risk parameters and therefore LGD.

**Table 8 – Overview requirements on validation posed by the Solvency Ordinance**

| Validation of own estimates | Section |
|---|---|
| **Obligation to have a robust system to validate the accuracy and consistency of rating systems and processes and estimating all relevant risk parameters. Demonstrate that internal validation process enables assessment of performance of internal systems.** | 147 (1) |
| **Regular comparison of realized risk parameters with own estimates, using historical data over a period as long as possible. Document methods and data. Update analysis and documentation at least annually. (Back testing)** | 147 (2) |
| **Use of other quantitative validation tools and comparison with relevant external data sources. (Benchmarking)** | 147 (3) |

| | |
|---|---|
| **Methods and data used for quantitative validation need to be consistent through time, document any changes** | 147 (4) |
| **Establish internal standards for situations where realized parameters differ significantly from own estimates. Standards need to take in account business cycles and other systematic variability in default experience.** | 147 (5) |

The Deutsche Bank does give an overview of what aspects there are to validation. (September 2003) This overview is quite comprehensive and includes most of the elements that are identified by other authors. Moreover it is in line with the requirements found in section 147 of the Solvency Ordinance.

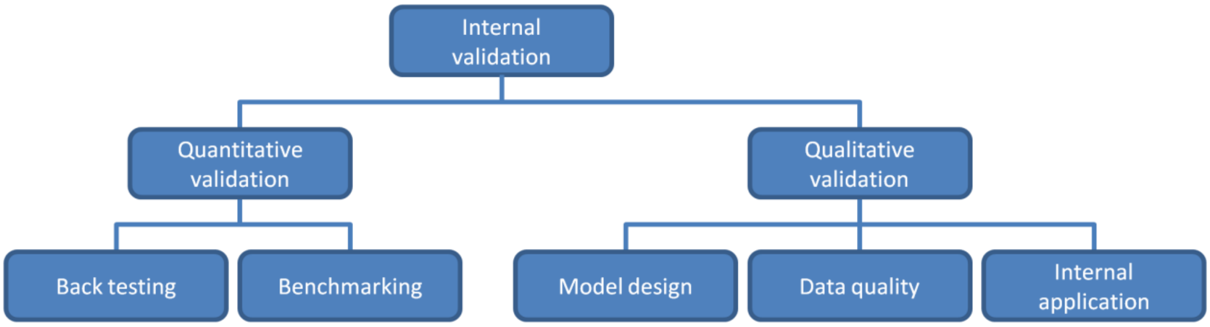


**Figure 11 – Aspects involved in validation based on the Deutsche Bank (2003)**

### 3.1.1 Quantitative validation

#### *Back testing*

Back testing means comparing the realized values for the risk parameters with the internal estimations. This gives some indication of the predictive power of the models over that particular time period. As a back test will be carried out so the next paragraph gives more details on back testing.

#### *Benchmarking*

Benchmarking is comparing the internal estimations with other relevant data. This could be data from rating agencies or other commercial providers. The usefulness of benchmarking depends very much on the choice of a suitable benchmark. (Deutsche Bundesbank, 2006, p. 64) For LGD it is particularly difficult to compare with relevant external data sources. Main problems are differences in default definition, biases in the external data sample and different measures of losses. (Basel Committee on Banking Supervision, 2005, p. 72) Even if the data was comparable banks are very reluctant to share it due to competitive considerations.

### 3.1.2 Qualitative validation

Qualitative methods are complementary to quantitative methods. Qualitative validation serves to make sure that quantitative methods are applicable and in some cases need to be completed before quantitative validation.

#### *Model design*

Before model design can be checked models must be documented transparently and thoroughly. The effect of the risk drivers must be identified separately. It must be assessed whether these effects are economically plausible. Model design is more important when data is limited.

#### *Data quality*

A correct and sufficiently large data history is a prerequisite for the development of any reliable model. Attention should be paid to data integrity and consistency.

#### *Internal application*

Internal application is also referred to as use test. This means evaluating the banks internal processes to see how the results of the models are used internally. If the bank uses the models in its own decision making this signals confidence in the models to the regulator.

## 3.2 Back testing

Back testing means comparing the realized values for the risk parameters with the internal estimations.

In this section performance criteria that assess the performance of a model will be discussed. Again it is important to pay attention to the differences between regression problems such as LGD models and binary classification problems such as PD models or the separate cure model in the current Volkswagen Bank LGD model. Both are discussed here because the currently used LGD model by Volkswagen Bank has sub models that include both types.

For the regression problems the performance criterion is goodness of fit. Van Gestel and Baesens (2009) identified for this criterion calibration and precision. This separation into calibration and precision will be used in this research.

Table 9 – Performance criteria and their application

| Performance criterion | Applicable to LGD models (regression problems) | Applicable to PD models (binary classification problems) |
|---|---|---|
| Calibration | ✓ | ✓ |
| Precision | ✓ | |
| Discriminatory power | | ✓ |

### 3.2.1 Model stability

One can view on stability is viewing stability as a property of a model. A stable model adequately models the cause-effect relationship between risk drivers and risk parameters. Therefore a stable model gives stable outcomes for the estimations for risk parameters as it is not affected by incidental correlations in the data set. (Deutsche Bundesbank, September 2003, p. 62)

There is however one conceptual problem with this idea of stability. We can never be sure that we have found a causal relationship between risk drivers and risk parameters. Therefore it is impossible to tell if a model is really stable or if this is caused by incidental correlation and making it impossible to determine stability. In practice a model is called stabile if it works well for a long period of time without many alterations. (Linker, 2009)

### 3.2.2 Data stability

Stability can be assessed for the data set. One can check if the contracts that are used to back test the model have similar characteristics to those used to develop the model. (Castermans, Martens, Van Gestel, Hamers, & Baesens, 2007, p. 7) This is important because it is very likely that a model will not work well if there are changes in the data set caused by changes in external conditions.

### 3.2.3 Calibration

Calibration measures the deviation between observed values and those predicted by the model. A model is well calibrated if this deviation is small. (Deutsche Bundesbank, September 2003, p. 62)

#### *Classification problem calibration*

In case of a classification problem calibration means checking whether the prediction matches the realizations. In the case of for example a PD model this means checking whether the number of predicted defaults corresponds to the number of realized defaults. (Van Gestel & Baesens, 2009, p. 270)

#### *Regression problem calibration*

This is part of determining goodness of fit such as described before. LGD calibration means verifying whether the predicted loss amounts correspond to the actual realized losses. (Van Gestel & Baesens, 2009, p. 270)

### 3.2.4 Precision

Precision is also part of goodness of fit. To asses precision for LGD models, one uses correlation measures between the a-priori estimates and the ex-post realizations. (Van Gestel & Baesens, 2009, p. 271)

In this report calibration and precision are determined separately to find the goodness of fit of a regression model. This also means that both do not measure entirely separate things but they are related.

### 3.2.5 Discriminatory power

The discriminatory power of for example a PD model is the capability to identify a-priori the occurrence of a default. (Deutsche Bundesbank, September 2003, p. 62) For the cure model this would be the ability to distinguish between a contract that will cure and one that will not.

## 3.3 Statistics for back testing

This section gives an overview of the statistics available for testing each performance criterion. This is partly based on the work of Castermans et al. (2007) who made an overview of statistics for PD back testing. For the statistics for LGD models Van Gestel and Baesens (2009) was used. To verify what statistics are used in practice an internal document by Van der Heijde (2008) was informative.

The table below is not meant to be comprehensive but to give an overview of statistics that are useful in practice for LGD and PD model back testing.

**Table 10 – Performance criteria and the statistics to measure them**

| Performance criterion | Statistic |
|---|---|
| **Data stability** | Stability Index<br>Chi-squared |
| **Calibration (binary classification)** | Binomial<br>Hosmer-Lemeshow |
| **Discriminatory power (binary classification)** | Coefficient of Concurrence<br>Gini |
| **Calibration (regression)** | Mean Squared Error<br>Mean Absolute Deviation<br>R-squared |
| **LGD precision (regression)** | Coefficient of correlation of rank ordering |

Per performance criterion the statistics are described in more detail in the appendix C. In this appendix attention will be paid how they can be calculated and how the results should be interpreted.

## 3.4 Data collection

*The previous sections dealt with identifying performance criteria and the accompanying statistics. Therefore it is now possible to start the actual back test of the current LGD model. In this section the data collection used for both the back test and the subsequent improvements will be described.*

### 3.4.1 Data source

The data set has been prepared by the data expert of the Risk Management department. The exact process has been described in detail in the LGD data preparation 2009 internal document. (Ekkel, 2009) This section will give a brief overview of the data collection relevant for this research.

Data for each contract is stored in the Escort information system. This system is quite comprehensive as it is used for both keeping records for business processes in the front office as well as accounting. Client data is entered at the moment an application is made for a contract. As a contract is accepted this system keeps track of payments.

All the data was extracted from those two systems. It was processed using Access 2003, using queries data was filtered and records were combined.

### 3.4.2 Data set

- The data set contains all defaulted financial lease and hire purchase contracts that have a default begin date between January 1st 2006 and December 31st 2008.
- Contracts have either cured before July 1st 2009 or have not cured. In this way contracts will have had sufficient time to cure if that was to happen. Therefore only ended contracts were selected as only for these contracts it is possible to determine the amount of loss with certainty.

### 3.4.3 Missing values and exclusions

In principle all contracts that have a default date between the dates specified have been included. There were however two considerations.

- Some contracts have a status that indicates they are still active. If this is the case they should not be included in the data set as the outcome of the contract and possible loss cannot be determined yet. This was the case for *** contracts and these have been excluded from the data set.
- *** contracts had missing values for the exposure at default. It was not possible to determine the exposure reliably in another way so these contracts have been excluded from the data set.

### 3.4.4 Data sets

#### *Total data set*

The total data set refers to the data set that includes all contracts with a default date in the specified period and are not excluded as described before. This data set includes *** contracts.

#### *Back test data set*

This set is a subset of the total dataset. It is limited to all contracts that have a default start date between January 1st 2008 and December 31st 2008. In this way there is no overlap with the contracts that have been used to develop the current model. In this dataset there are *** contracts.

### *Development set*

This data set contains 70% of all defaulted contracts of the total data set. The selection of contracts was done at random. This set will be used to develop the improved model and contains *** contracts

### *Validation set*

The remaining 30% of defaulted contracts that are not part of the development set. This set contains *** contracts and will be used to determine the performance of improved models on new data.

### *Best estimate LGD data set*

This data set contains all contracts of the total data set. Added are the exposure, loss and if the contract has cured at monthly intervals after the default date.

## 3.5 Back test results

*In this paragraph the results of the back test will be discussed. For each statistic mentioned here a more detailed description and interpretation can be found in appendix C.*

### 3.5.1 Data stability

The data stability was assessed for all the variables that are used in the current LGD model. The chi-square and stability index were determined for each variable. Sometimes a filter was applied to the variable before the test, because in the model that variable was only used on for a particular sub group of contracts as a result of the tree structure of the model. The two sets were only different on two variables, the composition in terms of hire purchase and financial lease contracts and the amount of contracts that are past the original end date.

- In the back test sets the fraction of hire purchase and financial lease contracts is significantly different from the modeling set. The fraction of financial lease contracts is higher in the back test set.

**Table 11 – Product types in the 2008 modeling set and back test set**

<<Figure confidential>>

- The fraction of contracts which are past the original end date has changed significantly. In the back test set this fraction is much smaller than in the development set.

**Table 12 – Contracts past original end date in the 2008 modeling set and back test set**

<<Figure confidential>>

- The other variables did not show significant differences between the 2008 modeling set and the back test set. The stability analysis for all variables can be found in appendix D.

### 3.5.2 Cure model calibration

To assess the calibration of the cure model the Hosmer-Lemeshow statistic was used. This statistic compares the expected and realized number of contracts in different pools. In this case the pools were the six different model predictions.

The Hosmer-Lemeshow statistic checks the calibration for different pools of exposures. The statistic has a chi-square distribution where the degrees of freedom is equal to the number of pools.

The statistic shows that for each pool there is a significant difference between the realized and the predicted number of recoveries. This means that the cure model is not calibrated well.

**Table 13 - H.L. Statistic for back test set**

<<Figure confidential>>

This is confirmed when the average probability of cure per pool is plotted. The lines for the predicted and realized values are quite far apart.

<<Figure confidential>>

Figure 12 – Average predicted probability of cure and average realized probability of cure for the 6 different categories of contracts. Each category belongs to a group of similar model predictions.

### 3.5.3 Cure model discriminatory power

The CoC statistic and Gini coefficient were determined to assess the discriminatory power of the cure model. Because the model gives the exact same prediction of the probability of cure for many different contracts, a method was used in which pairs of predictions and realizations were compared. This method is described in more detail in appendix C, section C.3.

The CoC statistic takes values between 0.5 and 1. The lowest value would be expected to result from a model which chooses randomly between predicting a cure and a non-cure.

Table 14 – Discriminatory power for the cure model

<<Figure confidential>>

The CoC value found in the back test is low. This means the model only slightly outperforms a purely random model.

### 3.5.4 Loss given non-cure model calibration

To assess the calibration of this model the differences between the prediction and realized values of loss given non-cure need to be determined. This is done using the mean squared error and the mean absolute deviation.

The R-squared statistic gives the amount of variance that is explained by the model. All values can be compared to those of a model that predicts the average LGN for each contract.

Table 15 - Calibration for the loss given non-cure model

<<Figure confidential>>

The LGN model performs very similar to a model that predicts the mean LGN for all cases. It performs slightly better on the MSE but slightly worse on the MAD statistic. At the time of development the model still outperformed the model that assumes mean LGN for all cases.

Also the R-squared is low indicating that the model explains little of the sample variance.

### 3.5.5 Loss given non-cure model precision

The precision of a model is measured by determining the correlation of rank ordering between the model estimates and the realizations. In this case it was done using the coefficient of correlation of the rank ordering.

Table 16 - Precision for the loss given non-cure model

<<Figure confidential>>

The correlation coefficient is quite low, indicating a low correlation between the model estimates and the realizations. This means that the precision of the loss given non-cure model is not very high.

### 3.5.6 Back test conclusion

- The back test data set is different from the model development set in terms of product composition. There are relatively more financial lease contracts in the back test set. Also there are fewer contracts that are past their original end date in the back test set. This will have had a negative effect on the predictive ability of the model on the back test set.
- The cure model is not calibrated well as there is a significant difference between the predicted and realized number of recoveries, which is shown by the Hosmer-Lemeshow statistic.
- The cure model does not discriminate well between contracts that will cure or not. It only slightly outperforms a random model as can be seen from the low CoC value.
- The loss given non-cure model is not calibrated well. It performs similar to a model that predicts the average LGN. Also the amount of variance explained by the model is low as the R-squared shows.
- The correlation between loss predictions and realizations is quite low. This means that the model scores reasonably low on precision.

## 3.6 Current LGD model problems

There are several problems with the current LGD model. To start there is the performance issue that was identified in the back test.

- Low predictive power of the current LGD model. The back test shows that the predictive power of the current LGD model is not very good. In fact this was also the problem with the previous model. The back test of the previous model showed very similar results as the current back test.

Besides statistical performance issue there are also other considerations.

- The current model has been created on a data set that contained an error. During this research an error was identified in the data set. Some contracts which showed a loss of almost ***% of the EAD had no actual loss. In these cases the relation with the client had been extended by signing a new contract. The remaining exposure was not booked on the new contract but was shown as a loss on the old contract.
  This error has large consequences for the LGD model, because now 4.4% of the contracts in the total dataset have a no loss instead of a very big loss.

- The model has several hard cut-off points between different categories. This is not a problem if the variable is dichotomous but can be hard to explain for a continuous variable. For example in the loss given non-cure model a small age difference of 1 year could mean very different predictions for the expected loss. In reality it is not likely that a small age difference would mean such a large change in loss expectation. To a lesser extend this holds the variables vintage of the contract, months remaining and loan to value.

- Loan to value is considered to be an important variable in the loss given non-cure model. The current model only checks if the loan to value is more or less than ***% and only for contracts which are past their original contractual lifetime. Volkswagen Bank management feels this variable could provide more information in a model if it is used in another way. Because loan to value is not used for all contracts in the loss model, the current model provides no incentive to steer for less risky contracts in terms of loan to value because there is hardly any effect on LGD and therefore capital requirements.

  To add if a stress test scenario is carried out in which collateral values drop, the current model will continue to give the same loss predictions despite significant changes in collateral values.

- There is doubt whether the choice of variables used in the model is optimal. (Ernst & Young, 2008) This is in line with the expectations of the Risk Management department which expects that other variables could be more useful in predicting LGD. Also because the error in the dataset described earlier there may also be different relations between the variables and LGD.

## 3.7 Conclusion

This chapter explored how the performance of LGD model can be measured. With this knowledge an actual back test was carried out and the results of this were presented. At the end an overview of the problems that were identified is given. In doing this the third and fourth research question are answered.

3. *How does the current Volkswagen Bank LGD model perform in terms of predictive power?*
   a. *How can the Volkswagen Bank LGD model be back tested as part of validation?*
   b. *What criteria and accompanying statistics can be found to test the performance of a LGD model according to literature?*
   c. *What regulatory requirements for back testing are specified by the German Solvency Ordinance?*

Validation is a broad assessment of the capabilities of a risk model. This includes both quantitative and qualitative aspects, back testing forms together with benchmarking the quantitative part of validation. The Solvency Ordinance prescribes that back testing is carried out regularly to compare the realized and predicted values.

The following table gives a summary of the performance criteria and useful accompanying statistics to measure them:

**Table 17 - Performance criteria and the accompanying statistics**

| Problem type | Performance criterion | Statistic |
|---|---|---|
| **Binary classification** | Calibration | Hosmer-Lemeshow |
| | Discriminatory power | CoC, Gini |
| **Regression** | Calibration | MSE, MAD, R-squared |
| | Precision | Coefficient of correlation of rank order |

4. *What problems can be identified by the business and from a back test of the current Volkswagen Bank LGD model?*

The back test showed that the predictive power of the current LGD model is not very good and it has been developed on a dataset that contained an error. Further the model has hard cut-offs which are hard to explain intuitively. There is doubt whether or not the right variables have been chosen and the expectation is that loan to value should have a more important role.

# 4 Univariate analysis

*As the name of this chapter implies it will describe the results of the univariate analysis that was carried out asses the relations between the independent variables and the probability of cure or the loss given non-cure. The results of this univariate analysis forms part of the input for the next chapter in which the actual LGD model improvements are made by answering a sub question about what risk drivers could be included in the improved LGD model.*

<<Chapter confidential>>

# 5 Improvements

*This chapter deals with the most central part of this thesis: finding improvements for the LGD model. It has been written to give a good insight into how the improvements have been found. This is done by describing the preceding analysis and the choices made in the actual creation of the improved model .The model found in this way is also tested to see what its predictive performance is both on the data that has been used to develop it and new data.*

<<Chapter confidential>>

# 6 Other improvements

*To enable the actual implementation of the improved LGD model several related tasks need to be carried out. These are outside the scope of the original research objective of improving the LGD model and will therefore be treated in a separate chapter.*

<<Chapter confidential>>

# 7 Conclusions and recommendations

*This final chapter will present the conclusions and recommendations.*

<<Chapter confidential>>

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# Appendix A: Definitions

This appendix gives additional definitions and those treated in chapter 2 of this report.

## Best Estimate LGD (BELGD)

Expected loss for exposures that already have a default status given economic circumstances, exposure status and additional unexpected losses during the recovery period. (Solvency Ordinance, section 132 (9))

## Cure

A contract cures if it becomes a normal contract again because it recovers from arrears and the bank considerers the obligor is no longer considered unlikely to pay. (Linker & Van Baal, Report LGD model 2008 VW Bank, 2008, p. 11)

## Default

According to the Solvency Ordinance, Section 125, article 1 default has occurred for a particular obligor if any of the following two events has occurred:

- The institution has material reason to consider that the obligor is unlikely to pay its credit obligations in full to the institution or any group enterprise belonging to the group of institutions or financial holding group to which the institution belongs without recourse by the institution to actions such as realizing the security (if held)
- The obligor is past due more than 90 successive calendar days on any material part of its overall credit obligation to the institution or to a group enterprise belonging to the group of institutions or financial holding group to which the institution belongs.

Events that should be taken as indications of unlikeliness to pay are the following. These are set out against the default reasons identified by Volkswagen Bank. (Linker & Van Baal, 2008, p. 11)

| Solvency Ordinance | Volkswagen Bank Category |
|---|---|
| Value adjustment | |
| Sell at material loss | |
| Distressed restructure | Individual payment deadline granted, obligor in debt restructuring |
| Filed obligor bankruptcy | Bankruptcy obligor |
| Obligor in bankruptcy or similar | Confiscation by tax authority, bankruptcy obligor |
| Other | Untraceable, repossession , cancelled by VW Bank, decease of contractor, loss of an object, actual loss, migration obligor to foreign country, false information, embezzlement |

## Down turn factor

According to section 132 (3) the bank should use LGD estimates that are appropriate for an economic downturn if those are more conservative than the long-run average. Multiplying the LGD estimate with the downturn factor gives the LGD estimation for an economic downturn.

## Exposure at Default (EAD)

The amount outstanding at the moment of default.

### Loss Given Default (LGD)

Loss given default is the expected ratio of the loss on an exposure due to the default of a counterparty to the amount outstanding at default. (Solvency Ordinance, section 132 (1))

### Loan to value (LtV)

The ratio between the exposure and the value of the collateral. In case this is determined for a contract in default it is the ratio between the exposure at default and the theoretical value of the collateral.

### Loss

According to section 126 of the Solvency Ordinance loss means economic loss. This definition includes material direct and indirect costs associated with collecting.

### Risk driver

Independent variables in a model that models a certain risk parameter as a dependent variable. These risk drivers are presumed to have a causal effect on the risk parameter value. In the report these are also referred to as variables.

### Risk parameter

Parameters used in Internal Ratings Based approach, such as for example PD, LGD, CCF and EAD.

# Appendix B: LGD model developed in 2007

<<Appendix confidential>>

# Appendix C: Statistics for measuring performance explained

## C.1 Data stability

### *Stability index*

The stability index assesses the equivalence of two data sets. If these are similar it means that the data set is stabile and has not changed over time. In case of a back test there are the development data set and the back test data set. Data should be divided into m buckets. $D_i$ denotes the percentage of cases in bucket *i* in the development set, and $B_i$ denotes the percentage in bucket *i* in the back test set. The stability index is calculated using the formula below.

$$SI = \sum_{i=1}^{m} (D_i - B_i) \times ln\left(\frac{D_i}{B_i}\right)$$

Advantages are the ease of calculation and this statistic is easy to understand intuitively. Problem is defining when to conclude that both data sets are different and a significant shift has taken place. An arbitrary choice of values for the stability index for which to conclude that data sets are dissimilar has to be made. Commonly applied rules are (Castermans, Martens, Van Gestel, Hamers, & Baesens, 2007, p. 8):

- SI ≤ 0.1: no significant shift
- 0.1 ≤ SI ≤ 0.25: minor shift
- SI > 0.25: major shift

### *Chi-squared test*

The chi-square is a widely used non-parametric test. The statistic works very similar to the stability index. Again data is divided in m buckets. In case a back test is carried out $D_i$ denotes the percentage of cases in bucket *i* in the development set, and $B_i$ denotes the percentage in bucket *i* in the back test set.

$$\chi^2 = \sum_{i=1}^{m} \frac{(D_i - B_i)^2}{B_i}$$

(Cooper & Schindler, 2008, p. 484)

Advantage here is that the values for the statistic to conclude that both data sets are different are clearly statistically defined. Because the statistic has a Chi-square distribution with m-1 degrees of freedom, critical values for which to conclude dissimilarity of data sets can be found. If the value for the statistic found exceeds the critical value the hypothesis that both sets are similar should be rejected. This means the back test data set is not similar to the development set and a significant shift has taken place. Critical values can be lookup up from tables in which the degrees of freedom and probability are set out for the chi-square distribution.

A disadvantage is that the conclusion of the statistic could depend on the number of buckets used. (Castermans, Martens, Van Gestel, Hamers, & Baesens, 2007, p. 4) For different numbers of buckets the test could give different conclusion.

## C.2 Calibration for binary classification problem

Because these statistics are commonly used for PD models the examples given and explanation uses probability of default as well. However PD can be replaced by probability of cure and these can be used for testing the cure sub model of the LGD model.

### *Binomial test*

The binomial test assumes that defaults are binomially distributed with a probability equal to PD and the number of them equal to the number of contracts. It tests the probability that the actual number of defaults occurred under the assumption that the PD is correct. If the probability of occurrence of the realized number of defaults is very small than it should be concluded that the PD is not correct. (Deutsche Bundesbank, September 2003, p. 71)

*p* is the probability of default, *n* denotes the number of contracts and *m* is the number of defaults observed. The probability of exceeding that observed number of defaults is given by the following formula.

$$P(X > m) = \sum_{k=m}^{n} \frac{n!}{k!\,(n-k)!} \times p^k \times (1-p)^{n-k}$$

If this probability calculated is below a chosen significance level than it should be concluded that the model is not calibrated well.

### *Hosmer-Lemeshow*

The Hosmer-Lemeshow test checks the calibration for different pools of exposures. Suppose there are *k* pools, $n_i$ contracts in pool *i*. Predicted probability of default for pool *i* is $p_i$ and $d_i$ is the number of realized defaults in pool i. The statistic is (Linker & Van Baal, 2008, p. 19):

$$HL = \sum_{i=1}^{k} \frac{(n_i p_i - d_i)^2}{n_i p_i (1 - p_i)}$$

This statistic is chi-squared distribution with k degrees of freedom. If the value of the statistic exceeds the critical value it should be concluded that the model is not calibrated well. In that case the average probabilities of default per pool are significantly different for the predicted and realized set.

## C.3 Discriminatory power for binary classification problem

The explanation is done as if it was described for a PD model but it holds for a cure model as well. In terms of PD estimation and realization there are four possibilities:

| | Default | No default |
|---|---|---|
| **Default predicted** | Correct alarm | False alarm |
| **Non-default predicted** | False non-alarm | Correct non-alarm |

Ideally the correct alarm rate would be high while the false alarm rate is as low as possible. In that case the model has the best discriminatory power as it can distinguish very well between defaults and non-defaults.

### *ROC curve*

ROC curve is short for 'Receiver Operator Characteristics Curve'.  The ROC curve plots the false alarm rate against the hit rate, the correct alarm rate. It does so for all cut-off values. Cut-off values are values of the model outcome at which we decide between an alarm or no alarm.  About the values below the cut-off value we conclude that there is no alarm predicted by the model, the values higher than the cut-off value means an alarm. By changing the cut-off value the hit rate and false alarm rate vary. To get a complete view of those for all cut-off values it is possible to plot them. This is what is done in the ROC curve. The curve starts at the origin, and ends in the point (1,1).

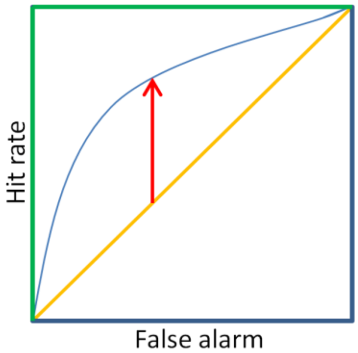


**Figure 13**

Ideally there would be no false alarms so that the model has perfect discriminatory power. This means an ROC curve with a right angle like the green line in the figure, where the false alarm rate is always 0% and the hit rate is 100%.

A model that has zero predictive power would have an ROC curve represented by a 45° straight line like the yellow line. This is what is expected to happen if the model randomly chose between an alarm or no alarm.

In practice a model will score between the random and the perfect model, the closer to the perfect model the better. (Christodoulakis & Satchell, 2008, p. 29) In this figure the blue curve gives ROC curve for a hypothetical model.

Sometimes it is convenient to aggregate the entire curve into one summary statistic. All of these are based on the idea that being closer to the perfect model means that the surface area under the ROC curve is larger.

### CoC

CoC or Coefficient of Concurrence is similar to the AUROC statistic. It expresses the area below the ROC curve as a fraction of the area under the perfect model graph. AUROC means Area under ROC curve. Both take an interval from 0.5 to 1, where the lowest value indicates zero discriminatory power and the highest perfect discriminatory power. However when comparing two models on this statistic it could be that a model with the highest CoC has a lower discriminatory power for a particular cut-off point. So an CoC value should be interpreted with some care. (Christodoulakis & Satchell, 2008, p. 31)

### Kolmogorov-Smirnov

It is also possible to look at classification error rate using the Kolmogorov-Smirnov statistic. This statistic measures the maximum distance between the ROC curve and the 45° straight line. (Deutsche Bundesbank, September 2003, p. 70) In the figure this is denoted by the red arrow. A larger distance means a steeper ROC curve. This indicates better discriminatory power.

### *CAP curve*

CAP curve is an abbreviation for Cumulative Accuracy Profiles curve. This curve is much like plot of a cumulative distribution function. It plots the fraction of all clients on the x-axis against the fraction of all defaults on the y-axis for all cut-off points. The curve starts at the origin, and ends in the point (1,1). As the fraction of cases increases the fraction of defaults should increase too if the model is working properly.

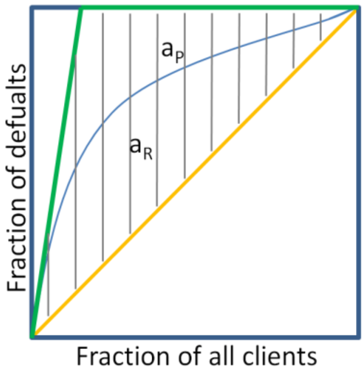


**Figure 14**

A model with perfect discriminatory power would identify all defaults first, so the fraction of defaults goes to 100% as fast as possible. This is represented in the figure by the green line.

A model with zero discriminatory power would be represented by a 45° straight line. In the figure this is the yellow line.

In practice the CAP curve of any model will be between the lines that represent perfect discriminatory power and zero discriminatory power. (Christodoulakis & Satchell, 2008, p. 31)

Again there is a statistic that aggregates the information from the CAP curve into a single summary statistic. This is based on the fact that a larger the area under the CAP curve means that the model has a better discriminatory power.

### Gini coefficient

The Gini coefficient is also referred to as accuracy ratio. It is calculated as the fraction of the area between the CAP curve and the 45° straight line over the area between the perfect model line and the 45° straight line. In the figure this means dividing the area marked by $A_R$ over the triangular area between the green and yellow lines.

This coefficient varies between 0 and 1, where 1 indicates perfect discriminatory power. (Satchell & Xia, 2008, p. 116)

### *Relationship between Gini coefficient and CoC*

It has been shown by Engelmann et al. (2003) that the Gini coefficient and AUROC statistics are essential equivalent. The Gini coefficient is a linear transformation of the AUROC. If one of them is known the other can be calculated using the formula below.

$$Gini = 2 \times (CoC - 0.5)$$

### *Dealing with identical estimates*

If multiple contracts are given the same estimate for PD or probability of cure this will create a "plateau" in the ROC or CAP curve. (Gupton, 2005, p. 78) This is a problem because in this case the curve depends on the arbitrary ordering of the contracts that have the same predicted value. Depending on the order of the contracts the curve will vary. This makes it impossible to use ROC curves or CAP curves for models that give many identical predicted values.

It is however possible to calculate a CoC value. This can be done by comparing pairs. A pair consisting of a prediction and a realization is compared with another pair. There were two groups of contracts made from which the pairs were compared. The first group contained the contracts that did cure, the second one contains those which did not.

If the pair with the highest prediction has also the highest realization this is concordance between those pairs. If this not the case the pairs are discordant. In case both predictions are the same this is seen as a tie regardless of the realization. (Linker, 2009) Suppose there are $n$ pairs, $n_c$ denotes the number of concordant pairs and $n_t$ the number of ties.

$$CoC = \frac{n_c + 0.5 \times n_t}{n}$$

In this way it is possible to determine a CoC value even if the model has identical estimations for different contracts.

## C.4 Calibration for regression problems

### *Mean Squared Error*

Mean squared error or MSE measures how much an estimate differs from the realized value. It does this evaluation for each contract. Suppose there are $n$ observations and $X_i$ denotes the estimated value for observation $i$. $Y_i$ gives the realized value of observation $i$. MSE is then calculated as follows:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(X_i - Y_i)^2$$

If the MSE is zero it means that the model has perfect accuracy. In that case the model estimates are exactly similar to the realization for each contract and there is no error. Generally higher values of MSE mean a worse model but the value in itself is meaningless. However the values can be used when comparing two models on the same data set. In that case the model with the lowest MSE is the best model. A good model to compare against is a model that assumes the mean for all predictions.

### *Mean Absolute Deviation*

The term mean absolute deviation or MAD can be used to refer to a measure of statistical dispersion. However is can also be used to assess how much estimates differ from the realized values very much like the MSE. Suppose there are *n* observations and $X_i$ denotes the estimated value for observation *i*. $Y_i$ gives the realized value of observation *i*. MAD is then calculated as follows:

$$MAD = \frac{1}{n}\sum_{i=1}^{n}|X_i - Y_i|$$

Again a MAD value of zero indicates perfect accuracy. The interpretation is similar to the MSE. The higher the MAD the worse is the model performance. Values are only meaningful in comparing different models on the same data set. Again for example a model that always predicts the average.

### *R-squared*

R-squared measures the amount of variance that is explained by a particular model. This gives an indication of the fit of the model on the data. It is defined as the fraction of the explained variance over the total variance.

Values can range from 0 to 1, where 0 indicates that the model does not explain any variance, while 1 indicates that all the variance is explained by the model.

## C.5 Precision for regression problems

LGD precision can be determined by looking at the correlation between ranks of the model estimates and the ranks of realized values.

Ranks are given ascending where the largest value gets the lowest rank. So the highest value in the set is given rank 1. In case of tied ranks the average rank is used for all values that are the same.

An example of this ranking is given below.

| Observation | Rank |
|---|---|
| 1 | 1 |
| 0,8 | 2,5 |
| 0,8 | 2,5 |
| 0,2 | 4 |
| 0,1 | 5 |

A statistic that can be used to determine the correlation between ranks of the model predictions and realizations is the coefficient of correlation. Assume *X* denotes the rank of estimated values and *Y* the rank of realized values. The coefficient of correlation is then found using the following formula. (Hull, 2007, p. 144)

$$\rho = \frac{E(XY) - E(X) \times E(Y)}{SD(X) \times SD(Y)}$$

The values of the correlation coefficient are in the interval between -1 and 1. Negative values denote negative correlation and positive values indicate positive correlation. If the precision of a LGD model is very good the coefficient of correlation approaches 1. Less precise models achieve lower values for the coefficient of correlation.

## Appendix D: Data stability back test

Taken from the excel file used to calculate the statistics for the back test: back_test_v9_macro_limited.xlsm

<<Appendix confidential>>

## Appendix E: Overview back test results

Taken from the excel file used to calculate the statistics for the back test: back_test_v9_macro_limited.xlsm

<<Appendix confidential>>

## Appendix F: Abbreviations of legal forms of counterparties in financial lease contracts

| Abbreviation | Name | Description |
|---|---|---|
| **BV** | Besloten Vennootschap | Limited liability private company |
| **CV** | Commanditaire Vennootschap | Limited partnership |
| **EZ** | Eenmanszaak | Sole trader |
| **FA** | Vennootschap Onder Firma | Partnership under common firm |
| **NP** | Natuurlijk Persoon | Natural person |

## Appendix G: Results univariate analysis probability cure

<<Appendix confidential>>

## Appendix H: Results univariate analysis loss given non-cure

<<Appendix confidential>>

## Appendix I: Default reason abbreviations

| Abbreviation | Full reason |
|---|---|
| **BSL** | Confiscation by tax authorities |
| **BTR** | Individual payment deadline granted |
| **COV** | Client untraceable, without known place of housing |
| **FAI** | Bankruptcy of client |
| **ING** | Collateral seized |
| **OPZ** | Cancellation by Volkswagen Bank |
| **OVC** | Death of client |
| **TLS** | Collateral total loss |
| **VBL** | Migration of client to foreign country |
| **VIG** | Client committed forgery |
| **VRL** | Loss of collateral, embezzlement |
| **WSP** | Client in debt restructuring |

## Appendix J: CHAID algorithm used by SPSS

Entirely based on the SPSS 15.0 helpfile

# Notation

The following notation is used throughout this appendix unless otherwise stated:

| | |
|---|---|
| Y | The dependent variable, or target variable. It can be ordinal categorical, nominal categorical or continuous. If Y is categorical with J classes, its class takes values in C = {1, …, J}. |
| Xm, m=1, …, M | The set of all predictor variables. A predictor can be ordinal categorical, nominal categorical or continuous. |
| ℏ={**x**n,yn} Nn=1 | The whole learning sample. |
| wn | The case weight associated with case n. |
| fn | The frequency weight associated with case n. Non-integral positive value is rounded to its nearest integer. |

# CHAID Algorithm

The following algorithm only accepts nominal or ordinal categorical predictors. When predictors are continuous, they are transformed into ordinal predictors before using the following algorithm.

### Merging

For each predictor variable X, merge non-significant categories. Each final category of X will result in one child node if X is used to split the node. The merging step also calculates the adjusted p-value that is to be used in the splitting step.

1. If X has 1 category only, stop and set the adjusted p-value to be 1.

2. If X has 2 categories, go to step 8.

3. Else, find the allowable pair of categories of X (an allowable pair of categories for ordinal predictor is two adjacent categories, and for nominal predictor is any two categories) that is least significantly different (i.e., most similar). The most similar pair is the pair whose test statistic gives the largest p-value with respect to the dependent variable Y.

4. For the pair having the largest p-value, check if its p-value is larger than a user-specified alpha-level α merge . If it does, this pair is merged into a single compound category. Then a new set of categories of X is formed. If it does not, then go to step 7.

5. (Optional) If the newly formed compound category consists of three or more original categories, then find the best binary split within the compound category which p-value is the smallest. Perform this binary split if its p-value is not larger than an alpha-level α split-merge .

6. Go to step 2.

7. (Optional) Any category having too few observations (as compared with a user-specified minimum segment size) is merged with the most similar other category as measured by the largest of the p-values.

8. The adjusted p-value is computed for the merged categories by applying Bonferroni adjustments.

### Splitting

The “best” split for each predictor is found in the merging step. The splitting step selects which predictor to be used to best split the node. Selection is accomplished by comparing the adjusted p-value associated with each predictor. The adjusted p-value is obtained in the merging step.

1. Select the predictor that has the smallest adjusted p-value (i.e., most significant).

2. If this adjusted p-value is less than or equal to a user-specified alpha-level α split , split the node using this predictor. Else, do not split and the node is considered as a terminal node.

### Stopping

The stopping step checks if the tree growing process should be stopped according to the following stopping rules.

1. If a node becomes pure; that is, all cases in a node have identical values of the dependent variable, the node will not be split.

2. If all cases in a node have identical values for each predictor, the node will not be split.

3. If the current tree depth reaches the user specified maximum tree depth limit value, the tree growing process will stop.

4. If the size of a node is less than the user-specified minimum node size value, the node will not be split.

5. If the split of a node results in a child node whose node size is less than the user-specified minimum child node size value, child nodes that have too few cases (as compared with this minimum) will merge with the most similar child node as measured by the largest of the p-values. However, if the resulting number of child nodes is 1, the node will not be split.

## Appendix K: Bootstrap method for LGD conservative margin

```
Sub Bootstrap()

    Dim Number_Portfolio As Double
    Dim Portfolio_Size As Double
    Dim Original_Number_Contracts As Double
    Dim i As Double
    Dim j As Double
    Dim x As Double
    Dim Sum_EAD As Double
    Dim Sum_LGD_EAD_realized As Double
    Dim Sum_LGD_EAD_model As Double

    Worksheets("Bootstrap").Range("F:H") = ""


   'Determine the number of contracts in the worksheet
    Original_Number_Contracts =
Worksheets("Bootstrap").Range("A:A").Cells.SpecialCells(xlCellTypeConstants).Count - 1

    'Determine size of simulated portfolio
    Portfolio_Size = WorksheetFunction.RoundUp(1 * Original_Number_Contracts, 0)

    'Determine number of replications
    Number_Portfolio = 1000


    For i = 1 To Number_Portfolio
    Sum_EAD = 0
    Sum_LGD_EAD_realized = 0
    Sum_LGD_EAD_model = 0

        For j = 1 To Portfolio_Size
        x = WorksheetFunction.RandBetween(1, Original_Number_Contracts)
        Sum_EAD = Sum_EAD + Worksheets("Bootstrap").Range("B1").Offset(x, 0).Value
        Sum_LGD_EAD_realized = Sum_LGD_EAD_realized +
(Worksheets("Bootstrap").Range("B1").Offset(x, 0).Value *
Worksheets("Bootstrap").Range("C1").Offset(x, 0).Value)
        Sum_LGD_EAD_model = Sum_LGD_EAD_model +
(Worksheets("Bootstrap").Range("B1").Offset(x, 0).Value *
Worksheets("Bootstrap").Range("D1").Offset(x, 0).Value)
        Next j

    Range("F1").Offset(i, 0) = i
    Range("G1").Offset(i, 0) = (Sum_LGD_EAD_realized / Sum_EAD)
    Range("H1").Offset(i, 0) = (Sum_LGD_EAD_model / Sum_EAD)

    Next i

End Sub
```

## Appendix L: Files used

<<Appendix confidential>>