On modelling the electricity futures curve

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Management summary

The electricity industry has changed considerably during the last decade. With new market participants entering the market of former monopolists and the introduction of derivatives, it has become increasingly important to develop accurate price models for these contracts, both for risk management and valuation purposes.

In this study we statistically analyze the German electricity future market. Research particularly aimed at modelling the EEX futures curve is still scarce, while innovative new approaches have been developed. This research creates an overview of the different approaches and selects a model that is the best candidate to model the German electricity futures curve. With the illiquidity of the EEX option market in mind, the following research question was formulated:

> Which price models are best suited to model the German electricity futures curve, taking into account our wish to have closed-form option pricing formulas?

Based on our study of the (German) electricity market and the performed data analysis on the EEX futures prices, we require good futures curve models to:

- Include seasonal patterns. Prices are higher for contracts delivering electricity during winter months.
- Allow the specification of a complex volatility structure. Volatility depends on the length of the delivery period, the time to delivery and the time of the delivery period. Accurately modelling this complex structure is critical, also for option pricing.
- Create a good fit to the initially observed futures curve.
- Produce analytical, closed-form option price formulas.

Based on these requirements, we consider the simplified direct swap model proposed by Benth & Koekebakker (2008) to be the best candidate. The main advantages of this model are that:

- The model does not rely on a non-explicit relation between spot and futures prices nor on smoothing algorithms to derive the futures price dynamics. Only observations of actually traded futures contracts are used.
- A perfect fit with initial futures curve is created.
- It is possible to specify the complex volatility structure and still have analytical, closed-form option pricing formulas.

Drawbacks of the approach are:

- Only market data of the futures contracts that cannot be decomposed into smaller contracts can be used (atomic swaps).
- We cannot infer the spot price dynamics from the swap price dynamics.
- We need a lognormal specification of the atomic swaps in order to have analytical, closed-form option pricing formulas. It is shown that this specification cannot fully capture the fat tails of the log-returns.

Acknowledgement

Little over a year ago, I had the opportunity to do a summer internship at the headquarters of Nuon N.V., one of the largest energy corporations in the Netherlands. With great help of my dear friend Rick Bitter, I was able to land a position at the (at least for me) most exciting department of the company; Energy Trade & Wholesale. During the two-and-a-half months I spend there I was able to experience the international, exciting and thriving business of trading electricity personally. I would like to thank Andreas Gelfort, Nanne Visser 't Hooft, Rick Bitter and Roderick Timmer for giving me the opportunity to experience and learn all about the practise of trading electricity in a relaxed, fun and instructive environment that inspired me to do this research.

I would also like to thank Reinoud Joosten. As my supervisor, he immediately warned me for the comprehensiveness and difficulty of this research, but I nevertheless went along and started the research, soon to find out that his warning words were justified. After several months of reading many articles, analyzing data and writing I came to the conclusion that the original research plan was too much work and perhaps too ambitious for a bachelor's thesis. I found understanding and cooperation from both my bachelor supervisor Reinoud Joosten and Arun Bagchi from the department of Mathematics at the University of Twente to continue the remainder of the original research plan for my master research. I would like to thank both of them for being flexible and thinking along and Reinoud Joosten in particular for the time and effort he has put in reading and evaluating this thesis. Laslty, I would like to thank Kolja Loebnitz for reading this thesis and giving useful comments to further improve this work.

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1 Introduction

1.1 Motivation

The electricity industry has undergone big structural changes over the last two decades. Traditionally, electricity companies were regulated or state-owned monopolies governing the generation, transmission, distribution and retail of electricity. In this regulated setting, power prices changed rarely and did so in a deterministic way.

With the 1989 Electricity Act, the United Kingdom was the first country in Europe to create a system of independent regulation, opening the electricity market by stages from 30% in 1990 to 100% in 1998. Many countries and regions followed and although the speed and scope of the reforms varies across geographical locations, all liberalization processes are based on the same underlying concept: the separation of the potentially competitive activities of generation and retail from the natural monopoly activities of transmission and distribution, as well as creating an electricity wholesale and retail market. As a result of this restructuring, prices are now set by the fundamental powers of supply and demand.

In these new, liberalized electricity markets, national and international parties have joined the formerly exclusive group of market participants, creating new risks as well as new opportunities for utility companies, distributors and consumers alike. Electricity wholesale markets are now the centres of an increasing amount of trading activity in spot contacts (short-term delivery of electricity). Because of the large price risk involved in trading spot contracts and the wish to hedge (price) risk in general, other contingent claims such as futures, forwards and options have been introduced to the electricity market. These contracts are traded bilateral, i.e., Over The Counter (OTC), and via organized energy exchanges such as the Nordic Power Exchange (Nordpool) in Scandinavia and the European Energy Exchange (EEX) in Germany. Due to the increasing trading activity, it has become increasingly important for all active trading parties to develop price models for the contracts they buy and sell, both for risk management and valuation purposes. The specific characteristics of electricity and electricity contracts, however, make that many well-known price models developed for both stock and fixed-income markets do not fit observed electricity prices very well. This has led to a lot of research activity in understanding the dynamics of electricity price processes and in developing price models that are able to describe these dynamics accurately.

Almost all scientific contributions in this research area so far focus on modelling the Scandinavian price curves because of the high liquidity, long price history and persistency. The German electricity market is less liquid, has a much shorter price history and lacks the persistence of the Scandinavian market and is therefore subject to a far smaller amount of theoretical studies. The studies on the German electricity market mainly focus on modelling the spot price by applying or modifying spot price models that were developed and calibrated using Scandinavian market data. Literature on the explicit modelling of the EEX futures is scarce; although there are a few studies on the pricing of derivatives based on spot price models (see Burger, Klar, Müller and Schindlmayr (2004)). It is interesting to take one step back and analyze the German (futures) market on its own. We will study the important features of the EEX futures curve and investigate how this curve can be modelled best.

1.2 Goal of this research

It is our ultimate goal to accurately model the German electricity futures curve. An important issue we have to take into account when evaluating the German electricity market is the lack of liquidity of the options traded at the German power exchange, the EEX. Option quotes are therefore not very reliable and ideally we would like to have accurate future price models that give analytical, closed-form option pricing formulas.

To do this, we need to investigate the (German) electricity market and its characteristics, examine the different approaches to electricity price modelling, map the existing electricity price models, perform the necessary data analysis and actual fit candidate price models to historical data and see which one performs best. Since this is too elaborate for a bachelor's thesis, we choose to split up the research into two parts. The first part (this report) focuses on the analysis of the German electricity market, the mathematics behind price modelling and the different approaches proposed to model electricity prices. We try to find the (classes of) models that are the best candidates to accurately describe the unique features of the German electricity market and allow for closed form option pricing. The second part of the research will be done in the near future and covers the actual fitting of the selected model(s) by estimating the model parameters using historical EEX price data and stochastic filtering techniques to find out which model performs best.

1.3 Research question & methodology

Taking into account the motivation and goal of this research, the following research question is chosen:

> Which price models are best suited to model the German electricity futures curve, taking into account our wish to have closed-form option pricing formulas?

From the short introduction it might already be clear that identifying these models is no easy task in the relative young, unique and continuously changing electricity industry. To be able to answer the formulated research question we start with an investigation of the (German) electricity market. We consider the important developments and discuss the implications of the liberalization processes to place this research into perspective. We discuss the important characteristics of the spot and futures market and identify the unique features of electricity and investigate how these features influence the dynamics of electricity price processes. Identifying these features and their influence gives us some direction when comparing different types of price models.

To see if and how the identified features influence the spot and futures price dynamics of the German electricity market, we analyze spot and futures price data from the German wholesale market, the European Energy Exchange (EEX). We identify the structure of German electricity price processes and identify the important factors we have to take into account when modelling German electricity price processes.

In order to understand, analyze and develop price models we present an overview of the development of price modelling for more mature markets such as stocks and interest rates since many of the proposed commodity price models are (modified) versions of models proposed for these mature markets. By also presenting a theoretical study of derivative pricing and discussing the relation between spot and futures prices we will have the mathematical baggage needed to weigh all the scientific contributions that have been made to this research area over the past decade.

Lastly, we will discuss several different types of electricity price models that have been proposed over the past decade. With the background knowledge of the first section, the identification of the important factors and features of the German electricity market and the overview of mathematical concepts and pricing mechanisms we will be able to select the model that is most likely to accurately model the futures curve and produces closed-form option pricing formulas.

1.4 Outline of the paper

In Section 2.1 we start with a description of the main activities of the electricity industry and discuss the liberalization processes across Europe and Germany in particular in Section 2.2. The unique features of electricity and their relation to price process behaviour are pointed out in Section 2.3. We end with the structure and products traded at power exchanges in Section 2.4.

The empirical data analysis of the German electricity market will be performed in Section 3. With data from the European Energy Exchange (EEX) we will analyze the German spot (Section 3.1) and futures market (Section 3.2). In Section 3.3 we summarize our main findings.

Section 4.1 then will introduce several approaches to the mathematical modelling of price processes. The developments of stochastic modelling in the stock and fixed income market will subsequently be discussed in Section 4.2. Because we are looking for models that produce analytical, closed-form option pricing formulas, we provide the mathematical theory behind derivative pricing in Section 4.3. Section 4.4, lastly, discusses classical theory on the relation between spot and futures prices and we will see if these theories are expected to hold on the electricity market.

With all the theory and analysis from Sections 2, 3 and 4 we discuss several electricity price models in Section 5. Two approaches prevail for the modelling of the electricity futures curve; the spot approach (Section 5.1) and the futures approach (Section 5.2). The (dis)-advantages for both approaches will be discussed.

In Section 6 we will present the conclusions of this research.

2 The electricity industry

Electricity is an integral part of life in our modern society and, as such, we heavily rely on the availability of this commodity whenever and wherever we want to. As our society develops further, the secure supply of environmentally friendly generated electricity at competitive prices becomes more and more important. This section will provide a basic overview of the electricity industry, with a special focus on Germany. We will start with a brief discussion of the activities that make up the electricity industry: generation, transmission, distribution and supply of electricity. Then the liberalization processes that radically transformed the structure of the industry over the last decades will be discussed, followed by a discussion of the unique characteristics of electricity and the consequences for electricity price processes. We end this section with the structure of trading at power exchanges.

2.1 From energy to electricity

Despite the enormous changes to the world since the first power station started generating and transporting electricity around 1890, the electricity industry is still about the same thing: transforming primary energy sources into electricity and transporting and supplying it to consumers. This path can be divided into four activities: generation, transmission, distribution and supply. Figure 2.1 schematically shows how the first three activities are linked.

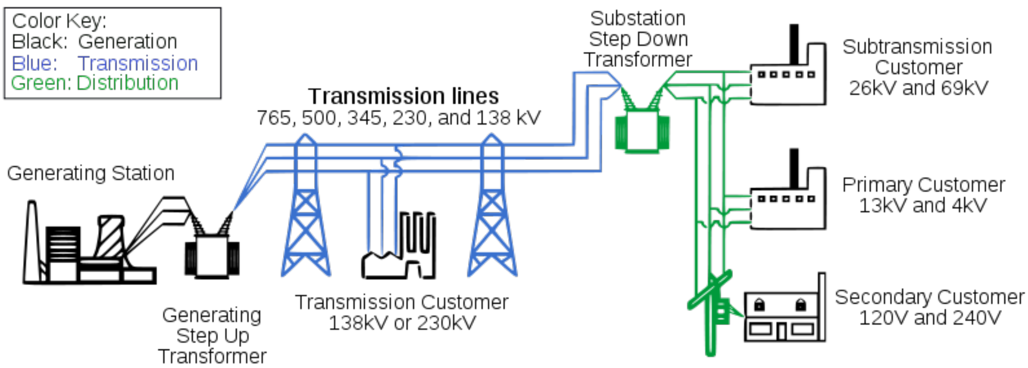

Fig 2.1: Generation, transmission and distribution of electricity.

2.1.1 Generation

The generation process consists of turning non-electrical energy sources into electricity, executed by power stations with large electromechanical generators. Heat engines that primarily drive these generators are fuelled by chemical combustion, nuclear fission, the kinetic energy of flowing water and a growing number of other technologies. Conventional fuels such as fossil fuels (coal, gas and oil), nuclear fission and the kinetic energy of flowing water are still the most used fuels in today's power plants, but technological developments and growing concern over environmental impacts have led to the introduction of other natural sources as generator fuel. Examples are wind and solar power, wood, geothermal heat, biomass and waste.

Germany is a true giant looking at electricity generation and consumption in Europe. With generated electricity amounting to 640,000 GWh in 2008 and an installed generation capacity that almost doubles that figure, it leaves all other European countries behind.

Traditionally, Germany heavily relies on hard coal and lignite (brown coal) as fuel for their power production. These two sources together supplied almost 44% of the energy for power production in 2008. Despite lignite being a low-cost and domestic resource, environmental issues place limits on further expansion. With domestic production declining for the past several decades, Germany is increasingly relying on the import of hard coal for the supply of solid fuels.

Another important source for the generation of electricity is nuclear energy. Nuclear energy as a source of fuel has lost market share since 2000, but in 2008 still amounted to 23,3%. The declining market share is mainly due to legislation aimed at gradually phasing out nuclear plants.

The fourth largest energy supplier for power generation is natural gas accounting for about 13,0% in 2008.

The most remarkable transition in Germany is the rising production share of wind power. With the highest installed wind capacity in the world, wind turbines represented 13% of total installed capacity in 2004. Because wind power is primarily used for peak demand, it 'only' represented 6,5% of total electricity generated in 2008. The development of wind power in Germany, however, is very important for the market development globally.

Because the output of wind turbines depends on the weather, wind power is an intermittent source of electricity. The increasing share of wind power in the source of supply mix has some implications on grid management and provided a reason for the development of other, non-intermittent, but still renewable, sources such as biomass, solar power and geothermal heat. The development of these new renewable sources of electricity in Germany is the most advanced in Europe due to favourable legislation. Germany achieved its goal of advancing renewable energy and together with wind power, these renewable energy sources accounted for 14% of total electricity generated in 2008.

Figure 2.2 shows German electricity generation by energy source in 1990 and 2008.

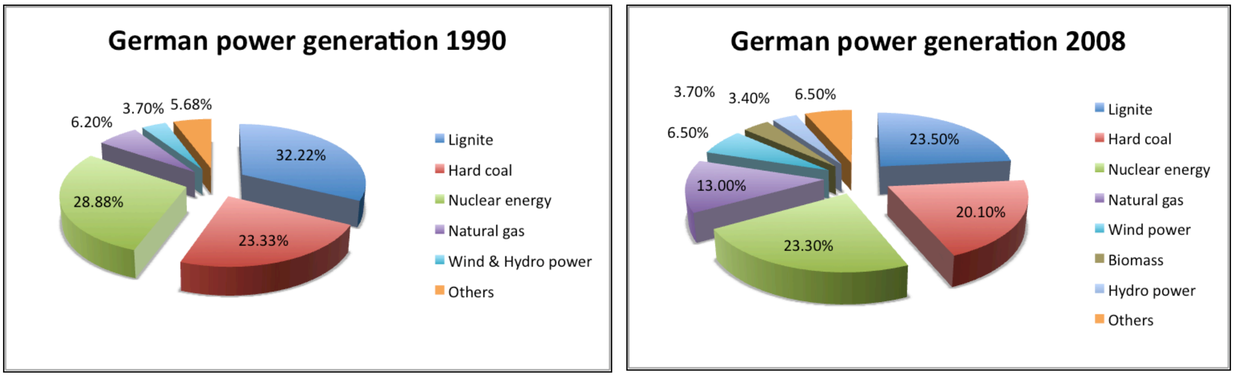

Fig 2.2: German power generation by energy source in 1990 and 2008. Source: BMWI, ENS.

2.1.2 Transmission

Electric power transmission is the transfer of electricity. Via an elaborate network of power lines, electricity from generation plants is transported at high voltage (to cover long distances and reduce power losses) to substations near populated areas. The network of power lines is often referred to as the transmission network or "transmission grid". Because it is practically impossible to store electricity in large quantities, the transmission network must be very reliable. By using multiple redundant lines between points on the network, power can be routed through a large variety of routes based on economical factors such as the economics of the transmission path and non-economical factors such as power line breaks. This assures the immediate, safe and stable transport of sufficient quantities of power to the substations at minimal power loss.

When it comes to the safe and secure supply of electricity, the German high voltage transmission grid is one the best around. With over 1.7 million kilometres, the grid is Europe's largest and most reliable, resulting in the shortest length of outages resulting from breakdowns (see Figure 2.3). Because of the sheer size of the transmission system, Germany is divided into four safety cells, or balancing zones. In this setting, a major blackout would turn off the lights in only one of the four zones. Within each balancing zone, a system operator is responsible for the stability of the system by keeping the electricity frequency and voltage constant, maintaining safety limits, expanding the network and balancing the continuously changing supply and demand. In Germany, these system operators are without exception subsidiaries of the large utility companies currently active in Germany. Figure 2.4 shows the four balancing zones.

Within Europe the transmission grids are more and more connected together in the so-called international grid. The international connection increases profitability of all generators connected to it and further improves the security of supply. Being centrally located, Germany's high voltage grid plays an important role as a transit grid for cross-border exchange of electricity.

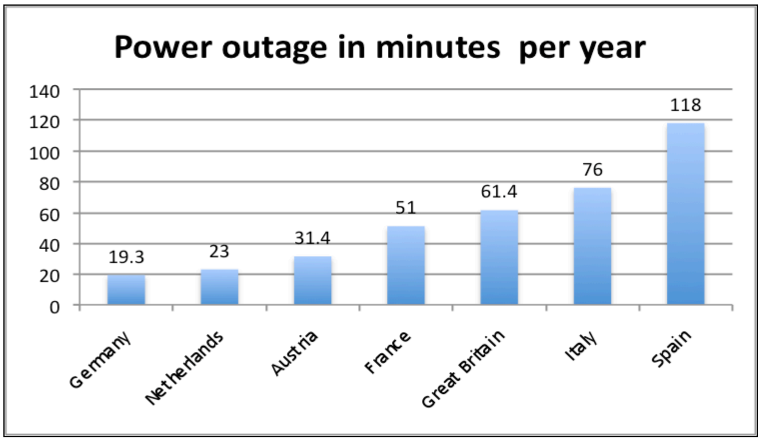

Fig 2.3: Power outage in minutes per year due to breakdowns across Europe.
Source: 'Facts about the German electricity grid', Vattenfall 04-2009[1].

Fig 2.4: Balancing zones in Germany after liberalization.
Source: 'Facts about the German electricity grid', Vattenfall 04-2009[1].

[1] See www.vattenfall.com/germangrid/downloads/Electricity-grid-facts.pdf.

2.1.3 Distribution

Electricity distribution is the final physical stage of delivering electricity to consumers. The distribution grid consists of substation transformers, medium- and low-voltage power lines, and low-voltage transformers distributing electricity regionally. At the substations near populated areas, transformers decrease the voltage of the transferred power to lower levels and via medium- and low-voltage power lines the electricity is transported to low-voltage transformers after which it is transported to the consumer.

Due to the high costs of developing, maintaining and operating transmission and distribution networks, the transmission and distribution network are generally considered to be natural monopolies.[2]

2.1.4 Retail

The fourth and last activity in the sequence is the retail of electricity to consumers. In the traditional electricity industry, one monopoly entity owns and operates the entire chain of activities, resulting in consumers having no choice of supplier. With the liberalization of the electricity industry, which will be discussed in the next section, competition is encouraged and companies compete to win their market share and remain in business. In this setting, consumers can choose the electricity provider of their liking.

2.2 Liberalization

The 'traditional' electricity industry we knew for nearly a century has undergone major structural and operational changes over the last two decades. Politicians, economists and industry specialists have abandoned the general consensus that generation, transmission and distribution of electricity are best handled centrally and argue that free and competitive markets are more efficient at delivering electricity to consumers. In this subsection we will shortly describe the 'traditional' electricity industry after which we will discuss the ongoing liberalization processes across Europe and the implications thereof in more detail. We end with a discussion of the liberalization process in Germany.

2.2.1 The traditional electricity industry

In the days of Thomas Edison and his first commercial power plant delivering Direct Current to customers in lower Manhattan, small companies generated and delivered electricity in municipal areas to customers who were willing to pay the price. With the introduction of Alternating Current in 1896, it was possible to transport electricity over long distances. This system was efficient, fast and more reliable than Direct Current and it quickly became the standard. During the first decades of the 20th century more and more people were connected to the electricity network and it gradually became a national strategic asset, like coal and oil.

Besides the strategic nature of electricity and the natural monopoly aspects of transmission and distribution networks, economies of scale at generation plants and the technical challenge of coordinating generation and transmission activities were the main reasons for vertically integrating the industry. As a result of the vertical integration prices had to be controlled to protect consumers. In the case of privately owned utility companies, as was the case in the United States and Canada, the government enforced economic regulation in order to prevent the companies of exercising their monopoly power. Regulation aimed at setting reasonable prices, monitor costs, ensuring quality, securing supply and offering incentives to be efficient. In Europe most utility companies were state-owned monopolies. Either way, we see that governments

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2 A natural monopoly exists in a particular market if a single firm can serve that market at lower cost than any combination of two or more firms. Definition from the Organization for Economic Co-operation and Development (OECD), www.OECD.org

played a huge role in the development, structure and operation of the traditional electricity industry. Vertically integrated companies own and operate all of the nation's generation facilities as well as the transmission and distribution networks. Consumers have no choice of supplier and prices are state regulated, rarely change and do so in a deterministic way.

2.2.2 Transformation of the traditional industry

Over the last decades it has become increasingly difficult for politicians, economists and industry specialist to uphold the view that a system of one vertically integrated company is the only way to structure the electricity market. Successful deregulation and privatisation of other infrastructural industries, increasing plant efficiency and especially the realization that the potential competitive activities of generation and supply can be separated from the transmission and distribution activities led to the belief that a free, competitive market could also be beneficial to the electricity industry. It is believed that liberalization of the electricity sector should make the industry more responsive to changes in business and technology, attract private investment, promote technical growth, lead to lower electricity prices, increase efficiency and improve customer satisfaction as different parties compete with each other to win their market share and remain in business (Bajpaj & Singh, 2004). In the liberalized electricity sector prices are thus set by the fundamental powers of supply and demand.

The 1989 Electricity Act issued by the government of the United Kingdom, opening up the electricity market in stages towards 100% in 1998, marked the beginning of the ongoing liberalization processes across Europe. In December 1996 the European Union adopted the 96/92/EC directive 'concerning common rules for the internal market in electricity', which was expired by the 2003/54/EC directive. With these directives the wish to create an internal electricity market that guarantees the best conceivable level of competition and free choice of supplier for consumers was formalised. The main requirements of these directives were the following.

- **Unbundling of activities**
  At vertically integrated companies, the potential competitive activities of generation and supply have to be both managerially and legally separated from the transmission and distribution activities to avoid discrimination and competition-distortion. The transmission and distribution network thus remain natural monopolies.

- **Opening up generation and supply activities for competition**
  The activities of generation of electricity and the supply of electricity to consumers should be opened up for competition. At the wholesale level, new market participants must be able to enter the market to buy and sell electricity.

- **Non-discriminatory third party access to network**
  Non-discriminatory rules on access to the transmission and distribution network of third parties have to be created.

- **Instalment regulating authority**
  An independent and competent national authority charged with settling disputes on contracts and negotiations has to be installed.

Although the directives require the generation and supply activities to be legally and managerially separated from transmission and distribution activities (legal unbundling), it is not (yet) required to house these activities in separate entities (ownership unbundling). The opening up of generation and supply activities for competition created the need for organized market places where time varying (stochastic) demand and supply are continuously balanced. With the existence of such a market place and non-discriminatory access to transmission and distribution networks, even companies with no generation assets whatsoever can become electricity providers.

The need for organized market places to trade electricity led to the emergence of two main types of markets: power pools and power exchanges. Power pools refer to public initiatives where participation is mandatory; there is no trading activity except for trading via the power pool. Power exchanges, on the contrary, are private initiatives to create an efficient market and participation is voluntary and a bilateral market usually coexists together with the exchange. Power exchanges are nowadays preferred over power pools because they are efficient in matching supply and demand at the lowest prices without compromising the reliability of the system. Two of Europe's largest electricity markets, Scandinavia (Nordpool) and Germany (EEX), are organized by power exchanges. Due to the specific characteristics of electricity we see that power exchanges usually consist of several submarkets. These specific characteristics and the different submarkets of power exchanges will be discussed in the next two sections, respectively.

Other issues identified by the directives concern tariffication, market power in electricity generation, environmental protection, security of supply, generating capacity and different degrees of market opening between Member States.

The directives should, in theory, prevent vertically integrated companies from using control of their transmission network to reduce competition. The level of unbundling currently required by the directives (legal unbundling), however, leaves the incentives for curbing competition intact. Apart from the refusal of third parties accessing the network, generating companies that also operate the network have additional tactics such as charging unreasonably high access and service fees and discriminatory access requirements to hinder access of competing generators. The fact that many vertically integrated companies opted for legal unbundling instead of ownership unbundling raises the question whether or not these companies were able to manipulate the legislative and regulatory process in favour of this weaker form of unbundling.

Although Member States gradually transferred the directives into national law in the last decade, it thus remains questionable if it has led to competitive electricity markets. Set up by the European Commission in 2003, the European Regulators Group for Electricity and Gas (ERGEG) is charged with advising and assisting the European Commission on the creation and smooth functioning of internal energy markets. In the organization's annual report of 2008[3] it is stated that the biggest result of opening up the electricity sector is the consumers' right to choose their supplier. Competition in retail of gas and electricity, however, is almost non-existent and insufficient unbundling is still a major obstacle for competition and security of supply.

It is clear that there is still al long way to go and that a lot of effort from both European and national regulators is needed to address the key issues to grow towards an internal energy market. The process has not ended with directive 2003/54/EC and the European Commission urges stricter measures concerning the unbundling of activities, consumer protection and national authorities.

2.2.3 Liberalization process in Germany

With the adoption of the 'Energiewirtschaftgesetz von 1998', or National Energy Act 1998, Germany transferred the 1996/92/EC directive of the European Union into national law. With this act the electricity market in Germany was seamless and completely liberalized. In this section we discuss the traditional and liberalized German electricity industry.

2.2.3.1 Traditional German electricity industry

Traditionally, Germany was one of only a few countries in Western Europe without a broad government monopoly in the electricity sector. The German electricity sector could be characterized as a mixture of public, private and mixed economy companies. Within a framework of territorial monopolies electricity supply companies were solely active in their own 'franchise' area. German supply companies are still classified according to the size of the area in which they are active.

Large network energy supply companies operate on a supra-regional level. Before 1998, eight network energy supply companies generated about 79% of total generated electricity. These eight companies also operated transmission networks and five of them (RWE, VEW, EnBW, HEW and BEWAG) were large vertically integrated companies. The remaining three

[3] www.energy-regulators.eu

(PreussenElektra AG, VEAG and Bayerwerk AG) were solely active in the generation and transmission business.

'Regional energy supply companies' operate on the regional level. Before 1998 around 80 regional companies were responsible for 10% of the electricity generated in the country. These companies delivered electricity from the large network supply companies and their own production facilities to end consumers, and acted as distributor for smaller 'municipal utilities'.

These 'municipal utilities' supply end consumers in their respective municipalities with electricity, gas and water. In 1997 around 900 municipal utilities generated 11% of the total generated electricity.

2.2.3.2 Liberalized German electricity industry

The National Energy Act 1998 made an end to almost a century of territorial monopolies in the German electricity sector. By implementing a complete and immediate liberalization for industrial consumers and households, the German government went beyond the requirements of the 1996/92/EC directive. Upon this new legislation, large network supply companies reacted immediately with a massive wave of mergers, starting already in 1997 with the merger of 'Badenwerk' and 'Energieversorgung Schwaben' into the new network supply company EnBW. These mergers traverse the European Union's objective to create a market with the best possible level of competition. The EU, however, did not hinder the market concentration due to the mergers and acquisitions whatsoever, mainly focusing on the legal unbundling of activities. Table 2.1 presents an overview of the number of utility companies active in Germany and respective market shares before and after the process of liberalization.

Since 1997, the number of network energy supply companies decreased from eight companies with a market share of 79% of total generated electricity to only four companies with a massive 95.2% market share in 2004 due to the mergers described earlier. These mergers also implied the concentration of the number of system operators operating the transmission network from eight in 1997 to four in 2004. Although less dramatic, the number of companies active in the retail of electricity to consumers also decreased, whereas the market share of the four largest retailers increased to 72.8% in 2004. All in all we thus see that the German electricity industry exhibits significant market concentration after the process of liberalization was started.

Even though the liberalization of the German electricity market started about ten years ago, German power consumers continue to face high electricity prices. One of the possible reasons has to do with the abuse of market power, which for the remainder of this paper is defined as the ability to profitably shift up electricity prices above competitive levels. The German federal cartel office recently announced a forthcoming investigation into electricity producers and the wholesale market. They will be trying to work out why electricity prices remain high and in some cases are even rising, even though oil, gas and coal prices have fallen sharply. It is suspected that prices are kept artificially high by shutting down power plants to limit the supply of electricity, but this will be hard to prove.

A second possible explanation for the high electricity price is the lack of independent system operators. The same network supply companies generating 95% of the total generated electricity operate the entire transmission grid. It is argued that this gives them a huge advantage over independent producers, who struggle to gain fair access to the network and do not have the information about supply and demand across the grid.

As a possible third reason for high prices some name the inordinate political power of the big utility companies, e.g. by appointing former ministers to paid advisory or supervisory boards.

All in all we see that the German electricity market is far from being a free, competitive market. Perhaps change is coming with the investigation of the German federal cartel office and the European Commission forcing utility companies to unbundle even further by using their antitrust powers.

	Before the process of liberalisation	After the process of liberalisation	
	1997	1999	2004
Generation (not capacity)	- 8 network energy supply companies with 79% of electricity production: RWE, VEW, EnBW, BEWAG, HEW, PreussenElektra AG, BayerwerkAG, VEAG - Others with 21%: - regional energy supply companies with 10%; - municipal utilities with 11%	- 6 network energy supply companies with 73.8%: RWE: 28.4%; E.ON: 24.7% EnBW: 7.2% VEAG: 8.9% HEW: 2.6% Bewag: 2.1 % - Others with 26.2%: - municipal utility - regional producers - new local producers	- 4 network energy supply companies with 95.6%: RWE: 38.7% E.ON: 26.5% EnBW: 13.8% Vattenfall Europe: 16.2% - Others with 4.4 %: - municipal utility - regional producers - new local producers
Transmission	- 8 network energy supply companies with 100% in their territories	- 100% share by 6 network energy supply companies	- 100% share by 4 network energy supply companies
Distribution (low voltage power supply)	- 80 regional energy supply companies - 900 municipal utilities	- regional energy supply companies - municipal utilities	- 50 regional energy supply companies - 700 municipal utilities
Sales to end consumers	- 5 network energy supply companies (RWE, VEW, EnBW, BEWAG, HEW) are also active in sales (51%-59% in 1995 (!) including by capital shares concerning regional energy supply companies) - 80 regional energy supply companies - 900 municipal utilities	- 6 network energy supply companies with 61.6%: RWE: 29.1% E.ON: 18.5% EnBW: 6.1% VEAG: no data HEW: 4.8% Bewag: 3.1% - Others with 38.4% - municipal utility - regional producers	- 4 companies with 72.8%: RWE: 16.8% E.ON: 22.1% EnBW: 19.5% Vattenfall Europe: 14.4% - Others with 27.2 % - 700 municipal utilities - regional producers

Table 2.1: Market structure Germany before and after liberalization (Brandt, 2006).

2.3 Electricity: A unique commodity

Besides the homogenous nature of commodities, the main difference between commodities and pure financial products like stocks, bonds and other marketable securities is that commodities actually represent physical consumption goods. Because electricity is homogenous and is used for consumption, it is generally considered a commodity. There are, however, some specific characteristics that distinguish electricity from other commodities, even from the related commodities of natural gas and oil. In this subsection we will discuss the main characteristics of electricity and see how these characteristics relate to empirical observations of historical price data.

2.3.1 Non- storability

One of the unique features of electricity when we compare it with other commodities is its non-tangible nature. It is not a material element of nature itself, but rather an engineered product through the use of material elements. Electricity is often described as the delivery of conducted energy to consumers for direct consumption. This 'direct consumption' reveals one of the most unique and influential properties of electricity: non-storability.

At this moment in time we are not able to store large quantities of electricity in an economically efficient way. The only fairly efficient storage possibilities are hydro storage basins, although the storage capacity is small compared to the aggregate demand of electricity. This lack of efficient storage possibilities leads to less flexibility in operating the electricity market since electricity must be produced at the same rate as consumed to avoid a network collapse. In electricity markets, sudden shocks in the demand or supply of electricity due to unscheduled plant outages, changing weather conditions or other unforeseen events can therefore not be balanced by stored electricity or by cutting consumption. Because of the unpredictable nature of the occurrence of these events, the delicate balance between supply and demand can suddenly be disturbed. Although there are some generation plants that can quickly ramp production up or down to balance supply with demand, it is far more (cost) efficient for most plants to operate on a fixed level instead of being switched on and off regularly. It is thus fair to say that the supply of electricity is inelastic to changes in demand, especially at peak levels. On the other side, electricity demand is price inelastic. Consumers don't think about using electricity and because of the lack of real-time metering consumers cannot react to high prices by cutting consumption. Shocks to the system therefore have a large impact on electricity prices, especially on day-ahead spot markets (delivery of electricity the next day).

As a result of the lack of flexibility, large price spikes are observed in the spot market. Price spikes are a sudden steep increase of prices, almost directly followed by a steep decline to some average level. Figures 2.5 and 2.6 present the hourly spot prices and the base load spot price (daily average of the 24 hourly spot prices) at the European Energy Exchange (EEX) respectively. The spiky behaviour (both negative and positive) is clearly visible in both figures, but is more pronounced within the hourly spot data (note the difference in the y-axis scale for the two figures!). This is as expected because the base load spot price data is the average of the hourly data and is therefore smoothed. At 07/11/2006 we can see an example of an extreme hourly price spike: the price for one megawatt, delivered the next day between 18.00h and 19.00h, skyrocketed to € 2,436.

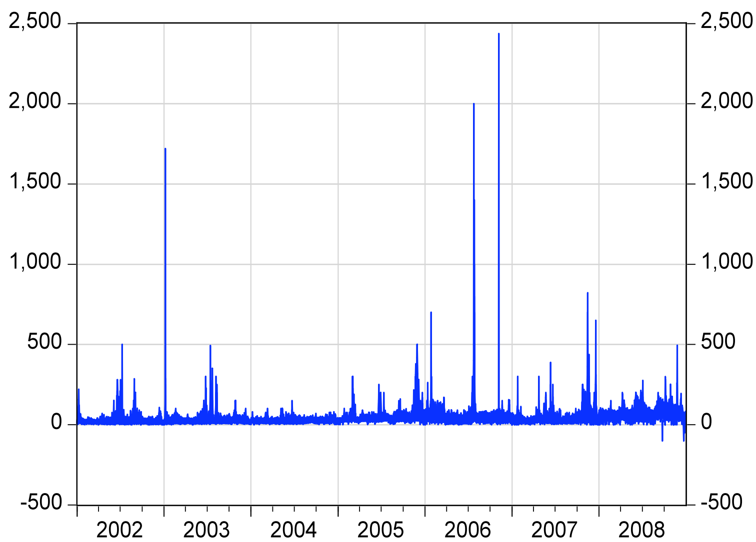

Fig 2.5: EEX hourly spot price, measured in Euros per MWh, January 2002 – December 2008.

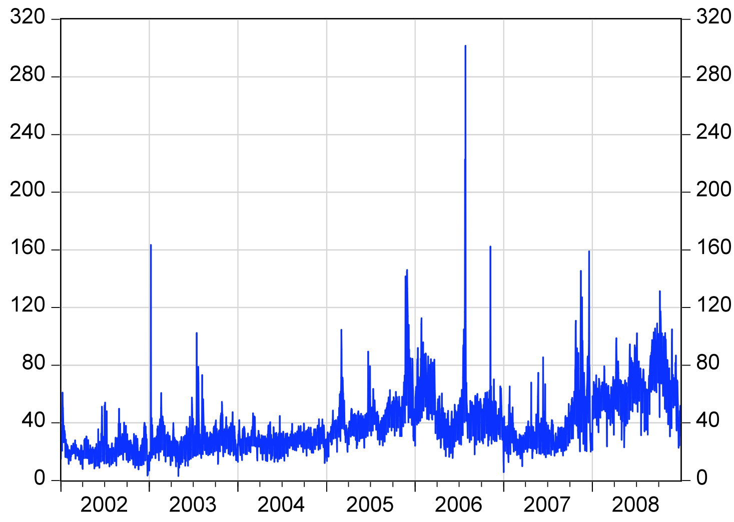

Fig 2.6: EEX base load spot price, measured in Euros per MWh, January 2002 – December 2008.

2.3.2 Periodic behaviour

In the previous subsection we discussed the dependence of the price of electricity on the level of demand at every point in time. Combined with the fact that electricity can not be stored, it is well known and rather obvious that electricity demand exhibits seasonal patterns due to weather conditions such as the temperature and the number of daylight hours, see for example Eydeland & Geman (1998) and Pilipovic (1998). In the colder winter season, for example, people tend to turn up the heating and switch on the lights earlier when compared to the warmer summer season. In warm summers, on the other hand, people use large amounts of electricity to power air-conditioners. In some countries even the supply of electricity exhibits seasonal patterns. Hydro plants, for example, heavily rely on the amount of snow melting and rain falling. This predictable periodic behaviour in the supply and demand of electricity is transferred to the periodic behaviour of spot prices. See for example Cartea & Figueroa (2005) for the seasonal pattern observed in the England and Wales spot market.

Although different patterns occur in different markets, generally periodic behaviour is visible within a calendar year (seasonal), a week (intra-week) and within a single day (intra-day). Within a calendar year we observe seasonal patterns related to different weather conditions for different seasons. Intra-week and intra-day patterns are related to business activity. On business days electricity prices are higher than during the weekends and holidays. Within a day we observe significant price differences in price levels on so called peak and off-peak hours. Peak hours are generally defined from 8.00am until 8.00pm, although this differs for different exchanges.

We do not expect the same periodic behaviour for spot and futures prices. Spot prices are expected to exhibit intra-day, intra-week as well as seasonal periodic behaviour. Futures prices, on the other hand, are not expected to exhibit intra-day and intra-week patterns. Electricity futures are based on the average spot price during a specific delivery period. In Germany only futures contracts with delivery periods equal to one month, one quarter or one year are traded for several months, quarters and years ahead. We therefore only expect to see seasonal periodic behaviour related to the specific delivery period of the contract.

Periodic price patterns with different periods are thus expected for both spot and futures prices. When modelling these price processes we should therefore not model the dynamics purely by a random walk, in which price processes are allowed to evolve 'freely'. We have to incorporate these structural and predictable price patterns into the models (Lucia & Schwartz, 2002).

2.3.3 Mean reversion

Due to the lack of extensive historical data of competitive electricity markets, there is no agreement on the long-run behaviour of electricity prices. However, Pilipovic (1998) argues that mean reversion is most suitable to model electricity prices. As opposed to stock price modelling with the use of a geometric Brownian motion (see Section 4.2.1), where prices are allowed to move 'freely', electricity prices tend to converge to some long run equilibrium (not necessarily stationary) dictated by the marginal cost of production. A sudden price shock to a geometric Brownian motion (GBM) will have a permanent effect because all subsequent price changes are uncorrelated (see Figure 2.7). From a simple visual comparison of Figures 2.5 and 2.6 with Figure 2.7 we see that this feature makes the GBM itself not very suitable for electricity spot price modelling. With a mean reversion model sudden price shocks might be observed in the short run but will not have a permanent effect on the price level in the long run, as is observed in the electricity market. Economic reasoning for this mean reversion is given by adjusted supply: high prices will attract high cost producers to the market, putting a downward pressure on the price and vice versa.

Characteristic times of mean reversion have a magnitude of several days or at most weeks and can be explained by changing weather conditions or recovery from plant outages. Many scientific contributions support mean reversion for commodity and electricity price modelling; see for example Cortazar & Schwartz (1994), Schwartz (1997) and Geman & Roncoroni (2006).

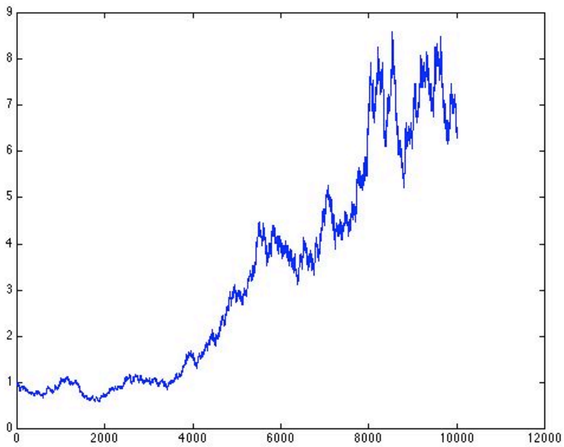

Fig 2.7: Example of a generated path of geometric Brownian motion.

2.3.4 Extreme, time varying volatility

In finance, the volatility of a price process refers to the degree of unpredictable change over time of the process and is defined as the standard deviation of the price returns. Being a unit-less measure, relative and absolute price movements in different markets with potentially very different base values can be easily compared, higher volatility indicating higher uncertainty.

The electricity spot market is characterized by extreme volatility and we already saw that the inelastic supply and demand the volatile, spiky behaviour. The positive relation between electricity prices and demand also acts as fuel for volatility, especially in times when there is a shortage of electricity. The increase in demand determines the use of more expensive energy sources for the production of electricity, thus increasing the marginal costs of production. The marginal costs rise exponentially depending on the use of wind, nuclear, hydro, coal, oil or gas fuelled power. At times of extreme shortage, the few generators that can still provide electricity can even act as monopolists, asking exorbitant prices. At times when demand is very low, the opposite occurs. Inflexible generation plants have a hard time getting rid of their electricity. To prevent the costly event of shutting down their generation plant, electricity is dumped at very low price levels, sometimes even becoming negative. This combination of factors leads to very high and time varying volatility (heteroskedasticity) observed in the electricity spot markets. Average daily volatility on electricity spot markets vary between 10% and 50%, depending on the market considered and on price levels, whereas oil and gas average daily volatilities are only 3% and 5%, respectively.

Because electricity futures depend on the electricity price during the delivery period, they generally are less volatile when compared to spot prices. In his seminal article, Samuelson (1965a) formulated the relation between the volatility of a futures contract and the time to maturity of the contract. He proposes and empirically tests that the volatility of a futures contract increases when the time to maturity approaches zero. This proposition is well known as the 'Samuelson hypothesis" or "Time To Maturity (TTM) hypothesis". This maturity effect is consistent with the notion that with the expiration date coming closer and closer, more information about market conditions at the expiration date becomes available, which leads to an increase of volatility (Anderson & Danthine, 1983). In markets where this hypothesis holds, the accurate valuation of options and other derivatives on these futures require that estimates of the volatility of futures prices should depend on the time until maturity (Bessembinder, Coughenour *et al.*, 1996). Evidence for the hypothesis to hold has been found in many futures markets, including electricity; see Allen & Cruickshank (2002) and Nakamura, Nakashima *et al.* (2006). It indicates that sudden abnormal shocks have greater influence on short-maturity contracts than on contracts with more time to maturity. Pricing models should therefore have a non-constant volatility function that at least depends on the time to maturity of the contract.

2.4 Electricity trading on power exchanges

Trading is about buying and selling goods, services or both and electricity trading is no exception. As an important part of the liberalization process, opening up the generation and retail activities to competition drove the establishment of electricity wholesale markets. In Section 2.2 we briefly introduced the two types of markets that emerged: power pools and power exchanges. Since the German electricity wholesale market is organized as a power exchange with a coexisting bilateral market, we will focus our attention on trading at power exchanges.

Several types of contracts, previously unknown to the electricity industry are now traded by a large variety of market participants. Generating companies may now enter bilateral contracts (non-standard contracts between two parties) to supply generated power to distributors and large industrial consumers, or sell power to a power exchange in which power brokers, speculators and industrial consumers also participate.

Power exchanges usually consist of multiple submarkets. Depending on the maturity, contracts are either traded on the spot or derivatives market. The essence of trading on the spot market is the ready delivery and acceptance of the goods sold. The derivatives market is often used to reduce market price risk and comprises contracts on some underlying asset with longer time to maturity. Contracts can be settled either physically (with actual delivery of electricity) or financially (no actual delivery of electricity, cash settlement).

This subsection will provide a structural overview of typical power exchanges by discussing the spot and derivatives market. Furthermore, the different types of contracts traded at these markets will be discussed. We end with a discussion of the German power exchange, the EEX. General pricing theory for the products traded at power exchanges will be discussed in Section 4.

2.4.1 Spot market

A commodity spot market (or cash market) is generally a market in which commodities are sold for cash and are delivered immediately, i.e., these contracts are settled physically. In order for spot markets to function properly, the infrastructure to conduct the actual delivery of the goods must be in place. Because of the immediate nature and high transaction costs, spot markets are usually organized exchanges on which only standardized products can be traded.

The electricity spot market is not organized as the immediate delivery market described above. Due to the non-storability of electricity, the immediate delivery of electricity is only possible in exceptional cases. The electricity spot market can be divided into two markets: a day-ahead spot market and an imbalance market for 'immediate' delivery.

Trading on the day-ahead spot market is organized on power exchanges where supply and demand are matched for each hour of the following day. Supply on the spot market stems from surplus production that cannot be sold long-term. Similarly, users obtain quantities of electricity not accounted for by long-term contracts due to unforeseeable demands. On the other hand, electricity generators buy spot electricity when delivery obligations cannot be fulfilled or when the market price lies below the own variable production costs.

The products traded on the day-ahead spot market are standardized and because trading rules apply to both sides of the transaction, a power exchange is a neutral market place. With respect to the delivery hours of a day-ahead spot contract, two load types are typically traded: base load (delivery of electricity the entire following day) and peak load (delivery of electricity during peak hours the following day). Competition between generators, distributors, speculators and large industrial consumers occurs when buy and sell bids are submitted to the exchange. Each bid specifies the quantity and the minimum price (maximum price) at which one is willing to sell (buy) the electricity. Directly after the deadline for submitting a bid, the exchange matches supply with demand and publishes the resulting system prices for each hour of the next day. Only bids from sell side parties with a minimum price below the system price and bids from buy side parties with a maximum price above the system price will effectuate and are settled at the system price. This procedure is called a uniform pricing auction and the system price corresponds to the production costs of the marginal power plant, i.e., the last power plant needed to cover the most recent level of demand.

The imbalance market is a market for the immediate sale and delivery of electricity. A supplier of electricity can enter this market when, for whatever reason, there is an immediate shortage of power in their obligation to supply a certain amount of electricity to the network. Since electricity is practically non-storable, a situation like this requires immediate action and an imbalance market facilitates this type of trading. Prices on this market are significantly higher than the prices on the day-ahead spot market and mostly hourly products are traded.

2.4.2 Derivatives market

Due to the very volatile behaviour of electricity spot contracts, market participants trading these spot contracts are exposed to several risks of which market price risk, counterparty risk and volume risk are the most important ones. With longer-term contracts previously unknown to the electricity industry, the derivatives market supports the participants in managing these risks. The longer-term contracts promise the delivery of spot electricity (the underlying asset) during some future period of time (delivery period) and are therefore also known as derivatives. Derivatives are traded for three reasons:

- Hedging
- Arbitrage
- Speculation

Hedgers either produce or need electricity and use derivatives to secure the position from market price risk, counterparty risk, etc. The sale of a futures contract, for example, can be used to hedge for falling electricity prices by locking the price that paid for the asset (the futures price). Arbitrageurs take advantage of price differences between e.g. futures that are traded on the exchange and similar contracts traded outside the exchange by buying the cheaper contract and selling the expensive one at the same time. As opposed to hedgers, speculators try to gain profits by taking on price risks. They provide liquidity for participants with contrary market strategies.

Many derivative contracts are used in electricity trading, but the most common ones are futures, forward and option contracts. Other contracts such as contracts for difference, swaptions, bulk contracts, cross commodity derivatives and many more exist, but lie outside the scope of this research. We will focus on standardized electricity contracts traded on exchanges and will not discuss the non-standardized, mostly bilateral negotiated contracts.

2.4.2.1 Futures contracts

A typical futures contract is a standardized, transferable and obligatory contract to buy or sell a specified quantity of the underlying asset at a particular future point in time (maturity) for a specified price contracted today (futures price). Futures are mainly used to decrease market price risk, i.e., they are used to fix the price to be paid (received) for the delivery (supply) of the underlying asset at some future point in time. The risk to both buyer and seller of the contract is symmetrical and unlimited, i.e., the amount lost and gained by each party is equal and opposite.

The maturity, quality and quantity of the underlying asset are standardized. The only negotiable aspect of the contract is the fixed price paid for the underlying asset at the future point in time: the futures price. This standardizing of contracts is done to facilitate trading on futures exchanges.

The futures price at time t for the delivery of the underlying asset with spot price process $S(t)$ at maturity of the contract T, is denoted by $f(t,T)$. The payoff (Φ) of such a contract at maturity T is given by

$$\Phi = S(T) - f(t,T).$$

From this relation we see that in order to avoid arbitrage, the futures price negotiated at time T for delivery of the underlying asset at T (immediate delivery) must be equal to $S(T)$. Any other price would lead to free money (arbitrage) by selling the higher priced product and buying the other product.

Because the negotiated futures price has to be paid at a future point in time, there are no initial costs of entering a futures contract. Due to changing market conditions, the value of a particular futures contract, however, does change over time. Suppose, for example, that we previously entered a futures contract to receive the underlying asset at December 1, 2009 for €100. Now assume that the current futures price for that particular contract equals € 110. When we now sell our futures contract for the new price we would make € 110- €100= €10.

The value of each futures contract is marked to market at the end of every trading day. This means that financial positions are valued based on the current fair market price as we did in the simple example above. Differences between last day's value and the current value are settled immediately and the gain or loss of a position is added or withdrawn from a so-called margin account owned by the holder of the position. Because the risk for both parties in theory is unlimited, exchanges use these margin accounts to guarantee that contracts are honoured.

Because it is easy to buy and sell futures on exchanges, it is uncommon for futures contracts to end with the actual delivery of the underlying asset. Sellers and buyers usually cancel out their obligation by an offsetting purchase or sell.

There is a significant difference between the typical futures described above and the electricity futures that are traded on power exchanges. With typical futures the underlying asset is bought/sold and delivered at the same future point in time, the maturity of the contract. For electricity futures the maturity and delivery of the contract do not coincide. Instead of one particular delivery date, electricity futures deliver electricity over a period of time, known as the delivery period. Futures contracts for different lengths of the delivery period are traded, but the most common ones are contracts delivering electricity during one month, one quarter or one year. In other words, electricity futures require the delivery of a specified quantity of electricity (measured in MWh) during a future period of time. A December 2009 futures contract, for example, is a contract delivering one MW during every hour of the delivery period December 2009. The electricity futures price at time t of such a contract is denoted by $f(t,T_1,T_2)$, where T_1 and T_2 mark the beginning and end of the delivery period, respectively. When we let $S(i)$ denote the base spot price at time i, then the payoff of a long position in an electricity futures contract is

$$\sum_{i \in \{T_1,T_2\}} S(i) - (T_2 - T_1) f(t,T_1,T_2).$$

We thus see that the underlying asset of electricity futures is not the spot electricity price at a specific future point in time, but rather the arithmetic average of the hourly spot prices during the delivery period. This fact complicates the modelling of electricity futures prices. Classical theories on pricing futures all define a relation between the underlying asset at a specific future point in time and the futures price. In Section 4.4 we will discuss the spot-futures relation further.

2.4.2.2 Forward contracts

Like a futures contract, a forward contract is an agreement between two parties to buy or sell an asset at a certain time in the future for a price contracted today. Unlike futures contracts, forwards are normally traded bilaterally, i.e., in over-the-counter (OTC) markets and contract parties usually customize the contract in order to make it fit their needs. Because they are private agreements, there's always the risk that a party may default on its side of the agreement. Futures contracts, as we saw, are marked to market on a daily basis and margin accounts and clearinghouses are used to guarantee the transaction, drastically lowering the probability of default to close to zero. As with futures, buyers and sellers of forward contracts can cancel out their obligation by taking an offsetting position, although this is more difficult due to the customized, non-standard specifications of forward contracts.

Throughout this paper we will assume deterministic interest rates. As a consequence, forward and futures prices will be equal for contracts with the same maturity and the same underlying asset (Hull, 2006). For the remainder of this report we will focus on futures contracts traded on power exchanges.

2.4.2.3 Option contracts

There are two basic types of options; a call and a put. A call option gives the holder the right, but not the obligation, to buy the underlying asset at a certain date in the future (expiration date T) for a certain price contracted at the initiation of the contract (strike price K). A put option gives the holder the right, but not the obligation, to sell the underlying asset at the expiration date for the strike price. Options like these, where the holder can only exercise the option at the expiration date, are known as European options. Numerous other types such as American (exercise possible at any time up to expiration) and Asian (payoff depends on the entire path of underlying process during lifetime of option) exist, but are beyond the scope of this research.

The fact that the holder is not obliged to execute the transaction distinguishes options from forward and futures contracts, where the holder is obliged to buy or sell the underlying asset. At the expiration date the option is either exercised or expires worthlessly. Because of this right to exercise or not, option contracts do not come for free. The option premium paid to the seller of the option is done at the negotiation of the contract. In Section 4.3 we will discuss how the option premium can be calculated.

Electricity options are predominantly traded on exchanges. Basically there are two types of options available in the electricity market: European style options with electricity futures as the underlying asset and Asian style options with electricity spot price as the underlying asset. The focus of this research is on European style options on futures with either the average base or peak load spot electricity as the underlying asset. Options are usually available for a wide range of delivery periods and strike prices.

As with normal European call options, the holder of a European call option on an electricity future has the right, but not the obligation, to buy the futures contract for the strike price at the expiration of the option. With the notation of the electricity futures contracts given in the previous subsection, we see that the payoff function at the option's expiration date T of a European call option on an electricity futures contract delivering electricity between T_1 and T_2 is given by

$$N_{MW} \max(f(T,T_1,T_2) - K, 0),$$

where N_{MW} is the number of delivery hours of the underlying futures contract (remember: futures contract deliver one MW for every delivery hour). Derived similarly, the payoff function of a European put option at the expiration date T is given by

$$N_{MW} \max(K - f(T,T_1,T_2),0).$$

2.4.3 Trading at the EEX

In the early stages of the liberalization in Germany, energy contracts were negotiated exclusively bilaterally. Very soon, however, the need for a central, coordinated and standardized trading platform emerged and in June 1999 the "European Energy Exchange (EEX)" was founded in Frankfurt am Main. Actual trading started at June 2000 and in August of that same year the Leipzig based "Leipzig Power Exchange (LPX)" was founded. In July of the year 2002 both exchanges merged to establish the new Leipzig based EEX AG which has been the only energy exchange in Germany ever since.

Traded commodities at the EEX are gas, coal, electricity and CO_2 allowances. The trading is location-independent, anonymous and conducted electronically via the XETRA trading system (spot market) and the Eurex trading system (derivatives market). Although the amount of electricity traded at the EEX increases every year, a large part of the electricity contracts is still negotiated bilaterally because market information is obtained from the disclosure of the contractual partner(s). Traders are therefore not always interested in the anonymity of trading on an exchange.

Electricity trading on the EEX can be done on two separate markets: a spot and a derivatives market. Both markets have seen an increase of participants over the last years amounting to 145 participants on the spot market and 132 participants on the derivatives market as of January 2009.[4] In addition to power suppliers, institutions such as industrial enterprises, banks, power traders and financial service providers are becoming increasingly active and are licensed as trading participants on the EEX.

2.4.3.1 EEX spot market

With an increasing traded volume of 154,4 TWh in 2008, which corresponds to approximately 28% of total consumption in Germany, the EEX spot market is an active market for trading short term electricity products. The EEX spot market is comprised of two submarkets: a day-ahead and an intra-day spot market.

On the EEX day-ahead spot market, traded instruments comprise a physical fulfilment on the next day(s). Individual hourly contracts and hourly block contracts (a period of several hours) as well as special contracts for base load, peak load and weekends are traded through the uniform EEX auction, which is held on every trading day. Market participants must submit their bids before noon, after which the EEX matches supply and demand functions and publicises the system price for every hour of the following day around 12.15h. In September 2008, as the first exchange in Europe, the EEX introduced negative electricity prices.[5] Since then, day-ahead hourly spot prices are bounded by a minimum price of € -3,000.0 and a maximum price of € 3,000.0 per MW and are quoted with one decimal point.

On the intra-day spot market, introduced in September 2006, electricity up until 75 minutes before the start of the delivery period can be traded continuously. Usually hourly contracts are traded and prices are quoted in Euro per MWh with two decimal digits, bounded by a minimum price of € -9,999.00 and a maximum price of € 9,999.00.

Because the aggregate amount of electricity traded on the intra-day spot market is very small compared to the amount of spot electricity traded on the day-ahead market (6,000 GWh vs. 138,328 GWh in 2009), the day-ahead spot market will be referred to as *the* spot market for the remainder of this report.

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[4] http://www.eex.com/en/document/43531/D_Unternehmensbroschuere_2009_final.pdf
[5] http://ockenfels.uni-koeln.de/uploads/tx_ockmedia/2008-04-14_stromtip.pdf

2.4.3.2 EEX derivatives market

On the EEX derivatives market, the long-term contracts that are traded comprise of futures and European style options on these futures. Actual transactions on the derivatives market are realized by matching anonymous executable orders. Buyers and sellers enter buy and sell orders specifying the price and quantity of a particular contract into the EEX system. The EEX displays the order in the participant's order book and executes the order when possible. With an increasing number of new trading participants with unchecked credit standings the counterparty risk becomes more and more important. To reduce the risk to participants, the EEX is itself the central counterparty ensuring the fulfilment of all transactions.

The following power futures can be traded on the EEX derivatives market: [6]

- Phelix base futures (cash settlement)
- Phelix peak futures (cash settlement)
- German base load futures (physical settlement)
- German peak load futures (physical settlement)
- French base load futures (physical settlement)
- French peak load futures (physical settlement)

Because there is far more liquidity in cash settled futures, we will focus our analysis solely to these contracts.

The underlying security of the Phelix Base futures is the average EEX index of Phelix Base for all delivery days of the delivery period. The EEX index of Phelix Base is the average of the auction prices of the hour contracts traded on the EEX spot market for the delivery hours between 0:00 am and 12:00 pm of each delivery day. For the Phelix peak futures, the underlying security is the average EEX index of Phelix Peak for the business days of the delivery period. The EEX index of Phelix Peak is the average of the auction prices of the hour contracts traded on the EEX spot market for the delivery hours between 8:00 am and 8:00 pm on each delivery day.

With respect to the delivery period of the futures contracts, three types are traded at the EEX; months (month futures), quarters (quarter futures) and years (year futures). Although there is a base and a peak load version for every delivery period, we will not consider peak load futures in this paper. Not all futures contracts are traded all the time. At maximum, the next six calendar months, the respective next seven calendar quarters and the respective six calendar years can be traded. The cash settled month futures are settled on the exchange day following the last day of trading of the contract, which is the day for the last delivery day of the delivery month on the EEX spot market. In other words, during the delivery period the contract can still be traded. A special feature of EEX quarter and year futures is cascading. With cascading, two trading days before the start of the delivery period, the original future is replaced by three monthly futures in the case of a quarter futures contract and by three month and three quarter contracts for the year futures contract. Prices are quoted in Euro per MWh with two decimal points.

Call and put option contracts traded on the EEX have Phelix base load futures described above as the underlying security and can only be exercised at the maturity (also the last trading day) of the option, i.e., only European options are traded. The maturity is fixed according to a certain scheme, but usually it is the third Thursday before the start of the delivery period. At maturity the option either expires worthlessly or the holder exercises the option and the corresponding futures position is recorded. Only options on the respective five next Phelix Base Month Futures, the respective six next Phelix Base Quarter Futures and on the respective three next Phelix Base Year Futures can be traded. EEX options refer to precisely one futures contract and prices are quoted in Euro per MWh with three decimal points.

[6] http://www.eex.com/en/document/4430/Konzept_Strom_Release_engl_01C.pdf.

3 Data analysis

In this section data from the EEX spot and futures market will be analyzed. We will investigate both price processes and see if and how the specific characteristics of electricity discussed in Section 2.3 are manifested in the German electricity market. Section 3.1 analyzes the spot market, followed by Section 3.2 discussing the EEX futures market. Section 3.3 will lastly present a short summary of our findings and will conclude on the important features to take into account when modelling EEX price processes.

3.1 EEX spot price analysis

Our spot price analysis focuses on the day-ahead spot contracts traded on the EEX. Our day-ahead spot dataset consists of daily observations of the 24 hourly prices covering the period between January 1st 2002 and December 31st 2008, amounting to 61.368 observations. The data were downloaded from the homepage of the EEX.[7] To simplify the terminology, the day-ahead spot market is referred to as the spot market from now on. Figure 3.1 presents a plot of the raw data.

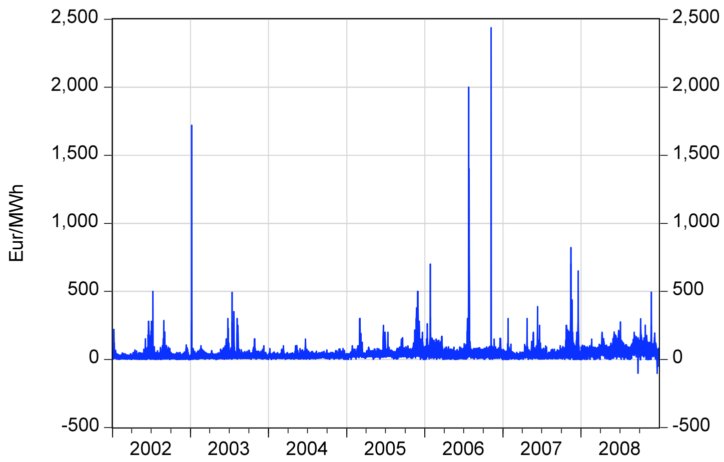


Fig 3.1: EEX hourly spot prices in Eur / MWh between January 2002 – December 2008.

3.1.1 The data at a first glance

It is good to start the data analysis with a visual inspection of the raw data. We look for (linear) trends, periodic behaviour, shifts in price levels and other non-stationary behaviour.

The first feature that catches our attention is the volatile and spiky behaviour exhibited by the hourly spot prices. Extreme price spikes are regularly observed and span the entire length of our data set, with the exception of 2004, were no extraordinary price spikes seem to have occurred. In most cases the price spikes were only short lived, falling back to normal levels within one or two days.

Secondly, we see negative hourly prices during the fourth quarter of 2008. As was mentioned in Section 2.4.3.1, the EEX made it possible for the prices of hourly spot contracts to become negative as of September 2008. The negative prices observed in Figure 3.1 mainly occurred during night hours and were caused by overcapacity of windmills. German law obliges the network-operators to use the electricity from the mills. Together with low demand, the lack of possibilities to freely dispose the electricity and the high start-up costs of power plants, this leads to a situation where paying the consumer for every Megawatt consumed is the only available option.

[7] www.EEX.com

We also see some preliminary evidence of volatility clustering in the data set. Volatility clustering, as noted by Mandelbrot (1963), is the tendency of large changes to be followed by large changes, positive or negative, and small changes to be followed by small changes. Our dataset clearly shows the clustering of price spikes, leading to very volatile periods, followed by stable periods of significant lower volatility. Volatility clustering should not be confused with mean reversion, which is also clearly visible from Figure 3.1. Mean reversion is the tendency of the process to return to a long equilibrium price level, whereas volatility clustering refers to the pattern that a period with high volatility is followed by a relative stable period and has nothing to do with the price level.

From Figure 3.1 it is hard to see if the data exhibits a deterministic (linear) trend due to the frequency of the observations. Smoothing the hourly data set by calculating the arithmetic average of the 24 hourly prices (the Phelix Base index), might help us to spot a possible trend. The base spot price is plotted in Figure 3.2.

Up until 2004 we see that, on average, the spot prices ranged from €20 to €40. In this period, large and relatively many price spikes are observed in 2003, whereas 2002 and 2004 show more stable price levels. In 2005 we see a significant price increase with spot prices increasing towards €70 in December. Both fundamental factors such as increasing fuel prices and the implementation of CO_2 emission trading in January 2005, and the exercise of market power are named as factors underlying this significant price increase (Lang & Schwarz, 2006). Lang & Schwarz conclude that up until 2005, fundamental factors explain most of the price movement and merely establish the average level and long-term price trends for the spot market in Germany. The large influence of CO_2 allowance prices on the spot price is caused by the fact that fossil fuels make up a large part of the German energy mix for the generation of electricity, as was discussed in Section 2.1.1. The drop in prices observed from March 2006 onwards was mainly due to a massive drop in CO_2 allowance prices. Rising fuel and CO_2 allowance prices underlie the steep price increase that is observed in 2008 (Böhm, Haas, Huber & Redl, 2009). It is thus fair to say that the overall rise of the German base spot price between 2002 and 2008 is for the vast part directly related to these fundamental factors.

To visualize the distribution of the base spot prices, a histogram is plotted in Figure 3.3. It is clear that the distribution of the spot prices is right-skewed, i.e., the right tail is longer indicating that the mass of the distribution is concentrated on the left of the probability density plot. When we plot the histograms of the individual years that make up our data set, a similar right-skewed distribution is observed for each year.

From a modelling perspective we prefer to work with persistent price patterns and symmetrically distributed time series. The price movements discussed above and the fact that the spot prices appear to be heavily skewed make that financial analysts often transform the time series to meet these statistical requirements.

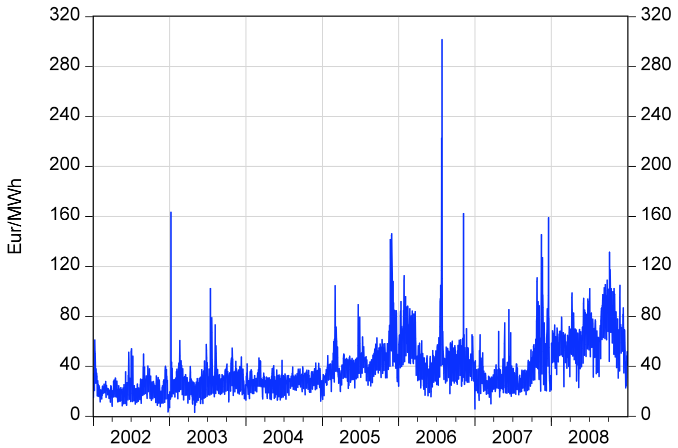


Fig 3.2: EEX base spot prices in Eur / MWh between January 2002 - December 2008.

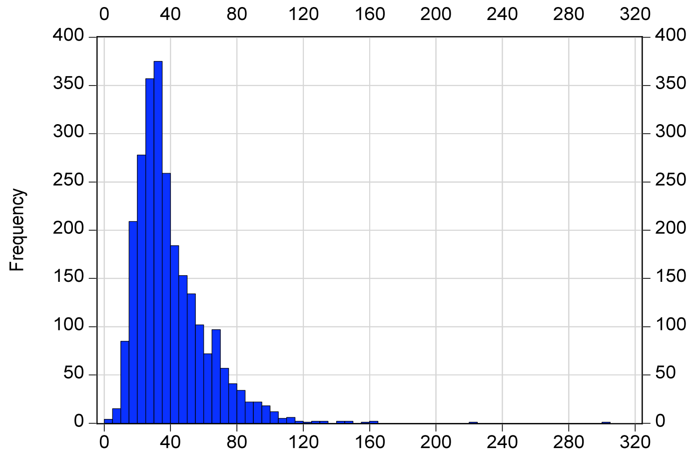


Fig 3.3: Frequency histogram of EEX base spot price between January 2002 - December 2008.

3.1.2 Data transformation

Data transformation usually refers to the application of an invertible, deterministic and continuous mathematical function to each point in a data set and is applied so that the transformed dataset satisfies the assumptions of a statistical procedure. Another possible reason to transform the dataset is to improve the interpretability or appearance of graphs. The averaging of hourly spot prices done in the previous section, for example, was a very simple example of data transformation to improve the interpretability of a graph.

The logarithmic transformation is slightly more sophisticated and highly popular in applied data analysis. It boils down to taking the natural logarithm of each data point in the set and is used to shrink the right tail of a positively skewed distribution, rendering the distribution more symmetrical. When applying a logarithmic transformation we have to be aware of the fact that the logarithm of negative numbers is not defined. By adding a positive number such that all data points become positive, we can work around this problem. As an illustration of the logarithmic transformation and its effect on the symmetry of the distribution, Figures 3.4 and 3.5 present the transformed data set and the corresponding frequency histogram. We see that the variance of the logarithmic transformed base spot price is more stable and the distribution has become far more symmetrical compared to the untransformed base spot price series depicted in Figure 3.3. Note that we only applied the logarithm to the base spot price, strictly containing positive prices.

Instead of looking at absolute or logarithmic transformed price levels, financial analysts often investigate the return of price process. Return series are easier to handle than price levels because returns have more attractive statistical properties. With returns we look at the relative changes in the variable and that allows us to compare variables across different periods of time and directly with other variables with potentially very different base values. The simple return of a price process $S(t)$ is defined as

$$r_t = \frac{S(t) - S(t-1)}{S(t-1)}.$$

The continuous compounded return, or log return is defined as

$$r_t = \ln\left(\frac{S(t)}{S(t-1)}\right) = \ln\big(S(t)\big) - \ln\big(S(t-1)\big).$$

With the log return we again have to be aware of negative data points. For small changes over short periods of time, log returns are approximately equal to simple returns, as is seen by the relation

$$r_t = \ln\left(\frac{S(t)}{S(t-1)}\right) = \ln\left(1 + \frac{S(t) - S(t-1)}{S(t-1)}\right) \approx \frac{S(t) - S(t-1)}{S(t)}.$$

Unless specified otherwise, we will apply the log return and not the simple return when we empirically test price returns. Log returns have the advantage of being time additive (multi period log returns are just the sum of individual log returns) and are mathematically convenient because logs and exponents are easy to manipulate with calculus. Figures 3.6 and 3.7 present the log return base spot price series and the corresponding frequency histogram, respectively. From Figure 3.6 we immediately see that there are no sudden price movements and that the process is even more stable (less fluctuation of the variance over time). The frequency histogram depicted in Figure 3.7 is far more symmetrical than the frequency histogram of the base spot price, although it appears to be less symmetrical compared to the frequency histogram of the logarithmic transform.

Except for the section where we analyze the intra-day periodic behaviour, we will focus on the base spot price and the log return of the base spot price for the remainder of this study. We thus accept the loss of information caused by switching from hourly quoted prices to the arithmetic average of these prices. Furthermore, it makes sense to analyze the base spot price since it serves as the underlying asset for all the derivatives we consider and we believe that this choice will not influence the general outcome of our data analysis.

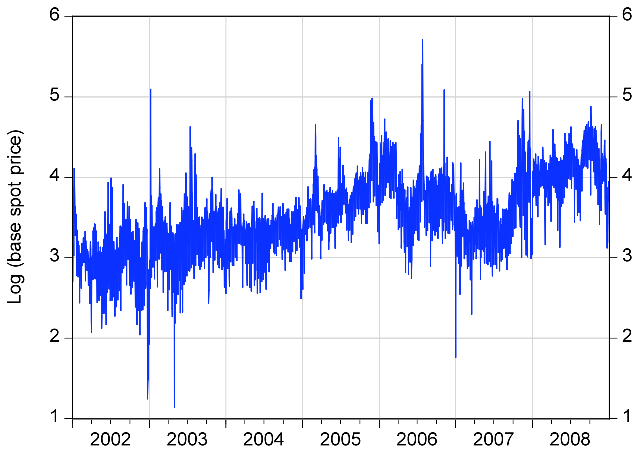


Fig 3.4: Logarithmic transformation of the EEX base spot price between January 2002 – December 2008.

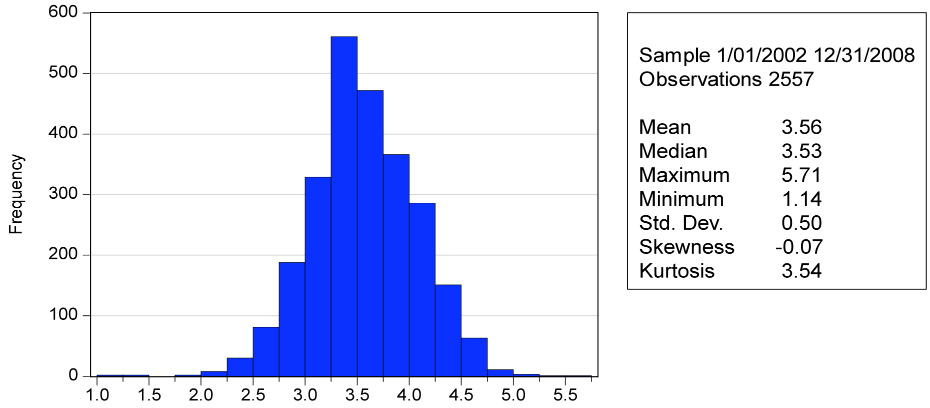


Fig 3.5: Frequency histogram and descriptive stats of the logarithm of EEX base spot price between January 2002 - December 2008.

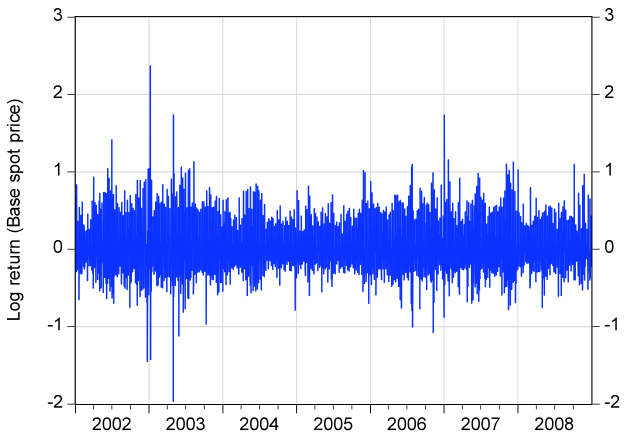


Fig 3.6: Logarithmic return of the EEX base spot price between January 2002 – December 2008.

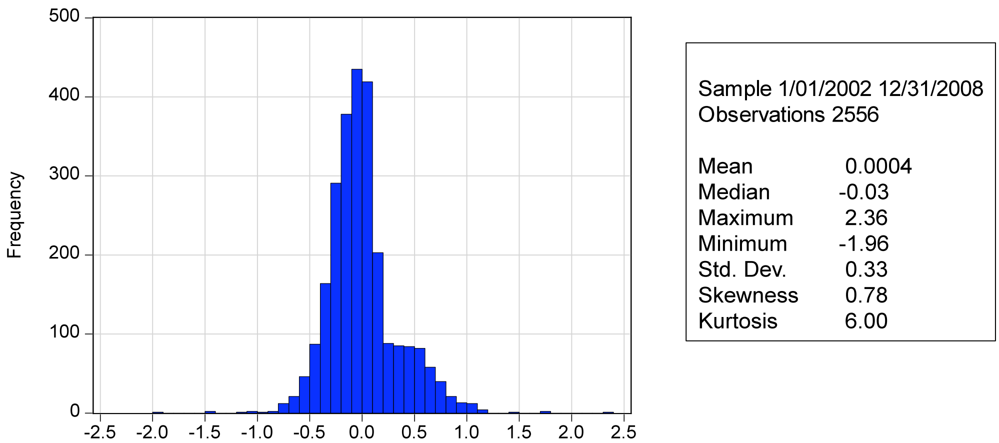


Fig 3.7: Frequency histogram and descriptive stats of the log return of EEX base spot price between January 2002 - December 2008.

3.1.3 Descriptive statistics

At the basis of virtually every quantitative data analysis are the descriptive statistics. It provides basic summaries of the sample and measures. In this section we will provide several measures of central tendency and dispersion using the statistical program Eviews 6 to get a general overview of the data analyzed. In the following two subsections we will mathematically define some of the measures used in this study (as defined by Eviews) and discuss the descriptive statistics of the EEX spot market.

3.1.3.1 Mathematical definitions

The sample mean is calculated as the average value of the N spot prices S available in the data set, i.e.,

$$\hat{\mu} = \frac{1}{N}\sum_{i=1}^{N} S(i).$$

The sample standard deviation is equal to the square root of the sample variance and is a measure of the dispersion of the data set and is defined as

$$\hat{\sigma} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(S(i) - \hat{\mu}\right)^2}\,.$$

The skewness of a data set tells us something about the asymmetry around the mean of the data set. The familiar bell-shaped curve of the normal distribution is symmetric around the mean and has skewness equal to zero. A positive skew indicates that the distribution has a long right tail and a negative skew implies that the distribution has a long left tail. The sample skewness is defined and calculated as

$$\hat{SK} = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{S(i) - \hat{\mu}}{\hat{\sigma}\sqrt{(N-1)/N}}\right)^3.$$

The kurtosis of a data set is as a measure of the peakedness or flatness of the distribution of a data set. The kurtosis of the normal distribution is 3. If the kurtosis exceeds 3, we say that the distribution is peaked (leptokurtic) relative to the normal distribution and more of the variance is the result of infrequent extreme deviations (fat tails); if the kurtosis is less than 3, the distribution is flat (platykurtic) relative to the normal distribution and more of the variance is the result of frequent modestly sized deviations. The sample kurtosis is defined and calculated as

$$\hat{K} = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{S(i) - \hat{\mu}}{\hat{\sigma}\sqrt{(N-1)/N}}\right)^4.$$

The Jarque-Bera (JB) test is used as a measure of normality. It tests the joint null hypothesis of the skewness and excess kurtosis (kurtosis-3) being zero. The test statistic is based on the sample skewness and kurtosis and is defined as

$$JB = \frac{N}{6}\left(\hat{SK}^2 + \frac{(\hat{K}-3)^2}{4}\right),$$

where N is the sample size. Under the null hypothesis of normality, the JB test statistic is distributed as χ^2 with two degrees of freedom. The reported P-value is the probability that the statistic exceeds (in absolute value) the observed value under the null hypothesis. A small probability value leads to the rejection of the null hypothesis of a normal distribution at a certain significance level.

3.1.3.2 Descriptive statistics EEX spot market

The descriptive statistics of the base spot price S(t) and the log returns Ln(S(t)/S(t-1)) are shown in Table 3.1. We see that both the base spot price and the log returns have high levels of standard deviation, indicating that both series are highly dispersed and therefore very volatile. Both series are positively skewed and exhibit high kurtosis compared to the normal distribution. Based on our data sample, we therefore find evidence that both spot price levels and log returns are not normally distributed. The log return series is, however, far more symmetrical (smaller skewness) and less peaked (smaller kurtosis) compared to the base spot price series as was already seen from the comparison of Figures 3.3 and 3.7. As a formality, the JB test statistic and corresponding P-value are calculated. Table 3.2 shows the results. From these values it is evident that the null-hypothesis that the data set is normally distributed is rejected for the price levels and the log returns.

	Mean	# Obs.	Std. Dev	Min	Max	SK	K
Panel A: entire sample							
S_t	40.16	2557	21.75	3.12	301.54	2.16	15.21
$Ln(S_t/S_{t-1})$	0,0004	2556	0,329	-1,96	2,37	0.78	6.00

Table 3.1: Descriptive statistics for the base spot price and the logarithmic return.

	JB-statistic	P-value
S_t	17888.08	0.000
$Ln(S_t/S_{t-1})$	1222.56	0.000

Table 3.2: Jarque-Bera test statistic and P-value for base spot price and logarithmic return.

3.1.4 Non-stationary behaviour

From a modelling perspective we prefer to work with stationary time series, especially for forecasting. Stationary time series are series whose statistical properties such as mean, variance and autocorrelation are all constant over time which makes it relatively easy to predict: you can simply assume that the statistical properties of the process in the future are the same as they have been in the past. Non-stationary time series, on the contrary, show (stochastic) trends, periodic behaviour, volatility clustering and other non-stable behaviour. As a general rule, non-stationary time series are unpredictable and cannot be modelled or forecasted. Results obtained by using non-stationary time series may be flawed and may indicate a relationship between two variables when there does not exist one for example. Non-stationary time series can, however, be rendered approximately stationary by mathematical transformations that remove trends (de-trending), periodic patterns and other non-stationary behaviour. The predictions of the "stationarized" series can then be untransformed by inverting the mathematical transformations to obtain predictions for the original series.

From our discussion of the characteristics of electricity in Section 2 it should come as no surprise that we cannot assume electricity time series to be stationary since periodic behaviour and time varying volatility are to be expected. A thorough investigation of non-stationary behaviour in (spot) electricity time series is therefore critically important. In this section we will take a close look at possible trends, periodic behaviour and time varying volatility of EEX spot prices.

3.1.4.1 Trend analysis

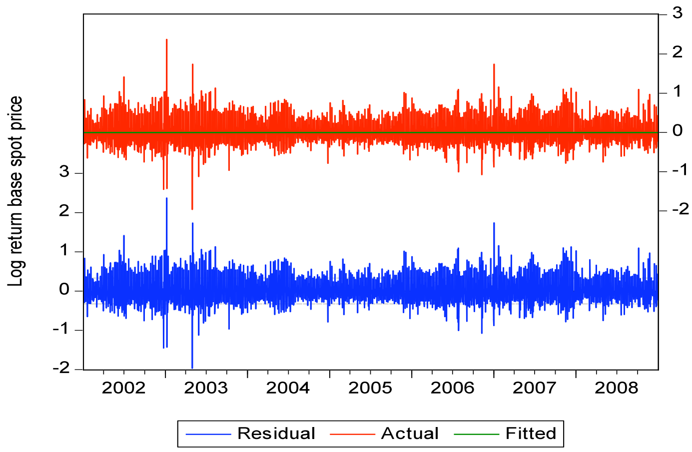
Although many would render the concept of a trend in a dataset self-evident, there is no logical algorithm for extracting it. Usually, ad hoc operations are used to extract the trend component and de-trend the original data. Examples of commonly used methods are fitting a straight line and calculating moving means with a predetermined time scale. However, these methods do not suit non-linear time series very well. More sophisticated methods are nonlinear regression and the application of sophisticated mathematical filters such as the Hodrick-Prescott filter.

With the nature of electricity and the first visual inspection of the data described in Section 3.1.1 in the back of our mind, we believe it will be difficult to describe the trend of the base spot price pure stochastically. Long-term fundamental price drivers such as fuel costs and CO_2 allowances drive long term spot prices and since these factors are highly stochastic themselves, it seems very difficult to describe the long term trend in spot prices purely by stochastic processes (i.e., without the use of exogenous data on oil and CO_2 prices). Add to that the changing regulatory environment in which power generators operate (remember the introduction of CO_2 allowances in 2005) and we see that estimating a long run trend, if present at all, will be very difficult to achieve. The log-returns time series visualized in Figure 3.6, on the other hand, does not seem to exhibit any trend at all. Values seem to oscillate around a long run mean, equal to zero. To check the postulation that the log-returns do not exhibit a trend, we performed a very simple linear regression analysis.[8] With the least squares method we estimated

$$\log - return = C(1) + C(2)t.$$

Figure 3.8 shows the graph of the actual series, the fitted line and the residuals (actual – fitted) and also provides a table with the coefficient estimates and p-values (marginal significance level) for the F-test testing the null hypothesis that the regression coefficient is zero against the two-sided alternative that it differs from zero. When, for example, the test is performed at a 5% significance level, a p-value lower than 0.05 is taken as evidence to reject the null hypothesis. The p-values in our regression models are far higher than any reasonable significance level. In other words: the probability of obtaining a test statistic at least as extreme as the one observed under the null hypothesis is very high. We therefore accept the null hypothesis that the regression coefficient is equal to zero; there is no statistical evidence of a (linear) trend in the log return data.

[8] Exponential, logarithmic and even the Hodrick-Prescott filter have also been applied, all with similar results: there's no trend in the log returns of the base spot prices during our sample period.



Variable	**Coefficient**	**P-value**
C(1)	0.001230	0.9248
C(2)	-6.50 E^{-7}	0.9413

Fig 3.8: Linear regression log return EEX base spot price.

3.1.4.2 Periodic Behaviour

In Section 2.3.2 we discussed periodic patterns in electricity price processes. As a result of the non-storability of electricity, temporary changes in the conditions of supply and demand for electricity on the spot market lead to significant price changes, even if they are perfectly expected. In this section we will analyse these predictable price patterns and investigate intra-day, intra-week and seasonal patterns of spot prices at the EEX. For the figures presented in this section, H1 refers to the first hour of a day (0:00 - 1:00), H2 to the second hour (1:00h – 2:00h), etc.

Intra-day

In this section we investigate the hourly EEX spot prices to see if a general periodic pattern is visible within a single day. To do this, we calculate the average for each of the 24 hours in a day during our entire sample period. With this approach, infrequent extreme observations might disturb the general pattern and we therefore apply an iterative procedure to remove these extreme observations. Please note that this procedure is solely applied to the data set for this particular subsection.

The procedure involves replacing those observations that lie outside a three standard deviation bandwidth from the mean and repeating this five times. We first calculate the sample mean and standard deviation and select the observations outside the three standard deviation bandwidth. These observations are then replaced by the median of all prices with the same hour, day and month as the extreme observations. Then the next iteration is performed. In total we performed five iterations.

Significant price differences between peak hours (weekdays 8.00 am – 8.00 pm) and off-peak hours (weekdays 8.00 pm – 8.00 am and weekends) are expected as a result of the different level of business activity throughout a day. Our investigation starts by investigating the hourly price pattern of a randomly picked date from our data set, December 13th 2005. The results are presented in Figure 3.9 and the expected pattern is clearly visible. Prices are significantly higher during peak hours, with a distinct peak around noon (H12) and at the start of the evening (H18). To see whether or not this pattern is typical for every year of our entire data set, Figure 3.10 plots the hourly averages for every year. Regardless of the year we are looking at, the same pattern appears: significant higher prices during peak hours. Lastly, Figure 3.11 pictures the average hourly price over the entire data set.

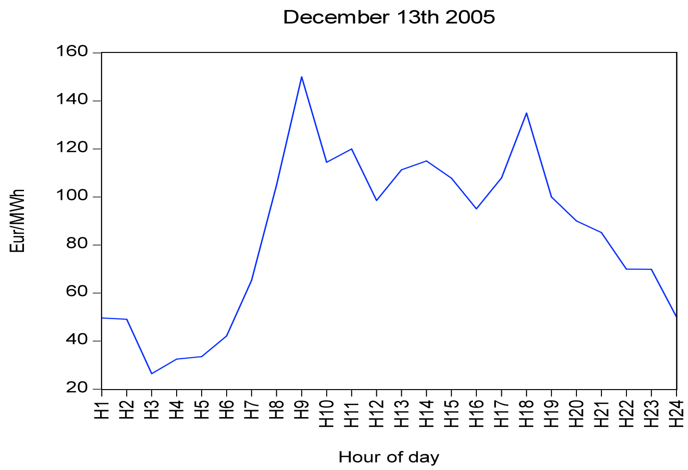


Fig 3.9: EEX hourly spot prices on December 13th 2005.

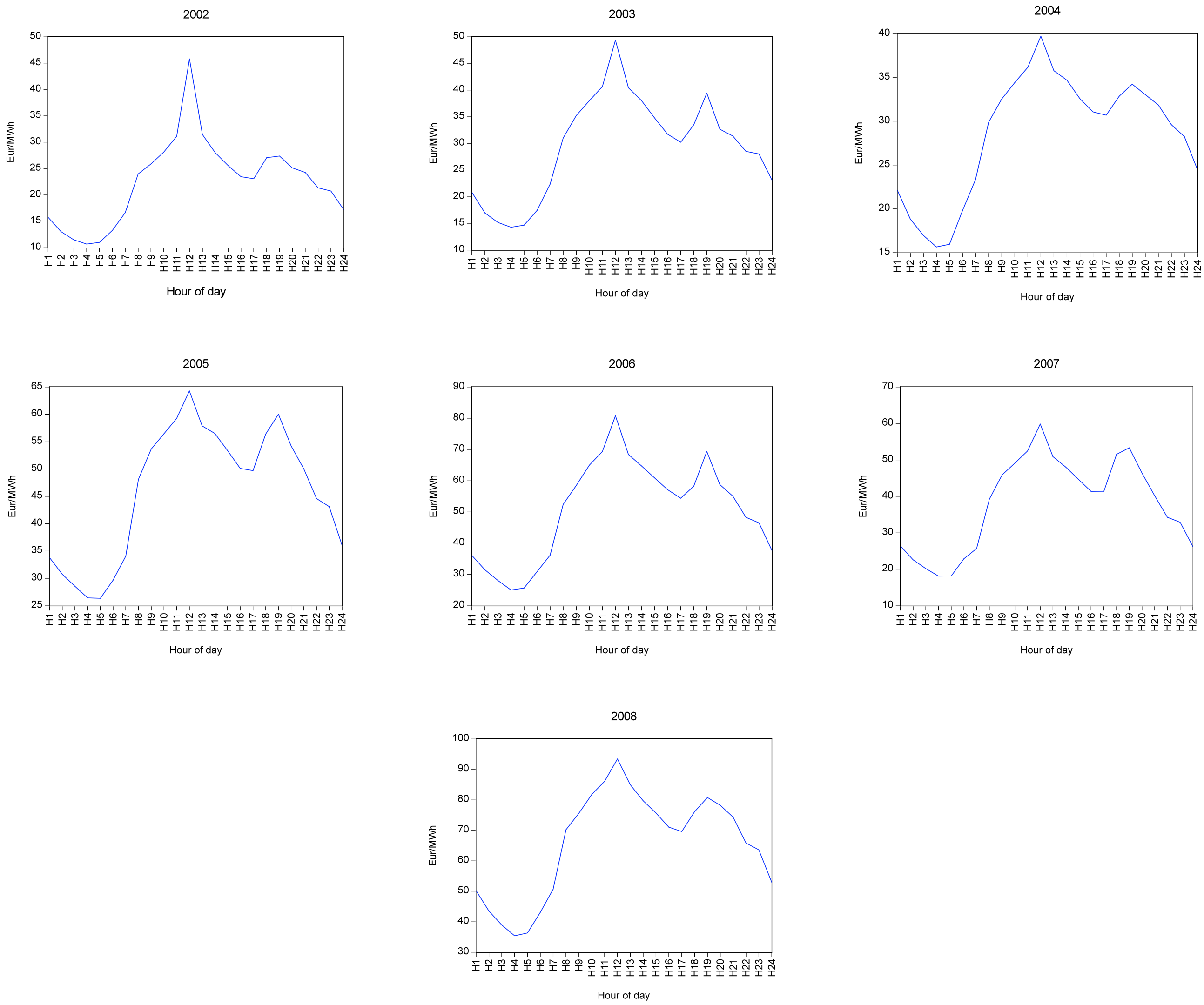


Fig 3.10: Average EEX hourly spot prices per year.

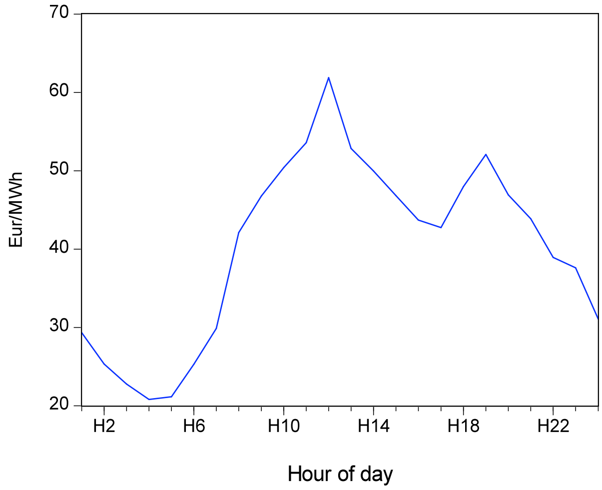
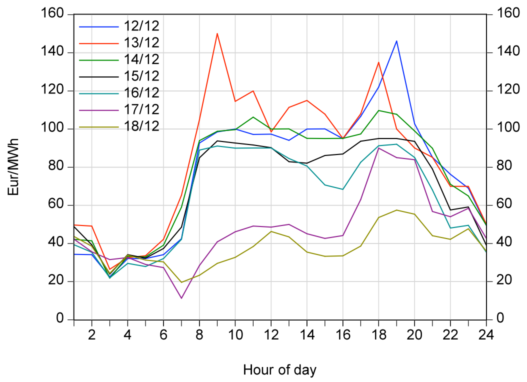


Fig 3.11: Average hourly EEX spot price between January 2002 – December 2008.

Intra-week

To check the periodic behaviour within a week we again start by looking at a typical week. We again expect to see a pattern related to business activity: significantly higher prices during business days. We picked the week starting at Monday December 12th 2005. The hourly price patterns during this week and the daily arithmetic averages of the hourly prices (base spot price) are presented in Figure 3.12. Roughly the same pattern appears when we plot the same variables over the entire sample period, see Figure 3.13. The hourly price pattern observed within a day is roughly the same throughout the week and resembles the intraday pattern discussed in the previous section. As expected, the price difference between weekdays and weekends are significant. Prices peak at the beginning of the week (Monday / Tuesday) and are on average declining towards the weekend with the lowest price levels reached on Sundays.



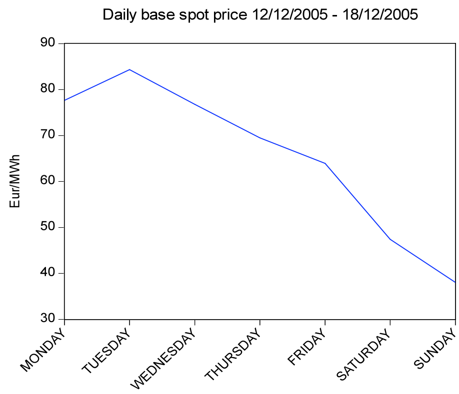


Fig 3.12: Hourly spot price pattern and base spot price during typical week.

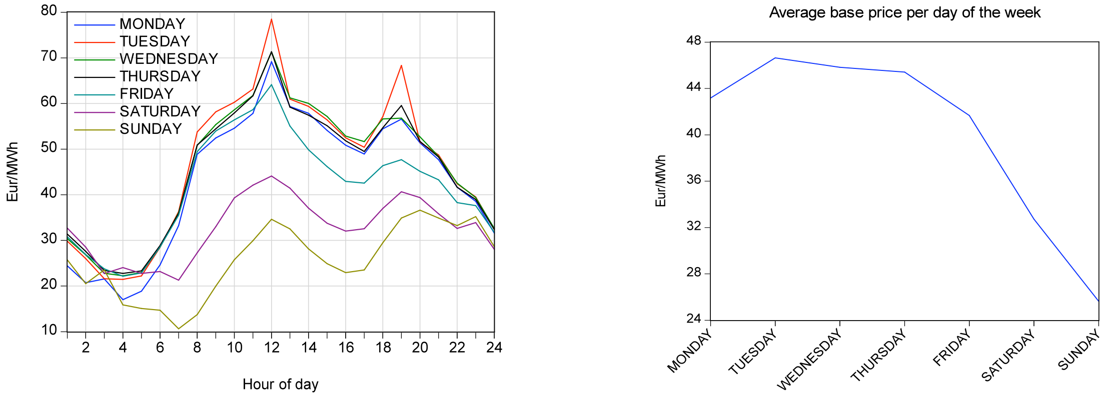


Fig 3.13: Average hourly spot prices and average base prices per day of the week over entire sample period.

A more formal investigation of intra-week price patterns is obtained by evaluating the correlation between lagged observations of the same series, also known as the autocorrelation. This mathematical tool is often used to find periodic patterns that are buried under noise and shows the similarity between observations as a function of the time separation between them. The autocorrelation of a periodic function has the nice property of being periodic itself, with the same period as the original function. We are thus looking for evidence of a periodic pattern with a period equal to 7 lags (one week) in the autocorrelation function of EEX spot prices that would confirm our postulate that spot prices exhibit intra-week periodic patterns.

Figures 3.14 and 3.15 present the autocorrelation function (ACF) for the differenced base spot price and the logarithmic base spot return up to 200 lags for the entire sample period.[9]
For each lag, the ACF returns the autocorrelation for observations that are separated in time by this number of lags (days). Note that the autocorrelation reaches values in the interval $[-1,1]$, a 1 indicating perfect correlation and -1 indicating perfect anti-correlation.

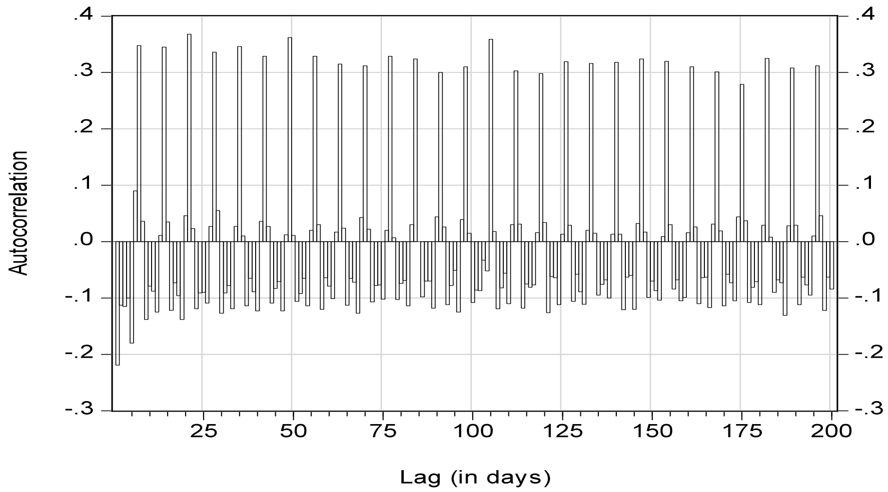


Fig 3.14 Autocorrelation function for differenced base spot prices between January 2002 – December 2008.

[9] Autocorrelation functions for the base spot price and logarithmic transformation of the base spot price showed similar periodic behaviour but are not presented in this report.

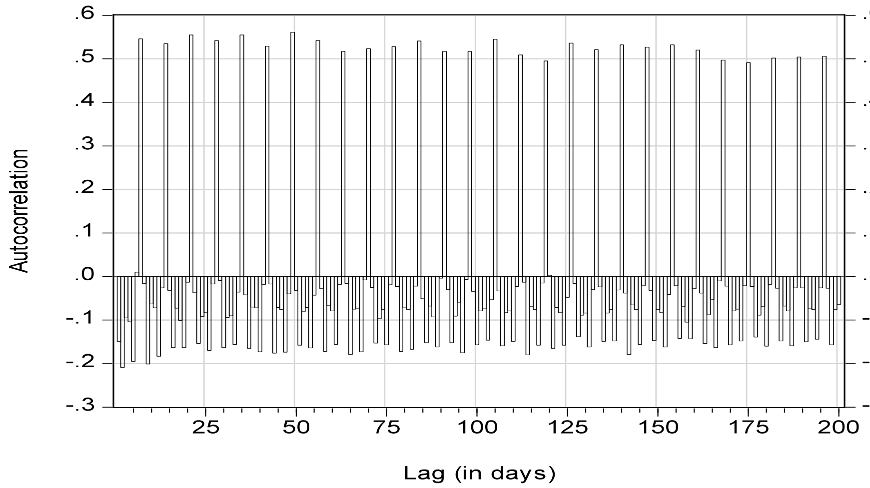


Fig 3.15: Autocorrelation function for log return base spot prices between January 2002 – December 2008.

For both series we clearly see a recurring and persisting pattern with a period equal to seven lags, in this particular case equal to seven lags. The high positive peaks occurring at lags that are a multiple of seven indicate that the differenced base spot prices and log return base spot prices on an a arbitrary day, respectively, are positively correlated to the variables exactly one week earlier. In the case of log returns, for example, this means that

$$\ln\left(\frac{S(t)}{S(t-1)}\right) \text{ for } t \in R^{+}$$

is positively correlated to

$$\ln\left(\frac{S(t-7k)}{S(t-1-7k)}\right) \text{ for } k \in \{1,2,3..\},\ t \in R^{+}.$$

To put it in other words: relative price changes move in the same direction as the relative price changes seven days earlier. We thus found compelling evidence for the expected intra-week periodic pattern of EEX spot prices.

Seasonal

Seasonal cycles differ for different markets. In the Californian power market, for example, summer month spot prices are significantly higher due to the use of air-conditioners. For the winter months in Germany, according to Janssen & Wobben (2009), the decrease of electricity demand due to the limited use of air-conditioning is outweighed by the increase of demand due to the increased use of lighting and heating. Furthermore, power plant fuels, particularly natural gas, are more expensive in the winter due to increased demand. We therefore generally expect winter spot prices to be above spot prices during summer months. To see if this predictable pattern is visible in our data set, Table 3.3 shows descriptive statistics for the average price levels according to the month of the year. The expected pattern of high winter prices is not observed uniformly. October, November and December show relative high average prices, as was expected. The average values for the summer months June, July and September, on the other hand, seem out of place. The relatively high prices during these months can possibly be explained by the obligatory yearly inspection of nuclear power generating units. These inspections are scheduled and distributed as evenly as possible during the low demand period between April and September and require the complete shutdown of the generating unit. As a result of this procedure, full nuclear capacity is available during winter in order to minimize price spikes caused by scarcity during the months with higher levels of demand. A negative side effect is the decrease of supply during summer months counteracting the otherwise significant summer price decrease. The potential of price spikes during summer months is therefore increased, leading to relative high summer prices.

To investigate the seasonal pattern more formally we combined the non-linear regression function for the logarithm of the base spot price proposed by Lucia & Schwartz (2002) with a linear trend function. The deterministic trend function that was estimated is given by

$$f(t) = \alpha + \beta * t + d_{Weekend} * D_{Weekend} + \sum_{Month=1..12} d_{Month} D_{Month},$$

where $D_{Weekend}$ and D_{Month} are dummy variables defined by

$$D_{Weekend} = \begin{cases} 1 & if \quad t = weekend \\ 0 & otherwise \end{cases} \quad \text{and} \quad D_{Month} = \begin{cases} 1 & if \quad t \in i^{th} - month \\ 0 & otherwise \end{cases},$$

and α, β, $d_{Weekend}$ and d_{Month} are the regression coefficients. The weekend dummy variable is implemented to account for the significant lower prices during weekends, discussed in the previous section. When there is a seasonal effect in the logarithm of the base spot prices, we should see at least one monthly dummy variable with a significant regression coefficient.

The significant regression coefficients (at a 5% significance level) are presented in Table 3.4. As was expected we observe significant coefficients for the intercept α, trend coefficient β and weekend coefficient $d_{Weekend}$. For the months April, May, August and December we also observe significant regression coefficients. So there is evidence of some seasonal pattern for the EEX spot price. The seasonal pattern, however, is not as clear and obvious as the intra-day and intra-week patterns observed earlier. At the 1% significance level, only the intercept α, trend coefficient β and weekend coefficient $d_{Weekend}$ are significant so there is no evidence of a seasonal pattern at the 1% significance level. A plot of the fitted non-linear regression function and the residual (actual – fitted regression function) is presented in Figure 3.16.

	Mean	# Obs.	Std. Dev	Min	Max
Panel A: base spot price, sub-sample according to month of the year					
January	38,71	217	20,77	5,80	163,46
February	40,10	198	18,73	11,46	95,93
March	37,62	217	18,51	9,02	104,60
April	36,07	209	17,16	7,94	98,68
May	32,31	217	14,96	3,12	74,67
June	39,29	210	19,84	8,74	94,38
July	43,48	217	30,41	9,72	301,54
August	36,77	217	15,03	10,40	86,68
September	43,63	210	21,58	14,27	108,92
October	45,71	217	23,22	10,90	131,40
November	47,04	210	28,21	7,69	162,25
December	41,23	217	22,41	3,47	158,97

Table 3.3: Descriptive statistics base spot price according to month of year.

	Estimate	Std. Error
Significant coefficients		
α	3.181	0.025
β	0.0004	9.17 E-06
$d_{Weekend}$	-0.422	0.014
d_{Apr}	-0.08	0.033
d_{may}	-0.26	0.033
d_{Aug}	-0.089	0.032
d_{Dec}	-0.095	0.032

Table 3.4: Significant regression coefficients f(t).

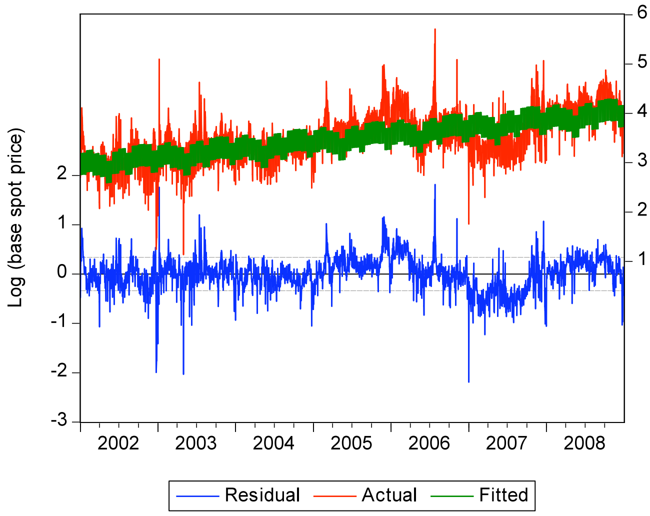


Fig 3.16: Logarithmic base spot price, fitted regression function and the residual.

3.1.4.3 Time-varying volatility

Besides the predictable patterns discussed in the previous section, there are other short term, unpredictable factors that play a deciding role in electricity prices on daily and hourly intervals. These factors are primarily concerned with weather conditions affecting the supply and demand of electricity. Temperature is probably the most influential factor. It determines the demand for cooling or heating and spot prices alter accordingly. The amount of rain falling also influences spot prices, especially in countries with a large hydro-electric production share. Long periods of drought, on the other hand, can lead to insufficient levels of cooling water in German rivers and could prevent generation plants from functioning at full capacity. This occurred, for example, on June 12 2007, when the base spot price jumped to € 85.41. In the last few years, wind intensity has also begun to play an important role in electricity pricing. The total capacity of windmills has increased dramatically in Germany. Since wind energy comes for free (when the infrastructure is installed, of course), European directives call for the prioritized utilization of wind energy regardless of the current spot market price. Strong winds can therefore lower spot prices. In times of little or no wind, on the other hand, the more expensive conventional generation plants must increase production and spot prices rise accordingly.

Although general weather forecasts can be established for seasons, weeks and sometimes even specific days, the actual conditions can only be seen on the day itself. The accuracy of the forecasts, however, does converge to actual weather conditions as time advances. Since electricity spot prices are determined one day prior to the actual delivery, forecasts are of great importance and play a deciding role in pricing spot electricity.

All of these factors contribute to the time varying, volatile behaviour observed for spot prices. In our discussion of the descriptive statistics of the log returns we already saw that the average daily volatility over the entire sample period equaled 32,9% (see Table 3.1), which is exceptionally high. In this section we will see how the spot volatility varies over time. We do this by calculating simple rolling moving average historical volatilities. For each day the standard deviations of the last 30, 90, 180 and 365 observations of the log returns prior to that day are calculated. This gives us four sequences of daily moving average historical volatilities with different periods for our entire data set. Figure 3.17 shows the daily historical volatilities series.

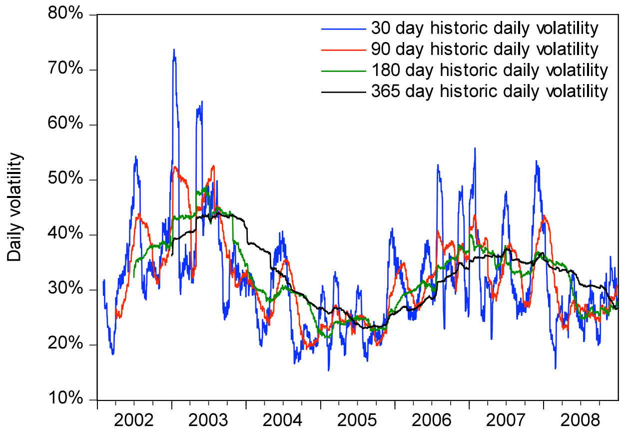


Fig 3.17: Rolling moving average daily historical volatility of EEX log returns.

As was expected, volatility is far from constant over time. Daily volatility behaves quite violently and with significant amplitude, reaching values between 15.5% and as much as 74.5%. It shall be clear that assuming constant volatility for the base spot price is usually not a justifiable assumption. To check whether the volatility level is also dependent on the price level we construct a scatter plot of the rolling moving averages of the base spot price and the corresponding rolling moving average of the sample standard deviations. Figure 3.18 shows the scatter plots for 30, 90, 180 and 365 day moving averages. From these plots we can clearly see that the volatility level is indeed dependent on the base price level. Higher levels for the standard deviation are observed for higher levels of the average base spot price.

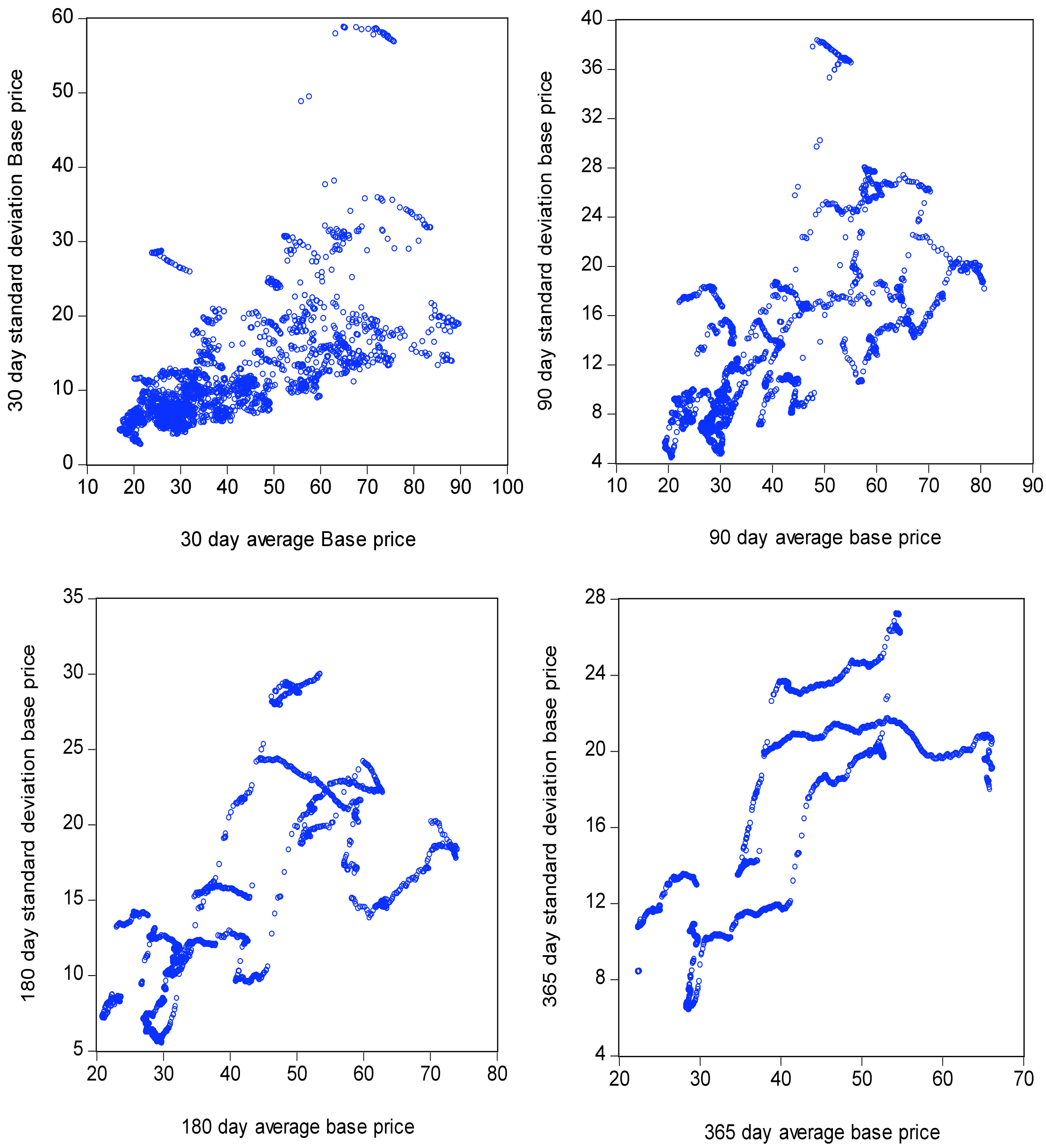


Fig 3.18: Scatter plots of 30, 90, 180 and 365 day moving averages with respect to the moving average standard deviations.

3.2 EEX futures price analysis

Our futures data set comprises settlement prices of all the traded Phelix base load futures contracts from July 1, 2002 to December 31, 2008, totalling 1644 trading days. All the data were downloaded from the EEX homepage. Up until October 1, 2003 on average seven month, seven quarter and three year futures were traded on an arbitrary trading day. From this date onwards we on average recorded seven month, seven quarter and six year contracts for every trading day. This adds up to a total of 11.895 quotes for a total of 87 month futures, 12.024 quotes for 37 quarter futures and 8.907 quotes for 13 year futures. The data set has no missing values but at the end of our sample period some spurious prices are observed. For several month and quarter futures quotes are equal to € 0,01 for a long period of time. These prices do not reflect real trading activity and are only observed for contracts with a very long time to maturity. These price entries were removed from the data set. For the remainder of this report we will therefore work with 11.622 observations for 86 monthly contracts, 11.691 observations for 34 quarterly contracts and 8.907 quotes for 13 yearly contracts.

Figures 3.19, 3.20 and 3.21 present the raw data for the futures contracts with delivery periods equal to a month, a quarter and a year respectively. Since futures contracts are only traded for a limited amount of time, these plots show discontinuities caused by the start / end of trading periods.

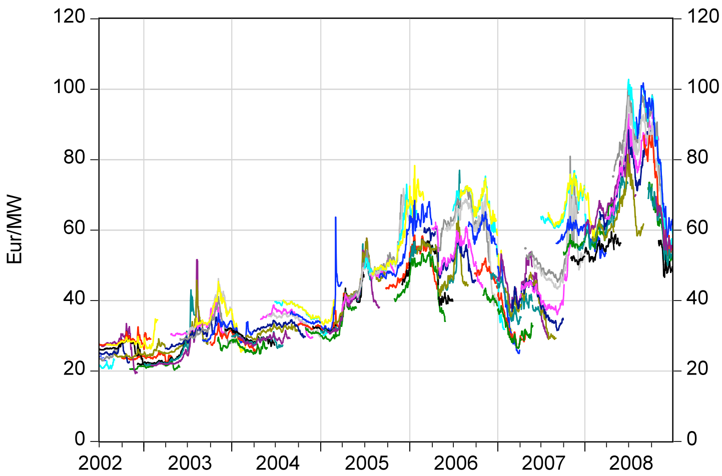


Fig 3.19: EEX month futures raw data between July 2002 - December 2008.

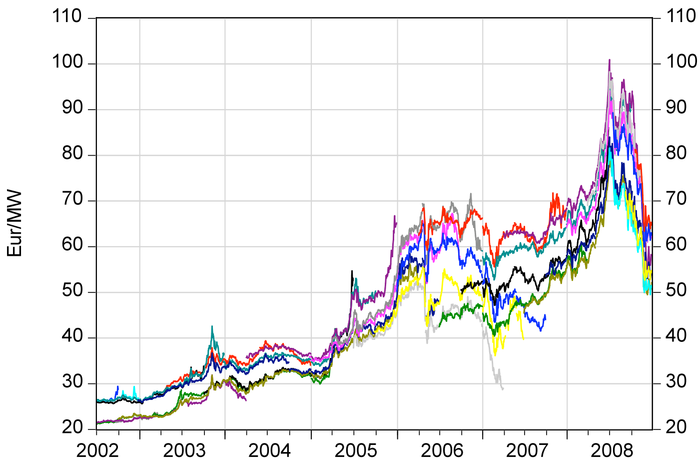


Fig 3.20: EEX quarter futures raw data from July 2002 - December 2008.

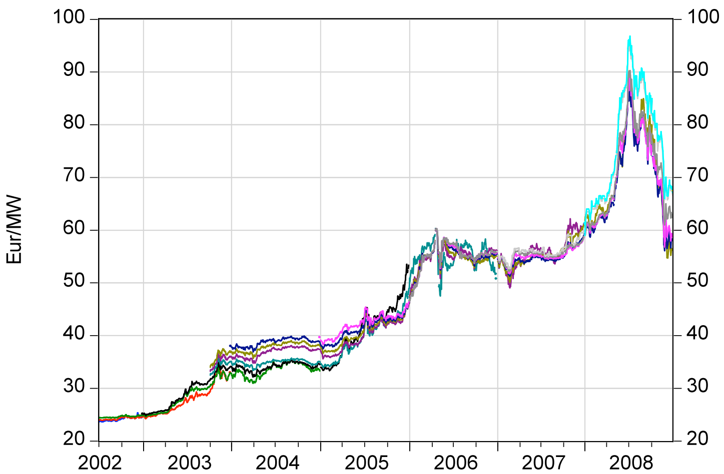


Fig 3.21: EEX year futures raw data from July 2002 - December 2008.

3.2.1 The data at a first glance

The first thing that strikes when we look at the three figures depicting the futures contracts traded at the EEX is the fact that the very volatile short lived price movements (spikes) observed in the spot market are not transferred to the futures market, or at least not directly. Lower futures price volatility is explained by the fact that short-term changes in supply and/ or demand of electricity have large and immediate effects on spot prices, but have a much smaller effect on futures prices. Futures prices depend on the average spot price during the delivery period, dampening the price movements of the spot. As a result, futures prices are far less sensitive to the arrival of (generally short lived) shocks to the system. Futures prices will not be significantly influenced when these shocks occur before the start of the delivery period, unless these shocks are expected to persist and influence spot prices during the delivery period.

All three figures exhibit a similar path and show a clear upward trend during our sample period. During 2005, the first quarter of 2006 and 2008 we see particularly large price shifts for all futures contracts, independent of the length of the delivery period. The introduction of CO_2 allowances and increasing fuel prices are the most likely cause for the steep increase of prices in 2005, averaging € 15 per MWh. During the first half of 2008 prices even seem to have increased exponentially, rapidly falling down again to price level reached in January 2008. To illustrate the co-integration of the electricity futures prices with fuel prices, Figure 3.22 shows the Brent Crude Oil 'spot' price for 2004 up until August 2009. The resemblance with the price movements visible on the EEX futures market is striking. The steep increase in 2005 and the first quarter of 2006 are clearly visible. Even more striking is the almost exact copy of the price pattern observed in 2008. Furthermore, both economic theory and empirical research suggest that natural gas and oil price are related. With natural gas and oil being substitutes in consumption as well as rivals in production, changes in supply or demand of the oil price drove changes in the natural gas price, but the converse did not appear to occur. This asymmetrical relationship might be explained by the relative size of the two markets. The crude oil price is determined on a world market, whereas the natural gas markets are regionally orientated. With gas prices following the oil price, we thus see that fuel prices for a large part explain the long term price movement of electricity futures traded at the EEX.

When we look at the differences between the month, quarter and year futures we observe that price volatility seems to decrease with the length of the delivery period, i.e., month futures exhibit more volatile behaviour compared to quarter futures, which are more volatile than year futures. This can be explained by the fact that electricity futures have the average spot price during the delivery period as the underlying asset. Month futures will be more sensitive to changes in supply and/or demand during the delivery period than quarter and year futures are because prices are averaged over a shorter period of time. Another reason for the volatile behaviour of month futures might be the fact that month futures, as opposed to quarter and year futures, remain tradable during the delivery period. As a result, price shocks during the delivery period will not only directly affect the spot price but will also affect the futures price of the contract with delivery in this period.

From Figures 3.19, 3.20 and 3.21 we also observe some preliminary evidence for the Samuelson hypothesis (increased volatility when time to maturity decreases) discussed in Section 2.3.4. The price processes indeed seem to become more volatile when the delivery period comes closer.

It thus seems that the volatility of futures prices is far from constant across time and product. Based on our first visual inspection, we expect the volatility to depend on the time to maturity as well as the length of the delivery period. The fact that both time to maturity and length of the delivery period show an inverse relation with volatility (volatility decreases with time to maturity and length of delivery period) and the fact that a shorter delivery period implies shorter time to delivery, makes it hard to separate the individual effects of these two time variables. The inverse relation is consistent with mean reversion in the underlying asset, in this case the electricity spot price. Very high (low) electricity spot prices will result in very high (low) futures prices for contracts close to the start of the delivery period. But mean reversion suggests that a very high (low) price today will be followed by a decrease (increase) of the price in the near future. Prices of futures contracts with longer time to maturity will therefore not increase (decrease) as much as short-term contracts, since a price reversal is expected in the future.

Fig 3.22: Brent crude oil price between 2004 and 2009.

3.2.2 Descriptive statistics

As with our discussion of the spot price, we will focus our analysis of the futures contracts on the price levels and the logarithmic returns. In this section will present the descriptive statistics and because of the extensive futures data set we split up our discussion of price levels and the returns in two sections.

3.2.2.1 Descriptive statistics of price levels

The total sample of price levels consists of 32.220 observations. All prices are denoted in Euros per MWh and besides the price level, we observe the following information for every single observation:

- **Trading day**
  To be able to calculate the time to delivery we report the date the price level was recorded for each observation.

- **Time to Delivery (TTD)**
  For each observation in our data sample we observed the difference in calendar days between the start date of the delivery period and the trading day the price was recorded. All base Phelix futures contracts traded at the EEX start delivering electricity on the first day of the month of their respective delivery periods. So, the October contract starts delivering on October 1, the Quarter 3 contract starts at July 1 and year contracts always start delivery at January 1, for example.

- **Length of Delivery period (DP)**
  For each observation we also reported the exact number of delivery days of the corresponding contract. With this information we will be able to see if volatility changes with the length of the delivery period.

- **Time of Delivery period (TODP)**
  In order to see if futures price levels exhibit seasonal behaviour we report the delivery month(s) for each observation. Since we look at seasonal behaviour within a calendar year, the time of year is only reported for month and quarter contracts.

With respect to the statistics we present the mean, number of observations (# Obs.), standard deviation (Std. dev.), minimum, maximum, average length of the delivery period $\overline{DP}$ and the average time to delivery $\overline{TTD}$.

The descriptive statistics for all contract price levels for the entire sample period are given in Table 3.5, panel A. The level of the mean and the range (maximum – minimum) suggests that the price levels are right-skewed. The average length of the delivery period of all the contracts traded during our sample period was around 5 months and the average time to delivery was well over one year.

In panel B we rearranged the data into sub-samples according to the length of the delivery period. We see that the average price level is increasing with the length of the delivery period. The upward trend in futures prices is the most likely cause. Year contracts are traded far longer than month contracts and therefore incorporate the higher expected price level for the entire future year(s), whereas month contracts are only traded for 6 months ahead. We also note that price levels for all delivery products are right-skewed, where the right-skewness is more pronounced for contracts with shorter delivery periods (and consequently shorter time to delivery). By looking at the standard deviation we can see that contracts with a shorter delivery period and shorter time to delivery show larger variability in prices, as was expected.

Panel C and D show the descriptive statistics with respect to the delivery month of the contract for all the month and quarter contracts, respectively. With these calculations we look for evidence of seasonal periodic behaviour in the price levels of the futures. When we look at the average price level of the month contracts in panel C, we observe significant higher price levels during the colder months from October to March. The level of the standard deviation is also significantly higher during these months, also when we compare the relative standard deviation (sample standard deviation divided by the sample mean). The relative standard deviation from October to March averages 38,1%, whereas it averages 35,3% from April to September. The same pattern is visible from the statistics for quarter contracts presented in panel D, i.e., higher price and standard deviation levels during quarter 1 and 4. The relative standard deviation for quarter 1 and 4 averages 36,3%, whereas it averages 34,3% for quarter 2 and 3. The difference between the (relative) standard deviation for month and quarter contracts again confirms our earlier findings of decreasing volatility with the length of the delivery period and time to delivery.

	Mean	# Obs.	Std. Dev	Min	Max	$\overline{DP}$	$\overline{TTD}$
Panel A: entire sample							
All	46,21	32220	16,72	19,27	102,75	145	434
Panel B: sub-sample according to length of delivery period (DP)							
Month	45,03	11622	17,44	19,27	102,75	30	79
Quarter	45,75	11691	16,93	21,15	100,93	91	329
Year	48,36	8907	15,21	23,65	96,80	365	1034
Panel C: month contracts, sub-sample according to time of delivery period (TODP)							
January	51,92	1025	19,99	26,13	102,75	31	82
February	51,21	990	19,42	25,35	101,75	28	84
March	45,01	963	16,08	23,62	86,88	31	81
April	40,52	944	13,68	21,70	73,50	30	83
May	36,61	909	11,89	20,24	59,50	31	81
June	41,69	935	14,38	21,78	75,36	30	87
July	43,18	948	16,20	21,53	92,55	31	81
August	40,36	934	14,44	20,60	81,43	31	73
September	43,84	959	16,66	22,90	90,23	30	73
October	45,35	980	17,78	22,26	93,49	31	72
November	50,49	1010	19,74	19,27	101,94	30	74
December	48,26	1025	18,40	21,81	97,78	31	77
Panel D: quarter contracts, sub-sample according to time of delivery period (TODP)							
1. Quarter	51,32	2957	18,61	26,00	100,93	90	327
2. Quarter	40,18	2918	13,64	21,15	81,74	91	327
3. Quarter	42,55	2846	14,71	21,15	86,09	92	328
4. Quarter	48,73	2970	17,72	25,62	97,50	92	333

Table 3.5: Descriptive statistics of price levels of futures contracts traded at the EEX between July 2002 and December 2008. Contracts are quoted in Euros per MWh and average Delivery Period $\overline{DP}$ and average Time to Delivery $\overline{TTD}$ are quoted in days. Each day during a delivery period represents 24 MWh.

3.2.2.2 Descriptive statistics of logarithmic returns

Going from price levels to logarithmic returns, one observation per traded contract is lost. With a total of 133 contracts traded during our sample period, our data set shrinks to 32.087 observations. As we did for the price levels, we observe the trading day, time to delivery, length of the delivery period and time of the delivery period for each observation. Please note that in order to annualize the standard deviation of log returns (the volatility), the sample standard deviation is multiplied by $\sqrt{250}$, since 250 is the average number of trading days in our sample.

Descriptive statistics of the logarithmic returns for all contracts during the entire sample period are given in Table 3.6, Panel A. We see that the overall mean is just slightly positive, suggesting a very small overall positive drift for futures returns traded at the EEX. Returns take on values in the range between -28.68% and 42.15% and together, the mean and range suggest a right-skewed distribution for the returns. The average annualized volatility across all contracts is equal to 22.2%. Average delivery period and average time to delivery are 145 and 434 days, respectively.

In panel B we sorted the data according to the length of the delivery period. As we saw with the analysis of futures price levels, panel B suggests right skewed returns for all delivery periods, where the skewness is more pronounced for contracts with shorter delivery periods (and consequently shorter time to delivery). Average volatility is clearly decreasing with the length of the delivery period; again reinforcing our postulate that volatility and the length of the delivery period are inversely related. Average volatility is highest for month contracts, averaging 29.92% with average time to delivery equal to 78 days. For quarterly and yearly contracts volatility averaged 18.21% and 13.46% with on average 328 and 1033 days to delivery, respectively.

	Mean	# Obs.	Std. Dev	Min	Max	$\overline{DP}$	$\overline{TTD}$
Panel A: entire sample							
All	0,0002	32087	0,2220	-0,2868	0,4215	145	434
Panel B: sub-sample according to length of delivery period (DP)							
Month	-0,0002	11536	0,2992	-0,2868	0,4215	30	78
Quarter	0,0004	11657	0,1821	-0,0836	0,0994	91	328
Year	0,0005	8894	0,1346	-0,0705	0,0884	365	1033

Table 3.6: Descriptive statistics of logarithmic price returns of futures contracts traded at the EEX between July 2002 and December 2008. Average Delivery Period $\overline{DP}$ and average Time to Delivery $\overline{TTD}$ are quoted in days. The sample standard deviation (volatility) is annualized by multiplying the sample standard deviation with the square root of the average number of trading days in a year, 250.

3.2.3 Periodic behaviour

For futures contracts only seasonal periodic behaviour is expected. Prices are based on monthly, quarterly or even yearly averages of spot prices, hence no intra-day or intra-week price patterns are observed. In Section 3.2.2.1 we introduced the four different time variables that were recorded for every observation in our data set. When analyzing seasonality not all time variables are expected to be related to the seasonal behaviour of electricity futures prices. Seasonality corresponding to the trading date, for example is not expected. Electricity futures have the average spot price of a fixed future period of time as the underlying asset and all the information concerning this future period is already incorporated in the futures price from the day it starts trading. This, of course, does not mean that futures prices do not change until maturity. Both the time to delivery (TTD) and delivery period (DP) have no seasonal influence since contracts with different TTD and DP are traded throughout the year and have no relation to calendar times.

The time of the delivery period (TODP), on the other hand, is related to seasonal behaviour. Table 3.5 already presented the statistics for the month and quarter contracts according to the time of the delivery period and significant price changes were found for contracts delivering in different months. By averaging all the quoted futures prices for each traded contract separately and plotting it against the delivery period of the contract, we can visualize the structure throughout the sample period. Figures 3.23 and 3.24 present the structure for monthly and quarterly contracts respectively. From the figures we observe a seasonal pattern with a period equal to one year. Looking at the individual years in our data sample, the same seasonal pattern is observed for month and quarter contracts. Average prices are higher for contracts delivering electricity during the colder first and fourth quarter.

The futures price returns give a similar picture. Table 3.7 presents the descriptive statistics of month contracts, sorted according to the calendar time of the delivery period (TODP). Price returns are positive from June until November and negative from December to May, indicating that prices on average rise from June to November to peak in November / December and gradually fall down again to hit the lower prices levels in the summer months with lower demand. From Table 3.7 we also see some evidence of seasonal volatility. With DP and TTD quite similar for all month contracts mutually, the influence of these time variables on the volatility level should also be quite similar across months and quarters respectively. Differences in the volatility might therefore be an indication of seasonal behaviour of volatility.

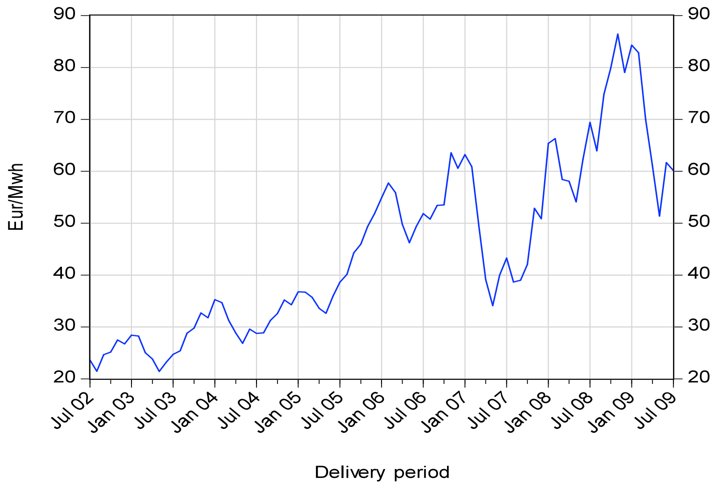


Fig 3.23: Average futures prices for month futures with respect to delivery period.

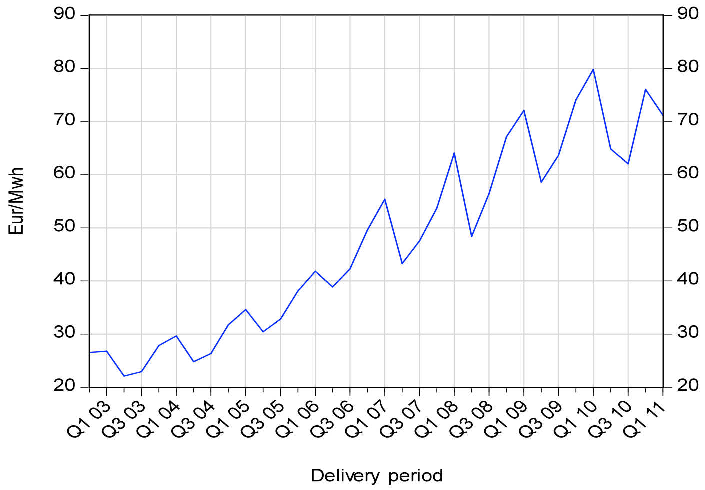


Fig 3.24: Average futures prices for quarter futures with respect to delivery period.

	Mean	# Obs.	Std. Dev	Min	Max	$\overline{DP}$	$\overline{TTD}$
Panel A: month contracts, sub-sample according to time of delivery period (TODP)							
January	-0,0014	1018	0,2938	-0,1680	0,1477	31	81
February	-0,0009	983	0,2828	-0,1092	0,0978	28	83
March	-0,0008	956	0,3622	-0,1830	0,4215	31	80
April	-0,0005	937	0,2567	-0,0725	0,1006	30	82
May	-0,0004	902	0,2513	-0,0857	0,0929	31	80
June	0,0001	928	0,2669	-0,0919	0,0793	30	86
July	0,0006	940	0,3460	-0,1547	0,2107	31	81
August	0,0002	926	0,3621	-0,2868	0,2708	31	72
September	0,0007	952	0,2785	-0,1070	0,0981	30	72
October	0,0007	973	0,2343	-0,0902	0,0698	31	72
November	0,0000	1003	0,3142	-0,1503	0,1922	30	74
December	-0,0008	1018	0,3065	-0,1423	0,0860	31	76

Table 3.7: Descriptive statistics of month futures logarithmic price returns according to the time of the delivery period (TODP). The full sample consists of all traded month contracts at the EEX between July 1, 2002 and December 31, 2008. Average Delivery Period $\overline{DP}$ and average Time to Delivery $\overline{TTD}$ are quoted in days. The sample standard deviation (volatility) is annualized by multiplying the sample standard deviation with the square root of the average number of trading days in a year, 250.

3.2.4 Time varying volatility

Looking again at the time variables we see that we can distinguish two time components on which the volatility of a futures contract might depend. From our discussion of the descriptive statistics of the base spot price and log returns, we already saw that volatility seems to decrease with the length of the delivery period. This effect is called the term structure of volatility. From an economic point of view it is clear that contracts with a longer delivery period are less volatile, since the arrival of news such as plant outages, fuel price shocks and weather conditions generally influence only specific periods of time. For contracts with long delivery periods the effect on the price will average out with opposite news during other periods, leading to lower volatility.

The other time variable of influence is the time to delivery (TTD). The effects of TTD on the volatility of futures prices are also known as the Samuelson hypothesis, which was discussed in Section 2.3.4. The Samuelson hypothesis states that the volatility of a futures contract increases as the time to maturity approaches zero. To see the effect(s) of time to delivery on the descriptive statistics of the returns we sorted the data according to TTD. The results are presented in panel A of Table 3.8. Please note that the negative TTD corresponds to month contracts only, since these contracts remain tradable during the delivery period. We observe a slight negative overall drift for contracts close to delivery (TTD <50) and an even smaller positive trend for contracts with longer time to delivery. Significant differences between sub-samples are observed for the average volatility. Contracts close to delivery exhibit significant higher levels of volatility. Especially the contracts with negative TTD exhibit high volatility, averaging 45.91%. Volatility is steadily declining for increasing TTD, averaging 13.86% for contracts with average TTD equal to 1033 days.

In order to see the effect(s) of time to delivery on the average volatility for different delivery periods, we repeated the calculations for monthly, quarterly and yearly contracts separately. The results of these calculations are given in panels B, C and D respectively.

The three panels show the same pattern: decreasing volatility when TTD increases. Looking at the level of the average volatility we again see clear differences for contracts with different delivery periods. Month returns exhibit significant higher levels of volatility compared to quarterly and yearly contracts. Returns of quarterly contracts, on their turn, are more volatile compared to yearly contracts. Panels B, C and D therefore confirm our postulate that the volatility of futures prices are indeed inversely related to DP and TTD.

	Mean	# Obs.	Std. Dev	Min	Max	$\overline{DP}$	$\overline{TTD}$
Panel A: sub-sample according to time to delivery (TTD)							
(← – 0)	-0,0013	1546	0,4591	-0,2868	0,4215	30	-15
[0 – 50)	-0,0007	3741	0,3331	-0,1547	0,1298	63	25
[50 – 100)	0,0003	3817	0,2326	-0,0854	0,0901	66	74
[100 – 300)	0,0005	9113	0,1910	-0,1024	0,0994	87	170
[300 – 500)	0,0004	4528	0,1631	-0,0746	0,0991	147	400
[500 – →)	0,0005	9342	0,1386	-0,0752	0,0744	285	1092
Panel B: month contracts, sub-sample according to time to delivery (TTD)							
(← – 0)	-0,0013	1546	0,4591	-0,2868	0,4215	30	-15
[0 – 50)	-0,0010	2688	0,3657	-0,1547	0,1298	30	25
[50 – 100)	0,0003	2688	0,2412	-0,0854	-0,0901	30	74
[100 – 150)	0,0005	2718	0,1956	-0,1024	0,0624	30	124
[150 – →)	0,0002	1896	0,2132	-0,0902	0,0855	30	169
Panel C: quarter contracts, sub-sample according to time to delivery (TTD)							
[0 – 100)	0,0003	1724	0,2283	-0,0836	0,0896	91	51
[100 – 200)	0,0006	1803	0,1865	-0,0615	0,0745	91	149
[200 – 300)	0,0007	1813	0,1762	-0,0762	0,0994	91	249
[300 – 400)	0,0004	1800	0,1720	-0,0738	0,0991	91	349
[400 – →)	0,0004	4527	0,1659	-0,0752	0,0838	91	528
Panel D: year contracts, sub-sample according to time to delivery (TTD)							
[0 – 100)	-0,0001	458	0,1848	-0,0591	0,0651	365	53
[100 – 200)	0,0005	489	0,1578	-0,0685	0,0466	365	149
[200 – 400)	0,0009	832	0,1519	-0,0705	0,0884	365	301
[400 – 600)	0,0004	953	0,1469	-0,0634	0,0699	365	496
[600 – →)	0,0005	6162	0,1232	-0,0645	0,0732	365	1358

Table 3.8: Descriptive statistics of month and quarter futures logarithmic price returns according to time to delivery (TTD). The full sample consists of all traded contracts at the EEX between July 1, 2002 and December 31, 2008. Average Delivery Period $\overline{DP}$ and average Time to Delivery $\overline{TTD}$ are quoted in days. The sample standard deviation (volatility) is annualized by multiplying the sample standard deviation with the square root of the average number of trading days in a year, 250.

3.3 Summary and conclusion

3.3.1 EEX spot prices

Our analysis of the EEX spot prices is based on hourly observations of the EEX spot price from 01/01/2002 until 31/12/2008, totalling 61.368 observations. We investigated the behaviour of the base spot price (2557 observations) and the logarithmic return (2556 observations). Our main findings are summarized below.

- Co-integration: long-term developments of EEX spot prices are co-integrated with fundamental price drivers such as fuel costs and CO_2 allowances. Determination of a deterministic trend for base spot price without using these fundamental variables is considered very difficult. For the log-return series no significant trend was found; the log returns oscillate around the mean (zero) during the entire sample period.

- Skewed distribution: the spot prices are not normally distributed. The frequency histogram shows a fat and long right tail, indicated by a positive skew (2,16) and excess kurtosis (12,21). Although more symmetrical, both the log base spot price and the log return are also not normally distributed according to the Jarque-Bera test statistic.
- High volatility: the time series of the EEX spot price exhibits high price spikes. As a result we observe a very high daily volatility for the EEX spot, averaging 32,9% (520% annualized) during our sample period.
- Time varying volatility: besides the high level of volatility we found that both the intensity and the frequency of price spikes change with time. As a result we observe hectic, unstable periods (average 30-day daily volatility around 50%) alternated with relative calm, stable periods (average 30-day daily volatility around 25%). This volatility clustering is also known as heteroscedasticity.
- Mean reversion: EEX spot prices rapidly return to 'normal' levels after a price spike is observed.
- Periodic behaviour: for the EEX we investigated the possible intra-day, intra-week and seasonal predictable pattern.
    - Intra-day: a clear and persistent pattern was observed for the hourly prices within a day. Prices are significantly above the base spot price on peak hours and below this level on off-peak hours.
    - Intra-week: we also found compelling evidence of a predictable intra-week pattern. On Monday and Tuesday prices are relatively high, dropping gradually to the lowest level, which is reached on Sunday. A stable autocorrelation function with high autocorrelation values and a clear and persistent periodic pattern endorses these findings.
    - Seasonal: although far less evident and clear compared to the intra-day and intra-week pattern we found evidence of a seasonal pattern using non-linear regression. The results are in accordance with the expected pattern of (slightly) higher spot prices during winter months.

3.3.2 EEX futures prices

The analysis of the EEX futures prices is based on observations of all the traded futures contracts for every trading days from 01/07/2002 until 31/12/2008. A total of 11,622 prices for 86 month contracts, 11,691 prices for 34 quarter contracts and 8,907 prices for 13 year contracts were observed. For each of the observations the date, time to delivery, length of the delivery period and time of the delivery period was recorded. As with the spot price we investigated the price level and the logarithmic return. Our main findings are summarized below.

- Co-integration: as was expected and already discussed for the spot prices, we see that the futures price levels are co-integrated with the fundamental price drivers such as fuel costs and CO_2 allowances.
- Level of volatility: the EEX futures prices clearly show lower levels of volatility. Short term changes or shocks in demand and/or supply have less influence on futures prices since futures prices are based on the average spot prices during the delivery period and the fact that these abnormal shocks are generally short-lived. The average annual volatility equals 22,2 %, significantly below the 520% average annual volatility of spot prices.
- Changing volatility: volatility of futures contracts change with
    - The length of the delivery period: the volatility of the EEX futures contracts decreases with the length of the delivery period. Across the entire sample, the annualized volatility of month contracts is 30%, compared to 18% and 13% for quarter and year futures, respectively.
    - The time to delivery: we re-arranged the observations according to the time to delivery and found that the longer a contract is away from the start of the delivery period the lower the volatility. This is known as the Samuelson hypothesis.

    - Time of the delivery period: by re-arranging the data according to the time of the delivery period we observed significant differences for the annual volatilities for different times of the delivery period. With the length of the delivery period and the average time to delivery being constant, these differences in volatility might be explained by the seasonal behaviour of volatility.
- Periodic behaviour: the EEX futures prices exhibit a seasonal pattern. Average prices are significantly higher during winter months. This pattern is observed for both month and quarter futures price levels as well as log returns.

3.3.3 Conclusion data analysis

Our data analysis shows that many of the specific properties of electricity price processes discussed in Section 2.3 are also true for the EEX spot and futures data. Although the two types of processes are closely linked and have many properties in common, it will prove to be difficult to find models that can accurately capture both spot and futures price dynamics. When modelling electricity price processes we therefore have to decide which process we want to model or which product we want to price. Since it is our goal to accurately model the futures price process, we are interested in the different approaches to the modelling of these prices. As we will see later on, this can be done by either modelling the spot price dynamics and deriving the corresponding futures prices, or by modelling the futures price dynamics directly. Both approaches have advantages and disadvantages and it is important to be aware of them. When we choose the first approach, for example, including very complicated mathematical functions to model the price spikes observed for the spot prices might unnecessarily complicate things.

In this section we found that in order to accurately model the EEX futures curve, price models should take care of the seasonal pattern that was observed and above all must be able to capture the complex, time varying volatility structure of the futures prices.

4 Mathematical background

In the previous two sections we presented a general overview of the (liberalized) electricity market and analyzed spot and futures prices from the EEX. We saw that with the liberalization process, a market place was created where generators, distributors, speculators, suppliers and large industrial consumers are able to trade many different types of electricity products on both spot and derivative markets. Due to the increasing trading activity, it has become increasingly important for all active trading parties to develop price models for the contracts they buy and sell, both for risk management and valuation purposes. This section will focus on the mathematics needed to understand, apply and develop electricity price models.

The first section will review three different modelling approaches. The second section will subsequently treat the developments of stochastic modelling in the more mature stock and fixed income markets. We end with some theory on derivative pricing and a discussion of the relationship between spot and futures prices.

4.1 Modelling approaches

When modelling price processes one can choose between three approaches (Anderson, 2004):

- Fundamental approach.
- Stochastic approach.
- Hybrid approach.

4.1.1 Fundamental approach

The fundamental approach uses fundamental variables to construct a model that fits historical price data as accurately as possible. It requires a high level of understanding and insight into the variables that are believed to influence prices and often results in complex, non-linear relationships between the driving variables. Due to the fact that the fundamental approach is based on real, often observable, market variables these types of models are suitable for (short term) forecasting.

In the electricity industry, fundamental approaches are based on competitive equilibrium models for the electricity market. Prices are derived from a model for the marginal generation cost of electricity and the expected consumption of electricity. Initially, these models were primarily constrained to autoregressive effects and price responses to fluctuations in fuel prices, demand, weather conditions and transmission constraints (Kosater, 2006; Vehvilainen & Pyykkonen, 2004; Rambharat, Brockwell & Seppi, 2005; Nogales, Contreras, Conejo & Espinola, 2002). Karakatsani & Bunn (2008) suggest that these factors should be complemented with aspects of plant dynamics, risk measures, market design effects, agent learning and strategic behaviour (e.g. exercising of market power). Advantages of the fundamental approach are the tractability of the factors and the fact that economic reasoning can be used to deduce properties of the factors. On the other hand, comprehensive data sets that are laborious to maintain are required and fitting the observable initial futures curve is hard. The forecasting power of fundamental models for electricity prices is also questionable. With continuously changing variables such as weather conditions being of great importance, forecasting electricity prices based on fundamental models usually do not look further than one week ahead.

4.1.2 Stochastic approach

The stochastic approach is, in principle, not concerned with actual (fundamental) price drivers but focuses on modelling the stochastic processes that represent prices directly. Historical price data provide estimates for the model parameters in such a way that the estimated models fit the historical data as accurately as possible. With these theoretical price models we are able to derive prices for derivative contracts such as European options with the stochastically modelled price process as the underlying asset.

From observations of the electricity market, we suggest that the movements of electricity spot and futures prices are at least random to some extent, and can thus be modelled with stochastic models. The stochastic approach works with explicit formulas for the electricity price processes. In some specific cases it is even possible to obtain closed form solutions for European options. The construction of stochastic models is in general easier than constructing fundamental models, but no rigorous economic motivation for the parameters have yet been given in the electricity market. A problem that is often encountered is the lack of long historical time series that would allow for more accurate parameter estimation. Also the continuous structural and regulatory changes in the electricity market can have large effects on price levels, making it hard to estimate the influence of these market changes on the parameter values. Nevertheless the stochastic modelling of electricity price processes is a very active research topic.

4.1.3 Hybrid approach

Authors on hybrid electricity spot price models believe that the combination of the fundamental approach with the stochastic approach allows the inclusion of important behaviour particular to the electricity spot price. Anderson & Davison (2008), for example, develop a so-called switching model in which the tendency of price spikes to cluster and persist can be a result of weather conditions, load conditions, a shortage of supply or more likely, a combination of these contingencies.

In this report we choose to focus our attention on the stochastic modelling of electricity prices. The fundamental and hybrid approach both require large data sets and assume specific economic relationships in the marketplace which makes the price projections very sensitive to violations of the assumptions. This implies that there is a significant modelling risk when the fundamental or hybrid approach is applied. The fact that we choose to focus on the stochastic approach, however, does not mean that the variables of stochastic models are not related to fundamental variables per definition. We only choose to describe models purely stochastically, i.e., without using any external data other than the history of realised electricity prices. Chapter 5 will provide an overview of several stochastic electricity price models.

4.2 Development of stochastic modelling

In this section we discuss the developments in stock and fixed-income markets. As we will see later on, many of the models used to model electricity price are modified versions of well-known models developed for these more mature markets. Knowing and understanding the underlying modelling principles is critical to review and understand electricity price models.

4.2.1 Stochastic stock price modelling

One of the founding fathers of the discipline we nowadays call mathematical finance is the French mathematician Louis Bachelier (1900). He is credited for being the first person to mathematically describe the Brownian motion, a stochastic process named after the Scottish botanist Robert Brown, who studied the motion of pollen suspended in water in 1827. Five years before the famous paper of Albert Einstein (1905) on Brownian motion, Bachelier derived the distribution function of the stochastic process underlying the Brownian motion, the Wiener process. Using the Wiener process, Bachelier presented the first stochastic model for random stock prices. For a formal definition and properties of a Wiener process the reader is referred to appendix A. Note that in today's literature the terms standard Brownian motion and Wiener process both are used to describe a stochastic processes satisfying the conditions of definition 1, appendix A. In this report we will use the terms Brownian motion and Wiener process interchangeably.

A long period of theoretical and empirical research followed the work of Bachelier, mostly during the second half of the 20th century. The work of Samuelson (1965a), Mandelbrot (1966) and Fama (1970) led to the famous Efficient Market Hypothesis (EMH). The EMH asserts that prices of traded assets in financial markets already reflect all known information. Otherwise, predictable price movements could lead speculators to risk-free profits. In *efficient markets* these so-called arbitrage opportunities cannot exist and speculators cannot expect to consistently outperform the market. Though being a true cornerstone of modern financial theory, the EMH is controversial and often disputed because substantial and lasting inefficiencies are observed in the market. This has led to the development of alternative theories, especially by behavioural finance economists.

One obvious shortcoming of modelling asset prices with Brownian motion is the fact that negative price levels might occur. This is due to the normality of the Wiener process (see appendix A). To overcome this shortcoming, Samuelson (1965b) formulated the price movements of a single stock by a lognormal probability model and the geometric Brownian motion (GBM) was born. Because of its importance for stochastic (stock) price modelling we will define the GBM explicitly.

Definition 4.1: *The process $X(t)$ is said to follow a geometric Brownian Motion (GBM) if it satisfies the following Stochastic Differential Equation (SDE):*

$$\begin{cases} dX(t) = \alpha X(t)dt + \sigma X(t)dW(t) \\ X(0) = a \end{cases}.$$

Or, equivalently

$$\begin{cases} \dfrac{dX(t)}{X(t)} = \alpha dt + \sigma dW(t) \\ X(0) = a \end{cases}.$$

From the last equation we see that the return of $X(t)$ is equal to the sum of two terms. The first term, αdt, is called the drift term of the process where the deterministic variable α represents the expected return per unit of time. The stochastic second term, $\sigma dW(t)$, adds randomness to the deterministic drift term via the volatility (standard deviation of returns) σ and the Wiener increment $dW(t)$. This Wiener increment can be interpreted as the difference of the Wiener process at an infinitesimal small period of time, i.e.,

$$dW(t) = W(t + \Delta t) - W(t).$$

To find the solution $X(t)$ of the SDE, we need a very important concept used in stochastic calculus, called Itô calculus. Named after Kiyoshi Itô, it extends the methods of calculus to stochastic processes such as Brownian motion. One of the most important results, Itô's lemma, can be found in Appendix B.

By using Itô's lemma, we derive the solution for a geometric Brownian motion $X(t)$ as:

$$X(t) = X(0)e^{(\alpha - \frac{1}{2}\sigma^2)t + \sigma W(t)}.$$

Using the moment generating function of the Gaussian distribution we can then derive the expected value as

$$E[X(t)] = X(0)e^{(\alpha - \frac{1}{2}\sigma^2)t} E\left[e^{\sigma W(t)}\right] = X(0)e^{(\alpha - \frac{1}{2}\sigma^2)t} e^{(\frac{1}{2}\sigma^2)t} = X(0)e^{\alpha t}.$$

Due to the normality of the Wiener process appearing in the exponent of the solution, we see that processes that are said to follow a Geometric Brownian Motion are log-normally distributed (the log of the process is normally distributed) and will thus give strictly positive prices (the exponential function returns strictly positive values). From the expected value we see that the price process is unbounded and, on average, is growing over time. Because of its simplicity, the GBM is still the most widely used process for stock price modelling and was used by Fischer Black and Myron Scholes to derive their famous option pricing formula (Black & Scholes, 1973).

One of the disadvantages of lognormal Ito diffusion processes (of which GBM is an example) is its inability to capture spikes and jumps in price levels often observed in financial markets. Merton (1973) and Cox & Ross (1976) introduced jumps into stochastic models for stock price processes. Merton developed a jump-diffusion model in which the abnormal volatility (shocks due to rare events) in the price is modelled by a Poisson process while the 'normal' volatility is still governed by a diffusion process such as geometric Brownian motion. Cox and Ross modelled stock price movement by a pure jump model with a random Jump component as the only random source, i.e., no driving Wiener process. More recent research on jump-diffusion models was amongst many others done by Duffie, Pan & Singleton (2000). Opposed to the GBM case, the resulting stock price processes will not be continuous in time, which allows one to include the occurrence of rare events into the modelling of the asset price.

Another way to introduce extra randomness is by making volatility stochastic. Pioneering work of Hull & White (1987), Stein & Stein (1991) and Heston (1993) led to the development of stochastic volatility models. In these models the volatility process σ itself is modelled by a stochastic differential equation with its own Wiener process as a random source.

4.2.2 Stochastic interest rate modelling

Several properties of the GBM are not satisfactory when developing stochastic interest rate models. The unbounded growth over time of price processes, for example, is not assumed to describe the development of interest rates and commodity prices very well.

Research on the modelling of the dynamics of interest rates focuses on either modelling the short rate (equivalent to modelling spot prices) or the forward rate (equivalent to modelling futures/forward prices). The short rate, denoted by $r(t)$, is the annualized interest rate at which an entity can borrow money for an infinitesimally short period of time from time t. The forward rate, denoted by $f(t,T)$, can be interpreted as the annualized interest rate, contracted at t, over the infinitesimal small interval $[T, T+\Delta T]$. From these definitions it can be easily seen that the short rate is in fact a forward rate where the time of contracting coincides with the start of the interval on which the interest rate is effective, i.e., $r(t) = f(t,t)$.

Literature reveals a large number of different stochastic models for the short rate. The general assumption is that the dynamics of the short rate $r(t)$, under the risk-neutral martingale measure Q (see Section 4.3), is given by the stochastic differential equation

$$dr(t) = \mu(t, r(t))dt + \sigma(t, r(t))dW(t),$$

where μ and σ are given functions and $dW(t)$ a Wiener increment under Q. Some of the most popular models are presented in Table 4.1. If a parameter is time dependent this is written out explicitly.

	Dynamics
Vasicek	$dr(t) = a(b - r(t))dt + \sigma dW(t)$
Cox-Ingersoll-Ross	$dr(t) = a(b - r(t))dt + \sigma\sqrt{r(t)}dW(t)$
Hull-White (extended Vasicek)	$dr(t) = (\Theta(t) - a(t)r(t))dt + \sigma(t)dW(t),\ a(t) > 0$
Hull-White (extended CIR)	$dr(t) = (\Theta(t) - a(t)r(t))dt + \sigma(t)\sqrt{r(t)}dW(t),\ a(t) > 0$

Table 4.1: Popular short rate models

To avoid the unbounded growth over time of the GBM, Vasicek (1977) introduced a *mean reverting* process. It has the property of reverting to the long run level b, with speed of reversion governed by the strictly positive parameter a. The economic rationale behind modelling with a mean reverting process is that when the short rate becomes too high it will eventually slow the economy down which will bring the short rate down again and vice versa. The Cox-Ingersoll-Ross (CIR) model (Cox, Ingersoll & Ross, 1985) can be seen as an extension of the Vasicek model where they allow the volatility to be proportional to the short rate level. Hull & White (1990) extended both the Vasicek and the CIR model by letting some or all parameters become time dependent, creating an infinite dimensional parameter vector that makes these types of models particularly appealing for fitting initial term structures.

The main advantage of interest models where the short rate is the only explanatory variable is that it is often possible to obtain analytical formulas for bond prices and other interest rate derivatives. The main drawbacks of short rate models are, however, that it is unreasonable to assume that the entire money market can be explained by only one variable, the short rate. Furthermore, it is hard to obtain a realistic volatility structure for the forward rates without introducing a very complicated short rate model. This has led to the proposal of models that use more than one state variable. The Heath-Jarrow-Morton (HJM) framework (Heath, Jarrow & Morton, 1992) is probably the most important and influential one.

Instead of using the short rate as the only explanatory variable, the HJM framework chooses the entire forward rate curve (forward interest rate values for all time periods that are considered) as their (infinite dimensional) state variable. It is important to notice that the HJM framework does not imply a specific model for the interest rates like the short rate models presented earlier. It is a framework that can be used for analyzing interest rates. The HJM framework differs from short rate models in the sense that HJM-type models automatically capture the full dynamics of the entire initial forward rate curve, i.e., the initial term structure.

The key to the techniques developed by Heath, Jarrow and Morton is that an arbitrage free framework for the stochastic evolution of the entire forward rate curve can be created, where the forward rates are fully specified through their volatility functions. This is expressed through the famous HJM drift condition and implies that within the HJM framework no estimation for the drift parameter(s) is needed once the volatility function is specified.

Although highly appealing from a theoretical point of view, the HJM framework often leads to models which are non-Markovian and can even have infinite dimensions, which makes it hard or even impossible to get analytical solutions. For certain choices of volatility structures, however, this problem can be tackled so that the resulting models can be expressed entirely by a finite-state Markovian system, making it computationally feasible. The mathematical formulation of the HJM framework can be found in Appendix C.

4.3 Derivative pricing

In connection with the first stochastic model for stock prices, Bachelier (1900) was also the first to consider the mathematical pricing of stock derivative contracts. Since then, markets have developed and a large number of derivatives are actively traded on many different markets nowadays. The most common and actively traded contracts are forwards, futures and options. To derive prices for these derivative contracts we need some mathematical tools and concepts that will be discussed in this section. We will also describe different derivative pricing models, with the Black- Scholes model (Black & Scholes, 1973) of course being the most famous one.

Following the work of Bachelier, Samuelson (1965b) considered long-term stock options and used the Geometric Brownian Motion (GBM) to model the random behaviour of the underlying stock. Based on this underlying process, he modelled the random value of the option at the exercise date. His model required the two non-observable parameters α and β as its inputs. Parameter α reflects the expected rate of return of the stock and β represents the discount factor at which the option value at expiration is discounted. Because both parameters are not observable in the market, different observers might propose different values for the parameters, depending on their degree of risk aversion. Because of this, the option price model developed by Samuelson did not offer unique option prices because of the differences in risk aversion of market participants.

To attack the problem of non-unique option prices, Black & Scholes (1973) used a completely different approach that would lead to the famous and widely used Black-Scholes (BS) option pricing formula. Their approach is based on the dynamically hedging (replicating) the option under consideration. They propose that, given certain simplifying assumptions, the cost of this replicating strategy is known in advance. In an arbitrage-free market these costs thus equal the option's price.

The first concept that we have to grasp in order to understand and apply the BS model is the concept of no-arbitrage. Derivative pricing is often based on replicating the final payoff of the derivative with a so-called self-financing trading strategy. We define a trading strategy as a stochastic process adapted to the filtration F_t. This means that the trading strategy can only be based on the information that is available at time t. A self-financing trading strategy then is a trading strategy that requires no further investments after the initial investment. Now, an arbitrage possibility is defined as a self-financing trading strategy that has zero initial cost, a non-negative end wealth P-almost surely and a positive end wealth with positive probability. In other words, an arbitrage opportunity has non-negative end value with probability 1, a positive probability of ending up with a positive end value, all at zero initial cost.

In constructing their famous option-pricing model, Black and Scholes constructed such a self-financing trading strategy that replicated the payoff of a European option. They assumed stock prices to follow a continuous process, a lognormal distribution of the stock price at any point in time, constant interest rate, constant volatility and a stock that does not pay out dividends. We already saw that the GBM model fits these characteristics and Black and Scholes showed that the only theoretical pricing function $\Pi(t) = F(t,S(t))$ that is consistent with the absence of arbitrage opportunities is when $F(t,S(t))$ is the solution to the following boundary value problem:

$$\begin{cases} F_t(t,S(t)) + rS(t)F_s(t,S(t)) + \dfrac{1}{2}S(t)^2\sigma^2(t,S(t))F_{ss}(t,S(t)) - rF(t,S(t)) = 0 \\ F(T,S(T)) = \Phi(S(T)) \end{cases},$$

where r denotes the deterministic risk-free interest rate, $\sigma(t,S(t))$ represents the volatility of the underlying asset $S(t)$, the subscripts denote the partial derivatives of the pricing function and $\Phi(S(T))$ is the payoff of the derivative at maturity T.

Although subject to some weak points concerning the assumptions underlying the model, the reader can feel safe knowing that the pricing equation developed by Black and Scholes is really the 'correct' equation. Luckily enough, there is an alternative argument for the derivation of the solution to this rather horrible pricing equation.

This alternative argument for the pricing of derivatives was developed by Cox, Ross & Rubinstein (1979) and is called risk neutral valuation. They observed that if the parameters of the option-pricing model developed by Samuelson (1965b) were based upon the same assumptions of the Black-Scholes model, they would produce consistent option prices. This observation led them to the conclusion that both approaches were equivalent, although one required the input of two variables dependent on the level of risk aversion of the investor, whereas the other did not. They concluded that these variables must somehow cancel and that, as long as they reflect the same degree of risk aversion, the degree of risk aversion does not affect option prices. If the parameters can reflect any degree of risk aversion, Cox, Ross and Rubinstein brilliantly suggested that the parameters then could also be based on the assumption of no risk aversion at all. Investors with no risk aversion are called risk-neutral investors and don't require excess return for the risk they are taking. Those investors would discount the expected payoff at the risk-free interest rate and do not require excess return for the risk they are taking. It is important to understand that the risk neutral valuation approach does not assume that investors de facto live in a risk neutral world. The derived formula only says that the value of the derivative can be calculated as if we all life in a risk neutral world. In fact, investors can have any risk preference they like, as long as they prefer a larger amount of money to a lesser amount.

The derivation of risk neutral pricing formulas requires the calculation of risk neutral probabilities. These are 'artificial' probabilities such that the theoretical price of the option is equal to the expectation of the option payoff under this new, risk neutral, probability measure.

This new probability measure is often denoted by Q and is closely linked to the concepts of equivalent probability measures and martingales.

Definition 4.2: *A probability measure* Q *is equivalent to the probability measure* P *for a process* X *if for all possible outcomes* a *of* X *it holds that*

$$P(a) > 0 \text{ if and only if } Q(a) > 0.$$

Definition 4.3: *A stochastic process* $X(t)$ *is called a* F_t*-martingale with respect to the probability measure* Q *if the following conditions hold:*

- X is adapted to the filtration $\{F_t\}_{t\geq 0}$
- For all t

$$E^Q\left[|X(t)|\right] < \infty.$$

- For all s and t with $s \leq t$ the following relation holds:

$$E^Q\left[X(t)|F_t\right] < \infty.$$

The first condition simply states that we can observe the value $X(t)$ at time t; the second condition is just a technical condition. The third condition, however, is the most important one. It says that the expected value of a future value of $X(t)$ with respect to the probability measure Q, given the information available to us at time s, is equal to $X(s)$.

In 1979, Harrison & Kreps (1979) combined the concepts of equivalence and the martingale and showed that a market is arbitrage free if, and only if, there exist an equivalent martingale measure. They also showed that in an arbitrage free market the theoretical price $\Pi(t)$ for any derivative claim $\Phi(S(T))$ with maturity T on the underlying asset $S(t)$ is given by:

$$\Pi(t) = S_0(t)E^Q\left[S_0(T)^{-1}\Phi(S(T))|F_t\right],$$

where Q is a martingale measure with $S_0(t)$ as the numeraire. The numeraire refers to the basic standard by which values are measured. In many cases we measure the value by the time value of money, represented by a bank account. When we take the time value of money as our numeraire, i.e.,

$$S_0(t) = S_0(0)\exp(\int_0^t r(s)ds),$$

we get the risk neutral valuation formula which we will define next.

Definition 4.4: *Assuming the existence of a short rate* $r(t)$*, the arbitrage free, risk neutral valuation formula* $\Pi(t)$ *for a derivative claim* $\Phi(S(T))$ *at maturity* T *takes the form*

$$\Pi(t) = E^Q\left[\exp(-\int_t^T r(s)ds)\Phi(S(T))|F_t\right],$$

where Q *is a martingale measure with the bank account as the numeraire.*

When we take the short rate of interest to be a constant, it can be easily seen that the risk neutral valuation formula reduces to

$$\Pi(t) = \exp(-r(T-t))E^Q\left[\Phi(S(T))\middle|F_t\right].$$

With the general results of the risk neutral valuation approach we are now able to calculate theoretical prices for many different types of derivatives. The price of a European call option on a stock derived in this way, for example, can be shown to be consistent with the rather horrible partial differential equation we saw earlier and leads to the famous Black-Scholes option pricing formula. For a complete proof of the Black Scholes equation the reader is referred to Björk (1998).

Definition 4.5: *The Black-Scholes option price formula for a European call option at time t on the stock $S(t)$ with strike price K, and maturity T is given by the formula*

$$\Pi(t) = S(t)N(d_1(t,S(t))) - \exp(-r(T-t))KN(d_2(t,S(t))),$$

where $N()$ is the cumulative distribution function for the standard normal distribution, given by

$$N(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x}\exp(-\frac{t^2}{2})dt$$

and

$$d_1(t,S(t)) = \frac{1}{\sigma\sqrt{T-t}}\left\{\ln\left(\frac{S(t)}{K}\right) + \left(r+\frac{1}{2}\sigma^2\right)(T-t)\right\},$$

$$d_2(t,S(t)) = d_1(t,S(t)) - \sigma\sqrt{T-t}.$$

Definition 4.6: *The Black-76 option price formula for a European call option with exercise date T and exercise price K on a futures contract with underlying asset $S(t)$ and delivery date T_1 is given by*

$$\Pi(t) = F(t,f(t,T_1)) = \exp(-r(T-t))\big(f(t,T_1)N(d_1) - KN(d_2)\big).$$

$N()$ is again the cumulative distribution function of the standard normal distribution described earlier, where

$$d_1(t,f(t,T_1)) = \frac{1}{\sigma\sqrt{T-t}}\left(\ln\left(\frac{f(t,T_1)}{K}\right) + \frac{1}{2}\sigma^2(T-t)\right),$$

and

$$d_2(t,f(t,T_1)) = d_1(t,f(t,T_1)) - \sigma\sqrt{T-t}.$$

Since the electricity futures and options we are interested are derivatives, we can use the risk neutral valuation approach to derive the pricing functions for these products. Remember that $f(t,T_1,T_2)$ denotes the futures price of an electricity futures contract on the average delivery spot price between T_1 and T_2 contracted at time t, and K is the strike price of a European option with expiration date T with the futures contract $f(t,T_1,T_2)$ as the underlying security. Remember that the payoff function of a call option at expiration date T is given by

$$N_{MW}\max\big(f(T,T_1,T_2) - K,0\big),$$

where N_{MW} is the number of delivery hours of the futures contract. Applying the risk neutral valuation approach to the option with payoff function given above results in the following expression for the price $\Pi(t)$ of the option:

$$\Pi(t) = F\big(t, f(t,T_1,T_2)\big) = \exp\big(-r(T-t)\big)N_{MW}E^Q\Big[\max\big(f(T,T_1,T_2)-K,0\big)\Big|F_t\Big].$$

From the expression we see that the expected value is to be taken under the risk neutral probability measure Q. Therefore, we only need to specify the dynamics of the futures price under this risk neutral measure. We do not need the futures price dynamics under the 'real world' (objective) probability measure P at all. The only role played by the objective measure P is that it determines which events are possible and which are not. All the price models proposed in Section 5 will therefore be stated under the risk neutral probability measure Q only. The question that remains is how the expected value of the futures price under the risk neutral measure Q can be calculated. Generally this boils down to numerous calculations of tedious integrals. The 'change of numeraire' technique developed by Geman, El Karoui & Rochet (1995) can, however, drastically reduce the computational work needed.

Despite the ongoing development in derivative pricing, quantitative pricing remains a challenging task. Not always do derivative pricing models result in nice behaved formulas that can be solved analytically. In many cases numerical techniques such as Monte Carlo simulation are necessary to obtain a satisfying solution. In this research we are only interested in models that result in closed-form (analytical) option price formulas that are of the Black-Scholes type. This means that the underlying price process must be log-normally distributed.

4.4 Relation between spot and futures prices

Following the developments in the fixed income markets, recent literature reveals two approaches for the modelling of electricity futures: modelling spot prices and inferring the futures price process, or modelling the electricity futures directly. The latter approach has focused on the application of the Heath-Jarrow-Morton (HJM) framework to the electricity market and will be discussed in Section 5. For the 'spot price approach' to work, a relationship between electricity spot and futures prices has to be derived. This section will present some classical theory and results concerning this relationship and we will see whether these approaches are suitable to be applied to electricity and electricity contracts.

4.4.1 Assumptions and notation

There roughly are three classical views concerning the relationship between spot and futures prices for commodities (Fama & French, 1987). The first theory is linked to storage costs and benefits of holding the commodity, whereas the second and third approaches derive a relationship between futures prices and expected spot prices. But before we will dive into the derivation, we need to make the following assumptions concerning the market and its participants:

- There are no transaction costs involved in the trading business.
- The market participants are subject to the same tax rate on all net trading profits.
- The market participants can borrow and lend money at the same risk-free rate of interest.
- The market participants take advantage of arbitrage opportunities as they occur.

Furthermore, the following notation will be used:

$T-t$: Time to maturity of the forward or futures contract
$S(t)$: Price of the asset underlying the forward or futures contract today

$f(t,T)$: Negotiated contract price to be paid at maturity T
r: Risk-free interest rate per annum, expressed with continuous compounding

4.4.2 Cost of Carry approach

In a market where the spot is liquidly traded, we can perfectly hedge (replicate) a short position in a futures contract by a long position in the spot, financed by borrowing at the risk free rate r. This strategy is known as the buy-and-hold strategy and defines the futures price as

$$f(t,T) = S(t)\exp\big(r(T-t)\big).$$

From this well known relation we see that the futures price converges to the underlying spot price when the time to delivery $T-t$ approaches zero.

In the buy-and-hold strategy it is required to buy and store the underlying spot. For stocks this will not effect the relation, but for most commodities it will. Commodities are physical consumption assets and these usually do not provide income such as a dividend yield we often see with stocks. Actually, commodities are often subjected to significant storage costs. Let U denote the present value of all the storage costs, net of income, during the life of the forward contract. Now consider a forward contract on a consumption asset with price $S(t)$ with storage costs equal to U. Using the notation above, where $T-t$ is the time to maturity, r is the risk-free rate and $f(t,T)$ is the forward price we would expect the relationship between $f(t,T)$ and $S(t)$ to be given by:

$$f(t,T) = \big(S(t)+U\big)\exp\big(r(T-t)\big).$$

To verify the equation, we first consider the case when $f(t,T) > \big(S(t)+U\big)\exp\big(r(T-t)\big)$. In this case, investors can buy the underlying asset and short sell (selling an asset you do not own) the forward contract, hereby creating an arbitrage. When we consider $f(t,T) < \big(S(t)+U\big)\exp\big(r(T-t)\big)$, however, we see that there is nothing to stop this inequality from holding because owners of a consumption commodity keep such an asset for its consumption value, not necessarily because of its investment value. They are therefore reluctant to sell the commodity and buy the futures contract because the futures contract cannot be consumed. The only thing we can conclude for the relationship between spot and futures prices for consumption assets is

$$f(t,T) \le \big(S(t)+U\big)\exp\big(r(T-t)\big).$$

From a price modelling perspective, we are obviously not that happy with this inequality relation. By finding a way of incorporating the benefits of holding the physical asset rather than the futures contract into the relation, we might end up with equality. The benefit of holding the physical asset is often referred to as the convenience yield γ and was introduced for storable commodities by Brennan & Schwartz (1985). The cost of carry, c, measures the storage costs plus the interest that is paid to finance the asset minus the income earned on the asset. The relationship between spot and futures prices can now be summarized in terms of the cost of carry and the convenience yield and is defined as follows:

$$f(t,T) = S(t)\exp\big((c-\gamma)(T-t)\big).$$

4.4.3 Risk neutral expectation approach

The second pricing theory explains the price of a futures contract in terms of the expected future spot price. To derive the mathematical expression, consider an investor to be long one futures contract, contracted at t. The payoff of this position at maturity T is equal to

$$S(T)-f(t,T).$$

Since a futures contract is a derivative, we can apply the risk neutral valuation approach discussed in the Section 4.3 with a constant risk free rate of interest. Hence, since a futures contract is entered at zero cost, the risk neutral pricing formula becomes

$$\Pi(t)=\exp\left(-r(T-t)\right)E^{Q}\left[S(T)-f(t,T)\middle|F_t\right]=0,$$

where Q is the risk neutral martingale measure with the time value of money as the numeraire. The futures price $f(t,T)$ is already determined at time t and can therefore be taken outside the expectation, i.e.,

$$\Pi(t)=\exp\left(-r(T-t)\right)E^{Q}\left[S(T)-f(t,T)\middle|F_t\right]=\exp\left(-r(t-t)\right)E^{Q}\left[S(T)\middle|F_t\right]-\exp\left(-r(T-t)\right)f(t,T)=0$$

$$\exp\left(-r(T-t)\right)f(t,T)=\exp\left(-r(T-t)\right)E^{Q}\left[S(T)\middle|F_t\right],$$

which implies

$$f(t,T)=E^{Q}\left[S(T)\middle|F_t\right].$$

4.4.4 Risk premium Approach

To evaluate the risk premium approach we first need to discuss the rational expectation hypothesis that has also been considered to relate futures and spot prices. This hypothesis states that the futures price is the best prediction of the spot price at delivery, or, in mathematical terms,

$$f(t,T)=E\left[S(T)\middle|F_t\right].$$

A quick look at this formula reveals that the rational expectation hypothesis is identical to the risk neutral expectation approach when $Q=P$. It is not to be expected that the rational expectation hypothesis holds for reasons we already discussed for the risk neutral expectation approach. The theory of normal backwardation argues that producers of commodities wish to hedge their revenues and are therefore willing to accept a discount on the expected future spot price. When we define the risk premium as

$$P(t,T)=f(t,T)-E\left[S(T)\middle|F_t\right],$$

we see that the theory of normal backwardation argues that the futures are traded at a discount when compared to the expected future spot price leading to a negative risk premium, or, in mathematical terms,

$$f(t,T)\leq E\left[S(T)\middle|F_t\right],$$

which implies

$$P(t,T)\leq 0.$$

The existence of a risk premium in the electricity market can be economically explained by a significant difference in the degree of risk aversion between market participants on the supply and demand side of futures contracts. When suppliers of futures contracts are relatively risk averse, they are willing to accept a discount on the expected spot price at delivery to secure their revenues, with a negative risk premium as a result. When buyers of futures contracts are relatively risk averse, on the other hand, they are willing to pay a premium on the expected spot price at delivery to buy a futures contract resulting in positive risk premiums.

For electricity generators (who generally supply futures contracts) it does not make sense to fix the price for their entire output. By ramping up or down the output of power plants they are able to benefit from price fluctuations, especially in the short term. Load Serving Entities (LSE's), on the other hand, distribute and deliver electricity to consumers. They are usually on the demand side of futures contracts and do not have much flexibility to adjust the demand according to the price. Furthermore, they do not want to bear the risk of a high price spike occurring in the near future because these spikes lead to higher electricity prices. Hence, it does make sense for LSE's to fix the price and lock in as much as expected future demand in the futures market. In general we can thus say that we expect the risk premium to be positive, since the buyers of futures contracts are expected to be more risk averse than the seller, especially for short-term contracts.

Recent research confirms our preliminary thoughts. In their research on the Pennsylvania, New Jersey and Maryland (PJM) market, Longstaff & Wang (2004) find evidence for a positive risk premium in the very short-term market.

4.4.5 Application to the electricity market

In this section we will discuss whether the classical approaches can be used to define the relationship between electricity spot and futures prices. The buy-and-hold strategy used in the cost of carry approach requires the storage of the underlying spot. It should come as no surprise that this is practically impossible for electricity due to the non-storable nature. Furthermore, the measurement of the convenience yield is a delicate task since it is not directly observable.

The risk neutral expectation approach may be more convenient and give more flexibility to start with. However, we should notice that electricity futures contracts deliver electricity over a period of time instead of at a fixed date. Consider a futures contract delivering electricity at a constant flow during the delivery period (T_1, T_2). The constant flow is defined as

$$\frac{S(t)}{T_2 - T_1}.$$

Assuming constant interest and settlement at maturity of the contract, the price of this futures contract becomes

$$f(t, T_1, T_2) = E^Q\left[(T_2 - T_1)^{-1}\int_{T_1}^{T_2} S(u)du \middle| F_t\right].$$

We thus have to find the risk neutral measure Q. However, because real time electricity is not traded (only day-ahead) we must conclude that the electricity market is incomplete. As a result, there will not be a unique risk neutral equivalent probability measure and therefore no unique relation between the futures and spot price. By using historical price data the probability measure Q the market is using can be determined, but there is no explicit relation between the futures contract and the expected spot price at delivery. As a conclusion, we cannot derive the futures dynamics based on arbitrage arguments since real spot electricity cannot be traded and stored.

We already saw that the rational expectation hypothesis is identical to the risk neutral expectation approach when we choose P as our risk-neutral probability measure. We therefore face the same problems with the risk premium approach as we did with the risk neutral expectation approach: no unique risk neutral probability measure, hence no unique price. To find the probability measure that is consistent with the market, the market price of risk (risk premium) is estimated from historical data. The probability measure to be used is selected by calibration in such a way that the expressions for the futures contracts derived above hold under this new, risk-adjusted probability measure.

Altogether we see that although some of the classical approaches provide some intuition for the electricity market, there is still no explicit relation between the futures and the (average) spot price. Historical data analysis on the differences between futures prices and the average spot prices of the respective delivery period can provide estimates for the prevailing risk premium in the market but this usually is very difficult. Additional assumptions have to be made to develop models that are consistent with observed market prices.

5 Electricity price models

In this section we will discuss the two stochastic approaches to electricity futures price modelling and discuss several different applications for both approaches. With the basic knowledge on electricity and the (German) electricity market, the data analysis of the German spot and futures market and the mathematical and economical foundation for the derivation of price models provided in the previous sections, we will then be able to choose the best candidate for the modelling of the German electricity futures curve, given our wish for closed-form option pricing formulas.

Modelling electricity futures price processes can be done by following either the 'spot price approach' or the 'futures price approach'. The first approach is based on the accurate modelling of the underlying electricity spot price process, from which the corresponding futures price process is derived by applying one of the relations between spot and futures prices discussed in Section 4.4. The 'futures price approach' tries to model the futures price dynamics directly and is based on the Heath-Jarrow-Morton (HJM) framework (Heath, Jarrow & Morton, 1992) developed for the fixed income market and already discussed in Section 4.2.2.

The first section will provide an overview of electricity spot price models that also have been used as a basis of the spot price approach to the modelling of electricity futures prices. In the second section we will discuss the futures price approach and its applications.

5.1 Spot price approach

The spot price approach defines the spot price dynamics explicitly and derives the corresponding futures price dynamics. A wide variety of spot price models have been introduced in the literature and we unknowingly already discussed some (versions) of them when we talked about the stock and fixed income markets. From the previous sections it was hopefully made clear that electricity price processes exhibit unique features and directly applying spot price models developed for the more traditional markets will therefore generally not lead to satisfying results for the electricity market. The spot price models discussed in this section try to capture (some of) the unique features of electricity spot price processes.

5.1.1 Mean reversion

As one of the characterizing features of many commodity prices, including electricity, practically every spot price model includes mean reversion. Originally introduced by Vasicek (1977) for specifying interest rate dynamics, it was soon adapted for modelling (storable) commodities. Schwartz (1997) provides economic reasoning for the mean reverting nature of commodity prices. He states that relatively high prices lead to increased supply since high-cost suppliers now enter the market, putting a downward pressure on the prices and vice versa, provided that there is no excess capacity which can serve as a barrier to entry. As we saw from our data analysis of the EEX spot market in Section 3.1, electricity prices show clear mean reverting behaviour.

The Ornstein-Uhlenbeck diffusion process is one of the simplest mean reverting models and is defined by

$$dX(t) = \kappa\big(\mu - X(t)\big)dt + \sigma dW(t),$$

where $\kappa > 0$ is the rate of mean reversion, μ is the long term mean of the process, σ is the volatility and $dW(t)$ a Wiener increment. Schwartz (1997) modelled commodity log prices $\ln\big(S(t)\big)$ with this type of model. The one-factor model by Schwartz is defined by

$$dS(t) = \kappa\big(\mu - \ln\big(S(t)\big)\big)S(t)dt + \sigma S(t)dW(t).$$

One of the possible issues with this particular model, however, is the fact that it implicates that the volatility of future returns converge to zero and future prices will converge to a fixed value when maturity increases (Schwartz, 1997). From our data analysis of EEX spot prices we know that this is not a desired feature.

In the same paper, Schwartz also investigates a two-factor model for the development of the spot price. Besides the stochastic spot price, this multi-factor model has a second factor that is also modelled by a stochastic differential equation. He chooses to model the convenience yield as a second factor and defines his two-factor model as:

$$\begin{aligned} dS(t) &= (\mu - \delta)S(t)dt + \sigma_1 S(t)dW_1(t) \\ d\delta(t) &= \kappa(\alpha - \delta)dt + \sigma_2 dW_2(t), \end{aligned}$$

where the increments of the Wiener process are correlated with correlation coefficient ρ, i.e.,

$$dW_1 dW_2 = \rho dt.$$

Compared to Schwartz' one-factor model, this model implies that the volatility of futures will decrease with maturity but will converge to a value different from zero. Schwartz concludes that empirical evidence on oil forward curves implies that this property is more desirable.

5.1.2 Periodic behaviour

The mean reverting models introduced by Schwartz (1997) are in theory able to capture the mean reverting nature of electricity prices. To further enhance the possible fit to market data, Lucia & Schwartz (2002) add a periodic component to a mean reverting model. They investigate a one-factor model for the base spot price defined by

$$\begin{aligned} S(t) &= f(t) + X(t) \\ dX(t) &= -\kappa X(t)dt + \sigma dW(t), \end{aligned}$$

and a one-factor model for the logarithm of the spot price making use of the same stochastic differential equations. This model is defined by

$$\begin{aligned} \ln\big(S(t)\big) &= f(t) + X(t) \\ dX(t) &= -\kappa X(t)dt + \sigma dW(t). \end{aligned}$$

We see that in both cases the dynamics of $X(t)$ are modelled by an Ornstein-Uhlenbeck process with μ equal to zero (i.e., zero long-run mean). The deterministic function $f(t)$ is used to incorporate periodic effects (intra-week and seasonal). In their study of the Nordic electricity market they conclude that the first model gave the better fit to futures prices.

In the second part of their paper they also investigated two-factor models based on the two-factor model of Schwartz & Smith (2000). Schwartz & Smith stochastically model the price dynamics as having a short-term mean reverting component and a long-term equilibrium price level. The two-factor model for the base spot price proposed by Lucia & Schwartz (2002) is defined by:

$$\begin{aligned} S(t) &= f(t) + X(t) + \varepsilon(t) \\ dX(t) &= -\kappa X(t)dt + \sigma_X dW_X(t) \\ d\varepsilon(t) &= \mu_\varepsilon dt + \sigma_\varepsilon dW_\varepsilon(t) \\ dW_X(t)dW_\varepsilon(t) &= \rho dt \end{aligned}$$

A similar two-factor model for modelling the logarithm of the spot price, is defined by:

$$\ln S(t) = f(t) + X(t) + \varepsilon(t)$$
$$dX(t) = -\kappa X(t)dt + \sigma_X dW_X(t)$$
$$d\varepsilon(t) = \mu_\varepsilon dt + \sigma_\varepsilon dW_\varepsilon(t)$$
$$dW_X(t)dW_\varepsilon(t) = \rho dt$$

The mean reverting models (with and without periodic component) we considered so far are not able to capture the observed spikes and jumps in the electricity spot price process. In Section 3.1 we observed that these price spikes are the cause of the high level of volatility and the high kurtosis (fat tails). Two modifications to the mean reverting, periodic models can be applied to capture the spike behaviour: adding jump components or introducing stochastic volatility.

5.1.3 Jump diffusion

The introduction of jump terms implies the adding of at least one extra stochastic factor and several extra parameters, which makes the estimation and fitting to market data more difficult. One of the simplest jump diffusion processes is given by Clewlow & Strickland (2000) and is defined by the equation

$$dS(t) = \kappa\left(\mu - \phi K_m - \ln\left(S(t)\right)\right)S(t)dt + \sigma S(t)dW(t) + KS(t)dN,$$

where κ is again the mean-reversion rate, μ is the long-run mean of $\ln\left(S(t)\right)$, ϕ represents the average number of jumps per year, K_m is the mean jump size, σ denotes the spot price volatility, K is a log-normally distributed jump and dN denotes a Poisson process. A particular shortcoming of this model, though, is that the occurrence positive and negative jumps are independently distributed over time due to the Poisson process. It is therefore not possible to ensure that a large upward jump is shortly followed by a downward jump, a pattern we see in electricity markets. It instead uses the mean reversion rate to force price spikes back to the mean. Increasing the mean reversion rate to ensure a fast return to the mean of price spikes leads to unrealistic mean reversion during more stable periods. One way to solve this problem is by applying so-called regime-switching models. Regime switching models divide the time series into separate phases or regimes with different underlying processes. The occurrence of a jump in electricity spot prices can then be considered as a change to another regime. The switching mechanism is usually governed by a random variable that switches between regimes which themselves are driven by independent stochastic processes.

Further research on (multi-factor) jump-diffusion models was performed by Deng (1999) and Geman & Roncoroni (2006).

5.1.4 Stochastic volatility

Another approach to capture high kurtosis is to introduce the stochastic volatility as a second stochastic factor. In general, these models can be expressed as

$$dS(t) = \mu S(t)dt + \sigma(t)dW_1(t)$$
$$d\sigma(t) = a(t, S(t), \sigma(t))dt + b(t, S(t), \sigma(t))dW_2(t)$$
$$dW_1(t)dW_2(t) = \rho dt$$

From this structure we see that we can fit these types of models to particular markets and price data by choosing the deterministic functions a and b in such a way that the volatility structure is fitted as accurately as possible.

Several authors have proposed different choices for the functions a and b. Hull & White (1987) model the squared volatility (variance) as a Geometric Brownian Motion, Stein & Stein (1991) model the volatility with the standard Ornstein-Uhlenbeck process, whereas Heston

(1993) models the variance with a mean-reverting process similar to the Ornstein-Uhlenbeck process.

5.1.5 Conclusions spot price approach

The spot price models discussed in the previous section have all been considered as a basis to derive the futures price dynamics and several studies have acknowledged their ability to model (several) specific features of electricity spot price processes. However, the spot price process is not our main interest. It is our goal to find a set of models for the German electricity futures curve that allow for analytical, closed form option pricing formulas. In this context, introducing very complicated mathematical functions to accurate model the observed spot price spikes, as is done by Deng (1999) for example, will unnecessarily complicate the derivation of the futures prices further on which could make the model unusable in practise. These spikes are not observed for the EEX futures market and might be accounted for by the high volatility, without imposing an extra factor.

The main drawback of the spot price approach in general is that it is very hard to specify the spot price dynamics in such a way that the theoretical futures prices are consistent with observed market prices, as was discussed in Section 4.4.5. Many additional assumptions, that might not be true in practice, must be made to derive the futures price dynamics and it is very difficult to create a fit with the observed initial futures curve. Furthermore, this approach may lead to very complicated expressions for risk management and option pricing, where numerical methods are called for (Benth & Koekebakker, 2008).

5.2 Futures price approach

Instead of using the spot price and assuming a relation between spot and futures prices, the futures price approach models the dynamics of electricity futures prices directly based on the Heat-Jarrow-Morton (HJM) framework (Heath, Jarrow & Morton, 1992)(see appendix C). Instead of using the short rate dynamics to derive the forward rate dynamics, the HJM framework in fixed income markets directly models the forward rate dynamics and uses the observed initial forward rate curve as a condition, creating a perfect initial fit.

There are two types of applications of the HJM framework for futures price modelling. The first one models the instantaneous-delivery futures contract (no delivery period, delivery on fixed maturity time) and subsequently derives the corresponding dynamics for futures delivering electricity during the delivery period (T_1,T_2). The second approach models the electricity futures with a delivery period directly. We will now discuss both types.

5.2.1 Instantaneous delivery approach

A direct analogy of the HJM framework to electricity markets would be to assume that the futures/forward price dynamics for futures delivering electricity at a specific point in time, under the risk-neutral martingale measure Q are modelled by either one of two types of models (Koekebakker & Ollmar, 2005).

<u>Type 1:</u> Futures price dynamics in which the volatility function(s) are deterministic (but time dependent) and independent of the futures price level. The dynamics are given by

$$df(t,T)=\sum_{i=1}^{K}\sigma_i(t,T)dW_i(t),$$

where K is the number of factors included in the model, W_i, $i=1..K$ are independent Brownian motions under the martingale measure Q and $\sigma_i(t,T)$, $i=1..K$ are time dependent volatility functions. Notice that since it is costless to enter in a futures or forward contract, it must have zero expected return under the martingale measure Q (Black, 1976). This implies that the drift parameter $\alpha(t,T)$ is set equal to zero and therefore disappears from the expression for the dynamics.

The solution is given by

$$f(t,T) = f(0,T) + \sum_{i=1}^{K} \int_{0}^{t} \sigma_i(s,T) dW_i(s),$$

where $f(0,T)$ denotes the observed initial curve. Using the Ito isometry (Bjork, 1998, Ch. 4)

$$E\left[\left(\int_{a}^{b} g(s) dW(s)\right)^2\right] = \int_{a}^{b} E\left[g(s)^2\right] ds,$$

we easily see that this implies that the futures prices are distributed as:

$$f(t,T) \sim N\left(f(0,T), \sum_{i=1}^{K} \int_{0}^{t} \sigma_i^2(s,T) ds\right).$$

<u>Type 2:</u> Futures price dynamics in which the volatility function(s) are deterministic (but time dependent) and proportional to the futures price level. The dynamics are given by

$$\frac{df(t,T)}{f(t,T)} = \sum_{i=1}^{K} \sigma_i(t,T) dW_i(t),$$

with solution

$$f(t,T) = f(0,T) \exp\left(-\frac{1}{2} \sum_{i=1}^{K} \int_{0}^{t} \sigma_i^2(s,T) ds + \sum_{i=1}^{K} \int_{0}^{t} \sigma_i(s,T) dW_i(s)\right).$$

Similar to the derivation of the Geometric Brownian motion in Section 4.2.1 and again using the Ito isometry, we see that the futures prices are distributed as

$$\ln\left(f(t,T)\right) \sim N\left(\ln\left(f(0,T)\right) - \frac{1}{2} \sum_{i=1}^{K} \int_{0}^{t} \sigma_i^2(s,T) ds, \sum_{i=1}^{K} \int_{0}^{t} \sigma_i^2(s,T) ds\right).$$

Because we have the freedom to choose the volatility function(s) as we like, we are able to construct a wide variety of futures price dynamics. To illustrate the fact that the HJM framework itself does not imply a specific model for the electricity prices but rather provides a framework for analyzing them, it can be easily shown that the mean reverting one-factor spot price models proposed by Lucia & Schwartz (2002) discussed in Section 5.1.2, are consistent with the HJM models discussed in this section with one factor $(K = 1)$ and a volatility function defined by

$$\sigma_1(t,T) = \sigma \exp\left(-\kappa(T-t)\right),$$

where σ and κ are positive constants. From this equation we see that the volatility function in these models is decreasing with maturity T, approaching zero when $T \to \infty$.

Besides Koekebakker & Ollmar (2005), Bjerksund, Rasmussen & Stensland (2000) and Keppo, Audet, Heiskanen & Vehvilinen (2004) also considered the HJM framework for the electricity market. They all consider modelling a continuum of instantaneous-delivery futures contracts (no delivery period!), as was specified above. This all seems to work out quite nicely, but we have to be aware of the fact that the dynamics derived according to this procedure are the dynamics for instantaneous-delivery futures and not for the futures delivering electricity over a period of time, the ones actually traded. As a result, when fitting the model to data from electricity markets, a continuous curve of instantaneous-delivery futures contracts has to be created from the observed curve of real electricity futures prices by using some smoothing algorithm.[10]

A one-factor model by Bjerksund *et al.* (2000) developed using this approach is given by

$$\sigma_1(t,T) = \frac{a}{T-t+b} + c,$$

with a, b and c positive constants. With good estimates of the parameters, we see that this volatility function produces a sharply falling volatility curve when T becomes larger, approaching c as $T \to \infty$. This is a more desirable property than the volatility function approaching zero when $T \to \infty$.

Although we can recapture the dynamics of the actually traded contracts, Benth & Koekebakker (2008) argue that the implied dynamics will become very complicated. Furthermore, the analytical tractability for option pricing is lost. They show using arbitrage arguments that, with some technical conditions on the coefficient functions, when the instantaneous-delivery futures dynamics are modelled by the one factor model

$$df(t,T) = \sigma(t,T)f(t,T)dW(t),$$

the implied dynamics for electricity futures delivering electricity during the period (T_1,T_2), which they refer to as swaps because the delivery of electricity over a period of time resembles a swap (swapping fixed for floating cash flows), is given by

$$df(t,T_1,T_2) = \sigma(t,T_2)f(t,T_1,T_2)dW(t) - \int_{T_1}^{T_2} \delta_2\sigma(t,u)\frac{\hat{w}(\tau;T_1,T_2)}{\hat{w}(\tau;T_1,u)}f(t,T_1,u)du dW(t).$$

Here, δ_2 denotes partial differentiation with respect to the second variable and $\hat{w}(\tau;T_1,T_2)$ is related to the settlement of the contract. The important thing to notice about this equation is the fact that the swap dynamics $(df(t,T_1,T_2))$ are not multiplicative and not even Markovian since the swap dynamics depend on the dynamics of all other swaps delivering electricity over the period (T_1,u), $T_1 < u < T_2$. Furthermore, we see that the only way we end up with lognormal swap dynamics (needed for Black-Scholes type analytical closed-from option pricing formulas!), is when σ is not a function of the maturity, i.e., $\delta_2\sigma(t,u) = 0$. We then obtain the lognormal swap dynamics:

$$df(t,T_1,T_2) = \sigma(t)f(t,T_1,T_2)dW(t).$$

Our data analysis of EEX futures prices showed that the volatility of EEX futures depends on the length of the delivery period, the time to delivery and the time of the delivery period. Since we are looking for accurate future price models producing analytical closed-form option pricing formulas, we must therefore conclude that the HJM approach using instantaneous-delivery futures does not fit our needs since we cannot have lognormal swap dynamics and a desirable volatility structure at the same time.

[10] See for example Adams & van Deventer (1994) for an approach based on the maximum smoothness criterion

5.2.2 Direct swap approach

Because the instantaneous-delivery contracts are not traded and the difficulties pointed out, it is tempting from a HJM-framework perspective to model the swap dynamics directly. Benth & Koekebakker (2008) developed this approach and argue that modelling these swaps directly allows one to utilise the information present in swap prices without relying on some ad hoc smoothing algorithm.

A problem encountered when modelling the swap dynamics directly with the HJM-framework, is that the swap dynamics are specified for all delivery times, i.e., it is assumed that the market trades in futures with delivery at all times between today and some specified future point in time. The resulting dynamics under the equivalent risk-neutral martingale measure Q are such that there are no arbitrage opportunities by trading futures with different times to maturity. Since we are dealing with swaps delivering electricity over a period of time, this no-arbitrage requirement becomes rather complicated and is very hard to achieve in practise when we want flexible, tractable models for option pricing (Benth & Koekebakker, 2008). In this setting, Benth & Koekebakker show that a lognormal model for the swap dynamics cannot satisfy the no-arbitrage condition and at the same time possess a volatility structure depending on the delivery period. In conclusion, the only lognormal model for the swaps dynamics, which is arbitrage-free, is again given by

$$df(t,T_1,T_2) = \sigma(t) f(t,T_1,T_2) dW(t).$$

The assumption restricting us from achieving our goal of having lognormal swaps dynamics with a maturity dependent volatility structure is the assumption that we want to model swaps consistently for all possible delivery periods. Benth & Koekebakker show that when we relax the no-arbitrage condition to hold only for actual traded swaps, we can have lognormal swap dynamics and a desirable volatility structure.

This approach focuses on the stochastic modelling of traded swap contracts that cannot be decomposed into other traded swap contracts. Given the stochastic dynamics of these "atomic swaps", a no-arbitrage condition is used to derive the dynamics of the swaps that can be composed by different atomic swaps. In the specific case of the EEX, where monthly, quarterly and yearly contracts are traded, monthly futures are the smallest contracts and therefore become the atomic swaps.

In a lognormal model, the one factor atomic swap dynamics under the equivalent risk-neutral martingale measure Q is given by

$$df(t,T_1,T_2) = \sum(t,T_1,T_2) f(t,T_1,T_2) dW(t),$$

where $\sum(t,T_1,T_2)$ is a continuously differentiable and positive function representing the volatility, $f(t,T_1,T_2)$ is the price of an atomic swap contract delivering electricity during the delivery period (T_1,T_2) and $dW(t)$ a Wiener increment. When we assume settlement at maturity of the contract, the swap volatility associated with the instantaneous-delivery futures volatility function $\sigma(t,T)$ as

$$\sum(t,T_1,T_2) = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} \sigma(t,u) du.$$

By this equation, well known volatility functions proposed for instantaneous-delivery futures in many different (commodity) markets can now be related to electricity futures contracts. Benth & Koekebakker (2008) investigate six different volatility functions, all inspired by instantaneous-delivery futures models. Similar to our results, they find a strong maturity effect for the volatility that cannot be modelled with a simple negative exponential function. They also find evidence of seasonal changing volatility and state that the two effects together are best captured using an additive volatility function.

By using the change of numeraire technique developed by Geman, Karoui *et al.* (1995), they also derive an expression for European options. In the case of a lognormal specification of $f(t,T_1,T_2)$ with volatility function $\sum(t,T_1,T_2)$, the price $\Pi(t)$of a European call option with maturity τ and strike K on the swap $f(t,T_1,T_2)$ with $\tau \leq T_1$ is given by,

$$\Pi(t) = \exp\left(-r(T-t)\right)\left(f(t,T_1,T_2)N(d_1) - KN(d_2)\right).$$

Here $N()$ is the cumulative distribution function for the standard normal distribution, given by

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp(-\frac{t^2}{2})dt,$$

and

$$d_1(t,\tau) = \frac{\ln\left(\frac{f(t,T_1,T_2)}{K}\right) + \frac{1}{2}\int_t^{\tau} \sum^2(s,T_1,T_2)ds}{\sqrt{\int_t^{\tau} \sum^2(s,T_1,T_2)ds}},$$

$$d_2(t,\tau) = d_1(t,\tau) - \sqrt{\int_t^{\tau} \sum^2(s,T_1,T_2)ds}.$$

5.2.3 Conclusions futures price approach

Comparing the two futures price approaches that were discussed, we must first conclude that the approach using instantaneous-delivery futures will not satisfy our needs, as was already pointed out in Section 5.2.1. The direct swap approach, on the other hand does fit our needs. The simplified 'market model' using only atomic swaps that was proposed by Benth & Koekebakker (2008) allows us to specify the desired volatility function and produces Black-Scholes type, closed-form option pricing formulas for a lognormal specification of the atomic swaps. The fact that we can only use data from the atomic swaps is a disadvantage of this particular model because the market information present in the discarded observations is lost. We believe, however, that because relative little contracts overlap, not that much information is lost. A disadvantage of the futures price approach in general is the fact that it is not possible to infer spot prices from the futures price dynamics. However, this is of minor care to us since we are interested in the futures market and not the spot market per se. Lastly, Benth & Koekebakker show that although the lognormal specification of the swap dynamics produces analytical, closed-form option pricing formulas and allows for a volatility structure that depends on the delivery period, the lognormal specification is not able to fully capture the fat tails of the log returns.

6 Conclusion

At the start of the research the following research question was formulated:

> Which price models are best suited to model the German electricity futures curve, taking into account our wish to have closed-form option pricing formulas?

To answer the question we first presented an overview of the specific properties of electricity and the implications on price processes. Stylised facts of electricity spot price processes include price spikes, periodic behaviour with different periods, mean reversion and time varying high levels of volatility. Futures prices are expected to exhibit periodic patterns and show lower, but still time varying volatility.

From our data analysis we conclude that the EEX price processes exhibit many of the stylised facts. Our main findings for spot prices include very high levels of volatility and intra-day as well as intra-week periodic patterns. We also found some evidence for seasonal patterns, although less explicit. The main finding of our analysis of the EEX futures prices is that the level of volatility depends on the length of the delivery period, the time to delivery and the time of the delivery period. In order to accurately model the EEX futures price curve our candidate model must be able to capture this complex volatility structure.

Given our aim of stochastically modelling the electricity futures price and based on the extensive number of scientific contributions that were read, we conclude that electricity futures prices are stochastically modelled using either the spot price approach or the futures price approach.

With the spot price approach we rely on a relation between the spot and futures prices to derive the futures price process. However, due to the non-storability of electricity we found that there is no explicit relation between spot and futures prices. Fitting the parameters as well as creating an accurate fit of the model with observed data is very difficult. Furthermore, it may lead to very difficult expressions for option prices, where numerical methods are needed. Therefore, we believe that the spot price approach is not ideal to model the EEX futures curve.

As an alternative to the spot price approach two types of the futures price approach were discussed: the instantaneous-delivery futures approach and the direct swap approach. We found that with the instantaneous-delivery approach we only end up with analytical, closed-form option pricing formulas by modelling the volatility as being independent of the delivery period. Because we showed that the volatility structure of EEX futures prices is a complex function depending on the delivery period we discard the instantaneous-delivery futures approach.

Finally, the model that we consider to be the best candidate to the modelling of the EEX futures price curve, while producing closed-form option pricing formulas is the direct swap approach proposed by Benth & Koekebakker (2008). With this approach we do not rely on non-explicit price relations and smoothing algorithms but only use the information of the EEX futures market prices. We can furthermore specify the complex volatility structure, create a perfect initial fit and still have analytical, closed-form option pricing formulas for a lognormal specification of the swaps.

There are, however, also some disadvantages of this approach. We can only use market data for those traded swap contracts that cannot be decomposed into other traded swap contracts, the atomic swaps. We thus loose the information contained in other prices, but we believe that because relative little contracts overlap, not that much information is lost. We furthermore need the lognormal specification of the atomic swaps in order to have closed-form option pricing formulas. In an application to futures price data from Nordpool, the lognormal model is not able to capture the fat tails of the log returns, leaving room for improvement. A disadvantage of the futures price approach in general is the fact that it is not possible to infer spot prices from the futures price dynamics. However, this is of minor care to us since we are interested in the futures market and not the spot market per se.

In future works we will analyze the complex volatility structure of futures prices further, will actually fit the direct swap approach to the EEX futures data using stochastic filtering techniques and find out which model performs best.

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Appendix A – Stochastic Modelling

This appendix will present some basic results from stochastic modelling. For a more detailed review we refer to Bjork (1998).

When modelling asset prices in continuous time, the most complete and elegant theory is obtained if we use diffusion processes and stochastic differential equations as our building blocks. A stochastic process $X(t)$ is said to be a diffusion equation (or difference equation) if its local dynamics can be approximated by a stochastic difference equation of the following type:

$$X(t+\Delta t) - X(t) = \mu\big(t, X(t)\big)\Delta t + \sigma\big(t, X(t)\big)Z(t).$$

With $Z(t)$ a normally distributed disturbance (noise) term which is adapted to the filtration generated by the process up until time t, while μ (drift term) and σ (diffusion term) given deterministic functions. We say that a process is adapted to the filtration F_t if its future value is independent of the history of the process up until time t. To model the normal disturbance term $Z(t)$ we need to specify a Wiener process.

Definition A.1: *A stochastic process* $W(\;)$ *is called a Wiener process if the following conditions hold:*

1. $W(0) = 0$
2. the process W has independent increments, i.e., if $r < s \le t < u$ then $W(u) - W(t)$ and $W(s) - W(r)$ are independent stochastic variables.
3. For $s < t$ the stochastic variable $W(t) - W(s)$ has the Gaussian distribution $N\big(0, t - s\big)$.
4. W has continuous trajectories.

Remark A.1: We use the notation $N\big(\mu, \sigma^2\big)$ for the Gaussian distribution with expected value μ and variance σ^2.

With the definitions of the Wiener process and the stochastic difference equation we can now write:

$$X(t+\Delta t) - X(t) = \mu\big(t, X(t)\big)\Delta t + \sigma\big(t, X(t)\big)\Delta W(t),$$

where $\Delta W(t)$ is defined by

$$\Delta W(t) = W(t+\Delta t) - W(t).$$

Our goal is to find an expression for the diffusion equation that describes the infinitesimal changes of the process $X(t)$. It is then tempting to divide by Δt and let Δt go to zero. However, it can be shown that with probability 1 a Wiener process trajectory is nowhere differentiable which implies that the time derivative of a Wiener process does not exist. We therefore just let Δt go to zero and we obtain the expression

$$\begin{cases} dX(t) = \mu\big(t, X(t)\big)dt + \sigma\big(t, X(t)\big)dW(t) \\ X(0) = a \end{cases}.$$

This stochastic differential expression is the building block for all the analysis done in this paper and the models can all be considered to be variations of this expression.

Appendix B – Itô's Lemma

Theorem B.1 (Itô's Lemma): *Assume that the process* $X(t)$ *has a stochastic differential given by*

$$dX(t) = \mu(t)dt + \sigma(t)dW(t),$$

Where μ and σ are adapted processes, and let f be a $C^{1,2}$-function. Define the process Z by $Z(t) = f(t, X(t))$. Then $Z(t)$ has a stochastic differential given by

$$df(t, X(t)) = \left\{ \frac{\delta f}{\delta t} dt + \frac{\delta f}{\delta x} dX(t) + \frac{1}{2} \frac{\delta^2 f}{\delta x^2} \left(dX(t) \right)^2 \right\},$$

where we use the following formal multiplication table.

$$\begin{cases} (dt)^2 = 0 \\ dtdW(t) = 0 \\ \left(dW(t) \right)^2 = dt \end{cases}$$

Appendix C – HJM framework

In this appendix we will discuss the Heath-Jarrow-Morton (HJM) framework (1992) for the stochastic modelling of interest rate dynamics. Before we discuss the HJM itself we first have to understand how short rates, forward rates and zero-coupon bonds are defined and related.

Definition C.1: *The short rate, the instantanuous forward rate and zero-coupon bond are defined as follows*

> *Short rate*
> The short rate, denoted by $r(t)$, is the annualized interest rate at which an entity can borrow money for an infinitesimally short period of time from time t.
>
> *Instantaneous Forward rate*
> The forward rate, denoted by $f(t,T)$, can be interpreted as the annualized interest rate, contracted at t, over the infinitesimal small interval $(T, T+\Delta T)$.
>
> *Zero-coupon bond*
> A zero-coupon bond with maturity date T is a contract that guarantees the holder 1 Euro to be paid on the date T. The price at time t of a zero-coupon bond with maturity T is denoted by $P(t,T)$.

Definition C.2

1. The instantaneous forward rate $f(t,T)$ can be defined by

$$f(t,T) = -\frac{\delta \log P(t,T)}{\delta T}.$$

2. The instantaneous short $r(t)$ is can be defined by

$$r(t) = f(t,t).$$

As an immediate consequence of these definitions we see that

$$P(t,T) = \exp\left(-\int_t^T f(t,s)ds\right).$$

We now turn to the HJM framework itself and start by assuming the following for the forward rate dynamics.

Assumption C.1:

Under the martingale measure Q we assume that the forward rates are specified as

$$\begin{cases} df(t,T) = \alpha(t,T)dt + \sigma(t,T)dW(t) \\ f(0,T) = f^*(0,T) \end{cases},$$

where W is a Wiener process under the measure Q, α represents the drift of the forward rates, σ represents the volatility of the forward rates and $f^*(0,T)$ is the observed initial forward rate curve.

Since we modelled the forward rate dynamics directly under the martingale measure Q, we automatically have arbitrage free prices and we don't have the problem of checking whether the market is arbitrage free. We do, however, now have two expressions for the same zero-coupon bond price today.

$$P(0,T) = \exp\left(-\int_0^T f(0,s)ds\right)$$

$$P(0,T) = E^Q\left[\exp\left(-\int_0^T r(s)ds\right)\right]$$

In order for these two formulas two hold simultaneously a famous relation between α and σ in the forward rate dynamics was developed. This relation is known as the HJM drift condition.

Proposition C.1 (HJM drift condition) under the martingale measure Q, the processes α and σ must satisfy the following relation, for every t and every $T \geq t$.

$$\alpha(t,T) = \sigma(t,T)\int_t^T \sigma(t,s)'ds$$

From this proposition we see that when we specify the forward rate dynamics under the martingale measure Q, we can freely specify the volatility structure. The drift parameters are then uniquely determined

Schematically, the use of the HJM model can now be written as follows:

1. Specify, by own choice, the volatilities $\sigma(t,T)$
2. The drift parameters of the forward rates are then uniquely determined by

$$\alpha(t,T) = \sigma(t,T)\int_t^T \sigma(t,s)'ds \quad .$$

3. Go to the market and observe today's forward rate structure

$$\left\{f^*(0,T); T \geq 0\right\}.$$

4. Integrate the forward rate dynamics to get the forward rates as

$$f(t,T) = f^*(0,T) + \int_0^t \alpha(s,T)ds + \int_0^t \sigma(s,T)dW(s) \quad .$$