Cross-sectional stability of tidal inlets: influence of basin geometry and basin friction

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# Cross-sectional stability of tidal inlets: influence of basin geometry and basin friction

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Pictures on cover page: schematisation of 2DH model setup and graphic of the Kelvin and Poincare modes. See chapter 3 for context.

# Abstract

An inlet cross-section is considered in equilibrium when there is no net import or export of sediment from the inlet channel, and cross-sectionally stable when small deviations from this state return the inlet to the equilibrium. It is common to assess the cross-sectional stability of tidal inlets using zero dimensional pumping-mode (PM) models in combination with the stability concept of Escoffier (1940). It is currently unknown what the influence of basin geometry and basin friction is on the stability of tidal inlets. The objective of this study is to investigate the effect of basin geometry and friction on the hydrodynamics and cross-sectional stability of tidal inlets and to evaluate the validity of using PM models to assess inlet stability.

This is done by formulating an idealised, linear, horizontal depth-averaged two dimensional (2DH) model for single inlet systems. The model is applied on the Frisian and Texel inlet systems. Both systems are part of the Dutch Wadden Sea inlets. Both inlets are schematised as three adjacent compartments: the ocean, inlet and basin. Each compartment is characterised by a width  $W_j$ , length  $L_j$ , depth  $H_j$  and offset to the system centre  $\delta_j$ . Derived characteristics are the basin surface area  $A_b = W_b L_b$ , basin aspect ratio  $S_b = \frac{W_b}{L_b}$  and inlet cross-sectional area  $A_i = W_i H_i$ . The model is forced by an incoming Kelvin wave in the ocean compartment.

Formulating the model required choosing representative values for the ocean compartment and the amplitude of the incoming Kelvin wave to ensure a desired tidal range in front of the inlet mouth, amongst others. Bottom friction in the inlet and basin was determined using an iterative method to assess the value of Lorentz' linear friction coefficient, which involves a velocity scale. A PM-model was formulated accounting damping through friction in the inlet and radiation damping.

Regarding hydrodynamics, it was found that the frequency of maximum tidal amplification due to Helmholtz-mode resonance is sensitive to basin geometry. This is attributed to tidal wave propagation through the basin. It was furthermore found that the PM model corrected for radiation damping better predicts the trend of the 2DH model at short inlet channels. This suggest that radiation damping is more important for short inlets. The Texel inlet system is much more dissipative than the Frisian inlet system.

Regarding cross-sectional stability it was found that basin geometry has a greater influence on large systems. The aspect ratio of the basin is found to have a profound influence on inlet stability at large basins. In the Texel inlet case, the case-study presented in chapter 6 showed that the aspect ratio of the basin could even result in the absence of a stable root.

The influence of two damming projects in the Wadden Sea, which altered the basin geometry, are studied in case studies. The case studies showed that the damming of the Lauwerzee in 1969 causes the Frisian inlet channel to diminish in size. Basin depth has a negligible effect on the predicted stable inlet cross-sectional area. The PM- and 2DH model results show similar predictions.

The Texel inlet is predicted to increase its cross-sectional area as a response to the closure of the Zuiderzee, assuming that the closure of the Zuiderzee led to a higher average basin depth. Basin depth has a large influence on the predicted cross-sectional area. Retaining the average basin depth before closure, the stable cross-sectional area will even slightly decrease, while a very steep increase in depth could lead to a cross-sectional area which is twice as big as before the closure. It is concluded that in such systems, basin depth is the most uncertain and important parameter for determining the stable cross-sectional area. It is concluded that the PM model is only valid for relatively small or deep basins.

Keywords: tidal inlet, inlet stability, pumping mode, 2DH, basin geometry, basin friction

# Preface & acknowledgements

We will never cease from exploration And at the end of all our exploring Will be to arrive where we started And know the place for the first time.

T.S. Eliot, Four Quartets 1942

What lies in front of you is not a document fit for poems. It is a scientific work, the result of applying the scientific method. Its subject is specialised, idealised and technical. Its contents objective and rational. It is only in this preface that the author is allowed to express personal thoughts. As the culmination of the academic student's career and usually the largest single piece of research it is common to profess experience gained and difficulties mastered. Little is so straightforward as doing science. We start where others ended, claiming a little bit of "unexplored land" every time using increasingly ingeneous methods. The formidity of these methods allow for great progress but disarticulate knowledge of the practitioner. Especially so for students, fundamental understanding of the *how* is not a first requirement to gain abundant results from the many advanced models to choose from.

Notwithstanding how much fun it is to study, write and build a model — even debugging, in moderation, has its charm — and how challenging it can be to understand and apply the mathematics underlying it, the greatest challenge is to keep track of *what* it is you're doing. After all, being a student is more than getting at the end of the line of a particular section of science. This is why I took the liberty of including this small poem. To add a little bit of me, human, to place where it not supposed to be. It is a great challenge to rediscover time and time again our own situation, to see ourselves embedded in history, tradition and beliefs that give us direction and inspiration. Even more so for those of the exact sciences. **Acknowledgements** First of all, I would like to acknowledge my supervisors Suzanne, Pieter, Henk and Ronald for their guidance and valuable feedback, without which this thesis would not have seen the light of day. I would like to especially thank Pieter for his invaluable expertise and time. He has the gift of inspired teaching and a contagious enthousiasm.

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Koen Berends Enschede, Wednesday 15<sup>th</sup> August, 2012

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# CHAPTER

# Introduction

From an engineering perspective, the boundaries between ocean and land are of great interest. The coastal area accommodates many functions. Often the coastal zone is part of the nearcoast ecological zone. For low-lying countries the coastal zone is an important line of defence against the sea, and many countries have sea-ports or other important navigational routes near or through the coastal zone. Proper management of these functions requires knowledge of the natural system.

## **1.1** Barrier coasts

Not all coasts can be described as a closed, single-line front that clearly separates land from water. While some coast are like this — e.g. the Western Dutch coastline — others appear jagged such as the Scandinavian peninsula. Yet others have such a large coastal zone that the line between water and land is rather diffuse - not clear where the sea begins and land ends.

Almost 15% of the world's coastlines consist of barrier coasts, many of which have barrier islands (de Swart and Zimmerman, 2009). Barrier islands are large morphological features with lengths ranging up to 50 km and widths up to several km — and support several functions including agriculture and population. Examples of well-studied barrier island systems are the Dutch, German and Danish Wadden coastline (e.g. van der Vegt et al. (2007); Herman (2007), see figure 1.2), the Ria Formosa system in southern Portugal (e.g. Pacheco et al. (2010)), the Venice Lagoon (Tambroni and Seminara, 2006) and systems on the east coast of the United States (e.g. the Beaufort Inlet

in North Carolina (Hench and Luettich, 2003)). Barriers coast are characterised by an inner basin that lies behind the outer coastline; the basin connected to the sea by a relative narrow inlet. By way of this inlet, the inner basin co-oscillates with the tidal movement of the sea.

## **1.2** Tidal inlet systems

The tidal inlet system contains distinct features that are schematically shown in figure 1.1. A typical system consists of several morphological systems that are created by the dynamics of the tides and wave-induced littoral drift along the coasts (de Swart and Zimmerman, 2009). Akin to fluvial delta's, the ebb-tidal delta is formed from sediment transported out off the basin during ebb. A similar feature is found at the basin-side of the inlet. This flood tidal delta is characterised by channels and local topographic highs (Hayes, 1980). The tidal divide is the boundary between two adjacent systems, characterised by low flow velocities and an elevated bed level (Vroom and Wang, 2012). The tidal inlet itself is the conduit between the open sea or ocean and a back-barrier basin.

From an engineering point of view one of the most important questions is whether or not an inlet will remain open on the long term. The cross-sectional area of the inlet is determined by the hydrodynamics of tidal inlet system. According to Escoffier (1940) an inlet cross-section is stable if the maximum flow velocity in the inlet is equal to a certain equilibrium velocity and if any deviation from the stable situation causes the system to react in such a way, that the former stability is once again attained. The underlying thought behind this concept is the balance of two opposing mechanisms; wind-wave induced littoral transport on the one hand which transport sediment into the inlet, and tide-induced sediment transport clearing the inlet of sediment on the other hand.

After major events such as heavy storms, or after human intervention such as damming part of the tidal basin, the bathymetry and hydrodynamics of tidal inlets systems are known to change in such a way that the tidal inlet is forced towards a new state (van de Kreeke, 2004). Knowledge of the processes that play a role in inlet stabilisation is key to many engineering applications, such as building jetties to stabilise the inlet or dredging to keep the inlet open.

The tidal inlet itself is a relatively short and narrow channel. Therefore, the currents in the channel are driven by the hydraulic gradient between sea and basin, instead of progression of the tidal wave (Cln. Brown in CEM (2001)). Following this observation, currents in the inlet have been modelled by using a relatively simple oscillation model (e.g. (Escoffier, 1940; Keulegan, 1967; van de Kreeke, 1990)), which do not take basin geometry and bottom friction in the basin into account. This model will be referred to as the *pumping mode* (PM-) model. A mathematical description of such an approach is given in chapter 2. It is postulated (Brouwer et al., 2012a)



Figure 1.1: Sketch of the features of a tidal system

that the validity of using pumping mode models to assess inlet stability depends on geometry and bottom friction of the basin.

More recent, advances in numerical modelling and increase of computational power have led to detailed studies of the two-dimensional flow field (e.g. Hench and Luettich (2003); Herman (2007); Tran et al. (2012)), but systematic studies of parameters pertaining to basin geometry or bottom friction in the basin are still not feasible. An idealised semi-analytical two-dimensional (2DH<sup>1</sup>) model such as the model presented by Roos et al. (2011) provides a computationally attractive alternative.

# 1.3 Research objectives

#### 1.3.1 Knowledge gap

It is common to study inlet stability using pumping-mode models, limiting possibilities to study the influence of the geometry of the back-barrier basin on stability. However, it is postulated (Brouwer et al., 2012a) that the validity of using pumping mode models to assess inlet stability depends on geometry and bottom friction of the basin.

It is currently unknown what the influence of basin geometry and basin friction is on the stability of tidal inlets.

<sup>&</sup>lt;sup>1</sup>2DH is two-dimensional in the horizontal plane.



Figure 1.2: The location of the wadden sea barrier Islands.

#### 1.3.2 Research objective

The objective of this study is to investigate the effect of basin friction and basin geometry on the hydrodynamics and morphodynamics of the single inlet systems and to evaluate the validity of using PM-models to assess inlet stability.

This objective will be achieved by modifying the idealised 2DH model presented by Roos et al. (2011) to be used for tidal inlet systems and assessing the influence of two-dimensional parameters on hydro- and morphodynamic indicators. The initial analyses will be performed using the (highly) schematised Texel and Frisian inlet systems as basis. In a separate case-study, the influence of human intervention in both systems on inlet stability will be studied.

#### 1.3.3 Research questions

The aim is to answer the following questions with respect to the model setup:

- 1. How can the 2DH hydrodynamic model be formulated for a single-inlet system?
- 2. How can inlet morphodynamics be incorporated in the 2DH model?

With respect to the model results we want to know how two-dimensional variables affect hydrodynamics and morphodynamics of the inlet system and channel.

- 3. In what way does the 2DH model reproduce system hydrodynamics in comparison with tidal resonance with respect to the PM-model, and what is the influence of the physical mechanisms of radiation damping, bottom friction and basin geometry?
- 4. In what way does the 2DH model predict inlet stability and how sensitive is the stability of the inlet to parameters and processes added by the 2DH model?

Two case-studies introduced in section 2.4- the Texel and Frisian inlet systems, provide tangible illustration. In an idealised setup, the effect of human intervention - e.g. damming of large parts of the basin - on basin hydrodynamics will be studied. Consequently, the stable cross-sectional area — if it exists — will be predicted in order to answer the following question:

5. What is the effect of large-scale damming in tidal systems of the Dutch Wadden Sea on system dynamics, with emphasis on inlet channel stability?

## 1.4 Research approach & outline

#### 1.4.1 Methodology

To systematically research the effect of the basin geometry and basin friction of the basin first the hydrodynamics, and secondly the morphodynamic stability is studied. For reasons mentioned in section 1.3.1 an idealised 2DH-model is used. The results from this model are systematically compared with a pumping-mode or PM model. The PM-model is presented in chapter 2. The 2DH model is presented in chapter 3 and described in more detail in Appendix A. The basic systems used in the studies on hydro- and morphodynamics are the Frisian and Texel inlet systems. To minimise the deviation from the PM-model and clarify comparison between the two models in the hydro- and morphodynamic studies, the aspect ratio and offset of the basin are set to default<sup>2</sup> values of 1 and 0 respectively.

On the subject of hydrodynamics the focus is on resonant amplification<sup>3</sup> of the tidal wave in the basin. Specifically, the focus is on the so-called Helmholtz or eigenfrequency resonance which is most likely to occur in tidal inlet systems the size of the Wadden Sea inlets. Besides that, it allows for direct comparison with the PM-model. The characteristics of spatial structure of the basin studied are the **aspect ratio**<sup>4</sup> ( $S_b$ ) and **depth**  $H_i$  — both cannot be modelled using the

<sup>&</sup>lt;sup>2</sup>An aspect ratio  $S_b$  of 1 signifies a basin with equal length and width. An offset of 0 signifies that the inlet is placed exactly at the centreline of the basin

<sup>&</sup>lt;sup>3</sup>The measure for amplification is introduced in chapter 3

<sup>&</sup>lt;sup>4</sup>Defined as the ratio of the width and length of the basin. Please see chapter 3.

PM model. To study the relative effect of radiation damping the **length**  $(L_i)$  and **width**  $(W_i)$  of the inlet are varied. Both can be modelled using the PM-model, one directly  $(L_i)$  and the other indirectly as part of the inlet cross-sectional area  $A_i$ . Known effects of radiation damping are that the inlet length in the PM-model must be taken longer to account for parts of the ocean being directly affected by oscillation of the basin, and friction in the inlet should be increased.

On the subject of morphodynamic stability the frequency of the incoming tidal wave is fixed at the frequency of the M2 tidal constituent. The effect of **basin area** — keeping the shape constant — on the predicted cross-sectional area  $A_i$  is studied to assess the general comparison between 2DH and PM model prediction, both with default and effective inlet length. Consequently, the **basin aspect ratio** and **basin depth** are varied to assess the effect of the spatial structure on inlet stability. Furthermore, because the 2DH model allows for a different bottom friction coefficient per compartment, the effect of bottom friction in the basin is studied.

Two case-studies are presented to study the influence of spatial structure of the basin on inlet stability in a practical case. The two case studies are introduced in section 2.4. They are used as basic systems throughout the report. The effect of drastic reduction of the basin size of both systems is studied with respect to the the estimated cross-sectional area.

#### 1.4.2 Report outline

The report is outlined as follows:

**Chapter 2: Theoretical Background** In this chapter the concept of inlet stability and the theoretical background of the PM-model are introduced. Furthermore the two case-studies are introduced.

Chapter 3: 2DH Single Inlet Model: theoretical background and methodology This chapter introduces the new 2DH model. Central to this chapter are the first two research questions. Necessary design choices and limitations are discussed - a more technical discourse is found in Appendix A.

Chapter 4: Hydrodynamic properties of tidal inlet systems This chapter is centred around the third research question. Here basin free surface elevation as simulated by the pumping mode model is compared to results from the 2DH model. Differences are explained in the context of the influence of bottom friction and basin geometry. Additional dynamics exposed by the 2DH model are highlighted.

**Chapter 5: Morphodynamic stability of the inlet channel** This chapter is centred around the fourth research question. Here the sensitivity of the predicted cross-sectional area of the inlet to bottom friction and basin geometry is estimated. The results are compared to pumping mode model predictions.

**Chapter 6: Case studies** Two case studies are presented: the closure of the Lauwersea in 1969 and the closure of the Zuiderzee in 1932. The stable cross-sectional area after closure is determined, and the sensitivity to parameters is assessed.

**Chapter 7: Discussion** This chapter discusses some limitations of the 2DH model, the stability concept and methodological choices.

**Chapter 8: Conclusions & Recommendations** In this chapter the research questions are answered, and some recommendation for future research are made.

# CHAPTER 2

# Theoretical Background

T<sup>HIS</sup> chapter introduces the concept of inlet stability, the pumping mode (PM) model, the damping mechanisms of bottom friction and radiation damping and introduces the two basic inlet systems.

## 2.1 Inlet stability

The basic concept presented in all stability studies is that inlet stability depends on two mechanisms; import and export of sediment. Import of sediment mainly takes place because of wind-wave induced littoral transport — which like a conveyor belt moves sediment along the coast. At inlets in the coast the sediment deposits. The tides act as main agent of sediment export, flushing the inlet twice every tidal cycle. Weather conditions such as storms might at once deposit large amounts of sediment into the inlet mouth, or create an additional inlet in an hitherto closed coast. From an engineering perspective there is a strong need for relatively simple relationships to assess stability of an inlet. Two related approaches are those of Escoffier (1940) and prism-gap relationships O'Brien (1931, 1969). Both approaches assume that there exists an equilibrium velocity (Escoffier) or tidal prism for which no net sediment erosion or deposition will take place.

Tidal prism relationships directly relate the tidal prism to cross-sectional area of the inlet. The tidal prism is defined as the amount of water coming in at ebb or flood. Accounting only for one tidal component, the tidal prism can be estimated by integration of the discharge or velocity curve

$$u_{max} = \frac{\pi P}{A_i T} \tag{2.1}$$

where P is the tidal prism  $[m^3]$ ,  $A_i$  the cross sectional area of the inlet  $[m^2]$ ,  $u_{max}$  the maximum velocity per tidal cycle in the inlet  $[ms^{-1}]$  and T the tidal period in seconds. Equations that relate the tidal prism to the equilibrium cross-sectional area are usually of the form

$$A = mP^n \tag{2.2}$$

where n is a dimensionless constant and m a constant with its dimension dependent upon the value of n. This equation is usually attributed to O'Brien (1931). Many studies have subsequently been performed to estimate the constants using measured data. An overview of the use of (2.2) is given by Stive and Rakhorst (2008) and D'Alpaos et al. (2009). Regression analysis to determine the value of the constants in (2.2) sprouted several relationships which are more or less applicable to a selected set of inlets, e.g. the 'Furkert-Heath' relationship for inlets on the New Zealand coast (Hume and Herdendorf, 1988).

A less abstract approach is that of Escoffier (1940). The key assumption is that for a certain depth-and cross-sectional averaged maximum velocity, the inlet will be in (dynamic) equilibrium. His original assumption that this velocity is about  $1 \text{ ms}^{-1}$  has been sustained in literature, though Kraus (1998) mentions that this mainly regards exposed inlets. Sheltered inlets can be stable at lower velocities because of smaller littoral drift. This highlights that the empirical nature of such approaches, while attractive because of their seeming simplicity, do not take into account littoral drift, grain sizes, vertical processes and sediment transport processes.



Figure 2.1: Sketch of Escoffier's (Escoffier, 1940) stability concept for tidal inlets. Inlets with velocities lower than the equilibrium velocity tend to decrease in cross-sectional area, while those with higher velocity increase. This system tendency - denoted by the arrows in the figure - leaves a stable and unstable root.

# 2.2 Pumping mode model

In a simplified approach, a tidal embayment can be modelled as a 'mass-spring-system' with external forcing. Analogously, this system behaves much like when you hold a spring in your hand which has a mass attached to it. By moving your hand up and down, external forcing is applied and the mass starts to co-oscillate with the movement of your hand. This kind of problem is wellknown in engineering sciences as the damped and forced simple oscillator. The main assumptions



Figure 2.2: A sketch of a schematized tidal inlet system showing the parameters of the pumping mode model

in this model are that the free surface  $\zeta$  has spatially uniform movement in the basin and the flow velocity u is uniform over the inlet. The physical system is governed by two equations. Figure 2.2 shows a sketch of the model parameters. The first is conservation of mass for the basin;

$$A_b \frac{\partial \zeta_b}{\partial t} = -A_i u_i \tag{2.3}$$

where  $A_b$  is the surface area of the basin [m<sup>2</sup>],  $\zeta_b$  the free surface elevation of the basin [m],  $u_i$  the flow velocity in the inlet [ms<sup>-1</sup>] and  $A_i$  the cross-sectional area of the inlet [m<sup>2</sup>]. The second equation is the conservation of momentum in the inlet channel:

$$\frac{\partial u_i}{\partial t} + \frac{r_i}{H_i}u = -g\frac{\zeta_o - \zeta_b}{L_i} \tag{2.4}$$

where g is the gravitational acceleration  $[ms^{-2}]$ ,  $\zeta_o$  the free surface elevation of the ocean [m],  $L_i$ the length of the inlet [m],  $H_i$  the depth of the inlet and  $r_i$  a friction coefficient  $[ms^{-1}]$ . The external forcing  $\zeta_o$  is assumed to be a sinusoidal oscillation with amplitude  $Z_o$  and angular frequency  $\sigma$ . Therefore both equations can be combined to the well-known Helmholtz equation

$$\frac{\partial^2 \zeta_b}{\partial t^2} + \frac{r_i}{H_i} \frac{\partial \zeta_b}{\partial t} + \sigma_0^2 \zeta_b = \sigma_0^2 \zeta_o \tag{2.5}$$

which is known as the damped and forced harmonic oscillator with

$$\sigma_0 = \sqrt{\frac{gA_i}{A_bL_i}} \tag{2.6}$$

The non-transient solution to this equation is:

$$\zeta_b = \Re \left( Z_o \hat{\zeta}_b e^{i\sigma t} \right), \quad u_i = \Re \left( i\sigma \frac{A_b}{A_i} Z_i \hat{\zeta}_b e^{i\sigma t} \right)$$
(2.7)

where  $Z_o$  is the amplitude of the tidal wave at the inlet mouth,  $\hat{\zeta}_b$  the basin amplitude function and  $u_b$  is the depth-average flow velocity in the inlet. The amplitude function  $\hat{\zeta}_b$  and modulus of the amplitude function  $|\hat{\zeta}_b|$  - which returns half the tidal range - is given by

$$\hat{\zeta}_b = \left(-\frac{\sigma^2}{\sigma_0^2} + i\frac{r_i}{H_i}\frac{\sigma}{\sigma_0^2} + 1\right)^{-1}, \quad |\hat{\zeta}_b| = \left(\sqrt{\left(1 - \frac{\sigma^2}{\sigma_0^2}\right)^2 + \left(\frac{r_i}{H_i}\frac{\sigma}{\sigma_0^2}\right)^2}\right)^{-1}$$

Consequently, by taking the derivative of  $\hat{\zeta}_b$  with respect to  $\sigma_0$ , the equation for maximum amplitude including friction is derived:

$$\sigma_{max} = \sqrt{\sigma_0^2 - \frac{1}{2} (\frac{r_i}{H_i})^2}$$
(2.8)

The value for the friction coefficient  $r_i$  introduced in (2.4) is found using the Lorentz' linear friction coefficient. For a sinusoidal tidal signal r is expressed as

$$r = \frac{8c_d u_{max,i}}{3\pi} \tag{2.9}$$

with the constant dimensionless drag coefficient  $c_d = 2.5 \times 10^{-3}$  and  $u_{max}$  the maximum depthaveraged tidal velocity. For applications in ocean models,  $u_{max}$  is usually estimated to be equal to the maximum velocity of a tidal wave:

$$u_{max,i} = Z_o \sqrt{\frac{g}{H_i}} \tag{2.10}$$

Velocities in tidal channels potentially exceeds this estimation significantly. In a semi-nonlinear approach, the friction coefficient is determined iteratively, and equation (2.9) is used as a first guess. Please see section 3.2.1 for a discussion on this subject.

# 2.3 Radiation damping

Two other large scale processes contribute to damping, i.e. radiation damping and horizontal flow separation (Maas, 1997). Flow separation can be parameterised through entrance/exit losses

(Brouwer et al., 2012b), but is not considered in this study. Radiation damping is represented in the 2DH model, and can be parameterised in the PM model by simultaneously adjusting the length of the inlet and the bottom friction parameter. Physically, radiation damping occurs when amplification in the tidal basin causes waves to be radiated back into the sea (Maas, 1997). The following parameterisation of radiation damping follows (P.C. Roos, personal communication). Recalling (2.7), the complex amplitude of the free surface elevation of the basin is the product of the amplitude of the ocean in front of the inlet  $Z_o$  and the amplitude function of the basin  $\hat{\zeta}_b$ . With radiation damping  $Z_o$  is also influence by the oscillation of the inlet. By assuming that  $Z_o$  is the result of the superposition of two waves — one forced, incoming wave and one scattered wave from the inlet — Roos, following Buchwald (1971), concluded that the effect of the scattered wave on  $A_{eq}$  can be expressed as follows

$$\zeta_b = \Re \left( (Z_o + Z_{o,s}) \hat{\zeta}_b e^{i\sigma t} \right) = \Re \left( Z_o \hat{\zeta}_{b,eff} e^{i\sigma t} \right)$$
(2.11)

where  $\hat{\zeta}_{b,eff}$  is the amplitude function corrected for radiation damping:

$$\hat{\zeta}_{b,eff} = \left(\sqrt{\left(1 - \frac{\sigma^2}{\sigma_{\theta,eff}^2}\right)^2 + \left(\frac{r_{i,eff}}{H_i}\frac{\sigma}{\sigma_{\theta,eff}^2}\right)^2}\right)^{-1}, \quad \sigma_{\theta,eff} = \sqrt{\frac{gA_i}{A_b L_{i,eff}}} \tag{2.12}$$

where the effective length  $L_{eff}$  and effective bottom friction  $r_{eff}$  are expressed as

$$L_{i,eff} = L_i + \frac{H_i}{H_o} \frac{W_i}{\pi} \left(\frac{3}{2} - \Gamma - \ln \frac{\pi W_i}{\lambda}\right), \quad \frac{r_{eff}}{H_i} = \frac{L_i}{L_{i,eff}} \frac{r_i}{H_i} + \frac{H_i}{H_o} \frac{\sigma W_i}{2L_{eff}}$$
(2.13)

where  $\Gamma$  is Eulers' constant (0.5772...) and  $\lambda = \frac{2\pi\sqrt{gH_o}}{\sigma}$  the wavelength of the forcing wave.

## 2.4 The Wadden Sea inlets

The barrier coast protecting the Dutch Wadden Sea (figure 1.2) contains several tidal inlet systems. Two of them are used as a case study in chapter 6, and serve as basic systems for the analyses of chapters 4 and 5.

#### 2.4.1 Importance of the Wadden Sea

The Wadden sea has been UNESCO world heritage since 2009, being "one of the last remaining natural large-scale intertidal ecosystems, where natural processes continue to function largely undisturbed" (UNESCO, 2012). However, land subsidence in the near future due to the extraction of natural gas is estimated to be up to 48 cm by 2050 (Nederlandse Aardolie Maatschappij B.V., 2005), while the studies by Elias (2003a) and Oost (1995) have shown that its inlets still adapt to changes due to the closure of the Zuiderzee (1932) and Lauwerzee (1969). Knowledge of hydrodynamics and morphodynamics of such systems is vital to protect and preserve valuable systems

such as the Wadden Sea. In his review of the historial data of the Wadden Sea, Oost (1995) gives an anthology of sources that highlights the importance of understanding the interaction between basin hydrodynamics and inlet morphodynamics. In the second half of the 15<sup>th</sup> century people of Holland complained about loss of land due to increased tidal amplitude, blaming it on the increasing cross-sectional area of the inlets (Oost, 1995). Oost, citing Sha (1989), remarks this might be due to the increase of the tidal basin resulting from the storm surge of 1477, increasing the tidal prism. This illustrates how changes in either basin or inlet geometry affect each other and are of significant importance to people surrounding the basins. Aside from storms, changes in the tidal prism can also be initiated by human intervention. Two cases stand out: the closure of the Zuiderzee in 1932 and the damming of the Lauwerszee in 1969. Figure 2.3 gives an overview of the current and former basins. Table 2.1 sums up the characteristics of the inlets.

Table 2.1: Characteristics of Wadden Sea inlets prior to human intervention

Inlet System	Basin			Inlet			
	Р	$A_b$	$H_b$	$W_i$	$H_i$	$L_i$	$Z_o$
Frisian Inlet	$0.31 \ 10^9 \ {\rm m}^3$	$126~{\rm km^2}$	4 m	$3.2 \text{ km}^1$	$7 \mathrm{m}$	$3 \mathrm{km}$	$1.25 \mathrm{~m}$
Texel Inlet	$0.79 \ 10^9 \ { m m}^3$	$4000~\rm km^2$	$4.5 \mathrm{m}$	$4.5 \mathrm{~km}$	12.3 <sup>2</sup> m	$14 \mathrm{km}$	$0.7 \mathrm{~m}$

#### 2.4.2 Frisian Inlet

The Frisian Inlet or "Friesche Zeegat" is the usually defined as the area between the barrier islands of Ameland and Schiermonnikoog. It features two main channels; the Pinkegat channel serving the relatively small Wieremurwad basin<sup>3</sup> and the Zoutkamperlaag channel. Despite the name, the Lauwers inlet east of Schiermonnikoog was not the channel draining the Lauwerszee, though it had until 1550 A.D. (Oost, 1995). In the following I will restrict the definition of the Frisian Inlet to the Zoutkamperlaag-Lauwerszee system, treating the Engelsmanplaat high as watershed between the Zoutkamperlaag and Pinkegat systems. This assumption was also used by van de Kreeke

<sup>&</sup>lt;sup>1</sup>While the channel between Ameland and Schiermonnikoog is about 10 km wide, the Zoutkamperlaag channel is estimated to be about 3.2 km wide. The cross-sectional area of 24.500 m<sup>2</sup> reported by van de Kreeke (2004) lead to a depth of about 7.7 m

<sup>&</sup>lt;sup>2</sup>It is assumed that the Marsdiep was at a dynamic equilibrium prior to the closure of the Zuiderzee. According to Elias (2003a), the tidal prism of the Texel inlet increased with 26% after the closure. Its current prism is just over  $1 \times 10^9$  m<sup>3</sup>. The tidal prism before closure (around 1926) is assumed to be approximately  $0.79 \times 10^9$  m<sup>3</sup>. The maximum velocity in the inlet is assumed to be  $1 \times \text{ms}^{-1}$  - following Escoffier's theorem (Escoffier, 1940). The cross-sectional area is then calculated using the following formula (from van de Kreeke (2004)):  $A_{eq} = \frac{\pi P}{u_{eq}T}$  where P is the tidal prism and T the period of the M2 tidal wave. The inlet is schematised as a rectangular box allowing calculation of the 'effective' depth given the width.

<sup>&</sup>lt;sup>3</sup>Having a basin size of about  $52 \text{ km}^2$  (Maas, 1997)

(2004). The closure of the Lauwerszee in 1969 brought about several changes in the system. Its surface area was reduced by about 30% from  $125 \text{ km}^2$  to  $90 \text{ km}^2$  (van de Kreeke, 2004). Soon after closure of the inlet high sedimentation rates were reported in the basin leading to reduction of the cross-sectional area and depth of the inlet (Oost, 1995; van de Kreeke, 2004).

#### 2.4.3 Texel Inlet

The Texel inlet is the largest tidal inlet of the Wadden Sea, located between the island of Texel and the shore of Holland. It is characterised by a pronounced ebb-tidal delta - the "Noorderhaaks" shoal - and the deep Marsdiep channel. Prior to the damming of the former Zuiderzee, the Texel Inlet was one of two inlets draining the basin - the Vlie inlet its companion. The Eierlandse Gat inlet was both now and before the closure, separated from the Vlie and Marsdiep systems by a tidal divide (Elias, 2003b). After closure, the Vlie and Marsdiep systems seem to have reverted from a double inlet system to two single inlet systems with only limited transport between them (Ridderinkhof, 1988). Recently van de Kreeke et al. (2008) argued that equilibrium cross-sectional areas of inlets in a double-inlet system separated by a topographic high — a 'Wantij' — approach those that would be expected using two single-inlet systems. The damming of the Zuiderzee had pronounced consequences. The mean tidal range at Den Helder increased significantly and suddenly from 1.15 m to about 1.35 m, the tidal prism increased<sup>4</sup> from around 600  $10^6$  m<sup>3</sup> to 1100  $10^6 \text{ m}^3$  and the Marsdiep channel depth increased (Elias, 2003a,b), while the basin size decreased dramatically. Elias (2003b), based on expert judgement, suspects that it will still be many decades until a new dynamic equilibrium is reached, while Kragtwijk et al. (2004) think that it will take a least a century.

<sup>&</sup>lt;sup>4</sup>The increase of the tidal prism is also partly attributed to sea level rise from 1870 onward (Elias, 2003a).



Figure 2.3: The basins of the Texel and Frisian inlet before and after basin reduction. The Zuiderzee was dammed in 1932, the Lauwerszee in 1969.

# CHAPTER 3

# 2DH Single Inlet Model: theoretical background and methodology

THIS chapter contains a description of the single inlet or 2DH model, its derivation from the so-called 'Taylor' model (Taylor, 1921), the solution method for several compartments and implementation for single inlet systems. The mathematical description of the model in this chapter is concise; for a more elaborate description please see Appendix A.

## 3.1 The Taylor model

Taylor (1921) solved the problem of tide propagation in semi-enclosed tidal basins. In the following decades, his solution has been expanded to allow for multiple compartments (Godin, 1965), energy dissipation at the closed end (Hendershott and Speranza, 1971), bottom friction (Rienecker and Teubner, 1980), horizontal viscosity (Roos and Schuttelaars, 2009) and depth variations in longitudinal and lateral directions (Roos and Schuttelaars, 2011). This study is restricted to the extension of Taylor's model for multiple compartments, including bottom friction. Assuming that horizontaly viscous effects and advective terms can be neglected, constant density and  $H \gg \zeta$ ,



Figure 3.1: Top view of the semi-enclosed basin

the linear shallow water equations in the f-plane are formulated as follows

$$\frac{\partial u}{\partial t} - fv + \frac{ru}{H} = -g\frac{\partial\zeta}{\partial x}$$
(3.1a)

$$\frac{\partial v}{\partial t} + fu + \frac{rv}{H} = -g\frac{\partial\zeta}{\partial y}$$
(3.1b)

$$\frac{\partial\zeta}{\partial t} + H(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x}) = 0$$
(3.1c)

and boundary conditions

$$v = 0 \text{ at } y = 0 \text{ and } y = W \tag{3.2}$$

$$u = 0 \text{ at } x = 0$$
 (3.3)

where f is the Coriolis parameter  $f = 2\Omega \sin \theta \ [s^{-1}]$  with latitude  $\theta$  and  $\Omega$  the angular frequency of Earth's rotation, H the uniform depth [m] and r the linear bottom friction coefficient  $[ms^{-1}]$ . The velocity components u, v are given in  $[ms^{-1}]$ , time t in [s] and spatial dimensions x, y in [m]. Figure 3.1 shows a sketch of the system. At the open boundary at x = L the system is forced by an incoming Kelvin wave.

Equations (3.1a)-(3.2) allow for typical wave solutions known as Kelvin and Poincaré modes, valid for an infinite channel. Examples of these modes are plotted in figure 3.2. When the additional boundary condition (3.3) is imposed, the system becomes closed on one end and, as a result, the boundary conditions can be met by neither Kelvin nor Poincaré modes alone. Taylor (1921) solved this problem by proposing a superposition of two Kelvin waves and an infinite number of Poincaré

modes. The resulting problem pertaining to free surface elevation is described as follows

$$\zeta(x,y,t) = \Re \left\{ Z_f \left( \sum_{n=1}^{\infty} \left( \alpha_n \hat{\zeta}_n(y) \right) e^{ik_n x} + \hat{\zeta}_f(y) e^{ikx} + \alpha_r \hat{\zeta}_r(y) e^{-ikx} \right) e^{-i\sigma t} \right\}$$
(3.4a)

$$u(x, y, t) = \Re \left\{ Z_f \left( \sum_{n=1}^{\infty} \left( \alpha_n \hat{u}_n(y) \right) e^{ik_n x} + \hat{u}_f(y) e^{ikx} + \alpha_r \hat{u}_r(y) e^{-ikx} \right) e^{-i\sigma t} \right\}$$
(3.4b)

$$v(x,y,t) = \Re \left\{ Z_f\left(\sum_{n=1}^{\infty} \left(\alpha_n \hat{v}_n(y)\right) e^{ik_n x} + \hat{v}_f(y) e^{ikx} + \alpha_r \hat{v}_r(y) e^{-ikx}\right) e^{-i\sigma t} \right\}$$
(3.4c)

where  $\alpha_r$  and  $\alpha_n$  are the (complex) amplitudes of the reflected Kelvin and Poincaré modes respectively, relative to the amplitude of the forced wave  $Z_f$ . The other symbols are wavenumber k for Kelvin waves and wavenumber  $k_n$  for the n-th Poincaré mode  $[m^{-1}]$ , angular frequency  $\sigma$   $[s^{-1}]$ , the amplitude functions  $\hat{\zeta}_{n,r,f}$ ,  $\hat{u}_{n,r,f}$ ,  $\hat{v}_{n,r,f}$  of the Poincaré, reflected and forced (incoming) Kelvin waves. From a practical perspective the infinite summation in (3.6a) must be truncated. A finite summation is used instead, in combination with a collocation method to determine the as of yet unknown relative amplitudes  $\alpha_r$ ,  $\alpha_n$ . The collocation method involves choosing m+1 points along x = 0 where m is the number of Poincaré modes and formulating for these locations the equations resulting from (3.6b) and the boundary conditions. The resulting system of linear equations can be solved by matrix algebra:

$$\begin{bmatrix} \hat{u}_r(y_1) & \hat{u}_1(y_1) & \hat{u}_2(y_1) & \cdots & \hat{u}_m(y_1) \\ \hat{u}_r(y_2) & \hat{u}_1(y_2) & \hat{u}_2(y_2) & \cdots & \hat{u}_m(y_2) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \hat{u}_r(y_{m+1}) & \hat{u}_1(y_{m+1}) & \hat{u}_2(y_{m+1}) & \cdots & \hat{u}_m(y_{m+1}) \end{bmatrix} \begin{bmatrix} \alpha \\ \alpha_1 \\ \vdots \\ \alpha_m \end{bmatrix} = - \begin{bmatrix} \hat{u}_i(y_1) \\ \hat{u}_i(y_2) \\ \vdots \\ \hat{u}_i(y_m + 1) \end{bmatrix}$$

Figure 3.2 shows that there are two kinds of Poincaré modes: trapped and free. In absence of bottom friction, free poincare waves have a real wavenumber, which returns in a sinusoidal spatial structure. Trapped waves are characterised by an imaginary wavenumber, and have only a sinusoidal structure in one direction. Whether the wavenumber is imaginary or complex depends on the tidal frequency  $\sigma$ , latitude  $\theta$  and the depth H and width W of the basin. Without friction, there are a finite number of free modes and infinite number of trapped modes. In reality, most seas are to narrow to allow for free Poincaré modes at all.

When bottom friction is included the clear distinction between trapped and free waves is lost since the wavenumber is no longer strictly real or imaginary. This results in both kind of waves showing behaviour characteristic of the other (trapped waves propagating in one direction).



Figure 3.2: A 'shapshot' of a Kelvin, a free and a trapped Poincaré wave at an arbitrary moment in time. Damping is through bottom friction.

# 3.2 Extension to multiple compartments

Whereas a single compartment features a single, closed boundary condition at x = 0, multiple connected compartments require that same boundary to be at least partly open. At closed (dry) boundaries no-flow condition (3.5a) is applied. At open (wet) boundaries, two matching boundary conditions are necessary. These are the equal-flux (3.5b) and equal-surface elevation (3.5c) conditions:

$$H_j u_j(x_c, y_c, t) = 0$$
 (3.5a)

$$H_{j}u_{j}(x_{c}, y_{c}, t) - H_{j+1}u_{j+1}(x_{c}, y_{c}, t) = 0$$
(3.5b)

$$\zeta_j(x_c, y_c, t) - \zeta_{j+1}(x_c, y_c, t) = 0$$
(3.5c)

Where  $x_c, y_c$  are the coordinates of points along the boundary. An example with two compartments is sketched in figure 3.3. All compartments, with exception of the first, have two 'families' of modes. A family of modes is defined as one Kelvin mode and m Poincaré modes that propagate or decay in the same direction. For any but the first compartment the Taylor problem (3.6) is extended;

where  $\hat{\zeta}^+, \hat{u}^+, \zeta^+$  represent the family of modes which has one Kelvin mode propagating in the positive *x*-direction and a set of Poincaré modes decaying or propagating in the positive *x*-direction.



Figure 3.3: Modes in multiple compartments. Small arrows denote the propagation direction of Kelvin waves and free Poincaré modes. Big arrows the decay direction of trapped Poincaré modes.

Unknown still are the relative amplitudes of the respective modes represented by  $\alpha$ , which is a vector containing the unknown amplitudes  $\alpha$ . They can be found using the collocation method.



Figure 3.4: Placing of collocation points. Red points denote a closed boundary, blue points an open boundary.

The collocation method for multiple compartments involves (i) choosing a set of predefined points — referred to as 'collocation points'— at the boundaries where two compartments connect and (ii) building a system of linear equations that can be solved by matrix inversion. The placing of these collocation points is subject to some limitations. A collocation point should be placed at every the corner of the compartments and the spacing of the points should be equal along the boundaries. A point can be placed at a closed or open boundary. Figure 3.4 shows an example. At closed points only the no-flow boundary condition applies with modes from 1 compartment.

$$\boldsymbol{\alpha}_{\boldsymbol{j},\boldsymbol{c}}\hat{u}_{\boldsymbol{j}}(\boldsymbol{x}_{c,c},\boldsymbol{y}_{c,c}) = 0 \tag{3.7}$$

where  $x_{c,c}, y_{c,c}$  are the coordinates of the 'closed' collocation points. At open point two conditions apply, the 'no flow' and matching free-surface:

$$\Sigma \boldsymbol{\alpha}_{\boldsymbol{j},\boldsymbol{o}} \hat{u}_j(x_{c,o}, y_{c,o}) H_j = \boldsymbol{\alpha}_{\boldsymbol{j+1},\boldsymbol{o}} \hat{u}_{j+1}(x_{c,o}, y_{c,o}) H_{j+1}$$
(3.8a)

$$\boldsymbol{\alpha}_{\boldsymbol{j},\boldsymbol{o}}\hat{\zeta}_{\boldsymbol{j}}(\boldsymbol{x}_{c,o},\boldsymbol{y}_{c,o}) = \boldsymbol{\alpha}_{\boldsymbol{j+1},\boldsymbol{o}}\hat{\zeta}_{\boldsymbol{j+1}}(\boldsymbol{x}_{c,o},\boldsymbol{y}_{c,o})$$
(3.8b)

Equations (3.7) and (3.8a) form a system of linear equations which is solved using matrix inversion.

#### 3.2.1 Iterative determination of bottom friction

The 2DH model allows for a different bottom friction coefficient per compartment. Bottom friction based on Lorentz' linearisation;

$$r_j = \frac{8c_d U_j}{3\pi} \tag{3.9}$$

where  $c_d$  is a drag coefficient [-],  $r_j$  the friction coefficient [ms<sup>-1</sup>] in compartment j and  $U_j$  a maximum velocity representative for the compartment j [ms<sup>-1</sup>]. It is common to use the maximum velocity of the tidal wave as a measure;

$$U_j = Z_o \sqrt{gH_j^{-1}} \tag{3.10}$$

where  $Z_o$  is the amplitude of the tidal wave [m], g the gravitational acceleration [ms<sup>-2</sup>] and  $H_j$ the depth of the compartment. For the ocean compartment in the case of the Frisian inlet, a depth of 20 m and tidal amplitude of 1.25 meter in front of the inlet corresponds to a maximum velocity of 0.875 ms<sup>-1</sup>. The relatively shallow basin and inlet would in turn receive a much higher bottom friction coefficient. However, it is argued that the dynamics of the inlet-basin system invalidate such an approach, since the flow in the inlet mainly results from the pressure gradient between ocean and basin, rather than progression of the tidal wave (see also Cln. Brown in (CEM, 2001)). The effect of iterative determination is shown in figure 3.5, and compared with the friction determined using (3.10) for the Frisian inlet system. Results show that iteration leads to lower friction coefficients, especially in the basin. The trends are similar, with a decreasing friction coefficient with increasing depths. It is concluded that friction cannot be determined using (3.10). For this reason bottom friction is determined iterativily. For every iteration, the velocity measure per compartment  $U_j$  is calculated from the average velocity amplitude in the compartment with area  $A_j$ 

$$U_j = \frac{1}{A_j} \iint \sqrt{\hat{u}_j^2 + \hat{v}_j^2} \, dx \, dy$$

and the friction coefficient  $r_j$  is re-evaluated. In this way, the inlet and basin compartment friction coefficients are iteratively determined. The ocean compartment not, since this would results in rather subjective results; its results greatly depend on the length of the ocean cell.



Figure 3.5: The effect of iterative determination on the friction coefficient is demonstrated the Frisian inlet system ( $A_b = 126 \text{ km}^2$ ,  $L_i = 3 \text{ km}$ ,  $W_i = 3 \text{ km}$ ,  $H_b = 4\text{m}$ ,  $H_i = 7.5 \text{ m}$ ,  $S_b = 1$ ,  $\delta_b = 0$ ) and M2-tidal frequency.

#### 3.2.2 Abrupt corner problem

The boundary between j and j + 1 has, at the side of compartment j, two boundary conditions. It should be noted that this delivers a discontinuity in the system. The solution method solves this problem by superposition of Kelvin and Poincaré waves, i.e. a superposition of continuous functions. This results in a phenomenon analogous to the so-called *Gibbs phenomenon* - well known in the field of signal processing in Fourier analysis. The influence of this phenomenon can be suppressed by increasing the number of modes, but nonetheless results in peaks at the corners in the u flow field. A possible solution is to include horizontal viscosity and forcing a no-slip condition at the closed boundaries; Roos and Schuttelaars (2009) have extended Taylor's problem to include viscous effects. However, this is greatly increases the complexity of the solution method. Moreover, the extension to multiple compartments proved to be rather difficult - if not impossible (P.C. Roos, personal communication).



Figure 3.6: Schematisation of an idealised single inlet system showing the dimensions of the compartments, direction of velocity components u, v, amplitude Z and free surface elevation  $\zeta$ 

# 3.3 Inlet schematisation

Inlet systems are schematised as a three-box model. Each compartment is defined by a set of parameters, which are the length  $L_j$ , width  $W_j$ , depth  $H_j$  and offset  $\delta_j$  from the inlet centre line - all in meters. Figure 3.6 shows a schematic of the system. Derived parameters are the surface area of the basin  $A_b$ , the cross-sectional area of the inlet  $A_i$  and the shape of the basin  $S_b$ ;

$$A_b = W_b L_b, \quad S_b = \frac{W_b}{L_b}, \quad A_i = W_i H_i$$

The shallow water equations for this system are solved using the 2DH-model described in chapter 2.

# 3.4 Model implementation

The 2DH model is implemented in MatLab. The linear system of equation is solved by matrix inversion - which results in the amplitudes of all the waves in the system. Parameters of the system are summarized in table 3.1.

#### 3.4.1 Dimensions of ocean compartment

The depth of the ocean compartment is taken at 20 meters, which is assumed to be representative for the North Sea. The length of the ocean compartment has arbitrarily been chosen at 100 km.
System Dimensions	$\{D_j, W_j, L_j, \delta_j\}$	see table $2.1$		
Length of ocean compartment	$L_o$	$100 \mathrm{km}$		
Width of ocean compartment	$W_o$	$290~\mathrm{km}$		
Depth of ocean compartment	$D_o$	20 m		
Latitude	θ	52°		
Tidal Frequency	$\sigma$	$1.405 \times 10^{-4} s^{-1}$		
Tidal amplitude of incoming wave	$Z_{f}$	see section $3.4.2$		
Collocation point spacing	[-]	see section 3.4.3		

Table 3.1: Parameters of the 2DH single inlet model

The width of the ocean compartment should be chosen so, that the M2-tidal wave passing in front of the inlet mouth closely resembles a Kelvin wave. Good resemblance was found at a width of 290 km.

#### 3.4.2 Amplification measure

Resonance is defined as amplification of the tidal elevation oscillation, causing the basin to have higher amplitudes than the tidal amplitude itself. The amplification factor is accordingly defined as

$$F = \frac{\langle |Z_b| \rangle}{|Z_o|} \tag{3.11}$$

where  $\langle |Z_b| \rangle$  is the spatially averaged amplitude of the free surface in the basin [m] and  $Z_o$  the amplitude of the free surface [m] just outside the inlet.  $|Z_o|$  is determined so that is resembles the tidal amplitude of the Frisian and Texel inlet systems — 1.25 and 0.7 m respectively. This requires knowing what amplitude  $Z_f$  must be used as input of the 2DH model in order to obtain the desired  $Z_o$  (see figure 3.7). To this end, the model is run without the inlet — a 'Taylor-problem' set-up — and run for different values of the forcing frequency and forcing amplitude. Through frictional damping and phase differences, controlled by  $\sigma$ , the amplitude in front of the inlet mouth  $Z_o$  differs.

#### 3.4.3 Model resolution

The collocation method used to solve the linear system of equations involves having to define a set of collocation points. The spacing of the collocation points is ultimately dependent upon the geometry of the inlet; i.e. there is a minimum number of collocation points over the inlet. It was found that the solution method required an equidistant set of collocation points. The result



Figure 3.7: The amplitude in front of the inlet  $Z_o$  differs from the amplitude of the amplitude of the incoming wave  $Z_f$  through frictional losses and phase differences.

is that inlet width can only be varied with discrete steps, i.e. only along the collocation points. The standard collocation point spacing used is 1000 m for hydrodynamic analyses and 500 m for stability analysis.

# CHAPTER 4

## Hydrodynamic properties of tidal inlet systems

This chapter shows how basin geometry, basin friction and inlet geometry affect hydrodynamic properties. The two systems that are used as a basis for the parameter variation are schematised versions of the Frisian and Texel inlet systems. Their characteristics are repeated in table 4.1 - a more detailed introduction of the systems has been given in section 2.4. The amplification factor of the basin — introduced in section 3.4.2 — is defined as the ratio of the tidal amplitude in front of the basin without inlet  $Z_o$  and the spatially-averaged tidal amplitude in the basin  $Z_b$ . To minimise the deviation from the PM-model and clarify comparison between the two inlet systems in the hydro- and morphodynamic studies, the aspect ratio and offset of the basin are set to default<sup>1</sup> values of 1 and 0 respectively. An amplification factor lower than 1 indicate a *damped* response, while a value higher than 1 indicates an *amplified* response.

Inlet System	Basin					Inlet			
	Р	$A_b$	$H_b$	$S_b$	$\delta_b$	$W_i$	$H_i$	$L_i$	$Z_o$
Frisian Inlet	$0.31 \ 10^9 \ {\rm m}^3$	$126 \ \mathrm{km^2}$	4 m	1	0	3.2 km	7 m	$3 \mathrm{km}$	$1.25 \mathrm{~m}$
Texel Inlet	$0.79 \ 10^9 \ {\rm m}^3$	$4000 \ \mathrm{km^2}$	$4.5 \mathrm{m}$	1	0	$4.5~\mathrm{km}$	$12.3 \mathrm{~m}$	$14 \mathrm{km}$	$0.7 \mathrm{~m}$

Table 4.1: Characteristics of Wadden Sea inlets prior to human intervention

<sup>&</sup>lt;sup>1</sup>An aspect ratio  $S_b$  of 1 signifies a basin with equal length and width. An offset of 0 signifies that the inlet is placed exactly at the centreline of the basin



Figure 4.1: The effect of **bottom friction in the basin** on amplification of the tidal wave. White-Blue colors denote a *damped* response, while yellow-red colors denote *amplification*.

## 4.1 Basin Friction

Section 3.2.1 detailed how bottom friction is included in the 2DH model. To study the effect of friction in the basin, the friction coefficients in the ocean and inlet compartment are kept constant — i.e. no iterative determination. The values of the friction coefficients are determined using (3.10). Subsequently the drag coefficient  $c_d$  of the basin is varied. The results are shown in figure 4.1.

The response of the Frisian inlet system shows much higher amplification at lower friction values. In addition, eigenmode resonance also generates positive amplification rates (see section 4.4). The Texel inlet system also shows some amplified response at low friction coefficients, but at values higher than  $c_d = 3.5 \times 10^{-4}$  no amplified response at all is visible. The main resonant frequency — i.e. the frequency where amplification is maximal — lowers as the drag coefficient increases. Similar behaviour is known from the PM-model for friction in the *inlet*. Recalling (2.8), the resonant frequency is equal to

$$\sigma_{max} = \sqrt{\sigma_0^2 - \frac{1}{2} (\frac{r_i}{H_i})^2}$$

where  $\sigma_0$  is the resonant (eigen) frequency when friction in the inlet is not included. Results from figure 4.1 show that the resonant friction is affected by friction in the basin as well. The Frisian inlet systems also shows higher resonant frequencies. These are associated with eigenmode frequencies.



Figure 4.2: The effect of **depth of the basin** on amplification of the tidal wave. White-Blue colors denote a *damped* response, while yellow-red colors denote *amplification* 

## 4.2 Basin Geometry

The characteristic parameters of the basin were introduced in section 3.3. In this section, the influence of the depth  $H_b$  and aspect ratio  $S_b$  of the basin are studied. The value of the linear friction coefficient for the inlet and basin compartments is determined using iteration as described in section 3.2.1.

The depth of the basin is varied between 3m and 50m. Results are shown in figure 4.2. The frequency of the tidal wave with largest amplification as predicted by the PM-model<sup>2</sup> is plotted alongside the 2DH-model results for comparison. The Frisian inlet system shows a resonant frequency associated with the Helmholtz frequency. The PM model predicts a higher resonant frequency than the 2DH model results show. At small depths the 2DH resonant frequency lowers.

The Texel inlet system shows barely any amplification, and only at low frequencies. In the previous section, it was shown that the Texel system only has some amplification at very low drag coefficients. In general, it is concluded that the Texel inlet system is too large and dissipative to result in amplification through forcing from a relatively narrow inlet. At higher frequencies there are resonant frequencies that while still in the damped response regime, show higher amplification rates. These are associated with eigenmode frequencies of the basin.

The aspect ratio of the basin  $S_b = \frac{W_b}{L_b}$  is varied between 1/14 and 14. Results are shown in figure 4.3. The frequency of maximum amplification lowers as the aspect ratio elongates. For the

 $<sup>^{2}</sup>$ Both the standard model as the model with the effective length and friction correction for radiation damping. Please see chapter 2 for more detailed information on this model



Figure 4.3: The effect of **aspect ratio of the basin** on amplification of the tidal wave. White-Blue colors denote a *damped* response, while yellow-red colors denote *amplification* 

M2 tidal frequency, this mean that for  $S_b \leq 0.13^3$  there is no tidal amplification. The Texel inlet system shows no amplification at all, which is attributed to the basin being dissipative. The top of the curve of maximum amplification of the Frisian inlet system is slightly offset with respect to the default basin  $S_b$ . This is discussed in more detail in section 4.4.

<sup>&</sup>lt;sup>3</sup>Since  $S_b = \frac{W_b}{L_b}$ , this means that the length is approximately 7.7 times larger than the width.



Figure 4.4: The effect of **length of the inlet** on amplification of the tidal wave. White-Blue colors denote a *damped* response, while yellow-red colors denote *amplification* 



Figure 4.5: The effect of **width of the inlet** on amplification of the tidal wave. White-Blue colors denote a *damped* response, while yellow-red colors denote *amplification* 



Figure 4.6: The effect of **inlet offset** on amplification of the tidal wave. White-Blue colors denote a *damped* response, while yellow-red colors denote *amplification* 

#### 4.3 Inlet geometry

The characteristic parameters of the basin were introduced in section 3.3. In this section the dimensions of the inlet are varied. The length of the inlet is varied between 1 km and 20 km — the results are shown in figure 4.4. The width of the inlet between 1 km and the width of the basin — the results are shown in figure 4.5. Finally the offset of the inlet with respect to the basin centre-line is varied between -5 and 5 km. To this end  $\delta_b$  is changed, while the inlet is kept in place. The results are shown in figure 4.6.

Results shows that an increase in length lowers the highest amplification-frequency. The PMmodel predictions shows that the corrected PM-model much better predicts the trend produced by the 2DH model at smaller lengths. The slight over-prediction is attributed to the depth of the basin — at higher depths the corrected PM-model is expected to *under-predict* the highest amplification-frequency. The Texel inlet system shows no amplification at all, which is attributed to the basin being too shallow — referring to figure 4.2.

The influence of inlet width on tidal amplification is in the case of the Frisian inlet system much better predicted by the PM model results. Higher widths lower the highest amplification-frequency. Once again, the Texel inlet system shows no amplification at all, which is attributed to the basin being too shallow — referring to figure 4.2.

The 'harbour-paradox' mentioned by Terra et al. (2005) is not visible. The harbour paradox describes a particular kind of nonphysical model behaviour. Some models show an *increase* of amplification at lower inlet widths, while a decrease of amplification is expected. The reason for this is that damping of the amplification in these model is (partly) due to radiation damping. At small widths, the effect of radiation damping becomes smaller (see equations (2.13)). Models with radiation damping as an important damping mechanism might as a result show an increase of amplification due to a decrease of radiation damping. Flow separation in the horizontal pane near the inlet mouths is proposed as the countering mechanism to this 'paradox' (Terra et al., 2005). However, since the 2DH model does not include flow separation and does not show the harbour paradox, it can be concluded that radiation damping is not the major damping mechanism.

The influence of the offset of the inlet is similar to the influence of basin shape. The frequency where amplification occurs is closest to the PM-prediction at zero offset. When the offset increases — in either direction — this limit falls to lower frequencies.

### 4.4 Eigenmodes resonance and radiation damping

Results presented in the previous sections mentioned higher eigenmode resonance patterns in the higher frequence range. In this section the eigenmode resonance is studied. In particular, ei-



Figure 4.7: An overview of eigenmodes in a closed and semi-enclosed basin

genmode theory is applied to see if low eigenmodes can be responsible for the curved shape of the Helmholtz frequency in, e.g. figure 4.3. Furthermore, results of the 2DH model for the ocean compartment are filtered to show the influence of the tidal inlet system on the ocean compartment.

**Eigenmodes** Helmholtz resonance is associated with the basin surface elevation oscillating spatially uniform. As such it is the lowest eigenmode, also referred to as the 'pumping mode'. Higher eigenmodes are related with the wavelength and the geometry of the basin, i.e. resonance occurs if a standing wave can develop. In closed basins, this happens when there is an antinode  $(\frac{\delta \zeta}{\delta x} = 0$  $\forall$  t) at both ends. These eigenmodes are sometimes (e.g. by Maas (1997)) referred to as 'sloshing' modes. In semi-enclosed or open-ended basins the eigenmodes have an antinode at the closed end and a node ( $\zeta = 0 \forall$  t) at the open end. If the wavelength of the forcing are so that these conditions are met (see figure 4.7), resonance occurs. Since the wavelength is dependent on basin geometry and forcing frequency, resonance occurs at certain dimensions of the basin. in a two-dimensional approach, resonance can occur over the width and length of the basin. In closed basins, resonance occurs if the wavelength in the basin equals  $\frac{n}{2}L$  with n = [1,2,3..] and L can be either the width



Figure 4.8: Colors denote the friction value in the *basin* after iterative determination. High values are associated with high flow velocities, while low values are associated with low velocities in the basin. Diagonal lines are the *theoretic* frequencies of higher eigenmode resonance. Each lines shows a different eigenmode (different *n*-values, see text). The curved line is the Helmholtz resonance from the 2DH model. The straight, vertical red line shows the default basin aspect ratio, while the dashed, vertical magenta line highlights the offset of the resonance symmetry.

or length of the basin. In open-ended basins resonance occurs if the wavelength equals  $\frac{n}{4}L$  with n = [1,3,5..]. Ignoring friction, the forcing frequency that excites resonance over the length of the basin is given by

$$\sigma_r = \frac{2\pi}{\beta L_b} \sqrt{gH_b} \tag{4.1}$$

where g is the gravitational accelation,  $H_b$  the depth of the basin,  $L_b$  the length of the basin and  $\beta$  the fraction for open or closed basins discussed above. To find resonance over the width of the basin,  $L_b$  should be replace with the width of the basin,  $W_b$ . Equation (4.1) is applied to figure 4.3 to compare the theoretical resonant frequency with the observed one. However, instead of the amplification factor, a colormap showing the friction in the basin after iterative determination is plotted. Results are shown in figure 4.8. Open-ended basin eigenmode resonance is only considered for resonance over the length — the sectional plane over the x-axis (see figure 3.6) — because the inlet is on this profile.

Results show that the resonant patterns observed in the other figures in this chapter can indeed

be attributed to higher eigenmodes. Furthermore, higher-eigenmode resonance is related to high friction coefficient values in the basin. The lowest eigenmode — the helmholtz or pumping mode — is on the other hand associated with very low friction values. Since the friction is determined following flow velocities in the basin, Helmholtz resonance is associated with low flow velocities. Since Helmholtz resonance involves a spatially uniform movement of the basin — so the surface elevation gradients are very small — this is according to expectations.

Figure 4.8 shows that the curved shape of the 2DH-Helmholtz resonance line is not directly related with a higher eigenmode. This confirms that the resonant pattern shown in figure 4.3 is indeed the resonance resulting from the Helmholtz mode and not the Helmholtz mode 'blending in' with a higher eigenmode. Instead, the curve of the Helmholtz resonance line is explained to result from propagation of the tidal wave, with friction in the basin determining the degree of curvature.

Finally, figure 4.8 shows that the off-set of the Helmholtz curve coincides with the off-set of the eigenmode. Intuitively, one would expect the centre of the symmetry to be at an aspect ratio of  $S_b = 1$ . Theoretically, this would also be expected for the eigenmodes if there is only closed-basin resonance. However, since the sectional plane over the x-axis also allows for open-ended basin resonance the centre of symmetry is located at an aspect ratio of 2 — where  $W_b = 2L_b$ . This follows from equation (4.1). The Helmholtz curve centre of symmetry is also at  $S_b = 2$ . This observation seems to confirm that the Helmholtz mode is in someway affected by the propagation of the tidal wave.

**Radiation damping** Finally, figure 4.9 shows radiation damping in the ocean compartment. The 2DH model was run for the Frisian inlet system with an open and closed inlet. The results for the tidal range of the ocean compartment were substracted to produce figure 4.9. Results show that radiation damping for the Frisian Inlet system is about 10 to 20 centimeters.

### 4.5 Conclusions

From the results presented in this chapter the following conclusions can be drawn

- Helmholtz or pumping mode resonance is sensitive to basin friction and geometry. This is attributed to propagation of the tidal wave.
- In general, the PM-model over-predicts the frequency of maximum amplification. The effective parameter correction to account for radiation damping does a better job predicting the reaction to changes in inlet geometry. The over-prediction is attributed to the role of basin depth and basin friction.



Figure 4.9: Colors denote the friction value in the *basin* after iterative determination. High values are associated with high flow velocities, while low values are associated with low velocities in the basin. Diagonal lines are the *theoretic* frequencies of higher eigenmode resonance. Each lines shows a different eigenmode (different *n*-values, see text). The curved line is the Helmholtz resonance from the 2DH model. The straight, vertical red line shows the default basin aspect ratio, while the dashed, vertical magenta line highlights the offset of the resonance symmetry.

• The difference between the two systems show that the Texel inlet basin is much more dissipative than the Frisian inlet.

# CHAPTER 5

## Morphodynamic stability of the inlet channel

THIS chapter shows how basin geometry and basin friction affect the velocity in the inlet. In particular, the focus is on how these characteristics affect the point of stability according to Escoffier. The concept of inlet stability was introduced in chapter 2. The two systems that are used as a basis for the parameter variation are the Frisian and Texel inlet system. Their characteristics can be found in table 4.1. In varying the parameters the frequency of the forcing wave is fixed at the M2 tidal constituent, which is the major constituent for both systems. Consequently the cross-sectional area of the inlet  $A_i$  is varied along with another parameter. The stability line that is found using Escoffier's theorem (Escoffier, 1940) at 1 ms<sup>-1</sup> from the 2DH model is compared with the line found from the PM-model to assess the influence of the spatial structure of the basin, bottom friction and iterative computation of bottom friction on inlet stability.

## 5.1 Manner of morphological change

There are three manners in which the cross-sectional area  $A_i$  can be increased or decreased; (1) variable inlet width at fixed depth, (2) variable depth at fixed width and (3) variable depth and variable width. The latter requires some additional assumptions regarding the inlet cross-sectional aspect ratio  $S_i$  — the ratio between width and depth of the inlet. It is possible to assume geometric similarity following O'Brien and Dean (1972); van de Kreeke (2004) — resulting in a fixed ratio of inlet width and inlet depth — but such an assumption is not based on research since the *manner* by which the inlet changes never has been studied systematically van de Kreeke (2004). Through the



Figure 5.1: The effect of different manners of morphological change of the cross-sectional area of the inlet. The Frisian inlet before closure had a cross-sectional area of about 22500 m<sup>2</sup>. The shaded area along the curves of the 2DH model results denote uncertainty related to where in the inlet to measure the flow velocity.

friction term in the PM-model it is also possible to represent different manners of morphological change. The effect of different manners of morphological change is plotted in figure 5.1. The cross-sectional area of the Frisian inlet basin before basin reduction — which is the default basin in chapters 4 and 5. It shows that while the response of the velocity curve to different manners of change are quite different from one another, the predicted stable cross-sectional areas are not so far apart. This is to be expected though, since variation is initiated from the default system, which is already close to stability and per definition results in identical model outputs regardless of the manner of morphological change.

The choice for a method is, without scientific research on the subject, rather subjective. To evaluate the uncertainty of the predicted stable cross-sectional area with respect to the manner of morphological change, all three methods are used in the study of the influence of basin friction in figure 5.2. Results show that the uncertainty due to the manner of morphological change is relatively low — with the exception of the 'fixed width' method in the Texel inlet system. Higher basin drag coefficients  $(c_d)$  values than  $3 \times 10^{-3}$  show that no stable root exists (see section 2.1). Values for the basin  $c_d$  lower than  $3 \times 10^{-3}$  show two roots within the limits of the linear model. In the actual tidal inlet systems, studies of Oost (1995) and Elias (2003a) of the two systems do not mention significant changes in channel width in either channels. It is known however, that the Texel inlet channel deepened and the Frisian inlet channel decreased in depth in the past decades (see section 2.4). This arguments against using a fixed-depth. On the side of computational effectiveness, the fixed-width method circumvents a limitation of the 2DH model



Figure 5.2: This figure shows the line when flow velocities in the inlet equal 1ms-1 for a given inlet cross-sectional area and basin friction drag coefficient [-]. Three different manners of morphological change of the inlet channel are plotted (see section 5.1). Please note the different scales for the Frisian and Texel inlet system. See section 5.2 for a discussion on the figures. Friction is not determined iteratively (see section 4.1)

regarding resolution: the freedom of changing the width of the channel is limited by the spacing of the collocation points, and a decrease of spacing results in a significant increase of computational time. Motivated by above consideration, the 'fixed-width' method is used in the following sections.

## 5.2 Basin friction

The influence of the basin friction is shown in figure 5.2. Please note that while figure 5.2 and figure 5.1 bear resemblance, they show very different relationships. The difference between the different methods of morphological change were discussed in section 5.1. To study the effect of friction in the basin, the friction coefficients in the ocean and inlet compartment are kept constant — i.e. no iterative determination. Both the Frisian and Texel inlet systems results shows a negative relationship of the basin  $c_d$  with the stable cross-sectional area of the inlet  $A_i$ . Lower friction coefficients allow for greater cross-sectional areas. The unstable root (see section 2.1) is only visible in the Texel inlet results for the fixed-width. It is expected that the unstable roots of the other relationships fall within the range of  $H_i < 3m$  — which we assume cannot reliable be assessed with a linear model.



Figure 5.3: The influence of **aspect ratio of the basin** on the flow velocity in the inlet.

## 5.3 Basin geometry

The characteristic parameters of the basin were introduced in section 3.3. In this section, the influence of the depth  $H_b$  and aspect ratio  $S_b$  of the basin are studied. Furthermore, the offset of the inlet with respect to the basin centreline is studied.

The depth of the basin is varied between 3 and 50 meters. Results for the Frisian Inlet (figure 5.4) show that basin depth has no or little influence. The current — default — inlet is situated right of the stable root line. This indicates that the inlet is too large, and will diminish in size. The Texel inlet results show that basin depth has a profound influence on inlet stability. The point where no stable roots exist at all is at a depth of about 3.5 meters. The current inlet is positioned on the left of the stable root line, meaning the inlet is too small, but right of the unstable root line — ensuring that system tendency will result in an open cross-section. Were it on the left of the unstable root line, the inlet would ultimately close according to Escoffier's theorem.

The aspect ratio of the basin (see figure 5.3) seems to have little influence based on the flow velocity in the inlet for the Frisian inlet system. The Texel inlet shows a response to basin aspect ratio similar to the response to basin depth. At an aspect ratio of about 0.47, no stable root exists anymore. This denotes a length which is about 2.2 times larger than its width.

The offset of the inlet also has little influence on the Frisian inlet system. In the extremes the stable cross-sectional area increases very slightly.

Generally, the Frisian inlet is not very sensitive to basin geometry, Figure 5.6 shows that increasing the surface area  $A_b$  of the system, keeping everything else constant, does influence the stable cross-sectional area. Since the main difference with the Texel inlet is the size, it is probable that the size of the basin is a measure for the relative influence of basin geometry.



Figure 5.4: The influence of **depth of the basin** on the flow velocity in the inlet.



Figure 5.5: The influence of **offset of the inlet** with respect to the basin centreline on the flow velocity in the inlet.

## 5.4 Conclusions

Based on results presented in this chapter the following conclusions are drawn:

- The manner of morphological change of the inlet channel has little influence on the relationship between basin friction and the stable inlet cross-sectional area.
- Basin friction influences the both the Frisian and Texel inlet system. A lower basin drag coefficient  $c_d$  leads to greater cross-sectional area's.
- Basin geometry generally does not influence the Frisian inlet, but it does influence the Texel inlet system. It is concluded that the surface area  $A_b$  is the main parameter that influences the relative influence of basin geometry.



Figure 5.6: The influence of surface area of the basin on the flow velocity in the inlet.

• For the Texel inlet system, values for basin depth  $H_b$  and basin aspect ratio  $S_b$  have been found at which no stable cross-sectional area exists.

# CHAPTER 6

## Case Study

This chapter presents two case studies pertaining to the basin reduction of the Frisian and Texel inlet systems, which were introduced in section 2.4. The Frisian Inlet had the Lauwerszee dammed of as part of the Deltawerken project in 1969. The Texel inlet used to drain the Zuiderzee with the Vlie inlet until the closure of the Zuiderzee in 1932. Both case studies are approached in the same way. First the situation as it was before human intervention is presented. Then, the effects of human intervention — the damming of a part of the basin in both cases — are assessed. Finally, the sensitivity of the results to parameters are evaluated. Similarities and differences are discussed.

## 6.1 The damming of the Lauwerszee

The Frisian Inlet is located between the barrier islands of Schiermonnikoog and Ameland. Its main channel is the Zuidkamerlaag; the smaller Pinkegat channel located West of the Engelsmanplaat is not considered to be part of the Frisian Inlet in this study — please see section 2.4 for a discussion on this topic. In 1969 the Lauwerszee eastuary was dammed of as part of the Deltawerken-programme. Figure 6.1 shows the situation before the damming of the Lauwerszee — which nowadays is known as the Lauwersmeer. The bathymetry data is from the OpenEarth repositories<sup>1</sup>.

<sup>&</sup>lt;sup>1</sup>website: https://publicwiki.deltares.nl/display/OET/OpenEarth — openDAP server: http://opendap.tudelft.nl/



Figure 6.1: The Frisian Inlet system of 1967

#### 6.1.1 Effects of the damming

Direct effects of the damming were (i) a reduction in basin size and (ii) an aspect ratio change. Thirdly, because the Lauwerszee has relatively more flats than channels compared to the rest of the basin, the average basin depth will increase as a result of the damming. This effect is hard to quantify though, since it is unknown what is the best way to calculate average basin depth from bathymetric data. Please see section 7.1 for a discussion on this topic and section 8.2 for recommendations on how to deal with this deficit. Oost (1995) furthermore reports the following indirect effects: a partial fill of the inlet and basin and a shift in the eastern watershed.

Based on results from chapter 5 the reduction in basin size would lead to a smaller crosssectional area, while reduction in average depth will not result in much change. The basin before and after the damming is schematised as shown in table  $6.1^2$ . The representative depth of the basin as a consequence of the damming of the Lauwersea is unknown. Therefore, three scenario's are used to describe the current (2012) situation. In the first, there are no changes in the basin depth. The second has 0.5m increase in average depth and the third 1m increase of basin depth.

<sup>&</sup>lt;sup>2</sup>These data are from literature, not derived from bathmymetric data. See (Maas, 1997; van de Kreeke, 2004)

Year	$L_b$	$W_b$	$A_b$	$H_b$	$S_b$	$\delta_b$	$W_i$	$L_i$
1967	$9~{\rm km}$	$14 \mathrm{~km}$	$126~{\rm km^2}$	4 m	1.6	-6 km	$3 \mathrm{km}$	$3~{ m km}$
2012	$6 \mathrm{km}$	$15 \mathrm{km}$	$90 \ \mathrm{km^2}$	4 m	2.5	-6.5 km	3 km	3km

Table 6.1: The Frisian inlet before and after basin reduction

#### 6.1.2 Model results

The PM and 2DH models were run for the system before and after the damming. The friction coefficient resulting from the iterative calculation in the 2DH model was used as friction coefficient for the inlet in the PM model. Results are shown in figure 6.2. The inlet has a cross-section of 2.2  $\times 10^4 \text{m}^2$  before reduction. The PM model predicts a cross-section of 2.3  $\times 10^4 \text{m}^2$ , and the 2DH model a cross-sectional area of 2.0  $\times 10^4 \text{m}^2$  before reduction.

Both models predict a reduction in inlet-cross sectional area after the damming of the Lauwerszee. The PM model predicts the inlet cross-sectional area will shrink to about  $1.7 \times 10^4 \text{m}^2$  while the 2DH model predicts a stable cross-sectional area of about  $1.5 \times 10^4 \text{m}^2$ . This is just outside the limits estimated by van de Kreeke (2004), who based on an exponential fit of observed data predicted a cross-sectional area between 1.2 and  $1.6 \times 10^4 \text{m}^2$ . The right hand graph in figure 6.2 shows the sensitivity of the predicted stable cross-sectional area to spatial variables. Basin depth was varied between 3 and 50 meters, inlet length between 1 and 15 km, the aspect ratio between 2.5 and 0.4 and the offset between being entirely on the left, in the centre and entirely on the right.

Results show that the predicted stable cross-sectional area is sensitive to inlet length, inlet offset and basin depth. The aspect ratio has little to no influence. The 2DH results closer resemble PMmodel results when the basin is very deep ( $H_b > 50$ m). This is expected based on hydrodynamics. Without friction, the wavenumber [ms<sup>-1</sup>] of the tidal wave is given by

$$k = \frac{\sigma}{\sqrt{gH_b}} \tag{6.1}$$

Where g is the gravitational acceleration  $[ms^{-2}]$  and  $H_b$  the depth of the basin [m]. Greater depths leads to a smaller wavenumber and greater wavelengths. For relatively small basins, a large wavelength means the basin is more likely to osscilate spacially uniform. It is for this reason PM-models assume small and deep basins. Results from figure 6.2 shows that shallower basins will lead to smaller cross-sectional area's for the Frisian inlet system.



Figure 6.2: The left hand figure (a) shows the Escoffier curves for the Frisian inlet system before and after closure of the Lauwerszee according to the PM and 2DH model. The right hand figure (b) shows the sensitivity of the stable root of the 2DH model to several parameters.

## 6.2 The damming of the Zuiderzee

The Zuiderzee was an inland sea in the Netherlands which was dammed in 1932 as part of the Zuiderzee werken. The former Zuiderzee area now accomodates several fresh-water lakes — the largest of which are the IJsselmeer and Markermeer — and several polders including the entire province of Flevoland. The former Zuiderzee was serviced by two inlets - the Marsdiep or Texel inlet and the Vlie inlet. Figure 6.3 shows the situation before and after damming.

#### 6.2.1 Effects of damming

After closure, the former double inlet system reverted to two single inlet systems, with only limited exchange of water volume between them (Ridderinkhof, 1988). Direct effects for the Texel inlet was a substantial *increase* of the tidal prism with about 20% (Elias, 2003a). This is attributed to the increased reflection of the tidal wave at the Afsluitdijk dam, compared to the relatively weak reflection in the former Zuiderzee which was dominated by bottom friction. Likewise to the closure of the Lauwerzee (see section 6.1), the basin after closure had relatively more channels than flats and the mean depth of the basin is increased. The morphological adjustment to this sudden non-equilibrium state shows an increase of the volume of tidal flats (Elias, 2003a) and an increase of the depth of the tidal inlet (Elias and van der Spek, 2006). Table 6.2 shows the schematised inlet systems before and after closure.



Figure 6.3: The basins of the Texel and Frisian inlet before and after basin reduction. The Zuiderzee was dammed in 1932

#### 6.2.2 Model results

The PM and 2DH models were run for the system after the damming. The friction coefficient resulting from the iterative calculation in the 2DH model was used as friction coefficient for the inlet in the PM model. Results are shown in figure 6.4. Before closure the inlet had a cross-sectional area  $A_i$  of  $5.5 \times 10^4 \text{m}^2$ . Because the system *before* closure was a double inlet system — which is beyond the scope of this study — the models are ran only for the situation after the closure. The Texel inlet is assumed to be a single inlet system based on Ridderinkhof (1988).

Because the Texel inlet system directly after closure had a higher channel to tidal flat ratio, the average depth increased. The model is run both with the assumed average depth before closure  $(H_b = 4.5\text{m})$  and with a significant larger depth of  $H_b = 10\text{m}$ . Figure 6.4 shows that 2DH and PM models show very different results at the shallower depth. Increased depth increases similarity between the two models. A similar result was seen in the Frisian inlet case study.

The right hand graph in figure 6.2 shows the sensitivity of the predicted stable cross-sectional area to spatial variables. Basin depth was varied between 3 and 10 meters, inlet length between 1 and 20 km, the aspect ratio between 2.5 and 0.3 and the offset between being entirely on the left,

	$L_b$	$W_b$	$A_b$	$H_b$	$S_b$	$\delta_b$	$W_i$	$L_i$
1927	$100 \mathrm{km}$	$40~{\rm km}$	$4000~\rm km^2$	$4.5 \mathrm{~m}$	0.4	-20km	$3 \mathrm{km}$	$14 \mathrm{km}$
2012	$17 \mathrm{~km}$	$40 \mathrm{km}$	$680 \ \mathrm{km^2}$	$4.5 \mathrm{m}$	2.4	-20km	$3 \mathrm{km}$	$14 \mathrm{km}$

Table 6.2: The Texel inlet system before and after basin reduction



Figure 6.4: Escoffier diagrams showing the expected result of the damming of the Zuiderzee with the PM and 2DH models.

in the centre and entirely on the right. Results show that the predicted stable  $A_i$  is indeed very sensitive to basin depth. This suggest that the PM-model can only be used for a system the size of the Texel inlet basin at large depths ( $H_b > 10m$ ). Also, compared to the case of the Frisian inlet system the PM-model is more sensitive to the inlet length than the 2DH model. This is explained from the fact that resulting from basin area size, frictional losses in the basin are larger in the Texel inlet case. Therefore spatial variables pertaining to the basin are relatively more important than those pertaining to the inlet. Note that the offset of the inlet is considered a variable pertaining to the basin<sup>3</sup>. Results also show a sensitivity of the PM-model to inlet offset. This is due to the fact that the friction factor in the PM-model is determined from the 2DH model.

### 6.3 Conclusions

The results from the Frisian inlet case study shows that the effect of the damming of the Lauwerzee is similar for both models. The 2DH model consequently predicts lower stable cross-sectional areas,

<sup>&</sup>lt;sup>3</sup>Technically, the offset of the *basin* is varied, not the one of the inlet. Therefore, inlet offset is the offset of the inlet with respect to the basin

which is mainly attributed to basin depth. The predicted cross-sectional area of the 2DH model are within the limits van de Kreeke (2004) had determined from an exponential fit of observations, while the PM model predicts a slightly larger cross-sectional area.

The 2DH results from the Texel inlet show that the damming of the Zuiderzee leads to an increase in inlet cross-sectional area. This is in agreement with observations (Elias, 2003a). 2DH results are very sensitive to basin depth. Since this is the most uncertain parameter — as compared to aspect ratio and inlet offset, it can be concluded that investigating how to choose an appropriate basin depth has priority. Please see chapter 8 for recommendations on this subject.

# CHAPTER 7

## Discussion

A <sup>2DH</sup> single inlet model was developed to study the influence of basin geometry and basin friction on hydrodynamic properties and morphodynamic stability of tidal inlet systems. The results are compared with a pumping-mode (PM) model to assess the validity of using PM-models to asses inlet stability. This chapter elaborates on several points, both from a model-technical nature as well as physical considerations:

## 7.1 Physical limitations of the 2DH model

The 2DH model knows several limitations. Discussed are uniform compartment depth, entrance/exit losses, tidal constituents and linearisation of the depth-averaged shallow water equations.

The 2DH model uses a uniform depth per compartment. The most characteristic feature of tidal inlet systems — the channel-shoal bathymetry — can therefore not be modelled. Hence there is the need to come up with a representative basin average depth, which might not be simply the mean depth of the basin, keeping in mind the role of basin bottom friction. Much the same goes for the schematisation of the inlet. As can bee seen in figure 6.1 the inlet consist of a deep channel and relatively shallow sides. It would require another study to know how measured bathymetry can best be schematised. This schematisation would not necessarily be the same for the PM and 2DH model, and ideally be calibrated with observed data. Such analysis was beyond the scope of this study.

A process that is not represented in the 2DH model are entrance/exit losses. These losses are

commonly attributed to an asymmetry between inflow and outflow, and can be modelled with an extra friction term (Maas, 1997). A recent study (Brouwer et al., 2012b) studied entrance/exit losses as a partial mechanism to achieve stable *double* inlet systems.

The only tidal constituent used in this study is the M2 semi-diurnal lunar tide. While this is the dominant constituent along the Waddensea coast, others are present too. In particular the presence of a spring-neap cycle could lead to higher maximum velocities in the inlet and consequently lead to higher cross-sectional areas.

An important assumption in the linearisation of the depth-averaged shallow water equations is that the amplitude scale is larger than the depth scale. In seas like the North Sea this is a valid assumption. Depending on the tidal range in the basin and inlet, the shallow depths used in these symptoms might stretch the applicability of the linear model. Other nonlinear mechanisms that is e.g. sloping basin bottoms which may have a large influence on stability (Maas, 1997).

## 7.2 The stability concept

The stability concept of Escoffier (1940) used might be a crude oversimplification. His estimation of a stable velocity of  $1\text{ms}^{-1}$  has been commonly used in literature. van de Kreeke (1992) reviewed the use (and misuse) of Escoffier's concept in literature, specifically in combination with tidal prism - cross-sectional area (AP) relationships. He argues that the latter is not necessarily an alternative to the first, but they can complement each-other. In fact, van de Kreeke (1992) argues that while the equilibrium velocity is approximately  $1\text{ms}^{-1}$ , the exact value depends on littoral drift, sediment characteristics, wave climate and tidal period. In light of this argument, AP-relations can be used to establish a better equilibrium velocity line suited to a specific tidal inlet system. An example of this is given by Tung (2011) for tidal inlet systems in Vietnam. As a result, the equilibrium velocity curve is no longer constant. Such approaches however, require the combination of measurements and modelling.

## 7.3 The measure for amplification

The amplification measure was introduced in section 3.4.2. Resonance was defined as the amplification of the tidal oscillation, causing the basin to have higher amplitudes than the tidal amplitude itself. Recalling the amplification factor:

$$F = \frac{\langle |Z_b| \rangle}{|Z_o|} \tag{7.1}$$

The averaging over the entire basin clearly favours the pumping mode resonance. Other forms of resonance are of the higher eigenmodes, associated with half or quarter wavelengths. This kind

of resonance would have one or more nodes and antinodes within the basin. Resonance would then be high in the antinodes, and low in the nodes. Averaging over the entire basin might flatten this pattern. The choice for this kind of measure was made for two reasons. First, Helmholtz or pumping mode resonance is the most likely kind of resonance to occur in tidal inlet systems. Given the long wavelengths of the tidal wave, most tidal basins are to small to illicit eigenmode resonance. The second reason is comparison between the 2DH and PM model. Eigenmode resonance cannot be modelled using the PM model. Instead, the response as modelled in the PM approach is per definition 'basin average' — i.e. following from the model assumptions.

### 7.4 Morphological change

Section 5.1 introduced three different manners of morphological change. For lack of arguments (e.g. from literature) to support either method the choice was made for the method which had width fixed. In a system with barrier islands which are inhabited it can be argued that the inlet sides are reinforced, e.g. with sea dikes. In any case, the choice for only one method introduces some uncertainty. Nonetheless, it is expected that since the PM-model used the same method for morphological change, the influence on results is limited.

## 7.5 Computational limitations of the 2DH model

The computation time of the 2DH model is relatively short. Depending on the size of the basin it is in the order of 0.3 to 1 second per iteration with a collocation point spacing of 1000 m. Iterative determination of bottom friction requires several iterations. Using under-relaxation and a precision of  $r \pm 10^-4$  there are approximately 5-6 iterations needed. Increasing the number of collocation points (smaller spacing) or increasing the size of the basin the computation times are increased. The collocation points itself present the main technical limitation of the 2DH model, since the freedom to vary geometry is ultimately bound to the location collocation points. It was found that methods to be flexible with collocation points spacing greatly influence the outcome of results and should be avoided. Increasing the resolution is therefore the only way to have more flexibility in varying e.g. inlet width. Which in turn (greatly) increases computational time.

# CHAPTER 8

## Conclusions and Recommendations

The aim of this study is to investigate the effect of basin friction and basin geometry on the hydrodynamics and morphodynamics of the single inlet systems and to evaluate the validity of using PM-models to assess inlet stability. In this chapter the research questions posed in section 1.3.3 are answered, and recommendations for future research are presented.

## 8.1 Conclusions

It was found that the stable cross-sectional area in terms of the concept of Escoffier (1940), is sensitive to both basin geometry and basin friction. The case study shows that the PM and 2DH model show reasonable good agreement in the case of the Frisian inlet system, but not in the case of the Texel inlet system. It is concluded that the PM-model approach is only valid for relatively small or deep systems. Furthermore, the inclusion of radiation damping in the PM-model shows a better agreement with the 2DH model regarding hydrodynamic properties.

1. How can the 2DH hydrodynamic model be formulated for a single-inlet system? The 2DH model, based on the original model of Taylor (1921) is expanded to multiple compartments and includes bottom friction following Roos and Schuttelaars (2011). The model solves the depth-averaged shallow water equations with no-flow boundary conditions on closed sides and matching flow and surface elevation conditions on open sides using a collocation method. The inlet system is schematised using three compartments — ocean, inlet and basin — with parameters

width  $W_j$ , length  $L_j$ , depth  $H_j$  and offset  $\delta_j$  for every compartment. To system is forced by an incoming Kelvin wave at the open side of the ocean compartment. The amplitude of the incoming Kelvin wave is chosen so, that the amplitude in front of the inlet was kept a desired value. The friction term is linearised using Lorentz' linearisation. The resulting linear friction coefficient involves a velocity scale, which is determined using an iterative method per compartment.

2. How can inlet morphodynamics be incorporated in the 2DH model? It was found that the concept of Escoffier (1940) could be simply incorporated in the 2DH model outputs. The choice of where in the inlet the velocity should be measured had little influence on the outcome of the stability calculations. The model allows for three manners of morphological change of the inlet cross-section: fixed inlet width, fixed inlet depth or variable width and depth. Morphological change using fixed inlet width was chosen and consequently used throughout the study.

3. In what way does the 2DH model reproduce system hydrodynamics with respect to tidal resonance with respect to the PM-model, and what is the influence of the physical mechanisms of radiation damping, bottom friction and basin geometry? Both resonance resulting from the Helmholtz or pumping mode and higher eigenmodes are found using the 2DH model. It was found that the PM model predicted higher resonant frequencies for the Helmholtz mode, which is attributed to basin friction. It was found that the frequency of maximum tidal amplification due to Helmholtz-mode resonance is sensitive to basin geometry. This is attributed to tidal wave propagation through the basin. It was furthermore found that the PM model corrected for radiation damping better predicts the trend of the 2DH model at short inlet channels. This suggest that radiation damping is more important for short inlets. Basin friction is found to be mainly a damping mechanism.

4. In what way does the 2DH model predict inlet stability and how sensitive is the stability of the inlet to parameters and processes added by the 2DH model? It was found that the PM model generally predicts higher values of the stable inlet cross-sectional areas than the 2DH model. The aspect ratio of the basin is found to have a profound influence on inlet stability at large basins. In the Texel inlet case, the case-study presented in chapter 6 showed that the aspect ratio of the basin could even result in the absence of a stable root. Basin geometry generally does not influence the Frisian inlet, but it does influence the Texel inlet system. It is concluded that the surface area  $A_b$  is the main parameter that influences the relative influence of basin geometry. In large systems, such as the Texel inlet system, basin geometry has a large influence on results.

5. What is the effect of large-scale damming in tidal systems of the Dutch Wadden Sea on system dynamics, with emphasis on inlet channel stability? The case studies of chapter 6 showed two different responses to the damming of inlet systems. Both responses were qualitatively in agreement with observations. The PM model predicts the inlet cross-sectional area will shrink to about  $1.7 \times 10^4 \text{m}^2$  while the 2DH model predicts a stable cross-sectional area of about  $1.5 \times 10^4 \text{m}^2$ . This is just outside the limits estimated by van de Kreeke (2004), who based on an exponential fit of observed data predicted a cross-sectional area between 1.2 and 1.6  $\times 10^4 \text{m}^2$ . The Texel inlet is predicted to increase its cross-sectional area as a response to the closure of the Zuiderzee, assuming that the closure of the Zuiderzee led to a higher average basin depth. Basin depth has a large influence on the predicted cross-sectional area. Retaining the average basin depth before closure, the stable cross-sectional area will even slightly decrease, while a very steep increase in depth could lead to a cross-sectional area which is twice as big as before the closure. It is concluded that in such systems, basin depth is the most uncertain and important parameter for determining the stable cross-sectional area. The PM- and 2DH model results show large differences. It is concluded that the PM model is only valid for relatively small basins.

### 8.2 Recommendations

This study used an idealised linear hydrodynamic model to assess the influence of basin geometry and basin friction on inlet stability. Within the known limitations of the idealised model, several recommendations for future research are given.

- Investigate how a representative basin geometry can be constructed from bathymetric data. The Open Earth project has historical bathymetry data available free to use. With the help of historical cross-sectional areas (e.g. mentioned by van de Kreeke (2004)) it should be possible to calibrate the schematisation. Based on the results from the case study, the focus should be on determining an appropriate basin depth and inlet length. Possible this is not straightforward. The challenges for basin depth are how to deal with the differences in elevation between tidal flats and channels. Since most of the flow goes through the channels, it is probable that the appropriate basin depth is lower than the geographic average. Determining inlet length might also not be very straightforward, as the main channel often extends far into the basin.
- Establish and calibrate a tidal prism cross-sectional area (AP) relationship for the Wadden Sea inlets, that incorporates sediment characteristics and sediment transport due to littoral drift. An earlier estimation of an AP relationship is given Stive and Rakhorst (2008), but it remains unclear how the parameters were derived. Furthermore, it is recommended to

explicitly couple the AP relationship to the Escoffier approach with the aim of finding an expression for the equilibrium velocity in the inlet.

• It was found that where in the inlet cross-sectional velocity was measured, had little influence on the stability curve. This confirms that the flow in the inlet is well approximated by widthaveraged one dimensional model, as suggested in literature. Therefore it is recommended to consider the option of replacing the compartment inlet with a 1DH model. Potentially, such an approach could circumvent the 'Gibbs-phenomena-like abrupt corner problem mentioned in section 3.2.2 by applying a distribution function over the open/wet boundary.

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# List of frequently used symbols

# Roman characters

$A_b$	Surface area of the basin $[m^2]$
$A_i$	Cross-sectional area of the inlet channel $[\mathrm{m}^2]$
$c_0$	Shallow water wave celerity $[ms^{-1}]$
$c_d$	Drag coefficient [-]
f	Coriolis Frequency $[s^{-1}]$
F	Amplification of the basin [-]
g	Gravitational Acceleration $[ms^{-2}]$
Η	Compartment Depth [m]
k	Wave Number $[m^{-1}]$
L	Compartment Length [m]
P	Tidal Prism $[m^3]$
r	Linear friction parameter $[ms^{-1}]$
R	Rossby Deformation Radius [m]
S	Compartment Aspect ratio [-]
W	Compartment Width [m]
$Z_f$	Amplitude of incoming Kelvin wave [m]
$Z_n$	Amplitude of Poincaré wave (n=[1,2]) [m]
$Z_o$	Amplitude in front of inlet mouth [m]
$Z_r$	Amplitude of reflected Kelvin wave [m]

### **Greek characters**

- $\alpha$  Reflection factor [-]
- $\beta$  Discrete wavenumber of Poincaré mode [m<sup>-1</sup>]
- $\delta$  Compartment offset from system centreline [m]
- $\zeta$  Free surface elevation [m]
- $\hat{\zeta}$  Free surface elevation amplitude [m]
- $\theta$  Latitude (degrees)
- $\sigma$  Angular Velocity [s<sup>-1</sup>]
- $\sigma_0$  Eigenfrequency [s<sup>-1</sup>]
- $\Omega$  Angular rotation rate of the Earth [s<sup>-1</sup>]

## Subscripts

- b Basin compartment
- f Incoming Kelvin wave
- i Inlet compartment
- j Compartment number
- $n \quad n^{th}$  Poincare mode [n=1,2,...]
- o Ocean compartment



## Mathematical background - 2DH model

**Basic equations** We start from the linearised shallow water equations, accounting for inertia, Coriolis, friction and a pressure gradient. The base set of equations is given as follows

$$u_t - fv + \frac{ru}{H} = -g\zeta_x \tag{A.1}$$

$$v_t + fu + \frac{rv}{H} = -g\zeta_y \tag{A.2}$$

$$\zeta_t + H(u_x + v_y) = 0 \tag{A.3}$$

and boundary condition

$$v = 0 \quad at \quad y = 0 \quad and \quad y = W \tag{A.4}$$

where (A.1) and (A.2) are the momentum equations in the x, y directions respectively and (A.3) is the continuity equation. These equations express the variables in terms of each other, i.e. v is a function of the time-derivative of u and the x-derivative of  $\zeta$ , which is somewhat of a inconvenience for the purpose of finding a solution. Therefore, these equations are re-arranged to express them as functions of only themselves. The result will be the (frictional) Klein-Gordon equation for  $\zeta$  and two polarisation equations for u and v. Rewriting the basis equations lead to the frictional Klein Gordon and polarisation equations are found. The Klein-Gordon equation is given as:

$$(\Phi^2 + f^2)\zeta_t - gH\Phi\nabla^2\zeta = 0 \tag{A.5}$$

The polarisation equations are given as

$$\left[\Phi^2 + f^2\right]u = -g\Phi\zeta_x - fg\zeta_y \tag{A.6}$$

$$\left[\Phi^2 + f^2\right]v = gf\zeta_x - g\Phi\zeta_y \tag{A.7}$$

with

$$\Phi = \frac{\partial}{\partial t} + \frac{r}{H}$$

#### A.1 Wave solutions for an infinite channel

**Ansatz for**  $u, v, \zeta$  In the following section ansatz solutions for the variables u,v and  $\zeta$  are substituted in the Klein Gordon equation and polarisation equations. To ease reading, the following variable is

$$s = -i\sigma + \frac{r}{H}$$

The variable  $e^{i(kx-\sigma t)}$  is a complex wave function The ansatz solutions are

$$\zeta(x, y, t) = \hat{\zeta}(y)e^{i(kx-\sigma t)} \tag{A.8}$$

$$u(x, y, t) = \hat{u}(y)e^{i(kx-\sigma t)}$$
(A.9)

$$v(x, y, t) = \hat{v}(y)e^{i(kx-\sigma t)}$$
(A.10)

with wave number  $k \ [m^{-1}]$  and angular frequency  $\sigma \ [s^{-1}]$  and amplitude functions  $\hat{\zeta}, \hat{u}, \hat{v}$  still unknown. Substituting the ansatz in the Klein Gordon equation lead to the the frictional eigenvalue problem.

$$\hat{\zeta}_{yy} + \alpha^2 \hat{\zeta} = 0 \tag{A.11}$$

with

$$\alpha^2 = \frac{i\sigma s^2 + i\sigma f^2}{gHs} - k^2$$

This eigenvalue problem should be combined with a boundary condition to form a well-posed problem. The boundary condition for  $\hat{\zeta}$  is obtained by substituting the ansatz in the polarisation equation (A.7). For v = 0 at y = 0 and y = W it follows that

$$s\hat{\zeta}_y - ifk\hat{\zeta} = 0 \tag{A.12}$$

**Expressions for velocity components** For the velocity functions, the ansatz solutions are substituted in the polarization equations.

$$[s^{2} + f^{2}]\hat{u} = -g\left(ik\sigma + f\frac{\partial}{\partial y}\right)\hat{\zeta}$$
(A.13)

$$[s^{2} + f^{2}]\hat{v} = g(ikf - \Phi\frac{\partial}{\partial y})\hat{\zeta}$$
(A.14)

Ansatz for amplitude functions The eigenvalue problem (A.11) with boundary condition (A.12) has the general solution

$$\hat{\zeta}(y) = A_1 \cos \alpha y + A_2 \sin \alpha y \tag{A.15}$$

substitution of (A.15) in (A.12) to evaluate the boundary conditions gives two equations for y = 0 and y = W and two unknown coefficients  $A_1$  and  $A_2$ . This system of equations and unknowns can be expressed in matrix form:

$$\begin{bmatrix} -ifk & \alpha s \\ -ifk\cos\alpha W - \alpha s\sin\alpha W & \alpha s\cos\alpha W - ifk\sin\alpha W \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = 0$$
(A.16)

From this follows that either one of the matrices should be zero. The coefficient matrix should be non-zero, since allowing  $A_1$  and  $A_2$  to be zero would return trivial solutions, i.e. it would render the entire solution thread zero. The multiplication of the first row of the matrix with the coefficient matrix does return a valuable expression for  $A_2$ , which will be necessary later on

$$A_2 = \frac{ifk}{\alpha s} A_1 \tag{A.17}$$

The system of equations (A.16) is homogeneous. Non-trivial solutions only exist if the determinant is zero:

$$(\alpha^2 s^2 - f^2 k^2) \sin \alpha W = 0$$

substituting the definition for  $\alpha^2$  and factorizing yields

$$(s^{2} + f^{2})(\frac{i\sigma s}{gH} - k^{2})(\sin \alpha W) = 0$$
 (A.18)

which is the 'three-root solution' well known in these type of problems.

**Dispersion relations** From this equation the Kelvin and Poincaré wave dispersion relations are determined. The first root are the inertial waves which, in the absence of friction, have the same angular frequency as the Coriolis parameter but do not exist if friction is included. The second root returns the Kelvin wave dispersion relationship

$$k = \pm \sqrt{\frac{i\sigma s}{gH}} \tag{A.19}$$

and the third root returns the dispersion relationship for Poincaré waves

$$k = \pm \sqrt{\frac{i\sigma s^2 + i\sigma f^2}{c_0^2 s} - \beta^2}$$
(A.20)

with

$$\beta = \frac{n\pi}{W}$$

For Poincaré modes it follows from the root that  $\alpha = \beta$ . For Kelvin waves it follows that  $\alpha = R_f^{-2}$  with the frictional Rossby deformation radius  $R_f$ 

$$R_f^2 = \frac{sgH}{i\sigma f^2} \tag{A.21}$$

With the dispersion relations, the amplitude functions can be derived. For Kelvin modes, application of the dispersion relation and definition for  $A_2$  (A.17) it follows that  $A_2 = \pm i A_1$ . Therefore, the amplitude function can be expressed as a complex exponential. This returns for Kelvin modes

$$\hat{\zeta} = A_1 e^{\pm iy/R_f} \tag{A.22}$$

**Solutions** Now, by combination of the general solution for  $\hat{\zeta}$ , the definition for  $A_2$  and  $\alpha$  the solution for the free surface elevation for Kelvin and Poincaré waves can be developed.

**Kelvin modes** The free surface equation for Kelvin waves can be determined by substituting the equations for  $\hat{\zeta}$  and  $\zeta$  and choosing an arbitrary amplitude  $Z_0$  for  $A_1$ . This returns

$$\zeta(x, y, t) = \Re \left\{ Z_0 e^{-iy/R_f} e^{i(kx - \sigma t)} \right\}$$
(A.23)

using the earlier derived equations for the velocity amplitude functions (A.13) and (A.14) by substituting k from the dispersion relation and the ansatz for  $\hat{\zeta}$ . Care should be taken to take the correct combinations of signs for the two waves. Substituting the parts and multiplying both sides with -i and  $\sqrt{s}$  allows to write the solutions in the following compact notation

$$\hat{u} = \pm i \sqrt{\frac{ig\sigma}{Hs}} \hat{\zeta} \tag{A.24}$$

The remaining velocity component v is zero everywhere.

**Poincaré modes** The free surface equation for Poincaré waves can be determined by substituting the equations for  $\hat{\zeta}$  and  $\zeta$  and choosing an arbitrary amplitude  $Z_n$  for  $A_1$ . This returns

$$\zeta_n(x, y, t) = \Re \left\{ Z_n \left( \cos \beta y + \frac{ifk}{\beta s} \sin \beta y \right) \left( e^{i(kx - \sigma t)} \right\}$$
(A.25)

The solutions for the velocity components u and v are found by substituting an ansatz similar to  $\zeta$  - i.e.  $u = \hat{u}e^{i(kx-\sigma t)}$  and  $v = \hat{v}e^{i(kx-\sigma t)}$  in the polarisation equations. This results in

$$u_n(x, y, t) = \Re \left\{ Z_n \left( \frac{if\sigma}{sH\beta} \sin\beta y - \frac{igk}{s} \cos\beta y \right) e^{i(kx-\sigma t)} \right\}$$
(A.26)

$$v_n(x, y, t) = \Re \left\{ Z_n \left( \frac{-i\sigma f^2}{s^2 H\beta} + \frac{g\beta}{s} \sin \beta y \right) e^{i(kx - \sigma t)} \right\}$$
(A.27)

#### A.2 Properties of Kelvin and Poincaré modes

#### A.2.1 Properties of Kelvin modes

In a channel two Kelvin wave can exist going in opposite direction. Recalling equations (A.19) and (A.22), Kelvin modes can have either a positive or negative wave number k and deformation radius  $R_f$ . Let these be defined as follows:

$$k_k^+ = +\sqrt{\frac{i\sigma s}{gH}}, \quad R_f^+ = \sqrt{\frac{sgH}{i\sigma f^2}}$$
$$k_k^- = -\sqrt{\frac{i\sigma s}{gH}}, \quad R_f^+ = -\sqrt{\frac{sgH}{i\sigma f^2}}$$

In the Northern Hemisphere, waves always propagate having the coast on the right side. In the defined coordinate system, Kelvin modes traveling in 'negative x direction' have wave number  $k_k^-$  and decay in negative y-direction, therefore requiring the same sign in the Rossby deformation radius. More general, the two Kelvin modes can be described as follows:

$$\zeta_{L}^{+} = Z e^{-i(y+\delta)/R_{f}^{+}} e^{i(k^{+}x-\sigma t)}$$
(A.28)

$$\zeta_{1}^{-} = Z e^{-i(y+\delta)/R_{f}^{-}} e^{i(k^{-}x-\sigma t)}$$
(A.29)

Similarly, the velocity component switches signs depending on the direction of the mode - see (A.24):

$$u_{L}^{+} = \hat{u}^{+} Z e^{-i(y+\delta)/R_{f}^{+}} e^{i(k^{+}x-\sigma t)}$$
(A.30)

$$u_{k}^{-} = \hat{u}^{-} Z e^{-i(y+\delta)/R_{f}^{-}} e^{i(k^{-}x-\sigma t)}$$
(A.31)

The implications of these formulas is sketch in figure A.1



Figure A.1: The sign of Kelvin modes in a channel on the Northern Hemisphere. The waves propagate with the coast on their right

#### A.2.2 Properties of Poincaré modes

The wave number of Poincaré waves was given in (A.20). While the Kelvin mode wave number is, without friction, by definition is real, this is not the case with Poincaré modes. This happens if referring to (A.20).

$$\frac{i\sigma s^2 + i\sigma f^2}{c_0^2 s} < \beta^2$$

Substituting  $c_0^2$  for gH, this can be recast as

$$\frac{i\sigma s^2 + i\sigma f^2}{(n\pi)^2 gs} < \frac{H}{W^2}$$

While the right-hand sight will always be smaller than anything for infinite wide channels (as considered in the previous chapter) - in the problem considered where poincare modes are 'trapped' between two boundaries. In absence of bottom friction, free poincare waves have a real wavenumber, which returns in a sinusoidal spatial structure. Trapped waves are characterised by an imaginary wavenumber, and have only a sinusoidal structure in one direction. Whether the wavenumber is imaginary or complex depends on the tidal frequency  $\sigma$ , latitude  $\theta$  and the depth H and width W of the basin. Without friction, there are a finite number of free modes and infinite number of trapped modes. In reality, most seas are to narrow to allow for free Poincaré modes at all.

When bottom friction is included the clear distinction between trapped and free waves is lost since the wavenumber is no longer strictly real or imaginary. This results in both kind of waves showing behaviour characteristic of the other (trapped waves propagating in one direction).

As with Kelvin waves, there exists a positive and negative wavenumber.

$$k_n^+ = +\sqrt{\frac{i\sigma s^2 + i\sigma f^2}{c_0^2 s} - \beta^2}$$
$$k_n^- = -\sqrt{\frac{i\sigma s^2 + i\sigma f^2}{c_0^2 s} - \beta^2}$$

From the solutions (A.25)-(A.27) it can be seen that the mode decays in positive x-direction if the wave number is  $k_n^+$ . See figure A.2 for a sketch of the problem. Again, more general poincare waves for the problem are described as follows:

$$\zeta_n^{\pm} = \Re \left\{ Z_n \left( \cos \beta y + \frac{ifk^{\pm}}{\beta s} \sin \beta y \right) \left( e^{i(k^{\pm}x - \sigma t)} \right\} \right. \\ \left. u_n^{\pm} = \Re \left\{ Z_n \left( \frac{if\sigma}{sH\beta} \sin \beta y - \frac{igk^{\pm}}{s} \cos \beta y \right) e^{i(k^{\pm}x - \sigma t)} \right\} \right. \\ \left. v_n^{\pm} = \Re \left\{ Z_n \left( \frac{-i\sigma f^2}{s^2H\beta} + \frac{g\beta}{s} \sin \beta y \right) e^{i(k^{\pm}x - \sigma t)} \right\} \right\}$$



Figure A.2: The sign of Poincare modes in a channel.