“The impact of take-or-pay contracts on the profitability of a combined heat and power plant”

Bachelor Thesis

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Preface

In front of you lies my bachelor thesis. This thesis has been written in the framework of the bachelor study Industrial Engineering and Management at the University of Twente.

In mid April 2012, after an unforeseen series of events, I ended up at a presentation from EnergyQuants at the University of Twente on decision models for risk in the energy market. I was still looking for a bachelor assignment, and the energy market seemed an interesting subject.

A month later, I started my assignment at EnergyQuants with the task to build a decision model for a combined heat and power plant. If I ever thought I would never use the few things I learned about Matlab ever again, I was wrong.

After almost four months of reading, formulating, writing, testing, debugging and sometimes starting all over again, the model eventually worked as it should. I finally understand why IT/programming projects at the government always take up twice the time as planned. Sometimes, lines of code almost write themselves, while at other times you can stare at the screen without making a single line of progress just wondering why the output of the model is not as it should be.

Even though the weather sometimes made it hard to concentrate, and an untraceable typo in the code of the model has the potential to ruin your entire day, I experienced the months working on my thesis as a useful and an educational experience. I learned a lot on model building and testing, the energy market, financial aspects of a power plant and many other things.

I would like to thank my supervisors at EnergyQuants, Alexander Boogert and Henk Sjoerd Los, for allowing me to do my assignment at EnergyQuants and for their support and patience when I was writing this thesis. Furthermore I would like to express my gratitude to my supervisor from the University of Twente, Reinoud Joosten, for his support.

Management Summary

This report contains a study on the influence of take-or-pay contract on the profitability of a combined heat and power plant (CHP). A combined heat and power plant is a power plant that next to electricity also generates usable heat. CHP plants can for example be used for district heating, the horticulture industry and industrial processes that require heat. A contract with a take-or-pay (TOP) clause gives the recipient the obligation to consume a minimum and/or maximum amount of gas stated in the contract. If this clause is violated, the recipient has to pay a penalty for the difference between the consumed amount and the contracted amount.

One of the issues with using a contract with a TOP-clause for a CHP plant is the fact that over time the heat requirement of the installation is not always in line with the electricity requirement. In recent years, there has been a downward trend in electricity prices which makes it questionable whether these CHP plants should run to produce electricity. This uncertainty on power production has an impact on the consumed amount of gas and creates uncertainty about future levels of gas consumption.

It could be possible that the current structure of the TOP-contracts is not suitable anymore in the new situation. If this is true, it requires modifications of the contracts in order for CHP plants to stay profitable. Such information is important for gas suppliers and (future) CHP plant owners.

In order to gather insights on this problem, we have built an extension to an existing power plant model from EnergyQuants which can simulate a CHP. We have formulated a mathematical model which calculates the optimal distribution of starts and fuel consumption over the project horizon. Furthermore, we have improved the power plant model in order to be able to work with a deterministic heat demand.

With this new model, we have run scenarios with realistic price data for the years 2009-2011. These scenarios yield the following results:

- When take-or-pay constraints force a CHP to consume an amount of fuel which is 5% percent higher or lower than the optimal fuel amount, this leads to an average value decrease of 1,28%.

- When take-or-pay constraints force a CHP to consume an amount of fuel which is 10% percent higher or lower than the optimal fuel amount, this leads to an average value decrease of 4,55%.

- When take-or-pay constraints force a CHP to consume an amount of fuel which is 20% percent higher or lower than the optimal fuel amount, this leads to an average value decrease of 18,20%.

- A take-or-pay contract that forces the amount of fuel that has to be consumed to be higher than the optimal amount, has more impact on the value of a plant than a constraint that force the fuel consumption to be less than the optimal amount.

- There is no clear preference for a contract with daily, quarterly or yearly gas prices.

We suggest validating these results for more years, as fluctuations exist within the three years in our sample. This could mean that our sample data is not a representative sample for all data. Also, further research can be conducted on the addition of CO_2 demand, heat storage and a power grid connection.

Table of Contents

1 Introduction

This section introduces the topic of this study and the context in which the study has been conducted. Section 1.1 gives a description on the company where the thesis is conducted. Section 1.2 addresses the reasons for this study and the problem statement. We discuss the methodology of this thesis in Sections 1.3 – 1.5.

1.1 Company description

EnergyQuants is a consulting firm that develops quantitative decision support models and risk management tools for the commodity sector. They provide software and consulting mainly to the commodity markets and to the energy market in particular.

The company was established in January 2011, and combines the knowledge of two experts active in the energy and financial sector since 2000. Both are experienced from the viewpoint of both the utility and the energy consulting. The company is based in the Netherlands and has an international focus (EnergyQuants).

1.2 Problem identification

In recent years, the Dutch government has started initiatives to increase the efficiency of power production and the reduction of CO_2 emissions. One of these initiatives is the promotion of decentralized energy production. An example of decreasing CO_2 output with decentralized energy production is the application of a combined heat and power plant (CHP). A CHP plant produces both energy and usable heat from coal, gas or another power source. In the Netherlands, there are a number of CHP plants for the agricultural industry using gas to produce energy, heat and CO_2 for greenhouses.

The gas used for these CHP plants is usually contracted with large suppliers and most of the times these contracts contain a take-or-pay (TOP) clause. A take-or-pay clause gives the recipient the obligation to consume a minimum amount of gas stated in the contract, or pay a penalty for the difference between the consumed amount and the contracted amount. One of the issues with using a CHP plant is the fact that over time, the heat requirement of the installation is not always in line with the electricity requirement. In recent years, there has been a downward trend in electricity prices which makes it questionable whether these CHP plants will run to produce electricity. This uncertainty on power production has an impact on the consumed amount of gas and creates uncertainty about future levels of gas consumption.

It could be possible that the current structure of the TOP-contracts is not suitable anymore in the new situation. If this is true, it requires modifications of the contracts in order for CHP plants to stay profitable. Such information is important for gas suppliers and (future) CHP plant owners.

This information leads to the formulation of the following problem statement:

"Are take-or-pay contracts still suitable in the future with the current trend in electricity prices?"

1.3 Research questions

In order to get more insights in the problem statement, we look at the economic value of a CHP plant. To do this, we build a model that optimizes the running pattern of a CHP plant. This thesis is divided into two parts: the derivation of a mathematical model formulation to calculate the economic value of a CHP with multiple constraints and an application to real life.

EnergyQuants already has a functional implementation of a power plant optimization problem for a simple power plant. This plant model is based on linear programming and built in Matlab. A CHP plant is a special kind of a power plant, and the optimization of a CHP plant is an extension of the current power plant optimization model. Additions that have to be made in the existing model are long term restrictions such as yearly take-or-pay constraints and the total number of starts the CHP plant is allowed to make in a year. Next to that, the current formulation has to be extended to be able to work with a deterministic heat demand. When these additions have been made in the optimization model, realistic parameter settings have to be used as input variables in order to make an analysis of the impact of different take-or-pay contracts.

With this in mind, the following two research questions have been formulated:

1. *"Can the existing power plant model of EnergyQuants be extended in order to be able to optimize a CHP plant?"*

2. *"What is the impact of take-or-pay contracts on the profitability of a CHP plant?"*

In order to answer this research question, the following four sub questions have been formulated:

1. *How can a yearly number of allowed starts be implemented in the existing plant model?*
2. *How can take-or-pay contracts be implemented in the existing plant model?*
3. *How can deterministic heat demand be implemented in the existing plant model?*
4. *What are realistic cost elements and parameter settings for a CHP?*

These questions will be answered in the following sections. After the first three research questions have been answered, the current optimization model will be adapted for a CHP plant. The results of question four will be used as input variables in this model, after which the results will be analyzed to be able to give an answer to second research question in Section ten. Section eleven will summarize the results and give recommendation for further research.

1.4 Scope and Limitations

The results of this thesis are not only useful for gas suppliers and CHP plant owners, but should also give insights to EnergyQuants for valuating gas- and CHP projects. The model that will be extended is a model built by EnergyQuants. Inputs for this model are gas- and electricity prices. Deterministic scenarios for gas- and electricity prices will be used in this thesis and will be made in cooperation with EnergyQuants.

The original power plant model is built in Matlab in combination with Excel. As we will use this model as a basis to build upon, we will also build the extension in Matlab and Excel.

The model is based on linear programming and this could lead to a large-scale problem if we want to analyze a long period with several constraints. To decrease the computation time, the problem is divided in sub-problems. However, this might lead to a solution which is close to, but not exactly the maximum value. This is deemed acceptable.

The plant we modeled has no outgoing connection to the power grid, which means it has to consume al produced energy itself. This has influence on the choice of CHP installation. Also, the model we build has no separate heat storage or boiler, which has influence on the run pattern.

Apart from the electricity and heat demand, greenhouses can also have a need for CO_2 to increase the growth of certain crops. In this model usable CO_2 output for greenhouses is not modeled to reduce complexity.

1.5 Methodology

Section 2 gives a background on linear optimization problems and how these will be solved in Matlab. Section 3 describes the current power plant optimization model used by EnergyQuants and Section 4 will describe the differences between a normal power plant and a CHP plant.

In Section 5 to 7, the first three sub questions will be addressed and additions to the current model will be described. When the first three sub questions have been answered, a model can be built to simulate a CHP plant. The final additions to the model are described in Section 8.

In Section 9, we will describe and estimate realistic parameter settings for our plant model. We will run the model with these parameters settings for three different years, after which the results of these scenarios will be analyzed in Section 10.

The research will be concluded in Section 11, where we will answer both research questions and give recommendations for further research.

2 Theoretical background

In this section, the theoretical background of the thesis is discussed. We will give a short overview on linear programming problems and how we will solve these problems in Matlab.

2.1 Linear programming

Linear Programming is a way of describing and solving mathematical problems. It is a scientific approach to decision making and usually involves the use of one or more mathematical models.

The model we will use in this thesis is a prescriptive or optimization model. A prescriptive model “prescribes” behavior for an organization that will enable it to best meet its goal. The components of a descriptive model include

- Objective function(s)
- Decision variables
- Constraints

An optimization model seeks to find the values of the decision variables that optimize the objective function among the set of all values for the decision variables that satisfy the given constraints. (Winston, 2004, p. 2)

Small LP problems can be solved by hand. However, as the number of decision variables increases, the number dimensions of the feasible region increase as well and a computer is needed to solve the problem. In this thesis, we will use Matlab to solve the LP problems. This is done by a custom Matlab file called lp_solve. A more detailed description of lp_solve can be found in Appendix 13.

2.2 LP problems

In this thesis, we will work with two different LP models. The first model is the plant optimization model which calculates the optimal value of the CHP over a smaller period for a given number of starts and a given fuel interval. This model is a mixed integer linear problem. A more in depth description of this model can be found in Section three.

The second LP model determines the optimal combination of starts and fuel consumption. This model uses the output from the first LP model as input variables and tries to find a feasible optimal combination. This model is a 0-1 integer linear problem and will be described in Section five and six.

2.3 Milp & 0-1 IP problems

An LP problem in which all variables are integers is called a pure integer LP (IP). MILP problems are LP problems which where some of the variables are required to be integers. A 0-1 IP problem is an IP where all variables are required to be either 0 or 1. The LP obtained by omitting all integer or 0-1 constraints on variables is called the LP relaxation of the IP. In order to solve an IP problem, first the optimal value of the relaxation of the LP has to be calculated. The feasible region for any IP must be contained in the feasible region for the corresponding LP relaxation. (Winston, 2004, pp. 475-477)

In practice, most IP's are solved by using the technique branch-and-bound. Branch-and-bound methods find the optimal solution to an IP by efficiently enumerating the points in a sub problem's feasible region (Winston, 2004, pp. 512-524).

3 Current power plant optimization model

In this section, the current power plant optimization for a normal power plant used by EnergyQuants will be described. In order to build a CHP plant optimization model, extensions will be made taking this model as a framework.

3.1 Constraints and parameters

The current model used by EnergyQuants has been built for power plants using gas as input to produce electricity. By using hourly gas- and electricity prices as input parameters, the model computes when the plant should be turned on or off and on what power production level the plant should run. The model takes several constraints into account which are relevant for a normal power plant, but are not sufficient to model a CHP plant. The constraints which are already implemented are:

- Minimum power production
- Maximum power production
- Minimum power efficiency
- Maximum power efficiency
- Minimum hours per run
- Minimum hours between two successive runs
- Maximum number of starts
- Ramp-up rate (maximum upwards difference in hourly power production rate)
- Ramp-down rate (maximum downwards difference in hourly power production rate)
- Minimum amount of gas that has to be consumed
- Maximum amount of gas which can be consumed

Other input parameters in the model are the time frame over which the model makes a calculation, the prediction period for the gas and electricity prices, maintenance periods, fixed cost per startup, fuel usage per startup, and the starting position of the plant (on or off). When the plant is on at time *t*, it means that the plant is consuming fuel at time *t* to generate power. When the plant is in idle mode, it means that the plant is not consuming any fuel.

To reduce the complexity and computation time of the model, the valuation period gets divided into smaller periods. By default these periods are weeks (168 hours), but can any number of hours. For each smaller period, the optimal set of decision variables will be calculated by lp_solve and a corresponding optimal value will be given.

3.2 Optimal value

The state of the plant at hour t is defined as X_t. Available values for X_t are:

$X_t = 0$ when the plant is idle at time t

Minimum production level $\leq X_t \leq$ maximum production level when the plant is on.

If we look at a plant in the horticulture, CO_2 becomes another important cost aspect. Depending on the crops in the greenhouse, a horticulturist also needs CO_2 for a better harvest. Sometimes a CHP will produce all the CO_2 needed for the crops, while at other times there is a shortage and CO_2 has to be bought from external sources. If the CHP produces too much CO_2, it will also cost money because of the limited emission rights of the whole horticulture industry.

If we take this into account, the value of the plant becomes

$$\sum_{t=1}^{t=N} X_t * \left(e_t - \frac{g_t}{\eta}\right) - \alpha_t * CO_{2,t}^{in} - \beta * CO_{2,t}^{out}$$

Where e_t is the electricity price at time t, g_t is the gas price at time t and η is the efficiency. The variable α_t is the cost of buying CO_2at time t, β is the cost of CO_2 emissions, and $CO_{2,t}^{in}$ and $CO_{2,t}^{out}$ are the amounts of CO_2 which are either bought or emitted at time t.

Because α_i, β, $CO_{2,i}^{in}$ and $CO_{2,i}^{out}$ are dependent on a lot of other variables like the type of crops, the total emission rights of the industry, the type and size of the plant and many other variables, they are very hard to estimate correctly. Because of this, we have decided not to take these variables into account in our research. For further research, this might be an interesting topic.

When we take the above into consideration, we can rewrite the value of the plant as:

$$\sum_{t=1}^{t=N} X_t * \left(e_t - \frac{g_t}{\eta}\right)$$

In this thesis, we refer to the above formula when we optimize the total value of a CHP plant.

4 What is a CHP plant?

A CHP plant is a special kind of power plant which produces both energy and usable heat instead of just energy. Because of this, the efficiency of CHP plants is higher than the efficiency of traditional ones. In this section, we will give a background on different types of CHP plants.
It is important to know the differences between CHP installations, because different plants require a different mathematical formulation (Weber, 2005).

CHP applications mainly use two types of CHP installations: steam turbines and gas turbines. We will choose to work with these installations as they are the most common in the CHP industry. Also, they already partly modeled by Weber (Weber, 2005, pp. 97-106).

4.1 Steam Turbines

Steam turbines are available in sizes from under 100 kW to over 250 MW and are widely used for combined heat and power (CHP) applications. Unlike gas turbine and reciprocating engine CHP systems where heat is a byproduct of power generation, steam turbines normally generate electricity as a byproduct of heat (steam) generation. A steam turbine is captive to a separate heat source and does not directly convert fuel to electric energy. The energy is transferred from the boiler to the turbine through high pressure steam that in turn powers the turbine and generator. This separation of functions enables steam turbines to operate with an enormous variety of fuels, from natural gas to solid waste, including all types of coal, wood, wood waste, and agricultural byproducts (sugar cane bagasse, fruit pits, and rice hulls). In CHP applications, steam at lower pressure is extracted from the steam turbine and used directly or is converted to other forms of thermal energy.

Steam turbine CHP systems generally have low power to heat ratios, typically in the 0.05 to 0.2 range. This is because electricity is a byproduct of heat generation. Hence, while steam turbine CHP system electrical efficiency may seem low, it is because the primary objective is to produce large amounts of steam. However, the effective electrical efficiency of steam turbine systems is high, because almost all the energy difference between the high-pressure boiler output and the lower pressure turbine output is converted to electricity. This means that total CHP system efficiencies are generally high and approach the boiler efficiency level. Steam boiler efficiencies range from 70 to 85 % depending on boiler type and age, fuel, duty cycle, application, and steam conditions

Steam turbines differ from reciprocating engines and gas turbines in that the fuel is burnt in a piece of equipment, the boiler, which is separate from the power generation equipment, the steam turbo generator. As mentioned previously, this separation of functions enables steam turbines to operate with an enormous variety of fuels.

The primary locations of steam turbine based CHP systems are industrial processes where solid or waste fuels are readily available for boiler use. In CHP applications, steam extracted from the steam turbine directly feeds into a process or is converted to another form of thermal energy. Steam engines are mainly seen in the chemicals, primary metals, and paper industries. Pulp and paper mills are often an ideal industrial/CHP application for steam turbines. Such facilities operate continuously, have a high demand for steam, and have on-site fuel supply at low, or even negative costs (waste that otherwise would have to be disposed of) (Energy Nexus Group, 2002).

CHP applications use two types of steam turbines: non-condensing and extraction.

4.1.1 Non-Condensing (Back-pressure) Turbine

The non-condensing turbine (also referred to as a back-pressure turbine) exhausts its entire flow of steam to the industrial process at conditions close to the process heat requirements.

The term “back-pressure” refers to turbines that exhaust steam at atmospheric pressures and above. The specific CHP application establishes the discharge pressure. The most typical pressure levels for steam distribution systems are 50, 150, and 250 psig. District heating systems most often use the lower pressures, and industrial processes use the higher pressures. Power generation capability reduces significantly when steam is used at appreciable pressure rather than being expanded to vacuum in a condenser. Discharging steam into a steam distribution system at 150 psig can sacrifice slightly more than half the power that could be generated when the inlet steam conditions are 750 psig and 800°F, typical of small steam turbine systems. A graphical representation of a back pressure turbine can be seen in Figure 1 (Energy Nexus Group, 2002).

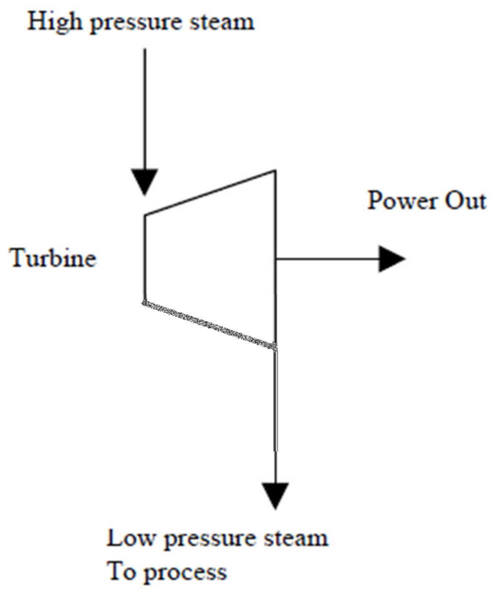

Figure 1: Back-Pressure Turbine (Energy Nexus Group, 2002)

4.1.2 Extraction Turbine

The extraction turbine has openings in its casing for extraction of a portion of the steam at some intermediate pressure before condensing the remaining steam. The extracted steam may be used for process purposes in a CHP facility or for feed water heating as is the case in most utility power plants.

The steam extraction pressure may or may not be automatically regulated. Regulated extraction permits more steam to flow through the turbine to generate additional electricity during periods of low thermal demand by the CHP system. In utility type steam turbines, there may be several extraction points, each at a different pressure corresponding to a different temperature. The facility's specific needs for steam and power over time determine the extent to which steam in an extraction turbine is extracted for use in the process.

With these choices the designer of the steam supply system and the steam turbine has the challenge of creating a system design which delivers the (seasonally varying) power and steam which presents the most favorable business opportunity to the plant owners. A graphical representation of an extraction condensing turbine can be seen in figure 2 (Energy Nexus Group, 2002).

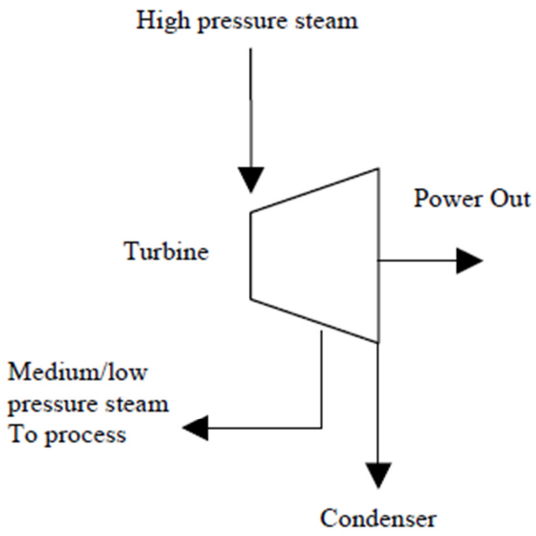

Figure 2: extraction condensing steam turbine (Energy Nexus Group, 2002)

4.2 Gas Turbines

Gas turbines are available in sizes ranging from 500 kW to 250 MW. Gas turbines can be used in power-only generation or in combined heat and power (CHP) systems. Gas turbines operate on natural gas, synthetic gas, landfill gas, and fuel oils. Plants typically operate on gaseous fuel with a stored liquid fuel for backup to obtain the less expensive interruptible rate for natural gas. Gas turbines produce a high quality (high temperature) thermal output suitable for most combined heat and power applications. High-pressure steam can be generated or the exhaust can be used directly for process drying and heating. This high-quality exhaust heat can be used in CHP configurations to reach overall system efficiencies (electricity and useful thermal energy) of 70 to 80 percent.

The oil and gas industry commonly uses gas turbines to drive pumps and compressors. Process industries use them to drive compressors and other large mechanical equipment, and many industrial and institutional facilities use turbines to generate electricity for use on-site. When used to generate power on-site, gas turbines are often used in combined heat and power mode where energy in the turbine exhaust provides thermal energy to the facility.

The majority of the simple-cycle gas turbine based CHP systems are operating at a variety of applications including oil recovery, chemicals, paper production, food processing, and universities. Simple-cycle CHP applications are most prevalent in smaller installations, typically less than 40 MW.

A typical commercial/institutional CHP application for gas turbines is a college or university campus with a 5 MW simple-cycle gas turbine. Approximately 8 MWh of 150 psig to 400 psig steam (or hot water) is produced in an unfired heat recovery steam generator and sent into a central thermal loop for campus space heating during winter months or to single-effect absorption chillers to provide cooling during the summer.

Within a gas turbine, atmospheric air is compressed, heated, and then expanded, with the excess of power produced by the expander (also called the turbine) over that consumed by the compressor used for power generation. Consequently, it is advantageous to operate the expansion turbine at the highest practical temperature consistent with economic materials and internal blade cooling technology and to operate the compressor with inlet air flow at as low a temperature as possible. As technology advances permit higher turbine inlet temperature, the optimum pressure ratio also increases.

A gas turbine based system is operating in combined heat and power mode when the waste heat generated by the turbine is applied in an end-use. For example, a simple-cycle gas turbine using the exhaust in a direct heating process is a CHP system, while a system that features all of the turbine exhaust feeding a heat recovery steam generator and all of the steam output going to produce electricity in a combined-cycle steam turbine is not (Energy and Environmental Analysis, 2008).

5 Startups

In this section, we answer the first sub question; *how can a yearly number of allowed starts be implemented in the existing plant model?*

First, we will give a short explanation on startups and why they have to be modeled. After that, a mathematical representation and the implementation in Matlab will be discussed.

5.1 Relevance of startups for a CHP plant

When working with a TOP-contract, most gas suppliers sell their gas at a price which is day specific. However, electricity prices vary a lot on different hours during a normal day. There is a peak around lunchtime and an even higher peak around 18:00 o'clock. Because of this, it depends on the hour whether it is profitable to turn on a CHP plant. Figure 1 shows the energy price (euro per MWh) for the fifth of January in the price scenario made together with EnergyQuants.

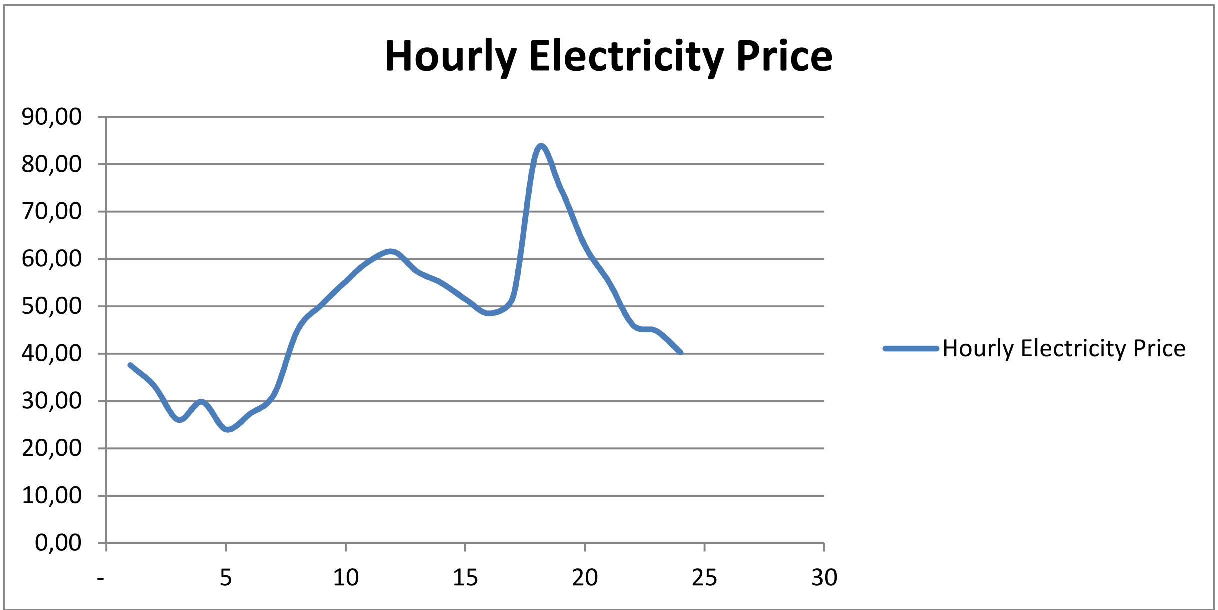

Figure 3: Estimation energy prices 5-1-2012

In the optimal situation, it would be possible to change the status of your CHP on an hourly basis. However, this needs a lot of monitoring and currently is not technically possible. Next to that, CHP plants have a minimum up-time for each run. When a CHP has been turned off, it also needs a cool down time before it can start up again. In the coming sections, we will work with a minimum up and down time of twelve hours. However, this is variable.

If we assume that the plant is only profitable during peak hours in electricity prices, and has minimum up- and down time of twelve hours, the optimal solution is to have one run of twelve hours each day over the period in which the plant generates the most profit. However, startups cost a fixed amount of money and a fixed amount of fuel each time. Next to that, there usually is a maximum number of starts incorporated in the financial contract. Startups also have impact on the wear and tear of the CHP plant. Because of this, it might be better to only use a limited number of startups during a week, month or year.

5.2 Mathematical representation of startups

In order to make a mathematical representation of the problem with a limited number of startups, we break the problem up into smaller periods of weeks. We do this in order to decrease the problem complexity and computation time. We compute for each week the total value of the plant for a different number of starts. Afterwards, the optimal number of starts each week is selected using an additional optimization

When there only is a limited number of a starts each year, you cannot perform a start every day. Instead, with a minimum up- and down time of twelve hours, the CHP plant will start between zero and seven times a week. The current power plant model is already able to compute the weekly optimal value for a given number of starts. If we compute this over *m* weeks, with *n* different number of start possibilities, this gives us an *m* by *n* matrix with the optimal value for each possible number of starts for every week. Selecting the optimal amount of starts can be done with the following mathematical model.

$i = Weeknumber$
$j = Number\ of\ starts$
$r_{i,j} = Number\ of\ starts\ j\ in\ week\ i$
$V_{i,j} = Value\ in\ week\ i\ with\ j\ starts$
$S_{i,j} = Decision\ variable\ that\ equals\ 1\ if\ the\ CHP\ uses\ j\ starts\ in\ week\ i$
$R = Maximum\ number\ of\ starts$

$$Max\ z = \sum_i \sum_j V_{i,j} * S_{i,j}$$

$$s.t. \quad \sum_i \sum_j r_{i,j} * S_{i,j} \leq R$$

$$\sum_j S_{i,j} = 1 \qquad (\forall i)$$

$i, j, r_{i,j}, R\ are\ integers$
$S_{i,j} = 0\ or\ 1$
$R > 0$
$r_{i,j} \geq 0$
$i = 1,2, \ldots [number\ of\ weeks]$
$j = 0,1, \ldots [maximum\ starts\ per\ week]$

The goal is to maximize the total value of the CHP. This is done by multiplying a set of decision variables, $S_{i,j}$, with a set of corresponding values, $V_{i,j}$.

The first constraint bounds the maximum number of starts that can be made over the entire valuation period. The second constraint ensures that when the CHP starts j times in week i, it cannot start $j + x$ times in the same week.

5.3 Startups in Matlab

Unfortunately, the lp_solve algorithm described in Section 3.2 cannot work with this mathematical formulation. This is because the current mathematical formulation is not exactly in the form

$$Max\ v = f^T * x$$
$$s.t.\ \ A * x \leq b$$

In order to do so, we have to transform $V_{i,j}$ into a vector *f*, transform $S_{i,j}$ into a vector *x*, and rewrite the constraints as matrices. In this section, we will give an example on how this is done.

Making a vector of $V_{i,j}$ by removing one dimension looks quite difficult, but is easily done by placing all subsequent rows under each other. This will transform an *m* by *n* matrix into an *m***n* vector where the old element $V_{i,j}$ now corresponds with $f_{(j-1)*m+i}$.

We will give an example by looking at the first four weeks of the year, with five different starting options. The Matlab model creates the following output for $V_{i,j}$.

$$\begin{bmatrix} 0 & 79030 & 80644 & 82045 & 83445 \\ 0 & 76462 & 78100 & 79545 & 80990 \\ 0 & 73895 & 75556 & 77046 & 78535 \\ 0 & 70989 & 72675 & 74208 & 75742 \end{bmatrix}$$

In this matrix, the rows are week one to four, and the column are zero to four startups. Because the price model gives the same energy prices in the first four weeks of January, the values of $V_{i,j}$ are the same in each column, if we transform this matrix like we described above, we can create the vector *f*;

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 79030 \\ 76462 \\ 73895 \\ 70989 \\ 80644 \\ 78100 \\ 75556 \\ 72675 \\ 82045 \\ 79545 \\ 77046 \\ 74208 \\ 83445 \\ 80990 \\ 78535 \\ 75742 \end{bmatrix}$$

Now we have our vector *f*, we want to know what to do with our decision variables. In the current model, these are described by action matrix $S_{i,j}$, which has the same size as $V_{i,j}$. This matrix needs to be transformed into vector *x*, with the same length as vector *f*.

We build this vector by again placing all subsequent rows under each other, as we did with the previous matrix. We now have made a vector where the old element $S_{i,j}$ corresponds with $x_{(j-1)*m+i}$.

Next, we rebuild the constraints of the mathematical model. We start with the constraint:

$$\sum_i \sum_j r_{i,j} .* S_{i,j} \leq R$$

This constraint ensures that the sum of the number of starts each week does not exceed the total number of allowed starts. This constraint can be written as one equation where $A * x$ is the left hand side and *b* is the right hand side of this equation.

The right hand side of our constraints, *b,* equals the maximum number of allowed starts, R. Our set with decision variables, *x,* is currently a vector of size *m*n* by 1 and corresponded with $S_{i,j}$. Because we have to satisfy $A * x \leq b$, we know *A* is a 1 by *m*n* matrix. The values in *A* are the weights for the decision variables which in our example equal $r_{i,j}$.

The weight for each decision variable in this constraint should be equal to the number of starts of the corresponding element in vector *x.* For the example we used above, this gives us the following matrix for *A*:

$$[0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 1 \quad 1 \quad 1 \quad 2 \quad 2 \quad 2 \quad 2 \quad 3 \quad 3 \quad 3 \quad 3 \quad 4 \quad 4 \quad 4 \quad 4]$$

This matrix consists of a sequence of *n* (the number of weeks) times each possible number of starts. For any other number of weeks or number of start possibilities*,* the matrix A is created in the same way as shown in the example above.

For example, if we rewrite this as a linear constraint, we get

$$\begin{gathered} 0x1 + 0x2 + 0x3 + 0x4 + 1x5 + 1x6 + 1x7 + 1x8 + 2x9 + 2x10 + 2x11 \\ +2x12 + 3x13 + 3x14 + 3x15 + 3x16 + 4x17 + 4x18 + 4x19 + 4x20 \ \leq R \end{gathered}$$

Next, we have to define *A* and *b* for the weekly starting constraint,

$$\sum_j S_{i,j} = 1$$

To avoid confusion with the previous constraint, we will call *A* and *b* respectively *Aeq* and *beq*.

As with the previous constraint, we first describe this problem with an example where we use four weeks and five starting options. Because *m* and *n* are still of the same size, the decision variables are the same vector *x.*

In the mathematical formulation, $S_{i,0} + S_{i,1} + S_{i,2} + S_{i,3} + S_{i,4}$ had to equal 1 for each *i*. this means that the vector *b* consists out of ones, and is a vector with size *m* (the number of periods)

In this example, *beq* becomes the vector, $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

With any other number of weeks *m,* of starting iterations, *n*, *beq* will always be a vector of size *m* and will be filled with ones.

Aeq equals the left hand side of the equation, and always is a matrix of size *m* by *m*n*. This is because there are *m* equations that have to be solved for *m*n* decision variables.

For each *i*, the row of matrix *Aeq* should correspond with $S_{i,0} + S_{i,1} + S_{i,2} + S_{i,3} + S_{i,4}$

This means that these five elements should get a 1, and the other elements in the row should be zero. This is in accordance to the values $S_{i,j}$ can be.

In this example, *Aeq* becomes the matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Aeq will always be a size *m* by *m*n* matrix with the same pattern as the matrix from our example.

If we combine these matrices into a set of linear equations of the form $A * x \leq b$, we get:

$x1 + x5 + x9 + x13 + x17 = 1$ constraint week 1
$x2 + x6 + x10 + x14 + x18 = 1$ constraint week 2
$x3 + x7 + x11 + x15 + x19 = 1$ constraint week 3
$x4 + x8 + x12 + x16 + x20 = 1$ constraint week 4

To finalize our example, we have to combine the constraints above. The LP formulation to calculate the optimal division of starts over the periods becomes:

$Max\ v = f^T * x$

s.t.
$0x1 + 0x2 + 0x3 + 0x4 + 1x5 + 1x6 + 1x7 + 1x8 +$
$2x9 + 2x1 + 2x11 + 2x12 + 3x13 + 3x14 + 3x15 +$
$3x16 + 4x17 + 4x18 + 4x19 + 4x20 \leq R$ Max # starts constraint

$x1 + x5 + x9 + x13 + x17 = 1$ constraint week 1
$x2 + x6 + x10 + x14 + x18 = 1$ constraint week 2
$x3 + x7 + x11 + x15 + x19 = 1$ constraint week 3
$x4 + x8 + x12 + x16 + x20 = 1$ constraint week 4

$0 \leq x \leq 1$
$x = 0\ or\ 1$

With all this in mind, we have built a Matlab file which rebuilds all the matrixes from the mathematical model given in Section 5.2 and such that we can use the function lp_solve. It uses the output from the lp_solve file to create a vector with the optimal starts for each period. This vector can be used as input variable to determine the optimal value over the valuation period.

5.4 Analysis

Using the new Matlab file, we can run different scenarios in order to see the impact of the new model on the plant output. To save time, we have chosen to run these scenarios for a month instead of a whole year. We have done this, because the effects over a month are similar to the effects over a year, but on a smaller scale. Apart from measuring the value of the plant, the runtime has been monitored in order to be able to see what the impact is of splitting the problem into smaller periods. For all our scenarios in this section, the maximum power generation level is 100 MW and the minimum power generation level is 10 MW. The minimum run time and minimum idle time are both 12 hours. The start state of the plant each period is idle.

Scenario 5.1 has no limitation on the total number of starts and the possibility to start zero to seven times a week. Because the plant has an on and off time of twelve hours, this is the maximum amount that still leads to a different optimal solution.
In Scenario 5.2, we decrease the number of total starts to fifteen. The maximum number of starts each week is seven.
In Scenario 5.3, the total number of starts allowed is unbounded, but the number of starts each week is brought back to a maximum of three, which effectively decreases the total allowed number of starts to a maximum of fifteen starts that month.
In Scenario 5.4, we do not split up the problem into smaller sub problems of a week, but instead do a calculation over a whole month. The total number of allowed starts will be fifteen as in Scenario 5.2. Because the problem is not split into sub-problems, the total value will probably be higher.

The results of these scenarios can be found in Appendix 1.

From the results from Scenario 5.1, we see that the optimal solution with infinite starts would be to make a run every day, as we already stated in Section 5.1. Week five only has three starts, because it consists out of the 29th, 30th and 31st of January and not a whole week. If we run the model for a whole month, we thus have 31 starts.

In Scenario 5.2, it can be seen that when the total amount of starts goes down, Matlab calculates that it is more profitable to make longer runs instead of turning the plant off for a longer period. This means that the plant will also run during hours where it actually will make losses. As a result, the consumed amounts of gas and produced energy are higher, but the total profit is lower.

Scenario 5.3 gives all weeks an equal amount of starts. The amount of consumed gas and created energy surprisingly is the same. However, because the division of the starts is less optimal than in Scenario 5.2, the optimal value is slightly lower. Because in this scenario we only have four different starting options instead of eight, the amount of decision variables is halved. We can see that this leads to an expected decrease in runtime.

Scenario 5.4 calculates the optimal value over a whole month. This indeed gives a slightly more optimal value for the plant. The value computed Scenario 5.2 is 99.87% of the optimal value in Scenario 5.4. However, Scenario 5.4 requires over eleven times more time to compute this outcome, as it is not split up into sub problems. If this scenario is run over a whole year, the increase in runtime will be even higher. Appendix 9 contains a table with runtimes for different valuation periods without splitting it into smaller periods.

5.5 Conclusion

In this section we gave a mathematical representation of a power plant with a finite number of allowed starts. This formulation has been implemented in the power plant optimization model in Matlab, and we have analyzed different scenarios. From these scenarios, we can conclude that a bounded number of starts can lead to a lower optimal value, but a higher gas consumption and energy production. However, the latter is specific for this case, as with in other price scenarios, the model might choose to lengthen the idle time instead of the run time. This will result in a lower optimal value, but also lower gas consumption and energy production
we can also conclude that splitting the problem into smaller sup problems gives a suboptimal value which is extremely close to the optimal value, while decreasing the running time of the model with a significant amount.

6 Take-or-pay

In this section, we give an answer on the second research question; *How can take-or-pay contracts be implemented in the existing power plant model?*

We give a short explanation on take-or-pay contracts and their influence on CHP plants. After that, a mathematical representation and the implementation in Matlab will be discussed. Finally, an analysis will be given with the new Matlab model.

6.1 Relevance of take-or-pay contracts for a CHP plant

A contract with a take-or-pay clause gives the recipient the obligation to consume a minimum amount of gas, or pay a penalty for the difference between the consumed amount and the contracted amount. There can also be a maximum allowed of fuel that may be consumed. In the ideal situation, the optimal fuel consumption of a CHP plant is within the range of the take-or-pay clause. Because the energy price is volatile and dependent on a lot of external factors like the weather, this is not always the case. In cold winters or hot summers, more energy is used to respectively warm and cool houses and offices than average. This leads to a higher energy price, which means that running the CHP will be profitable frequently. When the winters are less harsh or the summers are relatively cool, energy prices will be lower, which means the CHP will be less frequent in the money.

In the scenario described above, the owner of a CHP plant will probably get an optimal value if he uses more gas than allowed in the take-or-pay contract. In the second scenario, the plant might only be in the money for small periods during the year, meaning the CHP plant owner probably needs to use less fuel than in the take-or-pay contract in order to get the optimal value.

In both scenarios, the optimal solution cannot be used, as it violates the take-or-pay clause of the contract. In the next section, we will describe a mathematical model which takes take-or-pay constraints into account and distributes the fuel in an optimal way over different periods in the project.

6.2 Mathematical Formulation with take-or-pay constraints

In order to make a mathematical representation of the problem with extra fuel constraints, we again split the valuation period into smaller periods. We use the mathematical formulation from Section 5 and add several extra constraints. These include maximum fuel consumption and minimum fuel consumption. This means that we also need an extra variable for the amount of fuel consumed. Next to that, the matrix with optimal values for a number of starts each week needs to be expanded with a dimension for the fuel used. The same happens with our set of decision variables.

This gives us the following mathematical formulation.

$i\ = Weeknumber$

$j = Number\ of\ starts$
$k = fuel\ consumption\ interval$
$r_{i,j,k} = Number\ of\ starts\ j\ in\ week\ i\ with\ k\ fuel\ consumption$
$f_{i,j,k} = Fuel\ consumed\ in\ week\ i\ with\ j\ starts\ and\ fuel\ consumption\ interval\ k$
$V_{i,j,k} = Value\ in\ week\ i\ with\ j\ starts\ and\ fuel\ consumption\ interval\ k$
$S_{i,j,k} = Decision\ Variable\ that\ equals\ 1\ if\ CHP\ uses\ j\ starts\ in\ week\ i\ with\ fuel$
$consumption\ interval\ k$
$R = Maximum\ number\ of\ starts$
$Fmax = Maximun\ Fuel\ consumption$
$Fmin = Minimum\ Fuel\ consumption$

$$Max\ z = \sum_i \sum_j \sum_k V_{i,j,k}.* S_{i,j,k}$$

$$s.t. \qquad \sum_i \sum_j \sum_k r_{i,j,k} * S_{i,j,k} \leq R$$

$$\sum_i \sum_j \sum_k f_{i,j,k} * S_{i,j,k} \leq Fmax$$

$$\sum_i \sum_j \sum_k f_{i,j,k} * S_{i,j,k} \geq Fmin$$

$$\sum_j \sum_k S_{i,j,k} = 1 \qquad (\forall i)$$

$i, j, k, r_{i,j,k}, R\ are\ integers$
$S_{i,j,k} = 0\ or\ 1$
$R > 0$
$r_{i,j,k}, f_{i,j,k}, Fmin, Fmax \geq 0$
$i = 1,2, \ldots [number\ of\ weeks]$
$j = 0,1, \ldots [maximum\ starts\ per\ week]$
$k = 1,2, \ldots [maximul\ fuel\ interval]$

This model looks a lot like the model from Section 5.2, but there are some changes. First of all, there is an extra dimension for the fuel consumption, *k*. Also, two constraints have been added to bind the maximum amount of fuel and minimum amount of fuel consumed over the whole -contract period. The maximum and minimum amount of fuel that can be consumed corresponds with the values in the take-or-pay contract.

6.3 Take-or-Pay in Matlab

As in the previous section, these extra constraints have to be altered in order to work with the lp_solve algorithm.

First of all, we will have to add another input variable to the model, the fuel used each week. This transforms our two-dimensional matrix $V_{i,j}$ into the three dimensional matrix $V_{i,j,k}$ where k stands for a fuel interval. This fuel interval consists of a minimum and maximum value for the allowed amount of fuel consumption

In the ideal situation, the amount of fuel consumed would be calculated with interval steps of 1 MWh. This way, the fuel used each week will be allocated in an optimal way. However, this will lead to a huge increase in the number of decision variables, as it is not exceptional if the amount of gas in a take-or-pay contract is in the range of 1000MWh to 10000MWh. Because of this, the intervals have to be larger in order to give an acceptable computation time.

Because a take-or-pay contract limits both the maximum and minimum amount of fuel that can be consumed, it would seem logical to add a dimension for the maximum and minimum amount of fuel that can be consumed each week.

If we choose to calculate the value each week for n different amounts of minimum and maximum fuel consumption, we create two more dimension in our model and we obtain n^2 times more decision variables.

Instead, we choose to describe the maximum and minimum fuel consumption with fuel intervals. We choose a fixed number of intervals for each scenario, and the maximum and minimum fuel consumption for each interval is scaled to the total number of intervals. This seems reasonable as they are related variables. If we use n different intervals, we obtain n times more decision variables instead of n^2 times more, as we only create one extra dimension.

The maximum amount of fuel that can be consumed per period is either the maximum amount of fuel that the plant can consume if the plant runs on maximum capacity during the whole period, or the maximum amount of fuel allowed to consume over the whole valuation period. However, it is unlikely that the second scenario is realistic, especially for projects with a longer valuation period. The first scenario will only occur when either the energy price is extremely volatile over different periods or when the constraint on the minimal amount of fuel forces to plant to run on total capacity during the whole period. Also these scenarios are highly unlikely to occur.

By setting the maximum amount of fuel consumption allowed each week to a lower amount than the constraints described above, we will create either smaller intervals, or less intervals of the same length. The first option leads to a better solution and the second option leads to less computation time, which both are improvements for the model. However, by setting the maximum amount of fuel consumption allowed each week too low, we have the risk that it will be below the optimal value for certain periods.

To cover the problem stated above, we chose to split the fuel options into equal intervals between zero and twice the average amount fuel allowed each week. As will be shown in the analysis, this still covers the optimal amount of fuel consumed for each week for our practice scenarios. For example; when the average amount of fuel allowed each week is 10000MWh and we use four different fuel iterations, the model will give the optimal value for the intervals [0-5000 MWh], [5001 -10000 MWh], [10001-15000 MWh] and [15001-20000 MWh]. The scenario with a maximum amount of 0 MWh does not have to be run, as it is being calculated in the zero starts option when the start state is off, or is infeasible when the start state is on.

The current model is already capable of calculating the optimal value for a given period, with a given number of starts and a given minimum and maximum fuel. This optimal value corresponds with our $V_{i,j,k}$. Once again, we will have to transform this three-dimensional matrix to a one-dimensional vector. We will used the same heuristic as we used in Section 5.3, but because we have one dimension more, we will have to do it twice. This transforms the size *m* by *n* by *p* matrix $V_{i,j,k}$ into vector *f* with size $m * n * p$, where the old element $V_{i,j,k}$ corresponds with the new element

$$f_{(j-1)*m*p+(i-1)*p+k)}$$

Now we have our vector *f,* we will do the same with our set of decision variables $S_{i,j,k}$.We will transform this matrix to the vector *x* with size $m * n * p$, where the old element $S_{i,j,k}$ corresponds with the new element

$$x_{(j-1)*m*p+(i-1)*p+k}$$

Next, we will look at the extra constraints formulated in the previous section, starting with

$$\sum_i \sum_j \sum_k f_{i,j,k} \;.*\; S_{i,j,k} \leq Fmax$$

$$\sum_i \sum_j \sum_k f_{i,j,k} \;.*\; S_{i,j,k} \geq Fmin$$

These constraints ensure that the optimal value will not exceed the minimum and maximum values of the take-or-pay contract. These constrains are very similar to the maximum number of starts constraint, and can be modeled in the same way.

Our vector *b* for these constraints is equal to the right side of the equations, which are $Fmax$ and $Fmin$. Because our set of decision variables, *x*, has size *m*n*p* by 1*,* the size of *A* is 2 by *m*n*p.* The value assigned to both rows off *A* is the amount of fuel consumed in the optimal solution for the *i, j* and *k* of the corresponding decision variable.

The last constraint which has to be added, is the weekly fuel constraint which ensures that each week, only one fuel interval can be chosen for the optimal solution. As said in the previous section, this constraint can be combined with the weekly start constraint and can be written as:

$$\sum_j \sum_k S_{i,j,k} = 1$$

To construct *Aeq*, we will look at an example with four weeks, three fuel intervals and two starting options. This gives us 4*3*2=24 decision variables.

In the mathematical formulation $S_{i,1,1} + S_{i,1,2} + S_{i,1,3} + S_{i,2,1} + S_{i,2,2} + S_{i,2,3}$ has to equal 1 for all *i.* Like in the previous section, our *beq* is equal to the right side of the constraints, equaling a vector of size *m* (the number of periods) filled with ones.

Aeq equals the left hand side of the equation, and always is a matrix of size *m* by *m*n*p*. This is because there are *m* equations that have to be solved for *m*n*p* decision variables.

For each *I,* the row of matrix *Aeq* should describe $S_{i,1,1} + S_{i,1,2} + S_{i,1,3} + S_{i,2,1} + S_{i,2,2} + S_{i,2,3}$, meaning these values will receive a one. All other values in the row will be zero.

In this example, *Aeq* becomes the following matrix.

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Aeq will always be a matrix of size *n* by *m*n*p,* with the same pattern as the matrix above.

If we combine these matrices into a set of linear equations of the form $A * x \leq b$, we get:

$x1 + x2 + x3 + x13 + x14 + x15 = 1$	Constraint week 1
$x4 + x5 + x6 + x16 + x17 + x18 = 1$	Constraint week 2
$x7 + x8 + x9 + x19 + x20 + x21 = 1$	Constraint week 3
$x10 + x11 + x12 + x22 + x23 + x24 = 1$	Constraint week 4

Together with the constraints for the maximum number of starts and the minimum and maximum amounts of fuel, we can extend the LP model to calculate the optimal combination of starts and fuel consumption each week.

With the information above, we have extended the Matlab file to be able to work with the extra dimension and to rebuild all the matrixes from the mathematical model given in Section 6.2 to prepare them for the function lp_solve. The new Matlab file does not only give a vector with an optimal starting sequence, but will also give a vector with the optimal fuel consumption each week. Together, these two vectors can be used to determine the optimal value over the total valuation period.

6.4 Analysis

In this section, we run the improved model for several scenarios to get data on the impact of the new constraints. In order to save time, the scenarios all have been run for a month. We run several scenarios where the take-or-pay maximum is below the optimal amount and several scenarios where the take-or-pay minimum is above the maximum amount.

In the previous section we ran a scenario without fuel and start restrictions. This scenario had a value of 367.865, and a gas consumption of 67.695 MWh. We take this scenario as a base case for the other scenarios.

6.4.1 Maximum fuel lower than optimum fuel amount

In the first set of scenarios we will set the limits of the take-or-pay contract to a minimum of 50000 MWh and a maximum of 60000 MWh. In order to see the influence of the take-or-pay constraints and chosen interval method, we choose to leave the number of starts unbounded.

The results of these scenarios can be found in Appendix 2

As we already expected, a higher number of intervals increases the optimal value. This is because when the intervals are smaller, the probability that the solution is close to the optimal solution gets higher. However, for a small number of intervals this might not always be the case, as we can see in Scenario 6.4. Even though there are more intervals, the optimal value becomes smaller. However, if we keep increasing the number of intervals, we see that the optimal value increases as well.

With smaller intervals, the model can make more precise computations. This can also be seen in the fact that in the latter scenarios, the amount of consumed gas keeps coming closer to the maximum allowed amount by the take-or-pay constraint. When the intervals are larger blocks, the amount of consumed gas is not exactly the maximum allowed amount.

Another interesting fact is that the optimal value only decreases with 1,63%, while the amount of gas decreases with 11,88%. We will look at this in the coming sections, when we will run more realistic scenarios.

6.4.2 Minimum fuel higher than optimum fuel amount

For the second set of scenarios, we set the minimal fuel higher than the optimal amount. We set the minimum amount of fuel to 75000 MWh and the maximum amount to 85000 MWh In order to see the influence of the take-or-pay constraints and chosen interval method, we choose to leave the number of starts unbounded.

The results of these scenarios can be found in Appendix 3.

In these scenarios, we also find better results when we increase the number of intervals. We also see that for smaller intervals, the optimal solution sometimes is lower than the solution with a lower number of intervals. At the same time, these solutions also tend to use less starts than the more optimal solutions. This is fixed when the interval size is chosen small enough.

The maximum value in our example does not differ much from the optimal value without a take-or-pay clause. The optimal value decreases with 2,10% while the amount of gas increases with 10,94%. When we run more realistic scenarios, we will find out whether this is unique for this scenario, or whether a large difference in fuel consumption always leads to a small decrease in the optimal value.

6.5 Conclusion

In this section we gave a mathematical representation on how you can allocate your fuel in an optimal way when you are bounded to a minimum or maximum amount. This formulation has been implemented in the power plant optimization model in Matlab and scenarios have been run to analyze different scenarios. From these scenarios, we can conclude that it is possible to come close to an optimal solution for the fuel allocation without overloading the model with decision variables.

7 Deterministic heat demand

In this section, we answer the third research question; *how can deterministic heat demand be implemented in the existing power plant model?*

We first describe the importance of heat for CHP plants. Next, we will describe the relation between heat production and power production for the plants described in Section 3. After that, different scenarios will be run in order to analyze the additions to the Matlab model.

7.1 Relevance of heat demand for a CHP plant

Heat production is a crucial part of a CHP plant, as it is one of the two generated output values. In a CHP plant, heat is not just a side product, but a main product of which a certain amount is needed for processes. For example, in the horticulture, the heat is needed for climate control in greenhouses. Without the heat generation, a CHP is just a normal power plant. CHP Heat is also used for district heating of university campuses and city district heating. It can also be used for industrial processes which require large amounts of heat.

7.2 Relation between power production and heat production

The amount of heat that can be produced by a CHP is dependent on the power production and vice versa. The relation between the power production (P^{EL}) and heat production (Q^{HT}) differs for different CHP types. In this section, we describe this relation for three types of turbines: a back-pressure steam turbine, an extraction condensing steam turbine and a gas turbine. These are the same CHP installations as we described in Section 4. For this description, we use the mathematical model described by Weber. (Weber, 2005)

7.2.1 Back-pressure steam turbine

Back-pressure steam turbines are often used in CHP if power and heat are needed simultaneously and in rather stable shares, since they produce electricity and heat in a constant ratio. The amount of heat that can be produced is dependent on the power production, and the amount of power produced is dependent on the amount of heat that is generated. This relation can be written in linearized form as follows:

$$P_{t,s}^{EL} = S^{EL} * Q_{t,s}^{HT}$$

In this constraint, $P_{t,s}^{EL}$ stands for the energy output of at time *t* in state *s*, $Q_{t,s}^{HT}$ is the heat output of the plant at time *t* in state *s* and S^{EL} describes the power to heat ratio, which is a plant specific variable. The power to heat ratio is the amount of power generated per unit of generated heat. As stated in Section 4, this ratio is usually between 0.05 and 0.2 for a back pressure steam turbine.

The fuel consumption for a back-pressure turbine is only dependent on the power generation, and not on the heat generation. However, as can be seen in in the equation, the heat generation has influence on the power generation (Weber, 2005).

7.2.2 Extraction condensing steam turbine

Extraction condensing steam turbines are a more complex turbine type, because of its higher flexibility. Such turbines offer the possibility to produce power and heat in an at least partly variable ratio, by extracting steam of a conventional steam turbine. The operation area of an extraction condensing steam turbine can be shown in a PQ chart. This chart shows all possible combinations of the produced electricity (P) and the produced heat (Q).

A graphical representation of a typical PQ chart is given in Figure 4.

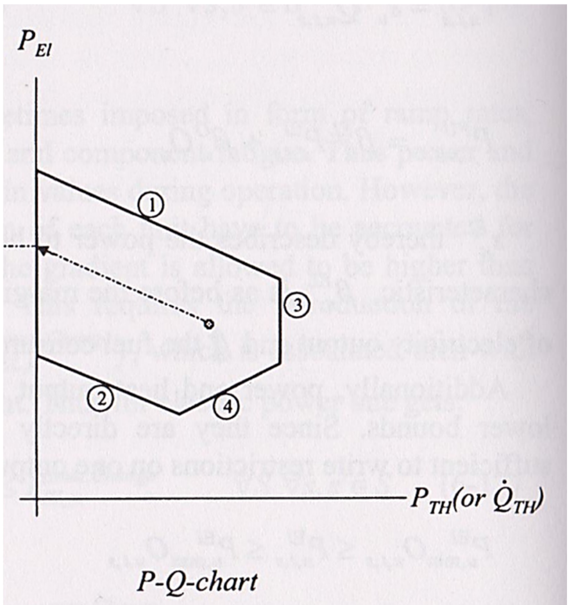

Figure 4: PQ Chart (Weber, 2005)

As can be seen in the PQ-Chart, the feasible region is defined by 5 lines, of which one is the y-axis which describes the amount of produced electricity, P^{EL}. The intersection of the y-axis with line 1 is the maximum power output of the plant with zero heat output. The intersection of the y-axis with line 2 is the minimum electric output of the plant with zero heat output.

The slopes of line 1 and 2 are dependent on the efficiency of the power production e^P and of the efficiency of heat production e^Q. These relate to the electric power reduction due to heat production, which is the ratio e^P / e^Q.

Line 3 models a maximum heat outlet, due to the limited heat exchanger capacity. This is a given number, which is plant specific.

Finally, line 4 provides a plant specific minimum power to heat ratio, $S^{EL}_{u,min}$ which is caused by the maximum ratio between the extraction and the condensing flow.

In linearized form, these constraints can be written as

$$\frac{1}{e^P} * P^{EL}_{t,s} + \frac{1}{e^Q} * Q^{TH}_{t,s} \leq P^{EL}_{max}/e^P \qquad (1)$$

$$\frac{1}{e^P} * P_{t,s}^{EL} + \frac{1}{e^Q} * Q_{t,s}^{TH} \geq P_{min}^{EL}/e^P \qquad (2)$$

$$Q_{t,s}^{HT} \leq Q_{max}^{HT} \qquad (3)$$

$$P_{t,s}^{EL} \geq S_{min}^{EL} * Q_{t,s}^{HT} \qquad (4)$$

In these constraints, P_{max}^{EL} stand for the maximum power production of plant *u* and P_{min}^{EL}stands for the maximum power production of plant *u.*

Unlike with the back-pressure steam turbine, the fuel consumption of the extraction condensing steam turbine is dependent on both the electric power production and the heat production. (Weber, 2005)

7.2.3 Gas Turbines

Gas turbines have as main goal to produce electric power, and heat production is a welcome side effect. The model descriptions of the gas turbine are almost similar to that of the back-pressure steam turbine. However, a gas turbine usually has as the option that the heat generated can alternatively be taken through a heat exchanger to produce useful heat or be directly released to the environment through an auxiliary cooling system. This leads to the following linearized equation.

$$P_{t,s}^{EL} \geq S^{EL} * Q_{t,s}^{HT} \qquad (5)$$

If no cooling system is available, all heat will be used, and the constraint is the same as with the back-pressure steam engine. Compared to the back-pressure steam engine, a gas turbine has a much higher power generation efficiency, and a much lower heat to power ratio.

7.3 Heat demand in Matlab

In the Matlab model, we choose to model the extraction condensing steam turbine. Because heat demand and power demand of most CHP applications do not have a steady ratio over different seasons, we expect that this turbine will yield the best results. We assume that there is no excess power production which can be sold to the electricity network.
Next to that, the extraction condensing steam turbine is the most complicated to model, and can be easily transformed into a model for the other turbines.

7.3.1 Modeling the PQ Chart

In order to model the heat demand, we need to define the feasible region the model can operate in. This region is given by the PQ chart described in Section 7.2.2. In order to model this, Weber has added an extra variable for the heat produced on time *t* to his mathematical formulation. However, over a whole year this leads to 8760 extra decision variables which will complicate the mathematical model considerably.

To avoid having these extra decision variables, we made some assumption that the plant will never generate more heat than necessary. This is because heat production has an influence on fuel consumption, which costs money. Because of this, the heat demand is an input variable which is known beforehand, instead of a decision variable that has to be optimized. In this case, the produced heat is the main product of the CHP, and heat demand always needs to be met. Electricity generation is a byproduct which is only generated if it is needed for heat generation or when it is more profitable than buying electricity. The consequence for the model is that it is not possible to extend the formulation with a heat storage device. However, this is not part of this thesis and can be an interesting topic for further research.

Taking the above into account, we can see that $P_{t,s}^{EL}$ is the only unknown variable in the first constraint of the PQ chart. This means that we can rewrite (2) into

$$P_{t,s}^{EL} \leq P_{max}^{EL} - Q_{t,s}^{TH} * \frac{e^{P}}{e^{Q}}$$

By solving this equation for every t, we acquire a maximum value for $P_{t,s}^{EL}$ for every t.

In the same way, we can transform (3) into

$$P_{t,s}^{EL} \geq P_{min}^{EL} - Q_{t,s}^{TH} * \frac{e^{P}}{e^{Q}}$$

Which gives us a minimum value $P_{t,s}^{EL}$ can become at time *t.*

The minimum value $P_{t,s}^{EL}$ can become is not only bounded by constraint (3), but also by constraint (5).

Because both constraints must apply, this means that our hourly minimum value of $P_{t,s}^{EL}$ is equal to the maximum of both constraining values.

Whit this in mind, we do not have to make an extra decision variable for the produced heat, but we have to transform the hourly minimum and maximum allowed power production with the formulas mentioned above and replace the old vectors P_{min}^{EL} and P_{max}^{EL} with the newly computed values.

The last constraint which bounds the heat production has not been modeled as a constraint in the LP model, but as a check before the model starts running. Because $Q_{t,s}^{HT}$ and Q_{max}^{HT} are both known beforehand, the model will first run a feasibility check whether this constraint is true for every *t.*

7.3.2 Modeling the must-run constraint

Because we do not have a decisions variable and constraints for the heat productions anymore, the model will look at the value of $e_t - \frac{g_t}{\eta}$ (where e_t is the electricity price at time t, g_t is the gas price at time t and η is the efficiency of power production) for choosing when it is the most profitable to turn the plant on and off. However, the model also has to satisfy the heat demand because else the horticulturist will not have a warm greenhouse. This can be solved by adding a constraint to the model which forces the CHP plant to run at all times in order to produce enough heat. This is not an optimal solution, as it forces the model to also run at times where it is not profitable and neither needed for heat generation.

This has been solved by changing the lower bound values of the decision variable for the plant status. For the plant status, the upper bound equals one, meaning the plant is running. The lower bound for this variable is zero, meaning the plant is off.
Instead a vector with only zeros, we have changed the lower bound to a vector with value zero if there is no heat demand at hour *t* and value 1 if there is a heat demand at hour *t.* This forces the plant to run in times when there is heat demand, while it will look at the value of $e_t - \frac{g_t}{\eta}$ at times when there is no heat demand.

7.3.3 Modeling the new optimal value.

The choice to not work with an extra decision variable for the heat production has influence on the way the optimal value for each iteration is calculated. Because the heat demand and efficiency for heat production are given, we can compute the amount of fuel which for heat production during each period. Because the fuel constraints still apply, the maximum and minimum amount of fuel that can be used each iteration to generate power has to be subtracted by the amount of fuel consumed for heat production. The total value of the plant has to be subtracted by the amount of fuel consumed for heat production times the fuel price.

7.4 Analysis

In this section, we run the model for different scenarios to see what the impact of our method is. First we run a scenario with a steady heat production each hour. After that, we run a scenario with periods without heat demand. Next, we will try to explore the other boundaries of the PQ Chart. Finally, we will run scenario with random heat demand.

In all scenarios we work with the possibility to start zero to seven times a week and with 6 fuel intervals. For all scenarios, $P^{EL}_{max} = 100$, $P^{EL}_{min} = 10$, $e^P = 50\%$, $e^Q = 95\%$ and $S^{EL}_{min} = 0{,}3$. Unlike with the other scenarios in the previous sections, the start status each week is set to on to satisfy the must run condition. This a problem we will take a look at in the coming sections. The results of these scenarios can be found in Appendix 4

In Scenario 7.1, the heat demand is 10MWh/h. This means that the minimum amount of power that has to be generated each hour equals the maximum of $P^{EL}_{min} - Q^{TH}_{t,s} * \frac{e^P}{e^Q}$ and $S^{EL}_{min} * Q^{HT}_{t,s}$ which equals the maximum of 4.74 and 3, which is 4.74. if we look in our excel output sheet, we see that this is indeed the scenario. Our maximum power that can be generated should be equal to $P^{EL}_{max} - Q^{TH}_{t,s} * \frac{e^P}{e^Q}$ = $100 - 10 * \frac{0{,}5}{0{,}95} = 194{,}7$. According to our excel sheet, this is true. The maximum fuel consumption each hour is 200 MW. The total fuel consumption is dependent on the power production, the heat production and the efficiency of both productions. In this scenario $\frac{35282}{0{,}5} + \frac{7440}{0{,}95} \approx 78395$, which is also correct.

In order to verify the must-run constraint only accounts for periods where there is a heat demand, we constructed a ten day period without any heat demand in Scenario 7.2. These are day 8 to day 17, which equals the whole second period, and the first three days of the third period. When we take a look at the results, we see that in these periods the plant makes several starts, and is not bounded by the must run constraint. However, the number of starts is one lower than we would expect at first. This is because the plant status is always on at the start of the week, which the model currently does not count as a start. We will look into this in the coming sections.

During the periods where the heat demand does not equal zero, the model behaves as it should, and the conclusions from Scenario 7.1 also apply for this scenario.

In Scenario 7.3, we force the minimum allowed power production to line 4 in the PQ Chart. When the heat demand is 20, $P_{min}^{EL} - Q_{t,s}^{TH} * \frac{e^P}{e^Q}$ equals -0,53 and $S_{min}^{EL} \; * Q_{t,s}^{HT}$ equals 6. The excel sheet confirms this. The maximum amount of fuel that can be produced equals 89,5 MW, and the maximum fuel consumption is never higher than 200 MW. The formula for the total fuel consumption also applies.

In Scenario 7.4, the heat demand is not the same each hour, but a random integer between 0 and 20. When we rearrange the outcomes for the minimum and maximum allowed hourly energy production, we get the following table. These numbers correspond with the formulas mentioned in Section 7.3

Heat Demand	Minimum Power	Maximum Power
0	10	100
1	9,5	99,5
2	8,9	98,9
3	8,4	98,4
4	7,9	97,9
5	7,4	97,4
6	6,8	96,8
7	6,3	96,3
8	5,8	95,8
9	5,3	95,3
10	4,7	94,7
11	4,2	94,2
12	3,7	93,7
13	3,9	93,2
14	4,2	92,6
15	4,5	92,1
16	4,8	91,6
17	5,1	91,1
18	5,4	90,5
19	5,7	90,0
20	6,0	89,5

Table 1: boundaries for hourly Minimum and Maximum fuel for given Heat Demand

Finally we have also run a scenario in which the heat demand during one hour was higher than the maximum heat production of the plant. In this scenario, the validity check does its work, and stops the model from running by giving an error code that the plant cannot produce enough heat to meet the demand.

7.5 Conclusion

In this wection, we wanted to model a deterministic heat demand in the existing model. Instead of adding extra decision variables, we defined a feasible region for all combinations of power- and heat production. This region has been defined with linearized constraints and has been modeled in Matlab. Afterwards, several scenarios have been run to analyze different scenarios. From these scenarios we can conclude that the model behaves as predicted in Section 7.3. We can also see that an increase in heat demand has a drastic influence on the profitability of the plant. This however might be specific for this scenario. We will look as this in the coming sections, where we will run a model with more realistic settings.

8 Finishing the model

The new model currently takes the starts, fuel and heat constraints into account for every week. However, because the model splits the problem in smaller sub-problems instead of running one big calculation, there are small complications when these sub solutions are combined. In this section, we will discuss these problems and optimize the model further.

8.1 Problems in the current model

In order to reach a reasonable computational time, the former model of EnergyQuants splits the problem into smaller sub problems. However, this is at the cost of small inaccuracies. These inaccuracies are:

- If period *i* ends in an idle state, and period *i+1* starts in a running state, this is not counted as a startup.
- The runtime or idle time between two periods can be less than the minimum run or idle time.
- The model does not take ramping constraints into account when a run takes place over multiple periods.

Some of these problems have already surfaced in scenarios in the previous sections.

The first two errors have an impact on the feasibility of the running pattern of the plant, as they might lead to more starts than allowed or an infeasible on/off sequence. The effects of the ramping error are assumed to be marginal, and more complicated to resolve. Because of this we choose to only look at the first two problems.

8.2 Mathematical formulation

To solve the startup problem, we decided to add a new variable that describes the state op the plant during a period at time *t = 0* and at the end of the period. We do not add a variable for how long the plant has been in that state, which means this model does not solve the second problem.

With this extra variable, we can make a new mathematical formulation. We have used the model from Section 6 and rebuild it to be able to cope with an extra dimension for the state of the plant. This yields the following model.

$i \ = Weeknumber$
$j = Number\ of\ starts$
$k = fuel\ consumption\ interval$
$l \ = state\ of\ the\ plant\ at\ t = 0\ and\ the\ end\ of\ the\ period$
$\quad l\ equals\ 1 = on - on$
$\quad l\ equals\ 2 = on - off$
$\quad l\ equals\ 3 = off - on$
$\quad l\ equals\ 4 = off - off$
$r_{i,j,k,l} = Number\ of\ starts\ j\ in\ week\ i\ with\ fuel\ consumption\ interval\ k\ and\ plant\ state\ l$
$f_{i,j,k,l} = Fuel\ consumed\ in\ week\ i\ with\ j\ starts, fuel\ consumption\ interval\ k\ and$

$plant\ state\ l$

$V_{i,j,k,l} = Value\ in\ week\ i\ with\ j\ starts, fuel\ consumption\ interval\ k\ and\ plant\ state\ l$

$S_{i,j,k,l} = Decision\ Variable\ that\ equals\ 1\ if\ CHP\ starts\ j\ times\ in\ week\ i\ with\ fuel$
$consumption\ interval\ k\ and\ plant\ state\ l$

$R = Maximum\ number\ of\ starts$

$Fmax = Maximun\ Fuel\ consumption$

$Fmin = Minimum\ Fuel\ consumption$

$$Max\ z\ = \sum_i \sum_j \sum_k \sum_l V_{i,j,k,l} * S_{i,j,k,l}$$

$$s.t. \qquad \sum_i \sum_j \sum_k \sum_l r_{i,j,k,l} * \ S_{i,j,k,l} \leq R$$

$$\sum_i \sum_j \sum_k \sum_l f_{i,j,k,l} * \ S_{i,j,k,l} \ \leq Fmax$$

$$\sum_i \sum_j \sum_k \sum_l f_{i,j,k,l} * S_{i,j,k,l} \geq Fmin$$

$$\sum_j \sum_k \sum_l S_{i,j,k,l} = 1 \qquad (\forall i)$$

$$S_{i,j,k,1} + S_{i,j,k,3} + S_{i+1,j,k,3} + S_{i+1,j,k,4} = 1$$

$$S_{i,j,k,2} + S_{i,j,k,4} + S_{i+1,j,k,1} + S_{i+1,j,k,2} = 1$$

$i, j, k, l, r, R\ are\ integers$

$S = 0\ or\ 1$

$R\ > 0$

$r_{i,j,k,l}\ , f_{i,j,k,l}, Fmax, Fmin\ \geq 0$

$i = 1,2, \dots [number\ of\ weeks]$

$j = 0,1, \dots [maximum\ starts\ per\ week]$

$k = 1,2, \dots [maximul\ fuel\ interval]$

$l = \{1,2,3,4\}$

In the old model, the state at *t = 0* is the same for all periods. While the state at *t = 0* has no direct influence on the value or the fuel consumption of the plant, it has influence on the possible states of the following hours. For example, if *t = 0* is in idle state and *t = 1* is in a running state, it counts a start in that period and the plant has be in a running status for at least the minimum run time. However, if *t = 0* was also in a running state, it would not have counted as a start and there would be no minimum runtime for that run.

If we want to model this correctly, it means that when the plant ends a week in a running state, this should also be the case at *t = 0* for the next week. The same goes for when the plant ends a period in idle state. This can be modeled with three constraints, of which one is already in the model.

The first constraint which is already in the model is

$$\sum_{l} S_{i,j,k,l} = 1$$

This constraint ensures that the plant cannot be at to different states l at the same time during week *i*. The first new constraint in the model states that when plant ends running, it cannot be idle at *t = 0* in the next week. The second constraint ensures that if the plant ends idle during period *i*, it cannot be running at *t = 0* in period *i+1*.

Together, these three constraints force the model to choose a feasible solution which combines the end state in period *i* and the start state in period *i+1*.

8.3 Implementation in Matlab

The model described in Section 8.2 links the end state of one period to the beginning state of the next period. This is an improvement to the old model, but does not give a solution for the minimum run time and minimum idle time. This can be solved by making a model which gives the start- and end position more states which not only define the running state, but also for how many hours the plant has been running or has been idle. By creating the right constraints, the minimum run and idle times can always be satisfied. However, this model looks very much like a model where the whole valuation period is calculated at once, instead of being split up in smaller periods. This model increases the calculation time considerably as can be seen in Scenario 5.4.

Because the considerable increase in calculation time, the small size of the error, and the fact that this error only occurs in scenarios without a constant heat demand. We choose to solve this problem by creating a new file in Matlab that detects any errors before the model gives the final output, and use a heuristic to fix these errors after the optimization.

This new Matlab file makes a difference between two scenarios: A run which is shorter than the minimum runtime, and an idle time which is shorter than the minimum idle time. For both these scenarios are different ways to make the output feasible.

8.3.1 Run between periods smaller than minimum run time

There are several options to fix an infeasible solution yielded by the model where there is a runtime that is smaller than the minimum run time. These are:

1. Do not make the short run, but instead make it extra idle time
2. Decrease the idle time before run with the slack hours of the short run
3. Decrease the idle time after the run with the slack hours of the short run
4. Remove idle time between the run and the previous run
5. Remove idle time between the run and the next run

Options 2 and 3 both have two sub scenarios. In Scenario 2.1 and 3.1, the idle time before/after the short run is more than the minimum idle time plus the slack hours of the short run. In Scenario 2.2 and 3.2, the runtime cannot be corrected in the idle time before or after the run.

The influences of these solutions on the output are stated in Table 2.

Solution	Effect on # starts	Effect on Fuel Consumption	Problem with heat demand?
1	-1	--	Maybe
2.1	0	+	No
2.2	0	?	Maybe
3.1	0	+	no
3.2	0	?	Maybe
4	-1	++	No
5	-1	++	No

Table 2: effects of different solutions on the short run problem

The solutions that change the least to optimal solution from the model are 2.1 and 3.1. However, these are only feasible if the maximum fuel constraint allows for extra fuel consumption. If the yielded solution from the model is already too close to the maximum amount of allowed fuel, these solutions will be skipped by our heuristic. These solutions are also only possible when the idle period before (2.1) or after (3.1) the run has excess idle time that can be converted to runtime.

The effects of Solution 2.2 and 3.2 are very hard to predict, as their changes will create new infeasibilities in the periods next to the original infeasible period. These infeasibilities again have to be solved until there is a run with excess runtime or an idle period with excess idle time. Because of the number of small changes these solutions can bring and the complexity of the changes that have to be made in the optimal solution, we have decided to not use these options in our heuristic.

When Solution 2.1 and 3.1 are not feasible, the model has to choose between Solution 1, 4 and 5. If the amount of consumed fuel is equal or close to the maximum allowed amount, Solution 4 and 5 become infeasible and the model will choose Solution 1. If the amount of consumed fuel is equal or close the minimum allowed amount, only Solution 4 and 5 a feasible. The heuristic will calculate both solutions and choose the most profitable solution.

If the amount of fuel is not close to the minimum or maximum amount, the model will calculate the outcome of Solution 1, 4 and 5, and choose the most profitable one.

A special scenario exists when there is a heat demand during the short run and the fuel consumption is already close or equal the maximum allowed of fuel. This is a scenario that cannot be made feasible by the model, as it will either violate the heat demand (solution 1) of the fuel constraint (Solution 2-5). The chances that this scenario will occur are very small.

8.3.2 Idle time between periods smaller than minimum idle time

In the second scenario, there is shorter idle time between periods than the minimum idle time. For this problem, there also are 5 solutions

1. Instead of an idle period, the plant keeps running
2. Decrease the runtime before the idle time with the slack hours of the idle time
3. Decrease the runtime after the idle time with the slack hours of the idle time
4. Remove the run between the idle time and the previous idle time
5. Remove the run between the idle time and the next idle time

Option 2 and 3 both again have two sub scenarios. In Scenario 2.1 and 3.1, the runtime before/after the short idle time is more than the minimum runtime plus the slack hours of the short idle time. In Scenario 2.2 and 3.2, the slack of idle time cannot be corrected in the runtimes surrounding the short idle time.

The impact of these solutions on the output are stated in Table 3.

Solution	Effect on # starts	Effect on Fuel Consumption	Problem with heat demand?
1	-1	++	No
2.1	0	-	Maybe
2.2	0	?	Maybe
3.1	0	-	Maybe
3.2	0	?	Maybe
4	-1	--	Maybe
5	-1	--	Maybe

Table 3: effects of different solutions on the short run problem

The heuristic for this situation is almost equal to the heuristic of the previous section. However, with the exception of Solution 1, all these solutions might create a problem with the heat demand, which leads to an infeasible solution. However, it is unlikely that these scenarios will occur often.

Like in the previous section, Solution 2.1 en 3.1 are the preferred solutions, as they change the least to the optimal value, fuel consumption and on/off sequence. Solution 2.2 and 2.3 are skipped because of their complexity and the choice for Solution 1 or Solution 4 or 5 depends on the fuel consumption and impact on the profitability of the plant.

8.4 Analysis

In order to see the impact of the chosen heuristic, we created fictional scenarios in order for all options to be modeled. Because the input variables on energy prices are manipulated in order to force the model to generate different scenarios, it is almost impossible to compare the results on economic value. Because of this, we only compared the differences on fuel consumption, energy production and number of starts. For all scenarios, the maximum energy production level is 100 MWh/h, and the minimum level is 10 MWh/h. The minimum run time and minimum idle time are both 12 hours. The efficiency of energy production is 50%

8.4.1 Run between periods smaller than minimum run time

Scenario 8.1

In this scenario, we simulated a scenario with a runtime of 6 hours between two periods. The idle time before the run is 18 hours, and the idle time after the run is 12 hours. Fuel consumption is not close to either the maximum or minimum value.

Scenario 8.2

In this scenario, we simulated a scenario with a runtime of 6 hours between two periods. The idle time before the run is 12 hours, and the idle time after the run is 18 hours. Fuel consumption is not close to either the maximum or minimum value.

Scenario 8.3
In this scenario, we simulated a scenario where the consumed fuel is close to the minimum allowed amount. The idle time before and after the short run is 12 hours.

Scenario 8.4
In this scenario, we created a scenario where the consumed fuel is to the maximum allowed amount. The idle time before and after the short run is 12 hours.

The results of these scenarios can be found in Table 4.

Scenario	Impact on Starts	Impact on fuel consumption	Impact on Energy production
8.1	0	+120	+60
8.2	0	+120	+60
8.3	-1	+235	+120
8.4	-1	-1205	-600

Table 4: results Scenarios 8.1 - 8.4

In the Scenario 8.1 the chosen solution by the heuristic is to fix the shortage in runtime in the idle time before the run. Because in the old solution it is not profitable to turn on the CHP during these hours, the heuristic will choose to run at minimum power during these hours. This leads to an increase in energy production of the minimum power times the changed hours. The increase in fuel consumption times the efficiency equals the increase in energy production. Scenario 8.2 is equal to Scenario 8.1, except that the run in lengthened to the idle time after the run instead of before.

Scenario 8.3 forces the model to choose to change the idle time before or after the run into runtime. In this scenario, the chosen idle time has a length which is equal to the minimum idle time. The results of the heuristic are that the amount of starts decreases with 1. The energy production increases with the minimum power times the minimum run time, while the fuel consumption equals the power production divided by the efficiency minus the fixed amount of fuel used per start.

Scenario 8.4 forces the model to shift the short on run to idle. This leads to one less start. The effect on power generation in this scenario is equal to the length of the short run times the maximum power. If the plant does not run on maximum power during the period, this amount will also be lower. The decrease in fuel consumption equals the power production divided by the efficiency minus the fixed amount of fuel used per start.

8.4.2 Run between periods smaller than minimum run time

In order to see the impact of the chosen heuristic, we created fictional scenarios in order for all options to be modeled. The parameters for these scenarios are the same as in Section 8.4.1

Scenario 8.5
In this scenario, we simulated a case with an idle time of 6 hours between two runs in different periods. The run before the idle time is 18 hours, and the runtime after the idle time is 12 hours. Fuel consumption is not close to either the maximum or minimum value.

Scenario 8.6
In this scenario, we simulated a case with an idle time of 6 hours between two runs in different periods. The run before the idle time is 12 hours, and the runtime after the idle time is 18 hours. Fuel consumption is not close to either the maximum or minimum value.

Scenario 8.7
In this scenario, we simulated a case where the consumed fuel is close to the minimum allowed amount. The runtime before and after the idle period is 12 hours.

Scenario 8.8
In this scenario, we created a case where the consumed fuel is to the maximum allowed amount. The runtime before and after the idle period is 12 hours.

Scenario 8.9
In the last scenario, fuel consumption is close is close to the maximum allowed amount. The run time before and after the idle period is 12 hours. However, there is a heat demand during both runs

The results of these scenarios can be found in Table 5.

Scenario	Impact on Starts	Impact on fuel	Impact on Energy production
8.5	0	-1200	-600
8.6	0	-1200	-600
8.7	-1	+115	+60
8.8	-1	-2405	-1200
8.9	0	0	0

Table 5: results scenario 8.5 – 8.9

In the first two scenarios, the heuristic can choose the two preferred solutions. The effect of these solutions is that the energy production decreases with the converted hours (6) times the maximum power generation level. If the plant does not run on maximum power during the converted period, this amount will be lower. The decrease in fuel consumption equals the power production divided by the efficiency.

In Scenario 6.3, the consumed fuel is close the minimum allowed amount, so the heuristic will choose to make a longer run instead of having idle time. This leads to an increase in energy production of the minimum power times the changed hours. The increase in fuel consumption times the efficiency minus the fuel consumption per start equals the increase in energy production.

In Scenario 6.4, the consumed fuel is close the maximum allowed amount, but the shortage in idle time cannot be fixed in the runtime before or after the shortage. The heuristic will than choose to skip either the run before or after the short idle run, depending on their economic value and heat demand. The energy production will decrease with the runtime of the skipped run times the power generated each hour. In our example this is the maximum amount, but this can be lower. The decrease in fuel consumption equals the power production divided by the efficiency minus the fuel cost per startup.

Scenario 6.5 shows an infeasible scenario, where no solution can be chosen; partly changing the states of the previous or next run will lead to the same infeasibility, converting the idle to run violates the fuel constraint, and converting the runs before or after to idle time violates the heat demand. However, these scenarios are unlikely to occur.

8.5 Conclusion

In this section, we wanted to find solutions for some inconsistencies and infeasibilities in the model. Most of these errors are caused by splitting the model into smaller periods.

Some of the problems can be solved manually, by making a heuristic on how to react on different infeasible scenarios. Because this is not an optimization, it is not guaranteed that these fixes will give back the optimal solution. However, the solutions they yield will most of the times be feasible.

Even though we did not find a heuristic for every infeasibility, we have showed that it is possible to improve the model without adding significant computation time. In a model where ramping constraints are taken into account, a same kind of heuristic can be used to make the model feasible.

9 Realistic Settings

In this section, we will discuss realistic parameter settings for our research. For this model, there are three sets of parameters: The plant specific parameters (Maximum power etc.), contract parameters (allowed fuel consumption and starts) and project specific parameters (Fuel prices etc.).

9.1 Plant specific parameters

Plant specific parameters are parameters which are different for each CHP. In our model, these parameters are

- Minimum power
- Maximum power
- Power generation efficiency
- Minimum run time
- Minimum idle time
- Fixed costs per start
- Fuel usage per start
- Maximum Ramp up
- Maximum Ramp down
- Heat generation efficiency
- Minimum power to heat ratio

For the parameters above, we tried to find settings for a common CHP plant in the horticulture industry. However, in this industry there are a lot of different types of greenhouses which all have different heat and energy requirements. Because of this, we contacted the Dutch association for CHP, Cogen, for an average case description for a CHP in the Dutch horticulture industry.

According to Cogen, an average CHP plant has a power capacity of 2MW. Unlike with a gas engine, a steam turbine can perform reasonably on part load (U.S. Environmental Protection Agency, 2008). However, in the literature, we have not found what the minimum part load of an extraction condensing steam turbine is. Because a steam turbine needs a certain amount of fuel to be able to produce steam, we estimate this amount to be 20% of the maximum capacity.

Because the heat demand and power demand vary during different periods in a year, we have modeled an extraction condensing steam turbine. For a CHP of this size and type, the electrical efficiency can be up to 37%. The thermal efficiency for a CHP this size and type is around 75%. A realistic minimal power to heat ratio for a steam turbine is 0,2 (Energy Nexus Group, 2002)

Steam turbines require long warm-up periods in order to obtain reliable service and prevent excessive thermal expansion, stress and wear. The warm-up time for a steam turbine usually lies between 1 hour and 1 day, depending on the size of the CHP plant. (U.S. Environmental Protection Agency, 2008). Because Steam Engines come in the size range of 0,5 – 250 MW, we estimate the minimum idle time for our plant to be around 2 hours. We estimate the minimum runtime to be approximately the same as the minimum idle time.

In the literature, we did not find information regarding fixed or fuel costs of startups for a small steam engine CHP plant. As can be read in the next sections, our scenario works with a continuous heat demand. This means that the plant will always be on. If a heat storage device is added to the model, these costs become relevant. However, because the size of this CHP is relatively low compared to the size range of a CHP, we expect these costs and gains to be negligible.

As already mentioned in the previous sections, ramping constraints are not taken into account in this thesis. We assume the CHP can ramp instantly

To conclude, the most important parameters are summarized in Table 6.

Parameter	Value
Minimum power generation level	0,4 MW
Maximum power generation level	2 MW
Power generation efficiency	43%
Minimum run time	2 hours
Minimum idle time	2 hours
Heat generation efficiency	75%
Minimum power to heat ratio	0,2

Table 6: CHP parameters

9.2 Contract Parameters

Important input parameters for our research are the parameters in the contract between the gas recipient and the supplier. These parameters are the values of the take-or-pay clause, which equal the minimum and maximum allowed fuel consumption, and the allowed number of starts.

Instead of estimating these values, we will first run all price scenarios to obtain optimal values for the fuel consumption and the number of starts. Next, we will change these variables in order to find out what their impact is on the total value of the CHP plant. We will simulate different take-or-pay values by increasing the minimum allowed fuel consumption or by decreasing the maximum allowed fuel consumption. In a scenario with constant heat demand, the number of starts is not a constraining value, as the plant always has to be running in order to meet the demand. However, when a heat storage device is added, the allowed number of starts becomes a more relevant parameter.

9.3 Project Parameters

Situation parameters are parameters which are specific for the time and place of the project. In our model, these parameters include the gas prices, the energy prices, and the heat demand.

In order to simulate realistic scenarios gas prices and energy prices, we obtained historical data on the hourly trading prices for energy and gas in 2009, 2010 and 2011. These three years will be three different scenarios in our analysis.

In 2009, the average energy price was 39,16 euro per MWh and the average gas price as 12,19 euro per MWh. In 2010, these price were 45,38 euro and 17,41 euro. In 2011, these prices were 52,03 and 22.64 euro. A graph of the energy prices and gas prices can be found in Appendix 11

In 2009, the ratio of average energy price divided by the gas price times the efficiency is 1.38. In 2010 this was 1.12 and in 2011 this was 0,99. Because of these numbers, we expect the plant to be more profitable in 2009 than in 2010 and 2011.

In the literature we did not find a realistic heat demand for a smaller CHP in the horticulture. Instead, we look at another type of heat demand: the hourly heat generation of a district heating installation in the Netherlands. We scaled this heat demand with the size of our average plant (2 MW) to estimate the hourly heat demand for a smaller CHP in the horticulture. A graph of this heat demand can be found in Appendix 12.

9.4 Conclusion

In this section, we discussed the input parameters for our research. In the next section, we will run several scenarios for different years and different take-or-pay values in order to see the impact of these variables on the total value of the CHP plant.

10 Analysis

In this Section, we will analyze the impact of different take-or-pay values and different price scenarios. We will combine the constructed model and the realistic settings from the previous sections and we will run different scenarios for different years. With the results of these scenarios, we will give an answer to the second research question; "*What is the impact of take-or-pay contracts on the profitability of a CHP plant?*"

10.1 Scenario descriptions

To analyze the impact of different take-or-pay values in different price scenarios, we run scenarios with the parameters settings from Section 9. For each year, we first ran a scenario without fuel constraints. This yields the optimal solution and fuel consumption over that year. Next, we will run scenarios in which the fuel consumption is forced to be 5%, 10% of 20% higher or lower than the optimum amount.

For most scenarios, the number of fuel intervals is equal to twenty. This ensures an accurate fuel usage. Because there always is a heat demand in the scenarios we run, we have set the number of possible starts and the state options both to one. This means the plant will only try iterations with zero starts, and with state options "on-on". Appendix 10 shows a histogram with calculation times for each iteration for the scenario described above. Because our scenarios do not have starts incorporated, the number of starts and minimum run and idle time are no longer constraints. This speeds up the model enormously.

For some scenarios, twenty fuel intervals took too much calculation time in the combination LP (over 10 minutes). Most of these scenarios are run with a 20% lower amount of fuel. In order to get results for these scenarios, we set the number of fuel intervals to a lower level. This might lead to a slightly less optimal solution.

Different delivery contracts exist within the gas industry. We approximate them with a contract with a daily gas price, a contract with a quarterly gas price, and a contract with a yearly gas price. We have run scenarios with all three contract types in order to find out whether there are relevant differences on the total value.

In order to see whether there is impact between a contract with a daily gas price, a quarterly gas price, or a yearly gas price, we ran scenarios for all three scenarios. Lastly we ran a scenario for each year with the average quarterly price of all three years.

The results of these scenarios can be found in Appendix 5 to 8.

As can be seen in the scenario results, it is hard to say which type of contract yields the highest value for the gas recipient. In 2009, a quarterly gas price would have resulted in the highest value. In 2010, a daily gas price would have yielded the most income and in 2011, a yearly gas price would have generated the most income. Because there is no clear preference from our scenarios, we have taken the average of each year over all contract types and summarized this in the next sections

10.2 Results 2009

A summary of the average results over 2009 are given in the table below. More detailed results of all scenarios over 2009 can be found in Appendix 5. If we do not take fuel constraints into account and we look at the fuel consumption over the three different contract types, the average optimal fuel consumption over 2009 was 33549 MWh with an optimal value of 123.418 euro. If we change the take-or-pay values of the contract, we gain the following results.

2009			
Fuel change	**Optimal Value (123.418)**	**Average percent value change**	**Average percent value change per percent fuel change**
5% higher	122.190	-0,99%	-0,20%
10 % higher	118.383	-4,08%	-0,41%
20% higher	N/A	N/A	N/A
5% lower	122.379	-0,84%	-0,17%
10% lower	119.813	-2,92%	-0,29%
20% lower	110.165	-10,74%	-0,54%
5% range average	122.285	-0,92%	-0,19%
10% range average	119.098	-3,50%	-0,35%
20% range average	110.165	-10,74%	-0,54%

Table 6: Average results 2009

The first column describes the deviation in fuel consumption from the optimal amount. The second column shows the optimal value related to the fuel consumption in the first column. The third column shows the difference in value to the optimal value, and the fourth column shows the decrease in value per percent change of fuel consumption.

In 2009 the average gas price was a lot lower than the average electricity price. Because of this, the plant is in the money for a large proportion of the valuation period. As a result, the optimal amount of fuel consumption is close to the maximum amount. Because of this, we could not run a scenario in which the take-or-pay constraint forces the model to use 20% more fuel than the optimal amount.

As we can see in the table above, the impact of fuel changes on the total value gets relatively higher when the percentage of fuel change gets higher. When the take-or-pay value forces the consumed fuel to be within a 5% range of the optimal fuel, the impact on the total value is less than 1 percent.

10.3 Results 2010

A summary of the average of the results over 2010 are given in the table below. More detailed results of all scenarios over 2010 can be found in Appendix 6. If we do not take fuel constraints into account and we look at the fuel consumption over the three different contract types, the average optimal fuel consumption over 2010 was 29742 MWh with an optimal value of 29.133 euro. If we change the take-or-pay values of the contract, we gain the following results.

2010			
Fuel change	**Optimal Value (29.133)**	**Average percent value change**	**Average percent value change per percent fuel change**
5% higher	28.502	-2,16%	-0,43%
10 % higher	26.896	-7,68%	-0,77%
20% higher	19.743	-32,22%	-1,61%
5% lower	28.457	-2,32%	-0,46%
10% lower	26.921	-7,59%	-0,76%
20% lower	20.943	-28,11%	-1,40%
5% range average	28.480	-2,24%	-0,45%
10% range average	26.908	-7,63%	-0,76%
20% range average	20.343	-30,16%	-1,51%

Table 7: Average results 2010

As in 2009, the impact of fuel constraints on the total value gets higher as the consumed fuel differs more from the optimal value. Also, it seems that the effect of constraining the minimum and the maximum allowed fuel consumption have approximately the same result, with an exception in the 20% range. In this range, an increase in fuel consumption leads to a total value which is 4.11 percent lower than a 20% decrease in fuel consumption

10.4 Results 2011

A summary of the average of the results over 2011 are given in the table below. More detailed results of all scenarios over 2011 can be found in Appendix 7. If we do not take fuel constraints into account and we look at the fuel consumption over the three different contract types, the average optimal fuel consumption over 2010 was 24331 MWh with an optimal value of -53.456 euro. If we change the take-or-pay values of the contract, we gain the following results.

2011			
Fuel change	**Optimal Value (-53.456)**	**Average percent value change**	**Average percent value change per percent fuel change**
5% higher	-53.856	-0,75%	-0,15%
10 % higher	-54.871	-2,65%	-0,27%
20% higher	-58.836	-10,06%	-0,51%
5% lower	-53.794	-0,63%	-0,13%
10% lower	-54.724	-2,37%	-0,24%
20% lower	-58.723	-9,85%	-0,49%
5% range average	-53.825	-0,69%	-0,14%

10% range average	-54.798	-2,51%	-0,25%
20% range average	-58.780	-9,96%	-0,50%

Table 8: Average results 2011

The percentages in the summarized table over 2011 look a lot like the percentages over 2009. Also here, staying within a 5% range of the optimum fuel consumption will lead to a decrease in value of less than 1 percent. For all ranges, there hardly is any difference whether the minimum or the maximum fuel consumption is constrained.

10.5 Results Overall

If we take the average of the results in the previous three sections, we can make an estimation of the impact of fuel constraints on the total value of the plant. The combined average values over 2009, 2010 and 2011 can be found in the table below.

Average Results		
Fuel change	**Average percent value change**	**Average percent value change per percent fuel change**
5% higher	-1,30%	-0,26%
10 % higher	-4,80%	-0,48%
20% higher	-21,14%	-1,06%
5% lower	-1,26%	-0,25%
10% lower	-4,29%	-0,43%
20% lower	-16,23%	-0,81%
5% range average	-1,28%	-0,26%
10% range average	-4,55%	-0,46%
20% range average	-18,20%	-0,91%

Table 9: Average results overall

Overall, we can see that the impact of changing the fuel constraint gets higher when the consumed fuel deviates more from the optimal solution. When the fuel constraints force the fuel consumption to be 5 percent higher or lower than the optimal amount, the total value decreases on average by 1,28%. When the consumed fuel has to be 10% higher or lower than the optimal amount, the value decrease with an average of 4.55%. When the constraints force the fuel consumption to be 20% higher or lower than the optimal amount, the total value decreases with an average of 18,20%.

In the table, we can see that a fuel consumption which is higher than the optimal value has a higher impact on the total value of the project than constraining the maximum allowed fuel consumption.

In 2010, the percentage at which the value decreases is 2-3 times higher as in 2009 and 2011. A logical cause for this effect would be a higher standard deviation in the energy of gas prices. However, these standard deviations are higher in 2009 and lower in 2011. We have not found the reason why these percentages where this different from the other years. Because all other variables were constant, it must have something to do with the energy or gas prices.

If we compare the results from 2009, 2010 and 2011, it seems like there is a downward trend in profitability of a CHP. We should take into account that even though there is a negative value in 2011, heat has been generated which otherwise had to be generated with a boiler. If this downwards trend continues in the next few years, a CHP as we modeled will not be profitable in the future.

We have also run scenarios where the average quarterly gas price over three years has been taken as an indicator for the gas price in the contract. The results of these scenarios can be found in Appendix 8. However, these results did not give new insights on the results we already have.

10.6 Conclusion

In this section, we have analyzed the results of several realistic scenarios in order to answer the question *"What is the impact of take-or-pay contracts on the profitability of a CHP plant?"*

The main conclusions we can draw are:

- When take-or-pay constraints force a CHP to consume an amount of fuel which is 5% percent higher or lower than the optimal fuel amount, this leads to an average value loss of 1,28%.
- When take-or-pay constraints force a CHP to consume an amount of fuel which is 10% percent higher or lower than the optimal fuel amount, this leads to an average value loss of 4,55%.
- When take-or-pay constraints force a CHP to consume an amount of fuel which is 20% percent higher or lower than the optimal fuel amount, this leads to an average value loss of 18,20%.
- A take-or-pay contract that forces the amount of fuel that has to be consumed to be higher than the optimal amount, has more impact on the value of a plant than a constraint that force the fuel consumption to be less than the optimal amount. We have not found why this is the case.
- There is no clear preference for a contract with daily, quarterly or yearly gas prices

From our results, we can see that the impact on the value of a CHP increases as the take-or-pay constraints force the plant to deviate further from the optimal amount of fuel. Next to that, being forced to consume more gas than needed has a higher impact on the profitability than using less than the optimal fuel amount

For different years, the impact of take-or-pay constraints is not always the same. In our scenarios, the impacts in 2010 are higher than in 2009 or 2011. We have not found a cause for this phenomenon in our data.

11 Conclusions and Recommendations

11.1 Conclusions

Our report started with two research questions:

1. *"Can the existing power plant model of EnergyQuants be extended in order to be able to optimize a CHP plant?"*

2. *"What is the impact of take-or-pay contracts on the profitability of a CHP plant?"*

In order to make the existing power plant model able to optimize a CHP plant, a 0-1 IP problem has been formulated which distributes the number of starts and fuel over different periods in the valuation period. This mathematical model has been implemented in Matlab to extend the already existing model. Next to that, a mathematical formulation on heat demand has been modeled and implemented in Matlab. To make the model work as it should, several other additions and fixes had to be made, which are described in Section 8.

The new model is capable of working with a heat demand and can distribute starts and fuel in an optimal way over different periods in the valuation period. In order to be able to give a satisfactory answer to the second research question, realistic price data for the years 2009-2011 have been used. With this data, scenarios have been run in order to see the impact of different take-or-pay values on the total value of a CHP.

- When take-or-pay constraints force a CHP to consume an amount of fuel which is 5% percent higher or lower than the optimal fuel amount, this leads to an average value loss of 1,28%.

- When take-or-pay constraints force a CHP to consume an amount of fuel which is 10% percent higher or lower than the optimal fuel amount, this leads to an average value loss of 4,55%.

- When take-or-pay constraints force a CHP to consume an amount of fuel which is 20% percent higher or lower than the optimal fuel amount, this leads to an average value loss of 18,20%.

A take-or-pay contract that forces the fuel consumption to be higher than the optimal amount has a higher impact on the value of the plant than when less fuel can be consumed. However, the latter might lead to an infeasible scenario where there is not enough fuel to meet the heat demand.

If we look at the absolute value of the CHP, we can observe that the value decreases each year. Because our sample size consist of only three years, it is hard to say whether this is a trend or a coincidence. However, if it is a trend, CHP installations like we modeled might not be profitable anymore in the near future and it might cost less money to generate heat with a simple boiler.

11.2 Recommendations for further research

In this report we considered the years 2009, 2010 and 2011 as historical data. In this sample, 2010 reacts more sensitive on changes in fuel consumption than the other two years. It might be interesting to extend the sample size over more years in order to validate the results, as it is not certain whether 2010 was an exceptional year. If this is the case, our results might not be representative. When a larger sample is used, we can also see whether the downward trend is a coincidence in our data, or a real trend.

Another interesting subject of further research is the CO_2 component of a CHP plant, which is already mentioned in Section 3.2. CO_2 demand can have impact on the capacity at which the plant has to run, and thus on the economic value of a power plant. Next to that, CO_2 emission rights have impact on the total value of the CHP plant.

Most modern CHP plants can have a heat storage system, or a separate boiler next to the CHP. This gives a CHP the possibility to turn off during periods where producing energy is not profitable, while heating a greenhouse with heat from the storage. This leads to a more optimal solution. When a heat storage system is modeled, the number of starts a CHP can make becomes a relevant factor.

Finally, other types of CHP plants can be interesting to model. Our current model assumes no connection to the power grid, which means that all the energy which is generated has to be used. When a power grid connection is present, generated energy can be sold. This means that there is less need for a variable power to heat ratio, so other CHP types become an interesting option to model.

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Appendices

Appendix 1: results Scenario 5.1 – Scenario 5.4

Scenario 5.1: unbounded number of starts

Week#	#starts	Value	Produced electricity	Consumed Gas
1	7	87.324	7.610	15.255
2	7	85.003	7.610	15.255
3	7	82.683	7.610	15.255
4	7	80.042	7.610	15.255
5	3	32.814	3.330	6.675
Total	31	367.865	33.770	67.695

Runtime = 12.79 seconds

Scenario 5.2: maximum 15 starts

Week#	#starts	Value	Produced electricity	Consumed Gas
1	2	80.644	8.660	17.330
2	2	78.100	8.660	17.330
3	2	75.556	8.660	17.330
4	6	78.810	7.820	15.670
5	3	32.814	3.330	6.675
Total	15	345.924	37.130	74.335

Runtime = 14.00 seconds

Scenario 5.3: maximum 3 starts per period

Week#	#starts	Value	Produced electricity	Consumed Gas
1	3	82.045	8.450	16.915
2	3	79.545	8.450	16.915
3	3	77.046	8.450	16.915
4	3	74.208	8.450	16.915
5	3	32.814	3.330	6.675
Total	15	345.657	37.130	74.335

Runtime = 7.95 seconds

Scenario 5.4: 15 allowed starts, but only 1 period.

Week#	#starts	Value	Produced electricity	Consumed Gas
1	2	80.609	8.670	17.350
2	1	76.744	8.750	17.505
3	5	80.142	7.910	15.845
4	5	77.421	7.910	15.845
5	2	31.455	3.420	6.850
Total	15	346.370	36.660	73.395

Runtime = 161.83 seconds

Appendix 2: Results Scenario 6.1 – Scenario 6.8

Scenario 6.1: 2 fuel intervals

Week#	#starts	Value	Produced electricity	Consumed Gas
1	7	87.324	7.610	15.255
2	7	85.003	7.610	15.255
3	7	82.683	7.610	15.255
4	0	0	0	0
5	3	32.814	3.330	6.675
Total	24	287.823	26.160	52.440

Runtime = 18.62 seconds

Scenario 6.2: 3 fuel intervals

Week#	#starts	Value	Produced electricity	Consumed Gas
1	7	87.324	7.610	15.255
2	7	85.003	7.610	15.255
3	7	82.683	7.610	15.255
4	0	0	0	0
5	3	32.814	3.330	6.675
Total	24	287.823	26.160	52.440

Runtime = 28.14 seconds

Scenario 6.3: 4 fuel intervals

Week#	#starts	Value	Produced electricity	Consumed Gas
1	7	87.324	7.610	15.255
2	7	85.003	7.610	15.255
3	7	79.016	5.983	12.000
4	7	77.121	5.983	12.000
5	3	31.167	2.564	5.142
Total	31	359.631	29.749	59.652

Runtime = 54.33seconds

Scenario 6.4: 5 fuel intervals

Week#	#starts	Value	Produced electricity	Consumed Gas
1	7	87324	7.610	15.255
2	7	84369	7.183	14.400
3	7	82158	7.183	14.400
4	7	71353	4.783	9.600
5	3	32564	3.078	6.170
Total	31	357.768	29.835	59.825

Runtime = 63.54seconds

Scenario 6.5: 6 fuel intervals

Week#	#starts	Value	Produced electricity	Consumed Gas
1	7	87.325	7.610	15.255

2	7	85.003	7.610	15.255
3	7	79.016	5.983	12.000
4	7	77.121	5.983	12.000
5	3	31.167	2.564	5.142
Total	31	359.631	29.749	59.652

Runtime = 76.92 seconds

Scenario 6.6: 7 fuel intervals

Week#	#starts	Value	Produced electricity	Consumed Gas
1	7	85.788	6.840	13.714
2	7	83.666	6.840	13.714
3	7	81.544	6.840	13.714
4	7	79.422	6.840	13.714
5	3	29.488	2.196	4.407
Total	31	359.908	29.555	59.265

Runtime = 104.08 seconds

Scenario 6.7: 10 fuel intervals

Week#	#starts	Value	Produced electricity	Consumed Gas
1	7	87.324	7.610	15.255
2	7	84.369	7.183	14.400
3	7	79.016	5.983	12.000
4	7	77.121	5.983	12.000
5	3	32.564	3.078	6.170
Total	31	360.394	29.835	59.825

Runtime = 298.28 seconds

Scenario 6.8: 20 fuel intervals

Week#	#starts	Value	Produced electricity	Consumed Gas
1	7	86.580	7.183	14.400
2	7	83.090	6.583	13.200
3	7	81.034	6.583	13.200
4	7	78.979	6.583	13.200
5	3	32.204	2.821	5.656
Total	31	361.886	29.751	59.656

Runtime = 1379.60 seconds

Appendix 3: Results Scenario 6.8 – Scenario 6.16

Scenario 6.9: 2 fuel intervals

Week#	#starts	Value	Produced electricity	Consumed Gas
1	1	79.030	8.790	17.585
2	7	83.551	8.379	16.793
3	7	81.046	8.379	16.793
4	7	78.542	8.379	16.793
5	3	32.466	3.593	7.201
Total	25	354.634	37.520	75.165

Runtime = 25.08 seconds

Scenario 6.10: 3 fuel intervals

Week#	#starts	Value	Produced electricity	Consumed Gas
1	2	72.237	11.192	22.394
2	6	83.880	7.820	15.670
3	7	82.683	7.610	15.255
4	7	80.363	7.610	15.255
5	3	32.814	3.330	6.675
Total	25	351.976	37.562	75.249

Runtime = 27.65 seconds

Scenario 6.11: 4 fuel intervals

Week#	#starts	Value	Produced electricity	Consumed Gas
1	1	79.030	8.790	17.585
2	7	83.551	8.379	16.793
3	7	81.046	8.379	16.793
4	7	78.542	8.379	16.793
5	3	32.466	3.593	7.201
Total	25	354.634	37.520	75.165

Runtime = 40.09 seconds

Scenario 6.12: 5 fuel intervals

Week#	#starts	Value	Produced electricity	Consumed Gas
1	4	78.178	10.067	20.153
2	6	83.880	7.820	15.670
3	6	81.514	7.820	15.670
4	7	80.363	7.610	15.255
5	2	28.664	4.316	8.641
Total	25	352.599	37.632	75.389

Runtime = 58.58 seconds

Scenario 6.13: 6 fuel intervals

Week#	#starts	Value	Produced electricity	Consumed Gas
1	4	86.048	10.067	20.153

2	6	83.551	7.820	15.670
3	6	81.046	7.820	15.670
4	7	80.363	7.610	15.255
5	2	25.676	4.316	8.641
Total	25	356.683	37.632	75.389

Runtime = 55.32seconds

Scenario 6.14: 7 fuel intervals

Week#	#starts	Value	Produced electricity	Consumed Gas
1	5	80.740	9.584	19.193
2	5	77.965	9.584	19.193
3	7	82.683	7.610	15.255
4	7	80.363	7.610	15.255
5	3	32.814	3.330	6.675
Total	27	354.565	37.718	75.572

Runtime = 77.29 seconds

Scenario 6.15: 10 fuel intervals

Week#	#starts	Value	Produced electricity	Consumed Gas
1	7	86.048	8.384	16.803
2	7	83.551	8.379	16.793
3	7	81.046	8.379	16.793
4	7	78.542	8.379	16.793
5	2	28.664	4.316	8.641
Total	30	357.851	37.837	75.823

Runtime = 118.33 seconds

Scenario 6.16: 20 fuel intervals

Week#	#starts	Value	Produced electricity	Consumed Gas
1	7	86.048	8.384	16.803
2	7	83.551	8.379	16.793
3	7	81.046	8.379	16.793
4	7	78.542	8.379	16.793
5	3	30.967	3.953	7.921
Total	31	360.154	37.474	75.103

Runtime = 704.85 seconds

Appendix 4: Results Scenario 7.1 – Scenario 7.4

Scenario 7.1: constant heat demand of 10 MWh/h

Week#	#starts	Value	Produced Electricity	Produced Heat	Consumed Gas
1	0	34.840	7.993	1.680	17.755
2	0	32.470	7.993	1.680	17.755
3	0	30.100	7.993	1.680	17.755
4	0	27.730	7.993	1.680	17.755
5	0	10.126	3.309	720	7.375
Total	0	135.266	35.282	7.440	78.395

Runtime = 12.94 seconds

Scenario 7.2: no heat demand on day 8-17

Week#	#starts	Value	Produced Electricity	Produced Heat	Consumed Gas
1	0	34.840	7.993	1.680	17.755
2	6	83.684	7.690	0	15.410
3	2	51.260	7.952	960	16.925
4	0	27.730	7.993	1.680	17.755
5	0	10.126	3.309	720	7.375
Total	8	207.640	34.937	5.040	75.220

Runtime = 55.81 seconds

Scenario 7.3: constant heat demand of 20 MWh/h

Week#	#starts	Value	Produced Electricity	Produced Heat	Consumed Gas
1	0	-15.520	7.683	3.360	18.904
2	0	-17.782	7.683	3.360	18.904
3	0	-20.045	7.683	3.360	18.904
4	0	-22.308	7.683	3.360	18.904
5	0	-11.219	3.184	1.440	7.884
Total	0	-86.874	33.918	14.880	83.499

Runtime is 13.22 seconds

Scenario 7.4: Random heat demand between 0 and 20

Week#	#starts	Value	Produced Electricity	Produced Heat	Consumed Gas
1	0	36.893	8.151	1.588	17.975
2	0	32.546	8.156	1.627	18.026
3	0	25.571	8.066	1.784	18.010
4	0	25.255	8.082	1.721	17.976
5	0	10.326	3.380	685	7.481
Total	0	130.591	35.837	7.405	79.468

Runtime is 13.77 seconds

Appendix 5: Results realistic Scenario 2009

Daily gas price

2009 Daily Gas Price							
Fuel constraint	**Value**	**Fuel Consumption (MWh)**	**Energy production (MWh)**	**Heat Production (MWh)**	**Starts**	**Runtime (sec)**	**Remarks**
No constraint	121.951	33.774	12.392	3.718	0	133.63	
5% higher	120.758	35.463	13.118	3.718	0	152.35	
10% higher	116.588	37.153	13.845	3.718	0	146.54	
20% higher	N/A	N/A	N/A	N/A	N/A	N/A	Infeasible due to max fuel
5% lower	120.755	32.086	11.665	3.718	0	136.48	
10% lower	118.009	30.397	10.939	3.718	0	143.12	
20% lower	107.414	27.013	9.484	3.718	0	175,32	8 fuel intervals

2009 Daily Gas Price						
change in fuel constraints	**Value Decrease**	**Value Decrease %**	**Value decrease per percent change**	**Percentage of value decrease per percent change**	**Value decrease per percent change compared to previous point**	**Percentage Value decrease per percent change compared to previous point**
5% higher	-1.193	-0,98%	-238,6	-0,20%	-238,6	-0,20%
10% higher	-5.363	-4,40%	-536,3	-0,44%	-834	-0,68%
20% higher	N/A	N/A	N/A	N/A	N/A	N/A
5% lower	-1.196	-0,98%	-239,2	-0,20%	-239,2	-0,20%
10% lower	-3.942	-3,23%	-394,2	-0,32%	-549,2	-0,45%
20% lower	-14.537	-11,92%	-726,85	-0,60%	-1059,5	-0,87%

Quarterly gas price

2009 Quarterly Gas Price							
Fuel constraint	**Value**	**Fuel Consumption (MWh)**	**Energy production (MWh)**	**Heat Production (MWh)**	**Starts**	**Runtime (sec)**	**Remarks**
					0		
No constraint	125.065	33.148	12.122	3.718		129.28	
					0		
5% higher	123.837	34.806	12.835	3.718	0	140.18	

10% higher	120.320	36.463	13.548	3.718	0	154.58	
20% higher	N/A	N/A	N/A	N/A	N/A	N/A	Infeasible due to max fuel
5% lower	124.120	31.490	11.409	3.718	0	130.02	
10% lower	121.600	29.829	10.695	3.718	0	129.49	
20% lower	111.783	26.510	9.268	3.718	0	147.85	

2009 Quarterly Gas Price						
change in fuel constraints	**Value Decrease**	**Value Decrease %**	**Value decrease per percent change**	**Percentage of value decrease per percent change**	**Value decrease per percent change compared to previous point**	**Percentage Value decrease per percent change compared to previous point**
5% higher	-1.228	-0,98%	-245,6	-0,20%	-245,6	-0,20%
10% higher	-4.745	-3,79%	-474,5	-0,38%	-703,4	-0,56%
20% higher	N/A	N/A	N/A	N/A	N/A	N/A
5% lower	-945	-0,76%	-189	-0,15%	-189	-0,15%
10% lower	-3.465	-2,77%	-346,5	-0,28%	-504	-0,40%
20% lower	-13.282	-10,62%	-664,1	-0,53%	-981,7	-0,78%

Yearly gas Price

2009 Yearly gas price							
Fuel constraint	**Value**	**Fuel Consumption (MWh)**	**Energy production (MWh)**	**Heat Production (MWh)**	**Starts**	**Runtime (sec)**	**Remarks**
No constraint	123.238	33.726	12.371	3.718	0	132.82	
5% higher	121.975	35.415	13.097	3.718	0	145.56	
10% higher	118.241	37.100	13.821	3.718	0	144.08	
20% higher	N/A	N/A	N/A	N/A	N/A	N/A	Infeasible due to max fuel
5% lower	122.262	32.040	11.646	3.718	0	166.58	
10% lower	119.831	30.351	10.920	3.718	0	165.48	
20% lower	111.299	26.974	9.468	3.718	0	121.50	12 fuel intervals

2009 Yearly gas price						
change in fuel constraints	**Value Decrease**	**Value Decrease %**	**Value decrease per percent change**	**Percentage of value decrease per percent change**	**Value decrease per percent change compared to previous point**	**Percentage Value decrease per percent change compared to previous point**
5% higher	-1.263	-1,02%	-252,6	-0,20%	-252,6	-0,20%
10% higher	-4.997	-4,05%	-499,7	-0,41%	-746,8	-0,61%
20% higher	N/A	N/A	N/A	N/A	N/A	N/A
5% lower	-976	-0,79%	-195,2	-0,16%	-195,2	-0,16%
10% lower	-3.407	-2,76%	-340,7	-0,28%	-486,2	-0,39%
20% lower	-11.939	-9,69%	-596,95	-0,48%	-853,2	-0,69%

Appendix 6: Results realisctic Scenario 2010

Daily gas price

2010 Daily Gas Price							
Fuel constraint	**Value**	**Fuel Consumption (MWh)**	**Energy production (MWh)**	**Heat Production (MWh)**	**Starts**	**Runtime (sec)**	**Remarks**
No constraint	30.158	30.071	10.837	3.718	0	133.27	
5% higher	29.428	31.575	11.446	3.718	0	147.65	
10% higher	27.733	33.080	12.093	3.718	0	144.78	
20% higher	20.030	36.088	13.386	3.718	0	152.54	
5% lower	29.372	28.567	10.152	3.718	0	132.55	
10% lower	27.707	27.061	9.505	3.718	0	138.80	
20% lower	21.416	24.049	8.210	3.718	0	197.72	12 fuel intervals

2010 Daily Gas Price						
change in fuel constraints	**Value Decrease**	**Value Decrease %**	**Value decrease per percent change**	**Percentage of value decrease per percent change**	**Value decrease per percent change compared to previous point**	**Percentage Value decrease per percent change compared to previous point**
5% higher	-730	-2,42%	-146	-0,48%	-146	-0,48%
10% higher	-2.425	-8,04%	-242,5	-0,80%	-339	-1,12%
20% higher	-10.128	-33,58%	-506,4	-1,68%	-770,3	-2,55%
5% lower	-786	-2,61%	-157,2	-0,52%	-157,2	-0,52%
10% lower	-2.451	-8,13%	-245,1	-0,81%	-333	-1,10%
20% lower	-8.742	-28,99%	-437,1	-1,45%	-629,1	-2,09%

Quarterly gas price

2010 Quarterly gas price							
Fuel constraint	**Value**	**Fuel Consumption (MWh)**	**Energy production (MWh)**	**Heat Production (MWh)**	**Starts**	**Runtime (sec)**	**Remarks**
No constraint	29.115	29.568	10.583	3.718	0	129.77	
5% higher	28.527	31.047	11.219	3.718	0	146.10	
10% higher	27.057	32.525	11.854	3.718	0	144.00	
20% higher	20.260	35.486	13.128	3.718	0	280.35	

5% lower	28.525	28.089	9.947	3.718	0	124.03	
10% lower	27.111	26.608	9.310	3.718	0	133.04	
20% lower	21.402	23.650	8.038	3.718	0	98.97	12 fuel intervals

2010 Quarterly gas price						
change in fuel constraints	**Value Decrease**	**Value Decrease %**	**Value decrease per percent change**	**Percentage of value decrease per percent change**	**Value decrease per percent change compared to previous point**	**Percentage Value decrease per percent change compared to previous point**
5% higher	-588	-2,02%	-117,6	-0,40%	-117,6	-0,40%
10% higher	-2.058	-7,07%	-205,8	-0,71%	-294	-1,01%
20% higher	-8.855	-30,41%	-442,75	-1,52%	-679,7	-2,33%
5% lower	-590	-2,03%	-118	-0,41%	-118	-0,41%
10% lower	-2.004	-6,88%	-200,4	-0,69%	-282,8	-0,97%
20% lower	-7.713	-26,49%	-385,65	-1,32%	-570,9	-1,96%

Yearly gas price

2010 Yearly gas price							
Fuel constraint	**Value**	**Fuel Consumption (MWh)**	**Energy production (MWh)**	**Heat Production (MWh)**	**Starts**	**Runtime (sec)**	**Remarks**
No constraint	28.125	29.586	10.590	3.718	0	122.46	
5% higher	27.552	31.066	11.227	3.718	0	132.83	
10% higher	25.897	32.549	11.865	3.718	0	385.13	
20% higher	18.938	35.507	13.137	3.718	0	136.66	
5% lower	27.474	28.106	9.954	3.718	0	125.27	
10% lower	25.944	26.618	9.314	3.718	0	128.24	
20% lower	20.010	23.668	8.046	3.718	0	137.39	12 fuel intervals

2010 Yearly gas price						
change in fuel constraints	**Value Decrease**	**Value Decrease %**	**Value decrease per percent change**	**Percentage of value decrease per percent change**	**Value decrease per percent change compared to previous point**	**Percentage Value decrease per percent change compared to previous point**
5% higher	-573	-2,04%	-114,6	-0,41%	-114,6	-0,41%

10% higher	-2.228	-7,92%	-222,8	-0,79%	-331	-1,18%
20% higher	-9.187	-32,66%	-459,35	-1,63%	-695,9	-2,47%
5% lower	-651	-2,31%	-130,2	-0,46%	-130,2	-0,46%
10% lower	-2.181	-7,75%	-218,1	-0,78%	-306	-1,09%
20% lower	-8.115	-28,85%	-405,75	-1,44%	-593,4	-2,11%

Appendix 7: Results realisctic Scenario 2011

Daily gas price

2011 Daily gas price							
Fuel constraint	**Value**	**Fuel Consumption (MWh)**	**Energy production (MWh)**	**Heat Production (MWh)**	**Starts**	**Runtime (sec)**	**Remarks**
No constraint	-53.865	24.363	8.345	3.718	0	128.64	
5% higher	-54.306	25.582	8.869	3.718	0	138.31	
10% higher	-55.373	26.800	9.393	3.718	0	135.47	
20% higher	-59.760	29.238	10.441	3.718	0	254.40	10 fuel intervals
5% lower	-54.262	23.145	7.821	3.718	0	139.90	
10% lower	-55.242	21.921	7.295	3.718	0	134.79	
20% lower	-59.412	19.485	6.247	3.718	0	158.82	12 fuel intervals

2011 Daily gas price						
change in fuel constraints	**Value Decrease**	**Value Decrease %**	**Value decrease per percent change**	**Percentage of value decrease per percent change**	**Value decrease per percent change compared to previous point**	**Percentage Value decrease per percent change compared to previous point**
5% higher	-441	-0,82%	-88,2	-0,16%	-88,2	-0,16%
10% higher	-1.508	-2,80%	-150,8	-0,28%	-213,4	-0,40%
20% higher	-5.895	-10,94%	-294,75	-0,55%	-438,7	-0,81%
5% lower	-397	-0,74%	-79,4	-0,15%	-79,4	-0,15%
10% lower	-1.377	-2,56%	-137,7	-0,26%	-196	-0,36%
20% lower	-5.547	-10,30%	-277,35	-0,51%	-417	-0,77%

Quarterly gas price

2011 Quarterly gas price							
Fuel constraint	**Value**	**Fuel Consumption (MWh)**	**Energy production (MWh)**	**Heat Production (MWh)**	**Starts**	**Runtime (sec)**	**Remarks**
No constraint	-53.404	24.331	8.331	3.718	0	124.68	
5% higher	-53.783	25.548	8.854	3.718	0	128.96	
10% higher	-54.784	26.766	9.378	3.718	0	136.86	
20% higher	-58.595	29.202	10.425	3.718	0	131.85	

5% lower	-53.726	23.114	7.808	3.718	0	122.54	
10% lower	-54.658	21.898	7.285	3.718	0	125.74	
20% lower	-58.602	19.464	6.238	3.718	0	92.53	12 fuel intervals

2011 Quarterly gas price						
change in fuel constraints	**Value Decrease**	**Value Decrease %**	**Value decrease per percent change**	**Percentage of value decrease per percent change**	**Value decrease per percent change compared to previous point**	**Percentage Value decrease per percent change compared to previous point**
5% higher	-379	-0,71%	-75,8	-0,14%	-75,8	-0,14%
10% higher	-1.380	-2,58%	-138	-0,26%	-200,2	-0,37%
20% higher	-5.191	-9,72%	-259,55	0,49%	-381,1	-0,71%
5% lower	-322	-0,60%	-64,4	-0,12%	-64,4	-0,12%
10% lower	-1.254	-2,35%	-125,4	-0,23%	-186,4	-0,35%
20% lower	-5.198	-9,73%	-259,9	-0,49%	-394,4	-0,74%

Yearly gas price

2011 Yearly gas price							
Fuel constraint	**Value**	**Fuel Consumption (MWh)**	**Energy production (MWh)**	**Heat Production (MWh)**	**Starts**	**Runtime (sec)**	**Remarks**
No constraint	-53.098	24.300	8.318	3.718	0	125.23	
5% higher	-53.478	25.515	8.840	3.718	0	134.09	
10% higher	-54.457	26.730	9.363	3.718	0	136.88	
20% higher	-58.153	29.163	10.409	3.718	0	127.22	
5% lower	-53.394	23.084	7.795	3.718	0	125.10	
10% lower	-54.273	21.868	7.272	3.718	0	173.87	
20% lower	-58.156	19.438	6.227	3.718	0	85.61	12 fuel intervals

2011 Yearly gas price						
change in fuel constraints	**Value Decrease**	**Value Decrease %**	**Value decrease per percent change**	**Percentage of value decrease per percent change**	**Value decrease per percent change compared to previous point**	**Percentage Value decrease per percent change compared to previous point**
5% higher	-380	-0,72%	-76	-0,14%	-76	-0,14%

10% higher	-1.359	-2,56%	-135,9	-0,26%	-195,8	-0,37%
20% higher	-5.055	-9,52%	-252,75	-0,48%	-369,6	-0,70%
5% lower	-296	-0,56%	-59,2	-0,11%	-59,2	-0,11%
10% lower	-1.175	-2,21%	-117,5	-0,22%	-175,8	-0,33%
20% lower	-5.058	-9,53%	-252,9	-0,48%	-388,3	-0,73%

Appendix 8: quarterly average gas price over 3 years

2009

2009							
Fuel constraint	**Value**	**Fuel Consumption (MWh)**	**Energy production (MWh)**	**Heat Production (MWh)**	**Starts**	**Runtime (sec)**	**Remarks**
							1 Jan to 30 Dec
No constraint	-23.978	21.498	7.125	3.718	0	127.62	
5% higher	-24.343	22.573	7.587	3.718	0	128.83	
10% higher	-25.070	23.650	8.051	3.718	0	127.48	
20% higher	-27.782	25.808	8.978	3.718	0	93.73	12 fuel intervals
5% lower	-24.253	20.423	6.663	3.718	0	139.51	
10% lower	-25.159	19.345	6.199	3.718	0	90.74	12 fuel intervals
20% lower	-28.665	17.193	5.274	3.718	0	346.72	12 fuel intervals

2009						
change in fuel constraints	**Value Decrease**	**Value Decrease %**	**Value decrease per percent change**	**Percentage of value decrease per percent change**	**Value decrease per percent change compared to previous point**	**Percentage Value decrease per percent change compared to previous point**
5% higher	-365	1,52%	-73	0,30%	-73	0,30%
10% higher	-1.092	4,55%	-109,2	0,46%	-145,4	0,61%
20% higher	-3.804	15,86%	-190,2	0,79%	-271,2	1,13%
5% lower	-275	1,15%	-55	0,23%	-55	0,23%
10% lower	-1.181	4,93%	-118	0,49%	-181,2	0,76%
20% lower	-4.687	19,55%	-234,35	0,98%	-350,6	1,46%

2010

2010							
Fuel constraint	**Value**	**Fuel Consumption (MWh)**	**Energy production (MWh)**	**Heat Production (MWh)**	**Starts**	**Runtime (sec)**	**Remarks**
No constraint	28.584	29.740	10.657	3.718	0	125.87	
5% higher	27.842	31.229	11.297	3.718	0	133.30	

10% higher	26.116	32.715	11.936	3.718	0	132.77	
20% higher	17.193	35.693	13.216	3.718	0	69.84	10 fuel intervals
5% lower	28.003	28.253	10.017	3.718	0	126.21	
10% lower	26.285	26.765	9.377	3.718	0	84.48	12 fuel intervals
20% lower	20.307	23.789	8.098	3.718	0	309.39	12 fuel intervals

2010						
change in fuel constraints	**Value Decrease**	**Value Decrease %**	**Value decrease per percent change**	**Percentage of value decrease per percent change**	**Value decrease per percent change compared to previous point**	**Percentage Value decrease per percent change compared to previous point**
5% higher	-742	-2,60%	-148,4	-0,52%	-148,4	-0,52%
10% higher	-2.468	-8,63%	-246,8	-0,86%	-345,2	-1,21%
20% higher	-11.391	-39,85%	-569,55	-1,99%	-892,3	-3,12%
5% lower	-581	-2,03%	-116,2	-0,41%	-116,2	-0,41%
10% lower	-2.299	-8,04%	-229,9	-0,80%	-343,6	-1,20%
20% lower	-8.277	-28,96%	-413,85	-1,45%	-597,8	-2,09%

2011

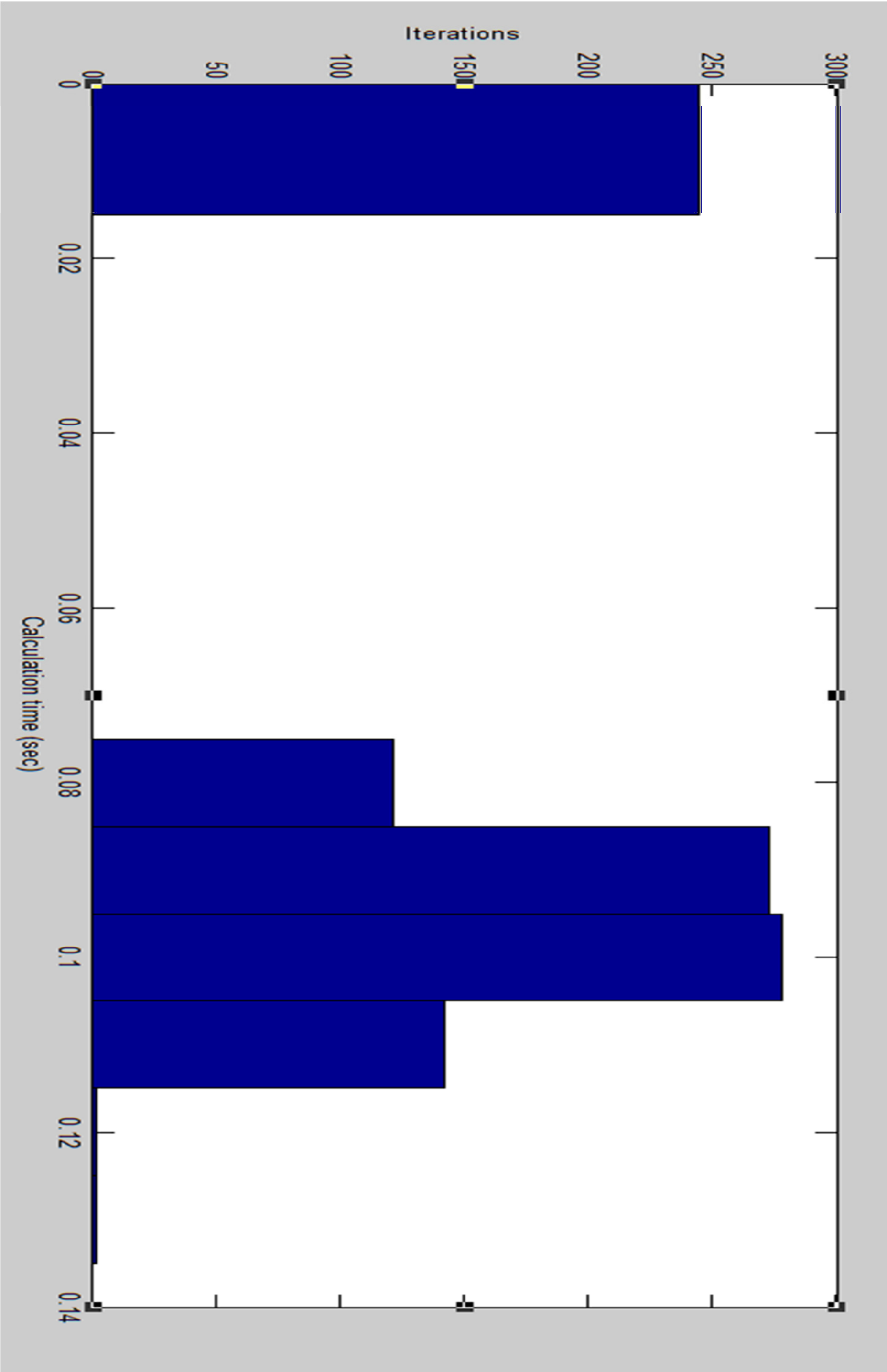
2011							
Fuel constraint	**Value**	**Fuel Consumption (MWh)**	**Energy production (MWh)**	**Heat Production (MWh)**	**Starts**	**Runtime (sec)**	**Remarks**
No constraint	106.877	36.282	13.470	3.718	0	140.26	
5% higher	104.944	38.096	14.250	3.718	0	157.66	
10% higher	N/A	N/A	N/A	N/A	N/A	N/A	Infeasible due to max fuel
20% higher	N/A	N/A	N/A	N/A	N/A	N/A	Infeasible due to max fuel
5% lower	105.936	34.467	12.690	3.718	0	141.78	
10% lower	103.645	32.652	11.909	3.718	0	143.13	
20% lower	94.700	29.021	10.348	3.718	0	960.38	

2011						
change in fuel constraints	**Value Decrease**	**Value Decrease %**	**Value decrease per percent change**	**Percentage of value decrease per percent change**	**Value decrease per percent change compared to previous run**	**Percentage Value decrease per percent change compared to previous run**
5% higher	-1.933	-1,81%	-386,6	-0,36%	-386,6	-0,36%
10% higher	N/A	N/A	N/A	N/A	N/A	N/A
20% higher	N/A	N/A	N/A	N/A	N/A	N/A
5% lower	-941	-0,88%	-188,2	-0,18%	-188,2	-0,18%
10% lower	-3.232	-3,02%	-323,2	-0,30%	-458,2	-0,43%
20% lower	-12.177	-11,39%	-608,85	-0,57%	-894,5	-0,84%

Appendix 9: impact of smaller runs on total run time

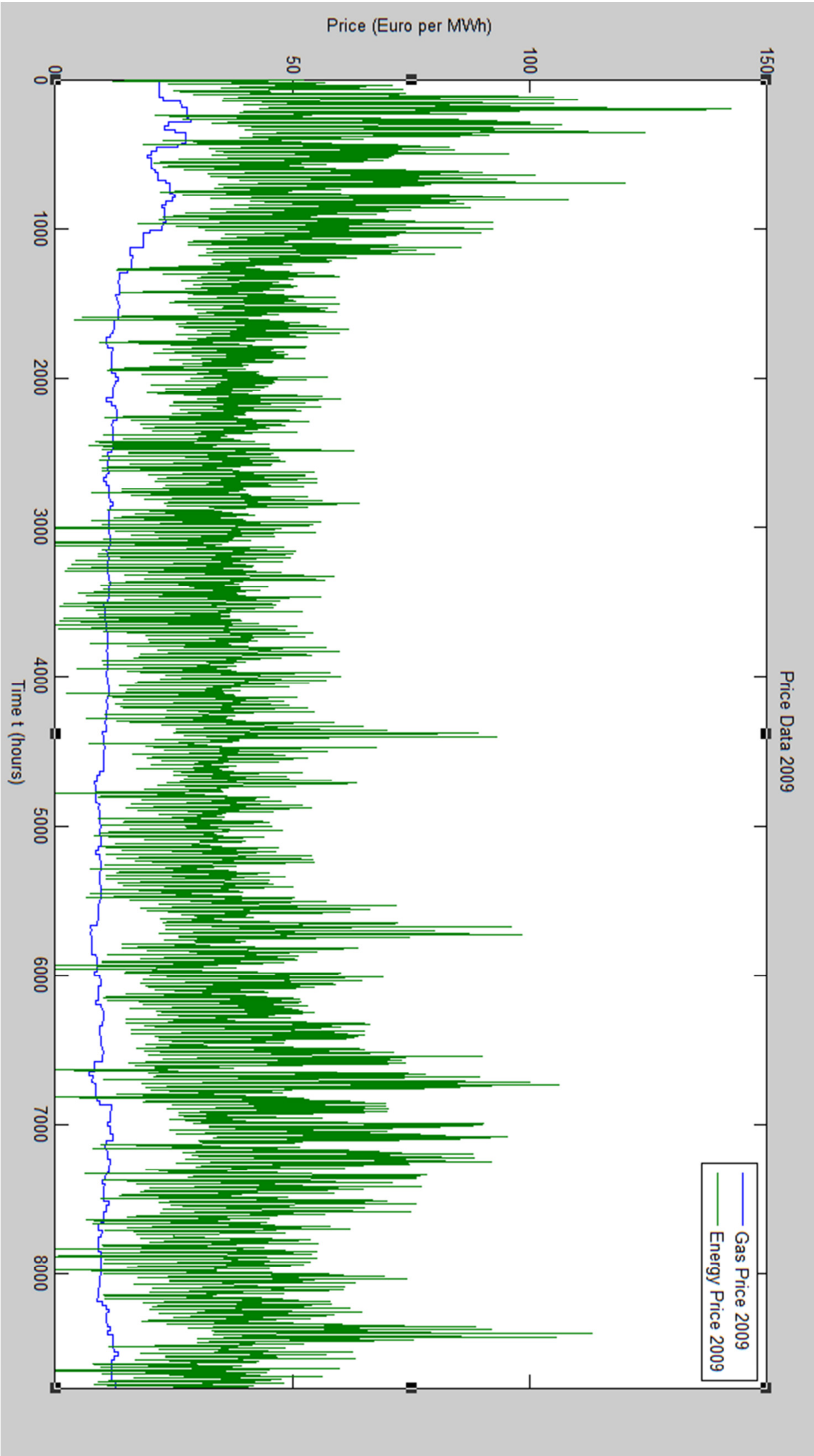
Valuation horizon	Starts	Runtime with periods of 1 week (sec)	Runtime in 1 period (sec)	Value multiple periods	Value 1 period	Value difference
1 week	7	8.81	8.81	87.324	87.324	0%
2 weeks	7	14.52	27.45	163.079	163.170	0,06%
3 weeks	7	20.41	34.14	235.790	236.045	0,11%
4 weeks	7	26.45	56.31	305.698	306.273	0,19%
5 weeks	7	34.12	65.00	355.828	356.695	0,24%
2 weeks	10	15.87	58.93	167.325	167.372	0,03%
3 weeks	10	22.67	133.18	240.258	240.425	0,07%
4 weeks	10	29.17	83.82	310.380	310.829	0,14%
5 weeks	10	36,34	85.16	360.642	361.296	0,18%

Appendix 10: Histogram iteration calculation time

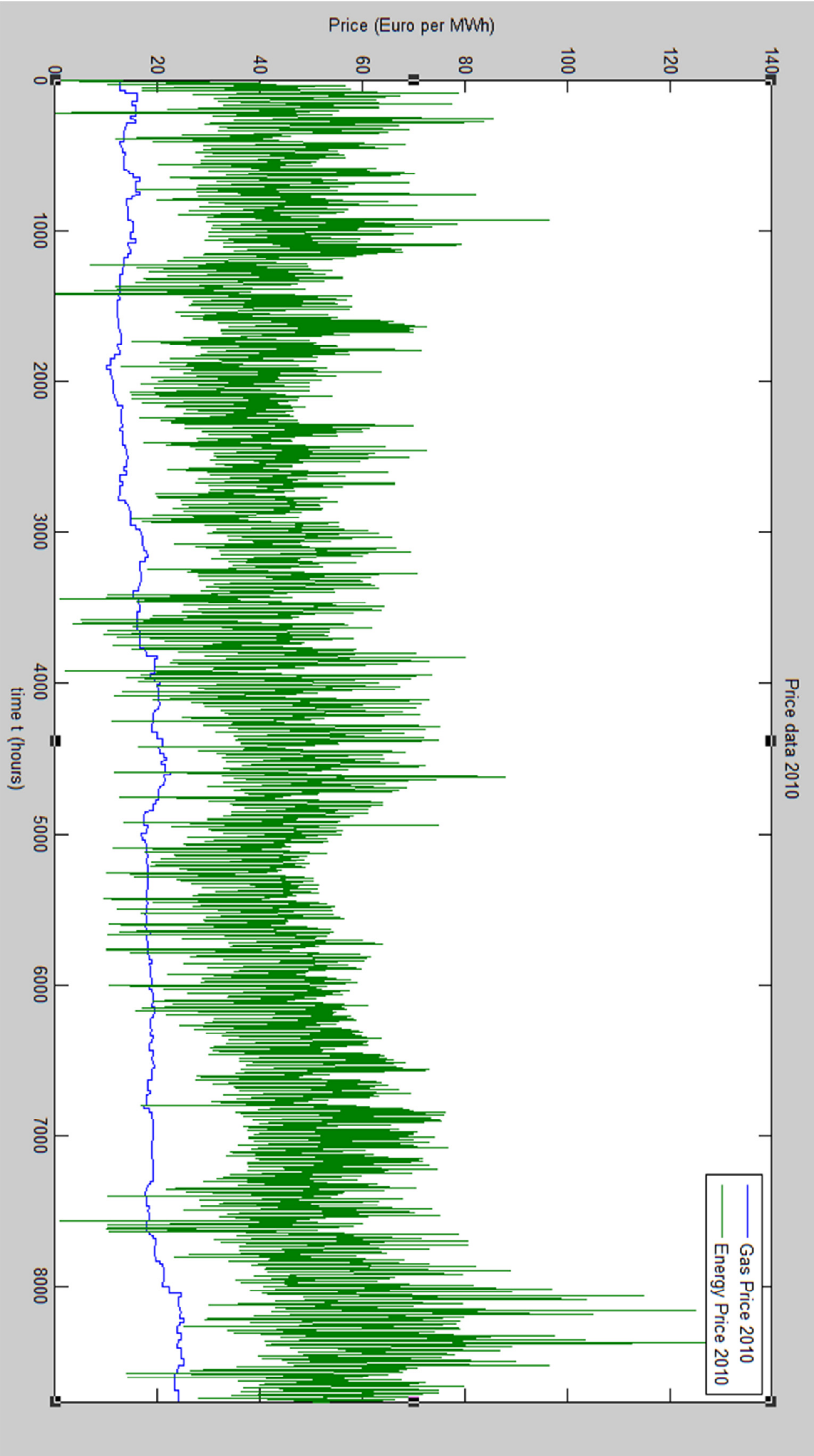


Appendix 11: Gas and Energy prices

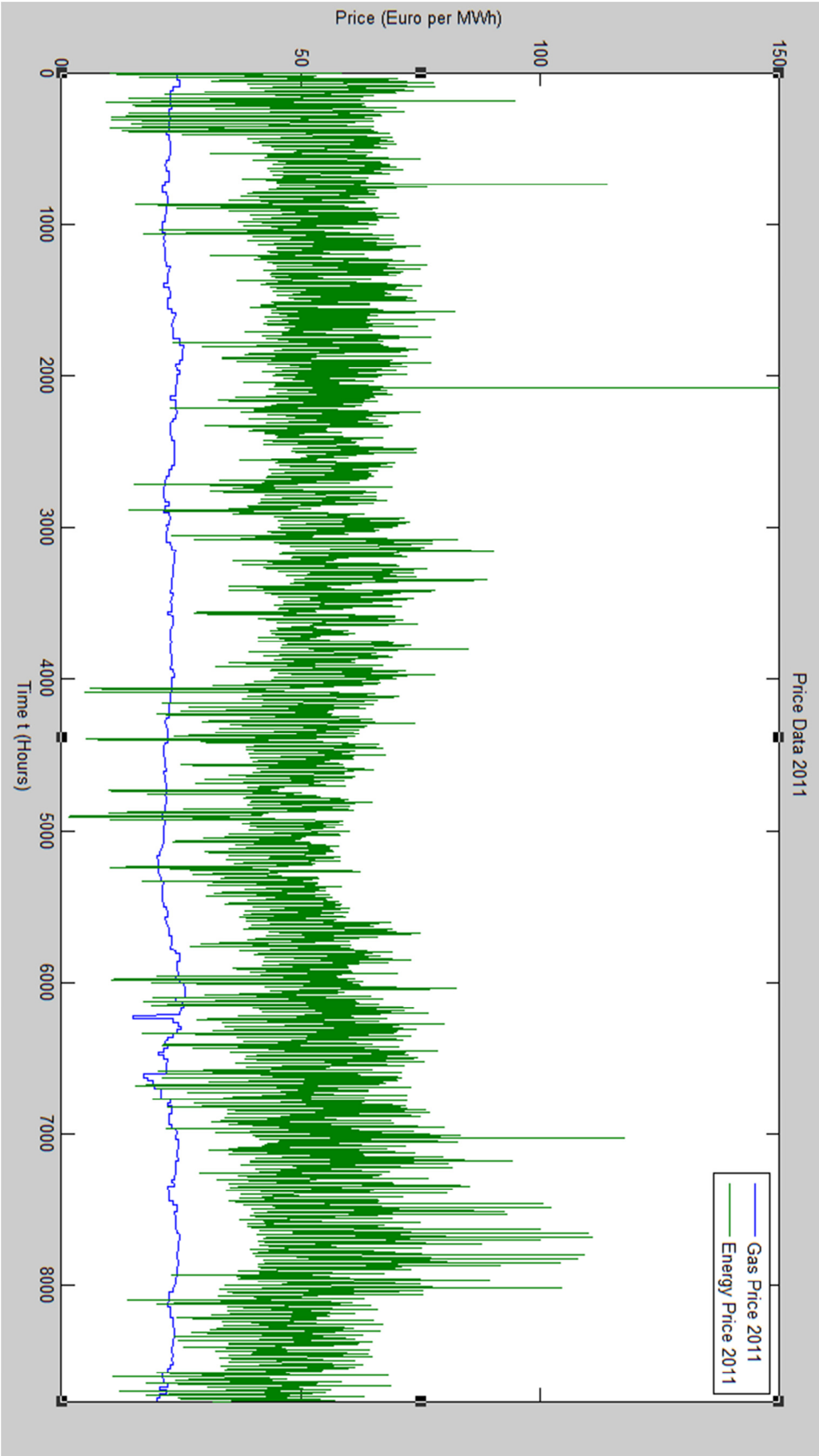
2009



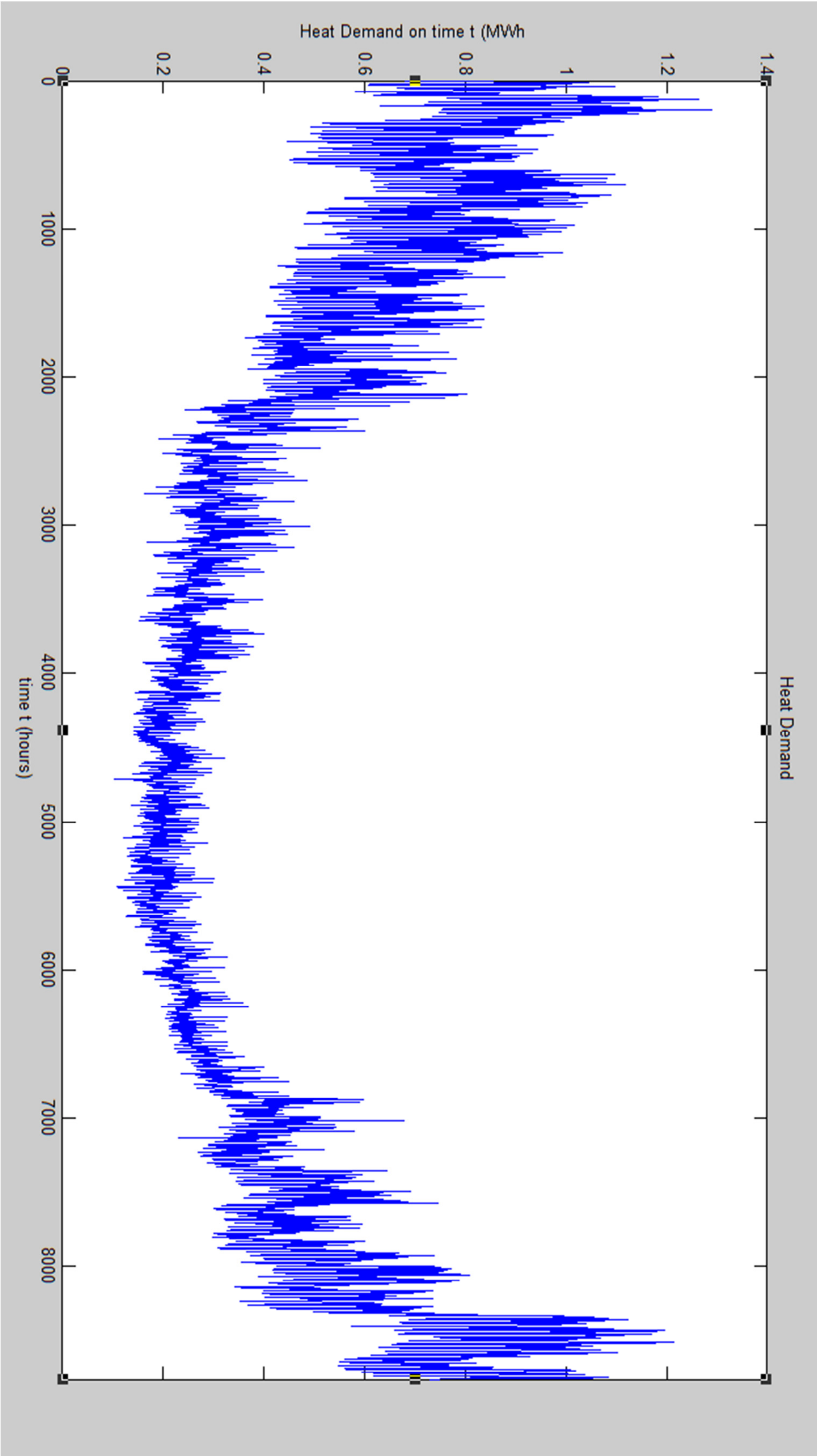
2010


Price data 2010
Price (Euro per MWh)
time t (hours)
Gas Price 2010
Energy Price 2010
0
20
40
60
80
100
120
140
0
1000
2000
3000
4000
5000
6000
7000
8000

2011


Price Data 2011
Price (Euro per MWh)
Time t (Hours)
Gas Price 2011
Energy Price 2011
0
50
100
150
0
1000
2000
3000
4000
5000
6000
7000
8000

Appendix 12: Heat Demand



Appendix 13: Description of LP_Solve

lp_solve is a custom Matlab file to solve mixed integer linear programming (MILP) problems of the following format.

$$
\begin{aligned}
&Max\ v = f^T * x \\
&s.t.\ \ A * x \leq b \\
&\qquad vlb \leq x \leq vub \\
&\qquad x(int)\ are\ integer
\end{aligned}
$$

In order to solve this MILP, lp_solve needs the following input:

f: n vector of coefficients for a linear objective function.
A: m by n matrix representing linear constraints.
b: m vector of right sides for the inequality constraints.
e: m vector that determines the sense of the inequalities:
e(i) = -1 ==> Less Than
e(i) = 0 ==> Equals
e(i) = 1 ==> Greater Than
vlb: n vector of lower bounds
vub: n vector of upper bounds.
xint: vector of integer variables.

In this description, $Max\ v = f^T * x$ is the objective function. The vector *x* is the set of decision variables and $A * x \leq b$ is the set of constraints ot the model.

When all input variables are given, lp_solve will try to find a feasible optimal solution consisting of optimal objective value, and the corresponding optimal set of decision variables. In case of the existing plant model, the decision variables are for each hour the running status of the plant (on or off), whether to turn the plant on or off, and the amount of electricity produced.

lp_solve is based on linear programming in Matlab and is currently used to determine the optimal value for a normal power plant. We will also use lp_solve to be able to optimally distribute the number of starts and fuel consumption over different periods during the valuation period.