

The application of real option analysis on a Gas-to-Wire investment scenario

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Abstract

With this study, we aim to increase understanding of the insights that real option analysis (ROA) has to offer, particularly in comparison to dynamic decision tree analysis (DTA). We point out the fundamental theoretical shortcoming of applying a constant discount rate in the latter approach, and explain how real options resolve this issue. Based on the fundamentals of risk-neutral valuation and replicating portfolio concepts, we address different perspectives on how to treat non-hedgeable risks in a real option framework. We adopt an integrated view combining option pricing and decision analysis, which is theoretically consistent and allows an assessment of both market risk and private risk.

To illustrate the practical application of real option analysis, we construct a model which determines the optimal time to switch from gas production to electricity generation directly at the wellhead (Gas-to-Wire). To deal with the path-dependent price paths in this investment problem, we use a combination of Monte Carlo simulation and a backwards regression algorithm. We construct forecasting models for natural gas and electricity prices. These models deal with the seasonal effects, price jumps, mean-reversion and time-varying volatility observed particularly in electricity prices. With a comparative study, we show that ROA provides results that significantly deviate from those yielded by DTA.

Keywords: *real option analysis, private risk, path-dependent, Monte Carlo simulation, time series analysis, Bermudan swap option, backwards regression, Gas-to-Wire*

Glossary

Glossary financial theory

American option	Option which can be exercised at any time during the lifetime of the option.
Arbitrage	Opportunity to make a risk-free profit at zero cost, making use of pricing inconsistencies in the market.
Bermudan option	Option which can be exercised at a number of pre-specified dates before maturity.
Black-Scholes formula	Formula used to calculate the arbitrage-free price of a European option under a set of restrictive assumptions.
Call option	Derivative which grants the right to buy the underlying asset at a previously specified price.
Capital Asset Pricing Model	Classic model used to estimate the return required by investors based on the risk-free interest rate and the correlation of the asset return with the prevailing market return.
Classic ROA	Real option approach which relies on market replication of the project, considering private risk as a source of tracking error.
Complete market	Market model in which every financial asset can be replicated with a set of other financial assets, where all agents are able to trade all assets and no transaction costs exists.
Convenience yield	Benefits and costs stemming from possessing a commodity compared to holding its financial equivalent, caused by the opportunity to profit from temporary shortages and storage costs.
Decision Tree Analysis	Method used to value a project with embedded flexibilities by incorporating decision points and probabilities of different scenarios, using a constant discount rate for all cash flows.
Derivative	Financial instrument which derives its value from that of an underlying asset, with the payoff depending on the specified conditions.
Discounted Cash Flow	Method used to value a project by discounting future cash flows at a constant rate in order to obtain the net present value.
Discount rate	Rate at which estimates of future cash flows are discounted, reflecting the time value of money and risk-adjustment.
Diversification	Reduction of risk by spreading investments, as such reducing the variance of the portfolio's return. Perfect diversification leaves the investor exposed only to market movements.
European option	Option which can only be exercised at the end of its lifetime.

Futures contract	Contract which obliges to buy/sell an asset at a certain point in the future, with the price to be paid determined today.
Geometric Brownian Motion	Stochastic process to model the behaviour of asset prices over time, assuming returns follow a normal distribution with constant parameters.
Hedging	Practice to reduce risk exposure by taking an offsetting position to a security. A perfectly hedged portfolio eliminates all market risk.
Heteroskedasticity	Term to describe varying variance over time. Unconditional heteroskedasticity does not depend on previous observations, conditional heteroskedasticity does.
Integrated ROA	Real option approach which assumes that the market is partially complete, valuing the project part which can be replicated with arbitrage pricing and the remaining part with subjective valuation.
Leverage	Increasing the potential return of an investment with debt or by using derivatives, at the cost of a higher risk.
MAD ROA	Real option approach comparable to the subjective approach, assuming that the replicating portfolio is a twin security worth the subjectively estimated value of the project.
Market risk	Part of project risks that can be replicated and hedged by financial instruments under the assumption of a complete market.
Ornstein-Uhlenbeck model	Extension of Geometric Brownian Motion that incorporates mean-reversion. As the simulated variable deviates more from its equilibrium level, the reverting effect becomes stronger.
Portfolio	The collection of investments held by an investor, which may include all forms of financial instruments.
Private risk	All project risks that cannot be hedged by market instruments, formally defined as the tracking error of the replicating portfolio.
Put option	Derivative which grants the right to sell the underlying asset at a previously specified price.
Real option	Valuation method which explicitly values flexibilities in real-world projects based on financial option theory.
Replicating portfolio	Portfolio consisting of financial instruments, which replicates the payoffs of a real project in all market states and at all times.
Return	Logarithm of the price at a certain time divided by the return of the previous time point, approximating the first difference of the price series over the specified interval.
Revised classic ROA	Real option approach which applies either option pricing or decision analysis, depending on the dominating type of project risk.

Risk-free rate	Theoretical rate which investors can earn without being subject to any risk. Often approximated by the return on government bonds with very low default risk.
Risk-neutral valuation	Valuation method which assumes an artificial risk-neutral price distribution, allowing to use the risk-free rate as drift of the underlying. Provides the same value as real valuation if the complete market assumption holds.
Security	Term to describe a financial instrument, such as a stock, a bond or a derivative.
Short-selling	Selling assets borrowed from a third party, with the intention of buying back identical assets at a later time to return to the third party. This practice allows to profit from price decline without making an initial investment, but has an unlimited downside potential if not hedged.
Spot contract	Contract to buy or sell an asset at the current time against the prevailing market price.
Subjective ROA	Real option approach which assumes the subjective estimate of the project value can be considered a replicating portfolio, using its NPV as basis for option valuation under the assumption of market completeness.
Swap option	Option which grants the right to swap one stream of cash flows for another stream of cash flows.
Time series	Series of observations over a period of time, such as prices or returns.
Twin security	(Hypothetical) security traded on the financial market that is perfectly correlated with the real project.
Vašíček model	Mean-reverting stochastic model to replicate the behaviour of the interest rate over time.
Volatility	Standard deviation of the return on an asset, being used as a measure of uncertainty of return.
Weighted Average Cost of Capital	Estimate for the average cost of capital, consisting of the cost of debt and the cost of equity proportional to their share of total capital.

Glossary energy market

30/30 ambition	Goal of EBN to have 30% of Dutch natural gas produced from small gas fields by the year 2030.
APX-ENDEX	Dutch energy exchange, on which both short- and long-term contracts on natural gas and electricity are traded.
Balancing market	Electricity market on which electricity is traded to correct for imbalances between supply and demand on the short term.

Connectivity	Rate at which the gas-containing volumes in a field are connected to each other, allowing a gas flow towards the well.
Energie Beheer Nederland	Dutch government-owned institute, involved in every gas winning operation in the Netherlands as a facilitating partner. Also has an advisory task towards the Dutch government.
Expansion factor	Indicates how much gas will expand when it is retrieved from the reservoir.
Gas-(Initially)-In-Place	(Initial) amount of gas present in a reservoir. Not all gas in a reservoir can be economically retrieved.
GasTerra	Major gas trading institute in the Netherlands, co-owned by Shell, Exxon, EBN and the Dutch state. Has the public task to buy gas produced at small gas fields when requested.
Gas Transport Services	Full daughter company of Gasunie, responsible for the transport of natural gas through the main transport networks. Also performs conversion operations.
Gasunie	Government-owned institute which owns and manages the Dutch main gas distribution network.
Gas intersection	Planned function of the Dutch gas transport network to serve as a logistic centre for the transport and storage of natural gas in the north-west of Europe.
Groningen gas field	Major gas field located in the province of Groningen. It is the largest natural gas field in Europe and one of the largest in the world. Also referred to as the Slochteren gas field.
Line-packing	Storing natural gas in the transport network under high pressure. Varying the pressure allows to store less or more gas in the pipelines.
Natural gas	Gas mixture containing hydrocarbons which has a high energetic value. It is found in underground reservoirs. Natural gas is used as an energy source both directly and as input to generate electricity. It is also used as feedstock in the chemical industry.
Nederlandse Aardolie Maatschappij	E&P operator jointly owned by Shell and Exxon, being the largest natural gas producer in the Netherlands and the sole exploiter of the Groningen gas field.
Nederlandse Mededingings-autoriteit Energiekamer	Competition regulator on the Dutch energy market, having the authority to enforce regulation on parties in the market. Also responsible for providing licenses to market parties.
Net-to-gross ratio	Part of the gross volume of a formation which can contain gas.
Permeability	Rate at which natural gas can flow through the porous rock formation.
Porosity	Percentage of a formation which can contain fluid or gas.

Reserve	Economically retrievable amount of gas in a reservoir.
Saturation	Percentage of porous volume in a formation filled with gas.
Small fields policy	A Dutch government policy to stimulate the development of small gas fields in order to reduce the burden on the Groningen gas field. The policy comprises a guaranteed sale of production, and fiscal advantages for projects at the North Sea.
Spark spread	Difference between the price of electricity and the price of the amount of input fuel required to generate the same amount of electricity.
Tail-end gas field	Gas field which is in a mature state of exploitation.
TenneT	Government-owned institute which owns and manages the Dutch electricity high-voltage transport network.
TTF	Virtual trading point for natural gas, which is facilitated by energy exchange APX-ENDEX.
Tubing	Tube placed in the well through which the gas flows from the reservoir to the surface. The diameter of the tubing determines the pressure and the friction level within the well.
Virgin gas field	Currently unexploited gas field containing a small amount of gas.
Volume	Gross volume of the formation containing gas, measured by multiplying the area of the formation with its thickness.

Acronyms and abbreviations

AC	Autocorrelation.
ADF	Augmented Dickey-Fuller.
APX	Amsterdam Power Exchange.
AR	Autoregressive.
ARMA	Autoregressive Moving Average
ARIMA	Autoregressive Integrated Moving Average.
CAPM	Capital Asset Pricing Model.
DCF	Discounted Cash Flow.
DTA	Decision Tree Analysis.
EBN	Energie Beheer Nederland.
EBT	Earnings Before Taxes.
ENDEX	European Energy Derivatives Exchange.
EPCCI	European Power Capital Cost Index.

EWMA	Exponentially Weighted Moving Average.
E&P	Exploration and Production.
GARCH	Generalized Autoregressive Conditional Heteroskedasticity.
GBM	Geometric Brownian Motion.
GIIP	Gas-Initially-In-Place.
GIP	Gas-In-Place.
HC	High-caloric gas.
LC	Low-caloric gas.
MA	Moving Average.
MAD	Market Asset Disclaimer.
MWh	Megawatt hour.
NAM	Nederlandse Aardolie Maatschappij.
NGL	Natural Gas Liquid.
NMa	Nederlandse Mededingingsautoriteit.
NPV	Net Present Value.
PAC	Partial Autocorrelation.
PRP	Programme Responsibility Partner.
ROA	Real Option Analysis.
UCCI	Upstream Capital Cost Index.
UOCI	Upstream Operating Cost Index.
VAR	Vector Autoregressive.
VEC	Vector Error Correction.
WACC	Weighted Average Cost of Capital.

Mathematical notations used

α	Real-world drift of an asset (excluding dividend payments).
$\hat{\alpha}$	Risk-neutral drift of an asset (excluding dividend payments).
α	Relative error of option value Monte Carlo simulation.
α'	Adjusted relative error of option value Monte Carlo simulation.
a_{ij}	Risk-neutral transition probability from state i to state j without considering time value.
α_{ij}^*	Risk-neutral transition probability from state i to state j when considering time value.

$ACV(h)$	Autocovariance of lag h between time series .
$ACVF^{(3)}$	Sum of autocovariances $ACV(h)$ to the third power providing autocorrelation-corrected standard error of skewness.
$ACVF^{(4)}$	Sum of autocovariances $ACV(h)$ to the fourth power providing autocorrelation-corrected standard error of kurtosis.
B	Variable indicating the size of a deterministic time trend for a single time step.
B_A	Amplitude parameter for annual seasonal function.
B_{SA}	Amplitude parameter for semi-annual seasonal function.
β	Correlation between asset and market return in CAPM.
c	Constant, used in several equations.
CF_t	Cash flow at time t .
$\chi^2_{(p,df)}$	Chi-square critical value with df degrees of freedom and significance level p .
d_1	Variable in Black-Scholes option pricing model.
d_2	Variable in Black-Scholes option pricing model.
df	Degrees of freedom.
D	Market value of debt in WACC.
D_i	Average daily effect of price series.
DO	Average diesel oil price.
DR	Exponential decline rate parameter of production rate.
Δ	Lag operator indicating the difference between an observation at time $t + 1$ and at time t .
e	Mathematical constant that is the base of natural logarithms.
E	Market value of equity in WACC.
$E(\cdot)$	Expected value of a variable.
$E_P(CF_t)$	Expectation of real-world cash flow at time t .
$E_Q(CF_t)$	Expectation of risk-neutral cash flow at time t .
ev_i	i th largest eigenvalue of matrix l_2 in Johansen procedure.
Exp	Expansion factor of gas in a reservoir.
ε_t	Random error term at time t .
λ	Market risk premium defined as the Sharpe ratio.
φ	Intensity of Poisson arrival process in jump diffusion model.

φ_D	Intensity of Poisson arrival process for a downward price jump.
φ_U	Intensity of Poisson arrival process for an upward price jump.
F_T	Price of futures contract maturing at time T .
FO	Average fuel oil price.
G_{ij}	Matrix in Johansen procedure.
h	Number of time lags indicating the number of time steps an observation lies before the observation at time t .
h_a	Number of autoregressive lags.
h_m	Number of moving-average lags.
H	Hurst exponent indicating the persistence of a time-series trend.
H_0	Indicator for null hypothesis in statistical tests.
i	Indicator for a certain state.
j	Indicator for a certain state other than i .
J	Average jump size in jump diffusion model.
JB	Jarque-Bera test statistic.
JB_{adj}	Jarque-Bera test statistic adjusted for autocorrelation.
$JH(rk n)$	Johansen procedure test statistic for rank rk and n time series.
δ	Dividend payment or net convenience yield.
κ	Mean-reversion rate in Ornstein-Uhlenbeck process.
l_1	$n \times rk$ matrix in Johansen procedure.
l_2	$rk \times n$ matrix in Johansen procedure.
μ	Total expected return of an asset including possible dividends or net convenience yield.
μ_J	Average jump size in jump diffusion model.
m_i	i th central moment of a time series.
M_i	Average monthly effect of price series, for month i .
n	Number of observations in a data set.
$N(\cdot)$	Cumulative normal distribution.
N/G	Net-to-gross ratio of a rock formation.
σ	Constant volatility, square root of variance of returns.
σ_t	Time-dependent volatility.
σ_J	Standard deviation of jump size in jump diffusion model.

$\emptyset_A$	Shifting parameter in annual seasoning function.
$\emptyset_{SA}$	Shifting parameter in semi-annual seasoning function.
P_t	Value of a tracking portfolio at time t .
PCC	Pearson correlation coefficient.
Por	Porosity of a rock formation.
p	Significance level.
p_{ij}	Transition probability from state i to state j .
q_{ij}	Discounted risk-neutral transition probability from state i to state j .
π	Mathematical constant π .
θ	Risk-neutral/arbitrage-free value of a derivative.
Q	Ljung-Box test statistic.
ρ	Correlation coefficient.
r_d	Required return on debt in WACC.
PR_0	Initial production rate of a gas field.
PR_t	Production rate of a gas field at time t .
r_e	Required return on equity in WACC.
r_f	Risk-free interest rate.
r_m	Market return.
rk	Rank in Johansen procedure.
R^2	Coefficient of determination.
R_t	Residual at time t .
R_{it}	Residual for state i at time t .
R_{jt}	Residual for state j at time t .
RSS	Residual sum of squares.
s_i	Input variable in an artificial neural network.
Sat	Saturation level of a rock formation.
SG	Sigmoid function in an artificial neural network.
$\bar{S}$	Average asset price over a certain amount of time.
S_e	Price of electricity.
S_g	Price of natural gas.
S_t	Price of the underlying asset at time t .

SS_t	Spark spread at time t .
Π	$n \times n$ matrix in Johansen procedure, being the product of l_1 and l_2' .
Γ_h	$n \times n$ matrix in Johansen procedure used to remove autocorrelation at lag h .
t	Time point expressed in days, also used as indicator of the value of another value at time t .
$t_{(p,df)}$	Critical value of t-distribution with significance level p and df degrees of freedom.
T	Maturity date of an option or contract, expressed as the total number of time steps.
Tax	Corporate tax rate.
V_t	Vector containing deterministic parameters in Johansen procedure.
Vol	Volume of a rock formation.
ω	Parameter reflecting weighted long-term volatility in GARCH model.
W_t	Wiener process at time t .
w_i	Weighting variable with indicator i .
WD	Weighting variable for diesel oil price
WF	Weighting variable for fuel oil price.
X	Prespecified strike price of an option.
X_t	Strike price of an option at time t .
$\bar{x}$	Average return over a certain amount of time.
x_t	Logarithmic return of an asset at time t .
$x_{e,t}$	Logarithmic return of electricity spot price at time t .
$x_{g,t}$	Logarithmic return of gas spot price at time t .
x_j	Payoff occurring only in state j .
$\gamma_{i,j}$	Binary variable which has a value of 1 if $i = j$ and 0 if $i \neq j$.
y	Output of artificial neural network.
Y_t	Vector describing n time series.
z_n	Variable in Hurst test, describing the summed deviation from the mean up to point n .
$Z(x_t)$	Point t in joint distribution of return x_t .
Z_{it}	Vector in Johansen test.

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Chapter 1

Introduction

The energy industry is becoming increasingly complex and uncertain. Operators in the industry face more difficult investment decisions, calling for advanced and flexible decision making tools. In this study the application of real option analysis is researched, which structures real world investment decisions in a way similar to financial options. To bring real option analysis into practice, in this study we construct an option model for an investment problem in the field of Gas-to-Wire production.

We provide a background of the historical, present and future state of the Dutch energy industry. Energy producers have to deal with developments in legislation, markets, technology and resources. The involved uncertainties may present profitable opportunities, but more than ever require quick and adequate responses to changing conditions. From this perspective, it is interesting to consider the role that real option analysis could play in future decision making. The purpose of this study is to increase insight in the role real option analysis could have in the valuation of projects, particularly in the energy industry. We formally state our research goals in a number of research questions. Finally, we explain the methodology we use for this study.

1.1. Background

At the time of writing this thesis, the Dutch energy market was going through a series of important developments. As a consequence, exploiters of gas fields in the Netherlands face uncertainties, opportunities and problems not encountered before. Decision making in such an environment is difficult, requiring support from advanced decision tools. Such a decision tool is real option analysis. By modelling real investment problems in an option framework, it explicitly addresses managerial flexibility with respect to changing and uncertain market conditions. In this study we focus on the application of real option valuation in the energy industry.

The subsurface of the Netherlands is rich in natural gas. The Groningen gas field is the largest field in all of Europe and is in the top ten of largest fields worldwide (Correljé *et al.*, 2003). Dozens of medium-sized gas fields also contribute to the amount of gas significantly. The Netherlands have made heavy use of this natural resource, having met the bulk of energy requirements with natural gas for the last decades. The exploitation of gas fields has been a major source of income for the Dutch state. Historically the large Groningen gas field helped providing all the natural gas demanded in the Netherlands. After decades of production its supply capacity has decreased significantly. Soon, the field will no longer be able to cover the gap between demand and the supply of other Dutch gas fields. To maintain the balancing function of the Groningen gas field as long as possible, the government stimulates the development of other gas fields in the Netherlands with protection and fiscal measures. The Groningen gas field holds natural gas which contains relatively little hydrocarbons, making it a low-caloric gas. Other Dutch gas fields often contain high-caloric gas. The main transport network and many applications are fitted to the composition of the Groningen gas, posing difficulties when the percentage of production from other fields increases (Energiekeuze, 2011). Energie Beheer Nederland (financial government partner participating in all gas field exploits) has set the goal to produce 30 billion m³ of natural gas from fields other than the Groningen field in 2030, the so-called 30/30 ambition.

Not only in the Netherlands, but also globally, fossil fuel sources suffer from strong depletion. In particular the mass-scale exploitation of oil has notable effects; we approach the point the demand for oil will exceed supply permanently. To be able to still meet energy demand in the future more sources need to be found. Hydrocarbon resources that were previously unattractive economically (e.g., oil/gas fields at sea and in difficultly accessible formations) are exploited now or will be in the near future, when production of the easier accessible fields is declining. Technological developments as well as market conditions are important for the exploitation of such fields.

Another major trend is the transition to renewable energy. To stimulate such developments, producers of renewable energy have a preferred position in the energy market; they are stimulated fiscally and the energy they sell has priority over conventional energy. However, renewable energy sources often have an intermittent output. To ensure that demand can be met at all times, natural gas has an important balancing function due to its flexible and fast production opportunities. As such, an increasing share of renewable energy affects the role of natural gas (Roadmap 2050, 2011). European energy markets were previously largely government-controlled, since about a decade ago the European Union has been striving for an integrated and liberalised energy market. The Dutch energy market has been open since 2004, yet competition is still quite underdeveloped. It is uncertain how liberalisation of the energy market will develop and what effects this will have.

Under these circumstances companies seek innovative methods to develop fields with low economic attractiveness on the first view, such as marginal or almost depleted fields. A possible

manner to exploit such fields is Gas-to-Wire production, which is the generation of electricity from natural gas by placing a motor or turbine at the field itself. Laying pipelines and compressing natural gas is then not required. Relatively high investments are required for Gas-to-Wire, therefore the profitability of such a project strongly depends on correct responses to uncertainties. In this study, we will assess the application of real option valuation on Gas-to-Wire production.

1.2. Research purposes

Real option analysis is a valuation technique based on financial option theory, treating real world projects as if they were financial assets. It distinguishes itself from traditional techniques by explicitly incorporating managerial flexibility during the project and the effect of altering the risk profile due to decisions made. The main goal of this study is to increase understanding of real option valuation and the possible merits it has regarding the valuation of flexibility compared to those of other decision tools. We perform an extensive literature study to assess the different views on real options.

A central issue in real option theory is the distinction between market and private risk; as options are valued under the assumption that the risk of price changes in the underlying asset can be hedged, in principle only risks that are liquidly traded on the market are eligible for option valuation. We address the presence of non-hedgeable (private) risk in real projects, and how such uncertainties can be treated in a real option structure. Another core aspect we research is the adjustment of discount rates to the changing risk profile of a real option. We describe how cash flows can be adjusted properly for risk, explaining how risk-neutral valuation can be applied for this purpose.

We illustrate the application of real option valuation with a simple real option model applied to Gas-to-Wire production, determining the economically optimal point to switch from gas production to electricity generation. The construction of this model shows how we can deal with the theoretic issues in a practical setting. The main goal of constructing the model is to compare the insights it offers compared to traditional decision making tools. We refrain from drawing strong conclusions about the real-world attractiveness of Gas-to-Wire; reliable data for this innovative production method is limited, and results are strongly influenced by project-specific factors.

The value of the real option at a given point in time partially depends on the prices of natural gas and electricity respectively. We try to increase insight to the nature of the behaviour of these price series. Issues that we consider are the theoretical behaviour of the series, diagnostic testing on historical price series, modelling techniques used in commodity pricing and estimating the risk-neutral drift. For the option model presented in this study we attempt to create realistic price models, without attempting to optimally reflect the observed behaviour.

1.3. Research questions

The main goal of the study is to answer in which respects real option analysis can provide additional value compared to the *state-of-the-art* decision tools that are applied in practice. We pose the main research question here:

1.3.1. Main question

How does the insight in the value of flexibility stemming from real option analysis on Gas-to-Wire production compare to the insight obtained from dynamic decision tree analysis?

1.3.2. Sub-questions

We pose seven sub-questions which together answer the main question. Table 1 provides an overview of the chapters in which these sub-questions are treated. To make the comparison between ROA and DTA, we must first specify what definition of ROA we use. For this, we address two core issues for which no trivial answers are available. Theoretically, private risks are not viable for option pricing; we study several viewpoints on how ROA can deal with such risks (question 1). The other issue is how we can adjust the discount rate towards the changing risk profile of a project (question 2). Answering these two sub-questions lays the groundwork of our real option model. To apply ROA on a Gas-to-Wire scenario, we must work out a structure for the investment problem. Our focus lies on the behaviour of the price series of natural gas and electricity (question 3), and how these characteristics can be modelled. We also consider aspects on the technical side of production for both regular gas production and Gas-to-Wire (question 4), but do so in less detail. After identifying the separate processes, we see how we can combine them in a real option model (question 5). We test how the real option model distinguishes itself by comparing it with a similarly structured decision tree model. We run several scenarios with both models, so that we can compare their results (question 6). Finally, we aim to provide some useful suggestions for further research, which could further increase the insights gained from ROA (question 7).

- 1) *How can we address the presence of private risk in a project in a real option framework?*
- 2) *How can we account for the changing risk profile of a flexible project in discounting?*
- 3) *How can we model the behaviour of price series for natural gas and electricity?*
- 4) *What are the technical characteristics of gas- and Gas-to-Wire production?*
- 5) *How can we construct a real option for valuing Gas-to-Wire production?*
- 6) *How do the results of real option valuation compare to dynamic decision tree analysis with respect to the value of flexibility?*
- 7) *What further research can be performed to increase insight in the application of real option valuation in practical settings?*

Sub-question	Chapter	Subject
1	Chapter 3	Real option theory
2	Chapter 3	Real option theory
3	Chapter 4	Introduction to price behaviour of energy commodities
	Chapter 5	Diagnostic testing of historical data
	Chapter 6	Description of modelling techniques
	Chapter 7	Modelling and estimation of price models
4	Chapter 4	General issues of Gas-to-Wire option
	Chapter 8	Model-specific issues of Gas-to-Wire option
5	Chapter 8	Construction
6	Chapter 9	Simulation setup and results
	Chapter 10	Conclusions
7	Chapter 11	Suggestions for further research

Table 1: Overview of the treatment of sub-questions per subject and chapter.

1.4. Workflow

In this section we describe the chronological steps we take in this study. The first step of the project is to increase insight in how the Dutch gas and electricity markets work. This includes a description of the physical production processes, legislation and the main parties involved. As the exploitation of a gas field can take multiple decades, it is important to have an insight in both

current and future developments in the energy market. The main information sources we use are standard works and internal sources within the Petroleum Geosciences department of TNO, complemented with information made publicly available by the main parties in the market.

A great body of literature on real option analysis exists. Different schools of real option theory co-exist, each with their own view on what real options are and how they should be applied. We try to distinguish between these approaches and their implications, without attempting to explicitly compare them. Instead, we deduce the most suitable approach by testing their fits with option pricing theory. For this purpose, we pay special attention to the concept of risk-neutral valuation in option pricing, replicating portfolios and the distinction between market and private risk.

The behaviour of commodity price series is known to be notably different from that of financial stocks. In particular electricity prices follow a unique and complex pattern. We assess literature concerning the characteristics of these price series to gain more insight in their behaviour. Based on this information, we perform several diagnostic tests to check whether the actual behaviour of historical price data is consistent with theory. For each test we provide a theoretical introduction. We investigate a number of techniques in price series modelling. These techniques are used as building blocks of the eventual price series model. Based on the results of the diagnostic tests and the characteristics of the techniques assessed, we build price models for natural gas and electricity. We estimate their parameters based on the available price data.

After assessing the theoretical issues we perform research on the characteristics of a Gas-to-Wire project in a real option framework. Notable issues that we treat are the uncertainty of the reservoir size, physical production constraints and the required investments. We combine the price series models and several other components with models on well productivity and reservoir size into a real option model. We also build a dynamic decision tree on the same investment problem, to compare the performance and differences of both valuation techniques. Afterwards, we perform scenario to test for several developments we deem uncertain. The study is completed with a conclusion and recommendations for further research. These focus on both the application of real option analysis in general and the performance and possible improvements of the Gas-to-Wire model.

Chapter 2

Introduction to natural gas and energy in the Netherlands

To provide a background on the natural gas industry, we give a brief introduction in this chapter. First we discuss the physical nature of natural gas and its main applications. Then we give an overview of natural gas in the Netherlands, describing its importance in a historic, present and future perspective. In particular, we pay attention to the role of the major Groningen gas field. Also we explain the production process of natural gas, with a focus on exploitation in the Netherlands.

After describing the role of natural gas in the Netherlands in general, we go in more detail about the Dutch energy market. We describe the market structure for both natural gas and electricity. We pay attention to the transition from a national, government-controlled market to an integrated and liberalised European energy market, and the issues that play a role in this transition. Also we discuss some important trends, such as the transition to renewable energy and the preservation policy of the Groningen gas field.

We describe the main parties involved in both the natural gas and electricity market, linking their roles in flowcharts. In a separate section, we explain what Gas-to-Wire production is and the position it has in the energy market. Finally, we explain the pricing processes for gas and electricity. We describe how prices are established, what characteristics they have and how future developments might affect prices.

2.1. Properties of natural gas

Natural gas, as referred to in this study and as found in the Netherlands, is a gas mixture which consists mainly of several hydrocarbon gases (particularly methane) and some other gases (NaturalGas.org, 2011a). It is a combustible gas mixture; its high energetic value makes it a useful energy source. Natural gas can be converted into electricity, but it is also used directly by end-users. Some of its main applications by end-users are heating of buildings, water heating and cooking. Finally, natural gas is also used as a feedstock in the chemical industry. Natural gas is a relatively clean resource, causing less pollution than other fossil fuels (Eurogas, 2007; United States Environmental Protection Agency, 2007). Burning natural gas emits only minor amounts of soot, and releases far less carbon dioxide into the atmosphere than other fossil energy resources. When speaking in the context of energy sources, natural gas is often simply dubbed gas. The terms ‘gas’ and ‘natural gas’ are used interchangeably in this study.

Natural gas is found beneath the surface, accumulated in porous geological formations which lie under a denser layer of rock (NaturalGas.org, 2011a). To release the gas, a hole must be drilled through this layer of rock, allowing the gas to reach the surface. As the gas is usually under high pressure, it flows up by itself through the well. After being retrieved, the gas is led through a purification process in order to make the mixture feasible for transportation and usage. The composition of natural gas depends on its origin. In Table 2, we show the components usually present in natural gas and compare them to those present in Groningen gas (Van Thillo, 2008). The precise composition determines the energetic value of the gas (NaturalGas.org, 2011a). The Wobbe index relates the relative density of gas to its caloric value, and serves as an indicator for the interchangeability of fuel gases. Natural gas must be processed in such a way it can be offered in a certain composition. If the gas mixtures differ too much in quality, this would pose problems for the installations relying on natural gas. In practice this means the gas in a distribution system must always have a Wobbe index within a certain margin.

Component	Percentage Typical composition	Percentage Groningen gas
Methane	70-90 %	81.2 %
Higher hydrocarbons	0-20 %	3.6 %
Carbon dioxide	0-8 %	0.9 %
Oxygen	0-0.2 %	0 %
Nitrogen	0-5 %	14.3 %
Hydrogen sulphide	0-5 %	0 %
Rare gases	Trace	Trace

Table 2: Typical composition of natural gas and composition of Groningen gas.

2.2. Natural gas in the Netherlands

In 1959, a major gas field was discovered in the province of Groningen (Correljé *et al.*, 2003). As one of the largest gas fields in the world, it had the potential to supply the country with energy for decades. Upon its discovery the Dutch state decided to construct a nationwide gas transport network to make use of this newfound resource (Van Overbeeke, 2001; GasTerra, 2008). Nowadays most buildings in the Netherlands are heated using natural gas, accounting for 78% of the direct natural gas consumption by Dutch households (Jeeninga, 1997; Energiewereld.nl, 2009). The remaining consumption consists of heating water and cooking. Beside the large direct consumption of natural gas, it is also the main source for generating electricity in the Netherlands. According to the Dutch Central Bureau of Statistics (CBS), in 2010 62.3% of the total amount of electricity generated in the Netherlands stemmed from natural gas (Centraal Bureau voor de Statistiek, 2011). In Table 3 the shares of energy sources in electricity generation are provided.

For the Dutch government, the exploitation of natural gas has been a major source of income, stemming from direct income, profit taxes and royalties on natural gas (Correljé *et al.*, 2003).

Aside from the large share natural gas has in electricity generation, natural gas is also of importance to guarantee a stable electricity output towards end-users (Tröster *et al.*, 2011). Nuclear plants and coal-fired power stations are inflexible in responding to changes in short-term demand. Renewable energy sources are dependent on varying weather conditions such as wind and sunlight, and therefore provide an intermittent output. Furthermore, it is difficult, energy-inefficient and expensive to store an overproduction of electricity (He, 2007). In order to respond to changing demand patterns quickly, the generation of electricity from gas is a necessity. Electricity can quickly be generated from natural gas by undergoing a relatively simple process. Storage facilities and the gas transport system itself allow storing large amounts of gas for fast electricity generation when required.

Source	% of Dutch electricity generation
Natural gas	62.3 %
Coal	18.5 %
Other fossil fuels	3.7 %
Nuclear energy	3.4 %
Renewable energy	9.4 %
Other sources	2.7 %

Table 3: Percentage of Dutch electricity generation by energy source (CBS, 2010).

Several chemicals can be made from natural gas, which in turn are used to create products such as plastics, plywood, paints etc. Also natural gas becomes increasingly important as a fuel source for cars. In Table 4 we categorise the worldwide use of methanol, the main raw material produced from natural gas (Wesseling *et al.*, 1991).

Source	% of methanol application
Methanol	36 %
Raw material to produce esters	22 %
Solvents and diverse products	22 %
Acetic acid	11 %
Fuel	9 %

Table 4: Percentage of worldwide applications of methanol (Wesseling *et al.*, 1991).

With the increasing collaboration between member states of the European Union, the role of the Dutch gas industry has become increasingly important from a European perspective as well. The Dutch gas reserves are the biggest of the European Union. The Netherlands currently export about two-thirds of their produced gas (Centraal Bureau voor de Statistiek, 2012). However, by the time of 2025 the government expects the Netherlands to be a net importer of gas (Rijksoverheid.nl, 2012b). The government plans for the Netherlands to function as ‘gas intersection’ in the future, using the Dutch transport network and storage capacity to play a central role in importing and exporting gas in the northwest of Europe (Van der Hoeven, 2009a).

2.3. Natural gas production in the Netherlands

The first step in discovering natural gas fields is seismological research. Gas fields in the Netherlands are usually found between 2000 and 4000 meters beneath the surface (Total E&P, 2012). By generating artificial vibrations and analysing the returning echoes, geophysicists can map the geological structure of a certain area. This is an important input to establish the probability of the presence of a gas field. If a high probability of a substantial volume of natural gas within a

certain structure is interpreted, an exploration well must be drilled to prove that gas is actually present (NaturalGas.org, 2011b). With an exploration well a more accurate estimate of the reservoir's gas volume and gas quality can be obtained (Gas-Initially-In-Place: GIIP). In case the discovered natural gas reserve can be produced in an economically feasible manner, production wells are drilled on the site. Some processing of the gas is performed directly at the wellhead (NaturalGas.org, 2011c). Water and particulate solids (e.g., sand, salts) are removed on the spot, as these could damage or cloak the pipelines. Sand and other solid particles are removed by using scrubbers. Associated water is removed by a dehydration process.

After the initial filtering of the retrieved natural gas, it is transported to a production facility by making use of low-pressure pipelines. Often these facilities are located on or nearby the field. In the facility heavier hydrocarbons, such as ethane, propane and butane are condensed in a separator by changing the temperature and pressure, so that they can be retrieved as individual commodities (NaturalGas.org, 2011c). These separate end-products are called natural gas liquids or NGLs (Energy Information Administration, 2006). If hydrogen sulphide, carbon dioxide or other acidic gases are present, they must be removed from the mixture. If sulphides are obtained, they can be used for the production of sulphur. Finally, traces of other matters such as helium and mercury should also be separated from the gas mixture. Once the gas has been processed to a mixture with a certain percentage of methane (roughly 80% for the low-caloric network and about 90% for the high-caloric network (Hoezoandergas, 2012)) it is compressed, so that it can be transported through the high-pressure transport network (NaturalGas.org, 2011e). The Nederlandse Gasunie requires a pressure of 66 to 80 bar when receiving the gas on their transport network (Gasunie, 2008). Varying the pressure allows to store less or more gas in the network depending on demand. This storage method is called line-packing. More dense regional distribution networks connect the transport network to the end-users.

The demand for natural gas is significantly higher during winters, because of its application to heating (Aalbers *et al.*, 2007; NaturalGas.org, 2011d). Also within smaller time frames demand patterns can be observed, for example differences between day and night. Produced gas can be stored when there is no immediate demand. The transport network itself provides an amount of storage capacity due to its high pressure. Further, underground reservoirs are used to store gas. In general, more gas than demanded is produced from the primary gas fields in the summer, while in the winter demand exceeds production (Nederlandse Aardolie Maatschappij, 2011). This allows meeting peak demand during cold days when production capacity is insufficient. The pressure of a gas field decreases when it is gradually depleted. A compressor or injection well may be required to increase the pressure of the produced gas artificially in order to meet the required pressure of the transport network, this results in higher marginal costs (Energeia, 2011). Also the location and size of a gas field determine the costs involved. Each field requires wells to be drilled, the placement of installations and connecting the site to the gas distribution network. Therefore, the amount of gas and the prevailing gas price must be sufficient to make the extraction of gas from a small field economically feasible. Exploiting gas fields on sea requires higher investments than on land. The profitable extraction of smaller gas fields is strongly dependent on the technologies available (TNO, 2008). Most gas fields have a lifetime between ten and thirty years. At the end of production, the installations should be dismantled and the environment should be brought back into its original state (Shell, 2006). Safety measures must be taken to ensure that no remaining gas escapes to the surface or to another subsurface formation (Barclay *et al.*, 2002). After inspection and approval of the measures taken the site is returned to its original owner.

2.4. Characteristics of the Dutch energy market

The Dutch energy market has quite a complex structure with interaction between the private and public domain. Since the Electricity Act (1998) and Gas Act (2000) were introduced in the Netherlands, the energy market shifted from a government-controlled market to a more liberal one (Nederlandse Mededingingsautoriteit, 2012d). This liberalisation was in compliance with European Union guidelines, seeking to reach an open European energy market in the long term. Since 2004 Dutch consumers have been free to decide which energy company provides them with gas and electricity, while the companies can set their own prices (Rijksoverheid.nl, 2012d). Despite attempts to create an open market, several factors give cause for continued government intervention on the market. We briefly describe some of the main factors in this section.

2.4.1. Importance of energy to society

Energy is a driving factor for a nation's society and economy. It is therefore important that a reliable and stable energy supply exists (Ministerie van Economische Zaken, Landbouw & Innovatie, 2011). The governmental policy-makers have to consider multiple factors to guarantee such a supply. There are certain risks involved when importing energy from politically unstable countries, relying on inflexible energy sources, making use of energy sources with unpredictable output and so on. Also some long-term visions would likely not be taken into consideration in a truly liberal market (Correljé *et al.*, 2003). The government therefore retains a central role in the energy market as a policy maker.

2.4.2. Monopoly positions

Though the number of energy companies has increased in recent years, the traditional ones retain a powerful position in the market (Europa.eu, 2007; Sia Partners, 2011). Energy companies fall under the supervision of the Dutch competition regulator (Nederlandse Mededingingsautoriteit or NMa) to prevent abuse of their position and to ensure that the energy market functions properly. For example, the regulator obliges companies to present sufficient information to end-users (Nederlandse Mededingingsautoriteit, 2012d). Another monopolistic aspect of the energy market is that the same transport- and distribution networks are used for all gas and electricity. It would be unfeasible if every energy company would have to construct its own network. As a consequence, the transport- and distribution networks are in the hands of so-called 'natural monopolists'. These parties are subject to regulation as well.

2.4.3. Stimulating exploitation of small gas fields

Searching for and exploiting gas fields requires large investments, and is always paired with uncertainty of the amount of gas that can be retrieved. To stimulate producers to discover and exploit small gas fields, the government obliges the (partially state-owned) gas trading company GasTerra to buy all natural gas produced for a fair price and under reasonable conditions (Rijksoverheid.nl, 2012c). One of the reasons for this 'small fields policy' is to slow down the exploitation of the Groningen gas field. The term 'small fields' refers to all gas fields other than the Groningen gas field; the actual field size may be substantial. The policy helps to preserve the function of the Groningen gas field as swing producer and long-term natural gas reserve (Correljé *et al.*, 2003).

2.4.4. Transition to renewable energy

As the reserves of fossil fuels are expected to be depleted during the following decades, other sources of energy must be assessed to meet energy demand in the future. In addition, the European Union has set goals to strongly reduce carbon dioxide emissions (Ministerie van Economische Zaken, Landbouw & Innovatie, 2011). For these reasons, a large-scale transition from the current energy sources to renewable energy is planned. The government intervenes to stimulate

the development and use of renewable energy sources, enabling them to compete with the cheaper fossil fuels.

2.4.5. Licensing of gas producers

Producers must obtain a license from the government if they wish to explore a certain area for gas, exploit a gas field or store gas (Rijksoverheid.nl, 2012c). As searching for and exploiting gas fields are activities that may have consequences for the environment and the safety of the nearby-living residents, gas producers are not allowed to do so without formal approval of the Dutch state. For each license application, a study is performed to assess the consequences of proposed activities.

2.4.6. Licensing of energy companies

As energy companies have a responsibility towards society, they are subject to regulation and must obtain a license before entering the market. Some of the requirements an energy company has to meet are financial stability, providing clear information to end-users and offering reasonable payment arrangements before shutting down the energy supply to an end-user (Nederlandse Mededingingsautoriteit, 2012d).

2.5. Parties in the Dutch energy market

2.5.1. Gasunie

The N.V. Nederlandse Gasunie, often referred to simply as Gasunie, is a state-owned company which possesses and manages the Dutch main gas transport network. Their core tasks are building and maintaining the gas transport network, and the transportation and storage of natural gas (Gasunie.nl, 2012). Tasks related to transporting gas are performed by Gas Transport Services, a 100% subsidiary company of Gasunie. Gasunie operates two main transport networks, one for high-caloric gas (HC) and one for low-caloric gas (LC) (Correljé *et al.*, 2003; Rijksoverheid.nl, 2012a). The natural gas from the Groningen gas field is LC, other Dutch fields provide HC. LC is delivered to households and exported to other countries, while HC is used by large industrial parties and for generating electricity by energy companies. Gas Transport Services can convert HC to LC by adding nitrogen. Conversion the other way around is merely an administrative swap: LC is not physically turned into HC. Besides owning the Dutch gas transport network, Gasunie also (co-)owns several networks in other European countries. In these countries, the activities of Gasunie are subject to the regulations of the respective countries.

2.5.2. Regional gas transmission system operators

There are several regional parties which own and control the gas distribution network in a certain area (Rijksoverheid.nl, 2012a). These regional networks have a high density, require a low gas pressure (some 8 bar), and distribute gas from the main transport network to the end-users. The law obliges these parties to be legally independent from the energy companies (Gaslicht.com, 2011). As some energy companies in the past also owned regional distribution networks, they had to place these networks in hands of independent entities. The Dutch state has a majority share in all these parties. Gas Transport Services charges a transport fee to regional gas transmission system operators. As most gas is produced in Groningen, this transport fee depends on the distance from their gas field, hence transportation becomes increasingly more expensive the farther away from Groningen. The transmission system operators recharge this fee to end-users.

2.5.3. GasTerra

GasTerra is a former part of the Nederlandse Gasunie. In 2005, it was split off from the Gasunie, as European guidelines prescribe that the transport and sale of gas should be carried out by

separate entities (GasTerra, 2012b). Gasunie retained the task of managing the transport network, while GasTerra took over the task to trade in natural gas. While the Gasunie is completely owned by the Dutch state, only half of the shares of GasTerra remained in the hands of the state (10% directly, 40% via Energie Beheer Nederland). The other shares are owned by Royal Dutch Shell (25%) and ExxonMobil (25%) (GasTerra, 2012a). GasTerra buys natural gas directly from the producers, consequently selling it to energy companies, industrial users or exporting it to other countries. They are obliged by law (specifically, Article 54 of the 2000 Gas Act) to procure gas from small field producers against a reasonable price (Van der Hoeven, 2009b; Energie Beheer Nederland, 2012b). GasTerra has had a fixed annual profit of 36 million euros for years; they correct their contractual prices at the end of the year to obtain this profit (GasTerra, 2009).

2.5.4. Other gas traders

In addition to GasTerra, several other gas traders (shippers) are active in the Netherlands. These traders only entered the market recently as a result of the liberalisation; GasTerra is still the major gas trader in the Netherlands (Van der Hoeven, 2010). Unlike GasTerra, other traders have no obligation to procure gas obtained from small fields. All traders must obtain a license before being allowed to trade in natural gas.

2.5.5. Nederlandse Aardolie Maatschappij

The Nederlandse Aardolie Maatschappij (NAM) is the largest producer of natural gas in the Netherlands, producing about 75% of Dutch gas (Nederlandse Aardolie Maatschappij, 2012). The company is owned by Shell (50%) and ExxonMobil (50%). NAM is the single exploiter of the Groningen gas field (in partnership with EBN), and as such operates the largest natural gas reserve of the Netherlands. Half of the gas they produce stems from the Groningen gas field, 25% comes from smaller fields in the Netherlands, and 25% is produced from fields in the North Sea.

2.5.6. Other E&P-operators

Besides NAM, there are several other parties active in the discovery and exploitation of gas fields in the Netherlands. Such parties are called Exploration & Production-operators (E&P-operators or simply operators). As the Groningen gas field is exploited solely by NAM, the remaining operators are active on other Dutch gas fields lying both on- and offshore.

2.5.7. Energie Beheer Nederland

Energie Beheer Nederland B.V. (EBN) is a company owned by the Dutch state, functioning as a partner in the discovery and exploitation of natural gas. EBN is not involved in these activities as an operator, but rather as a financial partner. EBN also facilitates Dutch E&P-operators with its expertise and knowledge (Energie Beheer Nederland, 2012a). When an operator obtains a licence allowing it to explore for natural gas in a certain area, by law it can request EBN to participate in the exploration process (Energie Beheer Nederland, 2012b). In this case, EBN will obtain a 40% share in the participation, aiding the operator in the searching process. They are subject to the same risk-and-reward profile as the operator. By splitting the costs, the producer has a lower financial threshold for starting the exploration process. After an operator has acquired a production license, EBN usually participates on a 40% basis as well. Unlike the exploration process, the operator has no say in this; only the government can decide to withhold EBN from participation (Energie Beheer Nederland, 2012b). The operator and EBN again are subject to the same risk-and-reward profiles. In case EBN did not participate in the exploration process, 40% of the costs for this process will be reimbursed. Besides their participation in a large number of producing gas fields, EBN has an advisory role towards the policy makers of the government as well. Further EBN owns a 40% share in GasTerra, and therefore has an influence in the trade of natural gas.

2.5.8. Energy companies

Energy companies sell gas and/or electricity to end-users. The companies purchase natural gas from gas traders, electricity is bought from an intermediate trading party (Programme Responsibility Party), which is often owned by the energy company itself (Antwoordvoorbedrijven.nl, 2012). Energy companies operating in the Netherlands fall under the regulation of the NMa, but are free to set their own consumer prices. In recent years, new energy companies have entered the market, increasing the competition in the energy market. Most energy companies own installations to generate electricity, but electricity can be procured from other producers or from abroad as well. When an energy company sells renewable energy without owning installations to produce it, they must purchase certificates representing a share in renewable energy production. The multiple roles of energy companies make it ambiguous which role is referred to when speaking of an energy company. Therefore, in this study we only use the term 'energy company' when referring to its role as supplier of gas and electricity to end-users connected to the distribution network, such as households and small industrial users. For its other roles, we use the terms 'electricity producer' and 'Programme Responsibility Partner' (trader) respectively.

2.5.9. TenneT

TenneT TSO B.V. (hereafter TenneT) manages the Dutch high-voltage network (TenneT, 2008). It can be seen as the counterpart of Gasunie responsible for the electricity transport network, operating in a highly similar structure. TenneT is completely owned by the Dutch state (TenneT, 2012). It is responsible for the safe and effective transportation of electricity throughout the Netherlands.

2.5.10. Regional electricity transmission system operators

Low-voltage regional distribution networks are connected with the high-voltage network of TenneT, allowing the distribution of electricity to households and other end-users. These regional networks are owned and managed by several parties. The Dutch state has a majority share in all transmission system operators. Energy companies are no longer allowed to own a regional distribution network. The transportation costs recharged to end-users are the same for all regions in the Netherlands (Gaslicht.com, 2011).

2.5.11. Electricity producers

Electricity can be produced from several sources, all requiring a specific installation. In many cases, these installations are owned or partially owned by energy companies, but independent producers exist as well. A legal distinction is made between regular electricity and renewable electricity. When energy companies sell renewable electricity to consumers, they do not necessarily own such installations themselves. However, they need to purchase certificates from a producer of renewable energy, to prove that the electricity sold stems from a renewable energy source (Wetten.overheid.nl, 2012).

2.5.12. Programme Responsibility Parties

In the Dutch energy market, electricity is traded by an intermediate party between producers and energy companies, named a Programme Responsibility Party or PRP (TenneT, 2011). These parties make transactions for the supply of electricity. By reporting these transactions to TenneT on a daily basis, TenneT can measure and settle the difference between the actual and the transacted amount of electricity. PRPs must obtain a license from TenneT in order to be active as a trader. There are two types of PRPs: parties that are only allowed to trade, and parties that are responsible for the connections with the distribution network as well. Most energy companies have their own PRP to purchase electricity (Antwoordvoorbedrijven.nl, 2012).

2.5.13. Nederlandse Mededingingsautoriteit Energiekamer

The Nederlandse Mededingingsautoriteit, abbreviated to NMa, is the Dutch competition regulator. Its 'Energiekamer' (Energy department) is the department of NMa responsible for the regulation of the Dutch energy market (Nederlandse Mededingingsautoriteit, 2012a). Its tasks are described in the Electricity Act and Gas Act. Among its core tasks are the licensing of parties active in the energy market, setting tariffs, evaluating the effectiveness of the transport and distribution networks, and monitoring the developments in the energy market (Nederlandse Mededingingsautoriteit, 2012c). The NMa Energiekamer has an influence on all parties involved in the energy market, from the initial discovery processes to the final supply of electricity and gas to end-users. The goals of the regulator are to protect consumers, stimulate an open energy market and ensure that government policies are carried out.

2.6. Value chain of the Dutch energy market

Having described the main parties involved in the Dutch energy market in the previous section, a brief overview will be provided of how these parties are linked together. In Figure 1 we provide an overview of the Dutch gas market. About 75% of Dutch natural gas is produced by the NAM and 25% by other operators. EBN has a stake in virtually all gas exploitations. Produced gas can be sold to GasTerra or other gas traders; recall that GasTerra is obliged to purchase gas from small fields against a fair price. Natural gas is delivered to one of the two main transport networks (owned and managed by Gasunie) under high pressure: one with high-caloric gas, one with low-caloric gas. These transport networks also allow for storage. From the main transport network, LC is distributed further to end-users by means of low-pressure regional distribution systems, owned and controlled by regional operators. Natural gas consumed by end-users is traded via energy companies. LC is also exported to other countries directly via the high-pressure network. HC can be sold to energy producers, who subsequently generate electricity from the gas. Also HC can be directly delivered to large industrial users. The HC transport network allows for export to other countries as well.

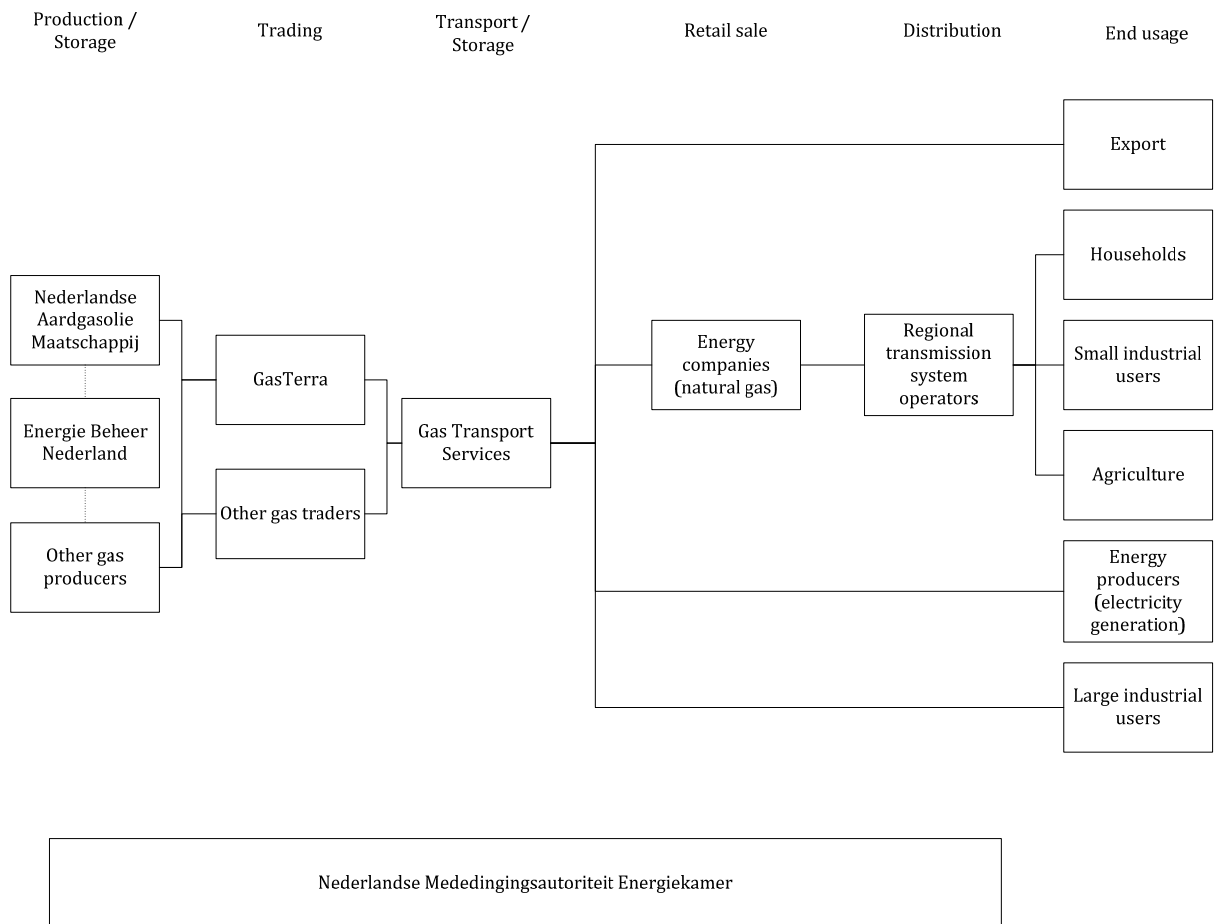


Figure 1: Value chain of the Dutch natural gas market.

In Figure 2 we provide an overview of the Dutch electricity market. The structure of the Dutch electricity market is quite similar to that of the gas market. A major difference however is the absence of storage facilities, so that a constant balance between all parties must be established. As a consequence, the actual behaviour of gas and electricity markets is quite different, despite their comparable market structures. Electricity is produced from several sources, with a relevant distinction made between renewable and non-renewable sources. Programme Responsibility Parties buy the produced electricity on a daily basis, consequently selling it to energy companies, industrial users or other countries. TenneT is responsible for the transportation of electricity over the high-voltage network, after which regional operators transfer it to end-users using a denser low-voltage network.

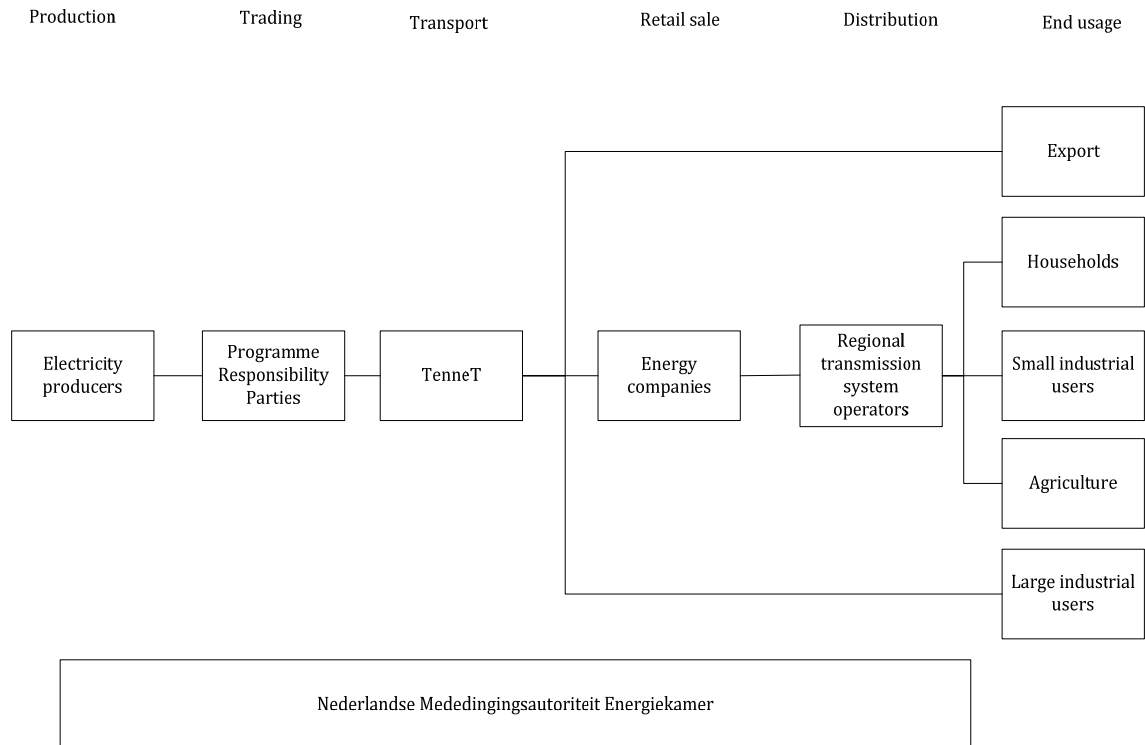


Figure 2: Value chain of the Dutch electricity market.

2.7. Gas-to-Wire

Gas-to-Wire is a term used to describe the process of generating electricity from natural gas at or close to the source (Thomas & Dawe, 2003). This is considered a different production method than the generation of electricity from gas at a centralized power plant. Gas-to-Wire places a gas motor or a gas turbine close to the gas field, allowing to convert gas into electricity directly. Generally the more efficient but heavier gas motors are placed on land, while the less efficient gas turbines are used at sea because of their lower weight which needs a lighter supporting structure. From here on, a gas motor or –turbine will be referred to as ‘generator’.

Gas-to-Wire is a potential alternative to regular gas production for exploiting smaller fields in particular, which may not be worth the investments required to connect to the main gas transport network (Van den Berg, 2011; ABTechnology, 2012). Correljé *et al.* (2003) claim that the economic feasibility of exploiting a minor gas field is often dependent on the distance to the transport network. It might be possible to exploit a marginal gas field which is not economically feasible with regular exploitation (called ‘virgin fields’), or a gas field which is an advanced stage of depletion (also called a ‘tail-end field’). The electricity generated can be sold to energy companies, but also be used to power facilities close by, for example an installation at sea.

The short distance between the gas field and motor or turbine does not allow for storing large amounts of gas, therefore managing the gas supply to the generator is one of the difficult features of Gas-to-Wire production (Van den Berg, 2011). The generator is able to produce electricity in a flexible manner and is able to deal with monotone changes in gas supply on short notice. However, technical problems occur when disturbing pulses such as impurities in the gas are present in the gas supply.

According to Thomas & Dawe (2003), installing a pipeline connecting to the gas transport network and a wiring connecting to the electricity grid are almost equally expensive when the distance is the same. It is therefore important to consider the nearest distance to the main electric-

ity network and the main gas network when determining whether Gas-to-Wire is more profitable than regular gas production.

2.8. Energy trading in the Dutch energy market

2.8.1. Gas trading

When natural gas consumption in the Netherlands started, the price of natural gas was indexed to the oil price or that of oil-based products. The reason for this is that gas could be priced just below the price of oil equivalents, thereby stimulating the use of natural gas. This relationship still exists, however not as directly. The spot market for gas has become increasingly important since the liberalisation of the market (Clingendael International Energy Programme, 2008; Lewis, 2010), also pricing formulas referring to oil prices allow for deviation of the gas price. When active on the spot market, an operator can halt production or store gas while waiting for a favourable moment to sell.

The Title Transfer Facility (TTF) is a virtual trading hub for natural gas in the Netherlands, which was established by the Gasunie in 2002. The TTF is facilitated by the Dutch energy exchange APX-ENDEX, with APX responsible for short-term contracts and ENDEX for long-term contracts (Vlam & Custers, 2010). Prices on the TTF are determined by supply and demand (Gas Transport Services, 2012). Several contract lengths are available on the TTF, for example a day ahead, a month ahead, a quarter ahead and a year ahead. The total volume of gas present in the GTS transport network at a certain moment can be traded as many times as wanted, therefore the amount of gas traded at the TTF normally exceeds the physical volume by a factor called the 'churn factor'. With a churn factor of 3.66 in 2007, the TTF was considered a moderately liquid market (European Energy Regulators, 2007).

As largest trader in natural gas, GasTerra offers varying types of contracts next to the TTF. These contracts have prices indexed to crude oil or oil products, coupled to weights which determine the influence of these prices on the gas price (von Bannisseht, 2008b; GasTerra, 2012c). These formulas are notated in the form [*number of months over which the average price is taken – number of months between averaging period and contract period – number of months for which the average price is valid*]. The structure of a 6-2-6 contract is illustrated in Figure 3.

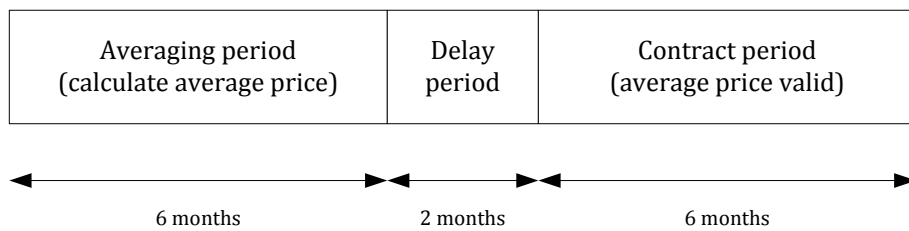


Figure 3: Structure of a 6-2-6 oil-indexed gas contract.

The pricing formula applied can be valid for one or multiple years. GasTerra also offers the possibility of fixed prices for the duration of a year, gas contracts based on the TTF price, and in some cases specific price arrangements. For large users, two oil-based price formulas can be used; the 3-0-3 and the 6-0-3 method. These contracts set a price for three months, based on the average fuel oil and diesel oil prices for the past three and six months respectively. The weighting variables for these prices can be adjusted by GasTerra. The following formula is applied:

$$S_g = c + WF * FO + WD * DO \quad (2.1)$$

with

S_g	as the gas price
c	as a fixed component
DO	as the average diesel oil price
FO	as the average fuel oil price
WD	as the weighting variable for diesel oil
WF	as the weighting variable for fuel oil

For suppliers to small users, the 6-2-6 method is applied. With this method the average fuel oil price over six months is taken, and set as contract price for the next six months with a delay of two months. The regional charge includes a fee which comprises the majority of the price. The following formula is used (Kofman & Ophuis, 2012):

$$S_g = \frac{22.3}{500} * FO + \text{Regional charge} \quad (2.2)$$

with

FO as the average fuel oil price

According to Aalten (2010), the constant and the weighting variables in the formulas GasTerra applies to determine prices are modified in such a way that they coincide almost perfectly with the TTF prices. This observation is in line with GasTerra's intention to abandon the direct price link to oil (Yamoah, 2007; Von Bannisseht, 2008a). The rationale behind this intention is that, since the liberalisation of the energy market, GasTerra cannot allow its prices to deviate much from the gas prices established at the exchange (Lomme, 2008). This would result in arbitrage opportunities and disturb the market.

2.8.2. Electricity trading

Electricity cannot be stored easily after it has been generated. When electricity is generated and cannot be used right away, it can be converted to another type of energy, allowing to regenerate electricity when required. Examples of this are storing electricity in a battery, or pumping water into a reservoir. Such conversions come at a cost, while a significant amount of energy is lost during the process as well (Evans & Guthrie, 2007). Therefore the industry continuously seeks to match supply to the energy demand as well as possible (Escribano *et al.*, 2002). The demand pattern for electricity is highly variable. Electricity prices on the spot market are set by the hour, or even on shorter time intervals. Electricity is traded on the Dutch energy exchange APX-ENDEX, comprising both spot contracts and future contracts. The spot market includes day-ahead contracts, agreeing to physically deliver electricity the next day against the specified price. Spot prices are set via a bidding procedure that involves minimum and maximum prices set by APX-ENDEX (Verkuyl *et al.*, 2005). A significant amount of electricity trading takes place outside the APX-ENDEX as well. The German energy exchange EEX, as the leading energy exchange in Central Europe, also plays a role of note in the Netherlands. The Netherlands are a net importer of electricity, with the shortage of Dutch production often covered via EEX import (Armstrong *et al.*, 2004). Furthermore, there is a specific balancing market for correcting short-term imbalances in electricity supply.

The variance of APX returns has shown a decreasing trend over the past years. Also large price jumps have occurred less. A possible explanation for this trend is the liberalisation and international integration of the energy market, allowing to strike a better balance between supply and demand (A. Huygen, personal communication, 30 March 2012). Another recent development is the ability to influence demand to some degree, enabling a better balance as well. At the same

time, there are also reasons to believe that variance will increase in the future. The increasing share of renewable energy makes the electricity production less predictable, the resulting imbalances could have a strengthening effect on variance in returns.

We can make a distinction between base-load and peak-load power plants (Cordaro & New York Affordable Reliable Electricity Alliance, 2008). Base-load power plants generate electricity at a constant rate. When chosen rationally, base-load power plants are able to generate electricity at the lowest marginal costs. Physical constraints play a role as well; not all power plants are able to adapt their output in a flexible and/or efficient way. Energy companies are contractually bound to meet customer demand. When base-load production falls short to meet demand, peak-load plants are therefore activated to fill the gap between supply and demand (He, 2007). The amount of time a peak-load plant is running can vary strongly (depending on their marginal costs and flexibility), from producing on a daily basis to only several hours a year. When demand increases prices rise, which can render power plants with higher marginal costs economically feasible. However, meeting demand has priority over maximising marginal profit from peak-plant production. When production is unprofitable, only the demanded output is provided. When production is profitable, the plant could produce at maximum capacity and sell the excess production on the spot market.

Chapter 3

Real option theory

The previous chapter provided a background regarding the state of the Dutch energy market. It is now time to introduce the concepts of real option theory. Chapter 3 starts with an overview of traditional valuation methods. We point out which shortcomings these methods have, and how real options address them. The rationale behind real option analysis is illustrated by explaining how the famous Black-Scholes option pricing model can be applied to real world projects. We relax the strict assumptions of this model later on.

As option pricing theory relies on risk-neutral valuation, we provide an explanation of this concept. Option pricing theory only applies to liquidly traded financial instruments. Therefore it should be possible to construct a replicating portfolio of financial instruments equivalent to the value and risk profile of the real project. We describe several approaches on this subject. Due to the inability to hedge against non-traded risks, real option theory distinguishes between private and market risk. We dedicate a section on how different types of uncertainty can be dealt with.

Finally we treat various issues in real option pricing, especially where it differs from standard financial options. Volatility is noted to be particularly hard to estimate in real options when having multiple sources of uncertainty. Co-dependencies may exist in the investment problem, notably between the size of the project and the investments required. Other issues considered are the effect of competition, the absence of a fixed maturity date, and suboptimal decision making.

3.1. Traditional valuation methods

A large number of valuation methods exists to quantify the attractiveness of investments. In this section we will discuss the main valuation methods and their properties briefly, to allow for comparison with the real option methodology explained later in this chapter. Drury (2008) identifies four valuation methods widely applied in practice, namely the Accounting Rate of Return, the payback method, the discounted cash flow analysis, and the Internal Rate of Return.

The Accounting Rate of Return (ARR) divides the estimated annual profit by the average investment (the initial investment minus final salvage value), thereby obtaining the expected return of the project. The payback method calculates the time required to earn back an initial investment. The most commonly used valuation methods are based on discounted cash flow (DCF) techniques. Using such methods, the cash flows during the lifetime of the project are identified and consequently discounted at a rate reflecting both the time value of money and the riskiness of the project. Net present value (NPV) or traditional DCF analysis assumes that future cash flows are deterministic, as soon as the production decision is made. To reflect both the time value and the riskiness of the project, a constant discount rate is applied to future cash flows. Usually the Weighted Average Cost of Capital (WACC) of the firm is used as discount rate. The discounted sum of cash flows is the present value of the project. If this value is bigger than 0, then it is a signal to accept the project. The Internal Rate of Return (IRR) applies discounting on the cash flows, seeking the discount rate which sets the discounted benefits equal to the required investment. Stated otherwise, it provides the maximum discount rate which provides a nonnegative project value. Often both NPV and IRR are applied to obtain both a realistic value of the project and an upper bound discount rate.

The traditional valuation methods have some important shortcomings. The ARR and the payback method do not take into account the time value of money and the riskiness of the project, yet these properties are important to investors. The payback method also does not provide an expected return, making it unsuitable as a standalone tool to base investment decisions on. A flaw in IRR is that multiple solutions are available when positive and negative cash flows alternate, with only one solution being economically relevant. Traditional DCF assumes that decisions are irreversible, with new information getting available at a later time not altering the cash flows or intermediate decisions made. This is often not realistic. For example, when no gas is discovered in a certain area, it would make no sense to build a production facility. Yet such decisions can be made in traditional DCF (Prasanna Venkatesan, 2005), making this method unfit for projects with embedded flexibilities.

Compared to the traditional methods discussed before, a more realistic approach is (static) Decision Tree Analysis (DTA). Project options are defined at decision nodes to allow managerial flexibility, while other nodes reflect uncertain events with certain probabilities assigned. Future cash flows are discounted, so DTA can be seen as an enhanced version of DCF (Piesse *et al.*, 2004). A flaw in this approach is that the decisions made over time alter the risk profile of the project, which conflicts with applying a single risk-adjusted discount rate used to calculate the present value (Brandão *et al.*, 2005). For example, the learning phase of a project could be less risky than the production phase. Investors will expect to receive compensation in line with the degree of risk they are exposed to. Section 3.2 goes in more detail about the issue of discounting.

In advanced applications of valuation methods, often a combination of DTA, Monte Carlo simulation and decision optimisation algorithms is applied (Smith & McCardle, 1999). The decision tree serves as an input of state variables for the simulation, allowing to model both decisions and uncertain events. Using these state variables, at each endpoint in the tree simulation can be

performed, with uncertainties being modelled stochastically. Algorithms can be used on the simulated paths to optimise the production decisions made given the information available at that time. A constant discount rate is applied to all cash flows; usually sensitivity analysis is performed to assess the effect of altering the discount rate. In this study we refer to such an approach as *dynamic* DTA. Some authors adopt a broad view on real options, which is discussed in Section 3.3. They state that the inclusion of managerial flexibility is sufficient to regard a valuation method as real option valuation, meaning that the description above could also be viewed as a real option approach (Dias, 2012a). Others strictly consider a real option as an application of classic option pricing. Table 5 provides a brief summary of the valuation methods discussed in this section.

Method	Calculation	Shortcomings
Accounting Rate of Return (ARR)	Average annual profit divided by average investment.	Does not value time, risk and investment size.
Payback method	Time required to earn back initial investment.	Does not value time, risk and investment size, provides no expected return.
Discounted Cash Flow (DCF)/ Net Present Value (NPV)	Project cash flows discounted with a rate reflecting both time value and risk.	Does not incorporate the flexibility to respond to new information.
Internal Rate of Return (IRR)	Maximum discount rate which provides a nonnegative return.	Does not incorporate the flexibility to respond to new information, provides no expected return.
Decision Tree Analysis (DTA)	Extension of DCF incorporating decisions and/or uncertainties.	Does not adjust the discount rate when the risk profile of the project changes.

Table 5: Summary of traditional decision tools and their main shortcomings.

3.2. Discount rate

The DCF and DTA methods require the estimation of a discount rate. This factor has a great impact on the NPV of a long-term project. In this section we address the theoretical background of discount rates, which is useful both to point out the theoretical flaw in DTA and its implications for risk adjustment. A discount rate should incorporate both the time value of money and compensation for uncertainty of future cash flows (Robichek & Myers, 1966). Unfortunately, it is difficult to obtain a discount rate which is able to accurately reflect all the risks the project is subject to (Mun, 2002). To mention some, a project's value can be influenced by inflation, the size of the company, credit risk, country risk, shareholder decisions, etc. Many random events can occur during the lifetime of a project, making it hard to derive a proper discount rate analytically. The most commonly used discount rate is the Weighted Average Cost of Capital (WACC). This is the average cost of capital for the company. In its basic form the WACC assumes that a company is funded with one source of equity and one source of debt, both demanding a single constant return. In practice companies can raise money from multiple sources with different expected returns (e.g., preferred stocks, warrants etc.), making the calculation of WACC more expansive. Note that interest costs are deducted from corporate profits, hence the inclusion of corporate tax in the equation.

$$WACC = \left(\frac{E}{E + D} * r_e \right) + \left(\frac{D}{E + D} * r_d \right) * (1 - Tax) \quad (3.1)$$

with

E	as the market value of equity
D	as the market value of debt
r_e	as the cost of equity
r_d	as the cost of debt
Tax	as the corporate tax rate

A company must at least earn the WACC to satisfy owners, stock holders and creditors. If a company fails to do so, rational investors would not be willing to invest. Hence, a project discounted at the WACC should at least have an NPV of 0 in order to be attractive to the capital providers. However, the project may have a different risk profile than the company as a whole, meaning that investors would require a different return (Mun, 2002; Smith, 2005). Even if the project is not funded separately, the project will alter the overall risk profile of the company. By estimating the correlation of the project with market risk, the discount rate can be adjusted better to a specific project (Constantinides, 1978; Magni, 2009).

To calculate the cost of equity models such as the Capital Asset Pricing Model (CAPM) are applied. We explain the CAPM in this section for its general applicability and minimal data requirement. Note that the CAPM is not necessarily the most accurate model to estimate the return required by equity holders. Other models, such as the three-factor model by Fama & French (1992), take into account more factors which influence return. Empirical evidence shows that such models provide more explanatory power on returns of diversified portfolios (Mun, 2002). The CAPM states that the expected return of an asset is equal to the risk-free rate plus a market risk premium depending on the relationship between the volatility of the asset's return and that of the market return (Sharpe, 1964; Merton, 1973b). The underlying reasoning of the model is that investors only care about the systemic risk (related to the movements of the market as a whole) of the asset, as all other risks can be diversified away. Diversification means that non-systemic risks are offset by holding many uncorrelated assets in a portfolio. The expected return under the CAPM is denoted as

$$r_e = r_f + \beta(E(r_m) - r_f) \quad (3.2)$$

with

r_e	as the cost of equity
r_m	as the market return
r_f	as the risk-free interest rate
β	as the correlation between variances of the asset return and market return

In mathematical form, the beta is described as $\beta = cov(x, r_m) / var(r_m)$. It can be viewed as a number describing the volatility of the asset relative to the volatility of the market. In other words, the beta is a measure for part of the asset's riskiness that cannot be removed through diversification. The risk premium for the asset is given by the term $\beta(E(r_m) - r_f)$.

Ang & Liu (2004) state that an appropriate discount rate for a project depends on the market return, the risk-free rate and the beta of the project. In contrast to the CAPM assumptions, these are all variable over time in reality. By incorporating stochastic forecasting models on these factors, we could obtain a more accurate present value (Geltner & Mei, 1995; Schulmerich,

2010). For traditional DCF, the use of a constant discount rate is often rationalised by assuming that the portfolio investment opportunity and the systemic risk exposure (i.e., the beta) remain constant over time (Merton, 1973b; Fama & Schwert, 1997). For decision trees and real options these assumptions are less valid, as embedded flexibilities changes the nature of risk (Trigeorgis, 1996).

As calculating a discount rate which accurately captures the uncertainties related to a specific project can be difficult, an alternative often used is to adopt a discount rate known to be applied to a similar project or a set of similar ones. Also management can subjectively set an internal hurdle rate, representing a minimal return required on a project in order to be accepted (Rutherford, 2001). Determining the discount rate in such a way ignores the preferences of the market, making the subjective approach flawed from a theoretical perspective.

Summary of discounting in project valuation

- In project valuation, future cash flows are discounted at a rate reflecting both time preference and uncertainties.
- The most common discount rate used is the WACC, which is used as a measure for the minimum return required by investors.
- The return required by shareholders is often calculated with the CAPM or a comparable model.

3.3. Introduction to real option analysis

Real option analysis (ROA) is a method to value real world projects by modelling decisions in an option framework. Its application is based on the option theory used to value options on financial assets (Luenberger, 1998). In finance, a standard option is the right, but not the obligation, to buy (call option) or sell (put option) an asset at a predefined strike price. This allows the holder of the option to defer the investment decision up to a certain date, waiting for new market information (i.e., the asset price) to arrive. A rational holder of an option will only exercise the option if the asset price exceeds the predefined strike price at the decision point. In real options, the term 'asset' should be viewed in a broad sense. It is the value of the project, should it be undertaken. If the option is not exercised before maturity, the investor loses the cost of the option itself. 'Classic' ROA uses an approach similar to that of financial options. When the underlying risk of a project behaves as if it is traded, we can apply option pricing theory on real investment decisions. Two conditions required to apply the theory are that the uncertainty associated with the project is market risk (we will treat different sources of uncertainty later on) and that the decision maker has the managerial flexibility to make investment decisions based on new information. As such, a real option explicitly incorporates managerial flexibility.

Real options address several aspects that are ignored in traditional decision making tools (Triantis & Borison, 2001; Van de Putte, 2003). A real option values flexibility because it includes the possibility to alter the course of the project at the decision points in order to maximise profit or minimise losses given the information available at that time (Copeland & Keenan, 1988; Mun, 2002; Brandão *et al.*, 2005). More specifically, a project is only undertaken if the NPV exceeds 0. Another distinction is that it adjusts the discount rate to the risk profile, which differs for the decision paths in a project with flexibility. Note that decision trees are able to model flexibility as well, but do not adjust the corresponding discount rates.

We will illustrate the analogy between financial options and real options by the Black-Scholes option pricing model, which is an application of risk-neutral pricing under strict assumptions (Black & Scholes, 1973). Despite the similarities, it should be noted the Black-Scholes formula is based on assumptions fitting financial options better than real options. Merton (1998), one of the founders of option pricing theory, warned against the application of option theory to real world problems. He stressed to consider the limitations of the model, and keep in mind what purpose it serves. The main limitations and assumptions of classic real option pricing will be assessed in detail later in this chapter, along with different approaches to deal with them. The Black-Scholes formula can be used to obtain the value of a European call or put option. The major assumptions of the Black-Scholes model are the following.

- No arbitrage opportunities exist.
- Cash can be borrowed and lent at a constant risk-free interest rate.
- Buying and short-selling of the underlying asset is unrestricted.
- No transaction costs exist.
- The underlying asset's price follows a lognormal distribution.
- The underlying asset does not pay dividends.

Under these assumptions we can create a hedged position, so that the value of the portfolio does not depend on the price of the underlying asset. We do this by constructing a portfolio consisting of the option, the underlying and cash (including negative amounts due to short-selling), so that price changes of the asset are offset by the other instruments. It is then possible to apply risk-neutral valuation.

Translated to real options, a call option is the possibility to undertake a project; a put option is the possibility to abandon it. The formulas below are modified versions of the original Black-Scholes formula for the value of call and put options at time t , including the effect of continuous dividend payments as well.

$$call = S_t e^{-\delta(T-t)} N(d_1) - X e^{-r_f(T-t)} N(d_2) \quad (3.3)$$

$$put = X e^{-r_f(T-t)} N(-d_2) - S_t e^{-\delta(T-t)} N(-d_1) \quad (3.4)$$

with

$N(\cdot)$ as the cumulative distribution function of a standard normal distribution, having an average of 0 and a standard deviation of 1 ,

$$d_1 = \frac{\ln\left(\frac{S_t}{X}\right) + (r_f - \delta + 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}},$$

$$d_2 = \frac{\ln\left(\frac{S_t}{X}\right) + (r_f - \delta - 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}}.$$

The meanings of the symbols in both financial and real options are provided in Table 6 (Leslie & Michaels, 1997).

Symbol	In financial options	In real options
X	Strike price	Present value of required expenditures to exercise the option
S_t	Stock price	Present value of expected net cash flows at t
t	Current time	Current time
T	Time to expiry	Time that decision is deferred
σ	Volatility of stock price	Volatility of present value of expected cash flows
r_f	Risk-free interest rate	Risk-free interest rate
δ	Fixed cash dividends	Costs to preserve the option

Table 6: Legend of symbols of Black-Scholes model in financial and real options.

In real options S_t represents the present value at time t of the expected net cash flows, should the option be exercised. The strike price X describes the present value of the expenditures required to exercise the option (Carlsson & Fullér, 2003). These costs are only incurred when the option is actually exercised, such as the costs to acquire an asset (call option) or to abandon a project (put option). The volatility σ is defined as the square root of the variance of the project returns, based on the free cash flows. Returns are assumed to follow a Geometric Brownian Motion (i.e., normally distributed and unrelated over time, standard deviation remains constant). The option value increases with volatility. This is because an option holder profits from favourable movements of the value of the underlying, while downside risk is limited to losing the option value.

In line with option pricing theory, cash flows in ROA are discounted using the risk-free interest rate r_f . This is the (theoretical) return required when an investment has no possibility of default. Before discounting, we apply risk-neutral probabilities to calculate the expected cash flows. Several issues in real options make the application of risk-neutral valuation less natural than for financial options. Generally project-specific risks cannot be hedged; higher risk calls for a higher discount rate. Also companies, unlike financial institutes, are generally unable to borrow at or near the risk-free rate.

For real options, 'dividends' δ represent the costs to preserve the option, or the money draining away during the lifetime of the option (Leslie & Michaels, 1997). Examples are payments to preserve production rights and money lost through competition. In practice, it is difficult to forecast and estimate the leakage of cash over the length of the option. Also these losses are generally not constant over time (Trigeorgis, 1996). Many real option practitioners therefore act as if no dividend payments exist (Bodén & Åhlén, 2007).

Flexibilities embedded in a project are usually not captured by a simple European call or put option. The project might comprise exercise- and abandonment decisions at different time points, multiple investment opportunities, strike prices variable over time, time-varying volatility, etc. (Trigeorgis, 1993a; Mun, 2002). Consequently, no analytical solutions can be found frequently. Instead, numerical approaches such as a binomial tree or simulation are required (Cortazar, 2000; Wood, 2007; Matsumoto *et al.*, 2010). These methods approximate the analytical

value by dividing the partial differentials of the option in many steps. Having multiple options on a project could be considered as a portfolio of options. In general, the value of such a portfolio will be nonadditive due to interdependencies between the options. This means that the total value of flexibility is different from the sum of individual option values; they can be sub- as well as superadditive (Trigeorgis, 1993b; Trigeorgis, 1996).

Real options are best suited for projects with large uncertainty and the managerial flexibility to respond to this uncertainty (Van de Putte, 2005; Kodukula & Papudesu, 2006). The value of flexibility is greatest when the NPV of the project without flexibility is close to 0, so that decisions are more likely to be taken during the course of the option. For decisions which are obviously good or bad beforehand, flexibility provides little additional value. Also when management does not have the opportunity to react to uncertainty, ROA is not the most suitable choice for valuation. The application of option pricing theory on real world projects was recognized not long after its introduction (Myers, 1977). However, the field of real option analysis is still very much in development. Existing literature is often not explicit in its approach, also assumptions and conditions tend to vary (Borison, 2003; Sáenz-Diez & Gimeno, 2008). Application in practice remains limited (K. Huisman, personal communication, 4 May 2012). For these reasons, there is no readily applicable framework for real option valuation.

Summary of real option theory

- Real option theory is based on an analogy between investment decisions and financial options, modelling business decisions as the right to make an investment.
- Real options explicitly address the value of flexibility in projects where management can flexibly respond to new information becoming available over time.
- The Black-Scholes assumptions are generally less valid for real projects than for financial assets.
- Often analytical solutions are not available for real options due to their complexity.

3.4. Risk-neutral valuation

A core concept in option pricing is risk-neutral valuation. This technique is widely applied to obtain the value of derivatives¹ by both academics and financial practitioners (Appeddu *et al.*, 2012). The Black-Scholes model is an application of risk-neutral valuation under strict assumptions. This section focuses on the main principles of risk-neutral valuation, providing an intuitive insight why risk-neutral valuation is used. For the mathematical properties of risk-neutral valuation, sources such as Luenberger (1998) and Bingham & Kiesel (2004) can be addressed.

To calculate the present value of an asset, one could take the expected return of an asset, and then discount it based on the preferences of the investor. However, it is difficult to estimate the future growth rate of an asset's value. Risk-neutral valuation provides a methodology which does not require estimating this rate (Miller & Park, 2002). The application of risk-neutral valuation requires two major assumptions. First, the market must be complete. This means that every good can be exchanged by any participant in the market without transaction costs (Merton, 1973a; Constantinides, 1978). Every agent has perfect market information, so no trader has an advantage through knowledge. Also short-selling and borrowing are unrestricted and can be

¹ A derivative is a financial instrument that derives its value from that of an underlying asset. All forms of options fall under the term 'derivative'.

done at the risk-free rate. Secondly, arbitrage opportunities are absent; there are no imbalances in the market which allow for the possibility of a risk-free profit at zero cost. When these assumptions hold, a derivative can be replicated by holding a linearly weighted combination of securities (Gisiger, 2010). As arbitrage opportunities do not exist, this linear combination must have the same value as the derivative. If this were not the case, an investor could buy the cheaper of the two and sell the more expensive one, thus making a risk-free profit without a cost (Tilley, 1992). Risk-neutral valuation provides the unique arbitrage-free price of the derivative based on this principle. It does so by using the artificial concept of risk-neutral probabilities. We provide an example based on Gisiger (2010) to explain this concept.

Suppose the economy can be in one of n states at time t , with a specific state denoted as j (in reality an infinite number of possible states may exist). For every state a unique so-called Arrow security is available, which pays off a positive amount x_j to the holder of the security when the asset reaches state j and zero otherwise. The real probability that the asset state will shift from an arbitrary state i to state j is denoted by p_{ij} , with the sum of probabilities being equal to 1. We do not assume interest yet. Each security has a price representing the value the market places on this state. This means that the price of the security needs not to be equal to $p_{ij} * x_j$. For example, a security paying off in a certain state could be perceived as a more valuable addition to one's portfolio, therefore being priced higher than its rationally expected payoff. The discrete payoff structure described here is illustrated in Figure 4.

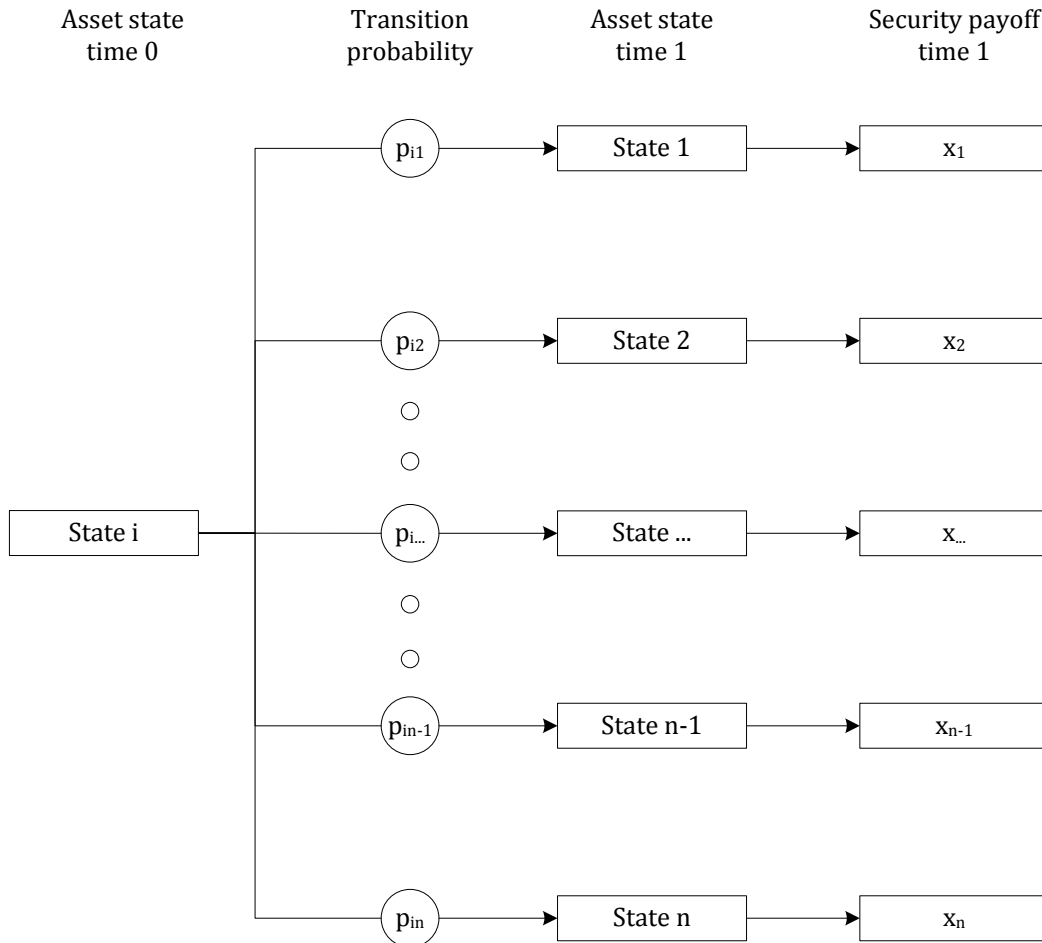


Figure 4: Example of discrete payoff structure in a fictive economy described by n Arrow securities (based on Gisiger, 2010).

Now we introduce a derivative, which returns the payoff of the security matching state j . This derivative can be considered as a portfolio of all Arrow securities. We price this derivative by using risk-neutral valuation, describing its value as θ_i . Contrary to what its name may indicate, risk-neutral valuation does not assume investors to be risk-neutral. All risk preferences of the market are incorporated in the actual security prices. As such, the real price of a security does not need to be consistent with the real probability of the asset ending up in the state j . Risk-neutral probabilities can be viewed as the sum of state prices compounded to 1; we denote these probabilities as a_{ij} . Multiplying each risk-neutral probability with the corresponding payoff x in state j provides the value of the derivative. Note that the real probabilities p_{ij} are not required for this; their information is incorporated in the security prices.

$$\theta_i = \sum_{j=1}^n a_{ij} * x_j \quad (3.5)$$

So far we assumed that an investor is indifferent between receiving money at time 0 or at a later time. In reality, money has a time value due to time preference of people (they prefer money now over money at a later point in time), leading to the existence of interest. If we would not discount future cash flows, an investor could short-sell the complete set of securities and use the received sum to purchase a risk-free bond, earning the risk-free rate as the price of the securities remain constant. As such, the investor could make a risk-free profit without cost, which contradicts the no-arbitrage assumption. Thus, future payoffs should be discounted at the risk-free rate r_f to obtain the arbitrage-free price of today. The discounted state prices at time 0 then sum up to $\frac{1}{1+r_f}$ instead of 1, we denote these new probabilities as a_{ij}^* . We describe the time 1 risk-neutral probabilities by q_{ij} , compounding the probabilities a_{ij}^* at the risk-free rate r_f to obtain a present-time probability measure summing up to 1. Introducing time value leads to the following set of equations:

$$q_{ij} = a_{ij}^* * (1 + r_f) \quad \forall i, j \quad (3.6)$$

$$\sum_{j=1}^n q_{ij} = 1 \quad (3.7)$$

$$\theta_i = \frac{1}{1 + r_f} \sum_{j=1}^n q_{ij} * x_j \quad (3.8)$$

From these equations it follows that under risk-neutral valuation the *expected* prices grow at the risk-free rate. This is a powerful concept, as it is no longer required to estimate the actual growth rate. The drift of the asset value is effectively removed, instead replacing it with the risk-free rate (Kat, 1998). The only parameter left to estimate is then the volatility. By for example filling in the Black-Scholes pricing formula, the implied volatility can be obtained.

In a complete market, risk-neutral valuation and valuation under perfect delta hedging (offsetting the effect of price movements with a combination of financial instruments held simultaneously, see Section 3.5) provide the same derivative price. A perfectly hedged portfolio is riskless and as such must provide a risk-free return. Delta hedging and risk-neutral valuation are therefore mathematically equivalent. This helps understanding why the rather artificial risk-neutral valuation principle also applies to the real world when hedging is possible. If the assumption of a complete market does not hold, the risk-neutral probabilities are not unique. As the derivative

in that case cannot be fully replicated by holding securities, no single arbitrage-free price can be obtained for the derivative (Gisiger, 2010). Instead the value of the derivative will lie between some lower and upper bounds. When calculating the risk-neutral value of an option, uncertainties which cannot be hedged are therefore theoretically not viable for risk-neutral valuation (Smith & Nau, 1995). Also other assumptions of the complete market often do not hold in practice. Such issues are sometimes addressed by assuming the market is *approximately* complete. In real option settings, the incompleteness of the market may be too substantial for such an assumption to hold. Sections 3.5 and 3.6 go in more detail about treating non-hedgeable risks.

Though the assumptions of risk-neutral valuation may sound strong, they need not to be more restrictive than those for DCF. Generally future cash flows are discounted with the WACC, for which the cost of equity is often calculated using the widely applied Capital Asset Pricing Model. The assumptions for the CAPM and risk-neutral valuation are the same (Birge & Zhang, 1998; Cudica, 2012). Thus, accepting the DCF methodology implicitly means that we accept the assumptions for risk-neutral valuation as well.

Summary of risk-neutral valuation

- Risk-neutral valuation is a technique to estimate the arbitrage-free price of a derivative in a complete market.
- It allows replacing the actual drift of an asset price by the risk-free rate, no longer requiring to estimate this parameter.
- The assumptions for risk-neutral valuation are strong, but often implicitly adopted in DCF as well.

3.5. Replicating portfolio concept

Risk-neutral pricing presumes a perfect hedge can be constructed for the portfolio held. If this is to hold in a real option setting, it is possible to construct a replicating portfolio for the project. In its most rigid form (classic ROA), this means that the project is replicated by a portfolio consisting of market-driven instruments that is exactly equivalent (Brennan & Schwartz, 1985). Hence, holding a portfolio consisting of financial instruments (e.g., cash, assets, derivatives) should provide the exact same payoff as the project itself at all times and in all states. If this is possible, we can also construct a perfect hedging portfolio by mirroring the replicating portfolio (short-selling may be required for this), allowing to apply risk-neutral valuation.

To retain the equivalence between the real project and the replicating portfolio, (continuous) adjustment of the portfolio might be required. We can do this under the assumption that no transaction costs exist in a complete market (Tilley, 1992). In reality, transaction costs are of course involved in trading. Constantinides (1986) justifies the assumption of no transaction costs by stating that the existence of transaction costs does not significantly alter the asset proportions held compared to the theoretical proportions. However, others argue the rigid complete market assumption in fact does significantly affect the validity of the theory (Mayshar, 1981; Haug & Taleb, 2011). At the very least, we should keep in mind the absence of transaction costs when applying the model (Merton, 1987). Another deviation from the complete market observed in practice is the presence of arbitrary opportunities arising due to market imperfections. Such opportunities tend to be quickly corrected by the market itself (Tham, 2001).

The replicating portfolio concept assumes that the holder is able to borrow and lend at the risk-free rate; generally only financial institutes are able to do so, or at least approach this rate. The risk-free rate is often estimated as the yield on a government bond with a very low default probability. However, there are no investments which guarantee a return with absolute certainty. The yield on government bonds varies over time. As financial options usually mature within months, it is often assumed that the rate remains constant. The longer the time to maturity, the weaker this assumption becomes. At the moment of writing this thesis interest rates are at a very low level, meaning that under risk-neutral valuation prices are expected to grow slowly.

In reality a replicating portfolio is rarely found for a real project, we further address this issue in Section 3.6. Where some real option authors strictly hold on to the requirement of being able to construct a perfectly replicating portfolio, other authors have loosened this principle. They assume that a replicating portfolio can also be derived by subjectively estimating the market value of the project (Amram & Kulatilanka, 2002; Brealey *et al.*, 2008). They justify the subjectively derived asset value by adopting a shareholder view. Valuation assesses how much a project contributes to the value of the firm, thereby considering the project itself as if it were a traded asset (Borison, 2003). They value the project with traditional DCF (hence without incorporating flexibility) to obtain a subjective estimate of the market value of the twin security. Some authors deem this value to be the best unbiased estimate of the project (we call this assumption Market Asset Disclaimer or MAD), and is consistent with the assumptions of DCF. Although the underlying of a real option is generally not liquidly traded, we may chose to treat it as if it were a financial asset. The rationale is that we seek the arbitrage-free value of the project, as this is comparable to the added value of the project to the market value of the company (Benaroch & Kaufmann, 1999). Wrongly valuing the project would eventually result in arbitrage opportunities which are corrected by the market. Although the subjective- or MAD approaches are not as restrictive as the classic approach, the assumption of market completeness remains intact.

Another real option approach is to consider the market to be partially complete (Smith & Nau, 1995; Smith, 2005). Arbitrage pricing is then explicitly considered to be applicable only on the part of the project that can be replicated with financial instruments. Subjective probabilities are applied on the part for which no portfolio can be constructed. A difficulty with this integrated approach is that it requires individually assessing each source of risk to see whether it can be replicated or not. In this study we adopt the integrated approach, mainly for its theoretical consistency. We expand on this and other approaches in Section 3.8, but first we need to address some other theoretical issues in ROA.

Summary of replicating portfolio concept

- To be able to apply risk-neutral pricing, it should be possible to construct a portfolio of market instruments which replicates the payoff of the project for all states and times.
- Some authors have loosened the replicating portfolio condition, stating that the project itself can be considered as a traded asset from a shareholders perspective. Others consider the market partially complete, and apply arbitrage pricing only on the part which can be replicated.
- In practice it is rarely possible to construct a perfectly replicating portfolio for a real world project. The degree to which this affects the validity of real option analysis is still debated.

3.6. Types of uncertainty

Following the replicating portfolio concept described in Section 3.5, in classic and integrated ROA we should make a distinction between uncertainties that can be replicated by financial instruments and uncertainties that cannot. Some different perspectives on types of uncertainty and how to deal with them in a real option framework are discussed in this section.

When assessing uncertainty in future cash flows, one can distinguish between market risk and private risk. Market risk is risk that can be replicated by financial instruments. Individual companies have no influence on it. Market information is revealed over time, thereby solving uncertainty. An example of market risk is the risk caused by changing commodity prices, and can be hedged by taking a position in these assets. Private risk is more complex to define, as it should contain all sources of uncertainty that cannot be replicated by financial instruments (Amram & Kulatilanka, 2002; Piesse *et al.*, 2004). Some authors only consider technological uncertainty. This is a source of uncertainty which stays the same when no learning actions are undertaken, such as determining the size of a gas field (Hooper III & Rutherford, 2001). Such definitions do not cover all non-replicable risks. A type of uncertainty like the weather does not fall under the definition of technological uncertainty (as we cannot resolve it by undertaking action) but also is no market risk (as it is not correlated with the economy). The weather would be viewed as private risk, unless a derivative exists on the market which *exactly* reflects the uncertainty associated with it. The reason for this is that a synthetic portfolio mimicking the project would then include this derivative, and as such can be hedged. As option theory assumes that perfect hedging is possible, only risks that can be hedged should be included in option pricing. The introduction of new types of derivatives allows more and more risks to be viewed as market risk. In practice it may therefore be difficult to clearly distinguish between types of risk.

An issue often not assessed in literature is that risk may be correlated with the market, but that no derivative exists (Kaufman & Mattar, 2002). Considering this risk to be uncorrelated would then be incorrect, but it is no market risk either. Smith (2005) argues to treat the correlated part of such risks as market risk and the uncorrelated part as private risk. Merton (1998) provides a formal definition of private risk, stating that private risk can be measured as the tracking error of the portfolio representing the underlying asset (Amram & Kulatilanka, 2002). Mathematically the tracking error can be defined by $dS_t/S_t - dP_t/P_t$, where S_t is the project value (the underlying) and P_t is the value of the tracking portfolio. Applying this definition, the difference between the value of the replicating portfolio and the value of the underlying asset is considered private risk. As such, we can measure private risk objectively.

In the gas industry, projects often comprise a large degree of private risks as well as market risks. Typically the size of the reserve is the main source of private risk, while the gas price is the main source of market risk. Two approaches are used to rationalise incorporating a certain degree of private risk in the classic approach. We may assume that private risk is only minor after the option has been exercised, and will not have a great impact on the payoff (i.e., the market is approximately complete). The tracking error then increases with the amount of private risk. The second approach is to include such uncertainties in the valuation process, but assume that they can be hedged as well. We might be able to diversify away private risk by trading it with comparable risks, even though these are not publicly traded (Mattar & Cheah, 2006). For example, the amount of gas present in a field is uncertain. If this risk could be traded with other fields subject to similar uncertainty, the risk could theoretically be traded and levelled out. In such a manner risks uncorrelated with the market could be hedged as well. Due to the absence of a liquid market for such contracts, the assumption that private risk can be hedged in a manner comparable to market risk is controversial, but it might apply to a certain degree.

Summary of types of uncertainty in real options

- Market risk is risk that can be replicated with financial instruments. Private risk can be defined as the tracking error between such a portfolio and the actual project.
- Classic option valuation depends on the assumption of a complete market, meaning that only risks traded in the market can be included. Other real option approaches relax these assumptions.
- Private risk may be assumed hedgeable or neglectable relative to market risk, but it increases the tracking error when applying only risk-neutral valuation.
- In practice, it can be difficult to distinguish risks due to correlation with the market. Many risks therefore lie somewhere between private risk and market risk.

3.7. Risk adjustment

An essential characteristic of ROA is that it adjusts the discount rate to the varying risk profile of the project. Two approaches are possible for this (Triantis & Borison, 2001; Mun, 2002; Arnold & Crack, 2004). First, the discount rate adjusted for the riskiness of the project phase can directly be applied to the estimated future cash flows. Second, the future cash flows can be adjusted for their risk, obtaining their certainty equivalent instead. In such a way a risk-neutral distribution is created, allowing for risk-neutral valuation by discounting the adjusted cash flows at the risk-free interest rate. Both methods provide the exact same outcomes when correctly applied, risk-neutral valuation is usually preferred due to its easier implementation (Birge & Zhang, 1998; Mun, 2002). As an option is a leveraged instrument, it has a more risky profile than the underlying asset (Cudica, 2012; Dias, 2012b). When working with real probabilities, the discount rate must therefore be consistent with the risk profile of the *option* to obtain the same value as with risk-neutral valuation.

Suppose that we have an asset which provides a variable return x . To obtain the risk-neutral value of an option, we should remove the risk premium from the actual expected growth rate (denoted as α) of this asset (Tilley, 1992; Trigeorgis 1993b). The risk-neutral drift of the underlying asset is the risk-free rate r_f in case the equilibrium return μ without paying dividends is earned (Cox & Ross, 1976). When the asset pays out constant dividends δ , we add these to the growth rate to obtain the equilibrium return (Trigeorgis, 1996). From the CAPM, it follows that the growth rate plus the dividend payment is equal to the risk-free interest rate plus the market risk premium. The individual terms need not to be equal, only their sum. The following relation can be described for the equilibrium return μ of the asset (Quigg, 1993; Dias, 2012b):

$$\mu = \alpha + \delta = r_f + \lambda \sqrt{\text{var}(x)} \quad (3.9)$$

The market price of risk λ is defined by the Sharpe ratio (Constantinides, 1978; Sharpe, 1994; Saénz-Diez & Gimeno, 2008):

$$\lambda = \frac{E(x - r_f)}{\sqrt{\text{var}(x - r_f)}} \quad (3.10)$$

From this definition, it follows that the Sharpe ratio in fact measures the excess return received for the extra volatility the investor is subject to compared to a risk-free investment. When r_f is constant this reduces to simply the volatility of the asset. The Sharpe ratio assumes that returns

are normally distributed, but when standard deviation is not too large the result can also be used for non-normally distributed returns. In reality investors care about skewness and kurtosis as well; statistics not reflected by the Sharpe ratio (Scott & Horvath, 1980; Sharpe, 1994). Further the Sharpe ratio implicitly adopts the CAPM assumptions, meaning that investors only care about non-diversifiable risk. As private risk is considered to be uncorrelated with the market, $\beta = 0$ and thus can be diversified away (Luenberger, 1998; Mattar & Cheah, 2006). Investors will then only require the risk-free return on private risk, even though the risk in itself may be substantial. Note that reducing private risk leads to better investment decisions, as such increasing value to investors. When adopting the viewpoint that private risk cannot be diversified away in practice, we could also discount private risks at a higher rate (Mattar & Cheah, 2006).

The risk-neutral cash flows E_Q can be obtained by deducting the market premium from the real cash flows E_P . This market premium can be substituted by the second term in the CAPM (Smith, 2005; Samis *et al.*, 2007). It follows that the certainty-equivalent cash flows can be defined as (Trigeorgis, 1996; Saénz-Diez & Gimeno, 2008):

$$E_Q(x_t) = E_P(x_t) - \lambda\sqrt{\text{var}(x)} = E_P(x_t) - \beta(E(r_m) - r_f) \quad (3.11)$$

After reducing the estimated cash flows with the market risk premium, we can discount the certainty-equivalent cash flows E_Q at the riskless interest rate, thereby obtaining the present value (Schwartz & Trigeorgis, 2001).

3.8. Integrated risk-neutral approach

As seen in Section 3.6, several approaches are available regarding handling private risk. We cannot point out a clear dominant view, as the manner chosen to deal with private risk depends on the vision of the modeller on real options. Borison (2003) distinguishes five real option approaches, which differ regarding their perspective on dealing with both types of risk. In Table 7 we provide a brief overview of the properties of these approaches. We only focus on their background, ignoring differences such as the techniques applied, etc. In this study we choose to work with the integrated approach, because we believe it has the strongest theoretical foundations when we are required to deal with both types of risk.

Cox *et al.* (1985) provide a description of a market model which we assume to hold for the real option approach we consider in this study. They adopt the viewpoint of a rational and well-diversified shareholder as described in the CAPM framework. This point of view complies with the theory of option valuation. Furthermore, the shareholder approach is in line with maximising the market value of the company, which we consider to be a rational objective for real option valuation. Shareholders agree with the subjective assessment of management of private risk, while management discounts these cash flows with the risk-free risk to reflect the risk-neutral position of shareholders towards private risk. Under these assumptions, the integrated method results in a single theoretically correct option price.

The integrated approach explicitly distinguishes between market risk and private risk, treating the first based on capital market information (i.e., risk-neutral valuation) and the latter based on subjective estimates. As such, we consider it as a combination of classic ROA and DTA. We believe that this view is theoretically most consistent when the model should be able to deal with both types of risk. Note that the approach reduces to pure option pricing or -decision analysis when only one type of risk is considered. Smith (2005) opts for a fully risk-neutral integrated approach on real options. Under the assumption that private risks are uncorrelated with the market, the real probability distribution is consistent with the risk-neutral approach. This fol-

lows from having a beta of 0 in the CAPM, so that shareholders require no additional return on private risk. The real probability is equivalent to the real distribution in this case, as $\hat{\alpha}_{prv} = \alpha_{prv} - \lambda\sqrt{var(x)} = \alpha_{prv} - 0$. For non-market risks correlated to the market, the probability distribution lies between the real distribution and the risk-neutral distribution. We do this by subtracting the risk premium from the drift of the private risk for the part correlated to the market. The conditional drift for correlated private risk is obtained by

$$\alpha_{prv}(prv|mkt) = \alpha_{prv} - \beta(\alpha_{mkt} - r_f) \quad (3.12)$$

ROA approach	Key assumption	Properties
Classic approach	A replicating portfolio consisting of market instruments can be constructed for the project, so that risk-neutral valuation can be applied.	▪ Theoretical foundation consistent with option pricing. ▪ Applies objective arbitrage pricing approach. ▪ Allows only for valuation of projects strongly dominated by market risk.
Subjective approach	A subjective project value is estimated, for which the option value is calculated using risk-neutral valuation.	▪ Does not explicitly replicate a project with market instruments. ▪ Assumption of a replicating market portfolio is retained. ▪ Subjective judgment is theoretically inconsistent with arbitrage pricing assumptions.
MAD approach ²	The subjective estimate of the project without flexibility is considered a twin security which replaces the replicating portfolio.	▪ Provides a rationale for application on most investment problems. ▪ Does not require distinguishing between types of risk. ▪ Accuracy depends on how well assumption of market completeness holds.
Revised classic approach	When market risk dominates the project risk-neutral valuation is used on the entire project, when private risk is dominating decision analysis is used.	▪ Recognises option pricing is not applicable to private risk. ▪ Requires quantifying the amounts of market- and private risks. ▪ Crude approximation when both types of risk are significant.
Integrated approach	The market is partially complete with respect for the project, so that a replicating portfolio can be constructed for the embedded market risks and decision analysis is used on private risks.	▪ Combines arbitrage pricing with decision analysis to assess all types of risk. ▪ Combination of pricing techniques is theoretically consistent. ▪ Requires distinguishing between all sources of market- and private risk.

Table 7: Summary of real option approaches with respect to private risk.

² The subjective approach and MAD approach are largely comparable, but MAD distinguishes itself by providing a more fundamental rationale by introducing a hypothetical twin security.

3.9. Estimating volatility

In the Black-Scholes framework, the square root of variance measures the volatility σ of the returns provided by the asset (Hull, 2008). Volatility is considered to be the most influential factor in the valuation of a real option, as an increase in volatility increases the upside potential of the investment (Piesse & Van de Putte, 2004). The volatility used in a real option model is the volatility of the returns of the underlying project, which in turn are affected by multiple uncertainties.

Many analytical approaches are available to estimate volatility. The simplest method is to take the standard deviation of the returns as volatility, keeping it constant over time. More advanced techniques allow for incorporating clustered volatility (non-constant volatility correlated over time) and mean-reverting behaviour. We will address such techniques in Chapter 6 of this thesis. Monte Carlo simulation is a numerical method to compute volatility which is particularly useful when dealing with multiple sources of uncertainty (Van de Putte, 2005). We can model each source of uncertainty individually, including possible co-dependencies. We can then derive the volatility of the project from the obtained stochastic free cash flows. In real-world practice, *ad hoc* approaches are common as well (Mun, 2002). An example is to base the volatility estimate on management assumptions, fitting the volatility distribution based on their experience and insights. Obviously such estimates may not be in agreement with market estimates.

3.10. Variable strike price

When considering a financial option within the Black-Scholes framework, there is no dependency between the strike price X and the stock price S_t ; the strike price remains constant over time (Margrabe, 1978). For a real option however, we may expect the required investments to undertake the project increase over the course of years. Also a relation may exist between the value of the project and the development costs for the project. A larger gas field would require more production wells and pipelines for example, resulting in higher development costs than for a smaller field. For real options, a time-dependent strike price X_t is usually required. Another co-dependency between X_t and S_t could be technological development. Through technological innovation, the efficiency of gas exploitation increases. This feature could be modelled by introducing a negative correlation factor between X_t and S_t over time. Also we can distinguish between incremental development and radical development. Small improvements are made continuously, but at times also technologic breakthroughs take place which greatly improve the efficiency of a process.

3.11. Competition

An aspect that could influence the value of the real option is competition. Without competition, only the market and the actions of the producer itself can affect values. As the holder of a financial option has the exclusive right to exercise it, absence of competition (e.g., by holding a patent or owning exploitation rights) is requisite to apply standard option valuation to real business situations. When we need to include strategic decision making in a real option framework, we may do so by combining option pricing with game theory (Huisman, 2000). Game theory assumes that each player will rationally attempt to maximise his profit, given his beliefs about the choices the other players will make. The solution to the problem is a Nash equilibrium, a state in which a player cannot improve his payoff by unilaterally changing his strategy given his beliefs of the other players' strategies (Shoham & Leyton-Brown, 2009). Actions are chosen in such a way that they are in line with the perceived probability that other players choose certain actions (Varian, 1992).

In this study we ignore the influence of competition in the valuation. We assume that operators are price takers, whose actions have no effect on the price of commodities. Since operators of marginal fields only have a limited share in the market, we consider this to be a reasonable assumption. Once a producer has obtained the exclusive exploitation rights of a field, producers are no longer competing for the same resource. This also limits the applications of the game theory when considering a single field. The phase before obtaining production rights is in fact competitive, but is not incorporated in our option model. Possibly the ongoing liberalisation of the energy market will increase competition, increasing the need to model competition.

Incorporating a game structure which accurately reflects the competition and strategic decisions may strongly increase the complexity of a real option model. Among the aspects that we could consider are asymmetries of competing firms, leader- and follower strategies, production capacity decisions, investments for technological improvement, etc. In general, we can assume that competition will decrease the value of an option (He, 2007).

3.12. Unspecified exercise and maturity dates

The possible exercise- and maturity dates for a real option situation may not be defined as clearly as for a financial option (Miller & Park, 2002). Project phases often do not have a strict deadline, but rather an estimated length which is subject to uncertainty as well. A delay may also exist between the decision to exercise the option and the actual exercise, for example when a production facility needs to be constructed. Another problem with the absence of an explicit expiration date for a real option is that decisions may be postponed indefinitely, leading to a time horizon for which no reasonable forecasts can be made. As option value increases with time, there is a risk of overvaluing the options value with expiration dates lying far in the future (Adner & Levinthal, 2002). Also it might be theoretically optimal to defer exercising the option forever when the growth rate of the asset exceeds the discount rate. A solution to prevent such problems is to set an expiration date for the option internally (Adner & Levinthal, 2002).

3.13. Suboptimal exercise policies

A common point of criticism on real options is that it assumes an optimal exercise policy when calculating the option value, where 'optimal' means maximising expected value given current information. One may argue that managers do not necessarily make optimal decisions, and as such the option value could be lower than the value calculated under optimal exercise policies. Though this is a defensible point of view, Copeland & Tufano (2004) reason that the obtained value is useful in practice. They draw an analogy with financial options, which in reality are not necessarily exercised at the optimal time either. Yet they are still valued as if they were optimally exercised. This is because we would not obtain the correct market price when based on suboptimal decisions. Pricing an option too low would create arbitrage opportunities. It is then possible to buy the option and short-sell the underlying to make a risk-free profit (Natenberg, 1994; Bingham & Kiesel, 2004). Though this might not be directly possible with real options, arbitrageurs would eventually be able to profit when projects are undervalued.

Another point brought up by Copeland & Tufano (2004) is the merit of 'option thinking'. Option thinking encourages rational decision making, basing decisions only on expected future cash flows. The awareness of management that flexibility is present in the project and can be addressed by making intermediate decisions is an important acknowledgement. The optimal exercise value gives an indication what additional value can be obtained through managerial flexibility. Thus, although the real option value overstates the value compared to suboptimal decision making, the obtained value still has meaning in practice. Careful interpretation of the model and the resulting value are necessary to correctly appreciate the result, though.

Real option modelling in Gas-to-Wire production

Where the previous chapter focused on the theory of real option valuation, in this chapter we go in more detail describing the issues that play a role for the application of a real option on Gas-to-Wire production. First, we describe the problem that will be modelled in the option framework. The goal of the option model is to find the economically optimal time to switch from gas production to electricity production for a tail-end gas field.

Two technological uncertainties are particularly important for the real option model: the distribution of the reservoir size and the productivity of the well. We provide some background information on these variables, along with the distribution chosen for them. Another key factor for the option value is to understand how energy prices behave. We also assess the role of legislation. The corporate tax rate has a direct effect on the cash flows, as does the royalty producers have to pay on natural gas exploitation. The government seeks to stimulate the development of smaller gas fields, several measures could be implemented for this in the future.

4.1. Problem description

In this section we describe the global setup of our investment problem. The goal of the option model is to find the optimal time to shift from gas production to electricity production. Two sorts of fields are considered feasible for Gas-to-Wire production: virgin gas fields and tail-end gas field. As tail-end fields are already producing gas, the infrastructure to connect the field to the transport network is present. For virgin gas fields the investments for gas production should also be made, including the drilling of the well. Also the uncertainties regarding the reservoir size and well productivity are significantly larger for virgin fields. For these reasons, we apply the option model on a tail-end gas field in this study. In principle the model can also be applied on virgin fields with little modifications. During the lifetime of the option, several investments can be made. We will treat these in more detail later in this chapter. When the pressure in the reservoir becomes too low, a compressor should be installed to bring the pressure up to the desired level for the transport network. When the decision is made to switch to electricity production, this requires connecting a generator to be installed on the spot and set up the wiring between the generator and the electricity transport network.

4.2. Uncertainty about reserve size

A major source of technological uncertainty in natural gas exploitation is the amount of gas that can be obtained from the field (Haskett, 2003). This uncertainty can be reduced by performing research, such as drilling exploration wells to check whether gas is present at that location (Laughton *et al.*, 2004). However, uncertainty about the exact size of the field is still present when production has started. This uncertainty decreases during exploration, but even when production is ended some minor uncertainty might remain. In Figure 5, we provide an example of how the uncertainty distribution narrows down as more activity is undertaken. Note that the estimated mean can change as well when more information becomes available, while after the production phase of course only a marginal amount of gas remains.

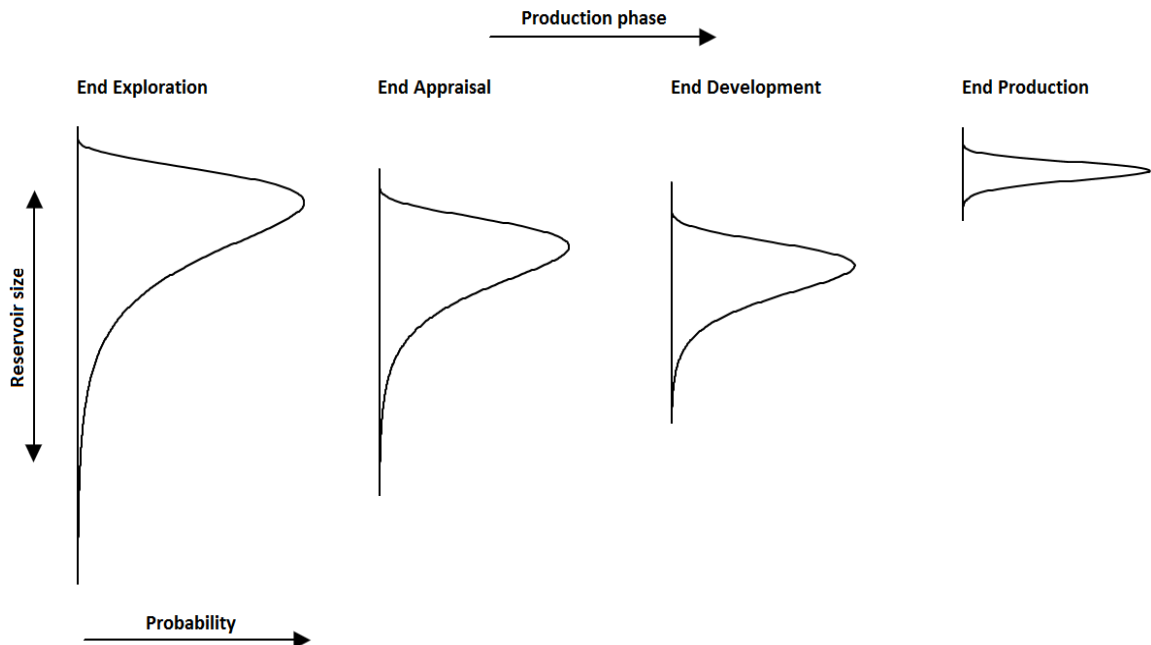


Figure 5: Reduction of reserve uncertainty distribution over time (based on Haskett, 2003). The estimated mean changes over time as well.

The quantity of gas present in a tail-end reservoir (Gas-In-Place or GIP)³ can be estimated with the formula

$$GIP = Vol * N/G * Por * Sat * Exp \quad (4.1)$$

with

Vol as the volume of the formation
N/G as the net-to-gross ratio
Por as the effective porosity of the formation
Sat as the gas saturation
Exp as the expansion factor

Here, the volume is the area times the thickness, or the gross volume of the structure. The net-to-gross ratio indicates which volume of the reservoir can contain gas, since gas is only found in parts of the structure. The effective porosity is the percentage in the rock formation that can contain fluid or gas (the void space in the rock layer). The gas saturation is the percentage of the porous volume filled with gas, the other part being water. Finally, the expansion factor shows how much the volume of gas will increase to compare the volume to that under surface conditions. Not all Gas-In-Place can be retrieved from the field; GIP multiplied with a recovery rate gives the reserve that can actually be economically won.

The GIP formula presented here is a simplified one; much more detail can be incorporated (see for example Dake, 2001). By modelling the uncertainty distribution of the individual variables (usually as lognormal random variables), a measure for the uncertainty of the quantity is obtained. The product of many lognormal random variables is also lognormal. Instead of individually modelling each variable as random, a (truncated) lognormal distribution is often used to model the reserve size (Kaufman, 1992). When the variables are modelled independently, correlation effects between the variables should be assessed.

A simplifying assumption we make in our model is that of perfect connectivity. This means that the gas from the entire reservoir is able to flow to the well unhampered, so that the full reserve can in principle be retrieved. In reality amounts of gas are often separated by dense rock formations, requiring multiple wells or field-enhancing techniques to allow the gas to reach the surface.

As a tail-end field is nearing the end of its production, the remaining uncertainty distribution should be small relative to the uncertainty at the beginning. We assume that the actual reserve size can be obtained at the start of the option life. This means we can draw a value from the uncertainty distribution, which we then consider deterministic.

4.3. Well productivity

There are two forms of constraints which limit the output of a gas reservoir; reservoir constraints and well constraints. A well has a production capacity which is mainly determined by the diameter of its tubing and the pressure difference between the reservoir and the surface. When the field contains sufficient reserves and the pressure is high enough, the limiting factor in production is the well flow capacity. The production rate is then simply the maximum amount of gas that can flow to the surface. But when the reserves fall below a certain level, the pressure becomes too low and the well no longer produces at its maximum capacity.

³ For virgin gas fields the term Gas-Initially-In-Place (GIIP) would be used, the formula remains unchanged.

Several techniques exist to improve the conditions of the field after reservoir constraints arise, increasing the production capacity of the well. These allow exploiting the field quicker and/or assessing more parts of the reservoir. Water or gases (such as CO₂) can be injected in the reservoir, which increases the pressure again and drives the natural gas towards the well (Oldenburg *et al.*, 2001). Another technique is hydraulic fracking, which is to create fractures in the rock formation by using pressurised liquids, thereby improving the speed at which the gas can flow through the reservoir (Montgomery & Smith, 2010).

Production capacity does not necessarily increase linearly with the number of wells. When multiple wells are placed, the natural gas flow is divided over these wells. This might result in production levels lower than maximal, depending on the amount of gas present in the reservoir and the distance between the wells. In general, for marginal gas fields which are eligible for Gas-to-Wire production a single well is sufficient, provided that the field has high connectivity. The benefit of adding more wells usually does not outweigh the required investment. In this study, we therefore presume that only a single production well is active.

When gas flows to the surface, it may contain water. When the gas pressure is insufficient, this water can form a layer at the bottom of the well, causing pressure against the upwards flow. If this pressure becomes too large, it stops the gas flow. A particular danger lies in temporarily shutting down the gas production. The water particles in the well will then drop down, after which the upwards pressure may be insufficient to restart production. A minimal rate of production may therefore be required to prevent the well from collapsing. This implies that continuous production should take place at the final stage of exploitation.

The main factor determining the output of a production well is the pressure of the reservoir. If the pressure is sufficiently high, the well is able to produce at maximum capacity. Another important factor is the permeability, which is the rate at which the gas can flow through the porous rock formation (Le Gallo *et al.*, 1998). The gas flow from the reservoir to the surface goes through a tube placed in the production well (PetroStrategies, Inc., 2012). A choice the operator must make is the diameter of the tubing in the well. A large diameter allows for a gas flow with less friction. When the pressure drops below a certain level, the initial tubing can be replaced with a smaller one to increase the pressure in the tubing at the cost of increasing friction (Schlumberger Limited, 2012). By varying the inflow rate of the well, a balance between pressure and production rate can be struck also.

A decline curve can be used to model the decrease of production over time (Palacio & Blasingame, 1993; Agarwal *et al.*, 1998). The traditional decline curves are hyperbolic, harmonic (a special case of the hyperbolic function) or exponential functions (Arps, 1944; Petrocenter, 2012). The exponential production function declines the fastest and therefore provides the most conservative estimate, while the harmonic function provides the least conservative. For wells with a production history it is easier to determine a distribution than for virgin gas fields. Historical production information can then be used to estimate the development of the production rate. The gas fields which are potentially suitable for Gas-to-Wire production are generally slowly declining with a relatively constant output for a long time, making a harmonic function the best fit of the three (J. Breunese, personal communication, 18 July 2012). Figure 6 shows the development of production rate over time using a harmonic production function. The harmonic production function is given by

$$PR_t = PR_0 / (1 + DR_t * t) \quad (4.2)$$

with

t	as the production day
PR_0	as the initial production rate
PR_t	as the production rate at time t
DR_t	as the decline rate parameter

The decline rate is dependent on the production rate at time t : $DR_t = (DR_0/PR_0) * PR_t$. It follows that the decline rate decreases when the production rate drops. In the case of electricity generation, the generator capacity can be lower than the production rate of the well, meaning that production is lower than indicated by the harmonic function. To reflect this capped production level, we calculate the decline rate by inserting the generator capacity instead of the well capacity if the latter is higher, leading to a slower decline. To avoid having to solve the equation for PR_t at every time step, we approximate the production rate by using PR_{t-1} as input for the decline rate at time t .

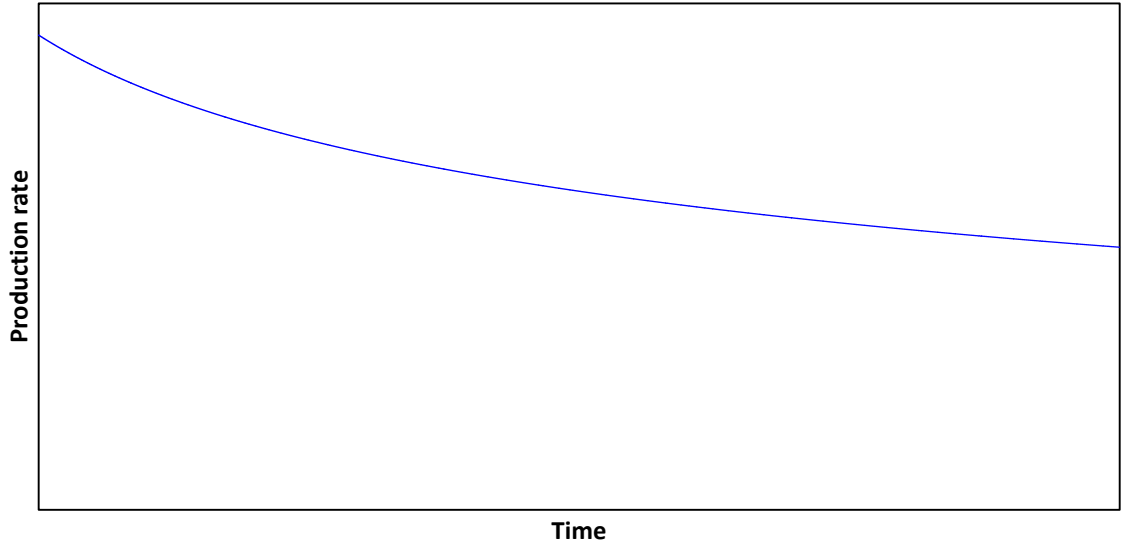


Figure 6: Example of a harmonic production function developing over time, using a 7.3% annual decline rate. Production can be seen to decline at a decreasing rate.

4.4. Price series for energy commodities

In this section we discuss some properties of energy prices, which may be relevant for our model. In Chapter 5 we perform diagnostic tests to evaluate these characteristics; here we provide a more general discussion. Stock price forecasts are often modelled based on the Black-Scholes model. This model states that prices follow a Geometric Brownian Motion (GBM) over time; a normal distribution of price changes with volatility being constant over time. There is a general consensus this model does not accurately describe the behaviour of price series of commodities (Deng, 2000). The price of a commodity tends to revert back to a (possibly time-varying) long-term equilibrium level (Deaton & Laroque, 1992; Pao, 2007). Some possible explanations for this so-called ‘mean-reversion’ observed for energy commodities are the balance between supply and demand, economic planning and the cyclic development of exploration techniques (Anderluh, 2007). Producers may strive to maintain a price level which maximises profit in the long run, withholding commodities from the market in times of low prices (Dias, 2004). The force reverting back to the mean becomes stronger the farther away from the equilibrium level.

Another property which is not appropriately modelled with the GBM is the occurrence of price jumps (Merton, 1976). Jumps are most notably present in electricity prices and form a significant risk in the energy market if not properly managed. The tails of a normal distribution do not capture such behaviour and therefore predict extreme price changes much more seldom than observed in practice. A sudden extremely sharp rise in energy prices can emerge due to an event such as a temporary supply shortage, for example caused by an outage or failure of the transmission infrastructure (Blanco & Sonorow, 2001; Knittel & Roberts, 2005). Another explanation is the occurrence of periods with unexpectedly high demand (Christensen *et al.*, 2011). The weather often plays a role in this. End-users generally pay a fixed contract price for electricity, making their demand inelastic with regard to the spot price. As the price tends to revert back quickly to its previous level after a jump, such an event is also called a spike in the series.

The demand for energy is not constant, but varies over time both in the short and in the long term. In the gas demand, a clear seasonal pattern can be observed (Energy Information Administration, 2011). Due to its application for heating, the demand for gas in the winter months is much higher than in the summer months. Electricity also has a seasonal demand pattern (Koopman *et al.*, 2005). In the winter demand for electricity is higher, as more hours of lighting are required and more indoor activities take place. It is also possible that a higher electricity demand exists during the summer due to the use of air conditioning, yet this strongly depends on which market is analysed (He, 2007). Demand patterns can also be noted on shorter terms. Particularly because of industrial activity, a difference in energy demand between week and weekend days exists (Nogales *et al.*, 2002). Furthermore, energy demand is higher during the day than during the night, even when comparing the hours of the day differences are observed.

When considering the price of a commodity in finite supply as a function of marginal cost plus an interest rate, in the long term a U-shape of the price over time could be expected (Slade, 1982). Initially costs decrease due to technological development, thereby lowering the marginal costs. However, when resources are gradually depleted, it becomes more difficult to produce the commodity, hence increasing the marginal cost. At a certain point in time, the increased difficulty in production will outweigh technological development, which causes an upward drift in price from then on. Natural gas production has existed for decades and gas becomes more difficult to win, indicating that gas production is currently in the upward slope.

As most electricity in the Netherlands is generated using natural gas, possibly a statistical relationship exists between gas and electricity prices. The overall demand for energy affects both prices, while gas and electricity prices may affect each other directly as well. Though one would generally expect both prices to move in the same direction, a negative relationship is a possibility as well. When other energy sources are used relatively more to generate electricity (such as an increased share of renewable energy), this will decrease the demand for natural gas and thus may decrease prices, while the electricity price could be unaffected.

The GBM assumes that volatility remains constant over time. In reality, the volatility of energy prices often turns out to be variable over time. This feature, dubbed heteroskedasticity, exists in a conditional form (depending on previous values) and an unconditional form (Alexander, 2001). In the unconditional form periods of high and low volatility can be predicted, in the conditional form this is not possible. For energy commodities usually unconditional heteroskedasticity can be observed, for example as a result of seasonal patterns.

4.5. Sales strategies

In this study we assume that all gas trading takes place based on oil-indexed contracts. This is the most common method in the current market. We assume that the producer can choose between a 3-0-3 and a 6-0-3 contract after the previous contract expires. Due to the convergence between GasTerra prices and TTF prices (Section 2.8.1), we consider the publicly available TTF prices to be equivalent to the GasTerra prices. We presume that all electricity to be sold on the daily spot market, where prices are quoted for the next day. The producer then makes a production decision on a daily basis. In Chapter 7 stochastic spot price models for gas and electricity are constructed.

Aside from the contracts defined in this study, several other contract forms exist. This means that in reality other sales strategies than the ones described in the model can be used. For example, futures contracts with several maturities can be traded. Over-the-counter contracts can be made between the producer and a third party, setting their own parties and conditions. For electricity also a special balance market exists. At this market, electricity is traded to correct imbalances at short notice (Schiffers, 2010). In swing contracts no fixed amount to be delivered is specified, instead leaving some slack for uncertainty in supply and demand. Finally, it is possible to sell gas or electricity on foreign markets as well, though this imposes some restrictions.

The possibility of trading in futures contracts could influence the sales strategy particularly when the market is in backwardation or contango (McCormack *et al.*, 2003). In backwardation, the prices of futures lie below the expected spot price. In such a case one could decide to sell the commodity now and buy the future contracts to retrieve the commodity cheaper at a later date. In contango, the price of a futures contract lies above that of the expected spot price. One could then make a profit could by physically holding the commodity and sell futures on the commodity. This strategy is especially applicable to natural gas, as it can be stored easily and at low costs.

4.6. Distance to transport network

An important consideration when deciding whether or not to invest in a gas field is its distance to the gas- or electricity transport network. Connecting the field to the transport network requires substantial investments, whether it is gas or electricity that is transported. The costs of transporting gas or electricity to the main grid are dependent on the distance to the (nearest) connection point, the surface it must cross, whether the grid must be laid in sea or on land etc. The most straightforward models for this determine the Euclidian distances between the gas field and the chosen connection point to supply the gas or electricity. We may use this simple approach to obtain a rough estimate for the costs, but a more detailed cost profile should be determined individually for each project. A possibility that could be considered in future studies is the development of central hubs, to which several gas fields can connect. From this central hub further transport could take place, reducing the costs to connect to the grid for the producer. Such hubs currently do not exist. In this study, we assume a distance of 10 kilometres to the electricity transport network, which is considered a moderate distance.

4.7. Generator properties

The current price of a gas turbine or motor required for Gas-to-Wire production is about 8 million euro. A single generator can convert 50.000 m³ to electricity daily. It is expected to be able to produce ten to fifteen years, after that a big revision of about 3 million euro is required. The minimum input required is about half of its capacity, under this level the generator has difficulties to remain operating. Though generators are flexible units, they are still subject to limitations and constraints which decrease the production levels compared to theoretically optimal produc-

tion. He (2007) identifies some physical constraints on electricity-generating facilities. As Gas-to-Wire is a recent innovation, only rough estimates about generator properties are available.

- **Efficiency:** The efficiency of a generator is related to the capacity utilised. Often a generator reaches optimal efficiency when producing at its maximum capacity. A generator has an efficiency of 35 to 40%, meaning that up to 65% of the energetic value of natural gas is lost in the generation process (Janssen *et al.*, 2012). In consultation with TNO we assume a generator with a slightly higher efficiency of 45%.
- **Ramp rate:** The gas supply to a generator needs to change monotonically. Therefore, a certain amount of time is required to change the generation capacity from one level to the other. Compared to other power plants, this rate is quite short: within minutes the desired rate can be reached. When a generator produces for an entire day, the ramp rate can be ignored without strong consequences.
- **Minimum up time and down time:** Though generators are flexible, the supply of gas cannot instantly be opened or stopped. For this reason, a generator has to keep running for a certain amount of time when started, and must stay down for a certain amount of time when shut down as well. Again, when production decisions are made for daily production we believe this aspect can be neglected.
- **Startup and shutdown cost:** Usually some costs are made when starting up or shutting down a generator. A distinction can be made between a straight cost component and a fuel cost component. These costs are influenced by several other factors as well. In this study we incorporate startup- and shutdown costs in the variable costs.
- **Breakdown rate:** Generators are subject to unexpected breakdowns, requiring reparation before production can be continued. It is not known when a generator will break down and how long reparation will take. Such events are often modelled by means of a Poisson process (or another distribution fitting the characteristics of the breakdown rate), allowing for simulation of random breakdown events and uncertain repair times. For a well-maintained generator the breakdown rate is low, we consider it to be zero.
- **Maintenance rate:** Apart from unexpected breakdowns, a generator needs regular maintenance. Unlike reparations, maintenance can be planned in advance. The maintenance rate can be modelled as a fixed percentage of down time. Downtime is roughly 2% a year; we assume that maintenance takes place during non-production days. The annual maintenance costs lie about 15% of the purchasing cost of the generator.

4.8. Dutch tax regime

When calculating the free cash flows related to the projects, it is necessary to pay attention to the tax regime (Sureth, 2002; Kretzschmar *et al.*, 2005). Changes in taxation affect the value of a real option; Kretzschmar *et al.* (2005) state that a dynamic tax forecast significantly impacts the option value, as unlike for financial options the tax effect on the asset and the real option cannot be separated. We provide a short overview of the Dutch tax regime on gas producers in this section. The current profit tax rate is 20% on annual profits up to € 200,000 and 25% for all profits over that amount (See Article 22, Wet op de vennootschapsbelasting 1969, 2012). Over the past years, a decreasing trend in Dutch profit tax rates can be observed. In a 2011 report, KPGM states that they believe a worldwide trend of decreasing corporate tax rates has come to an end, and no further sharp decreases should be expected (Kannekens & Campbell, 2011).

Currently fiscal losses can be subtracted from the taxable profits over the previous year (carry back) and the nine following years (carry forward), as described in Article 15ab of the Wet op de vennootschapsbelasting 1969 (2012). In the oil and gas industry, the carry forward principle is particularly important for compensating costs made during the exploration phase and setting up the production installation. The carry back principle is especially applicable for the costs incurred during the abandonment phase after the field is exploited. Intermediate losses can also be deducted, but the losses incurred before and after the production phase are generally most significant. Under Dutch legislation, carry back has priority over carry forward when both are applicable (Lips, 2010).

Taxable income is often defined as the Earning Before Taxes (EBT). This income is, in its most basic form, given by

$$\text{Earnings Before Tax} = \text{Revenues} - \text{Expenses} \quad (4.3)$$

Expenses consist not only of operation costs, but also overhead costs (e.g., administrative costs), interest costs, depreciation, amortisation etc. Tax regulation should be applied to properly allocate such costs on a specific project. In particular the effect of depreciation on free cash flows can be significant. Most commonly, depreciation is done by taking the difference between the initial investment and the expected salvage value (Belastingdienst, 2012). This difference is divided by the economic or technical life time, whichever is shorter. A maximum of 20% of the investment can be subtracted from profit annually. After a depreciation method has been chosen, it must be applied consistently (Jorissen *et al.*, 2009). In the option model, we perform depreciation linearly in accordance with the technological lifetime of the investment.

Apart from the profit taxes (which are paid to the Ministry of Finance), gas producers are also obliged to pay a 12% royalty of their annual revenue to the Ministry of Economic Affairs, as the natural gas resources are property of the Dutch state. Gas producers exploiting marginal fields at the North Sea are allowed to deduct 25% of their investment costs from the profit over which they are supposed to pay profit taxes to the state (Rijksoverheid, 2012c). With this measure, the Dutch government seeks to stimulate the development of gas fields which are otherwise economically unattractive.

4.9. Legal developments

A factor which potentially has a great impact on the profitability of exploiting minor gas fields is legislation (Mun, 2002). This section highlights some relevant legal developments in the industry. Due to their uncertain nature these are not included in the model, but may serve to interpret the results based on current legislation. We expect that new regulation and/or fiscal measures will be favourable towards innovative exploitation of minor gas fields. The rationale behind this is that the government wishes to stimulate exploitation, but under the current conditions this is hardly feasible from an economical point of view.

No clear legislation yet exists on the direct generation of electricity from small gas fields. Where there is a guarantee that gas produced from small fields can be sold to GasTerra, in the electricity market there is no comparable rule or trading party. Also it is not known yet how the generated electricity will be put to use. For example, electricity generated from the fields could be used to stabilise the intermittent output of windmills.

In the framework of renewable energy biogas may become an issue. This gas can be mixed with natural gas, thereby partly fulfilling the demand of natural. Further electricity produced from renewable sources has preference over electricity generated from other sources. As the gov-

ernment attempts to stimulate the use of renewable sources, the effect of their policies might affect the conventional production of natural gas (European Commission, 2008).

The small fields policy is related to the balancing function of the Groningen gas field, which meets the demand which is not met by other Dutch gas fields. In roughly fifteen years the Groningen gas field will no longer be able to meet the remaining demand completely. The Netherlands will then have to import natural gas as well. When this happens, there is a realistic possibility that the current small fields policy will undergo a significant transformation to adapt to the new situation.

It is common practice that produced gas is compressed and subsequently delivered to the main transportation network. For minor gas fields, we can imagine that such fields will deliver directly to the low-pressure distribution network or a specific client. This form of exploitation does not fall under the small fields policy, hence producers are not protected by means of a guaranteed sales contract. When direct production for low-pressure networks will be introduced, new regulation could be implemented.

Diagnostic testing of price data

Diagnostic testing is an important concept in modelling price series. When applied on historical data, it provides insight in the characteristics of the series. This insight allows picking suitable techniques for the forecasting models. After modelling is completed, we can also apply diagnostic testing to check if assumptions have been satisfied. In this chapter our focus lies on the testing the properties of the raw data. Appendix II shows the results of diagnostic testing on the residuals of the price models as constructed in Chapter 7.

We perform diagnostic tests for normality, autocorrelation, stationarity, mean-reversion, time effects, correlation and cointegration between gas and electricity series. A brief theoretical introduction is provided for each test performed. We include graphs and plots to visually illustrate the behaviour of the series.

5.1. Purpose of diagnostic testing

In finance, the price development of an asset is often described as a time series: a sequence of price observations over time. When forecasting the development of a certain price series, the model should capture the features incorporated in the actual series. Before attempting to model a price series, we should perform several diagnostic tests on the available historical data to gain insight in the properties of the series. Based on these properties, we can make an underpinned choice for the modelling techniques to be applied. After a model has been constructed, diagnostic testing again is required to test whether theoretical assumptions are met. A significance level of 0.05 or 0.01 is most common in diagnostic testing (Keuzenkamp & Magnus, 1995). In this chapter we adopt a significance level of 0.05 for all tests applied. As the purpose of the tests is mainly to gain insight into the behaviour of the series, we feel that a level of 0.01 would be too strict. Several tests are performed with the commercial software package EViews 7.

5.2. Historical data sets

We perform diagnostic testing on the historical data for gas and electricity prices. The sample containing gas prices was obtained from the website of APX. It contains daily gas prices (All-Day Index, expressed in euro per MWh) from 2 January 2006 up until 31 December 2011. APX calculates these prices by taking the volume-weighted average price of all orders delivered during the day (APX-ENDEX, 2012). As such, no prices are available for days during which no transactions took place (356 in total). Only the first two years of the set include daily prices for every day of the year. For this reason, not all week days are equally represented, and small gaps exist between consecutive prices. For analytical convenience we ignore these gaps; we act as if all prices follow each other up directly. An alternative would be to fill the gaps with estimated prices, using techniques such as interpolation or some forecasting model. However, this way we would make assumptions about the behaviour of the series beforehand, contradicting the purpose of this chapter. Still, we realise that the use of discontinuous data may influence the results of our tests, particularly when time lags play a role. We therefore check whether the full data set has the same distribution as the 2006-2007 subset, as the latter contains no gaps. For this, we use Kuiper's test,⁴ which tests the null hypothesis that the empirical cumulative distributions of both sets are equivalent. Its test statistic is defined by (Stephens, 1965):

$$\left(\sqrt{\frac{nn'}{n+n'}} + 0.155 + \frac{0.24}{\sqrt{\frac{nn'}{n+n'}}} \right) \times \left(\sup_{-\infty < S_t < \infty} (cdf_{full}(S_t) - cdf_{sub}(S_t)) - \inf_{-\infty < S_t < \infty} (cdf_{full}(S_t) - cdf_{sub}(S_t)) \right) \quad (5.1)$$

with n and n' as the number of observations in the full set and the subset respectively, cdf as the empirical cumulative distribution function, and S_t as the price at time t . We find that our calculated test value of 4.138... exceeds the 5% critical value of 1.747 (Stephens, 1970). We therefore reject H_0 , and find that the subset and the full set have different empirical distributions. This result does not lead us to definite conclusions (with large samples even small differences may be significant), but the difference might be caused by the inconsistencies within the full set. We use the 2006-2007 subset to double-check the outcomes of our tests, when we find notable differences we report these in the results.

⁴ Kuiper's test is closely related to the Kolmogorov-Smirnov test. In comparison, it is better able to deal with tail deviations and patterns in the data, making it more fitting for the gas price series we compare. It requires no assumptions about the distribution of the data, or dividing data in bins such as the chi-square test.

In 2005, the Dutch electricity exchange APX closed access to historical electricity prices, making data available only to paying members (Enreadt, 2005). We received a set of electricity prices from APX for the purpose of this study. The sample includes daily electricity prices (base prices) from 1 November 1999 until 23 March 2012; prices are quoted for all days. Prices are expressed in euro per MWh. It should be noted that electricity prices are also quoted intra-daily (hourly or a custom time interval). Prices fluctuate significantly during a day, so that in practice trading on the spot market consists of more than trading on a daily basis. Intra-daily data were not available for this research.

5.3. Visual observation of price series

As the first testing step, we informally assess the series by visually analysing their pattern over time. This gives us an indication of the relevant properties of the series. Also it may provide us with information not picked up by formal tests. We provide plots of the daily gas price (Figure 7) and the daily electricity price (Figure 8) of the available data. Figure 9 combines both graphs in one plot, including only days for which both prices are quoted. The calendar years on the x-axes are shown on fixed 400-day intervals (gas) and 1000-day intervals (electricity); remember that for the gas series not all years have an equal amount of data.

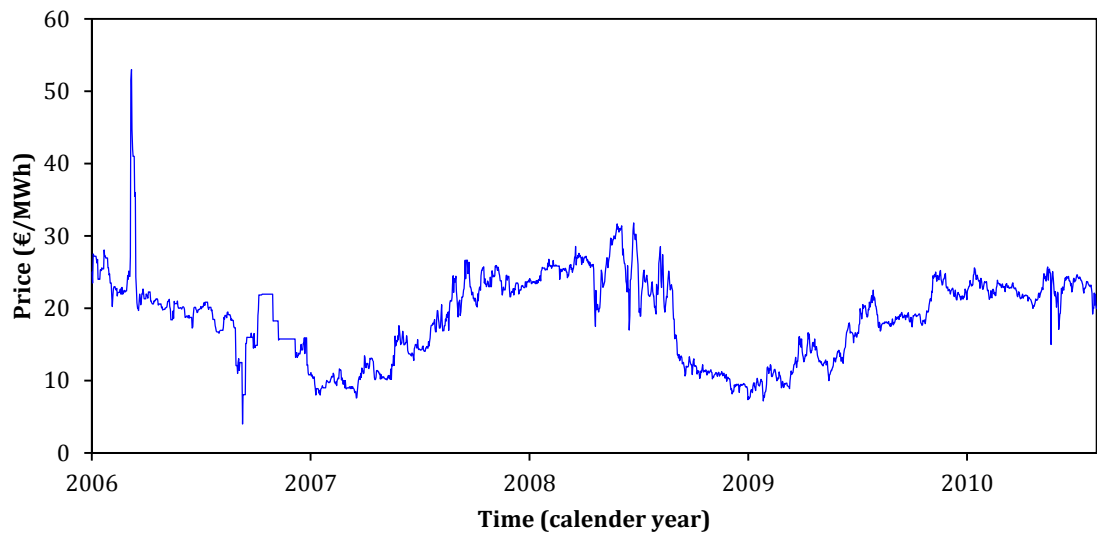


Figure 7: Gas prices APX 2006-2011. In the long terms prices seem to fluctuate around 18 €/MWh. Periods of high and low volatility appear to alternate.

The price series in Figure 7 appears to fluctuate around a certain equilibrium level, moving only little in the long run. A wave-like shape can be observed. The time frame is too short however to see whether it is a recurring sine-like pattern, a slow mean-reverting process, or perhaps completely random. The wave is too long to be explained by annual seasonal effects, with roughly three years between the peaks. Periods of high and low volatility can be distinguished. On the left side of the graph a remarkable price spike can be observed. We informally checked some possible explanatory factors such as the weather, electricity prices, and incidents in the industry, but found no qualitative explanation why this particular spike occurred.

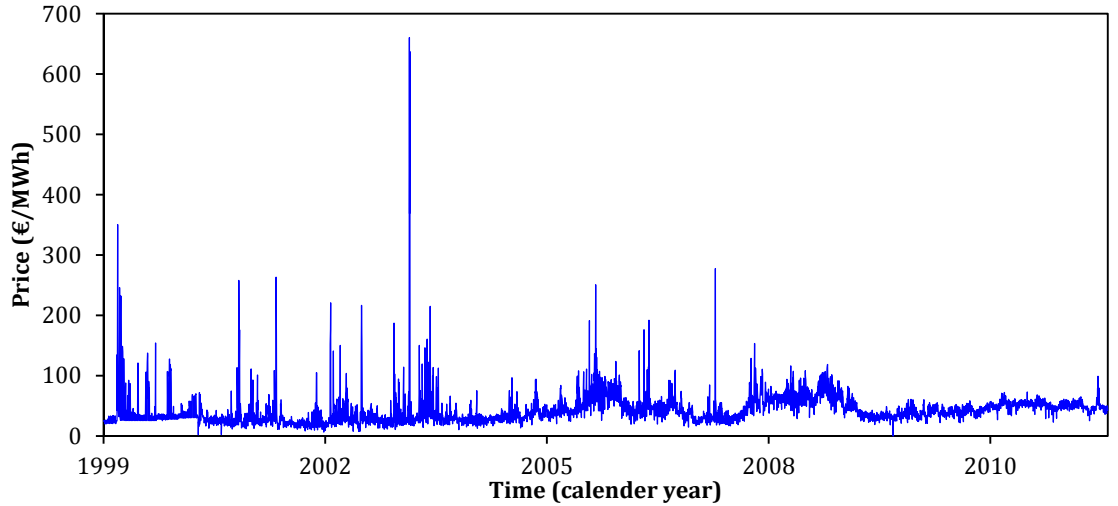


Figure 8: Electricity prices APX 1999-2012. The spikes can be clearly distinguished from the regular price levels. Volatility can be seen to decrease over the years. Closer observation reveals a mild seasonal pattern.

Consistent with the theoretic assumptions of electricity price behaviour, the price series for electricity as shown in Figure 8 is highly volatile. Occasionally large spikes are observed, with prices far exceeding the average price level for a short period of time. Further a recurring wave pattern may be observed, possibly indicating seasonal effects and mean-reversion.

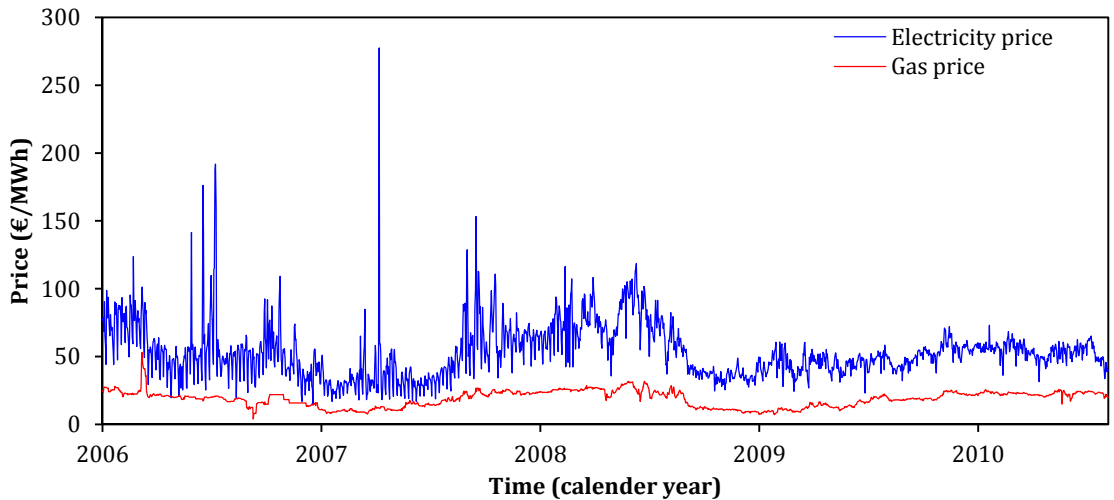


Figure 9: Gas price and modified electricity price series 2006-2011. Peaks and dips seem to coincide at some points (e.g., between 2008-2009), but the electricity prices are seen to be much more volatile.

By taking the first difference of a price series, we obtain the (raw) return series. Diagnostic testing is generally performed on return series rather than on price series, as these have several analytical properties not found in price series (Quantitivity.wordpress.com, 2011). Often logarithmic returns are used in econometric analysis; these approximate the value of raw returns for small price differences. Logarithmic returns are defined by the expression $x_t = \ln(S_t/S_{t-1})$. Unlike a prices series, a logarithmic return series provides values of an equal measure. If prices are assumed to be lognormally distributed, then their corresponding logarithmic returns are normally distributed. Logarithmic returns can be summed to obtain the compounded return over a period of time, thereby retaining the normality property. Finally, logarithmic returns are mathematically convenient when performing operations such as differentiation and integration.

A shortcoming of logarithmic returns is that they cannot be calculated when negative prices exist.⁵ When we use the term ‘return’ in this study, we always refer to the logarithmic return.

In Figure 10 and Figure 11, we plot the daily logarithmic returns for natural gas and electricity respectively. The gas returns appear to show some volatility clustering, containing periods of highly volatile returns as well as periods of low volatility. Some rare spikes in returns can be clearly distinguished in Figure 10. The electricity returns in Figure 11 exhibit a strongly volatile pattern. Spikes occur quite frequently. After the liberalisation of the Dutch energy market in 2004, the return pattern becomes notably less extreme. In general, the volatility of electricity returns seems to have decreased over the years.

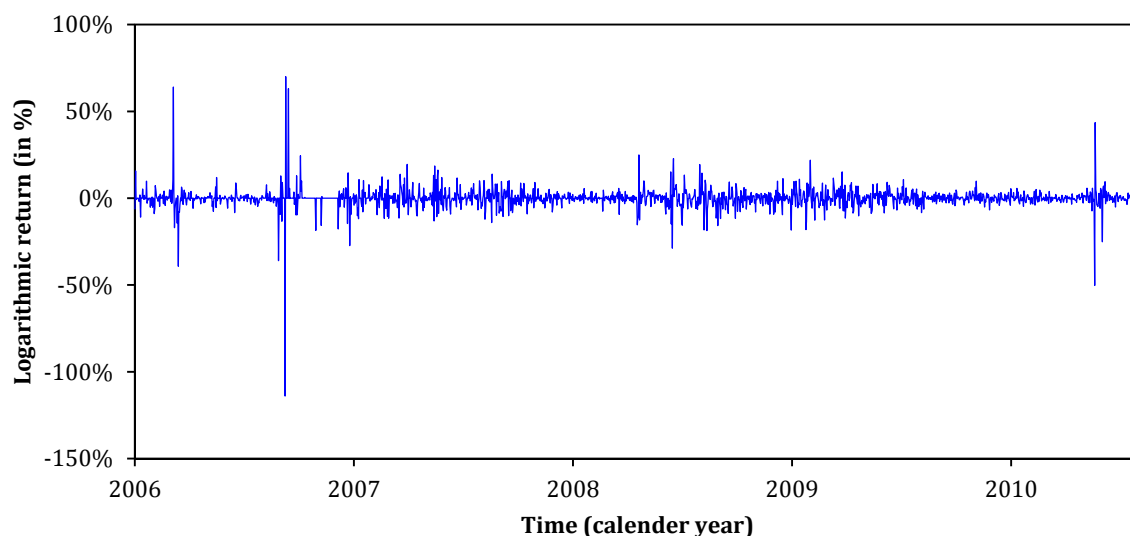


Figure 10: Daily logarithmic gas returns APX 2006-2011. Alternating periods of high and low volatility can be distinguished.

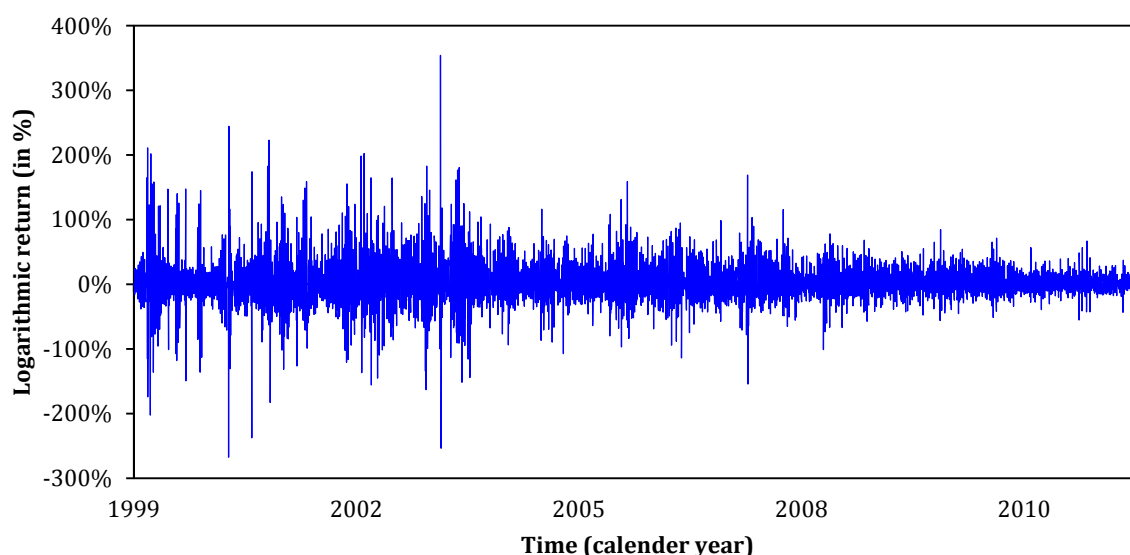


Figure 11: Daily logarithmic electricity returns APX 1999-2012. The decreasing trend in volatility can be clearly observed here.

⁵ Negative prices can occur in electricity pricing, but are not present in the APX sample.

5.4. Stationarity

5.4.1. Theory of stationarity

An important aspect to consider when modelling a time series is stationarity. Intuitively, a stationary process is a data-generating process that remains the same over time. Stationary series have convenient analytical properties not present in nonstationary series, allowing to perform several statistical tests which are not applicable to nonstationary series (Alexander, 2001; Mishra *et al.*, 2010). We can make a distinction between strong stationarity and weak stationarity. A return series $x_1, \dots, x_t$ is strongly stationary if the joint distribution $Z(x_1), \dots, Z(x_t)$ is the same as the joint distribution $Z(x_{1+h}), \dots, Z(x_{t+h})$, where t is a finite number of points and h can be any integer (Myers, 1989). In other words, the distribution is independent of the point in time; any time frame of t points should have the same distribution. A series is considered weakly stationary if a covariance between $Z(x_t)$ and $Z(x_{t+h})$ exists which depends solely on h for any $Z(x_t), Z(x_{t+h})$. This implies that both the expected value and the variance of $Z(x)$ do not depend on the point in time. Weak stationarity is measured by taking the first difference of the series and creating a new series from it (i.e., $\Delta x_t = x_{t+1} - x_t$ for all t), applying a stationarity test on the new series. When a time series happens to be nonstationary, it is usually transformed in such a way it becomes weakly stationary. This can be done by inserting variables which explain certain movements in the series, such as a linear trend. A well-known test to check a series for weak stationarity is the augmented Dickey-Fuller (ADF) test. Other stationarity tests include the original Dickey-Fuller test and the Philip-Perron test (Wang & Tomek, 2004). We apply the ADF test as it often provides the best results for finite samples. We explain the ADF test based on the original test and its later extension.

The original Dickey-Fuller test checks for the presence of autocorrelation ρ between x_t and x_{t-1} . The series is nonstationary if such a correlation is found (Yaffee & McGee, 1999). Three versions of the test exist: testing for a lagged term, a lagged term with an intercept (i.e., a constant allowing for a mean other than 0) and a lagged term with an intercept and a deterministic time trend. Not accounting for a constant or trend could substantially reduce the power of the test when it is actually present in the series. Incorrect inclusion of these factors can also reduce the power of the test. The original Dickey-Fuller test presumes that residuals follow a Geometric Brownian Motion, causing a misfit when higher-order autocorrelation is present in the series. The augmented Dickey-Fuller test builds on the original test, but removes autocorrelation for higher lags up to a specified number. The test requires specifying a number of h lags to be set, while the real number of lags is usually unknown. A common method for this is to use an information criterion, comparing the extra explanatory value of adding a lag to the cost of adding an extra parameter. Calculating an information criterion helps to select the optimal number of lags. The augmented Dickey-Fuller test is performed on the following model.

$$\Delta x_t = c + Bt + (\rho - 1)x_{t-1} + w_1\Delta x_{t-1} + \dots + w_{h-1}\Delta x_{t-h+1} + \varepsilon_t \quad (5.2)$$

with c as a constant, B as a deterministic time trend, ρ as the correlation between x_t and x_{t-1} , h as an indicator for the lag size, w_{h-1} as a weight parameter, Δx_{t-h+1} as a lag term and ε_t as the error term at time t .

The critical values from the t-distribution cannot be applied for the Dickey-Fuller test. Instead, the test has a table with its own critical values. The test statistic of the augmented Dickey-Fuller test is given by dividing the estimator of $\rho - 1$ by its own standard error. The test defines the null hypothesis as $\rho = 1$. If this is the case, it follows that $x_t - x_{t-1} = \varepsilon_t$ (ignoring deterministic and lag effects), which means that the series follows a random walk and as such is not stationary

(Hurn, 2009). If the null hypothesis is rejected (which happens when the test statistic is smaller than the critical value, the test statistic is a negative number), this means that no unit root is found in the series, so the series is stationary. The augmented Dickey-Fuller test can be performed in EViews, the program determines the optimal number of lags according to the selected information criterion.

5.4.2. Results of testing for stationarity

We test the return series for stationarity, performing the augmented Dickey-Fuller test in EViews. The test is performed for a constant and a linear trend. The linear trend is particularly relevant for the electricity price series, which appears to have a decreasing trend in the size of returns. The results of the stationarity tests are presented in Figure 12 and Figure 13. For both series, the null hypothesis of nonstationarity is soundly rejected at the 5% level. Hence, both return series appear to be stationary. We checked whether the tests would provide different outcomes when assuming no linear trend, but for both series the null hypothesis was still rejected. Further we tested for the 2006-2007 subset for gas, for which H_0 is also rejected. These results not shown here. As the test results indicate that both return series are stationary, we can apply other diagnostic tests on them without further modification.

Null hypothesis: <i>Gas returns</i> has a unit root		
Exogenous: Constant, Linear trend		
Lag Length: 12 (Automatic – based on AIC, maxlag=24)		
Augmented Dickey-Fuller test statistic	t-statistic	Prob.*
Test critical values:		
1% level	-14.35455	0.0000
5% level	-3.963084	
10% level	-3.412275	
	-3.128070	
*MacKinnon (1996) one-sided p-values.		

Figure 12: EViews results of augmented Dickey-Fuller test on gas return series. The test statistic is smaller than the critical values, indicating that the series is stationary.

Null hypothesis: <i>Electricity returns</i> has a unit root		
Exogenous: Constant, Linear trend		
Lag Length: 31 (Automatic – based on AIC, maxlag=31)		
Augmented Dickey-Fuller test statistic	t-statistic	Prob.*
Test critical values:		
1% level	-16.55546	0.0000
5% level	-3.960078	
10% level	-3.410804	
	-3.127197	
*MacKinnon (1996) one-sided p-values.		

Figure 13: EViews results of augmented Dickey-Fuller test on electricity return series. The test statistic is smaller than the critical values, indicating that the series is stationary.

In Section 5.5, we also test the *price* series for stationarity, finding that the electricity price series is stationary as well. We want to explain the implications these results could have for our diagnostic tests. The stationarity of the electricity price series indicates that we could apply some diagnostic tests directly on this series. This may be preferable, because with the conversion from prices to returns some information is lost. However, the gas price series is not stationary, for comparability we continue testing on the return series. Also, in general literature describes tests only for return series, these may require modification for testing on price series. Finally, our interest is in constructing price-generating processes, so that we are more interested in the properties of the return series.

5.5. Cointegration

5.5.1. Theory of cointegration

When two or more price series appear to be moving in the same direction in the long term, these series could be cointegrated (Mandal & Van de Weide, 2011a). This means that the series share a common stochastic drift; the spread between the series is mean-reverting. If two or more non-stationary time series have a linear combination that is stationary, these series are cointegrated. When one or more series are stationary, cointegration cannot exist. Two series are considered cointegrated if they are both nonstationary, but there exists a linear combination Y_t that is stationary (Engle & Granger, 1987):

$$Y_t = S_{g,t} - wS_{e,t} \quad (5.3)$$

with Y_t as the linear combination, $S_{g,t}$ and $S_{e,t}$ as the gas and electricity prices respectively, and w as a weighting parameter. Obviously we could use a more general description of the right-hand side series as well.

Several tests exist for identifying cointegration relationships. The Engle-Granger test is strongly based on the augmented Dickey-Fuller test (Engle & Granger, 1987). A problem with this test is that w in (5.3) will be wrongly estimated when the series are in fact not cointegrated. Regression will then result in a value of w minimising the error, causing the null hypothesis to be rejected too often. A further disadvantage of the Engle-Granger test is that it only allows testing for a single cointegration relationship at a time (Alexander, 2001). A test that allows testing for multiple cointegration relationships simultaneously is the Johansen procedure. This procedure is commonly applied when testing for cointegration (Sørensen, 2005). In its general form, Y_t is described as a vector of n nonstationary time series. These are cointegrated when $rk < n$ stationary linear combinations exist, where rk stands for the rank number. The Johansen procedure is performed on the following model (Johansen, 1991; Segura & Braun, 2004):

$$\begin{aligned} \Delta Y_t &= \Pi Y_{t-h} + \Gamma_1 \Delta Y_{t-1} + \dots + \Gamma_h \Delta Y_{t-h+1} + wV_t + \varepsilon_t \\ \Delta Y_t &= \mathbf{l}_1 \mathbf{l}'_2 Y_{t-h} + \Gamma_1 \Delta Y_{t-1} + \dots + \Gamma_h \Delta Y_{t-h+1} + wV_t + \varepsilon_t \end{aligned} \quad (5.4)$$

with h as the lag order, w as a weighting parameter, V_t as a vector containing deterministic regressors (such as a constant, trend, seasonal effects etc.), Γ_1 to Γ_h as $n \times n$ matrices and ε_t as the error term at time t . If the hypothesis holds that the series are cointegrated, it follows that $\mathbf{l}_1 \mathbf{l}'_2 = \Pi$. Π is an $n \times n$ matrix being the product of $\mathbf{l}_1$ and $\mathbf{l}'_2$ ($\mathbf{l}'_2$ is a scaled matrix of $\mathbf{l}_2$), which are matrices of the size $n \times rk$ and $rk \times n$ respectively. To find the cointegration rank, the number of linear correlations between ΔY_t and Y_{t-1} after removal of autocorrelation is tested. First, we introduce three new variables:

$$\begin{aligned} Z_{0t} &= \Delta Y_t \\ Z_{1t} &= (\Delta Y'_{t-1}, \dots, \Delta Y'_{t-h+1}, V_t, 1)' \\ Z_{ht} &= Y_{t-h} \end{aligned} \quad (5.5)$$

From these terms, matrices G_{ij} are defined as

$$G_{ij} = T^{-1} \sum_{t=1}^T Z_{it} Z'_{jt} \quad (5.6)$$

Here, T is the number of observations in the series and i and j are state indicators having a value of 0, 1 or h . $\mathbf{Z}_{0t}$ and $\mathbf{Z}_{ht}$ are regressed on $\mathbf{Z}_{1t}$ using least squares regression, yielding residuals R_{0t} and R_{ht} respectively. In a sense, this regression filters out the deterministic effects and disturbances considered noise. The residual sum of squares is denoted as

$$RSS_{ij} = \frac{1}{T} \sum_{t=1}^T R_{it} R'_{jt} \quad (5.7)$$

The residual sums of squares can be used to estimate Π , namely by $RSS_{hh}^{-1} RSS_{h0}$. The matrix $\mathbf{l}_2$ is estimated as the eigenvectors of the residuals which represent a cointegration relationship. The corresponding eigenvalues ev_i for each column of eigenvectors can be found by solving

$$\left| ev_i - RSS_{hh}^{-1/2} RSS_{h0} RSS_{00}^{-1} RSS_{0h} RSS_{hh}'^{-1/2} \right| = 0 \quad (5.8)$$

The maximum likelihood function for rk is given by

$$L_{max}^{-2/T}(rk) = |RSS_{00}| \prod_{i=1}^{rk} (1 - \widehat{ev}_i) \quad (5.9)$$

This function is maximised when all eigenvalues up to and including ev_{rk} are nonzero, while all higher eigenvalues are 0. The Johansen procedure is a sequential test, starting at testing the hypothesis of 0 cointegrating relations and moving up to the point it fails to reject the hypothesis. The test statistic for the presence of rk cointegration relations is given by

$$JH(rk|n) = -T \sum_{i=rk+1}^n \ln(1 - ev_i) \quad (5.10)$$

Due to the nonstationarity of the time series eligible for the procedure, the Johansen procedure has its own table with critical values, to which the calculated test statistic is compared. When multiple cointegration relations are found, the forecasting series can be modelled using a Vector Error Correction (VEC) model. A VEC model is essentially a VAR model as briefly explained in Section 5.10, expanded with terms to account for the cointegration relationship (Engle & Granger, 1987). The results of the Johansen test help establishing such a model.

5.5.2. Results of cointegration testing

Before testing the price series for possible cointegration, first we must test the price series (instead of the return series tested previously) on stationarity, again using the Augmented Dickey-Fuller test. For series to be cointegrated, both series must be nonstationary, but a stationary linear combination of both should exist. We show the results of the tests in Figure 14 and Figure 15. At the 5% level, the t-statistic for the gas price series is higher than the 5% critical value; it follows that the price series for gas is not proven to be stationary. For the electricity price series, the hypothesis of nonstationarity is rejected at the 5% level, indicating that the electricity price series is stationary. Because one of the series is stationary, cointegration between both price series cannot exist. No further testing is required.

Null hypothesis: <i>Gas prices</i> has a unit root		
Exogenous: Constant, Linear trend		
Lag Length: 13 (Automatic – based on AIC, maxlag=24)		
	t-statistic	Prob.*
Augmented Dickey-Fuller test statistic	-2.722192	0.2276
Test critical values:	1% level	-3.963084
	5% level	-3.412275
	10% level	-3.128070
*MacKinnon (1996) one-sided p-values.		

Figure 14: EViews results of augmented Dickey-Fuller test on gas price series, indicating nonstationarity.

Null hypothesis: <i>Electricity prices</i> has a unit root		
Exogenous: Constant, Linear trend		
Lag Length: 30 (Automatic – based on AIC, maxlag=31)		
	t-statistic	Prob.*
Augmented Dickey-Fuller test statistic	-7.020379	0.0000
Test critical values:	1% level	-3.960078
	5% level	-3.410804
	10% level	-3.127197
*MacKinnon (1996) one-sided p-values.		

Figure 15: EViews results of augmented Dickey-Fuller on electricity price series, indicating stationarity.

5.6. Autocorrelation

5.6.1. Theory of autocorrelation

Autocorrelation is the correlation between two observations in the same time series, where the lag indicates the number of time steps between both observations (Taylor, 2008). It is of interest to test for autocorrelation of returns, as it gives insight in how returns are related over time. For example, negative autocorrelation indicates that positive returns tend to be followed by negative returns and vice versa, such information can be used when constructing the forecasting model. To see whether returns exhibit volatility clustering, an autocorrelation test can be performed on the *squared* returns (Mandal & Van der Weide, 2011b). Alternating positive and negative returns may have similar volatilities, but this is only observed when all tested observations have the same sign.

A simple autocorrelation test is the Box-Pierce test. Its test statistic is the weighted sum of squares of autocorrelation for each lag. The idea behind the test is that for a large number of observations, the test statistic would be distributed according to a chi-square distribution. The test was later improved by Ljung & Box so that the test statistic would more closely resemble the chi-square distribution (Ljung & Box, 1978). Nowadays the Ljung-Box test is the most common autocorrelation test. We provide the Ljung-Box test statistic (also known as Q-statistic) in Equation (5.11).

$$Q = n(n+2) \sum_{i=1}^h \frac{\hat{\rho}_i^2}{n-i} \quad (5.11)$$

where n is the sample size, h the number of lags being tested, and $\hat{\rho}_i$ the sample autocorrelation at lag i , defined as

$$\hat{\rho}_i = \sum_{t=h+1}^n x_t x_{t-h} / \sum_{t=1}^n x_t^2 \quad (5.12)$$

where x_t describes the observation at time t . Note that the test statistic is additive: if autocorrelation is significant for a certain lag length, it is also significant for all greater lag lengths. The null hypothesis states that data are *i.i.d.*, and is rejected for the chosen significance level p if the test statistic exceeds the critical value as specified in the chi-square distribution: $Q > \chi^2_{(p,df)}$, where the degrees of freedom df are equal to the number of lags h .

One of the issues when performing the Ljung-Box test is the question up to which number of lags the data should be tested. Testing for an insufficient number could potentially ignore correlations between data, yet testing for too many lags negatively affects the robustness of the test. Burns (2002) suggest that the number of lags should be no more than 5% of the length of the series. Due to day-of-the-week effects described in literature, the autocorrelation test should at least include 7 lags, corresponding to the 7 days in a week. We can imagine autocorrelation effects spanning much larger time frames as well though, for example annual effects.

After a time series is modelled, residuals should have no clear pattern remaining. Instead only random errors should occur. Therefore, no autocorrelation between the error terms should exist. The Ljung-Box test can be applied on the residuals to test whether this condition is satisfied.

5.6.2. Results of testing for autocorrelation

We test the return series for the presence of autocorrelation in EViews. We present the test results in the form of a correlogram. Autocorrelation (denoted as 'AC') is tested up to 36 lags. In addition, also partial correlations are tested (denoted as 'PAC' in the correlogram). The partial correlations represent the autocorrelation remaining after taking removing the effect of autocorrelation for smaller lags. For each lag the Ljung-Box test statistic is provided ('Q-stat' in the correlogram), indicating whether autocorrelation is present at that lag. The probabilities in the correlogram ('Prob. H_0 ') show the probability that the null hypothesis of no autocorrelation holds. Because the Ljung-Box test statistic is additive, a significant correlation for a certain lag means that autocorrelation is significant for all higher lags as well. The significance of the individual correlation coefficients is indicated by dotted lines, EViews uses $\pm 2/\sqrt{n}$ as the 5% standard error bounds.

The correlogram for gas returns (Figure 16) provides high Ljung-Box Q-stats and low corresponding probability values, indicating that significant autocorrelation is present. The first autocorrelation term is negative and quite notable; all other autocorrelation terms are generally insignificant and appear to be random in sign. However, performing the same test on the 2006-2007 subset provides a very comparable autocorrelation pattern. Of course the subset, since it comprises a substantial part of the full set, has a strong effect on the pattern of the full set as well. Still, this result may indicate some pattern for which we have no explanation. We assess the strong first-lag negative autocorrelation+ in Chapter 7 by estimating a mean-reverting model, which by nature incorporates negative correlation between returns. Higher-lag autocorrelation is not addressed further, as it is generally insignificant and lacks a clear pattern.

The correlogram for electricity returns (Figure 17) shows a repeating autocorrelation pattern, each time having six consecutive negative autocorrelations followed by one large positive autocorrelation term. This pattern can be explained by the large price differences between week- and weekend days. The positive seventh lag correlation is mainly caused by the strongly negative returns occurring on most Saturdays; from Equation (5.12) it follows that this yields highly positive autocorrelation estimates. For lags other than (a product of) seven, these negative returns may have either a positive or negative effect on autocorrelation, depending on the sign of the return we compare it to. The fact that most lags show negative correlation indicates that the

series in general exhibits negative autocorrelation, and that only the weekend day effect breaks this pattern. Partial correlations are particularly notable within the same week, but the weekly effects remain observable also for larger lags. The Ljung-Box test indicates that the autocorrelation is significant. Note that, opposed to the gas price series, most higher-lag autocorrelation is significant as well. In Section 7.3 we assess this autocorrelation pattern by incorporating binary variables for each day of the week.

We also perform the Ljung-Box test on the squared returns of gas and electricity to check for volatility clustering. The corresponding correlograms are not displayed in this study. The squared gas returns are significantly autocorrelated, indicating volatility clustering. Especially the first and sixth lag display strong autocorrelation. It is difficult to provide a qualitative explanation for this effect, partially because the irregularity of price data. We will not address the observed sixth lag autocorrelation further. Autocorrelation is also significantly present in the squared electricity returns, displaying a weekly pattern as well.

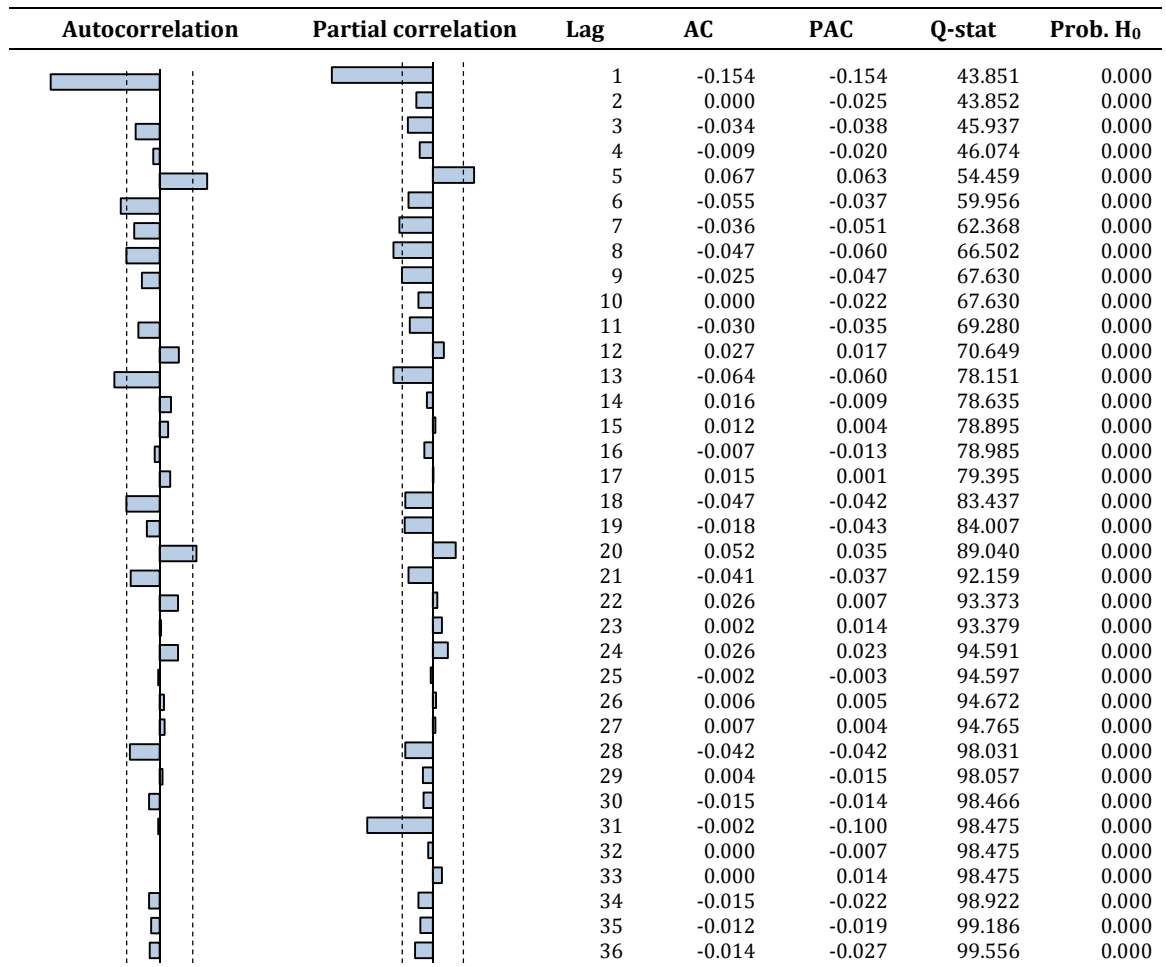


Figure 16: Correlogram for gas returns, testing for autocorrelation up to 36 lags. Significant autocorrelation is present in the sample; the dotted lines indicate the 5% standard errors (± 0.047). Higher lags appear to have random signs.

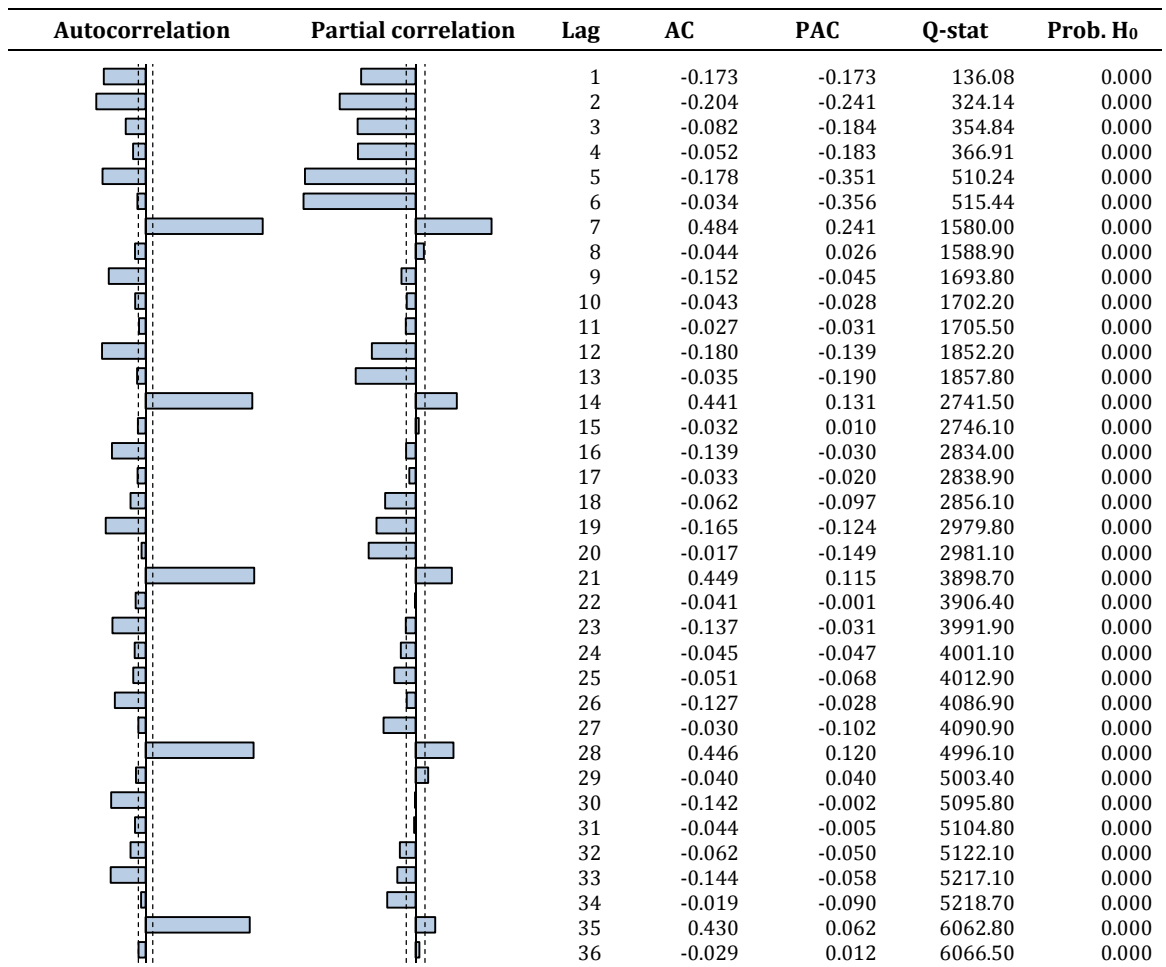


Figure 17: Correlogram for electricity returns, testing for autocorrelation up to 36 lags. Significant autocorrelation is present in the sample; the dotted lines indicate the 5% standard errors (± 0.030). A weekly pattern can be observed.

5.7. Time effects

5.7.1. Theory of time effects

Time effects are predictable deviations of a price series based on a time indicator. As the demand for gas and electricity varies depending on the day of the week and the time of the year, it is possible that these differences are reflected in their prices. Accounting for such time effects improves the accuracy of the forecasting models. We expect time effects for electricity to be stronger due to its lack of storage possibility. Storage of gas makes it possible to mitigate demand effects, particularly in the short term for which storage costs are low.

A visual observation of the price series can give a first indication of daily or seasonal effects. We can observe daily differences by a repeating weekly pattern in the graph. Alternatively, we can construct chronological time series for each day of the week, plotting all seven series in the same figure. In this way daily differences can be observed more clearly; we can see whether daily prices consistently differ from each other. The seasonal effect can be observed as an annual wave pattern, we may note a semi-annual peak as well.

To formally test whether time-related patterns are present in a time series, Franses (1992) suggests first estimating an autoregressive model for the entire series, and then checking whether this model fits the subsets sorted on time indicators as well. If the hypothesis that the parameter

estimates are the same for all subsets is rejected, it is implied that a time pattern is present in the series. When applying a conventional F-test on the residuals of the models estimated for the entire series and the subsets, assumptions of normality and GBM property of the error term need not to be valid. To test for time effects, we perform a simple regression test based on He (2007). We test two regression models against each other, one with and one without a time effect:

$$S_t = \bar{S} + \varepsilon_t \quad (5.13)$$

$$S_t = \bar{S} + \gamma_{i,j} D_i + \varepsilon_t \quad (5.14)$$

where S_t is the asset price at time t , $\bar{S}$ is the average price of the entire sample, $\gamma_{i,j}$ is a binary variable indicating the day or month, D_i is a variable which sets the equation equal to the daily or monthly average (i.e., the average deviation from the mean) and ε_t as the error term with an $N(0, \sigma^2)$ distribution. Except for the term D_i both models are identical. The expected value of the prices forecasted by (5.13) is given by (5.15), the forecasts of (5.14) are given by (5.16).

$$E[S_t] = \bar{S} \quad (5.15)$$

$$E[S_t | \gamma_{i,j} = 1] = \bar{S} + D_i \quad (5.16)$$

To test whether the time effects are significant, we compare the performances of models (5.13) and (5.14) in an F-test, calculating the F-value as

$$F_value = \frac{\left(\frac{RSS1 - RSS2}{v_2 - v_1} \right)}{\frac{RSS2}{n - v_2}} \quad (5.17)$$

where RSS1 stands for the squared sum of the residuals for (5.13), RSS2 stands for the squared sum of the residuals for (5.14), v_1 stands for the number of variables in (5.13), v_2 stands for the number of variables in (5.14), and n stands for the number of data points in the sample to which both models are applied. The null hypothesis is that the factor D_i is not significant (i.e., $D_i = 0$), and will be rejected if the calculated F-value exceeds the specified critical value.

5.7.2. Results of testing for time effects

We test the historical data sets for daily and monthly deviations. As already seen in Section 5.6, autocorrelation in the electricity series strongly resembles a weekly pattern. Holidays are also known to have different and predictable deviations; these effects are not tested for because we expect them to have limited influence on the cash flows of the project. To test whether seasonality is present in the set of gas prices, first we perform a visual test to see the monthly deviations from the average price level. In Figure 18 we provide a plot of these deviations. We note that the prices in January, November and December are higher than during the rest of the year. For the rest of the year, differences are usually small and without a clear pattern. Though prices are higher in the winter, a wave-like seasonal pattern cannot be observed from this data. Regressing a sine function on the deviations yields an R^2 of 0.57, indicating a rather poor fit. Figure 7 also does not show repeating seasonal patterns. Following the procedure of He (2007), we formally test the significance of monthly deviations. When the calculated F-value exceeds the critical value, H_0 : *the seasonal parameter has no significance influence*, is rejected. As can be seen in Table 8, the monthly deviations that are significant at the 5% level are January, April, August, November and December. In our gas price model, we will address monthly deviations with binary variables, rather than fitting a sine function.

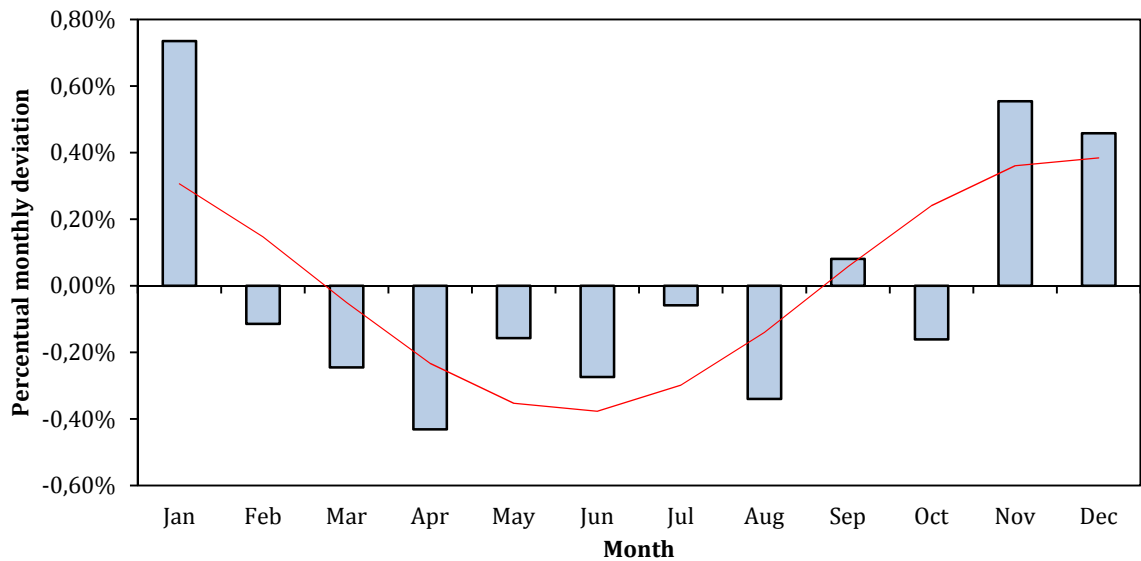


Figure 18: Average monthly deviations from average gas price 2006-2011, with fitted sine function. The winter months show notably higher average prices, but no clear sine pattern can be distinguished.

Month	F-value	5% Critical value	Result
January	31.62	3.90	Reject H_0
February	0.90	3.90	Not reject H_0
March	0.04	3.90	Not reject H_0
April	4.80	3.90	Reject H_0
May	2.32	3.90	Not reject H_0
June	3.22	3.90	Not reject H_0
July	0.71	3.90	Not reject H_0
August	12.99	3.90	Reject H_0
September	0.48	3.90	Not reject H_0
October	1.13	3.90	Not reject H_0
November	17.45	3.90	Reject H_0
December	10.64	3.90	Reject H_0

Table 8: F-test for monthly effects in gas prices. Deviations are significant for 5 months at the 5%-level.

Next, we test the daily deviations for the gas prices, starting with a visual observation. As 2006 and 2007 are the only years which include price data for every day, we plot the gas price series per day for these years in chronological order. This way, every day of the week is equally represented in the graph. From Figure 19, the daily prices appear to follow a highly similar path. There is no visual indication that any day has a significantly different pattern than the other days. Figure 20 shows the percentual deviations from the average price level. We note that weekend prices are somewhat lower, but most deviations are less than 1% of the price. We perform the F-test to assess the presence of daily deviations. In Table 9 we show that no significant daily effects are present in the analysed gas price data. We therefore will not include daily deviations in the gas price model.

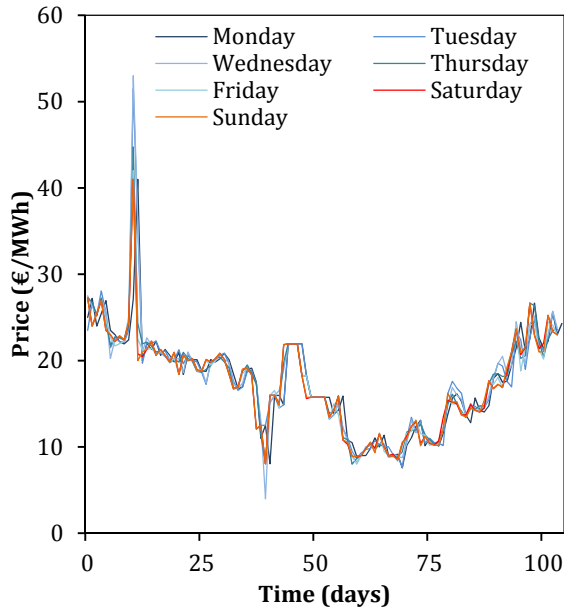


Figure 19: Gas prices 2006-2007 sorted by weekday. No significant deviations can be observed.

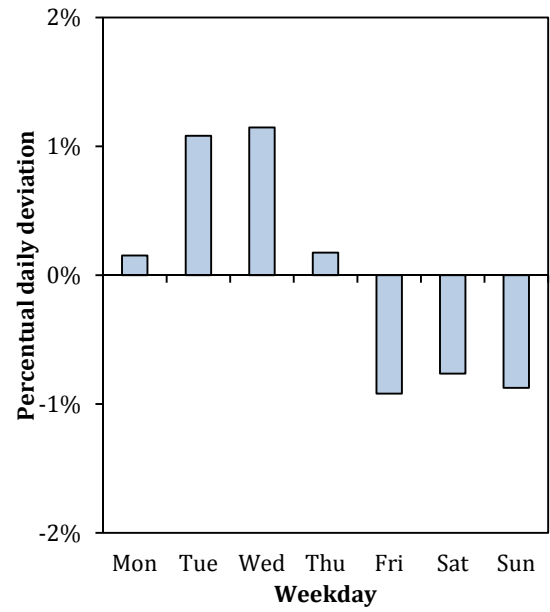


Figure 20: Average daily deviations from average gas prices 2006-2011.

Day	F-value	5% Critical value	Result
Monday	0.00	3.93	Not reject H_0
Tuesday	0.10	3.93	Not reject H_0
Wednesday	0.10	3.93	Not reject H_0
Thursday	0.00	3.93	Not reject H_0
Friday	0.09	3.93	Not reject H_0
Saturday	0.06	3.93	Not reject H_0
Sunday	0.08	3.93	Not reject H_0

Table 9: F-test results for daily effect in gas prices 2006-2011. No significant deviations are found.

We perform the same tests for the electricity prices between 1999 and 2012. In Figure 21, the average monthly deviations of the total average are given. A single wave-like pattern can be observed during the year; the R^2 of the regressed sine function is 0.83. The largest differences are observed for the months January, June and September; we try to address these deviations by estimating a semi-annual sine function as well in the modelling phase. From Figure 8 it may be difficult to observe the seasonal pattern due to its high volatility. The F-test results of testing for monthly effects in electricity prices are provided in Table 10. Six deviations are considered to be significant. Naturally deviations are not significant for every month, as periods of moderate demand provide prices close to the mean. The fact that half of the deviations are significant indicates incorporating a seasonal effect in the forecasting model would improve its accuracy.

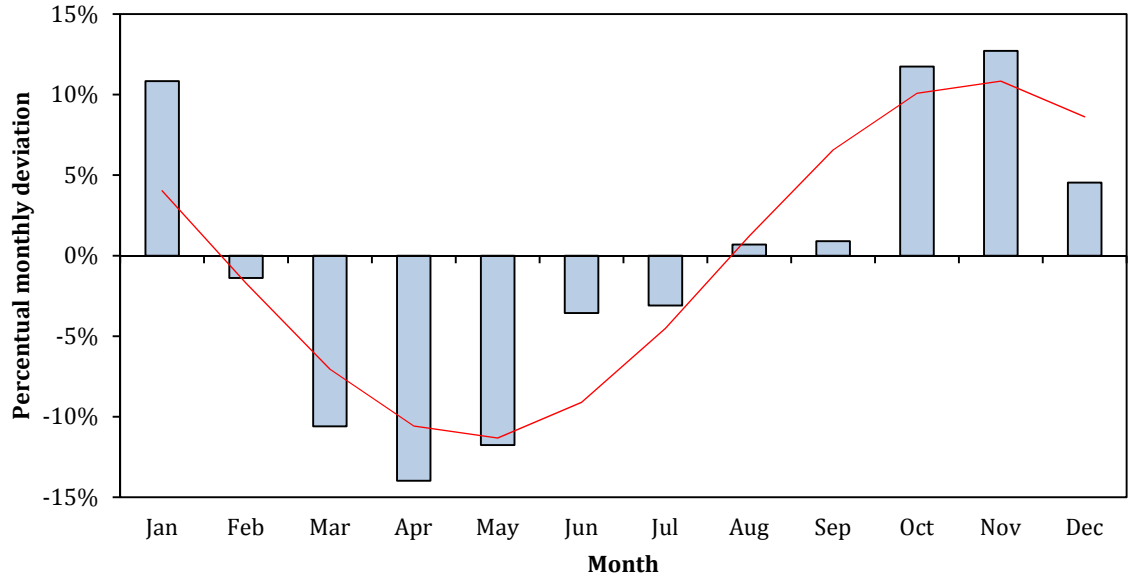


Figure 21: Average monthly deviations from average electricity prices 1999-2012, with fitted sine function. A sine-like pattern can be observed, differing most notable for January, June and September.

Month	F-value	5% Critical value	Result
January	7.83	3.86	Reject H_0
February	0.43	3.87	Not reject H_0
March	31.44	3.87	Reject H_0
April	46.90	3.87	Reject H_0
May	20.12	3.87	Reject H_0
June	1.95	3.87	Not reject H_0
July	0.89	3.87	Not reject H_0
August	0.01	3.87	Not reject H_0
September	0.14	3.87	Not reject H_0
October	18.97	3.87	Reject H_0
November	18.01	3.87	Reject H_0
December	2.79	3.86	Not reject H_0

Table 10: F-test results for monthly effects in electricity prices 1999-2012. Six months have significantly deviating prices.

Finally, daily effects in the electricity series are tested. As the spikes in electricity prices have a strong effect on the scale when plotting the daily series, we set an upper bound of € 100 for plotting Figure 22. Prices above this bound were removed from the sample to make the differences between daily price series better observable.⁶ As for the gas price series we only show the 2006-2007 subset; the high volatility of the series would make it difficult to observe the differences for larger samples. We note that the price series for weekend days are consistently below the week day price series. The average daily deviations of the prices clearly show this, as can be noted in Figure 23. We perform an F-test on the full data set to formally research the differences between daily electricity prices. The results in Table 11 indicate that all differences are statistically significant at the 5% level, except for Friday.

⁶ The removal of prices causes some minor shifts between the graphs, as extreme prices are more often observed during week days.

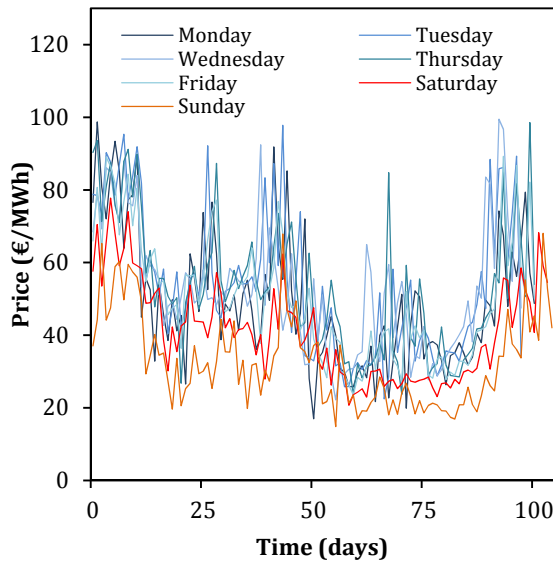


Figure 22: Electricity prices 2006-2007 sorted by weekday. Weekend prices are notably lower.

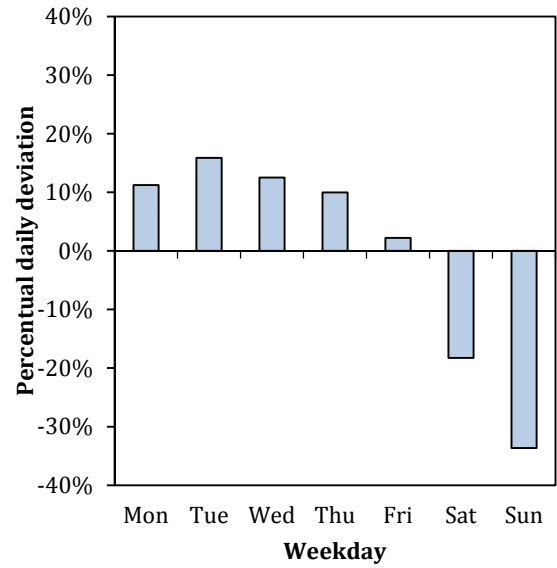


Figure 23: Average daily deviation from average daily electricity price 1999-2012.

Day	F-value	5% Critical value	Result
Monday	12.43	3.87	Reject H_0
Tuesday	32.44	3.87	Reject H_0
Wednesday	17.47	3.87	Reject H_0
Thursday	21.76	3.87	Reject H_0
Friday	1.50	3.87	Not reject H_0
Saturday	185.73	3.87	Reject H_0
Sunday	983.37	3.87	Reject H_0

Table 11: F-test results for daily effect in electricity prices 1999-2012. Six daily deviations are statistically significant.

5.8. Mean-reversion of price series

5.8.1. Theory of mean-reversion

A number of definitions exist for the concept of mean-reversion (Exley *et al.*, 2004). In this study, when using the term mean-reversion, we refer to a process which exhibits negative correlation between differences over disjoint intervals. Several tests exist to indicate whether a series is mean-reverting or not. However, we did not find a generally accepted test, perhaps due to the absence of a single prevailing definition of the process.

Visually, we can observe mean-reversion in plots of the price series when prices stay close to a long-term trend, and revert stronger to this trend when prices drift away from the trend. A graph of the return series should show alternating periods of positive and negative returns, with smaller returns around the mean and larger returns when deviating from the trend line. According to Fama & French (1988), mean-reversion in the stationary component of price series can be observed by the presence of negative autocorrelations in returns. If the price series is stationary, this is another indicator that mean-reversion exists. A more formal test to indicate mean-reversion is to calculate the Hurst exponent (Cajueiro & Tabak, 2005; Sánchez Granero *et al.*, 2008). The origins of this approach lie in the field of hydrology, but it was later applied in finance as well. The intuition behind this test is to relate the rate of decrease in autocorrelation to increasing time lags. The test model is described by

$$cn^H = \frac{\max(z_1, \dots, z_n) - \min(z_1, \dots, z_n)}{\sqrt{\frac{1}{n} \sum_{t=1}^n (x_t - \bar{x})^2}} \quad (5.18)$$

with

$$z_n = \sum_{t=1}^n x_t - \bar{x}$$

x_t describes the return at time t and $\bar{x}$ is the average return for the subseries of length n . c is a constant and H is a number called the Hurst exponent, with $0 < H < 1$. In the test, first the right-hand side of Equation (5.18) is calculated. Then we use least squares regression to calculate c and H . When the Hurst exponent is a number < 0.5 , the series is considered to be mean-reverting as it has no persistent trend (Qian & Rasheed, 2004). The closer the Hurst exponent is to 0, the stronger the mean-reverting effect. Note that the denominator in (5.18) represents the volatility of the series, so that the range in the numerator is proportionally rescaled.

5.8.2. Results of testing for mean-reversion of price series

Visually, we observe that the price series stays close to the average price levels in the long run (see Figure 7 and Figure 8). Prices deviate from these levels, but over the course of years move only little on average. The gas price is shown to revert slowly, having periods of years in which the price is constantly above or under the average price level. The electricity price seems to be reverting to its mean quicker. From the correlograms presented in Section 5.6, it follows that negative correlations are found in both gas and electricity returns. This is an indicator for the existence of mean-reversion. Stationarity of the electricity price series has been proven in Section 5.4, this was not the case for the gas price series. We calculate the Hurst exponent for the gas and electricity series, using the entire sample sets. We show the results of the tests in Table 12. Both estimates for H are smaller than 0.5, indicating that both series are mean-reversion. The Hurst exponent for the electricity series is stronger than for the gas series. This shows that the reverting effect is stronger for the electricity series, which is in line with the observed visual patterns.

Series	Hurst exponent (H)	Result
Natural gas	0.41494	$H < 0.5$, so the series is mean-reverting
Electricity	0.20172	$H < 0.5$, so the series is mean-reverting

Table 12: Estimated Hurst exponents for gas and electricity price series, both indicating mean-reversion.

5.9. Normality

5.9.1. Theory of normality

A simplifying assumption often made in price series modelling is that returns follow a normal distribution. If returns are normally distributed, they can be modelled with the Geometric Brownian Motion. For this reason the return series are often tested for normality (Alexander, 2001). The test is also applied to the residual after constructing a model, to check whether the error term can be modelled as a Geometric Brownian Motion.

Normality can be visually observed by plotting the returns in a histogram, which should resemble a bell-shaped curve. Some well-known formal tests for normality are the Shapiro-Wilk test, the Kolmogorov-Smirnov test and the Jarque-Bera test. Frain (2007) states the Jarque-Bera test outperforms the other normality tests in most cases. In financial data analysis it is the most

commonly applied test. We perform the Jarque-Bera test in this study as well, due to the autocorrelation we proved in Section 5.6 some modifications are required. The Jarque-Bera test can be performed to test whether a data set has the skewness and kurtosis matching a normal distribution (Jarque & Bera, 1987; Alexander, 2001). A normal distribution has a skewness of 0 and a kurtosis of 3. Under the condition that returns are independent of each other (i.e., no autocorrelation), the standard errors of these moments are approximated by $\sqrt{6/n}$ and $\sqrt{24/n}$ respectively, where n stands for the number of observations and should be sufficiently large. The Jarque-Bera test statistic JB is given by

$$JB = \frac{n}{6} \left(\text{Skewness}^2 + \frac{1}{4} (\text{Kurtosis} - 3)^2 \right) \quad (5.19)$$

with

$$\text{Skewness} = \frac{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^3}{\left(\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 \right)^{3/2}}$$

$$\text{Kurtosis} = \frac{\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^4}{\left(\frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2 \right)^2}$$

H_0 is a joint hypothesis stating that the skewness is 0 and the excess kurtosis (i.e., the deviation from 3) is 0. The test statistic has a chi-squared distribution. When the test statistic exceeds the specified critical value, we reject the null hypothesis. Hence, normality is then rejected based on skewness and kurtosis. In EViews, the Jarque-Bera test statistic is readily available, combined with the probability that H_0 is not rejected (i.e., non-normality is not proven).

As stated before, the estimates of the standard errors are invalid when returns are autocorrelated. One approach would be to first remove autocorrelation from the series, and then apply the Jarque-Bera test on the new series. A disadvantage of this approach is that it first requires specifying a model which removes autocorrelation, while we have not started modelling yet. Alternatively, it is possible to adjust the estimated variance of skewness and kurtosis for autocorrelation. Following the approach of Lobato & Velasco (2004), we modify the Jarque-Bera test statistic to account for the variance stemming from correlation. First we calculate the autocovariance (ACV) for all $n - 1$ possible lags h . Autocovariance is defined as

$$ACV(h) = \frac{1}{n} \sum_{t=1}^{n-|h|} (x_t - \bar{x})(x_{t+|h|} - \bar{x}) \quad (5.20)$$

Consequently, these autocovariances are taken to the power of 3 (for skewness) or 4 (for kurtosis) and summed:

$$ACVF^{(3)} = \sum_{h=1-n}^{n-1} ACF(h)^3$$

$$ACVF^{(4)} = \sum_{h=1-n}^{n-1} ACF(h)^4 \quad (5.21)$$

The second, third and fourth central moments of the series are required as well, these are given by

$$m_i = \frac{1}{n} \sum_{t=1}^n (x_t - \bar{x})^i \quad (5.22)$$

for $i = 2, 3, 4$

Finally we define the adjusted test statistic, here called JB_{adj}

$$JB_{adj} = \frac{n * m_3^2}{6 * ACVF^{(3)}} + \frac{n * (m_4 - 3m_2^2)^2}{24 * ACVF^{(4)}} \quad (5.23)$$

Like in the original test, the test statistic is compared to the critical value from the chi-squared distribution, and we reject normality when the test statistic exceeds the critical value. Caution should be taken when interpreting the test result; for modelling purposes the result *only* holds meaning when the model actually succeeds in fully removing autocorrelation. The adjusted test does not reflect the actual distribution of the raw data set. Bao (2012) endorses the method proposed by Lobato & Velasco (2004), but warns that bias-correction in finite samples may not be sufficient, particularly when testing on a large excess kurtosis.

5.9.2. Results of testing for normality

EViews combines a plotted histogram with descriptive statistics of its distribution, and calculates the Jarque-Bera test statistic for the series. Due to the presence of autocorrelation in the series, these figures do not say much about their normality; we only add Figure 24 and Figure 25 to provide some insight in the actual distributions of the return series. Performing the adjusted Jarque-Bera test yields test statistics of 24,148 for gas returns and 1,838 for electricity returns. In Table 6 the results of the tests are shown. At the 0.05 significance level, electricity returns are shown to follow a normal distribution after removing autocorrelation. For gas returns this is not the case. We note that these values are much lower than the Jarque-Bera test statistics shown in Figure 24 and Figure 25, indicating that the distributions would approach normality better after removing autocorrelation. In the case of gas returns the GBM is still considered a misspecification. The consequence of these findings is that the GBM can be applied for the electricity price model, granted that autocorrelation is fully removed. For the gas price model this is insufficient, further refinements are then required to apply the GBM in a statistically sound manner.

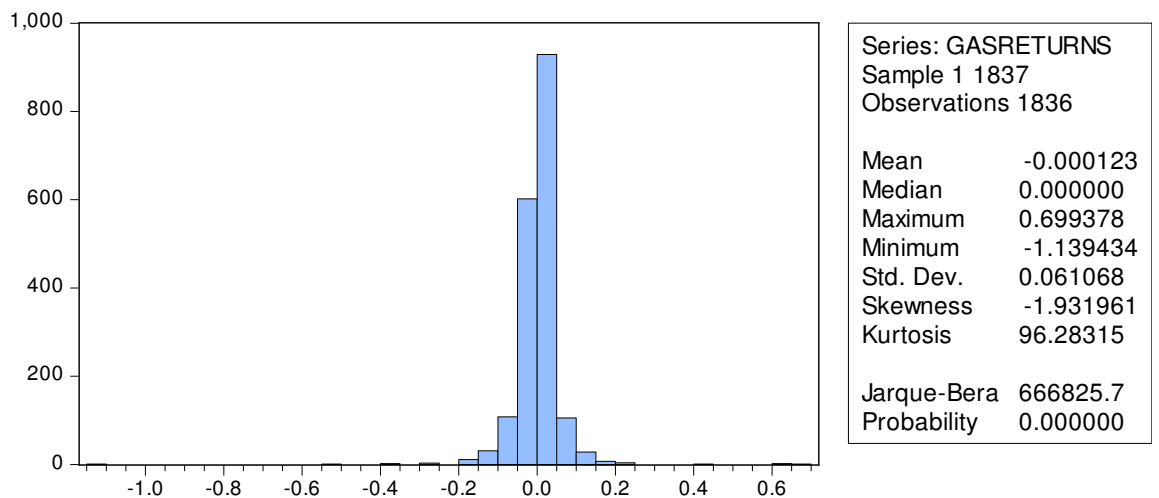


Figure 24: Histograms and descriptive statistics of gas returns 2006-2011.

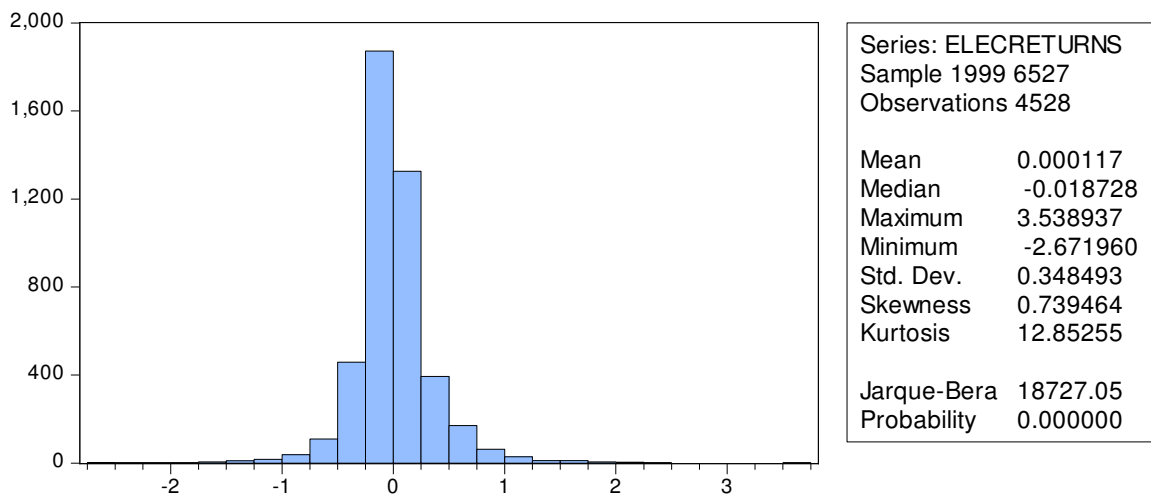


Figure 25: Histogram and descriptive statistics of electricity returns 1999-2012.

Series	Adjusted Jarque-Bera test statistic	Chi-square critical value at p=0.05	Result
Gas returns	24148	1935	Reject H_0 , returns not normally distributed.
Electricity returns	1838	4683	Not reject H_0 , returns not proven to be non-normally distributed.

Table 13: Results of adjusted Jarque-Bera test on gas and electricity returns.

5.10. Cross-correlation

5.10.1. Theory of cross-correlation

Returns of an asset are often not independent of the movements of other assets in the market; they tend to be related in some way. A common measure for such a relationship is the correlation coefficient, which is the linear co-dependency between two variables. To distinguish correlation between times series from autocorrelation within a time series, the former is also referred to as cross-correlation. In the case of gas and electricity prices, the fact that natural gas serves as physical input for producing electricity is a reason to suspect they are co-dependent. Correlation can be positive or negative. If correlation is positive, one asset moving up in price means that the other asset is likely to move up in price as well. For negative correlation this relationship is inverse. A cross-correlation of 0 does not necessarily mean that the series move independent of each other, but interdependency may be lagged or described by some nonlinear relationship (Yule, 1926). A correlation test should be performed always on the return series. Calculating the correlation between the price series would place a very high weight on early returns. This is because prices are basically an integral of returns up to time t , i.e., $S_t = S_0 + \int_{i=1}^t x_i$. Therefore the correlation of price series has little practical value for forecasting models.

We can make a distinction between unconditional correlation (constant over time) and conditional correlation (time-varying). Unconditional correlation between two return series can only exist in case the series are jointly covariance-stationary, meaning that the covariance between

returns $r_{g,t}$ and $r_{e,t-h}$ depends only on the lag h (Alexander, 2001). We can test this property by first estimating a bivariate vector autoregression model (VAR)⁷, which is defined as

$$\mathbf{Y}_t = \begin{bmatrix} c_g \\ c_e \end{bmatrix} + \begin{bmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{bmatrix} \mathbf{Y}_{t-1} + \begin{bmatrix} \varepsilon_{g,t} \\ \varepsilon_{e,t} \end{bmatrix} \quad (5.24)$$

where $\mathbf{Y}_t$ is the vector containing the gas and electricity returns at time t , c_g and c_e are constants for the gas and electricity series respectively, $\varepsilon_{g,t}$ and $\varepsilon_{e,t}$ are error terms, and $A_{1,1}$ to $A_{2,2}$ are parameters to be estimated. As the model can be rewritten as a set of linear equations, we can estimate them independently with ordinary least squares regression. After doing so, we can solve the quadratic equation (5.25).

$$1 - \text{tr} \begin{bmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{bmatrix} z + \begin{vmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{vmatrix} z^2 = 0 \quad (5.25)$$

where $\text{tr} \begin{bmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{bmatrix}$ is the trace of the matrix (the sum of the main diagonal), $\begin{vmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{vmatrix}$ is the determinant of the matrix and z can be a real or a complex number (Lay, 2006). If at least one solution of the quadratic equation lies outside the unit circle in the complex plane (a circle with a radius of 1), the series is jointly covariance-stationary. Only then it is possible to meaningfully calculate unconditional correlation between the returns.

The Pearson correlation coefficient (PCC) can be calculated to check whether the series are correlated (Rodgers & Nicewander, 1988; Hull, 2010; Ruiz *et al.*, 2012). The Pearson correlation coefficient is given by

$$PCC = \frac{\frac{1}{n} \sum_{t=1}^n (x_{g,t} - \bar{x}_g)(x_{e,t} - \bar{x}_e)}{\sqrt{\frac{1}{n} \sum_{t=1}^n (x_{g,t} - \bar{x}_g)^2} * \sqrt{\frac{1}{n} \sum_{t=1}^n (x_{e,t} - \bar{x}_e)^2}} \quad (5.26)$$

where n is the number of observations, $x_{g,t}$ and $x_{e,t}$ are gas and electricity returns at time t respectively, and the barred returns represents the mean. The sample distribution of the PCC follows a Student distribution with $n - 2$ degrees of freedom, its test statistic is given by

$$t_value = \frac{PCC}{\sqrt{\frac{1 - PCC^2}{n - 2}}} \quad (5.27)$$

Even if series are jointly covariance-stationary, the PCC outcome may not be meaningful. This is because the correlation coefficient is calculated under the assumption that observations are *i.i.d.* and normally distributed (Yule, 1926; Hanssens *et al.*, 2003). If autocorrelation is present in the series the variance of the joint distribution is increased, which is then incorrectly reflected in the PCC (Stephenson, 1997). Also due to time effects lagged correlation effects can be present; calculating the PCC with a time lag may influence results. Stated formally, the calculated cross-correlation is then assumed to be a function of autocorrelation. If the series are not normally distributed, the significance test provides inaccurate test statistics, so we cannot state with certainty whether correlation is significant or not. Therefore, time series should be detrended and normalised first to establish a truly meaningful cross-correlation (Podobnik & Stanley, 2008; Horvatic *et al.*, 2011).

⁷ The univariate version of this model (AR) is briefly explained in Appendix I.

Aside from unconditional correlation, more complex dependencies can exist between the series as well. Tests for such dependencies are not performed in this study, but we briefly mention some for completeness. A technique to discover nonlinear relations is Transfer Function Identification (Hanssens *et al.*, 2003; Borghers & Wessa, 2012). This involves calculating the so-called half slope ratio. The value of this ratio allows estimating transformation parameters which adjust time series in such a way that non-linear relations become linear. Correlation can then be estimated based on the adjusted series. Authors such as Conlon *et al.* (2008) and Plerou *et al.* (2008) apply Random Matrix Theory to discover complex interdependencies between time series. This approach consists of comparing the eigenvalues of an estimated correlation coefficient matrix to those of randomly generated matrices, checking for statistically significant deviations between them to filter out interdependencies that are not directly observable.

5.10.2. Results of testing for cross-correlation

As the return series for natural gas does not include a value for each date, first we remove the matching non-trading days from the return series for electricity. For the remaining data, the VAR(1) model as described in (5.24) is estimated. The following quadratic equation is obtained:

$$1 + 0.28438z + 0.01960z^2 = 0$$

Solving this equation yields real solutions of -5.98... and -8.52..., which both lie outside the unit root circle. Hence, cross-correlation can be estimated for the series. As the series exhibit autocorrelation and non-normality, this coefficient says little about the actual sign, magnitude and significance of the correlation between both series.

We obtain a Pearson correlation coefficient of 0.0760 or 7.6%. We perform a t-test to assess the significance of this coefficient. Though the return series for natural gas is not normally distributed, the large sample size of 1837 might be sufficient for the test to obtain a reasonable estimate of significance. The t-test yields a t-value of 2.3065. The 5% critical value is 1.6525, hence the cross-correlation is statistically significant at this level, under the assumption that cross-correlation is a function of autocorrelations.

5.11. Price jumps

5.11.1. Theory of price jumps

A price jump as referred to in this study is a large price deviation from the prevailing price level, which can be either positive or negative. What deviation is considered 'large' is up to debate; no formal definition of a jump exists. Mathematically a jump is considered to be a point or set of points at which the return function is not continuous (Merton, 1976). For an infinitely small time interval, the GBM only allows for infinitely small movements, while a jump is an instantaneous large moment on an infinitely small time interval. Unfortunately, this mathematical definition provides no guidance on how to distinguish jumps in a discrete time series.

A simple practical definition of a jump is a price which differs from the unconditional mean by three times the standard deviation or more, informally dubbed the 3σ procedure (Mancini & Renò, 2006; He, 2007).⁸ When prices are generated by the GBM, such extreme price levels should occur very rarely. The presence of jumps therefore implies fatter tails than observed in a normal return distribution. Note that jumps by itself increase the standard deviation of the series, causing the 3σ border to increase. As jumps are rare events, it is difficult to model a gener-

⁸ The same procedure may be applied on the return series, but makes it more difficult to distinguish jumps from other processes such as mean-reversion.

ating process for them based on a limited amount of observations. The available historical data may contain too few jumps to estimate their frequency, expected size and volatility.

5.11.2. Results of testing for price jumps

Using the 3σ procedure, we distinguish seven prices meeting this criterion in the gas price series, out of a total of 1837 observations. This corresponds to about 0.38% of the series. We show the filtered series containing only the jumps in Figure 26. The filtered series consists of seven subsequent days with unusual high prices. Therefore, only a single extreme return in the series causes the jump, with the prices following after the jump returning towards a more moderate level. Based on only a single event, no generating process can be estimated for the series. Also, the effect of such rare jumps on the project value will be limited. For these reasons, we do not explicitly consider jumps in the gas price model.

We perform the same procedure on the electricity price series, as illustrated in Figure 26. For the electricity price series 53 jumps are found, accounting for roughly one percent of the sample. Jumps cause very high standard errors for skewness and kurtosis, indicating why the adjusted Jarque-Bera test proves normality and at the same time we note a large number of jumps. The number of jumps indicates that they play a significant role in the price process for electricity. Recall that the series is more volatile in the earlier years than in later years, meaning that the average standard deviation does not reflect the standard deviations of more recent data. When only the later years were considered, standard deviation would be smaller, perhaps resulting in more prices being regarded as jumps which fall within the 3σ boundaries now. In Chapter 7, we estimate a jump diffusion process to account for jumps in our electricity price model.

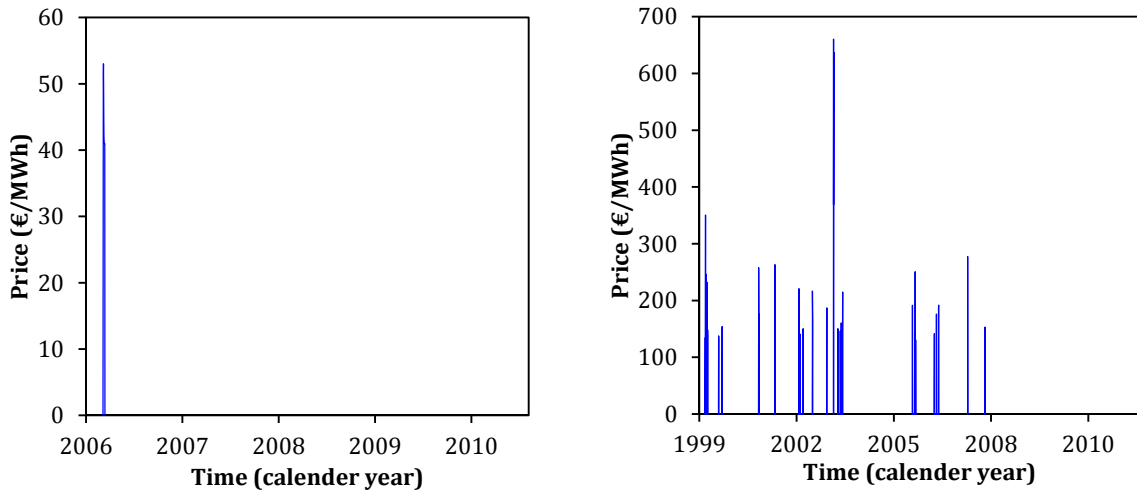


Figure 26: Jumps filtered from gas price sample 2006-2011 (left) and from electricity price sample 1999-2012 (right).

As a key reason to separate jumps is to improve the normality property of a time series, we redo the normality test on the series without the jumps. Autocorrelation for both series remains significant and largely unchanged, so we again apply the adjusted Jarque-Bera test. For the gas series, JB_{adj} drops notably to 20,663; this value still far exceeds the critical value. For the electricity sample, JB_{adj} in fact sharply increases to 30,831, so that we must now reject the hypothesis of a normal distribution. The absence of jumps strongly decreases the standard errors in the test statistic, causing the remarkable difference. Still, removing jumps should improve the normality property (the regular JB confirms this presumption, providing a new value of 8,485). These

seemingly inconsistent results raise questions about the performance of the adjusted Jarque-Bera test, even though Bao (2012) warned for distortions in case of a large excess kurtosis.

5.12. Chapter summary

In this section we provide an overview of the diagnostic tests performed in this chapter. Recall the purpose of these tests was to increase insight in how the historical price series behave. Based on this knowledge, we determine our focus for the modelling techniques relevant for our price models. Table 14 shows the key results and their implications for modelling for all tests.

Test	Key results	Implications
Stationarity	Both return series are stationary.	Several statistical tests can be applied on these series meaningfully.
Cointegration	Electricity price series is stationary, so no cointegration relationship can be established.	Possible co-dependency cannot be described by cointegration.
Autocorrelation	Gas prices contain significant autocorrelation, no clear pattern was found.	Autocorrelation effects should be removed from the series.
	Electricity prices contain significant autocorrelation, a clear weekly pattern was observed.	Autocorrelation in electricity prices caused by time effects.
Time effects	Gas: No significant differences in day prices. Prices in winter months significantly higher than average, no smooth seasonal pattern.	Gas price model should account for monthly deviations.
	Electricity: Weekend prices significantly lower than average. A statistically relevant seasonal pattern is observed.	Electricity price model should account for daily deviations and seasonal pattern.
Mean-reversion	Both price series are mean-reverting; the effect is stronger for electricity prices than for gas prices, being rescaled for the difference in volatility.	Price models should incorporate mean-reversion.
Normality	Gas prices are not normal after removing autocorrelation.	The GBM is applicable for electricity after removing autocorrelation. Results are not valid when autocorrelation is not completely removed.
	Electricity prices are normal after removing autocorrelation.	
Cross-correlation	The return series are jointly covariance-stationary, calculated in the presence of autocorrelation. The cross-correlation is 7.6%.	Cross-correlation can be calculated, but has little practical meaning due to violated assumptions.
Price jumps	Gas prices contain almost no jumps.	Gas jumps are ignored in modelling.
	Electricity prices contain many jumps.	Electricity jumps should be estimated by a separate process.

Table 14: Summary of diagnostic tests.

Techniques for modelling energy prices

There are many models and techniques available to model the behaviour of price series. In this chapter, we briefly discuss some main models are, as well as several important processes often used to extend basic models. For incorporating mean-reversion, we discuss the Ornstein-Uhlenbeck process. We treat the jump diffusion model for its ability to cope with unpredictable price jumps. To account for deterministic time effects, we describe the use of binary variables and the sinusoid time function. GARCH is explained as it can be applied to reflect time-varying volatility. We include a brief overview of techniques that were researched, but not used in this study. To conclude the chapter, we explain how we can estimate the risk-neutral drift of commodity prices.

6.1. Geometric Brownian Motion

The Geometric Brownian Motion (GBM) is the most basic model to generate a price path (Hull, 2008). In the Black-Scholes framework, asset prices are expected to follow the GBM. Often a linear trend is also added, reflecting an expected long-term return. Returns following the GBM with a linear trend are defined as

$$x_t = \alpha dt + \sigma dW_t \quad (6.1)$$

with

x_t	as the return at time t
α	as the deterministic drift
σ	as the volatility or standard deviation
W_t	as a Wiener process representing a continuous-time random walk

In the long term a price series following (6.1) is expected to follow the deterministic drift α , but the path moves randomly due to the Wiener process incorporated. The larger the volatility relative to the drift, the stronger the prices may deviate from the drift. The drift and volatility are both assumed to be constant, so that the return process does not change over time. The return process is normally distributed; the rationale behind this assumption is that returns are influenced by a large number of independent random variables. This makes their sum approximately normally distributed by the Central Limit Theorem. Samuelson (1965) provides a proof for price changes being normally distributed in a market with perfect information. The empirical validity of the GBM is widely debated (Mandelbrot, 1963; Luenberger, 1998; Marathe & Ryan, 2005).

6.2. Ornstein-Uhlenbeck process

Processes may not follow a random walk as suggested by the GBM, but rather fluctuate around some long-term mean. The Ornstein-Uhlenbeck process is a modification of the geometric Brownian motion that incorporates mean-reversion (Uhlenbeck & Ornstein, 1930), described by

$$x_t = \kappa(\bar{S} - S_t)dt + \sigma dW_t \quad (6.2)$$

In this model, $\bar{S}$ represents the mean price, S_t represents the price at time t and κ represents the rate at which the price returns to its long-term average. This equation shows that the larger the difference between price at time t and the long-term price level is, the stronger the movement back towards the mean becomes. The term $\bar{S} - S_t$ increases in size the more S_t deviates, causing a stronger effect. If the process reverts to an equilibrium level which changes linearly over time, we can include this by replacing $\bar{S}$ by $(\bar{S} + \alpha S_t dt)$. The Ornstein-Uhlenbeck process is often incorporated in more complex models, allowing to model mean-reverting behaviour of multiple variables such as prices, interest rates and volatilities.

6.3. Jump diffusion model

The jump diffusion model assumes that the occurrence of price jumps follows a Poisson process, while the mean and volatility of the jump size can be modelled with a lognormal distribution (Merton, 1976; Tankov & Voltschkova, 2009). The Poisson assumption implies that price jumps occur independently of each other and have an identical distribution. In Equation (6.3) we show how the Poisson jump is modelled (Craine *et al.*, 2000). The last term of the equation is an addition to the standard GBM process. This extension is used to correct the GBM for the fat tails in returns that are often observed in finance.

$$x_t = \alpha dt + \sigma dW_t + Jdq \quad (6.3)$$

Here the jump size J is drawn from a normal distribution $J \sim N(\mu_J, \sigma_J^2)$ and dq is a Poisson counter with intensity φ , so that $\text{Prob}(dq = 1) = \varphi dt$.

A shortcoming of the original jump diffusion model as developed by Merton (1976) is that it does not incorporate mean-reversion (Cartea & Figueroa, 2005). Therefore after a jump occurs the price is not corrected, but it rather remains at its new level. Including a mean-reverting term allows the price to return quickly to its equilibrium level after a jump, which is a more realistic representation of spikes observed in energy prices. This means we should replace the first term of (6.3) with that of the Ornstein-Uhlenbeck process described in Equation (6.2). De Jong & Huismans (2002) state that the mean-reverting parameter κ is generally overestimated in order to correct for reversion after jumps. We can avoid this problem by separating the jump model from the model describing the normal market state, i.e., estimating Jdq on a data set containing only the filtered jumps. It is often difficult to estimate the parameters of a jump diffusion model in an accurate manner (Gijbels *et al.*, 2005). Particularly how to return to the regular price behaviour after a jump occurs is a notable issue. Authors have proposed several *ad hoc* approaches such as correlated positive and negative jumps, special mean-reverting parameters for jumps, etc., yet no uniform approach exists on how to estimate and model a jump generating process.

6.4. Binary variables

As stated before, the price of electricity is subject of many predictable patterns. Such patterns can be assessed by including binary variables $\gamma_{i,j}$ in the model, having a value of 1 if the current state i equals a certain state j (e.g., day, week, season) and a value of 0 if not (Knittel & Roberts, 2005):

$$\gamma_{i,j} = \begin{cases} 0, & i \neq j \\ 1, & i = j \end{cases} \quad (6.4)$$

This way, we can apply parameters associated with a certain state when suitable, and ignore them if other parameters are more fitting. As such, the model can quickly respond to changing states. A disadvantage of including binary variables is that it quickly expands the number of parameters we need to estimate. Also, particularly when applied to larger time frames, shock adjustments can occur when changing from one time frame from the other, while the actual transition process is often smoother (Lucia & Schwartz, 2002).

6.5. Sinusoid time function

The seasonal pattern observed in energy prices is often seen to follow a wave-like pattern. This pattern can be explained by the strong relation between temperature and the energy demanded for heating and cooling. As temperature changes gradually over the year, so does the price. A way to model this smooth seasonal effect is by incorporating a sinusoid function (Pilipovic, 1998). When we observe multiple wave patterns (e.g., energy prices are high both during winter and summer), we can capture this effect by adding another sinusoid function with another frequency. We provide a generic example of a sinusoid function below. It contains a deterministic time trend, an annual wave pattern and a semi-annual wave pattern.⁹

⁹ Leap years cause minor shifts when applying this formula. These can be accounted for by using the number 365.25 instead of 365, but we believe the effect in absolute price differences is minor for a 20-year forecast.

$$Bt + B_A \left(\sin \left(\frac{2\pi t}{365} + \phi_A \right) \right) + B_{SA} \left(\sin \left(\frac{4\pi t}{365} + \phi_{SA} \right) \right) \quad (6.5)$$

with

B	as a deterministic time trend
t	as the time point
B_A	as the annual amplitude parameter
B_{SA}	as the semi-annual amplitude parameter
ϕ_A	as the annual shifting parameter
ϕ_{SA}	as the semi-annual shifting parameter

6.6. Co-dependency modelling

When a linear relationship exists between certain factors in the models (such as cross-correlation between two price series), we can model this by correlating the Wiener processes (or another random process) of the different models. Only one Wiener process is then truly random, with the depending processes being partially correlated with this process. This relationship is described by Heston (1993). From his model, we can derive the following relation:

$$dW_{1,t} = \sqrt{\rho} dW_{2,t} + \sqrt{1 - \rho} dW_{3,t} \quad (6.6)$$

where ρ represents the correlation between $dW_{1,t}$ and $dW_{2,t}$; $dW_{2,t}$ and $dW_{3,t}$ are independent GBMs. Note that the third GBM is not applied in the model, but serves as a dummy to correlate the other two GBMs.

The correlation model described in Equation (6.6) is a basic one. More complex techniques allow for the incorporation of non-linear dependencies and time-varying relationships. Such models are not used in this study, but merely described as reference. Silvennoinen & Teräsvirta (2008) and Hull (2010) identify several classes of co-dependency models. Models such as Vector Error Correlation Model (VECM) and multivariate GARCH directly estimate a coefficient covariance matrix which describes co-dependencies within the model. Another approach is to describe risk factors in several independent equations, correlating the dependent variables with these equations. We mention these so-called multi-factor models in Appendix I. With the application of copulas, complex marginal distributions of variables are transformed into well-known distributions. For example, the Gaussian copula transforms a marginal distribution in a standard normal distribution. The joint distribution can then be estimated more easily (Meucci, 2011). The copula chosen has a strong effect on the estimated co-dependency between the actual distributions, so often multiple copulas should be tested (Venter, 2002).

6.7. Constant volatility

The simplest and most common approach to estimate the volatility of a time series is to calculate the standard deviation of the time series, and apply this number as the volatility for every forecasting step. In this way, a constant and unconditional estimate for volatility is obtained. We provide the formula for constant volatility estimation below (Alexander, 2001).

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2} \quad (6.7)$$

where n is the number of observations, x_i represents the i_{th} observation and $\bar{x}$ represents the average of observations. The assumption of constant volatility might oversimplify the dynamics of price moments. Volatility is known to fluctuate over time in many cases. Often quiet and active periods of trading alternate, causing periods of low and high volatility. Applying the average volatility then structurally provides incorrect estimates. Choosing an appropriate time window for the estimation of volatility is important; too small a window provides inaccurate results, while too large a window may include data no longer relevant for forecasting (Hull, 2008).

6.8. GARCH

A technique allowing to describe volatility that varies over time is the GARCH model, standing for Generalized Autoregressive Conditional Heteroskedasticity (Bollerslev, 1986). GARCH allows for volatility to vary over time and revert back to a long-term mean (Alexander, 2008; Zivot, 2008). Also it helps to explain fat tails in returns and asymmetric effect in volatility changes. These characteristics make GARCH suitable to incorporate several aspects of volatility often observed in practice.

When estimated on historical data, a GARCH model is basically an autoregressive moving average (ARMA) model applied on the squared residuals of a model (Zivot, 2008). We briefly explain the properties of an ARMA model in Appendix I. A general GARCH model contains p moving average terms (ARCH terms) and q autoregressive terms (GARCH terms). Large ARCH terms indicate a quick response to market movements, large GARCH terms account for a more persistent volatility. The variance equation for a GARCH(p, q) model is given by Equation (6.8).

$$\sigma_t^2 = \omega + \varepsilon_t + \sum_{i=1}^p w_i \sigma_{t-i}^2 + \sum_{j=1}^q w_j \varepsilon_{t-j}^2 \quad (6.8)$$

with

σ_t	as the volatility at time t
ω	as the weighted long term mean of volatility
p	as the number of ARCH lags
q	as the number of GARCH lags
ε_t	as the error term at time t
w_i	as weighting variable for the i_{th} ARCH lag
w_j	as weighting variable for the j_{th} GARCH lag

Restrictions on the model are that $\omega \geq 0$, $w_i > 0$ and $w_j > 0$. This is to ensure that the model always returns a positive volatility. To obtain a stationary process, an additional restriction we impose is that $\sum_{i=1}^p w_i + \sum_{j=1}^q w_j < 1$. If the sum of weights is equal to or exceeds 1, there is no mean-reverting force in the model, likely causing volatility estimates to explode or converge to 0 (Ravanelli *et al.*, 2005; Hull, 2008). If the GARCH model is stationary, we may still obtain temporary extreme estimates though (Kluppelberg & Lindner, 2010). Some authors suggest setting the parameter ω at 0, stating that it is unnatural to assume a lower bound on variance (Lindberg, 2011). However, in the long term the expected value of a stationary GARCH model converges to a variance of $\omega / (1 - \sum_{i=1}^p w_i - \sum_{j=1}^q w_j)$. When applying the model for long term forecasts, it therefore makes sense to set the convergence level equal to the long-term variance, an estimation technique known as variance targeting (Francq *et al.*, 2009).

A difficulty with GARCH models is that they must be well-calibrated in order to properly reflect the actual volatility, which ideally is done by using a large amount of historical data (Alexander, 2001). In the absence of such data, the structure of the volatility changes could be wrongly specified, and provide estimates worse than a constant volatility model would. Disturbances in the historical data can have a significant effect on the estimates. The most applied form of GARCH is the GARCH(1,1) model. Higher lag models are generally not preferred because their coefficients are less robust and more difficult to estimate, usually obtaining just a local optimum (Alexander, 2001; Zivot, 2008). This is because higher lag models often have many local minima and maxima.

6.9. Other modelling techniques

This chapter described several modelling techniques suitable for forecasting models on commodity prices. We researched several other techniques which are not used for the models in this study. To increase understanding of these techniques and for their possible use in future research, they are described in Appendix I. We provide a very brief description of them here.

- **ARIMA model:** AR models place a certain weight on past observations as input for the forecast, MA models do the same for past error terms. AR and MA terms are often combined in a single forecasting model, called an ARMA model. Additionally, the lag differences between past observations may have explanatory power too. An ARMA model including differenced terms (denoted as I) is called an integrated model (ARIMA).
- **Exogenous variables:** Can be added to a forecasting model to explicitly include the effect of a certain variable. This way, we can address qualitative insights in future developments. When exogenous variables can be forecasted with accuracy, they can reduce the residuals of the model.
- **Multiple-factor model:** Allows for several parameters to be modelled stochastically, such as the convenience yield and the interest rate. Such extensions may better approximate reality. Multiple-factor models are more difficult to estimate than single-factor models, usually requiring algorithms such as the Kalman filter to estimate unobservable parameters. Also we may require historical futures contract prices.
- **Regime-switching model:** Distinguishes two or more regimes corresponding to a certain state. Each regime has its own distribution, which can function independently of the other distributions. A transition matrix contains the Markov probabilities for shifting from one regime to the other. Regime-switching models allow for better parameter estimation when clearly different market states can be distinguished.
- **Artificial neural network:** Connects a set of exogenous input variables in a hidden layer and performs mathematical operations to match these inputs with the desired output. As such, it can establish complex (mathematical) relationships between input variables otherwise unobservable.
- **Wavelet transformation:** A technique which decomposes a series in multiple series of different frequencies, making use of an algorithm to do so. The wavelet transformation is particularly applicable to transform a nonstationary series into a number of stationary series.

6.10. Estimating risk-neutral drift with futures prices

It is difficult to estimate the real long-term drift of an asset. The reason for this is that it requires measuring a factor that is not observable because it lies in the future. The application of risk-neutral valuation eliminates the need to estimate this parameter. For stocks, under the risk-neutral measure, the expected price grows at the risk-free rate, leaving out the need to estimate the real-world drift α . However, this method is generally not applicable for commodities. Users who physically hold a commodity may be able to profit from temporary shortages. This so-called *gross* convenience yield fluctuates over time, and is based on an inverse relation with inventory levels (Gibson & Schwartz, 1990). Further, when physically holding a commodity, storage costs decrease the return value. Possible costs when holding a commodity are the costs for the storage facility, maintenance, insurance, etc. Deducting the storage costs from the convenience yield provides a cash flow comparable to a dividend payment: $\delta = \text{gross convenience yield} - \text{storage costs}$, sometimes referred to as the *net* convenience yield. When using the term convenience yield in this study, we refer to its meaning of net convenience yield. We need to account for this dividend-like payment (usually but not necessarily positive)¹⁰ when estimating the drift. We illustrate this procedure with a set of equations (Trigeorgis, 1996). Say that the total expected return μ for an investor holding the commodity is given by

$$\mu = \alpha + \delta \quad (6.9)$$

This return is equivalent to the risk-free rate plus the market risk premium as defined in the Sharpe ratio (recall Section 3.7), so

$$\mu = r_f + \lambda\sigma \quad (6.10)$$

From setting equal Equation (6.9) and (6.10)

$$\alpha + \delta = r_f + \lambda\sigma$$

it follows that

$$\alpha - \lambda\sigma = r_f - \delta$$

We know that the risk-neutral drift $\hat{\alpha}$ of an asset is equal to its real drift minus a market-risk premium, see Equation (3.11). Thus, the risk-neutral drift of a dividend-paying asset is given by

$$\hat{\alpha} = \alpha - \lambda\sigma = r_f - \delta \quad (6.11)$$

A convenient method to determine δ is to assess futures contracts on the commodity (Trigeorgis, 1996; Luenberger, 1998; Casassus, 2004). A futures contract (or simply ‘futures’) is an agreement between two parties to trade an underlying asset at a specified maturity date for a specified price. Futures are standardised contracts traded on the exchange. Settlement of the contract may take place physically or financially, the contracts are often traded many times before maturity.

No costs are involved to enter into a futures contract, except for the transaction costs. In a liquid market, the futures price will therefore be adjusted so that the present value of all cash flows is equal to 0. First we will consider this mechanism while ignoring dividends. To prevent arbitrage opportunities, the futures price should be equal to the expected spot price at maturity (Mandler,

¹⁰ In particular commodities held only for investment purposes, such as precious metals, are known for having negative net convenience yields.

2003). If this were not the case, a risk-free profit could be made by taking a position in the futures contract and an inverse position in the underlying. If the futures price exceeds the current spot price plus the risk-free return until maturity, the investor is cheaper off by buying the underlying now, missing out only on the interest rate had the money been invested in a risk-free bond instead. It follows that the futures price discounted at the risk-free rate must equal the current spot price.

For commodities, the relationship between spot price and futures price is often more complex due to the convenience yield (Trigeorgis, 1996; Dinceler *et al.*, 2005). As the commodity is not physically held when holding a futures contract, the ‘dividend’ component is not included in its pricing. When futures contracts on commodities in plentiful supply¹¹ are liquidly traded, their real values are therefore equivalent to the risk-neutral expectation of the spot price at time T . Hence, we can obtain the risk-neutral growth rate when prices of futures contracts with different maturity dates T are available. Say that the prices of two futures contracts are available, with maturity dates T_1 and T_2 respectively (with $T_2 > T_1$). Expressed as a function of the spot price and the risk-neutral drift, the values of these contracts are then given by:

$$\begin{aligned} F_2 &= S e^{\hat{\alpha} T_2} \\ F_1 &= S e^{\hat{\alpha} T_1} \end{aligned} \tag{6.12}$$

It follows that the risk-neutral drift $\hat{\alpha}$ between date 1 and 2 is

$$\hat{\alpha} = \frac{\ln(F_2/F_1)}{T_2 - T_1} \tag{6.13}$$

The spot price can be considered as a special case of a futures contract, namely a futures contract at maturity (future and spot prices converge to the same level at the maturity date to avoid arbitrage). So, in Equation (6.13) F_1 may be substituted with S_t as well. The calculated drift depends on the futures contracts used in the equation. When many futures contracts with different maturities are available, a futures curve can be constructed which represents the risk-neutral price development over time. The curve of the expected real spot price lies above the futures curve by a risk premium.

¹¹ A commodity with a finite nature can be plentiful in supply as well. In this context, ‘plentiful’ refers to a commodity which can be easily obtained by any market participant.

Construction of price forecasting models

Based on the diagnostic tests and the techniques treated in the previous chapters, we now construct the price forecasting models for natural gas and electricity. Consistent with option pricing theory, we adopt a risk-neutral approach. The risk-neutral drifts are estimated based on the available futures contract information.

The gas price model is relatively simply, keeping in mind only average contract prices are required eventually. We use binary variables for monthly deviations, and an Ornstein-Uhlenbeck process for the stochastic part. The forecasted prices follow the behaviour of the actual series closely. Significant higher-lag autocorrelation remains present in the residuals.

We combine five techniques to construct the electricity price model. It contains a sinusoid function and daily binary variables to reflect time effects, a mean-reverting stochastic process, a GARCH model for volatility, and a jump-diffusion model for price jumps. The price model does not fully remove the autocorrelation pattern. In the discussion of this study we provide some suggestions to improve the electricity price model.

We attempt to calculate a meaningful cross-correlation, applying the constructed models to remove time effects and autoregressive trends as much as possible. However, lagged correlations reveal that a significant time pattern is present in the correlation between the series. Therefore we choose not to use the lagged correlation, instead assuming no linear dependency exists between both series.

7.1. Estimating risk-neutral drifts

Following the approach described in Section 6.10, we use futures contracts to estimate the drift of gas and electricity prices. We obtained the publicly available future prices of 4 April 2012 from the ENDEX website. Future contracts of several lengths are traded, with maturities ranging from a week ahead to several years ahead. However, the number of contract lengths available is still quite limited, making it difficult to accurately estimate all effects that theoretically could be derived from futures price data. ENDEX applies an averaging method to obtain a value for quarterly, seasonally or yearly futures contracts. To prevent arbitrage opportunities, they make corrections on the monthly prices (APX-ENDEX, 2011). This procedure causes some marginal deviations between monthly contracts and contracts over a longer period of time.

In order to estimate the long-term drift, we do not include seasonal or quarterly contracts; the seasonal effects incorporated in these contracts hamper the analysis of long-term effects. Futures on natural gas are available until 2018, futures on electricity until 2017. The future curves plotted in Figure 27 include the average spot prices over 2011 and the subsequent calendar year futures. For the year 2012 no calendar year futures prices are available, we determine the value of this year with interpolation.¹² We add a linear trend line to represent the average risk-neutral drift.

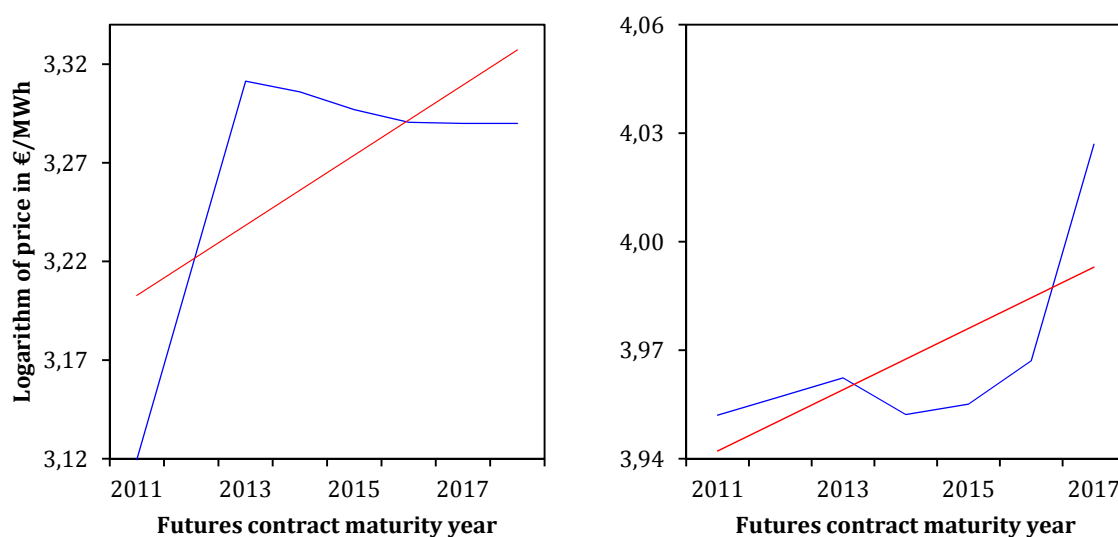


Figure 27: Futures curves of natural gas (left) and electricity (right) in blue, with a regressed linear trend line in red.

The futures prices of natural gas clearly lie above the current spot price, but are expected to decrease slowly after 2013, ending in an almost flat line. This might indicate that the current demand for gas is high relative to production, so investors prefer to acquire natural gas in the nearby future over the same commodity at a later time. The electricity price is expected to increase only little over the coming years, before sharply increasing between 2016 and 2017. It is unknown what causes this sudden rise and whether this trend is expected to continue in the years after.

The exploitation of a gas field usually lasts longer than the five or six years represented in the futures curves. Additionally, the ENDEX is considered only a moderately liquid market. These factors make it difficult to determine a long-term trend. Ideally the risk-neutral drift would sim-

¹² Taking the average spot prices over 2012 until 4 April would not represent the average of the year, as this set would be biased towards the winter prices.

ply follow the futures curve, yet extrapolating the curve is difficult due to the irregular patterns of both curves. Instead of extrapolating the curve, we take the average trend as the risk-neutral drift. This means that the simulated prices will differ from the observed risk-neutral process in the short term and provides a conservative estimate after those years. Absent the insight how prices are expected to develop in the long term, taking the average drift seems to be a justified decision. The average drift is calculated by filling in Equation (6.13) for all maturities following each other (e.g., 2014/2013, 2015/2014 etc.) and taking the average value. The drift between 2011 and 2013 is assumed to be constant. Completing this procedure gives an annual risk-neutral drift of 2.435% for the gas price and 1.249% for the electricity price. Dividing these numbers by 365 gives the daily drift. These outcomes are quite close to some long-term forecasts of real drifts, as presented in Section 8.3. This may indicate that the average trend is closer to reality than the extrapolated trend. We will test for an alternative high-drift scenario as well.

7.2. Modelling gas price series

We model the gas price series with a deterministic function describing monthly effects and a stochastic part modelled with the Ornstein-Uhlenbeck model. In the option model, we assume that natural gas is sold via contracts based on the average gas price of three or six months. The main characteristic of the gas price series is therefore to provide realistic averages, with short-term behaviour being less important. Ornstein-Uhlenbeck appears to be a suitable choice for this. It includes the mean-reversion property, without requiring to estimate short-term deviations.

Following Lucia & Schwartz (2002), we decompose the movements of prices in a deterministic time pattern and a stochastic process. Diagnostic testing revealed the existence of some monthly effects; the deterministic part can take account of these movements. First the series must be deseasonalised, and then we can estimate the Ornstein-Uhlenbeck model on the newly created series. As stated in the test results, no clear seasonal pattern can be distinguished. For this reason, we account for seasonality by including binary variables $\gamma_{i,j}$ for each month i . The monthly price effects are denoted by M_i , representing the deviation from the logarithm of the price. In Table 15 the estimated monthly effects are presented.

Parameter	Estimate
M_1	0.138
M_2	-0.021
M_3	-0.046
M_4	-0.081
M_5	-0.030
M_6	-0.051
M_7	-0.011
M_8	-0.064
M_9	0.015
M_{10}	-0.030
M_{11}	0.104
M_{12}	0.086

Table 15: Estimated monthly deviations from average logarithmic gas price.

We now estimate the Ornstein-Uhlenbeck process for the deseasonalised price series. Smith (2010) performed an empirical test on the accuracy of several estimation methods on the Ornstein-Uhlenbeck model, and concluded that least squares and maximum likelihood provide the best results overall, but are relatively poor on estimating the mean-reversion parameter. Meth-

ods such as jack-knife maximum likelihood¹³ can provide more accurate results on this aspect, but perform poorly overall. To estimate the parameters of the gas series we apply the least squares method as proposed by Van den Berg (2010). The following values are obtained:

Parameter	Estimate
$\bar{S}$	2.867
κ	0.015
σ	0.063

Table 16: Parameter estimates of logarithmic Ornstein-Uhlenbeck process for stochastic gas price.

To incorporate the risk-neutral drift of the price, the equilibrium price level must increase over time. We do this by adding the risk-neutral drift calculated in the previous section to the equilibrium price at every time step. In Figure 28, we show two sample paths randomly generated with the forecasting model, with the logarithmic series converted back to real prices. The blue graph represents the historical price series. It can be seen that the actual price series includes periods of clustered volatility, which is not reflected well in the forecasted series.

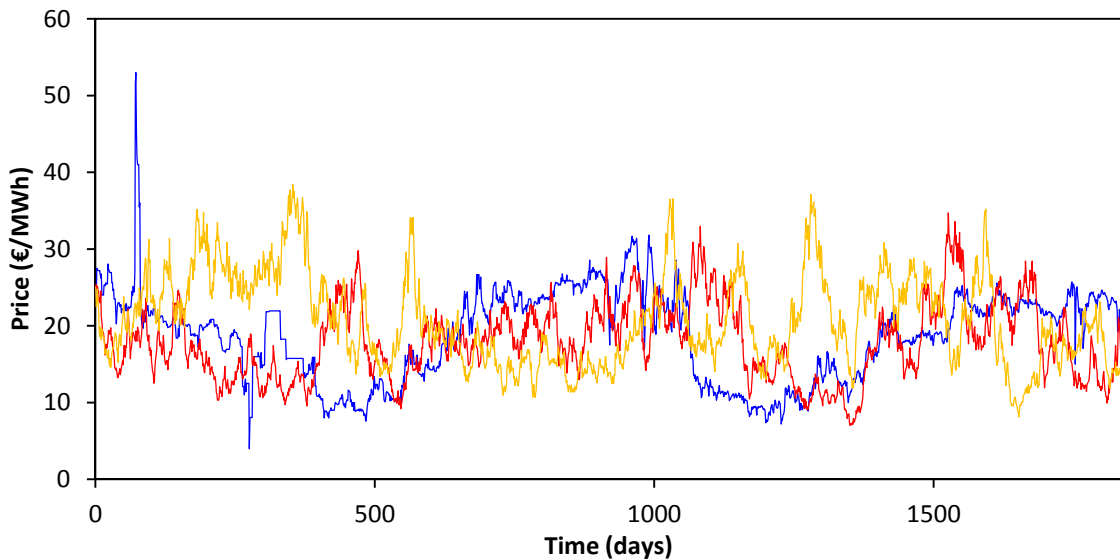


Figure 28: Randomly simulated gas price paths compared to historical gas price series (blue). Apart from the lack of volatility clustering, behaviour seems comparable.

In Figure 29, we plot a graph using the stochastic part of the previous day's price (i.e., the price minus the time effect) as input for each day-ahead forecast. The blue line represents the actual data, the red line the forecast. Again, the differences in volatility can be observed. Overall the simulated series seems to follow the behaviour of the actual series quite closely. Following He (2007), we perform 5000 price simulations to compare the first four moments of the simulated series to those of the actual series. We show the results in Table 17; they indicate a stronger right-side tail than observed in the historical data. The positive risk-neutral drift is used instead of the negative historical drift, explaining the higher mean value.

¹³ With jack-knifing, the parameters are recalculated leaving out one or more observations, thereby estimating the bias and standard error of the statistic.

Moment	Historical price series	Simulated price series
Mean	18.769	21.783
Standard deviation	5.871	7.570
Skewness	0.111	0.991
Kurtosis	0.613	1.485

Table 17: Comparison between moments of historical and simulated gas prices.

We obtain the residuals of the model by taking the difference between the expected value of the day-ahead forecast and the actual price. We perform residual testing in Appendix II, we present our main findings here. Autocorrelation is reduced and insignificant for the first two lags, but significant autocorrelation remains for higher lags. Particularly the 9th lag shows noticeable autocorrelation. Including an MA(9) term to the model removes significant autocorrelation, but it is unlikely this autocorrelation is a structural effect. Instead, removing the seven price jumps depicted in Figure 26 also removes significant autocorrelation. The Jarque-Bera test still rejects normality of the residuals; some outliers strongly affect their distribution. The R^2 when applying day-ahead forecasting on the actual series is 0.96.

More advanced volatility modelling techniques such as GARCH or a mean-reverting stochastic volatility might be able to reflect heteroskedasticity better. Since contract prices are based on average prices over at least three months and a large number of simulations is performed, we believe that the assumption of constant volatility will not have a strong effect on the option value.

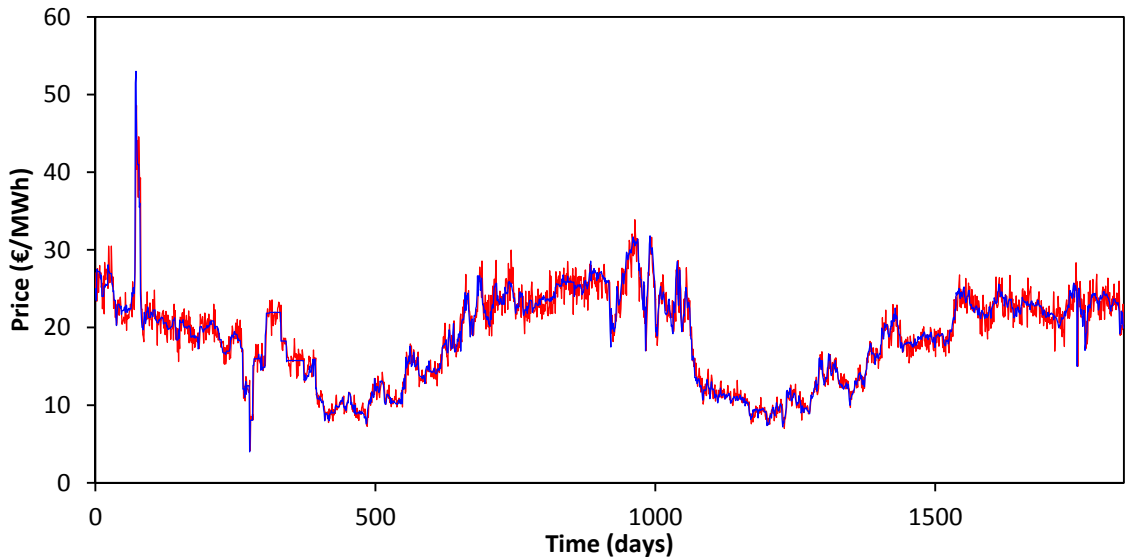


Figure 29: Randomly simulated day-ahead forecast of gas price compared to historical gas price series. The forecasted series follows closely, but is more volatile at some points.

7.3. Modelling electricity price series

The first issue we address is the part of the data set that we should use for parameter estimation. The volatility has shown a decreasing trend over the years, but some developments (see Section 2.8) indicate that volatility may increase again in the future. Using a too recent time window might underestimate volatility. We note that the pattern before the liberalisation of the Dutch energy market in 2004 is more fickle than afterwards. Due to the structural difference between both markets, we estimate parameters based on the historical prices from 2004 to 2012, assuming that the behaviour of future prices is approximated by an average of this period.

The series from 2004 to 2012 were tested again for mean-reversion and stationarity, resulting in the same conclusions as in Chapter 5. The price series is still mean-reverting and the return series still satisfies the stationarity property. We do not show the results of these tests here.

Diagnostic testing in Section 5.7 revealed that significant differences in daily- and monthly prices exist. We again decompose movements in a deterministic time pattern and a stochastic pattern. Pilipovic (1998) proposes a sine function to account for seasonal patterns, or a double sine function if both a winter and a summer peak exist in the prices. We estimated both the sine and double sine function; the double sine function provided a higher adjusted coefficient of determination and is therefore preferred. Following He (2007), we add binary variables to account for the day of the week:

$$B_A \left(\sin \frac{2\pi t}{365} + \phi_A \right) + B_{SA} \left(\sin \frac{4\pi t}{365} + \phi_{SA} \right) + \sum_{i=1}^7 \gamma_{i,j} D_i \quad (7.1)$$

where B_A , ϕ_A , B_{SA} , ϕ_{SA} and D_i (for $i = 1, \dots, 7$) are constant parameters to be estimated.¹⁴ In this equation, $B_A \left(\sin \frac{2\pi t}{365} + \phi_A \right)$ represents the annual trend, $B_{SA} \left(\sin \frac{4\pi t}{365} + \phi_{SA} \right)$ the semi-annual trend and D_i is the average daily deviation from the weekly average. Finally, $\gamma_{i,j}$ is a binary variable representing the day of the week.

We linearly regress Equation (7.1) on the price series, estimating all parameters simultaneously. A single constant is added as a shifting parameter, which must be removed afterwards. Note that the constant only serves as a temporary intercept. We show the estimates of the parameters in Table 18. Representing only *average* deviations, the deterministic time function does not completely remove time effects. We show an example of the deterministic seasonal component (in red) in Figure 30, compared to the actual log of the price series in 2007.¹⁵ It can be seen that the seasonal trend follows the actual behaviour of the series, showing a sharp price decrease during weekends and a slight wave-pattern over the year.

Parameter	Estimate
B_A	0.109
ϕ_A	2.197
B_{SA}	-0.039
ϕ_{SA}	0.101
D_1	0.096
D_2	0.152
D_3	0.138
D_4	0.122
D_5	0.085
D_6	-0.084
D_7	-0.290

Table 18: Parameter effects of time function for logarithmic electricity price.

¹⁴ A deterministic linear trend is also added in other applications, but this would not allow for incorporating a risk-neutral drift based on futures contracts.

¹⁵ A single year allows observing the time trend better in a graph. The year 2007 was chosen for roughly lying in the middle of the sample.

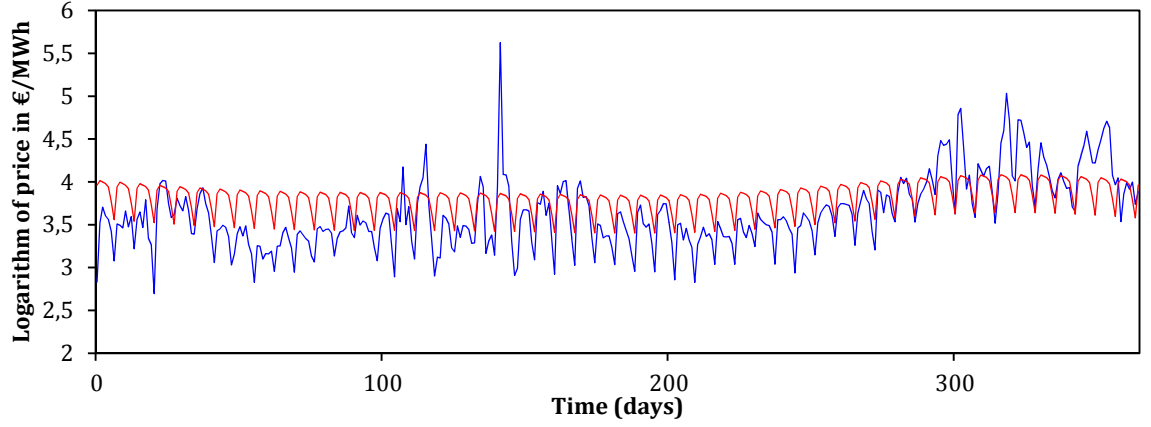


Figure 30: Historical logarithmic electricity price series and estimated seasonal trend for the year 2007. The comb-like pattern of the historical series is strongly reduced after correcting for the time trend.

For the stochastic part, several modelling techniques are available. A general approach is to distinguish between a normal and extreme behaviour of the series, either with different regimes or by adding jumps. He (2007) compares the performance of several stochastic models, concluding that regime-switching models provide results closer to the historical data than jump diffusion models. However, a jump diffusion model also captures the behaviour of the electricity prices quite well. In this study we estimate a jump diffusion model on the deseasonalised price series, as it does not require using an algorithm for the estimation of the parameters. For the purpose of this research a model *reflecting* the behaviour of the commodity is sufficient; we do not seek to optimise with respect to historical data.

We filter out jumps from the deseasonalised price series by removing all prices more than three standard deviations away from the mean, obtaining 16 jumps. This is quite a small amount (about 0.5% of the total), making it difficult to accurately derive the intensity of arrival. In addition, jumps only occur in the early years of the sample. To allow for both upwards and downwards jumps, we adopt a dual jump diffusion model. The processes for upward jumps ($> 3\sigma$) and downward jumps ($< -3\sigma$) can then be modelled independently, allowing for better parameter estimation. Under the assumption that the entire trajectory of the process is known (which is reasonable for a sufficiently large sample), Makhnin (2008) states that the intensity of jumps occurring can be estimated as

$$\hat{\varphi} = \frac{1}{T} \sum_{t=1}^T I(t \leq T) \quad (7.2)$$

with T as the total number of observations and I as a function providing a value of 1 if a jump occurs at time t , and 0 otherwise. This is simply the maximum likelihood estimator for the Poisson arrival parameter (Larsen & Marx, 2006). For the series above, this procedure results in an estimated average arrival time of 0.003658 for an upward jump and 0.001663 for a downward jump, denoted as φ_U and φ_D respectively. The homogenous Poisson process implies that both arrival processes are constant over time and all jumps occur independently of each other. We assume that prices to revert back to the equilibrium price level after a jump, that is $\bar{S} + \hat{\alpha}t$. Further we add the restriction that only a single jump can occur during a day. The probability of a jump occurring is then the cumulative Poisson distribution for 1 jump up until an infinite number of jumps, calculated as 1 minus the chance of no jump occurring. The probability of a positive price jump occurring at any given day is then

$$1 - \frac{e^{-\varphi_U} \varphi_U^0}{0!} = 0.0037 \dots$$

and for a negative jump it is

$$1 - \frac{e^{-\varphi_D} \varphi_D^0}{0!} = 0.0017 \dots$$

We take the average price and standard deviation as the mean jump size and volatility respectively. In the model we treat jumps as *prices* instead of returns. The resulting jump size distributions are $N(5.087771, 0.213189)$ for the upwards jumps and $N(2.683667, 0.066571)$ for the downward jumps. We model the remaining deseasonalised series without jumps with an Ornstein-Uhlenbeck model, alike to the model estimated for the gas price. Estimation of the series yields the following results:

Parameter	Estimate
$\bar{S}$	3.775
σ	0.180
κ	0.131

Table 19: Parameter estimates of logarithmic Ornstein-Uhlenbeck process for electricity price.

In Appendix II we provide the results of diagnostic testing on the residuals of this model. Though reducing autocorrelation, except for the first lag autocorrelation is still significantly present in the residuals. Also the weekly effect remains observable, indicating that volatility is higher on weekdays as well. By estimating a GARCH model, we attempt to remove this autocorrelation. Equation (6.8) shows the structure of a GARCH model. The maximum likelihood estimator for a GARCH model is given by (Hull, 2008):

$$\prod_{t=1}^T \left[\frac{1}{\sqrt{2\pi\hat{\sigma}_t^2}} \exp \left(\frac{-R_t^2}{2\hat{\sigma}_t^2} \right) \right] \quad (7.3)$$

with $\hat{\sigma}_t^2$ as the estimator of GARCH variance at time t and R_t as the model residual. Equation (7.3) is maximised locally¹⁶ under the restrictions described in Table 20.

Restriction	Purpose of restriction
$\omega / \left(1 - \sum_{i=1}^p w_i + \sum_{j=1}^q w_j \right) = 0.180241^2$	Makes long-term expected variance converge to average variance.
$\sum_{i=1}^p w_i + \sum_{j=1}^q w_j < 1$	Ensures GARCH process is stationary.
$\omega, w_i, w_j > 0 \quad \forall i, j$	Ensures estimated variance is always positive.

Table 20: Restrictions on the GARCH estimation algorithm.

Moving average terms have a direct effect on the autocorrelation of a particular lag; autoregressive terms have a more lingering effect. Moving average terms are therefore better suitable to address autocorrelation stemming from weekly effects. As a starting point, we include a con-

¹⁶ The estimation algorithm used provides a local optimum depending on the starting values; parameter estimations may vary when trying to reproduce the result.

stant and seven moving average terms in the GARCH model to account for the weekly effect. Then we add autoregressive terms up to seven lags to see if these yield any improvement. We use the Akaike information criterion as the decision rule for adding terms, we do not add new terms if these do not decrease the criterion any further. Using the confidence bounds proposed by Meko (2009), we remove terms which are statistically insignificant at the 5% level from the model, and maximise the maximum likelihood estimator again without the omitted variables. Following this methodology, we obtain a $\text{GARCH}((7),(1,6,7))^{17}$ model.

Parameter	Estimate
c	0.001
GARCH(7)	0.618
ARCH(1)	0.076
ARCH(6)	0.054
ARCH(7)	0.221

Table 21: Parameter estimates of $\text{GARCH}((7),(1,6,7))$ model.

Figure 31 shows the volatility estimates using the $\text{GARCH}((7),(1,6,7))$ model when applied on the historical data set, compared to the squared residuals of the model and the estimated constant volatility. We can see that the GARCH model is able to capture the behaviour of the residuals to a certain degree. Figure 32 compares randomly simulated volatilities using the constant volatility estimate and the GARCH model. The volatility clustering property of the latter model can be clearly distinguished. In Appendix II, we perform several diagnostic tests on the GARCH model. Our results indicate that the GARCH model improves the properties of the residuals and removes the weekly pattern in autocorrelation, but the residuals are still significantly autocorrelated and not normally distributed. More specifically, the model consistently underestimates the more extreme residuals. These extreme residuals are likely to stem from behaviour unexplained in the current model. This issue is not resolved in this study; the discussion in Chapter 11 includes some suggestions to better capture this behaviour.

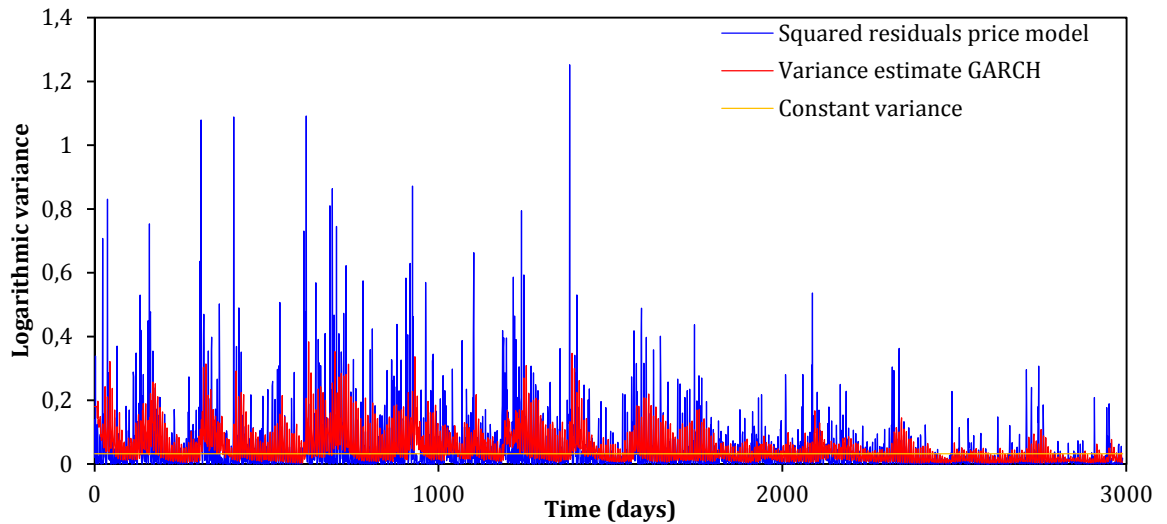


Figure 31: $\text{GARCH}((7),(1,6,7))$ model applied on residuals electricity price model. The GARCH variance fits the squared residuals to a certain degree, but does not capture the more extreme residuals. The improvement compared to constant volatility can be clearly observed.

¹⁷ In this notation only included (G)ARCH terms are mentioned, whereas the standard notation means ‘up to’ the specified number. For example only the 7th GARCH term is included in the model, not all terms from 1 to 7.

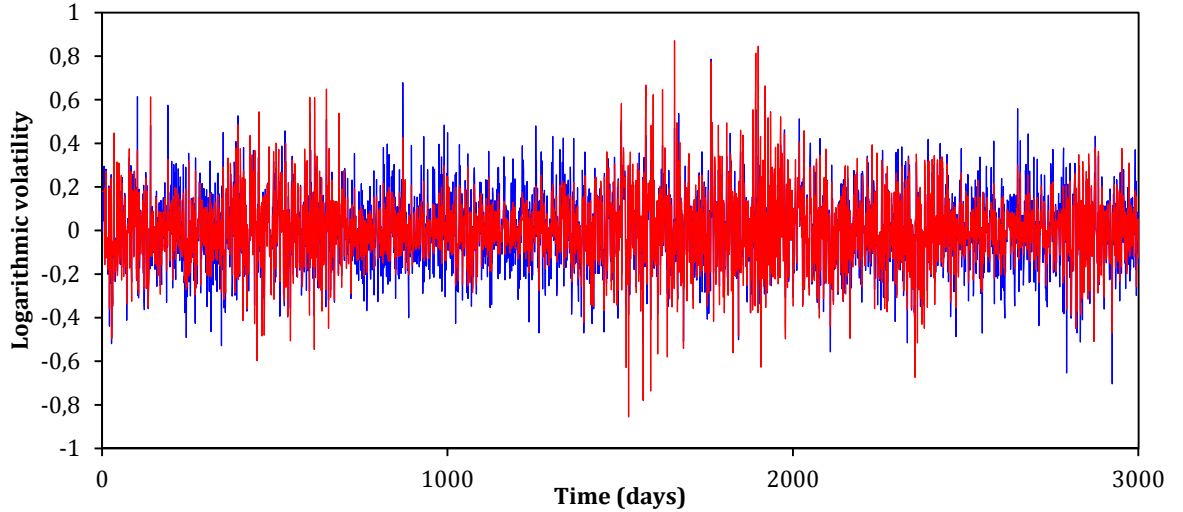


Figure 32: Comparison between logarithmic volatilities simulated with constant variance (blue) and GARCH model (red). The GARCH simulation clearly shows volatility clustering.

We construct the eventual price series by taking the deterministic time trend, and adding either the stochastic return or a jump to this trend. Random errors determine the movements in GARCH volatility. We cap the stochastic price at €1000, considering all prices above this level as outliers (see Section 8.3.6. for a rationale). We show two randomly generated electricity price series in Figure 33, compared with the historical price data of 2006-2009. The latter is represented by the blue graph. In Figure 34 we show the results of a simulation that uses the previous day historical price as input for the next day forecast and random errors for the GARCH volatility (not based on actual residuals). We exclude jumps from this series, as the main equation is not fit to these prices. As the volatility of the historical series decreased over the years and we expect forecasted electricity prices to be more volatile than this, in the last part of the graph it can be noted the simulated series exhibit more volatile behaviour. Table 22 compares the moments of the historical and simulated series. The model provides more skewed results with a higher kurtosis; this is likely to be attributed to the extreme variances GARCH occasionally returns. The effect on the mean and standard deviation is limited though.

Moment	Historical price series	Simulated price series
Mean	48.829	50.843
Standard deviation	20.253	21.943
Skewness	2.193	3.499
Kurtosis	12.883	31.512

Table 22: Comparison between moments of historical and simulated electricity prices.

The R^2 statistic of the model is 63.9%. At least part of this low explanatory power is explained by ignoring the decreasing trend of volatility, thereby consistently over- or underestimating volatility compared to historical values.¹⁸ We believe that the model as it stands now is sufficient for our research purpose, but for real-world applications better estimates are likely required. In Chapter 11 we do some suggestions to improve the performance of the electricity price model in future research.

¹⁸ As the expected error value is 0, the GARCH model has no effect on the explanatory power.

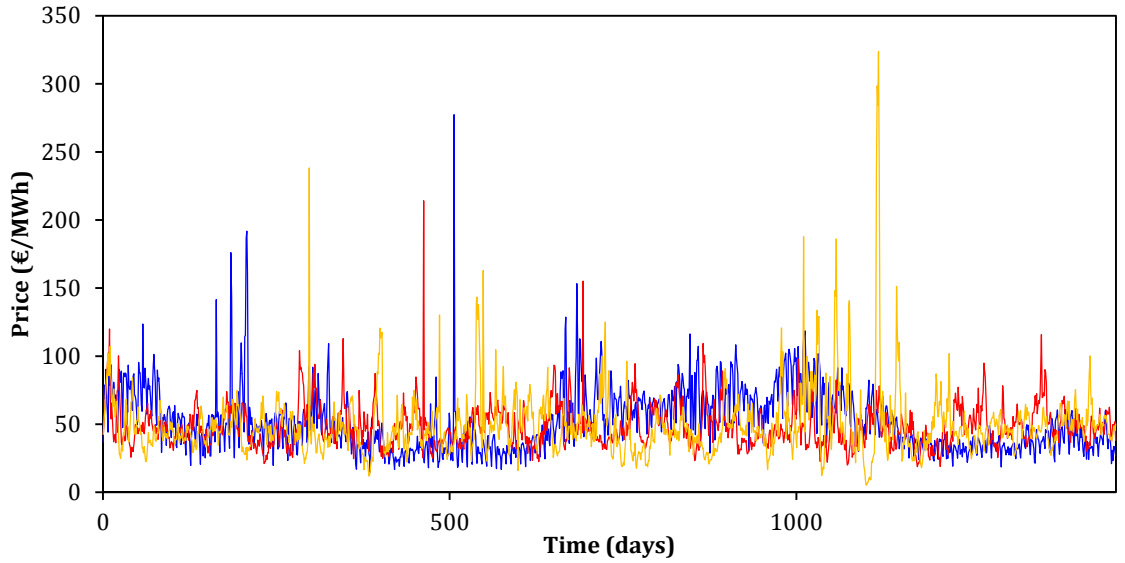


Figure 33: Sample electricity price paths compared to historical price series 2006-2009 (blue). The simulated graphs should represent the average behaviour of the historical series.

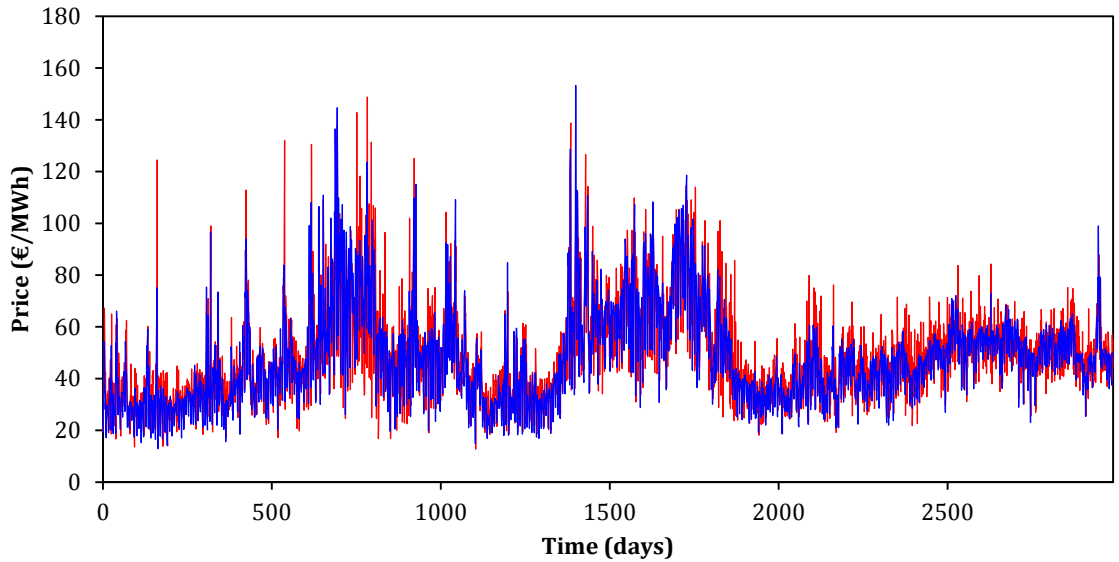


Figure 34: Day-ahead forecast compared to historical electricity price series without jumps 2004-2012. The simulated series tends to overestimate volatility towards the end.

7.4. Cross-correlation between return series

As stated in the previous sections, the constructed forecasting models do not fully remove autocorrelation effects in the time series. Also in Section 5.9 we show that the raw return series are not normally distributed when not corrected for autocorrelation, while the natural gas series remains non-normal even when autocorrelation would be fully removed. The series do therefore not meet the requirements described in Section 5.10 to calculate cross-correlation. Nevertheless we have strongly reduced autocorrelation and time patterns. Therefore we check whether a relevant correlation coefficient can be estimated, under the following two assumptions:

- The cross-correlation is a function of the remaining autocorrelation in the series.

- The number of observations is sufficient to calculate a reliable significance value for the non-normal series.

To be able to estimate correlation, we detrend the historical price series as much as possible. By directly applying our pricing models on the historical data and using the historical drifts, we remove the time effects and the mean-reverting effect. We divide the obtained returns by their volatility estimates to scale them better. After performing these operations, the resulting series for electricity is close to normally distributed. The series for gas contains a number of extreme values which strongly skew its distribution. To assess this issue, we remove ‘extreme’ outliers from the sample. We define extreme outliers as observations which are removed from the first and third quartile by more than five times the interquartile difference. Following this procedure we remove 51 outliers from the adjusted gas series. Note that this approach increases the accuracy of the correlation estimate, but at the same time ignores unexplained movements in the series. Some theoretical validity is traded in to obtain a more meaningful coefficient. After removing the outliers, the series is normally distributed.

Calculating the correlation coefficient yields a value of 5.77%, which is statistically significant at the 5% level. Due to the infrequencies in the data set, the assessment of lagged correlation does not exactly equal the actual time lag in days. Still, calculating the lagged correlation (with the electricity series following the gas series with lag $t + h$) for 50 lags reveals a clear time pattern, meaning that the constructed series are still not sufficiently detrended. The estimated coefficient could therefore be caused by time effects unaccounted for in the model. For this reason, we must assume that no cross-correlation exists between the series. Improving the properties of the model or assessing non-linear relationships is required to account for possible co-dependencies between the series.

Constructing the real option model

In this chapter, we construct the real option model for Gas-to-Wire production. We consider an investment problem where gas is produced initially, and we can shift to electricity generation every year. The daily production decision for electricity generation is based on the spark spread.

We explain the structure of both the real option model and the dynamic decision tree. In most respects the latter is the same as the real option model, differing on the discount rate applied and the use of real probability distributions instead of risk-neutral ones. We provide an overview of the key assumptions in the final section of the chapter.

In Section 8.2, we find estimates for the parameters used in the simulation model. This includes the risk-free rate, the WACC, the drifts for market prices and cost developments, and the investment costs.

We choose Monte Carlo simulation as the method to calculate the option value, mainly because its ability to deal with path-dependent payoffs. Also it is flexible and insightful for others, allowing for further modifications and expansion. To be able to calculate the American option value with simulation, we have to compute the expected continuation values. We use the Longstaff-Schwartz algorithm to perform a backwards regression on the simulated price paths.

8.1. Real option structure

In this section, we outline the main properties and assumptions of the option model. Section 8.7 provides a more extensive overview of the key assumptions. We apply the option model on a tail-end field. This means that the gas production facilities have already been established, and pipelines connecting the production site to the transport network are laid as well. A single production well is active on the field. We set the maturity of the option at twenty years, starting at the beginning of 2012 and maturing at the end of 2031. We consider amounts of exploitable gas remaining after maturity lost; in practice our simulated fields are always abandoned before maturity. We can make the decision to switch from gas production to electricity generation at the start of every year; the decision goes into effect immediately. The decision is irreversible, meaning that we cannot switch back to gas production after exercising the option.

Gas is produced continuously. Every three months, we choose to fix either the six-month average (6-0-3 contract) or the three-month average (3-0-3 contract) for the next three months, whichever offers the higher price. Electricity produced is sold on the spot market on a daily basis. We base the production decision on the spark spread, which should be greater than the pre-specified threshold in order to produce. The electricity produced is sold the same day. We draw the amount of Gas-in-Place from a lognormal distribution, considering GIP an uncorrelated private risk that is resolved before applying the option model. Determining the reserve can be viewed as a decision tree placed before the option model (Dias, 2012c). Investment- and production costs are private risks with a correlation to the market; we further assess this property in Section 8.4.3. We assume the growth of costs to be deterministic. The gas price and electricity price are the market risk in the real option, and are modelled by stochastic processes.

8.2. Decision tree structure

To compare the performance of the real option with that of other dynamic evaluation tools, we construct a decision tree model as well. We structure this model in mostly the same way as the real option model, basically only differing on the point of risk adjustment. Risk-neutral simulation and –discounting as seen in ROA implicitly risk-adjusts cash flows to the project’s risk profile, because the risk-neutral drift is equivalent to the real drift minus the market risk premium. This is a feature not reflected in the DTA; recall that a decision tree only uses real probabilities and a single discount rate incorporating risk preference (see Section 3.2). This means that we use the real probability distribution for price simulation instead of risk-neutral simulation. More specifically, we use the real drift α instead of the risk-neutral drift $\hat{\alpha}$. Further we replace the risk-free discount rate by the WACC. DTA makes no distinction between market risk and private risk. Every cash flow is estimated according to its real probability distribution and discounted at the same rate. For the remainder, the real option model and the decision tree are the same. The daily production decisions and the investment decisions made are equal. We use the same random number streams for both simulations, as we perform them parallel.

8.3. Parameter values

8.3.1. Discount rates for risk-neutral and real valuation

In a risk-neutral framework we discount future cash flows at the risk-free interest rate. This is the return which supposedly can be earned without a probability of default. Literature usually suggests using the return on United States treasury bonds with equal length as the option as the risk-free interest rate, being approximately risk-free as they never defaulted. As we consider the exploitation of a Dutch gas field, Dutch government bonds are the most natural choice for the risk-free rate. At 3 July 2012 the annual return on a 20-year Dutch government bond was 2.672% (Forexpros, 2012).

At the moment of writing this thesis, interest rates are at a historical low point. It is unlikely that the current interest rate will be representative for the lifetime of the option. The reason why we still choose this interest is that it is not clear how a stochastic interest rate would affect the risk-neutral drift estimated for the market prices. This is because the risk-neutral drift for a commodity is calculated with the risk-free interest rate and the convenience yield; recall that $\hat{\alpha} = r_f - \delta$. We cannot estimate a stochastic process for δ with our data. With sets of historical futures prices and interest rates available, it would be possible to assess the changes in dividend and drift in relation to the r_f . We could then model r_f and $\hat{\alpha}$ stochastically.

Several discounting approaches exist (Lewis *et al.*, 2009). We can make arguments for both continuous and discrete compounding, with continuous compounding being somewhat more conservative. Also combinations of discrete and continuous compounding are used. In the model, we apply discrete annual compounding. For the decision tree, we use the WACC as the discount rate. As the simulation is not based on an actual case, we must presume a WACC. Reports of E&P operators indicate that most of them use a WACC between 10 and 15% (Ramos, 2008; Kokin & Dzuba, 2009; Hira & Wood, 2012). In this study, we take a WACC of 12.5% as discount rate.

Valuation method	Discount rate
Real option	2.672%
Decision tree	12.500%

Table 23: Risk-neutral and real annual discount rates.

8.3.2. Real drift of gas and electricity prices

For the real option model, we estimated the risk-neutral drift for the gas and electricity prices based on the prices of APX ENDEX futures contracts. The decision tree requires estimating real drifts. We use the 2030 outlook from the European Commission to estimate these drifts (European Commission, 2003). We discuss this approach along with some other possibilities in this section.

A common method to estimate the real drift of prices is to extrapolate the average drift of historical data (Pindyck, 1999). This method has several disadvantages. Most importantly, it does not incorporate insights in future developments which could alter the price. Also, the choice and length of the time window used may have a great effect on the estimate. Applying extrapolation on our full historical data sets, the annual real drift would be 4.27% for the electricity price and -4.49% for the gas price. Considering the positive drifts observed in futures contracts, the negative estimate for gas is a counterintuitive result. These findings would only be consistent in case of a large negative convenience yield, meaning that the costs of storing natural gas would have to exceed the benefits of holding it by far.

From Equation (3.9) it follows that we could also estimate the real drift by adding the market premium to the risk-neutral drift that we obtained from the futures contracts. This approach would be most consistent with real option theory, and allow for the most valid comparison between ROA and DTA. However, to estimate the market premium we would need a data set containing both futures- and spot prices, which was not available for this study.

Some large research institutes and companies present outlooks in which they reflect on long-term developments in the energy industry. Some of them provide an expected price development as well. Usually these studies have a global or continental focus; detailed country-specific outlooks are rare. As long-term price developments are strongly influenced by macroeconomic forces, we believe there is no objection to use Europe-oriented studies to estimate Dutch price developments. The European Commission (2003) forecasts an average annual drift of 2.80% for

the European gas price up to 2030, measured as percentage of the prevailing gas price. They make use of a model based on world supply and -demand and a correlation with the oil price. From the findings presented in the European Commission study *Roadmap to 2050*, it follows that the expected annual drift for electricity (measured as percentage of the prevailing electricity price) until 2030 is 1.72% (European Commission, 2011). This drift is estimated based on a scenario where current electricity policies are continued, new policies stimulating electricity generation from renewable energy could drive the drift up to 2.35%.

Annual drift	Real option	Decision tree
Drift gas price	2.43%	2.80%
Drift electricity price	1.25%	1.72%

Table 24: Risk-neutral and real price drifts for natural gas and electricity.

8.3.3. Growth rate of costs

We estimate the real growth rate of costs based on cost indexes of the industry. The Upstream Capital Cost Index (UCCI) is an index which tracks the development in capital costs associated with oil and gas projects (IHS Indexes, 2012). The portfolio includes 28 diversified projects around the world, and measures costs stemming from labour, facilities, material and equipment. The European Power Capital Cost Index (EPCCI) is a similar index on the costs of constructing power-generating facilities. Finally, the Upstream Operating Cost Index (UOCI) provides an estimate for the operating costs in the oil- and gas industry. A comparable index for the power industry is not available, but the UOCI may well reflect the operating expenses involved with Gas-to-Wire, since both involve the processing of gas. Annual index figures are available since 2000, estimating the trend of these series yields effective annual growth rates of 5.49% (UCCI), 7.27% (EPCCI) and 5.59% (UOCI). The indexes have shown a sharper increase in costs during the last years. A possible explanation for this is that costs in the energy industry tend to be correlated to oil prices, which have also grown strongly during the last years.

To obtain the cost drifts for the real option, we assess the correlation between the cost drift (which is considered to be private risk) and the market. We fill in the CAPM described in Equation (3.8). To estimate the beta, we calculate the correlation between the cost index returns and the market returns.¹⁹ We take the Dutch AEX as a proxy for the market; we obtain annual values for this index from 1Stock1.com (2012). We calculate the correlation between the annual logarithmic returns for the AEX and those of the UCCI, EPCCI and UOCI. This yields beta estimates of -0.01, -0.02 and 0.00 respectively, indicating that costs are almost uncorrelated to the market. The average market return for the AEX is difficult to forecast, particularly considering the turbulent crisis years. The long-term historical return might be the best estimate for a long-term forecast, as it is hard to predict global developments over the course of 20 years. The average effective AEX return since the founding of the index in 1983 was 9%, we take this rate as the market return (Jansen, 2012). In a risk-neutral framework, the drifts for the costs of gas and electricity production are then given by filling in Equation (3.12), see Table 25 for the results.

We emphasise that this approach is merely an approximation showing how a drift for a correlated private risk process can be obtained, spurred by a lack of real cost data. When performed on an actual project, it would be better to estimate a project-specific development of costs and measure the correlation between this structure and the market return relevant for the project (for example the index on which the company's shares are traded). The estimation performed in

¹⁹ IHS Indexes does not specify a currency unit for their indexes. For theoretically consistent risk-neutral valuation costs should be correlated to an index in the same currency as the project pays off, otherwise introducing currency risk.

this section suffices to provide a rough estimate of cost development and illustrate the adjustment of correlated private risk, and should be viewed only as such.

Growth rate	Real option	Decision tree
Growth rate gas costs	5.57%	5.49%
Growth rate electricity costs	7.41%	7.27%
Growth rate operating costs	5.59%	5.59%

Table 25: Annual cost drifts for decision tree and real option.

8.3.4. Investment costs

During the lifetime of the option, we can make several large investments. It is important to distinguish these costs from the operational costs; investment costs are deductible from the profitable tax over multiple years (with the exception of abandonment), thereby influencing the free cash flows of the project. We consider the following investments.

- **Compressor:** A compressor is required for gas production when the pressure at the wellhead falls below a certain level. We do not include a function of wellhead pressure in the simulation model. Instead, we make the investment decision for a compressor based on the production rate.
- **Gas-to-Wire investment:** When we make the decision to start generating electricity, a generator should be placed on the field. Also the generator should be connected to the transport grid, this requires wiring to be laid. Together we consider these investments to be the strike price of the option.
- **Abandonment:** When the field is no longer commercially exploited, an investment must be made to abandon the field. This investment differs from the other investments in that it has no economic life time. As a consequence, abandonment costs cannot be depreciated. Carry back can be applied to deduct the expenses from the profit of the previous year. The costs for abandonment are strongly dependent on the properties of the field and should be determined field-specific.

We show our estimates for the investment costs in Table 26. Due to the lack of historical cost information of Gas-to-Wire and project-specific characteristics there is still large uncertainty in the required investments. Of the investments made, we assume that only the generator and compressor have salvage value. We assume that their values decrease linearly with time, and have no salvage value left once the technological lifetime of the investment has been reached. Note that these capital costs increase over time with the rates specified in the previous section.

Investment	Cost
Wiring (per kilometre)	€ 160,000
Generator	€ 8,000,000
Compressor	€ 5,000,000
Abandonment	€ 1,250,000

Table 26: Cost parameter of project investments in 2012.

8.3.5. Operating cost

For the Groningen gas field, production costs are about € 0.01 per cubic metre produced (Energy Charter Secretariat, 2007). The Groningen field is relatively easily to exploit, for the gas fields eligible for Gas-to-Wire costs are likely to be higher. For onshore production we assume a

standard cost of € 0.015 per cubic metre, including factors such as processing and transportation.

When natural gas needs to be compressed, this requires more energy, making production more expensive. The acquisition of a compressor serves as the trigger to change to the higher cost estimate. Though lower pressure requires more energy to bring the gas up to the required pressure, operating costs are assumed to remain constant for all levels of compression. Taking a set of constant physical properties, the amount of energy required to compress 50,000 m³ from 10 bar to 65 bar is about 4.385 MWh (N. González Díez, personal communication, 29-08-2012). We take the average 2011 electricity price as fixed price,²⁰ this results in an estimate of €0.005 per m³ for compression costs.

For Gas-to-Wire production, we estimate the costs for maintenance of the generator at 15% of the purchase cost annually. Operating costs are estimated at 1% of the generator costs under the assumption of continuous production at full capacity. In our model these costs, under current prices, are divided by 365 and the production capacity to obtain a daily cost price per cubic meter of gas exploited. All described costs are 2012 values, and increase with the operating cost rate specified in Section 8.3.3.

Operation	Cost
Gas production without compression	€ 0.015/m ³
Gas production with compression	€ 0.020/m ³
Gas-to-Wire	€ 0.004/m ³
Maintenance generator	€ 1,200,000/year
Generator revision	€ 3,000,000 (end lifetime)

Table 27: Estimates for operating costs (based on 2012 capital costs).

8.3.6. Price cap on electricity prices

As stated in Section 2.8, electricity prices are subject to a maximum price set by the APX. This price is based on the prevailing tariffs in the balancing market; as such they are changing over time and not publicly available. In Section 6.8 we point out that occasional extreme outcomes are inherent to the GARCH model. For this reason, we need to enforce a price cap in the simulation model (He, 2007). This price cap should be high enough to allow for extreme behaviour of the series (which is observed in historical series as well), but at the same time should consider the regulatory influence. Keeping in mind that the largest price observed in the historical data is €660, we set the price cap for the following 20 years at €1000.

8.3.7. Field, production and abandonment properties

We assume that the amount of GIP is 0.25bcm, which is an average size for a field eligible for Gas-to-Wire. We set the lower and upper bounds for the GIP at 0.2 and 0.3 bcm respectively. The production function for this field has an initial production rate of 53,000 m³/day and an annual decline rate of 5.48%. We set the threshold for abandonment for gas production at a production rate of 40,000 m³ per day. For electricity generation this threshold is 25,000 m³ per day, as this is the minimum capacity of the generator. Most small fields in the Netherlands contain high-caloric gas. We therefore take the average energetic value of Dutch high-caloric gas; 69 m³ of natural gas contains 1 MWh of energy (GasTerra, 2012d).

²⁰ The compression costs could be linked to the simulated electricity prices as well, but we expect this to have a limited effect on annual cash flows.

Parameter	Value
Field size	0.25 bcm
Initial production rate	53,000 m ³ /day
Annual decline rate	5.48%
Energetic value natural gas	0.014444 MWh/m ³
Abandonment rate gas production	40,000 m ³ /day

Table 28: Parameter settings for field properties.

8.4. Valuation methods for real options

The real option we present in this study can be exercised at a finite number of specified dates before maturity (the first day of each year), and as such is considered a Bermuda-type option. More specifically the real option can be viewed as a nonstandard Bermudan swap option, allowing to trade one stream of variable cash flows for another stream of variable cash flows by making a fixed payment. It is difficult to value an American or Bermudan option analytically (Joy *et al.*, 1996; Broadie & Glasserman, 1997; Gamba, 2002; Minqiang Li, 2009). This is because we need to find the optimal exercise policy to determine the option value. Further our real option contains some nonstandard components, such as multiple stochastic processes, mean-reversion and random jumps, increasing the difficulties of analytical valuation even more (Miller & Park, 2002; Law, 2007). But also most numerical methods, such as binomial trees, require complex modifications to deal with such properties.

An important feature of our real option is that its payoffs are path-dependent. Most valuing techniques have great difficulty valuing such options. Often analytical solutions are not available at all. The problem is that we need to solve for the asset value(s) and the optimal exercise policy simultaneously; the asset value itself depends on the exercise decision made. Monte Carlo simulation is well able to deal with path-dependency, making it the most natural choice for the presented problem (Barraquand & Pudet, 1996; Glasserman *et al.*, 1999; Cortazar, 2000). Also simulation is a flexible and insightful method, allowing for easy modifications and extensions. For these reasons we choose Monte Carlo simulation as valuation method for the real option.

Monte Carlo simulation is not without disadvantages. A problem with simulation is that many runs are required to approximate the value of the option, potentially requiring long simulation times. When an insufficient number of runs is made, the calculated value can also significantly deviate from the theoretical value. A lower- and upper bound must be calculated to quantify the approximation error. Further Monte Carlo simulation is a forward process, rather than a backwards calculation such as the binomial tree (Choudhury *et al.*, 2008). A disadvantage of this feature is that the value obtained is strongly influenced by the exercise policy chosen for the Bermudan option. In contrast, backwards valuation allows determining the optimal policy recursively. Section 8.5 shows how to deal with this issue by applying a stopping algorithm.

8.5. Longstaff-Schwartz algorithm

According to Carriere (1996), the optimal value of a Bermudan call option can be expressed as

$$call = \max \{0, S_1 - X_1, \dots, S_t - X_t\} \quad (8.1)$$

This value is obtained by adopting an optimal exercise policy, which cannot be determined with a forward Monte Carlo simulation. A solution to determine a suboptimal exercise policy of an

American or Bermudan option on a simulated price path is to run a stopping algorithm on it, providing a low-biased estimate for the option value. In this study we use the algorithm of Longstaff & Schwartz (2001). A stopping algorithm works backwards on the simulated price path from the maturity date. At each time step, we compare the exercise value and continuation value to see whether it is optimal to exercise the option or to wait. We perform regression analysis on future cash flows to obtain the *expected* values (Tilley, 1993). We do this because using the actual simulated cash flows would indicate perfect foresight (Broadie & Glasserman, 1997). The mathematical rationale behind using regression is that a function which can be differentiated twice can be approximated by a linearly independent set of basic functions (Pedersen, 1999). Longstaff & Schwartz (2001) regress future cash flows on state variables representing known information at point n . They only use in-the-money paths (paths for which the option is profitable to exercise), stating that this increases the efficiency of the algorithm and allows for better estimation of cash flows relevant for exercise decisions. The regression formula is provided by a set of basic functions. For example, we can use a polynomial function:

$$E_t^Q[CF_{t+1}] = c + w_1 S_t + w_2 S_t^2 + \dots + w_n S_t^n \quad (8.2)$$

Many other regression functions can be used as well, for example including cross products of several state variables. According to Longstaff & Schwartz (2001), in general, relatively simple regression functions provide good results when including a small number of state variables. For multiple state variables, the regression function chosen may have a much greater impact (Grau, 2008; Beveridge & Joshi, 2009). In this study we take the function in Equation (8.2) with $n = 3$. After obtaining the coefficients of the regression functions, a second simulation should be performed (Thom, 2009; Ware, 2011). The calculated exercise boundary may then be applied on the simulated state variables in a forward fashion, which is computationally more efficient. As the regression parameters are determined on the first simulation, the algorithm might be biased high due to the benefit of foresight (Andersen, 1999). Using a second simulation set ensures that the estimate is low-biased.

The Longstaff-Schwartz algorithm provides no measure on how close the lower-bound estimate is to the upper bound (Piterbarg, 2003; Joshi, 2006). Using regressed values results in a sub-optimal exercise policy, how close optimality is approximated depends on the quality of the regression. Algorithms for calculating the upper bound are also available in literature, but are more complex and computationally intensive (Broadie & Cao, 2008). We need to adopt the perspective of the option seller for this. The seller hedges against all possible exercise strategies by the buyer, and can profit when the buyer does not exercise optimally. Under optimal exercise the buyer- and seller prices converge to a single option price. Generally, lower bound algorithms are used to value derivatives, calculating the upper bound only once to gain insight in the deviation from the actual value. In this study we do not calculate the upper bound, but one should be aware that the Longstaff-Schwartz algorithm is used to obtain the lower bound only.

The real option model differs from a standard financial option in that future cash flows remain uncertain after exercise, while both the stock and strike price are known for a standard financial option when exercised. Hence, for the real option both the continuation- and exercise value should be estimates of the actual simulated cash flows, implying perfect foresight otherwise. In the case of uncertain exercise values, Piterbarg (2003) proposes to perform regression on the exercise values as well. As the continuation value is based on future exercise values, we should obtain estimates for exercise values first. We then perform backwards induction for the continuation values.

We apply the Longstaff-Schwartz algorithm on the decision tree model as well. Though we did not find such an approach in literature, we believe it to be most consistent with the real option model. Basically the algorithm provides a set of triggers used to optimise exercise decisions under uncertainty. We considered some other possibilities as well. A static decision tree would be obtained by placing all simulated cash flow paths in a single tree with equal probabilities assigned to each path. This approach essentially reduces the simulation results to a single tree containing the average cash flows; the chosen exercise date of the option is then based on the average. Compared to the real option model, a static decision tree undervalues flexibility. Alternatively, we could construct and solve a decision tree independently for each simulation, optimising the exercise decision with the benefit of perfect foresight. This method would overestimate flexibility value compared to the option model. We therefore believe that the backwards regression approach offers the best comparison.

Step	Description
Step 1	Generate i price paths of length n .
Step 2	For the simulated prices at point n , subtract strike price to obtain cash flow CF_n .
Step 3	Remove all calculated cash flows and corresponding asset values smaller than 0 before performing the regression analysis.
Step 4	From $n - 1$ to 0, regress S_n on CF_{n+1} to obtain the expected exercise value $E(CF_n)$ at point n .
Step 5	Regress S_{n-1} on discounted CF_{n+1} to obtain the expected continuation value at point n .
Step 6	Set the cash flow at time $n - 1$ at the actual exercise value when the expected exercise value is higher than the expected continuation value, and 0 otherwise.
Step 7	When the option is exercised at point $n - 1$, set the later positive cash flows for that price path equal to 0.
Step 8	Repeat steps 5 to 7 for $n - 2$ to 0.
Step 9	At time 0, divide the sum of the expected continuation values by i to obtain the option value.

Table 29: Stepwise description of modified Longstaff-Schwartz algorithm.

Summary of the Longstaff-Schwartz algorithm
▪ The Longstaff-Schwartz algorithm is a backwards regression on simulated price paths to determine the lower-bound option value based on expected continuation values. ▪ At each exercise point, a regression is performed on all in-the-money price paths to estimate the cash flow at the next point. Regression is used because investors do not have perfect foresight on price development. ▪ The algorithm replaces the estimated exercise value with the estimated continuation value if it is higher, eventually obtaining a cash flow matrix from which the lower-bound average option value at time 0 can be derived.

8.6. Spark spread as state variable and production threshold

For peak-load power plants, a common production decision rule is based on the so-called spark spread (He, 2007). This is the difference between the price of the commodity serving as input and the electricity price reduced with the percentage of the energetic value lost during the generation process. More formally, the spark spread is described as

$$SS_t = (S_{e,t}/\text{efficiency}) - S_{g,t} \quad (8.3)$$

where the prices of electricity ($S_{e,t}$) and gas ($S_{g,t}$) are both expressed in €/MWh

We define the ‘asset value’ in the model as the difference between the discounted sum of cash flows from gas production and from electricity production, excluding the strike price. The asset value can be considered as the added value Gas-to-Wire has, which may also be a negative value. The stochastic part of the cash flows is determined almost completely by the spark spread. For this reason, we use the average annual spark spread as the state variable on which the regression of future cash flows is based.

The spark spread also serves as a production threshold. Though the operator of a gas field is not required to buy gas as production input from the spot market, it still makes sense to base the production decision on the spark spread. This is because the market value of the firm is increased when transforming a commodity into a more valuable one (Fleten & Näsäkkälä, 2003). Thus, producing when the spark spread is positive means that value is created. Note that the value of unprocessed gas is lower than that of processed gas, meaning that a spark spread below 0 could still be economically attractive. An example of a randomly generated spark spread assuming a 45% generator efficiency is shown in Figure 35. Preliminary testing showed that setting the spark spread too rigid tends to lower the NPV, meaning that the effect of discounting outweighs the benefit of selling at higher prices. We set the spark spread threshold at -40, which eliminates production at the more extreme price levels (very low electricity price and very high gas price).

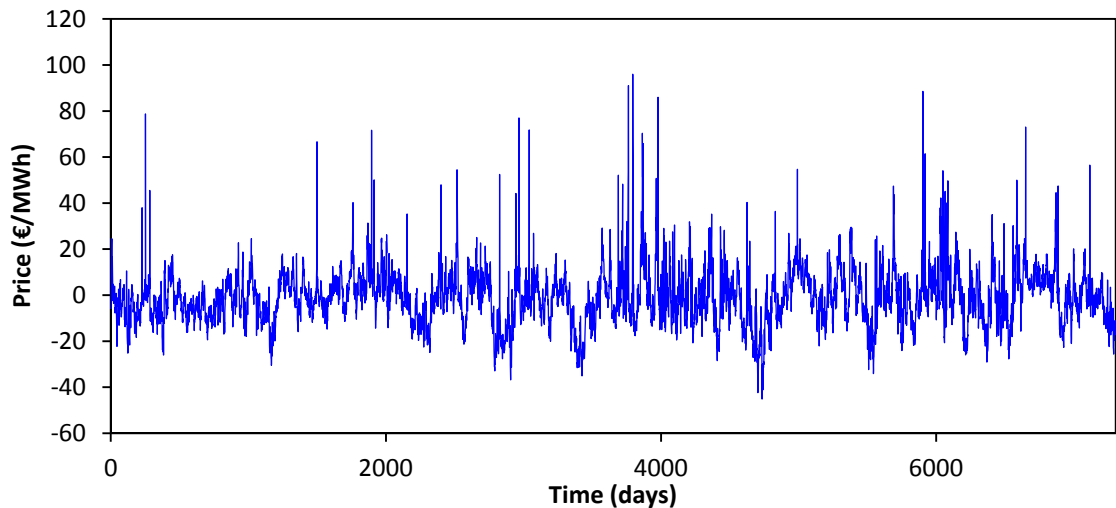


Figure 35: Randomly generated spark spread with 45% generator efficiency.

8.7. Key assumptions

In this section, we provide an overview of the key assumptions for our model.

Decision maker perspective

- Decision makers are rational and strive for profit maximisation based on the information available.
- We adopt the perspective of a well-diversified shareholder with regard to private risk, meaning that no additional risk premium is required.
- The capital market is complete with respect to market risk; private risk can be estimated subjectively. Investors agree with these subjective estimates.
- All investment decisions are made once a year.
- Decisions made are not affected by competition.

Gas production

- Gas production takes place on a continuous basis.
- Produced gas cannot be stored; gas is sold immediately after production.
- Each quarter, the producer makes a rational choice between a 3-0-3 and 6-0-3 contract, whichever yields the higher price.
- Produced gas can always be sold immediately (no demand constraints), with contract prices based on spot price averages.
- No gas is lost during transportation.

Electricity production

- The producer knows the day-ahead electricity price, and is able to respond to this information with daily production decisions.
- There are no demand constraints on the sale of electricity.
- Production decisions are based on a constant spark spread threshold.
- The building time to switch from gas to electricity production is zero.
- No electricity is lost during transportation.
- The energy lost during conversion from gas to electricity has no value.

Capital

- There are no capital restrictions: the operator has sufficient capital available to make all required investments at all times.
- The generator and the compressor have a salvage value that decreases linearly with their technological lifetime; all other investments have no salvage value.

Generator properties

- Maintenance activities do not interrupt production.
- Breakdowns do not occur.
- The generator can instantly reach maximum production capacity, as well as shut down instantly.
- The number of start-stops of the generator is unlimited.
- The properties of the generator remain constant over time.
- A revision of the generator is required at the end of its technological lifetime.

Tax

- We do not consider the effects of value added tax.
- Tax regulation remains constant over time. We treat other tax scenarios separately.
- Losses carried forward are equated with profits as soon as possible.
- Carry back has priority over carry forward.

Field properties

- The gas field exploited has perfect connectivity, meaning that the reserve can be fully exploited with a single production well.
- All gas present in the field is of the same mixture.
- Once the field is abandoned, it cannot be reopened for Gas-to-Wire production at a later stage.
- Abandonment takes place in the same year production of gas or electricity halts due to falling below the threshold value.

Simulation study

We perform a simulation study using the model established in the previous chapters. We explain the setup of the simulation study, describing the workflow which we follow to calculate the option value. We adopt a 95% confidence bound for the calculation of the lower bound option value, calculating the number of simulation runs required based on the variance of a sample simulation. Further, we provide a brief recap of the differences between the decision tree model and the real option model, showing how the used drifts and discount rate compare. Aside from the base case worked out in Chapter 8, we describe eight alternative scenarios for the purpose of scenario analysis. The final section of the chapter is dedicated to presenting and discussing the results of the simulation study.

9.1. Setup of simulation study

We perform a simulation study using the models we constructed in the previous chapters. We perform ROA and DTA parallel, calculating the cash flows for both methods in the same run. We use the same random numbers streams and the same daily production for both methods, so that cash flows can be compared. We use the simulation settings as described in Chapter 8, following the workflow specified in Table 30. We discount the net present values of the cash flows back to each possible year of exercise. This allows for comparison between exercise and continuation. After completion of the Longstaff-Schwartz algorithm, all cash flows are discounted back to time 0. In Appendix III we describe the more complex source coding used in the simulation model. The model itself also contains comments to clarify the structure of the functions used.

Step	Description
Step 1	Set simulation properties
Step 2	Generate random price paths for gas and electricity
Step 3	Calculate daily unconditional gas production
Step 4	Calculate daily conditional electricity production for each year
Step 5	Calculate daily revenue and operating costs
Step 6	Calculate annual earnings
Step 7	Make annual investment decisions
Step 8	Determine depreciation
Step 9	Determine carry back
Step 10	Determine carry forward
Step 11	Calculate free cash flows
Step 12	Store net present values of gas production and each option at exercise date
Step 13	Back to step 1, repeat workflow for next iteration

Table 30: Workflow of simulation run.

9.2. Approximation of true option value

An important choice in a simulation study is the number of iterations that we should perform. The more iterations, the closer the average option value approximates the ‘true’ option value (i.e., the theoretical value, would the set of partial differentials be solved). The same effect is obtained by reducing the variance of the simulated option values, but this is not always possible. Computation time increases linearly with the number of iterations. Projects for which the cash flows have a low variance require less runs than projects with high variance, as the true average value will be approached quicker (Plat, 2002). The mean-reversion property of the price series is a factor which we expect to limit the variance of the option value. To estimate the number of iterations required, we first do a sample simulation of 5,000 iterations to estimate the variance of the option value. Following Law (2007), we use Equation (9.1) to estimate the required number of runs based on a specified significance level and a relative error:

$$\min \left\{ df \geq n: \frac{t_{(p/2, df)} \sqrt{\frac{\hat{\sigma}^2}{df}}}{|\hat{\theta}|} \leq \alpha' \right\} \quad (9.1)$$

With

- df as the required number of runs/degrees of freedom
- n as the number of runs in the sample simulation
- $t_{(p, df)}$ as the critical value of the student distribution at p with df degrees of freedom
- $\hat{\sigma}^2$ as the estimated variance of the option value in the sample simulation
- p as the significance level
- α' as the adjusted relative error
- $\hat{\theta}$ as the estimate of the option value

The adjusted error is described by $\alpha' = \alpha / (1 + \alpha)$, where α is the specified relative error. We set both the significance level and the relative error of the value at 5%. The required number of runs is then 5,000, which is the smallest possible number given the size of the sample run. We presume this number is valid for all scenarios. We make a total of 10,000 runs for each scenario: 5,000 for calibration of the Longstaff-Schwartz algorithm and 5,000 for option valuation. A full simulation of 10,000 iterations takes four hours to complete with a 2.4GHz Intel Core i3 CPU.

9.3. Comparison between real option and decision tree analysis

To make a fair comparison between real option analysis and decision tree analysis, both make use of the same random number streams. Also daily production (i.e., the amounts of gas/electricity produced) is the same for both analyses. This way, valuation differences can be solely assigned to the probability distributions and the discounting approach. Table 31 shows in which aspects the real option analysis and decision tree analysis differ. Figure 36 shows how the discount rates and drifts compare for both methods. We can make some observations by looking at these rates. These effects are not necessarily applicable on ROA in general. Most obvious is the difference in discount rate, DTA values future both incoming and outgoing cash flows much less than ROA. The real option model discounts costs conservatively compared to the decision tree.

Rate	Real option analysis	Decision tree analysis
Drift price series (<i>Market risk</i>)	Risk-neutral, based on drift of futures prices.	Real, based on energy market outlooks.
Drift costs (<i>Private risk</i>)	Partially risk-neutral, based on correlation of public cost indexes with the market.	Real, estimated based on public cost indexes.
Discount rate	Risk-free rate, based on the return on a 20-year Dutch government bond.	WACC, based on an average WACC applied by E&P-operators.

Table 31: Differences between the real option and the decision tree. Note that the properties of the real option ensure that cash flows are implicitly adjusted for risk.

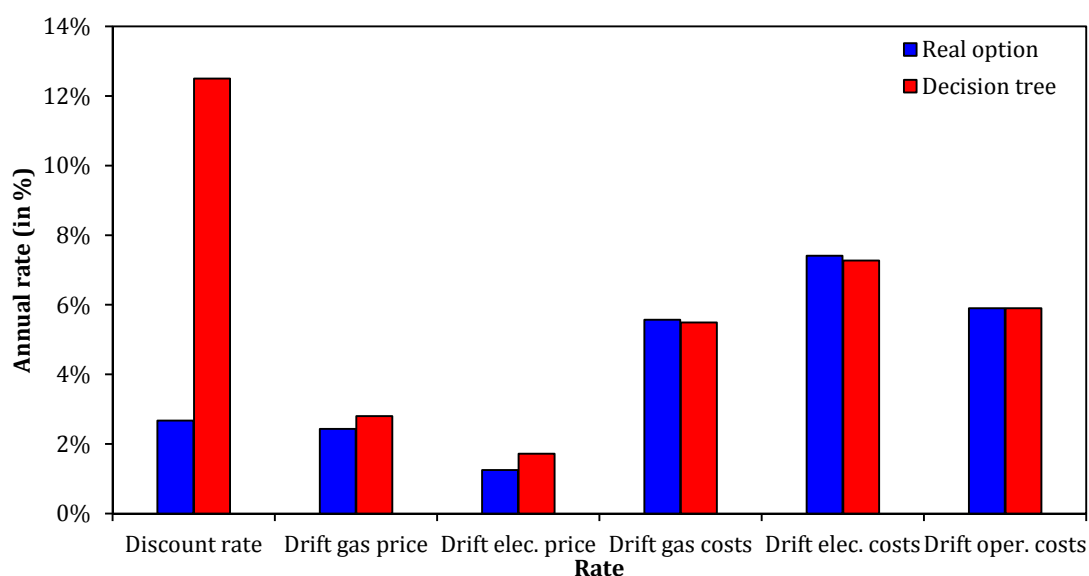


Figure 36: Comparison between drifts and discount rates of real option and decision tree. The real- and risk-neutral drifts are close to each other, but the discount rate for DTA is much higher than the risk-free rate. Therefore DTA places much less value on future cash flows.

9.4. Scenario analysis

The parameter estimates for the base case scenario are based on averages and current knowledge. Changing these parameters may have a strong effect on the value of the option. To account for the uncertainty of the estimates we perform scenario analysis, indicating what effect changing one parameter or a small number of parameters corresponding with some other scenario has on the option value. Next to the base scenario, we run eight other scenarios. We include the change of parameters that we consider least certain and may have a significant impact on valuation. Table 32 describes the scenarios that we test in this study. Note these scenarios are not necessarily consistent with their theoretical foundation. For example the risk-free rate should not be changed without re-evaluating the risk-neutral asset drifts as well. We ignore such considerations in the scenario analysis. We briefly explain why we test the chosen scenarios.

A prominent source of uncertainty is the reserve amount; each field has its own uncertainty distribution. Compared to the base scenario, we assume a relatively small and a relatively large reserve (with regard to Gas-to-Wire) to test the influence of field size on the added value of flexibility. The large reserve has an average of 0.5 bcm, with boundaries of 0.4 and 0.6. The small reserve has an average size of 0.1 bcm, with boundaries of 0.05 and 0.15. The production functions are modified to coincide with the adjusted size of the field. We take initial production rates of 70,000 (large reserve) and 45,000 (small reserve) with annual decline rates of 9.13% and 2.74% respectively.

The current corporate tax rate is quite low in a historical perspective. In case fiscal measures are taken to stimulate the exploitation of small fields, we can imagine that the effective tax rate for operators becomes even lower. In this low-tax scenario we assume that no royalties are paid, keeping income tax rates the same. On the other hand we think of a scenario with a high tax rate of 40%, which is on the high end of historical rates. Often higher rates are also paired with more depreciation opportunities. Carry back- and carry forward regulation remain unchanged in this scenario.

As can be noted from Figure 36, the corporate WACC is out of line with the risk profile of the project. In one scenario, we adopt a project-specific WACC to test how such a decision tree compares to ROA. We set this discount rate at 2.67%, which provides nearly the same NPV's as the real option model for the project without flexibility (i.e., only gas production).

The currently prevailing risk-free rate is historically low. In a time frame of twenty years, this rate is likely to fluctuate around some average rate higher than current rates; historically periods with extreme interest rates tend not to last for decades (Eichholtz & Koedijk, 1996). Since our model does not allow for stochastic interest rates, we must therefore assume a representative average rate. For this, we test a scenario with a constant risk-free rate of 5%.

Considering developments such as the growing world population, increasing living standards, increasing energy demand, sharply increasing oil prices and depletion of natural resources, we might expect a higher drift of market prices than our futures prices and outlooks suggest. We incorporate a scenario assuming a much stronger growth rate for gas- and electricity prices. We take an annual drift of 7% for both price series, for both DTA and ROA. Finally, we consider a scenario in which we increase the efficiency of the generator to 55%, anticipating on possible technological developments in Gas-to-Wire production.

Scenario	Change from base scenario	Purpose
Low tax	Royalty of 0%	Test for fiscal measures in favour of the exploitation of minor gas fields.
High tax	Flat corporate tax rate of 40%	Test for negative impact of tax regulation.
Moderate risk-free rate	Risk-free rate of 5%	Test for risk-free rate closer to average.
Project-specific discount rate	WACC of 2.67%	Test for performance of DTA with adjusted WACC.
High energy price	Risk-neutral drift of gas 7% Risk-neutral drift of electricity 7% Real drift of gas 7% Real drift of electricity 7%	Test for strong increase in energy prices.
Large reserve	Average reserve of 0.5 bcm Lower bound of 0.4 bcm Upper bound of 0.6 bcm Initial production rate of 70,000 Annual decline rate of 9.13%	Test for added value of flexibility for relatively large field.
Small reserve	Average reserve of 0.1 bcm Lower bound of 0.05 bcm Upper bound of 0.15 bcm Initial production rate of 45,000 Annual decline rate of 2.74%	Test for added value of flexibility for relatively small field.
Enhanced Gas-to-Wire	Generator efficiency of 55%	Test for technological development in Gas-to-Wire.

Table 32: Overview of alternative simulation scenarios.

9.5. Simulation results

In this section we discuss the results of the simulation study performed. First, we describe our application of the Longstaff-Schwartz algorithm. We applied the exercise policies determined with this algorithm on a second simulation, and briefly discuss the results and implications of the tested scenarios. Section 10.3 goes into more detail about the practical implications of the simulation study, in this section we focus more on the performance of the model and the observed differences between ROA and DTA.

The regression functions of the algorithm do not provide very accurate approximations of future cash flows. This is not an unexpected result; the current spark spread is a poor explanatory variable for the future spark spread due to its high volatility. This effect is strengthened by the long time horizon of the option. For most scenarios, the point of exercise provided by the algorithm is a date earlier than the optimal decision under perfect foresight. We provide an example of a regression function for a particular year in Figure 37. In this graph, the option is exercised if the line representing the expected continuation value falls below the line representing the expected exercise value. A unique set of these functions exists for each year.

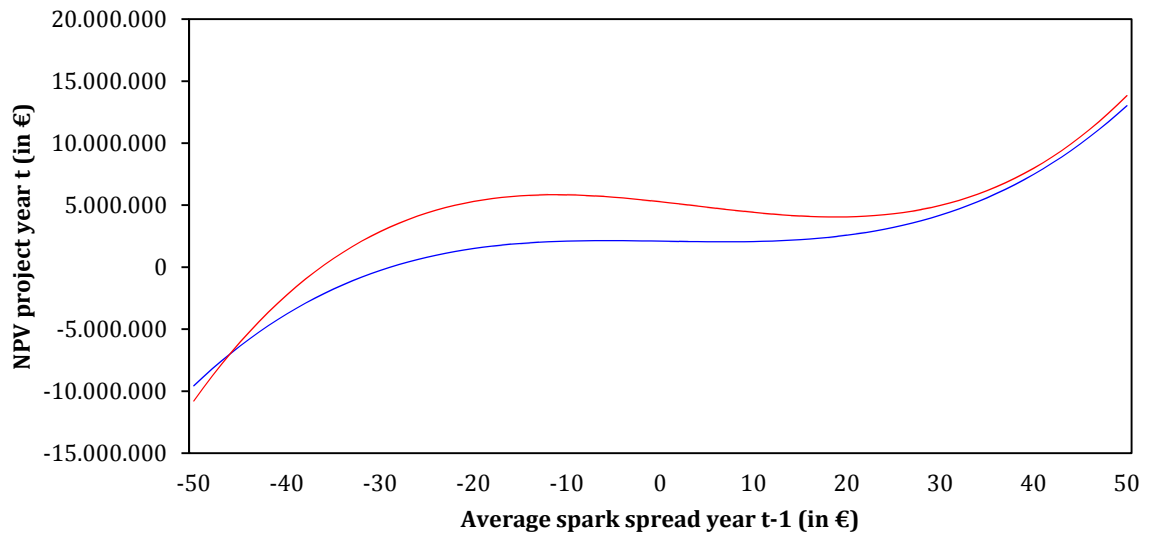


Figure 37: Functions of expected continuation value (red) and exercise value (blue) at year t . In this specific case it is almost always optimal to defer exercising the option.

Running the base scenario results in a real option value of € 727,203 for the opportunity to switch to Gas-to-Wire production. In 71% of the runs, the option is in-the-money at some point in time. The average NPV of gas production (i.e., the project without flexibility) is € 37,901,923, so that the swap option provides an additional value of 1.92%. In comparison, the decision tree gives us a much lower option value of € 35,343, which is 0.14% of the project value without flexibility. Both in an absolute and in a relative sense, DTA values the flexibility embedded in the project much lower. This can be explained by the fact that investments in DTA are more expensive relative to the discounted incoming cash flows following later.

We provide a summary of the main results for all scenarios in Table 33. Most notable is the scenario with the adjusted discount rate. With a relative value of 4.26% this option is clearly more valuable than its ROA counterpart, indicating that the risk profile of Gas-to-Wire production deviates significantly from that of gas production. The high energy price and the high efficiency scenarios strongly increase the option value. The latter is the only ROA scenario for which im-

mediate exercise is optimal. For the large reserve Gas-to-Wire becomes attractive only in a late production phase, after depleting most of the field. Consequently, added value from Gas-to-Wire becomes relatively smaller and is subject to more years of discounting. For these reasons the algorithm may have more difficulties finding the optimal exercise date.²¹ Note that DTA provides higher values than ROA; because few paths are in-the-money in this scenario, the policy estimated for DTA happens to activate some more profitable paths.

Generally, the options are exercised after two to five years of gas production. Provided that the spark spread at the time of exercise is sufficiently high, the investment required for Gas-to-Wire production can be earned back within a relatively short time. The fact that the generator usually has salvage value after abandonment plays a significant role in this; the surplus benefits of Gas-to-Wire only need to cover for depreciation. Because the options are generally exercised at a future date, the ROA approach yields a higher value of flexibility; the high discount rate applied by DTA places much less value on the cash flows stemming from Gas-to-Wire compared to the preceding cash flows from gas production.

Scenario	Real option value (% of project value)	Decision tree value (% of project value)
Base	€ 727,203 (1.92%)	€ 35,343 (0.14%)
Low tax	€ 3,015,089 (6.89%)	€ 141,387 (0.47%)
High tax	€ 609,345 (1.42%)	€ 25,617 (0.09%)
Moderate risk-free rate	€ 189,620 (0.55%)	n/a ²²
Project-adjusted WACC	n/a ²²	€ 1,616,308 (4.26%)
High energy price	€ 35,654,340 (79.76%)	€ 6,398,067 (21.47%)
Large reserve	€ 1,671 (0.00%)	€ 38,230 (0.08%)
Small reserve	€ 292,783 (1.43%)	€ 28,439 (0.19%)
Enhanced Gas-to-Wire	€ 12,722,615 (33.54%)	€ 2,614,0338 (9.85%)

Table 33: Simulated option values for all scenarios, both absolute and relative to the project value without flexibility.

²¹ With a small number of in-the-money paths, regression takes place on only a few paths. These paths therefore have a strong influence on the policy that is determined.

²² The scenario is only modified for one valuation method, hence should be compared to the base scenario.

Chapter 10

Conclusions

With this study we attempted to increase insight in both the theoretical and practical application of real option analysis, particularly in comparison to decision tree analysis. We illustrated the application of ROA with the construction of a simulation model on Gas-to-Wire production.

From a theoretical perspective, the real option approach has some notable merits compared to other valuation techniques. The main point in which ROA distinguishes itself is the adjustment of the discount rate to the risk profile of each project phase, which is most naturally done by applying risk-neutral valuation. This risk-adjustment solves a theoretical inconsistency present in DTA, therefore we prefer ROA from an academic point of view.

The limited application of real options in corporate practice may have several reasons. Risk-neutral valuation and risk-adjustment are concepts not easy to grasp for decision makers without a financial background. Also ROA is no decision tool generally applicable, working best for specific types of investment problems. Still, the stronger theoretical foundations, the objective approach of risk-neutral valuation and the adjustment to different risk profiles make real options able to provide better project valuations than DTA.

Our application of a real option model on the Gas-to-Wire investment scenario shows how adopting a corporate WACC in DTA may cause serious mispricing when valuing a project with a different risk-profile. Also adopting a project-specific discount rate provides deviating results from those of ROA due to changing risk which is not accounted for.

10.1. Theoretical evaluation of real option analysis

After more than thirty years of research on real options, academics still have not reached an agreement about what ROA stands for precisely. Definitions of ROA vary between a strict application of Black-Scholes modelling to basically any decision tool incorporating flexibility, making use of different rationales and techniques. The strict definition is consistent with option pricing theory, while looser definitions are fitted better to the properties of real projects. For this section, we use the integrated approach as adopted in this study as definition for ROA.

In comparison to traditional decision tools, real option analysis has some clear advantages. Most importantly, it values flexibility, allowing to respond to new information dynamically. In many studies, real options are compared with traditional tools. However, more advanced decision tools are able to deal with stochastic processes and decision optimisation as well as real options do. It would therefore be incorrect to state that real options bring a decisive advantage with respect to embedding flexibility in general. From an academic point of view, we prefer ROA over DTA; the latter is theoretically flawed due its application of a constant discount rate on projects with varying risk profiles. The beta in the CAPM is based on the covariance of the project returns with those of the market. As the covariance differs for each decision path, it is theoretically inconsistent to apply a single discount rate. In fact, the only fundamental aspect in which ROA differs from dynamic DTA is the risk-adjustment towards the different risk-profiles, which is done by applying risk-neutral valuation. ROA is therefore more consistent with pricing theory.

An assumption often made in financial option literature is that the risk-free rate, market return and beta are constant over time. This assumption is rationalised by stating that the investment opportunity of a single asset remains constant even when the risk-free rate and market return vary over time. Also when we consider a relatively short time span these parameters are more likely to be approximately constant. When considering multiple assets and/or a long time horizon the assumption of parameters being constant is difficult to uphold; treating the parameters as stochastic variables would results in a better fit with pricing theory.

The risk-neutral approach used for market risks in ROA has some favourable elements. It allows estimating the risk-neutral asset drift based on futures contracts, while the discount rate can be based on government bond yields. This objective approach incorporating all market information is theoretically superior over subjective estimation of real drifts and discount rates, which can be strongly influenced by personal beliefs and preferences.

Summary of theoretical evaluation ROA

- Definitions of ROA vary between strict and loose applications of option pricing, so that no general evaluation of its theoretical properties is possible.
- Real options are able to deal with flexibilities embedded in a project, but as a decision tool it is not unique in this respect.
- Unlike DTA, ROA treats the different market risk profiles of decision paths in a manner consistent with pricing theory.
- The integrated approach of ROA allows for a theoretically consistent valuation of both market and private risks.
- The assumption of a constant risk-free rate, beta and market return is not applicable well for ROA. We can address this issue by treating them as variables.

10.2. Practical evaluation of real option analysis

As stated in Section 10.1, dynamic decision trees are able to model decisions and uncertainties just as well as real options. In this respect the added value of real options is often overstated in literature, ignoring the more advanced decision tools that are used in practice nowadays. What remains regarding flexibility is the benefit of real option thinking; structuring a real option forces analysts to be more explicit about the nature of the flexibility and the assumptions made than other decision tools. The application of ROA should lead to investment decisions which are more in line with the expectations of investors, and consequently to decisions which help to maximise the value of the company. Nevertheless, how much the outcomes of ROA differ from those of DTA depends largely on the differences in risk profiles of the investment opportunities. If the risk profiles of the option paths are similar to each other, risk adjustment has a limited effect. A practitioner could then apply DTA with a project-specific discount rate to obtain similar results as with ROA.

We stress that real options are most useful when a series of conditions is met. Option value increases with volatility, provided that downside risk can be hedged. Therefore high uncertainty makes ROA more useful as a tool. A significant part of uncertainty should be market risk; otherwise the valuation method reduces to plain decision analysis without risk adjustment. ROA is useful when the expected value of the project without flexibility is close to zero. In that case decisions have a significant impact on its value. It is important that the decision maker actually has the opportunity to respond to new information becoming available in a flexible way. If this is not possible, an approach such as DCF may be more suitable.

A disadvantage of ROA is that the concept of risk-neutral valuation is not very intuitive, and therefore it may be hard to explain to decision makers without a financial background. Also quite strong assumptions are made for risk-neutral valuation. Their frequent violation makes it difficult to defend the application of real option valuation in practical settings. Though the calculation of the WACC required for traditional methods is partially based on the same principles, they become more prominent when applying risk-neutral valuation. These issues may to some extent explain why real option valuation has not been adopted on a large scale in practice so far.

Summary of practical evaluation ROA

- With regard to modelling flexibility, the main benefit of ROA may be that it requires a clear structure and explicit assumptions.
- The actual difference in option valuation with ROA and DTA depends on how much the risk profiles differ.
- Real options are most suitable for situations with large market uncertainty or an expected value close to zero, and the flexibility to respond to new information.
- The explicit adoption of risk-neutral valuation and the strong assumptions required for this method may be reasons why ROA has not obtained a prominent position in corporate practice.

10.3. Application of real option model to Gas-to-Wire production

Because of the inaccuracy of the cost estimates and our simplified approach towards the production curve, we are unable to draw strong conclusions about the actual economic attractiveness of Gas-to-Wire. We therefore restrict ourselves to the results of the simulation study, while keep-

ing in mind the investment problem we defined is partly theoretical. Under these reservations, Gas-to-Wire is moderately attractive for tail-end fields, provided that the generator has salvage value after abandonment. If this is the case, generally it is favourable to extend the lifetime of the field if the spark spread at the time is sufficiently large. The scenario with increased generator efficiency shows that Gas-to-Wire becomes much more attractive if generators could be improved.

In the proposed investment problem, private risk is limited. Many high-risk activities performed by E&P-operators such as exploration and well-drilling are not included in the scenario. As the corporate WACC is partially based on such activities, it makes sense that the absence of such activities should result in a lower project-specific discount rate. As a result, applying DTA with the corporate WACC understates the value of the option. When adopting a discount rate tailored to the project without flexibility, DTA overstates the value. This may be because the riskiness of Gas-to-Wire is higher than that of gas production (as electricity prices are more volatile); the adjusted discount rate fails to incorporate this characteristic. We would need a separate discount rate for Gas-to-Wire as well to solve this issue. In a more complex setting, we would also have to change these discount rates over time. When relying on subjective estimates, it is very hard to do this in such a way it coincides with market beliefs. From the scenario with the project-specific discount rate, it becomes clear how ROA can provide more accurate estimates due to risk-adjustment. The ROA approach should therefore be able to provide a valuation closer to the perception of a shareholder.

Discussion and further research

The purpose of this final chapter is twofold. In one part we focus on the application of real option analysis in general, and how it compares to other valuation techniques. In the other part we focus specifically on the Gas-to-Wire model presented in this study, and in which ways we could improve it.

The first point we assess is the validity of the assumptions applied on real options. We briefly recapitulate on what points they tend to fall short, how some of them can be improved and what consequences these assumptions may have on the practical application of real options. We pay special attention to dealing with non-diversifiable private risk. For complex projects including multiple optionalities, we consider how co-dependencies between multiple real options could be assessed in a simulation study.

We propose some extensions and improvements for the option model, particularly regarding improvement of the electricity price model. Further we recommend calculating an upper bound for the option value to increase insight in the accuracy and convergence rate of the model. Finally, we mention some techniques to reduce the computational burden of Monte Carlo simulation.

11.1. Validity of option theory assumptions

As we pointed out in this study, the assumptions made in financial option theory are generally less applicable to real projects. Two main reasons exist for this. Uncertainties that cannot be hedged with liquidly traded market instruments cannot be valued in a risk-neutral way. Also such uncertainties are often not solved simply by waiting, and management may not be able to respond to them. As a result, classic ROA often has a poor fit with real investment problems.

The corporate world is different from the financial world in some respects, which has an effect on the validity of option theory. Risk-neutral valuation requires the ability to short-sell assets, lend and borrow at the risk-free rate, continuously hedge the portfolio held, etc. Financial institutes approximate these assumptions much better than most companies. A perfect match with theoretical markets never exists, but we do not know to what degree mismatches affect ROA. From a theoretical perspective, the assumption of a constant risk-free rate, beta and market return is flawed when applying ROA, both due to the generally long time horizon and the changing risk profile. Further research could indicate how to model these parameters as stochastic and co-dependent processes, and what impact this would have on the option value. The approach to consider private risk as uncorrelated with the market and therefore risk-free is consistent with the CAPM, as long as the private risk can be diversified away. When an investor allocates a significant portion of his budget to a project, he may be unable to do so. In this case he would require a risk-premium for private risk. A subjective discount rate based on the utility function of the investor should then be used. We briefly describe this approach in Section 11.2.

We should stress that the shortcomings of assumptions are not exclusive to real option valuations; the CAPM or comparable models are used in DCF and DTA as well. The adoption of the risk-neutral valuation in option valuation makes these assumptions much more explicit though, both in the valuation method and in visibility. This could be a reason for the limited application of ROA in practice, failing to sufficiently defend or explain the assumptions to management.

11.2. Treatment of non-diversifiable private risk

One of the core difficulties of ROA is how to treat private risk. In this study we adopted a risk-neutral approach towards private risk. This view is consistent with a shareholders perspective, and should maximise value for this group. However, the company itself may be exposed to significant risks that cannot be diversified away, or the company may not even be publicly owned. The CAPM theory assumes that the decision maker acts as an agent for rational, well-diversified shareholders who only care about systemic risk. In reality, investment decisions are usually made by multiple decision makers with their own subjective beliefs and risk preferences. The decision maker may be an investor who needs to make a significant investment to participate, making him unable to diversify away private risk. In that case the risk-free discount rate no longer applies to non-systemic risk. Smith & Nau (1995) and Luenberger (1998) propose to perform so-called 'buying price analysis' in this case, making use of an personal exponential utility function to obtain the unique certainty equivalent of cash flows. This certainty equivalent is subsequently discounted at the risk-free rate. We can generalise this approach to deal with utility functions from multiple decision makers. Performing a comparative test between this approach and the risk-neutral approach would add precious insight to the role of ROA in this kind of investment problems.

11.3. Co-dependency between multiple optionalities

The model we constructed in this study contains a single optionality, namely to switch from gas to electricity production. In reality, often multiple flexibilities are embedded in a project. In the case of Gas-to-Wire production, such options could be to abandon the field before depletion, to

delay production, to expand production capacity, etc. The individual assessment of such options is possible. As stated before, the values of individual options are generally not additive due to co-dependency effects. Combining a small number of options in a single framework might be computationally feasible, but every optionality added will greatly increase the computational effort required. What is of interest for future research is how to interpret the values of individual options, so that the actual value of the project can be estimated more closely.

Gamba (2002) provides some structure for dealing with complex capital budgeting problems, mapping them as a sequence of simple real options, mutually exclusive options and independent options. This approach allows decomposing a complex option into a set of simple ones that can be solved independently. For options not falling in either of these categories, research on the co-dependency structure between individual options could increase insight in the matter. A possible procedure for such a study could be to simulate the values for a small number of individual options (using the same set of random numbers for all options) and estimate their joint distributions with techniques such as copulas or covariance matrices. We could then compare these results to a simulation model which allows combining multiple flexibilities.

11.4. Upper bound of option value

In this study we did not calculate the upper bound of the option value. For testing the convergence rate of the Longstaff-Schwartz algorithm and measure the deviation from the true value, it is useful to calculate the upper bound as well. Joshi (2006) provides a detailed description of an upper-bound algorithm, which may be applicable on the option presented in this study. An issue that needs further research is whether the algorithm is applicable on partially complete markets. As private risks cannot be hedged, upper-bound procedures based on hedging strategies may not fit our investment problem. We did not encounter a paper which addresses this issue. Based on the convergence rate between the upper- and lower bound, we could see how well the regression formula specified in the Longstaff-Schwartz algorithm succeeds in approaching the true option value.

11.5. Improvement of the electricity price forecasting model

The electricity price forecasting model still has room for significant improvement in capturing the behaviour of the actual price series. In particular, the model has difficulties dealing with large changes which on the other hand were not large enough to be marked as price jumps. A regime-switching model (see Appendix I) could bring an improvement over the mean-reverting model with jump diffusion. The distinction between the regular price process and jumps is rather arbitrary. In contrast, with a regime-switching model we can split the price processes in subsets which are more alike statistically. We optimise the decomposition of the series so that we can better estimate their properties. Also we expect regime-switching models to capture the observed volatility clustering better, so that we may no longer need a GARCH model. In a regime-switching model, the mean-reverting process with time effects that we constructed in this study could describe the normal regime.

Compared to the historical price series, our model consistently either over- or underestimates volatility. This is because we did not assume a volatility trend in our model, while the series in fact does show a clear decreasing trend. If we could obtain more insight in the future trend of volatility, we would be better able to model this parameter. Additional research on the developments in the electricity market is required to increase insight in these developments.

11.6. Co-dependency between price series

We did not find a convincing relationship between gas- and electricity prices. No cointegration relationship can exist due to the stationarity of the electricity price series we found. Our forecasting models do not sufficiently detrend the historical series to estimate a meaningful cross-correlation. Still, the graphs of the price series and the physical relationship between both commodities suggest that there is some form of (positive) co-dependency between the series. We could estimate cross-correlation better after removing the remaining autocorrelation. Still, linear dependency remains a limited concept. Joint distributions that allow for varying and non-linear relationships offer a more complete representation of co-dependency. If we find a positive relationship between the series, this would mean that the spark spread is smaller on average, resulting in a lower expected value added from Gas-to-Wire production.

11.7. Option model improvements

Our option model is influenced by two underlying stochastic processes, namely the GIP and the spark spread. Obviously we could treat many more factors as stochastic variables. We provide some suggestions, along with their implications for the model. Increasing the number of stochastic processes requires regression on multiple state variables for the Longstaff-Schwartz algorithm. The choice of the regression function then becomes increasingly important to determine the exercise policy. Often the regression must be tailored towards a specific problem. The advantage of a model which deals with multiple uncertainties results is that it provides a more generally applicable exercise policy. The alternative is to test for different scenarios and calculate their exercise policies independently.

We make use of a simplified production function requiring only the amount of GIP and the harmonic production parameters. A consequence of this approach is that we must fit the production function to each field, not able to derive a direct relationship between the two. Incorporating a model describing the physical flow of the reservoir would add a great deal of flexibility. We could then allow a much broader distribution of the GIP, instead of using different scenarios. Finally, the model is restricted to the currently prevailing regulation for carry back and carries forward. As corporate tax rates are generally related to the deduction possibilities, including this future would allow simulating a stochastic taxation approach. A possible manner for this would be to use several regulative scenarios which are triggered when the stochastic tax rate hits a certain bound.

11.8. Volatility reduction

Though the model presented in this study does not have an unusual high computation burden, Monte Carlo valuation in general is known for its long computation time and slow error reduction. For more complex problems the number of runs may become unfeasibly large. A manner to reduce the required number of runs is to decrease variance. Boyle *et al.* (2001), Grau (2008) and Cerrato (2009) propose some variance-reducing techniques applicable in the Longstaff-Schwartz framework. An important technique is using so-called 'control variates'. By introducing a stream of variables (the variate) positively correlated to the cash flow stream, we can make a linear combination of the two which has a smaller variance than the cash flows themselves. This new series requires less runs to obtain the same level of accuracy. Another well-known technique is quasi-Monte Carlo simulation, which replaces random numbers with streams of quasi-random numbers, which are actually deterministic values picked to minimise the discrepancy of the sequence. Finally, we could use a bootstrapping procedure, which resamples simulated distributions so that we can estimate standard errors of simulated parameters (Efron, 1980). This information helps to determine when we can stop the simulation.

Chapter 12

Bibliography

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Appendices

Appendix I: Other modelling techniques

This Appendix contains a short description of several modelling techniques, which we did not apply in this study, but may serve as background information or for possible application in further research.

Moving-average model

A moving-average (MA) model uses an average of past error terms ε_{t-j} and an estimated current error term ε_t to provide a forecast for the next observation (Alexander, 2001). A general moving-average model containing a constant c and q lags is described by

$$x_t = c + \sum_{j=1}^q w_j \varepsilon_{t-j} + \varepsilon_t \quad (\text{I.1})$$

A simple moving-average model assigns equal weights (denoted by w_i) to the past observations included. Sophisticated models often place more weight on recent events, particularly when larger time windows are used. A special case of a moving-average model is the exponentially weighted version of the model (EWMA). This model applies exponentially decreasing weight factors, thereby including all past observations in the forecast. Moving-average models allow for quick response to disturbances, providing relatively fickle outputs.

Autoregressive model

An autoregressive (AR) model uses weighted past observations x_{t-i} as forecasting input, adding a random term ε_t to the past observations. A general autoregressive model containing a constant c and p lags is described by

$$x_t = c + \sum_{i=1}^p w_i x_{t-i} + \varepsilon_t \quad (\text{I.2})$$

As observations are a sum of error terms ε up until time t , they incorporate all past error terms (Alexander, 2001). This in fact makes an autoregressive model an MA model with an infinite number of lags. Autoregressive models are suitable to model slowly declining effects; they provide relatively smooth results which are not strongly affected by short-term disturbances.

Autoregressive (integrated) moving average model

The concepts of moving averages and autoregressive terms can be combined in a single model. Such a model is called an ARMA model, containing p autoregressive terms and q moving-average terms. A generic ARMA model is shown in Equation (I.3).

$$x_t = c + \varepsilon_t + \sum_{i=1}^p w_i x_{t-i} + \sum_{j=1}^q w_j \varepsilon_{t-j} \quad (\text{I.3})$$

An ARMA model can be further expanded by including lags of the differenced series to the equation. Such a model is called an ARIMA model; it is particularly applicable when modelling a non-stationary series; the integrated part of the model can then remove this nonstationarity.

Exogenous model

An exogenous model bases forecasts on influencing variables, which have a certain effect on the output. When we attempt to forecast energy prices, such factors could be temperature, supply capacity, etc. The exogenous model allows for a qualitative forecast, where the influence of exogenous factors is made explicit (Harvey, 1989). Such variables are implicitly captured in the prices themselves, but may ignore future developments. For example, the development of a large new power plant increases the total supply of energy in the future, but the effect of such an event is not observable from currently available electricity prices.

A difficulty when using an exogenous model to forecast prices over the long term is that we must forecast the exogenous variables as well (Pindyck, 1999). For example, when we would include temperature as an exogenous variable, we require a temperature forecasting model to provide the necessary input. Exogenous models are often applied on historical data in order to explore or confirm causal relationships between factors. They are also applied particularly in short-term forecasting, as factors in the nearby future such as temperature can often be estimated with reasonable accuracy.

Multiple-factor model

A multiple-factor model can be used to incorporate the features of a price series in multiple equations (Alexander, 2008). As such risk is effectively decomposed in multiple factors. Multiple-factor models allow more than one variables to follow a stochastic pattern, usually two or three. The inclusion of such factors could bring significant improvements over one-factor models, provided that the risk factors to model are properly selected. An example of a two-factor model applied in this study is the jump-diffusion model.

A disadvantage of multiple-factor models is that they often include parameters which are not directly observable from the historical data. Estimation of the parameters can then become complex, generally requiring the use of algorithms such as the Kalman filter to obtain the parameter values (Harvey, 1989). In some cases, historical spot price data is not sufficient to estimate parameters. For example, we may require a set of historical future price data of futures with several maturities to estimate the stochastic behaviour of the convenience yield.

A well-known multiple-factor model for commodity prices is the three-factor model proposed by Gibson & Schwartz (1990). In this model, the price, the convenience yield and the risk-free interest rate are modelled as stochastic variables. The latter is modelled with the mean-reverting process as proposed by Vašíček in 1977 (Schwartz, 1997). The convenience yield is also assumed to follow a mean-reverting stochastic pattern; this process can be estimated based on the term structure of futures contracts.

Schwarz & Smith (2000) propose another model for the forecasting of commodity prices, which distinguishes between long-term movements and short-term deviations. The long-term movements reflect the path of the equilibrium price level, with the drift including the effects of inflation, decreasing supply, improving efficiency, etc. The short-term movements reflect deviations from the equilibrium price, for example as a consequence of the weather or supply disruptions. When market participants have the ability to store the commodity, they can correct these effects by adjusting the inventory levels. Short-term effects are expected to have a mean of zero, and are modelled by the Ornstein-Uhlenbeck process.

A way to estimate the drift in long-term equilibrium prices is by comparing future contracts with long maturities to spot prices. Such a relationship could be determined with the Kalman filter; we can apply this algorithm to estimate unobservable parameters (Harvey, 1989; Escobar *et al.*,

2003). In the absence of future prices it is more difficult to estimate parameters, as spot prices provide less information.

Regime-switching model

A regime-switching model distinguishes two or more regimes, with each regime modelling a distinct price behaviour reflecting a certain market state. For example, a regime-switching model allows to explicitly split jumps from mean-reversion. In a regime-switching model, the occurrence of events depends on the state the market is in. For us, this property would be useful particularly when modelling the frequent spikes in electricity prices. In this case, one regular regime and one spike regime would exist, both with their own mean and volatility. Most regime-switching models assume that each regime has a constant variance, as the decomposition of the series tends to (partially) remove volatility clustering (Mount *et al.*, 2006). The probability that the regime changes is described by a discrete Markov chain (meaning that any transition probability from one state to another only depends on the current regime), characterised by a matrix containing exogenous transition probabilities (Schindlmayr, 2005; He, 2007). A transition probability matrix for an n -regime model is provided in Equation (I.4).

$$\begin{bmatrix} p_{11} & \dots & p_{1n} \\ \vdots & & \vdots \\ p_{n1} & \dots & p_{nn} \end{bmatrix} \quad (\text{I.4})$$

When we make the probability of entering a spike regime small and the probability of leaving the regime large, the natural behaviour of spikes should be reflected, without the modelling of spikes corrupting the usual price behaviour. As the regimes have their own independent distribution, we cannot estimate parameters by using simple linear regression techniques. For each regime we should obtain a separate estimate that maximises the explanatory power for that specific state of the market, without being able to directly observe different regimes (Kobor *et al.*, 2005). Hamilton (1989) provides an algorithm that can be used to estimate the parameters of a regime-shifting model. Adding more regimes increases the accuracy of the model, but at the cost of having to estimate more variables (Hardy, 2001). A regime-switching model should have a proper balance between accuracy and the number of regimes.

Artificial neural network

An artificial neural network is a set of computational nodes which are interconnected through a layer structure (Szkuta *et al.*, 1999). We provide a generic example of a three-layer neural network in Figure 38.

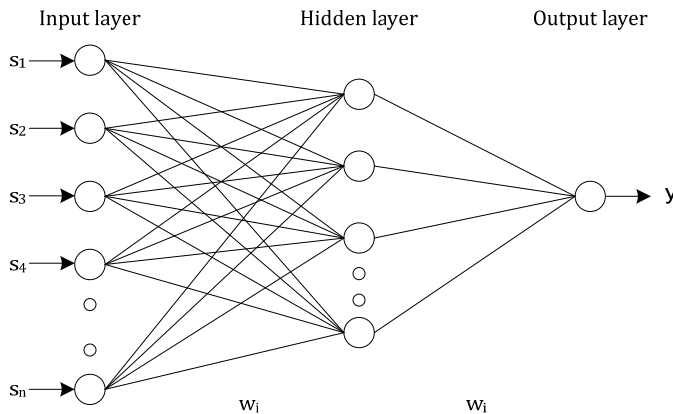


Figure 38: Generic example of a three-layer neural network (based on Szkuta *et al.*, 1999). Input variables enter the network from the left, and are transformed twice to obtain the output variable.

We can use multiple variables as input for a price forecasting network. For electricity, such variables (denoted as s_i) could be demand, the day of the week, temperature, previous prices, holiday periods, etc. These variables serve as a weighted input for the calculation units in the next layer, which then provides a new input for the next layer, eventually leading to an output variable (e.g., the price forecast). The calculation node first sums up the weighted input variables (Smith, 1997). Consequently, a nonlinear mathematical function called a sigmoid transforms this sum to a value between 0 and 1. A standard sigmoid function is provided in (1.5); other transformations are possible as well (Feng Gao *et al.*, 2000).

$$SG(s_i) = \frac{1}{1 + e^{-s_i}} \quad (1.5)$$

A training set of data is fed to the network to calibrate the weights of the inputs (Pao, 2007). By repeatedly running the network on the training set and measuring the error compared to the actual output, we can iteratively improve the model. This allows estimating complex functional relationships, even so complex that there may be no qualitative explanations available for them. After calibrating the weights, we can use the remainder of the data set to test the performance of the model.

A properly modelled neural network can provide good forecasting results. However, determining the number of input variables and calculating nodes must be done correctly. Also the training set we choose should be a correct representation of the whole data set. We can partly address this issue by using several training sets and choose the best result. Another issue in calibrating the model is the danger of over-fitting the set to a specific training set, thereby decreasing its forecasting accuracy (Feng Gao *et al.*, 2000). We could include a separate validation set to evaluate the network's performance during the training process. Finally, because the dynamics of the neural network are not easily understood due to its complex interrelated structure, it can be difficult to evaluate the constructed network (Smith, 1997). There are several programs available to construct neural networks, which have the ability to calculate the required number of nodes and the optimal weights. The performance of the network can be improved by adjusting settings to more accurately fit a specific problem.

Wavelet transformation

The wavelet transformation is a technique which decomposes a series in a set of different wavelet functions of the series (Graps, 1995). These wavelet functions have a more stable pattern than the series itself (Conejo *et al.*, 2005). The technique considers patterns observed over different time periods, allowing to include long-term and short-term patterns simultaneously (Kim *et al.*, 2002). Patterns with low frequencies are observed over a long-time window, patterns with high frequencies over a short-time window. We can use wavelet transformation to model the behaviour of nonstationary time series (Nason & Silverman, 1995). By applying the transformation to a price series, we can model patterns observed over several time intervals, such as seasonal-, weekly- and daily patterns. Decisions such as the number of wavelet functions to be derived and the formula used affect the accuracy of the forecast. Generally algorithms are used to perform the wavelet transformation.

Appendix II: Diagnostic testing of model residuals

Diagnostic testing of gas price model

To test whether the gas price model constructed in Section 7.2 is correctly specified, we perform some diagnostic tests on the residuals. We use the historical price at day $t - 1$ as input for the forecast, we obtain the residuals by subtracting the historical price for day t from the forecast for day t . We make use of the entire set of historical gas prices from 2006-2011.

First we apply the augmented Dickey-Fuller test to see whether the residual series is stationary and other tests can be applied; with a test statistic of -42.72 the series indeed proves to be stationary at the 5% level. Applying the Ljung-Box test shows that autocorrelation of the residuals is insignificant for the first tree lags at the 5% level, but remains significant for the higher lags. Particularly the autocorrelation for the ninth lag is notably high. We find comparable results for the squared residuals, though autocorrelation is significant from the second lag onwards here. The observed ninth lag autocorrelation is difficult to explain as any kind of time effect. Adding an MA(9) term to the model effectively removes significant autocorrelation from the residuals, but without any causal explanation this term is not suitable for inclusion in the forecasting model. Instead, we remove the seven jumps observed in the sample to test whether these cause the observed autocorrelation, as the residuals corresponding to these prices are notably high. Indeed, after this modification no significant autocorrelation is observed in the residuals. For the sample without autocorrelation, we can apply the regular Jarque-Bera test. Figure 39 shows that the hypothesis of normality is soundly rejected for the residuals. Most notably, some extreme residuals strongly affect the distribution of the residuals. This indicates that the gas price model is unable to capture some of the larger price deviations. Finally, we calculate R^2 for the gas price model applied on the full historical data set, indicating the proportion of variability in the sample explained by the model. We obtain $R^2 = 0.96$, meaning that the model explains a large part of the observed variability.

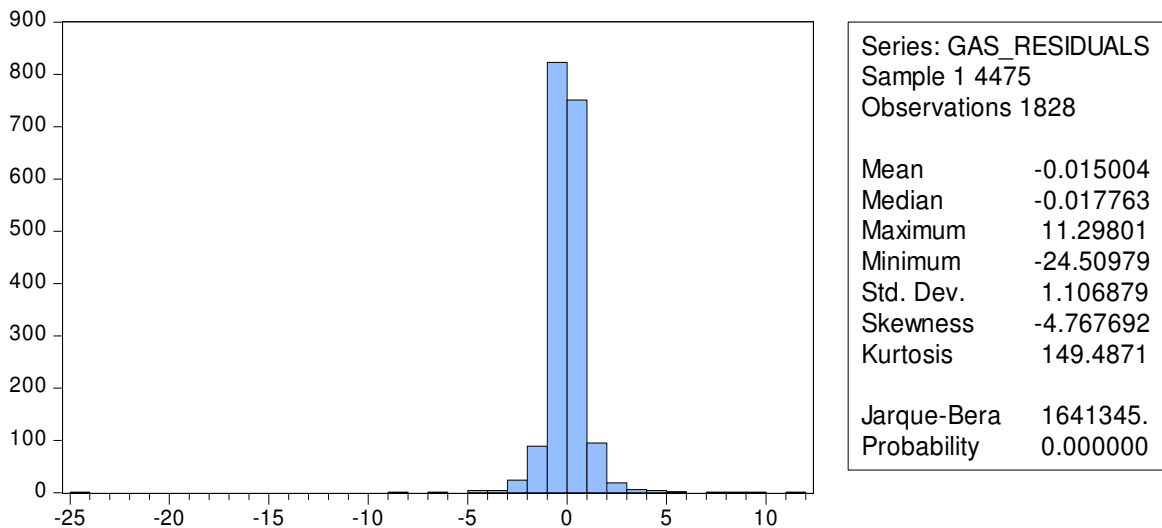


Figure 39: Normality test on residuals of gas price model excluding jumps. The series is not normally distributed.

Diagnostic testing of electricity price model

To test whether the electricity price model constructed in Section 7.3 is correctly specified, we perform some diagnostic tests on the residuals. The model uses the historical price at day $t - 1$ as input for the forecast, the residuals are obtained by subtracting the actual price for day t from

the forecast for day t . We do this for the daily prices between 2004 and 2012 with the exclusion of jumps, the same prices which were used to estimate the parameters of the model.

To see whether the residual series is stationary and other tests can be applied, we perform the augmented Dickey-Fuller test. With a test statistic of -3.15, this is the case at the 5% level. The Ljung-Box test shows that residual autocorrelation is removed for the first lag, but still significant for higher lags. Though reduced, we still observe the weekly pattern in the residuals, despite the inclusion of weekday effects in the model. The Ljung-Box test on squared residuals reveals significant autocorrelation for all 36 lags, indicating volatility clustering. This could indicate that not only the price level depends on the weekday, but volatility as well. With the power system operating closer to full capacity on weekdays, the electricity price could be more sensitive to short-term disturbances in supply and demand. An alternative explanation is that the measure of volatility simply increases with price, which tends to be lower on weekend days.

In Section 7.3 we also estimated a GARCH model. As this is a model fit to the squared residuals, the residuals we test here are the residuals of this set after applying GARCH, not those of the actual price series. The Ljung-Box test indicates that except for the first lag autocorrelation is still significant, though the model strongly reduced the degree of autocorrelation. The GARCH model is apparently successful in removing the weekly pattern. Alexander (2001) suggests two tests to assess the performance of the GARCH model. In the first test we divide the historical returns by their corresponding GARCH volatilities. The resulting series should then be normally distributed. Figure 40 shows that the actual distribution of this series resembles a normal distribution, but likely due to the presence of fat tails the Jarque-Bera test rejects the hypothesis of normality. In the second test we take the squared returns, divide these by their corresponding GARCH variances and check whether the resulting series is not autocorrelated. Applying a Ljung-Box test on this series reveals significant autocorrelation for all 36 lags tested. Also a mild weekly pattern remains visible.

Though the GARCH model improves the fit of volatility and largely removes the pattern in autocorrelation, it does not satisfy the theoretical properties. Possibly the addition of integrated terms can remove the remaining autocorrelation, making it an IGARCH model. However, the model residuals indicate there are still unexplained factors in the model. We would therefore prefer improvement of the model over fitting a complex volatility model to the residuals.

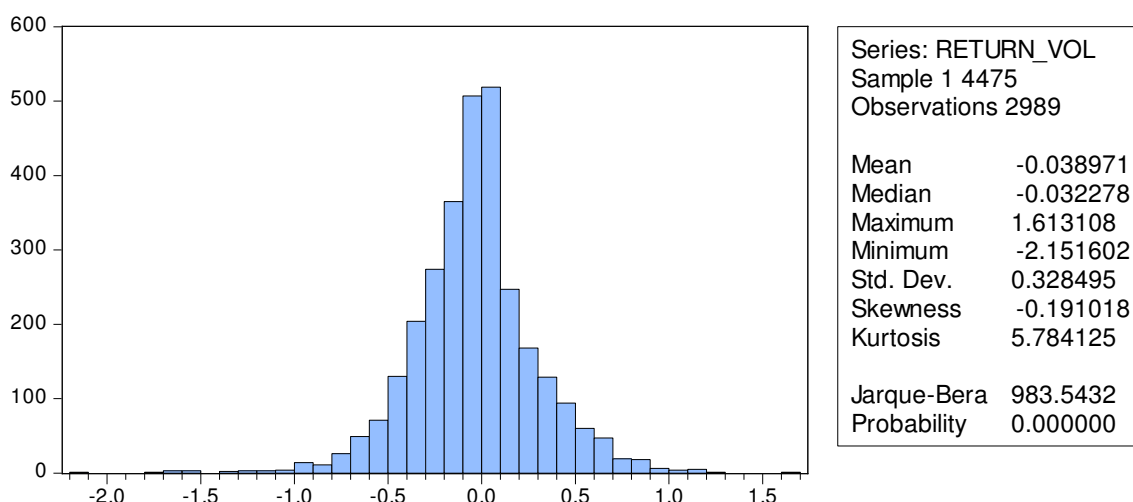


Figure 40: Normality test on electricity returns/GARCH volatility. The series is not normally distributed.

Appendix III: Source coding

In this Appendix, we describe some of the more complex coding in the Excel file “Real Option Model Gas-to-Wire”. For some functions a textual explanation suffices, other functions (particularly nested if-statements) are written out to illustrate their structure. We avoid the use of symbols in order to explain the intuition behind the statements more clearly.

Price simulation

The price simulations are divided in several processes, all expressed in logarithms. We calculate the deterministic time effects separately, being functions of the day of the week, month and total number of days. We calculate the mean-reverting part based on the mean-reversion speed, the equilibrium price level and the stochastic price part of the previous day. Finally, we add exponentials of the time effect and the stochastic part of the price to obtain the price. For electricity we calculate random jump probabilities. Whenever a jump occurs, we draw a price value from a separate distribution, replacing the other price processes. After a jump, the price is reset at the equilibrium level. We model the price generating process for natural gas as follows:

```
Calculate time effect at t
Calculate mean-reverting effect at t
Simulate random error at t
Calculate variance at t
ln(Return t) = Volatility + Mean-reversion
ln(StochPrice t) = ln(StochPrice t - 1) + ln(Return t)
ln(Price t) = ln(StochPrice t) + Time effect t
Price t = exp(ln(Price t))
```

The price generating process for electricity is modelled as follows:

```
if Jump size at t = 0 then
    Calculate time effect at t
    Calculate mean-reverting effect at t
    Simulate random error at t
    Calculate GARCH variance at t
    ln(Return t) = GARCH volatility + Mean-reversion
    ln(StochPrice t) = ln(StochPrice t - 1) + ln(Return t)
    ln(Price t) = ln(StochPrice t) + Time effect t
    Price t = exp(ln(Price t)) else
    ln(Price t) = Jump size at t
    Price t = exp(ln(Price t))
end
```

Production function Gas-to-Wire

The production function of electricity is conditional on the preceding gas production. Therefore, for each exercise year we calculate a separate production function. We calculate the production rate if at time t a number of conditions is met, namely: 1) the option must have been exercised, 2) the spark spread must exceed the threshold, 3) the reserve should exceed the minimum capacity of the generator, 4) Production rate should exceed the minimum capacity of the generator, and 5) Gas is still being produced at the time of option exercise (i.e., the field is not abandoned). Note that the coding distinguishes between day t (the actual day) and day n (the production day). See Equation (4.2) for the harmonic decline function.

```

if      Year day  $t \geq$  exercise date option          and
      Spark Spread > Threshold Spark Spread          and
      Reserve day  $t - 1 >$  Minimum capacity of generator and
      Production rate day  $n >$  Minimum capacity of generator and
      Gas production rate at option exercise date > 0 then

      min      Generator capacity
                Production rate at day  $n$ 

      else
      0
      end

```

The production rate at day n is given by

$$\text{Production rate at } n = \frac{\text{Initial production rate}}{1 + \text{Decline rate at } t} \quad (\text{II. 1})$$

with

$$\text{Decline rate at } t = \frac{\text{Initial decline rate}}{\text{Initial production rate}} * \text{Production rate at } n - 1 \quad (\text{II. 2})$$

We calculate the production days t by counting the number of days where production exceeds 0. In case gas production takes place before exercise, we multiply the number of years in between by 365 and add them to t .

Storing last production rate

This function stores the last production rate larger than 0, being used as input for calculating the harmonic decline rate parameter DR_t . We take the latest gas production value as input at the moment the option is exercised. The formula checks one day ahead to store the gas production value at the end of the previous year, serving as input for the first day of electricity production.

```

if      Year day  $t <$  Exercise date option          and
      Year day  $t + 1 =$  Exercise date option          then
      Gas production rate at day  $t - 1$               else

      if      Gas production rate at day  $t > 0$           then

```

```

Gas production rate at day t
else
Previous last production rate
end
end

```

Applying carry back

The cash flow column *Carry back* contains the amount of tax returned by offsetting tax payments of the previous year. The application of carry back requires that a fiscal loss (before applying carry back) is made in the current year, and a fiscal profit was made in the previous year (after applying carry forward).

```

if      Fiscal earnings year t - 1 - carry forward t - 1 > 0      and
      Fiscal earnings year t < 0      then
min
      Corporate tax year t - 1
      Corporate tax over abs(Fiscal earnings year t)
else
0
end

```

Storing carry forward amount

We calculate the carry forward amount for each year by taking the amount at the previous year, adding the fiscal loss (after carry back) and subtracting the carry forward amount offset against fiscal profits. Also, we subtract losses carried forward 9 years ago from the amount if they were not offset with profits in the meantime. The carry forward amount cannot fall below 0.

```

if      Fiscal earnings year t < 0      then
      if      Fiscal earnings year t-1 > 0      then
            if      abs(Fiscal earnings year t) > Fiscal earnings year t-1      then
                  Fiscal earnings year t-1 + Fiscal earnings year t      else
0
            end
            else
Fiscal earnings year t
            end
      else
0
      end
end

```

Applying carry forward

Carry forward can be applied when an amount of cumulative losses is carried forward, and a fiscal profit is made in the present year. We then reduce the fiscal profit with the amount of cumulative losses up to a fiscal result of 0.

max

min

Fiscal profit year t

Carry forward amount year t-1

0

Salvage value

If the field is no longer producing or if the final year of the option is reached, we can sell the generator or compressor. We assume linear depreciation, the machinery must be acquired less years ago than its technological lifetime to have a salvage value larger than 0. The logic for selling a compressor after exercising the option is slightly different, as we sell it immediately after the option is exercised.

Salvage value for compressor (gas production) and generator (electricity production):

```
if      Revenue year t = 0                                or
      Year t = 2031                                       and
      Year t - year investment ≤ Technological lifetime  and
      No salvage value before year t                     then
      (Technological lifetime - (Year t - year investment))/Technological lifetime * Investment value
else
      0
end
```

Salvage value for compressor (after exercising the option):

```
if      Year exercise                                       and
      Compressor bought before year t                     and
      Year t - year investment ≤ Technological lifetime  and
      No salvage value before year t                     then
      (Technological lifetime - Year t - year investment)/Technological lifetime * Investment value
else
      0
end
```

Truncated lognormal reserve distribution

We model the reserve amount (GIP) with a truncated lognormal distribution, which is a lognormal distribution with a lower limit and an upper limit. We scale the probability distribution up to provide a full distribution between the two boundaries. The inverse lognormal function we use in the model is described as follows:

Inverse lognormal distribution (Random truncated probability; Mean, Standard deviation)

Random truncated probability:

Cumulative lognormal distribution (Lower limit; Mean; Standard deviation) +

Random number *

(Cumulative lognormal distribution (Upper limit; Mean; Standard deviation) -

Cumulative lognormal distribution (Lower limit; Mean; Standard deviation))

Mean:

Mean value of lognormal distribution

Standard deviation :

Standard deviation of lognormal distribution

Monte Carlo simulation

We use Excel Visual Basics to code the Monte Carlo simulation. We provide the coding below. When pressing the 'Run Simulation' button on the worksheet 'Simulation Settings', the simulation runs for the specified number of iterations, storing the option NPV's and corresponding cash flows on the sheet 'Simulation Results'. When starting a new simulation, all old results are automatically cleared. The simulation runs in Manual calculation mode to prevent intermediate worksheet recalculation (Albright, 2011).

Function Simulation()

'Set calculation method to manual
Application.Calculation = xlManual

'Clears simulation results before every new simulation
Dim r As Range
Dim LastRow As Long
LastRow = Worksheets("Simulation results").Cells(Rows.Count, 1).End(xlUp).Row
Set r = Worksheets("Simulation results").Range("StoredSim").Resize(LastRow, 61)
r.ClearContents

'Performs number of iterations as specified on the worksheet "Simulation Settings"
Iterations = Range("Iterations")
For i = 1 To Iterations
Application.Calculate
Range("StoredSim").Resize(1, 61).Offset(i - 1) = Range("CalcSim").Resize(1, 61).Value2
Range("Counter") = i
Next i

'Set calculation method to automatic
Application.Calculation = xlAutomatic

'Shows an on-screen message when simulation is completed
MsgBox ("Simulation Complete")
End Function

Longstaff-Schwartz algorithm

We coded the Longstaff-Schwartz algorithm in Excel Visual Basics. The macro can be run from the sheet "Simulation Settings" by pressing the button "Update exercise policy", requiring at least the results of 1000 simulations to be stored. The algorithm performs regression on the last option year's cash flows, calculating and storing the expected cash flows and regression parameters. The algorithm works back until the first year of exercise. For every scenario the exercise policy should be updated, requiring a trial run for the new policy and a second run for valuation.

Sub Algorithm_exercise_policy()

```
Dim LastRow As Long
LastRow = Worksheets("Simulation results").Cells(Rows.Count, 1).End(xlUp).Row

If (LastRow - 10 < 1000) Then
    MsgBox ("Regression requires at least 1000 price paths")
Else

    'Set calculation method to automatic
    Application.Calculation = xlAutomatic

    'Clears stored calculated values before starting the algorithm
    Range("StoreExer").Resize(5000, 40).ClearContents

    'Update algorithm with simulated input
    Range("Input_LS").Resize(5000, 40) = Range("StoredSim").Resize(5000, 40).Value2

    For j = 1 To 20
        'Sorts nonzero cashflows for each year
        ActiveWorkbook.Worksheets("Exercise policy").AutoFilter.Sort.SortFields. _
            Clear
        ActiveWorkbook.Worksheets("Exercise policy").AutoFilter.Sort.SortFields.Add _
            Key:=Range("Year").Offset(0, 20 - j), SortOn:=xlSortOnValues, Order:=xlDescending, _
            DataOption:=xlSortNormal
        With ActiveWorkbook.Worksheets("Exercise policy").AutoFilter.Sort
            .Header = xlYes
            .MatchCase = False
            .Orientation = xlTopToBottom
            .SortMethod = xlPinYin
            .Apply
        End With

        'Stores regression coefficients and regressed formulas for each year
        Range("StoreRegress").Resize(1, 11).Offset(j - 1) = Range("CalcRegress").Resize(1, 11).Offset(j - 1).Value2
        Range("StoreExer").Resize(5000, 1).Offset(0, 20 - j) = Range("CalcExer").Resize(5000, 1).Offset(0, 20 -
            j).Value2
        Range("StoreCont").Resize(5000, 1).Offset(0, 20 - j) = Range("CalcCont").Resize(5000, 1).Offset(0, 20 -
            j).Value2
    Next j

    'Shows an onscreen message when exercise policy is determined
    MsgBox ("The exercise policy has been updated")
End If
End Sub
```