Graduation project AMC Amsterdam

Predicting bed census of nursing ward from hour to hour

Date
August 31th, 2011

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Management summary

Background
Nursing wards are suffering from continuous fluctuations in demand. This study proposes a decision support tool that provides insight in the fluctuations by predicting the hourly demand. This prediction contains both elective and non-elective patients. We related the number of elective patients present in the ward to the operating room schedule. The prediction of the demand supports the decision making in order to get into control of the fluctuations. The goal of the study was:

Design a decision support tool which determines the demand for beds in order to support the ward manager's strategic decision making on dimensioning wards and staff and the patient case mix and to support the tactical decision making on operating room planning and nurse staffing.

Approach
In order to gain insight in the aspects involved with bed capacity management and predicting the demand for beds we have analyzed the chirurgical nursing wards ‘G6Noord’ and ‘G6Zuid’ of AMC Amsterdam. Both wards are dedicated to gastro-intestinal patients. These wards are also used as a case study in order to validate the model we have developed as a basis for the tool.

We use a model described by Vanberkel et al. (2011) as a basis for our model. Vanberkel describes a model which determines the bed census resulting from elective patients on a daily basis. We extend this model by including non-elective patients and determining the bed census on an hourly basis. In order to support decision making in bed capacity management we introduce five performance indicators:

- *Demand percentiles*: Minimum number of beds required to fulfill the demand according to an α value.
- *Average operational bed occupancy*: The number of bed hours used in relation to the total number of bed hours available.
- *Variability in demand*: We use the coefficient of variance to measure the variability.
- *Misplacement rate*: Probability for a patient to be misplaced at another ward.
- *Rejection rate:* Probability of a patient to be rejected.

The performance indicators can be used for analyzing scenarios. We have run several scenarios and measured the performance of each scenario on the performance indicators. Based on this results we advice on the required bed capacity.

Results
We have analyzed the influence of adjusting the bed capacity of nursing wards G6NO and G6ZU on the performance on the performance indicators. The scenario with 23 beds at G6NO and 22 beds at G6ZU and the scenario with 22 beds at G6NO and 23 beds at G6ZU perform best on the criteria of a maximum rejection rate of 0.01. Implementing these scenarios will lead to an decrease in bed capacity of 3 beds and an increase of bed occupancy of 5%.

Recommendations

- Use the model on strategic and tactical level to support decision making.
- Integrate databases in order to make it easier to subtract data.
- Create a cyclic MSS in order to level the number of beds required and to reach a steady state.
- Improve the cooperation in planning of both beds and ORs.
- Better organize planning and storage of data to provide a more detailed MSS. This would probably improve the accurateness of the model output.

Preface

With this report I will finish the study Industrial Engineering and Management. That I will do my Master thesis project in health care was clear after I finished my Bachelor thesis in Deventer Ziekhuis. With Erwin as my supervisor for the Bachelor thesis, I got very interested in the situation in health care. Thank you for directing me into this area, because there are lots of opportunities to work on interesting projects, with this project as one of them.

The fact that it was very hard working, never was a problem for me, because I really liked to work on this thesis. But this project I would like to thank several people that helped me during the project. First I would like to thank my supervisors at the AMC Nikky and Aleida for their help during the project and especially the continuous constructive feedback. I never realized that supervisors put so much time and energy in supporting the student. Aleida, thank you also for the conversations at the end of a day of hard working. This helped me to relax and see the things with a clear view. Next I would like to thank Erwin for the support in general and with the programming in particular. It really helped me to start up the programming model. Further I want to thank my other colleagues at the department KPI (Quality and Process Innovation) at AMC. There always was a good atmosphere for both hard working and fun. Then I would like to thank all my family and friends. Last years I often had to skip family events and other events, because I needed to focus on the studies. Thank you all for always accepting this and never complain.

Finally I would like to thank my girlfriend Anna. She was always there to support me. Thank you!

Ferry Smeenk
Amsterdam, August 2011

1 Introduction

The ageing population, more advanced treatments and increasing patient expectations of high quality of care and service make the costs for health care rise each year. While costs are rising, the government is cutting back health care funding. This forces hospitals to cut costs such that efficiency becomes more and more important. Therefore, each department of the hospital therefore needs to improve the efficient usage of resources in order to reach the required cut backs. This need for efficiency also affects the nursing wards.

Patients stay on nursing wards for treatment or observation. The number of patients present at a ward (bed census) and the composition of the group of patients is continuously changing by arrival and departure of patients. Therefore, the wards are experiencing a lot of fluctuations in demand for beds and changes in nurse workload. A lack of insight in these fluctuations makes it difficult to adjust the employee schedule. The results are low overall bed occupancy and unnecessary salary costs for the nurses.

The demand for care mainly depends on elective patients from the OR-planning and the uncertain arrival of non-elective patients. Both elective and non-elective patients have uncertainty in their length of stay. To cope with the variability in the arrival process of patients, the ward managers make a provisional planning and postpone the actual scheduling of patients until the day of arrival. Therefore, the assignment of patients contains reactive planning. Because there is no quantitative analysis of the expected demand for care, the employee schedule is made based on historical development instead. A better prediction of the demand can lead to a demand driven planning. Possible improvements on nurse planning can be made by relating the employee schedule to the predicted demand.

A nursing ward serves several types of patients. The assignment of a patient group to a ward is based on medical grounds or through historical development. In case of overflow a patient is assigned to another ward, with possible negative influence on the quality of care. A patient case mix based on time characteristics, for instance variation in length of stay, might improve the overall bed occupancy. Given that older people have a higher likelihood of being hospitalized and of experiencing longer lengths of stay once they are in the hospital, it seems likely that in the nearby future the need to improve the utilization becomes even bigger.

The board of Academic Medical Center (AMC) in Amsterdam launched a project to approach the described situation by using analytical methods. This research is part of the project and will focus on the development of a management decision support tool to support bed capacity management and nurse staffing.

1.1 Research context

1.1.1 AMC

The Academic Medical Centre Amsterdam (AMC) is one of the eight academic medical centers in the Netherlands. As an academic medical centre AMC focuses on three main tasks: treatment of patients, medical scientific research and education. In 2010 AMC performed 375.000 outpatient treatments and 60.000 inpatient treatments. The number of nursing days was 200.000 with an average length of stay of 6.8 days. Besides serving the patients from the surrounding area, the AMC is serving patients from all around the Netherlands by offering top clinical care for several specialties.

The AMC is one of the eleven hospitals in the Netherlands that have a trauma center. A trauma center is coordinating the emergency care in the region.

1.1.2 Chirurgical nursing wards G6

The chirurgical nursing wards of 'G6' are used as a case study to test the model developed in this research. 'G6' consists of the nursing wards 'G6 Noord' (G6NO) and 'G6 Zuid' (G6ZU). Both wards hospitalize patients of the chirurgical department. G6NO is dedicated to 'General Surgery' and G6ZU is dedicated to both 'General Surgery' and 'Oral and Maxillofacial Surgery' ('*Mondziekten en Kaakchirurgie'*, MZK). Both wards have 30 certified beds, of which 24 beds-in-service. In 2010, each ward had approximately 700 patient admissions. About 20% of the patients originate from the Emergency Department (ED).

G6 hospitalizes patients from all over the Netherlands who stay in the AMC for gastrointestinal surgery. Gastrointestinal refers to the stomach and intestines ('*darmen*'). G6 also hospitalizes patients who stay for colorectal surgery. Colorectal refers to the rectum, anus and colon (the final section of the digestive system). The major part of the patients is diagnosed with cancer. Some other patients are diagnosed with Crohn's disease or Ulcerative Colitis. The preferred division of patients among G6NO and G6ZU is based on the diagnosis. Some patient types are preferably assigned to G6NO, other patient types are preferably assigned to G6ZU. The nursing ward to which the patient type is preferably assigned is further referred to as the dedicated ward.

1.2 Research objective

The research objective of the project is to develop a management decision support tool. Using this tool we advice the board of AMC on three aspects:

- *Dimensioning of nursing wards*
- *Patient case mix nursing wards*
- *Demand for care driven nurse scheduling*

Within the project this research focuses on the dimensioning of the wards and the mix of patient types. The demand for care driven nurse scheduling is outside the scope of this research and is looked at in a closely connected research. Because the tool developed in this research is input for the research on nurse scheduling we take into account important aspects of the nurse scheduling in order to merge the tools.

Dimensioning of nursing wards

The tool gives an hourly estimation of the expected demand for beds in the nursing ward. This information, in combination with the preferred score on the key performance indicators, is input for the decision on the number of beds to assign to a ward.

Patient case mix nursing wards

Each nursing ward serves several patient types, which have their own fluctuations in demand for care. These fluctuations might differ between patient types. We test whether an adjustment in the patient case mix will lead to better performance of the nursing wards on the key performance indicators.

The research objective is stated as follows:

Design a decision support tool which determines the demand for beds in order to support the ward manager's strategic decision making on dimensioning wards and staff and the patient case mix and to support the tactical decision making on operating room planning and nurse staffing.

1.2.1 Framework for planning and control

To explain the function of the different planning levels of the research objective we use the theoretical framework for health care planning and control (Hans, et al., 2011). The framework is shown in Figure 1. The framework consists of four managerial areas that are involved in health care delivery operations: Medical planning, resource capacity planning, materials planning and financial planning. The area of interest of the decision support tool is resource capacity planning. Resource capacity planning addresses the dimensioning, planning, scheduling, monitoring, and control of renewable resources. Within each managerial area the framework distinguishes four hierarchical levels of planning and control: strategic, tactical, offline operational and online operational. Within the resource capacity planning the decision support tool is focusing on strategic and tactical planning.

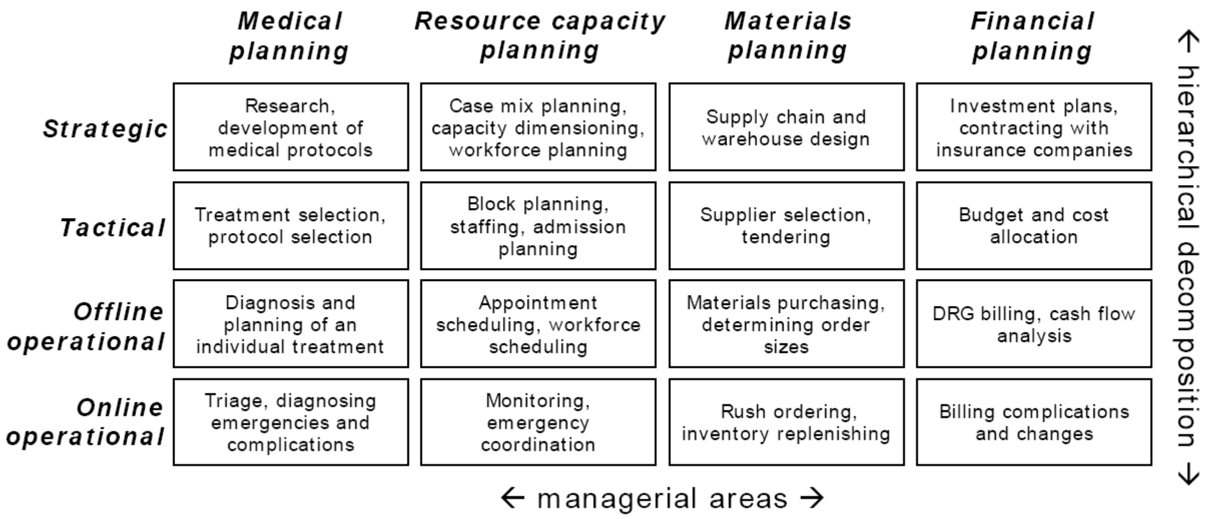

Figure 1: Example application of the framework for health care planning and control to a general hospital

Strategic resource capacity management

Decisions made on strategic level form the basis of a hospital and have a large impact on the other levels of the resource capacity planning. The strategic level concerns long term and structural decision making. The decisions are made by the hospital board and representatives of the specialties. The decisions are about case mix planning, capacity dimensioning and workforce planning. The case mix planning is about the type and the size of the patient groups. Capacity dimensioning is, among others, about the number of beds located in the wards. This is both about the total number of beds in the hospital and the number of beds assigned to a ward or a patient type. The workforce planning is about the amount of staff and the composition of the workforce.

Tactical resource capacity management

Decisions on the tactical level are largely influenced by the decisions made on the strategic level. On tactical level the results of these decisions are organized. The tactical planning takes place on intermediate term. Therefore the demand on which the decisions are based is uncertain and is often forecasted. The uncertainty in demand leads to a less detailed planning than the operational planning. Due to this demand uncertainty, tactical planning is less detailed than operational planning. The block planning is translating the production targets set at the strategic level into OR blocks assigned to specialties, the OR blocks are allocated to the specialties to perform elective surgeries. For staffing the number of staff members per shift is set, the actual scheduling of staff members is done on the operational level.

1.2.2 Performance Indicators

We have applied the tool to a test case consisting of two chirurgical nursing wards of AMC. We have run different scenario's as a basis for advice on the dimensioning and patient case mix of the chirurgical wards G6NO and G6ZU. To analyze the scenario's and to measure the performance of the model we use performance indicators. The performance indicators are commonly used in literature and are defined as follows:

- *Demand percentiles*: Minimum number of beds required to fulfill the demand according to an α value.
- *Average operational bed occupancy*: The number of bed hours used in relation to the total number of available bed hours.
- *Variability in demand*: The variation in demand for beds within a time period.
- *Misplacement rate*: Probability for a patient to be misplaced at another ward.
- *Rejection rate:* Probability that the admission request of a patient is rejected.

In section 4.5 we will explain the mathematical functions that we use to calculate the performance indicators.

1.2.3 Research questions

In order to reach the research objective and to structure the research we have formulated the following research questions.

1. Context analysis

- *What are the current planning process and the patient mix in the nursing wards and which aspects are influencing the demand for beds?*

The context analysis will provide information about the nursing wards and the planning processes in the nursing wards. We have done the context analysis in order to gain insight in the aspects required for the model. The context analysis is discussed in chapter 2.

2. Literature study

- *Which models are known to determine the demand for beds?*
- *Which steps are taken to model the demand for beds on an hourly basis?*

We review the literature on studies which determine the demand for beds. First we give an overview of models that are used on strategic level. Our special interest goes to models that incorporate the impact of the OR on the bed census in the nursing ward. We then focus on models that are used to predict the demand for beds on an hourly basis. The literature study is discussed in chapter 3.

3. Conceptualization of the mathematical model

- *How is the hourly demand for beds modeled?*
- *What is the mathematical formulation of the performance indicators?*

In chapter 4 we present the mathematical model which forms the basis of this research. The model determines the hourly demand for beds and incorporates both elective and non-elective patients. In section 4.5 we describe the performance indicators we use for testing the model and running experiments.

4. Test on case study

- *Is the hourly bed census model valid?*
- *What is the impact of adjusting the input parameters on the score on the performance indicators for the nursing wards G6NO and G6ZU?*

Section 5.1 describes how we use the data from the case study of nursing wards G6NO and G6ZU to validate the model. Further chapter 0 describes the scenarios we have run in order to gain insight in the impact of certain changes.

5. Managerial implications

- *What are the implications of the model for practice?*
- *What are the recommendations of the research?*

Finally, chapter 0 gives the conclusion of the research, discussion and the suggestions for further research.

2 Context analysis

This chapter describes the context of the bed capacity management of nursing ward G6 in the AMC. Section 2.1 describes the process of the nursing ward. Section 2.2 describes the bed capacity management. Section 2.3 describes the Operating Room planning which is the basis of the admission of elective surgery patients.

2.1 Process description nursing ward G6

The major part of the patients served at G6 are elective patients and are directly related to a planned surgery. Therefore, we describe the patient process on the basis of the elective surgery patient process. The elective surgery patients process consists of four parts. The pre-hospitalization process describes the steps taken before admission to the ward (section 2.1.1). The hospital stay consist of the pre-operative stay at the nursing ward (section 2.1.2), the surgical operation (section 2.1.3) and the post-operative stay at the nursing ward (section 2.1.4). In section 2.1.5 we will describe the non-elective patients and the elective non-surgery patients, and compare their patient process with the patient process of elective surgery patients.

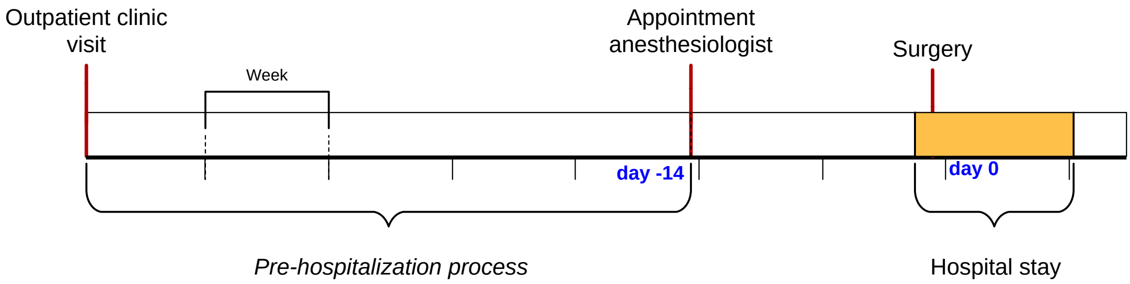

Figure 2: Elective surgery patient process

The major part of the elective patients of G6 is related to a surgery. The date and starting time of the surgery is known before the arrival of the patient and we will refer to this as a planned surgery. The day of surgery is the reference point (day 0), because the timing of the subsequent parts of the patient process is related to the time to surgery or the time after surgery.

2.1.1 Pre-hospitalization process

The hospital stay is triggered by the outpatient clinic visit and before the hospital stay the patient has to visit the anesthesiologist. The timeline of the pre-hospitalization process is shown in Figure 2.

Outpatient clinic visit

The elective surgery patients originate from the outpatient clinic. During the consultation at the outpatient clinic the physician decides to perform a surgical operation. The outpatient clinic contacts the planner of 'General Surgery' who registers the patient on the waiting list. Depending on the urgency of the surgery and the medical condition of the patient it will take about 3 months until surgery.

Appointment anesthesiologist

Two weeks before the planned surgery the patient visits the anesthesiologist. In preparation of this meeting the patient has to fill out a pre-assessment form. On this form the patient indicates his/her medical history and the medication currently using. During the meeting the anesthesiologist discusses the form. The anesthesiologist discusses the narcotics required for the surgical operation and in agreement with the patient the anesthesiologist decides on the type of narcotics that will be used.

2.1.2 Pre-operative patient process

The pre-operative process is the first visit of the patient to the nursing ward and it takes place before the patient goes for surgery. In this process the patient is prepared for surgery. The pre-operative patient process takes at most one day.

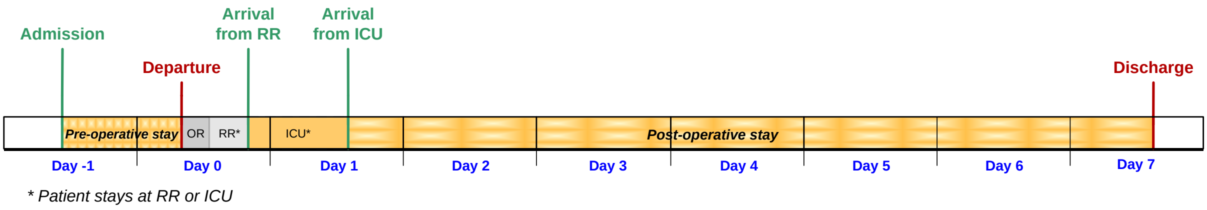

Figure 3: Hospital stay

Admission

Elective patient admissions are a result of planned surgeries. The planning process of the surgeries is described in section 2.3. The planned surgeries are taking place during weekdays. This results in a higher bed census during weekdays as shown in Figure 4. The average bed census is higher during weekdays than in the weekend and also the variation is higher.

Depending on the complexity and emotional impact of the surgery, the admission day is the day before surgery or the day of surgery. Most patients of General Surgery are diagnosed with cancer and they require complex and long surgical operations. Therefore, in general patients arrive the day before surgery. The planned admissions normally arrive between 10:30 and 11:00. The patients from MZK usually undergo a less complex surgery and therefore they arrive at the ward in the morning of the day of surgery. These patients have to arrive sober.

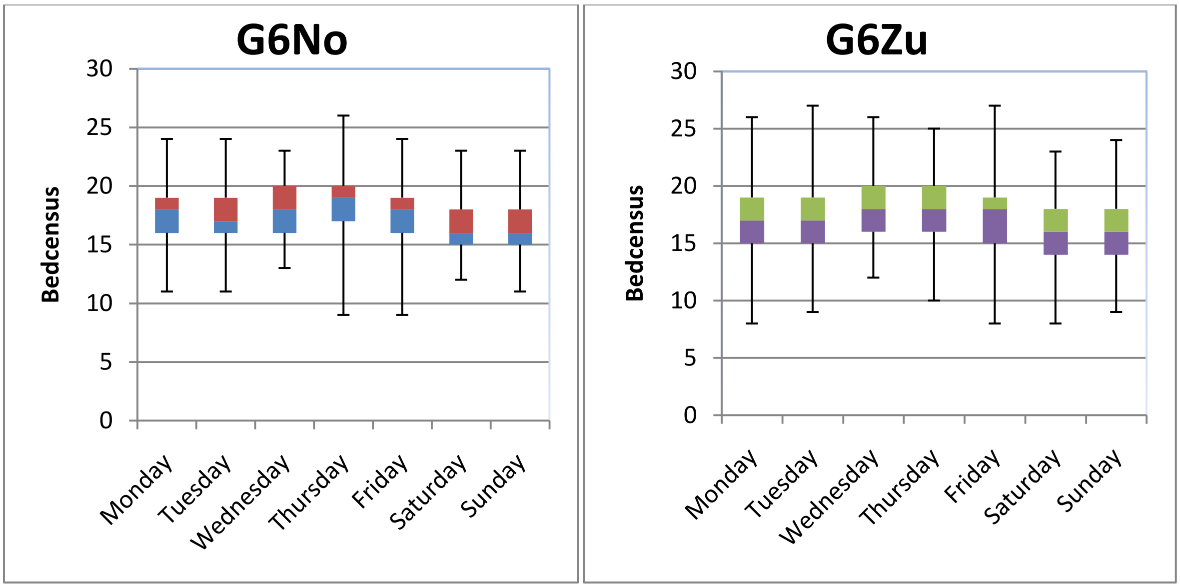

Figure 4: Average bed census per day of the week in 2010, Cognos

Pre-operative stay

After arrival of the patient at the ward the nurse and the intern perform the anamnesis. The anamnesis is a systematic approach of data collection in order to get insight in the patient's (medical) history and current condition. The anamnesis takes place in the admission room. After the anamnesis the patient is placed in his/her bed.

In preparation of the surgery the ward physician visits the patient. Also the operating surgeon visits the patient to get to know the patient and to discuss the patients' health condition. The anesthesiologist visits the patient to discuss the narcotics used during the surgery once more.

Pre-operative departure

The patient is transported to the operating complex by a nurse or nursing assistant and a nurse, nursing assistant or someone from Transportation Service. Transportation Service is the department that is responsible for transportation of patients within the AMC. For the surgery the nurses of the operating room (OR) place the patient on a special surgery bed. Depending on whether the patient continues to the Recovery Room (RR) or the Intensive Care Unit (ICU) the bed of G6 stays at the operating complex or will be returned to the ward.

Planned surgery on Monday

Patients planned for surgery on Monday have a different pre-hospitalization process. Because the surgical staff has low occupation in the weekend, the patients scheduled for surgery on Monday arrive on Friday, 13:30. After the anamnesis and the necessary visits, the patient can go home for the weekend. The patient returns to the ward on Sunday (20:00).

On Friday, the departure time of the patient depends on the availability of the operating surgeon and the anesthesiologist. When at 16:00 the operating surgeon or anesthesiologist has not seen the patient yet the nurse contacts them. When there is no time to meet the patient, for instance due to the interference of an emergency surgery, the patient might leave the ward without meeting one of them. In this case the operating surgeon or anesthesiologist will meet the patient on Monday in the early morning. The meetings all take place in the admission room; therefore, no bed is reserved for this patient on Friday.

2.1.3 Surgical operation

The surgical operations of elective patients are all planned surgeries. The surgery is generally scheduled in the early morning due to its length and complexity. Therefore the patients usually leave the ward at 8:00 to go to the operating room (OR). After the surgery the patient continues to either the RR or the ICU.

Operating Room (OR)

The surgery takes place in the OR-complex by a surgeon of the subspecialty related to the diagnosis. The day of surgery depends on the OR-days assigned to the subspecialty.

Recovery Room (RR)

In general, after surgery the patient continues to the RR. In the RR the patient's recovery of the surgery and narcotics is monitored. Most patients stay only for a few hours in the RR before they return to the ward. The nurses of G6 are responsible for bringing the patient back to the ward.

Intensive Care Unit (ICU)

Patients can go to the ICU planned or due to a complication occurred during surgery. For some types of surgeries, patients always have to continue to the ICU. Such an ICU stay is known before the patient enters the OR, a planned stay. When complications occur during the surgery, the surgeon can also decide to send the patient to the ICU instead of to the RR.

The length of stay (LOS) at the ICU depends on the patient's medical condition. In general the patient will return to the nursing ward the day after the surgery (day 1, Figure 3) but it can also take several days. For some patients there is an indication of the minimum number of days the patient has to stay at the ICU. For instance, a patient that is on respiration equipment will stay at least 2 days in the ICU before the patient can return to the nursing ward. Usually patients from the ICU return during the afternoon. The patient is brought back to the ward by nurses from the ICU or Transportation Service.

When there is no bed available at the ICU when a new patient arrives, the 'best patient' will be send to the patient's regular nursing ward. The 'best patient' is the patient who is in the best medical condition, and therefore has the lowest urge for high level care.

2.1.4 Post-operative patient process

Post-operative arrival

The patient normally returns from the RR during the afternoon or in the beginning of the evening on the day of surgery (day 0) or from the ICU in the afternoon on the day after surgery (day 1). In some cases a patient who is served in the ICU arrives in the late evening or during the night, because the ICU bed has to be cleared due to the arrival of an acute patient. Then nurses have to leave the ward and go to the ICU to get the patient.

Post-operative stay

After arrival at the ward the nurse explains to the patient the procedures until discharge. G6NO uses standardized procedures, called 'Fasttrack', which describe the steps in the patient process and at which moment in time these steps have to be taken.

Every morning the medical status of the patients is discussed during the physician's round ('*visite*'). There are two types of rounds: the physical round and the paper round. During both types of rounds the patient files ('*verpleegkundige status*') are discussed. Besides the discussion of the patient files the physician also physically visits the patient during the physical round. There are two types of physical rounds: the small round ('*kleine visite*') and the large round ('*grote visite*'). During the large round the physician takes more time to discuss the patient's situation.

Discharge

Most of the elective patients go home after discharge. Some patients will continue their treatment in a nursing home. Due to a lack of available beds in nursing homes these patients often have a LOS longer than medically necessary.

The decision to discharge is taken by the ward physician. Some general criteria on which the decision to discharge is based are that the patient eats and drinks enough and the intestines give signs of working. For some of the diagnoses a signal that the intestines are working is sufficient.

The serving nurse is responsible to arrange the aftercare before the discharge of the patient. On G6NO this is a task of the first-responsible nurse (the nurse who was responsible for the admission of the patient). This process preferably starts two days before the expected discharge. By sending a fax, the nurse contacts Transfer, the department that is responsible for arranging home care or a bed in a nursing home. After receiving the fax an employee from Transfer visits the patient for an intake talk in which they discuss the possibilities after discharge. The nurse fills in a letter for the home care.

The actions to be taken by the nurse before discharge are listed in a discharge checklist that is in the patient files. The different aspects on the list are briefly described below:

- *Discharge talk*: The nurse explains the situation after discharge and tells what to do in case of a complication.
- *Medication*: The prescriptions of medication have to be sent to the pharmacy.
- *Appointment outpatient clinic*: The nurse has to schedule an appointment in the outpatient clinic two weeks after discharge. The desk employee at the front office contacts the outpatient clinic and plans an appointment.
- *Letter to general practitioner*: The nurse has to write a letter for the general practitioner.

Medication project: Sure pil

G6NO is participating in a project called 'Sure pil'. The motivation to start this project is that prescribed medication caused complications during the hospitalization at nursing wards. The project tests whether involving pharmacists in the hospital stay will decrease the number of complications due to medication. This project focuses on elective patients with a LOS longer than 48 hours. The resident is responsible for the decision which medication to prescribe to a patient. In this project a pharmacist advices the resident on the medication.

An assistant-pharmacist visits the patient after admission. With the patient the assistant-pharmacist discusses the medication the patient is currently taking and based on this and the surgery he gives the resident advice on the medication for the patient. During the stay on the ward a pharmacist daily checks the prescribed medication of the patient. Before discharge the assistant-pharmacist again visits the patient and advices the resident on the medication the patient can take home.

Cleaning/preparing bed

After discharge of a patient the bed and the bed location cannot be reused before they are cleaned. Cleaning and preparation of beds is a task of the cleaning crew. They visit the ward twice a day (in the morning and in the afternoon) and they clean the used beds. Because there are always beds out-of-service, if necessary, a ward can take an out-of-service bed to replace a used bed that is not yet cleaned. In this case the nurses can clean the bed location.

2.1.5 Non-elective patient process

The patient flow of patients admitted to G6 is shown in Figure 5. The numbers originate from patient admissions in 2010, with first G6NO and second G6ZU and are taking from the hospital information system (ZIS). The figure shows that non-elective patients can go directly to the nursing ward or first go to the OR or the ICU. All non-elective patient arrivals to G6 are unplanned. Patients are announced just before arrival. Within the patient process at the ward there is no difference in comparison with the patient process of the elective patients.

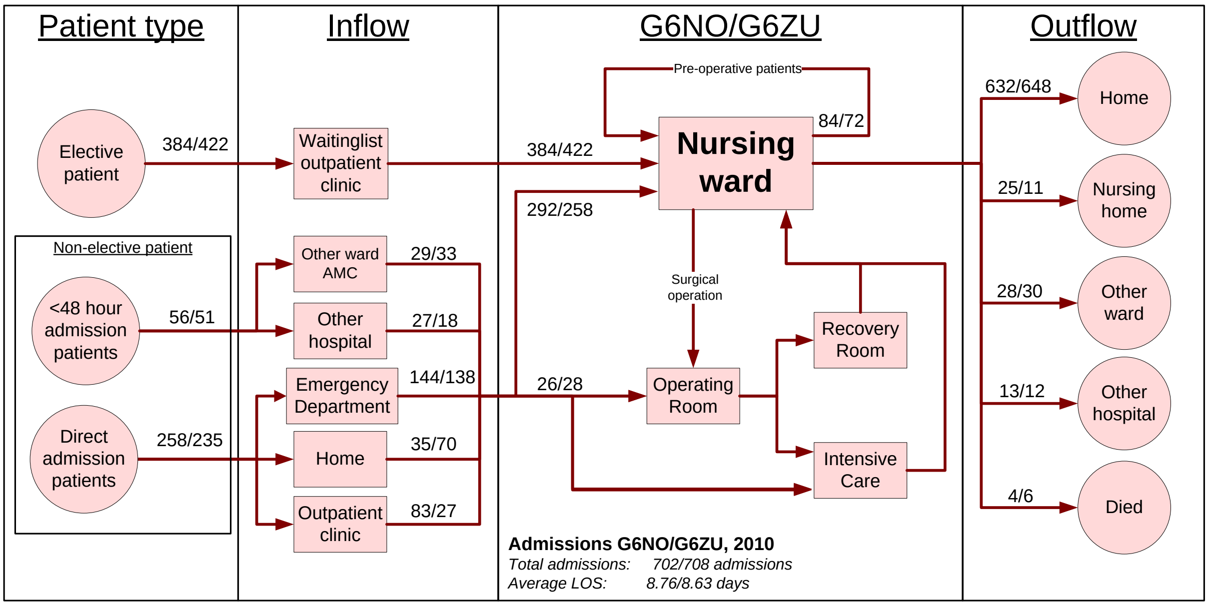

Figure 5: Patient flow of admissions to nursing ward G6, ZIS 2010

For non-elective patients the process is not always split up in two separated stays. Some patients do not go for surgery or already went for surgery before arrival at the ward. These patients only follow the described post-operative patient process (section 2.1.4). The following subsections describe the origin of non-elective patients.

Emergency Department

When there is patient at the ED of a diagnose type dedicated to G6, the specialist that is treating the patient calls the dedicated ward. The specialist discusses with the senior nurse whether it is possible to admit the patient to the ward. When the ward accepts the patient, the patient can be sent to the ward directly. When there is a complication of a patient that requires a surgical operation the patient is placed on the ORs urgency list (‘*spoedlijst*’). The patients on this list require a surgery within 24 hours.

Other hospital

Patients from other hospitals can be transferred to the AMC in case there are complications in their health situation. Also patients for some difficult types of surgeries are transferred to the AMC. These patients can either go directly to nursing ward G6 or first stay at the ICU or go for surgery. The policy of the AMC is that before returning to the original hospital these patients always first stay at a nursing ward within the AMC. Therefore, it is not allowed to go directly from the ICU to the original hospital.

Other ward AMC

Due to the patient mix, the nurses of G6 are used to hospitalize patients with relatively high demand for care in comparison to other nursing wards. Therefore, sometimes patients from other wards with complications are redirected to G6. Sometimes patients are admitted to G6 when there is no bed available at the dedicated ward.

Home

A patient served at ward G6 has a return guarantee. If a complication occurs within the first period after discharge from G6, the patient can always return to G6. The staff are familiar with the patient and feel responsible. The patient can go directly from home to the nursing ward, without visiting the ED. Independent of the current bed census, the patient is accepted. When all beds-in-service are occupied, this requires taking an additional bed into service, which is causing an increase of workload for the nurses.

Outpatient clinic

Patients visiting the outpatient clinic are sometimes instantly forwarded to a nursing ward. The specialist contacts the nursing ward, whether it is possible to admit the patient to the ward.

2.2 Bed capacity management

The discharge of patients is fully based on medical reasons. Therefore the discharges are not planned, and the bed planning fully consists of the admission scheduling. The stages of the admission scheduling are described in section 2.2.1. The bed census and the nursing workload are input for the admission scheduling and therefore we describe them in section 2.2.2.

2.2.1 Admission scheduling

Admission scheduling in general concerns the decision whether or not to admit a patient to the ward. In case of non-elective patients this also concerns the day and/or time of the day the patient is admitted. Admission scheduling occurs in three stages: weekly off-line planning, daily off-line planning and daily on-line planning[1].

Weekly off-line planning

The weekly off-line planning assigns the elective surgery patients of G6 to G6NO or G6ZU. The elective surgery patients are a result of the Operating Room planning (section 2.3). The head nurses of both wards discuss whether it is possible to accept the patients at the dedicated ward. The planning horizon of the weekly off-line planning is one week.

The surgeries of 'General Surgery' are planned one or two weeks in advance. The wards are informed on the arrival of an elective surgery patient on Thursday the week before the planned day of surgery. Therefore, every Thursday 8:00 the head nurses discuss the planned admissions of the upcoming week. The planned admissions of elective patients assigned to G6 are always accepted. All patients have a dedicated ward, the ward to which they are preferably assigned. On Thursday the head nurses decide, based on the expected length of stay of the current patients and the planned admissions of the upcoming week, whether the patient will be accepted by the dedicated ward or assigned to the other ward of G6. Because the surgeries are already planned, the head nurses have no influence on the arrival day of the patients.

Daily off-line planning

The daily off-line planning reconsiders the assignment of elective patients arriving that day. The planning is done by the head nurses and the planning horizon is one day. Daily, at 8:00, the head

[1] Off-line planning is a planning that takes a prior decision, assuming a planning horizon with a finite, fixed length. On-line planning is a planning that makes a series of decisions in time regarding planning horizons that (may) overlap.

nurses make an overview of the patients currently present at the ward and the planned arrivals of that day. The head nurses of both wards reconsider the provisional decisions made in the weekly off-line planning. Based on the current bed census and the expected nurse workload related to the current patient mix the head nurses decide whether the arriving patient is admitted to the assigned ward. The bed census is extensively described in section 2.2.2.

Daily on-line planning

The daily on-line planning concerns the decision whether to accept a non-elective patient. The planning is executed by a senior nurse or by the head nurse. Besides the acceptance of a patient, the senior nurse or head nurse can also decide on the day and moment of the day the patient can arrive at the ward.

Because the responsible nurse immediately has to decide about the acceptation of the patient, this cannot wait till the off-line planning the next morning. The senior nurse or the head nurse decides, based on the current bed census and the expected arrival and departure of patients, whether it is possible to accept a non-elective patient. Within the group of non-elective patients we can distinguish patient types that arrive directly or within a few hours and patient types that have to arrive within 48 hours or a couple of days. Patients from the ED, ICU, directly from home and from the outpatient clinic will arrive within a few hours after acceptation. Patients from other nursing wards have to arrive within 48 hours after acceptation and patients from other hospitals within a few days after acceptation. The arrival of patients from the ICU and the ED is described more thoroughly in the following subsections.

Intensive Care Unit patients

When a patient from the ICU is ready for transport to the ward the ICU calls the ward. Earlier that day, patients going to be discharged from the ICU that day, already appear in the hospital information system (ZIS) together with their destination. Therefore the head nurse checks every morning in ZIS whether a patient to be discharged from the ICU is assigned to G6. Often the physicians also inform the ward when a patient of one of the dedicated patient types is hospitalized on the ICU.

Emergency Department patients

The patients from the ED are managed by the bed coordinator of the ED. When a patient from the ED needs to be admitted to a nursing ward, the bed coordinator looks on the bed status overview in ZIS whether there is a bed available on the dedicated nursing ward. When there is no bed available, the patient will be assigned to another nursing ward. If there is no bed available in one of the other wards, the bed coordinator will call the dedicated ward whether it is possible to admit the patient.

Bed status overview ZIS

The bed status overview in ZIS displays the current bed occupancy of each nursing ward. The senior nurses are responsible for continuously updating this system. There is also a field to add comments. This can be about upcoming patients and reservation of the beds for upcoming patients. The bed status overview is recently implemented and is not yet working optimally.

The nursing wards have the possibility to close some of the beds-in-service for a while. Therefore, the number of beds-in-service can fluctuate. Nursing wards have their own responsibility in this decision. G6 is not using this method. However, it occurs that non-elective patients are not accepted, while not all beds-in-service are occupied. This because the senior experiences a high workload of the nursing team.

2.2.2 Bed census and nursing workload

The decisions in the admission planning (described in section 2.2.1) are mainly based on the number of patients in the ward (bed census) and the corresponding nursing workload. In general a patient is accepted if an in-service-bed and a nurse are available. To go into more detail on this decision we make a distinction between administrative bed census, physical bed census and nursing workload. These three types of census support the decisions made. The bed census types are based on the different situations that occur on the ward. The different situations that influence the different types of bed census are shown in Table 1 and described in the following subsections. After we have described the situations we will elaborate on the bed census types.

Warm bed

When a patient is physically using a bed at the ward, we call this a warm bed. The bed is in use by the patient and the patient requires nursing.

	Bed reserved	Bed occupied	Nursing workload
Warm bed	X	X	X
Off-ward bed	X	X	
Reserved bed	X		
Pre-operative patient	X		X

Table 1: Situations influencing bed census and workload

Off-ward bed

The off-ward bed is reflecting the situation of a patient that is in the OR or the RR. The patient is brought to the operating complex on his/her bed of the nursing ward. The bed of the nursing ward stays outside the OR, in the operating complex. In case the patient goes to the RR after surgery, the patient will be placed on the bed of the nursing ward again. If the patient continues to the ICU after surgery, the bed will be returned to the nursing ward later on the day. An off-ward bed is not available at the ward, and therefore this bed is occupied. No nursing is required, because the patient is not present at the ward.

Reserved bed

A bed is reserved if a patient of the ward is in the ICU or a patient is temporarily discharged. The wards reserve the bed, because they expect the patient to return to the ward. The reason to reserve a bed is that the returning day of the patient is unknown, and a reserved bed means a guaranteed bed when the patient has to return to the ward. Not reserving a bed can lead to rejecting the return of the patient or making an out-of-service bed operational (which leads to an increase in workload). In case of an ICU stay, G6ZU reserves the bed for a maximum of 3 days and G6NO keeps the bed reserved during the whole period of absence. Because the patient is not in the ward, no nursing is required.

Pre-operative stay

A pre-operative stay is an elective patient with the surgery scheduled on Monday. Since the process on Friday takes place in the admission room, the patient is not occupying a bed. But because it is known that the patient will return on Sunday evening, the bed is already reserved to ensure that the patient has a bed on Sunday. This patient requires nursing on Friday.

	Warm bed	Of-ward bed	Reserved bed	Pre-operative patient
Physical bed census	X	X		
Administrative bed census	X	X	X	X
Workload census	X			X

Table 2: Bed census types

Bed census types

Table 2 shows the different types of bed census. The physical bed census takes into account the moments that a bed is physically occupied by a patient. Therefore, this bed census type consists of the warm beds and the off-ward beds.

The administrative bed census is reflecting the number of beds that are not available for new arriving patients. In comparison with the physical bed census, this also takes into account the reserved beds and the pre-operative patient beds.

The workload census reflects the situations in which nursing is required. Therefore, it contains the warm beds and the pre-operative patients. In these situations, patients are physically present at the ward and therefore require nursing.

2.3 Operating Room planning

A surgery is performed by a surgeon in a certain OR. The surgeon is member of a subspecialty, and in general this subspecialty is operating the whole day in that specific OR. An extensive planning process takes place to plan the surgeries. The stages within this planning process are described in section 2.3.1. Section 2.3.2 describes the basic OR-day division of General Surgery and which of these patients are directed to nursing ward G6.

2.3.1 Stages Operating Room planning

The OR-planning consists of several stages taken by several actors of the AMC. The stages are shown in Table 3 and described in the following subsections.

	Planning stages	What	Who	When
1.	Yearly OR budget	Total yearly OR days per specialty	OR-centre	October
2.	Year planning	OR days to specialty	OR-centre	3 months in advance
3.	Sub specialty planning	OR days to subspecialty	Planning bureau specialty	6 weeks in advance
4.	Surgery planning	Provisional surgery schedule	Surgery planner sub specialty	Thursday week in adv.
5.	Week planning	Definite surgery schedule	OR-centre	Thursday week in adv.
6.	Day planning	Starting time of surgery	OR-centre	1 day in advance

Table 3: Stages OR planning

Yearly OR budget

The total number of OR-days that a division can use is based on the yearly OR budget. In July, the OR centre sends all operating divisions (B, C, D and E) the financial consequences of continuing with the current settings (usage of ORs). Also it sends a framework with regard to the upcoming financial year of the OR-centre. In cooperation with the management team operating room (MTOK) the divisions hand in a proposal for the budget. In October the division board of the OR-centre takes the decision on the final OR budget for each division.

Year planning

In the year planning the OR-centre assigns the available OR-days to the specialties. The OR-centre assigns only full OR-days to the specialties. The number of OR-days per specialty is based on the set yearly budget of each specialty.

The planning scope of the year planning is the upcoming 12 months. Monthly the planning period is adjusted by adding a new month at the end of the planning and taking out the past month. The year planning is 100% guaranteed for the upcoming 3 months. For month 4 till 12 the planning is guaranteed for 80%, which means that it can occur that this planning slightly changes within the next months and some OR-days will finally be assigned to another specialty.

Sub-specialty planning

The planning office 'General Surgery' monthly receives an updated version of the year planning. The planning office uses the OR-days assigned to Surgery, GIOCA ('*Gastro-Intestinaal Oncologisch Centrum Amsterdam'*) and Kidney transplants. The total number of OR-days can differ monthly. The planning office assigns a subspecialty to each OR-day. In section 2.3.2 we describe the OR-day division in more detail.

Surgery planning

Approximately 6 weeks before the day of surgery the sub specialties receive the schedule with the assigned OR-days from the planning office. For each subspecialty one of the surgeons is responsible for the planning of the surgeries (the surgery planner). The surgery planner plans surgeries in the available OR-days and assigns a surgeon to each surgery. This planning is done weekly, for the upcoming week. On Wednesday 12:00 the surgery planners have to hand over the planning for the upcoming week to the head of the surgery planners.

The patients are selected from the waiting list of the subspecialty. Every subspecialty has a waiting list of several weeks. Each patient on the list has his/her own specifics, for instance type of surgery, medical condition and urgency. Therefore, the patient selection is not based on first come first served.

Week planning

The week planning gives an overview of the planned surgeries in a certain week. At the latest Thursday 10:30 the OR-centre receives the schedule with surgeries for the upcoming week from the planners of the specialties. The OR-centre checks the proposed schedule with the availability of OR personnel and at 13:00 the OR-centre sends the definite week planning to the planners.

Day planning

Daily at 10:30 the OR program for the next day is set. This is done by representatives of anesthesiology, the RR, surgical assistants, anesthesia assistants and the day coordinator. The day coordinator is responsible for the surgeries of that day and is coordinating acute surgeries. During the meeting the exact starting times of the surgeries are set.

Confirmation start of surgery

On the day of surgery a planned surgery still can be cancelled. Therefore, during the day of surgery the final confirmation to start the surgery is given. Reasons for cancelling are the absence of necessary personnel due to unexpected sickness or the unavailability of the OR or surgeon due to the intervention of an emergency patient.

2.3.2 Basic OR day division General Surgery

Within the year planning the planning office General Surgery is responsible for planning the OR days of 'General Surgery', 'GIOCA' and 'Kidney transplants'. The total number of OR days per week differs weekly. The planner uses a basic assignment method which is based on 14 OR days a week for the division of the OR days among the sub specialties. This basic OR day division is shown in Table 4.

	Day of the week					
Sub specialty	Mon	Tue	Wed	Thu	Fri	Total
ADB infection			1			1
General						0
Colon	1		1		1	3
Endocrine				1		1
HPB	1	1		1	1	4
Mamma						0
Kidney			1			1
Oesophagus		1		1		2
Vascular	1				1	2
	3	2	3	3	3	14

OR-day General Surgery

OR-day GIOCA

OR-day Kidney transplants

Table 4: Basic OR day division General Surgery

If there are more than 14 OR days, the additional OR days are assigned to sub specialties based on the waiting list and the availability of surgeons. In some cases an OR day is assigned to two sub specialties. Both sub specialties will use a part of the OR day. The planning office also takes into account known (un)availability of the surgeons on certain days of the week. If a subspecialty cannot perform a surgery on an OR day due to the absence of surgeons, the OR day is assigned to another subspecialty. The original subspecialty will not receive compensation of another OR day. It happens that an OR block contains a surgery from another subspecialty then the subspecialty assigned to the block. This happens in confirmation between the planning office and both subspecialties.

In general two OR days are dedicated to patients originating from the outpatient clinic GIOCA (marked yellow in Table 4) and one OR day is dedicated to Kidney transplant surgeries (marked green in Table 4). In the year planning these OR days are also specifically assigned to GIOCA and Kidney Transplants. Additionally, GIOCA patients can also be assigned to other OR days.

OR-days of patients G6

Patients from the subspecialties 'ADB infection', 'General Surgery', 'Colon', 'HPB' and 'Oesophagus' are admitted to G6. Selecting these subspecialties in the basic division shows that daily patients from 2 OR-days are admitted to G6 (see Table 5).

	Day of the week					
Sub specialty	Mon	Tue	Wed	Thu	Fri	Total
ADB infection			1			1
General						0
Colon	1		1		1	3
HPB	1	1		1	1	4
Oesophagus		1		1		2
	2	2	2	2	2	10

Table 5: Basic OR days related to G6

3 Literature review

The aim of the model we develop is to predict the bed census of a ward on an hourly basis. In section 3.1 we describe papers that give a review on capacity management. Section 3.2 discusses the two main type of models to determine bed census and discusses studies that incorporate these models. Section 3.3 describes models that separately determine the bed census for elective and non-elective patients. Section 3.4 discusses studies that incorporate hourly effects in determining the bed census. Section 3.5 gives performance indicators taken from literature.

3.1 Capacity management

(Smith-Daniels, et al., 1988) provide a review, classification and analysis on capacity management in health care services. Among others they discuss articles on the size of the inpatient units and inpatient admission scheduling. More recent reviews are from Jack and Powers (Jack, et al., 2009) and Hulshof et al. (Hulshof, et al., 2011). Jack and Powers analyzed articles published between 1986 and 2006 about, among others, capacity management and utilization of resources. Hulshof et al. give a taxonomy for each hierarchical level of six clusters in health care. Among others, the article discusses the planning and control decisions taken in the hierarchical levels of the inpatient care service and gives an overview of related articles.

3.2 Bed census models

In literature there are two main type of models that determine the bed census: analytical models and simulation models. Analytical models are mathematical models that have a closed form solution. Analytic solutions isolate the outcome on the left-hand side of an equation, with all of the determinants on the right-hand side, so it is clear how the outcome depends on the inputs. An analytical model determines the steady state situation and an advantage of the model is the precision and the relatively short development time (Kolker, 2009). Because an analytical model describes a model with infinite capacity, it cannot incorporate blocking (rejections) of patients. Another disadvantage is that the described situation cannot be too complex. The complexity of the model is, amongst others, depending on the input of the model, which is deterministic or stochastic. Using deterministic data can simplify the model, but a drawback can be that it decreases the way the model reflects the reality. Due to the randomness in arrival and length of stay of a patient, stochastic data is better to use for predicting the bed census. Using deterministic data will lead to underestimation of the required number of beds. Simulation models are representations of a system and show the expected working of the system. Simulation models show how real processes evolve with time. A major advantage is that these models can incorporate the dynamic behavior of the processes. Besides this they can determine the capacity needs and show the effects of blocking patients. The disadvantages are that simulation models are inexact and that they take a lot of development time.

(Harrison, et al., 2005) developed a simulation model in order to determine the average bed occupancy and the observed variability in the occupancy in order to take strategic decisions. The admissions are modeled as a Poisson process and they distinguish between admission rates per day of the week. The discharges are modeled with a Binomial distribution with the number of patients discharged each day as a binomial variable. Akcali et al. (Akcali, et al., 2006) describe an analytical model with a network flow approach to optimize hospital bed capacity decisions. For optimizing the number of beds the model takes into account the hospital budget and the maximum number of days a patient is on the waiting list before being admitted to the hospital. (Mackay, et al., 2005) compare different types of models. They conclude that using the average LOS is not good. They give three reasons: the LOS profile is not Normally distributed by the high degree of skew, the LOS distribution is complex due to the different types of patients and reasons for unplanned or planned admission and the average LOS is unrelated to the time of day when the hospital is busy. Therefore, they advice a move towards more sophisticated modeling of hospital bed problems.

3.3 Elective and non-elective patients

Some of the studies separately determine the bed census resulting from elective patients and the bed census resulting from non-elective patients. Cardoen et al. (Cardoen, et al., 2010) review articles about operating room planning and scheduling. Amongst others they provide an overview of articles which integrate the ward in the operating room planning and scheduling. (de Bruin, et al., 2009) use an analytical model which distinguishes between scheduled and unscheduled admissions. An Erlang loss model with Poisson arrivals describes the bed census.

Other studies relate the number of elective surgery patients present in the nursing ward to the Master Surgery Schedule (MSS). They determine the impact of an MSS on ward occupancy. (Vanberkel, et al., 2011) describes an analytical model to relate recovering surgical patients' workload to the MSS. The model uses a stochastic variable to determine the number of patients resulting from an MSS block. The period a patient stays in the ward is determined by a stochastic variable describing the length of stay. Both variables are taken from empirical distributions. A three step model determines the total number of elective patients present in the ward. (Vanberkel, et al., 2011) does not take into account non-elective patients. (van Oostrum, et al., 2008) also take the resulting ward occupancy into account while developing the MSS. Their model uses mathematical programming to minimize the weighted sum of needed OR capacity and the peak demands of hospital beds. The demand for beds resulting from an OR block is determined by a decision variable indicating the number of surgical procedures assigned to the OR block and the requirement for a bed during the days after the surgery. (Beliën, et al., 2007) not only determines the impact of the MSS on the ward, but the main objective of the model is to determine an MSS which minimizes the expected shortage of beds. The models use stochastic numbers to determine the patients resulting from an OR block and a stochastic LOS. They compare the performance on solution quality and computation time of mixed integer programming models with heuristic approaches.

3.4 Hourly effects

The models described above all determine the bed census on a daily basis and do not incorporate hourly effects during the day. (Cochran, et al., 2006) propose a multi-stage stochastic methodology to balance the bed censuses of the entire hospital. They use both analytical and simulation models by first performing a queuing network analysis to achieve balanced targets and discrete event simulation is used to maximize the flow. For the simulation model they incorporate hourly effects for the group of direct admit patients. For this group they determine a unique number of arrivals per hour for each block of hours (length of block depending on the time of the day). For the ED admits they assume no hour of day pattern.

(Broyles, et al., 2010) use a statistical Markov chain approximation to determine the transient bed census of a nursing ward. They define an hourly expected arrival rate and the departures are included in the hour specific expected service rate. Both rates have hourly seasonality, and information about historical arrival and inventory levels, and contain all patient types. A Poisson process with as input variables the bed census at the start of the time slot, the expected arrival rate and the expected service rate determines the bed census at the end of the time slot. (Littig, et al., 2007) describe a statistical model that predicts the bed census for the upcoming 96 hours. They describe a three stage model which determines for four patient groups (surgical patients, ED patients, direct admit patients and transfers from other Unit) hazards which influence the patient inflow and the patient outflow. Using regression analysis the hazards result in the bed census.

3.5 Performance indicators

(DeLia, 2006) compares annual statistics with calculations for each day of the year. They conclude that annual statistics ignore day-to-day variation which results in a lower average occupancy level. Especially an analysis of long periods of high or low occupancy reveals differences in both statistics. An often used performance indicator is the occupancy level. In general an occupancy level of 85% is set to cover the demand fluctuations. (Green, et al., 2001) have studied the bed occupancy level as a primary determinant for bed capacity. They show that using the bed occupancy as a primary determinant to set the bed capacity will lead to excessive delays for patients admitted to small wards and wards with a high number of acute admissions. Therefore some researches also take into account other performance indicators. (Beliën, et al., 2007) use besides bed occupancy levels, the variance on the occupancy, the expected bed shortage and the probability of a shortage on each day of the period as performance indicators. They define the total expected bed shortage as the sum of the expected bed shortages on each day of the cycle time. This shortage is derived from the set bed capacity and the number of occupied beds. The variance of the number of occupied beds is determined on daily basis with the time period set to a week.

Another performance indicator that is often used is misplacement rate (Green, et al., 2001), (Harrison, et al., 2005). Misplacements are occurring when patients are placed in a nursing ward that is not the dedicated ward of this patient type. Some hospitals direct misplaced patients to the dedicated ward when there are beds available due to discharges. (Vanberkel, et al., 2011) uses the demand percentile to determine the required number of beds.

3.6 Contribution

The studies that are relating the bed census to the operating schedule only determine the impact of patients resulting from this schedule on the ward, non-elective patients are not taken into account. The studies that do combine both patient types to determine the patient demand on hourly basis use general distributions to determine the bed census. By determining the bed census following from both patient types and relating the non-elective patients to the MSS we will contribute to this research area.

4 Hourly bed census model

The model we have developed is an analytical approach to predict the bed census in nursing wards. The bed census we determine in our model is the administrative bed census (described in section 2.2.2). The time line of the model is in time slots (e.g. hours), for a set period of days. This period is a repetitive pattern throughout the year. The patient mix of a ward consists of elective patients and non-elective patients. First, we separately determine the bed census of both patient groups. Then we combine the groups to determine the bed census of the ward.

Elective patients

For our model, the group of elective patients only consists of patients undergoing a planned surgery. Other types of elective patients are placed in the group of non-elective patients. The elective surgery patients are a result of the surgery blocks of the Master Surgery Schedule (MSS). We build our model on the model of Vanberkel (Vanberkel, et al., 2011), which determines the impact of the surgery blocks on the wards on daily basis. In section 2.2.2 we describe which aspects of the model we use and which adjustments we made.

The basis of our model is the impact of a single surgery block on the bed census in the ward. Using the bed census of the single surgery blocks, we can determine the total bed census resulting from a single cycle of the MSS. Combining the overlapping days of multiple cycles we can determine the steady state bed census of elective patients during each day of the MSS.

A surgery block contains patients from a certain specialty with specialty specific characteristics. The number of patients that arrive from a block equals the number of patients that undergo surgery in this block, which is modeled as a random variable. The wards reserve a bed for the patients from the patients' admission to the ward until the discharge from the ward. The length of this period is depending on the specialty specific length of stay (LOS) distribution, modeled as a random variable. The arrival of the patients of a block is described by the admission process and the departure of patients by the discharge process.

The admission process describes the hourly bed census during the day before and the day of surgery. The number of surgeries in the surgery block determines the total number of patient admissions during these days and the admission day process determines the time slot of the admissions. The admission day process is the time slot specific admission probability of a single patient. The discharge process describes the number of patients still present in the ward during the days after surgery. To calculate the discharge process we use the starting census of the day, which is the number of patients from the block still present in the ward at the start of the day. The time slot specific discharge probability of a single patient determines the process during the day.

Non-elective patients

Each day of the week, non-elective patients are admitted to the ward. The arrival of this group of patients is unexpected and the treatment is often not fully known at arrival. We subdivide the non-elective patients in patient types with weekday specific characteristics. The arrival day forms the basis of the group. The time line of the period consists of a weekly repetitive pattern of non-elective arrivals. The basis of the model is the calculation of the impact of a patient type that arrives on a certain day of the week. With this impact we can determine the bed census resulting from a single week of arrivals. Combining the overlapping days of multiple weeks we determine the steady state bed census of non-elective patients per week.

To determine the impact on the ward of a patient type for a single arrival day, we start with the admission process. We describe the arrival with an inhomogeneous Poisson process, with a time slot specific arrival rate. The sum of all arrivals forms the starting census of the discharge process. The length of stay of a patient type is influenced by the arrival day. This because the availability of (human) resources for this unplanned patients is depending on the day of the week. The calculation of the discharge process is equivalent to the calculation of the discharge process for the elective patients.

Hourly bed census nursing ward
The last step in order to determine the hourly bed census of the nursing ward is to combine the elective and the non-elective patients for every day of the period.

4.1 Assumptions
Before we start with presenting the model, we will first give the assumptions of the model. This gives a description of the model. We subdivide the assumptions in assumptions for the admission process and assumptions for the discharge process.

Admission process
- Patient admissions take place at the begin of a time slot
- Elective patients are admitted to the ward at the day before or at the day of surgery
- Patient admissions are independent of each other
- Time slot admission of elective patients is independent of day of the week
- Non-elective patients arrive according to a time slot depending Poisson process

Discharge process
- Discharges take place at the end of a time slot
- Elective patients are not discharged during the day before surgery or the day of surgery
- Patient discharges are independent of each other
- Time slot of discharge is independent of the day
- Time slot of discharge is dependent of patient type

4.2 Elective patients
In line with Vanberkel (Vanberkel, et al., 2011) we take 3 steps to determine the bed census resulting from elective patients. The first step is to determine the demand resulting from a single surgery block of a patient type (section 4.2.2). The next step is to assign the patient types to the OR days according the MSS and to determine the impact of a single cycle (section 4.2.3). From the periods of a single cycle we calculate the steady state bed census resulting from all elective patients (section 4.2.4). First we will give the input for the elective patients model (section 4.2.1). The notation used in the model is described in Appendix A.

4.2.1 Model input
The day within the time line of the period is described by $q \in \{1,2,\dots,Q\}$, with Q equal to the number of days in the MSS and consisting of one or multiple weeks. Each day contains T time slots $t \in \{0,1,2,\dots,T-1\}$, with $T = 24$ to let the time slots represent hours. For instance time slot $t = 10$ contains the time period of a day between 10:00 and 11:00.

Elective surgery patients result from the MSS. The MSS is a repetitive pattern over the days of the period, with $q = 1$ and $q = Q$ as first and last day of the MSS. For each day of the MSS, a specialty j can be assigned to each available OR $i \in \{1,2,\dots,I\}$, with I as the total number of ORs. A surgery block that takes place in OR i on day q is indicated by $b_{i,q}$. Figure 6 shows an empty MSS.

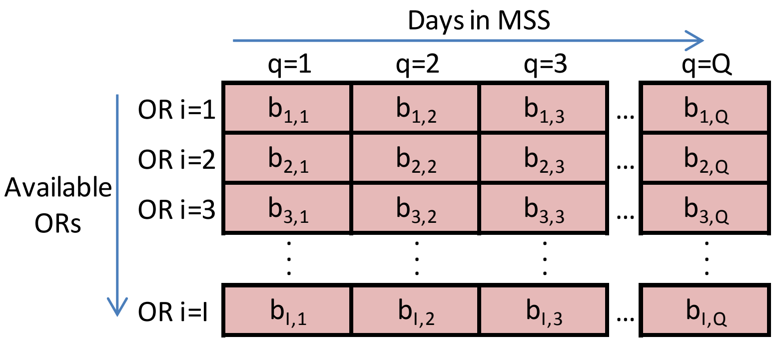

Figure 6: Example scheme of MSS

The number of patients resulting from a single surgery block of the MSS is given by the discrete distribution $c^j(y)$, with $y \in \{0,1,\dots,C^j\}$ and C^j as the maximum number of surgeries in one block of type j. For example, $c^1(3) = 0.33$ means that the probability that a block of type 1 contains 3 patients is 0.33. The time line of a block is shown in Figure 7. This time line contains days $n \in \{-1,0,\dots,L^j\}$. Day $n = 0$ is defined as the day of surgery. Day $n = -1$ contains patients that are admitted the day before surgery. L^j is the maximum LOS. The period taken into account for the LOS starts at the day of surgery and ends at the day of discharge. Patient admissions only take place during the day before and the day of surgery and are described in the admission process. Patient discharges only take place at the days after surgery and are described in the discharge process.

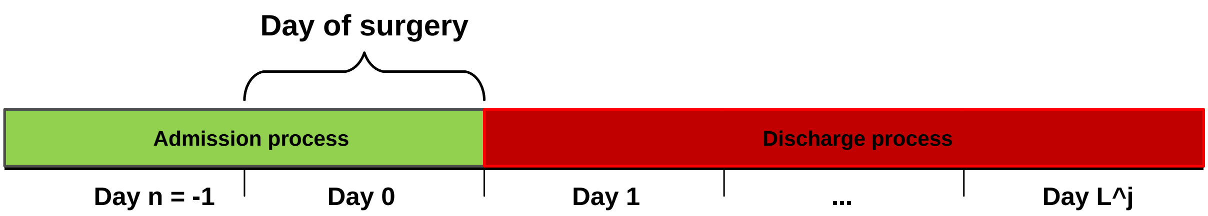

Figure 7: Time line of influence single surgery block on ward census

To determine the admission day of a patient we introduce g_n^j, which is the probability that a patient of type j is admitted at day n given this patient is admitted, with $g_{-1}^j + g_0^j = 1$. The time slot of admission is described by the admission day process. This process is mainly depending on the procedures to be taken till surgery and the availability of necessary medical staff. These procedures are reflected in the day process in the parameter $v_{n,t}^j$, which is the probability of a patient to be admitted in time slot t given this patient is not yet admitted. For example $v_{-1,12}^1 = 0.09$ means that for a patient of type 1, who is not yet in the ward just before time slot 12 on the day before surgery, the probability to be admitted during this time slot is 0.09. We calculate $v_{n,t}^j$ by dividing the probability of a patient to be admitted at the begin of time slot t of day n by the probability that the patient is not admitted before this time slot. Let $w_{n,t}^j$ be the probability that a patient of type j is admitted at the begin of time slot t given this patient is admitted today, with $\sum_{t=0}^{T-1} w_{n,t}^{j} = 1$, then $v_{n,t}^{j}$ is calculated as follows:

$$v_{n,t}^{j} = \frac{w_{n,t}^{j} g_n^j}{g_n^j \sum_{k=t}^{T-1} w_{n,k}^{j} + \sum_{u=n+1}^{0} g_u^j} \qquad for\; n = -1,0 \qquad (1)$$

Patients can be discharged during the days after surgery ($n = 1, \dots, L^j$). $P^j(n)$ is the probability that the LOS of a patient of type j is n days, with $\sum_{n=1}^{L^j} P^j(n) = 1$. The LOS distribution only describes the period after surgery and is not affected by the day of admission. Therefore a LOS of 5 days means discharge on day $n = 5$, independent of whether the patient is admitted the day before or the day of surgery. From $P^j(n)$ we determine s_n^j, which is the probability of a patient still present at the begin of day n to be discharged this day. We compute s_n^j as follows:

$$s_n^j = \frac{P^j(n)}{\prod_{m=1}^{n-1}(1 - s_m^j)} \qquad for\; n = 1 \dots L^j \qquad (2)$$

Many factors influence the time slot of discharge on the day of discharge. For instance, the starting time of the physician's round, procedures still to take place after the decision to discharge and the arrival time of the family or friends that come to pick up the patient. These factors are reflected in the discharge day process given by the parameter $z_{n,t}^{j}$, which is the probability of a patient of type j to be discharged at the end of time slot t of day n given this patient is still present at the begin of this time slot. We calculate $z_{n,t}^{j}$ by dividing the probability of a patient to be discharged at the end of time slot t of day n by the probability that the patient is not yet discharged at begin of this time slot. Let $m_{n,t}^{j}$ be the probability that a patient of type j is discharged at end of time slot t of day n given this patient is discharged this day, with $\sum_{t=0}^{T-1} m_{n,t}^{j} = 1$, then $z_{n,t}^{j}$ is calculated as follows:

$$z_{n,t}^{j} = \frac{m_{n,t}^{j} P^j(n)}{P^j(n) \sum_{i=t}^{T-1} m_{n,i}^{j} + \sum_{k=n+1}^{L^j} P^j(k)} \qquad for\; n = 1, \dots, \mathrm{L}^{\mathrm{j}} \qquad (3)$$

4.2.2 Bed census single surgery block

We describe the bed census at the begin of a time slot of a day resulting from a single surgery block with $h_{n,t}^{j}$. For example, $h_{3,12}^{1}(2) = 0.04$ means that the probability that 2 patients are present in the ward during time slot 12 of day 3 is 0.04. For the probabilities $h_{n,t}^{j}$ we have $\sum_{x=0}^{C^j} h_{n,t}^{j}(x) = 1$ for all $n \in \{-1, \dots, L^j\}, t \in \{0, \dots, T-1\}$.

We first calculate the admission process, describing the bed census during the day before and the day of surgery, and then we calculate the discharge process, which describes the bed census during the days after surgery. We use $a_{n,t}^{j}(x)$ to describe the admission process, with $a_{n,t}^{j}(x)$ as the probability of x patients admitted until and including the start of time slot t of day n. We use $d_{n,t}^{j}(x)$ to describe the discharge process, with $d_{n,t}^{j}(x)$ as the probability of x patients still in the ward at the begin of time slot t of day n.

Admission process

Using $c^j(y)$ and $v^j_{n,t}$ as model inputs, we can determine the admission process. The admission process is depending on the number of admissions. Therefore we calculate for each number of total admissions y the admission process $a^j_{n,t}(x|y)$, with $a^j_{n,t}(x|y)$ as the probability of x patients of type j admitted until and including the begin of time slot t of day n given y total admissions. Each time slot t of the days of the admission process the patients that are not yet admitted have a probability to be admitted during this time slot, $v^j_{n,t}$, and a probability not to be admitted during this time slot, $1 - v^j_{n,t}$. If there are g patients admitted until the begin of time slot t and y total admissions from this block, the probability of x patients admitted until and including the begin of time slot t is given by $\binom{y-g}{x-g}(v^j_{n,t})^{x-g}(1 - v^j_{n,t})^{y-x}$. Because at the start of the day before surgery no admissions have taken place yet, $g = 0$ for $t = 0$ of this day. We compute $a^j_{n,t}(x|y)$ for $n = -1,0$ by:

$$a^j_{n,t}(x|y) = \begin{cases} \binom{y}{x}(v^j_{n,t})^x(1 - v^j_{n,t})^{y-x} & for\, n = -1, t = 0 \\ \sum_{g=0}^{x}\binom{y-g}{x-g}(v^j_{n,t})^{x-g}(1 - v^j_{n,t})^{y-x} a^j_{n-1,T-1}(g|y) & for\, n = 0, t = 0 \\ \sum_{g=0}^{x}\binom{y-g}{x-g}(v^j_{n,t})^{x-g}(1 - v^j_{n,t})^{y-x} a^j_{n,t-1}(g|y) & for\, t = 1, \dots, T-1 \end{cases} \tag{4}$$

To determine the admission process of the block we have to multiply the admissions process for a given number of surgeries with the probability that exactly this number of surgeries are performed in this block. We compute $a^j_{n,t}(x)$ for $n = -1,0$ by:

$$a^j_{n,t}(x) = \sum_{y=x}^{C^j} a^j_{n,t}(x|y)c^j(y) \tag{5}$$

Discharge process

Discharges take place at the days after surgery, days $n = 1, \dots, L^j$. The probability for a patient, still present in the ward, to be discharged during a day is taken from the LOS distribution and described by s^j_n (section 4.2.1). With this we can determine the starting census of a day described by $d^j_n(x)$, which is the probability of x patients of type j still present in the ward on the begin of day n. For the day after surgery $d^j_n(x)$ equals the probability distribution for the number of patients that went for surgery, $c^j(x)$. This, because all admissions have taken place and discharges have not taken place yet. For the other days the starting position is depending on the discharges taken place during the previous days. Therefore for day n of the discharge process, the starting position x is calculated by the number of patients k present at the start of the day before, $n - 1$, multiplied by the probability of $(k - x)$ discharges during that day. This is given by the following formula:

$$d_n^j(x) = \begin{cases} c^j(x) & for\ n = 1 \\ \sum_{g=x}^{c^j} \binom{g}{x} \left(s_{n-1}^j\right)^{g-x} \left(1 - s_{n-1}^j\right)^x d_{n-1}^j(g) & for\ n = 2, \dots, L^j \end{cases} \tag{6}$$

We now have to determine the census at the start of the time slots of the day. This is determined by the time slot of discharge of the patients that are discharged this day. Many factors influence the time slot of discharge. For instance, the starting time of the physician's round, tasks still to do after the decision to discharge and the arrival time of the family or friends that come to pick up the patient. These factors are reflected in the discharge day process. The discharge day process gives for each time slot the probability to be discharged for a single patient that has to be discharged this day and is still present in the ward.

With the starting position, $d_n^j(x)$, of the day and the time slot specific discharge probability $s_{n,t}^j$ as input, we now can determine the number of patients present at the start of a time slot of a day. A patient still present at the ward at the begin of time slot t has a probability $s_{n,t}^j$ to be discharged during this time slot and a probability of $(1 - s_{n,t}^j)$ still to be present at the begin of the next time slot $(t + 1)$. The first discharge can take place at the end of time slot $t = 0$ of day $n = 1$. Because we determine the bed census at the begin of a time slot, this will be accounted for in the bed census of time slot 1. With a similar approach as for the starting position of the day we now compute the bed census at the start of time slot t for days $n = 1, \dots, L^j$ with:

$$d_{n,t}^j(x) = \begin{cases} d_n^j(x) & for\ t = 0 \\ \sum_{k=x}^{c^j} \binom{k}{x} \left(s_{n,t-1}^j\right)^{k-x} \left(1 - s_{n,t-1}^j\right)^x d_{n,t-1}^j(k) & for\ t > 0 \end{cases} \tag{7}$$

Hourly bed census surgery block

Given the admission and discharge process we can define the bed census at the begin of a time slot of a day, described by $h_{n,t}^j$ with:

$$h_{n,t}^j(x) = \begin{cases} a_{n,t}^j(x) & for\ n = -1,0 \\ d_{n,t}^j(x) & for\ n = 1, \dots, L^j \end{cases} \tag{8}$$

4.2.3 Single cycle MSS

With the bed census of the surgery bocks and the MSS as input we can determine the bed census resulting from a single cycle of the MSS. First we assign the bed census of the correct block type to an OR day. Then we determine the census for each day of the time line of the single cycle of a MSS.

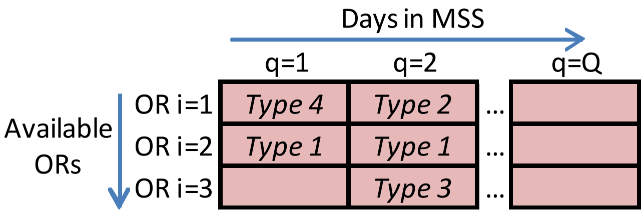

Figure 8: Example MSS

Assign block type to MSS

Figure 8 shows a part of an MSS. A patient type is assigned to the blocks $b_{i,q}$ of the MSS. The example contains 3 ORs for a time period of Q days. There are 4 different types of surgery blocks and surgeries are only planned on day $q = 1,2$. For example, type 2 assigned to block $b_{1,2}$ means that in OR 1 during day 2 surgical operations of the type 2 are performed. The MSS contains some empty blocks, which is corresponding with practice. To determine the bed census resulting from the cycle of the MSS we relate the bed census of the patient type to the OR day of the MSS, defined by surgery block $b_{i,q}$. If j denotes the specialty assigned to block $b_{i,q}$, then the distribution $\bar{h}^{i,q}_{m,t}$ for the number of patients of block $b_{i,q}$ still in the ward at begin time slot t of day m ($m \in \{0,1, \ldots Q, Q+1, Q+2, \ldots\}$) is given by:

$$\bar{h}^{i,q}_{m,t}(x) = \begin{cases} 0 & for\ m < q-1 \\ h^{j}_{m-q,t} & for\ m \geq q-1 \end{cases} \qquad (9)$$

Bed census Single Cycle Influence MSS

With the assignment of the blocks we now can determine the impact of a single cycle of the MSS on each day m of the time line of the Single Cycle Influence, $m \in \{0,1, \ldots, M\}$. The time line of the Single Cycle Influence starts on day $m = 0$, on which patients from the first day of the MSS (q=1) can arrive. M equals the largest value of $q + L^{j}$, which is the last day with a probability that there is still a patient from this MSS cycle present in the ward. Note that because the maximum length of stay differs per block type, the last patient in the ward does not have to be a patient from a block on the last day of the MSS. For the example MSS described in Figure 8, the impact on the bed census of the related nursing ward is shown in Figure 9.

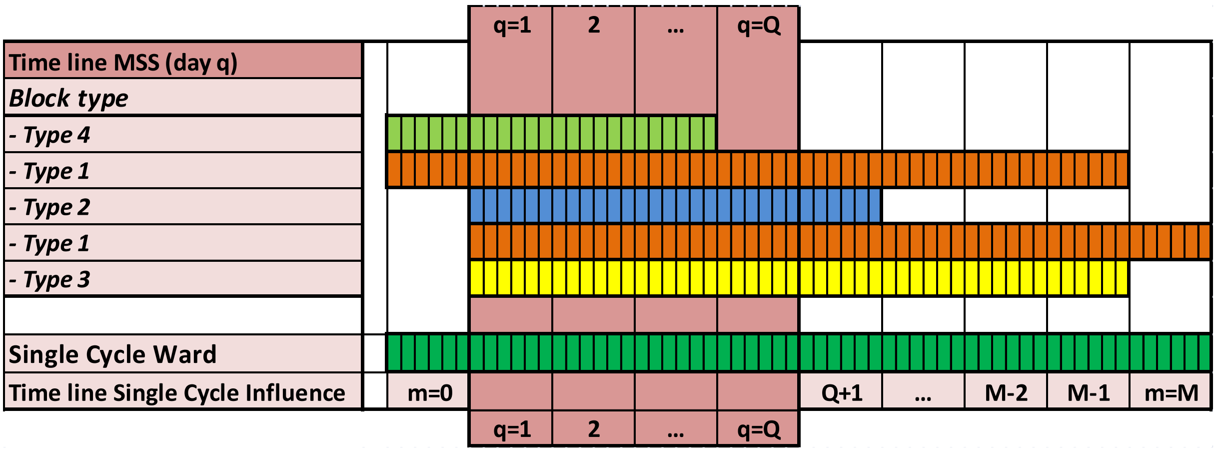

Figure 9: Time line Single Cycle of MSS

Given the impact of the blocks from the MSS we can determine $H_{m,t}(x)$, the probability that x patients are present in the ward at the begin of time slot t of day m from a single cycle of the MSS. To determine $H_{m,t}$ we have to combine the results of the single blocks. Because the impacts of the blocks are independent of each other we can use discrete convolutions (indicated by *) to combine the blocks. Let A and B be two independent discrete distributions. Then C=A*B is computed by:

$$C(x) = \sum_{u=0}^{U} A(u)B(x-u)$$

where U is equal to the largest x value with a positive probability that can result from A*B. Using convolutions, $H_{m,t}$ is computed by:

$$H_{m,t} = \bar{h}_{m,t}^{1,1} * \bar{h}_{m,t}^{1,2} * \dots * \bar{h}_{m,t}^{1,Q} * \bar{h}_{m,t}^{2,1} * \dots * \bar{h}_{m,t}^{I,Q} \qquad (10)$$

4.2.4 Steady State MSS

The impact of a single cycle of the MSS does not only comprehend the days of this cycle but also days outside this cycle. Figure 9 shows that during the days after the last day of the cycle there are still patients present in the ward. To determine the full impact of the cyclic MSS, we have to combine the impact of the consecutive cycles which affect a day of the MSS.

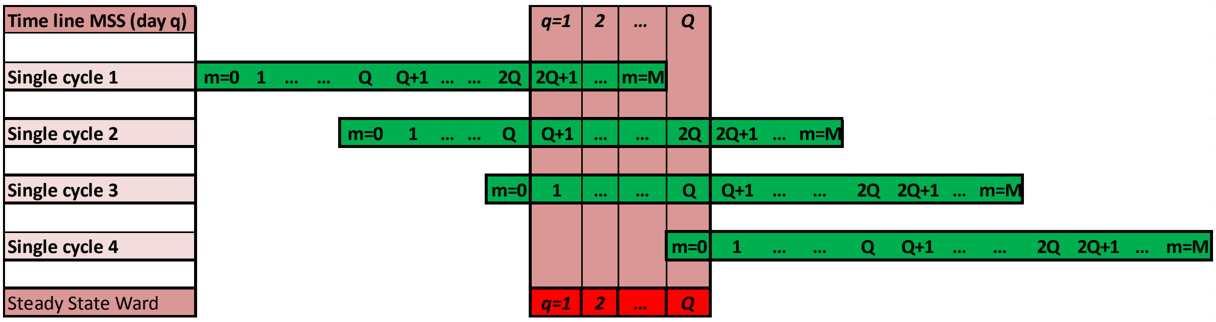

Figure 10: Overlapping cycles MSS

We divide the time line of the single cycle in periods of length Q to find the overlapping days, as shown in Figure 10. In total there are $\lfloor M/Q \rfloor$+1 cycles overlapping, with the last cycle only having day $m = 0$ as the overlapping day. We now combine for each time slot the census of the time slot of the corresponding days in order to determine the steady state for the time slots of the days of the MSS. Using convolution on the overlapping days we compute this by :

$$H_{q,t}^{SS} = H_{q-Q,t} * H_{q,t} * H_{q+Q,t} * \dots * H_{q+\lfloor M/Q \rfloor Q,t} \qquad (11)$$

4.3 Non-elective patients

We take three steps to determine the number of non-elective patients in the nursing ward. The first step is to calculate for all patient types the bed census resulting from patient arrivals during each day of the week (section 4.3.2). The next step is to calculate the result of patient arrivals during each day of the period (section 4.3.3). The last step is to determine the steady state bed census of the ward (section 4.3.4). We will start with describing the input for the non-elective model (section 4.3.1).

4.3.1 Model input

Non-elective patients are admitted to the ward during each day r of the week, $r \in \{1,2,\dots,R\}$ with $r = 1$ representing Monday and $R = 7$ representing Sunday. For each patient type $i \in \{0,\dots,I\}$ we determine the impact of arrivals of a single weekday, r, as shown in Figure 11. The arrival day specific admission day process, $\lambda_{r,t}^{i}$, is given as input, with $\lambda_{r,t}^{i}$ as the average number of arrivals in time slot t of day r. The admission day process reflects weekday specific aspects such as the opening hours of inpatient clinics related to the ward and weekday/weekend influences. The average number of admissions for a day is given by λ_{r}^{i}, with $\lambda_{r}^{i} = \sum_{t=0}^{T-1} \lambda_{r,t}^{i}$.

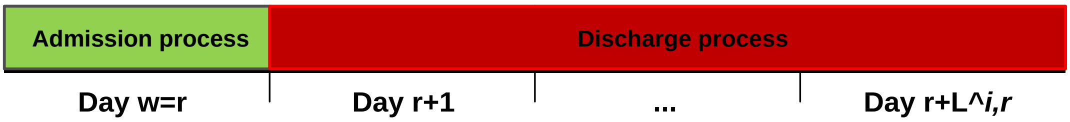

Figure 11: Time line non-elective block

The discharge process takes place at days $w = r+1, \dots, r+L^{i,r}$. The LOS distribution, $P^{i,r}(n)$ with $\sum_{n=1}^{L^{i,r}} P^{i,r}(n) = 1$, is dependent of the arrival day. This because the treatment of non-elective patients is often not fully known at arrival and can be delayed due to the lack of available (human) resources during the day of arrival and/or the first days after arrival. From the LOS distribution we can determine $\tilde{s}^{i,r}_{w}$, which is the probability of a patient admitted during day r and still present at the begin of day n to be discharged this day. We compute $\tilde{s}^{i,r}_{w}$ as follows:

$$\tilde{s}^{i,r}_{r+n} = \frac{P^{i,r}(n)}{\prod_{k=1}^{n-1}(1-\tilde{s}^{i,r}_{r+k})} \qquad for\ n = 1 \dots L^{i,r} \qquad (12)$$

The discharge day process is given by the parameter $\tilde{s}^{i,r}_{w,t}$, which is the probability of a patient of type i, admitted during day r, to be discharged at the end of time slot t of day w given this patient is still present at the begin of this time slot. We calculate $\tilde{s}^{i,r}_{w,t}$ by dividing the probability of a patient to be discharged at end of time slot t of day w by the probability that the patient is not yet discharged at begin of this time slot. Let $\tilde{m}^{i,r}_{n,t}$ be the probability that a patient of type i, admitted during day r, is discharged at end of time slot t of day n given this patient is discharged this day, with $\sum_{t=0}^{T-1} \tilde{m}^{i,r}_{n,t} = 1$, then $\tilde{s}^{i,r}_{w,t}$ is calculated as follows:

$$\tilde{s}^{i,r}_{r+n,t} = \frac{\tilde{m}^{i,r}_{n,t} P^{i,r}(n)}{P^{i,r}(n)\sum_{i=t}^{T-1} \tilde{m}^{i,r}_{n,i} + \sum_{k=n+1}^{L^{i,r}} P^{i,r}(n)} \qquad for\ n = 1, \dots, L^{i,r} \qquad (13)$$

4.3.2 Non-elective day

We describe the bed census at the start of time slot t of day w resulting from the non-elective patients of type i arrived on day r with $g^{i,r}_{w,t}(x)$. For this non-elective day block of type i we describe the admission process $\tilde{a}^{i,r}_{w,t}(x)$, for $w = r$, as x patients of type i admitted until and including time slot t of day r. We describe the discharge process $\tilde{d}^{i,q}_{w,t}(x)$, for $w = r+1, \dots, r+L^{i,r}$, as x patients admitted during day r still present in the ward at begin of time slot t of day w.

Admission process

The admission process is a Poisson process. A Poisson process assumes patients arriving independent of each other, which is the case for non-elective patients in nursing wards. For the first time slot of the arrival day we can use the Poisson process to determine $\tilde{a}^{i,r}_{w,t}(x)$. For the next time slots we multiply the probability of y admissions during this time slot with the probability of $(x-y)$ admissions during the previous time slots. We sum the probabilities for all possible positive values of y to reach x admissions. This is given in the following formula's for admission day $w = r$:

$$\tilde{a}_{w,t}^{i,r}(x) = \begin{cases} \dfrac{(\lambda_{r,t}^{i})^{x} e^{-\lambda_{r,t}^{i}}}{x!} & for\ t = 0 \\ \displaystyle\sum_{y=0}^{x} \frac{(\lambda_{r,t}^{i})^{y} e^{-\lambda_{r,t}^{i}}}{y!} \tilde{a}_{w,t-1}^{i,r}(x-y) & for\ t > 0 \end{cases} \tag{14}$$

Discharge process

The approach for the discharge process of the non-elective patients is equivalent to the approach for the discharge process of the elective patients. To determine the discharge process we first determine the bed census at the start of the day. Then we determine the day process of the discharges. For the first day of the discharge process the bed census at the start of the day equals the number of admissions the day before. This is given by a Poisson distribution with parameter λ_r^i as the average number of admissions during day r. For the other days we use $\tilde{s}_w^{i,r}$ as the probability of a patient still in the ward to be discharged during day w and (1-$\tilde{s}_w^{i,r}$) as the probability of a patient still in the ward not to be discharged during day w. With this we compute $\tilde{d}_w^{i,r}(x)$ as follows:

$$\tilde{d}_w^{i,r}(x) = \begin{cases} \dfrac{(\lambda_r^i)^x e^{-\lambda_r^i}}{x!} & for\ w = r+1 \\ \displaystyle\sum_{g=x}^{\infty} \binom{g}{x} \left(\tilde{s}_{w-1}^{i,r}\right)^{g-x} \left(1-\tilde{s}_{w-1}^{i,r}\right)^{x} \tilde{d}_{w-1}^{i,r}(g) & for\ r+1 < w \leq r + L^{i,r} \end{cases} \tag{15}$$

With the starting position of the day, $\tilde{d}_w^{i,r}(x)$, and the time slot specific discharge probability $\tilde{s}_{w,t}^{i,r}$ as input, we now can determine the number of patients present at the start of a time slot of a day. A patient still present at the ward at the begin of time slot t has a probability $\tilde{s}_{w,t}^{i,r}$ to be discharged during this time slot and a probability of $(1-\tilde{s}_{w,t}^{i,r})$ still to be present at the begin of the next time slot $(t+1)$. The first discharge of a day can take place at the end of time slot $t = 0$. Because we determine the bed census at the begin of a time slot, this will be accounted for in the bed census of time slot 1. With a similar approach as for the starting position of the day we now compute the bed census at the start of time slot t for days $w = \mathrm{r}+1, \ldots, \mathrm{r}+L^{i,r}$ with:

$$\tilde{d}_{w,t}^{i,r}(x) = \begin{cases} \tilde{d}_w^{i,r}(x) & for\ t = 0 \\ \displaystyle\sum_{k=x}^{\infty} \binom{k}{x} \left(\tilde{s}_{w,t-1}^{i,r}\right)^{k-x} \left(1-\tilde{s}_{w,t-1}^{i,r}\right)^{x} \tilde{d}_{w,t-1}^{i,r}(k) & for\ t = 1,.,T-1 \end{cases} \tag{16}$$

Bed census non-elective day

Given the admission and discharge process we can define the bed census at the begin of a time slot of a day, described by $g_{w,t}^{i,r}$ with:

$$g_{w,t}^{i,r} = \begin{cases} 0 & for\ w < r \\ \tilde{a}_{w,t}^{i,r}(x) & for\ w = r \\ \tilde{d}_{w,t}^{i,r}(x) & for\ w = r+1, \ldots, \mathrm{r}+L^{i,r} \end{cases} \tag{17}$$

4.3.3 Single period

Given the bed census resulting from a single patient type i from a single admission day r we now can determine the bed census resulting from patient admissions during each day r of the week, $r = 1, \dots, 7$. Figure 12 shows the time line resulting from non-elective patient admissions on each day of a Single Week Influence. We now combine the overlapping days using convolution:

$$G_{w,t} = g_{w,t}^{1,1} * g_{w,t}^{1,2} * \dots * g_{w,t}^{1,7} * g_{w,t}^{2,1} * \dots * g_{w,t}^{I,7} \tag{18}$$

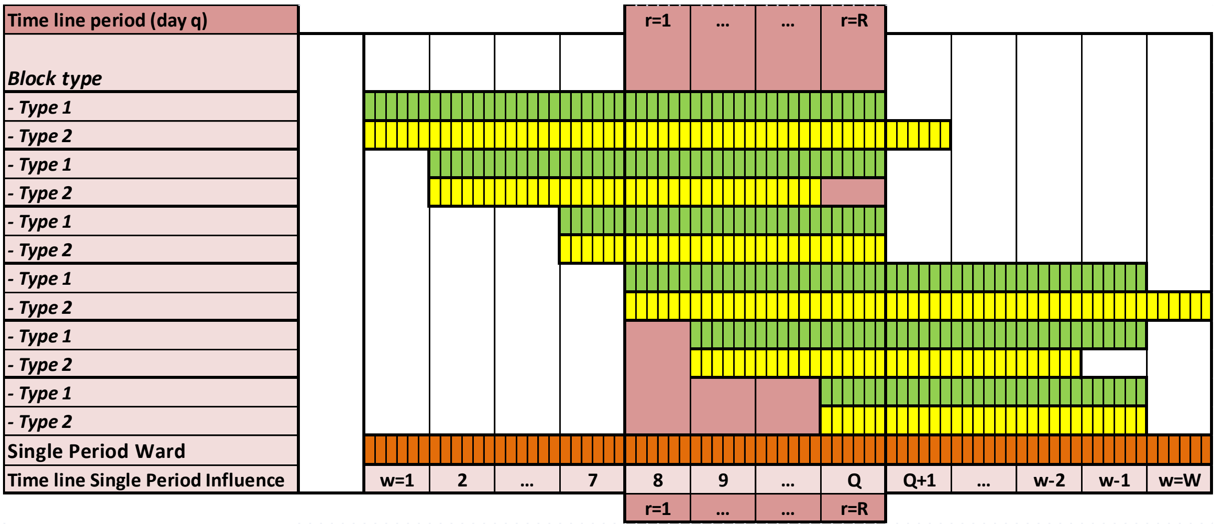

Figure 12: Time line Single Period

4.3.4 Steady state period

The last step is to determine the steady state bed census for a week. We will use the same approach as we do for the elective patients in section 4.2.4. We divide the time line of the Single Week Influence in periods of length $R = 7$ to find the overlapping days of multiple weeks, as shown in Figure 10. With W as the last day of the single period influence, in total there are $\lceil W/R \rceil$ cycles overlapping. We now combine for each time slot the census of the time slot of the corresponding days in order to determine the steady state for the time slots of the days of the week. Using convolution on the overlapping days we compute this by :

$$G_{r,t}^{SS} = G_{r,t} * G_{r+R,t} * G_{r+2R,t} * \dots * G_{r+\lfloor W/R \rfloor R,t} \tag{19}$$

4.4 Hourly bed census nursing ward

Given the demand probabilities of the elective patients and the demand probabilities of the non-elective patients we now can determine the demand probabilities of the nursing ward $Z_{q,t}(x)$ as the probability of x patients present in the ward at begin of time slot t of day q. To add the non-elective patients we first need to determine the day of the week of day q. Given that r is the day of the week corresponding to q, we compute $Z_{q,t}$ by:

$$Z_{q,t} = H_{q,t}^{SS} * G_{r,t}^{SS} \qquad for\ q = 1, \dots, Q\ with\ r = q\ (mod\ R) \tag{20}$$

An example of the output of the model is given in Table 6. For each time slot t of every day q of the time period the probability for a number of patients is given as output.

Nr of patients	0	1	2	3	4	5	6	7	8	9
Probability	0	0,00213	0,04473	0,19764	0,33711	0,27064	0,11573	0,02795	0,00382	0,00025

Table 6: Example output demand probabilities ward

4.5 Extensions and remarks

This section describes the extensions and remarks on the basic model.

4.5.1 Adjustment to daily bed census model of Vanberkel (2011)

The model described by Vanberkel (Vanberkel, et al., 2011) determines the impact of the MSS on the number of elective patients present in the ward on a daily basis. We have used this model as a basis to determine the impact of the elective patients on an hourly basis. This model describes the impact of elective patients from admission on the day of surgery until discharge from the nursing ward. In contrast to our model, patients are always admitted during the day of surgery.

The three step approach of Vanberkel is shown in Appendix B. The steps are similar to the three step approach we describe in our model (single block impact, single cycle MSS, steady state MSS). The daily census is based on admissions accounted for at the start of the day and discharges at the end of the day, shown in formula (21).

$$d_n^j(x) = \begin{cases} c^j(x) & for\, n = 0 \\ \sum_{g=x}^{C^j} \binom{g}{x} \left(s_{n-1}^j\right)^{g-x} \left(1 - s_{n-1}^j\right)^x d_{n-1}^j(g) & for\, n = 1, \ldots, L^j \end{cases} \tag{21}$$

To use this method as the bed census at the start of the day we have adjusted this formula by counting the admissions at the end of the day. The number of patients present at the start of the day after surgery now equals the number of total patient admissions, shown in formula (22). This formula is equal to formula (6).

$$d_n^j(x) = \begin{cases} c^j(x) & for\, n = 1 \\ \sum_{g=x}^{C^j} \binom{g}{x} \left(s_{n-1}^j\right)^{g-x} \left(1 - s_{n-1}^j\right)^x d_{n-1}^j(g) & for\, n = 2, \ldots, L^j \end{cases} \tag{22}$$

4.5.2 From demand probability distribution to bed census

The result of the basic model is the demand probability distribution of a ward. From this we can determine the bed census, which is restricted by the ward capacity. How to derive the bed census from the demand probability distribution is depending on whether we analyze one or two wards. In case of two wards patients can be misplaced to the non-dedicated ward. First we describe the approach for the analysis of one ward and then we describe the approach for two wards.

Single ward analysis

From the demand probabilities $Z_{q,t}(\mathrm{x})$ we can determine the bed censuses $\bar{Z}_{q,t}(x)$. The bed censuses takes into account the number of operational beds X. We determine $\bar{Z}_{q,t}(x)$ as follows:

$$\bar{Z}_{q,t}(x_k) = \begin{cases} Z_{q,t}(x) & for\ 0 \leq x < X \\ \sum_{x=X}^{\infty} Z_{q,t}(x) & for\ x = X \\ 0 & for\ x > X \end{cases} \tag{23}$$

Two wards analysis

In case we analyze two wards, we first need to incorporate misplacements before we can determine the bed census. Misplacements occur when the dedicated ward does not have capacity available to admit the new patient, and therefore the patients are admitted to another ward. We define the demand probability $Z_{q,t}^{k}(x_k)$ as the probability of x_k patients in ward k at the start of time slot t of day q. We define x_k as x patients in ward k and X_k as the operational bed capacity in ward k. We set $k = 1$ as the dedicated ward and $k = 2$ as the ward the patient is misplaced. The demand that exceeds the bed capacity of ward k is given by Δx_k, with $\Delta x_k = x_k - X_k$, and the number of available beds in ward k is given by ∇x_k, with $\nabla x_k = X_k - x_k$. We define the demand probability distribution of ward k after the misplacements as $\hat{Z}_{q,t}^{1}(x)$ and $\hat{Z}_{q,t}^{2}(y)$. We first determine $\hat{Z}_{q,t}(x, y|x_1, x_2)$ as follows:

$$\begin{array}{ll} \hat{Z}_{q,t}(x_1, x_2|x_1, x_2) = Z_{q,t}^{1}(x_1)Z_{q,t}^{2}(x_2) & for\ x_1 \leq X_1, x_2 \leq X_2 \\ \hat{Z}_{q,t}(X_1, x_2 - \nabla x_1|x_1, x_2) = Z_{q,t}^{1}(x_1)Z_{q,t}^{2}(x_2) & for\ x_1 \leq X_1, x_2 > X_2, \nabla x_1 \leq \Delta x_2 \\ \hat{Z}_{q,t}(x_1 + \Delta x_2, X_2|x_1, x_2) = Z_{q,t}^{1}(x_1)Z_{q,t}^{2}(x_2) & for\ x_1 \leq X_1, x_2 > X_2, \nabla x_1 > \Delta x_2 \\ \hat{Z}_{q,t}(x_1 - \nabla x_2, X_2|x_1, x_2) = Z_{q,t}^{1}(x_1)Z_{q,t}^{2}(x_2) & for\ x_1 > X_1, x_2 \leq X_2, \nabla x_2 \leq \Delta x_1 \\ \hat{Z}_{q,t}(X_1, x_2 + \Delta x_1|x_1, x_2) = Z_{q,t}^{1}(x_1)Z_{q,t}^{2}(x_2) & for\ x_1 > X_1, x_2 \leq X_2, \nabla x_2 > \Delta x_1 \\ \hat{Z}_{q,t}(x_1, x_2|x_1, x_2) = Z_{q,t}^{1}(x_1)Z_{q,t}^{2}(x_2) & for\ x_1 > X_1, x_2 > X_2 \\ \hat{Z}_{q,t}(x, y|x_1, x_2) = 0 & elsewhere \end{array} \tag{24}$$

From $\hat{Z}_{q,t}(x, y|x_1, x_2)$ we now can determine $\hat{Z}_{q,t}^{1}(x)$ and $\hat{Z}_{q,t}^{2}(y)$ as follows:

$$\begin{array}{l} \hat{Z}_{q,t}^{1}(x) = \sum_{x_1=0}^{\infty} \sum_{x_2=0}^{\infty} \sum_{y=0}^{\infty} \hat{Z}_{q,t}(x, y|x_1, x_2) \\ \hat{Z}_{q,t}^{2}(x) = \sum_{x_1=0}^{\infty} \sum_{x_2=0}^{\infty} \sum_{y=0}^{\infty} \hat{Z}_{q,t}(x, y|x_1, x_2) \end{array} \tag{25}$$

From the demand probabilities we can determine the bed census with formula 23.

4.5.3 Assigning patients with two dedicated wards

Some patient types do not belong to only one ward, but have two dedicated wards as shown in Table 7. This requires a method which assigns these patients to one of both wards. The method we introduce first determines the demand probability of the wards following from MSS blocks of single ward patient types and non-elective patients. Then we assign the patients following from MSS blocks of double ward patient types to the specific wards. This results in the demand probability distributions of the wards which contains all patient types. The assignment is based on the demand probabilities of the wards before the assignment and a set of decision rules. In the following sections we will describe the method in detail.

	Ward 1	Ward 2
Type 1	X	
Type 2		X
Type 3	X	X
Type 4	X	X
Type 5	X	
Type 6	X	
Type 7		X

Table 7: Example dedicated wards

Ward 1	Ward 2	Dummy ward 3
Type 1	Type 2	Type 3
Type 5	Type 7	Type 4
Type 6		

Table 8: Example dummy ward

We first determine the demand probabilities of the wards before the assignment method, described by $\tilde{Z}_{q,t}^{k}$. For both wards, ward 1 and ward 2, $\tilde{Z}_{q,t}^{k}$ consists of the single ward patient types and the non-elective patients and is determined as $Z_{q,t}$ in the basic model described in sections 4.2 until 4.4. Patient types that are dedicated to both wards are assigned to a dummy ward (ward 3) as shown in Table 8. We also determine $\tilde{Z}_{q,t}^{k}$ of the dummy ward, where $\tilde{Z}_{q,t}^{k}$ contains the double ward patient types. The next step is to divide the patients from the dummy ward among the two wards based on a set of decision rules resulting in $\bar{\bar{Z}}_{q,t}^{k}$. Examples of decision rules are:

- Equally divide the patients among both wards
- One of both wards is the preferred ward. Assign patients to the preferred ward until the bed capacity is reached, assign remaining patients to the other ward
- Level the bed occupancy for both wards
- Level the percentage of beds in use (in case of unequal number of beds for both wards)
- Assign patients to meet the nurse scheduling criteria for optimization

With the set of decision rules we need to have a unique solution of assigning the patients for each situation. Therefore we set a first order rule and add additional rules until we always have a unique solution. Based on the set of rules we now divide the patients among the wards. We do this for every time slot t of every day q of the time period.

An example of the decision is shown in Figure 13. We have $\tilde{x}_1$ as the number of patients in ward 1, $\tilde{x}_2$ as the number of patients in ward 2 and u as the number of patients in dummy ward 3. We set u_1 as the number of patients from dummy ward 3 assigned to ward 1 and u_2 as the number of patients assigned to ward 2. In order to assign the patients we set X_1 as the number of operational beds in ward 1 and X_2 as the number of operational beds in ward 2. For each combination of $\tilde{x}_1, \tilde{x}_2, u$ we now determine u_1 and u_2. Given u_1 and u_2 we can determine $Z_{q,t}^{1}(x_1|\tilde{x}_1, \tilde{x}_2, u)$ and $Z_{q,t}^{2}(x_2|\tilde{x}_1, \tilde{x}_2, u)$ as follows:

$$\begin{aligned}\bar{\bar{Z}}_{q,t}^{1}(\tilde{x}_1 + u_1|\tilde{x}_1, \tilde{x}_2, u) &= \tilde{Z}_{q,t}^{1}(\tilde{x}_1)\tilde{Z}_{q,t}^{2}(\tilde{x}_2)\tilde{Z}_{q,t}^{3}(u) \\ \bar{\bar{Z}}_{q,t}^{2}(\tilde{x}_2 + u_2|\tilde{x}_1, \tilde{x}_2, u) &= \tilde{Z}_{q,t}^{1}(\tilde{x}_1)\tilde{Z}_{q,t}^{2}(\tilde{x}_2)\tilde{Z}_{q,t}^{3}(u)\end{aligned} \qquad (26)$$

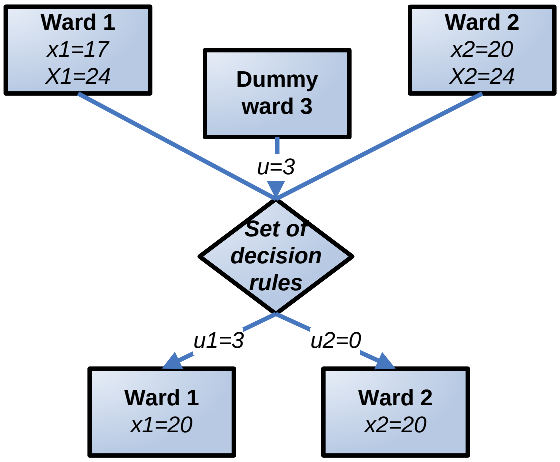

Figure 13: Example decision combination

To determine the demand probabilities after the division of the patients we need to sum over all combinations of $\tilde{x}_1, \tilde{x}_2, u$ as follows:

$$\bar{\bar{Z}}^1_{q,t}(x_1) = \sum_{\tilde{x}_1=0}^{\infty} \sum_{\tilde{x}_2=0}^{\infty} \sum_{u=0}^{\infty} \bar{\bar{Z}}^1_{q,t}(\tilde{x}_1 + u_1 | \tilde{x}_1, \tilde{x}_2, u)$$

$$\bar{\bar{Z}}^2_{q,t}(x_2) = \sum_{\tilde{x}_1=0}^{\infty} \sum_{\tilde{x}_2=0}^{\infty} \sum_{u=0}^{\infty} \bar{\bar{Z}}^2_{q,t}(\tilde{x}_2 + u_2 | \tilde{x}_1, \tilde{x}_2, u) \qquad (27)$$

From the demand probabilities we can determine the bed census $\bar{Z}^k_{q,t}(x)$ as described in section 4.5.2.

To clarify the method we have worked out two example sets of decision rules. In the first one the leveling is based on the bed occupancy and in the second one the leveling of the beds is based on the number of beds in use.

Leveling based on bed occupancy

The first example contains the following set of decision rules:

1. Level the bed occupancy for both wards
2. Assign patients to ward 1 until the bed capacity is reached, assign remaining patients to ward 2.

With this set of rules we level the percentage of beds in use for both wards and in case there is a patient left, this patient is assigned to ward 1. We set p_1 as the occupancy rate of ward 1 after assigning the patients, with $p = \frac{x_1+u_1}{X_1}$, and p_2 as the occupancy rate of ward 2 after assigning the patients, with $p_2 = \frac{x_2}{X_2}$. Then we determine the distributions for u_1 and u_2 as follows:

$$(u_1, u_2) = argmin\{p_1, p_2 | u_1 + u_2 = u\} \qquad (28)$$

Leveling based on number of beds in use

The second example contains the following set of decision rules:

1. Level the number of beds in use
2. Assign patients to ward 1 until the bed capacity is reached, assign the remaining patients to ward 2

With this set of rules we level the number of beds in use and in case there is a patient left, this patient is assigned to ward 1. In order to assign the patients we set Δw as the difference between the number of patients in ward 1 and ward 2, $\Delta w = x_1 - x_2$. The distributions for u_1 and u_2 are described as follows:

$$u_1 = \begin{cases} 0 & for\ \Delta w \geq u \\ \left\lceil \frac{x_1 + x_2 + u}{2} \right\rceil - x_1 & for\ -u+1 < \Delta w < u \\ u & for\ \Delta w \leq -u+1 \end{cases}$$

$$u_2 = \begin{cases} u & for\ \Delta w \geq u \\ \left\lfloor \frac{x_1 + x_2 + u}{2} \right\rfloor - x_2 & for\ -u+1 < \Delta w < u \\ 0 & for\ \Delta w \leq -u+1 \end{cases}$$

4.5.4 Pre-operative stay

A pre-operative stay is a patient scheduled for surgery on Monday and admitted on Friday, further explained in section 2.2.2. The patient is discharged later on this Friday and returns Sunday evening. The administrative bed census reserves a bed for this patient from the admission on Friday. To incorporate this, we have to adjust the admission process described in 4.2.2. Because the admission process differs from the general admission process applicable to the other days of the week we add the indices r for day of the week to the admission process. The admission process of the rest of the week remains equal, therefore we only distinguish two groups: surgeries on Monday and surgeries on other days of the week. Patient admissions of day -1 now shift to day -3 and the number of patients present during day -2 and -1 equal the bed census at the end of day -3. The adjustments are shown in formula (29) with $r = 1$ corresponding to surgery on Monday and $r = 0$ corresponding to surgery on other days of the week. The formula for the other days of the week remains as described in formula (1).

$$a_{n,t}^{j,r}(x) = \begin{cases} \sum_{y=x}^{c^j} a_{n,t}^{j,r}(x|y)c^j(y) & for\ r = 1, n = 0, -3 \\ a_{-3,T-1}^{j,r}(x) & for\ r = 1, n = -1, -2 \end{cases} \tag{29}$$

The adjustment of the admission day process for $r = 1$ is shown in formula (30), with the input variables $g_n^{j,r}$ and $w_{n,t}^{j,r}$.

$$v_{-3,t}^{j,r} = \frac{w_{-3,t}^{j,r} g_{-3}^{j,r}}{g_{-3}^{j,r} \sum_{k=t}^{T-1} w_{-3,k}^{j,r} + g_0^{j,r}} \qquad for\ r = 1 \tag{30}$$

That the admission process is no longer independent of the day, also affects the block census $h_{n,t}^{j}(x)$. Therefore we also add the indices r to the block census, resulting in $h_{n,t}^{j,r}(x)$. The extended admission process also affects the time line of the block, n, and the time line of the single cycle of the MSS. Adding the extra days before surgery result in $n \in \{-3, \ldots, L^j\}$ and $m \in \{-2, -1, \ldots, M\}$.

4.5.5 Half OR day

In some hospitals two specialties can be assigned to the same OR block $b_{i,q}$. To include these shared OR blocks, we assume that the time is equally divided among the specialties and the first specialty can perform surgeries in the morning and the second can perform surgeries in the afternoon. We introduce the indices p which indicate whether a block is taking place in the morning $(p = 1)$, in the afternoon $(p = 2)$ or the whole day $(p = 0)$. A surgery block now is defined as block $b_{i,q,p}$. The bed census of a patient type assigned to block $b_{i,q,p}$ is $h^j_{n,t,p}$. Because the number of surgeries $c^j(y)$ is related to the length of a surgery block and the admission process $a^j_{n,t}(x)$ is related to both the length of the block and the timing, we also need to adjust them accordingly resulting in subsequently $c^j_p(y)$ and $a^j_{n,t,p}(x)$. Adjusting the admission process results in the input variables $g^j_{n,p}$ and $w^j_{n,t,p}$.

4.5.6 Non-cyclic MSS

Not all hospitals are using a cyclic MSS for their surgery planning. Therefore we also describe an approach for a non-cyclic MSS. We do not have to adjust the model in order to determine the demand of a non-cyclic MSS. For a non-cyclic MSS we need to enlarge the time period of the data and we need to take into account a warm-up period to generate valid output data.

Because a non-cyclic MSS does not have overlapping cycles we need to run a warming up period before we can use the demand probabilities. This warming up period should be at least as long as the maximum LOS of the ward. This in order to not have included a possible arrival of a patient just before the start of the MSS. At the end of the period we cannot take into account the last day of the period. This because we might miss the patient admissions of planned surgeries the day after the cycle has ended. Due to the fact that a non-cyclic MSS requires a warming up period and the last day of the period cannot be used, the data period of the non-cyclic MSS is much longer than for the cyclic MSS. Where a cyclic MSS often consists of a period of two or four weeks, the non-cyclic MSS should describe a period of at least two months.

4.6 Performance indicators

To validate the model and to compare the effects of different interventions with each other we use performance indicators. In section 1.2.2 we have introduced the performance indicators and in this section we will describe the mathematical formulas to calculate the performances.

4.6.1 Demand percentiles

Hospitals determine the bed capacity to have sufficient coverage for a certain percentage of the time, for instance 85%. This is called the 85^{th} percentile of demand. The percentage is represented by the probability α. From the demand distribution of the ward, $Z_{q,t}(x)$, we can derive the cumulative distribution $\check{Z}_{q,t}(x)$, which is the probability that at the begin of time slot t of day n maximum x patients are present in the ward. $\check{Z}_{q,t}(x) = \sum_{k=0}^{x} Z_{q,t}(k)$. From $\check{Z}_{q,t}(x)$ we can determine the demand percentile represented by $D_{\alpha,q,t}(x)$, with x as the minimum number of beds required such that $\check{Z}_{q,t}(x)$ is larger or equal to α, given by:

$$D_{\alpha,q,t}(x) = \min\{x | \check{Z}_{q,t}(x) \geq \alpha\} \tag{31}$$

4.6.2 Average operational bed occupancy

The operational bed occupancy is the number of bed hours used in relation to the total number of bed hours. The total number of bed hours is based on the number of operational beds, for example G6NO contains 30 beds of which 24 are operational. We will determine the bed usage according to the administrative bed census described in section 2.2.2. The average bed occupancy of a nursing ward is determined for a certain period. For the period of the MSS the average bed occupancy is given by:

$$B^{occ} = \frac{\sum_{q=1}^{Q}\sum_{t=0}^{T-1}\sum_{x=1}^{X} x.\bar{Z}_{q,t}(\mathrm{x})}{Q.T.X} \qquad (32)$$

4.6.3 Variability in demand for beds

To measure the variability in the demand, we use the coefficient of variance. The general formula for the coefficient of variance is given by:

$$cv_x = \frac{S_x}{\bar{x}} \qquad with\ S_x = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i-\bar{x})^2}\ ,\bar{x} = \frac{\sum_{n=1}^{N} x_n}{N} \qquad (33)$$

For our model we calculate the coefficient of variance for each time slot, for each day and for the period. For each type of variance we provide the formulation. First we determine the standard deviation for each time slot in order to gain insight in the variation in the demand probabilities of a time slot. This is given as follows:

$$\mathrm{cv}_{\mathrm{q,t,x}} = \frac{\sigma}{\bar{\mathrm{x}}_{\mathrm{q,t}}} \qquad \sigma = \sqrt{\sum_{\mathrm{x}=1}^{\infty} \mathrm{Z}_{\mathrm{q,t}}(\mathrm{x}).\left(\mathrm{x}-\bar{\mathrm{x}}_{\mathrm{q,t}}\right)^2},$$
$$\text{where } \bar{\mathrm{x}}_{\mathrm{q,t}} = \sum_{\mathrm{x}=1}^{\infty} \mathrm{Z}_{\mathrm{q,t}}(\mathrm{x}).\mathrm{x} \qquad (34)$$

In order to gain insight in the variability during a day we measure the coefficient of variance of a day given by $cv_{q,x}$. We calculate $cv_{q,x}$ with $\bar{x}_{q,t}$ as the average demand for time slot t as follows:

$$cv_{q,x} = \frac{S_{q,x}}{\bar{x}_q} \qquad with\ S_{q,x} = \sqrt{\frac{1}{T-1}\sum_{i=0}^{T-1}\left(\bar{x}_{q,t}-\bar{x}_q\right)^2}$$
$$\bar{x}_q = \frac{\sum_{t=0}^{T-1}\bar{x}_{q,t}}{T} \qquad (35)$$

The coefficient of variance of a ward for a period containing Q days with $\bar{x}_j$ as the average demand on day j is determined as follows:

$$cv_x = \frac{S_x}{\bar{x}} \qquad with\ S_x = \sqrt{\frac{1}{Q-1}\sum_{j=1}^{Q}\left(\bar{x}_j - \bar{x}\right)^2} \qquad (36)$$
$$\bar{x} = \frac{\sum_{i=1}^{Q} \bar{x}_q}{Q}$$

4.6.4 Misplacement rate

Misplacements are patients that are assigned to a ward that is not the dedicated ward of this patient type. Misplacements occur when the dedicated ward does not have capacity available to admit the new patient, and other wards admit the patient. The misplacement rate is the probability of an arriving patient to be misplaced. In case of an analysis of a single ward there is no information about the demand in other wards of the hospital. Therefore it is not possible to determine the misplacement rate. When the demand of the ward exceeds the bed capacity, all demand beyond the capacity is included in the rejection rate described in section 4.6.5. The approach we describe is for analysis of two wards. First we will describe how to determine the misplacement rate for the elective patients and then for the non-elective patients because it slightly differs. We describe the misplacement rate for two wards.

Elective patients

For the elective patients we determine the misplacement rate per MSS block. Patient types that are dedicated to both wards are not included in the misplacements, because assigning the patient to the other ward is not a misplacement of this patient. Because misplacements only occur for patient types with one dedicated ward, we only need to describe the probability of patients misplaced from the dedicated ward to the other ward, where some patient types have ward 1 as dedicated ward and some patient types have ward 2 as the dedicated ward. First we determine the probability distribution for the number of arrivals, with $\hat{a}^j_{n,t}(k)$ as the probability of k arrivals of type j at the start of time slot t of day n, as follows:

$$\hat{a}^j_{n,t}(k|y) = \binom{y}{k}\left(g^j_n w^j_{n,t}\right)^k \left(1 - w^j_{n,t} g^j_n\right)^{y-k} \qquad (37)$$

$$\hat{a}^j_{n,t}(k) = \sum_{y=k}^{C^j} \hat{a}^j_{n,t}(k|y)\, c^j(y) \qquad (38)$$

By relating the patient type j to an OR block $b_{i,p}$ ($p \in \{1, \dots Q\}$) of the MSS we determine the arrivals of this block for a certain time slot t of day q as follows:

$$\hat{a}^{i,p,j}_{q,t}(k) = \begin{cases} \hat{a}^j_{-1,t} & for\ p > 1, q = p-1 \\ \hat{a}^j_{-1,t} & for\ p = 1, q = Q \\ \hat{a}^j_{0,t} & for\ q = p \\ 0 & elsewhere \end{cases} \qquad (39)$$

With the arrivals following from the single blocks of the MSS we can determine the total arrivals of patient type j for each time slot t of day q using convolutions:

$$\hat{a}^{j}_{q,t} = \hat{a}^{0,q,j}_{q,t} * \hat{a}^{1,q,j}_{q,t} * \dots * \hat{a}^{I,q,j}_{q,t} \hat{a}^{0,q+1,j}_{q,t} * \dots * \hat{a}^{I,Q,j}_{q,t} \tag{40}$$

We now use $\hat{a}^{j}_{q,t}$ to determine $M^{j}_{q,t}$, the probability of a patient to be misplaced when arriving at the start of time slot t of day q. In order to determine $M^{j}_{q,t}$ we first determine the misplacement probabilities given a combination of k, x_1, x_2 defined as $M^{j}_{q,t}(s|k, x_1, x_2)$, with s as the probability of a patient to be misplaced. We determine $M^{j}_{q,t}(s|k, x_1, x_2)$ as follows:

$$M^{j}_{q,t}(s|k, x_1, x_2) = \begin{cases} 0 & for\; x_1 \leq X_1 - k \\ \dfrac{mp}{k} & for\; x_1 > X_1 - k, x_2 < X_2, mp \leq \Delta x_2 \\ \dfrac{\Delta x_2}{k} & for\; x_1 > X_1 - k, x_2 < X_2, mp > \Delta x_2 \\ 0 & for\; x_1 > X_1 - k, x_2 = X_2 \end{cases} \tag{41}$$
$$with\colon mp = \min\{x_1 + k - X_1, k\}, \Delta x_2 = X_2 - x_2$$

Given the values for all $M^{j}_{q,t}(s|k, x_1, x_2)$ we now can determine $M^{j}_{q,t}$ from the bed censuses $\bar{Z}^{k}_{q,t-1}(x_k)$ derived in section 4.5.2. Because the bed census of a time slot includes the admissions of this time slot, we take the bed census of the previous time slot. Because this bed census includes the patients that are discharged before the start of the time slot our misplacement rate can be higher than the actual misplacement rate. We determine the misplacement rate per time slot as follows:

$$M^{j}_{q,t} = \sum_{k=0}^{C^j} \sum_{x_1=0}^{X_1} \sum_{x_2=0}^{X_2-1} M^{j}_{q,t}(s|k, x_1, x_2) \hat{a}^{j}_{q,t}(k) \bar{Z}^{1}_{q,t-1}(x_1) \bar{Z}^{2}_{q,t-1}(x_2) \tag{42}$$

We introduce $\bar{a}^{j}_{q,t}$ as the expected number of admissions of type j in time slot t of day q, with $\bar{a}^{j}_{q,t} = \sum_{x=0}^{C^j} x.\hat{a}^{j}_{n,t}(x)$. Given $\bar{a}^{j}_{q,t}$ and $M^{j,m}_{q,t}$ we can determine the average misplacement rate of type j on day q, $M^{j,m}_{q}$, and the average misplacement rate for type j for the time period, M^{j}.

$$M^{j}_{q} = \frac{\sum_{t=0}^{T-1} M^{j}_{q,t} \bar{a}^{j}_{q,t}}{\sum_{t=0}^{T-1} \bar{a}^{j}_{q,t}}$$
$$M^{j} = \frac{\sum_{q=1}^{Q} M^{j}_{q} \bar{a}^{j}_{q}}{\sum_{q=1}^{Q} \bar{a}^{j}_{q}} \tag{43}$$

Non-elective patients

For the non-elective patients we can use a similar approach. Because the admission process of the non-elective patients differs from the admission process of the elective patients $\hat{a}^{i,r}_{r,t}(k)$ is also determined differently. Because the ward will not have the same demand probability at arrival at the ward, we determine $\hat{a}_{r,t}^{i,r}(k)$, the non-elective patient arrival probability of type i on day r, for every week which can be determined as follows:

$$\hat{a}_{q,t}^{i,r}(k) = \frac{(\lambda_{r,t}^{i})^k e^{-\lambda_{r,t}^{i}}}{k!} \quad for\ q = r(mod\ R) \tag{44}$$

The other steps are similar to the steps taken for the elective patients, which results in an hourly misplacement rate per type $\widetilde{M}_{q,t}^{i,r}$, a daily misplacement rate per type $\widetilde{M}_{q}^{i,r}$ and a misplacement rate for the period for per type $\widetilde{M}^{i,r}$.

Ward

From the elective patients misplacement rate and the non-elective patients misplacement rate of the ward we can determine the total misplacement rate for the ward. The total average arrivals consists of all elective patient types j and all non-elective patient consisting of the combination of type i and arrival day of the week r. For the non-elective patients we have to We determine the misplacement rate of the ward as follows:

$$M = \frac{\sum_{j=0}^{J-1} M^j\, \bar{a}^j + \sum_{i=0}^{I-1}\sum_{r=1}^{R} \widetilde{M}^{i,r}\, \bar{a}^{i,r}}{\sum_{j=0}^{J-1} \bar{a}^j + \sum_{i=0}^{I-1}\sum_{r=1}^{R} \bar{a}^{i,r}} \tag{45}$$

4.6.5 Rejection rate

The rejection rate is the probability of an arriving patient to be rejected. Rejections are patients that cannot be admitted to the dedicated ward of this patient type nor one of the other wards. These patients are not admitted to the hospital and therefore are rejected. Rejections occur when both the dedicated ward and the other wards have reached their bed capacity. The rejection rates of elective patients is described by $R_{q,t}^{j}$ and the rejection rate of non-elective patients is described by $\tilde{R}_{r,t}^{i,r}$. We first describe the approach for a single ward analysis and then we describe the approach for a two wards analysis.

Single ward analysis

For a single ward patients are rejected when the bed capacity is reached at time slot of admission. We first determine the rejection rate for a time slot in order to determine the average rejection rate per day and the average rejection rate per time period. The rejection rate for a time slot is derived from $R_{q,t}^{j}(s|k,x)$ which is the rejection probability s for a given combination of k arrivals and x patients present in the ward. We give Δx as the number of available beds, with $\Delta x = X - x$. We determine $R_{q,t}^{j}(s|k,x)$ as follows:

$$R_{q,t}^{j}(s|k,x) = \begin{cases} 0 & for\ \Delta x \geq k \\ \dfrac{x+k-X}{k} & for\ 0 < \Delta x < k \\ 1 & for\ x = X \end{cases} \tag{46}$$

Given the values for all $R_{q,t}^{i,m}(s|k,x)$ we now can determine $R_{q,t}^{i,m}$. As explained in section 4.6.4 we take the bed census of the previous time slot to determine the rejection rate of a time slot. We determine the rejection rate per time slot as follows:

$$R_{q,t}^{i,m} = \sum_{k=0}^{C^j} \sum_{x=0}^{X} R_{q,t}^{i,m}(s|k,x) \hat{a}_{q,t}^{i,m}(k) \bar{Z}_{q,t-1}(x) \tag{47}$$

With the average arrivals $\bar{a}_{q,t}^{j}$ and $\bar{a}_{q}^{j}$ we can determine the average rejection rate of type j on day q, $R_q^{j,m}$, and the average rejection rate for type j for the time period, R^j.

$$R_q^j = \frac{\sum_{t=0}^{T-1} R_{q,t}^j \bar{a}_{q,t}^j}{\sum_{t=0}^{T-1} \bar{a}_{q,t}^j}$$

$$R^j = \frac{\sum_{q=1}^{Q} R_q^j \bar{a}_q^j}{\sum_{q=1}^{Q} \bar{a}_q^j} \tag{48}$$

With the same approach we can determine the rejection rates of the non-elective patients $\tilde{R}_{q,t}^{i,r}$, $\tilde{R}_q^{i,r}$ and $\tilde{R}^{i,r}$. From the elective patients rejection rate and the non-elective patients rejection rate we can determine the rejection rate for the ward. We determine the rejection rate of the ward as follows:

$$R = \frac{\sum_{j=0}^{J} R^j \, \bar{a}^j + \sum_{i=0}^{I} \sum_{r=1}^{R} \tilde{R}^{i,r} \, \bar{a}^{i,r}}{\sum_{j=0}^{J-1} \bar{a}^j + \sum_{i=0}^{I-1} \sum_{r=1}^{R} \bar{a}^{i,r}} \tag{49}$$

Two wards analysis

For a two wards analysis the formula for the rejection rate for a combination of bed census in the ward and the number of arrivals is different. Because now we have two wards we have to consider the bed census of both wards. We determine the rejection probability given a combination of k, x_1, x_2, $R_{q,t}^j(s|k, x_1, x_2)$, as follows:

$$R_{q,t}^j(s|k,x_1,x_2) = \begin{cases} 0 & for\ k \leq \Delta x_1 + \Delta x_2 \\ \dfrac{(x_1 + x_2 + k) - (X_1 + X_2)}{k} & for\ 0 < \Delta x_1 + \Delta x_2 < k \\ 1 & for\ x_1 = X_1, x_2 = X_2 \end{cases} \tag{50}$$

Given the values for all $R_{q,t}^j(s|k, x_1, x_2)$ we now can determine $R_{q,t}^j$ as follows:

$$R_{q,t}^j = \sum_{k=0}^{C^j} \sum_{x_1=0}^{X_1} \sum_{x_2=0}^{X_2} R_{q,t}^j(s|k,x_1,x_2) \hat{a}_{q,t}^j(k) \bar{Z}_{q,t-1}^1(x_1) \bar{Z}_{q,t-1}^2(x_2)\, k \tag{51}$$

The rejection rate per type per day and per period are similar as for the single ward analysis and are determined with formula (48). The rejection rate of the ward is determined with formula (49).

5 Proof of concept

This chapter contains the application of the model to the case study of the nursing wards of G6. In order to test the correctness of the conceptual model we have developed a programming model. We have applied this model to the case study. The model is developed in programming software of Delphi Embarcadero. In Section 5.1 we describes how we have verified and validated the model. In section 5.2 the input and output of the model are given in order to discuss interesting interventions in section 5.3. Section 5.4 contains the analysis of the scenarios we have run.

5.1 Verification and validation

First we have verified the model in order to assure that the model is correct and matches the mathematical model we have formulated in chapter 4. We have verified the model on two aspects: The single surgery block distributions and the convolutions.

5.1.1 Single surgery block

The first approach is used to verify the correctness of the calculations for the single surgery blocks for the elective patients and the single weekday block for the non-elective patients. For both patient types we have made example calculations in Microsoft Office Excel. The results for several example calculations where equal to the results of the Delphi model. This gives us an indication that the programming is according to the model.

5.1.2 Convolutions

To verify the convolutions for the steady state bed censuses we have used a computer program that predicts the daily bed census according to the model described by Vanberkel (2011). This program is developed in Visual Basic. As described earlier, we have used a similar approach as Vanberkel to determine the steady state. Although this model is calculating the bed census not on hourly basis but on daily basis, this will not affect the approach for the convolutions. We have compared the output of both the Vanberkel model and our Delphi model and they were equal.

5.1.3 Validation

For validation of the model we use the case study of G6. For both G6NO and G6ZU we have compared the results of the model with the actual realization. As a time period for both the realization and the model run we have used the year 2009. We have run the data for the full period and have neglected the first 8 weeks of the period which we use as a warm-up period. This corresponds with the maximum length of stay of a patient for our model (50 days after surgery). We also do not take into account the last three weeks of the period, because this is the period of Christmas. In total we have a period of 40 weeks (280 days). The input data for the distributions is taken from the whole year of 2009. The input data is derived from 3 different databases: 'OK Plus' containing OR data, 'LOCATI' which contains data from nursing wards, and the OR planning from the planning bureau. For both the model and the realization we only incorporate patients admitted in 2009. Patients admitted in 2008 with a stay in the ward in 2009 are taken out. Appendix D describes the input distributions derived from the data.

We compare the results of the model with the actual realization on basis of the performance indicators described in section 4.5. Table 9 shows the performance indicators of the realization and the model. The bed occupancy of the model for both wards is based on 24 operational beds. The average demand of the model is based on the maximum number of patients, which is affected by the maximum number of patients per surgery block/arrival day. We see that for both the bed occupancy and the average demand the model scores slightly lower than the realization. The CV of the model is based on the maximum number of patients. Each number of patients has a (small) probability of occurrence, which causes a larger CV for the model than for the realization. The table also shows the demand percentile. To determine the demand percentile for the realization we have compared for each time slot the demand percentile of the model with the actual realization of this time slot. The demand percentile in the table shows the percentage of time the realization of a time slot is equal or lower than the demand percentile of the model. For example for a certain time slot the 90% demand percentile of the model is 22 beds and the realization is also 22 beds. This time slot is counted true in the demand percentile. In the case the 90% demand percentile of the model is 22 beds and the realization is 23 beds the time slot is counted false in the demand percentile of the model.

	G6NO		G6ZU	
	Realisation	Model	Realisation	Model
Bed occupancy	0,76	0,75	0,73	0,72
Average demand	18,34	18,32	17,55	17,47
CV	0,08	0,22	0,10	0,22
Comparence demand percentile	85,32%	80%	85,15%	80%
	91,84%	85%	91,88%	85%
	96,67%	90%	96,88%	90%

Table 9: Performance indicators G6NO & G6ZU, 2009

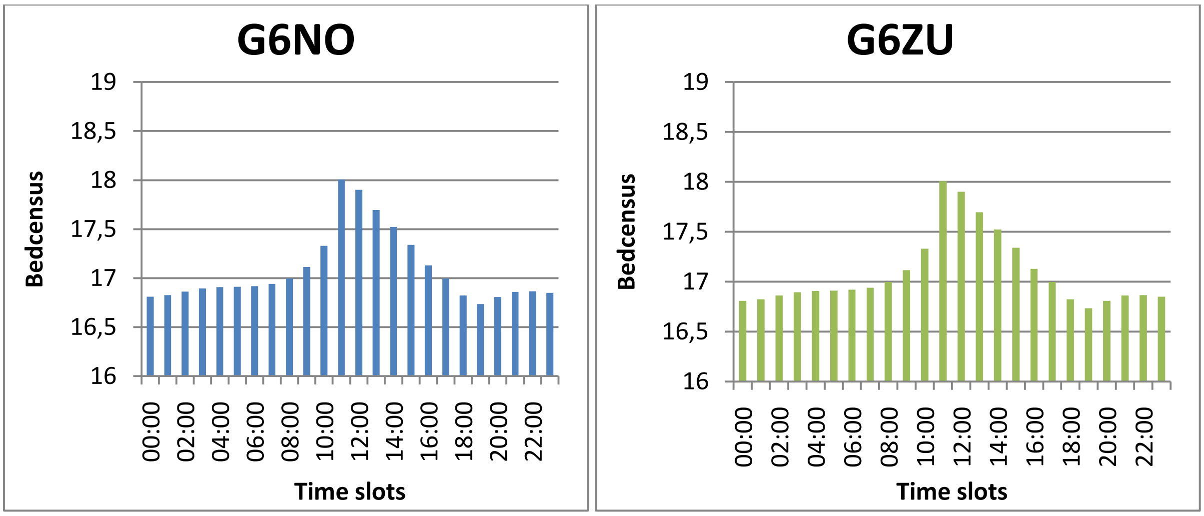

Figure 14: Average demand per time slot, Cognos 2010

Figure 14 shows the average demand in the ward per time slot. We see a peak demand during the beginning of the afternoon. The beginning of the evening also shows a small increase in demand. Figure 15 shows a similar pattern from the output of the model.

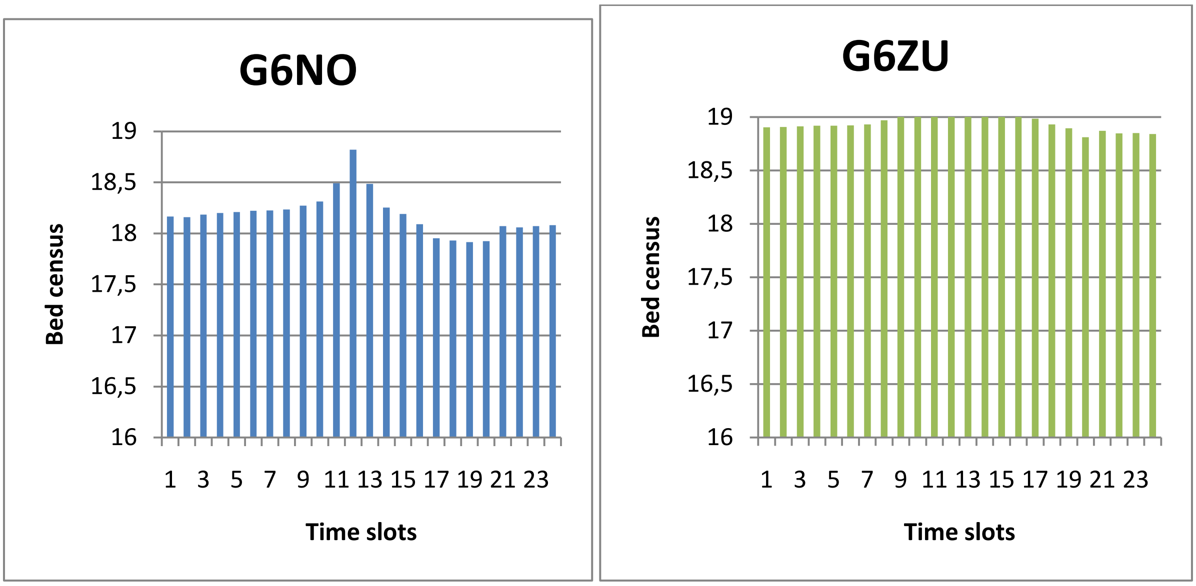

Figure 15: Demand for beds, model

5.2 Input & output model

The model we have developed gives several possibilities for running different scenarios. Due to the generic character of the model the major part of the scenarios can be run without adjusting the model itself, only by adjusting the input data. Therefore we first give a short overview of the input and output of the model in order to give insight in the opportunities in adjusting the input data.

Input model		
General settings	***Elective patient types***	***Non-Elective patient types***
- Period length, EL(Q),Non-El(R)	- Dedicated wards (w)	- Dedicated wards (w)
- Nr of time slots (T)	- Nr surgeries in block (c^j)	- Arrival day (r)
- Number of wards	- Admission day (g_n^j)	- Avg. admission ($\lambda_r^i v_{n,t}^j$)
- Master Surgery Schedule (patient type j)	- Admission time slot ($w_{n,t}^j$)	- Length of stay dist (P^j)
	- Length of stay dist. ($P^j(n)$)	- Discharge time slot ()
	- Discharge time slot ($z_{n,t}^j$)	-

Output model
- Ward demand per time slot ($Z_{q,t}$)
- Elective demand per time slot ($H_{q,t}$)
- Non-elective demand per time slot ($G_{r,t}$)
- Demand percentiles ($D_{\alpha,q,t}(x)$)
- Average operational bed occupancy (B^{occ})
- Coefficient of variation (cv_x)
- Rejection rate (R)
- Misplacement rate (M)

5.3 Interventions

In this section we describe interventions that can be run with the model. The described interventions are applied to the case study. For other wards or hospitals these interventions can also be interesting. For both

Adjust operational capacity

Test the influence of the number of beds on a ward on the performance indicators. Which number of beds on a ward satisfies the set criteria?

Combine G6NO and G6ZU to G6

AMC is planning to combine the chirurgical wards G6NO and G6ZU to one ward, G6. What is the impact on the performance? Which number of beds satisfies?

Admit elective patients in afternoon

Elective patients are admitted in the morning. Data analysis shows a peak demand around noon. Will changing the admission time of elective patients to the afternoon level the demand?

Discharge patients in morning

Will patient discharges in the morning level the demand?

Increase in emergency patients

The AMC is planning to open a large emergency center in order to become the regional emergency center. What is the impact on the ward if the number of emergency patients increases?

Higher utilization MSS

What is the effect of an increase in the utilization of the ORs?

Shorter LOS

Currently there are a lot of studies on decreasing the LOS of patients on nursing wards. What is the impact of a shorter LOS on the bed census of the ward?

5.4 Results

We have run the intervention 'Adjust operational capacity' described in section 5.3. The main performance indicator we test this. In total we have run 16 scenarios in which we have varied the number of beds on the ward. We have a scenario for each combination of beds with a maximum of 24 beds (which is the current operational capacity) and a minimum of 21 bed per ward. We have set 21 as the minimum because the average 90% demand percentile for G6ZU in the current situation is 21.06 beds (G6NO is 22.51 beds). The input for the number of beds is shown in Table 10. The other input variables and parameters were set equal for all scenarios.

Scenario	***1***	***2***	***3***	***4***	***5***	***6***	***7***	***8***	***9***	***10***	***11***	***12***	***13***	***14***	***15***	***16***
G6NO	24	24	24	24	23	23	23	23	22	22	22	22	21	21	21	21
G6ZU	24	23	22	21	24	23	22	21	24	23	22	21	24	23	22	21
Total	48	47	46	45	47	46	45	44	46	45	44	43	45	44	43	42

Table 10: Number of beds per scenario

As a main criteria to judge the performance of the scenario we use the rejection rate. The maximum rejection rate for AMC is set to 0.01 by the board of directors. The results of the scenarios are shown in Table 11. This table shows for each total number of beds the scenario that is performing best on the rejection rate.

Nr of beds	Scenario	Occupancy	Misplacement rate	Rejection rate
48	1	0.73	0.004	0.003
47	2	0.75	0.005	0.004
46	3	0.77	0.006	0.006
45	7	0.78	0.006	0.008
44	11	0.80	0.007	0.011
43	12	0.81	0.007	0.015
42	16	0.82	0.008	0.19

Table 11: Results scenarios

The misplacement rate of all scenarios is very low. This is caused by the fact that the patient mix of the wards contains a lot of patients types that are dedicated to both ward. These patient types will not be misplaced. For analysis we take out the scenarios that have a rejection rate higher than the 0.01 set by the board of directors. Therefore the minimum number of beds required is 45 beds. This means a reduction of 3 beds compared to the current situation.

Scenario	G6NO	Scenario	Occupancy	Misplacement rate	Rejection rate
4	24	21	0.78	0.007	0.008
7	23	22	0.78	0.006	0.008
10	22	23	0.78	0.006	0.008
13	21	24	0.78	0.006	0.009

Table 12: Performance indicators scenarios with 45 beds

Table 12 shows the results on the performance indicators of the scenarios with in total 45 beds. The occupancy level is 0.78 for all scenarios. The misplacement rate is lowest for scenario 7 and 10. These are also the scenarios were the number of beds are equally divided among the wards.

We advice to set the bed capacity of G6NO to 23 and of G6ZU to 22 or the bed capacity of G6NO to 22 and G6ZU to 23. This results in a decrease in the required total bed capacity of 3 beds and an increase in bed occupancy of 5%.

6 Conclusion and recommendations

This chapter discusses the conclusion (section 6.1) and the recommendations (section). Further we propose points for further research (section 6.4). The goal of the study was to develop a decision support tool to determine the hourly demand for beds in a nursing ward. The study is performed in AMC Amsterdam.

6.1 Conclusion

First we made a context analysis in order to gain insight in the process of the nursing ward in general and the bed management process in particular. While the admission planning of elective patients to the wards takes place in 3 steps, the actual decision to admit the patient to the ward is taken in the morning of the day of arrival. This decision is based on the current bed census.

We have developed a model in order to support the bed capacity management and nurse staffing. The model is based on the three-step approach described by Vanberkel et al. (2011). This approach relates recovering patients on the nursing ward to the Master Surgery Schedule and determines the demand for beds on a daily basis. We have developed a model that takes into account both elective and non-elective patients on an hourly basis. For the elective patients the model incorporates the patients' stay from admission the day before/of surgery until the discharge during the days after surgery. The arrival of non-elective patients is unexpected and therefore we include them with a weekday specific Poisson arrival process. The output of the model is the expected bed demand for a certain period in the steady state.

With the model one can run scenarios and analyze the outcome. To measure the outcome we have included the following performance indicators:

- *Demand percentiles*: Minimum number of beds required to fulfill the demand according to an α value.
- *Average operational bed occupancy*: The number of bed hours used in relation to the total number of available bed hours.
- *Variability in demand*: The variation in demand for beds during a time period.
- *Misplacement rate*: Probability for a patient to be misplaced at another ward.
- *Rejection rate:* Probability that the admission request of a patient is rejected.

6.2 Results

We have analyzed the influence of adjusting the bed capacity of nursing wards G6NO and G6ZU on the performance on the performance indicators. The criteria set by the board of directors of AMC is a maximum rejection rate of 0.01. The scenario with 23 beds at G6NO and 22 beds at G6ZU and the scenario with 22 beds at G6NO and 23 beds at G6ZU perform best. Implementing these scenarios will lead to an decrease in bed capacity of 3 beds and an increase of bed occupancy of 5%.

6.3 Recommendations

The recommendations are given below:

- Use the model on strategic and tactical level to support decision making.
- Set the bed capacity of G6NO to 23 beds and G6ZU to 22 beds or G6NO to 22 beds and G6ZU to 23 beds.

- Integrate databases in order to make it easier to subtract data.
- Create a cyclic MSS in order to level the number of beds required and to reach a steady state.
- Improve the cooperation in planning of both beds and ORs.
- Better organize planning and storage of data to provide a more detailed MSS. This would probably improve the accurateness of the model output.

6.4 Further research

For further research we suggest a few topics to extend the described model:

- Include the patients' absence to the ICU in the model.
- Include discharges during day of admission in order to include one day stay.
- Adjust the model in order to make it applicable to the operational level. With a model that incorporates data of actual patient arrivals, and not distributions taken from historical data, the model can be used for the on-line admission planning described in section 2.2.1.

Appendix A: Explanation notation of mathematical model

Indices

i	Operating room	$i \in \{1,2,\dots,I\}$
j	Patient type	$j \in \{1,2,\dots,J\}$
m	Day number in timeline	$m \in \{0,1,\dots Q,Q+1,Q+2,\dots.\}$
n	Day after surgery	$n \in \{-1,0,\dots,L^j;\forall j\}$
q	Day number in time cycle	$q \in \{1,2,\dots,Q\}$
SS	Steady state	
t	Time slot of a day	$t \in \{0,1,\dots,T-1\}$

Variables

x	Number of patients present in ward
y	Total number of patient admissions this day

Parameters

C^j	Max number of surgeries of type j in a surgery block
I	Number of operating rooms
L^j	Max length of stay of patient of type j
M	Number of days in the time line
Q	Number of days in the time cycle
T	Number of time slots in a day
$b_{i,q}$	Surgery block in operating room i on day q

Appendix B: Three step approach Vanberkel (2011)

Step 1 *Input*:

$c^j(x)$: probability distribution of specialty j completing x surgeries in one operating room block
d^j_n: is the probability that a patient, who is still in the ward on day n, is to be discharged that day

Output:

$h^j_n(x)$: probability distribution of x patients of a single operating room block of specialty j still in recovery n days after surgery

$$h^j_n(x) = \begin{cases} c^j(x) & \text{when } n = 0 \\ \sum_{k=x}^{C^j} \binom{k}{x} (d^j_{n-1})^{k-x}(1 - d^j_{n-1})^x h^j_{n-1}(k) & \text{otherwise.} \end{cases} \tag{2}$$

Step 2 *Input*:

$h^j_n(x)$: From Step 1
A Single MSS: Defines which day q specialty j operates in operating room I

Output:

$\bar{h}^{i,q}_m(x)$: probability distribution of x patients of the MSS still in recovery on day m.

$$\bar{h}^{i,q}_m = \begin{cases} h^j_{m-q} & \text{if } q \leqslant m < L^j + q, \\ \mathbf{0} & \text{otherwise} \quad \text{where } \mathbf{0}\text{, means } \bar{h}^{i,q}_m(0) = 1 \end{cases} \tag{3}$$

$H_m(x)$: probability distribution of x patients still in recovery on day m (result for a single MSS in isolation). Let * indicate a convolution then,

$$H_m(x) = \bar{h}^{1,1}_m * \bar{h}^{1,2}_m * \ldots * \bar{h}^{1,Q}_m * \bar{h}^{2,1}_m * \ldots * \bar{h}^{I,Q}_m \tag{4}$$

Step 3 *Input*:

$H_m(x)$: From Step 2
Repeating MSSs

Output:

$H^{SS}_q(x)$: the steady-state probability distribution of recovering patients on day q.

$$Hq^{SS}(x) = H_q * H_{q+Q} * H_{q+2Q} \ldots * H_{q+yQ} \tag{5}$$

Appendix C: Input data programming model

As input data for the validation and experimentation we use empirical data from the AMC. The data is derived from three different databases: OK Plus, LOCATI and the server of the planning bureau. We have integrated the data in order to derive the required distributions. The patient types we use as input for the model have ward specific characteristics. This because the ward can influence the length of stay of the patient. As in the model we separately build the data of the elective and the non-elective patients.

Elective patient distributions

Elective patients are resulting from a surgery block of the MSS. In the AMC the surgery blocks of General Surgery are sub divided in blocks of a subspecialty (section 2.3.2). Therefore we define the type of patient on the subspecialties of General Surgery. Because also half OR days occur, for some of the subspecialty we specified full and half OR days. Figure 16 shows the chirurgical surgery blocks we use for our model. As described in section 2.3.2 consists the MSS of the Chirurgical division of more than the presented blocks. We have only defined patient types for surgery blocks that contain patients who are related to G6. Sometimes also a patient from another type of block is assigned to G6, a misplaced patient, these patients are accounted for in the non-elective patient groups. This because these patients are not planned for G6, but are assigned to G6 due to a lack of available capacity on the dedicated ward. The patients normally are directed to the dedicated ward in the days after surgery.

In section 4.5.5 we describe the half OR days. For the G6 case study we do not differentiate in morning and afternoon OR block. This because about 45% of the half OR blocks has surgeries of different subspecialties taking place on the same half of the day, or two surgeries from the same subspecialty not taking place in the same half of the day. The data of patients related to a surgery performed in an OR block that is not corresponding with their subspecialty are added to the distribution of this OR block, while this is not their subspecialty. This because this happens only occasionally and we take the MSS blocks as a basis. Defining extra patient types will unnecessarily increases the complexity of the model, because this only occurs occasionally which will result in marginal number.

Type nr.	Subspecialty	Type nr.	Subspecialty
0	ADB infection	8	HPB
1	ADB infection half	9	HPB half
2	General	10	Mamma
3	General half	11	Mamma half
4	Colon	12	Oesophagus
5	Colon half	13	Oesophagus half
6	Endocrine	14	Vascular
7	Endocrine half		

Figure 16: Chirurgical surgery blocks

Number of surgeries (c^j)

The surgery probabilities (c^j) are estimated from OK plus. The database contains information about the urgency of the surgery in which it distinct in four groups: elective, urgent/semi urgent, planned after week planning and extra. For our elective patients distributions we have taken into account all groups except the urgent/semi urgent. This because these groups contain elective or semi-elective surgeries, who are planned on forehand. We derive the surgery probabilities per patient type from all surgery blocks of this patient type that have occurred during the period. An example of an overview of the number of occurrences for the patient types ‘Colon’ and ‘Colon half’ is shown in Figure 17. The frequency are converted in the surgery probability distribution, which is the input of the model. Because not always all patients of a block are directed to the same ward there is also a probability of zero patients resulting from this surgery block.

Subspecialty	Nr. Surgeries	Frequency	Prob. Surgery
Colon	0	18	0,075
	1	98	0,407
	2	115	0,477
	3	10	0,041
	Total:	241	1
Colon half	0	34	0,442
	1	31	0,403
	2	12	0,156
	Total:	77	1

Figure 17: Example surgery probability

Admission day (g_n^j)

The model distinguishes in admissions the day of surgery and the day before surgery. The admission probability for both days is given in Figure 18. The high probability of a patient to be admitted the day before surgery is normal for the patient types of G6 and is described in section 2.1.2. This due to the complexity and the impact of the surgery, described in The data study showed that approximately 5% of the elective patients are admitted a more than one day before surgery. According to the nurse managers of G6 these are either patients from other hospitals, or patients whose surgery is shifted a (few) day(s). Due to complexity reasons we have set this admission day to day before surgery and included the extra day as non-elective patient stay. The pre-operative patients stay on Friday is also included in the non-elective patients distribution. This due to the complexity of the adjustments to the programming model and the necessary extra parameter in the input data. We recommend to incorporate this in the model as described in section 4.5.4.

			Time Slot																							
Patient type	Adm. Day	Adm. Prob	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
Colon	-1	0,984	0	0	0	0	0	0	0	0	0	0,021	0,104	0,625	0,083	0	0,021	0,042	0	0	0	0	0,042	0	0	0
	0	0,016	0	0	0	0	0	0	0	0,2	0,2	0,6	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 18: Example admission day and time slot distribution

Admission time slot ($w^j_{n,t}$)

Figure 18 shows an example of the admission time slot distribution. The distribution is convenient with the regular admissions described in section 2.1.2. The day before surgery the patients are admitted at the end of the morning and admissions during the day of surgery take place in the early morning.

Length of stay distribution ($P^j(n)$)

The LOS distribution is given in days, with the day of surgery as a starting point. It is calculated as the difference between the day of surgery and the day of discharge. For example a patient that arrives on Monday 01.03.2010,went for surgery on Tuesday 02.03.2010 and is discharged the next week on Tuesday 09.03.2010 has a LOS of 7 days (9-2). The data set contained patients with a LOS equal to zero. These are patients that are discharged from the nursing ward during the day of surgery. Because the model does not allows discharges during the day of surgery, we count these discharges in the first time slot of the day after surgery. Therefore we have set the LOS of these patients to 1 day and the time slot of discharge to 0. We recommend to extend the model with discharges during the day of surgery. This to also include the absence from the ward during ICU stay and day admissions. The model requires a finite LOS in order to ensure convergence to a steady state. We have set a max LOS of 50 days for the patient types with a data set including patients with a LOS longer than 50 days. In total 6 patients had a LOS larger than 50 in 2010. We have included the additional LOS in the non-elective patients data. The presence of these patients, with sometimes a LOS longer than 100 days, is difficult to include in the model and can lead to an underestimation of the demand. Because the model is based on the administrative bed census (section 2.2.2) the patients bed stays reserved for a maximum of 5 days starting the day the patients is leaving the ward. We have set the discharge of these patients to the last time slot of day 5 after the last stay on the nursing ward. Possible return on the nursing ward after the 5 days period is included in the non-elective patients.

Discharge time slot ($m^j_{n,t}$)

The discharge time slot is set according the frequency of occurrences. Adjustments to the discharge time slot are described in the part about the LOS distribution.

Non-elective patients

Characteristics to base the clustering of non-elective patient on are the day of admission, admission process, LOS, origin and specialty. We have selected the day of admission and the specialty to cluster the non-elective patients. The reasoning for clustering on basis of day of admission is described in section 4.3.1. Table 13 shows the number of non-elective patients per specialty and their average LOS and variance in LOS. The average LOS and the LOS variance of General Surgery diverge from the others. Therefore we separate this from the other specialties. For both groups we create a patient type for each day of the week, resulting in 14 patient types. We have discussed with the head nurses of G6 about whether there is a criteria to split the large group of non-elective arrival. According to the head nurses the stay of the non-electives is hard to predict on forehand and is mainly determined by the occurring or absence of complications. Another patient type is the pre-operational patients that are admitted on Friday, as described in “Admission day” earlier in this section. In total we have 15 non-elective patient types.

G6NO	Num. patients	Avg LOS	Var of LOS
CHI	255	7,643	125,679
INT	9	1,111	0,611
ORT	4	1,25	0,917
VAA	4	1,5	3
MDL	3	1,333	0,333
GYN	1	1	-
CHP	1	0	-
LON	1	1	-
MZK	1	2	-
NEU	1	0	-
TRA	1	3	-
Tot. other	26		

G6ZU	Num. patients	Avg. LOS	Var of LOS
CHI	262	7,954	179,653
MZK	63	3,778	18,208
INT	11	4,909	10,491
MDL	2	7,5	0,5
KNO	1	2	-
LON	1	6	-
ORT	1	6	-
OTH	1	17	-
TRA	1	6	-
URO	1	6	-
Tot. other	82		

Table 13: Avg. LOS non-elective patient types, 2010

Average number of admissions per time slot ($\lambda_{r,t}^{i}$)

The average number of admissions is the number of occurrences during the year and the number of weekdays (for instance Monday) in a year. Note that the average number of admissions do not sum to 1 as the admission time slot distribution of the elective patients does.

Length of stay distribution ($P^{i,r}$)

The length of stay distribution of the non-elective patients is determined similar as the LOS distribution of the elective patients. Note that we determine a LOS distribution for every admission day to incorporated the influences of the admission day on the LOS. For the non-elective patients we have also set the max LOS equal to 50.

Discharge time slot ($\widetilde{m}_{n,t}^{i,r}$)

The discharge time slots of the non-elective patients are determined similar as the discharge time slots of the elective patients described above.

Appendix D: References

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