THE FOSTER AND HART MEASURE OF RISKINESS: PRACTICAL IMPLICATIONS

Master Thesis
Financial Engineering and Management
Job Arnold

Supervisory board:
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UNIVERSITY OF TWENTE.

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Management Summary

Goal

The goal of this paper is to provide a bridge between recent theoretical developments in the field of risk measurement and daily practice. It contains an investigation of the newly proposed Foster & Hart measure of risk, an acclaimed objective –independent of the decision maker– measure that identifies the critical wealth level for an unknown sequence of gambles.

FH measure

This critical wealth level is the amount of capital below which it becomes risky to accept the gamble, if faced with an unknown sequence of this gamble over a long period of time. This critical wealth can be seen as the barrier level that anyone investing in a risky asset should hold in order to avoid bankruptcy in the long term. Investors are able to avoid this by changing the participation rate of the gamble at every time step, following the so-called simple shares strategy.

The riskiness of the gamble, $R(g)$, is uniquely determined by the equation $\mathrm{E}\left[log\ \left(1+\frac{1}{R(g)}g\right)\right]=0.$

In fact it is the number that offsets the relative gross returns of the gamble in the long run, implied by a yield factor of 1. See Appendix A for a brief explanation of the measure and the rationale behind it.

Comparative study

A comparison between the industry standard, Value-at-Risk (VaR), and the FH measure shows that both measures depend on the underlying distribution of the gamble. VaR however does not take into account the long-term effects of holding a certain capital buffer and does also depend on an arbitrary confidence level.

Main findings

The FH measure is computationally easy and the equation is solved by means of a Matlab algorithm. Several simulations with these capital buffers have been performed and show that the FH measure is applicable and is indeed the 'threshold wealth' below which it becomes risky to accept a gamble if played for a long period of time.

Some assumptions behind the FH measure are however questionable in real-life investment settings and require further analysis, for instance the limitations of the simple shares strategy, the behavior of the FH measure for distributions with (short-term) negative expectation and the role of transaction costs.

There is still much to investigate before the FH measure might play an active role in risk management and regulation, but it seems to be a concept to take into account.

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1. Introduction

This paper explores whether a newly proposed risk measure, the Foster & Hart measure of riskiness, might be applicable in practice within financial institutions and if so, to what extent. The Foster & Hart (2009) paper follows a paper by Aumann & Serrano (2008) and the dissemination is slow. This paper is an attempt to close the gap between theory and practice and to make the measure comprehensible for practitioners. In the light of recent developments in the financial system the rise of an acclaimed objective measure of riskiness, independent on the decision-maker, seems to be a great promise.

The first signals of an emerging crisis came at the end of 2006 when many American house-owners faced problems in repaying their mortgage loans (due to high interest rates and tolerant conditions at which those loans were provided). Increasingly many of them were forced to leave their homes (foreclosures) forcing house prices to decline (Crouhy et al., 2008; Kalse, 2008). The collateral underlying asset backed securities became of less value than the loan itself, forcing banks to write down on their assets. Assets write-downs and credit losses reached $232 billion in April 2008 at which point the International Monetary Fund (IMF) estimated that financial losses stemming from the mortgage crisis might cost nearly $1 trillion (NY Times, 2008). In July 2009 the IMF stated that the global credit crunch had cost governments over $10 trillion already (Schifferes, 2009).

With large capital injections and bailouts many governments and the IMF tried to save the financial system. Ultimately this led to a financial crisis also involving governments. Both the costs of the capital injections and the increased public and regulatory awareness of governmental debts led to major budget cuts; the financial crisis had reached the real economy. Credit rating agencies downgraded sovereign debts of many countries; on August 5 Standard & Poor's cut the U.S. sovereign debt rating for the first time in history (Tabuchi, 2011). It became clear that even traditional safe havens, as U.S. sovereign bonds are not 'safe'; in fact they hold some risk. During the crisis there were times (2008-2009, early August 2011) that the ten-year U.S. Treasury bond yield dropped below the S&P 500's dividend yield (Sommer, 2011).

Risk is not a new concept, as Luhmann (1996) points out: "In the Middle Ages the term risicum was used in highly specific contexts, above all sea trade and its ensuing legal problems of loss and damage." In the late 17th century the term 'risk' appeared in the English language, around the time that insurance on cargo and ships rose at Lloyd's in London. Chinese and Babylonian traders already practiced the principle of distributing (diversifying) risk in the third and second millennium before Christ. With the development of probability theory the notion of risk changed towards an anticipation of returns, financially elaborated by Knight (1921) who pointed out the difference between 'risk' and ‘uncertainty’ and Hicks (1939) who introduced the allowance for risk in ‘anticipated’ returns (Markowitz, 1952).

In 1974 the Basel Committee on Banking Supervision was formed in response to the liquidation of the Herstatt Bank. It has to date published three Accords, the first of which focused on credit risk and required all affiliated banks to hold a capital buffer of 8% of risk-weighted assets. Basel II (2004) also included financial risk and operational risk, the capital buffer was held at 8% (BIS, 2006/2009). Basel III is a response to the current financial crisis and an attempt to strengthen these capital buffers (EC, 2009). The Committee had introduced tier-1 capital, as the amount a bank needs to hold in order to ‘remain a going concern’. In practice, retained earnings and parts of common equity were used as tier-1 capital. Nowadays the Committee proposes three changes related to tier-1 capital: an increase in minimum level, a stricter definition of common equity and a change in risk-weighted assets (Moody’s, 2010).

The principles behind the measurement of risk remain however unchanged, which means that the capital requirements are still based on risk-weighted assets. Banks are allowed to use several approaches in calculating operational, credit and market risk. The most well known approach for calculating market risk is the Value-at-Risk (VaR). Concepts similar to VaR are used in calculating other risk types. From a mathematical perspective the criticism on VaR is that VaR is not subadditive as any coherent measure of risk should be (Artzner et al., 1999), a concept that will be discussed in Section 3. Practical remarks vary from giving false confidence to leading to excessive risk-taking (e.g., Taleb, 1997; Einhorn, 2008).

Robert Aumann, Nobel Laureate in Economics, has (among others) explored the field of risk and economics. With Roberto Serrano he published a paper proposing an economic index of riskiness (Aumann & Serrano, 2008). Dean Foster and Sergiu Hart (the latter receiving his Ph.D. under the supervision of Robert Aumann) argued that this index lacked operational interpretation, leading them to a different measure of riskiness (Foster & Hart, 2009). Although Homm & Pigorsch (2010) argue that the AS index can be assigned an operational interpretation we focus on the Foster & Hart operational measure of riskiness (FH measure).

The measure identifies for every ‘gamble’ the critical wealth level below which it becomes "risky" to accept the gamble (Hart, 2009). Since all portfolios and projects can be seen as a gamble the FH measure could be applicable in current risk management. Instead of VaR based regulatory capital the ‘critical wealth level’ could be the minimal barrier level of capital that institutions should hold. This is only possible if the FH measure is applicable in practice by both banks and regulators. The central research question of this paper is therefore whether the FH measure is applicable in practice and to what extent it can be used as an alternative to current risk measures.

2. Research design

2.1 Research goal

Aumann & Serrano (2008) proposed an economic index of riskiness, lacking a clear operational interpretation (Foster & Hart, 2009). Foster & Hart (2009) modified the AS index to a different measure of riskiness, the FH measure. We are aware of the work of Bali et al. (2011), who have combined both into one generalized measure of riskiness, which is "able to rank equity portfolios based on their expected returns per unit of risk and hence yields a more efficient strategy for maximizing expected return of the portfolio while minimizing its risk". Since all three measures require the same data and can be computed by methods similar to the ones presented here, we focus on the FH measure.

In the light of these events the explicit research goal of this paper is to perform an explorative study, investigating whether the Foster & Hart measure is applicable as an alternative opposed to the current measures, and if so, to what extent. If a comparative study shows that the FH measure is applicable in practice, this paper should also address a list of actions needed before its actual acceptance and usage.

The goals of this paper can be summarized as follows:

- Provide a bridge, with two-way traffic, between theory and practice;
- Investigate whether the FH measure can be applied in practice;
- Compare current risk measures with the FH measure;
- Provide a list of actions needed before practical acceptance.

2.2 Research questions

From the research goal the central research question derived is formulated as follows:

How can the Foster & Hart risk measure be applied by (regulators of) banks that are member of or under jurisdiction of the Basel Committee on Banking Supervision?

This research question needs to be divided into sub questions to be able to answer it to its full extent and as unambiguously as possible (De Vaus, 2010). The first set of questions addresses theoretical concepts and definitions: risk, (coherent) risk measures and the FH measure in particular. The subsequent questions address the frame of reference (current measures and regulation) and a comparison. The last sets of sub questions cover several practical concepts: actual implications, (dis)advantages of (applying) the FH measure and computational issues.

Theoretical concepts and framework

1. What is risk and risk management?
2. What risk measures are commonly used?
3. What are the shortcomings of current risk measures?

The first question is about definitions and historical background of risk and risk management as an introduction towards risk measures, leading to Question 2 about current regulation and measures. The last question addresses their shortcomings and provides a bridge towards coherent risk measures and the Foster & Hart measure in particular.

Theoretical coherent risk measures and FH

4. Why the need for coherent risk measures?
5. What is the Foster & Hart risk measure?

These questions follow logically from the theoretical introduction ending with the shortcomings of current risk measures. It starts with a discussion of coherent risk measures and ends with a discussion of the new risk measure of Foster & Hart, accompanied by a brief discussion of the work of Aumann & Serrano and Bali et al.

Practical framework and applicability

6. What are the practical aspects for banks of the current measures and regulation?
7. What are the (dis)advantages of the FH risk measure compared to the current ones?
8. Is the FH measure applicable as risk measure in practice?

Before the computational component is addressed, Question 6 to 8 focus on the practical issues. What is the current practical frame of reference in regulation? What are possible (dis)advantages of the FH measure over current risk measures and is the FH measure applicable given this current practical framework?

Practical implications

9. What are the practical implications of using the FH as a risk measure for (regulators of) banks?

Question 9 answers the 'how' of the central research question, whereas Question 8 answers the 'whether'. It is directed towards the possible usage, in terms of computation and acceptance issues, based on both the theoretical and practical information of the previous sections.

Computations and application

10. If the FH risk measure is applicable, how does it perform compared to common alternatives for different distributions?

Question 10 is directed towards the actual practice: computation of risk for several distributions. This question is a logical consequence of the practical framework of the previous part.

3. Theoretical concepts and framework

This section provides a specific insight in the concepts of risk and risk management in a financial context. It is not a general or philosophical section on risk, nor is it a discussion of all mathematically relevant concepts on risk. The goal of this section is to answer Questions 1 through 3 and to set the frame of reference for the remaining questions.

3.1 The concepts of risk and risk management

What is Risk?

The standard definitions of risk focus on the downside of certain events. Webster's dictionary defines risk as "the chance of injury, damage or loss" and Oxford's dictionary defines it as "a situation involving exposure to danger". The latter defines the verb risk as "incur the chance of unfortunate consequences by engaging in (an action)".

In finance risk can be defined as "the quantifiable likelihood of loss or less-than-expected returns" (Investorwords.com) or "The chance that an investment's actual return will be different than expected." (Investopedia.com). Markowitz (1952) also stated that one should "include allowance for risk in "anticipated" returns". Risk is thus divided in losses and expected returns; expectations of returns on certain portfolios are thus essential in understanding financial risk.

Where risk refers to the probability of a loss, exposure means the possibility of loss (Horcher, 2005). Or as Oxford's dictionary states it, exposure is "the state of having no protection from something harmful". In other words risk is associated with quantifiable probabilities of losses and exposure is the actual state of being in a risky position.

A fundamental distinction is the distinction between risk and uncertainty, a distinction first described by Knight (1921). In this view risk is involved if randomness comes with objective probabilities (e.g., gambles with roulette or dice). A situation is uncertain if randomness is presented in alternative possible events (e.g., a horse race or insurance). Although the two types of randomness differ in the level of objectiveness, the hypothesis of probabilistic sophistication permits the application of probability theory in both cases (see Machina & Rothschild, 2008). Expectations of returns can therefore be analyzed with probability theory. Two remarks on expected values of portfolios are of great importance in the understanding of risk measurement.

1. The expected value of a portfolio means the statistical average and not the most likely value of a portfolio. The following gamble, for example, has an expected value of round 9999 euro, even though it is highly unlikely that it will result in a gain:

$$g = \begin{cases} -1 & p = 0.99 \\ 1{,}000{,}000 & p = 0.01 \end{cases}$$

2. Following Machina & Rothshild (2008) the first step in the economic characterization of risk is the shape of individual von Neumann-

Morgenstern utility functions representing individual preferences. Whatever the notion of 'riskier' means, it is clear that a certain payoff of $\bar{x} = \mathrm{E}[\tilde{x}]$ is less risky than a random wealth of $\tilde{x}$. If an individual always prefers the first to the latter he is said to be risk averse and has a concave utility function. A person with a convex utility function is said to be risk loving (Arrow, 1964 and Pratt, 1964).

Financial risk is involved in all transactions of a financial nature, the financing of investments, loans, and various other business activities. The main sources of financial risk are market risk, credit risk and operational risk (Horcher, 2005). There is however growing interest for other types of risk like liquidity risk (Hull, 2010). The Basel Accord defines the three major sources of risk as follows:

- Market risk is the risk that the value of an investment will decrease due to moves in market factors;
- Credit risk is the possibility that a bank's borrower or counterparty will fail to meet its obligations in accordance with agreed terms;
- Operational risk is the risk of loss resulting from inadequate or failed internal processes, people and systems, or from external events.

What is Risk Management?

According to Crocker (2003) risk is "endemic in our uncertain world", which shows once more the close relationship between risk and uncertainty. In both our personal and professional lives we need strategies to deal with risk exposure. Risk management can roughly be defined as dealing with risk. Financial risk management (subsequently referred to as risk management) can therefore be defined as dealing with uncertainties from financial markets (Horcher, 2005), and consists of three steps:

1. Assessment	**2. Mitigation & Control**	**3. Catastrophe planning**
-identification -quantification	-weighing costs & benefits -assign probabilities to alternative events	-loss reduction -minimize long-term effects

Figure 3.1 Consecutive steps in risk management
Sources: Hull, 2010; Horcher, 2005; Crocker, 2003

There are two general approaches to manage risk, risk decomposition and risk aggregation. Decomposing means that all risks are dealt with separately whereas risk aggregation involves diversification to reduce risks (Hull, 2010). Diversification in portfolio theory means that investors do not maximize discounted return, but select their portfolio thus that they simultaneously maximize the expected returns and minimize the variance of the portfolio (Markowitz, 1952).

The goal of risk regulators is "to make bankruptcy a highly unlikely event" (Hull, 2010). Assumable this goal is the same for risk managers across companies. Regulators therefore require banks to hold sufficient capital to absorb losses of very low probabilities. This capital level, the regulatory framework behind it and the measures involved are the subject of the next section.

3.2 Current risk measurement and regulation

Regulatory Framework: Basel Accords

The Basel Accords require banks to hold a capital of 8% of risk-weighted assets: all assets and off-balance exposures of banks weighted according to the risk they carry. The Basel Committee provides banks with certain methods of weighing the riskiness of certain assets. The first Basel Accord had significant weaknesses. For example, all loans to corporations had to carry a risk weight of 100% regardless of the credit rating of the underlying company. The accord also did not include default correlation and focused solely on credit risk (Hull, 2010). In 2004 the Basel Committee announced new rules and criteria (Basel II), including financial risk and operational risk, based on three so-called pillars:

- Minimum capital requirements (8% of risk-weighted assets);
- Supervisory review;
- Market discipline.

Since all calculations of risk are part of the first pillar, this article disregards the other two pillars. These core concepts are unchanged in the newly adopted regulations of Basel III, although the capital requirements are enhanced and as a result of the financial crisis the Basel Committee includes liquidity requirements in the new rules. As of 1 January 2015, banks will be required to meet the following new minimum requirements in relation to risk-weighted assets (RWA):

- 4.5% of common equity/RWAs;
- 6.0% of tier-1 capital/RWAs;
- 8.0% of total capital/RWAs.

Total capital is 8% of the sum of Credit risk RWA + Market risk RWA + Operational risk RWA. There are several choices for banks in calculating their risk-weighted assets. These choices are not discussed in detail, but will come forward in the practical discussion of current risk measures versus the Foster & Hart measure.

For credit risk banks can choose either one of:

1. The standardized approach (supervisor prescribes risk weights);
2. The foundation internal ratings based (IRB) approach (loss probability density function based);
3. The advanced IRB approach.

For market risk the preferred approach is Value-at-Risk or (VaR). For operational risk banks have again three choices:

1. The basic indicator approach;
2. The standardized approach;
3. The advanced measurement approach.

CURRENT RISK MEASURES

Markowitz stated that if the word risk were replaced by variance of return “little change of apparent meaning would result” (Markowitz, 1952). Risk measures based on standard deviation were historically widely used (Horcher, 2005). A major drawback is that these measures are not monotonic (Foster & Hart, 2009), meaning that the risk measure of a distribution X with all values lower than or equal to that of a distribution Y should be less than or equal to the risk measure of distribution Y. It is however an intuitive starting point for the measurement of risk: a low value indicates a small spread around the expected value, intuitively comparable to lower risk. However for describing the risk of events with low probabilities these measures are inappropriate (Artzner et al., 2004).

Many papers define risk in terms of change of values over time; Artzner et al. (1999) argue that risk is only related to the (variability of) future value of a portfolio, thereby introducing a ‘future net worth’ approach. They define a measure of risk as the minimum extra capital that makes the future value acceptable. Crucial to this concept is whether the future value belongs to the subset of *acceptable* risks, the definition of which is to be decided by regulators, exchange’s clearing firms or risk managers. It is apparent from these definitions that risk measures are used to determine the amount of capital to be kept in reserve.

The most widely used measure of risk is Value at Risk (VaR). This measure lies at the heart of risk measurement according to the Basel framework. It is developed to provide a single number that represents the entire risk of a portfolio. Both the measure and the value are called VaR, the loss level during a time T that will not be exceeded with a certainty of level α. The actual result of the computation of VaR depends on several factors: the probability function of losses (or profits), the choice of parameter α and the time horizon t.

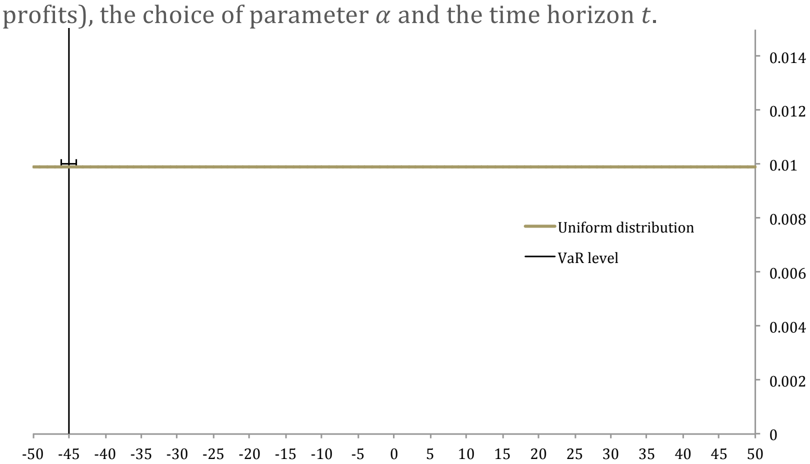


Figure 4.1 $\mathrm{VaR}_{0.95}$ for uniformly distributed asset

The most straightforward example to illustrate VaR is assuming an asset with random values uniformly distributed on the interval (-50,50). The VaR at a 95% confidence level ($\alpha = 0.05$) is -45, the loss level that is exceeded with a probability of 5% (Hull, 2010; Jorion, 2001).

Since VaR does not pay attention to the distribution in the tail of the probability function, there were several other methods derived from VaR focusing on the tail. For example the Expected Shortfall (ES) method, which measures the average future net worth of a position, conditional on the fact of a loss greater than the cutoff level α (Artzner et al., 2004; Acerbi & Tasche, 2002; Rockafellar & Uryasev, 2002). ES is therefore also called Tail/Average/Conditional VaR, or Expected Tail Loss. It answers the question: 'what is the expected loss if things go bad?' (Hull, 2010).

Banks have developed internal estimates of the capital needed for the risks they are exposed to besides to these theoretical and regulatory estimates, called economic capital. Based on this economic capital banks and investors calculate the risk-adjusted return on capital (RAROC) or the return on risk-adjusted capital (RORAC), both based on Value at Risk calculations (Hull, 2010).

3.3 Shortcomings of current risk measures

Although there has been much criticism lately on the current regulatory system for focusing primarily on risk of extreme events instead of consideration of risk in daily operations (e.g., Caballero & Krishnamurty, 2008; Artzner et al., 2004) this paper will to a large extent ignore regulatory deficiencies. The focus of this paper is primarily on practicality of risk measures and therefore will the focus of the discussion be directed towards regulatory-based measures of risk.

Even before the article of Artzner et al. (1999) on coherent measures of risk there was criticism on VaR for being too simplistic, dependent on underlying factors, and for being incomparable between companies (e.g., Beder, 1995). Grootweld & Hallerbach (2000) point out that VaR was originally intended as a diagnostic variable instead of a decision parameter. The industry advocated the use of VaR for its simplicity and flexibility (Hull, 2010). There is however a downside of the flexibility. Beder (1995) shows for three (real-life) portfolios that VaR results vary up to 140 percent between eight VaR approaches (different combinations of historical/Monte Carlo simulation, 100/250 trading days data set and RiskMetrics/BIS correlations).

Artzner et al. (2004) show that traders can 'spike' their position by taking advantage of the dependence of VaR on the confidence level, by entering positions with a high probability of a gain and a small probability of a huge loss. In these cases the relatively small VaR at the confidence level masks the (however small) probability of the huge loss. It is questionable whether VaR is a sound measure for portfolios with such probability distributions. Two simple examples show how problematic a single value for risk assessment may be if this number depends on too narrow constraints:

1. Consider the following asset:

$$g = \begin{cases} 1 & p = 0.99 \\ -1{,}000{,}000 & p = 0.01 \end{cases}$$

The expected value of this asset is round minus 9999 euro. How much capital is needed to manage the risk of a portfolio against its exposure? The VaR of this portfolio depends on the number of α, but even for a reasonably high value of say 1% the VaR of this specific asset is still 1 euro positive, although the downside risk of the unlikely extreme loss could mean bankruptcy for any agent entering this deal.

2. Artzner et al. (2004) show another problem of VaR with a portfolio consisting of a put and a call option with both 4% probability of a loss. At $\alpha = 0.05$ the VaR of both options is positive, although the combined portfolio certainly needs capital since both options cannot be in the money at the same time.

VaR ignores the tail of distributions, consider for example the following graphical representation of an arbitrary distribution and its $\text{VaR}_{0.99}$ level and note that the VaR level does not change if the probability mass of the tail shifts towards even less desirable loss levels.

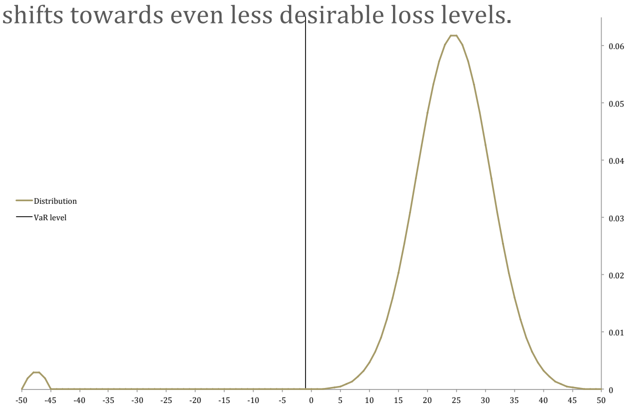


Figure 4.2 $\text{VaR}_{0.99}$ for arbitrary distribution with fat tail

The VaR of portfolios is easily manipulated, which is readily verified from the table below. Entering a combination with asset 2 offsets the undesirable VaR of the first asset, -100. The 99% VaR of the combination has 'improved' to 0, although the probability of an unfavorable event to occur has increased and the possible loss has increased by a multiple of around 1000.

Asset 1		Asset 2		Combined asset 1+2	
$\text{VaR}_{0.99}$	-100	$\text{VaR}_{0.99}$	100	$\text{VaR}_{0.99}$	0
p	**x**	**p**	**x**	**p**	**x**
0.011	-100	0.009	-100000	0.000099	-100100
0.989	1000	0.991	100	0.008901	-99000
				0.010901	0
				0.980099	1100

Table 4.3 $\text{VaR}_{0.99}$ for two assets and the combination of the two

Since VaR encourages the accumulation of shortfall risk, several alternatives for VaR were developed (Föllmer & Schied, 2010). These alternatives are however not present in current regulation and are in their turn criticized for their complexity (Hull, 2010), and for being dependent on a specific probability distribution (Artzner et al., 2004). There is an ongoing search for robust, but convenient risk measures that are both economically sound and practically operable.

4. Axiomatic risk measures

Section 3 made clear that within the Basel regulation VaR is the risk measure that is the corresponding industry standard, although it is often criticized. Especially the deficiency on shortfall risk has lead to research on other alternatives. As Föllmer & Schied (2010) point out such a search should start with specifying certain axioms ("a statement or proposition which is regarded as being established, accepted, or self-evidently true", Oxford Dictionary).

This section extends the theoretical framework with a discussion of coherent risk measures and the axioms underlying them and a discussion of the recently developed theories of Aumann & Serrano and Foster & Hart, the main subject of this paper.

4.1 Coherent risk measures

Artzner et al. (1999) suggested in their discussion of current risk measurement four properties that any so-called coherent risk measure should possess, derived from four axioms on acceptance sets, the set of acceptable risks according to a certain supervisor. They define these sets in terms of future net worth as introduced in Section 3.2, arguing that risk is the investor's future net worth $\Sigma_{1 \leq i \leq I} e_i \cdot A_i(T)$. This future net worth approach, based on market risk and therefore dealing with positions in certain currencies, considers one period of uncertainty $(0, T)$ for simplicity. The various currencies are numbered by $i, 1 \leq i \leq I$. The factor e_i denotes the "random number of units of currency 1 which one unit of currency i buys at date T" (Artzner et al., 1999). Since the initial positions are denoted by A_i, this automatically leads to the future net worth denoted as the sum of all positions providing $A_i(T)$ units of currency i at date T.

The axioms on acceptance sets and risk measures are closely related since Artzner et al. (1999) defines a measure of risk by "describing how close or how far from acceptance a [future] position is". The axioms need some additional notation, the set of states of nature is called Ω, and the final net worth of a position is a random variable denoted by X. The set of all risks is $\mathcal{G}$, the cone of nonnegative elements of $\mathcal{G}$ is denoted by L_+ and the negative elements of $\mathcal{G}$ are denoted by L_-. Finally, the set of final net worths, expressed in currency i, accepted by supervisor j is denoted in the generic notation $\mathcal{A}$. The set $\mathcal{A}$ is called the acceptance set. The four axioms on acceptance sets are as follows:

1. The acceptance set $\mathcal{A}$ contains L_+
2. The acceptance set $\mathcal{A}$ satisfies $\mathcal{A} \cap L_- = \{0\}$
3. The acceptance set $\mathcal{A}$ is convex
4. The acceptance set $\mathcal{A}$ is a positively homogeneous cone

By definition a measure of risk is a mapping from the set of all risks $\mathcal{G}$ into $\mathbb{R}$. The number $\rho(X)$ is the amount of capital that is needed to make a position acceptable, invested in a 'safe' position. If this number is negative, the amount can be withdrawn from the position. The axioms for risk measures are stated as follows, for all X and Y that are element of the set $\mathcal{G}$, all real numbers α and all positive values of t:

1. $\rho(X + c) = \rho(X) - c$ [translation invariance]
2. $\rho(X + Y) \le \rho(X) + \rho(Y)$ [sub-additivity]
3. $\rho(t \cdot X) = t \cdot \rho(X)$ [positive homogeneity]
4. $\rho(X) \le \rho(Y) \quad \forall Y, X \in \mathcal{G} \; with \; Y \le X$ [monotonicity]

The four axioms are quite straightforward and are widely accepted as logical assumptions for measures of risk. Property 1 states that any 'risk-free' invested capital that is added to the portfolio reduces the risk of the portfolio by the same amount. This property is also called the risk-free property (Artzner et al., 2004). The second property is based on the fact that diversification does not increase risks; the aggregation of two portfolios should equal or decrease the capital requirements. Property 3 states that the risk is independent of the unit of measurement, multiplying the amount invested or the outcomes, multiplies the risk the same multiplier. The last property, monotonicity, states that if a portfolio outperforms another portfolio it should always be considered as less risky and therefore the value of its risk measure should be smaller (Artzner et al., 1999; Hull, 2010).

A risk measure that satisfies all four axioms presented above is called coherent by definition. It is easy to show that VaR satisfies only three of these four conditions, because it is not sub-additive (e.g., Hull, 2010; Artzner et al., 1999; Rockafellar & Ursayev, 2002). Shortfall approaches are coherent, but as mentioned before, some are very dependent on the underlying probability function. Artzner et al. (1999, 2004) therefore plead for a generalized scenarios approach. They call it the "most general form of coherent risk measure" (Artzner et al., 2004, p. 401) and formulate the following procedure:

1. Calculate the average of the negative of the position's final net worth X, under each probability function belonging to $\wp$ (set of probability distributions of the (finitely man) states of nature);
2. Calculate the largest of all numbers found in the first step that corresponds to the formula $\rho(X) = \sup\{E_P[-X] | P \in \wp\}$.

There is an ongoing search for coherent measures of risk based on the four generally accepted axioms of Artzner et al. Recent research is heading towards so called convex measures of risk, and in particular entropic risk measures (which are convex, but not coherent, see Föllmer & Schied, 2010). The entropic risk measure, with parameter θ (risk aversion) is defined as (Föllmer & Schied, 2010):

$$\rho^{ent}(X) = \frac{1}{\theta}\log(\mathrm{E}[e^{-\theta X}]).$$

Convexity means, in terms of the axioms presented by Artzner et al. (1999), that a measure satisfies both the sub-additivity and the positive homogeneity property. Entropic risk measurement is based on the concept of entropy, derived from physics and is out of scope of this paper.

4.2 An operational measure of riskiness

Foster & Hart attempted to find an operational interpretation for the AS index, but were "led instead to the different measure of riskiness" (Foster & Hart, 2009). Riskiness is defined as a level of critical wealth below which it becomes risky to accept a gamble. The critical wealth is the capital buffer an investor should hold in order to avoid long-term bankruptcy if faced with an unknown sequence of the gamble.

Foster & Hart (2009) continued the search for a measure of riskiness with the following desiderata: objective, scale-invariant, monotonic, an operational interpretation and a simple formula.

Foster & Hart measure of riskiness

The measure is based on gambles g with finitely many values and limited liability, for the measure yields infinite riskiness for unbounded losses. The explanation of the measure requires some notation:

- $\mathrm{E}[g] > 0$
- $\mathrm{P}(x < 0) > 0,$ $\quad$ x *being outcomes of* g
- $L(g) = -min_i x_i > 0$ $\quad$ is the maximal loss of g
- $M(g) = max_i x_i > 0$ $\quad$ is the maximal gain of g
- $W_t \ (t = 1,2,\dots)$ $\quad$ is the wealth at time t
- $W_t = W_{t-1} + x$

For this wealth, in order to have a risk measure that indeed rejects risky gambles and accepts gambles below a certain threshold, there must be what Foster & Hart call a critical wealth function $Q(g)$ that gives a real valued positive number for each gamble g. This critical wealth function is used in a strategy s_Q that rejects a gamble g if $W < Q(g)$ and accepts it if $W \geq Q(g)$. This strategy yields no bankruptcy if the probability of wealth converging to zero over time is zero, that is $\mathrm{P}[lim_{t\to\infty} W_t = 0] = 0$. Irrespective of the initial wealth and the sequence of gambles offered in the future, the strategy s_Q guarantees that future wealth does not converge to zero (Foster & Hart, 2009).

The first theorem of Foster & Hart (2009) states that there exists a unique number $R(g) > 0$ for every gamble $g \in \mathcal{G}$ such that "a simple strategy s with critical-wealth function $Q(g)$ guarantees no bankruptcy if and only if $Q(g) \geq R(g)$ for every gamble $g \in \mathcal{G}$" (Foster & Hart, 2009). This condition states that the gamble is accepted if the critical wealth is higher or equal to $R(g)$ and gamble g is rejected for all wealth levels below it. $R(g)$ can therefore be seen as the minimal wealth that is needed to accept a gamble g and is thus a measure of riskiness of g. $R(g)$ is uniquely determined by the equation

$$\mathrm{E}\left[log\left(1 + \frac{1}{R(g)} g\right)\right] = 0.$$

Foster & Hart (2009) continue by showing that if only a proportion of a gamble is accepted this does not affect the riskiness of the gamble. It is possible in their 'shares setup' (where one could accept a fraction

$\alpha = W/Q(g)$ of a gamble) to 'stay in the game' even with wealth below the critical wealth level by accepting any nonnegative multiple of the original gamble. They prove that in the end this approach leads to a very sharp distinction between bankruptcy (which is avoided) and infinite wealth, where $R(g)$ is the threshold between the two.

The FH measure is thus an intelligible formula depending solely on the distribution of the gamble. Foster & Hart assume that bankruptcy means having wealth of zero, the literal sense of bankruptcy instead of possible regulatory or legal versions where losses may be limited. No bankruptcy and infinite growth are assumed to be preferred over bankruptcy. Another important assumption is that of limited liability; the measure yields infinite riskiness for unbounded losses, since $R(g) > L(g)$. Borrowing is not allowed in this setup.

Axiomatic approach regarding the FH measure

In 2011 Foster & Hart propose an axiomatic approach to their risk measure consisting of four axioms, the first two of which are straightforward and satisfied by many coherent measures of risk: distribution and scaling. The other two are derived from the concept of riskiness as wealth requirement: monotonicity and compound gamble:

1. If g and h have the same distribution then $Q(g) = Q(h)$ [distribution]
2. $Q(\lambda g) = \lambda Q(g)$ for every $\lambda > 0$ [scaling]
3. If $g \geq h$ and $g \neq h$ then $Q(g) < Q(h)$ [monotonicity]
4. Let $f = g + 1_A h$ be a compound gamble, where $g, h \in \mathcal{G}$ and A is an event such that g is constant on A, i.e., $g|A \equiv x$ for some x, and h is independent of A.

   If $Q(h) = Q(g) + x$ then $Q(f) = Q(g)$. [compound g]

The first axiom is straightforward and very widely accepted for measures of risk. The axiom of scaling (or scale-invariance in terms of the Foster & Hart 2009 paper) is the same as the positive homogeneity property following from Artzner et al. (1999), it states that the measure does not depend on the unit of measurement, rescaling the gamble results in the exact same rescaling of the risk measure. In financial literature this is referred to as law-invariance. Monotonicity is the same concept as explained before in Section 3.2 meaning that if a gamble g and a gamble h have the same distribution but one gamble outperforms the other, its risk measure should be smaller. The axiom of compound gamble is based on the wealth requirement and it states that if the current wealth plus the possible outcomes of the current gamble satisfies the wealth requirement for all consecutive gambles, then the current wealth is also appropriate for the compounded gamble (see Foster & Hart, 2011, pp. 5-7 for an illustration). The FH measure satisfies the four axioms; in fact it is the minimal function satisfying these axioms.

See Appendix A for an intuitive introduction and a short representation of the FH measure and the rationale behind it.

AUMANN & SERRANO INDEX

Aumann & Serrano (2008) follow an axiomatic approach in order to define riskiness. The starting point of Aumann & Serrano is a search for an objective measure of risk, not depending on the person that is exposed to it. The theoretical basis is the concept of risk-aversion as introduced in Section 3.1, since they are considering risk-averse decision makers, those who prefer less risky portfolios to more risky ones if all other things are equal.

It is important to note the difference that Aumann & Serrano (2008) make between desirability and riskiness. Desirability is based upon preferences of investors, whereas riskiness is objective. A 'riskier' investment can still be more desirable for a certain investor, for example in terms of expected value or maximum loss. A more risk-averse investor could find that a certain portfolio is too risky, while a less risk-averse investor may think that the opportunities of the portfolio outweigh the riskiness. The riskiness of the portfolio is the same for both investors (Aumann & Serrano, 2008). Although Aumann & Serrano start by describing their search for a 'measure', they continue by using the word 'index', which is more appropriate since the AS index is based on comparing gambles, opposed to a 'measure' that is defined separately for all gambles (Foster & Hart, 2009; Homm & Pigorsch, 2010).

The AS index is closely related to the concept of constant absolute risk aversion, that was introduced by Arrow (1965) and Pratt (1964). The utility function of a decision maker with a constant coefficient of absolute risk aversion α, is namely given by:

$$u(w) = -e^{-\alpha w}.$$

Aumann & Serrano (2008) try to find the cutoff level for which a gamble is rejected by every decision maker with smaller α and accepted by a decision maker with higher α. This cutoff level is the number $R(g)$ for which the following equation holds:

$$\mathrm{E}e^{g/R(g)} = 1.$$

We are aware of the work of Bali et al. (2011), which builds upon the constant relative risk aversion (CRRA) and the work of Homm & Pigorsch (2010) who connect the FH measure and the AS index by taking notion of the fact that the two are related to the adjustment coefficient (a quantity derived from ruin theory). These new contributions provide interesting solutions to certain limitations of the FH measure. These are however additions to the FH measure and do not change assumptions or variables behind the FH measure. We will not discuss these additions in this paper, but instead focus on the computation of the FH measure, since the line of reasoning remains intact.

5. Practical applicability

5.1 Current practice

The assumptions, axioms and variables behind the FH measure should be clear from the previous section, as is the regulatory framework of current risk measures within banks. To be able to convert all theoretical assumptions to a practical discussion it is necessary to understand daily practice. The information in this section is based on several interviews within the banking and risk management sector and on annual reports of major Dutch banks and Basel regulation. As introduced in Section 3.2 several approaches are allowed for banks to measure their risk, the table sums up the preferred approaches:

Risk	Approach	Confidence level	Distributions
Market risk	10-day VaR	α=0.99	historical data
Credit risk	1-year WCDR	α=0.999	PD, EAD, LGD
Operational risk	estimation	-	-

Table 5.1 Preferred regulatory approaches in calculating risk

For market risk the preferred approach is VaR, either derived from historical or Monte Carlo simulation. Most banks use the historical simulation approach; an approach that estimates the VaR for a certain market position based on the empirical distribution of the last n data points. The historical movements of an asset (profit and loss shocks) are used as a probability distribution for the current position. The time horizon of data depends on the specific asset and is approved by the regulator. A 10-day VaR with $\alpha = 0.99$ is required by the regulators for calculating market risk. The VaR for market risk is back-tested, which means that the daily VaR estimate is compared with the real-life results. If the actual loss exceeds VaR too often the regulator undertakes action.

For credit risk the approach is slightly different since it is based on a combination of models. The standard approach is PD*EAD*LGD, where PD stands for probability of default, EAD is the exposure at default and LGD is an abbreviation of loss given default. For each parameter banks develop models, also based on historical data. The worst credit default rate (WCDR), a measure that is very similar to VaR, is calculated given the three parameters described before. A 1-year WCDR with $\alpha = 0.999$ is required by the regulators for calculating credit risk. Although there is much criticism of historical data for lacking predictive properties, historical data are on the plus side credited for implying correlations: included in the historical data are the correlations of counterparty's default, which need not to be estimated or modeled.

The measurement of operational risk is even more complex, since it is based on highly infrequent events. The attention has shifted towards mitigation and control instead of risk measurement; managers need to estimate material risks of their operations. This is an arbitrary process with little data available; much is based on management interviews instead of historical data. Banks are therefore allowed by the regulators to use their own advanced measurements for operational risk.

Because of the limitations of statistical risk measurement based on historical data, there is increasing attention towards stress testing. Several low probability extreme events are simulated to see whether the (regulatory) capital of a bank is sufficient to survive. This is consistent with the findings within the field of operational risk where attention is heading towards understanding and preventing exposure at risks instead of purely measuring it. One of the major problems lying at the heart of the financial crisis is not the measurement of regular risk of common and well-understood products, but the measurement of highly complex financial products that have little historical data and are less understood (J. de Mulder, personal communication, April 19, 2012).

Within the Basel regulation banks are allowed to develop their own risk measurements, since the regulator understands that banks are much more aware of the type of products and risks that they are exposed to than the regulator could ever be. If a bank wants to use a different measure or an adapted measure it needs to perform parallel simulation runs of the adapted measure and the regulator imposed measure. The outcomes are evaluated and if the regulator is convinced that the internal measure is consistent and sound the regulator might allow it.

5.2 (DIS)ADVANTAGES OF DISCUSSED RISK MEASURES

The main goal of this paper is to investigate whether the FH measure is applicable. Actual advantages and disadvantages of this measure compared to other measures (as discussed in Section 3.2 and Section 4.2) should be the subject of a follow-up comparative study that elaborates on the mathematical soundness and financial implications of the results. It is, however, easily possible to compare several characteristics as a result of this study. Sometimes it is questionable whether a certain characteristic is a limitation or an advantage. This section first compares the two main measures of risk within this paper; VaR and the FH measure and concludes with a brief discussion of some questionable characteristics.

Distribution characteristics	**VaR**	**FH**
1. Distribution free	x	x
2. Continuous	x	discr. app.
3. Well-defined for unbounded losses	x	
4. Based on historical data	x	x
5. Independent of preferences	partly	x
6. Independent of arbitrary parameters		x
7. Riskiness > maximum loss		x
Axiomatic characteristics		
I. Distribution axiom	x	x
II. Translation invariant	x	
III. Subadditive		x
IV. Positive homogeneous	x	x
V. Monotonic	weakly	x
VI. Compound gamble		x

Table 5.2 Theoretical comparison of VaR and FH measure

Discussion of distribution characteristics

1. Both VaR and the FH measure are distribution free and thus applicable to any distribution; VaR by calculating a percentile of the distribution and the FH measure by calculating an expectation of 'log-returns'.
2. In the basic setup the FH measure is not applicable to continuous functions. This is however readily solved by discrete approximation.
3. The main disadvantage of the Foster & Hart measure is that it is not well defined for unbounded losses, a property that could be very problematic since many distributions in finance are unbounded from below (Homm & Pigorsch, 2011; Bali et al., 2011).
4. Both VaR and the FH measure are dependent of historical data with all possible limitations thereof as discussed in the previous section.
5. The most important advantage of the Foster & Hart measure over VaR is that it is 'objective'; it does not depend on preferences of users or arbitrary parameters such as a confidence interval.
6. The Foster & Hart measure always assumes riskiness greater than the maximum loss, opposed to VaR, which takes the $(1-\alpha)$ percentile of the underlying distribution as measure of risk.

Ad 3. Homm & Pigorsch (2011) and Bali et al. (2011) proposed modifications of the FH measure that suggest that it is possible to bound the measure from below within a similar setup as the FH measure.

Another remark is that the FH measure is designed and proven for gambles with finitely many outcomes. Riedel & Hellmann (2013) generalized the concept of critical wealth level to continuous distributions. Important to note here is that Riedel and Hellmann only study distributions that are bounded from below, actually showing that the limit for the critical wealth level is given by the maximum loss L(g).

If for a continuous random variable $\mathrm{E}[\log(1+X/L)\ ] \geq 0$, X representing the random variable and L representing the maximum loss, then the limit of the riskiness coefficient should be taken of the approximating discrete variables, which turns out to be L (Riedel & Hellmann, 2013). For all other cases (where the expectation is smaller than zero) the formula of Foster & Hart is applicable, as exemplified before in this paper.

Discussion of axiomatic characteristics

Both VaR and the FH measure violate one of the axioms of Artzner et al. (1999). Value at Risk is often criticized for not being subadditive, which means that the risk of a combined portfolio should be no greater than the sum of the individual portfolios (see for a discussion of this violation for example Hull, 2010). The Foster & Hart measure of risk is not translation invariant, a property that is logical in the Value at Risk setup: if a risk-free amount is added to the portfolio, its riskiness should decrease with the same amount. To see why it is also reasonable that this axiom should not hold in the Foster & Hart setup consider the following gamble:

$$g = \begin{cases} 120 & p = 0.50 \\ -100 & p = 0.50 \end{cases}$$

$R(g) = 600$, and for any reasonable choice of α the VaR of g is 100. If one adds 100 to this portfolio it follows immediately that one can lose nothing and gain 220 with equal probabilities. In this case its riskiness should be zero and not 500 (Foster & Hart, 2009). Indeed the VaR becomes zero for the new gamble. The last concept is that of monotonicity, both VaR and the FH measure are monotonic although Foster & Hart (2009) point out that VaR is only weakly monotonic with respect to stochastic dominance. For discussions on the concepts of stochastic dominance we refer to Hanoch & Levy (1969), Rotschild & Stiglitz (1970) or Hart (2011).

Besides all these theoretical and practical advantages of the Foster & Hart measure it is important to be aware of the following facts. The FH measure provides an idealized benchmark, since the measure might change if additional information is present for example on the sequence of gambles (Foster & Hart, 2009). The number $R(g)$ is a single figure representation of risk with all limitations that a single number can have. It is well known in the area of risk that it is hard to capture a difficult concept such as riskiness in a single figure. Finally it is good to mention that $R(g)$ provides no criteria for when to accept a gamble, only for when to reject. This is however from the viewpoint of regulators not problematic since they will be concerned with rejecting (too) risky projects instead of internal procedures of accepting certain projects.

5.3 Application of Foster & Hart measure

Before answering the question whether the FH measure is applicable, based on the theoretical and practical framework of previous sections, a brief comparison of current techniques and the FH measure is needed. So far this paper addressed axioms, properties, mathematical aspects and aspects from daily practice, but it did not address the reasoning behind the different measures of risk. Obviously the axioms of Sections 4.1 and 4.2 can be seen as reasoning behind the measures, but do these measures share the same objectives? All discussed measures are a representation of risk in a single value of a certain function, applied on a set of data or a distribution. But are they interchangeable by means of their goals?

First of all VaR answers the question "how bad can things get?', where Expected Shortfall answers the question 'if things go bad, how bad are they going to get?'. Following this line of reasoning the FH measure answers the question 'what total wealth is needed in order to prevent bankruptcy?' At first glance there is a difference in the VaR/ES rationale on the one hand and the FH rationale on the other hand. The connection between the two lines of questioning lies in the fact that the regulatory capital that a bank needs to hold is based on its Value-at-Risk. So both may be used as a measure for regulatory capital and based on their goals they are interchangeable. The amount of capital that is needed under both systems is part of the discussion in Sections 6 and 7.

It is still questionable whether the FH measure is practically applicable. That depends roughly on two criteria, whether the measure is mathematically sound and proven right and to what extent the measure is computationally complex. Since this paper only addresses the practical aspects, it assumes the FH measure to be mathematically true and well defined. For some comments on potentially problematic theoretical issues see Section 5.2. The computational complexity of the FH measure is not very high as partly pointed out in Section 4.2. It is distribution independent and depends solely on the values of the gamble and their respective probability mass:

$$\mathrm{E}(g) = \sum_i p_i \, x_i$$

$$\mathrm{E}\left[log\left(1 + \frac{g}{R(g)}\right)\right] = \sum_i p_i \log\left(1 + \frac{x_i}{R(g)}\right)$$

What is needed in order to solve the above equation is a probability distribution, comparable to what is needed in current VaR techniques. This can be achieved by applying historical simulation or Monte Carlo simulation or by a given distribution. The only difference between VaR and the FH measure lies in the fact that the FH measure needs a discrete function. Since it is current practice to estimate continuous functions underlying VaR by means of discrete simulation techniques this is hardly a problem. The actual calculation is however of greater concern. VaR is simply the $(1 - \alpha)$ percentile of a distribution of in the discrete simulation case the $n - \alpha n^{th}$ th scenario. The FH measure needs implicit solving of the equation, an iteration process that estimates $R(g)$ and optimizes it until the expectation equals zero. This might be done using software packages such as Excel's Solver or Matlab.

Two important issues must be kept in mind. First of all the FH measure is not defined for distributions with unbounded losses. An adaptation needs to be made for these cases (for instance with the findings of Riedel & Hellmann, 2013). Since these distributions (for example the normal distribution) are very common in the financial world, this is an issue to overcome in order to be able to apply the measure in real-life. Secondly, as pointed out in Section 5.2, the FH measure is developed and studied for discrete variables only.

Section 6 provides a brief comparison of VaR and the FH measure with use techniques as Matlab and Solver and an actual calculation of $R(g)$ with a simple data set. It focuses on the computational issues and issues for acceptance of any new measure and the FH measure in particular.

6. Practical implications

The previous section has shown that it should be mathematically possible to apply the Foster & Hart measure at least to data that is distributed with a discrete probability function. This section builds up towards the actual simulation and application of the FH measure in comparison to VaR for several distributions using Matlab simulation. The practical implications are discussed along two paths: the actual application in terms of computations or algorithms and the adaptations that banks and regulators might need.

6.1 Computation

Especially for the computation of market risk, for which usually historical data over a given time (a common choice is 501 trading days (Hull, 2010)) is used, the FH measure seems to be easily applicable since this sort of data is consistent with its requirements. For this purpose two computations are performed on 835 closing prices of Apple Inc. The actual calculations and the Matlab and Solver algorithms used can be found in the Appendix. Both methods are also tested for extreme cases and arbitrary and artificial distributions to get a first impression of the possible problematic situations involved in computing the FH measure by means of Matlab or Solver. This is used as input for the actual simulation and application in Section 7.

For the solving of the algorithm the most important part in software packages is the initial estimate of $R(g)$. For calculations by hand (not automated processes) it is wise to start with an extremely high value for $R(g)$ and a value close to $|L(g)|$. Since the underlying function approaches zero as it goes to infinity, the Newton method for approximation is not easily applicable and extremely dependent on the initial estimates for $R(g)$. Excel Solver is also at first sight extremely dependent on the initial estimate, for an artificial probability distribution this is shown in Appendix C. The Matlab script, found in Appendix B, is a good solution for the problems caused by Excel's Solver. It solves the easy and artificial cases easily and the script is not at all dependent on the initial values. For the lower bound the value $|L(g)| + e$ is chosen, the value e being the smallest number available within Matlab. For the upper bound any reasonable choice might be applied, for example $2|L(g)|$ or $L(g)^2$. It is logical to define at least the lower bound in terms of $L(g)$, since $R(g)$ is always strictly greater than $L(g)$ for discrete distributions. For the upper bound some tinkering is needed to fine-tune the most convenient value.

Two explicit cases are discussed here, the data and calculations can be found in Appendix C and D. The first case is an artificial distribution, created to find the extreme situations for which the packages used might experience problems. The artificial distributions of Appendix C create several problems for Excel Solver (the numbers correspond to the sections of the appendix):

1. For distributions with a large positive expectation the value of $R(g)$ is very close to $L(g)$, sometimes so close that Excel cannot find a solution;
2. The first estimate of $R(g)$ is of high importance, a wrong estimate leads to no solution or an infeasible one;
3. For normal distributions with a mean close to zero it is hard or sometimes even impossible to compute the FH measure;
4. The FH measure cannot deal with distributions with a negative expectation, under 'normal' market conditions this may not cause any problems; under the current situations this might be problematic.

Appendix D shows the results of applying the methods to the closing prices of Apple Inc. (AAPL). With Solver the target cell is the expectation of the FH measure its target value being zero. Changing the value of $R(g)$ (the initial value of which is set to $|L(g)| + 1$ in this case) solves the equation. As can be seen the 10-day $\text{VaR}_{0.99}$ is \$142337, opposed to $R(g)$ of \$72577. The solution of Excel equals the value of $R(g)$ found by means of the Matlab script of Appendix B.

The examples show that the FH measure can be applied to both artificial distributions and a portfolio consisting of stocks. This seems to imply that the FH measure is at least applicable for market risk. To verify whether this statement is actually true a more thorough comparative study of real-life portfolios is conducted in Section 7.

6.2 Acceptance and adaptations

The Foster & Hart measure is applicable for the same sort of data as VaR and the computations of the previous section show that the value of $L(g)$ can be computed by means of readily available software packages. There are however more aspects of importance for a new measure in order to be applied and accepted by both banks and regulators.

Banks are familiar with new and modified measures of risk since they are allowed under the Basel II/III regulation to apply their own measures if they can prove that these measures are consistent and sound. There are however technical limitations with respect to software packages. Since the data are exactly the same as for VaR calculations there does not seem to be technical problems in computing the FH measure. This should be verified within a follow-up comparative study. Besides the system integration of the algorithm two other aspects are of importance: the impact of the critical wealth level required by the FH measure and the intelligibility of the measure. The latter case is slightly more difficult than VaR but seems not overwhelming. The first aspect is interesting to investigate in further research. It might be the case that for several portfolios the FH measure requires less capital than current regulatory capital. The opposite is also possible in which case banks could refuse to apply the measure if their capital levels are too low or banks could accept both measures and brand themselves as applying to regulatory capital and the FH critical wealth if their capital levels are sufficient.

7. Application and computation of FH

The previous sections reveal two important findings: the FH measure seems to be applicable in real-world finance, but there are some serious issues that need to be overcome before it will be used in practice. This section elaborates on the work done in Section 6.1 where a basic algorithm was developed for computing the critical wealth level for discretely distributed random variables and some basic continuous distributions.

7.1 Requirements and conditions for the algorithm and Matlab script

Specifically the following issues are of importance:

- For some distributions (especially unbounded from below, continuous, but in some cases also the discretely approximated continuous random variables as shown in Section 6.1) the function for finding the critical wealth level cannot be solved;
- In typical cases where the expectation of the random variable is close to zero or else if the distribution is extremely skewed it is hard to solve the function without a good estimate of $R(g)$.

The complete algorithm for computing the critical wealth level should therefore distinguish several (basic) probability distributions and take the appropriate action.

To close the gap between highly complex mathematical proofs and practitioners a hands-on simulation may enhance understanding and intuition at the same time. In order to be able to convince practitioners that the 'magic number' for the critical wealth level is indeed the threshold between bankruptcy and an increase in wealth (on the long run) a simulation is added with this number. The user can fill out the number of time steps and the number of experiments with the Matlab script (Appendix E). The output, as shown in Appendix F, compares several values for the critical wealth, close to the FH theoretical value and gives the appropriate VaR number for comparison.

As found by computing the FH measure with simple algorithms and by means of simple techniques like Excel's Solver the values for the estimates are of great importance.

7.2 Binomial gambles simulated

At first the code has been written for computing the algorithm for the FH measure for basic gambles. This extends the understanding of the algorithm and simultaneously visualizes the theoretical assumptions and reasoning of Foster & Hart if it shows the same results. If the simulation yields results likewise to the results presented in the Foster & Hart paper (2009) this ratifies both the claims of Foster & Hart and the soundness of the algorithm.

Besides the basic algorithm, firstly mentioned in Section 6, this code has been extended with a simulation part. This allows the user to simulate the effects of applying the simple shares strategy over a given time horizon (t) and for a given number of experiments (m). The results of these simulations should be consistent with the findings of Foster & Hart, claiming that there is in the long run a sharp distinction between proportions $\alpha = W/Q(g)$ with $Q(g)$ smaller than $R(g)$ and proportions with $Q(g)$ greater than $R(g)$. This shows that in fact $Q(g)$ is the critical wealth level and a 'threshold' wealth for such gambles. This is formalized as follows:

$$\lim_{t\to\infty} P(W_t > W_0) = 0.5 \qquad if\ \alpha = 1$$

$$\lim_{t\to\infty} P(W_t > W_0) = 0 \qquad if\ \alpha < 1$$

$$\lim_{t\to\infty} P(W_t > W_0) = 1 \qquad if\ \alpha > 1$$

The core of the algorithm is the FH measure of risk, computed with two important arrays: the array of chances and the array of corresponding outcomes of the gamble. For a 50/50 gamble with outcomes -100 and +200 equally likely the arrays are {0.5, 0.5} and {-100,200} respectively. The notation of Section 5.3 has been used to shape the algorithm, creating a find zero function for the expectation with a loop that computes for all values and respective probability mass the value of the logarithm.

Two functions are active at the same time to compute the value of $R(g)$: the standard Matlab function fzero, used to find the zero of the expectation with estimates for $R(g)$ and the newly created function FH(Rg). The basic script is not different from the one shown in Appendix B and mentioned in Section 6. It is extended with code for a simulation as presented in Appendix E, which allows the user to fill in values for t and m and which presents the percentage of runs that end with a wealth at the final period that exceeds the initial wealth for ten different values for $Q(g)$. The results for basic discrete gambles are completely consistent with the results presented by Foster & Hart (2009). For the gamble presented in their paper (2009) the results of the simulation are presented in Appendix F. There is indeed a sharp distinction at the threshold wealth, which becomes clearer with more simulation experiments and runs. The appendix shows results for 1000x1000 and 10000x10000 experiments and runs.

7.3 Simulation of market risk

Market risk is an intuitive starting point for comparative means. The shares setup of Foster & Hart (2009) is explicitly mentioned to be interesting in this situation and the concept of investing in shares is widespread. Value at Risk is also specifically used in assessing market risk. If the algorithm can solve market risk distributions it should not be complex to extend it to credit risk, since the principles behind these two types of risk are approximately the same.

The typical distributions used in market risk are however at first sight a problem for the FH measure. Normal distributions with fat tails (high probability of extreme events, specifically those including high losses) and lognormal distributions are commonly used to model market risk. Some investments in market risk include the theoretical probability of limitless losses, for instance short calls. Theoretically the FH measure prescribes the maximum loss as the minimum capital requirement. As mentioned before Riedell and Hellmann (2013) found that the maximum loss is also the threshold wealth for continuous variables that are bounded from below.

Since all simulation packages use discrete approximation for continuous variables such as the normal distribution the first problem mentioned in the previous section might be no problem at all. The algorithm might well be able to find a real value for the critical wealth level. The same is true for computations based on real-life market data, which is common practice in historical simulation techniques.

Applying the algorithm to sixteen AEX stocks shows that it is possible (with 385 trading days of data) to compute the FH measure for these assets, results are given in the table below. All data were retrieved from finance.yahoo.com, for four stocks these data were insufficient or not available for this time horizon, and for five more stocks it is impossible to compute the critical wealth level in the given period since $E(g) < 0$. See Appendix G for all details. The numbers found for the remaining sixteen stocks are ran over again by hand in Excel, resulting in the same values, again proving that the algorithm is sound and able to find the critical wealth level for distributions that are discrete or discretely approximated.

The results for a simulation with $m = 5000$, $t = 5000$ and $W_0 = 1000$ (actually irrelevant since the entire outcome is independent of initial wealth as sh own before and by Foster & Hart (2009)) for the sixteen AEX funds are shown below (for a list of abbreviations see Appendix G):

	AGN	AH	AKZA	APAM	ASML	BOKA	DSM	HEIA	INGA	PHIA	PNL	RAND	REN	RDSA	UNA	WKL
Norm 10d VaR	196.72	78.77	138.77	240.20	124.32	148.09	106.15	91.57	239.41	124.88	1118.14	168.18	80.01	73.48	71.32	100.85
Hist 10d VaR	212.93	114.52	157.93	225.17	113.00	167.03	105.77	93.24	251.12	141.81	255.58	176.56	78.57	93.67	79.44	99.97
FH	211.35	74.13	133.89	970.97	67.91	226.94	94.07	54.22	420.58	92.04	498.34	217.17	52.83	108.31	57.60	120.09
Wt>W0 @0.8 FH	0.10	0.05	0.09	0.40	0.00	0.17	0.06	0.01	0.25	0.03	0.10	0.15	0.01	0.20	0.02	0.13
Wt>W0 @FH	0.49	0.49	0.51	0.52	0.51	0.50	0.49	0.50	0.52	0.51	0.50	0.50	0.50	0.49	0.50	0.49
Wt>W0 @1.2 FH	0.78	0.84	0.81	0.59	0.96	0.73	0.83	0.94	0.70	0.89	0.75	0.74	0.92	0.71	0.88	0.75

Table 7.1 Simulation results for 16 AEX stocks

This table shows that the historical 10-day VaR for investing 1000 euros in Unilever (UNA) is around 90 euros, where the critical wealth level of Foster & Hart is around 70. The fact that round 50% of the experiments lead to a situation for which $W_T > W_0$ shows that the critical wealth level is indeed the threshold value. Values of $Q(g) < R(g)$ lead to $\lim_{t\to\infty} P(W_t > W_0) = 0$ and the limit goes to one for values of $Q(g) > R(g)$, illustrated by the simulation results of $0.8 * R(g)$ and $1.2 * R(g)$.

The computation of wealth levels process applied is as follows:

1. Daily adjusted-closing prices are derived from finance.yahoo.com;
2. From the daily closing prices the daily returns are computed;
3. The daily returns are multiplied by the starting capital (W_0) to compute actual cash flows. (Since $Q(\lambda g) = \lambda Q(g)$ for every $\lambda > 0$ this could also be done as final step.);
4. The algorithm computes the critical wealth level for the assets;
5. As a comparative number the 10-day VaR with $\alpha = 0.99$ is computed both with historical simulation and normal approximation.

Likewise as executed for binomial gambles in Section 7.2 a simulation has been run with these numbers again showing a sharp distinction at the threshold wealth $Q(g) = R(g)$. This process follows these steps:

1. Fill in a number for t (the number of time steps) and m (the number of experiments), creating a $t * m$ matrix of ones;
2. Fill each column with a random stream of probable cash flows, derived from the matrix created in step 3 of the previous process;
3. Each column is treated as a different experiment, in a loop with initial wealth W_0 all t steps are followed and for each step the wealth at W_t is calculated by means of the shares strategy as explained in Section 4.2;
4. The final wealth W_T is stored in a results matrix and for all m experiments the final wealth is compared with the initial wealth;
5. The percentage of situations in which the final wealth exceeds the initial wealth is computed and stored as a single number for all m experiments;
6. By repeating this process for different values of $Q(g)$ around $R(g)$ the sharp distinction at $Q(g)$ can easily been shown, especially at relatively high values for t and m.

7.4 Sensitivity analysis

The FH measure solely depends on the distribution of the gamble. For market risk these distributions are empirical as discussed in previous sections. This implies that computing the critical wealth level still involves a non-objective component, namely the number of days of data. In the following table the values of the normal 10-day VaR, historical 10-day VaR and the FH critical wealth level for investing €1,000 in the AEX index are displayed for different days of data.

#Days	Norm 10d VaR	Hist 10d VaR	FH
100	54.25	69.72	72.17
200	63.24	68.17	48.44
300	70.39	79.44	146.25
400	91.59	108.50	151.40
500	91.64	112.03	*
600	87.43	104.21	1201.96
700	86.76	104.21	741.66
800	89.98	104.37	666.24
900	88.99	100.84	342.57
1000	90.74	100.84	166.53
...	...	...	...
2500	102.05	131.24	697.57

Table 7.2 Sensitivity to amount of data for VaR and FH compared for AEX

Note that the critical wealth level with 500 days of data is incomputable. This is due to the fact that $E(g) < 0$ for that period. The FH measure only exists if $E(g) > 0$ and $P(x < 0) > 0$ (x being outcomes of the gamble).

Also note that from $t = 600$ the critical wealth level is strictly greater than the VaR levels. This period incorporates the highly volatile market conditions in the credit crisis for which the FH measure accounts. VaR ignores the extreme events in this period(s), resulting in a stable capital requirement following VaR calculations. The graph shows the distribution of the daily returns for the 600-day period; the mean of the distribution is 7.3579E-05, the standard deviation is 0.011915 (about 162 times the mean).

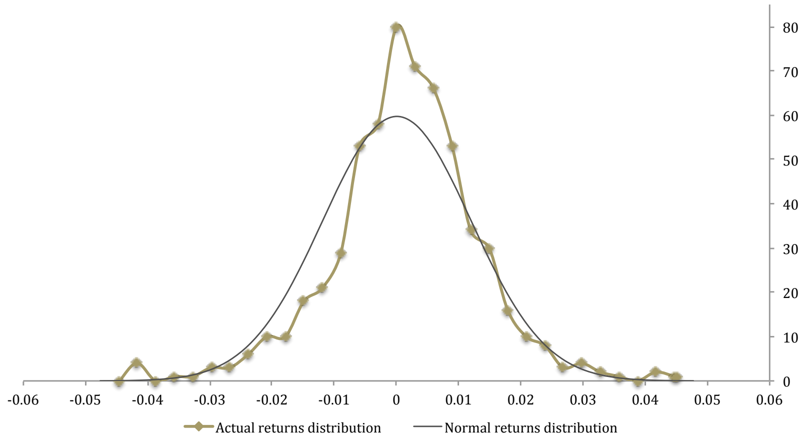


Figure 7.3 Distribution of 600 days AEX daily returns

Because VaR ignores the tail of the distribution it is more stable than the FH measure with respect to the amount of data used in computing it. It is highly disputable whether this is an advantage of VaR over the critical wealth level, since this shows the 'blind spot' of VaR as discussed in Sections 3.1, 4.2 and 5.2: it ignores the low-probability high-impact events in the tails of the distribution.

The table also reveals a problem that has been addressed before in practical discussion. For assets with unknown distributions (the ones needing historical data to compute parameters) the critical wealth level is rather dependent on the length of the data array used in computing it. It is impossible to say which value of the critical wealth level is the 'correct one' for the future. This is however not a shortcoming of the FH measure alone, but a consequence of the uncertainty involved in stock trading, and all risk measures suffer from this shortcoming.

For four stocks of the sixteen investigated in the previous Section 7.3 it is impossible to compute several critical wealth levels, due to the fact that for some periods $E(g) < 0$. For the other twelve a sensitivity analysis has been conducted for separate stocks and different days of data. In Appendix H the results for all stocks can be found, three examples are given here.

Stock: Aegon

#Days	Norm 10d VaR	Hist 10d VaR	FH
100	114.91	137.38	135.10
150	120.10	133.35	109.41
200	140.86	153.19	123.91
250	160.32	198.83	567.85
300	164.72	179.36	155.84
350	183.34	202.93	296.17
385	196.72	212.93	211.35

Stock: Heineken

#Days	Norm 10d VaR	Hist 10d VaR	FH
100	78.47	76.32	37.40
150	86.81	76.96	44.24
200	86.66	75.92	37.15
250	89.03	78.76	52.36
300	89.43	81.85	48.08
350	90.61	84.93	54.58
385	91.57	93.25	54.22

Stock: Wolters Kluwer

#Days	Norm 10d VaR	Hist 10d VaR	FH
100	69.24	87.19	102.15
150	63.44	77.08	60.03
200	83.55	87.19	50.00
250	89.09	97.30	160.96
300	86.23	92.05	94.39
350	95.75	97.30	132.56
385	100.85	99.97	120.09

Table 7.3 Sensitivity to amount of data compared for three stocks

These figures show again that it is impossible to conclude which value is the ‘correct one’. They may give the reader an impression of the differences between the critical wealth level and the VaR capital requirements for market risk. The results also show the dependence of these measures on the amount of data used as input. Consistent updating of the measures, as is current standard practice, seems to be the only logical ‘solution’ to this problem.

8. CONCLUSIONS

This section first answers the sub questions posed in the research design. The main findings and answer to the main research question follows subsequently.

THEORETICAL CONCEPTS AND FRAMEWORK

1. What is risk and risk management?
2. What are the current risk measures?
3. What are the shortcomings of current risk measures?

Financial risk is the quantifiable likelihood of loss or less than expected returns. The Basel Committee on Banking Supervision recognizes (in the current framework) three types of risk: market risk, credit risk and operational risk. Risk management covers the complete field of assessing and dealing with these risks.

The quantifiable pillar under Basel is the minimum capital requirement of 8% of risk-weighted assets, for which VaR (or measures closely related to VaR) is the preferred approach. VaR is advocated by the industry for its simplicity and intelligibility, but criticized from both science and practice for its manipulability and undesirable properties. Traders can create portfolios with acceptable VaR levels, but with low probability events of giant proportions.

THEORETICAL COHERENT RISK MEASURES AND FH

4. Why the need for coherent risk measures?
5. What is the Foster & Hart risk measure?

From a theoretical perspective VaR is criticized for violating an axiom of Artzner et al. (1999), namely the sub-additivity. It satisfies the other three axioms: positive homogeneity, monotonicity and translation invariance. The FH measure, discovered in search of an operational interpretation of the AS index, also violates one of these four axioms. This is in no way a major drawback, since this axiom is not relevant in the FH setup, as shown in Section 5.2.

The FH measure identifies the 'critical wealth level' below which it becomes risky to accept a gamble. It is designed to measure risk with a single number, based on facing an unknown sequence of the same gamble. This is consistent with, for instance, trading stocks.

$$E\left[log\ \left(1+\frac{1}{R(g)}g\right)\right]=0.$$

The risk in accepting a gamble g for which it is equally likely to gain 120 or lose 100 clearly depends on the amount of wealth one has. If one has only 100 or less, it is extremely risky to accept such a gamble. If one has for instance 1 million, it is not risky at all to accept the gamble. Foster & Hart show that there is a well-defined *critical wealth level* that separates between two situations: one for which it is risky to accept the gamble and the other for which it is not.

Practical framework and applicability

6. What are the practical aspects of current measures and regulation?
7. What are the (dis)advantages of the FH risk measure in comparison?
8. Is the FH measure applicable as risk measure in practice?

Banks are allowed by regulators to use their own approaches and models in calculating VaR (for market risk) or WCDR (for credit risk). Typically for market risk banks compute the 10-day VaR with $\alpha = 0.99$ and for credit risk a 1-year WCDR with $\alpha = 0.999$. There is growing attention towards stress testing since statistical risk measures have their limitations.

The advantage of the FH measure over VaR is that it is objective (independent of the decision maker), scale-invariant, monotonic, and has an operational interpretation and a simple formula. Besides that the FH measure incorporates the dynamics of repeated gambles, where VaR is criticized for being static.

On the downside both measures depend on the historical data for assets with unknown (or partially known) distributions. From a risk-averse perspective a major benefit of the FH measure is that it always assumes a critical wealth level that is greater than the maximum loss. The simulations show that this sometimes might lead to problematically large values for the amount of capital a bank needs to hold.

Several theoretical issues are, however, a point of interest. The FH measure is developed for discretely distributed variables and in this paper tested for discretely distributed or discretely approximated variables only. Continuous variables with bounded losses have a critical wealth level that is equal to the absolute maximum loss. For continuous variables with unbounded losses, the FH measure has no solution. This poses hardly any problem at all in practice: most software packages use discrete approximations and limitless loss is not a realistic assumption. However it remains an important theoretical issue to overcome.

The FH measure (and thus the AS index also) is applicable as a risk measure in practice. It is computable by means of a computationally easy algorithm within a commonly known package (Matlab).

Practical implications

11. What are the practical implications of using the FH as a risk measure?

The data that is needed in computing VaR is the same as needed for the FH measure, so in computing there are little practical implications. The FH measure is however slightly more complex than VaR, it is not easy to understand at first sight what the measure does and where the sharp distinction between ‘infinite wealth’ and ‘bankruptcy’ comes from. Appendix A might be a first step in the direction of intelligibility of this measure. To be accepted in practice banks or regulators should apply the FH measure for a long period besides current computations.

Computations and application

12. If the FH risk measure is applicable, how does it perform compared to common alternatives for different distributions?

    There is no general pattern with respect to the outcomes in comparing the critical wealth levels of the FH measure with the VaR levels. Both measures depend on the amount of data put in the computation; the FH measure reacts more to volatile situations. For some stocks, for example Heineken, the critical wealth level is less than the 10-day VaR for all periods of data used. For others the neglect of VaR for the tails of the distributions leads to an underestimation of the amount of capital needed in order to remain a going concern.

Main findings

How can the Foster & Hart risk measure be applied by (regulators of) banks that are member of or under jurisdiction of the Basel Committee on Banking Supervision?

1. The FH measure is certainly a type of measurement that might play a role in the future. Probably this will be in a modified form, because additional contributions still need to crystallize. It has desirable characteristics and is certainly a measure to take into account.

2. The FH measure is practically applicable and computationally quite easy. With standard software packages an algorithm can be used (for instance the one in the Appendix) to compute the critical wealth level. This requires only small adaptations of the current software and does not require any additional resources or investments.

3. Actual acceptance will take a long way: there is always skepticism towards new developments in a conservative world like the financial world. Besides that current trends shift towards stress testing instead of steady risk figures.

4. Despite the last statement of the previous conclusion: for as long as single number representations of risk are still used, the FH measure is of interest to anyone assessing risk, especially compared to the current measures used.

5. This paper showed that the FH measure is applicable, can be computed with little difficulty and the simulations show that indeed the claim of long-run avoidance of bankruptcy implies a critical wealth level of $R(g)$.

9. Further research and puzzles

The final section of this paper adds recommendations for further research and addresses some final puzzles that are still to be solved. Since the recommendations follow from conclusions and remarks discussed in this paper, they are simply given here as recommendations without further explanation (unless explicitly needed).

1. Conduct an extended research of the initial computation within market risk, with explicit attention towards the following situations:
   - Multiple real-life distributions for financial products;
   - Based on actual portfolios;
   - Specific attention for 'special cases' in computing the FH measure, for example: distributions with expectation close to zero, distributions with fat tails, distributions with extremely positive expectation, situations for which $R(g)$is very close to $L(g)$.
2. Compare the behavior of the FH measure and VaR thoroughly within market risk based on real-life data from a bank;
3. Conduct an extended comparison with other risk measures that are computationally easy and attractive for banks (for example Expected Shortfall, the Aumann-Serrano index);
4. Broaden the scope of Recommendations 1 through 3 by including other types of risk, especially credit risk;

From a theoretical point of view there are some puzzles to be solved:

5. Especially focus on continuous distributions, both with and without unbounded losses;
6. At first sight it seems to be counter-intuitive that for two completely differently distributed continuous variables with the same lower limit, the critical value is the same: $R(g) = L(g)$;
7. Also, focus on alternatives or setups with assets that have (sometimes) $\mathrm{E}(g) < 0$ or $\mathrm{P}(x < 0) = 0$. For many real-life financial assets this might be (for some time horizons) the case;
8. Extend the research theoretically by studying the papers of Homm & Pigorsch (2010) and Bali et al. (2011) in depth;
9. Investigate the effect of transaction costs on the applicability of the simple shares strategy (see Appendix I).

The scope of our research was directed towards banks, and although the regulatory framework for other financial institutions has many similarities with the regulation for banks there are however differences between for example Solvency II and Basel II. It is interesting to examine the practical implications for other financial institutions in more depth than we have done. The final recommendation is therefore:

10. Conduct an investigation that includes the practical framework for other financial institutions in order to analyze whether the FH measure or its close relatives are applicable for non-banking financial institutions as well.

REFERENCES

Acerbi, C., & Tasche, D. (2002). Expected Shortfall: a natural coherent alternative to Value at Risk. *Economic Notes 31*, 379-388.

Arrow, K. (1964). The role of securities in the optimal location of risk-bearing. *Review of Economic Studies 31*, 91-96.

Artzner, P., Delbaen, F., Eber, J.-M., & Heath, D. (1999). Coherent measures of risk. *Mathematical Finance*, 9, pp. 203-228

Artzner, P., Delbaen, F., Eber, J.-M., & Heath, D. (2004). Thinking Coherently. In Carr, P. (Ed.), *Derivatives Pricing: The Classic Collection* (p. 395-403). London: Incisive Financial Publishing Ltd.

Aumann, R. J., & Serrano, R. (2008). An economic index of riskiness. *Journal of Political Economy 116*, 810-836.

Bali, T. G., Cakici, N., & Chabi-Yo, F. (2011). A generalized measure of riskiness. *Management Science 57*, 1406-1423.

Bank of International Settlements. (1999). *International convergence of capital measurement and capital standards.* (BCBS 111) Basel: Bank for International Settlements.

Bank of International Settlements. (2006). *International Convergence of Capital Measurement and Capital Standards.* (BCBS 128) Basel: Bank for International Settlements.

Bank of International Settlements. (2009). *History of the Basel Committee and its Membership.* (BCBS History) Basel: Bank for International Settlements.

Bank of International Settlements. (2010). *Basel III: International framework for liquidity risk measurement, standards and monitoring.* (BCBS 188) Basel: Bank for International Settlements.

Bank of International Settlements. (2011). *Basel III: A global regulatory framework for more resilient banks and banking systems.* (BCBS 189) Basel: Bank for International Settlements.

Beder, T. S. (1995). VaR: seductive but dangerous. *Financial Analysts Journal 51*, 12-24.

Caballero, R. J., & Krishnamurthy, A. (2008). Collective Risk Management in a Flight to Quality Episode. *Journal of Finance 63*, 2195-2230.

COUNCIL REGULATION (EC) No 479/2009. Official Journal of the European Communities L145 (10-06-2009) pp. 1-9.

Credit crisis could cost nearly $1 trillion, IMF predicts. (2008, April 8). *The New York Times*. Retrieved December 17, 2011, from http://www.nytimes.com/2008/04/08/business/worldbusiness/08iht-imf.3.11771908.html

Crocker, K. J. (2003). Risk and Risk Management. In D. J. Meyer (Ed.), *The Economics of Risk*. Kalamazoo, MI: W.E. Upjohn Institute for Employment Research, (pp. 9-16).

Crouhy, Michel G., Jarrow, Robert A., & Turnbull, Stuart M. (2008). The Subprime Credit Crisis of 2007. *The Journal of Derivatives*, *16*, 81-110.

De Vaus, D. (2010). *Research Design in Social Research* (4th ed.). London: SAGE Publications Ltd.

Einhorn, D. (June/July 2008), Private Profits and Socialized Risk, GARP Risk Review

Föllmer, H., & Schied, A. (2002). Convex measures of risk and trading constraints. *Finance and Stochastics 6*, 429–447.

Föllmer, H., & Schied, A. (2010). Convex and Coherent Risk Measures. In R. Cont (Ed.) *Encyclopedia of Quantitative Finance.* Hoboken, NJ: John Wiley & Sons, (pp. 355-363).

Foster, D. P., & Hart, S. (2009). An operational measure of riskiness. *Journal of Political Economy 117*, 785-814.

Foster, D. P., & Hart, S. (2011). A wealth-requirement axiomatization of riskiness. (Hebrew University of Jerusalem, Working paper.) Retrieved March 27, 2012, from: http://math.huji.ac.il/~hart/papers/risk-ax.pdf

Grootweld, H., & Hallerbach, W. G. (2000). *Upgrading VaR from Diagnostic Metric to Decision Variable: A Wise Thing to Do?* (Report 2003 Erasmus Center for Financial Research, June).

Hanoch, G., & Levy, H. (1969). The efficiency analysis of choices involving risk. *Review of Economic Studies 36*, 335-346.

Hart, S. (2011). Comparing risks by acceptance and rejection. (Hebrew University of Jerusalem, Working paper.) Retrieved March 28, 2012, from: http://math.huji.ac.il/~hart/papers/risk-u.pdf

Homm, U., & Pigorsch, C. (2010). An economic index of riskiness: A comment. (University of Bonn, Working paper.) Retrieved February 25, 2012 from http://ect-pigorsch.mee.uni-bonn.de/

Horcher, K. A. (2005). *Essentials of financial risk management.* Hoboken, NJ: John Wiley & Sons.

Hull, J. C. (2010). *Risk Management and Financial Institutions* (2nd ed.). Boston, MA: Prentice Hall.

Jorion, P. (2001). *Value at Risk: The new benchmark for managing financial risk* (2nd ed.). New York, NY: McGraw-Hill.

Kalse, E. (2008, September 17). Kredietcrisis in vijf stappen. *NRC.* Retrieved December 17, 2011, from http://vorige.nrc.nl/nieuwsthema/kredietcrisis/article1987320.ece

Luhmann, N. (1996). *Modern Society Shocked by its Risks* (University of Hong Kong, Department of Sociology Occasional Papers 17), Hong Kong.

Machina, M. J., & Rothschild, M. (2008). Risk. In S. N. Durlauf & J. E. Blume (Eds.), *The New Palgrave Dictionary of Economics*. New York, NY: Palgrave Macmillan, (pp. 190-197).

Markowitz, H. (1952). Portfolio Selection. *Journal of Finance 7*, 77-91.

Moody's Analytics. (2012). *Basel III New Capital and Liquidity Standards – FAQs*. Moody's Analytics Inc. Retrieved February 18, 2012, from http://www.moodysanalytics.com/~/media/Insight/Regulatory/Basel-III/Thought-Leadership/2012/2012-19-01-MA-Basel-III-FAQs.ashx

Oxford dictionary (7th ed.). (2000). Hinsdale, IL: Penguin Press.

Pratt, J. (1964). Risk aversion in the small and in the large. *Econometrica 32*, 122-136.

Riedell, F., & Hellmann, T. (2013). *The Foster-Hart Measure of Riskiness for General Gambles.* (Institute of Mathematical Economics, Bielefeld Universität, Working Paper No. 474). Retrieved February 4, 2013, from: http://wwwhomes.uni-bielefeld.de/friedel/workingpapers.html

Risk. (n.d.). *Investopedia.com.* Investopedia ULC. Retrieved February 20, 2012, from http://www.investopedia.com/terms/r/risk.asp

Risk. (n.d.). *Investorwords.com*. WebFinance Inc. Retrieved February 20, 2012, from http://www.investorwords.com/4292/risk.html

Rockafellar, T. R., & Uryasev, S. P. (2002). Conditional value-at-risk for general loss distributions. *Journal of Banking & Finance 26,* 1443-1471.

Rockafellar, T. R., Uryasev, S. P., & Zabarankin, M. (2002). *Deviation Measures in Risk Analysis and Optimization.* (University of Florida, Department of Industrial & Systems Engineering Working Paper No. 2002-7). Retrieved February 24, 2012, from: http://ssrn.com/abstract=365640

Rothschild, M., & Stiglitz, J. (1970). Increasing risk: I. A definition. *Journal of Economic Theory 2*, 225-243.

Schifferes, S. (2009, July 31). '$10 trillion' credit crunch cost. *BBC News*. Retrieved December 17, 2011, from http://news.bbc.co.uk/2/hi/8177814.stm

Sommer, J. (2011, August 13). Whatever happened to 'risk free'. *The New York Times.* Retrieved December 17, 2011, from http://www.nytimes.com/2011/08/14/your-money/when-a-risk-free-investment-suddenly-is-not.html

Tabuchi, H. (2011, August 23). Moody's cuts Japan's rating one notch, citing its giant debt. *The New York Times*. Retrieved December 17, 2011, from http://www.nytimes.com/2011/08/24/business/global/japans-credit-rating-cut-by-moodys.html

Taleb, N. (April 1997), The Jorion-Taleb Debate, Derivatives Strategy. Retrieved February 20, 2012, from http://www.derivativesstrategy.com/magazine/archive/1997/0497fea2.asp

Weber, C. (Ed.). (2002). Webster's dictionary (4th ed., Vols. 1-4). Chicago, IL: Webster Press.

Foster and Hart measure of riskiness

Introduction

Dean Foster and Sergiu Hart proposed an operational measure of riskiness, inspired by the work of Nobel Laureate Robert Aumann and Roberto Serrano.

An important feature of the FH measure, compared to current measures, is that it is objective: it depends only on the distribution of the gamble and not on the decision maker.

Goal

The FH measure identifies the 'critical wealth level' below which it becomes risky to accept a gamble.

It is designed to measure risk with a single number, based on facing an unknown sequence of the same gamble. This is consistent with, for instance, trading stocks.

Measure

$$E\left[log\left(1+\frac{1}{R(g)}g\right)\right] = 0.$$

Rationale

The risk in accepting a gamble g for which it is equally likely to gain 120 or lose 100 clearly depends on the amount of wealth one has. If one has only 100 or less, it is extremely risky to accept such a gamble. If one has for instance 1 million, it is not risky at all to accept the gamble.

Foster and Hart show that there is a well-defined *critical wealth level* that separates between two situations: one for which it is risky to accept the gamble and the other for which it is not.

Simple shares strategy

If an investment allows an investor to accept any proportion of the gamble g, the investor accepts the proportion

$\alpha = \frac{W_t}{Q(g)}$, implying that the wealth at the next time will be

$$W_{t+1} = W_t + \alpha_t g = W_t + \frac{W_t}{Q(g)}g = W_t\left(1+\frac{1}{Q(g)}g\right)$$

Assuming that $Q(g)$ is 200, then the current wealth W_t will equally likely be multiplied by 1+120/200 = 1.6 or 1-100/200 = 0.5. In the long run, by the Law of the Large Numbers, the wealth will be multiplied about half the time by 1.6 and about half the time by 0.5. This means that there is a yield factor on average of $\gamma = \sqrt{1.6 * 0.5} < 1$ per period.

Foster and Hart show that the yield factor is larger than 1 if and only if the expectation of the log of the relative gross returns is larger than 0, thereby explaining the rationale behind the formula.

Appendix B – Matlab Script

```
%script to import xls data for Foster & Hart measure computation
%Job Arnold
%University of Twente

clear;
clc;

%import xls and declare global variables Pi and Vi
complete=xlsread('../VaR computation.xls','FHDATA','A2:B835');
global Pi;
global Vi;
Pi=complete(1:end,1);
Vi=complete(1:end,2);
if ne(size(Pi,1),size(Vi,1))
   warning('The input vectors are of different size!')
end

%define lower and upper bound for initial estimation of Rg
%Rglow is based on absolute value of Lg plus allowed error (initial
1E-09)
%Rghigh is based on absolute value of Lg squared
Lg=min(Vi);
Rglow=abs(Lg)+e;
Rghigh=abs(Lg)*10;

%call function solution Rg
Rgsolution=fzero(@FH,[Rglow,Rghigh]);

FH_wealth=Rgsolution;
FH_wealth




%script that calculates the FH measure for estimates of Rg
%Job Arnold
%University of Twente

function y = FH(Rg)

global Pi Vi;

sum = 0;

for i=1:1:size(Pi,1)
    row=Pi(i)*log10(1+(Vi(i)/Rg));
    sum=sum+row;
end

y = sum;
```

APPENDIX C – COMPUTATIONS WITH SOLVER FOR ARBITRARY DISTRIBUTIONS

				R(g) laag	**1503**
g				**R(g) hoog**	**1504**
p(i)	**x(i)**	**R(g) laag**	**R(g) hoog**		
0.01	-1500	-0.0269984	-0.025751878		
0.04	-50	-0.0005877	-0.000587337		
0.95	100	0.02657581	0.026558701		
	E[log(1+g/R(g))]:	-0.0010103	0.000219486		
E(g)	78				
VaR0.95:	-100				
VaR0.99	50				
R(g)	???				
mean	20				
stdev	20				
				Rg	1500
lowbound	-100				
upperbound	140				

n	Vi	Pi	E(g)	log
241		0.999999998	19.99999997	0.005714724
1	-100	3.03794E-10	-3.03794E-08	-9.10265E-12
2	-99	4.09567E-10	-4.05471E-08	-1.21449E-11
3	-98	5.50788E-10	-5.39772E-08	-1.61619E-11
4	-97	7.38854E-10	-7.16688E-08	-2.14516E-11
5	-96	9.8866E-10	-9.49113E-08	-2.83984E-11
6	-95	1.31962E-09	-1.25364E-07	-3.7497E-11
7	-94	1.75698E-09	-1.65156E-07	-4.93815E-11
8	-93	2.33344E-09	-2.1701E-07	-6.48631E-11
9	-92	3.09131E-09	-2.84401E-07	-8.49758E-11
10	-91	4.0851E-09	-3.71744E-07	-1.11034E-10
11	-90	5.38488E-09	-4.84639E-07	-1.44703E-10
12	-89	7.0805E-09	-6.30165E-07	-1.88088E-10
13	-88	9.28681E-09	-8.17239E-07	-2.4384E-10
14	-87	1.21502E-08	-1.05707E-06	-3.15287E-10
15	-86	1.58567E-08	-1.36368E-06	-4.06596E-10
16	-85	2.06424E-08	-1.7546E-06	-5.2297E-10
17	-84	2.68052E-08	-2.25163E-06	-6.7088E-10
18	-83	3.4721E-08	-2.88184E-06	-8.58352E-10

APPENDIX D – COMPARISON VAR AND FH FOR AAPL

HISTORICAL SIMULATION VAR - AAPL
FH COMPUTATION - AAPL

Data			VaR			FH		
AAPL			142336.92			72576.94		
			input:					
S0	610		Alpha	0.99				
Asset value	1000000		T	10				
N	834							
							Lg	55939.7499
			0.99th percentile		45010.8852		Rg	72576.9408
			10day VaR0.99		142336.917		E	3.8687E-10

DATA			VaR			FH		
DAY	Date	Adj Close	Scenarios	Portf. Value	Loss	Pi	Xi	FHM
834	4/25/12	610	664.1322196	1088741.344	-88741.34361	0.001199041	88741.34361	0.000415929
833	4/24/12	560.28	597.8149379	980024.4884	19975.51163	0.001199041	-19975.51163	-0.000167627
832	4/23/12	571.7	608.6372997	997766.0651	2233.934867	0.001199041	-2233.934867	-1.62803E-05
831	4/20/12	572.98	594.9846793	975384.7201	24615.27986	0.001199041	-24615.27986	-0.000215713
830	4/19/12	587.44	589.0429694	965644.2121	34355.78788	0.001199041	-34355.78788	-0.000333927
829	4/18/12	608.34	608.6393308	997769.3948	2230.605216	0.001199041	-2230.605216	-1.62556E-05
828	4/17/12	609.7	641.0925137	1050971.334	-50971.33401	0.001199041	50971.33401	0.000277024
827	4/16/12	580.13	584.7021793	958528.1628	41471.83715	0.001199041	-41471.83715	-0.000441207
826	4/13/12	605.23	592.8196605	971835.5091	28164.4909	0.001199041	-28164.4909	-0.000255748
825	4/12/12	622.77	606.6587352	994522.5168	5477.483232	0.001199041	-5477.483232	-4.08629E-05
824	4/11/12	626.2	607.8257272	996435.6184	3564.381643	0.001199041	-3564.381643	-2.62237E-05
823	4/10/12	628.44	602.5311601	987756.0002	12243.99981	0.001199041	-12243.99981	-9.6216E-05
822	4/9/12	636.23	612.454709	1004024.113	-4024.113117	0.001199041	4024.113117	2.81008E-05
821	4/5/12	633.68	619.1552274	1015008.569	-15008.56946	0.001199041	15008.56946	9.7882E-05
820	4/4/12	624.31	605.143806	992039.0263	7960.973749	0.001199041	-7960.973749	-6.05022E-05
819	4/3/12	629.32	620.5408726	1017280.119	-17280.11897	0.001199041	17280.11897	0.000111215
818	4/2/12	618.63	629.4125594	1031823.868	-31823.8679	0.001199041	31823.8679	0.000189335
817	3/30/12	599.55	599.6876332	983094.4807	16905.5193	0.001199041	-16905.5193	-0.000138089

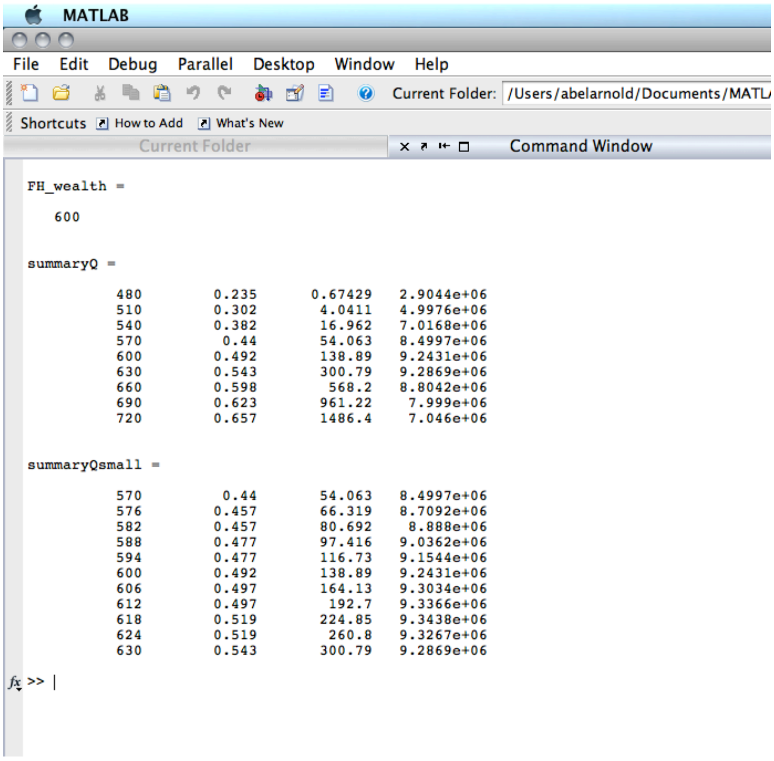
APPENDIX E – SIMULATION CODE MATLAB

```
%Value of FH critical wealth, used as middle point of simulation
values
Rg=FH_wealth;

%Simulation values for Q (critical wealth) based on Rg
Q=[0.8*Rg 0.85*Rg 0.9*Rg 0.95*Rg Rg 1.05*Rg 1.1*Rg 1.15*Rg 1.2*Rg];
Qs=[0.95*Rg 0.96*Rg 0.97*Rg 0.98*Rg 0.99*Rg Rg 1.01*Rg 1.02*Rg
1.03*Rg 1.04*Rg 1.05*Rg];

%Initilization of counters for simulation
num_cols = size(P,2);
num_rows = size(P,1);
num_Q = size(Q,2);
num_Qs = size(Qs,2);

%Storage matrix for results
resQ=zeros(num_Q,num_cols);
pctQ=zeros(num_Q,1);

resQs=zeros(num_Qs,num_cols);
pctQs=zeros(num_Qs,1);



%---
%SIMULATION FOR ALL VALUES OF Q

%Simulation loop for Q
for k=1:1:num_Q
    for i=1:1:num_cols
    W=W0;
        for j=1:1:num_rows
        W=W+(W/Q(k))*P(j,i);
        end
    resQ(k,i)=W;
    end
end

%Loop counting results after final period that exceeds initial wealth
for ii=1:1:num_Q
count=0;
    for jj=1:1:num_cols
        if resQ(ii,jj)>=W0
            count=count+1;
        end
    end
    pctQ(ii)=count/num_cols;
end

%Construct table with summary statistics
X_y=Q';

meanQ=mean(resQ,2);
medianQ=median(resQ,2);

summaryQ(:,1)=X_y(:,1);
summaryQ(:,2)=pctQ(:,1);
```

```
summaryQ(:,3)=medianQ(:,1);
summaryQ(:,4)=meanQ(:,1);

format shortg
summaryQ
%xlswrite('simulationresults.xls',summaryQ,Q,'A2')


%---
%SIMULATION FOR ALL VALUES OF Q_small

%Simulation loop for Q_small
for x=1:1:num_Qs
    for y=1:1:num_cols
    W=W0;
        for z=1:1:num_rows
        W=W+(W/Qs(x))*P(z,y);
        end
    resQs(x,y)=W;
    end
end

%Loop counting results after final period that exceeds initial wealth
for xx=1:1:num_Qs
count2=0;
    for yy=1:1:num_cols
        if resQs(xx,yy)>=W0
            count2=count2+1;
        end
    end
    pctQs(xx)=count2/num_cols;
end

%Construct table with summary statistics
X_y=Qs';

meanQs=mean(resQs,2);
medianQs=median(resQs,2);

summaryQsmall(:,1)=X_y(:,1);
summaryQsmall(:,2)=pctQs(:,1);
summaryQsmall(:,3)=medianQs(:,1);
summaryQsmall(:,4)=meanQs(:,1);

format shortg
summaryQsmall
%xlswrite('simulationresults.xls',summaryQsmall,'Qsmall','A2')
```

Appendix F – Results for G: {-120,200} with equal probabilities

{t, m} = {1000, 1000}



MATLAB

File Edit Debug Parallel Desktop Window Help

Current Folder: /Users/abelarnold/Documents/MATL

Shortcuts How to Add What's New

Current Folder Command Window

FH_wealth =

600

summaryQ =

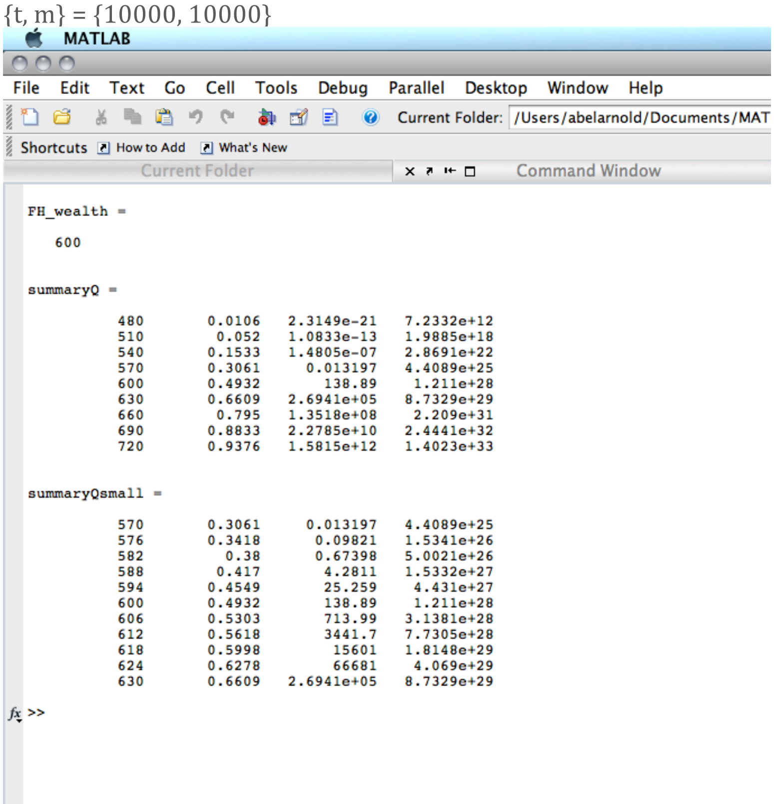
480	0.235	0.67429	2.9044e+06
510	0.302	4.0411	4.9976e+06
540	0.382	16.962	7.0168e+06
570	0.44	54.063	8.4997e+06
600	0.492	138.89	9.2431e+06
630	0.543	300.79	9.2869e+06
660	0.598	568.2	8.8042e+06
690	0.623	961.22	7.999e+06
720	0.657	1486.4	7.046e+06

summaryQsmall =

570	0.44	54.063	8.4997e+06
576	0.457	66.319	8.7092e+06
582	0.457	80.692	8.888e+06
588	0.477	97.416	9.0362e+06
594	0.477	116.73	9.1544e+06
600	0.492	138.89	9.2431e+06
606	0.497	164.13	9.3034e+06
612	0.497	192.7	9.3366e+06
618	0.519	224.85	9.3438e+06
624	0.519	260.8	9.3267e+06
630	0.543	300.79	9.2869e+06

fx >> |

{t, m} = {10000, 10000}



MATLAB

File Edit Text Go Cell Tools Debug Parallel Desktop Window Help

Current Folder: /Users/abelarnold/Documents/MAT

Shortcuts How to Add What's New

Current Folder Command Window

FH_wealth =

600

summaryQ =

480	0.0106	2.3149e-21	7.2332e+12
510	0.052	1.0833e-13	1.9885e+18
540	0.1533	1.4805e-07	2.8691e+22
570	0.3061	0.013197	4.4089e+25
600	0.4932	138.89	1.211e+28
630	0.6609	2.6941e+05	8.7329e+29
660	0.795	1.3518e+08	2.209e+31
690	0.8833	2.2785e+10	2.4441e+32
720	0.9376	1.5815e+12	1.4023e+33

summaryQsmall =

570	0.3061	0.013197	4.4089e+25
576	0.3418	0.09821	1.5341e+26
582	0.38	0.67398	5.0021e+26
588	0.417	4.2811	1.5332e+27
594	0.4549	25.259	4.431e+27
600	0.4932	138.89	1.211e+28
606	0.5303	713.99	3.1381e+28
612	0.5618	3441.7	7.7305e+28
618	0.5998	15601	1.8148e+29
624	0.6278	66681	4.069e+29
630	0.6609	2.6941e+05	8.7329e+29

fx >>

Appendix G – Market Risk simulation results for 16 stocks

Insufficient data: AF, CORA, DE, ULA
Negative trend: MT, FUR, KPN, SBMO, TNTE

	AGN	AH	AKZA	APAM	ASML	BOKA	DSM	HEIA	INGA	PHIA	PNL	RAND	REN	RDSA	UNA	WKL
Norm 10d VaR	196.72	78.77	138.77	240.20	124.32	148.09	106.15	91.57	239.41	124.88	1118.14	168.18	80.01	73.48	71.32	100.85
Hist 10d VaR	212.93	114.52	157.93	225.17	113.00	167.03	105.77	93.24	251.12	141.81	255.58	176.56	78.57	93.67	79.44	99.97
FH	211.35	74.13	133.89	970.97	67.91	226.94	94.07	54.22	420.58	92.04	498.34	217.17	52.83	108.31	57.60	120.09
Wt>W0 @0.8 FH	0.10	0.05	0.09	0.40	0.00	0.17	0.06	0.01	0.25	0.03	0.10	0.15	0.01	0.20	0.02	0.13
Wt>W0 @FH	0.49	0.49	0.51	0.52	0.51	0.50	0.49	0.50	0.52	0.51	0.50	0.50	0.50	0.49	0.50	0.49
Wt>W0 @1.2 FH	0.78	0.84	0.81	0.59	0.96	0.73	0.83	0.94	0.70	0.89	0.75	0.74	0.92	0.71	0.88	0.75

t=5000
m=5000

List of abbreviations:

AF	AirFrance/KLM
AGN	Aegon
AH	Ahold
AKZA	Akzo Nobel
APAM	Aperam
ASML	ASML
BOKA	Boskalis/Westminster
CORA	Corio
DE	DE Master Blenders 1753
DSM	DSM
FUR	Fugro
HEIA	Heineken
INGA	ING
KPN	KPN
MT	Arcelor Mittal
PHIA	Philips
PNL	Post NL
RAND	Randstad
REN	Reed Elsevier
RDSA	Royal Dutch Shell
SBMO	SBM Offshore
TNTE	TNT Express
ULA	Unibail/Rodemco
UNA	Unilever
WKL	Wolters Kluwer

APPENDIX H – SENSITIVITY ANALYSIS RESULTS

Stock: Aegon

#Days	Norm 10d VaR	Hist 10d VaR	FH
100	114.91	137.38	135.10
150	120.10	133.35	109.41
200	140.86	153.19	123.91
250	160.32	198.83	567.85
300	164.72	179.36	155.84
350	183.34	202.93	296.17
385	196.72	212.93	211.35

Stock: Boskalis Westminster

#Days	Norm 10d VaR	Hist 10d VaR	FH
100	79.78	73.18	31.47
150	88.92	94.11	40.26
200	111.49	117.74	85.83
250	121.05	145.82	207.99
300	120.76	144.49	114.12
350	133.30	164.85	128.03
385	148.09	167.03	226.94

Stock: Randstad

#Days	Norm 10d VaR	Hist 10d VaR	FH
100	107.78	153.29	62.86
150	115.14	151.98	64.03
200	126.64	149.24	99.66
250	140.63	163.05	179.66
300	143.19	161.83	118.26
350	156.32	165.00	187.26
385	168.18	176.56	217.17

Stock: Ahold

#Days	Norm 10d VaR	Hist 10d VaR	FH
100	61.24	66.62	30.06
150	66.12	71.66	66.00
200	72.31	95.31	81.56
250	75.04	113.40	151.67
300	76.71	117.69	151.62
350	78.08	115.10	105.30
385	78.77	114.52	74.13

Stock: DSM

#Days	Norm 10d VaR	Hist 10d VaR	FH
100	84.14	95.81	49.00
150	87.86	98.40	105.12
200	94.85	107.46	111.98
250	98.16	107.61	350.62
300	96.32	111.16	109.31
350	100.11	107.61	143.50
385	106.15	105.77	94.07

Stock: Reed Elsevier

#Days	Norm 10d VaR	Hist 10d VaR	FH
100	65.93	70.03	35.68
150	62.52	65.48	32.03
200	71.55	76.26	33.22
250	74.01	77.96	47.43
300	72.63	76.26	46.08
350	76.88	77.96	61.41
385	80.01	78.57	52.83

Stock: Akzo Nobel

#Days	Norm 10d VaR	Hist 10d VaR	FH
100	106.82	147.28	79.29
150	114.32	173.98	187.50
200	114.17	171.05	96.48
250	118.16	168.13	280.63
300	120.38	147.28	137.08
350	128.50	168.13	192.07
385	138.77	157.93	133.89

Stock: Heineken

#Days	Norm 10d VaR	Hist 10d VaR	FH
100	78.47	76.32	37.40
150	86.81	76.96	44.24
200	86.66	75.92	37.15
250	89.03	78.76	52.36
300	89.43	81.85	48.08
350	90.61	84.93	54.58
385	91.57	93.25	54.22

Stock: Unilever

#Days	Norm 10d VaR	Hist 10d VaR	FH
100	55.00	50.95	34.46
150	54.91	54.07	64.26
200	62.75	56.20	39.67
250	64.61	55.07	52.87
300	68.32	85.22	76.97
350	70.86	80.27	70.72
385	71.32	79.44	57.60

Stock: ASML

#Days	Norm 10d VaR	Hist 10d VaR	FH
100	122.70	137.62	68.73
150	126.67	114.74	142.83
200	134.45	114.30	79.38
250	131.78	114.74	90.20
300	126.08	114.30	84.25
350	123.91	113.86	79.74
385	124.32	113.00	67.91

Stock: Philips

#Days	Norm 10d VaR	Hist 10d VaR	FH
100	92.33	89.58	54.42
150	93.86	95.06	74.60
200	95.09	89.58	46.27
250	105.12	130.23	80.27
300	106.09	139.00	89.05
350	115.04	130.23	110.96
385	124.88	141.81	92.04

Stock: Wolters Kluwer

#Days	Norm 10d VaR	Hist 10d VaR	FH
100	69.24	87.19	102.15
150	63.44	77.08	60.03
200	83.55	87.19	50.00
250	89.09	97.30	160.96
300	86.23	92.05	94.39
350	95.75	97.30	132.56
385	100.85	99.97	120.09

APPENDIX I – EFFECT OF TRANSACTION COSTS

In the final discussions of this paper the effect of transaction costs for, for example stocks, posed some questions on the applicability of the simple shares strategy for gambles with relative returns (instead of fixed amounts) and (relative) entering costs. Three situations are discussed here as a starting point for further discussion and analysis.

FIXED TRANSACTION COSTS

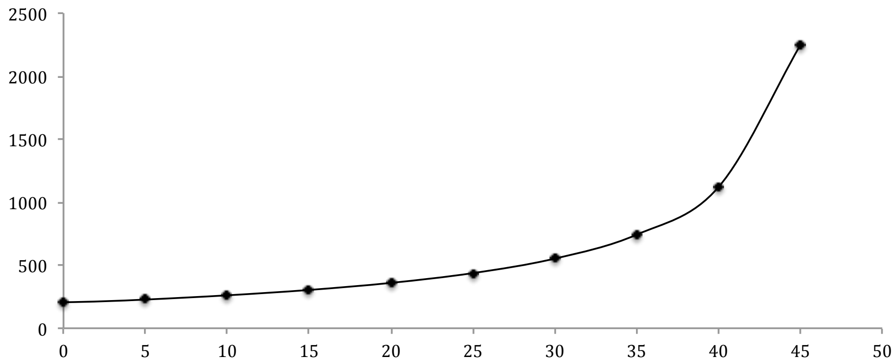
For gambles with fixed outcomes the effect of transaction costs is limited. Although there is no direct relationship between the transaction costs and the critical wealth level, the computation of the FH measure remains computationally easy. The transaction costs (c) are subtracted from the outcomes of the gamble, thus creating net results:

$$\mathrm{E}\left[log\left(1+\frac{g}{R(g)}\right)\right]=\sum_i p_i \log\left(1+\frac{x_i-c}{R(g)}\right)$$

Since the FH measure is not translation invariant the subtraction of a fixed amount does not decrease the riskiness by the same amount. Consider the following gamble:

$$g=\begin{cases}200-c & p=0.50\\ -100-c & p=0.50\end{cases}$$

The following graph and table depict the relationship between the transaction cost and the critical wealth level for g. Note that $\mathrm{E}(g)=0$ for $c=50$, hence $R(g)$ is non-existent.



c	0	5	10	15	20	25	30	35	40	45	50
R(g)	200	227.5	261.25	303.93	360	437.5	552.5	742.5	1120	2247.5	*

RELATIVE TRANSACTION COSTS

For gambles with relative costs (a percentage of the outcomes) the results are straightforward, since $R(g)$ is scale-invariant and thus $Q(\lambda g)=\lambda Q(g)$ for every positive lambda. Let τ represent the relative transaction cost, then $Q((1-\tau)g)=(1-\tau)\,Q(g)$.

Returns relative to transaction costs

The final problem addressed here is that of gambles with transaction costs and returns that relate to the amount invested (instead of fixed returns).

Consider the following situation for an investment in a stock:

$S_0 = 200$	transaction cost (spot price)
$W_0 = 1000$	initial wealth

$$g = \begin{cases} 0.1 \cdot S_0 & p = 0.50 \\ -0.05 \cdot S_0 & p = 0.50 \end{cases}$$

The returns for the first period are 20 or -10 with equal probability, and $Q_0(g) = 20$. This implies, by applying the simple shares strategy, an α of 50:

$$\alpha_t = \frac{W_t}{Q_t}, \quad \alpha_0 = \frac{W_0}{Q_0} = \frac{1000}{20} = 50.$$

Remark I It is impossible to participate for 5000%, since 50 shares of this stock cost 10,000, which is ten times the wealth at $t = 0$.

The stock price at $t = 1$ has increased (or decreased) by the same percentage as the payout of the previous period and equals 220 or 190. Assuming that the maximum proportion α at $t = 0$ is 5, the wealth W_1 is either 1100 or 950. Since also the critical wealth is changed by the same percentage it equals either 22 or 19. This means that α is constant for all t!

$$W_1 = \begin{cases} 1100 & S_1 = 220 & Q_1(g) = 22 \rightarrow & \alpha = 50 \\ 950 & S_1 = 190 & Q_1(g) = 19 \rightarrow & \alpha = 50 \end{cases}$$

Remark II With returns relative to the transaction cost (e.g., stock price) the participation rate is constant over time.

These puzzles are unsolved at this moment and require in-depth analysis of the simple shares strategy and the implications thereof for relative returns and transaction costs.