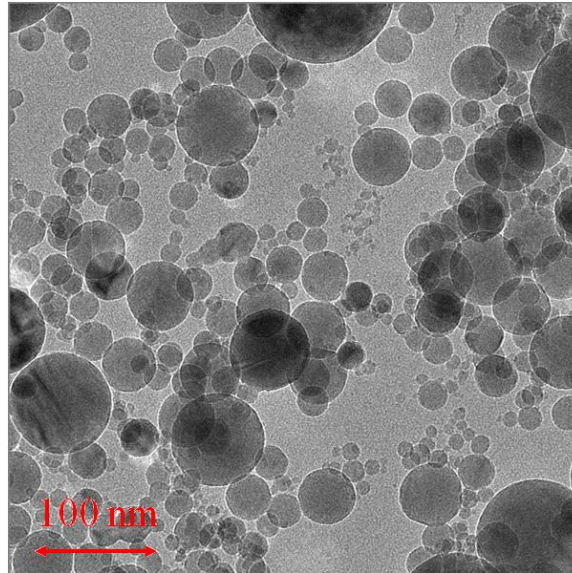


Investigation of methods to benchmark nanofluids



Nanoparticles dispersed in liquid

Bachelor thesis

Kevin van der Poel

Student bachelor Advanced Technology

Supervisor

Srinivas Vanapalli

Members of bachelor commission

Marcel ter Brake

Professor of EMS group

Niels Tas

TST group (external member)

Srinivas Vanapalli

EMS group

Date

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Abstract

Nanofluids, a suspension of nanomaterials in a basefluid, have shown promising improvements in their thermal properties compared to basefluids which are used for heat transfer nowadays. This interesting feature may offer unprecedented potential in cooling applications. Despite considerable theoretical and experimental research there still is no easy way to compare the nanofluid properties to its basefluid properties.

This thesis summarizes the basics of nanofluids, for example how nanofluids are created and what their (dis)advantages are. Furthermore, an overview of currently used methods to compare nanofluids to their corresponding basefluids is given. Continuing this work, a figure-of-merit is derived based on entropy for a constant temperature and heat flux boundary condition. The figure-of-merit allows for an easy comparison of the thermal properties of nanofluids to their basefluids.

Accordingly, the figure-of-merit is validated by comparing its value to literature values and their conclusions. Finally, measurements on thermal conductivity and viscosity will be performed to show that this figure-of-merit provides boundaries of other parameters for a nanofluid to be beneficial.

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I. Introduction

Efficient heat removal is one of the main challenges in numerous industries, including power generation, transportation, manufacturing and microelectronics. Currently existing cooling technologies are starting to be insufficient for the high amount of heat that needs to be removed. A good example are microchips which are becoming more important in everyday life as they are utilized in computer processors. Development in microchips focuses on making them smaller, therefore with higher heat flux density. In principle, the main limitation to this development is the rapid heat dissipation that is produced by the microchips. It is estimated that the next generation of computer chips will generate a localized heat flux over 10 MW/m^2 [1]. This heat flux requires sufficient cooling, which may be fulfilled by nanofluids [2].

Nanofluids are fluids in which nanometer-sized materials are dispersed, including nano-fibers, -tubes, -wires or -particles. The basefluid will give the starting point of the properties. The most common liquids used to create nanofluids are water, ethylene glycol and mixtures thereof. Water is used because of its low price, availability and safety. Ethylene glycol is used to work at temperatures below the freezing point of water.

The nanofluids exhibit better thermophysical properties compared to the basefluid, such as thermal conductivity, thermal diffusivity and convective heat transfer coefficients [3-5]. The developments in the field of nanofluids are still impeded by several factors, such as poor characterization of the suspensions, lack of fundamental understanding of the underlying mechanisms and discrepancy between research results.

It was found that nanofluids exhibit an increase in heat transfer characteristics but also an increase in viscosity, making the fluids flow resistance higher. However, there are many factors responsible for the changes in these properties. For example, the nanoparticles can differ in size, shape, and material. Furthermore, the concentration of the nanofluids could be varied and the surfactants used may leave traces of chemicals.

An important aspect of nanofluids, which influences the abovementioned factors is stability. Agglomeration of nanoparticles results in sedimentation of the particles and clogging in the heat exchanger, which decreases the nanoparticles' influence on the thermal properties. As such, a surface modifier is used to evenly disperse the nanoparticles in the basefluid. In this process the surface layer of the nanoparticles is modified to prevent the particles from clustering and settling. Xuan et al stated: "The common activators and dispersants are thiols, oleic acid, laurate salts. Selection of the suitable activators and dispersants mainly depends upon the properties of solutions and particles" [4]. Furthermore, nanofluids exhibit slightly different properties depending on the dispersion process during their manufacturing routes.

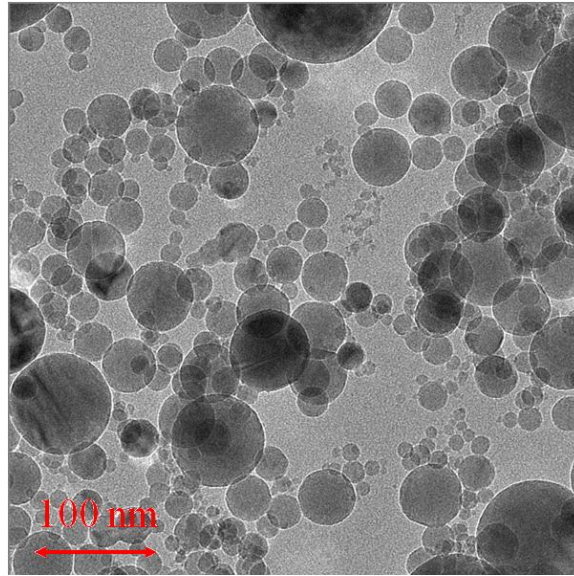


Figure 1. Transmission electron microscope (TEM) image of an Al_2O_3 /water nanofluid (0.06% volume concentration) [6].

It has been difficult to find a way to compare the nanofluid to the basefluid. [7]. Prasher et al. [8]. was among the first to analyze the benefits of nanofluids over basefluids. They suggested that the increase in thermal conductivity has to be at least four times the increase in viscosity with respect to the basefluid for it to be beneficial. With experimental evidence, Escher et al. [9]. demonstrated that the thermal conductivity has the lowest impact on the performance of a microchannel heat sink, which contradicts the statement made by Prasher et al. [8]. Other methods of comparison have been used, although they are not able to accurately predict the benefit of nanofluids. These methods will be discussed in more detail in subsequent chapters.

The biggest research into nanofluids at this moment is the EU funded NanoHex project. In this project, twelve leading European companies and research centers are working together to develop and optimize safe processes for the production of high performance nanofluids for the use in industrial heat management. Furthermore to create an analytical model to predict the thermal performance of the nanofluids. The nanofluids developed will be used in data centers and power electronics to demonstrate how nanofluids influence the reliability, reduce energy consumption, lower operating costs, cut carbon emissions and enable the development of more sustainable products and processes [10].

In this report, a figure-of-merit is calculated using a comparison of entropy production. This method is based on the use of the first and second law of thermodynamics simultaneously [11]. This method makes use of the irreversible production of entropy when a system transfers energy. When all losses are expressed in entropy it is possible to compare the results directly, where using other methods often use comparison between temperature and work, these have different dimensions. This figure-of-merit is calculated for constant heat flux and constant temperature boundary conditions and in both cases for laminar and turbulent flow. These figures-of-merit are used to compare nanofluids to their basefluids under the same conditions. The figures-of-merit will also be compared to other methods of comparison to see if they provide similar conclusions.

2. Convective heat transfer

There are three different modes of heat transfer namely; conduction, convection and radiation. For cooling purposes convection is mostly used because it is fast and can be controlled in a many ways. Where radiation is dependent of temperature and the emissivity of the material and conduction is dependent of the thermal conductivity, convection allows one to determine the rate of cooling by changing the flow speed.

Convection is the mechanism of heat transfer through a fluid in the presence of bulk fluid motion. This bulk motion will bring warmer parts and colder fluid in contact so the heat transfer will be faster.

There are different types of convection, natural and forced. Natural convection is caused by natural motion, such as the rising of a warm fluid. Forced convection means the fluid is forced to flow. In this case a pump is used and therefore it is forced convection. Forced convection is commonly used for cooling purposes.

There is also a difference between internal and external convection. When the fluid flow is confined in a pipe or duct it is called internal convection, whereas a flow around a pipe or on a plate is called external convection. Most commonly used in these kind of experiments is the internal flow [12].

The rate of convective heat transfer is expressed by Newton's law of cooling and is given by,

$$\dot{q} \left[\frac{W}{m^2} \right] = h_x (T_s - T_f) \quad (1)$$

This is the surface heat flux per square meter. To obtain the total heat, this is integrated over the surface resulting into

$$\dot{Q} [W] = h A_s (T_s - T_f) \quad (2)$$

Where:

h = Average convection heat transfer coefficient $\left[\frac{W}{m^2 K} \right]$

h_x = Local convection heat transfer coefficient $\left[\frac{W}{m^2 K} \right]$

T_s = Surface temperature [K]

T_f = Fluid temperature [K]

A_s = Heat transfer surface area [m^2]

For convective flow two different boundary conditions are considered, namely the constant heat flux boundary condition and the constant temperature. The fluid used for cooling will then be exposed to this constant heat flux, resulting in a linear rise in temperature of the fluid. A constant heat flux is very common among electronics. These will produce a constant heat rate when operating under steady state conditions.

In a constant temperature boundary condition the temperature difference between the fluid in the pipe and the surrounding temperature will decrease, therefore the heat transfer will decrease as well. This will result in a non-linear temperature profile. Both boundary conditions are illustrated in figure 2 with their corresponding temperature profiles. A constant temperature is common for a pipe running through a large quantity of a different fluid making that fluid have a stable temperature while the fluid in the tube will change its temperature. This can be used for heat exchangers.

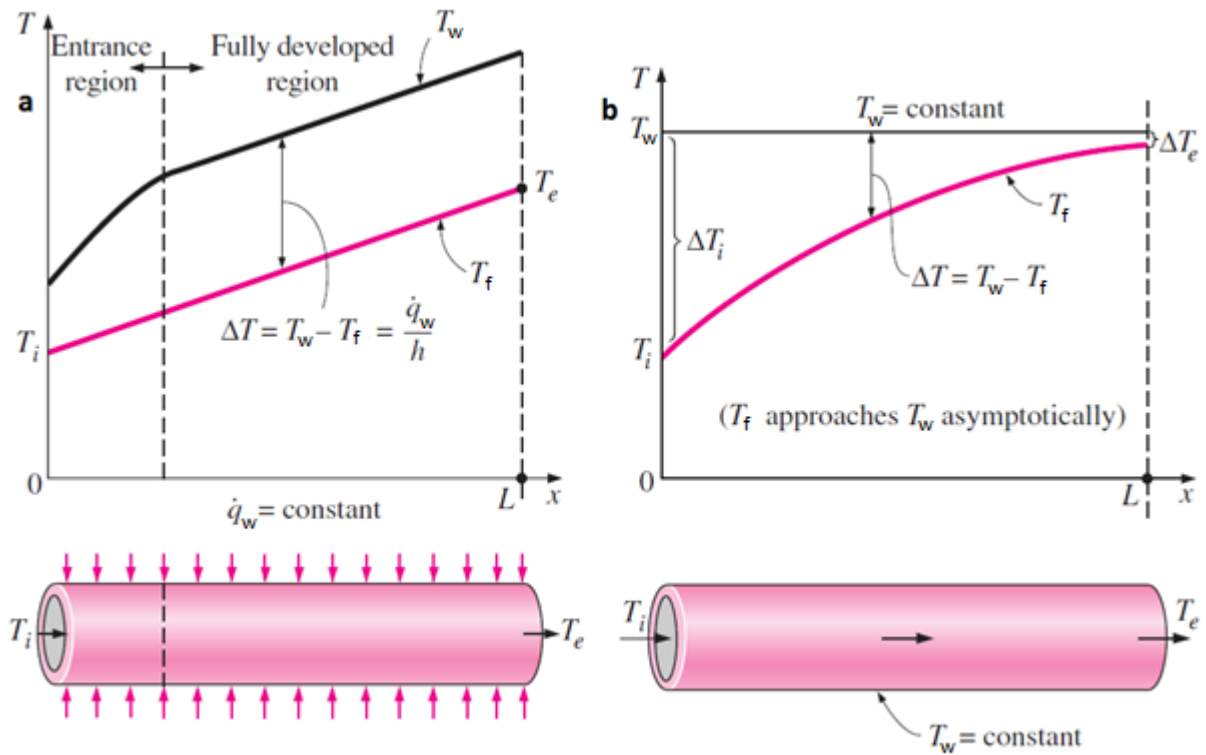


Figure 2. Temperature profile along the tube depending on the boundary conditions. a: Constant heat flux. b: Constant surface temperature [12].

A flow travelling through a pipe can be turbulent, laminar or a combination of both. A laminar flow is characterized by smooth streamlines and ordered motion and a turbulent flow is characterized by velocity fluctuations and highly disordered motion. To determine the regime of flow a characteristic Reynolds number is used, which is defined below. Because of the difference in the flow through the pipe for these two conditions the total heat transfer will be different.

When a fluid is forced to flow through a pipe or duct, it will be influenced by the walls of this concealment. The fluid in the pipe will travel with a certain velocity but the walls are standing still. This will create friction between the wall and the fluid right next to it and this will slow down the fluid. This slower fluid will in its turn slow down the fluid next to it. Because of this a velocity boundary layer will form as shown in Figure 3. This boundary layer will develop over a certain length of the tube and then will give a steady temperature profile, because the change in temperature over time will become constant due to the constant heat flux.

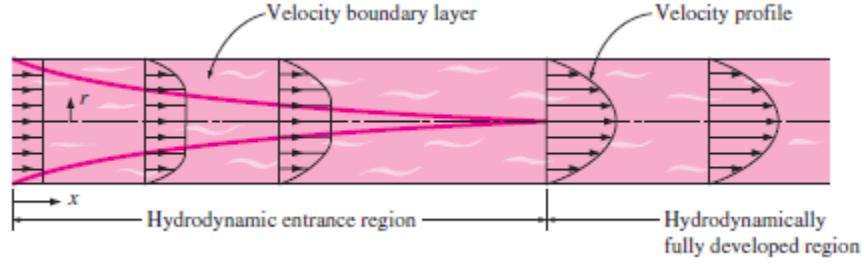


Figure 3. A velocity boundary layer in a tube [12].

This profile differs for a turbulent and laminar flow. The turbulent flow will have more motion between fluid layers and therefore the entrance region will be shorter and the velocity profile will be steeper at the walls and more flat in the middle.

For the temperature profile of the fluid there is a similar phenomenon as in figure 3. The temperature of the fluid at the wall changes much faster than that of the fluid in the middle. So a temperature profile will take form and also have its own entrance region. For the temperature boundary layer is also the case that the turbulent flow will have a smaller entrance region and a smaller angle at the walls because of the extra motions of the fluid in different directions.

The entrance regions of a laminar flow can be calculated using

$$L_{entrance\ laminar} = 0.05Re D. \quad (3)$$

Where Re is the Reynolds number and D is the tube diameter. This value can vary from the diameter of the tube to more than hundred times the tube diameter depending on the Reynolds number.

For a turbulent flow the entrance region is calculated using

$$L_{entrance\ turbulent} = 1.359Re^{1/4}. \quad (4)$$

And is approximately ten times the diameter, but can go up to forty times the diameter for very high Reynolds numbers.

As mentioned before the addition of nanoparticles will change the properties of the fluid. This influences how effective the fluid will be as a cooling agent.

The heat gained by the fluid as it flows along the tube increases the temperature of the fluid.

$$\dot{Q} = \dot{q}A = \dot{m}c_p(T_{out} - T_{in}) \rightarrow T_{out} = T_{in} + \frac{\dot{q}pL}{\dot{m}c_p} \quad (5)$$

Where A is the surface area and is equal to the perimeter multiplied with length of the tube. T_{in} is the temperature of the fluid going in to the tube, T_{out} is the fluid temperature coming out of the tube, \dot{m} is the mass flow, p the perimeter of the tube and c_p is the specific heat capacity. The rate of increase of the fluid temperature is obtained by taking the derivative of fluid temperature with respect to position x , which can be written as

$$\frac{dT_m}{dx} = \frac{\dot{q}p}{\dot{m}c_p} = \text{constant} \quad (6)$$

From the fluid temperature the wall temperature can also be found. The heat transfer between the fluid and the wall is given by Newton's law of cooling.

$$\dot{q} = h(T_{\text{wall}} - T_f) \quad (7)$$

This can be easily rewritten to find the wall temperature and inserting T_f results in

$$T_{\text{wall}} = T_{\text{in}} + \frac{\dot{q}px}{\dot{m}c_p} + \frac{\dot{q}}{h} \quad (8)$$

From this it can be seen that

$$\frac{dT_f}{dx} = \frac{\dot{q}p}{\dot{m}c_p} = \text{constant} = \frac{dT_{\text{wall}}}{dx} \quad (9)$$

So the rate of increase in wall temperature will be the same as the increase in fluid temperature.

A tube of 510 mm in length and 4 mm nominal inner diameter is considered in the calculations because flow experiments are carried out with this geometry. Figure 4 shows the fluid temperature profile along the length of the tube for heat input of 50 W and 100 W and for flow velocities of 0.5, 1, 1.5 and 2 m/s. Fluid properties of water are considered in these calculations.

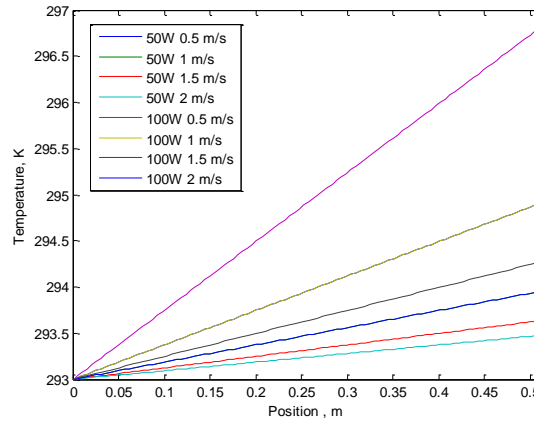


Figure 4. The fluid temperature at different heat inputs and different flow speeds.

From this plot it can be seen that when the speed increases the fluid temperature at the end of the tube will be lower. This is because the fluid heat capacity is a product of mass flow rate and specific heat capacity. So a larger flow rate means a larger heat capacity and hence lower exit temperature of the fluid.

It is also clear that when the heat is higher the temperature of the fluid will be higher at the end of the tube for the same flow rate.

Some lines in the plot are not visible because they are on top of each other. For example 50W at 0.5 m/s overlaps with 100W at 1m/s because the temperature increase will double as a result of double the heat but will half as a result of twice the velocity resulting in the same temperature value.

A similar plot can be made for the wall temperatures, as shown in figure 5.

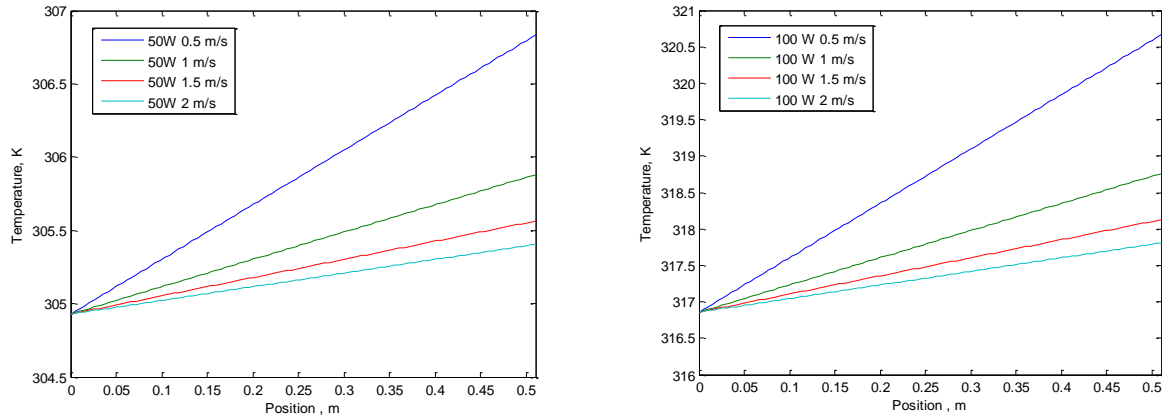


Figure 5. The wall temperature at different heat inputs and different flow speeds.

From these plots it can be seen that the inlet wall temperature increased with heat input. When the difference in temperature is compared to those of the fluid temperature these values are equal.

To see the influence of different parameters water will be compared to a fluid with the same properties of water but with a specific heat capacity that is 90% of that of water. Now only using a heater of 50W and a velocity of 1 m/s.

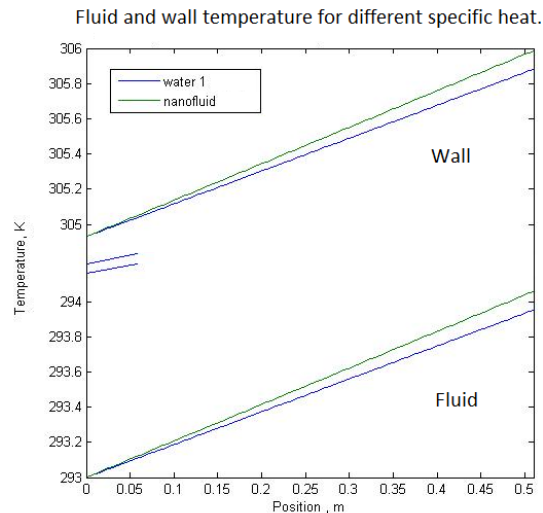


Figure 6. The fluid and wall temperature for water and a nanofluid with a specific heat capacity equal to 90% of that of water.

From this plot it can be seen that a lower specific heat will give a higher exit fluid temperature in the tube. The higher the value of the specific heat capacity the more energy is needed to heat the fluid. Therefore the 90% specific heat capacity fluid will have a higher temperature at the end of the tube.

The wall temperature shows the same behavior as for water. The temperatures are distributed in the same way and the increase in temperature is equal for the fluid and the wall.

Now the influence of thermal conductivity on the wall and fluid temperature is considered. A fluid is assumed with 110% of the thermal conductivity of water and all other properties are the same.

As equation 7 shows the thermal conductivity does not influence the fluid temperature. As a result of this the lines for the nanofluid and water are on top of each other in figure 7.

For the wall temperature things will be different.

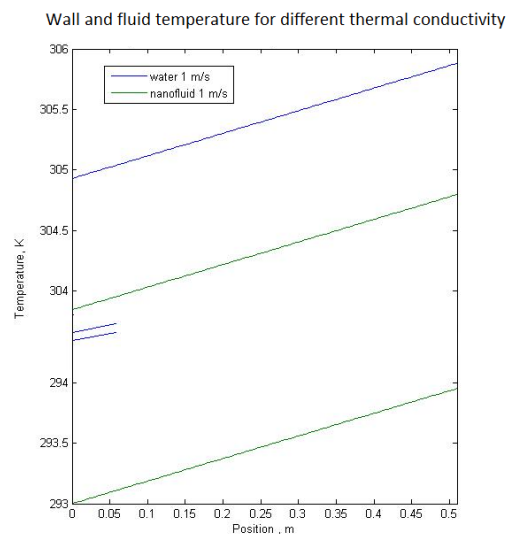


Figure 7. The wall temperature for different thermal conductivities.

The thermal conductivity changes the inlet wall temperature but rate of increase in temperature is not influenced.

Now a nanofluid is considered that has a specific heat capacity of 90% of that of water and a thermal conductivity that is 110% of that of water. This nanofluid will be compared to water to see how these two parameters influence the wall and fluid temperature.

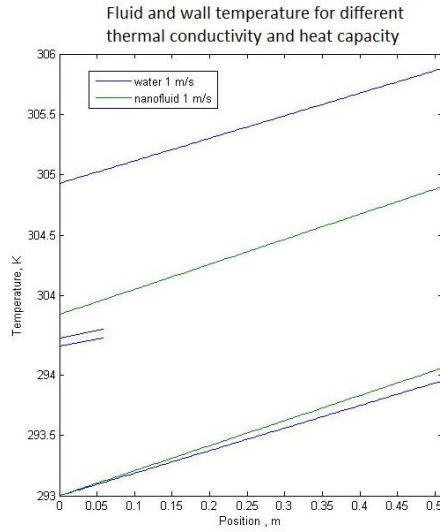


Figure 8. The fluid and wall temperature a nanofluid and water.

The difference in temperature between the wall and the fluid is 11 Kelvin at the end of the tube. This is less than a 4% difference meaning a very small difference in temperature. This assumption will be used in chapter 3.

The fluid temperature is lower for water in both cases. The difference in temperature is smaller when the flow is faster. To see which of the fluids has done the better cooling the wall temperature is needed.

The nanofluid has a lower starting temperature for the wall and the wall stays cooler for the nanofluid for the entire tube. The increase in thermal conductivity resulted in the lower starting temperature as can be seen in figure 7 and the lower specific heat makes the slope of the temperature increase higher. In this case the slope is not steep enough to cause the lines to intersect, making the nanofluid better. If the nanofluid has a lower specific heat or a smaller increase in thermal conductivity it is possible that the nanofluid is not beneficial.

When using forced convection the fluid has to move and to get the fluid moving a pump is used. This pump will use energy and therefore will be of importance for the efficiency of the total system. The pumping power can be calculated with [12]:

$$\dot{W} = \dot{V} \Delta P. \quad (10)$$

Where \dot{V} is the volume flow rate and the pressure drop is given by,

$$\Delta P = f \frac{L}{D} \frac{\mu v^2}{2}. \quad (11)$$

Where:

f = the friction factor

L = the tube length

v = the fluid flow velocity

μ = the viscosity

From this it can be seen that an increase in viscosity results in an increase in pumping power to retain the same speed. This means that for the fluid to flow the same speed, as is used in the figures 4-8, more energy is required.

In heat transfer studies there is a collection of nondimensional numbers used for comparing different factors to each other. Only two of these numbers are interesting at this point for this thesis, which are the Nusselt and the Reynolds number.

A commonly used nondimensional number in the heat convection studies is the Nusselt number (Nu). The Nusselt number represents the enhancement of heat transfer through a fluid as a result of convection relative to conduction. A higher Nusselt number indicates a more effective convection [12].

$$q_{conv} = h \Delta T \quad \text{and} \quad q_{cond} = \lambda \Delta T / L \quad (12)$$

Their ratio:

$$\frac{q_{conv}}{q_{cond}} = \frac{h \Delta T}{\lambda \Delta T / L} = \frac{hL}{\lambda} = Nu. \quad (13)$$

The Reynolds number is used to determine if a flow is dominated by its viscosity or density. With this ratio it can be determined if the flow is laminar or turbulent. For a flow in a tube the flow is laminar if $Re < 2300$ and turbulent if $Re > 10000$. In The region between 2300 and 10000 the flow is called a transition flow and can be turbulent or laminar depending on other factors such as pipe roughness.

The Reynolds number is given by [12]:

$$Re = \frac{\rho v D}{\mu}. \quad (14)$$

Where:

v = the velocity of the fluid

ρ = the density of the fluid

D = the diameter of the pipe

μ = the viscosity

In experiments with nanofluids the tubes used are of very small diameters and the fluids flows at low velocities, resulting mostly in a laminar flow.

One of the most important parameters in this thesis is entropy. Entropy is used in the second law of thermodynamics to identify the spontaneous changes among permissible changes. The first law is conservation of energy. The entropy of an isolated system increases in the course of a spontaneous change. Thermodynamic irreversible processes like cooling and free expansion of gasses are spontaneous processes and will therefore be accompanied by an increase in entropy. Because an irreversible process results in entropy production, the entropy production can be used to compare the energy lost in that irreversible process. This feature is used in this thesis to compare nanofluids and basefluids.

3. Benchmarking heat transfer fluids

There have been a lot of different ways to compare nanofluids to their basefluid. Some of these will be discussed in this chapter using some publications to show their derivation and results. The results will be used later on for comparison with theory, except the publication by Pawan et al. because Escher et al. already proved this method didn't work. Other methods will be discussed briefly.

When comparing nanofluids there are multiple parameters that should be considered. The thermophysical parameters, geometry and flow. For a nanofluid the thermophysical parameters differ from those of the basefluid. The viscosity, thermal conductivity and density are higher for the nanofluid where the specific heat is lower. For the geometry the tube length and diameter are of importance and for the flow the velocity of the fluid is of influence when comparing the fluids.

3.1 Comparing thermal conductivity to viscosity

Prasher et al.[8] was the first to find a way to compare fluids. They used alumina nanoparticles suspended in propylene glycol and dispersed them with ultrasonication. They created an equation starting by taking the Nusselt number

$$\frac{hL}{\lambda} = Nu. \quad (15)$$

For both the basefluid and nanofluid and divided these to get

$$\frac{D_{nf}}{D} = \left(\frac{\lambda_{nf}}{\lambda_{bf}} \right) \left(\frac{Nu_{nf}}{Nu_{bf}} \right). \quad (16)$$

Where D_{nf} is the tube diameter which gives the same heat transfer coefficient as the base fluid. The pressure drop in a fully developed flow is

$$\Delta p_{bf} = \frac{C \mu_{bf}}{D^4}. \quad (17)$$

Where C is a constant depended on mass flow and tube length, these are both kept constant. To maintain this constant value the diameter of the tube has to vary and will be larger for the nanofluid. Using the ratio of pressure drop and the formula for diameter ratio for the case where $h_{nf} = h_{bf}$ this gives

$$\frac{\Delta p_{nf}}{\Delta p_{bf}} = \left(\frac{\mu_{nf}}{\mu_{bf}} \right) \left(\frac{\lambda_{bf}}{\lambda_{nf}} \right)^4 \left(\frac{Nu_{bf}}{Nu_{nf}} \right)^4. \quad (18)$$

If $\frac{\Delta p_{nf}}{\Delta p_{bf}} > 1$ the nanofluid is worse than the base fluid for heat transfer. This formula shows that

Δp is more sensitive to changes in λ and Nu than to μ .

If the thermal conductivity of the nanoparticles is much higher than that of the fluid the Maxwell-Garnett effective medium model can be written as

$$\frac{\lambda_{nf}}{\lambda_{bf}} = 1 + C_\lambda \phi. \quad (19)$$

Where C_λ is the conductive enhancement coefficient a similar formula was found for viscosity

$$\frac{\mu_{nf}}{\mu_{bf}} = 1 + C_\mu \phi. \quad (20)$$

Inserting both these equations into the pressure drop ratio (equation 18) results in

$$(1 + C_\lambda \phi)^4 \left(\frac{Nu_{nf}}{Nu_{bf}} \right)^4 = 1 + C_\mu \phi. \quad (21)$$

Assuming $C_\mu \phi$ is very small by binomial expansion this equation reduces to

$$C_\mu = \frac{(1 + 4C_\lambda \phi) \left(\frac{Nu_{nf}}{Nu_{bf}} \right)^4 - 1}{\phi}. \quad (22)$$

Prasher et al. considered the case $Nu_{bf} = Nu_{nf}$ and got to $4C_\lambda = C_\mu$. They therefore concluded that the increase in viscosity has to be more than four times larger than the increase in thermal conductivity for nanofluid not to be beneficial.[8, 13]

3.2 COP

The coefficient of performance (COP) for a heat sink is defined as the ratio of the heat dissipated to the pumping power.

$$COP = \frac{\dot{q}}{P} \quad (23)$$

With this ratio the efficiency of the heat sink is calculated. The pumping power is the energy put into the system and \dot{q} is the heat removed. A higher COP means a more efficient system. The equations to calculate these parameters are mentioned in chapter 2.

3.3 Other methods

There have been many different ways to compare a nanofluid to its basefluid. Those not discussed above are mostly based on the heat transferred versus pumping power. The heat transferred is expressed in many different ways in the literature varying from differences in wall and fluid temperature to calculation of the heat transfer coefficient. The pumping power is calculated from equation 10 or measured from the pressure drop. These two can then be combined into a lot of different ways of comparison for example: transfer versus pressure drop, heat transfer coefficient versus pumping power, etc. These comparisons all have the same problem. They try to compare two values that are not the same unit. Thus temperature difference and pumping power can't be compared directly.

4. Comparison of nanofluid to the corresponding basefluid based on entropy production in the flow process

The analysis presented below is valid for any fluid regime provided the heat transfer and pressure drop correlations are known for that particular fluid regime and geometry.

The assumptions:

- Fully developed laminar flow
- Same velocity for all fluids
- Circular tube (can be compared to other geometries using the hydraulic diameter)

The entropy production rate due to flow resistance [14, 15] in a tube. A volume element of the channel is considered. The mass flow is constant and equal for the in- and out-flow. The second law of thermodynamics gives,

$$\dot{m}s_{out} = \frac{\dot{Q}}{T_f} + \dot{m}s_{in} + \dot{S}_i. \quad (24)$$

Where \dot{m} is the mass flow, s is the specific entropy, \dot{Q} is the heat flow into the fluid, \dot{S}_i is the entropy produced and T_f is the average temperature of the fluid. The heat absorbed by the fluid is expressed as,

$$d\dot{Q} = \dot{m} dh. \quad (25)$$

Where h is the specific enthalpy. Combining equation 24 and 25 results in,

$$T_f d\dot{S}_i = \dot{m} T_f ds - \dot{m} dh. \quad (26)$$

The pressure drop for a laminar flow is,

$$dP = -\frac{C}{2} \mu \frac{v}{D_h^2} dx. \quad (27)$$

Where C is a geometric constant determined by the shape of the channel (for a circular tube [16] $C = 64$), μ is the viscosity, D_h is the hydraulic diameter and v is the mean velocity in the channel.

Using the thermodynamic identity $TdS = dH - VdP$ and equation 26 and 27 this can be written as

$$d\dot{S}_i = \frac{\dot{m}}{\rho T_f} \frac{C}{2} \mu \frac{v}{D_h^2} dx. \quad (28)$$

Where ρ is the density of the fluid. Using $v = \frac{\dot{m}}{\rho A}$ and $A = \frac{\pi D_h^2}{4}$ (for a circular tube) the entropy production due to flow resistance results in,

$$\left(\frac{d\dot{S}_{\Delta P}}{dx}\right) = \frac{\pi C}{8 T_f} \mu v^2. \quad (29)$$

The entropy produces due to temperature difference between the tube wall temperature T_w and the fluid temperature is given by [14, 15],

$$\Delta S = \frac{\dot{Q}}{T_f} - \frac{\dot{Q}}{T_w}. \quad (30)$$

This can be written as

$$\Delta S = \dot{Q} \left(\frac{T_w - T_f}{T_f T_w} \right). \quad (31)$$

For a small temperature difference, as is shown in chapter 2, this results into,

$$d\dot{S} = \delta \dot{Q} \left(\frac{\Delta T}{T_f^2} \right). \quad (32)$$

Where \dot{Q} is the heat flow in the radial direction. For a fully developed flow heat flow is,

$$\delta \dot{Q} = h \pi D (T_w - T_f) dx = \pi Nu \lambda \Delta T dx. \quad (33)$$

Where h is the convective heat transfer coefficient, λ is the thermal conductivity of the fluid, D is the diameter of the tube and Nu is the Nusselt number equal to hD/λ . The Nusselt number is constant for a fully developed laminar flow. For a constant heat flux boundary condition, the Nu for a flow in a tube[16] is equal to 4.36 and for a constant surface temperature boundary condition this value is equal to 3.66. Inserting heat flow into equation 32, the entropy generation rate is,

$$\left(\frac{d\dot{S}_{\Delta T}}{dx}\right) = \frac{\pi Nu \lambda dT \Delta T}{T_f^2}. \quad (34)$$

In a steady flow, neglecting other heat losses, the energy gained by the fluid is equal to the energy transferred from the tube and is given by,

$$\dot{m} c_p dT = \pi Nu \lambda \Delta T dx. \quad (35)$$

Where \dot{m} is the mass flow rate of the liquid, c_p is the specific heat capacity of the fluid. Substituting ΔT into equation 34, it follows that

$$\left(\frac{d\dot{S}_{\Delta T}}{dx}\right) = \frac{\pi Nu \lambda}{T_f^2} \left(\frac{\dot{m} c_p}{\pi Nu \lambda}\right)^2 \left(\frac{dT_f}{dx}\right)^2. \quad (36)$$

This expression relates the entropy production rate to the temperature variation of the fluid in the tube.

4.1 Constant heat flux

For a constant surface heat flux q'' boundary condition, in the fully developed region the energy balance gives

$$\dot{m}c_p dT = q''\pi D dx. \quad (37)$$

And can be written as

$$\frac{dT}{dx} = \frac{q'' \pi D}{\dot{m} c_p} \neq f(x). \quad (38)$$

Substituting $\frac{dT}{dx}$ into equation 24 the entropy generation can be written as

$$\left(\frac{d\dot{S}_{\Delta T}}{dx} \right) = \frac{q''^2 \pi D^2}{T_f^2 Nu \lambda}. \quad (39)$$

The total entropy production rate is

$$\dot{S}_{tot} = \left(\frac{d\dot{S}_{\Delta P}}{dx} \right) + \left(\frac{d\dot{S}_{\Delta T}}{dx} \right). \quad (40)$$

For the condition in which the diameter of the tube and the flow velocity are considered not to vary, the differential change in entropy is given by,

$$d\dot{S}_{tot} = \left(\frac{\partial \dot{S}_{x,tot}}{\partial \mu} \right)_{\lambda=\text{const}} d\mu + \left(\frac{\partial \dot{S}_{x,tot}}{\partial \lambda} \right)_{\mu=\text{const}} d\lambda. \quad (41)$$

The nanofluid is beneficial compared to the base fluid when the differential of the total entropy production is negative.

$$d\dot{S}_{x,tot} = a_1 d\mu - b_1 \frac{d\lambda}{\lambda^2} < 0 \quad (42)$$

Where a_1 is equal to $\frac{\pi C}{8 T_f} v^2$ and b_1 is equal to $\frac{q''^2 \pi D^2}{T_f^2 Nu}$.

Rearranging and integrating on both sides gives the condition for the trade-off between the thermophysical properties of the nanofluid (nf) and the basefluid (bf) as a function of a_1 and b_1

$$\left(\frac{1}{\lambda_{bf}} - \frac{1}{\lambda_{nf}} \right) / (\mu_{nf} - \mu_{bf}) > \frac{a_1}{b_1}. \quad (43)$$

A nanofluid that satisfies the above inequality would produce lower entropy and hence would maintain a lower heat sink temperature compared to the base fluid for a fixed heat flux and

velocity of flow. From which can be concluded that the nanofluid is beneficial in a fully developed flow through a circular tube when the velocity of the fluid is constant.

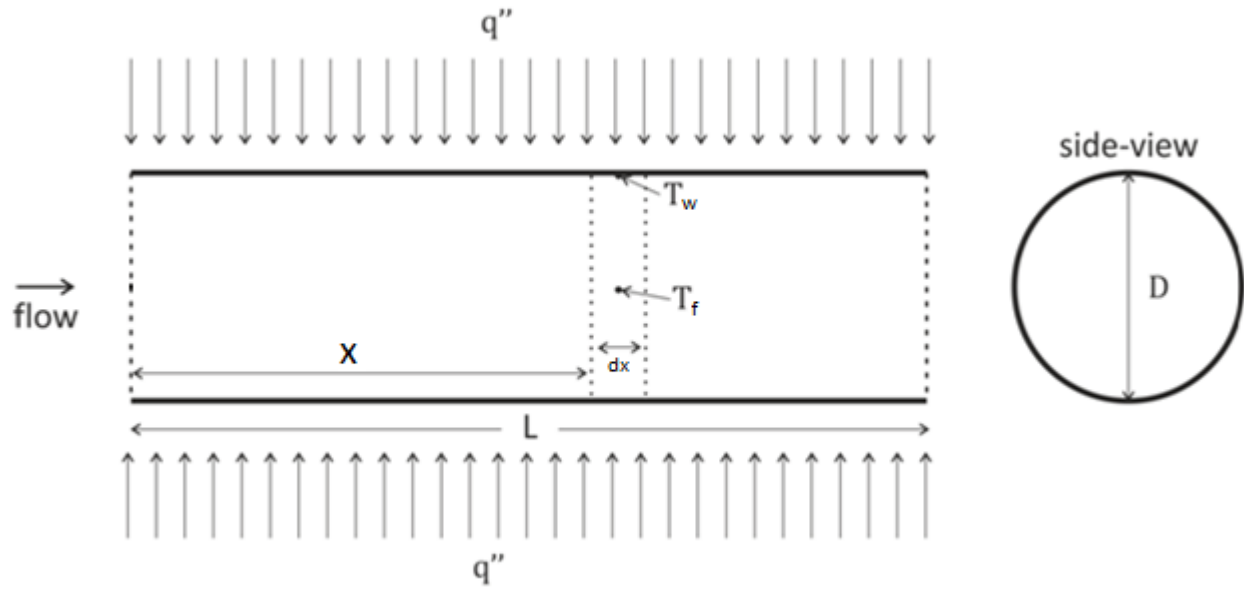


Figure 9. Schematic of a tube subjected to a constant heat flux and is being cooled by a fluid flow.

4.2 Constant temperature

For a constant temperature boundary condition the increase in mean temperature is dependent of the position in the tube. To find the temperature gradient for this boundary condition the energy balance is used.

The energy balance of a differential control volume gives

$$\frac{T_w - T_f(x)}{T_w - T_i} = \exp\left(-\frac{pxh}{\dot{m}c_p}\right). \quad (44)$$

Where T_i is the inlet temperature of the fluid. To get to the temperature gradient this has to be differentiated to x resulting into

$$\frac{1}{T_w - T_i} dT_f(x) = \frac{ph}{\dot{m}c_p} \exp\left(-\frac{pxh}{\dot{m}c_p}\right) dx. \quad (45)$$

And this can be rewritten as

$$\frac{dT_f(x)}{dx} = (T_w - T_i) \frac{ph}{\dot{m}c_p} \exp\left(-\frac{pxh}{\dot{m}c_p}\right). \quad (46)$$

This temperature gradient can be inserted in equation 36

$$\left(\frac{d\dot{S}}{dx}\right)_{\Delta T} = \frac{\pi Nu \lambda}{T_f^2} \left(\frac{\dot{m} c_p}{\pi Nu \lambda}\right)^2 \left(\frac{dT}{dx}\right)^2. \quad (47)$$

Resulting in

$$\frac{d\dot{S}_{\Delta T}}{dx} = \frac{\pi Nu \lambda}{T_f^2} (T_w - T_i)^2 \exp\left(-\frac{2\pi Nu \lambda x}{\dot{m} c_p}\right). \quad (48)$$

The x term in the exponent makes this entropy production dependent of x (which is not the case for constant heat flux). The figures below show the rate of production of entropy as a function of the distance x travelled. Figure shows this entropy production for water and a Fluidx this is an imaginary fluid with all the same properties of water with 1.2 times the thermal conductivity to show how the thermal conductivity influences the entropy production. From this figure it is clear to see the entropy production is not constant and if looked at closely it can be seen it is exponential as is expected from equation 48. This means the length of the tube will be of influence on the entropy production as a result of temperature difference.

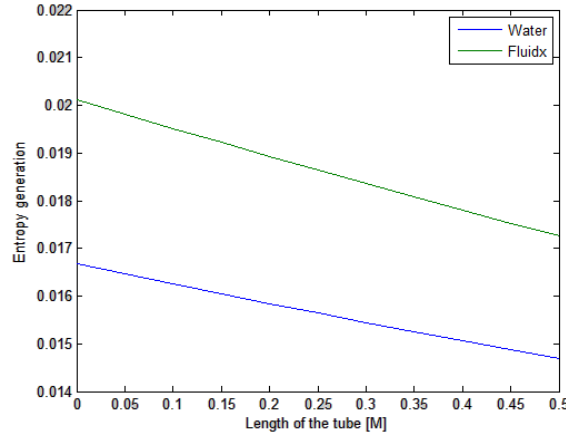


Figure 10. Entropy production versus the length of the tube.

To create an equation for comparing entropy production equation 34 will be integrated over the tube length. This results in

$$dS_{\Delta T} = \int \frac{d\dot{S}}{dx} dx = A \int_0^L e^{Bx} = \frac{A}{B} (e^{BL} - 1). \quad (49)$$

For convenience:

$$A = \frac{\pi Nu \lambda}{T^2} (T_w - T_i)^2 \quad \text{and} \quad B = \left(-\frac{2\pi Nu \lambda}{\dot{m} c_p}\right)$$

Where L is the length of the tube.

The entropy production due to flow resistance was given by equation 29

$$\frac{dS_{\Delta P}}{dx} = \frac{\pi C}{8 T} \mu v^2.$$

Also integrating this so both values can be added results into

$$dS_{\Delta P} = \int \frac{dS}{dx} dx = \frac{\pi C}{8 T} L \mu v^2. \quad (50)$$

The total entropy production is given by

$$dS_{total} = dS_{\Delta T} + dS_{\Delta P} = \frac{\pi C}{8 T} L \mu v^2 + \frac{A}{B} (e^{BL} - 1). \quad (51)$$

The complexity of this equation doesn't allow the creation of a figure-of-merit like the one given for constant heat flux. To see the comparison between two different fluids these should be calculated separately and the total entropy can then be compared to see which is the most efficient.

5. Comparison with literature

In order to validate the figure-of-merit derived in chapter 4, a comparison is made with research concerning nanofluids found in the literature. Two publications were selected where the data necessary to evaluate the figure-of-merit is available and this is inserted into equation 43. The conclusion of the publication was then compared to the conclusions found by calculating this figure-of-merit.

5.1 Escher et al.

The first publication is “On the cooling of electronics with nanofluids” by Escher et al. [9]. In this work the performance of a heat sink was measured using a fully developed laminar flow with a constant heat flux boundary condition. This heat sink had dimensions of 10x10 mm and the channels are put on side by side as shown in figure 11. Because of this, the number of channels is different for different channel sizes. The channels are not circular and therefore the geometric constant and the Nusselt number are a different value. The data from this publication are given in figure 12 and 13.

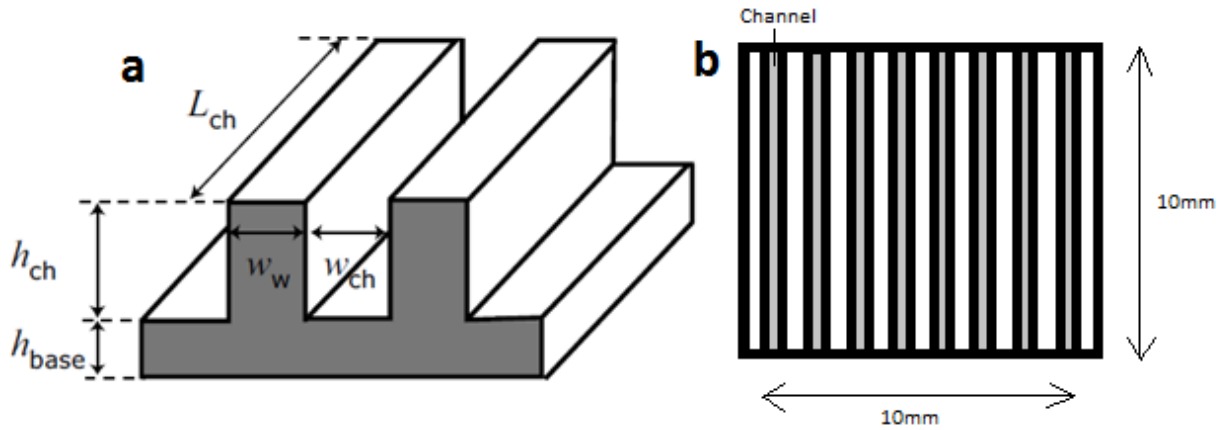


Figure 11.(a) Schematic of the heat sink used by Escher et al. (b) Top view of the heat sink with the parallel channels.

w_{ch} (μm)	w_w (μm)	h_{ch} (μm)	h_b (μm)
204	196	185	340
103	97	187	338
53	50	193	330

Figure 12. Different channel widths and their corresponding heights.

w_{ch} (μm)	$\phi_{vol}=0\%$ $\mu/(mPa\ s)$	$\phi_{vol}=5\%$ $\mu/(mPa\ s)$	$\phi_{vol}=16\%$ $\mu/(mPa\ s)$	$\phi_{vol}=31\%$ $\mu/(mPa\ s)$
50	0.95	1.27	2.2	–
100	1.06	1.21	2.23	9.9
200	1.03	1.16	2.5	10.6

Figure 13. Viscosity values measured for volume percentages of nanoparticles for different channel sizes.

From table 12 and table 13 the different channel sizes were taken. Escher et al. calculated the viscosity from the pressure drop, because of this there are small variations in the viscosities in table 13 for different channel sizes. The other values needed to compare the fluids are taken from Figure 14. Figure 14a shows a graph for the thermal conductivity at different temperatures, heat capacity and density as a function of particle concentration. From this graph the thermal conductivity was taken.

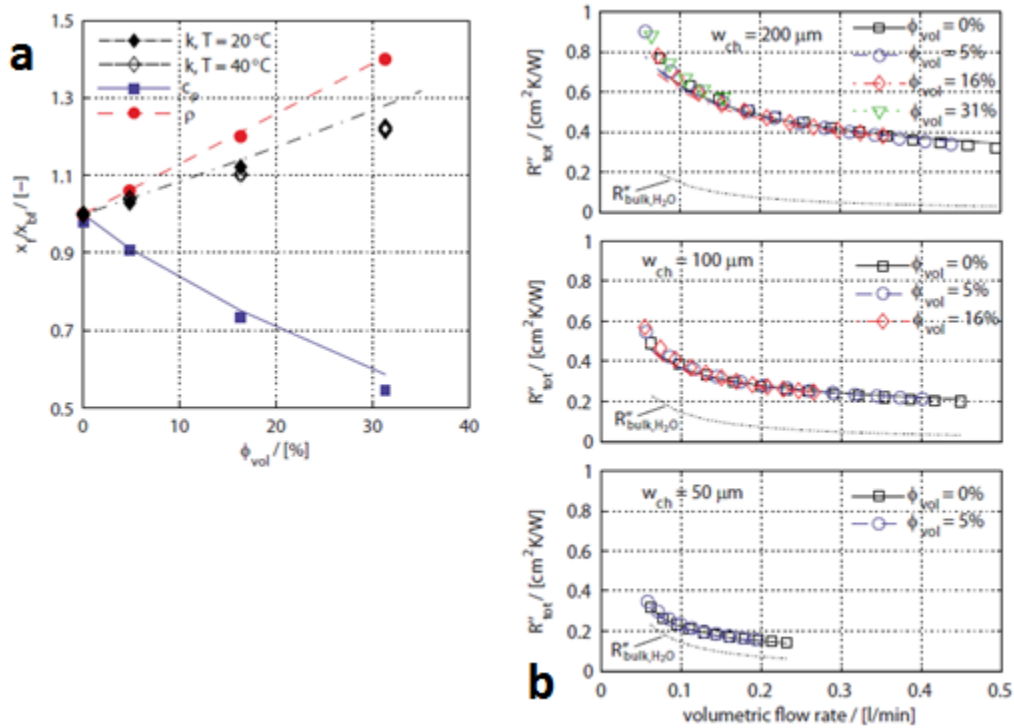


Figure 14. (a) Heat capacity and density ratio for different volume percentages of nanoparticles. (b) Thermal resistance for different flow rates and given for different volume percentages of nanoparticles.

The mass flow is determined by velocity density and the specific heat. If the velocity is constant the mass flow can also be assumed constant because the density increases a similar amount as the specific heat decreases so the product of these two will remain the same.

To determine the heat flux from the data, the procedure used is as follows.

In the article the thermal resistance is given by

$$R_{tot} = \frac{T_{heater,max} - T_{f,in}}{q''_{heater}} \quad (45)$$

Where R_{tot} is the total thermal resistance. This is defined by Escher et al. as the sum of the conduction thermal resistance through the silicon base, the convective heat flux from the channel wall and base to the fluid and the bulk thermal resistance to account for the limited heat capacity. The temperature difference $T_{heater,max} - T_{f,in}$ set in their experiments at a constant value of 20 Kelvin. Using the values from figure 14b for the thermal resistance the heat flux can be calculated from this equation.

The Nusselt number and geometric are different for noncircular channels. They are still constant because of the fully developed laminar flow, but do differ in value for the different channel sizes [12].

Table 1. Nusselt number and geometric constant for square shaped channels

Channel size [μm]	Nusselt number	Geometric constant
204x185	3.7	57
103x187	4.1	62
53x193	5.3	73

The calculations were made for different channel sizes and flow speeds using equation 43. The results are shown in table 2 and table 3.

Table 2. Left hand side (LHS) of the figure-of-merit for different channel sizes and volume percentages of nanoparticles.

LHS	50 μm	100 μm	200 μm
0 %	-	-	-
5 %	102.1	217.9	251.4
16 %	142.9	152.6	121.5

Table 3. Right hand side (RHS) of the figure-of-merit for different channel sizes and volume percentages of nanoparticles.

RHS	50 μm	100 μm	200 μm
0.1 L/min	$1.1 \cdot 10^6$	$1.8 \cdot 10^5$	$2.6 \cdot 10^4$
0.2 L/min	$3.3 \cdot 10^6$	$3.6 \cdot 10^5$	$5.9 \cdot 10^4$
0.3 L/min	-	$5.5 \cdot 10^5$	$9.3 \cdot 10^4$
0.4 L/min	-	$8.2 \cdot 10^5$	$1.4 \cdot 10^5$

For a channel size of 50 μm no values were provided in the text so these couldn't be calculated. The calculated numbers can now be compared and for the nanofluid to be beneficial the right hand side should be smaller. From these numbers it can be seen that none of the RHS values is smaller than the LHS values, meaning there is no channel size or flow speed for which the nanofluid would be beneficial. The temperature difference in this publication was 20 Kelvin this is about 7% of the total temperature. The figure-of-merit was derived for small temperature differences so will be less accurate in this case but still useable. Escher et al. reached a similar conclusion using the COP mentioned in chapter 3.

5.2 Pawan et al.

The second publication is “Entropy generation due to flow and heat transfer in nanofluids.” by Pawan et al. [17]. In this publication the entropy generation rate was used to compare different sizes of tubes. This was done for both a laminar and a turbulent flow using a constant heat flux boundary condition with a constant mass flow rate. For all channel sizes the Reynolds number was kept constant. This was achieved by changing the flow speed by the same power as the channel size so these values cancel in equation 14.

Pawan et al. started with a similar approach as this thesis.

$$\dot{S}'_{gen} = \frac{C_{1l}}{\lambda T^2} + \frac{C_{2l}\mu}{T\rho^2}. \quad (46)$$

Where C_{1l} and C_{2l} are constants defined as

$$C_{1l} = \frac{11}{48} q''^2 \pi D^2 \text{ and } C_{2l} = \frac{128 \dot{m}^2}{\pi D^4}$$

They used the ratio of entropy production of the nanofluid versus the basefluid and show in their publication this can be written as

$$\frac{\dot{S}'_{genNF}}{\dot{S}'_{genBF}} = \frac{\lambda_{BF} \rho_{BF}^2}{\lambda_{NF} \rho_{NF}^2} \times \frac{T_{BF}^2}{T_{NF}^2} \left(\frac{C_{1lNF} \rho_{NF}^2 + C_{2lNF} \mu_{NF} \lambda_{NF} T_{NF}}{C_{1lBF} \rho_{BF}^2 + C_{2lBF} \mu_{BF} \lambda_{BF} T_{BF}} \right). \quad (47)$$

For a nanofluid to be more effective this ratio should be smaller than unity.

Considered are a tube of 0.1, 1 and 10 mm in diameter. Pawan et al. used estimations for the values in equation 47 to simplify it.

Table 4. Values used for a 0.1 mm channel for reducing equation 47

Parameter	Value
Heat flux	100 W/m ²
Diameter	0.1 mm
Density	1000 kg/m ³
Mass flowrate	0.0001 kg/s
Thermal conductivity (basefluid)	1 W/m K
Viscosity (basefluid)	0.001 N s/m ²
Temperature	100 K

The values from table 4 result in $C_{1l}\rho^2 \ll C_{2l}\mu\lambda T$ adding the assumption $T_{BF} \sim T_{NF}$ equation 47 can be reduced to

$$\frac{\dot{S}'_{genNF}}{\dot{S}'_{genBF}} = \left(\frac{\rho_{BF}}{\rho_{NF}} \right)^2 \left(\frac{\mu_{NF}}{\mu_{BF}} \right). \quad (52)$$

Indicating that the nanofluid can still be better or worse, depending on the ratio of the density and viscosity of the nanofluid and basefluid.

Table 5. Values used for a 10 mm channel for reducing equation 47

Parameter	Value
Heat flux	100 W/m ²
Diameter	10 mm
Density	1000 kg/m ³
Mass flowrate	0.01 kg/s
Thermal conductivity	1 W/m K
Viscosity	0.001 N s/m ²
Temperature	100 K

Using these values and again the assumption $T_{BF} \sim T_{NF}$ equation 47 reduces to

$$\frac{\dot{S}'_{genNF}}{\dot{S}'_{genBF}} = \frac{\lambda_{BF}}{\lambda_{NF}}. \quad (53)$$

A nanofluid has always increased the thermal conductivity so this value will always be smaller than unity. Therefore in a 10 mm channel the nanofluid is always better according to Pawan et al.

The channel with a diameter of 1 mm showed no simplification and therefore no conclusions could be drawn from equation 47.

For comparison with the model derived in this thesis the values from this publication are taken for a particle concentration of 5 % (volume percentage). These data were taken from figure 15 that shows the data from different research, figure 16 shows the data used in the article and are needed for equation 43 and figure 17 shows all values found for viscosity by other researchers.

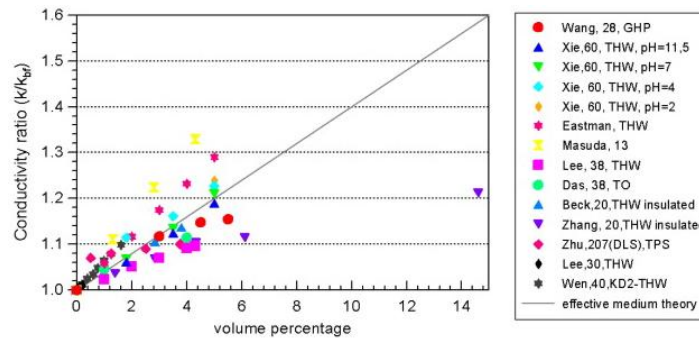


Figure 15. A number of research values for the thermal conductivity ratio for different volume percentages of nanoparticles.

Parameters	Values
T_{in}	300 K
$\Delta T = (T_{out} - T_{in})$	5 K
Length of channel	1 m
Basefluid (water) density	1000 kg/m ³
Basefluid conductivity	0.6 W/m K
Basefluid viscosity	0.001 N s/m ²
Basefluid C_p	4180 kJ/kg K
Particle (alumina) density	3900 kg/m ³
Particle conductivity	40 W/m K
Particle C_p	880 kJ/kg K

Figure 16. Different values for parameters used in the publication.

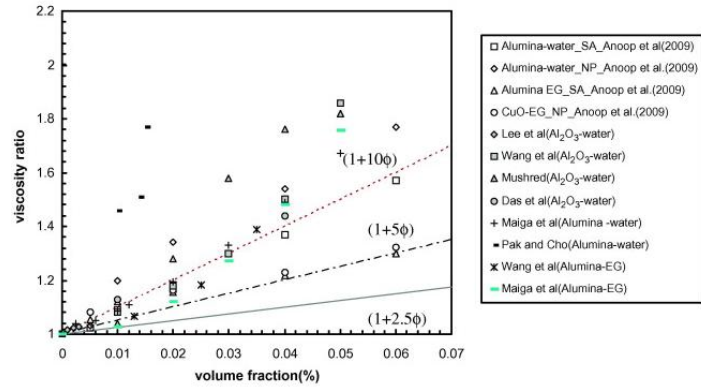


Figure 17. A number of research values for the viscosity ratio for different volume percentages of nanoparticles.

Using the values from the figures the left hand and right hand side of the figure-of-merit are calculated. These values are shown in table 6.

Table 6. The right and left hand side of the figure-of-merit for the different channel sizes.

Channel size	LHS	RHS
0.1 mm	210.8	$1.7 \cdot 10^{10}$
1 mm	210.8	$1.7 \cdot 10^6$
10 mm	210.8	169.8

Because only the tube diameter and the flow speed changed, the left hand side of the equation remains the same. Table 6 also shows that the increased channel size and flow speed have a major influence on the right hand side and therefore performance of the nanofluid. In this case the nanofluid is beneficial for the channel of 10 mm and not beneficial for the other two channels.

Pawan et al. concluded that for a channel with a diameter of 10 mm the nanofluid would be beneficial. Using the figure-of-merit derived in this thesis the same conclusion was found.

6. Measurements

Within the NanoHex project, University of Twente (UT) received several alumina nanofluids synthesized by other partners. The principal task of the UT within the project is to perform static and flow thermal measurements. It was shown in other studies at the UT that the nanofluids behave like any other classical fluid. Hence, knowledge of thermophysical properties such as density, heat capacity, thermal conductivity and viscosity is sufficient to predict the performance of the nanofluid. Flow measurements are not necessary to evaluate the fluids. Density and heat capacity are colligative properties and hence obeys the following mixture rule:

$$\rho_{nf} = x_v \rho_{par} + (1 - x_v) \rho_{bf} \quad (54)$$

$$c_{p,nf} = x_w c_{p,par} + (1 - x_w) c_{p,bf} \quad (55)$$

The subscripts part, bf and nf refer to the particle, base fluid and nanofluid respectively. ' x_w ' is the weight concentration of particles in the fluid. ' x_v ' is the volume fraction. Although there are classical models such as Maxwell-Garnett theory to predict thermal conductivity and viscosity of nanofluids, a wide discrepancy of the measured results with the model is reported in the literature [2, 3, 18]. This is because both these properties also depend on the size and morphology of the particle, the pH of the solution and the type of surfactant used to stabilize the fluid. Hence, it is necessary to measure the thermal conductivity and viscosity of the fluid.

Measurement of thermal conductivity

The thermal conductivity was measured using a transient hot wire method. The nanofluid is filled in a thermally isolated container and a probe is inserted into the fluid. This probe consists of a heater and a sensitive temperature sensor. During the measurement the heater generates a heat pulse and the thermometer measures the time evolution of the temperature of the probe. Because the heat will spread through the fluid and away from the probe it will cool the probe and from the speed of the cooling measured by a temperature sensor at the probe the thermal conductivity is calculated [18].

The apparatus to measure the thermal conductivity is shown in table 7. The container has two holes to measure two fluids simultaneously. The probe (needle) is fixed to the cap of the holder. The assembly after closing the cap is shown in figure 19. The entire unit could be inserted in a thermal bath filled with water as shown in figure 20. The temperature of the thermal bath could be adjusted with the thermostat.

The nanofluids were measured for about 2 to 2.5 hours and the probe measured the thermal conductivity every 15 minutes. So every fluid has a minimum of 8 data points. The fluids weren't tested longer because sedimentation occurred quite fast.



Figure 18. The isolated container with the probe next to it.



Figure 19. The closed assembly of the isolated container with the probe.

The viscosity is measured by a rotational type viscometer. This works by driving a spindle with a calibrated spring in the fluid. The drag of the fluid will deflect the spring and from this the viscosity is calculated [19].

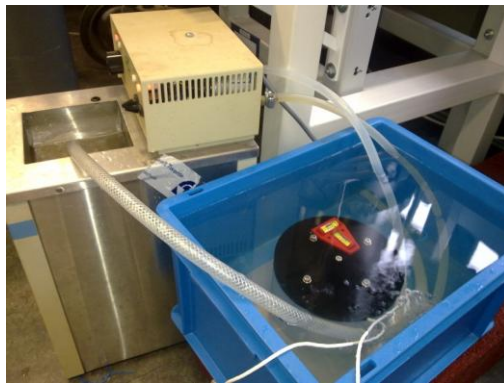


Figure 20. The container and the probe submerged in a bath of water.

Table 7 shows a list of alumina nanofluids that were measured. The base fluid consists of 50/50 wt % Ethylene Glycol (EG) to water mixture. This mixture is typically used as coolant because its freezing point is about -40°C . The nanofluids consist of either 9 or 4.5 wt % of particles in this basefluid and also some types of surfactants were varied in these fluids. The aim of preparation of these fluids is to investigate the stability of the particles in the fluid with the goal to have a high thermal conductivity and low viscosity of the nanofluid compared to the base fluid. Measured thermal conductivity and viscosity data for these fluids is also given in table 7. It is interesting to note that three fluids have a viscosity increase lower than 10 % compared to the base fluid with an increase in thermal conductivity in the range of 8-11 %. Typically the best nanofluids reported in the literature have thermal conductivity increase of about 10-15 % but with a high viscosity increase in the range of 50-100 %. Thus, these fluids might be interesting as a coolant.

Table 7. Measurements on the thermal conductivity of nanofluids using the KD2 probe.

Fluid name	Particle concentration	Basefluid	Temperature [K]	Thermal conductivity [W/(m*K)]	Viscosity [cp]
Basefluid 1	0%	50/50 wt % EG/H ₂ O	294	0.376	3.98
Nanofluid 1 (Basefluid 1)	9%wt Al ₂ O ₃	EG/H ₂ O 50/50 wt %	294	0.413	7.49
Nanofluid 2 (Basefluid 1)	9%wt Al ₂ O ₃ type III	EG/H ₂ O 50/50 wt %	294	0.409	7.8
Basefluid 2	0%	4.5 wt % PVP EG/H ₂ O 50/50 wt %	294	0.366	6.13
Nanofluid 3 (Basefluid 2)	9%wt Al ₂ O ₃	4.5 wt % PVP EG/H ₂ O 50/50 wt %	295	0.415	6.56
Nanofluid 4 (Basefluid 2)	9%wt Al ₂ O ₃	100 % H ₂ O	295	0.611	1.23

For these fluids the model is used to calculate the maximum flow speed for the nanofluid to be beneficial. A circular tube with a steady laminar flow and a constant heat flux boundary condition is assumed. The chosen values are listed in table 8

Table 8. The chosen values to compare the fluids.

Parameter	Value
Heat flux	10 W/cm ²
Tube diameter	4 mm
Temperature	298 K

For convenience, equation 43;

$$\left(\frac{1}{\lambda_{bf}} - \frac{1}{\lambda_{nf}} \right) / (\mu_{nf} - \mu_{bf}) > \frac{a_1}{b_1}$$

The right hand side is equal for all cases because the values to calculate these don't change

$$a_1 = \frac{\pi C}{8 T} v^2 = 0.084 v^2$$

$$b_1 = \frac{q''^2 \pi D^2}{T^2 Nu} = 0.013$$

The left hand side of the equation is different for the different fluids and are equal to:

Table 9. Left hand side of the figure-of-merit.

Fluid	LHS
Nanofluid 1	68
Nanofluid 2	56
Nanofluid 3	75
Nanofluid 4	94

For these fluids the corresponding maximum flow speed can now be calculated.

Table 10. The maximum flow speed velocity for different nanofluids.

Fluid	Maximum velocity [m/s]	Maximum mass flow [kg/s]	Maximum Reynolds number
Nanofluid 1	3.2	0.04	1755
Nanofluid 2	3	0.05	1580
Nanofluid 3	3.4	0.05	2010
Nanofluid 4	3.8	0.06	12333

The fluids show that the maximum velocities are high enough for the fluid to be beneficial. The Reynolds number are within a large part of the laminar region for the first and second fluid this is the result of a high viscosity, the fourth fluid with a similar maximum velocity has an eight times smaller viscosity and therefore eight times higher maximum Reynolds number. The Reynolds number for this fluid is well into the turbulent regime for which the figure-of-merit cannot be used. This calculation can also be done to calculate a tube diameter, temperature or heat flux to work with if the other parameters are known [19].

7. Conclusion

Based on entropy production a figure-of-merit was derived to compare performance efficiency of nanofluids to their corresponding basefluids, assuming a fully developed laminar flow through a circular channel with the same fluid velocity for the base and nanofluid and small temperature differences. The case of a constant heat flux boundary condition was tested for a laminar flow. In all of the tested cases the derived equation provided a simple way to determine the efficiency of a nanofluid compared to a basefluid. Furthermore changes in parameters could be compared easily because most of the time only one side of the equation changed. This figure-of-merit also shows what parameters influence the performance of the nanofluid in what way. The right hand side of the equation reduces to $\frac{v^2 C Nu}{8 q^2 D^2}$. For this to be as small as possible the heat flux (q) and diameter (D) should be increased while the speed of the fluid (v) should be decreased.

The constant temperature boundary condition did not result in such an easy figure-of-merit. Here the temperature gradient was not linear as it was for the constant heat flux, this resulted in a far more complex equation, in which wasn't possible to split the changing parameters for a nanofluid from the parameters of the test rig. It is still possible to compare the nanofluid to the basefluid using this equation by just inserting the values for both cases and comparing to total entropy produced from it. In this manner not a lot of work is required to compare the fluids, but this way it is not possible to see how different parameters influence the results without inserting different values.

Apart from deriving the figure-of-merit also some measurements on thermal conductivity and viscosity of alumina nanofluids are performed. All fluids showed an increase in thermal conductivity and viscosity compared to their basefluids. From these measurements the maximum mass flow rate and Reynolds number could be calculated for each of these fluids where the nanofluid and basefluid would produce equal entropy. If a flow below those values is used the nanofluids will be beneficial.

This shows that the figure-of-merit provides an easy way to compare nanofluids to their corresponding basefluid.

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