UNIVERSITY OF TWENTE

Performance of corporate bond indices – an approach inspired by Merton's model

Master thesis

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Management Summary

This thesis explores the application of using option pricing methodology in a firm value model in the evaluation of several different corporate bond types. More specifically, the aim of this thesis is to evaluate whether the application of market implied volatilities in a generalization of the Merton model results in a useful risk management tool for financial institutions. The corporate bonds returns have been obtained by modeling the corporate bonds as a nominal risk free corporate bond and equity derivatives. In order to capture the characteristics of the different types of corporate bonds put and call options have been used. The three types of bonds which have been modeled are investment grade, collateralized and high yield corporate bonds.

Currently used models require a lot of factors which are not directly observable in the financial markets and therefore need to be estimated. The proposed model requires none of these factors and still tries to capture the main characteristics of the risk distribution of corporate bonds by extending the Merton model. This extension involves a risk free component, a nominal bond, and a combination of equity derivatives for modeling the three different types of corporate bonds. As these three types of corporate bonds have different characteristics it is required to model each corporate bond with different equity derivatives.

We have tested the proposed model to three corporate bond benchmarks for portfolios of institutional investors. The three IBoxx benchmarks are a good representation of the complete spectrum of available bonds. The three types range from risky high yield bonds, investment grade bonds to low risk bonds protected by high quality collateral. An In-Sample test has been performed over the period 2006-2010 in order to evaluate the proposed model used for capturing the risk distribution of the three types of corporate bonds.

To test whether the model is not just fitted on the in sample period, also an Out-of-Sample test is performed over the year 2011. The outcome of the In- and Out-Of-Sample test shows that, although the results vary over the three types of corporate bonds, there is evidence that modeling corporate bonds with the proposed model adequately captures the behavior of the investment grade bond index and the high yield bond index, even in the volatile time frame of the last couple of years. The contribution of the model is however foremost the replacement of unobserved inputs with market factors, thereby reducing model risk. The performance of the model differs between the corporate bond types. As we do not use specific bond factors, but try to capture the risk in the bonds using equity price movements, the model might not be able to capture specific risks of the bonds. An explanation for the differences in performance therefore might be found in the difference in riskiness of the three bond types. The model performs best with the high yield corporate bonds, which might be explained by the fact that these bonds have a high sensitivity to the equity price

movements. The outcome of the proposed model for investment grade corporate bonds shows that in comparison to the high yield corporate bond model the behavior of this type of corporate bond is more difficult to capture. The behavior of collateralized corporate bonds is not adequately captured by the proposed model. This outcome shows that the proposed model needs an additional factor for capturing the specific risks of low risk bonds.

The performance of the proposed model has also been compared against one of the most common ways to model corporate bonds in an ALM context. The proposed model has the benefit in comparison to the current used methods that it needs no market factors to be estimated while it still has a strong intuitive appeal. A regression analysis showed that both the proposed model and the returns of the Euro Stoxx 50 index and risk free interest rates showed that they explain the same level of variance and the standard errors are also similar. However, the proposed model has the big advantage of a better description of the left tail of the distribution, important for risk measures. This shows that the proposed model could be a good alternative for analyzing portfolios of corporate bonds in an ALM or other risk management setting.

Preface

This thesis serves as the final assignment for the master Financial Engineering and Management at the Universiteit Twente. After finishing my bachelor Industrial Engineering and Management, I continued studying at the University of Twente but also started working for two days a week at Cardano. This allowed me to see how I could use the knowledge from my studies and get relevant work experience. Although there were a lot of benefits, I also sometimes struggled in the balance study versus work.

Although I am already working at Cardano for a number of years, I am glad to finish my masters and with this my student life. Without the support from my family and girlfriend this would not have been possible. Therefore, I would like to thank my parents, Jeroen and Marga, and my girlfriend Sanne de Wit. I am really grateful for the patience and never ending support you gave me.

Furthermore, I would like to thank my company supervisors and colleagues Bart Bos and Bart Heenk at Cardano for supporting me writing this master thesis. Your feedback was always positive and this greatly helped me in finishing this assignment.

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1. Introduction

In the world of finance many decisions are based upon a tradeoff between risk and return. As quantifying risk is important when deciding in which asset to invest in, econometrical analysis of risk therefore plays a large role. Current markets are under stress which also makes that the focus on risk management becomes stronger than ever. The primary focus of risk management within financial institutions has shifted from the traditional risk management but rather on the occurrence of extreme events, as Duffie and Singleton acknowledged (Duffie & Singleton, 2003).

Financial institutions, like pension funds or insurance companies, are managing large portfolios of assets. Many of these financial institutions have obligations to their stakeholders. In order to meet these obligations financial institutions need to diversify risk by investing in different assets. These portfolios consist of several different asset classes like equity, commodities, hedge funds, fixed income and derivatives. The choice of investing in a particular asset class also affects the expected return and the amount of risk of the portfolio.

For a financial institution the funding ratio or solvency ratio is one of the key financial indicators. The level of the funding ratio of pension funds influences the investment decisions. For example the Dutch regulator imposes in the FTK guidelines a minimum funding ratio of 105% to pension funds (De Nederlandsche Bank, 2007). If the funding ratio drops below this level then the pension funds are obligated to come up with a contingency plan in which they state how they will get a higher funding ratio in the next 5 years. The level of inflation indexing for pensions is often also related to the funding ratio and in insurmountable deficit pension funds have no choice but to cut promised pension.

Some of these financial institutions therefore invest a large part of their portfolio in well-diversified low risk investments. The rationale behind this strategy is that the minimum of the promised obligations is always covered. The most used low risk investments are high rated government bonds, which provide a natural hedge against the interest rate risk on their liabilities. The effect of a change in the level of interest rates on the price of a bond is exactly opposite to that of the price of the liabilities a financial institution is facing.

Bonds can be divided in government bonds and credit bonds. The difference between the latter two is that government bonds are issued by a sovereign and credit bonds are issued by a corporate. Financial institutions seeking a low risk investment will often choose for highly rated government bonds, because of the generally low probability of default. The higher certainty of getting the investment back is reflected by a lower default rate and has an effect on the expected return. The nominal yield on a government bond is lower than for example a company which has a higher probability of default.

Stock prices and interest rates both decreased a lot in the last few years mainly due to the financial crisis of 2008. The drop also had an effect on the funding ratio of pension funds. In the Netherlands many funding ratios dropped below or around the minimum nominal funding ratio imposed by the regulators and in some other countries pension funds are facing even lower funding ratios. This lower funding ratio necessitates making a higher return and hence requires closely monitoring the risks. Investors are therefore looking for other investment opportunities which will provide them a higher expected return.

2. Problem motivation

The most used alternative for investing in government bonds are corporate bonds, which will generally give a higher return. The higher return is a compensation for the fact that the risk of investing in corporate bonds is higher than that of bonds issued by a government. The financial crisis of the last few years showed that actual default rates of corporates are higher and defaults occur more clustered than predicted the models. As a result investors are measuring and managing risks from credit exposures more frequently and are more interested in the risk on a portfolio level.

The most frequently used approach to indicate the risks of holding an investment like corporate bonds is to measure the volatility of the bond price. A higher volatility makes an investment more risky because large price fluctuations are more likely to occur. Although the volatility of the investment is a convenient measure of risk it does not explain why a certain price has changed or which underlying factors are responsible for this price change. A common approach is to determine and estimate risk factors which will capture the risks that are driving the price change. The weight of the factor indicates the contribution of the factor to the total risk.

As each asset class endures different types and sizes of risk, the mapping of these asset classes to the risk factors is a challenging topic. Each risk factor represents a different kind of risk, e.g. interest rate risk, counterparty default risk. The total risk of the assets is determined by both the volatility of the factor and the weight of the factor in the model. A commonly used measure to analyzing these factors is through Factor Analysis. This method describes the observed types of risk in terms of market factors.

In finance most factor models are using a beta weighted linear combination of market risk factors. These underlying factors which are determining the price are unambiguously mapped to readily observable indices in the market, such as an equity beta to a market index or an interest exposure to swap rates.
But there are bond types for which the mapping to market factors is not straightforward like collateralized debt or high yield debt. For these asset classes mapping to risk factors is complex and tedious. Another data problem which often occurs is that asset managers are not willing to disclose exactly which assets they are holding. Financial institutions invest in a fund managed by an asset manager and get merely an indication of the exposure to some betas.

One of the problems with the risk measures from these types of factor models is that these risk measures are based on sensitivities to the risk factors, which are the portfolio's betas. These betas only refer to the undiversifiable part of the risk: the risk that cannot be hedged away by holding a large and diversified portfolio. Namely the sensitivity of these betas to the underlying risk factors will change as the underlying factors will change. These second order effects may become larger as the movements are larger. Also specific risks of a portfolio are not captured by these models.

Financial institutions often make use of third party software for their Asset Liability Management ('ALM') studies. Based on the outcome of the ALM studies, investment decisions will be made. Some of the models, like CAPM or Factor models, used in these software systems are making use of model assumptions and have limitations with respect to the use of these software systems. These limitations together with the described data problems cause for a need to investigate if there is a better way of looking at certain asset classes in ALM. One of the asset classes which are often confronted with data problem are corporate bonds. In this study the modeling of these corporate bonds for ALM studies is investigated.

The value of corporate bonds depends mainly on three drivers: the rate of return on risk free debt, additional security like collateral pledged and thirdly on the estimated probability of default. The return on riskless debt can found by looking to perceived risk free government bond and calculating its value with the interest curve of that particular moment. The terms agreed upon on the issuance of the corporate debt are available through the prospectus of the issued corporate bond. Estimating the probability of default imposes more difficulty. The exact moment of default and recovery rate of a corporate bond is upfront never known and therefore has to be estimated. Estimation and modeling of the expected probability of default imposes difficulties as it is a low frequency event and therefore the estimation is often based on a small number of observations. Many models like reduced form and firm value models which are made of capturing the behavior of corporate bonds struggle with determining the correct risk premium of a corporate bond due to the fact that the above probability of default is not known and has to be estimated.

A model which starts from the viewpoint that the main driver for value change of a corporate bond depends on the valuation of the underlying company is the Merton model (Merton, 1973). The intended approach for the proposed model is therefor to model a corporate bond as a risk free nominal bond plus a put option on the equity price. This is in line with the work of Robert Merton (Merton, 1973) and Black & Scholes which state that the liabilities are determined as the initial market price of assets less the initial market value of the equity (Black & Scholes, 1973). This model could intuitively provide a better fit than the traditional models used in ALM, because as by nature of the payoff of a bond, the return distribution of a credit portfolio is skewed. The reason for this skew is that the downward potential is larger than the upward potential. For this approach a non-linear factor model will be used as it contains options from which the return from holding the option varies non-linearly with the value of the underlying and other factors. As this model does not require a specific market factor for corporate bonds, this approach also offers the benefit that fewer factors are used to describe an economic environment and therefore it reduces the correlation or covariance matrix.

Within the financial market a lot of different types of corporate bonds exist. The scope of this research will be limited to the following asset classes; investment grade bonds, collateralized bonds and high yield bonds. These three types of corporate bonds are researched as they reflect three different but often used types of corporate bonds used by financial institutions and span the universe of this asset class.

The problem statement of this research is:

Develop a model for capturing the behavior of portfolios of corporate bonds in an Asset-Liability-Management context by making use of a risk free nominal bond and equity derivatives, in particular for high yield, investment grade and collateralized corporate bonds.

The research question is split up in smaller investigative questions to answer the research question. A common way to research the problem statement is dividing the research question in smaller investigative questions (Cooper & Schindler, 2008). This makes sure that all the information is found in order to answer the research question.

In order to develop a model which better captures the behavior of investment grade, collateralized and distressed debt the current methodologies will be researched what the strengths and weaknesses are and which methodologies are commonly used to model corporate bonds and measure the amount of risk. In this thesis we will apply the model to three portfolios represented by the well-known iBoxx benchmarks for the three debt types. An analysis of the most used theoretical models frequently used to model corporate bonds will be commented on its usage and limitations.

These questions resulted in the following investigative questions:

- Does using actual market implied volatility provide a better fit than using a longer term average implied volatility?
- Do the asset classes tested with the model capture the behavior of the corporate bond indices with all levels of equity returns?
- Does the proposed model work well in an ALM environment for all proposed corporate bonds?

3. Aims and contribution of this research

The aim of this research is to develop an alternative way of looking at corporate bonds used in Asset Liability Management (ALM) studies. In this study two main objectives are researched. The first objective is to develop a new, more intuitive approach to model corporate bonds by making use of a risk free bond and equity derivatives based on observables market factors and to comment on the usage of existing models in an ALM context in order to compare the outcome of the proposed modeling approach. The proposed model for capturing the behavior corporate bonds is described and compared against the outcomes of actual corporate bond indices used as a benchmark.

The second objective of this study is to establish whether the risk distribution that is derived from modeling the corporate bond returns with the proposed model can be used for risk management purposes. Therefore traditional approaches to map the asset classes into risk factors are examined and compared against the factors and other risk measures. The comparison of the models is made on the basis of the several ways to measure and comment on the risk arising from holding such an asset in a portfolio. The results of the new proposed model are examined on the fit of the total distribution and in tail measures.

The results will only be used in analyzing portfolios of corporate bonds and to comment on the characteristics of a portfolio of corporate bonds. The outcome of the proposed model will not be used for the pricing of individual corporate bond as the model is set up to comment on the characteristics of a portfolio of corporate bonds. Comments on the risk of one particular bond would require more and other information and therefore factors to model.

4. Basic concepts and literature research

In order to develop a model which captures the behavior of corporate bonds the most important characteristics of these corporate bonds had to be identified. Also an overview of the different approaches to model credits and determining of risk factors has to be researched.

Financial risks are the risks investors are facing in the financial markets by investing in products. These risks vary from fluctuations of interest rates, changes in credit rating or unexpected default. Credit risk is one of the most important risks investors are facing. The literature on the area of modeling credit risk can be divided in two main approaches structural models and reduced form models. Research on defining the risk measures of an asset show that a lot of effort has been made to understand which factors are driving the price of an asset.

4.1. Characteristics of corporate bonds

Before going in to more detail which type of data is used it is important to better understand the specifics of the bond types in the three IBoxx benchmarks. Therefore a short description of what a (corporate) bond is and which kinds of bonds are present in the market is given below.

4.1.1. Bonds

Bonds represent by far the most used fixed income instrument as an investment and are also the most liquid of the fixed income securities. A bond is an obligation by the issuer to pay money to the bond holder according to rules specified in the contact when the bond is issued. The bond normally pays out its face or par value at maturity. In addition most bonds pay out a periodic coupon payment. The coupon represents always a percentage of the notional of the bond. For example if a bond has a notional of €10,000, and with a 6% coupon, will pay out €600 as coupon.

4.1.2. Bond ratings

Although bonds will pay out a fixed coupon and face value, they are subject to default if the issuer is unable to meet his financial obligation or goes into bankruptcy. Rating agencies will therefore analyze and rate the issuer of the bond to reflect the probability of default. The two largest rating agencies in the world are Moody's and S&P[1]. The system of ratings is designed to allow investors make decisions and compare risk through a simple system of gradation. The assignment of a rating to a bond by one of the rating agencies is made largely on the financial status of the issuer. Next to the financial status, measured by financial ratios, do also other, more subjective variables could play a role. The table below shows the rating classifications of Moody's and S&P.

[1] www.moodys.com and www.standardandpoors.com

	Rating Moody	Rating S&P	Definition
Investment Grade	Aaa	AAA	Highest credit rating
	Aa2	AA	High grade
	A2	A	Medium grade
	Baa2	BBB	low medium grade
Speculative Grade	Ba2	BB	non-investment grade speculative
	B2	B	Highly speculative
Distressed	Caa2	CCC	Extremely speculative
	C	D	In default

Table 1. Classification of Moody's and Standard & Poor's long-term rating

4.1.3. Investment Grade Debt

As shown in the table 1 debt of an issuer of corporate debt could be considered 'investment grade' if a credit rating company, like Moody's or S&P, assigns a rating which is BBB / Baa or higher. The rating reflects the likelihood that an issuer of the bond will be able to meet the payment obligations to the investor. A higher credit rating indicates that a company or government has a lower risk on defaulting within a certain period.

Because investment grade debt is comprised out of the bonds with the highest ratings these bonds are often used for a low risk investment with fixed pay-off. As mentioned before the probability of meeting the financial obligations determines the interest rate spread on top of the risk free rate, therefore the fixed rate of these investment grade bonds will be lower than on speculative of distressed bonds.

4.1.4. Collateralized Debt

Collateralized debt means that the issuer has pledged cash or other eligible assets as collateral for their loan. Typically used collateral are inventory, accounts receivable, trademarks or intellectual property like patents. This collateral lowers the potential loss of an investor if the issuer is not able to meet the payment obligations. The value of the collateral is often upfront agreed upon with a haircut for potential lower selling amounts.

The collateral pledged to the bondholders ensures that the required return of the bond will be lower. The collateral ensures that in a default situation that the loss given default is reduced substantially. Also these kinds of loans could be issued when the corporate is not able to lend against a good price without the additional security. This often has a direct relation with the dire financial status of the corporate. One could argue that the most used example of collateralized debt is a mortgage. In case the lender does not repay their mortgage the bank of other issuer of the mortgage is entitled to claim the property and sell it on the market in order to get back their invested money or a partial payment.

4.1.5. High yield Debt

High yield debt is debt from which the issuer is already in default or highly likely that it will end up in that situation. Rating agencies often rate this kind of debt as CCC/Caa2 or lower. A common way of obtaining distressed debt is to buy them from their existing owners. These original investors often sell off their investments to others when a company enters in financial distress. The original investors are not willing to take the chance that they will end up with nothing and choose for the option of selling it for a lower amount to a new investor. The last few year hedge funds have been the largest buyers of such debt and skilled themselves in the packaging and restructuring of this type of debt.

In the years before the financial crisis in 2008 investors struggled with the fact that lending to normal corporates was no longer profitable enough. For the better part of the twentieth century companies were able to lend money directly to private investors via bonds or packages of bonds which resulted in a larger market with lower returns. Competition has caused that the return on corporate bonds is so low that a lot of investors argued that the price did not reflect the correct risk.

With the packaging of debt and sharing the individual risks a new investment opportunity arose. Hedge funds and other opportunists made it possible for a larger group of investors to invest in distressed debt. Normally the risk in investing in individual distressed debt was too large and only speculative investors are interested in this type of debt. The repackaging of this kind of debt meant that a new group of investors also was interested in investing and the size of the distressed market increased with a lot of secondary selling of these packages and distressed funds in which investors could invest.

4.2. Credit Risk

According to Giesecke's definition: *Credit risk is the distribution of financial losses due to unexpected changes in the credit quality of a counterparty in a financial agreement* (Giesecke, 2004).

Bond issuers may default on their financial obligations to the bond holders. Therefore bondholders demand extra compensation in the form of credit premium for the amount of credit risk they are facing.

Credit risk is believed to comprise out of two different components for a company, systematic risk and idiosyncratic risk. Systematic risk comes from common factors in the market and idiosyncratic risk is from specific factors of a company itself.

Credit risk can be seen as the excess risk that a bond holds over a risk free bond. This excess risk is expressed as the credit risk spread. A lot of studies have been performed in determining the factors that drive the credit risk spread. It is mostly said that the credit risk spread contains three main factors: a premium for the probability of default, a premium for liquidity and a premium for additional risks (Hull, 2006).

4.2.1. Probability of default premium

The probability of default premium is the premium which is required for compensation for the fact that the issuer of the bond may default. This premium consists of the probability of default of a bond, the probability of a rating change of a bond, the uncertainty in case of default how large the exposure and recovery of the bond is. The loss one expects over the lifetime of a bond, which is calculated as (Duffie & Singleton, 2003):

$$Expected\ Loss = exposure\ at\ default\ (EAD) * PD * LGD$$

Where PD= default probability and LGD = Loss given default

Probability of default is reflected in the credit rating that a company or an individual bond receives from a rating agency. These credit ratings are provided by rating agencies like Standard & Poor's, Moody's and Fitch. The highest credit rating (AAA) reflects that a company is less likely to default than a lower rated company (like BBB). The table below shows that a firm is more likely to default on the longer term than within a shorter period.

Table 2. Average cumulative default probabilities of different rating classes (in %) 1981-2009. Standard and Poor's, 2009

Years	1	2	3	4	5	10
AAA	0.00	0.03	0.14	0.26	0.39	0.82
AA	0.02	0.07	0.14	0.24	0.33	0.74
A	0.08	0.21	0.35	0.53	0.72	1.97
BBB	0.26	0.72	1.23	1.86	2.53	5.60
BB	0.97	2.94	5.27	7.49	9.51	17.45
B	4.93	10.76	15.65	19.46	22.30	30.82
CCC	27.98	36.95	42.40	45.57	48.05	53.41

4.2.2. Liquidity risk premium

Liquidity risk is the risk that if an investor faces if he wants to sell an asset that at that point in time no or limited buying or selling possibilities are available in the market. Liquidity and the price of a financial instrument have a negative relation. Instruments which are illiquid always trade at discounted prices, according to Bongaerts et al (Bongaerts, et al., 2009). Because of this risk investors demand a liquidity premium in order to receive compensation for running this risk.

4.2.3. Premium for additional risks

In addition to the risks mentioned in the last two paragraphs there are more risks that investors are requiring premium for. As Haesen & Houweling researched, the interest rate risk of corporate bond returns can be as large as two-thirds of the total risk (Haesen & Houweling, 2011). Other risk may occur due to market circumstances where investors or traders require risk premium for. Turbulent markets drive risk premiums upwards and even premiums for smaller risks might become significant.

4.3. Credit risk modeling

Many models have been developed in order to develop the best model for modeling the probability of default. There are many variations and extensions on theories and models which comment on the way probability of default should be determined and measured, but there are two main approaches in order to determine the default process, reduced-form models and structural models.

Merton composed a model based on the capital structure of a firm. This approach is considered the basis of the structural approach models. In this approach it is assumed that a company defaults at the bond at maturity time T if its assets value falls below the value of its debt or a specified value, both at time T. In this approach the equity of the firm becomes a contingent claim of the assets of the firm's assets value (Merton, 1974). Black and Cox extended this model by generalizing Merton's method into the first passage approach. In their model the firm defaults when the first time that the assets drop to a sufficiently low default boundary, regardless of this occurs at maturity time T.

Reduced form approach is set out by (Artzner & Delbaen, 1995), (Jarrow & Turnbull, 1995) and (Duffie & Singleton, 1999). Reduced-form credit models are models in which the default of a company is set by a default-intensity process. The most basic model states that the default intensity process is governed by the first arrival time of default τ of a Poisson distributed process with a constant mean arrival rate of λ. More advanced models incorporate the company's credit rating or other drivers of default to model the default intensity process more accurately. In these models the probability of default is set by looking at historical data and calibrating the model so it will reflect the right probability of default or intensity process. This implies that that the credits modeled should reflect the same type of debt as the used historical data.

One of the first reduced form models makes use of a Vector Auto Regression model. The Vector Auto Regression model is an econometric model which is often used to predict various types of variables. The VAR models are a generalization of the AR and ARMA models. Vector Autoregressive Models have been used for generating the economic scenarios and have been used in macro-economics since 1980. All the variables in a VAR are treated similarly by including for each variable an equation explaining its evolution based on its own lags and the lags of all the other variables in the model.

The goal of the proposed model is to develop an intuitive model which makes use of a risk free nominal bond and equity derivatives. As this approach is a generalization of the classic Merton model a more extensive description of structural of firm value models will be given in the next paragraph.

4.4. Structural or firm value credit models

Structural models try to model the inability of the issuer to repay the loan. Companies are in default when they cannot meet their financial obligations. The value of the company's assets and debt are used in order to determine if a company is in default. Black and Scholes (Black & Scholes, 1973) and Robert Merton (Merton, 1974) proposed a structural model for modeling corporate liabilities and the probability of default. In this model the liabilities are determined as the initial market price of assets less the initial market value of the equity. The default of a company occurs when, at the maturity of the debt (T), the issuer's assets at maturity (T) are less than the face value of the debt at that moment.

In the structural model of default probability of Black and Scholes is asset value of a firm (V) described as a stochastic process under which:

$$\mathrm{dV(t)} = (\mu - \gamma)\mathrm{dt} + \sigma\mathrm{dB(t)}$$

Where μ is the mean rate of return on assets and γ is the proportional cash payout rate, σ is the assets volatility and B(t) is a Standard Brownian motion with a unit variance parameter and a constant drift.

Let's set τ as the Default time. The firm is assumed to default at the bond with maturity date T, if the total market value of the firm is lower than the total amount of debt. Thus the default time τ is a discrete random variable given by

$$\tau = \begin{cases} T \ if \ V_T < D \\ \infty \ if \ else \end{cases}$$

Where D is the total amount of debt.

Figure 1 shows that the probability of default is the part of the distribution of the assets value which falls below the value of the liabilities.

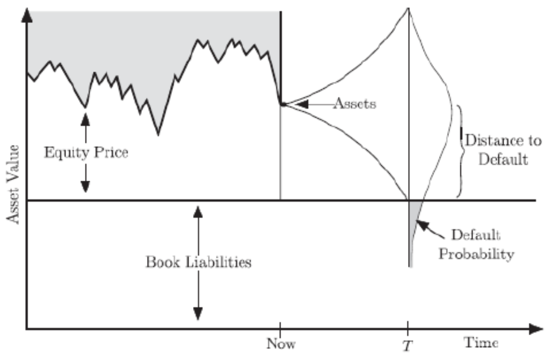

Figure 1. The Black-Scholes-Merton structural model of default **(Duffie & Singleton, 2003)**

Many models have taken the above mentioned structural model and have expanded and improved this model. For example when the value of the company's assets becomes less that the value of the debt a company is not in immediate default. Only when a company is unable to meet the promised financial obligations one can state that it is in default. Therefore Black and Cox (1976) proposed to a company is only in default when the assets drop below a certain set boundary.

The classic Merton model (Merton, 1974) predicts a positive correlation between corporate bond credit spread and stock return volatility. Many studies have empirically identified this link at both the aggregate and firm level. But for some stocks the positive correlation is far from perfect.

As the new model will be used to capture the behavior of three different types of corporate bonds it needs to be clear on which measures the new model will be tested. The next paragraph describes the risk measures on which the proposed model will be evaluated for three iBoxx corporate bond indices.

4.5. Determining risk measures

For determining if the proposed model adequately captures the behavior of the corporate bond types it is needed to look if the risk measures which are derived from the return distribution are in line with the risk measures derived from the traditional distribution risk measures. Therefor it is needed that the corporate bond returns from the proposed model are tested for their risk measures and could perform well. The proposed model needs to be tested if it shows the same level of variance and if the overall return distribution follows the same shape as the realized returns of the iBoxx benchmarks.

In order to comment on the risk measures of the proposed model a better understanding of these risk measures is needed. The proposed model provides good risk measures if the returns out of the model capture the behavior of the corporate bonds. For looking at the degree of risk in corporate bonds often is made use of the volatility of the price of an asset as the main indicator of the risk of an asset. This volatility is defined as the conditional standard deviation of the financial time series of (log-)returns. If the volatility of an assets return is high, the possible losses (or gains) are also high. In the past a lot of effort has been put in to understand which factors are driving the price of an asset. If these driving factors are known they can be used describing the risks of a corporate bond.

Another way to determine and quantify the risk of a portfolio by using a widely adapted measure of market risk called Value-at-Risk (VaR). The techniques used in VaR models was first developed by investment bank J.P. Morgan in its 'RiskMetrics' program but nowadays regulators make often use this measure. VaR is used to assess the level of loss that some low probability of being exceeded over a fixed time horizon, e.g. 99% confidence level. VaR models are set out to be used to compare the market risks of all types of activities. It provides a single measure that is easily understood by a great majority of people and do not ignore the risks arising from movements in the underlying risk factors and the specific risks of a portfolio (Duffie & Singleton, 2003). Market standard is that one takes a 99% confidence level and VaR can be defined in the following way according to (Weisner, 2010).

Value-at-Risk at time t and at level of significance 1-α is defined by the following expression.

$$P(r_t \leq VaR_t^{1-\propto}) = \propto,$$

$$where\ the \log return\ r_t\ is\ defined\ as$$

$$r_t = ln\ \frac{P_t}{P_{t-1}}$$

$$P_t\ is\ the\ price\ of\ an\ asset\ or\ portfolio\ of\ assets\ such\ as\ an\ index\ at\ a\ point\ in\ time\ t.$$

If a level of 99% confidence is taken the VaR can be determined by:

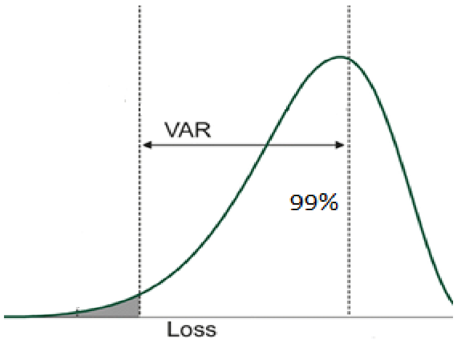

Figure 2. VaR model

Some of the VaR models used in ALM software assumes that returns are Normally distributed with N(μ,σ^2). Here also lies its weakness, as recent financial crises have shown, financial returns do not have to be Normally distributed. Duffie & Singleton state that whether or not the VaR of a portfolio is a relevant risk measure over a short time period depends on the liquidity of the portfolio and the risk of adverse extreme events which will result in net cash outflows or of severe disruptions in market liquidity (Duffie & Singleton, 2003). For this research we will therefor refer to historical simulation in order to comment on the VaR of the portfolios of corporate bonds.

Next to looking at the level of variance out of the proposed model and looking at the VaR of the portfolio's the characteristics of the return distribution need to be equal. If the actual return distribution of the benchmark is skewed or has fat tails the proposed model also needs to have a similar distribution.

5. Proposed model

This chapter will describe the how the three corporate bond types will be set up in the proposed model. Before describing each of the three corporate bond types separately a more high level description will be given.

The proposed model starts with a firm value model approach to model corporate bonds which is in line with the structural model as comprised by Merton, to model a corporate bond as a nominal risk free bond plus derivatives on the company's equity. For each of the three different types of corporate bonds a different set of equity derivatives will be used. By making use of actual market implied volatilities provided by major investment banks the proposed model does not have to estimate this parameter and next to setting the correct option strike no parameters have to be estimated. Because of the fact that no parameters have to be estimated this approach could intuitively provide a better fit than the mentioned reduced-form models which often make use of VAR and GARCH models to estimate the used parameters. The proposed model will be a non-linear factor model in terms of market factors because the proposed model contains equity options for which the return varies non-linearly with the value of the underlying. The proposed model splits the three corporate bond types in a two parts, namely (1) a risk free bond and (2) a set of equity derivatives depended on the characteristics of the corporate bond. The model will therefor take the following form for each of the type of corporate bonds modeled:

(1) In each of three types of corporate bonds the value of the risk free bond is calculated by:

$$P_0 = \frac{N}{\left(1+\frac{r(t_M)}{v}\right)^{vt_m}} + (c/v)\sum_i^M \frac{1}{\left(1+\frac{r(t_i)}{v}\right)^{vt_i}}$$

Where:

P_0= current price of the bond (t=0)
c = annualized coupon
v = annual frequency of coupon payments
N= notional of the bond
(t_M) = maturity of the bond (in years)
$r(t_i)$ = the spot-interest rate for a payment due at t_i (with t_i in years)

(2) The value of the equity derivatives used for determining the value the three corporate bonds are calculated by making use of the Black-Scholes option pricing formula for determining the value of the used call and put options:

$$Call_0(S,T,\sigma,r,K) = S_0N(d_1) - Ke^{-rt}N(d_2)$$

And

$$Put_0(S, T, \sigma, r, K) = Ke^{-rt}N(-d_2) - S_0N(-d_1)$$

With

$$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{2}$$

Where

Call= European call option price
Put= European put option price
S_0= stock price at t=0
T= time to maturity of option
σ= stock price volatility
r = continuous compounded risk free rate
K= strike price
N = the cumulative probability distribution function for a standardized normal distribution

The classic Merton model assumes that the value of a corporate bond depends on the value of the assets of the company. The proposed model does recognize this relationship as well, but assumes that this relationship is not straightforward if for example collateral is taken into account. By modeling the risk premium of a corporate bond with equity derivatives, the delta of these derivatives will vary as the value of the underlying company will change. The most prominent effect therefore is the fact that the corporate bond will be more sensitive to equity returns in a prolonged equity crisis, as the delta of the put option will be higher after a downturn of the equity markets.

The Merton model assumes that the volatility of the value of the assets of company (σ_V), is comprised out of the volatility of the equity of the firm (σ_E) and the leverage in the company ($\frac{E}{V} * N(d1)$).

$$\sigma_V = \sigma_E * \frac{E}{V} * N(d1)$$

The outcomes of the last crises show that the volatility of the assets may not be the best model for capturing the behavior of corporate bond returns. Also the Merton model assumes that the volatility of the risk free rate is 0, which clearly is not the case as the financial markets showed the past few years and is not to be ignored in less risky debt. The proposed model therefore deviates from the above stated relationship between the asset value and the value of its equity. By modeling the volatility of the company in such manner that it does not depend on a variable that is not found in the market the model should be able to improve the ability to capture the behavior of the corporate bond returns. The proposed model takes of the following functional form:

$$W = f(r_f, S, \sigma_E)$$

Here W denotes the value of the corporate bond and where the inputs are r_f the risk free interest rate, S is the spot equity price, σ_E is the implied volatility. The model is furthermore defined by the parameters set for the equity instruments, namely the number and nature of the instruments, the maturity of each instrument and the strike of each instrument.

Herewith we replace the value of the assets of the company and the leverage factor with the market valuation of the package consisting of these assets and the financing by the equity price (E) and the implied volatility (σ_E). The risk free interest rate (r_f) is for ease of implementation taken as the swap rates instead of government bonds. The market factors are now completely observable and therefor there is no need to have a further complexity added to the economic model.

This proposed model is not an entirely new approach for modeling corporate bonds. Previous attempts of using this approach estimated the volatility of the returns of the corporate bonds indices (Hull, et al., 2004) (Haesen & Houweling, 2011). This implied volatility is needed for valuing the equity options used in the model. This estimation of the implied volatility caused an additional variable which should be estimated and therefore further complicated the use of the model in economic scenario generation. This research uses the actual market implied volatility used by investment banks to trade their equity options. The market implied volatilities are obtained by averaging the implied volatility of major investment banks in Europe.[2] The data reflects the implied volatility used for pricing OTC derivatives on the Euro Stoxx 50. The range of the market implied volatilities at certain strikes provided by the investment banks consist of strikes between 70% - 130% as no the market outside this range is too small for investment banks to give quotes where parties can trade on. The fact that derivatives are bought and sold based on these implied volatilities ensures the correctness.

The way corporate bonds are modeled in the proposed model depends, in addition to the standard Merton model, on the characteristics of the type of corporate bonds. The asset classes which are going to be examined are investment grade corporate bonds, collateralized corporate bonds and high yield corporate bonds. These three asset classes are chosen as these are the most common in the portfolios of many large financial institutions. In the next paragraphs the three corporate bond models which require different derivatives are described.

[2] This data has been obtained by Cardano via the investment banks and therefore is not publically available. The data is available for verification of the results.

5.1. Modeling of investment grade corporate bonds

Investment grade corporate bonds are modeled in the proposed model as a risk free nominal bond and a sold put option. This approach is in line with the standard Merton model. If the value of the company decreases the value of the bond also will decrease. If the value of the company rises above the face value plus the coupon the corporate bond should not increase in value anymore. The sold put makes sure that if the company's value decreases also the value of the corporate bond will decrease.

For the modeling investment grade corporate bonds a risk free bond has been modeled with the duration equal to the average constituting bonds of the iBoxx euro investment grade corporate index plus a sold put option on the Euro Stoxx 50. In order to adequately reflect the index a series of 5 put options have been modeled each with a different maturity, e.g. 1 till 5 years. The received premium and the payoff of the put options are added to the return of the risk free bond.

The strike of the put option has been set at 70%. There is no specific reasoning for setting the 70%, other than that the 70% is the lowest strike available in the provided market data by the investment banks. The strike at 70% can be interpreted as the point from which the risk premium is no longer sufficient to cover the loss. Below the 70% the put option will mimic the loss an investor would make when a corporate bond will go into default. The options are modeled using a Black-Scholes model with the interest rate given by the market interest rates and the volatility used is the corresponding volatility for valuing an equity option on the Euro Stoxx 50 with a strike at 70%.

The proposed model for an investment grade corporate bond index can therefore be described by:

$$corporate\ bond = risk\ free\ bond - set\ of\ 5\ equity\ put\ options$$

In a formulaic form (for the complete definition of the functions P and Put, see above):

$$Value\ Bond_t = P_t(c = rf_0, v = annual, rf_t, N = 100, t_m = avg\ maturity\ index)$$
$$- \sum_{i=1}^{5} \frac{1}{5} * Put_{i,t}(S_t, T_i = 1,2,3,4,5\ year, \sigma_{i,t}, rf_t, K_i = 0.7 * S_0)$$

The volatility used as earlier mentioned is the actual implied volatility used in pricing the Euro Stoxx options by major investment banks in Europe. Because this implied volatility is given there is no need to estimate this parameter.

To give an indication of how the payoff of the corporate bond will behave as the value of the company decreases the below example is given. The actual proposed model investment grade corporate bonds will follow the same pay off. The graph shows that the maximum final payoff equals 108 which implies assumed fixed rate coupon of 8%. The sold put option has been set for the investment grade bond index at a strike of 70%, which is the lowest strike provided by the investment banks. The market for lower strikes is not big and liquid enough to obtain tradable quotes which could be used in this model. The level of 70% does not imply a 30% solvency of the average company, but the drop in the equity index value when actual defaults will start to take place. The pay-off profile of an investment grade corporate bond is shown in the below graph.

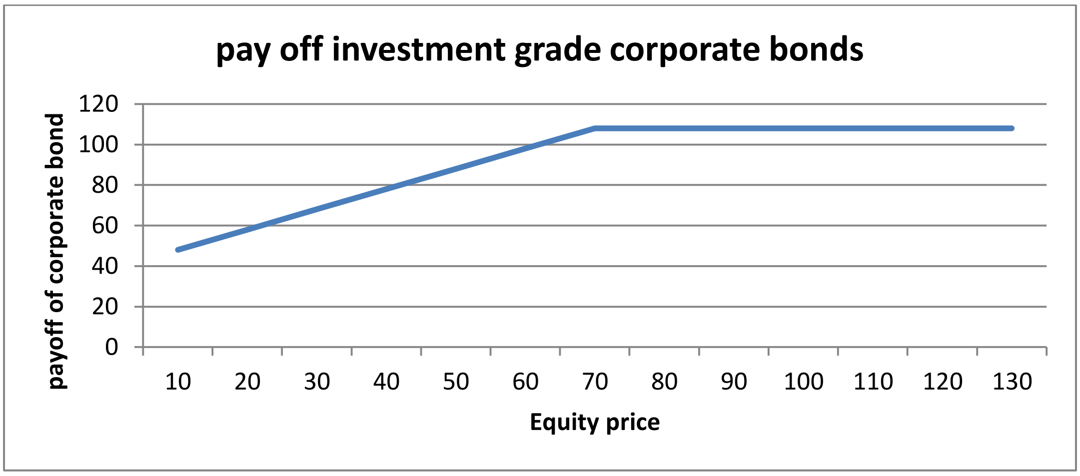

Figure 3. Value of corporate loan

As the corporate bond index comprises out of several bonds it is assumed that the actual payoff structure will be more smoothened. Generally speaking not all bonds will default at exactly the same time. The payoff structure is capped to the notional plus the interest. The part of the payoff above 100 reflects the risk free rate plus the risk premium which is received for being exposed to several risks.

Hence the below payoff structure shows how the proposed model with 5 put options with different maturities will look. The volatility is kept constant at 25% and risk free interest rate at 1.5%. Using the Black-Scholes option pricing formula it shows the payoff profile of a corporate investment grade bond index approximated by a staggered set of sold put options plus a risk free bond.

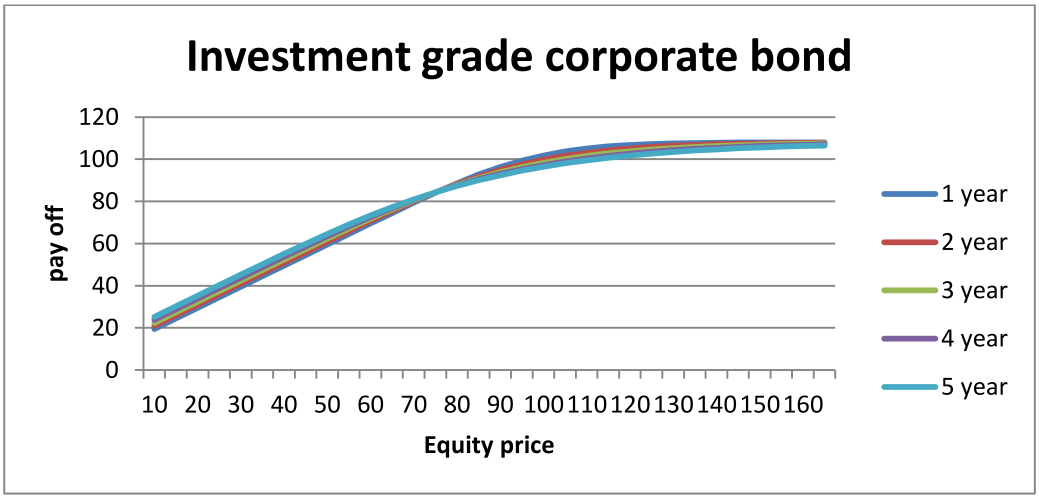

Figure 4. Investment grade corporate bond with different maturities.

5.2. Modeling of collateralized corporate bonds

Collateralized corporate bonds are modeled in the proposed model as a nominal risk free bond, a sold put option on the equity index and a bought put option on the underlying equity index. The nominal risk free bond and the sold put option make sure that if the value of the companies in the index decreases the value of the corporate bond also decreases. The minimum value of a collateralized bond will be the value of the collateral which has been pledged by the issuer. If the issuer is not able to meet his financial obligations, the holder of the loan may sell the collateral pledged and recover part of his loss. To reflect this corporate bond a bought put option makes sure the value of the corporate loan will never drop below the value of the collateral pledged. The below graph shows the payoff profile of the collateralized corporate bond.

For the proposed model a nominal bond has been modeled with the duration equal to the average constituting bonds of the iBoxx euro collateralized AAA corporate index, a sold put option on the Euro Stoxx 50 and a bought put option at a lower strike. The bought put option reflects the collateral which is posted for the corporate bonds. In order to reflect the index a series of 5 put options have been modeled each with a different maturity, e.g. 1 till 5 years. The received premium and the payoff of the put options are invested in the risk free bond. The strike of the sold option has been set at 70% and the bought put options have a strike at 30%. There is no specific reasoning for setting the 70%, other than that the 70% is the lowest strike available in the provided market data by the investment banks. Setting the strike of the bought put options at 30% is in line with common market practices for collateralized corporate bonds. The 70% can be interpreted as the point in which a company will be in default. The volatility used is the corresponding volatility for valuing an equity option on the Euro Stoxx 50 with a strike at 70%.

The proposed model for a collateralized bond index can therefore be described by:

$$\begin{aligned}&corporate\ bond\\&\qquad = risk\ free\ bond - set\ of\ 5\ equity\ put\ options\ strike\ 70\%\\&\qquad + set\ of\ 5\ equity\ put\ options\ strike\ 30\%\end{aligned}$$

In a formulaic form (for the complete definition of the functions P and Put, see above):

$$Value\ Bond_t = P_t(c = rf_0, v = annual, rf_t, N = 100, t_m = avg\ maturity\ index)$$

$$- \sum_{i=1}^{5} \frac{1}{5} * Put_{i,t}(S_t, T_i = 1,2,3,4,5\ year, \sigma_{i,t}, rf_t, K_i = 0.7 * S_0)$$

$$+ \sum_{i=1}^{5} \frac{1}{5} * Put_{i,t}(S_t, T_i = 1,2,3,4,5\ year, \sigma_{i,t}, rf_t, K_i = 0.3 * S_0)$$

The payoff of the collateralized corporate bonds as modeled in the proposed model is shown in the below graph as an example of how these corporate bonds will be modeled.

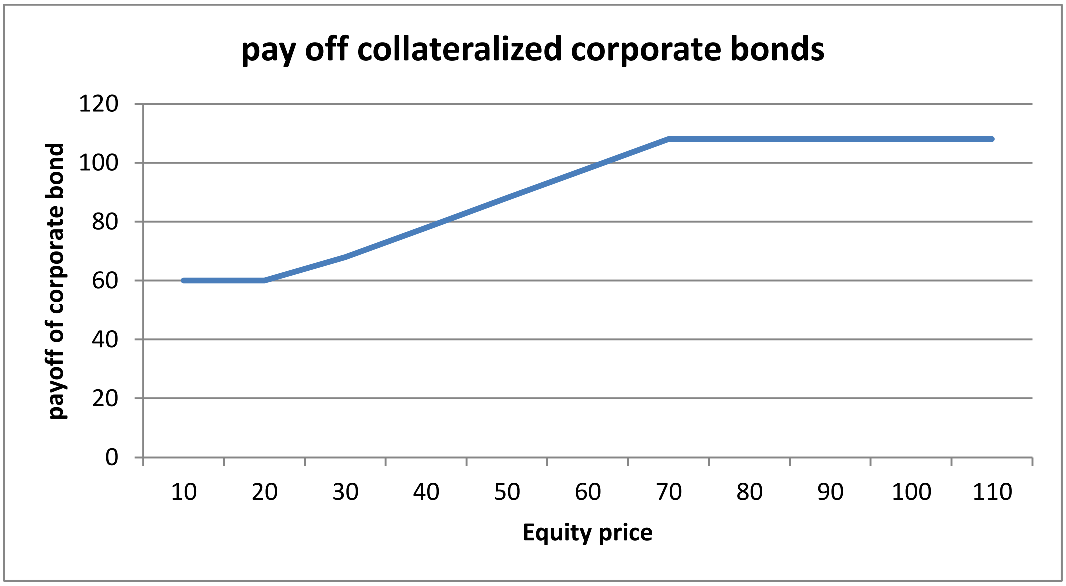

Figure 5. Pay off structure collateralized corporate bond.

If we look at different maturities for the two put options, keeping volatility (25%) and risk free interest rate (1.5%) the constant, the payoff of a corporate collateralized bond is more smoothened. The payoff is calculated using the Black-Scholes option pricing formula.

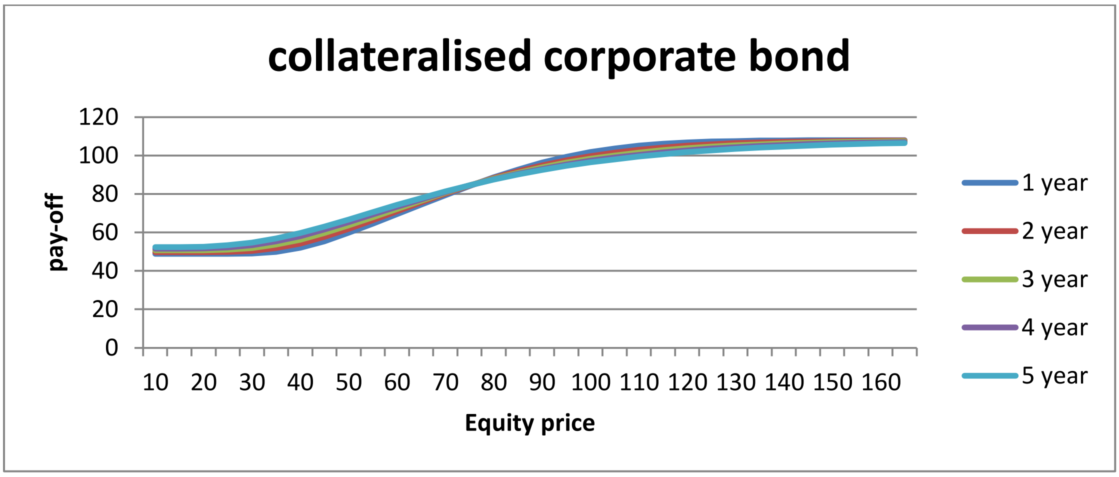

Figure 6. Pay off collateralized corporate bond throughout time.

5.3. Modeling of high yield corporate bonds

High yield corporate bonds are modeled in the proposed model as a nominal interest rate bond, sold put option and a bought call spread. The nominal risk free bond and the sold put option again make sure that if the value of the companies in the index decreases the value of the corporate bond also decreases. The fact that this is a high yield corporate debt means that the addition of the call spread reflects the additional return that may be made if the company recovers. This additional payoff compared to an investment grade corporate bond is shown by the fact that in this example the maximum payoff of 118 is possible, whereas the maximum return on an investment grade corporate bond is 108.

The proposed model is set up by making use of equity options which together will have the same payoff of that of high yield corporate bond returns. A sold put option with a strike of 70% will make sure that when the underlying value of the Euro Stoxx index will decrease the value of the corporate bonds will also decrease. In order to reflect the index a series of 5 put options have been modeled each with a different maturity, e.g. 1 till 5 years. The fact that high yield bonds in the actual market will sell below the clean market value is realized by a call spread. This call spread consists of a bought call with a strike at 70% and a sold call with a strike at 100%. There is no specific reasoning for setting the 70%, other than that the 70% is the lowest strike available in the provided market data by the investment banks. The strike at 70% can be interpreted as the point from which the risk premium is no longer sufficient to cover the loss. Below the 70% the put option will mimic the loss an investor would make when a corporate bond will go into default.

The proposed model for a high yield index can therefore be described by:

$$\begin{aligned}&corporate\ bond \\ &\qquad = risk\ free\ bond - set\ of\ 5\ equity\ put\ options\ strike\ 70\% \\ &\qquad + set\ of\ 5\ equity\ put\ options\ strike\ 30\%\end{aligned}$$

In a formulaic form (for the complete definition of the functions P, Put and Call, see above):

$$Value\ Bond_t = P_t(c = rf_0, v = annual, rf_t, N = 100, t_m = avg\ maturity\ index)$$

$$- \sum_{i=1}^{5} \frac{1}{5} * Put_{i,t}(S_t, T_i = 1{,}2{,}3{,}4{,}5\ year, \sigma_{i,t}, rf_t, K_i = 0.7 * S_0)$$

$$+ \sum_{i=1}^{5} \frac{1}{5} * Call_{i,t}(S_t, T_i = 1{,}2{,}3{,}4{,}5\ year, \sigma_{i,t}, rf_t, K_i = 0.7 * S_0)$$

$$- \sum_{i=1}^{5} \frac{1}{5} * Call_{i,t}(S_t, T_i = 1{,}2{,}3{,}4{,}5\ year, \sigma_{i,t}, rf_t, K_i = S_0)$$

The payoff of these high yield corporate bonds is shown in the below graph as an example of how these corporate bonds will be modeled.

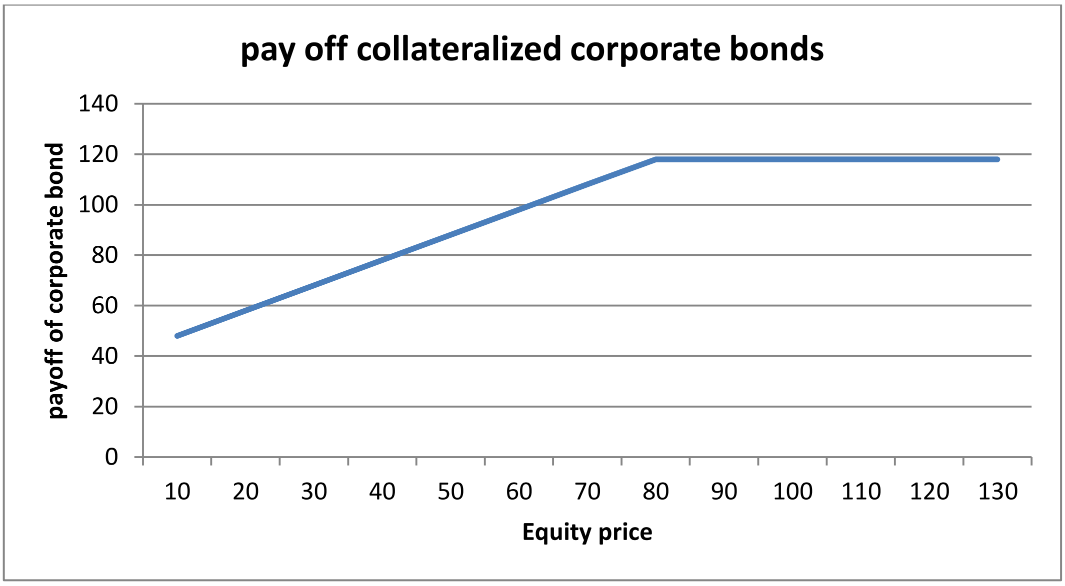

Figure 7. Pay off high yield corporate bond throughout time.

When buying distressed or high yield corporate bonds the extra return in comparison with a nominal bond is made by the fact that these bonds will sell at a lower bond price in comparison to other investment grade investments.

Keeping volatility (25%) and risk free interest rate (1.5%) constant the payoff of a high yield corporate bond using the Black-Scholes option pricing formula is shown below for the total portfolio of equity derivatives plus the risk free bond.

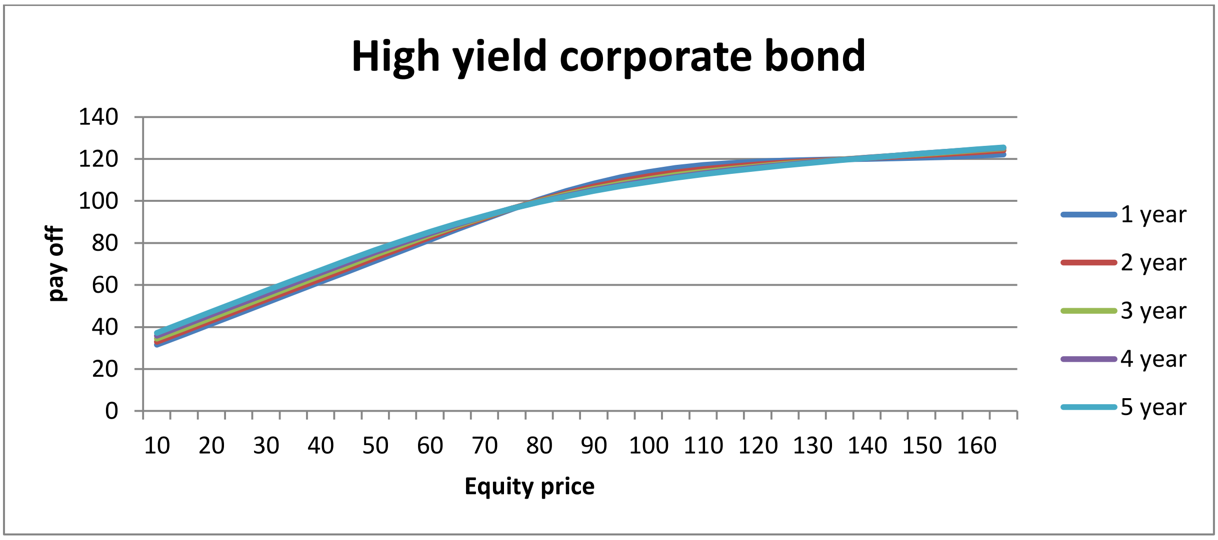

Figure 8. Pay off high yield corporate bond with different maturities.

6. Description of data

This chapter describes the data which will be used in the next chapter for researching the working and effectiveness of the proposed model. The only uncertain parameter in the proposed model will be the strike of the equity options, therefor the description of the used data will be the actual data used for testing the proposed model. The monthly corporate bond index returns are obtained from observable market corporate bond indices. As corporate bond index the Markit iBoxx indices are used. These indices are maintained by Markit. The iBoxx indices are the market leading fixed income benchmark indices. These indices are available for Euro, Sterling, USD and Asia. The indices used for this research are Euro indices comprised out of investment grade, collateralized and high yield corporate bonds.

For modeling the corporate bond returns and comparing them against actual market returns several sources of data have been used. For modeling the corporate bond returns risk free market interest rates, an equity index and its corresponding implied volatility have been used. These modeled returns are compared against actual corporate bond index returns.

6.1. Bond indices

As already stated we will apply the proposed model to three different types of corporate bonds. We have chosen for three indices from the iBoxx family, each representing the universe of this bond type. These indices comprise out of corporate loans which are based in the Euro zone. The historical monthly returns of these indices have been calculated from January 2007 till the end of 2011. Below graph describes the characteristics of the historical returns of the three indices.

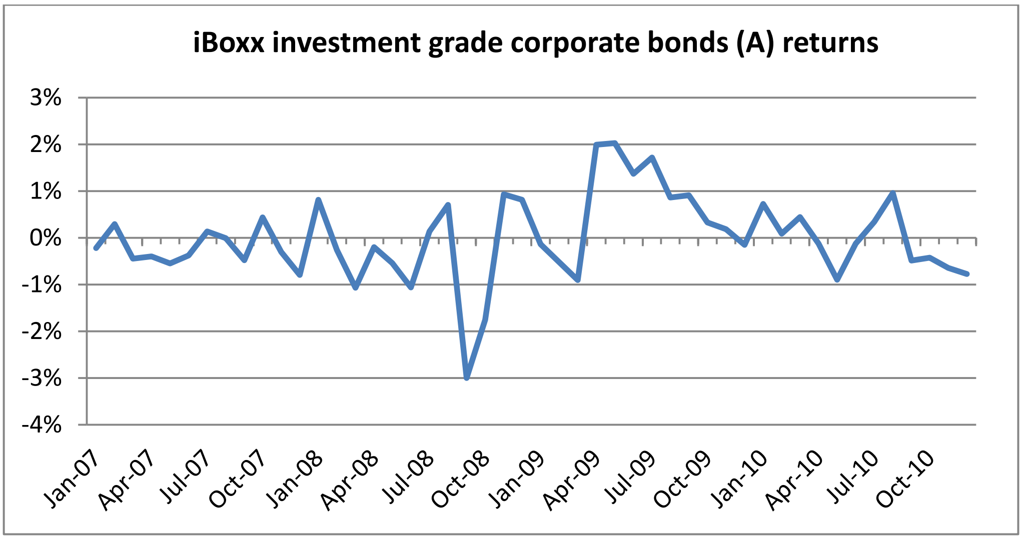

Figure 9. iBoxx investment grade corporate bond returns

For comparing the investment grade corporate bond returns the ‘iBoxx investment grade corporate bonds (A)’ is used. The returns in the graph above show that the returns made on the corporate bond index shows large movements in the period between August 2008 and December 2010. These euro corporate bonds also have an average maturity of 5 years with a rating of A.

For the modeling of the euro collateralized bonds, corporate bonds with an average maturity of 5 years and a rating AAA have been used. The reason why AAA bonds have been chosen is that with AAA bonds we now that the change in collateral value will be minimal as in this research the change in value of the underlying company will be tested. The iBoxx collateralized AAA index has been used, which comprises of corporate loans with a fixed coupon and which are secured by high quality been collateral.

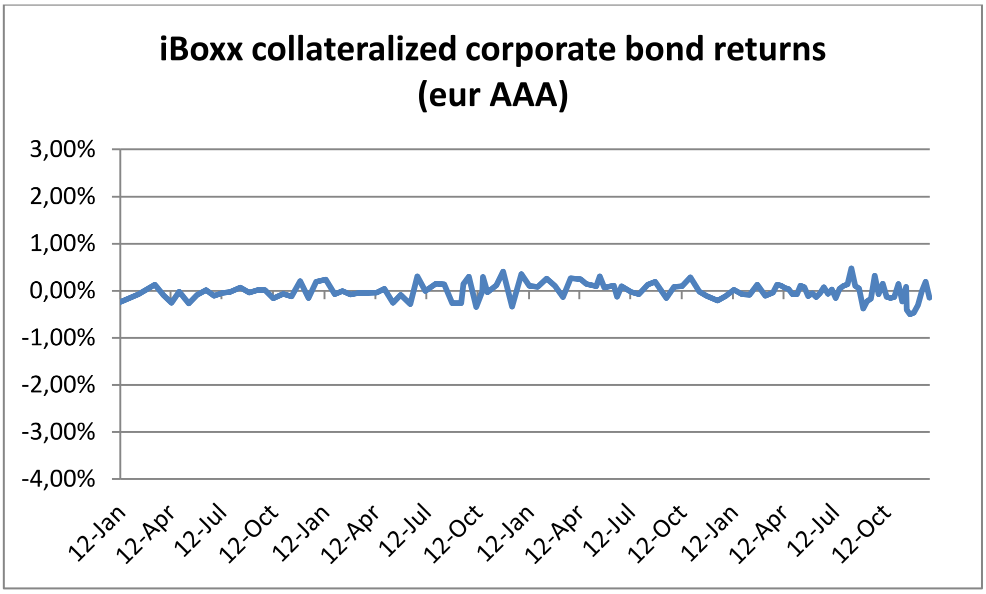

Figure 10. iBoxx collateralized corporate bond index returns

For the modeling of high yield corporate bonds, the ‘iBoxx euro high yield A’ has been used. This index comprises of high yield corporate bonds with an average maturity of 5 years and a rating B. For these types of bonds the ‘iBoxx Euro High Yield’ index has been used. This index comprises of corporate loans with a fixed coupon which a high risk and hence a high yield.

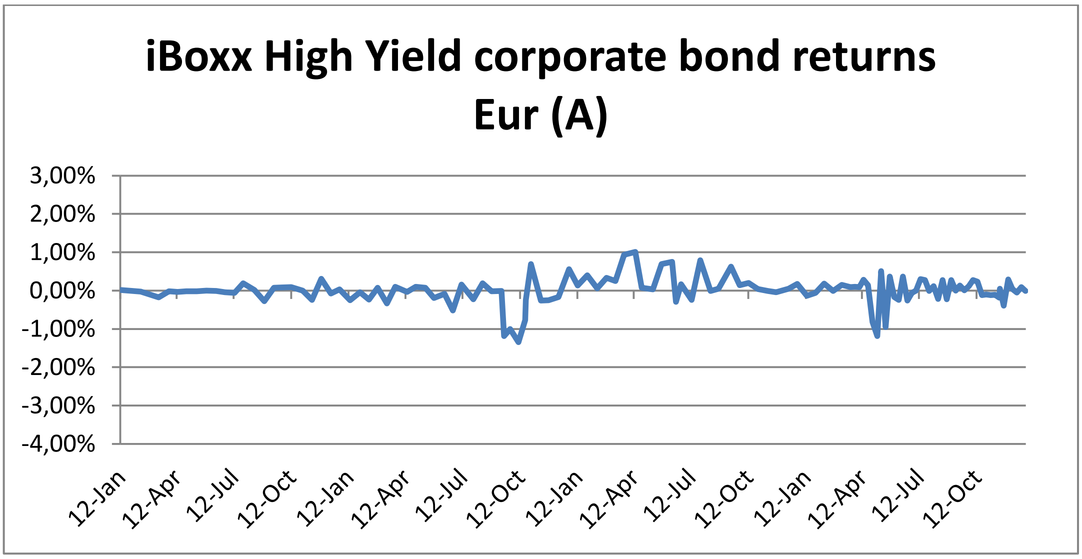

Figure 11. iBoxx high yield corporate bond index returns

6.2. Interest rates

The interest rates used in the model in order to price the nominal bonds and derivatives are the swap rates. This curve is commonly used in the market to fair value derivatives and risk free nominal bonds. Swap rates are the risk free interest rates which make the market value of the underlying swap at inception zero. Swap rates are viewed equal to the risk free rate, because these rates only apply to interest rate swaps which are collateralized and therefore are stripped of any other risk than the interest rate risk.

For euro denominated derivatives one should use the euro swap rates. As the used euro corporate bond indices is comprised out of corporate bonds with an average maturity of 5 years. The bonds and derivatives are modeled in such a way that the average maturity and duration equal that of the iBoxx bond indices. Therefore no swap rates are used beyond the 5 year point

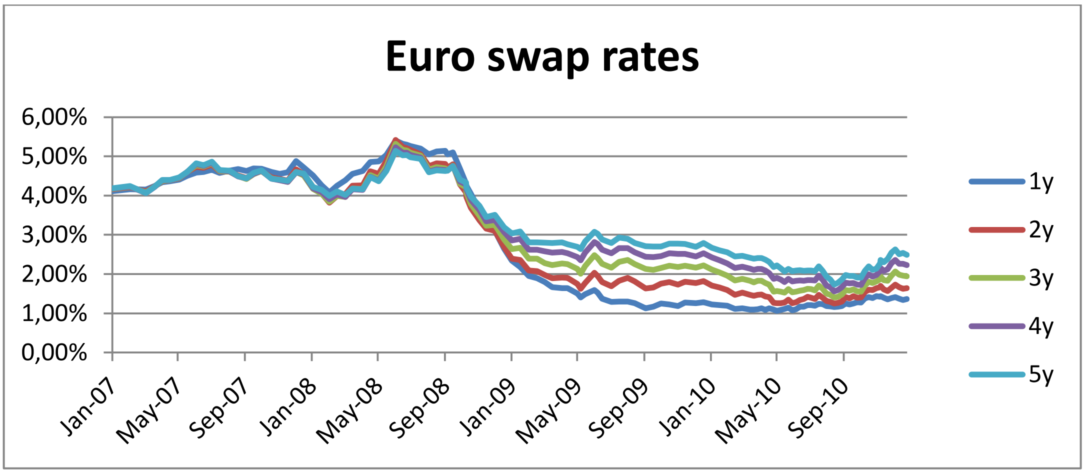

Figure 12. Euro swaprates

The interest rates show that up until the crises of 2008 the spread between the different tenors was not more than a few basis points. After the crisis the spread diverged as much as 2% between the 1 year and 5 year risk free swap rate.

6.3. Equity market

For modeling the corporate bonds derivatives are being used which have as underlying an equity index. Because the corporate bonds are bonds which are issued by European the companies' equity index used is the Euro Stoxx 50 index. This index is an often used benchmark for European equity. This index covers 50 stocks from several countries in Europe, such as Austria, Belgium, Germany, France and the Netherlands.

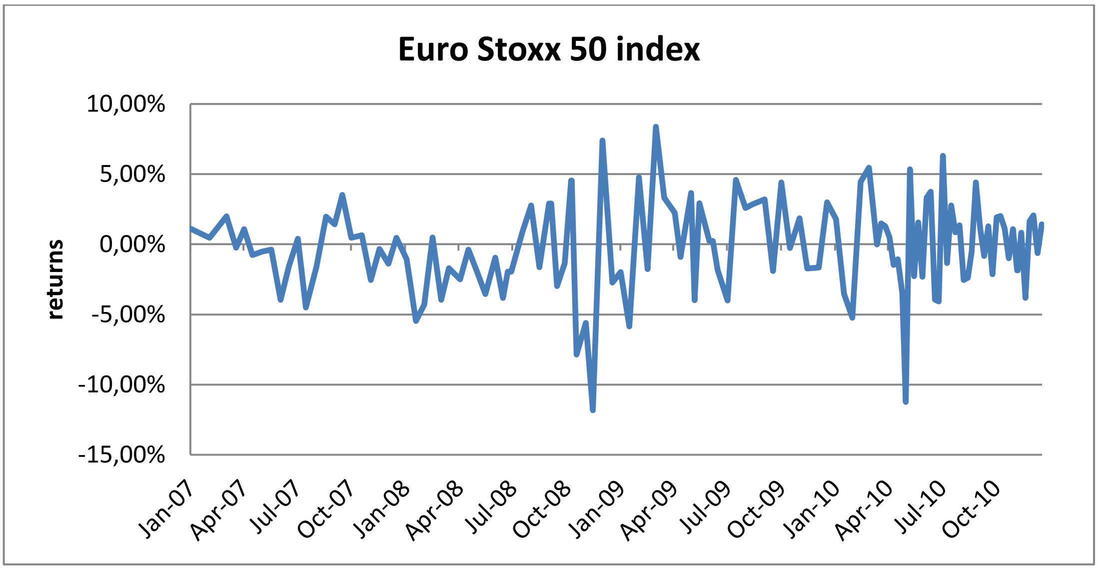

Figure 13. Euro Stoxx 50 index returns

The graph above shows that whereas corporate bond indices moved as much as -3% or +2%, the Euro Stoxx 50 index is showing -12% and +9%. This means that the equity index is more volatile than the iBoxx bond indices.

6.4. Implied volatility

The original Merton model and other asset value models in general assume that the debt and the assets are being modeled. The asset value of a company should therefore be used to model the corporate bonds with derivatives. The problem is that the asset value of a company is not observable in the market and therefore also not the volatility of the asset value (σ_A). The stock market shows equity values of a company and therefore also the equity volatility (σ_E).

Nickell, Perraudin and Varotto (2000) state that the asset value may be derived from the equity values by making use of the book value of the firm's liabilities (Nickell, et al., 2000). This is not public information for all the companies in the Euro Stoxx 50 index and if found this data is not published at a daily or weekly frequency. Bluhm, Overbeck and Wagner

(2003) also make reference in their approach where the equity value process is transformed into an asset value process by making use of Itô Calculus (Bluhm, et al., 2003). Hull, Nelken and White (2004) also showed that

$$E_o \sigma_E = \frac{\partial E}{\partial A} A_0 \sigma_A$$

Where E is defined as the value of the firm's equity and A the value of its assets, E_o and A_0 are the values at time zero and σ_E and σ_A are the instantaneous volatility of the company's equity and asset value at time zero (Hull, et al., 2004).

Both the approaches of (Bluhm, et al., 2003) and (Hull, et al., 2004) require additional modeling of the volatility or the volatility skews and the starting point of the proposed model was to try and avoid this complexity in the model.

The implied volatility used for valuing the options on the Euro Stoxx 50 index is shown in the below figure.

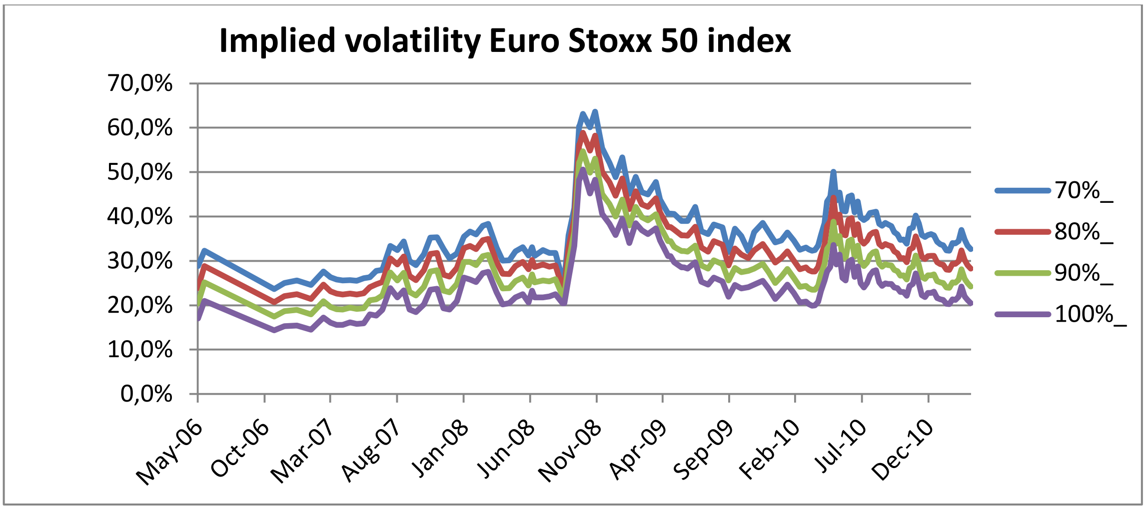

Figure 14. Implied volatility of Euro Stoxx 50 index with different strikes

The figure shows that in the period of the crisis the implied volatility has increased greatly. In august of 2008 the implied volatilities has risen by more than 40% to a level of more than 60%. After which markets did not return to the levels before the crisis. The average implied volatility in the below graph is approximately 25%.

7. In Sample test

In order to test if the proposed model captures the behavior of the corporate bonds the model is tested on the aforementioned iBoxx corporate bond indices. The risk measures derived from both the proposed model and the indices will be compared to be able to conclude this is the case. The hypothesis which is going to be tested is that modeling the three types of corporate bonds with the proposed model will capture the behavior of the corporate bonds.

H_0: $X_{model} = X_{indices}$

H_A: $X_{model} \neq X_{indices}$

Where X is the return distribution.

The above stated hypothesis looks if the proposed model adequately captures the behavior of the different kind of corporate bonds. Note that because of the very irregular probability distribution, it is not possible to use statistical tests for rejecting the null hypothesis.

Firstly an in-Sample test will be performed which will test the fit of the proposed model with the implied volatility obtained from major investment banks in Europe against the iBoxx returns. The strikes of the put and call options used in the model are calibrated to maximize the explained variance (R^2). This calibration showed that setting the strike at 70% for all three types of corporate bonds showed the best result. The In Sample test will be performed of the period 2006-2010.

The calibrated models will be compared to the iBoxx indices. In the next chapter an out-of-Sample test will be performed with the calibrated strikes over the year 2011 in order to see whether the model will provide good outcomes when it has been calibrated.

7.1. Testing methodology

A lot of literature has been written on describing the behavior of interest rates (Illmanen, 1995), equity markets (e.g. (Welch & Goyal, 2008)) and other studies on the predictability of corporate bonds (e.g. (King & Fuller, 1994)). The new proposed model will be tested by making use of linear regressions. Equal to the studies on interest rates, equity markets and other corporate bonds a least squares regression using end of month returns r_t on values of K explanatory variables $X_{1,t}$…,$X_{K,t.}$

$$r_t = \alpha + \beta_1 x_{1,t} + \cdots \beta_K x_{K,t} + \varepsilon_t$$

The aim is to describe the return over month t using the changes in the explanatory variables, in our case the changes in swap rates and the value of the equity options. Each regression starts from the first observation, January 2006. The number of observation of the regressions is 60, as we have 5 years of data. Each of the regressions is run separately for the investment grade, collateralized and high yield returns.

As shown by Ben Dor et al. (2007), the volatility of corporate bonds' excess returns over Treasury is not constant over time. They provide strong evidence that excess return volatility is higher when spreads are higher.

In chapter 8 we describe the In-sample tests will be performed by running the regression in month t. The obtained in-sample R^2 shows for each type of corporate bond how much of the variation in returns over the past t months could be explained by variation in the explanatory variables.

The use of implied volatility plays a large role in proposed model as one of the main advantages is that this parameter does not have to be estimated. In order to determine whether the use of the actual market implied volatility the outcome of the proposed model improves the model will also be run with a constant average implied volatility. The analysis of the actual market implied volatilities showed that for the Euro Stoxx 50 this equals approximately 25%, hence this figure will be used as a constant.

7.2. Proposed model with use of market implied volatilities

The three types of corporate bonds will be tested in the next sub chapters. The results of the model with the actual market implied volatilities are described by determining the R squared and is based on descriptive statistics like mean, standard deviation, kurtosis and skewness.

7.2.1. Investment grade corporate bonds

To compare the test results of the euro investment grade corporate bonds index the proposed model has been tested against the iBoxx euro investment grade bond index. As mentioned earlier the period which the In-Sample test will be performed is 1, January of 2006 until 31st of December 2010. The figure below shows iBoxx euro investment grade corporate index and the proposed model returns.

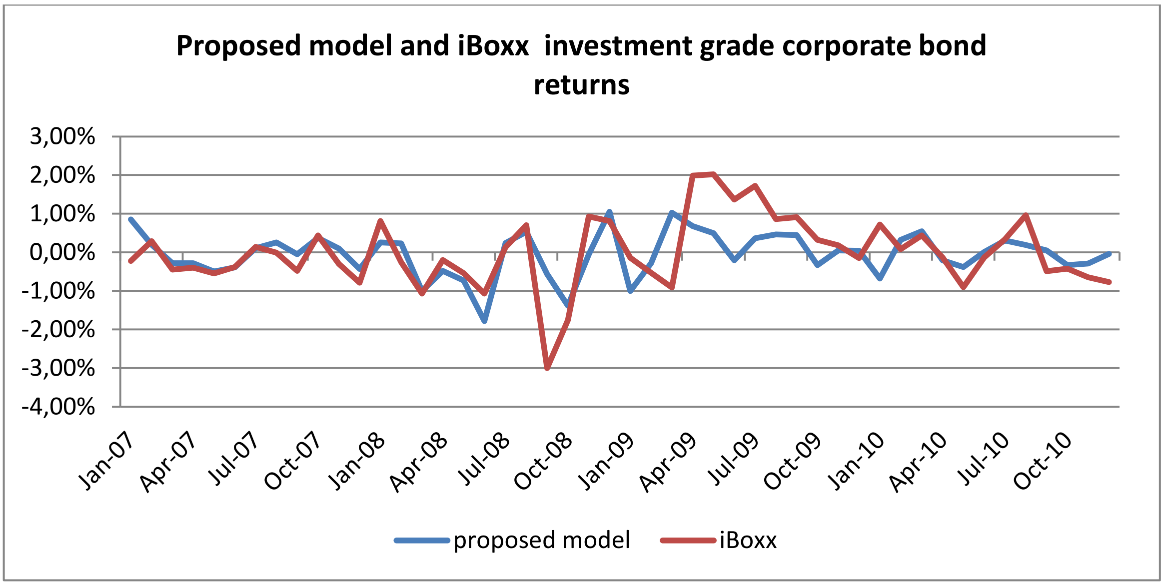

Figure 15. Proposed model and iBoxx investment grade corporate bond returns

The below graph show the differences between the iBoxx investment grade corporate bond index and the investment grade corporate bond returns from the proposed model.

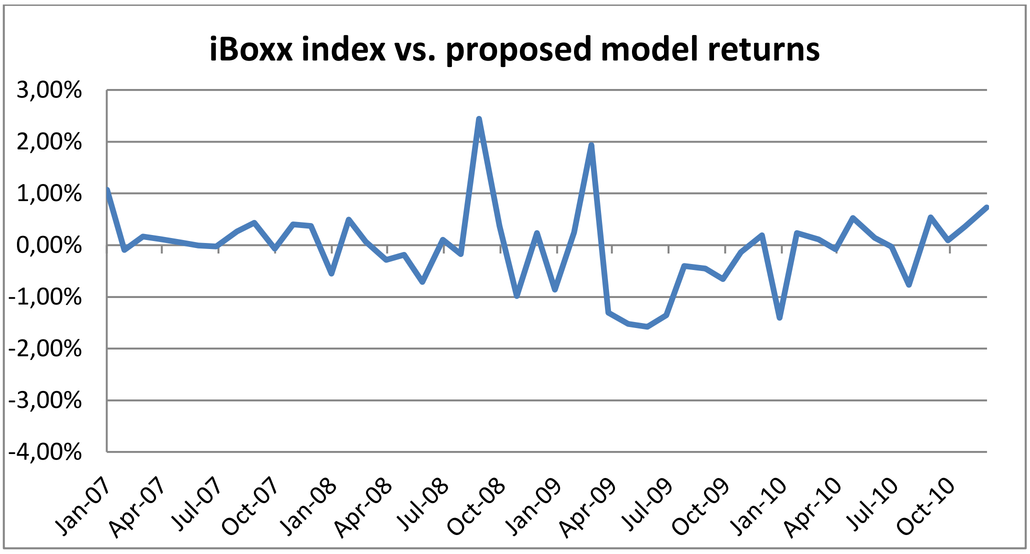

Figure 16. The difference between the iBoxx investment grade corporate bond index and the proposed model returns

If we compare the returns of the proposed model to the iBoxx investment grade index returns differences between 2% and -2% are observed. The largest differences occur in the months September 2008 and March 2009. The positive differences indicate that the proposed model shows higher returns than the iBoxx index returns. The period after March 2009 shows slightly negative differences.

Below the descriptive statistics are given for the both the iBoxx investment grade corporate bond index and the proposed model for investment grade corporate bonds with actual market implied volatility.

Table 3. Descriptive statistics iBoxx investment grade index and proposed model returns

Proposed model with implied vol		*Iboxx Euro investment grade*	
Mean	-0,12%	Mean	-0,01%
Standard Error	0,003	Standard Error	0,0014
Median	0,0005	Median	-0,0013
Standard Deviation	0,02	Standard Deviation	0,01
Kurtosis	8,9	Kurtosis	1,7
Skewness	-2,2	Skewness	-0,3

As seen both means are close to 0. The proposed model does show a higher volatility, as reflected by a higher standard deviation. The kurtosis of the proposed model is 8.9 which compared to the 1.7 of the iBoxx returns is higher.

A linear regression on both series of returns shows that the R^2 is 0.33. This R square shows that the explained portion of the variance is in line with expectations.

Table 4. Regression statistics iBoxx investment grade index and proposed model returns

Regression Statistics	
Multiple R	0,57
R Square	0,33
Adjusted R Square	0,3
Standard Error	0,008

If we compare both time series we see that the sign test shows that in 60% of all cases the proposed model shows the correct sign.

The statistics of the regression show that it is also interesting from a risk management perspective to look at the distribution of both series. The frequency table below shows the iBoxx investment grade corporate bond index returns and the proposed model returns for investment grade corporate bonds in the same period.

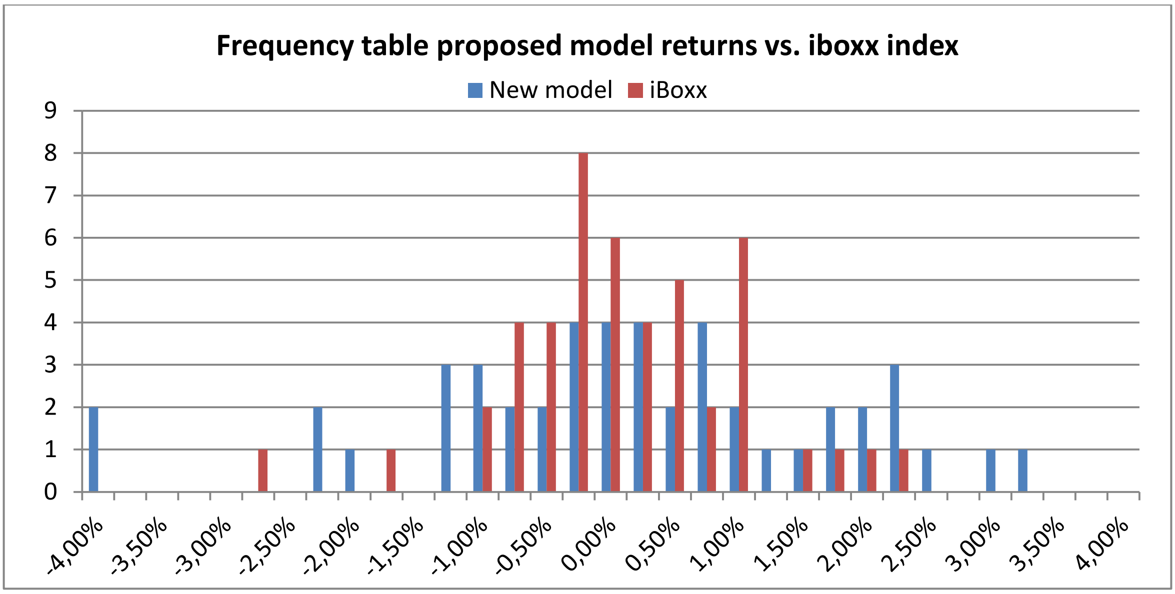

Figure 17. Frequency table iBoxx investment grade index and proposed model returns

The frequency table clearly shows that the proposed model is more evenly distributed. The weight of the distribution does not center as much around the mean as seen by the iBoxx investment grade corporate bond returns. This observation is backed by the higher standard deviation of the proposed model.

7.2.2. Collateralized corporate bonds

The collateralized corporate bonds are in a similar manner as the investment grade bonds tested against a benchmark. The collateralized corporate bonds in the proposed model are tested against the iBoxx euro collateralized AAA bond index. Again the period which the In-Sample test will be performed is 1, January of 2006 until 31st of December 2010. The figure below shows the returns of the iBoxx collateralized AAA index and the proposed model returns for collateralized corporate bonds.

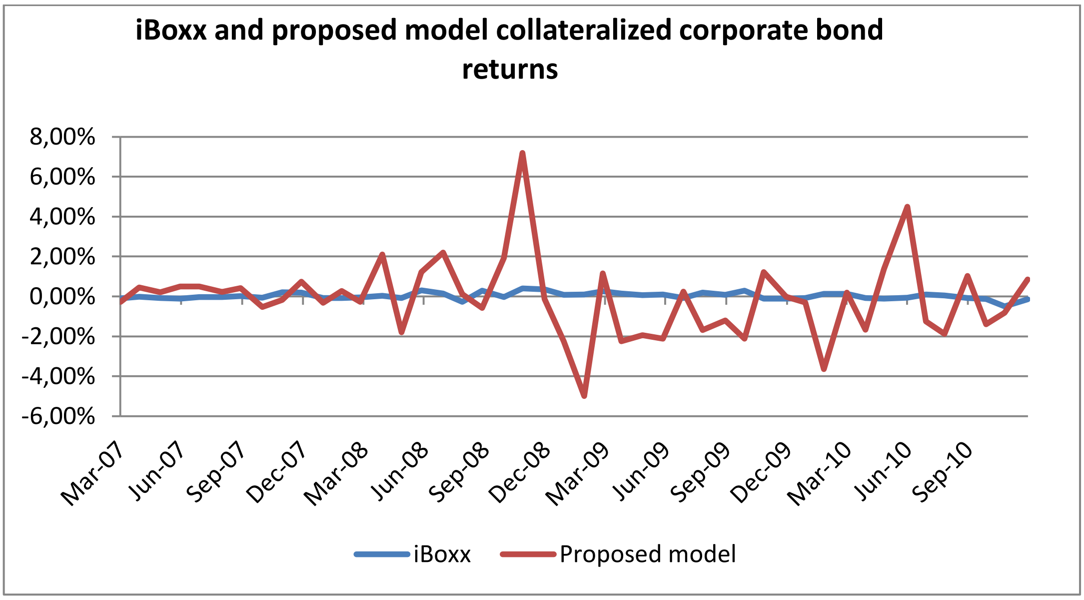

Figure 18. iBoxx and proposed model collateralized corporate bonds returns

The below graph show the differences between the iBoxx collateralized corporate bond index and the investment grade corporate bond returns from the proposed model.

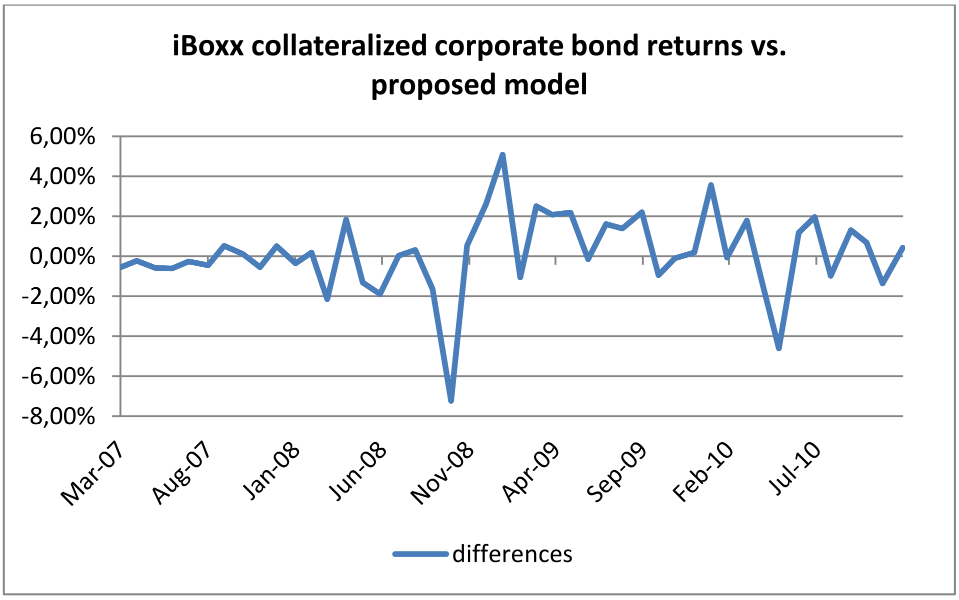

Figure 19. iBoxx collateralized corporate bond index versus proposed model returns

The returns of the iBoxx Collateralized AAA index show a less volatile pattern compared against the investment grade corporate bond returns. This is also expected due to the characteristics of the collateralized corporate bonds. The fact that they are collateralized means that in case of default the collateral may be sold by the bondholder in order to cover some of the losses.

The difference between the proposed model and the iBoxx collateralized index returns show a more volatile pattern than the proposed model compared to the investment grade corporate bond returns. The proposed model for collateralized corporate bonds seems to be unable to capture the behavior of the collateralized corporate bond index.

Below the descriptive statistics are given for the both the iBoxx index and the proposed model with the corresponding actual market implied volatilities.

Table 5. Descriptive statistics proposed model and iBoxx index returns

Proposed model collateralized corporate bonds		*iBoxx Euro collateralized AAA*	
Mean	-0,001	Mean	0,00002
Standard Error	0,003	Standard Error	0,002
Median	-0,001	Median	-0,001
Standard Deviation	0,02	Standard Deviation	0,095
Sample Variance	0,0004	Sample Variance	0,0001
Kurtosis	4,1	Kurtosis	0,09
Skewness	0,96	Skewness	0,56

The sign test shows that 47% of the time the proposed model shows the same sign as the returns of the iBoxx.

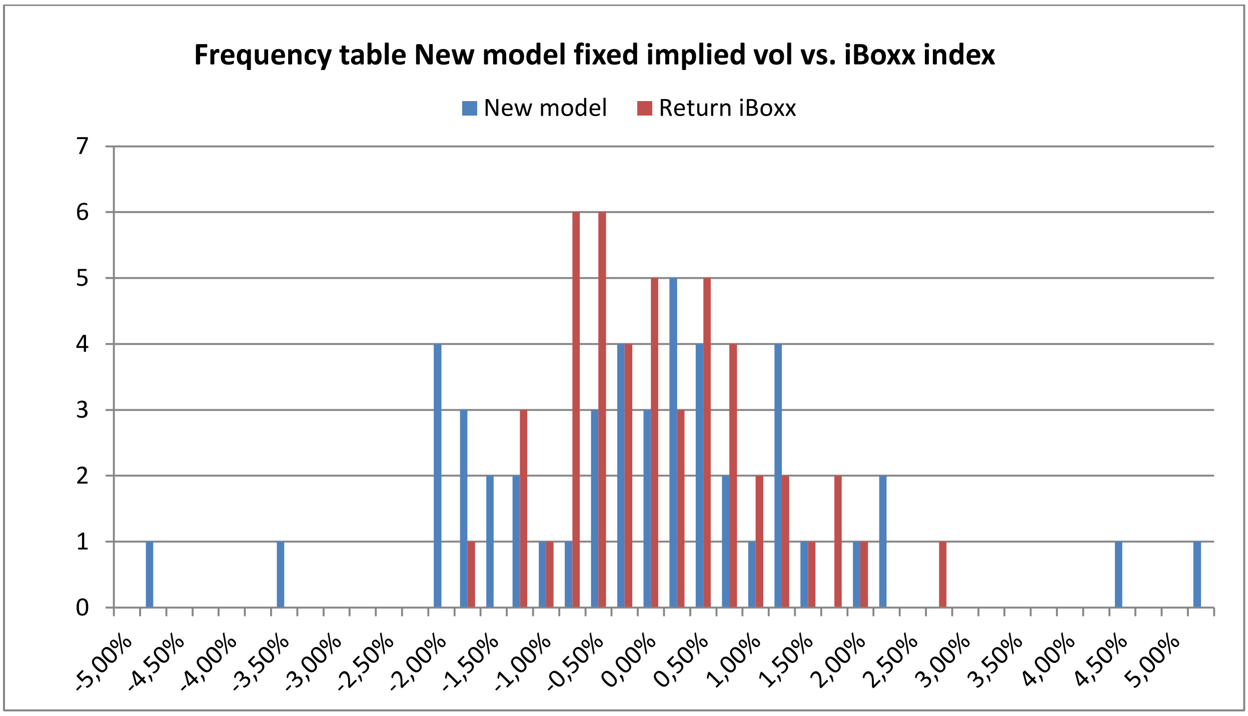

Figure 20. Frequency table proposed model with fixed implied volatility and iBoxx collateralized index returns

The frequency table shows that the proposed model seems to be more skewed than the actual iBoxx collateralized index returns. Both means seems to be of the same size and the volatility of both time series also are of the same magnitude.

Performing a linear regression on both time series shows that the R^2 of the proposed model with the implied volatility from the market is 0.5%. This R^2 is close to 0 which indicates that there is a lot of unexplained variance. The proposed model seems to be unable to capture the behavior of the collateralized corporate bonds.

Table 6. Regression statistics proposed model with fixed implied volatility and iBoxx index returns

Regression Statistics collateralized bonds with market implied volatility	
Multiple R	0,07
R Square	0,00
Adjusted R Square	-0,017
Standard Error	0,02

7.2.3. High yield corporate bonds

The high yield corporate bonds are compared against the iBoxx high yield corporate bond index. The period which the In-Sample test will be performed is 1, January of 2006 until 31st of December 2010. The figure below shows the returns of the iBoxx High Yield Euro corporate bond index and the proposed model returns for high yield corporate bonds.

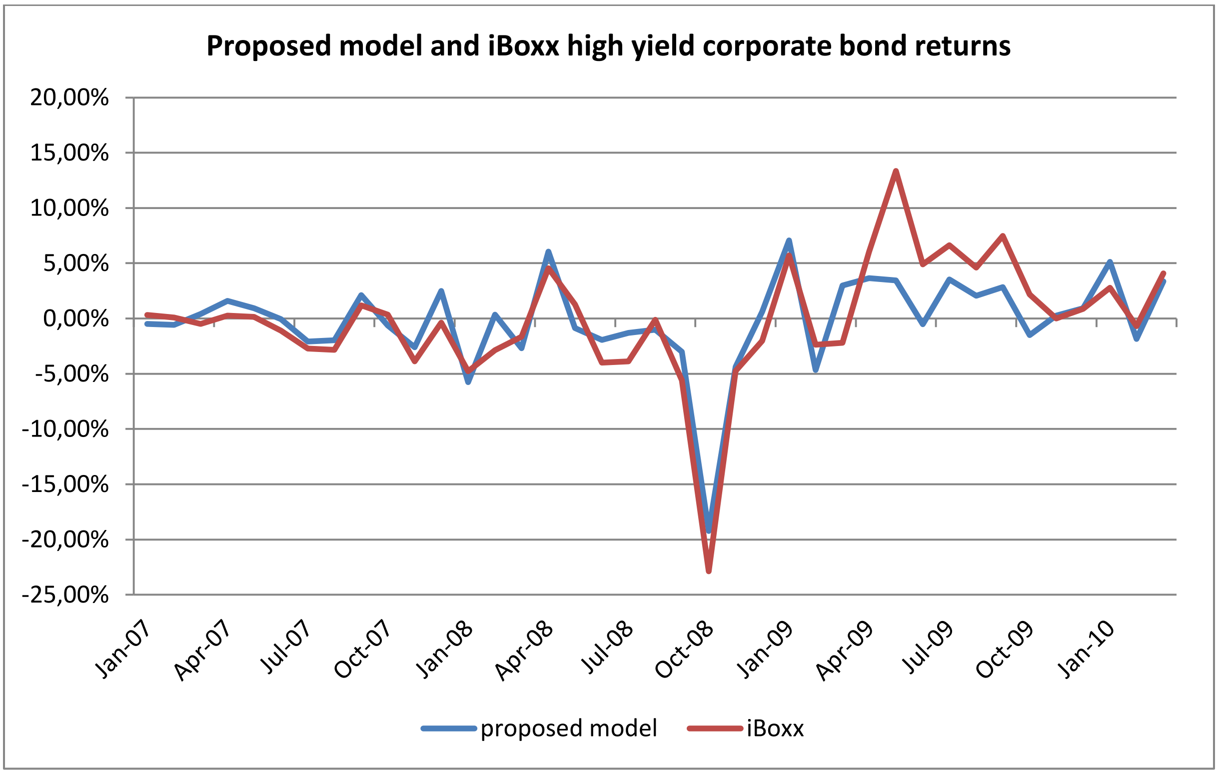

Figure 21. iBoxx collateralized corporate bond index returns

The figure above shows that the returns of the high yield euro corporate index have been extremely volatile. The crisis in 2008 has caused a return of -24%.

The below graph show the differences between the iBoxx high yield corporate bond index and the investment grade corporate bond returns from the proposed model.

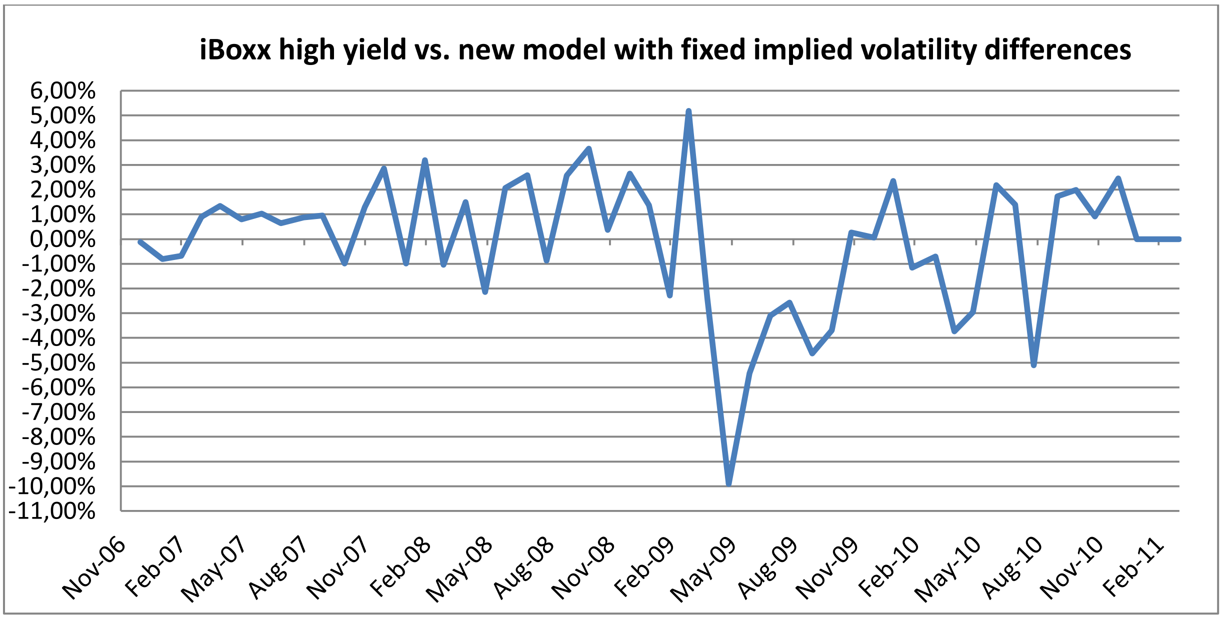

Figure 22. iBoxx high yield corporate bond index versus proposed model returns

The differences show that there are still months that the proposed model for high yield corporate bonds will not realize the exact same returns as the iBoxx high yield corporate bond index will make. The high spike at the end of March 2009 followed by the spike downwards could indicate that the equity market moved before the corporate bond market could react. The negative differences followed by the downward spike in April 2009 are hard to explain. It seems that the corporates of which the iBoxx consists show more positive returns that the proposed model shows.

Although the graph shows large differences of the proposed model for high yield corporate bonds versus the actual iBoxx high yield corporate bond index a lot of the total variance is captured. The sign test also shows that 72% of the returns of the proposed model will show the same sign as the iBoxx high yield corporate index.

The statistics of the proposed model returns and the actual euro high yield corporate index show that the means of the both indices are not identical. The proposed model has a mean which is more than 12 basis points more negative than the iBoxx Euro High Yield index.

Table 7. Descriptive statistics iBoxx collateralized corporate bond index and proposed model returns

proposed model		*iBoxx eur high yield index*	
Mean	-0,00191	Mean	-0,00079
Standard Error	0,00632	Standard Error	0,00745
Median	0,0027	Median	0,00095
Standard Deviation	0,043	Standard Deviation	0,051
Kurtosis	7,186	Kurtosis	8,371
Skewness	-1,91	Skewness	-1,483

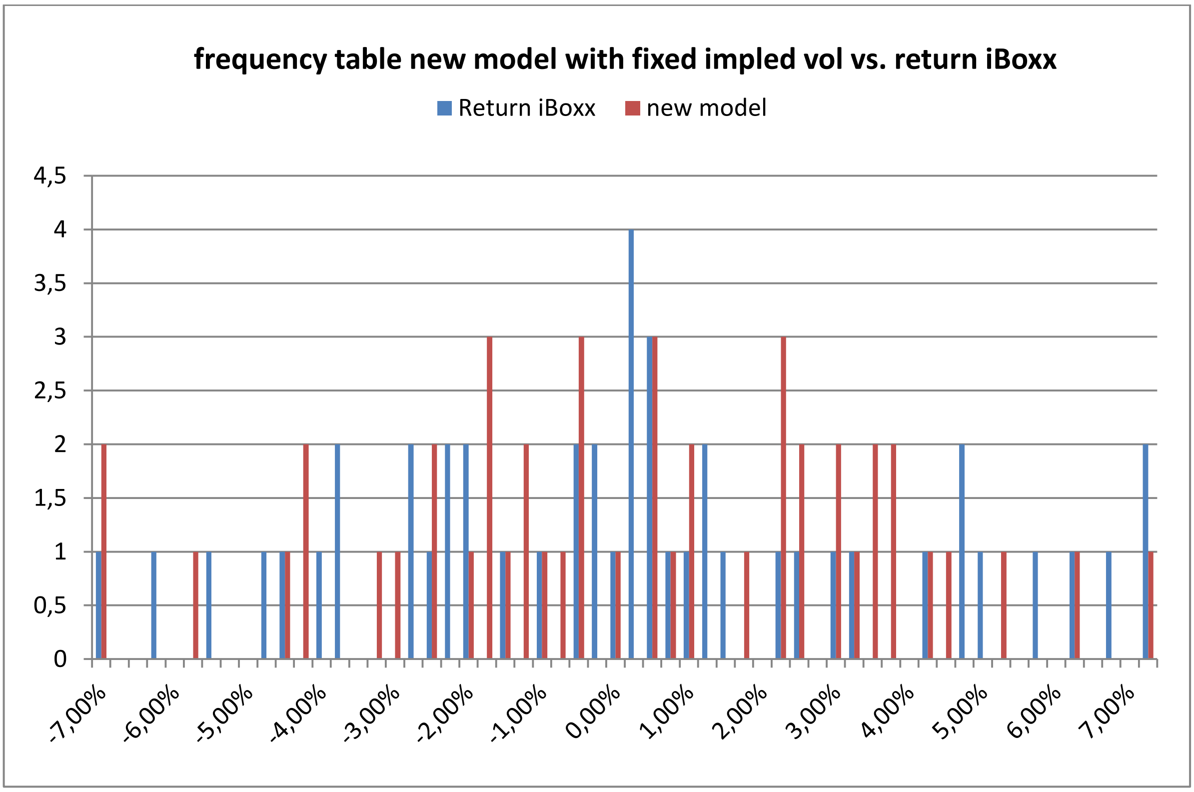

Figure 23. Frequency table proposed model and iBoxx collateralized corporate bond index returns

The above figure shows that compared to investment grade and collateralized corporate bonds the high yield returns have a more scattered pattern. The returns of the proposed model for high yield corporate bonds seem to be more in line with the actual iBoxx high yield corporate bond returns than the other two types of corporate bonds.

The linear regression between both series of returns shows that the proposed model seems to be capturing the behavior of the iBoxx high yield corporate bond returns. The R^2 shows that with 0.69 the proposed model captures a lot of the variance of returns of the actual iBoxx.

Table 8. Regression statistics proposed model and iBoxx collateralized corporate bond index returns

Regression Statistics	
Multiple R	0,83
R Square	0,69
Adjusted R Square	0,68
Standard Error	0,024

7.3. Proposed model with fixed implied volatility

The proposed model returns depend heavily on the used implied volatility for the modeling the sold put option. Therefore it is interesting to look if we do not use the implied volatility as observed by the market but would fit an average volatility if the outcome differs.

7.3.1. Euro investment grade corporate bonds

When the proposed model is tested with a fixed implied volatility the differences become slightly smaller as can be seen in the below graph.

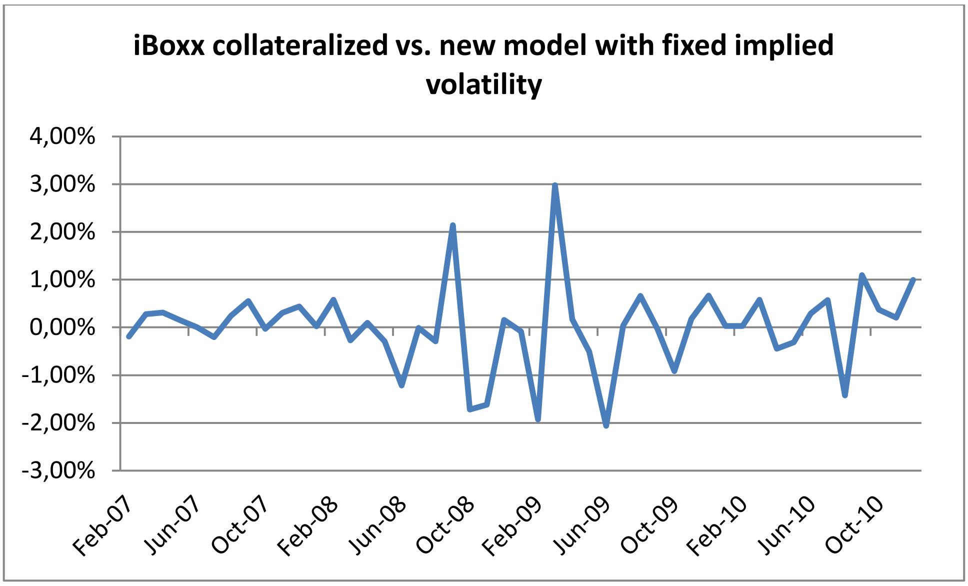

Figure 24 iBoxx collateralized corporate bond index versus proposed model returns

The sign test improves from 60% to 68% and the descriptive statistics also show more comparison with respect to using actual market implied volatilities.

Table 9 Descriptive statistics proposed model and iBoxx collateralized corporate bond index returns

Proposed model		*iBoxx euro investment grade*	
Mean	0,006%	Mean	-0,006%
Standard Error	0,0016	Standard Error	0,0014
Standard Deviation	0,011	Standard Deviation	0,009
Sample Variance	0,00012	Sample Variance	.00009
Kurtosis	1,96	Kurtosis	1,71
Skewness	-0,72	Skewness	-0,25

The frequency table in the below figure shows that the overall fit of the distribution is more in line with the actual observed iBoxx euro investment grade index returns than compared to the previous example with actual market implied volatilities.

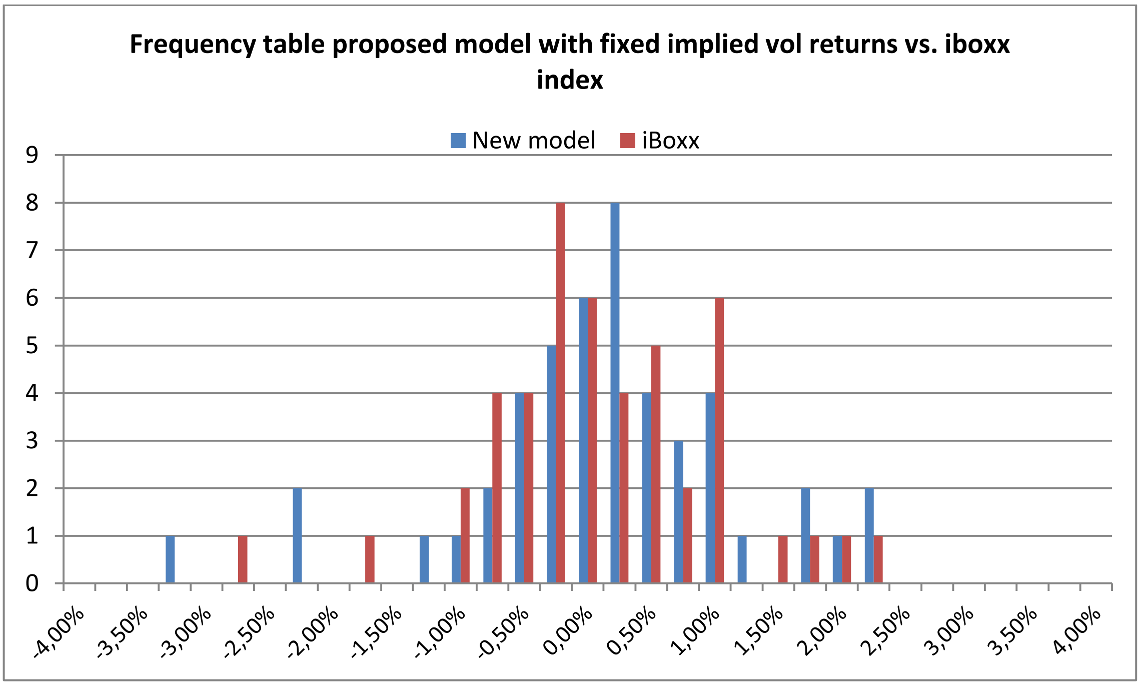

Figure 25. Frequency table proposed model versus iBoxx index

The linear regression shows that the R2 of the proposed model with a fixed implied volatility of 25% explains more of the variance than using actual market implied volatility on each of the data points. The R^2 now shows 0.36 versus the 0.32.

Table 10 Regression statistics proposed model and iBoxx collateralized corporate bond index returns

Regression Statistics	
Multiple R	0,60
R Square	0,36
Adjusted R Square	0,35
Standard Error	0,0075

7.3.2. Collateralized corporate bonds

Using the proposed model with the actual implied volatility of the Euro Stoxx showed that there still is a large amount of unexplained variance. Therefore the model will also be tested for collateralized bonds using a fixed average implied volatility.

The figure below shows the differences of the proposed model using a fixed average implied volatility against the actual iBoxx collateralized AAA index returns.

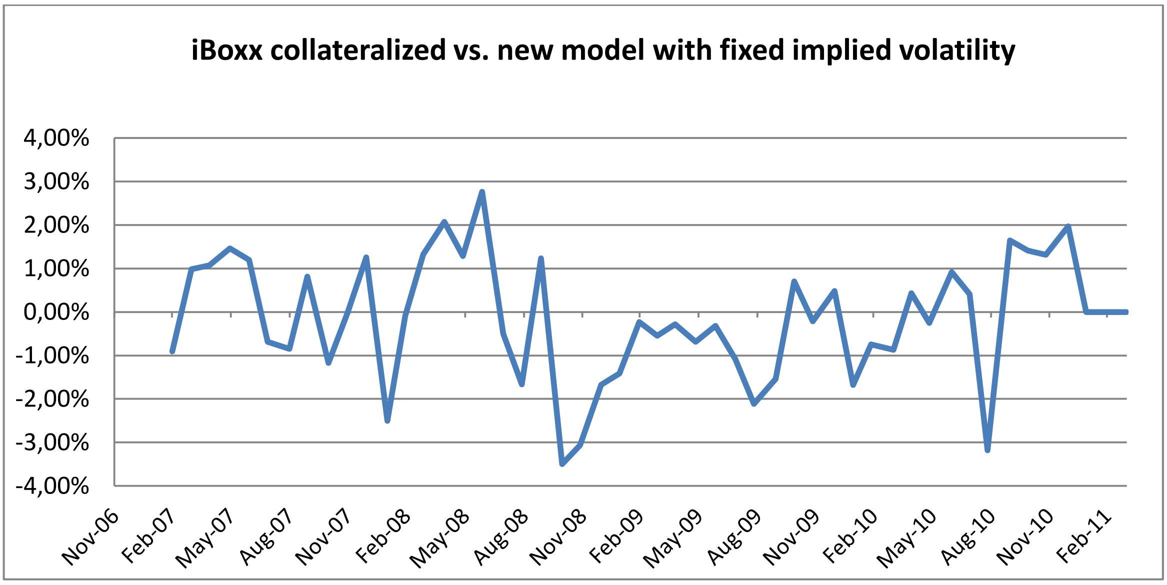

Figure 26. iBoxx collateralized corporate bond index versus proposed model returns

In comparison to the proposed model returns using the actual market implied volatilities we see that the proposed model using a fixed implied volatility does not shows smaller differences.

Table 11. Descriptive statistics iBoxx collateralized corporate bond index and proposed model returns

Model with fixed implied vol		*iBoxx Euro collateralized AAA*	
Mean	-0,0019	Mean	0,00002
Standard Error	0,0021	Standard Error	0,0014
Median	-0,001	Median	-0,001
Standard Deviation	0,014	Standard Deviation	0,06
Kurtosis	5,6	Kurtosis	0,9
Skewness	-1,4	Skewness	-0,77

The descriptive statistics of the comparison of the proposed model with a fixed implied volatility show that mean and volatility are lower and more in line with the actual iBoxx collateralized AAA index returns.

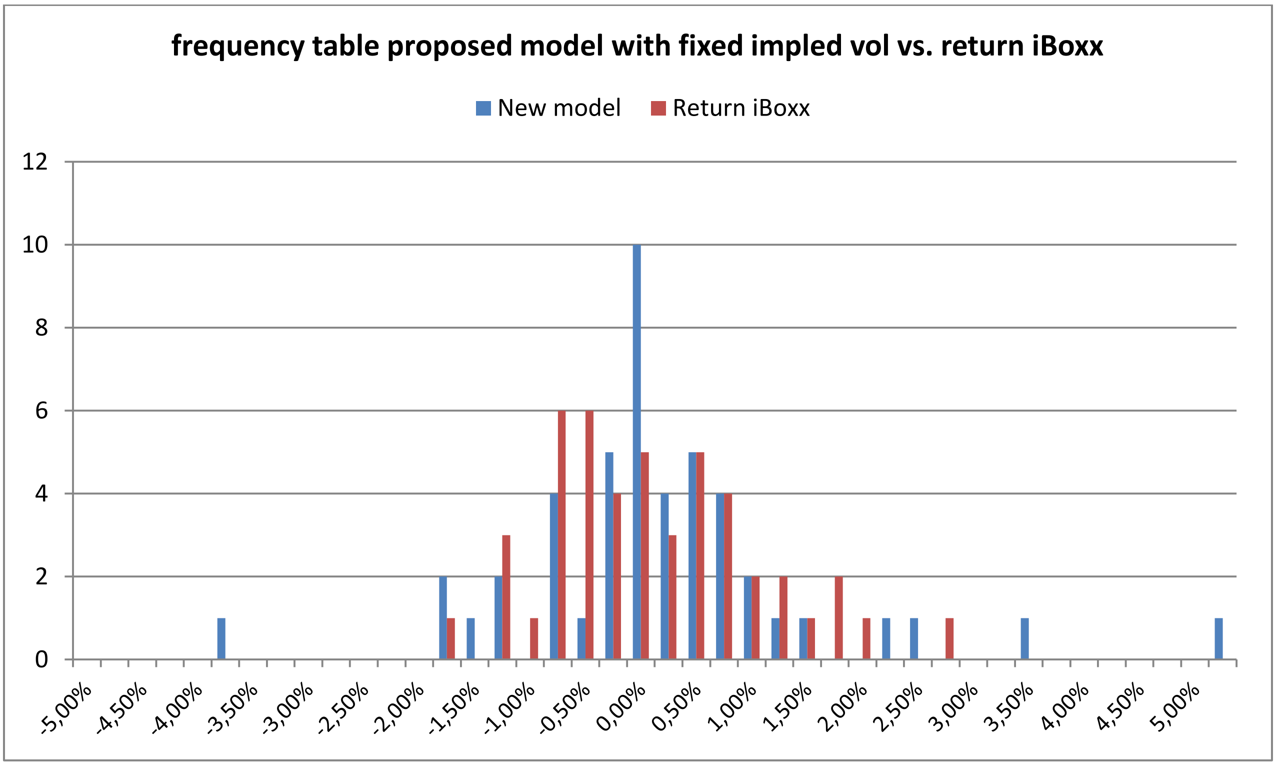

Figure 27. Frequency table proposed model for collateralized corporate bonds with fixed implied volatility versus iBoxx index

The sign test show 34% of the returns show the same sign in both the proposed model and the actual iBoxx returns.

The linear regression of the two series of returns shows that the R^2 is almost zero. Which again indicates that the proposed model does not capture the behavior of the iBoxx collateralized corporate bond index.

Table 12. Regression statistics proposed model and iBoxx collateralized corporate bond index returns

Regression Statistics	
Multiple R	0,077055
R Square	0,005938
Adjusted R Square	-0,01615
Standard Error	0,014429

7.3.3. High yield corporate bonds

The proposed model with the implied volatility of the market shows for high yields some large differences, -11% to +6%. Therefore is seems worthwhile to look if, just with investment grade returns if making use of an average implied volatility would improve the results.

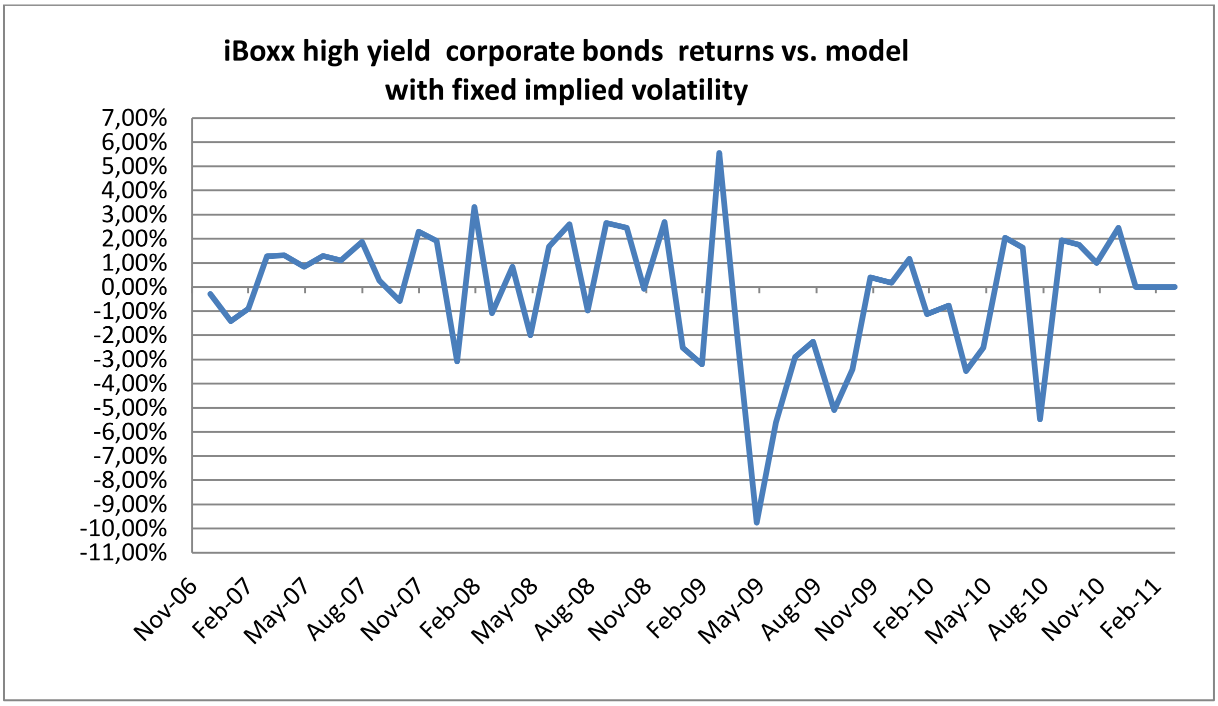

Figure 28. iBoxx high yield corporate bond index returns versus proposed model with fixed implied volatility

The figure above show that for the high yield bonds using an average implied volatility does not improve the differences. The most positive difference, still reads 4% and the most negative still shows -2,2%. However, it is clear from the graph that the model suffers from a time lag.

Also the sign test performed on both return series shows that in 70% of the cases both return series show the same sign. The proposed model with the actual market implied volatility showed a 72% score on the sign test. The use of the actual implied volatility scores with this 72% over 70% a slightly better fit.

Looking at the statistics shows that the returns with a fixed implied volatility show that the differences in statistics also didn't improve. The skewness of -2.3 versus -1.91 of the model with implied volatility show that the distribution gets skewed more, away from the iBoxx euro high yield index.

Table 13. Descriptive statistics proposed model en iBoxx high yield corporate bond index returns

proposed model		*iBoxx eur high yield index*	
Mean	-0,0034	Mean	-0,00079
Standard Error	0,0064	Standard Error	0,0075
Median	0,0042	Median	0,001
Standard Deviation	0,044	Standard Deviation	0,05
Kurtosis	8,8	Kurtosis	8,4
Skewness	-2,3	Skewness	-1,5

The below frequency table shows that the distribution of both the proposed model as the iBoxx returns are scattered.

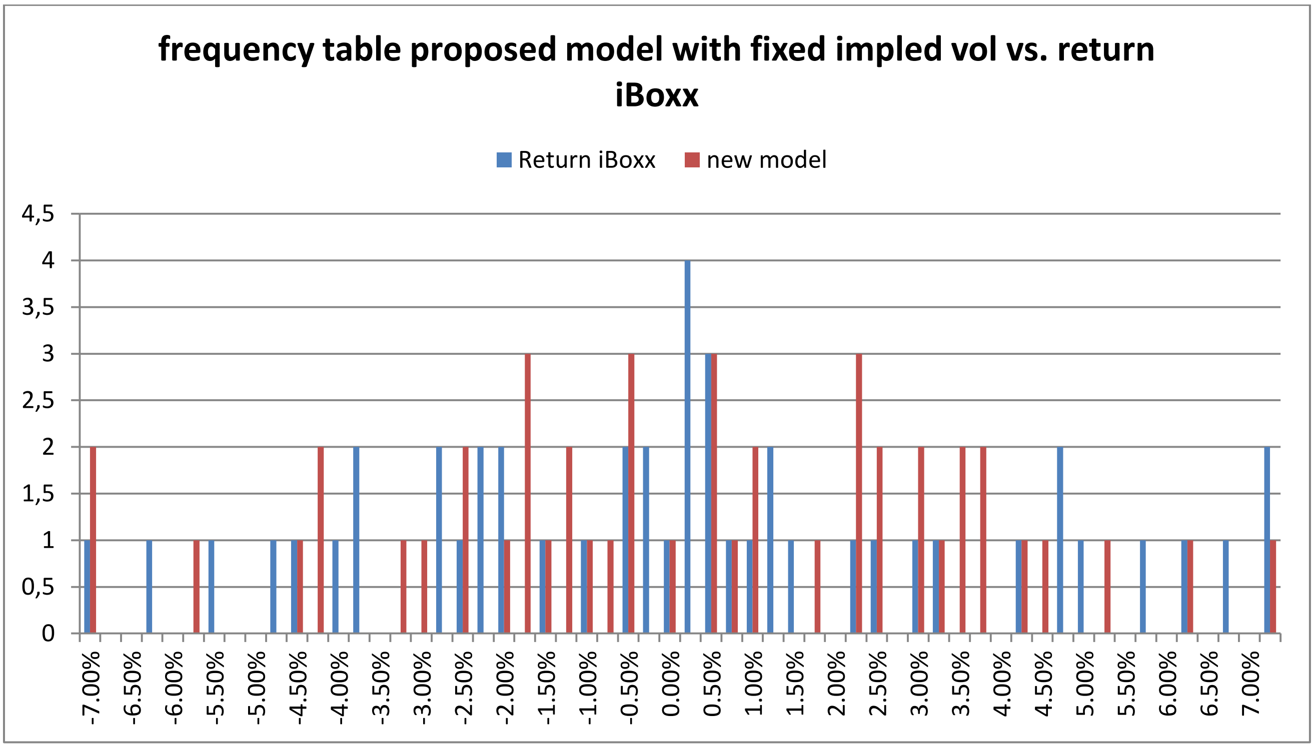

Figure 29 Frequency table proposed model for collateralized corporate bonds with fixed implied volatility versus iBoxx index

The linear regression of the two series of returns shows that the R^2 is almost 81%. Which again indicates that the proposed model does capture the behavior of the iBoxx collateralized corporate bond index.

Table 14 Regression statistics proposed model and iBoxx high yieldcorporate bond index returns

Regression Statistics	
Multiple R	0.81
R Square	0.68
Adjusted R Square	0.62
Standard Error	0.023

8. Out of Sample test

The In-Sample test which is made over the period 2006-2010 the strikes of the used options are calibrated to maximize the R^2. It is good practice to perform an Out-of-Sample test to look if the used model with the calibrated parameters will work well outside this period. The Out-of-Sample period is chosen to be January 2011 until 31st of 2011.

The Euro Stoxx index in this period has been very volatile. The below graph shows that the most negative monthly return from the end of July 2011 to August 2011 was -13,8%. Together with the other returns it shows that the Euro Stoxx 50 index was quite volatile during 2011.

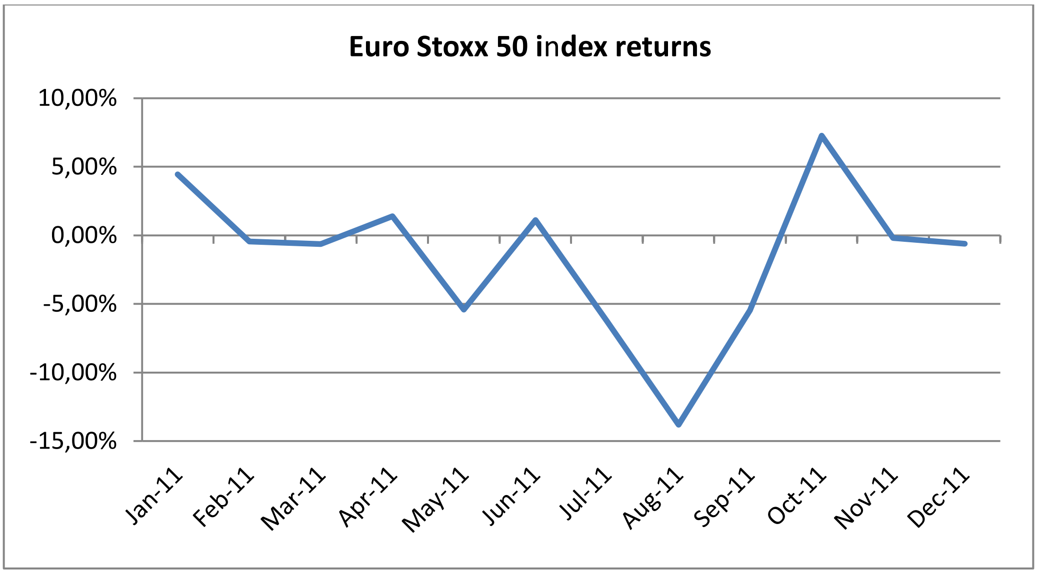

Figure 30. Euro Stoxx 50 index 2011 returns

If the monthly equity returns are volatile one would also expect that the implied volatilities will also show that the market changing. As can be seen in the below graph the implied volatilities rose quite rapidly during April and stayed at high levels throughout the rest of the year. The implied volatilities spiked as high as 74% in the months July and August.

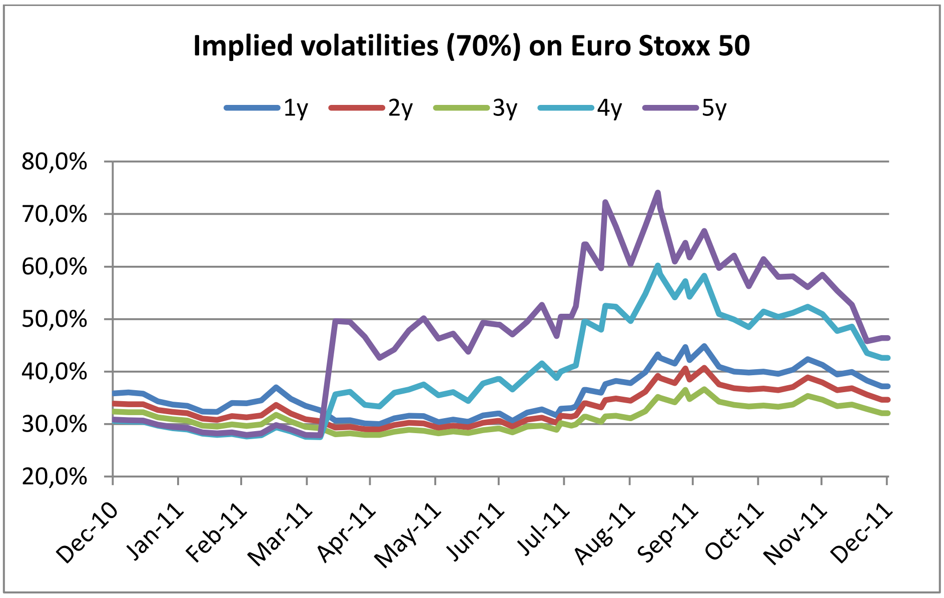

Figure 31. Implied volatility for different strikes of Euro Stoxx 50 index

The iBoxx corporate bond indices also showed some volatile patterns in 2011. In the below graphs all three bond indices monthly returns are shown. The high yield corporate bond index shows the largest returns, but the collateralized and investment grade corporate bond benchmarks remained relatively constant in comparison with the Euro Stoxx 50 index.

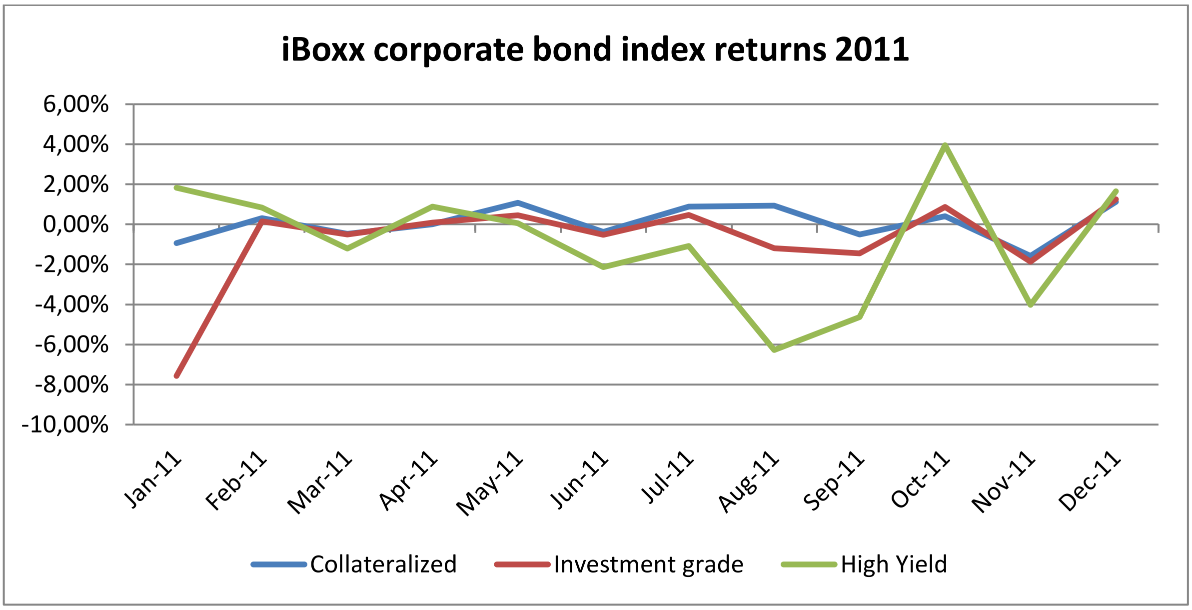

Figure 32. iboxx corporate bond indices returns in 2011

8.1. Proposed model with actual market implied volatility

The three types of corporate bonds will be tested in the proposed model with the calibrated strikes over the year 2011. The corporate bond benchmark indices from the iBoxx benchmarks are again used to compare the model monthly returns.

8.1.1. Investment grade corporate bonds

The investment grade corporate bonds show that in the volatile periods the investment grade model returns still show some relatively large differences of almost 4%. Between the period of February and July the proposed model seems to have a good fit with the actual iBoxx benchmark returns.

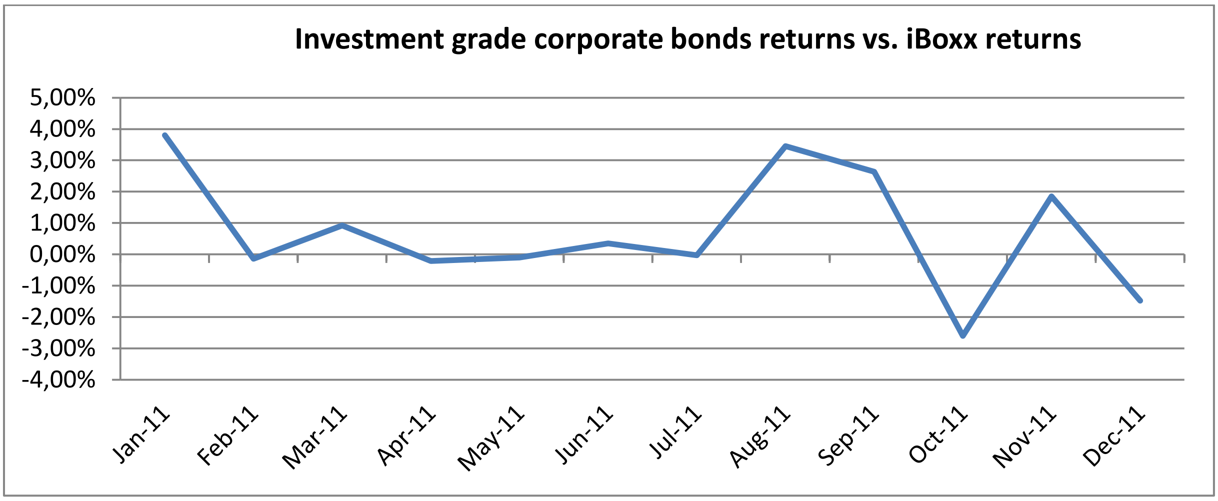

Figure 33 Proposed model and iBoxx investment grade corporate bond returns

The graph shows the differences between the both return series. The largest differences occur at the beginning of 2011 and in the second part of 2011.

8.1.2. Collateralized corporate bonds

The collateralized corporate bond returns show excellent differences in with respect to the iBoxx benchmark index. The differences move between 1% and -1%.

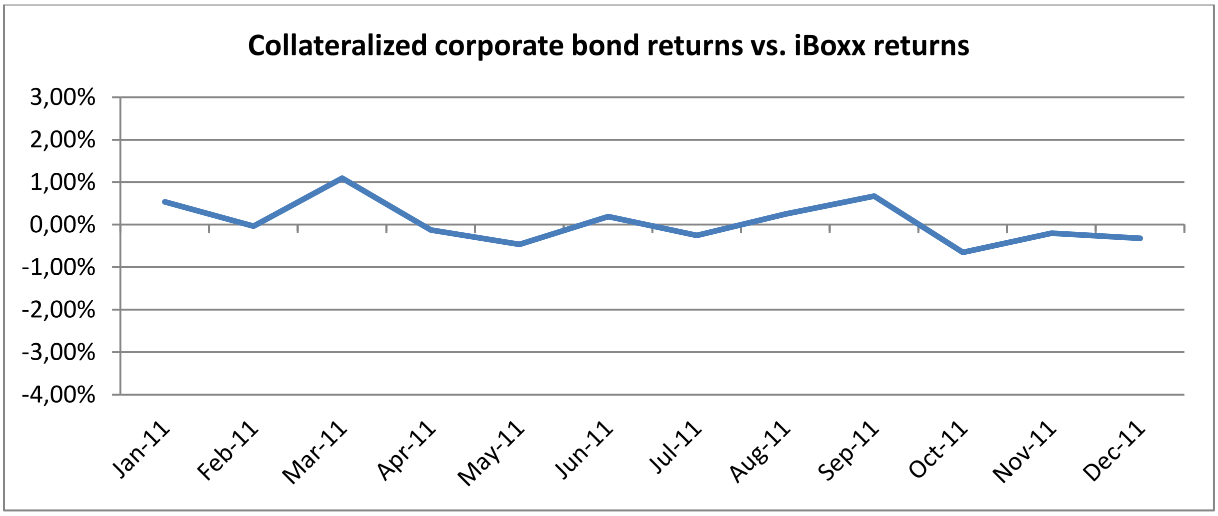

Figure 34. Proposed model with market implied vol and iBoxx investment grade corporate bond returns

8.1.3. High yield corporate bonds

The high yield corporate bond model with actual market implied volatilities shows that the differences from month to month vary between +4% and -3%. In the volatile months July and August were the Euro Stoxx 50 index showed the highest returns the high yield corporate model captures the behavior well.

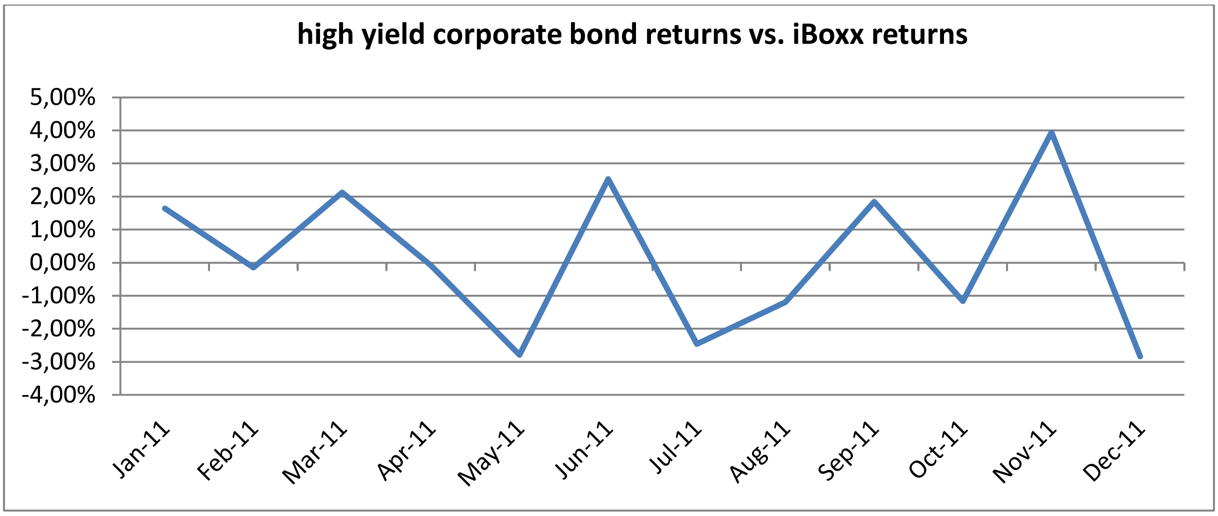

Figure 35 Proposed model with fixed implied vol and iBoxx investment grade corporate bond returns

8.2. Proposed model with fixed implied volatility

Just as in the In-Sample test the models are also tested with a fixed implied volatility of 25%. With the fixing of the implied volatility the models are tested on the sensitivity of the model to the implied volatility. For each of the three types of corporate bonds the monthly model returns are compared against the iBoxx benchmark index returns.

8.2.1. Investment grade corporate bonds

The investment grade corporate bond model with a fixed implied volatility shows similar differences as with the actual market implied volatility. The extreme values are less, about 1%, but the pattern is almost identically.

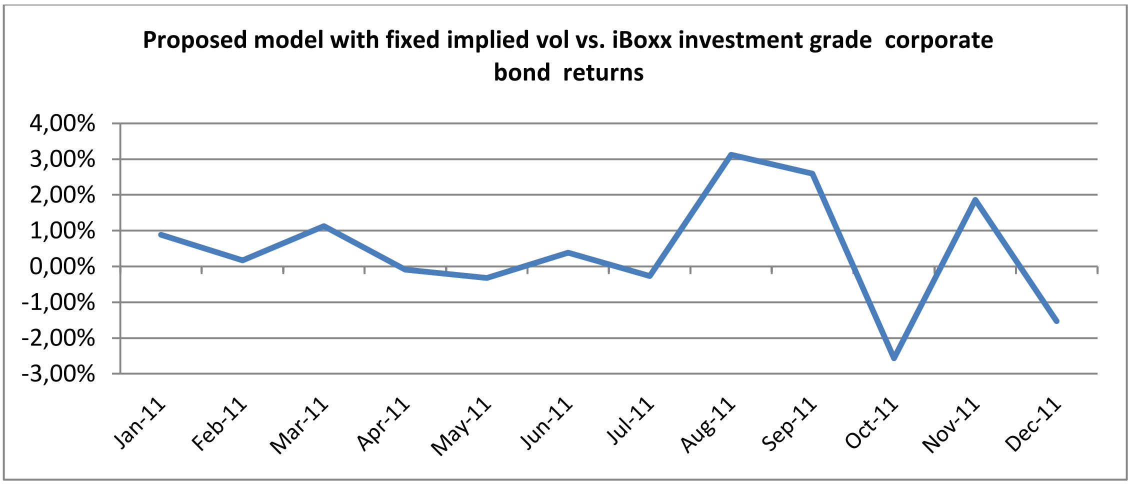

Figure 36. Proposed model versus iBoxx investment grade corporate bond returns

8.2.2. Collateralized corporate bonds

The collateralized corporate bond model shows also the same pattern as the model with the actual market implied volatilities. The pattern of the returns and the extreme values are relatively the same. In the volatile months July and August on the Euro Stoxx 50 index the model closely follows the benchmark index.

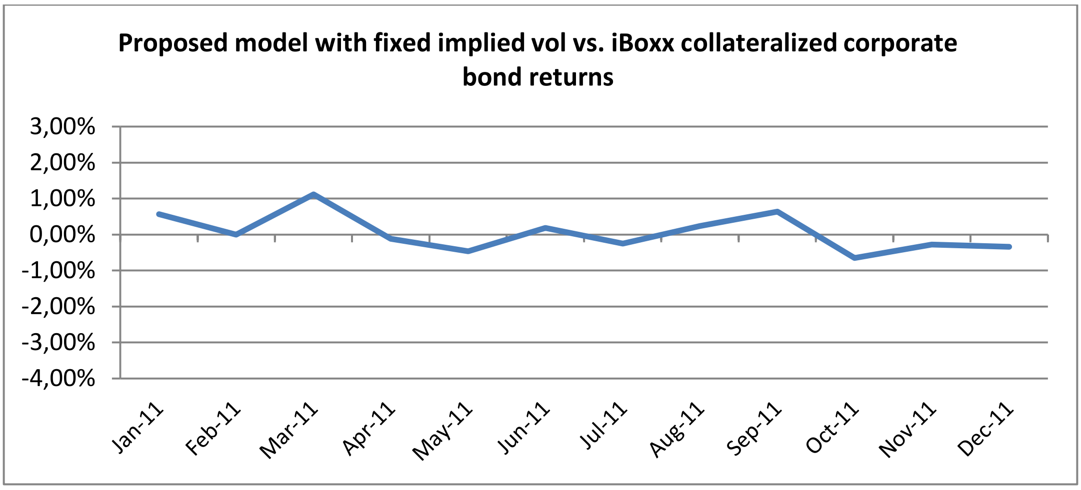

Figure 37. Differences proposed model versus iBoxx collateralized corporate bond returns

8.2.3. High yield corporate bonds

The high yield corporate bond model returns with a fixed implied volatility show that the differences with the iBoxx benchmark index are similar to the returns of the model with the use of the actual market implied volatilities. In the volatile Euro Stoxx 5 months the model with implied volatilities seems to provide a slightly better fit, but in the other months the fixed implied volatility high yield shows similar returns.

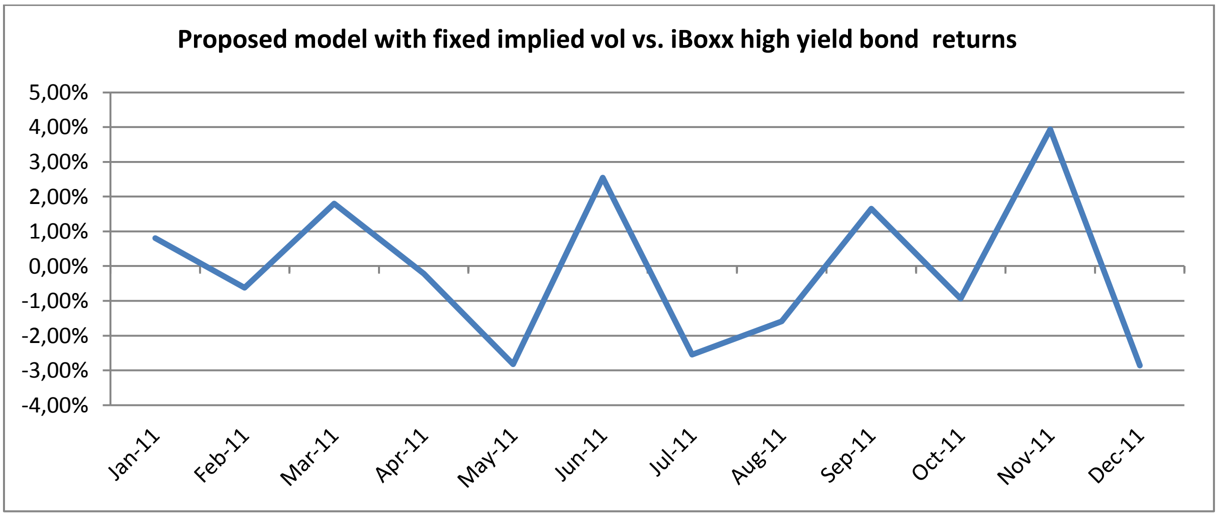

Figure 38. Differences Proposed model versus iBoxx high yield corporate bond returns

9. Conclusions and recommendations

Capturing the behavior of corporate bonds in a structural model requires modeling a nominal risk free bond with equity derivatives. These equity options normally require modeling the implied volatility of the underlying. By taking the implied volatility of the underlying directly from the investment banks which price these options the need of estimating the volatility disappeared.

Although the results differed, there is ample evidence that modeling a type of corporate bond with a structural model similar to the Merton model does give reasonable results even in the volatile time frame of the last couple of years. The differences in performance can mainly be attributed to the different types of bonds and their characteristics. The fact that the equity options are modeled on a representative equity index implies that the behavior of the corporate bond should be highly correlated with the underlying equity price of the company. If this correlation is low or even not present than the structural model will not capture the behavior of the corporate bond index.

If we look at the summary of how the three different types of bonds are modeled and their corresponding main sensitivity we see that the type of bond with the most sensitivity to equity is high yield corporate bonds. Collateralized bonds should mainly react to the volatility of the underlying index and investment grade should be sensitive to movements in the underlying index value.

Table 15. Overview of used derivatives and sensitivities of three proposed models

Type of bond	Type of options	Sensitivity
Investment grade	sold put	Delta, Vega
collateralized	sold put, bought put	Vega, (Delta)
high yield	sold put, bought call, sold call	Vega, +Delta, -Delta

The level of the strikes of each of the options is in the In-Sample test empirically determined. Because of the fact that the implied volatility is only given for 70% to 130% did give a restriction to the levels.

9.1. In-Sample results

The In-Sample analysis showed that the modeling of bonds requires different approaches for each of the types of bonds. The characteristics of the bonds required different types and number of options. The outcome of the actual implied volatility or taking an average implied volatility also differed a lot.

9.1.1. Investment grade euro corporate bonds

The in-Sample test results show that for the investment grade corporate bond there is a indication that it doe mimic these bonds as a nominal bond and a sold put option with a strike at 0.7 does mimic the actual iBoxx investment grade corporate index returns. Modeling the derivatives with the actual implied volatility given by the market showed that the model, with the sign test (60%) and the explained variance (R^2=0.32), seems to work reasonable in the time period of January 2006 to December 2010. Modeling these derivatives with a fixed average implied volatility of 25% improved the results, sign test (68%) and (R^2=0.36). An explanation could be that investors are looking more to at the longer term characteristics of the corporates than the short term movements of the equity prices.

9.1.2. Collateralized AAA euro corporate bonds

Collateralized bonds are bonds that have a pool of collateral assigned to them. In case the corporate defaults on a financial obligation an investor can sell the collateral before they join the general debtors for the other assets of the corporate. The value of a collateralized bond therefore does not also depend on the value of the underlying equity of the company but also on the type and value of the collateral.

In order to reflect the collateral posted with the corporate bond an extra derivative, a bought put option, with K=0.30, has been modeled next to the same put option, with strike K=0.70, as the investment grade corporate bonds. The bought put option reflects the collateral posted. The proposed model with the actual market implied volatility showed that the mean and standard deviation compared to the iBoxx collateralized AAA are not in line with the actual iBoxx index returns. The mean of the collateralized bonds is off by 12 basis points and the standard deviation shows that the proposed model is too constant in comparison with the iBoxx index, respectively 2%, versus 6.2%. The proposed model shows a positive skewness while the iBoxx index returns show that it has a negative skew in the same period. The Sign Test of 47% and the R^2 of 0.005 also indicate that trying to capture the behavior in this structural model with actual implied volatilities does not result in a good fit.

Modeling the collateralized bonds with the average implied volatility only slightly improves the results. The proposed model returns with an average implied volatility are just like the iBoxx returns negatively skewed but the mean and standard deviation of the proposed model are still showing large differences. The sign test (34%) and the R^2 of 0.005 still show that modeling this index with a structural model does not capture the behavior of the index.

Modeling the collateralized AAA euro corporate bonds with equity put options on the Euro Stoxx 50 does not give a good fit with the actual returns. It seems that investors in these types of corporate bonds are looking at another more dominant factor than the equity price. The type of collateral and its value could be a dominant factor.

9.1.3. High yield euro corporate bonds

For analyzing the euro high yield corporate bonds the iBoxx high yield index has been used for testing the proposed model. Modeling High yield debt with a nominal bond, a sold put option and a bought call spread showed results which indicate that it mimics the behavior of the corporate bonds. The proposed model with the actual market implied volatility has a sign test of 72% and the mean and standard deviation of the proposed model are in line the actual iBoxx returns. The linear regression shows that the variance of the iBoxx index is captured by the proposed model (R^2=0.68). The good fit with the proposed model shows that the high yield corporate bonds have positive and large correlation with the underlying equity price.

The result that the proposed model with an average implied volatility shows that without the actual implied volatilities the model captures less of the behavior of the corporate bond index. The sign test shows a small decrease to 70% and the mean and standard deviation also show less of a resemblance with the actual iBoxx high yield euro index returns.

Modeling the high yield corporate bonds with actual implied volatility from the market will produce the best results. This outcome showed that high yield corporate bonds returns are impacted by short term increased risk awareness, reflected by increased implied volatility.

9.2. Out of Sample results

The Out-of-Sample results show that the models over the period 2011 provide similar results as in the In-Sample period. The Euro Stoxx 50 index showed highly volatile monthly returns in that period. The corresponding implied volatilities also showed that after the first half year the rates spiked for at least three months. The Markit iBoxx benchmark indices showed less volatile patterns in the same periods. In the In-Sample period we saw similar behavior.

9.2.1. Investment grade euro corporate bonds

Capturing the behavior of investment grade corporate bond indices showed that the relationship between the proposed model and the benchmark indices was present and in the Out-of-Sample period the same relationship is present. The investment grade corporate bond returns show that the model does capture the behavior but that there are still some differences present.

9.2.2. Collateralized AAA euro corporate bonds

The collateralized corporate bond returns of the proposed model show that in the Out-of-Sample period the behavior of this index is better captured than in the In-Sample period. Were the model showed returns as high as 5% or 6% differences, the model follows the iBoxx returns in the year 2011 very well. The impact of the use of market implied volatilities seems to have only a little improvement, probably due to more stable conditions.

9.2.3. High yield euro corporate bonds

The high yield model in the Out-of Sample period show better results than in the period 2006-2010. The behavior of the index is better captured as the differences in returns are quite lower. The differences in the period end 2008 and beginning 2009 of 10% or -22% are in 2011 limited to -4%. This shows that in this period the High Yield corporate bond model performs better. The events in 2008 and 2009 were also more volatile and unpredictable but it still proof that the behavior of the corporate bond index is captured by the model.

9.3. Value-at-Risk

When looking at the VaR of the distributions of the proposed model returns we see that for investment grade and high yield corporate bond the Value-at-Risk is in line with the VaR calculated for the iBoxx benchmarks in that same period. For calculating the VaR (95%) for each of the types of corporate bond returns a portfolio equal to one million Euros was created. The below table shows the VaR for both the iBoxx benchmark portfolio as for the proposed model portfolio during the period of 2006-2010.

Table 16. VaR of portfolio of corporate bonds

	Investment grade	Collateralized	High yield
Proposed model	€ -22.500	€ -36.400	€ -93.200
iBoxx Benchmark	€ -17.600	€ -13.600	€ -73.600

In the Out-of-Sample period the VaR level of the proposed models were not breached by the actual iBoxx returns. In comparison with the VaR of the iBoxx benchmark the VaR for the investment grade and collateralized bonds was breached one time (investment grade € -18.700, collateralized € -16.500). Due to the small number of observation, we cannot draw conclusions, but the results are promising.

9.4. Proposed model against current ALM models

In order for the proposed model to work is to compare it with the current practices in ALM models. A common way for ALM software to model corporate bonds is to look at the relation between interest rates and stock indices and incorporate this relationship in their model. In the below two tables show the regression outcomes of both the proposed model against the iBoxx benchmark and the Euro Stoxx 50 index against the average risk free interest rates. The regressions that were performed had the following relationship

$$\text{return iBoxx}_t = \alpha + \beta_1 * \text{proposed model return}_t + \varepsilon$$

And

$$\text{return iBoxx}_t = \alpha + \beta_1 * \text{risk free interest rate differences}_t + \beta_2 * \text{Euro Stoxx 50 return}_t + \varepsilon$$

Table 17. Regression statistics of proposed model returns and that of ALM models

Regression Statistics proposed model and iBoxx		*Regression Statistics Euro Stoxx 50 and interest rates*	
Multiple R	0,57	Multiple R	0,53
R Square	0,33	R Square	0,28
Adjusted R Square	0,3	Adjusted R Square	0,27
Standard Error	0,08	Standard Error	0,05

The linear regression on the returns of the corporate bond index and the iBoxx benchmark showed that the R^2 is 0,33 and the linear regression on the returns of the Euro Stoxx 50 index and risk free interest rates showed that the R^2 is 0,28. The outcomes of both regressions show that both methods explain approximately the same level of variance. Together with the fact that the proposed model uses less unknown variables and therefore less factors which have to be estimated the proposed model could function as a good alternative for analyzing portfolios of corporate bonds in an ALM setting.

9.5. Recommendations

For capturing the behavior of corporate bond the proposed model uses equity derivatives. The below figure shows the average spot level of the underlying equity and the set strike as proxy for the level of leverage in the model. The level starts at 70% and varies over time as the equity market changes in value and the options expire and new options are modeled. One should interpret low values as a high sensitivity to equity prices, and a high value a low exposure to equity prices.

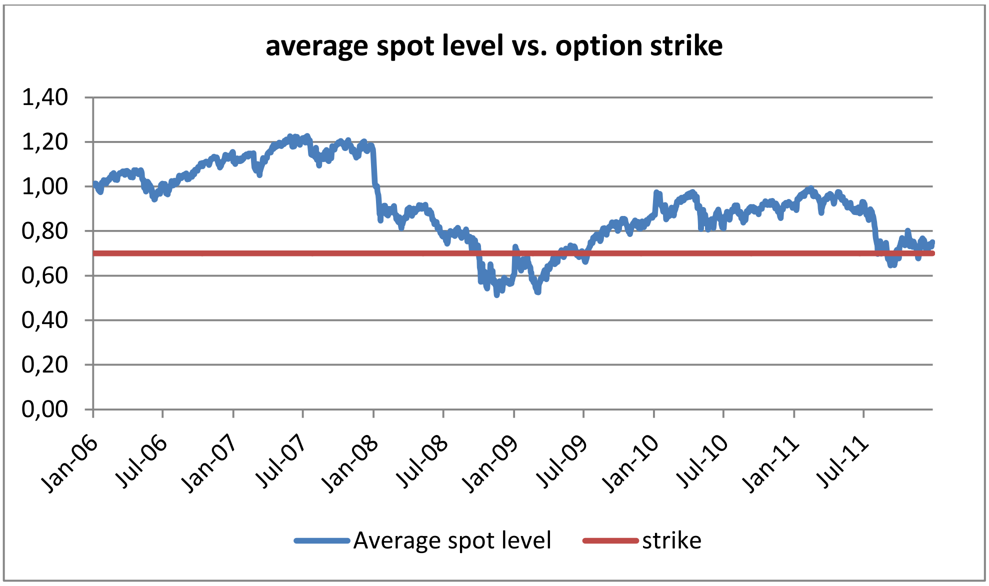

Figure 39. Average spot level Euro Stoxx 50 versus option strike of 70%.

One of the reasons this approach was tested is that upfront was believed that the relation between the company’s value and the value of its equity was not linear. The above figure clearly shows that was equity markets decline leverage factor also varies. This implies that the delta of the proposed model could become more constant. This research focused on capturing the behavior of corporate bonds in a generalization of the Merton model, but further research is recommended to how this relationship works in detail.

Also he proposed model has been tested for European corporate bonds. The behavior of corporate bonds in other markets, like in the United States, Japan or Great Brittan may be different. Looking if the proposed model also works in these markets is recommended before the model is going to be used in these markets.

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