UNIVERSITY OF TWENTE. SNS Asset Management

Master Thesis

Quantitative Portfolio Strategies

Beyond traditional theory? Low-risk as a useful factor for the construction of an equity portfolio.

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Abstract

The goal of this master's thesis is to improve the SNS Euro Equity Fund's current quantitative investment strategy by adding a low-risk factor. The purpose of this factor is to capture stocks with low-risk characteristics that deliver superior future expected stock returns. However, traditional finance principles assume that the stock market is efficient and that investors are rational and risk averse. Logically, this should result in a higher return for a more risky investment in equilibrium. Contrary to this intuitive reasoning, we provide convincing empirical evidence that low-risk stocks lead to considerable stronger future share price performance than high-risk stocks. The absolute returns of low-risk portfolios are substantially higher than those of high-risk ones. Moreover, their risk-adjusted returns statistically significantly outperform the risk-adjusted returns of high-risk portfolios in the long-run, regardless of whether we define risk as Conditional Value at Risk or historical volatility. This outperformance cannot be fully explained by the exposure to market risk and the current quantitative investment strategy of SNS Euro Equity Fund, therefore, adding a low-risk factor to this strategy seems worth considering. Additionally, we can conclude that the future expected performance of both the momentum and value strategy can be improved substantially by adding a low-risk factor, however, the effectiveness of the value strategy itself is debatable. To conclude, our results indicate that it is valuable to add a low-risk factor to the current quantitative investment strategy of SNS Euro Equity Fund in order to improve the expected future performance of an equity portfolio.

Preface

After enjoying life as both a student and athlete for more than seven years, I'm very proud to present my master's thesis which includes half a year of research in what perhaps is one of the greatest anomalies in financial markets.

First of all, I am very thankful to the equities team of SNS Asset Management that offered me this tremendous opportunity to be an intern in their team for six months. I owe special gratitude to my SNS supervisor Niels de Graaff, his enthusiasm, knowledge and experience in the field of portfolio management truly helped me achieving this result. Furthermore I would like to express my sincere appreciation to Hilde Veelaert and all colleagues who provided me the possibility to complete this thesis.

I would also like to thank my supervisors Reinoud Joosten and Toon de Bakker of the University of Twente for all the useful comments and remarks. Their generous guidance has invaluably shaped my thesis.

Last but not least, I would like to thank my family and friends, especially my parents for their support and encouragement during my time as a student at the University of Twente.

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Chapter 1

Introduction

SNS Asset Management (henceforth SNS AM) is the asset manager of SNS REAAL and manages 45.9 billion Euros year-end 2012. SNS AM uses so-called responsible investment criteria for all asset classes. The investing universe for all equity funds is based on fundamental policy principles and a wide range of social, ethical and environmental aspects, finally resulting in a socially responsible portfolio of stocks. The Portfolio Management department manages equity funds including the SNS Euro Equity Fund. Main goal of this department is to distinguish themselves by creating added value with a responsible portfolio.

Quantitative and qualitative factors serve as important inputs for the portfolio selection process. A tough question often arises: ‘How can these factors be improved?’. Recent academic papers and in-house research show that low-risk stocks often outperform high-risk stocks (i.e., have higher cumulative returns in the long run). This may be seen as counterintuitive and may contradict traditional theory, but are these findings also applicable to the investment universe of the SNS Euro Equity Fund? And if so, how to integrate this quantitative factor with the others to improve their portfolio in terms of outperformance? These are the main questions that will be answered in this thesis.

1.1 Research goal

This master’s thesis is conducted for the SNS Euro Equity Fund, this fund invests in stocks listed on the MSCI Europe Index. The fund managers attach great importance to four factors for the portfolio selection process: ‘price momentum’, ‘earnings momentum’, ‘value’ and ‘news’. The first three are based on quantitative criteria and the latter one is based on qualitative ones. The team is committed to improving their stock-picking strategies continuously, however, they still have the impression that the knowledge about the low-risk anomaly is underused in their investment process.

These findings made the fund managers think 'Why invest in high risk stocks, if the relation between expected return and risk is negatively correlated?'. This thought forms the basis for the idea of adding low-risk as a quantitative factor to the current ones. This factor tries to capture stocks with low-risk characteristics that deliver superior future (risk-adjusted) returns. The goal of this research is:

To improve the quantitative investment strategy by adding a low-risk factor.

The investment process of the SNS Euro Equity Fund and a definition of low-risk are outlined in the next sections in order to understand the goal and purpose of this research assignment.

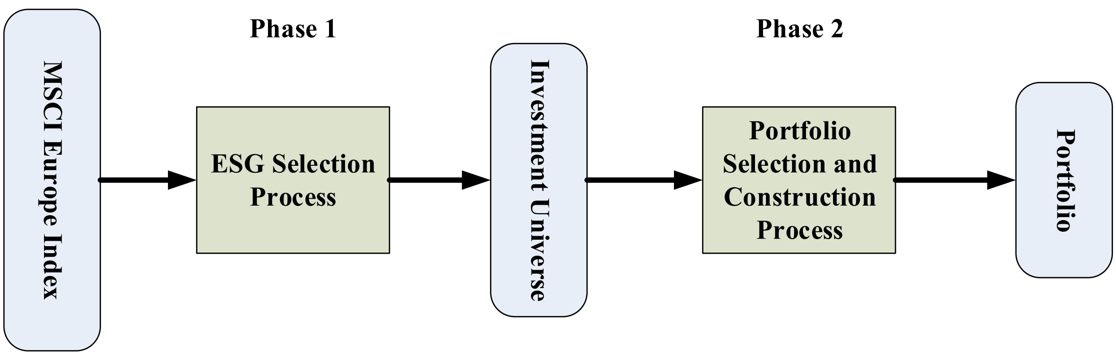
1.2 SNS Euro Equity Fund

The SNS Euro Equity Fund invests in European stocks listed on the MSCI Europe Index. The purpose of the investment process is twofold. Firstly, it attempts to create a so-called responsible investment universe. Secondly, it aims to select stocks from this universe that deliver superior future stock returns. The investment process consists of two phases:

- The first is the Economic Social and Government (henceforth ESG) Selection Process, resulting in an investment universe.
- The second is the Portfolio Selection & Construction Process, resulting in an equity portfolio.

This process is visualized by Figure 1.1.

Figure 1.1: Selection and construction process.



1.2.1 ESG selection process

The initial phase of this process is related to the philosophy of SNS in terms of social, ethical and environmental principles. The SNS AM ESG-department is continuously screening and evaluating stocks on their ESG-score by sector. Companies violating the fundamental policy principles are excluded from the investment universe. These fundamental principles relate to human rights, child labor, corruption, environmental contamination, other ethical principles and the development, production, use and maintenance of controversial weapon systems. Subsequently, selection of the 'best in class' stocks results in the top 40% of stocks by sector, scored solely on the ESG score (by means of a scorecard). Consequence is that the investment universe consists of approximately 40% of the MSCI Europe Index. However, the fund managers and the ESG-team can still add a company which does not belong to the best in class selection to the investment universe, if the company has products or operations which are perceived as characteristically sustainable. This is known as positive selection.

1.2.2 Portfolio selection & construction process

Main focus of this master's thesis is on the Portfolio Selection & Construction phase of the investment process. It is currently based on four factors: 'price momentum', 'earnings momentum', 'value' and 'news'. The first three factors are motivated by Asness et al. (2009, p.1), they emphasized the importance of value and momentum strategies in order to outperform the markets and found that: "Value and momentum ubiquitously generate abnormal returns for individual stocks within several countries, across country equity indices, government bonds, currencies, and commodities".

Momentum

Momentum is defined by Berger et al. (2009, p.1) as: "The tendency of investments to exhibit persistence in their relative performance. Investments that have performed relatively well, continue to perform relatively well; those that have performed relatively poorly, continue to perform relatively poorly". Jagadeesh & Titman (1993) concluded that past winners have the tendency to outperform past losers over an intermediate horizon. SNS distinguishes between price and earnings momentum.

[Confidential]

Henceforth, momentum refers to this combination of earnings and price momentum, unless mentioned otherwise and for clarity's sake, the momentum strategy is called a 'univariate' strategy instead of a bivariate strategy in this master's thesis.

Value factor

The value factor is based on the findings of De Bondt & Thaler (1985, 1987) that most people 'overreact' to unexpected, negative and positive news events, resulting in temporarily undervalued and overvalued companies. SNS uses the forward-looking price to earnings ratio in attempt to identify these companies. This measure uses the earnings forecast for next year and the current market price per share. The current market price per share is divided by the earnings forecast per share.

[Confidential]

Newsflow factor

SNS composes a newsflow factor based on qualitative analysis in order to avoid decisions purely based on quantitative data (the momentum and normalized value factor). The input used to determine the newsflow score are press releases, company visits, macroeconomic data, analysts' opinions and recent research papers. This qualitative kind of performance is analyzed by the portfolio managers and classified into quartiles based on a grading system.

[Confidential]

Portfolio construction

The portfolio construction process is primarily driven by the four factors explained in Section 1.2.2.

[Confidential]

The in-house made style monitor measures the current exposure to and performance of factors with respect to the investment universe and also serves as an important input for the construction of the portfolio. If there was lack of outperformance power of a certain factor in the last few months, the portfolio managers may decide not to overweight stocks with exposure to this factor.

SNS investment framework

SNS has a comprehensive investment framework, where the momentum, value and newsflow are the main quantitative and qualitative factors to assess stocks. The style monitor keeps track of the exposure to and performance of these factors. In practice, portfolio selection and construction decisions are made twice a month. The investment framework is visualized by Figure 1.3.

1.3 The low-risk anomaly

Markowitz (1952) assumes that the stock market is efficient and that investors are rational and risk averse. Logically, this should result in higher return for a more risky investment in equilibrium. The capital asset pricing model (CAPM), found by Sharpe (1964) and Lintner (1965) and based on the Modern Portfolio Theory, attempts to find the optimal portfolio based on a given risk profile. The predicted relationship between risk and return is better known as the security market line as shown in Figure 1.2. The higher the risk of the portfolio, the more return is required.

The low risk anomaly, however, stipulates that the relationship between risk and return might be flatter than predicted by the CAPM, since empirical analysis has unveiled evidence of a flat or even inversely related risk-return relationship in equity markets.

Figure 1.2: Relationship between beta and required return.

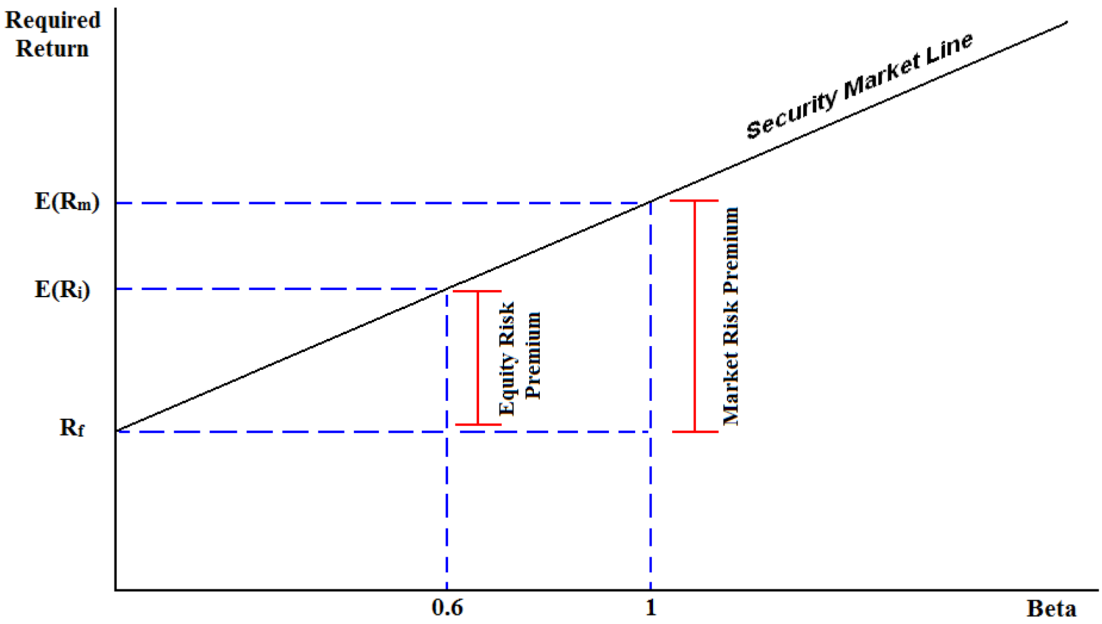


Figure 1.3: Investment framework.

[Confidential]

Black et al. (1972) concluded that the relationship between risk and return might be flatter than assumed by the CAPM. This finding is supported by Eugene Fama, he said that the relation between average return and beta is completely flat.[1] More recently, Baker et al. (2011) found that US stocks in the bottom volatility-quintile (i.e., low volatility stocks) have produced higher absolute returns than stocks out of the other volatility quintiles (i.e., more volatile stocks) in the long-run.

Similar results for emerging markets and developed non-US markets are confirmed by Dutt & Humphery-Jenner (2013). Baker et al. (2011) reported whether risk is defined as volatility or beta, low risk stocks consistently produced higher returns than high risk ones. Which risk measure is more fundamental is of practical interest, but results suggest that beta is more effective in large cap stocks and both act as drivers for small cap stocks.

1.4 Explanations for the low-risk anomaly

There are several different explanations for the low-risk anomaly, but we consider the next three as the most convincing ones. The first two are related to behavioral finance, whereas the latter is more rational.

First, Barberis & Huang (2012) argue that investors overestimate payoffs and underestimate risks of stocks with positively skewed payoff profiles, mostly high volatility or high beta stocks. The rationale behind this theory is the attractiveness of making large returns within a relatively short period of time. These stocks become overvalued and subsequently are more likely to underperform.

Second, Barber & Odean (2008) reported that attention-grabbing stocks (e.g., stocks in the news and stocks with extreme returns), mostly risky ones, are temporarily overbought and subsequently tend to underperform.

Third, Frazzini & Pedersen (2013) and Baker et al. (2011) pointed out that most portfolio managers cannot use leverage and that their performance is measured relatively to an index (benchmark) and, therefore, low-beta stocks are as risky as high-beta stocks, in a relative sense rather than an absolute one. Consequently, low-beta stocks are less likely to be purchased,

[1] According to Michael Peltz of *Institutional Investor* (1992)

because they are expected to underperform high-beta stocks. Less demand for low-beta stocks causes decreasing stock prices and increasing average returns.

With that in mind, let's illustrate this theoretical explanation with an example. Consider the security market line given by:

$$E(R_p) = R_f + \beta_p * [E(R_m) - R_f] \tag{1.1}$$

Where $E(R_p)$ is the required return of portfolio p, R_f is the risk free rate, β_p is the exposure of portfolio p to market-relative risk and $E(R_m)$ is the required return of market m. Suppose that the market's return is 14 percent, the risk free rate is 4 percent and that a low-risk portfolio has a beta of 0.8 and a high-risk portfolio has a beta of 1.2. For the low-risk portfolio the required return becomes:

$$E(R_p) = 0.04 + 0.8 * (0.14 - 0.04) = 0.12 \tag{1.2}$$

Or 12%. Subsequently, for the high-risk portfolio:

$$E(R_p) = 0.04 + 1.2 * (0.14 - 0.04) = 0.16 \tag{1.3}$$

Or 16%.

For the low-risk portfolio, to equal the return of the high-beta portfolio requires excess return of 4 percentage points. Suppose that, contrary to this equilibrium, the (expected) excess return of the low-risk portfolio is 3 percentage points. Fund managers might decide to continue investing in the high-risk portfolio with zero excess return to obtain 16 percent expected return instead of the 15 percent expected return of the low-risk portfolio.

Let's now make the simplifying assumption that we are able to borrow at the risk-free rate and that, again contrary to the security market line, the expected returns are indeed 15% and 16% for the low-risk and high-risk portfolio, respectively. Using 50 percent leverage on the low-risk portfolio would create a beta of 1.2 and still would equal risk exposure relatively to the market as prior to leverage. Expected profit of this leveraged low-risk portfolio would then be substantially higher than the non-leveraged high-risk portfolio. However, most portfolio managers cannot use leverage and their performance is measured relatively to an index and therefore they have no incentive to take advantage from such a mispricing.

However, the theme of this thesis is not the explanation of the anomaly, but the existence of it, and if it appears to exist, how to benefit from this phenomenon and how to make this anomaly applicable and useful for the investment strategy of the SNS Euro Equity Fund.

1.5 Research structure

The structure of this master's thesis can be constructed, now that we know what the low-risk anomaly is and how SNS composes their portfolio. Findings in the literature with regard to the low-risk anomaly are tested on mimicked equity portfolios. A more thorough analysis shows how low-risk interacts with the momentum and normalized value factor, and how the current strategy can be adjusted in order to improve the performance of the portfolio. Therefore the main question of this thesis is:

How can a low-risk factor be combined with the current quantitative investment strategy in order to improve the performance of an equity portfolio?

The main question is divided into sub-questions, these will help to solve the main question incrementally. The sub-questions are:

1. *Which low-risk strategies are applicable to the current quantitative investment strategy?*
2. *How can the performance of low-risk strategies be measured?*
3. *Does low-risk outperform high-risk in the SNS Euro Equity Fund universe and which low-risk strategy is associated with best future expected returns?*

Sub-questions 1, 2 and 3 determine pure low-risk performance. The next two sub-questions determine the performance of the current quantitative factors in combination with low-risk factors.

4. *Does low-risk add value as extension of the current quantitative investment strategy?*
5. *What is the importance of using distinct univariate quantitative investment strategies, like low-risk, momentum and value?*

1.5.1 Outline

These five sub-questions are elaborated subsequently in Chapters 2 till 6. In Chapter 2 we outline which risk-measure best suits the purpose of this research. Chapter 3 explains how the performance of an equity portfolio can be measured according to the literature. The findings in the previous two chapters serve as a basis for formation and performance measurement of low-risk portfolios in Chapter 4. Chapter 5 is dedicated to the question whether low-risk adds value to the current quantitative investment strategy in terms of performance. Chapter 6 shows the interaction between the low-risk, momentum and normalized value factor and proposes a new performance based quantitative investments strategy. Conclusions are drawn in Chapter 7, where the findings are summarized and an answer is given to the main question of this thesis. Finally, Chapter 8 is a discussion about the limitations of this research and comments on further research directions.

1.6 Scope

Restrictions of the scope of this research assignment are:

- Leverage and shorting constraint; fund managers cannot use leverage or short selling.
- Assumption is made that the current momentum and value factors are applied optimally.
- Currency risks are ignored, i.e., changes in stock prices are assumed to result from changes in the value of the stock.
- Transaction costs are not incorporated.

Chapter 2

Low-Risk Strategies

Goal of this chapter is to find measures which can serve as a proxy for risk and from which low-risk portfolios can be formed. It gives an overview of common low-risk strategies. Beside these existing strategies, we try to seek for an improvement in strategies which can expand the current body of literature. An important limitation is that the strategies must be tailored to the portfolio managers, so that it is applicable to their current decision-making process. The sub-question that is answered is 'Which low-risk strategies are applicable to the current quantitative investment strategy?'.

2.1 Risk measurement

Equity risk can be defined as the probability or uncertainty of losing equity capital. There are different methods of measuring this risk. The most common risk metrics are historic volatility and beta. Logically, different low-risk portfolios can be derived from these risk measures, the most obvious are: low-volatility, low-beta and a minimum variance portfolio. The first two are subsequently explained in the following sections. The latter is inapplicable, because they force to overweight low-risk stocks extremely, which is seen as an investment that is too risky from the portfolio manager's point of view.

Disadvantage of these methods is that they make no distinction between positive and negative returns. Upside gains are penalized the same as downside losses. An alternative risk measure is the Conditional Value at Risk and determines the average return on the portfolio in the worst X% of the cases. Positive upward swings are not penalized by this measure, while significant losses are.

2.1.1 Volatility

Numerous authors emphasize (e.g., Baker et al. (2011) and Jessop et al. (2011)) that a simple, but quite effective way to exploit the low-risk anomaly is to rank stocks into quantiles according to their historical volatility or historical beta over a given period of time. In this way they are able to compose portfolios based on the historical volatility or historical beta compared to other stocks within the market. The lowest volatility and lowest beta stocks are represented in the bottom quantiles. Quantiles are reclassified and rebalanced at the end of each month based on their revised volatility or revised beta, i.e., stocks are (partly) bought or sold. Baker et al. (2011) find that US stocks in the bottom quintile produced higher average absolute returns than stocks out of the other quintiles, regardless of whether risk is defined as volatility or beta. Nonetheless, results of Baker et al. (2011, p.47) suggest that: "Beta drives the anomaly in large stocks, but both measures of risk play a role in small stocks". Explanation can be found in the reasoning that benchmarked portfolio managers' focus disproportionately on large cap stocks.

Volatility

Historic volatility is a statistical measure for variation of price of a security over time and is derived from past historical observations of its market prices. It is primary used to assess the risk of tradable assets or portfolios. Volatility is determined over a specified period of time and enables the user to set the period of historical observations to own preferences with regard to the persistence of historic data. Volatility σ of stock i is mathematically represented by:

$$\sigma_i = \sqrt{\frac{1}{T-1} * \sum_{t=1}^{T} (r_{it} - \bar{r}_i)^2}\ , \qquad (2.1)$$

where $$\bar{r}_i = \frac{1}{T} * \sum_{t=1}^{T} r_{it}$$

and $$r_{it} = \frac{s_{it} - s_{i,t-1}}{s_{i,t-1}}\ .$$

s_{it} and $s_{i,t-1}$ are the prices of stock i at the end of day t and day t-1, respectively, and is adjusted for all corporate actions that effect an asset's price, such as stock splits and cash or stock dividends. Logically, r_{it} is the return of stock i over day t. The average historical return over T days is given by $\bar{r}_i$.

2.1.2 Beta

Beta is a market-relative measure of risk, it measures the tendency of asset's or portfolio's returns to respond to swings in the market. Low-beta stocks tend to be less volatile than the market and high beta stocks usually are more volatile than the market is. Main difference between volatility and beta, as risk measure, is that beta only measures the market-relative risk, whereas volatility measures total risk including its idiosyncratic risk (stock specific influences). Different coefficients of beta are explained in the Table 2.1.

Table 2.1: Beta Coefficients.

Beta Coefficient	Explanation
β<0	Negative beta means that the stock usually moves in opposite direction of the market (inversely correlated).
β =0	A zero-beta stock is insensitive and uncorrelated to the market.
0< β <1	Stock generally moves in the same direction, but with lower rates of change than the market.
β =1	Stock tends to move in the same direction and with the same rate of change as the market.
β >1	Stock generally moves in the same direction as the market, but with higher rates of change than the market.

Beta approximates the sensitivity of the stock's returns relative to the market's returns and is measured as the sample covariance between the return of the stock and the return of the market, divided by the market's variance of return (i.e., the covariance of market's return with itself):

$$\beta_i = \frac{\text{cov}[r_i, r_m]}{\text{cov}[r_m, r_m]} = \frac{\text{cov}[r_i, r_m]}{\text{var}[r_m]} \ , \tag{2.2}$$

where $\text{cov}[r_i, r_m] = \frac{1}{T-1}\sum_{t=1}^{T}(r_{it} - \bar{r}_i) * (r_{mt} - \bar{r}_m)$

and $\text{var}[r_m] = \sigma_m^2 \ .$

Volatility σ and return r of asset i and market m can be determined as explained in Section 2.1.1. The beta of asset i is given by β_i.

2.1.3 Conditional Value at Risk

Disadvantage of the previous risk measures is that they are not sensitive to tail information and, therefore, underestimate the probability of significant losses. Idea behind measuring tail risk is that former research (e.g., Baker et al. (2011), Dutt & Humphery-Jenner (2013), and Jessop et al. (2011)) have shown that more risk-taking activities are not rewarded by more ex post return. Generally, risk is determined by non-tail sensitive methods like volatility, beta or minimum variance. However, most assets exhibit non-normal returns, indicating that extreme returns are more likely to occur. This implies that it is important to consider this in addition to volatility or beta. A relatively simple way to measure left tail risk is Conditional Value at Risk (CVaR), sometimes also called Expected Shortfall.

CVaR measures the average return of the α% worst returns, it is determined with basic historical simulation. The main advantage of this approach is that it makes no assumption about any distribution on the stock returns. Other advantages are the intuitive simplicity and clearness. Suppose that we want to determine the $\text{CVaR}_{10\%}$ of a stock of the last 50 weeks based on weekly returns. Simply, this is the average return of the five worst weekly returns.

Acerbi & Tasche (2001) illustrate that the Conditional Value at Risk can be estimated in a few steps. First, sort n returns r_i in increasing order:

$$r_{1:n} \leq \ldots \leq r_{n:n} \tag{2.3}$$

Second, approximate the number of positive integer α% elements in your sample by:

$$w = \max\{m | m \leq n\alpha, m \in \mathrm{N}\}, \tag{2.4}$$

$$\text{where} \quad \mathrm{N} = \{1,2,\ldots\}.$$

Third, represent the set of α % worst returns by the least w outcomes:

$$\{r_{1:n}, \ldots, r_{w:n}\} \tag{2.5}$$

Fourth, estimate the Conditional Value at Risk of the α% worst returns for n number of returns:

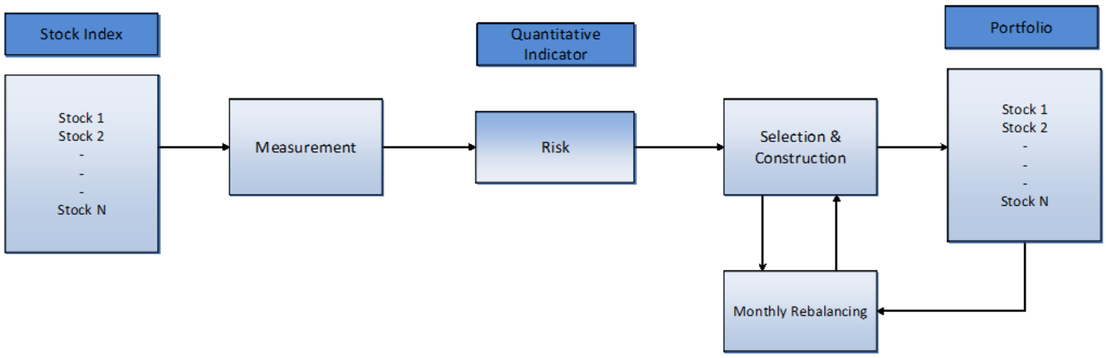
$$CVaR_n^{\alpha\%}(R) = \frac{\sum_{i=1}^{w} r_{i:n}}{w} \tag{2.6}$$

Which we call the Conditional Value at Risk of sample n at $\alpha\%$ level.

2.2 Portfolio formation methodology

The SNS Euro Equity Fund investment framework is rather complicated as can be seen in Figure 1.3, this complexity makes the reproduction of quantitative strategy based investment decisions impossible (i.e., a back test). Another constraint is the lack of historical data. Therefore, we introduce a simplified investment framework to be able to produce portfolios in order to back test the strategies. The framework to back test risk-based strategies is presented in Figure 2.1.

Figure 2.1: Simplified framework for risk-based strategies.



First, risk of all stocks out of the stock index is estimated based on one chosen risk measure. This *measurement* occurs on a monthly basis. Thereafter, stocks are ranked into q-quantiles where the lower quantile is a proxy for the least risky stocks and where the upper quantile is a proxy for the most risky ones.

Second, the *selection* phase selects stocks which match the specific risk profile of a strategy. Subsequently, matching stocks are included in the portfolio of this strategy (i.e., so-called long position) and non-matching stocks are not included at all (i.e., so-called neutral position).

Third, the actual portfolio is *constructed*. At the beginning of the back test an equal amount of money is invested in these stocks. Estimations of the stock's risk are updated at the end of each month throughout the back test. Based on the historic performance of the stocks in the portfolio and based on the updated stock's risk estimates, decisions are made regarding buying or selling stocks. By construction, equally-weighted portfolios partly sell stocks that outperform the portfolio and acquire a larger stake for stocks that underperform relative to the portfolio, assuming both still match the risk profile of the strategy.

This framework assumes that stocks are held for a period of at least 1 month and the portfolios are rebalanced once a month. This equally-weighted and frequently rebalanced portfolio construction seems a good approximation of reality.

For all risk-based strategies we chose to allocate stocks into 5-quantiles, also called quintiles. On the one hand this creates distinctiveness between the lower quintile portfolio and the upper quintile portfolio from a risk point of view and subsequently can provide a good approximation of differences in performance between low-risk and high-risk stocks, on the other hand we aim to create well-diversified portfolios in order to reduce the amount of unsystematic risk. Studies have shown (e.g., Statman (1987)) that you can eliminate most of your unsystematic risk maintaining a portfolio with at least 30 stocks. By creating quintiles, we easily fulfill this requirement.

Summarized, risk-based strategies are strategies which are based on ranking stocks monthly into quintiles based on one risk-measure.

2.2.1 Volatility strategy

Similar to Jessop et al. (2011), our first volatility strategy is based on the individual annualized 252-day stock volatilities. Although transaction costs are not incorporated in this master's thesis, main advantage of this strategy is that it requires relatively little rebalancing of portfolios resulting in relatively low transaction cost. Disadvantage of this strategy is that it is still quite persistent to historic data. As a result, it could have difficulties to adapt to changing business activities or changing market conditions. For that reason there is an inevitable trade-off between the persistence of volatility and transaction costs. However, this 252-day volatility strategy seems to exhibit a good balance between transactions costs and the adaptability to changing business activities or changing market conditions. In order to be able to estimate the difference in performance between strategies with different adaptive characteristics a more adaptive second strategy based on the individual annualized 63-day stock volatilities is created. [2]

[2] We discovered at an early stage that the transaction costs for strategies based on short historical periods (<=3 months) become unmanageably high. Therefore, from a practical point of view, no more strategies are build based on such short historical periods. Although these strategies will never be executed by SNS in real life, we are still interested in the difference in performance between these strategies and hence keep this strategy in this master's thesis.

2.2.2 Beta strategy

To allow comparison between the strategies (with the exception of the annualized 63-day volatility strategy), we set the period of historical data equal to one year. Hence, the beta strategy is based on individual 52-week stock betas. Weekly returns are used, instead of daily ones to increase the probability of accurately estimated betas of less liquid assets. The more historical data are included, the more persistent beta is and the more slowly a shift in beta is recognized. Hence, including longer estimation periods for the estimation of beta is more likely to be biased. On the contrary, including less historical data will increase the standard error of the estimation.

2.2.3 Conditional Value at Risk strategy

The Conditional Value at Risk strategy is based on the average historical returns of stocks in the worst 10% of the weeks. Again we set the period of historical data (approximately) equal to one year. For the sake of clarity, we use an estimation period of 50 weeks, resulting in the average return of the five worst weekly returns.

2.3 Conclusion

An answer is found to the question 'Which low-risk strategies are applicable to the current quantitative investment strategy?'. Different risk measures are used to compose low-risk strategies: 'volatility', 'beta' and 'Conditional Value at Risk'. The strategies are summarized in Table 2.2. The *indicator* depicts the risk measure used to compose the risk-based portfolios. All portfolios are equally-weighted and rebalanced once a month, i.e., stocks are held or (partly) bought or sold at the end of each month.

Table 2.2: Overview of low-risk strategies.

Strategy	Indicator
Volatility-based portfolio	252-day volatility
	63-day volatility
Beta-based portfolio	52-week beta
CVaR-based portfolio	50-week $CVaR_{10\%}$

Chapter 3

Performance Measurement

This chapter gives an overview of different methods of measuring equity portfolio performance. Performance is measured by the average annualized return and three risk-adjusted return metrics: alpha (determined by the capital asset pricing model and the Fama-French three-factor model), the Sharpe ratio and the Sortino ratio. The average annualized return is the return a portfolio achieves over a given period of time and is converted into an annual rate of return. Alpha, the Sharpe ratio and Sortino ratio are risk-adjusted returns, meaning that the return is adjusted for the risks taken. Although transaction costs are not incorporated, it is valuable for portfolio managers to evaluate the portfolio turnover rate in order to see how frequently stocks are bought and sold per period. This chapter gives an answer to the sub-question 'How can the performance of low-risk strategies be measured?'.

3.1 Average annualized return

The average annualized return a portfolio generates is the average return over a given period of time and is converted to an annual rate of return. It can be calculated by dividing the sum of returns by the count of these, the so-called arithmetic return. All portfolios are equally weighted, meaning that the return from month *t-1* to *t* of the portfolio is simply the average return of all the stocks in the portfolio for the same period. The arithmetic annualized return is given by:

$$r_p^{arithmetic} = \left(\frac{1}{T * N} \sum_{t=1}^{T} \sum_{i=1}^{N} r_{it} \right) * 12, \tag{3.1}$$

where $$r_{it} = \frac{s_{it} - s_{i,t-1}}{s_{i,t-1}}.$$

s_{it} and $s_{i,t-1}$ are the prices of stock i at the end of month t and month $t-1$, respectively, and is adjusted for all corporate actions that effect an asset's price, such as stock splits and cash or stock dividends. Logically, r_{it} denotes the return of stock i from month $t-1$ to t. Where $r_p^{arithmetic}$

is the average arithmetic annualized return of N stocks over T months. Note that we always lose one observation for the calculation of an initial stock return (i.e., *t-1*).

Intuitively, portfolio A with two historical monthly total returns of 15% and -15% equals total return of portfolio B with historical monthly returns of 5% and -5%. This would be true if the observations of returns in these financial time series could be treated as independent events, but this is not the case. Suppose you invest $100 dollar in both portfolios. Value at the end of month two for portfolio A and B is $100*1.15*0.85=$97.75 and $105*1.05*0.95=$99.75, respectively. This difference is substantial. This is known as geometric compounding. The geometric annualized return for equally-weighted portfolios is given by:

$$r_p^{geometric} = \left(\left(\frac{1}{N} \sum_{i=1}^{N} \left(\frac{S_{it}}{S_{i0}} \right)^{1/t} \right) - 1 \right) * 12 \tag{3.2}$$

Where s_{it} is the price of stock i at the end of month t, and s_{i0} is the price of this stock at the beginning of the investment. Logically, $r_p^{geometric}$ is the average geometric annualized return for portfolio p at the end of month t of N stocks.

3.2 Sharpe ratio

The Sharpe ratio uses the return of a portfolio over the risk-free rate. Next, this return is adjusted for the variation of price. The mathematical equation is as follows:

$$Sharpe\_ratio_p = \frac{E[R_p - R_f]}{\sigma[R_p - R_f]} \tag{3.3}$$

Where $E[R_p\text{-}R_f]$ is the expected return of portfolio p over the risk-free rate R_f.

Why is it in our case not appropriate to use the revised version by Sharpe (1994), where the risk-free return in the numerator and denominator is replaced by the return of the market? The answer is quite simple; this risk measure would always rank stocks with returns lower than the market below stocks with a return higher than the market (due to the negative numerator), regardless of the variation in price. For example, without leverage constraints, stock A with beta 0.5 and expected return of 5% can be leveraged such that it has twice the expected return of stock A with a beta of 1. Stock B has an expected return of 5.5% and a beta of 1. Both the leveraged stock as well as stock B have about the same risk exposure. Now suppose that the return of the market is 5.25%. The revised Sharpe ratio ranks Stock B above Stock A, while stock A performs a lot better, actually.

For simplicity's sake, we calculate the Sharpe ratio by:

$$\hat{Sharpe}\_ratio_p = \frac{\overline{R}_p}{\sigma[R_p]} \tag{3.4}$$

Where the realized average return $\bar{R}_p$ is divided by the standard deviation of returns of the portfolio. A higher Sharpe ratio suggests better risk-adjusted returns. Thus, higher is better.

3.3 Sortino ratio

Disadvantage of the Sharpe ratio is that it penalizes upward movements the same as downward movements. The Sortino ratio, introduced by Price and Sortino (1994), is primarily used to assess downside risks. Advantage is that large positive gains do not contribute to a more risky investment. The other way around, large negative gains cannot be eliminated by producing lots of very small positive returns. The Sortino ratio for portfolio *p* is given by:

$$Sortino\_ratio_p = \frac{E[R_p - R_f]}{DR_p} \tag{3.5}$$

Where $E[R_p\text{-}R_f]$ is the expected return of the portfolio over the risk-free rate and DR_p is the downside standard deviation and is calculated as follows:

$$DR_p = \sqrt{\frac{1}{\sum_{t=1}^{T} K_{pt}} \left(\sum_{t=1}^{T} \left(R_{pt} - R_u\right)^2 * K_{pt} \right)}, \tag{3.6}$$

$$\text{where} \quad K_{pt} = \begin{cases} 1, & R_{pt} < R_u \\ 0, & R_{pt} \geq R_u \end{cases}.$$

Where R_u is the user-specified required rate of return and R_{pt} denotes the return of portfolio *p* from month *t-1* to *t*. We set R_u to zero, indicating that negative returns are considered to be risky. However, taking the value of R_u into account, for simplicity reasons we calculate the Sortino ratio as:

$$\hat{S}ortino\_ratio_p = \frac{\overline{R}_p}{DR_p} \tag{3.7}$$

3.4 Alpha

Mainly, returns can be explained by the exposure to market, size and value risk. Alpha is the return which cannot be explained by the risks taken, also known as 'excess return' and often seen as the skills of the portfolio manager. The capital asset pricing model assumes market risk as the only source of systematic risk, whereas the Fama-French three-factor model assumes beside market risk, size and value as systematic risk sources. These models determine the exposure of a portfolio to several risk factors. As a result, the performance of the portfolio can be analyzed (i.e., positive alpha indicates that the portfolio outperforms the benchmark or stock index).

3.4.1 Capital asset pricing model

Intuitively, investors would require higher expected returns in exchange for more risk-taking activities. Sharpe (1964) and Lintner (1965) try to quantify this relationship, better known as the capital asset pricing model as already slightly touched upon in the first chapter. A number of simplifying assumptions are made; the stock market is efficient and investors are rational and risk averse. In this model the return of a stock or portfolio is solely based on its risk compared to market risk. This systematic risk is measured by beta. The CAPM equation is given by:

$$R_{pt} = \alpha_p + RF_t + \beta_p * (RM_t - RF_t) + \varepsilon_{pt}, \tag{3.8}$$

where $t = \{1, 2, \ldots, T\}$.

Where R_{pt} denotes the return of portfolio p from month t-1 to t, T are the number of months, RF_t is the risk-free rate, β_p is the exposure to market risk RM_t-RF_t and is determined by linear regression, α_p is the excess return and ε_{pt} is the error term which is assumed to be identically, independently and normally distributed with zero mean and variance σ^2. The market risk is defined as the expected return of the market over the risk-free rate. Essentially, exposure to market risk for a specific portfolio gives an extra reward on top of the risk-free rate.

However, Womack & Zhang (2003) state that CAPM models explanatory power is questionable and that the coefficient of determination (R^2) is about 85%. This means that 85% of the movements can be attributed to the movements of the market. On the other hand, there is still 15% variation in observed returns that remain unexplained, i.e., it is very likely that there are more explanatory variables.

3.4.2 Fama-French thee-factor model

Fama & French (1993) argue that stock returns depend, besides market risk, on a 'value' and 'size' factor. The main idea behind these additional factors is that it is unlikely that market risk is the only significant factor to address returns to.

Small companies are more sensitive to many risk factors, because they are less able to absorb financial losses. Therefore, small companies intuitively must earn more return. To represent this risk, they constructed a SmallMinusBig (*SMB*) size factor, where size refers to the stock's market capitalization. The SMB factor can be computed by subtracting the average return of the 30% largest stocks from the average return of the smallest 30% of stocks. Womack and Zhang (2003) indicate that the average historical annual SMB size premium is about 3.3%.

Companies with a high book to market ratio (book value divided by the current market value) are more risky than those with a low book to market ratio. The idea behind this factor is that high book to market ratios are caused by decreasing equity. This drop in equity is probably due to doubts about future earnings. The HighMinusLow (*HML*) value factor can be determined by subtracting the average returns of the 50% lowest B/M ratio stocks from the average returns of the 50% highest B/M ratio stocks. Womack and Zhang (2003) indicate that the average historical annual SMB size premium is about 5.1%.

The Fama-French three-factor model equation is:

$$R_{pt} = \alpha_p + RF_t + \beta_p * (RM_t - RF_t) + S_p * SMB_t + V_p * HML_t + \varepsilon_{pt}, \tag{3.9}$$

where $t = \{1,2,\dots,T\}$.

Where R_{pt} denotes the return of portfolio p from month $t-1$ to t, T are the number of months and in this equation β_p, S_p and V_p are the exposures to market risk, the size factor and the value factor, respectively. These are determined by linear regression. The error term ε_{pt} is assumed to be identically, independently and normally distributed with zero mean and variance σ^2. The excess return of the portfolio is denoted by α_p.

The differences between the CAPM and the Fama-French three-factor model are evident. A portfolio manager could be able to significantly outperform the market according to the CAPM by simply building a value and small-capped exposed portfolio. Actually, the portfolio is just more risky and therefore expected to generate more return. As a result, the predictive power of the Fama-French three-factor model is much better and adjusted R^2 increases to 95% in general. Analysis by the CAPM could rate a portfolio manager as outstanding, whereas analysis by the Fama-French three-factor model would rate this portfolio manager as average.

3.4.3 Welch's t-test

A significant positive alpha term indicates that a portfolio outperforms the market if we assume the given model to be correct. Therefore, active portfolio managers try to create a portfolio that produces a significant, positive alpha term such that:

$$\alpha_p > 0 \tag{3.10}$$

Anyhow, in our case, the differences in alphas between portfolios are arguable more relevant, since the portfolios are equally-weighted and the return on the market and other factors are subtracted from the European market (e.g., if the European market outperformed the MSCI Europe index, it is less likely to find a statistically significant positive alpha and more likely to find a statistically significant negative alpha), therefore it is more valuable and more interesting to test the alphas across quintiles.

The Welch's t-test[3] (Welch, 1947) is used to test whether there is any difference between the alphas of two portfolios having possibly unequal variances. Where the two-tailed test null hypothesis for portfolio i and j is given by H_0: $\alpha_i = \alpha_j$, and the alternative hypothesis by H_1: $\alpha_i \neq \alpha_j$. We do reject the null hypothesis when the probability that the observation occurs under this

[3] For this test portfolio returns are assumed to be normally distributed with variance σ^2.

hypothesis turns out to be less than 0.05. Subsequently we conclude that the alphas of the portfolios are statistically significantly different. The Welch's *t*-statistic is calculated as follows:

$$t = \frac{\alpha_i - \alpha_j}{\sqrt{\frac{s_i^2}{N_i} + \frac{s_j^2}{N_j}}} \tag{3.11}$$

Where α, s^2 and N are the alphas, variances and number of observations, respectively. With υ degrees of freedom given by:

$$\upsilon \approx \frac{\left(\frac{s_i^2}{N_i} + \frac{s_j^2}{N_j}\right)^2}{\frac{\left(\frac{s_i^2}{N_i}\right)^2}{N_i - 1} + \frac{\left(\frac{s_j^2}{N_j}\right)^2}{N_j - 1}} \tag{3.12}$$

3.5 Portfolio turnover rate

Transaction costs are not incorporated, because this phenomenon is out of scope of this research. Nevertheless, it is valuable for portfolio managers to evaluate the portfolio turnover rate in order to see how frequently stocks are bought and sold. Everything else being equal, a lower portfolio turnover rate is better because less transaction costs are incurred. The portfolio turnover rate is calculated as:

$$TR_{pt} = \frac{SB_{pt} + SS_{pt}}{V_{pt}} \tag{3.13}$$

Where TR_{pt} is the turnover rate of portfolio p over month t, SB_{pt} is the total value of stocks bought, SS_{pt} is the total value of the sold stocks and V_{pt} is the total value of the portfolio.

3.6 Conclusion

This chapter answers the question 'How can the performance of low-risk strategies be measured?'. It is obvious that the performance of strategies cannot be measured by just one performance indicator.

By using the CAPM, the excess return is calculated with respect to market risk. This model is able to attribute approximately 85% of the portfolio's movements to the movements of the market. On the other hand, there is still 15% variation in observed returns that remain unexplained. The Fama-French three-factor model seems more extensive trying to adjust returns for market, value and size risk and suffice to explain stock returns for approximately 95%. The Sharpe ratio adjusts returns for the variation of price. The Sortino ratio is a modification of the Sharp ratio and adjusts returns for the downside variation of price. A higher Sharpe or Sortino ratio suggests better risk-adjusted returns. These metrics will be applied in order to assess the performance of the low-risk strategies.

In addition, the portfolio turnover rate is determined to see how frequently stocks are bought and sold per month for each strategy.

Chapter 4

Performance of the Low-Risk Portfolios

In Chapter 2 we presented different risk-measures from which low-risk portfolios are constructed. Chapter 3 emphasized how the performance of the portfolios can be assessed. Here we back test low-risk portfolios in order to estimate future performance and give a conclusion with respect to the effectiveness of these strategies. The central question during a back test is 'What should we have done with this strategy at that historical point in time with those index constituents?'. Hereby we aim to answer the sub-question 'Does low-risk outperform high-risk in the SNS Euro Equity Fund universe and which low-risk strategy is associated with best future expected returns?'.

Note that the rows in the tables in this chapter reflect the performance of the strategies individually and consequently try to identify the low-risk anomaly, i.e., is there demonstrable evidence for a low-risk anomaly? While the columns reflect the strategy distinctiveness, i.e., is there detectable difference in performance between strategies?

4.1 Data

The SNS Euro Equity Fund invests in European stocks, most of them are represented in the MSCI Europe Index. Unfortunately, we cannot obtain more than 7 years of historical MSCI Index data. Due to this lack of data, we use an alternative but highly comparable one. This index, the STOXX Europe 600 Index, consists out of 600 European large, mid and small capitalization stocks and shares a large number of stocks with that of the MSCI Europe Index. We proceed doing analysis on this index with the underlying assumption that the behavior and performance equals that of the MSCI Europe Index, subsequently conclusions drawn from these results are also applicable to the MSCI Europe Index. We use STOXX Europe 600 Index data with a time series length of 19 years (1994-2013).

One problem associated with the low-risk strategies is that a stock must have been existing for at least X-days, where X depends on the number of trading days the risk-measure needs to estimate the stock's risk (e.g., 252 trading days for 252-day volatility). Therefore we are forced to exclude stocks which existed shorter than X-days. This approach induces survivorship bias, however, this kind of bias seems inevitable.

All data are downloaded on a monthly or weekly basis from Bloomberg. In total, there have been 1781 constituents of the STOXX600 Europe Index from 1994 to 2013 and 784 constituents of the MSCI Europe Index from 2006 to 2013.

4.2 Robustness

In this study, robustness refers to the stability and consistency of the strategy's excess returns over time. To assess this, the full sample spanning (Panel C) will be split into two subsample periods (Panels A and B), this is done in order to be sure that the results found on the oldest subsample (Panel A) are consistent on a more recent period (Panel B). These results give a proper indication of the performance of the strategies in more recent economic conditions and in the long-run. Both are of great importance for SNS, because it can make low-risk investing more explainable to investors. The periods are defined as follows:

- Panel A: 07-1994 to 05-2006.
- Panel B: 06-2006 to 07-2013.
- Panel C: 07-1994 to 07-2013.

Where we have set Panel B equal to the available historical MSCI Index data to examine the consistency of results, as measured by the STOXX Europe 600 Index, with the MSCI Europe Index for future research purposes of SNS. [4] It is important to note that a considerable period of time of Panel B is taken up by the credit crisis and its severe aftermath.

4.3 Results

In this section we analyze the full sample and subsample periods performance of the low-risk strategies, where each risk-measure serves independently as input for the construction of portfolios. As a result, stocks out of the index are four times classified into quintiles at the

[4] See Appendix C for a comparison between the linear regression results of the CAPM and Fama-French three-factor model on both indices. These results give us the presumption that, as assumed, the behavior and performance of the MSCI Europe Index matches with that of the STOXX600 Europe Index.

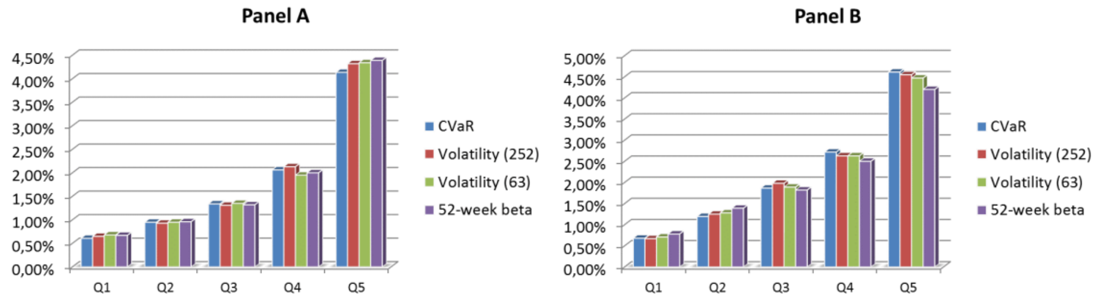
beginning of each month (i.e., 20 portfolios). First, the arithmetic and geometric returns are examined. Second, the Sharpe and Sortino ratios are evaluated, which are ratios that adjust the return of the portfolio for variation in price and downside variation in price, respectively. Third, with the help of regression analysis we estimate the alphas of the portfolios and the exposures to market, size and value risk. Note that we are particularly interested in the relationship between risk and return of the equally-weighted portfolios mutually. Hence it is important to shift attention to differences between portfolios alphas instead of focusing on raw alpha data which compares portfolios performance with respect to the European market, since factor data are derived from Kenneth R. French's website[5].

4.3.1 Absolute return

The absolute return can be expressed in arithmetic and geometric return. The geometric return will always be equal or less than the arithmetic return. In other words, to compensate for a loss requires more than an equal-size profit, in terms of percentage. The spread between the two depends on the dispersion of the distribution of portfolio returns. The higher the variation in portfolio returns, the greater the spread between the annualized arithmetic and geometric returns. The F-test proves that the variances between most quintiles are non-equal, suggesting that using geometric returns instead of arithmetic returns is worth considering.

Figure 4.1 shows the spreads in percentage points between the annualized geometric returns and the annualized arithmetic returns for each portfolio in Panel A and B. The spreads between these returns are certainly not equal among quintiles. To clarify, the average spread for Quintile 1 is 0.68%, while the average spread for Quintile 5 is 4.36%.

Figure 4.1: Spreads between the annualized arithmetic and geometric returns.



[5] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

The value of a portfolio at the end of period *t* is always reinvested in a new rebalanced portfolio for period *t+1*. Therefore, we cannot treat monthly returns as independent events. This fact, together with the huge difference in spreads between arithmetic and geometric returns among quintiles, provides us reasonable evidence in order to choose for geometric returns instead of arithmetic ones to reflect the absolute performance of the portfolios. Henceforth, we will use the term return instead of geometric return, unless mentioned otherwise.

Let's now have a closer look at the return of the risk-based portfolios. Table 4.1 reports these results. The *Q1-Q5* column contains the spreads between the average return of *Q1* portfolios (lowest risk firms) and *Q5* portfolios (highest risk firms). Clearly, for all strategies, each risk-measure is positively associated with future expected returns. In plain English, annualized returns are decreasing for more risky portfolios in the long-run. Next, we examine the properties of the portfolios formed by ranking on each risk-measure individually. In general, the bigger the spread between the average returns of *Q1* and *Q5*, the better the strategy works. In this case, the strategy is capable of selecting stocks with great future expected returns (*Q1*) and rejecting bad future performing stocks (*Q5*).

Table 4.1: Annualized geometric returns by risk-based portfolios for Panel A: July 1994-May 2006 and Panel B: June 2006-July 2013.

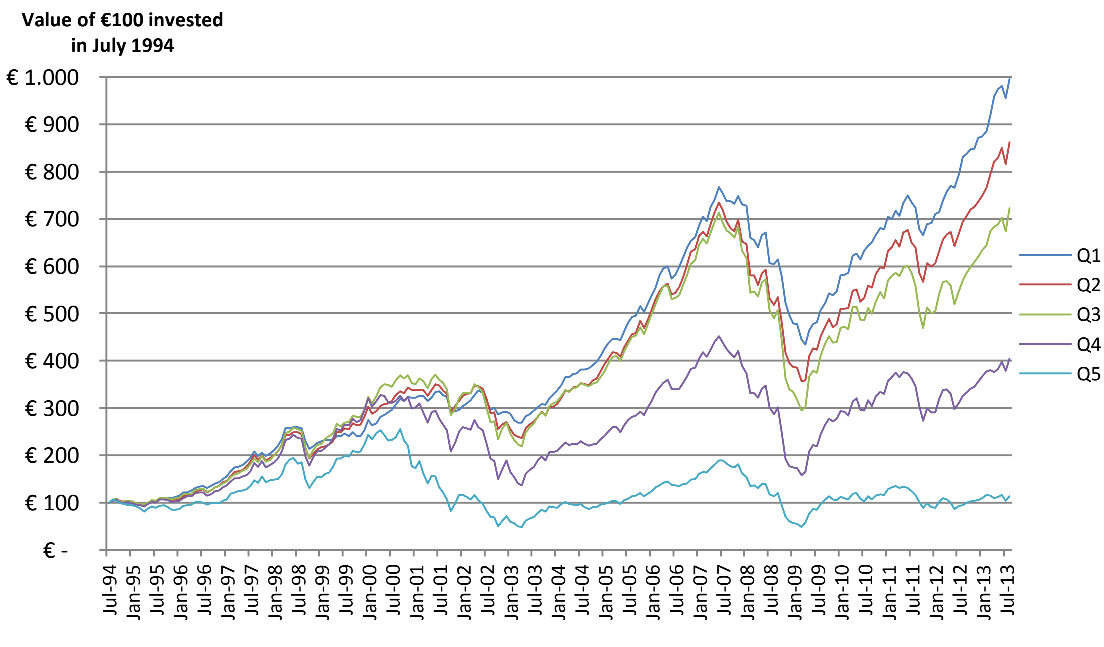
	Panel A					
	Q1 (low-risk)	Q2	Q3	Q4	Q5 (high-risk)	*Q1-Q5 (spread)*
***A.** CVaR$_{10\%}$*	14.81%	14.25%	14.12%	10.40%	2.60%	*12.21%*
***B**. 252-day volatility*	13.81%	14.13%	13.08%	9.56%	5.80%	*8.01%*
***C**. 63-day volatility*	13.60%	13.50%	12.71%	10.54%	5.50%	*8.10%*
***D.** 52-week beta*	12.44%	13.19%	14.19%	11.74%	4.49%	*7.95%*
	Panel B					
	Q1 (low-risk)	Q2	Q3	Q4	Q5 (high-risk)	*Q1-Q5 (spread)*
***A.** CVaR$_{10\%}$*	7.71%	6.57%	4.37%	2.31%	-2.81%	*10.52%*
***B**. 252-day volatility*	7.07%	5.09%	4.48%	3.71%	-2.21%	*9.28%*
***C**. 63-day volatility*	7.43%	5.01%	4.49%	2.73%	-1.95%	*9.38%*
***D.** 52-week beta*	5.09%	3.52%	2.57%	4.58%	2.51%	*2.58%*

For the univariate ranks on CVaR$_{10\%}$, spreads in both Panel A and Panel B are the greatest. Second best are the volatility-based quintiles. Regardless of whether we define risk as 252-day or 63-day volatility, both strategies perform practically equally. These results seem to be robust on Panel B, although the spread for the ranks on $CVaR_{10\%}$ slightly decreases whereas the spreads for the ranks on 252-day and 63-day volatility slightly increase. Clearly, beta-based quintiles lag behind in Panel A and largely fails to exploit the returns anomaly, as found between low-beta and high-beta portfolios over the period 1994 to 2006, in Panel B. As a result, the spread between the lowest and highest beta quintiles drop from 7.95% to 2.58% for Panel A and Panel B, respectively. These results suggest that this beta-strategy highly depends on a certain period of time and therefore its performance is not consistent over time. Another check to confirm that $CVaR_{10\%}$ is the best variable to exploit the low-risk anomaly is to analyze the returns of *Q1* and *Q2*. Again, they outperform the first and second quintiles out of the other risk-based portfolios.

Cumulative total returns

Obviously, absolute returns of the portfolios decrease as risk increases in the long-run. In fact, this implies that the cumulative total returns of the least risky portfolios are higher than that of more risky portfolios. An initial investment of €100 in the least risky portfolio in July 1994, based on the Conditional Value at Risk variable, grows to €1002.30 at the end of July 2013. On the contrary, the investment of €100 in the most risky portfolio is worth €111.33 at the end of July 2013. Figure 4.2 graphically represents the growth of a €100 investment in each $CVaR_{10\%}$-based quintile in July 1994.

Figure 4.2: Cumulative total returns by $CVaR_{10\%}$ quintiles, July 1994-July 2013.



Discriminatory power over time

A long-short position is created to reflect the absolute return of low-risk stocks over high-risk stocks throughout time. This investment strategy is composed by buying low-risk stocks (long position) and selling high-risk stocks (short position).

The results of this investment strategy are checked in different economic conditions. In general, we distinguish between bull and bear periods. A bull period is characterized by optimism and rising stock prices. On the contrary, a bear period is characterized by pessimism and a downward trend as regards stock prices. Mahue et al. (2009, p.2) summarize an algorithm to identify bull and bear phases of Pagan & Sossounov (2003) as follows:

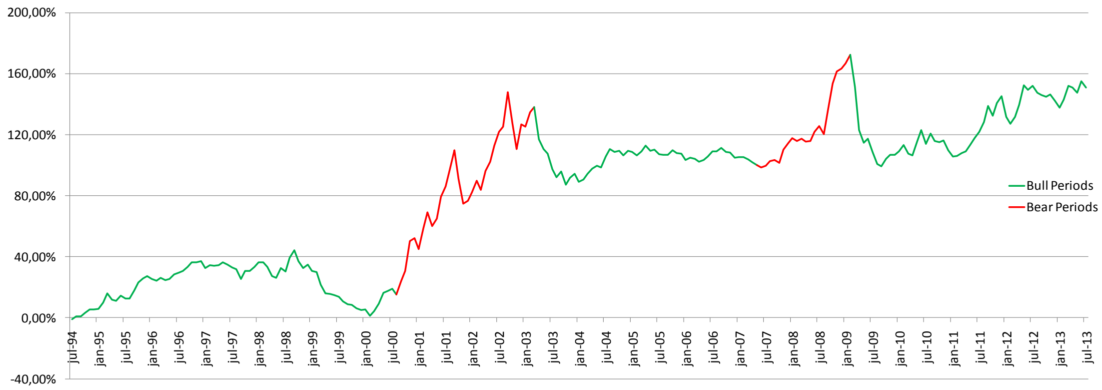
1. Identify the peaks and troughs by using a window of 8 months.
2. Enforce alternation of phases by deleting the lower adjacent of peaks and the higher of adjacent troughs.
3. Eliminate phases less than 4 months unless changes exceed 20%.
4. Eliminate cycles less than 16 months.

Following this algorithm results in:

- Bull periods: 07-1994 to 08-2000; 04-2003 to 05-2007; 03-2009 to 07-2013 (to date).
- Bear periods: 09-2000 to 03-2003; 06-2007 to 02-2009.

The total cumulative geometric return of the long-short portfolio is graphically represented in Figure 4.3, where the bull and bear periods are represented as green and red lines, respectively.

Figure 4.3: Total cumulative return by long-short (Q1-Q5) CVaR$_{10\%}$-based portfolio. July 1994-July 2013.



It is generally assumed that low-risk stocks tend to underperform relative to the market during bull markets, but outperform the market during bearish markets (e.g., CAPM). Remarkably, low-risk stocks do not seem to underperform relative to high-risk stocks during steady bull markets, although we do recognize a strong recovery period of high-risk stocks directly after a severe downward period. Especially, this recovery is recognized in the first half year after the depths of the dot-com bubble and the credit crisis. In the aftermath of the credit crisis, economic unstable conditions propelled the performance of low-risk stocks. Overall, low-risk stocks do not lose value during bull-markets, but gain during bearish markets. As a result, the absolute performance of low-risk stocks seems significantly better than the absolute performance of high-risk stocks in the long-run.

To conclude, these empirical results give us the idea that low-risk investing works. Univariately, the 10% Conditional Value at Risk is the strongest discriminative variable and 52-week beta is the weakest discriminator. Nevertheless, further research is needed to confirm this presumption, which is done in Sections 4.3.2 and 4.3.3.

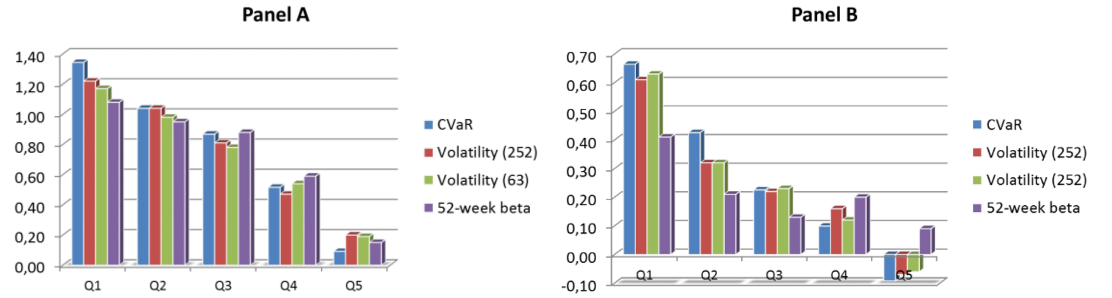
4.3.2 Sharpe & Sortino ratio

The Sharpe ratio uses the return of a portfolio over the risk-free rate. Next, this return is adjusted for the variation of price. Disadvantage of this ratio is that it penalizes upward movements the same as downward movements. The Sortino ratio, introduced by Price & Sortino (1994), is primarily used to assess downside risks. Advantage is that positive gains do not contribute to a more risky investment.

Logically, volatility increases for more risky portfolios. As a result, the Sharpe ratio decreases assuming equal returns. With the previous results in mind, it is not surprising that the Sharpe ratios are decreasing from *Q1* to *Q5*, as visualized by Figure 4.4. This risk-adjusted return of the $CVaR_{10\%}$-based *Q1* portfolio is superior, followed by volatility and beta portfolios.

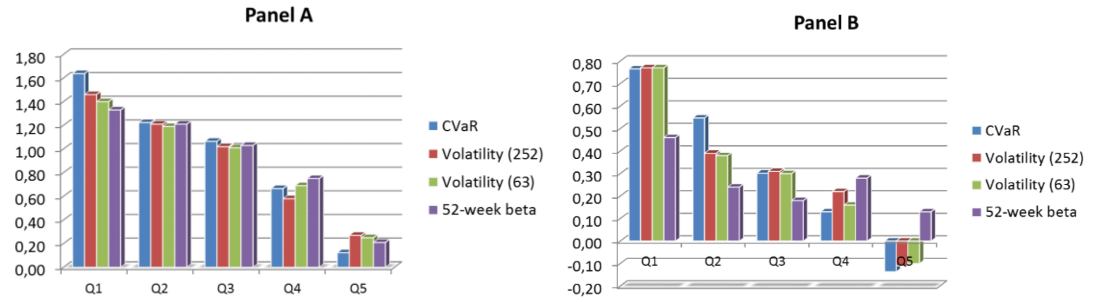
When the Sortino ratio produces similar results as the Sharpe ratio, return distributions are near symmetrical. Assuming equal means, positively skewed return distributions are associated with relative low downside variation in returns, while negatively skewed return distributions are associated with relative large downside variation in return. Thus, positively skewed distributions have a relative high Sortino ratio and negatively skewed distributions have a relative low Sortino ratio relative to the Sharpe ratio.

Figure 4.4: Sharpe ratio.



As Figure 4.5 illustrates, the Sortino ratio shows no dramatic changes with regard to the Sharpe ratio and give a similar picture as the results of the Sharpe ratio, indicating that the return distributions of the portfolios are practically symmetrical. Thus, downside risk-adjusted return of the $CVaR_{10\%}$-based *Q1* portfolio is also superior with respect to the other downside risk-adjusted returns of *Q1* portfolios.

Figure 4.5: Sortino ratio.



To conclude, the Sharpe and Sortino ratio confirm that $CVaR_{10\%}$-based portfolios outperform the others regarding discriminatory power.

4.3.3 Alpha

To discuss the results in more detail we perform regression analysis with time series data. Given the portfolio's time series of returns, alpha is the portion of returns that cannot be attributed to the risks taken, also known as 'excess return' or 'abnormal return' and often seen as the skills of the portfolio manager. Returns can be mainly explained by the exposure to market, size and value risk. If we find an alpha that is significantly different from zero, we do not reject the model, but we assume that this portfolio performs better or worse than the market due to the quantitative stock picking strategy. Anyhow, the differences in alphas between portfolios are arguable more relevant, since the portfolios are equally-weighted and the return on the market and other factors are subtracted from the European market (i.e., if the European market outperformed the index, it is less likely to find a statistically significant positive alpha and more likely to find a statistically significant negative alpha), therefore it is more valuable to test the alphas across quintiles.

The capital asset pricing model assumes market risk as the only source of systematic risk, whereas the Fama-French three-factor model assumes beside market risk, size and value as systematic risk sources.

Note that, for Panel B, weak statistical significance seems inevitable due to relatively large standard errors of mean alphas caused by a relatively small number of observations (Standard Error=$\sigma/\sqrt{n}$). Hence, strong statistical significance is harder to detect in Panel B compared to that of Panel A.

The return on the market, the risk-free rate, the *SMB* factor and the *HML* factor are from Kenneth R. French's website.[6] Notice that these numbers are based on the European market, but given in U.S. Dollar. All our stock return data are in Euro, therefore we have used the FX spot rate to convert the data from his website Kenneth R. French's website to Euro.

Capital asset pricing model

A higher absolute portfolio's return does not necessarily mean that this portfolio outperforms other ones. In a bear market, a defensive portfolio is supposed to produce higher absolute returns than an aggressive portfolio. In this section we adjust the portfolio's return for exposure to market risk.

In Table 4.2, for the univariate ranks on $CVaR_{10\%}$, volatility and beta, alphas are decreasing as riskiness increases. Logically, the exposure to market risk (beta) is increasing for more risky portfolios. In Panel A, the alphas for all first and second quintiles are statistically significant from zero, indicating that these portfolios outperform the (European) market. On the other hand, all Quintile 5 portfolios statistically significant underperform relative to the market. In Panel B, for the ranks on $CVaR_{10\%}$, alpha of Quintiles 1 and 2 are still statistically significant larger than zero, whereas the ranks on volatility only creates a significant alpha for Quintile 1. However, we cannot prove that any of the high-risk portfolios statistically significant underperform relative to the market. In contrast, the decrease of the alphas for *Q1* to *Q5* for the

[6] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

ranks on beta seems negligible, again confirming that the performance of the beta strategy appears unpredictable.

Table 4.2: Portfolios alphas estimated by the CAPM for Panel A: July 1994-May 2006 and Panel B: June 2006-July 2013.

		Panel A				
		Q1 (low-risk)	Q2	Q3	Q4	Q5 (high-risk)
A. *$CVaR_{10\%}$*	Alpha	6.44%**	4.44%**	3.17%*	-1.82%	-11.02%**
	Beta	*0.62*	*0.83*	*1.01*	*1.25*	*1.67*
B. *252-day volatility*	Alpha	5.40%**	4.44%**	2.38%	-2.69%	-7.86%*
	Beta	*0.63*	*0.81*	*0.98*	*1.26*	*1.69*
C. *63-day volatility*	Alpha	5.09%**	3.72%*	1.80%	-1.50%	-8.05%*
	Beta	*0.66*	*0.95*	*1.16*	*1.35*	*1.72*
D. *52-week beta*	Alpha	4.33%*	3.59%*	3.43%*	-0.42%	-9.67%**
	Beta	*0.59*	*0.81*	*0.99*	*1.24*	*1.76*
		Panel B				
		Q1 (low-risk)	Q2	Q3	Q4	Q5 (high-risk)
A. *$CVaR_{10\%}$*	Alpha	5.40%**	4.03%**	1.89%	0.09%	-4.11%
	Beta	*0.64*	*0.92*	*1.15*	*1.37*	*1.74*
B. *252-day volatility*	Alpha	4.76%*	2.56%	2.02%	1.48%	-3.52%
	Beta	*0.64*	*0.94*	*1.19*	*1.34*	*1.72*
C. *63-day volatility*	Alpha	5.09%**	2.47%	2.01%	0.49%	-3.33%
	Beta	*0.66*	*0.95*	*1.16*	*1.35*	*1.72*
D. *52-week beta*	Alpha	2.74%	1.02%	0.10%	2.23%	0.95%
	Beta	*0.69*	*0.98*	*1.13*	*1.34*	*1.69*

Note: ** and * denotes statistical significance at the 1% and at the 5% level, respectively. H_0: $\alpha_p=0$ and H_1: $\alpha_p \neq 0$.

The t-values of Welch's test[7] show that the alphas of *Q1* and *Q5* statistically significant differ within the univariate ranks on $CVaR_{10\%}$ for both Panel A and Panel B. For the ranks on 252-day and 63-day volatility, alphas of *Q1* and *Q5* for Panel A are statistically significant. Nonetheless, in Panel B, no statistical significance between portfolios alphas can be confirmed, but this is mainly due to relatively large standard errors of the mean alphas.

Although the most risky portfolios (*Q5*) show increasing alphas for Panel B compared to those of Panel A, the ranks on $CVaR_{10\%}$, 252- and 63-day volatility still exhibit considerable monotonically decreasing alphas for Quintile 1 to Quintile 5. On the contrary, beta-based quintiles seem to perform unpredictable and the results found on Panel A cannot be confirmed on Panel B and thus performance of this strategy lacks consistency over time. These results suggest that the strategies tested on Panel A are still effective and still perform well on Panel

[7] For results see Appendix A.2.

B, with exception of the beta strategy. Hence, we also perform regression analysis over the full sample period (Panel C).

Table 4.3 illustrates that the $CVaR_{10\%}$ strategy is still the most effective method to select stocks with best future expected returns and that the volatility and beta methods lack behind. Note that the differences between 252-day volatility and 63-day volatility are very small and both methods have nearly equal realized excess returns. Beta is much less effective than the other strategies, besides that it also lacks robustness on Panel B. Clearly, the t-values for the alphas means between low-risk and high-risk portfolios sharply increase within these strategies, as can be seen in Appendix A.3., confirming that the alphas between *Q1* and *Q5* are highly statistically significant different for Panel C.

Table 4.3: Portfolios alphas estimated by the CAPM for Panel C: July 1994-July 2013.

		Panel C				
		Q1 (low-risk)	Q2	Q3	Q4	Q5 (high-risk)
A. *$CvaR_{10\%}$*	Alpha	6.01%**	4.15%**	2.48%*	-1.28%	-8.52%**
	Beta	0.63	0.87	1.07	1.30	1.69
B. *252-day volatility*	Alpha	5.14%**	3.55%**	1.93%	-1.24%	-6.26%*
	Beta	0.63	0.86	1.06	1.29	1.70
C. *63-day volatility*	Alpha	5.07%**	3.06%**	1.66%	-0.94%	-6.32%*
	Beta	0.65	0.88	1.07	1.27	1.70
D. *52-week beta*	Alpha	3.59%*	2.35%	1.96%	0.43%	-5.54%*
	Beta	0.63	0.88	1.05	1.28	*1.73*

Note: ** and * denotes statistical significance at the 1% and at the 5% level, respectively. H_0: $\alpha_p=0$ and H_1: $\alpha_p\neq0$.

To conclude, the results for the univariate rankings on $CVaR_{10\%}$, 252-day and 63-day volatility for Panel A and Panel B suggest that the performance of these methods are consistent across time, alphas are decreasing as risk increases and this decrease (*Q1-Q5*) is statically significant, suggesting that low-risk investing works (if we assume the Capital Asset Pricing Model to be correct), regardless of whether we define risk as $CVaR_{10\%}$, 252-day or 63-day volatility. However, ranking on $CVaR_{10\%}$ seems superior to the ranks on volatility with regard to the selection of stocks with greatest future expected returns. The performance of the ranks on beta looks unstable across time and for that reason lacks robustness. To summarize, after correcting returns according to the capital asset pricing model, we can conclude that there exists a low-risk anomaly and that the univariate rank on $CVaR_{10\%}$ seems the best method to reveal this phenomenon.

Fama-French three-factor model

Fama & French (1993) argue that excess return can be explained by exposure to value and size risk. By construction, equal-weighted portfolios have a slight tilt to size, because small companies are overweight for a relative large proportion with respect to the capitalization-weighted market return. Equivalently, large companies are underweight with respect to the capitalization-weighted market return. Another reason to correct for size risk, intuitively, is the sensitivity of small companies to environmental factors. For that reason stock prices of small companies tend to be more volatile than those of large companies and therefore it seems plausible that high-risk portfolios are more exposed to small sized companies than low-risk portfolios. For those reasons it makes sense to correct the return of equal-weighted portfolios for the (excess) exposure to size risk. Beforehand, it seems a guess to make any statements about the exposure of risk-based portfolios to the value factor.

Table 4.4 shows the results of the time series regression analysis on the Fama-French three-factor model. For both periods, most of the portfolios are positively exposed to the size factor partly caused by construction. The exposure to this size factor is positively related to risk (i.e., on average, small capitalized stocks are more risky). In Panel A, for all portfolios, the exposure to the value factor is decreasing across quintiles. For Panel B, however, this tilt toward value has been reversed.

For Panel A, alphas for the fourth and fifth quintiles statistically significant underperform the market for the ranks on $CVaR_{10\%}$, whereas for the ranks on volatility only quintile four statistically significant underperforms the market. No quintiles of the beta strategy out- or underperform the market. $CVaR_{10\%}$ is the only method that is able to produce monotonically decreasing alphas along its quintiles.

For Panel B, average alphas shifted upwards and the severe underperforms of *Q5* portfolios has disappeared (this extreme underperformance is probably caused by the dot-com bubble burst). $CVaR_{10\%}$ Quintiles 1 and 2 outperform the market, while only Quintile 1 of the ranks on volatility outperforms the market. Again no excess return can be confirmed for the ranks on beta with respect to the market return and the quintiles of this strategy exhibit relatively flat average alphas. Alphas are monotonically decreasing as risk increases for ranks on $CVaR_{10\%}$. Ranking on volatility induces no consistent relation in alpha for *Q2* to *Q4*, but the alpha spread between *Q1* and *Q5* is considerably large. No clear downward trend for the excess return can be confirmed for *Q1* to *Q5* of the beta strategy. That is to say, the underperformance of 20%

highest beta stocks with respect to the 20% lowest beta stocks, from an absolute return point of view, is absorbed by the exposure to market risk and value and small capitalized stocks.

Table 4.4: Portfolios alphas estimated by the Fama-French three-factor model for Panel A: July 1994-May 2006 and Panel B: June 2006-July 2013.

		Panel A				
		Q1 (low-risk)	Q2	Q3	Q4	Q5 (high-risk)
A. *$CVaR_{10\%}$*	Alpha	**2.90%**	**0.48%**	**-0.03%**	**-4.31%***	**-11.71%****
	Beta	*0.69*	*0.92*	*1.09*	*1.33*	*1.79*
	SMB	*0.16*	*0.22*	*0.20*	*0.27*	*0.60*
	HML	*0.33*	*0.36*	*0.29*	*0.21*	*-0.02*
B*. 252-day volatility*	Alpha	**1.05%**	**0.53%**	**-1.95%**	**-6.04%****	**-5.83%**
	Beta	*0.73*	*0.88*	*1.06*	*1.38*	*1.78*
	SMB	*0.23*	*0.11*	*0.15*	*0.38*	*0.57*
	HML	*0.40*	*0.37*	*0.40*	*0.28*	*-0.28*
C*. 63-day volatility*	Alpha	**0.46%**	**-0.34%**	**-1.96%**	**-4.65%****	**-5.45%**
	Beta	*0.75*	*0.91*	*1.09*	*1.30*	*1.77*
	SMB	*0.24*	*0.16*	*0.17*	*0.22*	*0.64*
	HML	*0.42*	*0.38*	*0.35*	*0.28*	*-0.35*
D. *52-week beta*	Alpha	**-1.49%**	**-1.63%**	**-0.87%**	**-2.14%**	**-5.57%**
	Beta	*0.73*	*0.92*	*1.09*	*1.31*	*1.77*
	SMB	*0.35*	*0.24*	*0.25*	*0.27*	*0.33*
	HML	*0.53*	*0.48*	*0.39*	*0.13*	*-0.45*
		Panel B				
		Q1 (low-risk)	Q2	Q3	Q4	Q5 (high-risk)
A. *$CVaR_{10\%}$*	Alpha	**4.35%***	**3.77%****	**2.37%**	**1.11%**	**-1.56%**
	Beta	*0.74*	*0.96*	*1.12*	*1.32*	*1.55*
	SMB	*-0.05*	*0.19*	*0.27*	*0.53*	*0.66*
	HML	*-0.35*	*-0.14*	*0.09*	*0.21*	*0.70*
B*. 252-day volatility*	Alpha	**3.65%***	**2.33%**	**2.60%**	**2.46%**	**-0.99%**
	Beta	*0.74*	*0.97*	*1.15*	*1.29*	*1.53*
	SMB	*-0.06*	*0.17*	*0.30*	*0.56*	*0.61*
	HML	*-0.37*	*-0.13*	*0.12*	*0.19*	*0.71*
C*. 63-day volatility*	Alpha	**4.08%***	**2.22%**	**2.31%**	**1.72%**	**-0.86%**
	Beta	*0.76*	*0.99*	*1.15*	*1.27*	*1.53*
	SMB	*0.00*	*0.22*	*0.32*	*0.48*	*0.58*
	HML	*-0.35*	*-0.15*	*0.02*	*0.29*	*0.70*
D. *52-week beta*	Alpha	**1.74%**	**0.88%**	**0.64%**	**3.28%**	**3.22%**
	Beta	*0.79*	*1.01*	*1.11*	*1.26*	*1.51*
	SMB	*0.11*	*0.27*	*0.39*	*0.33*	*0.49*
	HML	*-0.38*	*-0.12*	*0.08*	*0.27*	*0.66*

Note: ** and * denotes statistical significance at the 1% and at the 5% level, respectively. H_0: $\alpha_p=0$ and H_1: $\alpha_p\neq0$.

Although alpha spreads between *Q1* and *Q5* have fallen over the last period, excess returns exhibit similar patterns for both periods for the ranks on $CVaR_{10\%}$ and volatility. These results are fairly similar to the regression results of the CAPM. Yet again, ranks on beta fail to perform consistent throughout time, therefore we cannot conclude that low-beta stocks outperform high-beta stocks. Next, we also perform a time series regression analysis over the entire 19-year period for all strategies, Table 4.5 reports these results.

For the total period, the differences between the alphas of *Q1* and *Q5* within the univariate ranks on $CVaR_{10\%}$ and volatility are highly statistically significant, indicating that low-risk still outperforms high-risk after analysis by the Fama-French three-factor model. These differences between low-risk and high-risk portfolios are 14.77% and 10.62% on average for the ranks on $CVaR_{10\%}$ and volatility, respectively. Ranking on beta induces a relative weak statistical significance compared to the ranking on $CVaR_{10\%}$ and volatility. Results of the Welch's t-test can be found in Appendix A.3.

Table 4.5: Portfolios alphas estimated by the Fama-French three-factor model for Panel C: July 1994-July 2013.

		Panel C				
		Q1 (low-risk)	Q2	Q3	Q4	Q5 (high-risk)
A. *$CVaR_{10\%}$*	Alpha	**5.50%****	**3.22%****	**1.39%**	**-2.23%**	**-9.27%****
	Beta	0.63	0.88	1.09	1.33	1.76
	SMB	0.07	0.19	0.22	0.34	0.59
	HML	0.11	0.19	0.22	0.19	0.13
B. *252-day volatility*	Alpha	**4.50%****	**2.49%***	**0.39%**	**-2.25%**	**-6.27%****
	Beta	0.64	0.87	1.07	1.34	1.78
	SMB	0.10	0.12	0.19	0.41	0.58
	HML	0.13	0.22	0.32	0.19	-0.04
C. *63-day volatility*	Alpha	**4.34%****	**2.02%***	**0.48%**	**-2.19%**	**-6.13%***
	Beta	0.66	0.89	1.08	1.29	1.78
	SMB	0.13	0.17	0.21	0.29	0.62
	HML	0.15	0.22	0.24	0.26	-0.08
D. *52-week beta*	Alpha	**2.47%**	**0.92%**	**0.57%**	**-0.34%**	**-4.82%***
	Beta	*0.65*	*0.89*	*1.07*	*1.31*	*1.78*
	SMB	*0.24*	*0.24*	*0.28*	*0.28*	*0.37*
	HML	*0.23*	*0.30*	*0.28*	*0.15*	*-0.18*

Note: ** and * denotes statistical significance at the 1% and at the 5% level, respectively. H_0: α_p=0 and H_1: $\alpha_p \neq 0$.

We can draw similar conclusions as in the last section of the CAPM analysis. Ranking on $CVaR_{10\%}$ seems to outperform the ranks on volatility. We cannot draw any conclusions about the difference in performance between the ranks on 252-day volatility and ranks on 63-day volatility. In contrast, ranking on beta induces no consistent relation in alpha.

4.4 Conclusion

This chapter gives an answer to the question 'Does low-risk outperform high-risk in the SNS Euro Equity Fund universe and which low-risk strategy is associated with best future expected returns?'. We can draw two conclusions from the univariate tests performed in this chapter.

First, the empirical evidence that low-risk leads to strong share price performance is convincing. For low-risk portfolios, the absolute returns, the Sharpe ratios, the Sortino ratios and the alphas estimated by the capital asset pricing model and the Fama-French three-factor model regression analysis consistently outperform high-risk portfolios in the long-run, regardless of whether we define risk as the Conditional Value at Risk or historical volatility.

Second, the Conditional Value at Risk is the best univariate variable with regard to the selection of stocks with best future expected returns and seems to exhibit the greatest discriminatory power followed by the 252-day and 63-day volatility. Comparing the latter two, both perform practically equally but we consider the ranks on 252-day volatility as the better method, because its turnover rate is significantly lower than that of the ranks on 63-day volatility[8]. Our analysis indicates that beta, however, fails to reveal this low-risk anomaly.

[8]For the Turnover Rates see Table A.2 and Table A.3 in Appendix A.

Chapter 5

Low-Risk as an Extension of the Current Quantitative Investment Strategy

The previous chapter determined pure low-risk performance and these results indicate that there exists a low-risk anomaly. However, 'Does low-risk add value as extension of the current quantitative investment strategy?'. That is the sub-question to be answered in this chapter. The current quantitative factors are the momentum (combination of earnings and price momentum) and the normalized value factor. But are these factors, unknowingly, selecting low-risk stocks and consequently benefit from this anomaly?

In this chapter we build a three-factor performance attribution model to capture performance generated by the current quantitative investment strategy of SNS, using the CAPM model plus the additional momentum and normalized value factor. It may be interpreted as a performance attribution model, where the coefficients indicate the proportion of the returns of the low-risk strategy ($CVaR_{10\%}$-based) attributable to these two quantitative investment strategies.

First, in order to achieve this, the monthly excess return of investing in cheap stocks relative to expensive ones (normalized value factor) and the monthly excess return of investing in the tendency of stocks to exhibit persistence in their relative performance (momentum factor) are estimated.

Second, we perform regression analysis on this model to assign the proportion of returns of $CVaR_{10\%}$-based portfolios attributable to the current quantitative investment strategy of SNS.

Finally, we draw a conclusion by answering the fourth sub-question.

5.1 Normalized value factor

SNS develops normalized forward-looking price to earnings ratios per sub-sectors, where a stock's ratio is compared with the average ratio in its sub-sector. To be able to determine the proportion of returns attributable to this factor, sub-sectors are mimicked according to the Bloomberg Industry Classification System. The forward-looking price to earnings ratios are collected from Thomson Reuters Institutional Brokers' Estimate System (IBES). SNS determines this ratio in a few steps:

[Confidential]

We construct the normalized value factor (*NVF*) as the equal-weight average return of the 50% highest normalized earnings yield (*NEY*) stocks minus the equal-weight average return of the 50% lowest normalized earnings yield stocks. This factor determines the performance of value stocks relative to growth stocks on a monthly basis.

5.2 Momentum factor

Momentum is defined by Berger et al. (2009, p.1) as "The tendency of investments to exhibit persistence in their relative performance. Investments that have performed relatively well, continue to perform relatively well; those that have performed relatively poorly, continue to perform relatively poorly". Jagadeesh & Titman (1993) concluded that past winners have the tendency to outperform past losers over an intermediate horizon. SNS distinguishes between price and earnings momentum.

Price momentum is the tendency to exhibit persistence in their relative price performance.

[Confidential]

Earnings momentum is the tendency to exhibit persistence in their relative profits (e.g., profit warnings are often followed by more profit warnings). A stock's earnings momentum can be mathematically represented as:

[Confidential]

5.3 Correlations between factors

Correlation between factors is checked for two reasons. First, correlation refers to a relationship between two variables. In our case, it refers to the relationship between the time series excess returns of factors. Two highly correlated factors indicate that it is likely that both factors are mainly profiting from the same stocks. In such a case, using two different factors is redundant. Second, in a multiple regression model, multicollinearity is the event in which explanatory variables are highly correlated. Although it does not affect the estimation of alpha, multicollinearity might cause inaccurate estimates of factor loadings. To examine correctness of factor loadings estimations of the *NVF* and *MOM* factor in the three-factor performance attribution model, correlation between time series excess return of these factors is checked. Table 5.1 reports the performance of factors and the correlations between their excess returns.

Table 5.1: Correlations between time series excess returns by factors.

Factor	Annualized Excess Return	Standard error	Panel A Correlations		
			RM-RF	NVF	MOM
RM-RF	8.37%	4.45%	1.00		
NVF	4.92%	1.49%	-0.16	1.00	
MOM	7.93%	4.07%	-0.35	-0.27	1.00
			Panel B Correlations		
			RM-RF	NVF	MOM
RM-RF	2.64%	6.11%	1.00		
NVF	-1.24%	1.97%	0.63	1.00	
MOM	4.88%	5.29%	-0.56	-0.74	1.00
			Panel C Correlations		
			RM-RF	NVF	MOM
RM-RF	6.22%	3.60%	1.00		
NVF	2.61%	1.20%	0.15	1.00	
MOM	6.79%	3.22%	-0.43	-0.44	1.00

The low correlations between the market factor *RM-RF*, the normalized value factor *NVF* and the momentum factor *MOM* in Panels A and C imply that multicollinearity will not substantially affect the estimates of the factor loadings. However, for Panel B, we have to be careful with the interpretations of factor loadings due to rather large negative correlation between the *MOM* and *NVF* factor. Note that the positive excess return of the normalized value factor for the 1994-2006 period has been reversed in Panel B.

5.4 A three-factor performance attribution model

This model quantifies the exposure of the $CVaR_{10\%}$-based portfolios to the current quantitative investment strategy of SNS. We employ this model to examine whether a low-risk factor can be used to improve their current strategy. For example, suppose that portfolio X has a statistically significant larger alpha than portfolio Y after performing regression analysis on this model. This indicates that the difference in alpha cannot be explained by the exposure to market-risk, the normalized value factor and the momentum factor. That is to say, the outperformance of portfolio X with respect to portfolio Y cannot be explained by the current quantitative investment strategy of SNS. Therefore the portfolio construction method used to build these portfolios add value in addition to their current strategy.

The three-factor performance attribution model equation can be mathematically represented as:

$$R_{pt} = \alpha_p + RF_t + \beta_p * (RM_t - RF_t) + V_p * NVF_t + M_p * MOM_t + \varepsilon_{pt}, \tag{5.1}$$

where $t = \{1, 2, \ldots, T\}$.

Where R_{pt} is the return of portfolio p from month $t\text{-}1$ to t, RF_t is the risk-free rate and RM_t is the return of the market. B_p, V_p and M_p are the exposures to market risk, the normalized value factor and the momentum factor, respectively. These are estimated by linear regressions. *NVF* is the excess return of cheap stocks relative to expensive ones, *MOM* is the excess return of past winners with improved prospects relative to past losers with worsened outlook, explanation of the normalized value factor and the momentum factor can be found in Sections 5.1 and 5.2, respectively. The excess return of the portfolio is denoted by α_{pt} and T are the number of months.

Regression results

Table 5.2 shows the results of the regression analysis for the risk-based portfolios ranked on $CVaR_{10\%}$ using the three-factor performance attribution model. As before, Quintile 1 is the proxy for the least risky portfolio and Quintile 5 is the proxy for the most risky one.

Table 5.2: Three-factor performance attribution model regression analysis for Panel A: July 1994-May 2006 and Panel B: June 2006-July 2013.

		Panel A				
		Q1 (low-risk)	Q2	Q3	Q4	Q5 (high-risk)
CVaR_{10%}	Alpha	**1.46%**	**0.31%**	**1.10%**	**-1.61%**	**-2.81%**
	Beta	*0.71*	*0.90*	*1.03*	*1.21*	*1.43*
	NVF	*0.51*	*0.53*	*0.45*	*0.37*	*-0.15*
	MOM	*0.21*	*0.12*	*-0.03*	*-0.21*	*-0.69*
		Panel B				
		Q1 (low-risk)	Q2	Q3	Q4	Q5 (high-risk)
CVaR_{10%}	Alpha	**4.01%**	**3.72%**	**2.88%**	**1.82%**	**0.28%**
	Beta	*0.78*	*0.96*	*1.07*	*1.20*	*1.36*
	NVF	*-0.44*	*-0.19*	*-0.05*	*0.52*	*0.67*
	MOM	*0.10*	*0.00*	*-0.17*	*-0.13*	*-0.52*

Note: ** and * denotes statistical significance at the 1% and at the 5% level, respectively. H_0: $\alpha_p=0$ and H_1: $\alpha_p \neq 0$.

In Panel A, the exposure to the *NVF* factor is generally decreasing across quintiles. For Panel B, however, this tilt toward value seems to be reversed. Logically, the behavior of the *NVF* factor partially matches the behavior of the *HML* factor as defined by Fama & French (1993). By definition, outperforming stocks have positive exposure to the price momentum factor. Since low-risk stocks have consistently outperformed the market, the exposure to (price) momentum is expected to be positive. This is confirmed by decreasing exposures for this factor from *Q1* to *Q5* for all periods.

The results of the regressions demonstrate decreasing alphas through *Q1* to *Q5* for Panel A and Panel B. The excess return spreads between *Q1* and *Q5* look stable over both periods, indicating that the low-risk factor consistently adds value to the current quantitative investment strategy over time. Therefore, adding a low-risk factor seems positively associated with future expected returns as extension to the current quantitative investment strategy.

That being said, we also perform time series regressions for Panel C to reduce the standard error of the mean alphas in order to increase the probability of detecting statistical significance between portfolios alphas. As expected, alphas are still (monotonically) decreasing through *Q1* to *Q5* with a yearly alpha spread averaging 5.83% as can be derived from Table 5.3. We can conclude, with respect to Panel C, that the excess returns of *Q1* and *Q2* statistically significant differ from that of *Q5* with t-values of 2.46 and 2.05, respectively (see Appendix A.4). This statistically significant difference also exists between the alphas of *Q1* and *Q4*.

Table 5.3: Three-factor performance attribution model regression analysis for Panel C: July 1994-July 2013.

		Panel C				
		Q1 (low-risk)	Q2	Q3	Q4	Q5 (high-risk)
CVaR$_{10\%}$	Alpha	**3.55%****	**2.42%***	**2.20%***	**-0.32%**	**-2.28%**
	Beta	*0.69*	*0.89*	*1.03*	*1.21*	*1.44*
	NVF	*0.27*	*0.36*	*0.36*	*0.36*	*-0.02*
	MOM	*0.20*	*0.10*	*-0.06*	*-0.20*	*-0.67*

Note: ** and * denotes statistical significance at the 1% and at the 5% level, respectively. H_0: $\alpha_p=0$ and H_1: $\alpha_p \neq 0$.

5.5 Conclusion

This chapter gives an answer to the question 'Does low-risk add value as extension of the current quantitative investment strategy?'. Based on the data of the past 19 years, we have shown that the excess return of low-risk portfolios cannot be fully explained by the current quantitative investment strategy. For the risk-based ($CVaR_{10\%}$) portfolios, the mean alphas of Quintile 1 and Quintile 2 statistically significantly differ from the mean alpha of Quintile 5, illustrating that adding a low-risk factor to the current strategy is worth considering. These findings seem to be consistent over time.

To conclude, the empirical evidence that low-risk leads to stronger share price performance additionally to the current quantitative investment strategy of SNS is convincing. Thus, low-risk seems to add value as extension of the current quantitative invest strategy.

Chapter 6

Interaction Effects between Factors

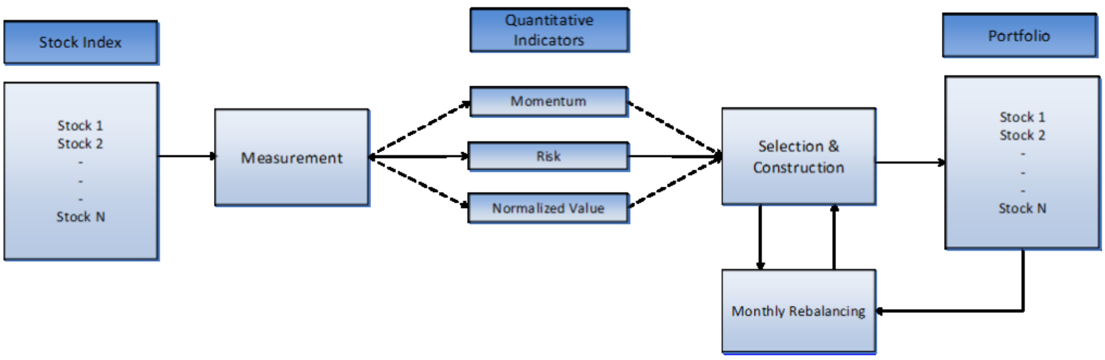
The previous chapters showed that the empirical evidence for the existence of a low-risk anomaly is plausible. In addition, this anomaly seems to be able to strengthen future expected returns in addition to the current quantitative investment strategy. The main goal of this chapter is to examine whether the performance of the normalized value factor or momentum factor differs depending upon the level of risk.

Firstly, we expand the simplified investment framework out of Section 2.2. The second section details how the low-risk and the normalized value factor interact. The third one discusses the interaction between low-risk and the momentum strategy. Finally, the sub-question 'What is the importance of using distinct univariate quantitative investment strategies, like low-risk, momentum and value?' will be answered.

6.1 Portfolio formation methodology: multivariate strategies

We introduced a simplified investment framework in Chapter 2 in order to be able to reproduce risk-based portfolios and subsequently estimate their performance. This framework is expanded in order to back test bi- or multivariate instead of univariate risk-based strategies. This expanded framework is visualized in Figure 6.1.

Figure 6.1: Simplified framework for multivariate strategies.



Here, the primary goal is to examine whether the performance of the normalized value factor or momentum factor differs depending upon the level of risk. To examine these interactions, the momentum, risk and normalized value indicators for all stocks out of the stock index are determined. This *measurement* occurs on a monthly basis.

Thereafter, stocks are ranked independently on each strategy into quartiles or quadrants[9], where the lower quartiles or quadrants are proxies for the stocks with best future expected returns and where the upper quartiles or quadrants are proxies for stocks with the worst future expected returns. Subsequently, we take the intersection of these quartiles or quadrants to form 16 equal-weighted portfolios.

Disadvantage of ranking on two variables is that it requires allocating stocks to a larger number of portfolios, this will reduce diversification inside these portfolios and increases the volatility of their profits. Hence, statistical significance will be harder to detect. To limit the effect of this phenomenon, we decided to form quartiles (16 portfolios) instead of quintiles (25 portfolios) for the risk and the normalized value factor.

For more information or explanation about this framework we refer to Section 2.2.

6.2 Low-risk and normalized value

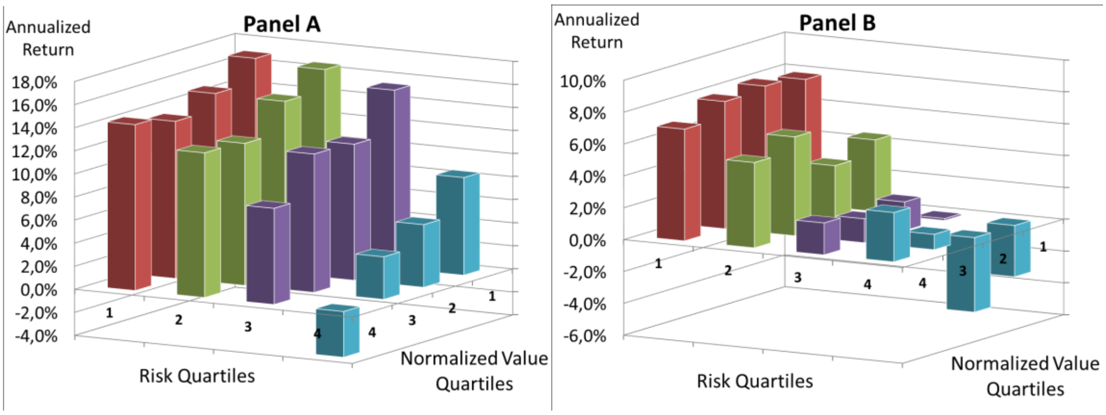
Section 5.3 shows that return of investing in cheap stocks relative to expensive ones has been reversed from positive excess return towards negative excess return. Fama and French (1993) argue that the excess return of investing in cheap stocks is attributable to a more risky investment. Since Panel B is mainly taken up by the credit crisis, it makes sense that expensive (less risky) stocks outperform cheap stocks (more risky). The results of ranking on risk and normalized value at once can be found in Appendix B and are visualized in Figure 6.2.

Ranking on risk while keeping value constant produces a considerable descent in average returns among expensive stocks for Panel A. This decent is much weaker, but still substantial, among cheap stocks. However, note that these relationships (spreads) have been reversed for Panel B compared to Panel A. Although the effectiveness of the univariate normalized value factor with regard to the selection of stocks with good future expected returns is debatable,

[9] By construction, the momentum strategy of SNS is divided into quadrants, see Section 1.2.2.

stocks with lower risk always have higher expected future returns among the normalized value factor, no matter what the performance of this strategy is.

Figure 6.2: Annualized returns for ranking on both risk and normalized value.



In both periods, varying normalized value quartiles within the lowest risk stocks produces very small spreads between cheap and expensive stocks of 2.62% and 0.85% for Panel A and Panel B, respectively. This indicates that the normalized value factor seems redundant for the lowest risk stocks in order to strengthen future expected returns. Clearly, the normalized value factor becomes far more effective among more risky assets. Spreads between cheap and expensive high-risk (*Q4*) stocks increase to 12.42% for Panel A and to -6.33% for Panel B.

To conclude, the empirical evidence that risk can serve as a valuable improvement to the univariate normalized value strategy is rather convincing. However, the effectiveness of the normalized value factor itself is debatable.

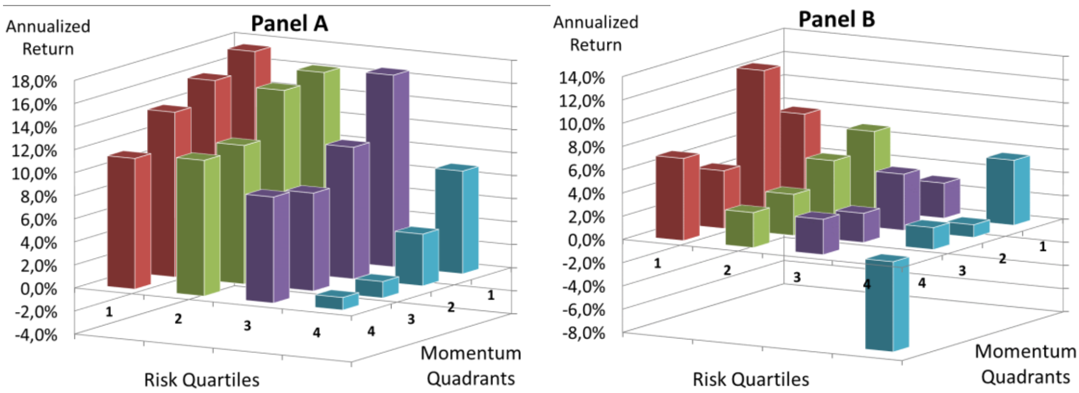
6.3 Low-risk and momentum

Note that the momentum factor is based on the price and earnings momentum. As a reminder, stocks with a relative positive price and relative positive earnings momentum belong to Quadrant 1, stocks assigned to Quadrant 2 have relative negative price momentums and relative positive earnings momentums. On the contrary, Quadrant 3 stocks have a relative positive price momentum and a relative negative earnings momentum. In Quadrant 4, earnings expectations and price momentum are both below average.

With that in mind, let's discuss the results of the ranks on risk and momentum which can be found in Appendix B and are visualized in Figure 6.3. For Panel A, if we keep momentum constant, decrease of returns are quite small within the first three risk quartiles. Across the most risky stocks, however, decrease of returns is much stronger, suggesting that the highest risk stocks in the ranks on momentum are less profitable. The spreads between risk *Q3* and *Q4* across the momentum quadrants are on average 7.46%. The improvements within the ranks on momentum for risk *Q1* to *Q3* stocks seem to be less effective.

Annualized returns are generally decreasing while varying momentum from Quadrant 1 to Quadrant 4 within a pre-set risk quartile. The spreads between Quadrant 1 and Quadrant 4 of the momentum strategy, while keeping risk constant, are 6.15%, 4.49%, 7.44% and 7.83% for risk *Q1* to *Q4*, respectively. Thus, the risk strategy produces better returns among strong momentums, indicating that stocks with a good momentum are generally more profitable within the sort on risk.

Figure 6.3: Annualized returns for ranking on both risk and momentum.



For Panel B, however, the spreads between high-risk (*Q4*) and low-risk (*Q1*) stock returns within the ranks on momentum seem arbitrary. Within momentum Quadrant 1 and 3, adding a risk factor produces very small spreads in average returns between high-risk and low-risk stocks, whereas the average annualized return spreads between high-risk and low-risk stocks within momentum Quadrant 2 and 4 are 11.45% and 14.82%, respectively. It is important to note that a considerable period of time of Panel B is taken up by the credit crisis and its severe aftermath and that the price momentum is positively related with the risk factor in times of uncertainty. Essentially, in that period, price momentum selects stocks with the same characteristics as the risk factor and therefore can be seen as an alternative risk-measure. On the contrary, in extremely rising stock markets, the price momentum is negatively related to the risk factor. Hence, in these extreme periods it might cause practical issues to combine the risk factor with the price momentum, due to a lack of stocks with both properties. As a result, insufficient stocks reduce diversification inside these portfolios and increase volatility of returns, seeing this in combination with the relatively short period of time of Panel B probably causes these arbitrary results. Anyhow, the empirical evidence that risk can serve as a valuable improvement to the momentum strategy is rather convincing.

Additionally, ranking on momentum produces a relatively weak downward trend in average returns for low-risk stocks. This downward trend, however, is much stronger for the most risky assets. These results imply that in times of uncertainty the momentum factor is more important among risky stocks than among low-risk stocks.

6.4 Conclusion

We draw a conclusion that answers the fifth sub-question 'What is the importance of using distinct univariate quantitative investment strategies, like low-risk, momentum and value?'. The findings in this chapter show that the bivariate strategies can be substantially more profitable than the univariate strategies.

While the risk factor and the momentum factor exhibit relatively high correlations[10], there is still incremental information from both. Within a momentum quadrant, the decline of future expected return is more pronounced among the most risky stocks, while this effect is weaker among less risky ones. On the other hand, the risk factor performs substantially better among stocks with a good momentum. Similarly, for the ranks on value, we can conclude that performance of this strategy could be improved by adding the risk factor, however, the effectiveness of the normalized value factor itself is debatable and the timing of using this factor successfully among more risky stocks is essential due to the lack of consistency over time. This timing is of less importance for the least risky stocks.

To conclude, a relative simple but effective way to improve the performance of the momentum and value strategy in the long-run is by just avoiding the most risky stocks. This strategy could be narrowed further depending on the preferences of the portfolio manager. However, note that it might cause practical issues to combine the price momentum with the risk factor in changing market conditions. The performance of univariate normalized value factor, however, is somewhat precarious. Investing in cheap stocks relative to expensive ones has been reversed from positive excess return towards negative excess return for Panel A and Panel B,

[10] Correlation coefficient between risk and momentum factor is 0.70 and 0.78 for Panel A and Panel B, respectively.

respectively. Therefore the normalized value factor lacks robustness and seems less useful than the momentum and risk factor.

Chapter 7

Conclusion

The goal of this master's thesis is 'To improve the quantitative investment strategy by adding a low-risk factor'. To achieve this goal, we answered the main question, 'How can a low-risk factor be combined with the current quantitative investment strategy in order to improve the performance of an equity portfolio?', incrementally in the previous chapters. Here we summarize the most important findings. Finally, we answer the main question of this research.

The first goal we set was determining the performance of pure risk-based portfolios within the investment universe of the SNS Euro Equity Fund. In order to be able to construct risk-based portfolios, risk of each stock is estimated by four different 'risk-measures': 252-day volatility, 63-day volatility, 52-week beta and Conditional Value at Risk at a 10% level. To summarize, the empirical evidence that low-risk leads to strong future share price performance is convincing. The absolute returns of low-risk portfolios are considerably higher than those of high-risk ones. Moreover, their risk-adjusted returns statistically significantly outperform the risk-adjusted returns of high-risk portfolios in the long-run, regardless of whether we define risk as the Conditional Value at Risk or historical volatility. These results are consistent over time. Beta, however, fails to reveal this low-risk anomaly. Additionally, our analysis indicates that Conditional Value at Risk is the best univariate variable with regard to the selection of stocks with best future expected stock returns and exhibits the greatest discriminatory power.

With respect to the ranks on volatility, our findings are consistent with the findings of Dutt & Humphery-Jenner (2013), Baker et al. (2011), and Sefton et al. (2011) that higher historical volatility leads to lower realized future returns in the long-run. However, unlike the latter two papers, we cannot find evidence that low-beta stocks outperform high-beta stocks consistently. This discrepancy seems to be caused by the fact that our study draws more attention to risk-adjusted returns over time, whereas those authors seem to consider robustness of (risk-adjusted) returns over time less important.

The second goal was to model a performance attribution model, where the proportion of returns generated by risk-based portfolios are attributed to the current quantitative investment strategy. We employ this model to examine whether adding a risk factor is worth considering. Our results show that returns of risk-based portfolios cannot be fully explained by the exposure to market risk and SNS's current quantitative investment strategy. Therefore, adding a low-risk factor seems positively associated with future expected returns additional to the current quantitative investment factors.

The latter results suggest that bivariate strategies could increase future expected returns. Hence, the third goal was to examine whether the performance of the normalized value factor or momentum factor differs depending upon the level of risk. While the risk factor and the momentum factor are correlated, there is still incremental information from both. Momentum strategies work, but are more pronounced among lower risk stocks. Similarly, the risk factor performs substantially better among stocks with a good momentum. In contrast to the momentum strategy, returns for the normalized value strategy seem unpredictable over time. Although this strategy can be improved significantly by adding the risk factor, the effectiveness of the normalized value factor itself is debatable. Hence, the timing of using this factor successfully is essential due to the lack of consistency over time.

Nonetheless, these findings show that bivariate strategies can be more profitable than univariate strategies and we can conclude that the future expected performance of both strategies can be improved substantially by adding a risk factor. However, one must wonder whether the normalized value factor itself can be exploited successfully. Therefore, we consider the combination of risk and momentum more valuable than the combination of risk and normalized value.

To conclude, a relatively simple but effective way to improve the performance of the current quantitative investment strategy in the long-run is by just avoiding the most risky stocks. This could be narrowed furthermore towards the best possible strategy, i.e., (cheap) low-risk stocks with a good momentum, depending on the preferences of the portfolio manager.

Chapter 8

Discussion

During this study we have encountered a number of issues that might require a more thorough analysis, from a practical as well as from a theoretical point of view. This chapter discusses about the most relevant limitations of this research and comment on further research directions.

First, a major concern during a back test is that the information must have been available at the time of a portfolio's construction. To avoid look-ahead bias, estimates of assets future risks must be made in order to build risk-based portfolios. We use four relatively simple historical approaches to predict an asset's future risk. Although these simple methods have been extensively tested and proven useful, there is some evidence that there might be room for improvement using more sophisticated forecasting models. Based on volatility, for example, Baillie et al. (1996) developed the Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity (FIGARCH) model. Advantage of this technique, with respect to ARCH or GARCH models, is the long memory property which fits the observed economic and financial time series better. In Appendix D we briefly discuss methods that can be used for further research on the low-risk anomaly.

Second, portfolio managers cannot use leverage and their performance is measured relatively to a benchmark. From a practitioner viewpoint, this leads to two implications:

- Low-beta stocks are as risky as high-beta stocks, in a relative sense rather than an absolute one. Exploiting the low-risk anomaly involves buying low-beta stocks and, hence, creates a mismatch between their portfolio's risk and their benchmark's risk. As a result, the monthly difference in return between the portfolio and the benchmark is relatively large. This discrepancy, called the ex post tracking error, is contractually restricted.
- The Achilles' heel of low-risk strategies is that they are especially profitable in the long-run. Low-risk stocks tend to severely underperform high-risk stocks during strong recovery periods (sudden market rebounds), but extremely outperform the market during bearish markets. This behavior does not help to meet the portfolio manager's yearly outperformance goals.

We believe that the use of a benchmark may hamper the implementation of low-risk strategies. Therefore, we think that more research is needed in order to create an incentive to exploit this phenomenon among benchmarked investors.

Last but not least, we also reserve a cautionary note, given the wide attention that low-risk investing has received over the last few years, one must wonder how this massive attention will affect future profitability of low-risk stocks.

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Appendix

A. Summary Statistics for the Low-Risk Strategies

A.1 Summary statistics of the performance of low-risk portfolios

Table A.1: Returns by 50-week $CVaR_{10\%}$-based quintiles for Panel A: July 1994-May 2006 and Panel B: June 2006-July 2013.

	Panel A					Panel B				
	Q1 (low-risk)	Q2	Q3	Q4	Q5 (high-risk)	Q1 (low-risk)	Q2	Q3	Q4	Q5 (high-risk)
A. Performance										
Avg. geometric return	14.81%	14.25%	14.12%	10.40%	2.60%	7.71%	6.57%	4.37%	2.31%	-2.81%
Avg. arithmetic return	15.42%	15.20%	15.46%	12.46%	6.73%	8.39%	7.77%	6.24%	5.02%	1.80%
Standard deviation	11.02%	13.71%	16.26%	20.14%	28.50%	11.61%	15.44%	19.35%	23.25%	30.47%
Sharpe ratio	1.34	1.04	0.87	0.52	0.09	0.66	0.43	0.23	0.10	-0.09
Sortino ratio	1.64	1.22	1.07	0.67	0.12	0.77	0.55	0.30	0.13	-0.14
Turnover rate (monthly)	30.36%	56.77%	60.08%	52.02%	31.65%	27.11%	51.88%	55.21%	49.84%	30.64%
A.1.CAPM										
Alpha	6.44%**	4.44%**	3.17%*	-1.82%	-11.02%**	5.40%**	4.03%**	1.89%	0.09%	-4.11%
t-statistics (Alpha)	3.92	3.04	2.32	-1.04	-2.97	2.80	2.90	1.07	0.04	-1.01
Beta	0.618	0.831	1.014	1.252	1.666	0.639	0.919	1.150	1.372	1.743
R^2	0.743	0.869	0.919	0.913	0.805	0.807	0.943	0.941	0.928	0.874
A.2.Fama-French 3 factor										
Alpha	2.90%	0.48%	-0.03%	-4.31%*	-11.71%**	4.35%*	3.77%**	2.37%	1.11%	-1.56%
t-statistics (Alpha)	1.93	0.41	-0.03	-2.54	-3.21	2.55	3.06	1.43	0.59	-0.47
Beta	0.693	0.921	1.092	1.334	1.788	0.736	0.959	1.124	1.315	1.549
SMB	0.161	0.215	0.200	0.268	0.599	-0.050	0.188	0.266	0.529	0.657
HML	0.327	0.361	0.288	0.207	-0.019	-0.349	-0.143	0.092	0.206	0.702
Adjusted R^2	0.812	0.925	0.946	0.928	0.835	0.854	0.957	0.951	0.955	0.918

Notes: The return on the market, the risk-free rate, the SMB factor and the HML factor are from Kenneth R. French's website (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). Notice that these numbers are based on the European Market, but given in USD. We have used the FX spot rate to convert these to EUR. ** denotes statistical significance at 1% and * denotes statistical significance at the 5% level where H_0: α_p=0 and H_1: $\alpha_p \neq 0$.

Table A.2: Returns by 252-day volatility-based quintiles for Panel A: July 1994-May 2006 and Panel B: June 2006-July 2013.

	Panel A					Panel B				
	Q1 (low-risk)	Q2	Q3	Q4	Q5 (high-risk)	Q1 (low-risk)	Q2	Q3	Q4	Q5 (high-risk)
A. Performance										
Avg. geometric return	13.81%	14.13%	13.08%	9.56%	5.80%	7.07%	5.09%	4.48%	3.71%	-2.21%
Avg. arithmetic return	14.46%	15.06%	14.39%	11.69%	10.12%	7.74%	6.34%	6.46%	6.34%	2.34%
Standard deviation	11.33%	13.55%	16.11%	20.43%	29.15%	11.59%	15.74%	19.94%	22.95%	30.12%
Sharpe ratio	1.22	1.04	0.81	0.47	0.20	0.61	0.32	0.22	0.16	-0.07
Sortino ratio	1.46	1.21	1.02	0.58	0.27	0.77	0.39	0.31	0.22	-0.11
Turnover rate (monthly)	21.39%	41.45%	45.73%	40.15%	26.13%	18.76%	35.82%	39.22%	35.60%	23.47%
A.1.CAPM										
Alpha	5.40%**	4.44%**	2.38%	-2.69%	-7.86%*	4.76%*	2.56%	2.02%	1.48%	-3.52%
t-statistics (Alpha)	3.09	2.91	1.42	-1.45	-2.02	2.43	1.77	1.11	0.58	-0.88
Beta	0.628	0.814	0.980	1.264	1.692	0.636	0.935	1.185	1.344	1.723
R2	0.726	0.853	0.875	0.904	0.795	0.800	0.940	0.941	0.914	0.875
A.2.Fama-French 3 factor										
Alpha	1.05%	0.53%	-1.95%	-6.04%**	-5.83%	3.65%*	2.33%	2.60%	2.46%	-0.99%
t-statistics (Alpha)	0.71	0.41	-1.39	-3.63	-1.56	2.14	1.75	1.57	1.18	-0.30
Beta	0.726	0.884	1.062	1.379	1.775	0.737	0.971	1.152	1.293	1.528
SMB	0.232	0.107	0.146	0.383	0.571	-0.058	0.174	0.300	0.560	0.611
HML	0.397	0.371	0.407	0.275	-0.284	-0.367	-0.129	0.120	0.185	0.707
Adjusted R2	0.825	0.909	0.923	0.933	0.835	0.853	0.951	0.953	0.944	0.918

Notes: The return on the market, the risk-free rate, the SMB factor and the HML factor are from Kenneth R. French's website (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). Notice that these numbers are based on the European Market, but given in USD. We have used the FX spot rate to convert these to EUR. ** denotes statistical significance at 1% and * denotes statistical significance at the 5% level where H_0: $\alpha_p=0$ and H_1: $\alpha_p \neq 0$.

Table A.3: Returns by 63-day volatility-based quintiles for Panel A: July 1994-May 2006 and Panel B: June 2006-July 2013.

	Panel A					Panel B				
	Q1 (low-risk)	Q2	Q3	Q4	Q5 (high-risk)	Q1 (low-risk)	Q2	Q3	Q4	Q5 (high-risk)
A. Performance										
Avg. geometric return	13.60%	13.50%	12.71%	10.54%	5.50%	7,43%	5,01%	4,49%	2,73%	-1,95%
Avg. arithmetic return	14.28%	14.45%	14.06%	12.49%	9.84%	8,14%	6,29%	6,38%	5,36%	2,52%
Standard deviation	11.62%	13.76%	16.30%	19.63%	29.18%	11,85%	15,89%	19,40%	22,98%	30,07%
Sharpe ratio	1.17	0.98	0.78	0.54	0.19	0.63	0.32	0.23	0.12	-0.06
Sortino ratio	1.40	1.19	1.01	0.69	0.25	0.77	0.38	0.30	0.16	-0.10
Turnover rate (monthly)	51.22%	95.80%	103.05%	91.84%	52.98%	46,59%	88,39%	95,70%	84,92%	49,42%
A.1.CAPM										
Alpha	5.09%**	3.72%*	1.80%	-1.50%	-8.05%*	5.09%**	2.47%	2.01%	0.49%	-3.33%
t-statistics (Alpha)	2.82	2.44	1.22	-0.86	-2.01	2.68	1.86	1.22	0.19	-0.83
Beta	0.643	0.828	1.009	1.217	1.682	0.657	0.950	1.158	1.345	1.720
R^2	0.722	0.858	0.906	0.908	0.783	0.818	0.951	0.950	0.914	0.874
A.2 Fama-French 3 factor										
Alpha	0.46%	-0.34%	-1.96%	-4.65%**	-5.45%	4.08%*	2.22%*	2.31%	1.72%	-0.86%
t-statistics (Alpha)	0.31	-0.28	-1.58	-2.80	-1.47	2.45	1.99	1.62	0.80	-0.25
Beta	0.747	0.910	1.089	1.299	1.772	0.755	0.991	1.154	1.265	1.528
SMB	0.243	0.164	0.169	0.221	0.644	0.001	0.218	0.320	0.476	0.579
HML	0.423	0.378	0.348	0.280	-0.352	-0.354	-0.149	0.017	0.292	0.698
Adjusted R^2	0.828	0.916	0.941	0.927	0.836	0.867	0.967	0.963	0.940	0.915

Notes: The return on the market, the risk-free rate, the SMB factor and the HML factor are from Kenneth R. French's website (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). Notice that these numbers are based on the European Market, but given in USD. We have used the FX spot rate to convert these to EUR. ** denotes statistical significance at 1% and * denotes statistical significance at the 5% level where H_0: $\alpha_p=0$ and H_1: $\alpha_p \neq 0$.

Table A.4: Returns by 52-week beta-based quintiles for Panel A: July 1994-May 2006 and Panel B: June 2006-July 2013.

	Panel A					Panel B				
	Q1 (low-risk)	Q2	Q3	Q4	Q5 (high-risk)	Q1 (low-risk)	Q2	Q3	Q4	Q5 (high-risk)
A. Performance										
Avg. geometric return	12.44%	13.19%	14.19%	11.74%	4.49%	5.09%	3.52%	2.57%	4.58%	2.51%
Avg. arithmetic return	13.11%	14.15%	15.51%	13.74%	8.88%	5.87%	4.91%	4.39%	7.08%	6.71%
Standard deviation	11.53%	13.83%	16.14%	19.86%	29.36%	12.46%	16.50%	19.03%	22.49%	29.27%
Sharpe ratio	1.08	0.95	0.88	0.59	0.15	0.41	0.21	0.13	0.20	0.09
Sortino ratio	1.33	1.21	1.03	0.75	0.21	0.46	0.24	0.18	0.28	0.13
Turnover rate (monthly)	36.55%	65.50%	71.49%	63.84%	34.18%	30.05%	56.38%	62.45%	57.01%	32.17%
A.1.CAPM										
Alpha	4.33%*	3.59%*	3.43%*	-0.42%	-9.67%**	2.74%	1.02%	0.10%	2.23%	0.95%
t-statistics (Alpha)	2.08	1.99	2.15	-0.25	-2.88	1.35	0.66	0.06	1.12	0.26
Beta	0.593	0.807	0.989	1.237	1.761	0.688	0.980	1.129	1.339	1.687
R^2	0.623	0.804	0.887	0.918	0.849	0.813	0.939	0.937	0.945	0.887
A.2 Fama-French 3 factor										
Alpha	-1.49%	-1.63%	-0.87%	-2.14%	-5.57%	1.74%	0.88%	0.64%	3.28%	3.22%
t-statistics (Alpha)	-0.92	-1.18	-0.68	-1.30	-1.76	1.01	0.67	0.43	1.92	1.03
Beta	0.733	0.919	1.090	1.311	1.772	0.793	1.014	1.107	1.264	1.506
SMB	0.349	0.244	0.248	0.272	0.330	0.106	0.268	0.390	0.334	0.485
HML	0.526	0.481	0.390	0.130	-0.454	-0.378	-0.122	0.078	0.271	0.656
Adjusted R^2	0.800	0.899	0.936	0.930	0.882	0.871	0.957	0.958	0.961	0.922

Notes: The return on the market, the risk-free rate, the SMB factor and the HML factor are from Kenneth R. French's website (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). Notice that these numbers are based on the European Market, but given in USD. We have used the FX spot rate to convert these to EUR. ** denotes statistical significance from α=0 at 1% and * denotes statistical significance at the 5% level where H_0: α_p=0 and H_1: $\alpha_p \neq 0$.

A.2 T-statistics for the comparison of portfolios Panel A and Panel B alphas

Table A.5: Alphas estimated by the CAPM for 50-week $CVaR_{10\%}$ ranked portfolios. H_0: $\alpha_i=\alpha_j$, H_1: $\alpha_i \neq \alpha_j$.

	Panel A					Panel B				
	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5
Q1	-	0.91	1.53	3.44	4.31	-	0.57	1.34	1.74	2.11
Q2		-	0.64	2.75	3.88		-	0.95	1.44	1.89
Q3			-	2.25	3.59			-	0.61	1.35
Q4				-	2.24				-	0.89
Q5					-					-

Table A.6: Alphas estimated by the Fama-French three-factor model for 50-week $CVaR_{10\%}$ ranked portfolios. H_0: $\alpha_i=\alpha_j$, H_1: $\alpha_i \neq \alpha_j$.

	Panel A					Panel B				
	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5
Q1	-	1.27	1.53	3.18	3.71	-	0.27	0.83	1.27	1.57
Q2		-	0.31	2.32	3.19		-	0.68	1.18	1.50
Q3			-	2.06	3.05			-	0.50	1.05
Q4				-	1.84				-	0.69
Q5					-					-

Table A.7: Alphas estimated by the CAPM for 252-day volatility ranked portfolios. H_0: $\alpha_i=\alpha_j$, H_1: $\alpha_i \neq \alpha_j$.

	Panel A					Panel B				
	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5
Q1	-	0.41	1.25	3.17	3.11	-	0.90	1.02	1.02	1.85
Q2		-	0.91	2.96	2.95		-	0.23	0.37	1.43
Q3			-	2.02	2.42			-	0.17	1.26
Q4				-	1.20				-	1.05
Q5					-					-

Table A.8: Alphas estimated by the Fama-French three-factor model for 252-day volatility ranked portfolios. H_0: $\alpha_i=\alpha_j$, H_1: $\alpha_i \neq \alpha_j$.

	Panel A					Panel B				
	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5
Q1	-	0.26	1.47	3.18	1.71	-	0.61	0.44	0.44	1.25
Q2		-	1.30	3.13	1.61		-	-0.13	-0.05	0.93
Q3			-	1.88	0.97			-	0.05	0.97
Q4				-	-0.05				-	0.88
Q5					-					-

Table A.9: Alphas estimated by the CAPM for 63-day volatility ranked portfolios. H_0: $\alpha_i=\alpha_j$, H_1: $\alpha_i \neq \alpha_j$.

	Panel A					Panel B				
	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5
Q1	-	0.58	1.41	2.62	2.99	-	1.13	1.23	1.45	1.89
Q2		-	0.90	2.25	2.75		-	0.22	0.69	1.37
Q3			-	1.45	2.31			-	0.50	1.23
Q4				-	1.50				-	0.80
Q5					-					-

Table A.10: Alphas estimated by the Fama-French three-factor model for 63-day volatility ranked portfolios. H_0: $\alpha_i=\alpha_j$, H_1: $\alpha_i \neq \alpha_j$.

	Panel A					Panel B				
	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5
Q1	-	0.41	1.24	2.27	1.47	-	0.93	0.80	0.87	1.31
Q2		-	0.92	2.07	1.30		-	-0.05	0.21	0.87
Q3			-	1.29	0.89			-	0.23	0.87
Q4				-	0.20				-	0.64
Q5					-					-

Table A.11: Alphas estimated by the CAPM for 52-week beta ranked portfolios. H_0: $\alpha_i=\alpha_j$, H_1: $\alpha_i\neq\alpha_j$.

	Panel A					Panel B				
	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5
Q1	-	0.27	0.34	1.78	3.54	-	0.68	0.97	0.18	0.43
Q2		-	0.06	1.63	3.48		-	0.39	-0.48	0.02
Q3			-	1.67	3.52			-	-0.79	-0.20
Q4				-	2.46				-	0.31
Q5					-					-

Table A.12: Alphas estimated by the Fama-French three-factor model for 52-week beta ranked portfolios. H_0: $\alpha_i=\alpha_j$, H_1: $\alpha_i\neq\alpha_j$.

	Panel A					Panel B				
	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5
Q1	-	0.06	-0.30	0.28	1.14	-	0.40	0.48	-0.63	-0.41
Q2		-	-0.40	0.24	1.14		-	0.12	-1.12	-0.69
Q3			-	0.61	1.37			-	-1.16	-0.74
Q4				-	0.96				-	0.02
Q5					-					-

A.3 T-statistics for the comparison of portfolios Panel C alphas

Table A.13: Alphas estimated for $CVaR_{10\%}$ ranked portfolios. H_0: $\alpha_i=\alpha_j$, H_1: $\alpha_i\neq\alpha_j$.

	Capital Asset Pricing Model					Fama-French 3 Factor Model				
	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5
Q1	-	1.14	2.12	3.86	4.78	-	1.46	2.62	4.37	5.14
Q2		-	1.10	3.07	4.27		-	1.36	3.46	4.53
Q3			-	2.09	3.69			-	2.29	3.86
Q4				-	2.33				-	2.45
Q5					-					-

Table A.14: Alphas estimated for 252-day volatility ranked portfolios. H_0: $\alpha_i=\alpha_j$, H_1: $\alpha_i \neq \alpha_j$.

	Capital Asset Pricing Model					Fama-French 3 Factor Model				
	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5
Q1	-	0.93	1.75	3.20	3.65	-	1.23	2.44	3.69	3.63
Q2		-	0.95	2.55	3.22		-	1.41	2.88	3.06
Q3			-	1.59	2.62			-	1.56	2.30
Q4				-	1.56				-	1.35
Q5					-					-

Table A.15: Alphas estimated for 63-day volatility ranked portfolios. H_0: $\alpha_i=\alpha_j$, H_1: $\alpha_i \neq \alpha_j$.

	Capital Asset Pricing Model					Fama-French 3 Factor Model				
	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5
Q1	-	1.17	1.96	3.05	3.57	-	1.43	2.38	3.56	3.50
Q2		-	0.90	2.20	3.02		-	1.12	2.59	2.84
Q3			-	1.40	2.56			-	1.64	2.30
Q4				-	1.66				-	1.31
Q5					-					-

Table A.16: Alphas estimated for 52-week beta ranked portfolios. H_0: $\alpha_i=\alpha_j$, H_1: $\alpha_i \neq \alpha_j$.

	Capital Asset Pricing Model					Fama-French 3 Factor Model				
	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5
Q1	-	0.62	0.84	1.59	3.10	-	0.87	1.10	1.53	2.60
Q2		-	0.22	1.05	2.78		-	0.23	0.78	2.15
Q3			-	0.86	2.67			-	0.59	2.05
Q4				-	2.10				-	1.66
Q5					-					-

A.4 T-statistics for the comparison of portfolios alphas estimated by the three-factor performance attribution model

Table A.17: Alphas estimated for 50-week $CVaR_{10\%}$ portfolios. H_0: $\alpha_i = \alpha_j$, H_1: $\alpha_i \neq \alpha_j$.

	Panel A					Panel B				
	Q1	Q2	Q3	Q4	Q5	Q1	Q2	Q3	Q4	Q5
Q1	-	0.56	0.18	1.43	1.29	-	0.13	0.48	0.82	1.14
Q2		-	-0.42	0.94	0.96		-	0.39	0.77	1.10
Q3			-	1.37	1.22			-	0.40	0.80
Q4				-	0.37				-	0.44
Q5					-					-

	Panel C				
	Q1	Q2	Q3	Q4	Q5
Q1	-	0.72	0.86	2.30	2.46
Q2		-	0.15	1.74	2.05
Q3			-	1.61	1.96
Q4				-	0.83
Q5					-

B. Ranking on Two Variables at Once: The Performance of Bivariate Strategies

B.1 Summary of the performance of bivariate portfolios

Table B.1: Ranking on $CVaR_{10\%}$ and momentum for Panel A: July 1994-May 2006 and Panel B: June 2006-July 2013.

	Panel A					**Panel B**				
	Q1 (positive mom.)	Q2	Q3	Q4 (negative mom.)	*Q4-Q1 (spread)*	Q1 (positive mom.)	Q2	Q3	Q4 (negative mom.)	*Q4-Q1 (spread)*
Q1 (low-risk)										
Annualized return	17.49%	16.01%	14.30%	11.34%	*6.15%*	7.75%	12.54%	4.96%	7.07%	*0.68%*
Standard deviation	12.70%	12.09%	13.19%	12.39%		12.00%	14.79%	12.23%	20.74%	
Alpha (CAPM)	8.66%	7.72%	5.42%	3.34%		5.46%	10.38%	2.73%	5.48%	
t-statistics (Alpha)	4.31	3.53	2.51	1.34		2.53	3.19	1.13	1.04	
Q2										
Annualized return	16.27%	15.76%	12.02%	11.78%	*4.49%*	6.85%	5.38%	3.53%	3.01%	*3.84%*
Standard deviation	15.85%	15.46%	16.19%	16.25%		17.00%	17.83%	16.59%	19.54%	
Alpha (CAPM)	5.74%	5.93%	1.87%	1.63%		4.40%	3.01%	1.06%	0.62%	
t-statistics (Alpha)	3.24	2.55	0.80	0.69		2.13	1.17	0.50	0.25	
Q3										
Annualized return	16.64%	11.43%	8.49%	9.20%	*7.44%*	4.83%	2.51%	3.04%	2.88%	*1.95%*
Standard deviation	18.20%	19.63%	18.56%	19.53%		20.59%	24.22%	19.66%	24.05%	
Alpha (CAPM)	5.40%	0.18%	-2.54%	-2.17%		2.54%	0.55%	0.71%	0.79%	
t-statistics (Alpha)	2.57	0.07	-1.01	-0.84		0.99	0.17	0.26	0.23	
Q4 (high-risk)										
Annualized return	8.92%	4.50%	1.43%	1.09%	*7.83%*	5.59%	1.09%	1.89%	-7.75%	*13.34%*
Standard deviation	23.08%	28.33%	23.99%	29.72%		23.46%	30.09%	26.11%	32.03%	
Alpha (CAPM)	-3.48%	-8.02%	-10.16%	-12.18%		3.79%	0.07%	0.18%	-8.71%	
t-statistics (Alpha)	-1.18	-1.70	-2.47	-2.74		0.93	0.01	0.04	-1.73	
Q4-Q1 (spread)	*8.57%*	*11.51%*	*12.87%*	*10.25%*		*2.16%*	*11.45%*	*3.07%*	*14.82%*	

Table B.2: Ranking on $CVaR_{10\%}$ and normalized value for Panel A: July 1994-May 2006 and Panel B: June 2006-July 2013.

	Panel A					Panel B				
	Q1 (cheap)	Q2	Q3	Q4 (expensive)	*Q4-Q1 (spread)*	Q1 (cheap)	Q2	Q3	Q4 (expensive)	*Q4-Q1 (spread)*
Q1 (low-risk)										
Annualized return	16.99%	14.98%	13.61%	14.37%	*2.62%*	7.86%	8.20%	7.99%	7.01%	*0.85%*
Standard deviation	12.28%	11.39%	11.80%	12.70%		13.93%	12.22%	12.55%	11.81%	
Alpha (CAPM)	8.59%	6.70%	5.09%	5.35%		5.56%	5.85%	5.67%	4.72%	
t-statistics (Alpha)	3.95	3.57	2.74	2.91		2.18	3.03	2.61	2.24	
Q2										
Annualized return	16.63%	14.90%	12.31%	12.53%	*4.10%*	4.50%	3.64%	6.20%	5.35%	*-0.85%*
Standard deviation	15.67%	15.33%	14.77%	16.22%		17.97%	17.21%	18.64%	15.92%	
Alpha (CAPM)	6.78%	4.67%	2.24%	1.91%		2.03%	1.14%	3.75%	2.88%	
t-statistics (Alpha)	2.81	2.54	1.29	1.04		0.99	0.69	1.79	1.49	
Q3										
Annualized return	15.43%	11.79%	11.99%	8.33%	*7.10%*	-0.12%	1.80%	1.50%	2.00%	*-2.12%*
Standard deviation	18.93%	18.23%	19.59%	19.17%		26.42%	22.38%	21.76%	19.85%	
Alpha (CAPM)	4.55%	0.40%	-0.04%	-3.44%		-1.96%	-0.47%	-0.78%	-0.37%	
t-statistics (Alpha)	1.62	0.21	-0.02	-1.79		-0.60	-0.18	-0.33	-0.15	
Q4 (high-risk)										
Annualized return	8.46%	5.43%	3.69%	-3.96%	*12.42%*	-3.22%%	-4.68%	0.97%	3.10%	*-6.33%*
Standard deviation	28.98%	27.58%	28.68%	28.27%		32.59%	27.66%	27.41%	28.93%	
Alpha (CAPM)	-4.56%	-7.88%	-9.78%	-17.15%		-4.04%	-6.33%	-0.72%	1.61%	
t-statistics (Alpha)	-1.03	-2.11	-2.47	-4.22		-0.80	-1.81	-0.20	0.33	
Q4-Q1 (spread)	*8.53%*	*9.55%*	*9.92%*	*18.33%*		*11.08%*	*12.88%*	*7.02%*	*3.91%*	

B.2 T-statistics for the comparison of portfolios alphas for the ranks on $CVaR_{10\%}$ and momentum

Risk ($CVaR_{10\%}$)	Momentum Q1	Q2	Q3	Q4
Q1	**P1**	**P2**	**P3**	**P4**
Q2	**P5**	**P6**	**P7**	**P8**
Q3	**P9**	**P10**	**P11**	**P12**
Q4	**P13**	**P14**	**P15**	**P16**

Table B.3: Alphas estimated, for Panel A, by the CAPM for the bivariate $CVaR_{10\%}$ and momentum strategy. H_0: $\alpha_i=\alpha_j$, H_1: $\alpha_i \neq \alpha_j$.

Portfolios	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15	P16
P1	-	0.31	1.10	1.66	1.09	0.89	2.20	2.26	1.12	2.47	3.47	3.30	3.39	3.26	4.12	4.27
P2		-	0.75	1.32	0.70	0.56	1.83	1.89	0.77	2.13	3.07	2.92	3.05	3.03	3.84	4.02
P3			-	0.63	-0.12	-0.16	1.12	1.18	0.00	1.49	2.39	2.25	2.43	2.59	3.36	3.56
P4				-	-0.79	-0.76	0.43	0.50	-0.63	0.85	1.66	1.53	1.76	2.13	0.00	3.05
P5					-	-0.06	1.32	1.39	0.12	1.69	2.69	2.52	2.68	2.73	3.56	3.74
P6						-	1.23	1.30	0.17	1.59	2.47	2.32	2.50	2.65	3.41	3.61
P7							-	0.07	-1.13	0.46	1.28	1.16	1.42	1.88	2.55	2.80
P8								-	-1.19	0.40	1.21	1.08	1.35	1.83	2.49	2.74
P9									-	1.50	2.42	2.27	2.45	2.60	3.37	3.57
P10										-	0.73	0.62	0.90	1.50	2.09	2.36
P11											-	-0.10	0.24	0.00	1.58	1.88
P12												-	0.33	1.09	1.65	1.94
P13													-	0.82	1.32	1.63
P14														-	0.34	0.64
P15															-	0.33
P16																-

Table B.4: Alphas estimated, for Panel B, by the CAPM for the bivariate $CVaR_{10\%}$ and momentum strategy. H_0: $\alpha_i=\alpha_j$, H_1: $\alpha_i \neq \alpha_j$.

Portfolios	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15	P16
P1	-	-1.26	0.84	0.00	0.35	0.73	1.45	1.46	0.87	1.26	1.37	1.16	0.36	0.93	1.06	2.58
P2		-	1.89	0.79	1.55	1.77	2.40	2.37	1.89	2.15	2.28	2.04	1.26	1.64	1.84	3.18
P3			-	-0.47	-0.53	-0.08	0.52	0.60	0.05	0.54	0.56	0.47	-0.22	0.45	0.50	2.05
P4				-	0.19	0.42	0.78	0.83	0.50	0.80	0.80	0.75	0.25	0.72	0.00	1.94
P5					-	0.42	1.13	1.16	0.56	1.01	1.08	0.91	0.13	0.75	0.85	2.41
P6						-	0.58	0.66	0.13	0.60	0.61	0.52	-0.16	0.49	0.55	2.07
P7							-	0.13	-0.44	0.13	0.10	0.07	-0.59	0.17	0.18	1.79
P8								-	-0.53	0.02	-0.02	-0.04	-0.66	0.09	0.09	1.66
P9									-	0.48	0.49	0.41	-0.26	0.41	0.46	1.99
P10										-	-0.04	-0.05	-0.62	0.08	0.07	1.55
P11											-	-0.02	-0.63	0.00	0.10	1.64
P12												-	-0.57	0.11	0.11	1.56
P13													-	0.55	0.59	1.93
P14														-	-0.02	1.19
P15															-	1.32
P16																-

B.3 T-statistics for the comparison of portfolios alphas for the ranks on $CVaR_{10\%}$ and normalized value

Risk ($CVaR_{10\%}$)	Normalized Value Q1	Q2	Q3	Q4
Q1	**P1**	**P2**	**P3**	**P4**
Q2	**P5**	**P6**	**P7**	**P8**
Q3	**P9**	**P10**	**P11**	**P12**
Q4	**P13**	**P14**	**P15**	**P16**

Table B.5: Alphas estimated, for Panel A, by the CAPM for the bivariate $CVaR_{10\%}$ and normalized value strategy. H_0: $\alpha_i=\alpha_j$, H_1: $\alpha_i\neq\alpha_j$.

Portfolios	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15	P16
P1	-	0.66	1.22	1.14	0.56	1.38	2.28	2.35	1.14	2.81	3.09	4.15	2.66	3.81	4.06	5.58
P2		-	0.61	0.51	-0.03	0.77	1.74	1.83	0.64	2.34	2.62	3.78	2.33	3.49	3.76	5.33
P3			-	-0.10	-0.56	0.16	1.12	1.22	0.16	1.75	2.01	3.19	2.00	3.11	3.40	4.98
P4				-	-0.47	0.26	1.23	1.33	0.24	1.86	2.12	3.31	2.06	3.18	0.00	5.05
P5					-	0.69	1.52	1.61	0.60	2.06	2.29	3.32	2.24	3.30	3.57	5.07
P6						-	0.96	1.07	0.04	1.60	1.85	3.06	1.92	3.01	3.31	4.89
P7							-	0.13	-0.70	0.71	0.92	2.19	1.42	2.46	2.78	4.39
P8								-	-0.79	0.57	0.77	2.02	1.35	2.35	2.68	4.28
P9									-	1.22	1.38	2.35	1.73	2.66	2.95	4.39
P10										-	0.17	1.41	1.02	1.97	2.31	3.90
P11											-	1.31	0.95	0.00	2.25	3.87
P12												-	0.23	1.06	1.44	3.05
P13													-	0.57	0.88	2.09
P14														-	0.35	1.68
P15															-	1.30
P16																-

Table B.6: Alphas estimated, for Panel B, by the CAPM for the bivariate $CVaR_{10\%}$ and normalized value strategy. H_0: $\alpha_i=\alpha_j$, H_1: $\alpha_i \neq \alpha_j$.

Portfolios	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10	P11	P12	P13	P14	P15	P16
P1	-	-0.09	-0.03	0.26	1.08	1.46	0.55	0.84	1.82	1.63	1.82	1.69	1.70	2.75	1.40	0.72
P2		-	0.06	0.40	1.35	1.85	0.74	1.09	2.07	1.92	2.17	2.01	1.83	3.05	1.58	0.81
P3			-	0.32	1.22	1.66	0.63	0.96	1.96	1.79	2.01	1.86	1.76	2.91	1.49	0.76
P4				-	0.91	1.34	0.32	0.64	1.73	1.53	1.73	1.58	1.60	2.71	0.00	0.59
P5					-	0.34	-0.59	-0.30	1.04	0.74	0.90	0.76	1.11	2.06	0.65	0.08
P6						-	-0.98	-0.68	0.85	0.51	0.66	0.51	0.97	1.93	0.46	-0.09
P7							-	0.31	1.48	1.24	1.43	1.29	1.42	2.47	1.05	0.40
P8								-	1.28	1.02	1.20	1.05	1.28	2.30	0.86	0.24
P9									-	-0.35	-0.29	-0.39	0.35	0.92	-0.25	-0.61
P10										-	0.09	-0.03	0.63	1.33	0.06	-0.37
P11											-	-0.12	0.58	0.00	-0.01	-0.44
P12												-	0.66	1.40	0.08	-0.36
P13													-	0.37	-0.53	-0.81
P14														-	-1.10	-1.32
P15															-	-0.38
P16																-

C. Comparison of Indices

Due to the lack of historical MSCI Europe Index data, we are forced to do data analysis on a different, but comparable index. This index, the STOXX600 Europe Index shares a large number of stocks with that of the MSCI Europe Index. For future research purposes of SNS, Panel B (06-2006 to 07-2013) on the STOXX600 Europe Index is set equal to the period of available historical MSCI Europe Index data. Here we are 'visually' checking the linear regression results of the CAPM and Fama-French three-factor model on both indices. Logically, results of the regression analysis on the MSCI Europe Index slightly differ from the regression analysis results on the STOXX600 Europe Index, Table C.1 and Table C.2 report the results of this analysis on the CAPM and Fama-French three-factor model, respectively.

Table C.1: CAPM regression analysis on both indices.

		MSCI Europe Index				
		Q1 (low-risk)	Q2	Q3	Q4	Q5 (high-risk)
A. *$CVaR_{10\%}$*	Alpha	**4,98%***	**3,86%****	**1,02%**	**-1,55%**	**-4,14%**
	Beta	*0,65*	*0,90*	*1,14*	*1,39*	*1,82*
B. *252-day volatility*	Alpha	**4,66%***	**1,83%**	**1,14%**	**1,48%**	**-4,96%**
	Beta	*0,62*	*0,93*	*1,18*	*1,35*	*1,80*
C. *63-day volatility*	Alpha	**4,86%***	**1,88%**	**1,55%**	**0,71%**	**-5,42%**
	Beta	*0,64*	*0,95*	*1,16*	*1,36*	*1,78*
D. *52-week beta*	Alpha	**2,88%**	**0,08%**	**1,22%**	**0,46%**	**-1,06%**
	Beta	*0,68*	*0,99*	*1,15*	*1,32*	*1,77*
		STOXX600 Europe Index				
		Q1 (low-risk)	Q2	Q3	Q4	Q5 (high-risk)
A. *$CVaR_{10\%}$*	Alpha	**5.40%****	**4.03%****	**1.89%**	**0.09%**	**-4.11%**
	Beta	*0.64*	*0.92*	*1.15*	*1.37*	*1.74*
B. *252-day volatility*	Alpha	**4.76%***	**2.56%**	**2.02%**	**1.48%**	**-3.52%**
	Beta	*0.64*	*0.94*	*1.19*	*1.34*	*1.72*
C. *63-day volatility*	Alpha	**5.09%****	**2.47%**	**2.01%**	**0.49%**	**-3.33%**
	Beta	*0.66*	*0.95*	*1.16*	*1.35*	*1.72*
D. *52-week beta*	Alpha	**2.74%**	**1.02%**	**0.10%**	**2.23%**	**0.95%**
	Beta	*0.69*	*0.98*	*1.13*	*1.34*	*1.69*

Note: ** and * denotes statistical significance at the 1% and at the 5% level, respectively. H_0: $\alpha_p=0$ and H_1: $\alpha_p \neq 0$.

Like the regression results on the STOXX600 Europe Index in Section 4.3.3, ranks on $CVaR_{10\%}$ exhibit monotonically decreasing alphas for increasing risk. In contrast to the STOXX600 Europe Index, ranking on volatility induces no consistent relation in alpha for *Q2* to *Q4* for the MSCI Europe Index, but the alpha spread between *Q1* and *Q5* is considerably large.

Table C.2: Fama-French three-factor model regression analysis on both indices.

		MSCI Europe Index				
		Q1 (low-risk)	Q2	Q3	Q4	Q5 (high-risk)
A. *$CVaR_{10\%}$*	Alpha	**3,87%***	**3,61%***	**1,50%**	**-0,33%**	**-0,81%**
	Beta	*0,75*	*0,94*	*1,11*	*1,30*	*1,55*
	SMB	*-0,08*	*0,10*	*0,18*	*0,44*	*0,64*
	HML	*-0,36*	*-0,11*	*0,12*	*0,30*	*0,98*
B. *252-day volatility*	Alpha	**3,45%***	**1,65%**	**1,76%**	**2,61%**	**-1,64%**
	Beta	*0,73*	*0,96*	*1,14*	*1,28*	*1,53*
	SMB	*-0,13*	*0,13*	*0,21*	*0,48*	*0,60*
	HML	*-0,39*	*-0,10*	*0,16*	*0,26*	*0,99*
C. *63-day volatility*	Alpha	**3,79%***	**1,72%**	**1,77%**	**2,07%**	**-2,12%**
	Beta	*0,74*	*0,98*	*1,15*	*1,26*	*1,51*
	SMB	*-0,07*	*0,17*	*0,21*	*0,41*	*0,56*
	HML	*-0,35*	*-0,10*	*0,02*	*0,36*	*0,99*
D. *52-week beta*	Alpha	**1,85%**	**-0,07%**	**1,94%**	**1,68%**	**1,83%**
	Beta	*0,79*	*1,02*	*1,10*	*1,22*	*1,52*
	SMB	*0,09*	*0,24*	*0,29*	*0,23*	*0,44*
	HML	*-0,38*	*-0,12*	*0,17*	*0,36*	*0,88*
		STOXX600 Europe Index				
		Q1 (low-risk)	Q2	Q3	Q4	Q5 (high-risk)
A. *$CVaR_{10\%}$*	Alpha	**4.35%***	**3.77%****	**2.37%**	**1.11%**	**-1.56%**
	Beta	*0.74*	*0.96*	*1.12*	*1.32*	*1.55*
	SMB	*-0.05*	*0.19*	*0.27*	*0.53*	*0.66*
	HML	*-0.35*	*-0.14*	*0.09*	*0.21*	*0.70*
B. *252-day volatility*	Alpha	**3.65%***	**2.33%**	**2.60%**	**2.46%**	**-0.99%**
	Beta	*0.74*	*0.97*	*1.15*	*1.29*	*1.53*
	SMB	*-0.06*	*0.17*	*0.30*	*0.56*	*0.61*
	HML	*-0.37*	*-0.13*	*0.12*	*0.19*	*0.71*
C. *63-day volatility*	Alpha	**4.08%***	**2.22%**	**2.31%**	**1.72%**	**-0.86%**
	Beta	*0.76*	*0.99*	*1.15*	*1.27*	*1.53*
	SMB	*0.00*	*0.22*	*0.32*	*0.48*	*0.58*
	HML	*-0.35*	*-0.15*	*0.02*	*0.29*	*0.70*
D. *52-week beta*	Alpha	**1.74%**	**0.88%**	**0.64%**	**3.28%**	**3.22%**
	Beta	*0.79*	*1.01*	*1.11*	*1.26*	*1.51*
	SMB	*0.11*	*0.27*	*0.39*	*0.33*	*0.49*
	HML	*-0.38*	*-0.12*	*0.08*	*0.27*	*0.66*

Note: ** and * denotes statistical significance at the 1% and at the 5% level, respectively. H_0: $\alpha_p=0$ and H_1: $\alpha_p\neq0$.

Although the differences between the ranks on $CVaR_{10\%}$ and the ranks on volatility for *Q1* seem to have become smaller, *Q2* of the ranks on $CVaR_{10\%}$ is the only second quintile that is able to produce statistically significant excess return on the CAPM and Fama-French three-factor model. Again, beta quintiles do not suffice to produce significant alpha with respect to the market and induce no relation in alpha for *Q1* to *Q5* on the Fama-French three-factor model.

Like the analysis on the STOXX600 Europe index, ranking on $CVaR_{10\%}$ seems to be the best variable for strong future expected stock returns, whereas beta fails to show that there exists a low-risk anomaly. Additionally, the difference between the 252-day and 63-day volatility strategy also seems negligible.

To conclude, the behavior of the MSCI Europe Index seems to match that of the STOXX600 Europe Index and therefore we can assume that the behavior of the STOXX600 Europe Index equals that of the MSCI Europe Index, also prior to 2006. These results came as no surprise, because these indices share a large number of stocks.

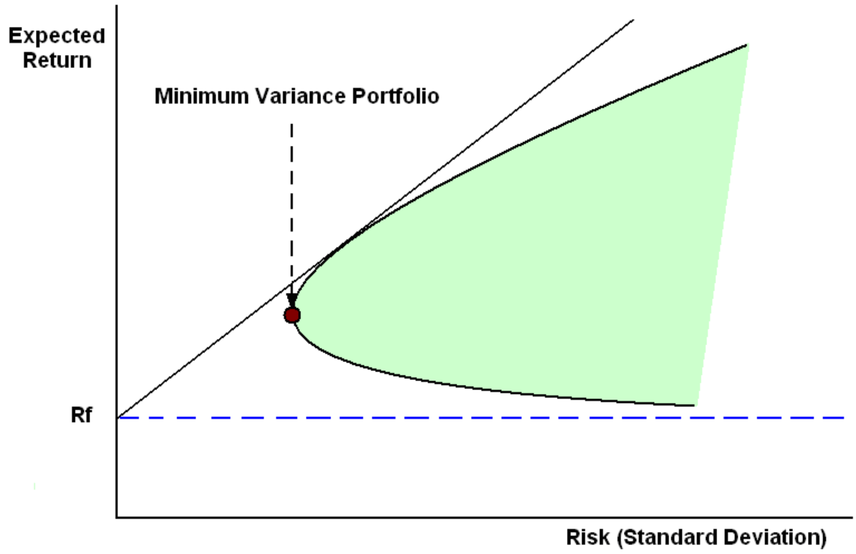
D. Further Research Directions

In this appendix we discuss literature that can be used for further research on the low-risk anomaly. First, the theory for the composition of a minimum variance portfolio is discussed. Second, we suggest an alternative approach to build tail-risk sensitive portfolios. Third, we introduce mathematical techniques to forecast volatility that might result in a more sophisticated method to build risk-based portfolios.

D.1 Minimum-variance portfolio

The minimum-variance method optimizes a portfolio with regard to variance. The idea behind this theory is straightforward. For example, the variance of a multiple asset portfolio consisting of low volatile securities can be more volatile than a portfolio consisting of more volatile securities, due to the covariances between security returns. Goal of this method is to find a portfolio with minimum variance. The minimum-variance portfolio is represented in Figure D.1.

Figure D.1: Minimum Variance Portfolio.



In this figure, the tangent line to the efficient frontier (upward curve of the green shaded area) is the combination of assets which optimizes the expected return for its level of risk. As a result, it has the highest expected Sharpe ratio.

Luenberger (2009) formulates the minimum variance optimization problem mathematically as:

$$
\begin{aligned}
&\min \sum_{i,j=1}^{n} w_i * w_j * \sigma_{ij} \\
&s.t. \\
&\sum_{i=1}^{n} w_i * \bar{r}_i = \bar{r} \\
&\sum_{i=1}^{n} w_i = 1 \\
&w_i \geq 0
\end{aligned}
\tag{D.1}
$$

This optimization problem finds the minimum variance portfolio out of an n assets investment universe. Where σ_{ij} is the covariance of two assets for $i,j=1,2,\ldots,n$, $w_i =1,2,\ldots,n$ represents the set of weights of the portfolio that sum to 1 (fully invested) and has an average return of $\bar{r}$. Weights of the assets must be equal or larger than zero, this additional constraint is attributable to the impossibility of short selling. However, the large numbers of zero elements make it difficult to solve this minimization problem, due to the non-invertibility of the covariance matrix.

For simplicity's sake, the minimum variance portfolio can be composed according to the single factor model of Clarke et al. (2010). They showed that the long-only minimum variance portfolio is strictly populated by stocks with betas lower than a specified threshold and can be determined analytically using a single factor model:

$$
w_i = \frac{\sigma_{LMV}^2}{\sigma_i^2} * \left(1 - \frac{\beta_i}{\beta_L}\right) \qquad \text{for all } \beta_i < \beta_L \tag{D.2}
$$

Where,

- w_i is the optimal weight for security i.
- σ^2_{LMV} is the variance of the long-only minimum variance portfolio.
- σ^2_i is the idiosyncratic variance for security i.
- β_i is the market beta for security i.
- β_L is the long-only threshold beta.

Individual weights depend on the volatility of the long-only minimum variance portfolio and the threshold beta. The cross-sectional variation in the individual weights is exclusively driven by its relative idiosyncratic variance and relative beta. High idiosyncratic variance drives the weight of the particular security towards zero, whereas a high beta excludes the security from the portfolio (if $\beta_i \geq \beta_L$). Thus securities are picked on their beta and weighted in inverse proportion to their volatility.

This minimization problem can be solved through a heuristic trial and error method or via linear programming. The heuristic trial and error method finds the long-only minimum variance portfolio by trying out various values for the long-only threshold beta which is continued until success, i.e., the long-only minimum variance portfolio is achieved. The long-only threshold beta can be approximated by taking one standard deviation below the mean beta, typical values of long-only threshold beta are 0.8.

D.2 Fat-Tailed Portfolio

Disadvantage of most risk measures is that they are not sensitive to tail information, this ignores the probability of significant losses. Idea behind measuring tail risk is that former research has shown that more risk-taking activities are not rewarded by more return. Generally, risk is determined by non-tail sensitive methods like volatility, beta or minimum variance. However, is tail risk rewarded by more return? Most assets exhibit non-normal returns, indicating that tail risk is important to consider in addition to volatility or beta. Relatively simple ways to 'unhide' tail information are skewness and kurtosis.

Skewness

Skewness measures the asymmetry of a distribution. Positively skewed distributions indicate long right tails and relatively large positive values. On the other hand, negatively skewed distributions have long left tails and relatively large negative returns. Population skewness of a securities return is measured by the third standardized moment:

$$\frac{1}{n} * \sum_{i=1}^{n} \left(\frac{r_i - \bar{r}}{\sigma} \right)^3 \tag{D.3}$$

Negative skewness often indicates a heavy left tail resulting in a more tail-risky investment.

Kurtosis

Kurtosis measures the 'peakedness' of a distribution and the heaviness of tails. The kurtosis of a normal distribution is 3, but often set equal to zero. This adjusted kurtosis is easier to interpret and is known as excess kurtosis.

Negative excess kurtosis is called platykurtic and has a wider and lower peak with thinner tails. Positive excess kurtosis is called leptokurtic and has smaller and higher peaks with fatter tails. The higher the kurtosis, the more risky a stock is in terms of equity risk. Population excess kurtosis of a stock's return is measured by the fourth standardized moment minus 3:

$$\left(\frac{1}{n}*\sum_{i=1}^{n}\left(\frac{r_i-\bar{r}}{\sigma}\right)^4\right)-3 \qquad \text{(D.4)}$$

D.3 Volatility Forecasting

The historical average volatility, simply the average volatility over an observed frequency, gives an indication of the riskiness of the stock in the future, but we have to realize that there is an inevitable trade-off between the interesting properties of volatility and the error term. Using a large number of observations will cause loss of volatility properties, too little data will cause a large error term. Unfortunately, this simple model produces relatively poor estimations of future volatility. Therefore we need a more specific estimator.

One of the characteristics of financial volatility is that it has a tendency to cluster, it exhibits persistency in its movements. This is first noticed by Mandelbrot (1963, p.418), he finds that: "Large changes in stock prices tend to be followed by large changes of either sign, whereas small changes tend to be followed by small changes of either sign". This characteristic provides the idea for creating mathematical techniques to forecast volatility, either for a single asset or a collection of assets.

Since the late 70's, different mathematical methods evolved to forecast financial volatility. Results presented by Engle (1993) suggest that financial market volatility is predictable when it varies over time. Volatility forecasts can be made either for a single asset or a portfolio of assets. These findings could help risk-averse fund managers to avoid assets whose volatilities are predicted to increase. But be aware of the fact that, as stated by Engle (1993, p.72):

“Volatility forecasting is a little like predicting whether it will rain; you can be correct in predicting the probability of rain, but still have no rain”. The best volatility forecasting model will minimize the error term. The error term is the difference between the estimated and realized values and can be defined as the Mean Squared Error (*MSE*):

$$MSE = \frac{1}{N} * \sum_{i=1}^{N} (h_t - \sigma_t)^2 \qquad \text{(D.5)}$$

Where h_t is the estimated volatility and σ_t is the realized volatility at time t for N observations.

EWMA model

The Exponential Weighted Moving Average (EWMA) historical volatility model method tries to recognize volatility clustering by giving more weightage to more recent observations. Although this model is not the most sophisticated one, the attraction is that it can offer relatively powerful volatility forecasts and is simple to implement, these two properties can be of great value for portfolio managers. EWMA is mathematically given by:

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)r_{t-1}^2 \qquad \text{(D.6)}$$

Where σ^2_t is today’s variance, λ is the decay factor and r^2_{t-1} is last period’s squared return. The factor loading must be chosen such that it minimizes the error term. JP Morgan’s Riskmetrics model uses λ=0.94 for daily volatility. The larger (smaller) the decay factor is, the more (less) weightage it assigns to historical observations.

GARCH Model

The Generalized Autoregressive Conditional Heteroscedasticity (GARCH) Model is a more sophisticated model to forecast short-term volatility. Autoregression means that the variance of tomorrow is a regressed function on itself, conditional comes from the fact that tomorrow’s variance depends on today’s variance. Heteroskedasticity means that the volatility is not constant over time. Andersen & Bollerslev (1998) concluded that, although numerous other studies suggested that ARCH cannot provide reliable volatility estimates, standard volatility models do provide accurate volatility forecasts on a short term horizon. GARCH gives, like the EWMA method, more weights on more recent information, difference is that GARCH is mean reverting (generalized part), meaning that the function decays towards its long-run average depending on its rate of decay (persistence). The GARCH (p, q) model refers to the fact that the regression is based on the number of steps in squared returns (*p*) and the number of steps in historical variances (*q*). The GARCH (p,q) model is given by:

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i r_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \qquad \text{(D.7)}$$

Where ω is the weighted long run variance and the persistency is given by $\alpha+\beta$. The infinite persistent process occurs when $\alpha+\beta=1$, meaning that the function will not return to its long-run average. Problems occur when extending the forecast horizon. In a stationary GARCH model memory decays exponentially fast. The model is, due to this property, not really suitable for the long-time vision of the portfolio managers.

FIGARCH Model

Baillie et al. (1996) developed an extension of the GARCH model, called Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity (FIGARCH) model. Main advantage of this technique is the long memory property which fits observed economic and financial time series better. In contrast to the stationary property of the GARCH (1, 1) process, Baillie et al. (1996, p.11) emphasized: “Where shocks to the conditional variance either dissipates exponentially or persist indefinitely; for the FIGARCH (1,d,0) model the response of the conditional variance to past shocks decays at a slow hyperbolic rate”. Further explanation of this model goes beyond the purpose of this appendix, but there are several software packages available which are able to determine the parameters of this model.