BACHELOR THESIS

MARCH 5, 2014

BIRDS OF A FEATHER FLOCK TOGETHER

An Analysis of Starling Swarm Intelligence in the StarDisplay Model

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Foreword

In your hands, you hold the result of a half year's work on my bachelor assignment, my thesis. My entire life my teachers, my parents and older brothers and have told me that school and later study would some day stop being easy. That I would see a time that I would have to start working for my goals instead of freewheeling my way through life.

They were right.

The past few months I have learned that for some things there is no shortcut, no easy way. But I have learned a far more important lesson. I have seen that most of the times, the long way, the hard way is the more valuable one. That there is great satisfaction to be obtained in doing something hard. And that working for your goals is really a lot of fun. So I would like to thank all the people that have been telling me this for all those years. I would like to thank them for their patience and their trust. A special word of gratitude to the gentlemen of my Exam Committee, each of whom has been a great motivator, both during my bachelor assignment and the bachelor itself, even if it didn't always show. I hope you will enjoy reading this thesis as much as I enjoyed writing it.

Abstract

Flocks of starlings have been known to show intricate aerial displays called murmurations. In the article "Self-organized aerial displays of thousands of starlings: a model", that was published in Behavioral Ecology as of the 11th of October 2010, H. Hildenbrandt, C. Carere and C.K. Hemelrijk explain the typical swarm behavior as the effect of self organization. They present the StarDisplay model, that can be used to describe the roosting behavior of a flock of starlings in terms of swarm intelligence. The model defines simple rules for the birds, in an attempt to mimic real life flight behavior. In this report, the effects of these rules are investigated. We have found three problems with the StarDisplay model and propose adaptations to solve these. The model uses Tait-Bryan angles to define the orientation of a bird in space. If the orientation is vertical, the heading and banking axes become parallel and the system loses one degree of freedom. This is solved by adding a driven gimbal or by using a different representation for the orientation. For this model, the Tait-Bryan angles suffice, so no adaptation is proposed. The updating mechanism for the interaction range lacks a lower boundary, but a radius can not be negative. Moreover, the radius does not stabilize when the desired number of partners is reached, but instead decreases. The absence of a lower boundary can be disregarded if the time step is sufficiently small. The limit radius is fixed with a minor alteration to the interaction range control. The banking angle control causes the birds to spin violently, instead of gradually banking toward their goal. In addition, the model makes use of infinite accelerations, which disagrees is physically impossible. Newton's second law for rotational motion is used to rewrite the updating mechanism. The proposed adaptations to the StarDisplay do not change the behavior of the model in the tested situation. They are an improvement because they impose boundaries and prevent discontinuities. Further research is needed to verify that the study done in the article can be reproduced using the adapted StarDisplay model.

1. Introduction

For ages, people have been intrigued by the swarming behavior of animals. Ant colonies, bee hives, schools of fish and flocks of birds all show a collective 'consciousness' that causes the population to behave as a single organism[4]. In popular culture, especially in science fiction, the concept of a hive mind has been exploited to great extent. The Borg from Star Trek have a hive mind and the television series Doctor Who features several alien races that communicate through a collective consciousness, such as the Cybermen and the Ood. In scientific context, this phenomenon is often referred to as swarm intelligence.[3] This term is typically used for systems of a large number of individuals that interact with each other and their environment to create a swarm. The behavior of the individuals is defined by simple rules that only work locally. These local interactions between the individuals lead to 'intelligent' global behavior.

Flocks of starlings have been known to show intricate aerial displays called murmurings. For a long time, the cause of this behavior has been subject of research.[2, 5] Even telepathy has been attributed to the animals to explain their flight patterns.[13] In the article "Self-organized aerial displays of thousands of starlings: a model"[8], that was published in Behavioral Ecology as of the 11th of October 2010, H. Hildenbrandt, C. Carere and C.K. Hemelrijk explain the typical flock behavior as the effect of self organization. In the article, they present the StarDisplay model, that can be used to describe the roosting behavior of a flock of starlings in terms of swarm intelligence.

The model defines the forces for each bird, drawing it toward its nesting place and maintaining comfortable flight. Moreover, it introduces interaction forces, controlling relative spacing and orientation of the birds to model the reaction of the birds to their neighbors. In their article, the team shows that the typical flock behavior can be modeled as a collection of self organized systems. In this report, we will make an analytic mathematical and physical analysis of the StarDisplay model and suggest improvements where it fails. The model will be reproduced and analyzed numerically using MATLAB R2012a.

2. The StarDisplay model

In this chapter, we will present a thorough analysis of the StarDisplay model. First, the system coordinates and the state space will be specified. After that, the forces that the birds experience will be evaluated individually. Here, the birds are considered as fixed wing aircraft. All forces are defined for a single bird.

The behavior of a single individual in the flock is determined by several variables. These are the velocity of the bird and its position with respect to the roost. Flock behavior is obtained by interaction between birds, which depends on their relative distance and orientation.

System coordinates

There are two relevant reference frames that are used in this model, the global coordinate system and a local one. Both coordinate systems are classic right handed Cartesian systems. In the global frame, the x-y-plane depicts the earth's surface and the z-axis points upward. The local reference frame is found by rotating the global frame around three elemental axes using the Tait-Bryan angles, as shown in Figure 2.1b[15]. The three angles that are used are the heading, α, the elevation, γ and the banking angle, β.

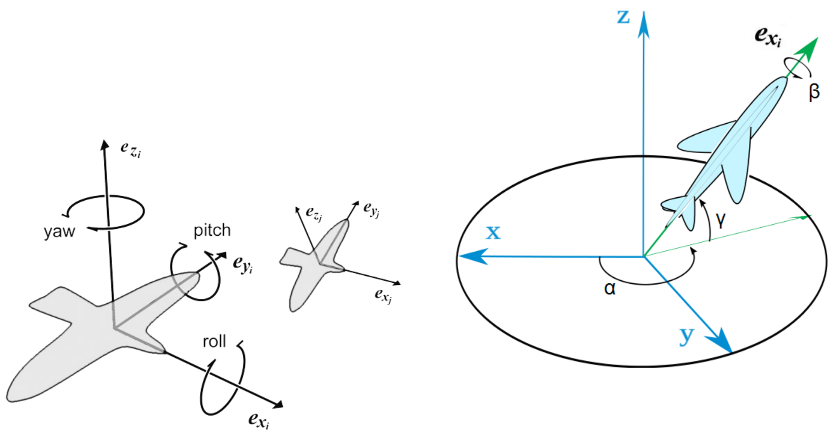
In the local reference frame, the forward direction for an individual i is called $\mathbf{e_{x_i}}$. The lateral and upward direction are called $\mathbf{e_{y_i}}$ and $\mathbf{e_{z_i}}$ respectively, where $\mathbf{e_{y_i}}$ is defined positive over the left wing of the bird, as can be seen in Figure 2.1a[8]. Rotation around the local upward axis $\mathbf{e_{z_i}}$ is called yaw, whereas rotation around the lateral axis $\mathbf{e_{y_i}}$ is called pitch. When the aircraft rotates around the longitudinal axis $\mathbf{e_{x_i}}$ it is called roll. The heading, elevation and banking convention will be used to specify the orientation of the bird in space, whereas the yaw, pitch and roll will be used to indicate the rotating action.

State space representation

The position and velocity of a bird is represented in the global reference frame by the position vector $\mathbf{p} = \left(\begin{smallmatrix} x \\ y \\ z \end{smallmatrix}\right)$ and velocity vector $\mathbf{v} = \left(\begin{smallmatrix} v_x \\ v_y \\ v_z \end{smallmatrix}\right)$. The orientation is represented by the angles α, β and γ. The heading and elevation depend on the elemental velocities by

$$\tan(\alpha) = \frac{v_y}{v_x} \quad \text{and} \quad \tan(\gamma) = \frac{v_z}{\sqrt{v_x^2 + v_y^2}}. \tag{2.1}$$

The banking angle β is an independent variable. It is assumed that for each moment in time, the acceleration of the bird depends only on the force applied to it. As will be explained later, the force that the bird experiences depends on the orientation of the bird, and therefore on β, as well as the position and velocity. This means that there are seven state space variables and the system can be written in state space representation as



(a) The local coordinate systems of two birds, i and j. The forward direction for an individual i is called $\mathbf{e_{x_i}}$, $\mathbf{e_{y_i}}$ is the lateral direction and $\mathbf{e_{z_i}}$ is the upward direction. Rotation around $\mathbf{e_z}$ is called yaw, rotation around $\mathbf{e_y}$ is called pitch and rotation around $\mathbf{e_x}$ it is called roll.[8]

(b) The local reference frame as shown in Figure 2.1a is found by rotating the global x-y-z frame by the Tait-Bryan angles. The angle α indicates the heading, γ the elevation and β is called the banking angle.[15]

Figure 2.1.: System coordinates.

$$\begin{pmatrix} \dot{\mathbf{p}} \\ \dot{\mathbf{v}} \\ \dot{\beta} \end{pmatrix} = f(\mathbf{p}, \mathbf{v}, \beta), \tag{2.2}$$

where $\dot{\mathbf{p}} = \mathbf{v}$. Using Newton's second law of motion we can write $\dot{\mathbf{v}} = \mathbf{a} = \frac{\mathbf{F}}{m}$, where $\mathbf{F}$ is the sum of all forces and m is the mass of the bird. The system can then be solved numerically, for instance, using the Euler method as follows:

$$\begin{aligned} \mathbf{v}(t + \Delta t) &= \mathbf{v}(t) + \frac{\mathbf{F}(t)}{m}\Delta t; \\ \mathbf{p}(t + \Delta t) &= \mathbf{p}(t) + \mathbf{v}(t + \Delta t)\Delta t; \\ \beta(t + \Delta t) &= \beta(t) + \beta_{in} - \beta_{out}. \end{aligned} \tag{2.3}$$

Here, Δt is the size of the time step. β_{in} and β_{out} are both angles that depend on Δt and control the banking angle, as will be explained later. The force $\mathbf{F}$ consists of two elements: $\mathbf{F}_{\text{Steering}}$ and $\mathbf{F}_{\text{Flight}}$. The flight force models the aerodynamics of the "soulless" bird and determines the reaction of the bird to the air. The steering force acts as the soul of the bird and controls the reaction to the environment. The steering force and the flight force are added to obtain the total of the experienced forces

$$\mathbf{F} = \mathbf{F}_{\text{Steering}} + \mathbf{F}_{\text{Flight}}. \tag{2.4}$$

Both $\mathbf{F}_{\text{Steering}}$ and $\mathbf{F}_{\text{Flight}}$ consist of several sub-elements and will be explained later.

Flight force

It is assumed that the flight dynamics of the birds follow the standard equations for fixed wing aircraft. These equations link the upward force, or lift L and the air friction, or drag D generated by the wings to the air speed v of the aircraft. Using the standard equations[1], we obtain

$$L = \frac{1}{2}\rho S v^2 C_L; \quad D = \frac{1}{2}\rho S v^2 C_D, \tag{2.5a}$$

where ρ is the density of the air, S is the surface area of the wings and C_L and C_D are respectively the lift and the drag coefficient. The air speed v is the absolute velocity of the bird and is defined as the norm of $\mathbf{v}$.

To obtain constant horizontal flight, the lift should equal the gravitational force and the thrust should equal the drag, balancing vertical and horizontal forces respectively. Using that, we can find the cruising speed v_0 at which a bird will fly at constant height. The thrust T_0 that has to be provided for constant speed can be found with this cruising speed by

$$L_0 = \frac{1}{2}\rho S v_0^2 C_L = mg; \quad D_0 = \frac{1}{2}\rho S v_0^2 C_D = T_0. \tag{2.5b}$$

The actual lift and drag forces are rewritten to minimize the number of variables and are defined as follows,

$$L = \frac{v^2}{v_0^2} L_0 = \frac{v^2}{v_0^2} mg; \quad D = \frac{C_D}{C_L} L = \frac{C_D}{C_L}\frac{v^2}{v_0^2} mg. \tag{2.5c}$$

The resultant flight force is found by adding lift, drag, thrust and gravitation. Here, the lift is directed towards the local upwards direction e_z and gravity towards the global downwards direction. Thrust points in the forward direction e_x and is opposed by the drag force, as is illustrated in Figure 2.2[7]. The resultant flight force is

$$\mathbf{F}_{\text{Flight}} = (\mathbf{L} + \mathbf{D} + \mathbf{T}_0 + m\mathbf{g}), \tag{2.6a}$$

where

$$\begin{aligned} \mathbf{L} &= L\mathbf{e}_z, \\ \mathbf{D} &= -D\mathbf{e}_x, \\ \mathbf{T}_0 &= T_0\mathbf{e}_x, \\ \mathbf{g} &= (0, 0, -g). \end{aligned} \tag{2.6b}$$

Both thrust and weight are assumed constant in this model, whereas lift and drag both depend solely on the velocity. The effect of the flight forces can therefore be evaluated by taking different initial velocities.

When flying in constant level flight with cruising speed, the lift and weight cancel each other by design, as do drag and thrust. The resulting force $\mathbf{F}_{\text{Flight}}$ is then zero and the bird will fly with constant velocity in the initial direction. The experiment is done by defining the initial position in the origin and the initial velocity equal to the cruising speed in x-direction. As can be seen in Figure 2.3, the bird continues moving in the x-direction with constant velocity.

If the initial velocity is lower than the cruising speed, gravity will outweigh the lift and the bird will descend. The drag will be smaller than the thrust, causing the bird to speed up. This effect is shown in Figure 2.4, where the initial position was defined in the origin with a velocity lower than cruising speed. As predicted, the velocity increases very quickly before stabilizing at the cruising speed. The height decreases and the bird slowly returns to level flight. If the initial velocity is higher than the cruising speed, the lift will be larger than the weight, causing the bird to rise. On the other hand, the drag will exceed the thrust, so that the bird will slow down. For small deviations from the cruising speed, the bird will react in a similar but opposite way as for a low initial velocity. For a very large initial velocity however, a problem arises. The elevation

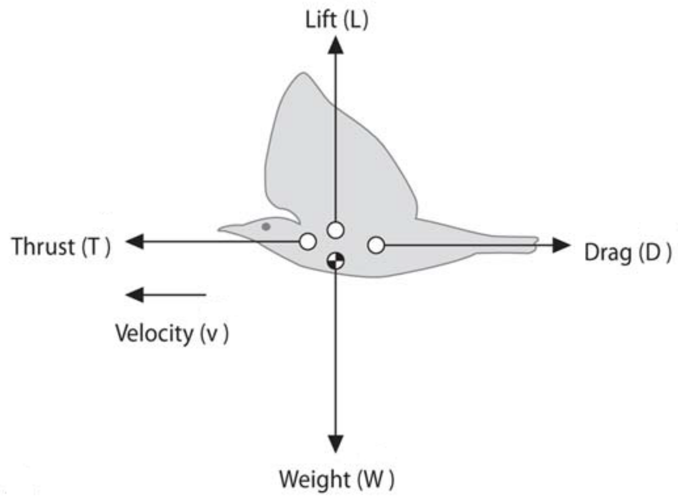
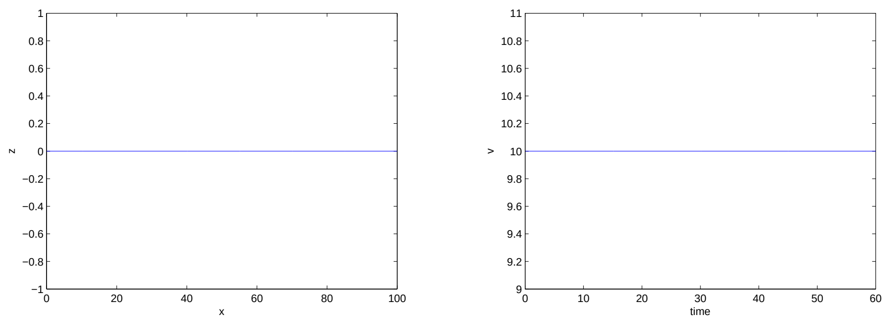


Figure 2.2.: The flight force $\mathbf{F}_{\text{Flight}}$ consists of four elements: lift, drag, thrust and weight. The lift is directed towards the local upwards direction e_z and gravity towards the global downwards direction. Thrust points in the forward direction e_x and is opposed by the drag force.[7]

increases until the bird flies straight upwards. At that point, the v_x and v_y are both zero, so the heading angle α is undefined. This is the result of a common problem that occurs when using Tait-Bryan angles, called gimbal lock. This is the loss of a degree of freedom that occurs when two of the three rotation axes align with each other. A solution to the problem of gimbal lock is given in Chapter 3.

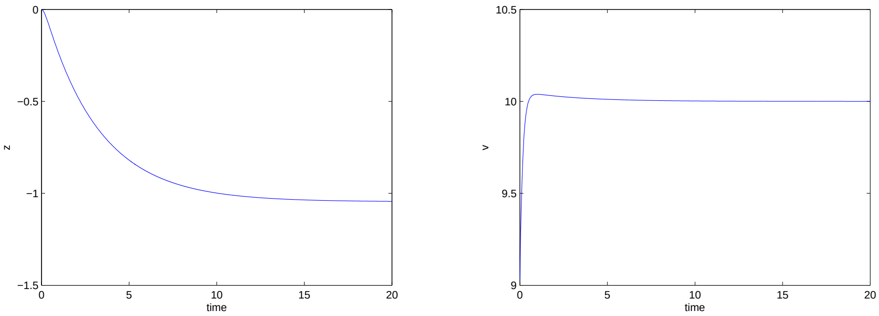
Banked turns

A relevant phenomenon in fixed wing aerodynamics is banking, in which the aircraft changes its roll to perform a lateral translation. As can be seen in Figures 2.1b and 2.6 [7], the banking angle β is defined as the angle between the local and global upward z axis. The lift is directed towards the local upward direction, so if β is not equal to zero,



(a) Position (b) Velocity

Figure 2.3.: When flying with cruising speed and in level flight, the flight force will be zero and the bird will continue with constant velocity in the initial direction. Here, the initial state is defined as $\mathbf{v} = (v_0, 0, 0)^T$, $\mathbf{p} = (0, 0, 0)^T$ and $\beta = 0$



(a) Position (b) Velocity

Figure 2.4.: When the bird has an initial velocity that is lower than the cruising speed, gravitational will be stronger than the lift and the bird will descend. The thrust will exceed the drag, so the bird will gain speed. The initial state is $\mathbf{v} = (v_0 - 1, 0, 0)^T$, $\mathbf{p} = (0, 0, 0)^T$ and $\beta = 0$, and v_0 is 10 ms^{-1}.

the lift will have a horizontal component, as is shown in Figure 2.6. The horizontal component will then act as a centripetal force and make the bird curve. It is notable that when flying with cruising speed while banking, the effective lift, L_{eff}, will be smaller than gravity and the aircraft will descend.

If the banking angle is constant, the centripetal force ensures circular motion. Since the effective lift is lower than the weight, the bird will drop and the resulting path will be a downward helix, as is shown in Figure 2.5. If both the angle and the velocity are known, we can use Newton's second law of motion to determine the radius of curvature of the helix by defining the centripetal acceleration $a_c$as

$$a_c = \frac{C_p}{m} = \frac{L\sin(\beta)}{m} = \frac{v^2 g\sin(\beta)}{v_0^2}, \tag{2.7}$$

where C_p is the centripetal force. The radius can be found by inserting this acceleration in the standard equation for circular motion,

$$r = \frac{v^2}{a_c} = \frac{v_0^2}{g\sin(\beta)} \tag{2.8}$$

This shows that the velocity with which a banked turn is made has no effect on the radius of curvature that is obtained.

Steering force

The steering force **F**steering is the sum of several forces that all represent a specific action. It encompasses both individual forces as well as the interaction between birds that creates the characteristic flocking behavior. Individual forces are the speed control, the attraction to the roost and a random force, all of which will be explained in the following sections. The interaction between birds will be elaborated upon later on.

Speed control

The forward velocity is controlled with a force $\mathbf{f}_\tau$ that brings back an individual to its cruising speed v_0 after it has deviated from it by

$$\mathbf{f}_\tau = \frac{m}{\tau}(v_0 - v)\mathbf{e}_x. \tag{2.9}$$

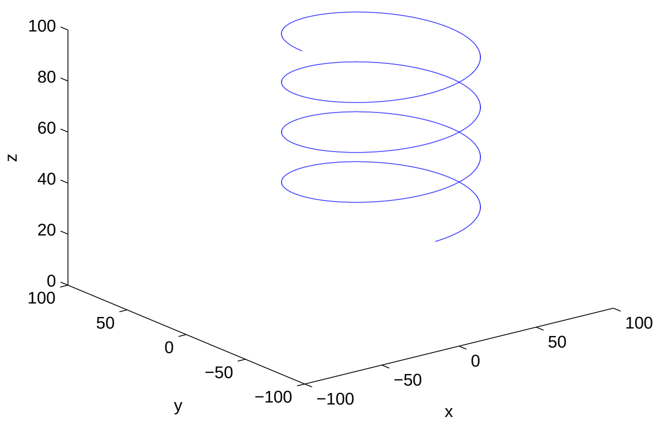


Figure 2.5.: If the banking angle is constant, there will be a constant centripetal force and the bird will fly in a circle. Since the effective lift is lower than the weight, the bird will also drop. The initial state $\mathbf{v} = (0, 2v_0, 0)^T$, $\mathbf{p} = (-25, 0, 100)^T$ and $\beta = 0.2$ results in a downward helix, as predicted.

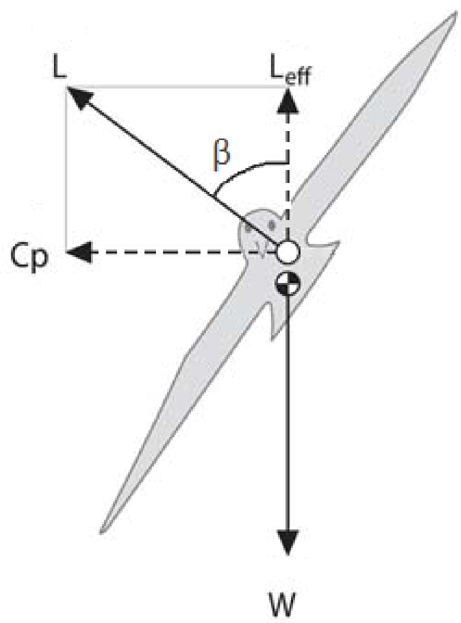


Figure 2.6.: If the banking angle β is not equal to zero, the lift has a horizontal component C_p that works as a centripetal force and pulls the bird into a circular path.[7]

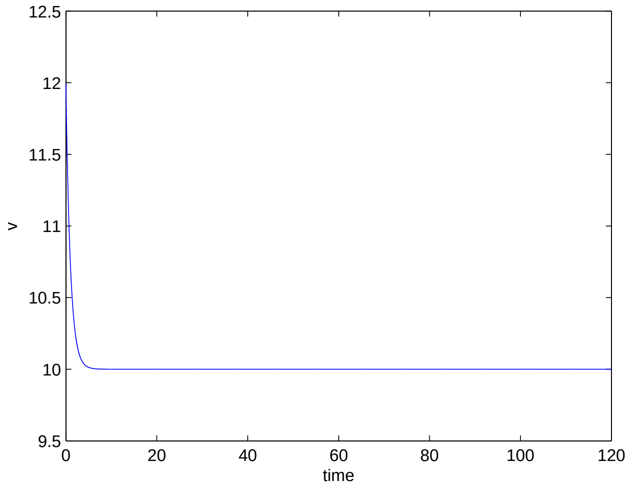


Figure 2.7.: If only speed control is applied, the velocity of a bird will approach the cruising speed exponentially. The initial state for this experiment was $\mathbf{v} = (v_0+2, 0, 0)^T$, $\mathbf{p} = (0, 0, 0)^T$ and $\beta = 0$ and only the speed control was modeled.

Here τ is the time constant for a deviated speed v returning to the cruising speed. This force is is always pointed in the forward direction by multiplying it by $\mathbf{e}_x$ The speed control depends only on the velocity of the bird. Using Newton's second law of motion, we can write this as the first order linear differential equation,

$$\dot{v} = \frac{(v_0 - v)}{\tau}. \tag{2.10}$$

The general solution for this equation is given as

$$v = C \exp(\frac{-t}{\tau}) + v_0, \tag{2.11}$$

where C depends on the initial velocity. This means the velocity will approach the cruising speed exponentially, as can be seen in Figure 2.7, where only the speed control has been taken into account. There can be seen that the system with initial state $\mathbf{v} = (v_0 + 2, 0, 0)^T$, $\mathbf{p} = (0, 0, 0)^T$ and $\beta = 0$ will return to the equilibrium position exponentially.

Altitude control

Roosting flocks tend to form a flat shape above the nesting place, which suggests that there is a certain height at which the birds prefer to fly (called the zero level). The altitude control force is proportional with the vertical distance from the zero level and always points downward,

$$\mathbf{f}_{alt} = -w_{alt}(\text{vertical distance})\mathbf{z}; \quad \mathbf{z} = (0, 0, 1)^T, \tag{2.12}$$

where w_{alt} is the weighting factor for this force. We assume that by vertical distance is meant the vertical position relative to the zero level, $z_i - z_0$, rather than the absolute distance. Using the vertical distance would result in a downward force, regardless whether the bird is flying above or below the zero level. The correct definition of the altitude control is

$$\mathbf{f}_{alt} = -w_{alt}(z - z_0)\mathbf{z}; \quad \mathbf{z} = (0, 0, 1)^T. \tag{2.13}$$

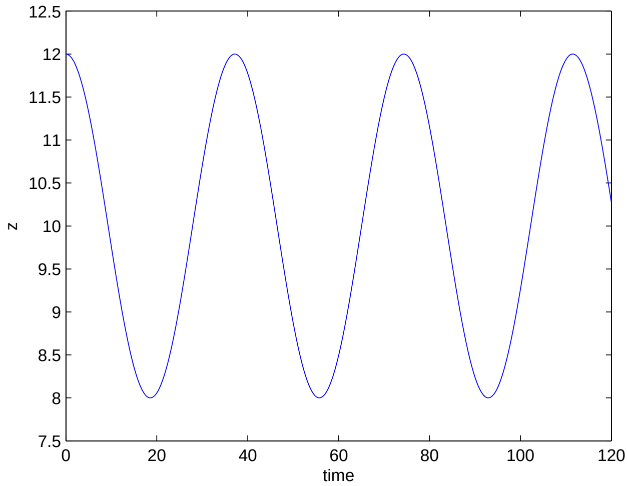


Figure 2.8.: When only altitude control is applied, the solution to the system will oscillate around the zero level. Here, the initial state of the system is given by $\mathbf{v} = (v_0, 0, 0)^T$, $\mathbf{p} = (0, 0, z_0 + 2)^T$ and $\beta = 0$

The altitude control depends only on the altitude of the bird. Since the force depends on the distance to the zero level, the force's effect can be described with the second order differential equation,

$$m\ddot{z} = w_{alt}(z_0 - z). \tag{2.14}$$

The solution for this equation is given as

$$z = C_1 \cos(\sqrt{\frac{m}{w_{alt}}}t) + C_2 \sin(\sqrt{\frac{m}{w_{alt}}}t) + z_0, \tag{2.15}$$

where C_1 and C_2 depend on the initial height z and vertical velocity v_z of the bird. This is the general solution for a simple harmonic oscillator, which means that when isolated, this force will cause the bird's altitude to oscillate around the zero level, as is shown in Figure 2.8.

Roost attraction

We observe the dynamics of a flock that gathers around their nesting place, or roost. Each individual is drawn towards the roost by the roost attraction force $\mathbf{f}_{Roost}$, which is defined as

$$\mathbf{f}_{Roost} = \pm w_{Roost} \left(\frac{1}{2} + \frac{1}{2} \langle \mathbf{e}_x, \mathbf{n} \rangle \right) \mathbf{e}_y. \tag{2.16}$$

It models the bird's desire to stay close to the roost, which is taken in the origin for simplicity. The roost attraction force $\mathbf{f}_{Roost}$ always points in the lateral direction $\mathbf{e}_y$. It is found by taking the dot product of the forward direction and $\mathbf{n}$, where $\mathbf{n}$ is the horizontal outward pointing normal unit vector of the roost boundary. Since both $\mathbf{n}$ and $\mathbf{e}_y$ are unit vectors, the result of the dot product lies between -1 and 1. The range is then transformed to $[0, 1]$ by halving the dot product and adding $\frac{1}{2}$. The sign is chosen to turn the bird towards the roost.

The roost attraction imposes two actions on the bird. Firstly, it turns the bird perpendicular to the roost, at which point the normal vector and $\mathbf{e}_y$ are orthogonal, so the dot product will be zero. Secondly, it works as a centripetal force, causing the bird to spiral around the roost before stabilizing to a perfect circular path. This is illustrated by Figure 2.9, where the bird was started of in $(20, 1, 10)^T$ with velocity $(10, 0, 0)^T$ and level flight.

The definition of this force is very gawky. The sign of the force is not defined mathematically, but is specified only in text. This can be solved easily by using a mathematical interpretation of the plus-minus sign. This is done by taking the negative dot product of $\mathbf{e}_y$ and $\mathbf{n}$. This turns the force towards the roost if it flying away from it and vice verse. The attraction force is then defined as

$$\mathbf{f}_{Roost} = -\operatorname{sgn}(\langle \mathbf{e}_y, \mathbf{n} \rangle) w_{Roost} \left(\frac{1}{2} + \frac{1}{2} \langle \mathbf{e}_x, \mathbf{n} \rangle \right) \mathbf{e}_y. \tag{2.17}$$

If the bird is flying away from or directly toward the roost, the dot product of $\mathbf{e}_y$ and $\mathbf{n}$ is zero and the bird will not change its direction. Both situations are unstable equilibria, so when other forces are taken into account, that problem will be solved automatically.

In steady state, the dot product is equal to zero, so the roost attraction force can be defined as

$$\mathbf{f}_{Roost} = -\operatorname{sgn}(\langle \mathbf{e}_y, \mathbf{n} \rangle) \frac{1}{2} w_{Roost} \mathbf{e}_y. \tag{2.18}$$

Using the same method used in Equations (2.7) and (2.8), we can find the steady state radius as

$$r = \frac{v^2}{a_c} = \frac{m v_0^2}{\|\mathbf{f}_{Roost}\|} = \frac{2 m v_0^2}{w_{Roost}} = 1600\text{m}. \tag{2.19}$$

The radius of the roost R_{roost} is defined as 150 meter, so a steady state radius of 1600 meter is not reasonable. This can be solved by taking w_{Roost} so that

$$r = \frac{2 m v_0^2}{w_{Roost}} = 150. \tag{2.20}$$

Random force

Errors in perception and steering are modeled with a random force $\mathbf{f}_\xi$,

$$\mathbf{f}_\xi = w_\xi \boldsymbol{\xi}, \tag{2.21}$$

where $\boldsymbol{\xi}$ is a random unit vector. This force bears great resemblance to a random walk in three dimensions. To ensure that the random force is independent of the integration time step Δt, the weighting factor w_ξ must proportional to $\frac{1}{\sqrt{\Delta t}}$. This will be proven for the one dimensional case in Appendix A.

We cannot resist pointing out one interesting property of the random walk. It has been proven that when provided enough time, the one dimensional random walk will almost surely return the origin. In fact, it will cross every point an infinite number of times. This phenomenon is called recurrence or the Gambler's Ruin.[10] The latter name is derived from the following analogue.

Imagine a gambler with a finite amount of money, playing a fair game against the bank. Eventually, the gambler will always lose to the bank. His money balance will follow a random walk and therefore will reach zero at some point in time and the gambler will have lost the game.

The random walk in two dimensions, where the sample space consist of a rectangular grid, is also recurrent. However,

"A drunk man will find his way home, but a drunk bird may get lost forever" -Shizuo Kakutani.

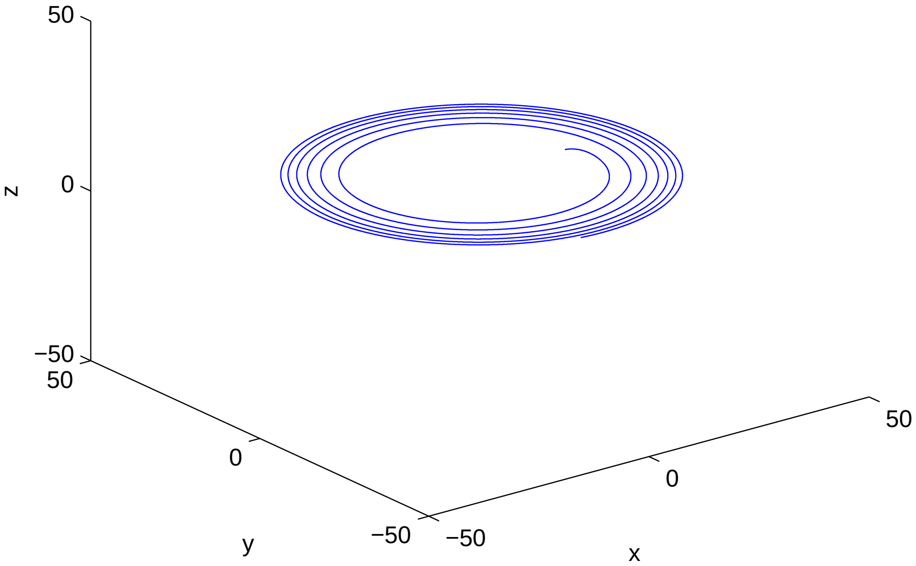


Figure 2.9.: The roost attraction causes the bird to turn its forward direction 'perpendicular' to the roost and spiral toward the steady state radius. In this experiment, the bird started of in $(20, 1, 10)^T$ with velocity $(10, 0, 0)^T$ and level flight.

The drunk man will visit every point on the grid infinitely many times and will eventually find his way home. The bird is not so lucky. Shizuo Kakutani[11], and many others[14, 12], prove that for higher dimensions, the path produced by a random walk is not always recurrent.

Interaction forces

The birds will flock together and start to move as a single organism due to the interaction forces between individual birds. Birds will only interact with others within their interaction radius, instead of the entire flock. The cohesion force makes birds attracted to each other, whereas the separation force will keep the birds at a comfortable distance from each other. The birds will adjust their flight in the same direction as their neighbors by introducing an alignment force. In the following sections, the subscript i denotes the subject individual i and j is an arbitrary bird. Variables that do not have a subscript are quantities, such as the mass m that are assumed constant among all birds.

Adaptive interaction range

Each bird only communicates with a limited number of its neighbors. This is obtained by defining an interaction range that is controlled by the number of individuals within radius R,

$$R_i(t+\Delta u) = (1-s)R_i(t) + s(R_{max} - R_{max}\frac{|N_i(t)|}{n_c}), \tag{2.22}$$

where $N_i(t)$ is the group of interaction partners, n_c is the desired number of partners, and R_{max} is the maximum radius within which the bird can observe its neighbors. The interaction range is updated with a time step Δu, which represents the reaction time of the bird and is not necessarily equal to the integration time step Δt. The range is updated by interpolating between the previous range and $(R_{max} - R_{max}\frac{|N_i(t)|}{n_c})$ with factor s. The group of interaction partners N_i consist of all birds that fall within the interaction range,

$$N_i \equiv \{j \in N; d_{ij} \leq R_i; j \neq i\}, \tag{2.23}$$

where N is the collection of all birds in the system and d_{ij} is the absolute distance between birds i and j.

The behavior of this adaptive interaction range is somewhat disruptive. Firstly, if the actual number of topological interaction partners $|N_i(t)|$ equals the desired number n_c, the interaction range will be updated by interpolating between $R_i(t)$ and 0, thus decreasing the range. The range ought to remain constant if N_i equals n_c. Secondly, if $N_i \gg n_c$, the interaction range can become negative, which is impossible for a radius. However, if the time step Δu is sufficiently small, the radius will decrease with small steps and approach the equilibrium at $|N_i(t)| = n_c$. The updating mechanism for the range will be rewritten to fix the limit radius. The adaptations to the StarDisplay model can be found in Chapter 3.

Separation

Separation is modeled by defining a force $\mathbf{f}_{s_i}$ away from the average direction to the bird's neighbors,

$$\mathbf{f}_{s_i} = -\frac{w_s}{|N_i(t)|}\sum_{i \in N_i(t)} g(d_{ij})\mathbf{d_{ij}}, \tag{2.24a}$$

where $\mathbf{d_{ij}}$ is the unit vector pointing from bird i to the neighbor j and w_s is the weighting factor for separation. The average direction $\sum_{j \in N_i(t)} g(d_{ij})\mathbf{d_{ij}}$ is a weighted

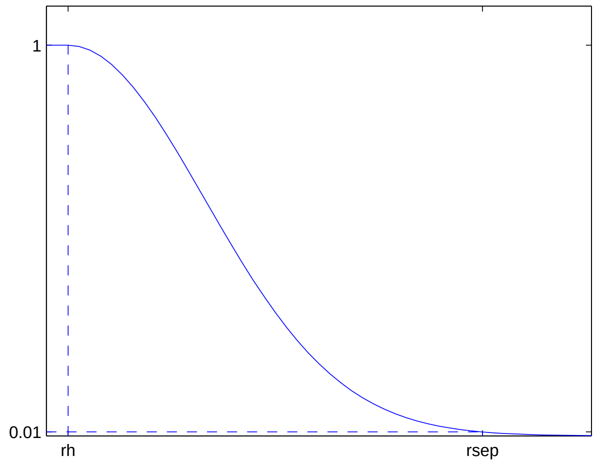


Figure 2.10.: The weighting factor $g(d_{ij})$, given in Equation (2.24b) determines how strongly a bird reacts to his neighbors. It is modeled so that it is almost zero at the edge of the separation zone r_{sep}. If the distance is smaller than r_h, there is no cohesion and birds repel each other.

average of the vectors $\mathbf{d_{ij}}$ with their respectable weighting factors $g(d_{ij})$. The weight that corresponds to the direction vector $\mathbf{d_{ij}}$ is a function of the distance d_{ij} and is given by

$$g(d_{ij}) = \left\{ \begin{array}{ll} 1 & d_{ij} \leq r_h \\ \exp(-\frac{(d_{ij}-r_h)^2}{\sigma^2}) & d_{ij} > r_h \end{array} \right\}, \tag{2.24b}$$

where r_h is the radius within which birds are not attracted to each other. The weighting factor $g(d_{ij})$ is a halved Gaussian with σ chosen so that $g(d_{ij})$ is almost zero at the border of the separation zone r_{sep}. Figure 2.10 shows how $g(d_{ij})$ is distributed.

The effect of the separation force is that the bird will move away from the neighbors within the interaction zone. A singularity occurs when the bird has no neighbors and N_i is equal to zero, so that the force is undefined. If there are no interaction partners, all interaction forces will be assumed zero.

Cohesion

Birds are attracted to each other by a cohesion force $\mathbf{f}_{c_i}$. This force depends on both the center of mass of the group of interaction partners and a degree of centrality C_i. The attraction force is given as

$$\mathbf{f}_{c_i} = C_i \frac{w_c}{|N_i^*|} \sum_{j \in N_i^*(t)} X_{ij} \mathbf{d_{ij}}, \tag{2.25a}$$

where C_i is the centrality of the bird and will be explained in Equation (2.27). Both N_i^* and X_{ij} tighten the criteria for interaction and will be evaluated in the next paragraphs. The force is weighted with a weighting factor w_c.

The center of mass of the group is defined as the average of the distance vectors of a birds neighbors. For the cohesion force, birds do not interact with neighbors that are either too close or that are located in their blind angle. Birds that are too close are not

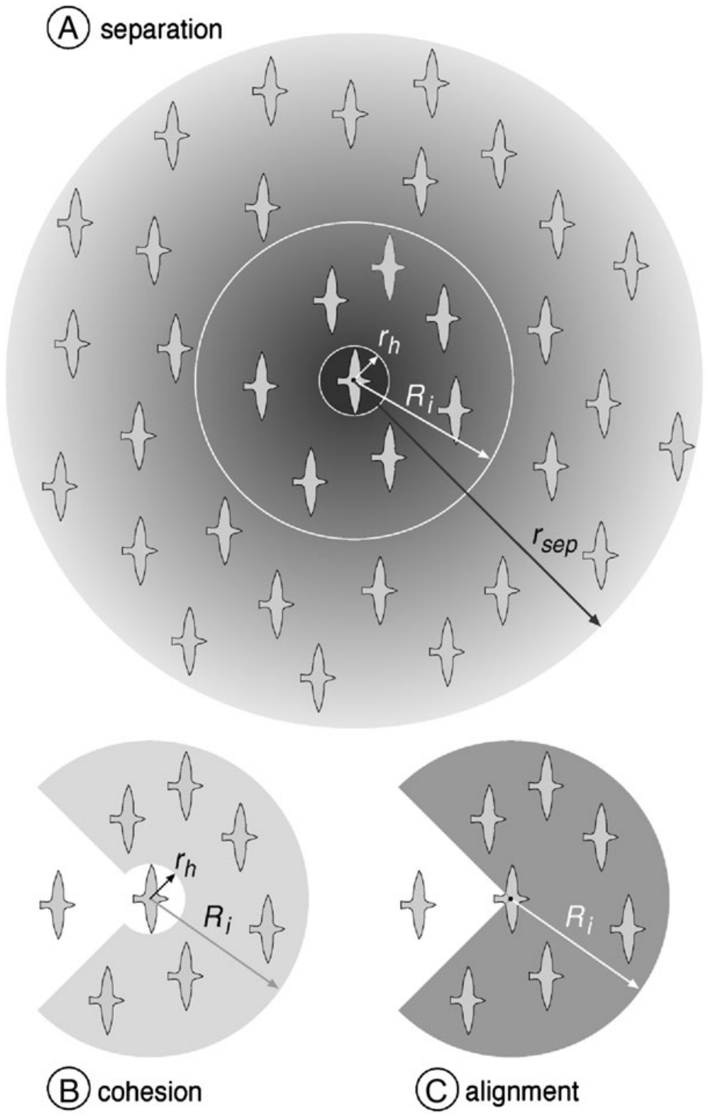


Figure 2.11.: The interaction ranges for separation, cohesion and alignment. For separation, the bird interacts with neighbors within R_i. Cohesion occurs within the same radius, but not with neighbors in the blind angle or that are closer than r_h. The birds that are within the range r_h are taken into account for the alignment force.[8]

counted when finding the center of mass and are omitted by multiplying with

$$X_{ij} = \begin{cases} 0 & \|\mathbf{d_{ij}}\| \leq r_h \\ 1 & \|\mathbf{d_{ij}}\| > r_h \end{cases}. \tag{2.25b}$$

Birds do not communicate with neighbors that they can not see, because they are located in the blind angle. For this, a different set of neighbors, N_i^* is defined as

$$N_i^* = \{j \in N_i; \text{bird } j \text{ not in 'blind angle' of } i\}. \tag{2.26}$$

The concept of centrality will make an individual on the edge of a flock move faster towards its neighbors than individuals that have a more central position. The centrality of a bird is defined as the distance to the center of mass of a sphere with radius N_G, which is twice the interaction radius. All birds within range are counted, including those that are in a bird's blind angle. The centrality of the bird is defined as

$$C_i = \frac{1}{|N_G|} \left\| \sum_{j \in N_G} \mathbf{d_{ij}} \right\|, \text{where} \quad N_G = \{j \in N; d_{ij} \leq 2R_i; j \neq i\}. \tag{2.27}$$

The effect of the separation and cohesion force is evaluated by starting two birds, one behind the other, in the same direction. The bird in the leading position does not interact with the other in terms of cohesion. The following bird experiences both separation and cohesion. The distance between the birds can then be described with the non linear differential equation,

$$\ddot{d}_{ij} = \frac{1}{m}(2w_s \exp(-\frac{(d_{ij} - r_h)^2}{\sigma^2}) - w_c). \tag{2.28}$$

The distance between the birds increases because of separation, which works on both birds and reduced due to cohesion, which is only experienced by one of them. Equation (2.28) is the result of adding two times (2.24a) to (2.25a). This non linear equation is very hard to solve explicitly. In Figure 2.12 can be seen that $g(d_{ij})$ is almost linear around the equilibrium position, so the solution for the differential equation can be found using the Taylor expansion at the equilibrium position,

$$\ddot{d}_{ij} = \frac{1}{m}(2w_s(0.500001 - 0.47037(d_{ij} - 1.67362) - w_c). \tag{2.29}$$

The general solution for this equation is given as

$$d_{ij} = C_1 \sin(3.429t) + C_2 \cos(3.429t) - 2.126w_c + 0.611. \tag{2.30}$$

The effect of the separation and cohesion force is shown in Figure 2.13. Here, two birds were started $2m$ apart on the x-axis, with their directions along the x-axis. It can be verified using Figure 2.12 that the linearisation is indeed valid for this interval.

Alignment

Birds experience a force $\mathbf{f}_{a_i}$ to align themselves with the forward direction of its interaction neighbors.

$$\mathbf{f}_{a_i} = w_a \sum_{j \in N_i^*} (\mathbf{e_{x_j}} - \mathbf{e_{x_i}}) \Big/ \left\| \sum_{j \in N_i^*} (\mathbf{e_{x_j}} - \mathbf{e_{x_i}}) \right\| \tag{2.31}$$

The alignment force is found by taking the sum of the difference in direction for all j and normalizing it. An interesting aspect of this definition is that a neighbor with a slightly different direction has the same impact as one that is aligned very differently, an effect that is the result of the normalizing action in Equation (2.31).

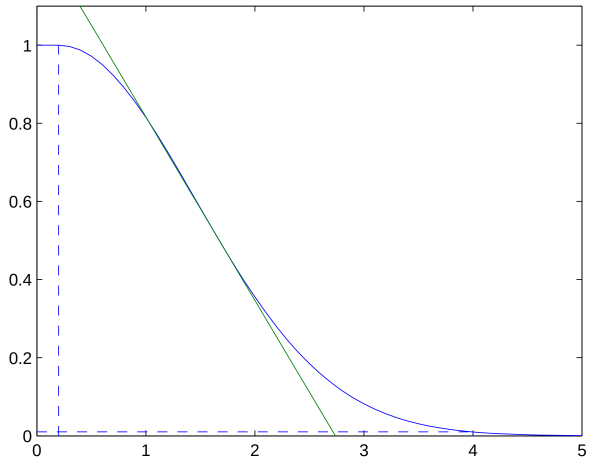


Figure 2.12.: The linear approximation of $g(d_{ij})$][The linear approximation of $g(d_{ij})$ around the equilibrium position. It shows that for small deflections, the linear approximation of the separation force suffices to find the behavior induced by separation and cohesion that is shown in Figure 2.13

Social force

The three social forces –separation, cohesion and alignment– are added to create the resulting social force $\mathbf{F}_{Social}$, that represents the interaction of a bird with the flock,

$$\mathbf{F}_{Social} = \mathbf{f}_s + \mathbf{f}_a + \mathbf{f}_c. \tag{2.32}$$

Steering force

We sum speed control (2.9), social force (2.32), roost attraction (2.16) and the random force (2.21) to obtain the resultant force on the bird induced by the flock, the roost and internal factors. The steering force is defined as

$$\mathbf{F}_{\text{Steering}} = \mathbf{F}_{Social} + \mathbf{f}_{\tau} + \mathbf{f}_{Roost} + \mathbf{f}_{\xi}. \tag{2.33}$$

Banking Angle Control

Figure 2.6 shows how a bird can use the imposed flight forces to its advantage by making a banked turn. The banking angle is controlled by two mechanisms. The inward control β_{in} changes the banking angle to make the bird turn in the direction of the steering force and the outward control β_{out} turns the bird back to level flight, to stop it from turning. The outward control depends on the actual banking angle and a weighting factor and is defined as

$$\tan(\beta_{out}) = w_{\beta_{out}} \sin(\beta)\Delta t. \tag{2.34}$$

The effect of the outward control is shown in Figure 2.14, where a single bird is given a initial banking angle and only outward banking is applied. It is shown that the bird makes a banked turn before rolling back to level flight. As the banking angle returns to level, the curvature of the path falls. Since in banking the effective lift is not enough to support the bird, it turns downwards and the altitude stabilizes only after the bird had returned to level flight.

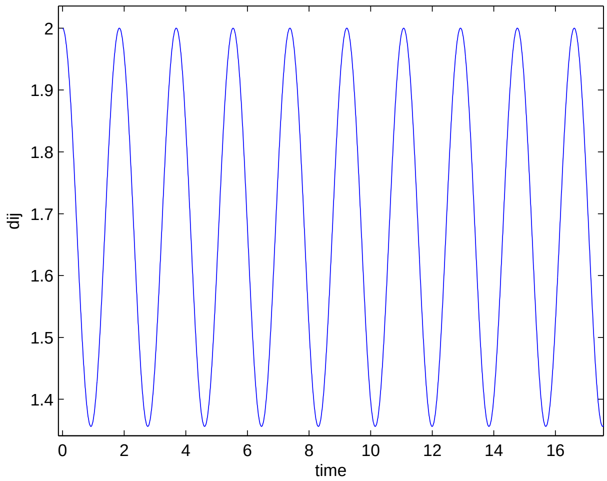


Figure 2.13.: The separation and cohesion force produce an oscillating solution. That is the result of the near linear behavior of the separation force around the equilibrium point. In this experiment, two birds were started $2m$ apart on the x-axis, with their directions along the x-axis.

The bird is turned in the direction of the steering force by updating the banking angle with the inward control angle, which is defined as

$$\tan(\beta_{in}) = w_{\beta_{in}} \|\mathbf{a}_{l_i}\| \Delta t, \tag{2.35}$$

where $w_{\beta_{in}}$ is the weighting factor for inward banking, Δt is the time step used in the Euler updating and $\mathbf{a}_l$ is the lateral acceleration caused by the steering force. The lateral acceleration is defined as the projection of the steering force on the local y-axis, divided by the mass and is multiplied by e_y to point it in the lateral direction,

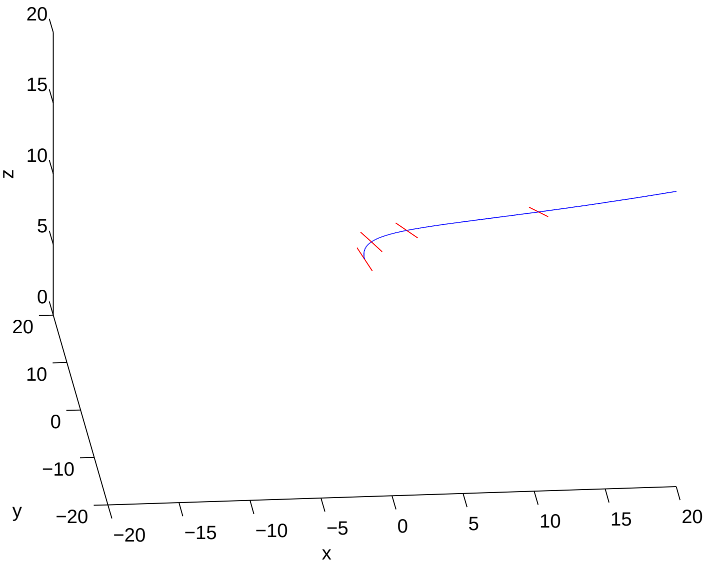
$$\mathbf{a}_l = \left(\frac{\langle \mathbf{F}_{\text{Steering}}, \mathbf{e}_y \rangle}{m} \right) \mathbf{e}_y. \tag{2.36}$$

Inward banking is shown in Figure 2.15, where a bird started in the origin and a constant steering force in the positive x direction is applied. The banking angle is updated with steps of almost one radian, which causes the bird to spin violently. This is the effect of the weighting factors of both the inward and outward control, where inward control is much too strong compared to outward. By changing the the relative weighting factors to make the outward control stronger, we obtain Figure 2.16. This shows the expected behavior, where the bird rolls into the turn until it is oriented toward the steering force and then returns to level flight. Another discrepancy in the inward banking control is that it is also zero for a bird that flies in the exact opposite direction of the steering force. However, this is an unstable equilibrium, so when other forces are taken into account, this will not be a problem.

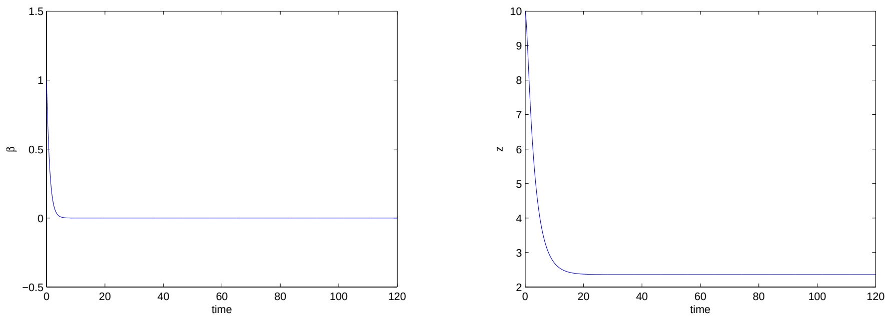
The banking angle is updated as is described in Equation (2.3), by adding inward control and subtracting outward control,

$$\beta(t + \Delta t) = \beta(t) + \beta_{in} - \beta_{out}. \tag{2.37}$$

Since both β_{in} and β_{out} do not depend directly on Δt, this is no proper Euler updating. Moreover, updating the banking angle directly implies that the angular velocity can change instantly. This implies an infinite acceleration, which is physically impossible. The banking control mechanism will be rewritten using a more physical approach in Chapter 3.



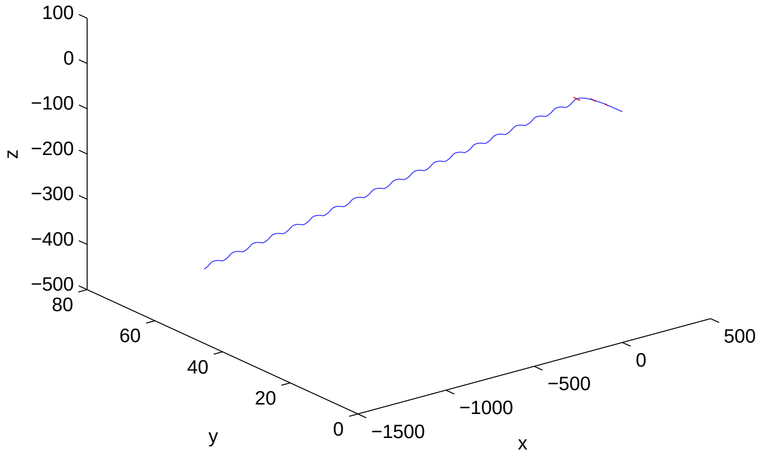
(a) 3D-plot



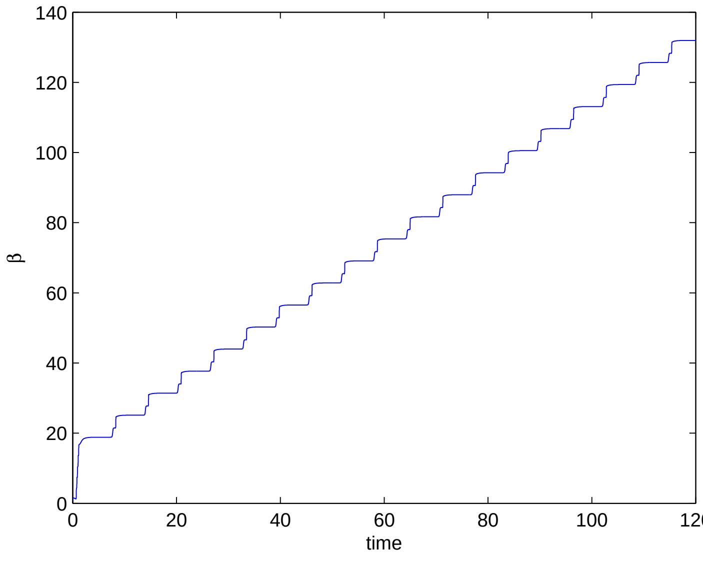
(b) Beta

(c) z

Figure 2.14.: A bird makes a banked turn before rolling back to level flight. The wings of the bird are shown. Here, the bird was given initial conditions $\mathbf{v} = (0, v_0, 0)^T$, $\mathbf{p} = (0, 0, 10)^T$ and $\beta = 1$. The bird starts off in the direction of the y-axis and turns toward the right. As the banking angle returns to zero, the bird stops turning and afterwards stabilizes its altitude due to the flight force.

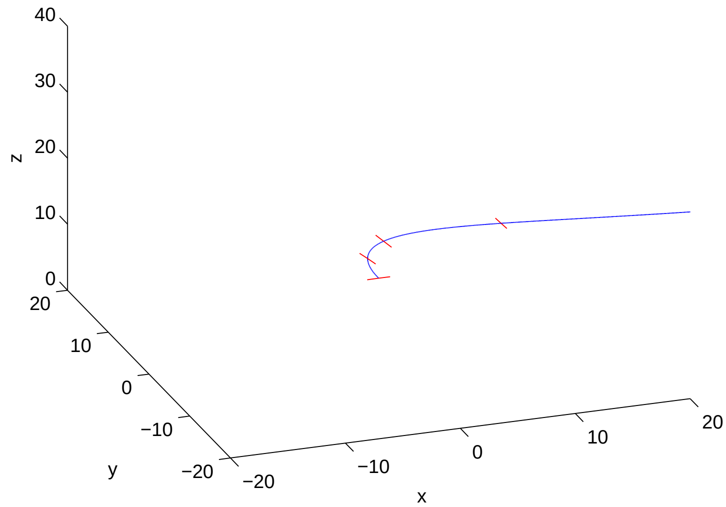


(a) 3D-plot

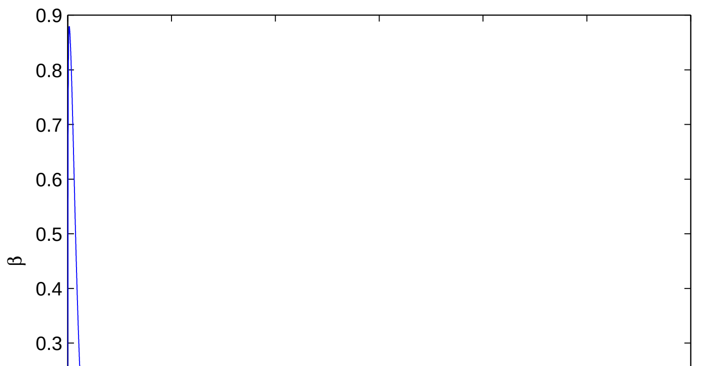


(b) Beta

Figure 2.15.: A bird responds to a constant force in the positive x direction. The initial state is given as $\mathbf{v} = (0, v_0, 0)^T$, $\mathbf{p} = (0, 0, 10)^T$ and $\beta = 0$. Because the weighting factor for inward banking is too big, the banking angle is updated with too large steps and the bird starts to spin.



(a) 3D-plot



(b) Beta

Figure 2.16.: A bird responds to a constant force in the positive x direction, after the relative weight for inward and outward control is changed. The initial state is again $\mathbf{v} = (0, v_0, 0)^T$, $\mathbf{p} = (0, 0, 10)^T$ and $\beta = 0$. The bird rolls toward the x axis and returns to level flight after it has orientated itself toward the steering force.

3. Improvements and alterations

In this chapter, we will propose adaptations to the StarDisplay model in order to solve problems that have been identified in Chapter 2. Firstly, s solution to the problem of gimbal lock will be presented. Secondly, the updating mechanism for the interaction range will be rewritten. Finally, the banking angle control will be redefined using Newton's second law for rotational motion.

Gimbal lock

Figure 3.1 shows two gyroscopes, that consist of three gimbals that are mounted inside each other to accommodate the three rotational axes. Imagine the bird fixed inside the plane that is defined by the blue ring, with nose and tail at the connection points with the green ring. The rings then represent the three Tait-Bryan angles, where orange is the heading, green the elevation and blue the banking. In the left figure, the gimbal is in an arbitrary position and the rotations are independent. In the right figure, all rings are positioned in the same plane, causing the orange and blue rotational axes to coincide. Rotation within this plane is now impossible, resulting in the loss of one degree of freedom. This means for the bird that changes in yaw can not be controlled. A solution to this problem is to mount the system in a fourth gimbal, that is controlled so that it is always aligned at 90 degrees with the blue gimbal. That way, rotation around the birds yaw axis is facilitated and the system is 'unlocked'. Other representations for the orientation of an object include using a rotation matrix, a single Euler axis and a rotation angle or quaternions.[9] All methods have their respective advantages. In the StarDisplay model, the birds orientation is controlled well enough that the problem of gimbal lock only occurs in extreme cases. Therefore, a representation in Tait-Bryan angles will suffice.

Interaction range

The updating mechanism for the interaction radius works disruptively. The radius should stabilize when the desired number of partners is met, but decreases instead. Since the radius will increase as soon as one more bird is excluded, the range will start to chatter, alternately taking the relevant neighbor into account and excluding it. The updating of the adaptive interaction range is defined as

$$R_i(t + \Delta u) = (1 - s)R_i(t) + s(R_{max} - R_{max}\frac{|N_i(t)|}{n_c}). \tag{3.1}$$

Since s depends on Δu, this is the numerical representation of the continuous differential equation

$$\dot{R} = w_R(R_{max} - R - R_{max}\frac{|N_i(t)|}{n_c}), \tag{3.2}$$

where w_R equals the ration between s and Δu. $\dot{R}$ should be zero when $|N_i(t)|$ equals n_c, but instead is equal to $-w_R R$, causing the radius the decrease exponentially toward

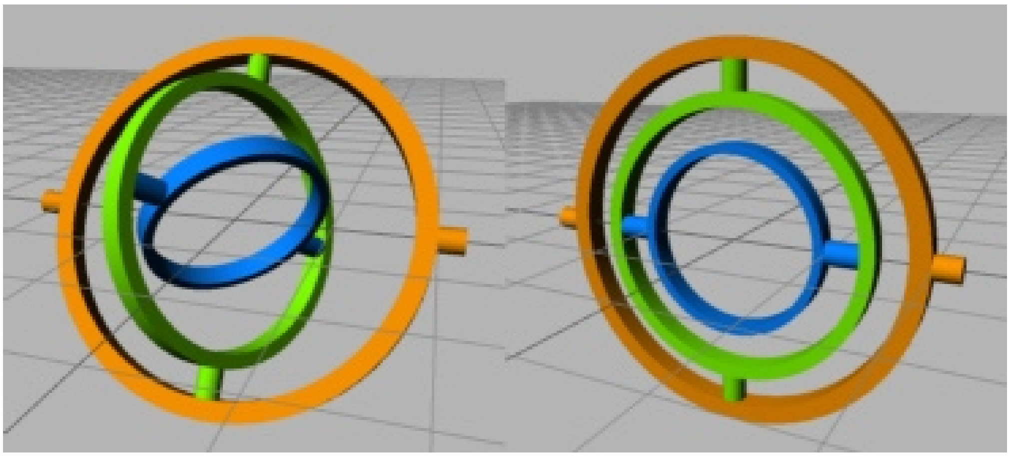


Figure 3.1.: The Tait-Bryan angles are illustrated using gyroscopes, consisting of three gimbals. The orange gimbal represents the heading, green the elevation and blue the banking. When the three rings align, as they do on the right, the system is locked in two dimensional rotation.[16]

zero. This can be solved by using a slightly altered version,

$$\dot{R} = w_R(1 - \frac{|N_i(t)|}{n_c})(R_{max} - R). \tag{3.3}$$

Now the derivative of the radius is zero when it meets the desired number of interaction partners. This, however, does not yet solve the problem of chattering. Since $|N_i(t)|$ is an integer, the radius can not be set so that it includes six and a half bird. This is easily solved by making the desired number of partners also an integer. The numerical representation of the interaction range control is

$$R_i(t + \Delta u) = R_i(t) + w_R(1 - \frac{|N_i(t)|}{n_c})(R_{max} - R)\Delta u. \tag{3.4}$$

Banking angle control

As was seen on page 21, the banking angle control mechanism has a few issues. The inward control is much too strong, causing the bird to spiral out of control. Moreover, the update mechanism is not physically meaningful.

Using Newton's second law for rotational motion, we can restore the physical meaning to the banking angle control. Newton's second law of motion states that the force that a mass experiences is proportional to its acceleration. The rotational parallel for this statement is

$$\tau = I\alpha; \tag{3.5}$$

where τ is the torque applied on a rotating object, I is its second moment of inertia and α is the angular acceleration. The angular velocity ω and the angle β are related to this by

$$\begin{aligned} \dot{\omega} &= \alpha \\ \dot{\beta} &= \omega, \end{aligned} \tag{3.6}$$

Now we define the torque so that it controls the banking angle. We choose to let the outward banking angle behave as a critically damped spring damper system. When the system is critically damped, it converges to its equilibrium as fast as possible without oscillating. The general equation of motion for undriven torsional harmonic oscillators is

$$I\ddot{\theta} + \Gamma\dot{\theta} + \mu\theta = 0, \tag{3.7}$$

where Γ is the rotational friction coefficient of the system and μ is the torsion constant. This is critically damped when Γ is defined as

$$\Gamma = 2\sqrt{\mu I}, \tag{3.8}$$

Rewriting this in terms of the torque this gives

$$\tau = \frac{-\Gamma\dot{\theta} - \mu\theta}{I} = -2\sqrt{\frac{\mu}{I}}\dot{\theta} - \frac{\mu}{I}\theta, \tag{3.9}$$

where the ratio $\frac{\mu}{I}$ determines the settling time for the system. Using this form, we define the outward banking torque as

$$\tau_{out} = -2w_{\beta_{out}}\omega - w_{\beta_{out}}^2\beta. \tag{3.10}$$

For the inward banking control, we define τ_{in} so that the angle stabilizes at such an angle that the lift covers the lateral force that the bird experiences. We assume that the bird flies with cruising speed, so that the centripetal force produced by the lift is defined as

$$C_p = L_0\sin(\beta) = mg\sin(\beta). \tag{3.11}$$

We define the desired banking angle so that the centripetal force is equal to the lateral projection of the steering force.

$$\beta_{des} = \arcsin\left(\frac{\langle \mathbf{F}_{\text{Steering}}, \mathbf{e}_y\rangle}{mg}\right) \tag{3.12}$$

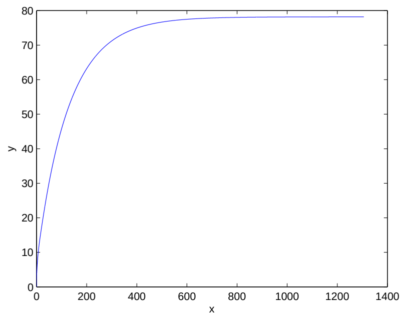
The inverse sine imposes boundaries at a 90 degree rotation, so if the desired centripetal force is larger than can be realized, the bird will not turn upside down. The inward banking torque is then defined to approach the desired angle as

$$\tau_{in} = -2w_{\beta_{in}}\omega - w_{\beta_{in}}^2(\beta - \beta_{des}). \tag{3.13}$$

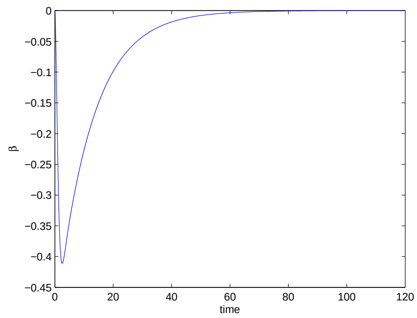
Following the parallel with translational motion, we define the update mechanism for the banking angle as

$$\begin{aligned} \omega(t+\Delta t) &= \omega(t) + \frac{(\tau_{in}(t) + \tau_{out}(t))}{I}\Delta t; \\ \beta(t+\Delta t) &= \beta(t) + \omega(t)\Delta t. \end{aligned}, \tag{3.14}$$

Figure 3.2 shows the response to a constant force in the positive x direction. Here, the initial state for the bird was $\mathbf{v} = (0, v_0, 0)^T$, $\mathbf{p} = (0, 0, 10)^T$, $\beta = 0$ and $\omega = 0$. As can be seen, the bird rolls into the curve and then turns back to level flight.



(a) Position



(b) Beta

Figure 3.2.: The response to a constant force in the positive x direction. The initial state for the bird is given as $\mathbf{v} = (0, v_0, 0)^T$, $\mathbf{p} = (0, 0, 10)^T$, $\beta = 0$ and $\omega = 0$.

4. Discussion

In their article "Self-organized aerial displays of thousands of starlings: a model"[8], H. Hildenbrandt, C. Carere and C.K. Hemelrijk present the StarDisplay model. In this model, the roosting behavior of a flock of starlings is explained in terms of swarm intelligence. The model defines simple rules for the birds, in an attempt to mimic real life flight behavior. The purpose of this report was to investigate the effect of these rules and propose adaptation wherever the behavior of the system was disruptive.

We have found that there are three problems with the StarDisplay model that were worth investigating. The problem of gimbal lock is a general issue in aircraft control dynamics, for which many solutions have been presented in literature[9, 6]. The orientation of the birds is restrained abundantly, so the problem will not occur in normal use of the model. The updating mechanism for the interaction range has been revised. In the StarDisplay model, the interaction radius would decrease whenever the desired number of partners was realized, instead of stabilizing at that point. The adaptive interaction range has been revised to eliminate these problems. Finally, the banking angle control was not working properly, causing the birds to spin violently. The updating mechanism has been rewritten in terms of Newton's second law for rotational motion. The banking angle is controlled so that the birds roll into the intended curve and return to level flight as fast as possible in the absence of a lateral force. The adapted forces are designed so that their effect is equal to that of the original ones, so the behavior of the system is unchanged. This is only true for small time steps. The adaptations improve the model by imposing boundaries on the forces and solving discontinuities. The adapted forces behave as the original for the isolated cases in which they were tested and designed. It is plausible that the discrepancies in the original model are negligible because most of them concern unstable equilibrium situations and there is enough noise in the system to disturb that and return the system to the stable situation. It is as yet untested whether the proposed changes have a significant effect on the model as a whole. The research in the article should be recreated using the new version of the StarDisplay model to verify that it still produces the typical starling flock shapes. Another interesting topic to investigate is the work that the birds do by following the behavioral rules of the model. The birds consume energy by their constant thrust, the steering force and the rotation around their roll axis. It would be interesting to investigate whether the starlings in the StarDisplay model have a realistic energy consumption pattern.

A. Random walk

The random walk in one dimension is defined formally by taking independent random variables $R_1, R_2, \dots$, where each R takes the values 1 or -1 with probability $\frac{1}{2}$. Define X as

$$X_{(n+1)\Delta t} = X_{n\Delta t} + \alpha R_n, \tag{A.1}$$

with

$$X_0 = 0. \tag{A.2}$$

Then $X_{n\Delta t}$ gives the position for time t, given that

$$t = n\Delta t. \tag{A.3}$$

The recurrent function (A.1) can be written in direct form as

$$X_{n\Delta t} = X_0 + \alpha \sum_{i=1}^{n} R_i = \alpha \sum_{i=1}^{n} R_i. \tag{A.4}$$

Using the independence of the random variables R_i and the fact that $\mathbb{E}(R_i) = 0$, the expectation of $X_{n\Delta t}$ equals

$$\mathbb{E}(X_{n\Delta t}) = \alpha \sum_{i=1}^{n} \mathbb{E}(R_i) = 0. \tag{A.5}$$

Therefore, the variance of $X_{n\Delta t}$ equals

$$\mathrm{Var}(X_{n\Delta t}) = \mathbb{E}(X_{n\Delta t} - \mathbb{E}(X_{n\Delta t}))^2 = \mathbb{E}(X_{n\Delta t}^2) \tag{A.6}$$

Once again using the independence of R_i, the variance is given as

$$\mathbb{E}(X_{n\Delta t}^2) = \alpha^2 \sum_{i=1}^{n} \sum_{j=1}^{n} \mathbb{E} R_i R_j. \tag{A.7}$$

If i and j are not equal to each other, there are four possible combinations that occur with equal probability, so the expectation of $R_i R_j$ is given as

$$\mathbb{E}(R_i R_j)_{i \neq j} = \frac{1}{4}(1 + 1 - 1 - 1) = 0. \tag{A.8}$$

On the other hand, if $i = j$, then $R_i R_j$ always yields 1, so the expectation is

$$\mathbb{E}(R_i R_j)_{i=j} = 1. \tag{A.9}$$

So the expectation of $R_i R_j$ is equal to 1. Filling this in in Equation (A.6) gives

$$\mathrm{Var}(X_{n\Delta t}) = \mathbb{E}(X_{n\Delta t}^2) = \alpha^2 \sum_{i=1}^{n} 1 = \alpha^2 n. \tag{A.10}$$

Now for scalability, or independence of the random of the time step chosen, $X_{n\Delta t}$ should have the property that the expectation and variance for given t are independent of Δt. The expectation is zero for every t, but the variance is given as

$$\mathrm{Var}(X_t) = \alpha^2 \frac{t}{\Delta t}. \tag{A.11}$$

This is only independent of T if α is taken so that

$$\alpha^2 \propto \frac{1}{\Delta t} \Rightarrow \alpha \propto \frac{1}{\sqrt{\Delta t}} \tag{A.12}$$

This means for the random force in the StarDisplay model that

$$w_\xi \propto \frac{1}{\sqrt{\Delta t}}. \tag{A.13}$$

B. Parameter values

Parameter	Description	Value
Δt	Integration time step	0.005 s
v_0	Cruising speed	10 m/s
m	Mass	0.08 kg
g	Gravitational constant	9.81 m/s^2
C_D/C_L	Drag-lift ratio	3.3
L_0	Default lift	0.7848 N
T_0	Default thrust	2.5898 N
τ	Time constant for speed control	1 s
z_0	Zero level	10 m
R_{roost}	Roost radius	150 m
Δu	Reaction time	0.05 s
s	Interpolation factor	0.005 s
R_{max}	Maximum interaction radius	100 m
n_c	Desired number of interaction partners	6.5
r_h	Radius of non-attraction	0.2 m
σ	Standard deviation of Gaussian $g(d_{ij})$	1.77 m
r_{sep}	Border of the separation zone	4 m

Weighting factor	Description	Value
w_{alt}	Altitude control	0.2 N/m
w_{Roost}	Roost attraction	0.01 N
w_ξ	Random force	0.01 N
w_s	Separation	1 N
w_c	Cohesion	1 N
w_a	Alignment	0.5 N
$w_{\beta_{in}}$	Inward banking	10
$w_{\beta_{out}}$	Outward banking	1

Bibliography

[1] Anderson, J. D. (2005). Introduction to flight (Vol. 199). Boston: McGraw-Hill.

[2] Ballerini, M., Cabibbo, N., Candelier, R., Cavagna, A., Cisbani, E., Giardina, I., Zdravkovic, V. (2008). Empirical investigation of starling flocks: a benchmark study in collective animal behaviour. Animal behaviour, 76(1), 201-215.

[3] Beni, G. (2005). From swarm intelligence to swarm robotics. In Swarm Robotics (pp. 1-9). Springer Berlin Heidelberg.

[4] Camazine, S. (Ed.). (2003). Self-organization in biological systems. Princeton University Press.

[5] Carere, C., Montanino, S., Moreschini, F., Zoratto, F., Chiarotti, F., Santucci, D., Alleva, E. (2009). Aerial flocking patterns of wintering starlings,Sturnus vulgaris, under different predation risk. Animal behaviour, 77(1), 101-107. ISO 690

[6] Evans, D. J. (1977). On the representatation of orientation space. Molecular Physics, 34(2), 317-325.

[7] Hemelrijk, C. K., Hildenbrandt, H. (2011). Some causes of the variable shape of flocks of birds. PloS one, 6(8), e22479.

[8] Hildenbrandt, H., Carere, C., Hemelrijk, C. K. (2010). Self-organized aerial displays of thousands of starlings: a model. Behavioral Ecology, 21(6), 1349-1359. ISO 690

[9] Ickes, B. P. (1970). A new method for performing digital control system attitude computations using quaternions. AIAA journal, 8(1), 13-17. ISO 690

[10] Kaigh, W.D.(1979). An Attrition Problem of Gambler's Ruin. The Mathematics Magazine, 52 (1), pp. 22–25

[11] Kakutani, S. (1942), A proof that there exists a circumscribing cube around any bounded closed convex set in R3, Annals of Mathematics (2) 43 (4): 739–741

[12] Montroll, E. W., Weiss, G. H. (2004). Random walks on lattices. II. Journal of Mathematical Physics, 6(2), 167-181. ISO 690

[13] Selous, E. (1931). Thought-Transference (or What?). BirdsConstable, London

[14] Spitzer, F. (1964). Principles of random walk (pp. 787-806). Princeton: van Nostrand.

[15] Wikimedia Commons, by Juansempere, 2009, Retrieved from http://commons.wikimedia.org/wiki/File:Plane.svg

[16] Retrieved from http://www.deepmesh3d.com/help/axis.htm