



# A light on the Shadow-Bond approach

The development of RI's new Commercial Banks PD model

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# Management Summary

In this thesis I describe the process of redeveloping the Commercial Banks Probability of Default (CBPD) model of Rabobank International (RI). This model had to be redeveloped since the ratings of the old model required too many overrides. Additionally, developing a new model was an opportunity to include new forward looking input parameters in the model. Together with my project team I developed the model with the Shadow-Bond approach, an approach aimed at mimicking S&P’s rating model. We had to choose this approach since there were not enough defaults in RI’s Commercial Banks portfolio to use the Good-Bad approach. While developing the model we made sure the model was developed in accordance with the guidelines set by the Quantitative Risk Analytics (QRA) department.

The first step we dealt with during the modelling process was the collection and preparation of data. We collected data from multiple sources and paid extra attention at the numerous assumptions made during the preparation to make sure we obtained a reliable dataset. After the preparation of the data we performed a regression on the constructed dataset to obtain a model. This model resembled the format of a scorecard; a PD could be calculated from the scores of a bank on a number of factors. The constructed scorecard consisted of 15 factors, of which the country factor had the largest weight. When calculating the capital impact of this model, we found that the new model was less conservative than the old model, since we observed an initial capital decrease of 10%. We found that the S&P ratings were less conservative during the crisis than RI’s old model ratings. Therefore, the constructed new model (which matches the S&P model) resulted in a capital decrease. Since RI prefers to keep this conservatism margin with respect to S&P we deviated from the QRA guidelines and performed an additional calibration on the model such that the new model mirrored the conservative level of the old model. The result of this calibration was the scorecard model shown in the table below.

Factor		Weight	Score
Country	100	100	100
Capital	100	100	100
Assets	100	100	100
Liabilities	100	100	100
Profitability	100	100	100
Efficiency	100	100	100
Market	100	100	100
Management	100	100	100
Technology	100	100	100
Environment	100	100	100
Regulation	100	100	100
Competition	100	100	100
Customer	100	100	100
Supplier	100	100	100
Other	100	100	100

Table 1: New Commercial Banks PD model

The factors in this model are selected based on their historical performance with exception of the factors Country and Capital. These factors are included based on the opinions of experts to make the model more forward looking. The weights of these factors were fixed at

████ and █████ respectively. The capital impacts of this model are █████ and █████ for RC and EC respectively.

After the construction of the model we tested the performance extensively. We defined the performance of the model as the extent to which the S&P ratings and the performed overrides were matched. We concluded that the new model performed slightly worse at matching S&P's ratings in comparison with the old model, but better at matching the overrides. Therefore we concluded that there are less expected future overrides with the new model. While performing an out of time test we concluded that the model does have trouble predicting extreme ratings, which might be a risk for RI.

Finally we performed additional research at a model called the Hybrid model. This model combines elements of the Good-Bad model and the Shadow-Bond model. The approach for constructing a Hybrid model can be used if there are too little defaults in the dataset to use the Good-Bad approach, but if it is not desired if the model is solely based on S&P ratings. This approach was however not an alternative for this developed Shadow-Bond model, since the Hybrid approach resulted in a model which was not stable.

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# 1 Introduction

Rabobank International (RI) is the international branch of the Rabobank Group. It has offices in 30 countries divided over the regions Europe, The Netherlands, The USA and South America, and Asia and Australia. The focus of its activities in these regions is on the food and agricultural sector.

RI is a commercial bank. This type of banking involves amongst others collecting deposits and granting loans (Hull, 2010). Credit risk is the largest risk faced by commercial banks, since loans and other debt instruments constitute the bulk of their assets (Lopez, 2001). Credit risk arises from the possibility that borrowers, bond issuers, and counterparties in derivatives transactions may default (Hull, 2010). Credit ratings can be used to assess the creditworthiness of counterparties.

Banks often use internal credit ratings to assess the creditworthiness of its counterparties. These ratings are calculated with internal credit rating models. During my internship at RI I have redeveloped such a model: the Commercial Banks Probability of Default model. This model is aimed at estimating the likelihood that a commercial bank will not meet its payment obligations (goes into default). In this thesis I will discuss the redevelopment of this model. Before I will do this, I will present some background information regarding the decision to redevelop this model. Additionally, I will elaborate on credit rating models and I will formulate the main question and corresponding research questions of my internship.

## 1.1 Background

As discussed, RI uses the Commercial Banks PD (CBPD) model to generate ratings concerning the creditworthiness of commercial banks. These ratings are generated for existing and new clients of RI. For new clients the ratings are used to determine the price of granting loans and for existing clients the ratings are used to determine the capital needed to cover the risks of these clients. Multiple types of financial institutions classify as commercial banks under RI's definition: investment banks, commercial banks (wholesale), commercial real estate funds, retail banks, custodians, private banks, asset managers, residential mortgage banks and universal banks (Herel, 2012). All these institutions are from now on referred to as commercial banks.

The most recent version of the CBPD model was developed by RI's Quantitative Risk Analytics (QRA) department in 2007 and is currently still used. QRA is amongst others responsible for the development of reliable quantitative risk models for RI's credit portfolio. QRA is also the department where this internship is performed. The models developed by QRA are used by departments all over the Rabobank Group. The CBPD model is mainly used by the Credit Risk Management Banks (CRMB) department. This department is responsible for the estimation of the creditworthiness of banks.

In June 2013 the managers of QRA and CRMB decided the CBPD should be redeveloped since the ratings generated with this model did not match their estimates of credit risk anymore. The most important reason for this mismatch is the financial crisis that started in 2008. This crisis has changed the financial system and these changes have not been implemented in the model yet.

The crisis made clear that liquidity is very important for the creditworthiness of banks (Kopitin, 2013). Liquidity is already present in the current CBPD model, but it must be analysed whether it should become more important in the new model. The crisis also illustrated the importance of the creditworthiness of countries banks are located in (Angeloni, Merler, & Wolff, 2012). Institutions located in countries with high creditworthiness are more likely to be bailed out successfully, which will decrease the probability of default of these banks. It must therefore be determined whether the country of a bank should also have an increased weight in the new model.

For this reason a team has been set up consisting of model developers from QRA and model users from CRMB who together will redevelop the current CBPD model. The model users from CRMB are called the experts from now on. Together with my supervisor Martin van Buren, I represent QRA in this model development team.

Now that the general background of the problem is given, I will give a short introduction to credit rating models such that the main question can be better understood.

## 1.2 Credit rating models

Credit risk can be quantified with credit ratings. A typical credit rating scale ranges from low to high ratings, where each rating represents a creditworthiness category. There are two types of credit ratings: internal and external ratings. Internal ratings are ratings which are generated by a bank and which are used within that bank only. External ratings are ratings generated by a credit rating agency as Moody's, S&P and Fitch and which are globally used. These rating agencies generate ratings for amongst others countries, firms and bonds.

Both internal and external credit ratings are generated with credit rating models. Credit rating agencies as Moody's, S&P and Fitch have their own models for generating external ratings. In contrast, the CBPD model which is going to be redeveloped during my internship is one of RI's internal rating models. The output of this model is a Rabobank Risk Rating. This rating is part of Rabobank's own rating scale. This scale consists of 21 ratings, R0 till R20, and ranges from good to bad creditworthiness. Each rating corresponds with a fixed default probability.

RI's current CBPD model is closely related to the Altman Z-score model (Altman, 1968). This model was one of the first credit rating models and was a linear scorecard with 5 predictors. From these 5 predictors a Z-score was calculated as a linear combination of the scores and the weights of these 5 predictors. Pompe and Bilderbeek (Pompe & Bilderbeek, 2005) performed research at the performance of different categories of financial ratios as predictors of defaults.

## 1.3 Main question and research questions

In this section I will describe the main question. In the background section I explained the current CBPD model is out-dated and should therefore be redeveloped. The main question follows from this redevelopment need:



*How should the new CBPD model be redeveloped such that it meets the requirements set by RI and the Dutch Central Bank (DNB)? What is the capital impact of this new model, and how does it perform in comparison with the old model?*

Heerkens identifies two types of problems (Heerkens, 2004): descriptive problems and explanatory problems. Descriptive problems are problems where one wants to describe an aspect of reality without trying to explain it (Heerkens, 2004). Explanatory problems are problems where an explanation of an aspect of reality is sought (Heerkens, 2004).

The main question can be split up in three sub questions. The first sub question asks to identify the relationship between the variables ‘model’ and ‘requirements’, therefore this part of the main question is an explanatory problem. In contrast, the second and third sub questions are concerned with the identification of the capital impact and the performance of the new model respectively. These sub questions are therefore descriptive. The main question is therefore a combination of an explanatory and two descriptive problems. The goal of answering this main question is giving shape at a model which can be implemented to calculate PDs of commercial banks.

In order to answer the main question I formulated research questions. Answering these questions will eventually result in an answer to the main question. The research questions I defined are shown in the table below.

#	Subject	Question
1	Requirements	What are the requirements for the new rating model?
2	Approaches	What model development approaches does RI have, and what approach should be used for this redevelopment process?
3	Modelling process	Given the chosen approach how can the new model be developed?
4	Capital impact	What is the capital impact of the new model?
5	Performance	How does the new model perform in comparison with the old model, and are there ways of improving this performance?

**Table 2: Research questions**

## 1.4 Outline

The research questions presented above form the backbone of this thesis. These questions are answered in different chapters. Research questions 1 and 2 are answered in Chapter 2. In Chapter 3 to 7 research question 3 is answered by describing the modelling process and in Chapter 8 the capital impact of the new model is calculated. In Chapter 9 a calibration is performed and in Chapter 10 the performance of the model is analysed. In Chapter 11 an alternative model is presented, which could result in a model with a better performance. Finally in Chapters 12 and 13 conclusions are drawn and discussed.

## 2 Requirements and approaches

In this chapter I will describe the requirements and approaches for developing a model at QRA. This chapter starts with an overview of these requirements. Thereafter I will give an overview of RI's different modelling approaches, I will discuss the methodologies of these approaches and will elaborate on the decision on what approach to use for the development of the CBPD model.

### 2.1 Requirements rating models

There are a number of requirements for the new CBPD model. These requirements are a combination of internal requirements set by QRA and external requirements set by the Dutch Central Bank and the Basel Committee. The combination of internal and external requirements is summarized in the general checklist of QRA (Opzeeland & Westerop, 2006). This checklist is shown below.

- The new rating model should be grounded on both historical experience and empirical evidence and should incorporate historical data as well as expert judgment.
- The historical data on which PD estimates are based should have a length of at least five years.
- The model must be developed with prudence.
- The outcomes of the model should be accurate and in line with available benchmarks such as external ratings.
- The model needs to be robust. To understand this requirement, one needs to know that the model is developed on a development dataset. Therefore the model is a result of the characteristics of this dataset. The requirement implies that changing this dataset a little should not result in a completely different model.
- The model must be logical / intuitive. This means that the model and its results make sense.

The managers of the QRA and CRMB departments have formulated two additional requirements for this CBPD model. These additional requirements are:

- The model must be forward looking in the sense of future portfolio composition and expected important factors.
- The capital impact resulting from a new model is not allowed to be too big.

The model we will develop during my internship must match these 8 requirements listed.

### 2.2 Modelling approaches overview

RI has three different approaches available for developing rating models: the Good-Bad approach, the Shadow-Bond approach and the expert based approach. The rating models resulting from these three approaches are scorecards. With these scorecards credit ratings for companies are calculated from a number of explanatory factors (as was the case with the Altman Z-model). Therefore this scorecard represents the relationship between the creditworthiness of counterparties and their scores on a number of factors.

The Good-Bad approach and the Shadow-Bond approach both make use of historic data and expert input to determine this relationship. The expert based approach does not use historical data, but relies on expert input solely. This approach does thus not meet the first requirement of the requirements listed in the previous section and is therefore only used when there is no historic data available to use the Good-Bad or Shadow-Bond approach. Since there is historic data for this model development available this approach is not preferred and will not be discussed further in this thesis.

The historic data used for the Good-Bad and Shadow-Bond approach depends on the model to be constructed. For the development of the CBPD model the dataset consists of historical data of commercial banks. This historical data consists of observations, i.e. snapshots of all information available of a bank at a certain date. Below the structure of an observation is shown.

Observation ID	Bank	Date	Explanatory variables	Creditworthiness information
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**Table 3: Observation structure**

As can be seen from the table an observation consists of five parts:

- Observation ID: Each observation has a unique identification code.
- Bank: The bank the observation is created from.
- Date: The explanatory variables and creditworthiness information of banks change over time. The date of the observation is the date the explanatory variables and the creditworthiness information are used for the observation.
- Explanatory variables: These are the variables which describe the state of the bank at the date of the observation.
- Creditworthiness information: This is an indication of the creditworthiness of the bank at the date of the observation. This information differs for the Good-Bad and the Shadow-Bond approach. For the first approach this information is given by a default indicator which can take the values of 1 and 0. For the Shadow-Bond approach this information is given by an external historic rating.

To determine the relationship between the explanatory variables and the creditworthiness of the observations, statistical analysis is performed. This statistical analysis involves performing a regression of the creditworthiness information on the explanatory variables. The regression technique differs for the Good-Bad and the Shadow-Bond approach. At the first approach a logistic regression is performed, while at the second approach a linear regression is performed. In order to understand both approaches, the concepts linear regression and logistic regression are shortly explained in the next section.

## 2.3 Methodologies approaches

In this section the methodologies of the Good-Bad and the Shadow-Bond approach are described. This section starts with a short explanation of linear regression by discussing the linear model. Thereafter logistic regression is explained by generalizing the linear model to a generalized linear model.

### 2.3.1 Linear regression

When a linear regression is performed, the assumption is made that the dependent variable has a linear relation with the explanatory variables (Heij, 2004). A typical linear model is given by the equation below.

$$y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_k x_k + \varepsilon \quad (1)$$

In this equation  $y$  is a vector of dependent variables,  $\alpha$  a constant,  $x_1 \dots x_k$  vectors of independent variables,  $\beta_1 \dots \beta_k$  the coefficients of these variables and  $\varepsilon$  a vector of random noise elements. The simplest approach for estimating a linear model is by applying the ordinary least squares (OLS) method. This method estimates the coefficients of the linear model such that the sum of the squared error terms is minimized. The result of minimizing these error terms is the OLS estimator of  $\beta$ :  $(X^T X)^{-1} X^T Y$ . In Appendix 14.1 it is shown that this estimator is indeed the estimator resulting in the lowest sum of squared errors. According to the Gauss-Markov theorem (Plackett, 1950), the OLS estimator for  $\beta$  is the best linear unbiased estimator (BLUE) if the following assumptions hold:

- The error terms have a mean of zero.
- The error terms are homoscedastic. This means that all error terms have the same finite variances.
- There is no correlation between the error terms.

### 2.3.2 Logistic regression

Linear regression can be used for modelling variables with a linear relationship with the explanatory variables. It is however less effective in modelling restricted or binary variables (Heij, 2004). For such dependent variables it is better to model a transformation of the dependent variable instead of the dependent variable directly. Models where a transformation of the dependent variable is modelled as a linear variable are called generalized linear models. The method for estimating these generalized linear models was introduced in 1972 by Nelder & Wedderburn (Wedderburn & Nelder, 1972) and developed further in 1989 by McCullagh & Nelder (McCullagh & Nelder, 1989). A generalized linear model consists of 3 components (Fox, 2008).

- A random component indicating the distribution of the dependent variable.
- A linear function of the regressors.
- A link function which links the expectation of the dependent variable to the linear function.

Binary variables can be modelled with a generalized linear model by making the assumption that the dependent variable is binomial distributed (the first component). The logarithm of the odd (third component) of such a binomial distributed variable is then modelled as a linear combination of the regressors (second component). The result of constructing such a generalized linear model is the logistic function. This function is shown in the equation below:

$$\pi(x) = \frac{e^{\beta x}}{1 + e^{\beta x}} \quad (2)$$

In this equation  $\pi(x)$  is the probability that the dependent variable has the outcome 0,  $x$  a vector of explanatory factors and  $\beta$  a vector with the coefficients of these factors. The probability that the outcome of the dependent variable is 1 is:

$$1 - P(Y = 0) = 1 - \pi(x) = \frac{1}{1 + e^{\beta x}} \quad (3)$$

The vector  $\beta$  can be estimated with maximum likelihood. The goal of this method is finding the coefficients such that the probabilities of the observed dependent variables are maximized. This approach is called logistic regression.

## 2.4 Good-Bad approach

Now that the methodologies of both approaches have been discussed, the Good-Bad approach can be explained. The first step of the Good-Bad approach is the construction of the observations as shown in Table 3. The creditworthiness information under the Good-Bad approach is given by a Good-Bad indicator which can take the values 'good' (0) and 'bad' (1). For all observation it is determined whether it is a 'good' or a 'bad'. An observation is classified as 'good' if the particular bank has not gone into default in the year after the observation date. If the bank did default in that year, the observation is classified as 'bad'. If for example an observation is created from SNS Reaal in March 2009, this observation is assigned a value of 0 ('good') if SNS Reaal was still performing in March 2009 and a value of 1 ('bad') if it has gone into default in this period. Since SNS Reaal did not default in this period the observation is marked as a 'good'. The choice for the observation period of one year comes from the fact that the model is aimed at estimating one year PDs.

The combination of explanatory variables and creditworthiness information (good/bad indicator) of the observations makes it able to perform a regression upon all observations. In this regression the good/bad indicator is the dependent variable. Since this variable is binary a logistic regression results in a better fit than a linear regression. For each observation it is calculated how big the probability of the observed 1 or 0 zero is, where the probability of an observed 0 is calculated with the formula below:

$$\pi(x) = \frac{e^{\beta x}}{1 + e^{\beta x}} \quad (4)$$

Maximum likelihood is used to estimate the vector  $\beta$  from the observations. From the estimated  $\beta$  the weights of the factors on the scorecard are determined. The weight of a factor is defined as the contribution of the coefficient of that factor to the sum of the coefficients of all factors. For example when  $\beta$  is a vector of three coefficients, the weight of the first factor is given by the equation:

$$w_1 = \frac{\beta_1}{\beta_1 + \beta_2 + \beta_3} \quad (5)$$

The sum of the weights of the different factors is therefore always 100%.

## 2.5 Shadow-Bond approach

The second approach to be discussed is the Shadow-Bond approach. The goal of this approach is to develop a model which matches the external ratings assigned to counterparties best (Vedder, 2010). Therefore this approach is aimed at constructing a rating model which generates ratings for companies that match their external ratings. One might ask why QRA wants to have such a model instead of using the external ratings directly, but this follows from the fact that for some companies which need to be rated by RI there are no external ratings available.

The observation structure and explanatory variables of an observation with the Shadow-Bond approach are the same as with the Good-Bad approach. The creditworthiness information is however different. Instead of determining whether each observation is a 'good' or a 'bad', the creditworthiness information of each observation is given by a historic external rating. It is checked for all observations what the external rating was at the date of the observation.

The guidelines prescribe S&P as the external rating agency (Vedder, 2010). The reason for this is that RI has a mapping table which makes it possible to translate S&P ratings into PDs as further explained by Jole (Jole, 2008). With this mapping table the S&P ratings can be translated into PDs. The creditworthiness information of each observation is then given by this PD. Just as was the case with the Good-Bad approach the creditworthiness information is then regressed on the explanatory variables. Since the dependent variable (the PD) is continuous, a linear regression can be performed.

However instead of regressing the PDs of the observations directly on the explanatory variables, the natural logarithms of these PDs are regressed as prescribed by the guidelines (Vedder, 2010). This is done to reduce the impact of observations with high PDs. Since the PDs associated with the S&P rating scale increase exponentially, observations with bad ratings have very high PDs. These observations would dominate the linear regression, which is not desirable since both good banks (low PDs) and bad banks (high PDs) need to be fitted well by the model. The regression formula therefore becomes:

$$\ln(PD) = \beta X + \varepsilon \rightarrow PD = e^{\beta x + \varepsilon} \quad (6)$$

In this equation  $\ln(PD)$  is a vector with logarithms of PDs,  $x$  a matrix of explanatory variables,  $\beta$  a vector of coefficients of these variables and  $\varepsilon$  a vector of noise elements. OLS is used to estimate  $\beta$ . The weights of the scorecard are then calculated with Equation 5.

According to Jensen's inequality equation the mean of values transformed with a concave function is lower than the transformed mean of the original values (Russell Davidson, 2004). Since the logarithm function is concave, this means that the average of the PD estimates will be lower than the average of the real PDs. The PD estimates generated with a model constructed with the Shadow-Bond approach are thus too optimistic. This is a weakness of the approach and additional research should be performed to find alternative approaches which do not have this drawback, for example non-linear regression techniques.

## 2.6 Choice for approach and reflection

At the time I joined the project team, it was already decided that the CBPD model was going to be redeveloped with the Shadow-Bond approach. In this section I will explain their arguments for making this choice, but will also give my personal reflection on it.

QRA generally prefers the Good-Bad approach over the Shadow-Bond approach (Vedder, 2010). The reason for this preference is that the Good-Bad approach is based on real creditworthiness information: defaults of counterparties. The Shadow-Bond approach in contrast is based on external ratings and is aimed at mimicking S&P ratings. Since these ratings represent the estimations of creditworthiness by this agency instead of the real creditworthiness, the Shadow-Bond approach can be thought of as modelling indirect creditworthiness which can be less reliable.

However to get reliable results with the Good-Bad approach enough bads (companies which went into default) are required. The minimum number of 'bads' to use this method is set at 10 by QRA (Piet, 2011). Since commercial banks do not default frequently there were not enough defaults in the development dataset to use the Good-Bad approach. Therefore the project team had to decide to use the Shadow-Bond approach.

Now I will give my reflection on this choice. I also prefer the Good-Bad approach over the Shadow-Bond approach since this approach is based on real default information. However, the project team had to make a decision which matches the guidelines. The guidelines prescribe that the Good-Bad approach could only be used if there are 10 'bads' in the dataset. I do not know exactly how many 'bads' were in the dataset, but apparently too little.

I think this choice might have been made too easily. QRA has a clear definition for a default, and thus also for what banks are 'bads'. Banks which have received government support have not defaulted according to this definition, such that these banks are not marked as 'bads'. However there is reason to believe that troubled banks will not continue to get government support in the future as also proposed by the Swiss Financial Market Supervisory Authority FINMA (FINMA, 2013). With this in mind, it would have been interesting to analyse whether enough banks which have gained government support could have been marked as bads to use the Good-Bad approach.

Furthermore, the S&P ratings used with the Shadow-Bond approach are backward looking in the sense that the ratings are assigned by S&P with the knowledge that banks in trouble will get government support. When using these S&P ratings to construct a model for rating banks in the future the assumption is made that banks in trouble will continue to receive government support in the future. Since this assumption might be invalid, it might be interesting to think of adjusting the Shadow-Bond approach such that the model will be more forward looking. A possible adjustment is downgrading the S&P ratings as if it were ratings without the possibility of government support.

### 3 The modelling process

In the previous chapter the methodology of the Shadow-Bond approach is described. In this chapter I will describe how we constructed a model with this approach. The process of constructing a model is called the modelling process and consists of 5 stages: the data collection stage, the data processing stage, the single factor analyse (SFA) stage, the multifactor analyse (MFA) stage and the testing stage. These different steps will be briefly discussed in this chapter and are visualized in the figure below.



**Figure 1: Modelling process**

The modelling process is performed by the project team consisting of experts, Martin van Buren (my internship supervisor) and myself. The first stage, the data collection stage, is mainly performed by Martin, the next three stages are mainly performed by myself. These four stages are discussed in more detail in the next four chapters. The testing stage however is not performed at the moment of writing this thesis. This stage is therefore only briefly described in this chapter.

#### 3.1 Data collection

The first stage of the modelling process is the data collection stage. In this stage the observations of the dataset are constructed. As discussed, an observation consists of the explanatory variables of a bank at a certain date and a corresponding historic S&P rating. The first step of creating the observations is identifying the explanatory variables of banks. For this reason the experts are asked to construct a list consisting of all risk drivers of banks. This list is referred to as the long list in the remainder of this thesis. The risk drivers on this list are referred to as factors from now on. The factors are the explanatory variables of the observations. The factor information of the observations is obtained from multiple sources.

#### 3.2 Data processing

In the data processing stage the dataset is prepared for the SFA and the MFA. The data processing stage consists of a number of steps. The most important steps of this stage are the cleaning of the data, the transformation of the factor values and the representativeness correction. The cleaning of the data involves the detection and replacement of missing factor values. The transformation is performed to make sure that all factor values are in the same range and the representative correction is performed to make sure that the model is representative for the banks which are rated by RI.



### 3.3 SFA

In the SFA stage the factors from the long list are tested on their standalone explanatory power of the PDs of the banks of the observations. This is done by calculating the Powerstats of the different factors. The higher the Powerstat of a factor, the higher its explanatory power.

### 3.4 MFA

In the MFA stage the model is constructed from the different factors. In contrast with the SFA stage, the combined explanatory power of a set of factors is calculated at the MFA stage. This way the interaction between the different factors is incorporated in the model. The model is estimated by performing a stepwise regression on the dataset. This is a technique for selecting the set of factors with the highest combined explanatory power. After the model is constructed, the confidence bounds of the different selected factors are analysed with a bootstrapping process.

### 3.5 Testing

The last stage of the model development process is the testing stage. In this stage a User Acceptance Test (UAT) is performed by the experts. The goal of the UAT is to test and judge the performance of the model by future end-users of the model (Opzeeland & Westerop, 2006). The experts performing the UAT have to comment on the performance of the model. These experts are not allowed to have been involved in the development stage since they could be biased in favour of the model (Vedder, 2010).

4 Data collection

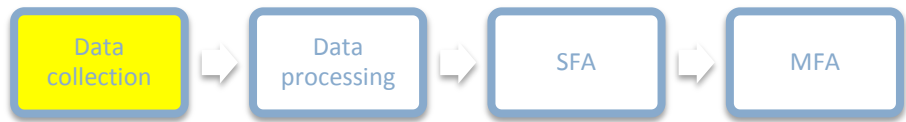


Figure 2: Data collection stage

The first stage of the development process is the data collection stage. The collection of data consists of three steps: the factor identification, the creation of the dataset and the first cleaning step. In the first step the factors with possible explanatory power are identified by the experts. These factors will be used as the explanatory variables in the regression. In the second step the dataset for the regression is constructed. In the third step the observations which do not meet the requirements for observations are removed from this dataset. The steps of the data collection stage are visualized in the figure below.



Figure 3: The three steps of the data collection stage

4.1 Factor identification

The first step of the data collection stage is the identification of factors with possible explanatory power for the PDs of banks. These factors are the explanatory variables described in Chapter 2. The experts involved in the development process were asked to identify the factors of commercial banks with explanatory power. In total they identified 10 factors. They identified two types of factors: financial ratios and qualitative factors. Qualitative factors indicate the quality of characteristics of banks that are less well measurable, but are judged by experts. Yet these qualitative factors are assigned a numerical value between 0 and 10. Financial ratios are objective and exactly measurable. These financial ratios can be calculated from the financial statements of a bank.

From the 10 factors identified by the experts, 5 factors are qualitative. These factors are shown in the table below.


**Table 4: Qualitative factors**

The first column of this table lists the unique identification codes of the qualitative factors. The second column contains the descriptions of these factors. As discussed, experts determine the scores of banks for these qualitative factors. The last factor [REDACTED] is the average of the factors [REDACTED] and [REDACTED]. This factor is included as a separate factor since it gives insight in [REDACTED].

Next to these qualitative factors the experts identified [REDACTED] financial measures with possible explanatory power. Although not all, the majority of these measures are ratios and therefore we will refer to these measures as financial ratios for the remainder of this thesis. The measures can be found in Appendix 14.2. Similarly to the qualitative factors, the financial ratios have their own unique identifiers. The financial ratios can be divided in 9 categories. Each category explains a different aspect of the financial performance of a bank. These 9 categories are cost efficiency, profitability, risk profile, portfolio quality, capital, funding, liquidity, size and diversification of business. The categories of the different ratios are shown in the second column of Appendix 14.2. As discussed in Section 3.1, the list consisting of the identified qualitative factors and financial ratios is called the long list. The financial ratios and qualitative factors on this list are referred to as factors.

## 4.2 Dataset creation



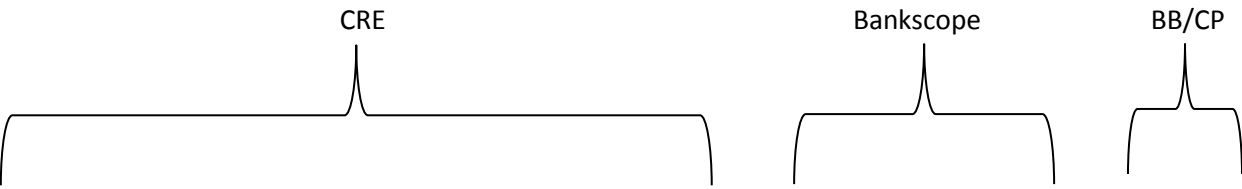
**Figure 4: Dataset creation**

The factors from the previous section are the explanatory variables from which the expected PDs of banks are calculated. To be able to do this, the relationship between these explanatory variables and the PDs must be determined. As discussed, this is done by performing a regression on a dataset. In this section it is first described how the different observations are created in general where after the process of matching historic S&P ratings at the observations is described in more detail.

### 4.2.1 Creation of observations

In Section 2.2 it is discussed that an observation is created from a bank at a certain date. Within RI's databases banks are identified by their World Wide IDs (WWIDs). Therefore the first two elements of an observation are the WWID of the bank the observation is created from and the date of the observation. Furthermore, an observation consists of values for the factors from the long list and a S&P rating. As described, the factors can be split up in qualitative factors and financial ratios. The qualitative factor values are downloaded from the Central Rating Engine (CRE) of RI. This is a database containing

qualitative rating assessments of banks. These assessments contain the scores of banks on the identified qualitative factors. The financial ratios of an observation can be calculated from the financial statements of the bank. Therefore for all banks of which qualitative rating assessments could be found in CRE the financial statements are downloaded from Bankscope, a database containing historic financial statements of banks. Finally, the historic S&P ratings of the observations are downloaded from Bloomberg or CreditPro. These ratings are then mapped to PDs as further explained by Jole (Jole, 2008). The dataset resulting from this procedure is shown in the table below.



WWID	Date	Q1	Q2	...		F1	...	F-end	PD
:	:	:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:	:	:

**Table 5: Dataset at the end of dataset creation step**

In this table the rows correspond with the observations in the dataset. In the first two columns are the WWIDs and dates of the observations. In the next ■ columns are the qualitative factor values of the observations. In the columns 'F1' to 'F-end' the different fields of the financial statements active at the observation date are shown. Finally, in the column 'PD', the PDs corresponding with the downloaded historic S&P ratings of the observations are shown.

In total CRE contains ■ qualitative ratings assessments of commercial banks. Of these ■ assessments we find ■ assessments of banks with unique WWIDs. Therefore on average each bank has ■ rating assessments in CRE. For each assessment the financial statements active at the date of the assessment are matched. For example if there is an assessment of a bank from February 2007 the financial statements of 2006 are matched if available at that time. If these statements were not yet available in February 2007 the statements of 2005 were matched. The reason for matching the most recent available statements instead of the statements of the year of the assessment is that when the model is used in practice one should also use the most recent statements. In the next section it is explained in more detail how the correct S&P rating is looked up and attached at the created observations.

#### 4.2.2 Attaching S&P ratings

The qualitative factors and financial ratios (which yet have to be constructed) form the explanatory variables in the regression equation. The other side of the equation is given by the logarithm of the PD corresponding with a historic S&P rating. These historic S&P ratings are also given in CRE for the different observations, however these ratings are not reliable and often missing. Therefore the correct historic ratings must be downloaded from other sources.

There are two sources for downloading historic S&P ratings: Bloomberg and CreditPro. Both databases contain historic S&P ratings over time. For each rating in CRE it is checked whether a historic S&P rating is available in one of these two databases. To be able to do this the CRE database must be linked with these two databases. This linking can be done by linking the names of the banks in CRE with the names of the banks in Bloomberg and CreditPro. There can however be minor differences in the exact names of the banks in these databases. For example ABN AMRO can be called 'ABN AMRO' in CRE and 'ABN AMRO S.A' in Bloomberg and/or CreditPro.

Therefore it is preferred to link these banks by a unique code, which is the same for a bank in all three databases. Bloomberg uses ISIN-codes to identify banks, whereas CreditPro uses CUSIP-codes. Since Bankscope lists ISIN codes of banks but not CUSIP-codes, we can only match the Bloomberg ratings directly with the observations through the ISIN codes. For this reason it is chosen to primarily use the Bloomberg database to obtain the historic ratings for the observations.

However, only historic S&P ratings of banks which are listed on a stock exchange can be found in Bloomberg. Ratings of unlisted banks can therefore not be downloaded from Bloomberg. The ratings for these banks are downloaded from CreditPro. If there is no rating present for a listed bank in Bloomberg, we will also check whether CreditPro does list a rating for that bank. The linking of the CreditPro database with the observations is done via bank names and countries of residence. For more details about this linking see Appendix 14.3. If there is no historic rating in both Bloomberg and CreditPro the current Bloomberg rating is mapped to an observation. If this is also not possible the S&P rating in CRE is used. If this rating is also not available no reliable rating can be attached such that the observation is useless and should be removed.

### 4.3 First cleaning step



**Figure 5: First cleaning step**

After the construction of the dataset the first cleaning step is performed. This is the last step of the data collection stage. In this step observations are removed which do not meet the requirements for observations set by QRA. This section will start with an overview of these requirements. Thereafter these requirements are discussed in more detail.

#### 4.3.1 Overview requirements

In this section the requirements for the observations are described. These requirements are given in the modelling guidelines of QRA (Vedder, 2010) and shown below:

- The rating used for an observation should be approved by the credit committee; therefore ratings which have never been approved should be removed.

- The rating used for an observation should be unaffected by parental support.
- Observations must be complete. Therefore observations without financial statements or without external rating should be removed.
- Observations are not allowed to be too old. Therefore observations constructed from too old qualitative ratings or too old financial statements should be removed.
- The time between two observations of the same bank must be at least ■ days.
- Observations with an external rating which indicates a default are not allowed.

#### 4.3.2 Rating status

The first requirement involves the statuses of the qualitative rating assessments used for the creation of the observations. These assessments can have three statuses: confirmed, approved and out-dated. When an assessment is generated it automatically gets the status 'confirmed'. Once it is approved by the credit committee it gets the status 'approved'. When an approved assessment is older than ■ year it gets the status 'out-dated'. The model has to be constructed from observations constructed from assessments that have ever been approved. Observations constructed from confirmed but not approved assessments are therefore removed.

#### 4.3.3 Parental support

The second requirement involves the parental support banks can enjoy. The model is aimed at estimating creditworthiness of counterparties on basis of their explanatory variables. Parental support is not present as a factor on the long list, but does influence S&P's external ratings. The reason for this is that parental companies can save its subsidiaries. Therefore the S&P ratings of the observations of banks with parental support are not representative for the creditworthiness of these banks. These observations should be removed. In CRE the qualitative rating assessments with and without parental support are shown. If there is a difference in those assessments for a particular bank the bank enjoys parental support and the observation constructed from the assessment is removed.

#### 4.3.4 Incomplete ratings

The third requirement states that observations should be complete. Observations without financial statement or without external rating are useless and should therefore be removed.

#### 4.3.5 Too old observations

The fourth requirement involves that observations need to be recent. This means that the rating assessment in CRE should be recent enough, and that the appended financial statements from Bankscope should also be recent enough. To determine the precise date bound we had to make a trade-off. On the one hand Basel requires internal rating models to be built on at least five years of data (BIS, 2006, p. 463), but on the other hand the more old data is used the worse the model reflects the current risk landscape. We decided to set the date bound at the day the old Commercial Banks model was used for the first time: ■■■■■. The reason for this choice was that for some qualitative factors

information was not available in CRE from before this date. Therefore choosing this date increases the data quality. Next to the removal of observations from ratings from before [REDACTED], observations with attached financial statements dating from years from before [REDACTED] were also removed.

#### 4.3.6 Time between observations

The time interval between observations of banks is variable. It can therefore be that a bank is rated twice in [REDACTED] days. These two observations of the same bank are thought of as being the same, and therefore as one observation with a double weight (Vedder, 2010). Since it is desired to have unique observations with equal weights, the older of the two observations is removed.

The assumption that two observations of the same bank with more than [REDACTED] days difference are independent can however be questioned. I think it is interesting to check the autocorrelation in the residuals of a series of observations from a bank, to determine whether the observations are really independent. This is also important for the validity of OLS, since the Gauss-Markov requires the residuals to be uncorrelated (Plackett, 1950). Further research should be performed at this topic.

#### 4.3.7 Defaults

Finally, observations with an S&P default rating are removed. The reason for this is that these observations disturb the regression too much. As discussed the regression in the MFA stage is performed upon the logarithm of the PDs. The values of the logarithms of the PDs corresponding with the non-defaulted ratings are roughly spoken in the range  $[-4, -9]$ , where the logarithm of 1 is 0 (a default). The few observations with a value of 0 influence the regression too much, such that these observations are removed. A drawback of this approach is that the constructed model will be too optimistic which might be a risk for RI.

#### 4.3.8 Overview

Before the first cleaning step the dataset consisted of [REDACTED] observations. In the table below an overview of the number of removed observations per cleaning step is given.

Cleaning step		Number of removed observations	
[REDACTED]		[REDACTED]	[REDACTED]
[REDACTED]		[REDACTED]	[REDACTED]
[REDACTED]		[REDACTED]	[REDACTED]
[REDACTED]		[REDACTED]	[REDACTED]
[REDACTED]		[REDACTED]	[REDACTED]
[REDACTED]		[REDACTED]	[REDACTED]
[REDACTED]		[REDACTED]	[REDACTED]
[REDACTED]		[REDACTED]	[REDACTED]
[REDACTED]		[REDACTED]	[REDACTED]

Table 6: Removal of observations

After the first cleaning step the dataset thus consists of [REDACTED] observations.

## 5 Data processing

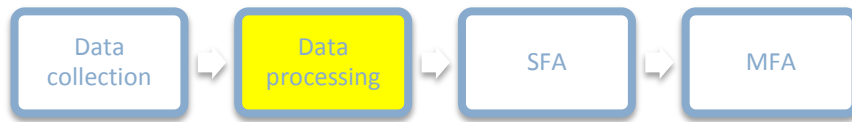


Figure 6: Data processing stage

In this chapter I describe the data processing stage, the second stage of the modelling process. This stage includes all steps necessary to prepare the dataset for the SFA and MFA. These steps are: inter- and extrapolation, calculation of financial ratios, taking logarithms, removal of factors, transformation of factors, and finally the representativeness correction. These steps are visualized in the figure below.

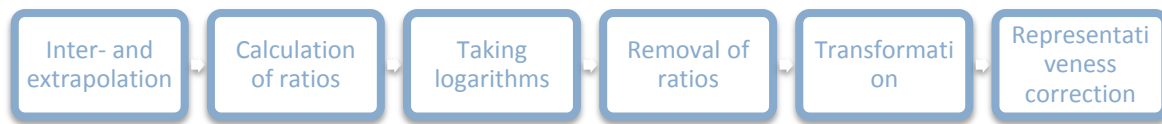


Figure 7: The steps of the data processing stage

### 5.1 Inter- and extrapolation

The dataset at the end of the data collection stage consists of observations consisting of qualitative factor values and financial statements fields. From these financial statements fields the financial ratio values must be calculated. However a lot of missing values occur in these financial statements fields and financial ratios can only be calculated from fields without missing values. Therefore we decided to estimate these missing financial statement values first such that we would be able to calculate more financial ratios later. We used inter- and extrapolation to estimate these fields. This process is called data filling and described in this section.

#### 5.1.1 Regular and exceptional fields

Before we started the data filling process, we had to find the missing values in the different financial statements fields. Recall that the financial statements are downloaded from Bankscope. The problem of detecting missing values arises from the fact that Bankscope does not recognise missing values in the different fields. Fields which are left blank in Bankscope automatically get assigned the value of zero. It is therefore not possible to distinguish missing values from fields with a value of zero in the financial statements.

For this reason we introduced the concept of regular and exceptional fields. Regular fields are fields which should be available for all banks, whereas exceptional fields do not have to be. Zeros at regular fields represent missing values, whereas zeros at exceptional fields represent fields with a value of zero. By definition missing values can thus only occur at regular fields. Only regular fields are therefore inter- and extrapolated.



An example of a regular field is the asset size of a bank. Each bank has an asset size. Missing values at this field are inter- and extrapolated. An example of an exceptional field is the total deposits size of a bank. Commercial banks have deposits, but investment banks and other financial institutions have not. Therefore not all banks contain values for this field and observed zeros can represent real zeros. This factor is therefore not inter- and extrapolated. We asked the experts determine for each financial statement field whether it is a regular or an exceptional field. In the fourth column of Appendix 14.4 this overview is shown.

### 5.1.2 Filling regular fields

To be able to use inter- and extrapolation there must be information of the missing value for other years than the year of the missing value. If this information is not available inter- and extrapolation cannot be used. The first step of the data filling process is therefore the creation of an overview of the available values over time for all banks for all regular fields. In the figure below an example of such an overview is shown. In this example the values on the regular field 'Total assets size' are shown for a bank. In total there are 5 observations from this bank ranging from 2007 to 2011. For these five observations the values on the total asset sizes (TA) fields are shown. The numbers are in billions.



**Figure 8: Total assets of a counterparty over time in billions**

As can be seen from the picture there are values available for the years 2007, 2008 and 2010. The field values for 2009 and 2011 are missing. These missing values should be inter- and extrapolated.

Interpolation of a missing value on a specific field can only be performed when there are values from before and after the observation to be interpolated. For the example above this means that the field value from 2009 can be estimated with interpolation. We decided to use the simplest interpolation technique available to estimate a missing value: linear interpolation. The formula for this technique is shown below.

$$F(n) = F(n - i) + i * \frac{F(n + j) - F(n - i)}{i + j} \quad (7)$$

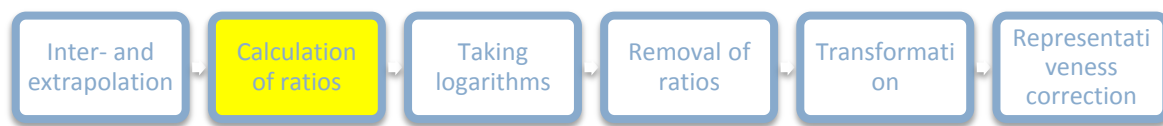
In this equation  $F$  is the field value of a bank as a function of  $n$ ,  $n$  is the year of the missing observation,  $i$  the number of years between the closest observation before  $n$  and  $n$  itself, and  $j$  the number of years between the closest observation after  $n$  and  $n$  itself. For the example above  $n$  is 2009 and  $i$  and  $j$  are both 1. The estimated field value thus becomes 1.2 billion.

Extrapolation is applied when there are only observations from the same bank from either before or after the missing observation. Such missing values can also be interpreted as missing edges. In the example above the missing value of 2011 is a missing edge. When extrapolation is applied the missing

field value is replaced by the field value of the observation that is the closest to the missing edge. For the example above the estimate of the missing field for 2011 thus becomes 1.3 billion. We preferred this approach over extrapolating trends because this latter approach can result in unrealistic values. For example a missing value for the factor asset size can become negative when a strong negative trend is observed at the observations before this missing value.

It is not possible to inter- or extrapolate when there is no field information available for any observation of the bank. In this case the missing values are left missing, which makes it not possible to calculate the financial ratios from these fields. In the next section it is described how the ratios are constructed from the different fields. It is then also discussed how is dealt with observations still containing missing values.

## 5.2 Calculation of financial ratios



**Figure 9: Calculation of factors**

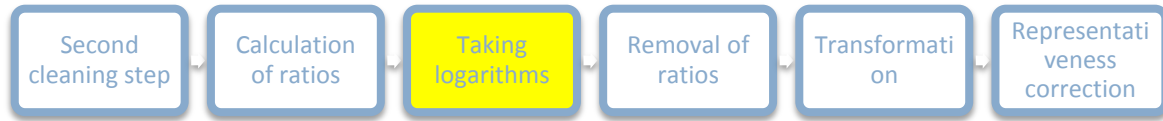
After the regular factor fields are filled the financial ratios are calculated for the observations. In the last column of Appendix 14.2 the formulas for the constructing of these ratios from the financial statement fields are shown. Recall that there are still some missing values in the financial statement fields. Since ratios cannot be constructed from regular fields containing missing values, not all ratios can be calculated. For example if the regular field ‘Liquid assets’ still contains a missing value for a certain observation, the financial ratio value ‘Liquid assets / Total assets’ of this observation cannot be calculated.

To deal with this issue we first calculated the financial ratios of which the fields did not contain missing values. Thereafter we identified the ratios which could not be calculated and decided to replace these ratios by medians of buckets of financial ratio values which could be calculated. The values in these buckets needed to be as representative as possible for the missing value. Therefore we decided to select only ratio values of observations of banks from the same country and same time period as the particular bank. The time period of an observation ranges from one year before the observation until one year after the observation. If there were more than 10 ratio values in the bucket we selected the median of this bucket as the best estimate of the missing ratio value. If there were less than 10 values in the bucket the missing value remained missing. Since there were also missing values in the qualitative factor values, we decided to use the same median replacement procedure for replacing these missing values.

The dataset at the end of the data collection stage consisted of 1000 observations. From these observations 100 observations had initially at least one missing value in either its qualitative factors or financial ratios and should thus be removed. By inter- and extrapolating the financial statements fields and applying median replacement for the qualitative factors and financial ratios we could recover 900

observations from these ████ observations. The dataset at the end of this step thus consists of ████ complete and ████ incomplete observations.

### 5.3 Taking logarithms of financial ratios



**Figure 10: Extending the long list**

After the calculation of the financial ratios, the long list is extended with the logarithms of the financial ratios. In Section 2.5 it is explained that with the Shadow-Bond approach a linear regression is performed on the logarithm of the PDs. This regression is performed in the MFA stage, where factors are selected on their linear explanatory power. Some financial ratios however do not have a linear relationship with the  $\log(\text{PD})$ . These ratios can have explanatory power, but will not be selected in the MFA stage, since the explanatory power is not linear. By taking the logarithms of these financial ratios and including them as additional factors on the long list, financial ratios with an exponential relationship with the  $\log(\text{PD})$  can still be selected in the MFA stage. The dataset after the taking of the logarithms is shown in the table below.

WWID	Date	Q1	...	<span style="background-color: black; color: black;">████</span>	R1	...	<span style="background-color: black; color: black;">████</span>	Log(R1)	...	<span style="background-color: black; color: black;">██████████</span>	PD
:	:	:	:	:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:	:	:	:	:

**Table 7: Dataset with logs**

In this table the first columns contain the qualitative factors (Q1, to ████), and the next columns contain the financial ratios (R1 to ████). Recall that the financial ratios are calculated from the financial statements fields. In the columns 'Log(R1)' to ██████████ the logarithms of the financial ratios 'R1' to ████ are given. As described, the logarithms of these ratios are also included in the MFA stage, where the factors with the highest combined predictive power are selected from all factors. For some financial ratio values it was not possible to calculate the logarithms. For example, it is not possible to take the logarithm of zero or a negative number. Therefore we decided to replace the logarithms of these values with missing values.

### 5.4 Removal of factors



**Figure 11: Removal of factors**

After the calculation of financial ratios there were still [REDACTED] observations containing missing values in the dataset. A number of these observations has its missing values only in logarithms of financial ratios. In this section it is analysed for each factor (logarithms of financial ratios included) how many observations have missing values on that specific factor. Factors with too many missing values are then removed from the dataset since too many observations are removed due to these factors.

There are no guidelines for the removal of factors. On the one hand we did not want to remove factors which could have explanatory power, but on the other hand we wanted as many observations as possible. We argued that generally speaking factors with many missing values have low explanatory power and could therefore be removed. Therefore we decided to remove factors with more than [REDACTED] missing values. In total [REDACTED] factors had more than [REDACTED] missing values. From these [REDACTED] factors, [REDACTED] factors were logarithms of financial ratios. Since the logarithms of the financial ratios are created from the original ratios, there is no loss of data when these logarithms are removed from the dataset. The only non-logarithm factor is the factor [REDACTED], indicating the [REDACTED]. Since there are many other factors on the long list describing the portfolio quality of a bank, we decided this factor could also be removed from the long list.

When removing these factors observations which had their missing values only at these factors have no remaining missing values anymore and are thus recovered. From the [REDACTED] observations with missing values [REDACTED] observations had their missing values only in at least one of these [REDACTED] selected factors. After recovering these observations [REDACTED] observations with missing values remained in the dataset. These observations were removed, resulting in a dataset of [REDACTED] observations upon which the final steps of the data processing stage will be performed.

## 5.5 Transformation of factors



**Figure 12: Transformation of factors**

After the removal of the observations with missing values, the financial ratios and logarithms of financial ratios are transformed. There are two reasons for this. The first reason is that the model should be intuitive, meaning that higher scores for the factors on the scorecard should result in a better rating, and thus in a lower PD. The second reason is that the weights of the factors on the scorecard should be interpretable. This means that the impact of a factor with a weight of 5% is as high as the impact of another factor with a weight of 5%. To achieve this factor values must have the same range and the same positive relationship with the creditworthiness. Therefore we transformed all factors to the [0, 10] range and transformed factors with a negative relationship into positive factors. In this section I describe how we performed this transformation.

### 5.5.1 Logistic transformation

The first step of the transformation process is the transformation of the different factors to the [0, 10] range. The qualitative factors are already in a [0,10] range, therefore only the financial ratios and logarithms of financial ratios need to be transformed. The financial ratios and logarithms of financial ratios (from now on factors) are transformed by applying a logistic transformation. This is the preferred transformation approach of QRA (Vedder, 2010). With this approach, the logistic function is used to map factor values to the [0, 10] range. The logistic cumulative distribution function (cdf) is given by the formula:

$$F(X \leq x) = \frac{1}{1 + e^{\text{Slope}(\text{Midpoint} - x)}} \quad (8)$$

In this formula the variable Midpoint is the mean of the logistic distribution function and the variable Slope a scalar parameter proportional to the standard deviation of the logistic distribution function (Vedder, 2010). The first step in performing the logistic transformation is finding the empirical cumulative distribution function (cdf) of the different factors. This function is constructed without the highest and lowest 5% factor values to reduce the impact of outliers as prescribed by the guidelines (Vedder, 2010). The formula used for the construction of the empirical cdf is shown below.

$$F_n(t) = \frac{\text{number of scores in the set} \leq t}{n} = \frac{1}{n} \sum_{i=1}^n 1\{x_i \leq t\} \quad (9)$$

Where  $n$  is the number of values in the set (the observations),  $x_i$  the  $i$ th factor value of the set and  $t$  a fixed number ranging from the smallest  $x_i$  to the highest  $x_i$ . The indicator  $1\{A\}$  indicates whether event  $A$  has occurred. In this formula  $A$  indicates whether a factor value is smaller than or equal to  $t$ . For each factor the empirical cdf is determined with this formula.

The next step is the fitting of the logistic cdf on the empirical cdf. This is done by using an iterative least squares algorithm in Matlab. This algorithm needs good starting points for the Midpoint and Slope. The guidelines prescribe the average of the 5% percentile factor value and the 95% percentile factor value as a good starting point for the Midpoint (Vedder, 2010). The starting value for the Slope is prescribed by the formula:

$$\text{Slope}_{\text{initial}} = \frac{2.994}{x_{0.95} - \text{Midpoint}_{\text{initial}}} \quad (10)$$

In this formula 2.994 is derived from solving  $F(X \leq x_{0.95}) = 0.95$  for the standard logistic function with the Midpoint and Slope equal to 0 and 1 respectively.

Once the starting values for the Midpoint and Slope are determined the Midpoint and Slope are estimated in Matlab. By plugging the estimated coefficients into Equation 8 the transformed values for the different factors can be calculated. By multiplying this value by 10 a value between 0 and 10 is obtained. This factor value in the [0,10] range is called the factor score. The temporary removed highest and lowest 5% factor values are also transformed with the estimated coefficients.

This procedure can be made clear with an example. The estimated Midpoint and Slope of factor [REDACTED] are 1.94 and 0.97 respectively. The factor score for a certain observation is 3.38. The factor score thus becomes:

$$F(X \leq 3.38) = \frac{10}{1 + e^{0.97(1.94-3.38)}} = 8.3 \quad (11)$$

### 5.5.2 Negative factors

Now all factors are in the [0,10] range the factors with a negative relationship must be transformed into factors with a positive relationship. This is done to make sure the model is intuitive. To be able to do this first the relationships of all factors need to be determined. The qualitative factors all have a positive relationship. For the financial ratios, the experts were asked to come up with a list with relationships. This list is shown in the fourth column of Appendix 14.2.

From this list the factors with negative relationships are identified. The scores of these factors must be transformed. This is done by subtracting these factor scores from the maximum factor score: 10. If the factor of the example from the previous section would have a negative relationship the factor score would become  $10 - 8.3 = 1.7$ .

The process of subtracting the factor score of negative factors from 10, can be integrated in the transformation to the [0, 10] range process. For the example from the previous section it is shown in the equation below how this can be done.

$$10 - \frac{10}{1 + e^{0.97(1.94-3.38)}} = \frac{10e^{0.97(1.94-3.38)}}{1 + e^{0.97(1.94-3.38)}} = \frac{10}{1 + e^{-0.97(1.94-3.38)}} \quad (12)$$

So by multiplying the Slope coefficients of the negative factors by - 1, the factors are transformed into positive factors. This way the transformation of factors to the [0,10] range and the transformation of negative factors to positive factors can be integrated in one step.

### 5.5.3 Reflection on transformation

In this section I will give my reflection on the transformation procedure described above. By fitting the logistic cdf to the empirical cdf of a factor, the assumption is made that the factor follows a logistic distribution with the regression parameters Midpoint and Slope. I think this assumption can be made for most of the factors, but am not sure whether it can for all. Therefore I decided to validate this assumption, by comparing the mean and standard deviation of the factor scores (the transformed values) with the mean and standard deviance of a uniform distribution. If it is true that the factors are logistic distributed these moments should match. The mean and standard deviation of a [0, 10] uniform distribution are 5 and 2.89 respectively. The average of the means and standard deviations of the transformed factors are shown in the table below. The complete list with means and standard deviation is shown in the last two columns of Appendix 14.5.

Moment	Average
Mean	5.07
Std. Dev.	3.09

**Table 8: Moments transformed values**

As can be seen from the table the average of the means is relatively close to the assumed 5. The standard deviation is however a little higher (3.09 against 2.89). This extra deviation comes from the fact that the factors are not really logistic distributed, but can also have other distributions.

For some factors there is however reason to believe that the factor does not follow a logistic distribution. For example factors can be U-shaped. This means that extreme factor scores are bad for the expected creditworthiness and average scores good, or the other way around. This can be true for growth or profit factors. Average growth or profit is a good sign, whereas extreme low or extreme high scores are a bad sign.

Since there is limited time to analyse these U-shaped ratios, they are transformed with a logistic transformation for now. This way explanatory power of U-shaped factors is lost, since they will not be selected in the MFA stage because they have no linear relationship with the PD. I think this is however a point for further research. It should be analysed whether and how these factors can be included in a scorecard model and how such a model can be implemented in the IT infrastructure of RI.

## 5.6 Representativeness correction



**Figure 13: Representativeness correction**

The last step of the data processing stage is the representativeness correction. This correction is performed because the model needs to be representative for the banks which are rated by RI. Recall that the observations are created from the qualitative rating assessments from CRE. These qualitative rating assessments are the ratings generated by RI and thus representative for the banks which are rated by RI. Many observations are however removed in the data cleaning step, therefore the obtained dataset is not equal to the initial dataset and thus not representative for the ratings generated by RI anymore. Therefore we decided to perform a representative correction on the obtained dataset (development dataset).

The first step we performed is the identification of banks rated by RI. These banks are the banks with qualitative rating assessments in CRE. The set of banks of which are assessments is called the representativeness set from now on. Most observations which were removed were removed because there was not enough information available, for example there was no Bankscope data or no historic S&P rating.

The availability of data is related to the size and country of a bank. For example there is a lot of information available of big banks from the US, whereas there is less information available of small Asian banks. For this reason we decided to perform a breakdown of the representativeness set and the development dataset to analyse the differences along these two dimensions. We started with the breakdown of the representativeness set. For the banks in this set we analysed in what region the banks were in and what the assets sizes of these banks were. In the table below the results of this analysis are shown.

	Size NA	Size < 100	Size > 100	Europe	Asia	NA
Bank 1						
Bank 2						
Bank 3						
Bank 4						
Bank 5						
Bank 6						
Bank 7						

Table 9: Breakdown of banks in CRE

As can be seen from the table there are 7 unique banks in CRE. From these 7 banks, 6 banks are located in Europe. Also the majority of these 6 banks consist of relatively small banks with asset sizes smaller than 100 billion. In the column 'Size NA' are the banks in CRE of which the assets size is not known.

The same table can be constructed for the development dataset. We analysed for all observations in the dataset in what region the bank is in and what the total assets sizes of that bank is. In the table below the results of this analysis are shown.

	Size NA	Size < 100	Size > 100	Europe	Asia	NA
Bank 1						
Bank 2						
Bank 3						
Bank 4						
Bank 5						
Bank 6						
Bank 7						

Table 10: Breakdown of banks in development dataset

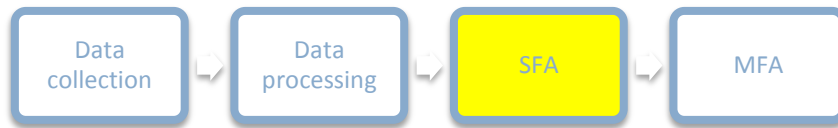
From this table it can be seen that the majority of the observations in the dataset are also from banks located in Europe. No big differences can thus be observed at first sight. We determined the weights per bucket by dividing the first table entry-wise by the second. These weights are shown in the table below.

	Size NA	Size < 100	Size > 100	Europe	Asia	NA
Bank 1						
Bank 2						
Bank 3						
Bank 4						
Bank 5						
Bank 6						
Bank 7						





## 6 Single Factor Analyse (SFA)



**Figure 14: SFA**

After the data has been collected and processed the Single Factor Analyse (SFA) is performed. In this stage the different factors are analysed on their standalone explanatory power. We did this by calculation the Powerstats of the different factors as prescribed by the guidelines (Vedder, 2010). In Section 6.1 the Powerstat concept is described, where after the Powerstats of the different factors are calculated in Section 6.2. In Section 6.3 more attention is given at factors with negative Powerstats.

### 6.1 Powerstat concept

The Powerstat concept is closely related to the Gini coefficient (Gini, 1912) and is a measure of the explanatory power of a factor. The Powerstat of a factor is calculated by comparing the scores of the factor with the scores of a factor with perfect explanatory power. The higher the Powerstat of a factor, the closer the factor to a factor with perfect explanatory power and thus the higher its explanatory power.

The first step in calculating the Powerstat of a factor is sorting the observations on basis of their factor scores. Then we determined for each observation what percentage of the observations has lower or equal scores at that specific factor. Then the PD is looked up for each observation and the sum of the PDs of the observations with smaller or equal factor scores is taken. This sum is then divided by the sum of the PDs of all observations. The result of this analysis is a series of points constructed from the different observations. Each point represents the percentage of observations with lower or equal factor scores of an observation against the sum of the PDs corresponding with these observations as a percentage of the sum of all PDs. The graph which can be constructed from these points is called the Power curve of a factor.

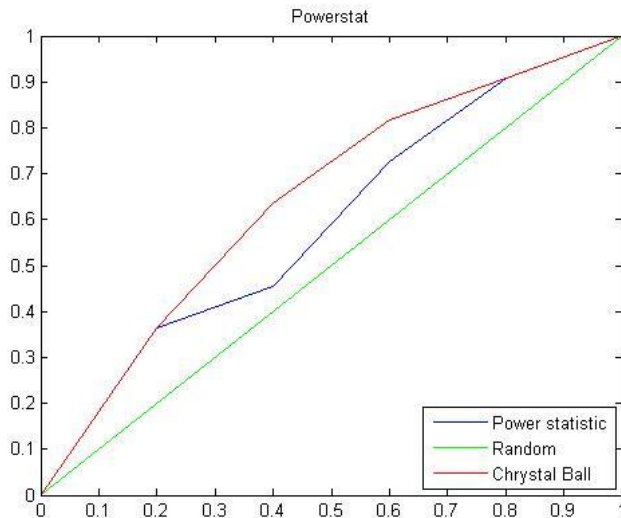
This approach can best be understood with an example. In the table below a dataset consisting of 5 observations is given.

#	Factor score	PD
1	3	0.2
2	4	0.05
3	6	0.15
4	7	0.1
5	8	0.05

**Table 12: Powerstat dataset**

This dataset contains 5 observations with factor scores and PDs shown in the last two columns respectively. The observations are sorted based on their factor scores. The factor score of the second

observation is 4. In total there are 2 observations with a factor score of 4 or lower, this is 40% of all observations. The sum of the PDs associated with these observations is 0.25 (0.2+0.05). This is 45.5% of the sum of all PDs (0.25/0.55). The point (0.4, 0.455) is therefore one of the points of the Power curve. By constructing the points corresponding with the other observations in the same way the Power curve shown as the blue line in the figure below is obtained.



**Figure 15: Powerstat, horizontal: cumulative proportion of observations, vertical: cumulative proportion of PDs**

Alongside the Power curve two other curves are plotted in the figure above. These curves are the Chrystal Ball curve (red line) and the Random curve (green line). The Chrystal Ball curve is the Power curve of a factor with perfect explanatory power. This curve can be constructed for a factor by sorting the PDs in decreasing order and the factor scores in increasing order as is done in the table below.

#	Factor score	PD
1	3	0.2
2	4	0.15
3	6	0.1
4	7	0.05
5	8	0.05

**Table 13: Chrystal Ball curve**

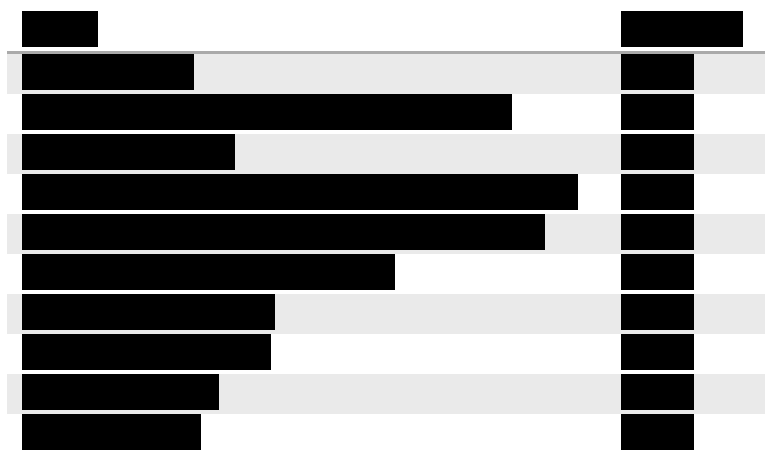
As can be seen from the table the lowest factor scores perfectly correspond with the highest PDs. If the Power curve of this table is constructed, the result is the Chrystal Ball curve shown in Figure 15. The other curve in this figure is the Random curve. This is the curve where the factor has no explanatory power at all. For such a factor the factor scores do not influence the PDs of the observations at all. The curve is therefore a straight line from (0,0) to (1,1).

After the construction of the three curves, the Powerstat can be calculated. The Powerstat is calculated as the ratio of the area between the Power curve and the Random curve and the area between the Chrystal Ball curve and the Random curve. The closer the Powerstat is to the Chrystal Ball curve, the

higher this ratio and thus the Powerstat is. The highest possible Powerstat (1) is achieved when the factor has perfect explanatory power. For the example in Figure 15 the Powerstat is 62,5%.

## 6.2 Results

Now the Powerstat concept is explained, the Powerstats of the different factors can be calculated. Since the Powerstat of a factor depends on the order of the sorted observations, and taking the logarithm of a series does not change the order of a series, the Powerstat of the original factor is the same as the Powerstat of the logarithm of the factor. In the table below the ten factors with the highest Powerstats are shown. The complete list of Powerstats is shown in Appendix 14.5.




**Table 14: Powerstats**

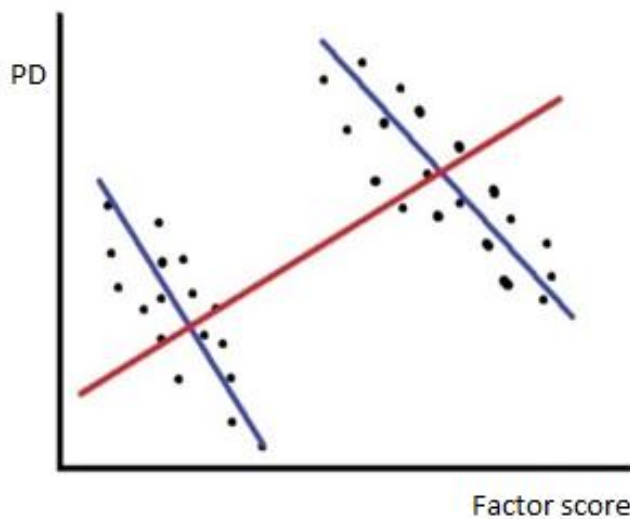
As can be seen from the table the country rating has the highest Powerstat. This means that this factor has the highest standalone explanatory power.

In the guidelines it is described that ‘After single factor analysis it must be decided which risk drivers will be taken to the multi-factor analysis’ (Vedder, 2010). There is thus no prescribed approach for selecting these factors. Therefore we had to make a choice about which factors to select. First we thought of removing all factors with a Powerstat lower than a certain threshold, but then we noticed that some factors had negative Powerstats. This was striking since this was by definition not possible because all factors were supposed to have positive Powerstats because of the transformation. Before we would remove these factors together with the other factors with Powerstats below the threshold we would like to know where these negative Powerstats came from. In the next section I will describe the analysis we performed at these factors.

## 6.3 Negative Powerstats

From the table in Appendix 14.5 it can be seen that some factors have negative Powerstats. These factors can be interpreted as factors with explanatory power the other way around. For these factors a higher score results in a lower expected creditworthiness for the observations. In Section 5.5.2 it is discussed that negative factors are transformed into positive factors during the transformation process. Therefore these negative Powerstats cannot be the result of negative relationships between the factor

scores and the creditworthiness. There is however an explanation for these negative Powerstats. This can best be understood with a figure.



**Figure 16: Factor scores with corresponding PDs**

In this figure the PDs of the observations are plotted against the factor scores of these observations for a factor with a negative Powerstat. As can be seen from the figure, the data points are distributed over two subsets. One subset corresponds with low factor scores and low PDs, the other subset corresponds with higher factor scores and higher PDs. When a regression would be performed on the complete dataset the result would be the red line. Since the slope of this red line is positive the relationship of this factor with the PDs is positive. This result is valid if this factor is the only explanatory variable of the PDs. Observations with low factor scores are located in the subset with low PDs, therefore the PDs of these observations are low. The same principle holds for observations with high factor scores.

However, when there is solely focused on either the first or the second subset, the relationship between the factor scores and the PDs is the other way around. Stated differently when only the first subset is taken into account, it can be observed that low factor scores correspond with high PDs and high factor scores correspond with low PDs. The same principle holds for the other subset. When a regression is performed on both subsets individually the relationship between the factor scores and the PDs would be negative as can be seen from the negative slopes of these blue lines.

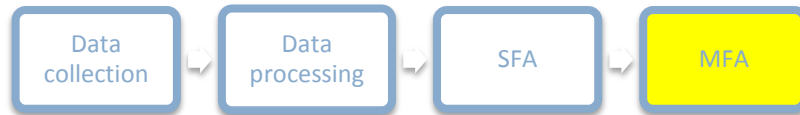
This can best be understood with an example. From Appendix 14.5 it can be seen that factor R39 '(Cash + government bonds) divided by total assets' has a Powerstat of [REDACTED]. The standalone explanatory power of the PD (reversed creditworthiness) of this factor is thus positive just as the slope of the red line in Figure 16. Therefore higher factor scores for this factor result in higher expected PDs for the corresponding observations. This is a counterintuitive result since one would expect banks with higher capital levels to have lower PDs.

Now this factor is combined with a binary factor, which indicates whether the country the bank is located in is creditworthy or not. Banks in countries with low creditworthiness in general need to have

higher capital levels to be active on the capital markets. These banks also have lower PDs in general than banks located in countries with high creditworthiness. The country factor explains both the higher factor scores and the higher PDs and thus in what subset the observation is in. The capital factor does not have to explain in what subset the observation is in now, but should only diversify between the observations in a subset. The sign of the capital ratio now becomes negative, such that banks with high capital levels in countries with low creditworthiness have lower PDs than banks with low capital levels in these countries.

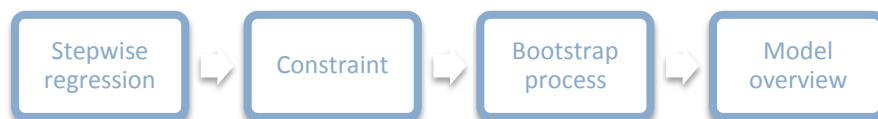
This example shows the power of combined explanatory power. Therefore factors with negative Powerstats (indicating negative relationships) should also be included in the MFA stage, since they can get positive explanatory power in a model with multiple explanatory variables. The same principle holds for factors with low but positive Powerstats. These factors can also have explanatory power in a model with other factors. Therefore we decided to include all factors in the MFA stage.

## 7 Multi Factor Analyse (MFA)



**Figure 17: MFA**

The last stage of the modelling process is the Multi Factor Analyse (MFA) stage. In this stage a model is constructed from the factors from the long list. We did this by performing a stepwise regression on the complete dataset. This regression approach is discussed in Section 7.1. During the project meetings experts stressed the importance of a liquidity factor in the model. Therefore we made sure that a liquidity factor was selected during the stepwise regression. In Section 7.2 I will describe how we did this. Furthermore one of the requirements of the model is that the selected factors are robust, which means that the confidence bounds of the selected factors are narrow and in the positive domain. Therefore we performed a bootstrapping process to determine the confidence bounds of the factors and to evaluate their robustness. This process is discussed in Section 7.3. In Section 7.4 an overview of the constructed model is given. The MFA stage is visualized in the figure below.



**Figure 18: The four steps of the MFA stage**

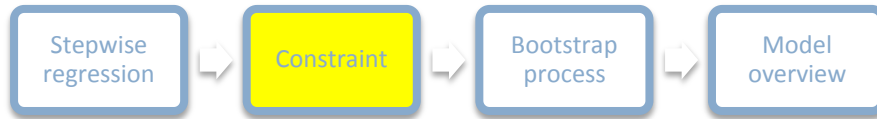
### 7.1 Stepwise regression

In total there are 114 factors (logarithms of financial ratios included) on the long list. The number of different models which can be constructed from these 114 factors is enormous. It is therefore not feasible to construct all different models and select the one with the highest explanatory power. For this reason the guidelines prescribe a stepwise regression, a heuristic approach for constructing a model with high explanatory power (Vedder, 2010). This way not all models have to be constructed. The steps of the stepwise regression approach are shown below:

1. Estimate the intercept only model.
2. Estimate all models with one additional factor and select from these models the models where the included factor has a significant negative coefficient (negative because high factor scores must result in low PDs).
3. Calculate the F-statistics of the selected models and select the model with the highest F-statistic.
4. Check whether the other factors in the model are still significant and negative and remove insignificant and/or negative factors.
5. Repeat step 2 to 4 until no factors can be found that are significant and negative.

The standard significance level at QRA for including factors is 0.05 (step 2). Also it is common at QRA to set the significance level for removing factors a little higher at 0.1. The reason for this is that significant factors for which the P-values slightly increase after the inclusion of other factors are not directly removed. Since we had no reason to deviate from these standard levels, we decided to use these significance levels.

## 7.2 Constraint factor



**Figure 19: Constraint**

During the expert meetings the experts stressed the importance of a liquidity factor in the model. Therefore we decided to make sure that a liquidity factor was incorporated in the model. The experts preferred factor **Liquidity** from the financial ratios from the liquidity category. They would like this factor to have a weight of 0.1 in the model, with the weight of the factor calculated as explained in Section 2.4.

We thought of two approaches of making sure this factor was included in the model. The first approach was performing a stepwise regression on the complete dataset and including factor **Liquidity** in the model afterwards. The second approach was selecting factors with a stepwise regression with the knowledge that factor **Liquidity** was already in the model. We preferred the second approach, because this way the combined explanatory power of the selected factors and factor **Liquidity** was higher. We made sure that factors with high combined explanatory power with factor **Liquidity** were selected during the stepwise regression by distributing factor **Liquidity** over the other factors and performing a stepwise regression on these adjusted factors. To understand this, recall the regression equation of the CBPD model:

$$\ln(PD) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_n x_n + \varepsilon \quad (13)$$

In this equation  $PD$  is a vector of PDs of the observations,  $x_1$  to  $x_n$  the  $n$  vectors of factor scores of the observations,  $\beta$  a vector of constants and  $\varepsilon$  a vector of random noises. In the regression the coefficient vector  $\beta$  is optimized. Now say the coefficient  $\beta_1$  is the coefficient of factor **Liquidity**, the factor which needs to be fixed with a weight of 0.1. Fixing the weight of this coefficient can be achieved by expressing this coefficient in terms of the other coefficients. If  $\beta_1$  should have a weight of  $\alpha$  percent,  $\beta_1$  can be expressed as:

$$\beta_1 = \frac{\alpha \sum_{i=2}^n \beta_i}{1 - \alpha} \quad (14)$$

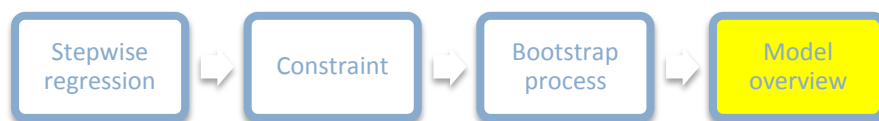
This equation holds because the sum of the other weights is equal to  $1 - \alpha$ . Now  $\beta_1$  is expressed in terms of the other  $\beta$ 's. When Equation 14 is substituted in Equation 13 the result is the formula below:





In this table the average weights of the factors can be found in the second column, the 5% and 95% confidence bounds of the factors in the third and fourth column respectively and the number of times the factors were assigned a negative weight in the last column. For the last three factors the confidence bounds of the weights are not completely in the positive domain. For the factor [REDACTED] the 95% confidence interval of the weight is [-0.6%, 2.4%]. Negative weights are not interpretable, but result from positive regression coefficients. Such positive regression coefficients are not desired since we want the factors in the model to have a negative relationship with the PDs (higher score should result in lower expected PD). From the last column of the table it can be seen that these factors were indeed assigned a positive coefficient a number of times. Therefore we concluded that these factors are not robust enough, and we decided to remove these three factors from the model.

## 7.4 Model overview



**Table 16: Model overview**

After the removal of the three unstable factors, we had a meeting with the experts. We asked their opinion about the selected factors. The experts agreed on the removal of the unstable factors from the model, but would like to have an additional capital factor included in the model. This capital factor should be forward looking and matching Basel’s focus on Tier-1 capital. Therefore we decided to include the factor [REDACTED] in the model. The experts believed this factor should get a weight of [REDACTED].

When we included this factor in the model the P-value of the factor [REDACTED] increased to [REDACTED]. Since the focus at QRA is on removing factors with little extra explanatory power and high P-values and [REDACTED] is quite high we decided to remove this factor from the model. The obtained model after the removal of this factor is shown below.

[REDACTED]	[REDACTED]	[REDACTED]	[REDACTED]
[REDACTED]	[REDACTED]	[REDACTED]	[REDACTED]
[REDACTED]	[REDACTED]	[REDACTED]	[REDACTED]
[REDACTED]	[REDACTED]	[REDACTED]	[REDACTED]
[REDACTED]	[REDACTED]	[REDACTED]	[REDACTED]
[REDACTED]	[REDACTED]	[REDACTED]	[REDACTED]
[REDACTED]	[REDACTED]	[REDACTED]	[REDACTED]
[REDACTED]	[REDACTED]	[REDACTED]	[REDACTED]
[REDACTED]	[REDACTED]	[REDACTED]	[REDACTED]
[REDACTED]	[REDACTED]	[REDACTED]	[REDACTED]
[REDACTED]	[REDACTED]	[REDACTED]	[REDACTED]
[REDACTED]	[REDACTED]	[REDACTED]	[REDACTED]
[REDACTED]	[REDACTED]	[REDACTED]	[REDACTED]

██████████		████	██████████	████	
██████████████████		████	██████████	████	

As can be seen from the table the weights of the selected factors range from for the country rating to for the factor . The last column of the table contains the notches impact of the factors. These notches impact must be interpreted as the number of Rabobank Risk Rating notches downgrade if the factor score goes from a 10 to a 0. A bank can thus have notches downgrade (for example rating goes from R9 to R15) if it moves from a country with a score of 10 (Switzerland) to a country with a score of 0 (Bangladesh).

As discussed in Section 7.3, the factors in the model need to be robust. For this reason we decided to perform an additional [REDACTED] bootstraps with the factors from the model. The factor [REDACTED] turned out to be negative in [REDACTED] bootstraps, all other factors were assigned only positive weights. Since this factor was included on expert basis, we did not remove this factor. This factor is however unstable. To check whether this is a problem we calculated the correlations of the different factor pairs. These correlations are shown in the table below.

# Table 18

Table 18: Factor pair correlations

The guidelines prescribe that absolute correlations between factor pairs are not allowed to be higher than 0.9 (Hoek, 2007). Although the *Perceived Attraction* has high correlations with the other factors, all correlations are below 0.9. Therefore we concluded this factor set is stable, and that the *Perceived Attraction* factors does not have to be removed from the model.

## 8 Capital impacts

Now that the new model is constructed the second part of the main question can be answered: *What are the capital impacts of this new model?* The capital impact of the new model is defined as the percentage increase or decrease in capital and is calculated for both regulatory capital (RC) and economic capital (EC) in this chapter. Since RI uses the AIRB approach its regulatory capital calculation is prescribed by Basel (BIS, 2006). This calculation together with the EC calculation is explained in Section 8.1. The capital impacts for both RC and EC are calculated and discussed in Section 8.2.

### 8.1 RC and EC calculation

RI's regulatory capital for credit risk is calculated with the formula:

$$RC = \sum_i EAD_i * LGD_i * (WCDR_{i,x} - PD_i) * MAM_i * \text{■} \quad (16)$$

In this formula the required capital is calculated as the sum of the product of  $EAD_i$ , the Exposure At Default (EAD), the Loss Giving Default (LGD), the Worst Case Default Rate (WCDR) minus the PD and the Maturity Adjustment Multiplier (MAM) of all loans. The EADs and the LGDs of the loans are calculated with special EAD and LGD models which are outside the scope of this thesis. The WCDRs and the MAMs are calculated with formulas explained in this section.

The WCDR is the percentage of defaults in one year on a large portfolio of exposures that will not be exceeded with a probability of  $x$  (Hull, 2010). This percentage can however also be calculated for a single loan. It is then the worst case default probability of a single loan. This probability is calculated with the Vasicek formula:

$$WCDR_{i,x} = N\left(\frac{N^{-1}(PD_i) + \sqrt{\rho_i}N^{-1}(x)}{\sqrt{1 - \rho_i}}\right) \quad (17)$$

In this formula  $N$  and  $N^{-1}$  represent the standard normal, and inverse standard normal distribution respectively and  $PD_i$  the probability of default of loan  $i$ . The correlation  $\rho_i$  between loan  $i$  and the market is the degree of systematic risk the loan has with the market. The variable  $x$  is a chosen confidence level and for RC set at  $\text{■}$  by Basel (BIS, 2006). The formula for the calculation of the correlation  $\rho_i$  of a loan is also prescribed by Basel and given below:

$$\rho_i = 0.12 * \frac{(1 - e^{-50*PD_i})}{1 - e^{-50}} + 0.24 * \left(1 - \frac{(1 - e^{-50*PD_i})}{1 - e^{-50}}\right) \quad (18)$$

As can be seen from the formula, the correlation of a loan only depends on the PD of the loan.

The Maturity Adjustment Multiplier (MAM) of a loan depends on the maturity and the PD of that loan. The formula for calculating the MAM of a loan is prescribed by Basel and given below:

$$MAM_i = \frac{1 + (M_i - 2.5) * b_i}{1 - 1.5 * b_i} \quad (19)$$

In which  $M_i$  represents the maturity of loan  $i$ , and  $b_i$  must be calculated with the formula:

$$b_i = 0.11852 - 0.05478 * \ln(PD_i)^2 \quad (20)$$

The Rabobank Risk Ratings of the different loans in RI's commercial banks portfolio can be calculated with the new model. Each RRR scale has a fixed default probability such that the corresponding PDs of the loans can be substituted in the formulas above. The regulatory capital needed for RI's commercial banks portfolio can then be calculated.

RI's economic capital calculation is comparable with its regulatory capital calculation, with the difference that the confidence level in the Vasicek formula is at [REDACTED] instead of [REDACTED]. Also the MAMs are calculated differently. For EC the MAMs are calculated with a separate model, which falls outside the scope of this thesis.

## 8.2 Impacts calculated

Now the capital calculations are explained, the impacts can be calculated. These impacts are calculated for RI's commercial banks portfolio by comparing the capital levels calculated with the ratings generated with the old model with the capital levels calculated with the ratings generated with the new model. In the table below the capital levels and corresponding impacts for RC and EC are shown.

[REDACTED]		[REDACTED]	[REDACTED]
[REDACTED]	[REDACTED]	[REDACTED]	[REDACTED]
[REDACTED]		[REDACTED]	[REDACTED]
[REDACTED]	[REDACTED]	[REDACTED]	[REDACTED]
[REDACTED]		[REDACTED]	[REDACTED]

Table 19: Capital impact,

As can be seen from the table, the capital impacts are quite severe. As a result of implementing the new rating model the regulatory capital will decrease with [REDACTED] and the economic capital level with [REDACTED]. Therefore it can be concluded that the new model is less conservative than the old model. Since one of the constraints of the new model is that its capital impact is not allowed to be too big, we have to perform a calibration on the model. This calibration will result in a more conservative model, such that the capital impact is less severe. In the next chapter it is described how this calibration is performed.

## 9 Calibration

In the previous chapter I described that the new model is less conservative than the old model, which is not desired. In this chapter I describe the process of performing a calibration on the new model such that the new ratings will become more conservative. Before this calibration can be performed we must analyse how it is possible that the capital impacts are negative. This difference is unexpected since the old model is also constructed with the Shadow-Bond approach and therefore also a replication of S&P's model. Since both the new and old model are replicates of S&P's model we expect the ratings of both models to be comparable and the capital impacts close to zero. In Section 9.1 I formulate three possible explanations of this problem and analyse where this difference may come from. Then I discuss three calibration approaches in Section 9.2 and calibrate the model with the selected approach in Section 9.3.

### 9.1 Rating comparison

In this section the ratings of the old model are compared with the S&P ratings. To be able to do this we mapped the S&P ratings to Rabobank Risk Ratings as explained further by Jole (Jole, 2008). Before the ratings are compared, one needs to know that experts have the possibility to override a model rating if they believe the rating does not match their own intuition. The ratings including overrides are the ratings which are used for the capital calculation of the previous chapter. Almost all overrides performed by the experts are overrides downwards (worse rating).

In total we could think of three possible explanations for the difference between the old model ratings and the S&P ratings:

- 1) S&P's rating model has become more optimistic since 2007 (the year the old model was developed).
- 2) Although the old model was a replicate of S&P's model, its ratings have been more conservative than S&P's.
- 3) The old model did match the S&P ratings, but that the negative capital impact is the result of the overrides downwards.

To determine which explanation (or combination of explanations) was correct we compared the old rating including overrides with the old model ratings and the S&P ratings. We analysed the differences by comparing the averages of the different ratings of the observations in the dataset. In the table below the results of this comparison are shown.


Table 20: Average PDs and ratings

From the table it can be seen that the averages of the old model ratings and old model ratings including overrides are indeed higher than the average of the S&P ratings. It can also be seen that the average old

model rating including overrides is higher than the average ratings without overrides. From this latter statement we can conclude that the negative capital impact is indeed also the result of the overrides and thus the third hypothesis holds. To better understand where this difference exactly comes from the average ratings over time are analysed for all the three different ratings. The new model ratings are also included in this comparison. The results are shown in the figure below.



**Figure 21: Average ratings over time**

In this graph the average rating per year of the four different models are shown on the y-axis. The higher the average rating of a model, the more conservative the model. The old model was developed in 2007. It can be seen from the figure that the average ratings of S&P (purple line) and the old model (red line) are close to each other in the years up to 2007. From 2007 until now the old model has been more conservative than S&P. This can be seen from the red line which is above the purple line for the years 2008 to 2013. The shape of the S&P line is comparable with the shape of the old model line, therefore it can be concluded that no dramatic changes have been performed at S&P's model. The difference between the old model ratings and the S&P ratings comes thus mainly from the fact that the old model has been more conservative than the S&P model in general although this model was aimed at mimicking the S&P ratings (hypothesis 2).

Now the difference between the S&P ratings and the old model ratings is explained it is also clear why the capital impact of the new model is negative. Since the new model is aimed at replicating S&P's ratings, and the old model has been more conservative than S&P was, the ratings of the new model are less conservative than the old model ratings. At this point we had to make a decision. The Shadow-Bond approach is aimed at mimicking S&P's rating model, but QRA does want to keep this extra conservatism margin with respect to the S&P ratings. Therefore we decided to calibrate the new model such that the

average of the new ratings is equal to the average of the old ratings including overrides. In the next section three approaches for doing this are described.

## 9.2 Calibration approaches

In this section I will describe three approaches for calibrating the model such that the average of the new ratings of the observations is equal at the average of the old ratings including overrides of these observations. These approaches are: an intercept correction, a new regression with constraint and a balanced new regression with constraint.

### 9.2.1 Intercept correction

The first approach is the simplest approach: an intercept correction. To understand this approach, see the new model in the equation below.

$$PD = e^{\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_{13} x_{13} + \varepsilon} \quad (21)$$

In this equation the 13 selected factors with corresponding coefficients are represented by  $x_1$  to  $x_{13}$  and  $\beta_1$  to  $\beta_{13}$  respectively. The intercept of the model is given by  $\beta_0$ . By increasing the intercept the average PD, and thus the average rating can be increased. The increase in intercept such that the average new rating matches the average old rating including overrides can be determined by trial and error.

### 9.2.2 Regressing with constraint

The second calibration approach is new to QRA and developed by myself. This approach involves performing a new regression with the constraint that the average new rating is equal at the average old rating. Since the average of the old ratings relates to the average of the logarithms of the old PDs the constraint becomes:

$$\sum_{all\ obs} \beta_0 + \beta_1 x_1 + \dots + \beta_{13} x_{13} = \sum_{all\ obs} \ln(PD_{old}) \quad (22)$$

This constraint is non-linear. Therefore the calibrated model cannot be estimated with least squares anymore, but a non-linear iterative solving algorithm in Matlab must be used. The advantage of this approach in comparison with the intercept correction approach is that the model matches the S&P ratings more closely. The reason for this is that with this regression not only the intercept but also the weights of the factors can be changed to optimize the performance of the model.

### 9.2.3 Balancing of weights

The usage of a non-linear solving algorithm in Matlab over a simple OLS regression brings additional advantages. One of these advantages is that the objective function can be adjusted. Instead of solely focusing on minimizing the squared error terms, additional objectives can be added. This was desirable since experts indicated that there was too much variance in the weights of the model. They believed the difference between the country factor with a weight of [REDACTED] and the market position factor with only a



weight of ■■■ was too high. To decrease this variance I developed a new model with an adjusted objective function. This adjusted objective function is shown below:

$$\text{Min.} \sum_{all\ obs} (\beta_0 + \beta_1 x_1 + ... + \beta_{13} x_{13} - \ln(PD_{S\&P}))^2 + \blacksquare \sum_{all\ w} -\ln(w) \quad (23)$$

In this objective function the first part is the same as the original objective function. In this part of the equation the sum of the squared errors between the observation scores and the logarithms of the PD is minimized. In the second part of the function the sum of the logarithms of the weights is maximized (this is done by minimizing this sum multiplied by -1). Recall that the weights  $w$  are calculated by dividing the coefficient  $\beta$  by the sum of  $\beta$ . The value ■ is set so that the performance of the model is not significantly reduced, but the variance in the weights is reduced.

### 9.3 Calibrated model

In this section I will make a choice for one of the three approaches described above. Since QRA has no preference of OLS over a non-linear solving algorithm, and the second approach results in a model with a better performance than the first model, I decided the second approach is the better approach of the first two approaches. To determine whether the third approach is better than the second approach, the Powerstat of the model obtained with the third approach is compared with the Powerstat of the model obtained with the second approach. Both Powerstats were ■■■. Therefore it can be concluded that the performances of both models are comparable.

Experts have indicated they prefer the balanced model over the constraint model. Therefore I think this model is a good improvement over the constraint model. At QRA there are however doubts whether this model can be implemented and whether it is accepted by the validation department of RI since it deviates from the standard approach. Nonetheless I choose this approach for the calibration at this point in the model development process. In the table below the model obtained with the third calibration approach is shown.

■■■	■■■	■■■
■■■	■■■	■■■
■■■	■■■	■■■
■■■	■■■	■■■
■■■	■■■	■■■
■■■	■■■	■■■
■■■	■■■	■■■
■■■	■■■	■■■
■■■	■■■	■■■
■■■	■■■	■■■
■■■	■■■	■■■
■■■	■■■	■■■
■■■	■■■	■■■
■■■	■■■	■■■
■■■	■■■	■■■

**Table 21: Calibrated model**

From this table it can be seen that the weights are indeed more balanced in the calibrated model than they were in the initial model. In the initial model the weights ranged from [REDACTED] to [REDACTED] whereas they only range from [REDACTED] to [REDACTED] in the calibrated model.

Now that the model is calibrated the new capital impact can be calculated. Since the model is calibrated on the old ratings including overrides, it is expected that the capital impact is close to zero. This expectation turned out to be correct. The capital impacts of the calibrated model are [REDACTED] and [REDACTED] for regulatory capital and economic capital respectively.

10 Model performance

In this chapter I will answer the last sub question: *How does the new model perform in comparison with the old model, and are there ways of improving this performance?* To be able to answer this question we first needed to determine how the performances of the new and old model are measured. Since both models are developed with the Shadow-bond approach, an approach for mimicking S&P’s ratings, the first performance measure is how good the ratings match the S&P ratings. In Section 10.1 I describe this analysis. The second performance measure we thought of comes from the fact that many old ratings had to be overridden by the experts. Since this is not desired we defined the second performance measure as the extent to which the ratings match the performed overrides. This analysis is performed in Section 10.2. We defined the third and last performance measure as the extent to which the new model can predict S&P ratings. Therefore we performed an out of time analysis in Section 10.3. In Section 10.4 we thought of a way of improving the model by analysing the country weight over time.

10.1 Comparison against S&P ratings

In this section I will analyse the performance of the new model ratings against the S&P ratings. I will start with a comparison of the averages of both ratings. Thereafter I will compare both ratings in more detail. In the table below the averages of the new ratings and S&P ratings of the observations in the dataset are shown.


Table 22: PDs and ratings new and S&P compared

From the table it can be seen that the average of the new ratings is higher than the average of the S&P ratings. The reason for this is that we have estimated the model with the constraint that the average new rating is equal at the average old rating including overrides. Since the old ratings including overrides are more conservative than the S&P ratings this difference is as expected.

The difference between the average new rating PD and S&P rating PD is however interesting. In Section 2.5 it was discussed that as a result of Jensen’s inequality theorem the mean of values transformed with a concave function is lower than the transformed mean of the original values. The transformed values in this case are the PDs. As can be seen from the table the mean of the new model PDs is lower than the mean of the S&P PDs (even though the new model ratings are more conservative than the S&P ratings). Since the PDs and not the ratings are used for RI’s capital calculations the risks faced on RI’s Commercial Banks portfolio is underestimated. This is a serious drawback of the Shadow-Bond approach, and further research should be performed at how to solve this issue.

Now that the averages of both ratings are compared, I compared the new model rating with the S&P rating for each observation individually. To be able to do this I mapped the S&P ratings to RRRs as explained by Jole (Jole, 2008). The matrix below is the result of this analysis.

# Figure 22

**Figure 22: Rating comparison new-S&P**

In this matrix for each observation its new model rating is shown in the vertical dimension and its S&P rating in the horizontal dimension. For example, the element ■ in the third row and first column indicates that there are ■ observations with a R3 rating of the new model and a R1 rating of the S&P model. From this table a single measure can be calculated for the performance of the new model: the average absolute notches difference between the new and S&P ratings. The average absolute notches difference is calculated by taking the sum of the absolute notches difference between the new ratings and the mapped S&P ratings and dividing this sum by the number of observations as shown in the formula below:

$$av\ abs\ notch\ dif = \frac{1}{Number\ of\ obs} * \sum_{all\ obs} abs(RRR_{New} - RRR_{S\&P}) \quad (24)$$

The average absolute notches difference is an alternative performance measure for the Powerstat. A drawback of the Powerstat is that it is a measure for the ability of a model to sort observations. A model has a high Powerstat if the observations with high model ratings have high S&P ratings. It can therefore be thought of as a relative measure, since the Powerstat of a model would not change if all new model ratings are downgraded with a notch. The average absolute notches is an absolute performance measure. If all new model ratings would be downgraded with a notch the average absolute notches would increase with approximately one notch. With the formula above we calculated the average absolute notches difference between the new model ratings and the S&P ratings. We also did this for the old model and the old model including overrides. The results are shown in the table below.


**Table 23: Absolute differences S&P**

From this table it can be seen that on average we observe 1 notches difference between the new model rating and the S&P rating. Also it can be seen that the old model and the old model including overrides perform slightly better in matching the S&P ratings than the new model. We analysed where this difference came from and found that this came from the imposed constraint for the new model. Without this constraint the new model performed better than the old model. Since we do not want to give up this constraint, we conclude that the performance of the new model cannot be improved at this point.

## 10.2 Comparison against overrides

One of the reasons for the redevelopment of the Commercial Banks model was that experts had to perform too many overrides on the old model ratings. Therefore it is interesting to analyse whether the new model ratings are closer to the overrides performed by the experts than the old model ratings were. To be able to test this we had to identify the performed overrides first. In total we identified 10 observations in the dataset with an override. For these observations the new and old model ratings were compared with the overridden ratings, resulting in two matrices as shown in Figure 22. From these matrices we calculated the average absolute notches differences. The results are shown in the table below.


**Table 24: Absolute differences overrides**

From this table it can be seen there are on average 1 and 1 notches difference between the new and old model ratings with the overridden ratings respectively. Therefore we can conclude that the new model is closer to the overrides, indicating that it should be less often overridden in the past. From this statement we can conclude that there are probably less overrides needed in the future.

## 10.3 Out of time analysis

After the analysis of the performance of the new model in matching S&P ratings and overrides we focused on the performance of the new model in predicting S&P ratings. Since there is no information about future S&P ratings, we estimated the calibrated model again on the dataset with only observations included from the years before 2012. With the estimated model we 'predicted' the ratings of the observations from 2012 and 2013. In total there were 10 observations from these years. These predicted observations are called out of time because their dates are from after 2011.



at the bad ratings. This result comes from the fact that the bulk of the observations has average ratings and the model is optimized for the bulk of the observations. Therefore it can be concluded that the model fails at predicting extreme (and especially bad) ratings. This is a serious treat for RI, and should be analysed how this feature can be solved. Perhaps it would be possible to attribute weights at the observations with bad S&P ratings.

## 10.4 Country weight over time

Now that the performance of the new model is compared with the old model the focus is on the second part of the research question: can this performance be improved. We limit ourselves thereby by focusing on whether the model can be made more forward looking with respect to the weight of the country factor. The reason for this is that we know this factor has become more important during the crisis (Angeloni, Merler, & Wolff, 2012). Furthermore S&P has announced in November 2011 that they have changed their rating methodology and that countries have become more important in their model (S&P, 2011). Therefore we will analyse in this section whether this change in country importance can be observed in the dataset and if so whether the model can be improved by adjusting the country weight for this matter. To understand how we performed this analysis, note the simplified regression equation below:

$$\ln(PD) = \alpha + \beta_1 x_1 + \beta_2 x_2 + \varepsilon \quad (25)$$

In this equation  $PD$ ,  $x$  and  $\varepsilon$  represents vectors of PDs, factor scores and error terms respectively. The length of these vectors is  $n$ : the number of observations in the dataset. The parameters  $\alpha$ ,  $\beta_1$  and  $\beta_2$  represent a constant and the two coefficients respectively. Now assume that the first factor  $x_1$  is the country factor which needs to be analysed over time. In total there are observations from 10 years in the dataset: 2000-2009. The country weight is analysed for each of these years. This is done by splitting the vector  $x_1$  in 10 new vectors:  $x_{1,1}$  to  $x_{1,10}$  with lengths equal at  $n$ . Each new vector gets the scores of the original vector if the observation is from the year of the new vector and a zero otherwise. In the table below it is shown how the dataset would look like for 10 observations from 10 different years.

	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
1	1	0	0	0	0	0	0	0	0	0
2	0	1	0	0	0	0	0	0	0	0
3	0	0	1	0	0	0	0	0	0	0
4	0	0	0	1	0	0	0	0	0	0
5	0	0	0	0	1	0	0	0	0	0
6	0	0	0	0	0	1	0	0	0	0
7	0	0	0	0	0	0	1	0	0	0
8	0	0	0	0	0	0	0	1	0	0
9	0	0	0	0	0	0	0	0	1	0
10	0	0	0	0	0	0	0	0	0	1

Table 26: Dataset country weight over time

In this table the 10 observations are shown in the vertical dimension. In the second table are the years of the observations. In the 10 different constructed vectors the  $X$ 's represents factor scores, where the empty fields represent zeros. It can be seen that the original vector is divided over the columns 2000 to 2009.

██████. To analyse the country weight over time for the development dataset the country factor is split up as shown in the table above. Then the calibrated model is estimated on this adjusted dataset, resulting in a model with █████ country weights and █████ other weights. The █████ country weights are shown in the figure below.

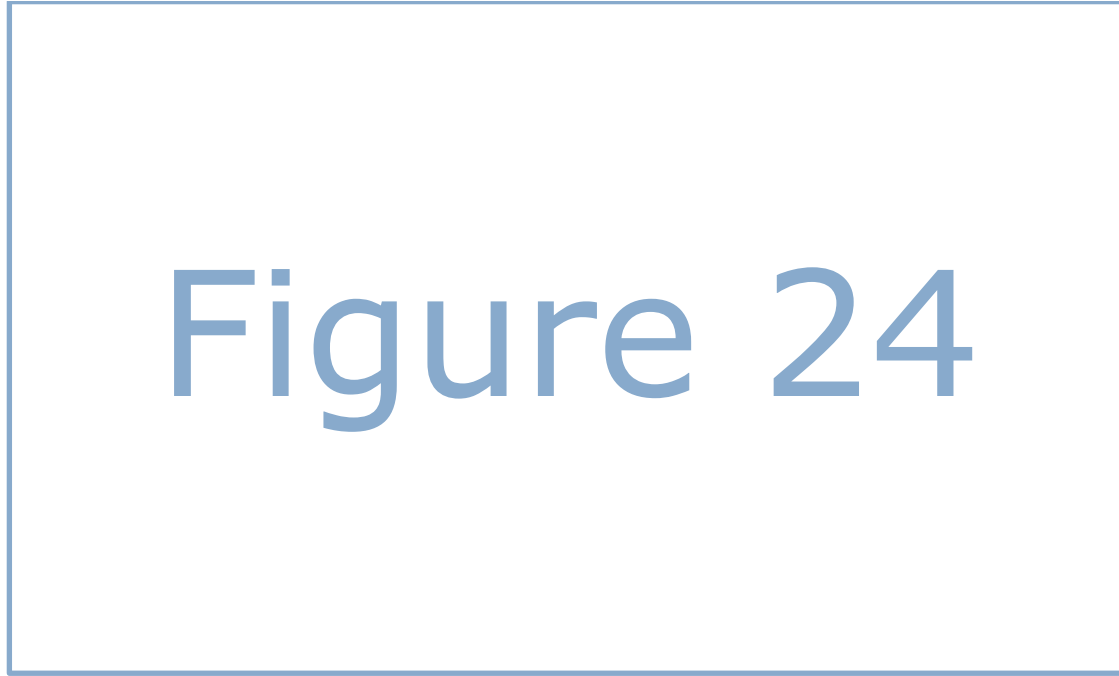


Figure 24: Country weight over time

From this figure it can be seen that the country factor has indeed become more important, as indicated by the increased weight. From █████ to █████ an increase in weight can be observed. Thereafter the weight stays roughly constant. These results match the hypothesis that the country factor has indeed become more important over the years. Since one of the requirements for the model is that it should be forward looking I think it is better to exclude old data of which it is shown that it is not representative for the short past and thus most probably for the future anymore. Therefore the calibrated model can be improved by excluding the country scores of old observations.

The improved calibrated model is estimated by dividing the country factor over two new country factors: one with scores for observations from before █████ and zeros otherwise and one with scores for observations from █████ and later and zeros otherwise. The weight of the factor with scores from before █████ is ignored in the weight balancing part of the objective function. The result is an improved calibrated model with a country weight of █████ instead of the initial █████. Also the other weights are slightly different. This improvement is however not implemented because it is not desired for the country factor to get even more weight.



## 11 Hybrid model


The Shadow-Bond approach is aimed at mimicking S&P's ratings. Since these ratings represent S&P's estimates of the creditworthiness of companies instead of the observed creditworthiness the Shadow-Bond model is constructed from indirect creditworthiness information. Furthermore observations with very bad S&P ratings are removed with the Shadow-Bond approach, since they influence the linear regression too much. These are two drawbacks of this approach, which makes it interesting to think of an alternative for this approach. In this chapter I will describe such an alternative approach. This approach is called the Hybrid approach and combines elements of the Shadow-Bond and Good-Bad approach. This approach still uses S&P ratings, but observations with bad S&P ratings no longer have to be removed. Also it is possible to include real defaults at the dataset, such that the model is no longer solely based on S&P ratings.

### 11.1 Methodology

The idea of the Hybrid approach is straightforward: the Shadow-Bond dataset is transformed into a Good-Bad dataset upon which a logistic regression is performed. This is done by transforming each observation with a PD into two new observations: a good and a bad. These two new observations are then assigned the weights  $1 - \text{PD}$  and  $\text{PD}$  respectively. For example when an observation has a S&P PD of 1%, there is a probability of 1% that the observation will become a default in the next year and thus become a bad. Therefore the weight of the bad created from this observation is 0.01. The weight of the good constructed from this observation is 0.99. In the tables below this transformation process is shown for a single observation.

WWID	Date	Expl. Var.	PD
1	1/1/2009		0.01

**Original dataset**



WWID	Date	Expl. Var.	1/0	Weight
1	1/1/2009		0	0.99
1	1/1/2009		1	0.01

**Transformed dataset, in the fourth column a 1 and 0 indicate a good and bad respectively**

Now the dataset is transformed a logistic regression can be performed on the dataset by maximizing the equation:

$$\text{Max} \sum_{i=1}^n \log(p_i) * w_i \quad (26)$$

In this equation  $p_i$  is the probability of the status (good or bad) of observation  $i$  and  $w_i$  the weight of this observation. If observation  $i$  is a good its probability is given with the formula:

$$\pi(x) = \frac{e^{\beta x}}{1 + e^{\beta x}} \quad (27)$$

If it is a bad, this probability is  $1 - \pi(x)$ . Since the dependent variable of all observations can only have the values 0 and 1, there are no longer observations with dependent variables which can be thought of

as outliers. Therefore observations with bad S&P ratings no longer have to be removed. Also since the dataset now consists of goods and bads, observed bads can be included in the dataset.

However we should be careful with including these bads. The transformed goods and bads in the dataset match probabilities of outcomes, whereas observed bads can be interpreted as outcomes of experiments, which is not the same. Therefore it is not possible to simply include these bads, since this would bias the dataset and would result in a model which is too conservative. Therefore not only bads should be included at the transformed dataset but also the goods matching the observed bads.

## 11.2 Model overview

Now the methodology of the Hybrid approach is explained, we will estimate a Hybrid model on the Shadow-Bond dataset. Since we do not know for all banks in the dataset whether they have defaulted or not, we limit ourselves to estimating a model on the transformed S&P dataset only. The explanatory variables of the model are the factors from the calibrated model. The regression is performed with the constraint that the average of the new ratings is equal to the average of the old ratings. The requirement that the weights of the model should be balanced is relaxed to better identify the differences in weights between the Hybrid and the original Shadow-Bond model. In the figure below the weights of the Hybrid and original Shadow-Bond model are shown.



**Figure 25: Weights compared**

The blue and red bars in the figure represent the weights of the Shadow-Bond and Hybrid model factors respectively. From the figure it can be seen that there are quite some differences in the weights of a number of factors. The largest difference is however in the country weight. This factor gets a weight of ■■■ in the Shadow-Bond model and only ■■■ in the Hybrid model. It can thus be concluded that the Hybrid model differs a lot from the Shadow-Bond model. To determine whether this model can be an alternative for the Shadow-Bond model it is analysed in the next section whether this model is stable.

### 11.3 Stability

The Hybrid model can only be a real alternative for the Shadow-Bond if it is stable. Therefore changing the dataset a little should not result in a completely different model. To test whether the Hybrid model is stable we removed the [REDACTED] goods and [REDACTED] bads corresponding with the [REDACTED] observations with the worst S&P ratings and estimated the Hybrid model again. In the figure below the obtained weights of the Hybrid model are compared with its original weights.



**Figure 26: Hybrid model weight comparison**

From this figure it can be seen that there are very large changes in the weights of the different factors. This is the result of removing only [REDACTED] from the original [REDACTED] observations. It can therefore be concluded that the Hybrid model is not stable at all. Therefore the Hybrid model is not an alternative for the Shadow-Bond model for this dataset, which only changes little if it is estimated on the dataset without the worst ratings.

However I think the Hybrid model can be an alternative for the Shadow-Bond model if there are more observations (goods/bads or observations with S&P rating) available. For the development of the Shadow-Bond approach we had [REDACTED] observations available with an average S&P PD of [REDACTED]. From these observations approximately [REDACTED] bads were constructed. According to the QRA guidelines the model resulting from the Good-Bad approach is deemed stable if it is estimated on at least [REDACTED] bads (Vedder, 2010). This difference in number of bads could explain the instability of the Hybrid model. Therefore I think the Hybrid model can be an alternative for the Shadow-Bond approach if the number of bads in the transformed dataset is higher. This can be achieved with including more observations with S&P ratings, or more observed goods and bads. Further research should be performed at this point.

## 12 Conclusion

In this chapter I will answer the main question described in Section 1.3. Below this question is shown again.

*How should the new CBPD model be redeveloped such that it meets the requirements set by RI and the Dutch Central Bank (DNB)? What is the capital impact of this new model, and how is its performance in comparison with the old model?*

The first part of this question involves the development process of this new CBPD model. We have developed this model with this Shadow-Bond approach, an approach for replicating S&P's rating model. We have chosen this approach over the Good-Bad approach because there were too little defaults in the portfolio to use the latter approach. QRA has clear guidelines for constructing a rating model with the Shadow-Bond approach. We made sure the new CBPD model was constructed according to these guidelines.

The modelling process started with the collection of data. We collected data from CRE, Bankscope, Bloomberg and CreditPro. After gathering as much data of Commercial Banks as possible we removed observations which did not match the requirements for observations set by QRA. The result was a dataset consisting of [REDACTED] observations. Then we had to deal with missing values. Initial [REDACTED] observations had at least one missing value, such that they could not be used for the model construction. After applying inter- and extrapolation, median replacement and removing factors with more than [REDACTED] missing values we reduced this number to [REDACTED], such that we ended with a dataset consisting of [REDACTED] complete observations.

To be able to construct a model from these observations we had to transform the factors values to the [0, 10] range. We did this by using the logistic function. We performed additional research at the validity of the assumption that factors are logistic distributed and found that in general this assumption holds. We also concluded that for U-shaped factors this assumption did not hold. The information given by these U-shaped factors is therefore not used in our model. After the transformation we assigned weights at the observations on basis of their assets sizes and countries. Then we calculated the Powerstats of the different factors and identified a number of factors with negative Powerstats. We performed additional research at this feature and discovered the power of combined explanatory power. Due to this result we decided to deviate from the guidelines and include all factors in the stepwise regression.

When we performed the stepwise regression initially [REDACTED] factors were selected. From these factors 3 factors deemed unstable and were removed. We calculated the capital impact of this model and found that the model was less conservative than the old model. Therefore we analysed the ratings of the new, old and S&P model more closely and found that RI's old model was more conservative during the latest financial crisis than S&P. For this reason we decided to perform an additional calibration on the model. We developed a new approach for calibrating a Shadow-Bond model which resulted in a model which was both more conservative and better balanced in terms of the weights of the input factors. The capital impact of this model was close to zero.

After the construction of the model we performed extensive research at the performance of the model. We concluded that the new model matched the S&Ps ratings almost as good as the old model, but that the new model was slightly better in matching the overrides. Furthermore we concluded that the new model has trouble predicting extreme ratings and that the model could be further improved by estimating the country factor only on recent observations.

Finally we proposed a hybrid model, which allows the inclusion of default data. However this model turned out to be unstable, since the factor weights changed heavily when a small number of observations were removed. Therefore this model was not an alternative for the Shadow-Bond model for this redevelopment process.

## 13 Discussion and future research

A number of points for discussion arise from this thesis. I will discuss these points in this chapter. The first point of discussion is the choice for the Shadow-Bond approach. As described in Section 2.6 we preferred the Shadow-Bond approach over the Good-Bad approach since there were not enough bads in the dataset to use this latter approach. However there were a lot of banks in the portfolio which have gained government support during the crisis. Since there is evidence that banks will not receive the same type of support in the future (FINMA, 2013) it could have been an improvement to change the definition of a bad such that there could have been enough bads in the dataset to use the Good-Bad approach. If there were still not enough bads in the dataset at this point, the Hybrid model could be an alternative. As discussed in Chapter 11 this model combines elements of the Shadow-Bond and Good-Bad approach and can provide outcome if there are too little bads to use the Good-Bad approach.

The second point for discussion is the forward looking performance of the Shadow-Bond approach itself. The S&P ratings used with this approach are backward looking in the sense that the ratings are assigned by S&P with the knowledge that banks in trouble will get government support. Since this assumption is doubtful for the future it might be interesting to think of approaches which are more forward looking. Also it might be questioned whether we fulfilled the objective of mimicking S&P's rating model in the best possible way. We fulfilled this objective by gathering data of a lot of factors and performing a stepwise regression on this data. For a number of factors which were selected by this regression it may be questioned whether these factors are real explanatory variables or perhaps effects of other factors which were not selected. It may be good to perform a qualitative study at S&P's rating model and verify the explanatory variables which are in the model developed in this thesis. This would make the model more forward looking. Now the parameters with the best historic performance are selected, although it is not given that these factors will also have the best performance in the future.

Also the regression technique of the Shadow-Bond approach can be discussed. Now a linear regression is performed upon the logarithms of the PDs. Since the logarithm function is concave and Jensen's inequality theorem states that the mean of values transformed with a concave function is lower than the transformed mean of the original values, the constructed model is too optimistic. We confirmed this hypothesis in Chapter 10 where we compared the averages of the model PDs and S&P PDs. We also concluded in that chapter that the model fails at predicting extreme ratings, which is also the result of performing an OLS. It would be interesting to analyse how these issues can be handled. Perhaps it is possible to introduce weights for extreme observations or use a non-linear regression technique instead of OLS. More research should be performed at these two points.

Another interesting point for discussion is that OLS assumes observations are independent. We did not check this for this model development. It would be interesting to test for autocorrelation in the residuals of the observations. If autocorrelation is then observed, additional improvements at the model should be performed. Also we have made the assumption that all factors are logistic distributed while transforming them. We concluded that for U-shaped factors this assumption did not hold. These factors are currently not selected in the MFA stage, but could have explanatory power. More research should be

performed at whether and how these factors can be included in a scorecard model and how such a model can be implemented in the IT infrastructure of RI.

## 14 Appendix

### 14.1 Derivation OLS estimator

The linear model is given by the equation:

$$Y = X\beta + \varepsilon \quad (28)$$

In this equation  $Y$  is a  $n \times 1$  vector of dependent variables. The variable  $X$  is a  $n \times m$  matrix of independent variables in which  $m$  is the number of independent variables in the model, and in which the first element may or may not be a constant. The variable  $\varepsilon$  is a  $n \times 1$  vector of noise elements with a mean of zero, same variance and no internal correlations. The ordinary least squares method minimizes the sum of the squared error terms. The error terms are represented by the vector  $\varepsilon$ . The squared vector of squared error terms is therefore  $\varepsilon^T * \varepsilon$ . (28) can be rewritten as  $\varepsilon = Y - X\beta$ . Substituting this formula in  $\varepsilon^T * \varepsilon$  results in the equation:

$$(Y - X\beta)^T * (Y - X\beta) \quad (29)$$

In order to get the OLS estimator of  $\beta$ , this equation has to be minimized. This can be done by taking the derivative with respect to  $\beta$  and setting this derivative equal to zero. Equation 29 can be rewritten as:

$$(Y^T - X^T\beta^T) * (Y - X\beta) = Y^TY - 2X^T\beta^TY + X^T\beta^TX\beta \quad (30)$$

The derivative with respect to  $\beta$  of this equation is:

$$2X^TY - 2X^TX\beta \quad (31)$$

Setting this equal to zero and solving for  $\beta$  gives:

$$(X^TX)\beta = X^TY \quad (32)$$

$$(X^TX)^{-1}(X^TX)\beta = (X^TX)^{-1}X^TY \quad (33)$$

$$\beta = (X^TX)^{-1}X^TY \quad (34)$$

The  $\beta$  in Equation 34 is the best estimate of the coefficients of the linear model.



This image is a highly abstract, black and white graphic. It features a series of horizontal black bars of varying lengths, creating a pattern that strongly resembles a barcode or a stylized, fragmented letter 'E'. The bars are arranged in a grid-like fashion, with some bars extending across the entire width of the image and others being shorter, creating a sense of depth and complexity. The overall effect is one of a dense, textured surface with a high degree of contrast between the black bars and the white background.

The image consists of a 16x16 grid of squares. Each square is either black or white. The pattern of black squares is as follows (rows 1-16, columns 1-16):

Row	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Note: 0 represents a white square, and 1 represents a black square. The pattern is a stylized letter 'A'.

Table 27: Financial factors

### 14.3 CreditPro mapping

For the observations without Bloomberg rating it is checked whether there is a rating in CreditPro. If there is, this rating is mapped to the observations. The mapping of the historic S&P ratings from CreditPro to the observations is done through the name of the bank in CRE and the country of residence of the bank. Since there can be banks with comparable names in different countries, this country of residence check provides extra certainty of the correctness of the match.

For the mapping names a number of alternatives is possible:

1. The CRE name can be mapped *exactly* to the name in CreditPro;
2. The CRE name is in full an exact match to a part of the name in CreditPro. An example is 'Credit du Nord' in CRE and 'Credit du Nord S.A' in CreditPro.
3. The CRE name is in full an exact match to a part of the name in CreditPro without dots and/or other additions (for example SA). An example is 'Credit du Nord SA' in CRE and 'Credit du Nord S.A' in CreditPro.
4. Alternative 2 and 3 but then the other way around.

Mappings via the last three alternatives are manually checked before accepted. Ratings acquired with non-matching mappings are removed.

	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	1
3	1	1	1	1	1
4	1	1	1	1	1
5	1	1	1	1	1
6	1	1	1	1	1
7	1	1	1	1	1
8	1	1	1	1	1
9	1	1	1	1	1
10	1	1	1	1	1
11	1	1	1	1	1
12	1	1	1	1	1
13	1	1	1	1	1
14	1	1	1	1	1
15	1	1	1	1	1
16	1	1	1	1	1
17	1	1	1	1	1
18	1	1	1	1	1
19	1	1	1	1	1
20	1	1	1	1	1
21	1	1	1	1	1
22	1	1	1	1	1
23	1	1	1	1	1
24	1	1	1	1	1
25	1	1	1	1	1
26	1	1	1	1	1
27	1	1	1	1	1
28	1	1	1	1	1
29	1	1	1	1	1
30	1	1	1	1	1
31	1	1	1	1	1
32	1	1	1	1	1
33	1	1	1	1	1
34	1	1	1	1	1
35	1	1	1	1	1
36	1	1	1	1	1
37	1	1	1	1	1
38	1	1	1	1	1
39	1	1	1	1	1
40	1	1	1	1	1
41	1	1	1	1	1
42	1	1	1	1	1
43	1	1	1	1	1
44	1	1	1	1	1
45	1	1	1	1	1
46	1	1	1	1	1
47	1	1	1	1	1
48	1	1	1	1	1
49	1	1	1	1	1
50	1	1	1	1	1
51	1	1	1	1	1
52	1	1	1	1	1
53	1	1	1	1	1
54	1	1	1	1	1
55	1	1	1	1	1
56	1	1	1	1	1
57	1	1	1	1	1
58	1	1	1	1	1
59	1	1	1	1	1
60	1	1	1	1	1
61	1	1	1	1	1
62	1	1	1	1	1
63	1	1	1	1	1
64	1	1	1	1	1
65	1	1	1	1	1
66	1	1	1	1	1
67	1	1	1	1	1
68	1	1	1	1	1
69	1	1	1	1	1
70	1	1	1	1	1
71	1	1	1	1	1
72	1	1	1	1	1
73	1	1	1	1	1
74	1	1	1	1	1
75	1	1	1	1	1
76	1	1	1	1	1
77	1	1	1	1	1
78	1	1	1	1	1
79	1	1	1	1	1
80	1	1	1	1	1
81	1	1	1	1	1
82	1	1	1	1	1
83	1	1	1	1	1
84	1	1	1	1	1
85	1	1	1	1	1
86	1	1	1	1	1
87	1	1	1	1	1
88	1	1	1	1	1
89	1	1	1	1	1
90	1	1	1	1	1
91	1	1	1	1	1
92	1	1	1	1	1
93	1	1	1	1	1
94	1	1	1	1	1
95	1	1	1	1	1
96	1	1	1	1	1
97	1	1	1	1	1
98	1	1	1	1	1
99	1	1	1	1	1

The image consists of a 20x5 grid of squares. Each square is either black or white. The pattern of black squares is as follows (rows are indexed 0 to 19 from top to bottom):

Row	Col 0	Col 1	Col 2	Col 3	Col 4
0	Black	White	Black	White	Black
1	Black	Black	Black	Black	Black
2	Black	White	Black	White	Black
3	Black	Black	Black	Black	Black
4	Black	White	Black	White	Black
5	Black	Black	Black	Black	Black
6	Black	White	Black	White	Black
7	Black	Black	Black	Black	Black
8	Black	White	Black	White	Black
9	Black	Black	Black	Black	Black
10	Black	White	Black	White	Black
11	Black	Black	Black	Black	Black
12	Black	White	Black	White	Black
13	Black	Black	Black	Black	Black
14	Black	White	Black	White	Black
15	Black	Black	Black	Black	Black
16	Black	White	Black	White	Black
17	Black	Black	Black	Black	Black
18	Black	White	Black	White	Black
19	Black	Black	Black	Black	Black

Table 28: Results fields analysis

[illegible]



Algeria	2000	100	Algeria	2001	100
Algeria	2002	100	Algeria	2003	100
Algeria	2004	100	Algeria	2005	100
Algeria	2006	100	Algeria	2007	100
Algeria	2008	100	Algeria	2009	100
Algeria	2010	100	Algeria	2011	100
Algeria	2012	100	Algeria	2013	100
Algeria	2014	100	Algeria	2015	100
Algeria	2016	100	Algeria	2017	100
Algeria	2018	100	Algeria	2019	100
Algeria	2020	100	Algeria	2021	100
Algeria	2022	100	Algeria	2023	100
Algeria	2024	100	Algeria	2025	100
Algeria	2026	100	Algeria	2027	100
Algeria	2028	100	Algeria	2029	100
Algeria	2030	100	Algeria	2031	100
Algeria	2032	100	Algeria	2033	100
Algeria	2034	100	Algeria	2035	100
Algeria	2036	100	Algeria	2037	100
Algeria	2038	100	Algeria	2039	100
Algeria	2040	100	Algeria	2041	100
Algeria	2042	100	Algeria	2043	100
Algeria	2044	100	Algeria	2045	100
Algeria	2046	100	Algeria	2047	100
Algeria	2048	100	Algeria	2049	100
Algeria	2050	100	Algeria	2051	100
Algeria	2052	100	Algeria	2053	100
Algeria	2054	100	Algeria	2055	100
Algeria	2056	100	Algeria	2057	100
Algeria	2058	100	Algeria	2059	100
Algeria	2060	100	Algeria	2061	100
Algeria	2062	100	Algeria	2063	100
Algeria	2064	100	Algeria	2065	100
Algeria	2066	100	Algeria	2067	100
Algeria	2068	100	Algeria	2069	100
Algeria	2070	100	Algeria	2071	100
Algeria	2072	100	Algeria	2073	100
Algeria	2074	100	Algeria	2075	100
Algeria	2076	100	Algeria	2077	100
Algeria	2078	100	Algeria	2079	100
Algeria	2080	100	Algeria	2081	100
Algeria	2082	100	Algeria	2083	100
Algeria	2084	100	Algeria	2085	100
Algeria	2086	100	Algeria	2087	100
Algeria	2088	100	Algeria	2089	100
Algeria	2090	100	Algeria	2091	100
Algeria	2092	100	Algeria	2093	100
Algeria	2094	100	Algeria	2095	100
Algeria	2096	100	Algeria	2097	100
Algeria	2098	100	Algeria	2099	100
Algeria	2100	100	Algeria	2101	100
Algeria	2102	100	Algeria	2103	100
Algeria	2104	100	Algeria	2105	100
Algeria	2106	100	Algeria	2107	100
Algeria	2108	100	Algeria	2109	100
Algeria	2110	100	Algeria	2111	100
Algeria	2112	100	Algeria	2113	100
Algeria	2114	100	Algeria	2115	100
Algeria	2116	100	Algeria	2117	100
Algeria	2118	100	Algeria	2119	100
Algeria	2120	100	Algeria	2121	100
Algeria	2122	100	Algeria	2123	100
Algeria	2124	100	Algeria	2125	100
Algeria	2126	100	Algeria	2127	100
Algeria	2128	100	Algeria	2129	100
Algeria	2130	100	Algeria	2131	100
Algeria	2132	100	Algeria	2133	100
Algeria	2134	100	Algeria	2135	100
Algeria	2136	100	Algeria	2137	100
Algeria	2138	100	Algeria	2139	100
Algeria	2140	100	Algeria	2141	100
Algeria	2142	100	Algeria	2143	100
Algeria	2144	100	Algeria	2145	100
Algeria	2146	100	Algeria	2147	100
Algeria	2148	100	Algeria	2149	100
Algeria	2150	100	Algeria	2151	100
Algeria	2152	100	Algeria	2153	100
Algeria	2154	100	Algeria	2155	100
Algeria	2156	100	Algeria	2157	100
Algeria	2158	100	Algeria	2159	100
Algeria	2160	100	Algeria	2161	100
Algeria	2162	100	Algeria	2163	100
Algeria	2164	100	Algeria	2165	100
Algeria	2166	100	Algeria	2167	100
Algeria	2168	100	Algeria	2169	100
Algeria	2170	100	Algeria	2171	100
Algeria	2172	100	Algeria	2173	100
Algeria	2174	100	Algeria	2175	100
Algeria	2176	100	Algeria	2177	100
Algeria	2178	100	Algeria	2179	100
Algeria	2180	100	Algeria	2181	100
Algeria	2182	100	Algeria	2183	100
Algeria	2184	100	Algeria	2185	100
Algeria	2186	100	Algeria	2187	100
Algeria	2188	100	Algeria	2189	100
Algeria	2190	100	Algeria	2191	100
Algeria	2192	100	Algeria	2193	100
Algeria	2194	100	Algeria	2195	100
Algeria	2196	100	Algeria	2197	100
Algeria	2198	100	Algeria	2199	100
Algeria	2200	100	Algeria	2201	100
Algeria	2202	100	Algeria	2203	100
Algeria	2204	100	Algeria	2205	100
Algeria	2206	100	Algeria	2207	100
Algeria	2208	100	Algeria	2209	100
Algeria	2210	100	Algeria	2211	100
Algeria	2212	100	Algeria	2213	100
Algeria	2214	100	Algeria	2215	100
Algeria	2216	100	Algeria	2217	100
Algeria	2218	100	Algeria	2219	100
Algeria	2220	100	Algeria	2221	100
Algeria	2222	100	Algeria	2223	100
Algeria	2224	100	Algeria	2225	100
Algeria	2226	100	Algeria	2227	100
Algeria	2228	100	Algeria	2229	100
Algeria	2230	100	Algeria	2231	100
Algeria	2232	100	Algeria	2233	100
Algeria	2234	100	Algeria	2235	100
Algeria	2236	100	Algeria	2237	100
Algeria	2238	100	Algeria	2239	100
Algeria	2240	100	Algeria	2241	100
Algeria	2242	100	Algeria	2243	100
Algeria	2244	100	Algeria	2245	100
Algeria	2246	100	Algeria	2247	100
Algeria	2248	100	Algeria	2249	100
Algeria	2250	100	Algeria	2251	100
Algeria	2252	100	Algeria	2253	100
Algeria	2254	100	Algeria	2255	100
Algeria	2256	100	Algeria	2257	100
Algeria	2258	100	Algeria	2259	100
Algeria	2260	100	Algeria	2261	100
Algeria	2262	100	Algeria	2263	100
Algeria	2264	100	Algeria	2265	100
Algeria	2266	100	Algeria	2267	100
Algeria	2268	100	Algeria	2269	100
Algeria	2270	100	Algeria	2271	100
Algeria	2272	100	Algeria	2273	100
Algeria	2274	100	Algeria	2275	100
Algeria	2276	100	Algeria	2277	100
Algeria	2278	100	Algeria	2279	100
Algeria	2280	100	Algeria	2281	100
Algeria	2282	100	Algeria	2283	100
Algeria	2284	100	Algeria	2285	100
Algeria	2286	100	Algeria	2287	100
Algeria	2288	100	Algeria	2289	100
Algeria	2290	100	Algeria	2291	100
Algeria	2292	100	Algeria	2293	100
Algeria	2294	100	Algeria	2295	100
Algeria	2296	100	Algeria	2297	100
Algeria	2298	100	Algeria	2299	100
Algeria	2300	100	Algeria	2301	100
Algeria	2302	100	Algeria	2303	100
Algeria	2304	100	Algeria	2305	100
Algeria	2306	100	Algeria	2307	100
Algeria	2308	100	Algeria	2309	100
Algeria	2310	100	Algeria	2311	100
Algeria	2312	100	Algeria	2313	100
Algeria	2314	100	Algeria	2315	100
Algeria	2316	100	Algeria	2317	100
Algeria	2318	100	Algeria	2319	100
Algeria	2320	100	Algeria	2321	100
Algeria	2322	100	Algeria	2323	100
Algeria	2324	100	Algeria	2325	100
Algeria	2326	100	Algeria	2327	100
Algeria	2328	100	Algeria	2329	100
Algeria	2330	100	Algeria	2331	100
Algeria	2332	100	Algeria	2333	100
Algeria	2334	100	Algeria	2335	100
Algeria	2336	100	Algeria	2337	100
Algeria	2338	100	Algeria	2339	100
Algeria	2340	100	Algeria	2341	100
Algeria	2342	100	Algeria	2343	100
Algeria	2344	100	Algeria	2345	100
Algeria	2346	100	Algeria	2347	100
Algeria	2348	100	Algeria	2349	100
Algeria	2350	100	Algeria	2351	100
Algeria	2352	100	Algeria	2353	100
Algeria	2354	100	Algeria	2355	100
Algeria	2356	100	Algeria	2357	100
Algeria	2358	100	Algeria	2359	100
Algeria	2360	100	Algeria	2361	100
Algeria	2362	100	Algeria	2363	100
Algeria	2364	100	Algeria	2365	100
Algeria	2366	100	Algeria	2367	100
Algeria	2368	100	Algeria	2369	100
Algeria	2370	100	Algeria	2371	100
Algeria	2372	100	Algeria	2373	100
Algeria	2374	100	Algeria	2375	100
Algeria	2376	100	Algeria	2377	100
Algeria	2378	100	Algeria	2379	100
Algeria	2380	100	Algeria	2381	100
Algeria	2382	100	Algeria	2383	100
Algeria	2384	100	Algeria	2385	100
Algeria	2386	100	Algeria	2387	100
Algeria	2388	100	Algeria	2389	100
Algeria	2390	100	Algeria	2391	100
Algeria	2392	100	Algeria	2393	100
Algeria	2394	100	Algeria	2395	100
Algeria	2396	100	Algeria	2397	100
Algeria	2398	100	Algeria	2399	100
Algeria	2400	100	Algeria	2401	100
Algeria	2402	100	Algeria	2403	100
Algeria	2404	100	Algeria	2405	100
Algeria	2406	100	Algeria	2407	100
Algeria	2408	100	Algeria	2409	100
Algeria	2410	100	Algeria	2411	100
Algeria	2412	100	Algeria	2413	100
Algeria	2414	100	Algeria	2415	100
Algeria	2416	100	Algeria	2417	100
Algeria	2418	100	Algeria	2419	100
Algeria	2420	100	Algeria	2421	100
Algeria	2422	100	Algeria	2423	100
Algeria	2424	100	Algeria	2425	100
Algeria	2426	100	Algeria	2427	100
Algeria	2428	100	Algeria	2429	100
Algeria	2430	100	Algeria	2431	100
Algeria	2432	100	Algeria	2433	100
Algeria	2434	100	Algeria	2435	100
Algeria	2436	100	Algeria	2437	100
Algeria	2438	100	Algeria	2439	100
Algeria	2440	100	Algeria	2441	100
Algeria	2442	100	Algeria	2443	100
Algeria	2444	100	Algeria	2445	100
Algeria	2446	100	Algeria	2447	100
Algeria	2448	100	Algeria	2449	100
Algeria	2450	100	Algeria	2451	100
Algeria	2452	100	Algeria	2453	100
Algeria	2454	100	Algeria	2455	100
Algeria	2456	100	Algeria	2457	100
Algeria	2458				



## Bibliography

- Altman. (1968). Financial Ratios, Discriminant Analysis and the Prediction of Corporate Bankruptcy. *The Journal of Finance*.
- Angeloni, C., Merler, S., & Wolff, G. (2012). Policy Lessons from the Eurozone Crisis. *The International Spectator*.
- BIS. (2006). *Basel II*. BIS.
- FINMA. (2013). *Resolution of global systemically important*. FINMA.
- Fox, J. (2008). *Applied Regression Analysis and Generalized Linear Models*. SAGE Publications.
- Gini, C. (1912). The origins of the Gini index. *Variabilità e Mutabilità*.
- Heerkens, H. (2004). *A methodological checklist for the high-tech marketing project*. Universiteit Twente.
- Heij, C. (2004). *Econometric Methods with Applications in Business and Economics*. Oxford University Press.
- Herel, M. V. (2012). *Expert meeting PD CB*. Rabobank.
- Hoek, B. V. (2007). *CB PD Working doc v 0 46 Gecapped NF*. Rabobank.
- Hull. (2010). *Risk management and financial institutions*. Pearson Education.
- Jole, M. v. (2008). *Mapping Rabobank Risk Ratings to S&P-ratings*. Rabobank.
- Kopitin, C. (2013). Managing risk of liquidity: A new dimension of risk management. *International Journal of Risk Assessment and Management*.
- Lopez, J. A. (2001). *Modeling Credit Risk for Commercial Loans*. Federal Reserve Bank of San Francisco.
- McCullagh, P., & Nelder, J. A. (1989). *Generalized Linear Models, Second Edition*. Chapman and Hall/CRC.
- Opzeeland, J. v., & Westerop, S. v. (2006). *General checklist PD models*. Rabobank.
- Piet, R. (2011). *PD model development methodology*. Rabobank.
- Plackett, R. L. (1950). Some theorems in least squares. *Biometrika*.
- Pompe, P. P., & Bilderbeek, J. (2005). The prediction of bankruptcy of small- and medium-sized industrial firms. *Journal of Business Venturing*.
- Russell Davidson, J. G. (2004). *Econometric theory and methods*. Oxford University Press.
- S&P. (2011). *Banks: Rating Methodology And*. S&P.

Vedder, R. (2010). *QRA Rating Model Development Guidelines*. Rabobank.

Wedderburn, R., & Nelder, J. (1972). Generalized Linear Models. *Journal of the Royal Statistical Society*.