POSITIONING POLICE EMERGENCY VEHICLES

DETERMINING FACILITY LOCATIONS AND ROUTING TECHNIQUES FOR FAST EMERGENCY RESPONSE

UNIVERSITY OF TWENTE.

POLITIE

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Determining facility locations and routing techniques for fast emergency response

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PREFACE

This report is the result of my internship at the *Politie Oost – Nederland* and part of the graduation assignment for my Master *Industrial Engineering & Management*, in the direction of *Production & Logistics*. This research could not be realized without the help of a few persons that I would like to thank in particular.

First of all, for giving me clear insights in different parts of the organization, I would like to thank Peter Teekman. As external supervisor for this thesis, he gave me all the help I needed to conduct my research. Besides the useful source of information, I would also like to thank him for the pleasant cooperation and I appreciate his continuous interest in my research.

Besides my external supervisor, I would also like to thank both of my supervisors of the University of Twente, Martijn Mes and Leo van der Wegen. I am grateful for their feedback, the sharing of their scientific knowledge and the fact that I could always turn to them for questions and answers.

I would also like to thank all the people from the base team IJsselstreek who provide me with information that I needed for this research. In particular, I would like to thank Patrick Fluyt for introducing me to the police, Geert de Jong for his helpful vision and ideas during my research, and Roeland Groen for a great cooperation.

Finally, I would like to thank my family and friends for supporting me during my research. With the writing of this preface, which is the last part of this report that I am writing, I realize that this is also the last part of my Master programme. I am grateful that I can look back to a great period as a student, with a lot of good memories.

Peter Muller

June 2014
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MANAGEMENT SUMMARY

Last year, the Dutch police adopted a major change, resulting in the need for closing a significant number of police stations in the nearby future. For the base team IJsselstreek, where this research is conducted, this means that at least one, probably two and maybe three out of four police stations need to close. One of the major impacts of closing a police station in the area is the assumed higher average response times for high-priority incidents, since not every location is covered anymore. This means that a different approach is needed to be able to cover the total area. During this research, the organization does not know which and how many police stations will close (a decision where this research can help), so different scenarios need to be analysed. Currently, three emergency vehicles are available for the base team IJsselstreek, but the organization is also interested in the consequences when less and more vehicles are used. This resulted in the following research goal:

'Give the base team IJsselstreek insight in the consequences of (i) using different police station locations and (ii) the number of emergency vehicles, with respect to the response times of high-priority incidents.'

We developed an emergency vehicle positioning model, where we maximize the expected coverage fraction, given a number of available vehicles and locations of police stations. This model is able to be solved to optimality within a reasonable amount of time when we generate a 48 hour plan. Moreover, when this model is used real-time, it can be solved again after an incident happens. This means that, when an incident happens and the nearest vehicle responds to it, the positioning model is able to reallocate the remaining vehicles in an optimal way.

For the expected demand, which is the input of the positioning model, we used historical data that includes all incidents from the years 2011 – 2013, registered with date/time groups, priority and coordinates. This enables us to create a forecast for the area of IJsselstreek, where we included weekly, daily and hourly patterns. We decided to divide the total area of IJsselstreek into 85 regular hexagons, resulting in a forecast for each hexagon. Furthermore we used a smart heuristic to get good estimates of the travel times between each pair of hexagons.

To test the developed positioning model in combination with the created forecast, we set up five sets of experiments to simulate for one year. Our key performance indicator is the on-time percentage of high-priority incidents, i.e., the percentage of prio 1 incidents that has a response time less than 15 minutes. It appeared that, when at least one police station needs to be closed, the best option is to close Eerbeek. Furthermore, there is no significant difference between the police stations in Lochem and Twello. Another conclusion is that, on average, an improvement of 5.4% in the on-time percentage can be achieved when applying the optimal positioning method, in comparison with an alternative way of positioning where all vehicles are standby at the police stations. Furthermore, the length of the shift changing time has a significant impact on the on-time percentage; going from 1 hour to 4 hours, results in an average decrease of 4.4%. Also the use of the proposed forecast is tested versus a simple forecasting method, where the performance of the extended forecast scores on average 3% better than the simple one. Finally we concluded that the addition of fairness constraints, i.e., prevent having some areas never be covered within 15 minutes, results in a loss in the average on-time percentage of 2.2%.

Based on this research, we recommend applying (i) the proposed forecasting method and (ii) the developed mathematical positioning model. Implementing the positioning model requires a smooth cooperation with existing *Geographical Information Systems* that the organization uses, so we recommend developing an integrated support tool. The forecasting method has room for improvement, since we aggregated all prio 1 incidents and we did not distinguish, for example, robberies from traffic incidents. Finally, options for further research include the behaviour of other vehicles, like motors, and the cooperation with neighbouring areas to get an overall optimal result.

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1. INTRODUCTION

Police reorganisation: a national police force

The Dutch police have been radically reorganised. The 25 regional forces plus the KLPD, each with its own chief have been replaced by a national police force, consisting of ten regional units. The national police force was launched on 1 January 2013.

Advantages of reorganising the Dutch police
The reorganisation of the Dutch police has a number of advantages:

- The police can work more as a unit if all police officers are subordinated to a single national police commissioner.
- As a result, they will be able to spend more time patrolling the streets and investigating crime.
- Officers will spend more time on policing because they will spend less time on paperwork.
- There will be less bureaucracy, for instance because it will be easier to lodge a criminal complaint.
- The various entities within the police will work together more promptly and effectively (especially in the area of computerisation).
- ICT, accommodation, purchasing and human resources can all be centralised. As a result, operational management overheads will be lower.

Organisation of the national police force
There will be changes to the police management structure and the division of responsibilities. There will be a single national police force, divided into ten regional units, a number of national units, such as a national criminal investigation unit, and a police service centre (responsible for nationwide operational management).

The Minister of Security and Justice will have full ministerial accountability for the national police force. The Minister will determine the budget and set the framework within which the national police force will work (for instance whether there will be quotas for imposing fines).

Authority over the police will not change. The mayor and the chief public prosecutor will still make local agreements about police deployment. Each municipality will draw up a public safety and security plan, which will serve as a basis for the mayor's management of the police.

National police commissioner
The national police force will be headed by a national police commissioner, who will be responsible for all ten regional police units.

Regional units
Ten regional units will be set up to carry out police tasks, each headed by its own chief.

National units
There will be one or more national units to carry out tasks that can best be organised at national level. For example, there will be a national arrest team and a national crime squad.

Police Service Centre for operational management
The new Police Service Centre will carry out the operational management tasks of the current forces and the Dutch Police Cooperation Facility.

As can be read above, from the first of January 2013, a major change in the organization of the Dutch police has been adopted. This resulted in a reduction of the number of corps from 26 to only one: The National Police. In addition, we now have ten new *regional departments*, divided into different *districts*. These districts are then divided into *base teams*, which are partly the former districts. The result is a whole new approach for each base team, because they have a much larger area to cover. Instead of covering only their own municipality, they now have to cover up to eight municipalities. The covering of activities include the surveillance and deployment of emergency units. Also the closing of different police stations contributes to the greater responsibility of each base team. For the emergency units, this means that they all start their shifts at one place and then have to cover the area. This requires a different approach of the organization and deployment of all units, including the emergency units, in order to be on time when necessary. More specifically, when there is only one base location where all units start, how should these units be deployed and how much of them are needed in order to achieve the general goal: ‘*For at least 90% of the high-priority incidents, a unit should be on the spot within 15 minutes.*’? In this research, we aim to give an answer to this question, where we focus on the emergency units, the most important units that react on high-priority incidents.

This chapter aims to give a clear description of the problem and the structure of the research. We start with an overview of (and our focus on) the organization in Section 1.1, followed by the motivation of this research in Section 1.2. Then we proceed with defining the scope in Section 1.3 and the research goal and research questions in Section 1.4. Finally, we show the structure of the research for the remaining chapters in Section 1.5.

1.1 ORGANIZATION

As mentioned before, currently the Dutch police consist of ten regional departments. The geographically largest department is *Oost Nederland*, where we focus on. This regional department is divided into five districts, where we only focus on the district *Noord en Oost Gelderland*. This district consists of six base teams, including *IJselstreek* where this research is done. See Figure 1 for an overview.

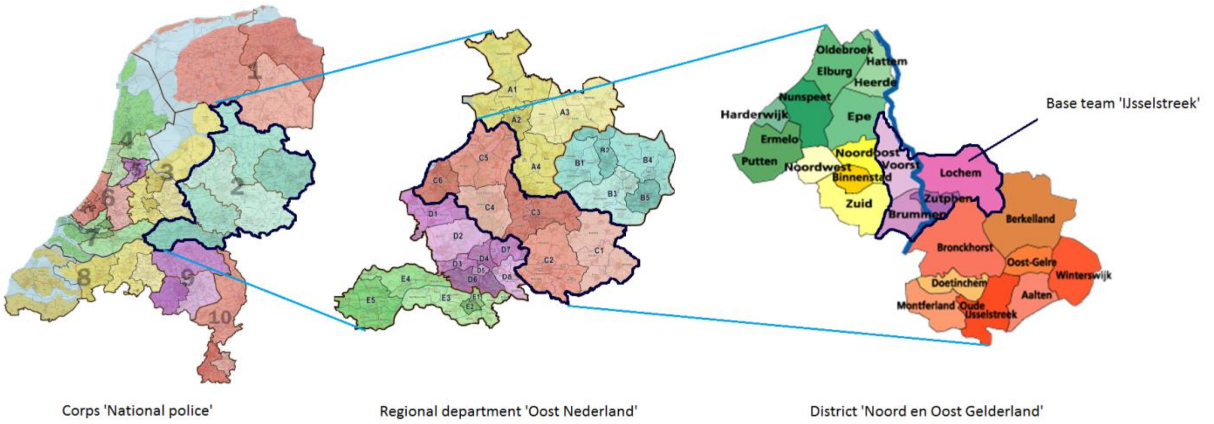


Figure 1: ***The focus on base team IJsselstreek.***

In the former organization of the Dutch police (before 2013-01-01), the district IJsselstreek consisted of four base teams, which were each located at each of the four municipalities in this area. The covering per base team consisted of so-called *circles*, which are imaginary circles where the midpoints are the physical police stations in each municipality and the radius is about 15 minutes of travel time. This means that one can easily draw these circles on a map and see which area is theoretically covered, see Figure 2. One can see that there are overlaps between the different circles and also areas which are not covered at all. These circles are used for the theoretical coverage of the area, indicating that at least 90% of the area is covered when all emergency units are located near the physical police station, while they wait for incidents to happen. This implies that the positioning of police stations is primarily based on geographical coverage. It is assumed that the higher the population, the higher the incident frequency. That is why police stations are located mostly around the centre of a municipality. However, there is no specific information used about incident patterns within the area of IJsselstreek.

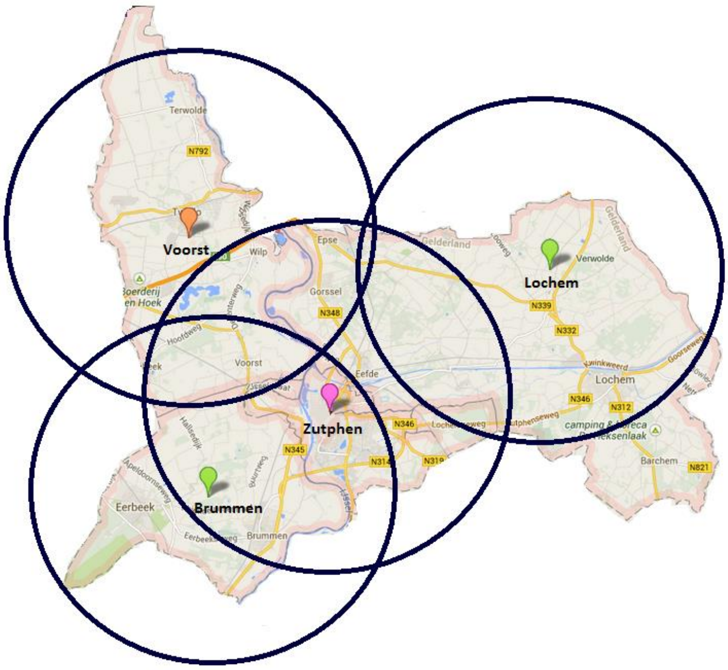


Figure 2: ***The former situation with four police stations where the circles represent the geographical coverage.***

In the new situation, there is one head station for base team IJsselstreek, located at Zutphen and in the nearby future the three other stations might close. This means that the circles from the former situation do not suffice anymore, because the drawing of one circle with midpoint Zutphen covers only about half of the total area. Therefore, another approach is needed where we can position emergency units in such a way that the total area is covered, even if we have only one police station in the area. In this case, total coverage refers to the general goal with respect to high-priority incidents, where a unit should be on the spot within 15 minutes for at least 90% of these incidents.

1.2 MOTIVATION

At this moment, there are still four police stations in the region of IJsselstreek. However, in the nearby future (within two years), at least two out of the four stations will be closed. Therefore, base team IJsselstreek is looking for another approach that can tackle the problem of covering the area with less physical police stations. In our research, we consider different scenarios. We consider the situation where only one police station is located in IJsselstreek and a set of options where two police stations are located in IJsselstreek. Furthermore, we are interested in the best way to position emergency vehicles for each location scenario. This means that, for example, when we have only one police station, located at Zutphen, we aim to give an answer to the question: How can we position the emergency vehicles in such way that the general goal for high-priority incidents is achieved with a minimum number of emergency vehicles, given the fact that there is one base location at Zutphen? When Zutphen is the only location, all units should start their shifts at Zutphen and, e.g., the emergency units will drive from Zutphen to their operating locations and drive back at the end of their shifts. The reason behind the uncertainty of closing one or two police stations is political, because the municipalities all want to keep a police station in their area. So instead of having only one police station in the area of IJsselstreek, located at Zutphen, there might also be a chance of having another police station in the area. Also from a geographical point of view, it might be wise to have, besides Zutphen, another base location, e.g., at the other side of the river IJssel. This means that the covering can then be done from two locations, which makes it perhaps easier to cover IJsselstreek, e.g., divided in two areas, west and east from the IJssel.

We have to keep the general goal of the police in mind with respect to the high-priority incidents (at least 90% within 15 minutes). This means that, starting from the moment that the incident is reported at the Emergency Control Centre (ECC), the police has less than 15 minutes to get a unit on the spot, where a unit is defined as 'any police officer that is on duty and equipped with the necessary outfit, tools, weapons, etc.'. Even though this goal is not established by law, it is a kind of standard of the Dutch police.

Now, the police are interested to know how they can achieve this general goal and how the emergency units should be organized with respect to the mentioned changes in the nearby future. So, keeping the future changes in mind, where should the police stations be located and how should the emergency units be positioned to guarantee that for at least 90% of the high-priority incidents a unit is on the spot within 15 minutes?

1.3 SCOPE

As mentioned in the previous section, the general goal of having for at least 90% of the high-priority incidents a unit on the spot within 15 minutes, regards any unit that is on duty and equipped with the necessary outfit, tools, weapons, etc. However, for the high-priority incidents, there is a number of units specialized in the handling of these incidents, called emergency units. An emergency unit consists of a specially equipped car and two police officers, driving around in specific areas or being standby somewhere in the area, e.g., at the police station. In the nearby future they should not only drive around, waiting for incidents to happen, but their tasks will be extended with other activities that can be interrupted when a high-priority incident occurs. So the emergency units are the most important units regarding the high-priority incidents. Moreover, even when a not-emergency unit is earlier on the spot, there is always an emergency unit on its way. Therefore, we focus on determining the best number of emergency units, and the positioning of these units.

Another important factor is the definition of 'high-priority incidents'. At the police, they categorize the incidents into four different categories, from 'prio 1' to 'prio 4', where prio 1 is the most urgent incident and prio 4 the least urgent one. The above definition of 'high-priority' regards only the prio 1 incidents, where we have the general goal of being for at least 90% of these incidents on the spot within 15 minutes. For the other categories there are also objectives, but these are less important. For example, for at least 90% of the prio 2 incidents, there should be a unit on the spot within 30 minutes. However, this target appears to be way less important than the prio 1 target. Therefore, we can completely omit them and fully focus on the prio 1 incidents. These are also the only incidents where the units are allowed to drive to with acoustic and light signals to be on the spot as fast as they can, taking legal regulations into account.

Regarding the response time of less than 15 minutes, the definition of response time is as follows:

> *'The response time starts when the incident is reported at the Emergency Control Centre and ends when a unit is on the spot.'*

This means that the response time consists of the time that the ECC is busy with the reporting of the incident and notifying the nearest unit (dispatch delay time), plus the time that a unit needs to react due to the breaking of other activities (reaction time), plus the time that a unit needs to get on the spot (travel time), see Figure 3 (Repede & Bernardo, 1994). This separation is important, because for the deployment of the emergency units we have to focus on the travel times instead of the response times. The dispatch delay time of the ECC as well as the reaction time of the emergency units cannot be influenced, so we have response time minus dispatch delay time and reaction time left for the deployment of emergency units to be on time. Therefore, we need some information about dispatch delay and reaction times first, so we can include this in the analysis.

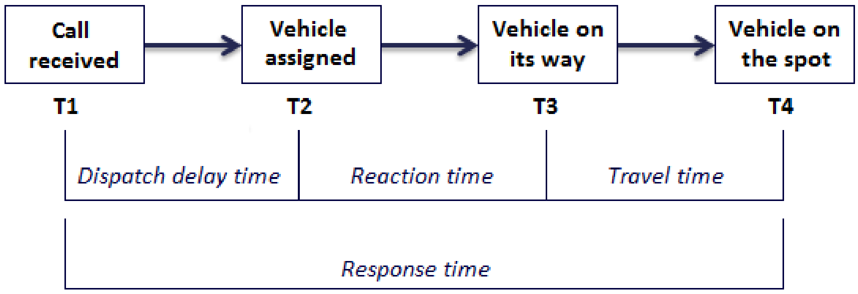


Figure 3: ***The response time of a prio 1 incident.***

Another restriction to our problem is that we have different scenarios for starting locations for the emergency units. We assume that a starting location is always one of the physical police stations. This means that, e.g., if we have only one police station in IJsselstreek, located at Zutphen, we assume that all units start and end their shifts at this location. It is given that the physical police station at Zutphen stays open in the future. This is a given fact, so we always have at least one starting location for emergency vehicles, located at Zutphen. This limits our solution space, but we can define a set of scenarios. Moreover, there is also consensus about the fact that at most two police stations will be open in the future. This reduces our solution space to the following two scenarios:

- Only one station, location: *Zutphen;*
- Two stations, locations: *Zutphen + a location of our choice.*

For both scenarios, we aim to solve the positioning of the emergency vehicles to optimality. The difference between the first and second scenario is the possibility to start and end shifts from a second location. For the second scenario, we want to introduce a second location of our choice. However, there are good internal reasons for the police to keep either the police station at Brummen or the police station at Voorst open. Without restricting us to these two possibilities, we do want to analyse them. However, either Brummen or Voorst might not be the most optimal location for a second police station, so we also want to analyse the scenario where we determine the optimal location for a second police station where can freely choose the location within the whole area of IJsselstreek. We can now define the problem as:

'Maximize the percentage of incidents for which an emergency unit is on the spot within 15 minutes.'

Ideally, the emergency vehicles are positioned nearby locations where incidents happen. Although knowing in advance exactly when and where incidents happen is unrealistic, demand can be estimated using historical data. This is known as forecasting. Positioning vehicles in order to serve as much demand as possible is in the scientific literature known as the Location Covering Problem (LCP). In Chapter 3 we discuss the roots of such problems and we also focus on possible methods to solve them.

1.4 RESEARCH GOAL/QUESTIONS

The goal of this research was already mentioned and is formulated as:

'Give the base team IJsselstreek insight in the consequences of (i) using different police station locations and (ii) the number of emergency vehicles, with respect to the response times of high-priority incidents.'

In order to operationalize this research goal, we define research questions that we need to answer to achieve this goal. The following questions will be answered during this research in chronological order:

1. *What does the (a) current and (b) desired situation look like with respect to the deployment of emergency units?*
   In Chapter 2 we want to give an overview of how the emergency units are currently organized and also what the desired situation should look like. Here we also want to sum up all the preconditions that we should deal with. This question is answered by discussions with personnel from the organization.

2. *What relevant literature is available about forecasting, location covering models and geographical constraints?*
   We aim to find relevant literature about these subjects by searching scientific literature databases and books. This provides us with the necessary knowledge and background for this research. This research question is answered in Chapter 3.

3. *What should a mathematical model look like in order to solve the positioning of emergency vehicles to optimality?*
   In Chapter 4 we aim to describe a mathematical model which is suitable for our situation. We find out what useful data is available at the police and, combined with the outcomes of the literature research, formulate a model that maximizes the coverage fraction. We also aim to give an answer to the question how to solve the formulated mathematical problem. At the end of this chapter, we have obtained a model that describes our problem, a method to solve this problem and a list of required input data.

4. *How should we generate the input for our model with respect to incident forecasting?*
   When we have an answer to the previous question, we know what data is available at the police and what the input data should look like. Then we can make a suitable forecast that fits our mathematical model. We want to answer this question with the knowledge gained from the previous research question and combine this with data gathered from the organization. We also want to describe a procedure how the forecast can be updated. This question is answered in Chapter 5.

5. *For each scenario, what is the best way for base team IJsselstreek to organize the deployment of emergency units?*
   In Chapter 6 we show the outcomes of the model and how this model can be used for base team IJsselstreek. We present results which give an answer to the question where to locate police stations, how many emergency vehicles are required and how to use the covering model in order to achieve the targets.

1.5 STRUCTURE

The structure of this report is as follows. The next chapter (Chapter 2) gives an analysis of the current, as well as the desired situation. This is followed by a literature research in Chapter 3, the description of the used model in Chapter 4 and the forecasting in Chapter 5. Then Chapter 6 describes the outcomes of the model, including the interpretation, and we close this report with the conclusion, recommendations and discussion points in Chapter 7.

2. SITUATION ANALYSIS

In this chapter we aim to give a clear overview of the current situation and the desired situation. The current situation is explained in Section 2.1, followed by different surveillance options in Section 2.2. We proceed with a first analysis of incident data in Section 2.3, where we look for spatial and temporal patterns. In Section 2.4, the time that the Emergency Control Center is busy handling an incident is analysed, known as the dispatch delay time. In Section 2.5 we analyse the incident handling time. An analysis of the geography of IJsselstreek can be found in Section 2.6 and we close this chapter with the desired situation in Section 2.7 and conclusions in Section 2.8.

2.1 CURRENT SITUATION

The emergency unit department is one of the core police forces that provides 24/7 emergency service where they react on incident calls. Emergency units are used for all types of incidents, categorized into four priority groups. Like we mentioned in Section 1.3, we only focus on the prio 1 incidents, since the objective of this group is by far the most important one. For the prio 2 incidents it is desired to be on the spot within 30 minutes, but the objective is not as important as for prio 1 incidents. Only for prio 1 incidents it is allowed to make use of acoustic and light signals, although there are situations where it is preferred not to use them, like robberies. Prio 1 incidents can be defined as:

'Incidents where immediate response of the police is necessary'

Examples of such incidents are burglaries, robberies, conflicts and collisions with serious injuries.

When an incident call is received at the general ECC, this ECC determines at first to which priority category it belongs. If the incident requires the deployment of an emergency unit, the ECC can pick the nearest emergency unit and gives the order to drive to the incident. The emergency units are equipped with an *Automatic Vehicle Location System* (AVLS), which gives a GPS location signal to the ECC. The ECC can see the current positions of these vehicles in the *Geographical Information System* (GIS), so they can make adequate decisions. In the nearby future, it is planned to equip more units with a GPS tracking system in order to make them also visible for the ECC. This is all part of the plan *'prio 1 voor iedereen'* (prio 1 for everyone), which implies that not only emergency units, but also the other units, should be able to react on prio 1 incidents.

When the ECC has assigned a certain emergency unit to drive to the incident, the emergency unit aborts the current activities and drives to the incident, like we already showed in Figure 3. When the emergency unit is currently busy with another high-priority incident, the ECC should assign another unit. When the emergency unit is on the spot of a high-priority incident, the necessary actions are performed and when the incident is solved, the emergency unit is again ready to be deployed for another incident. To complete the whole process of high-priority incidents, we add the handling time to the figure and call the whole route the *service time* (see Figure 4).

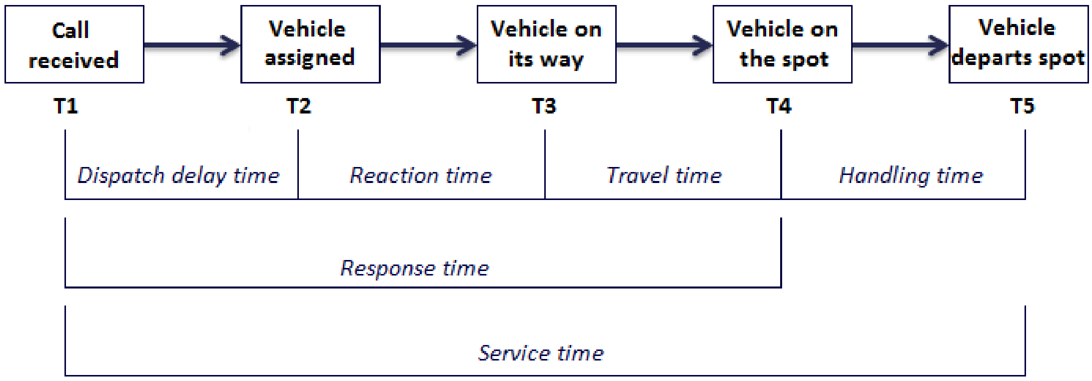


Figure 4: ***The service time of a prio 1 incident.***

2.2 SURVEILLANCE OPTIONS

An important distinction that we have to make is the difference between fixed starting locations of emergency units and their surveillance routes. Surveillance is a term that indicates a vehicle driving through a certain area, waiting for incidents to happen. However, surveillance has other benefits compared to waiting at the police station for an incident to happen. With surveillance, the police achieve:

- Visibility for the citizens: Citizens feel safer when they know there is police in their neighbourhood
- Probability of catching robbers/burglars in the act or witness other types of incidents
- Authority: When robbers/burglars see police driving around the area, there is a possibility they do not commit a crime

For simplicity, we call the surveillance routes ‘positioning’. Currently, the positioning is done based on driver experience. This means that there is no higher coordination level that tells them where to drive. When we focus on the possibilities of driving around the area in anticipation of possible incidents with unknown locations, we mention three options:

1. *Random surveillance.*
   Here the emergency unit is positioned in each area with almost the same probability. This means that a unit has no information about where or when incidents could happen in the area. The driver only knows how large the area is, where it is less smart to drive at the borders of the area because of the smaller covered area.

2. *Surveillance based on driver experience*
   This is basically what is currently done. Here the driver has some experience about the incident frequency from the area, implying that the emergency unit drives ‘more clever’ to certain areas. It is hard to measure the improvement from experience with respect to random surveillance, but we assume that this improves indeed the random surveillance option. Note that driver experience varies per driver and depends on, e.g., years of service in the area.

3. *Surveillance based on historical data*
   This option assumes that historical data about incident frequency from a specific area gives us information about the future incident frequency of this area. We can think of surveillance based on forecasting or information about future events in the area.

We can say that random surveillance is a special case of surveillance based on driver experience, where the driver has zero experience and no insight in the location of possible incidents. The most important distinction between surveillance based on driver experience (including random surveillance) and surveillance based on historical data, is the level of coordination. We distinguish two extreme points in the level of coordination: (1) coordination completely done from the top (emergency units follow direct orders where to drive) and (2) full self-control of the emergency units. In between these extreme points there are more options, like general control from the ECC to which municipality a certain unit has to drive, where the units control by themselves where to drive exactly within the municipality. Another possibility could be to provide the drivers with visualization of forecasted incidents, which can be helpful to make decisions about where to drive. In the current situation, there is limited control from the ECC to the emergency units; when the ECC notices a 'visually not so smart positioning' of the emergency units, they can decide to request a repositioning for one or more units.

2.3 INCIDENT DATA

For this research, we use data about prio 1 and prio 2 incidents that happened in the last three years (2011 – 2013), for the four municipalities within the region of IJsselstreek. Each incident is registered and we have per incident:

- Date/time that a call for an incident is received
- Date/time that the ECC gives order to the nearest available emergency unit to move to the incident
- Date/time that the relevant emergency unit arrived at the incident
- Date/time that the relevant emergency unit leaves the incident
- Priority of the incident (1 or 2)
- X and Y coordinates of the incident

Although we focus only on prio 1 incidents, we also requested prio 2 incidents, because when these incidents show the same patterns, we can make statistically more reliable forecasts due to the larger amount of data. Now, when we compare the available data with the data needed (from Figure 4), we see that we have four date/time groups (T1, T2, T4 and T5) from the diagram, where we only miss the date/time that a vehicle is on its way (T3). This data is not recorded at the police, because it is assumed that the reaction time is part of the travel time. In order to calculate the response times of the last three years, we do not need the date/time that a vehicle is on its way. We can just subtract the date/time that a call is received (T1) from the date/time that the relevant emergency unit is on the spot (T4). This gives us the so-called response time. When we count all the response times and divide the prio 1 incidents that are on time, i.e., response times less than 15 minutes, by the total number of prio 1 incidents, we get fractions of on-time incidents. See Table 1 for the results.

	2011			**2012**			**2013**		
	Total # incidents	# incidents on time	Percentage on time	Total # incidents	# incidents on time	Percentage on time	Total # incidents	# incidents on time	Percentage on time
Q1	130	103	79,2%	167	135	80,8%	168	132	78,6%
Q2	170	148	87,1%	173	135	78,0%	197	165	83,8%
Q3	164	115	70,1%	149	126	84,6%	208	172	82,7%
Q4	175	135	77,1%	176	143	81,3%	211	174	82,5%
	Total:		**78,4%**	**Total:**		**81,1%**	**Total:**		**82,0%**

Table 1: ***The number of high-priority incidents and their on-time percentages from the years 2011 – 2013.***

In Figure 5 it can be seen that in the past three years the target of 90% on time has never been met. The best on time percentage was achieved in the second quarter of 2011 and there is also not a clear improvement in the last quarters. The only promising conclusion that we can make from this data is the slight improvement each year, but that is not a strong conclusion with only three years of data available.

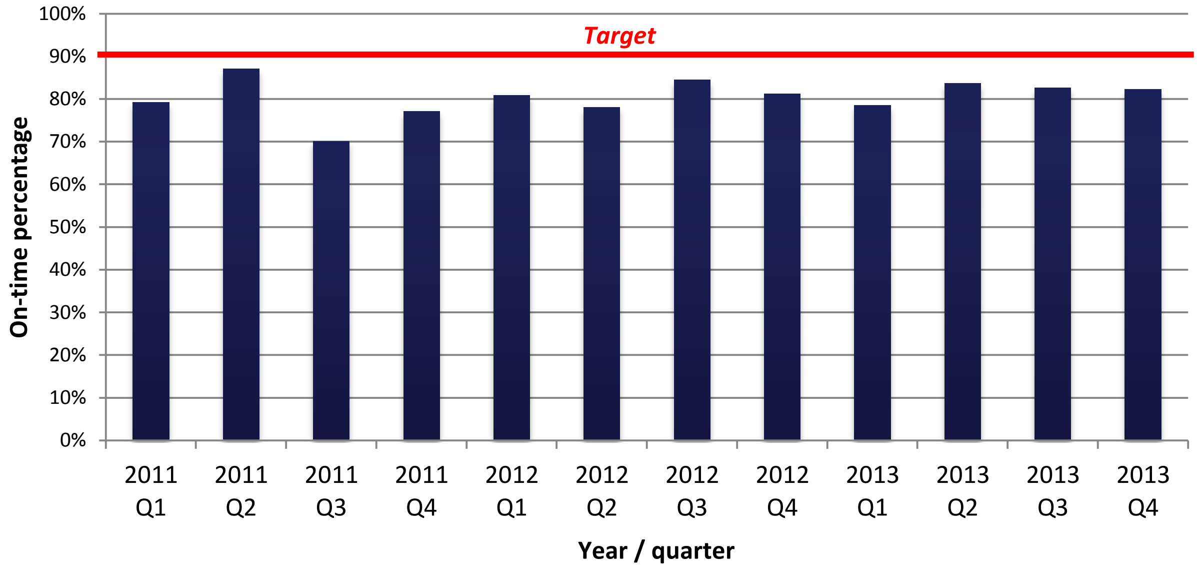


Figure 5: ***On-time percentages for each quarter in the years 2011 – 2013 compared with the target.***

If we compare each of the four municipalities, we see that there is a difference in the on time percentage between the municipalities. From Figure 6, we can conclude that it is easier to be on time within the municipality of Zutphen than within one of the other three municipalities. For the municipality of Zutphen, the 90% on time target is even met in the years 2011 and 2013.

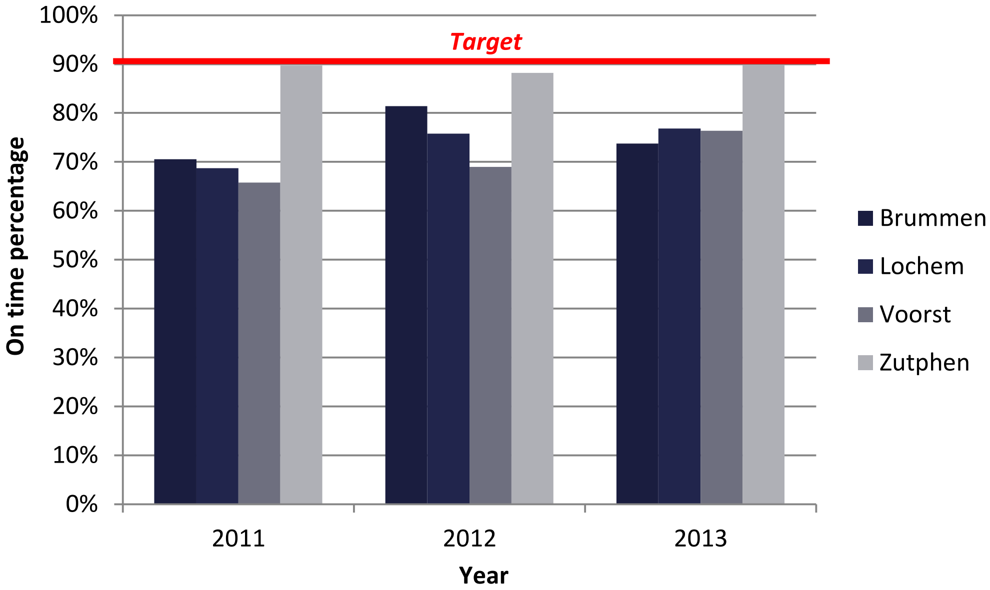


Figure 6: ***On-time percentages per municipality for the years 2011 – 2013.***

When we focus on the frequency of the occurrence of incidents, we can recognize patterns. We focus on forecasting of incidents in Chapter 5, but we aim to give an analysis of the available data in this section to emphasize the different patterns and the need for a good forecast. Therefore we aim to define in this section the dependent and independent factors that influence the number of incidents. This means that we first look for different spatial and temporal incident distributions from the available data and then test if there are dependencies. For example, when we recognize weekly patterns and daily patterns, we want to test if the time of the day is dependent of the day of the week. We start this analysis with the following hypotheses:

1. There are different incident distributions for each subarea within IJsselstreek.
2. There are yearly seasonal fluctuations, where the amount of incidents is probably higher at the darker days, i.e., between October and March.
3. There are weekly seasonal fluctuations, where the amount of incidents is probably higher at the weekends.
4. There are daily seasonal fluctuations, where the amount of incidents fluctuates during a day.
5. The prio 1 incidents follow the same distributions as the prio 2 incidents.

When we take a look at Figure 7, we can see that the first hypothesis can be accepted. This figure shows the incident distribution in the region of IJsselstreek for a whole year (2013), indicating that there are clearly subareas where incidents are more likely to happen than other subareas, as we expected. All subareas of IJsselstreek are showed with the number of high-priority incidents happened last year. The areas are not equally sized, but one can see at one glance that there are clear differences in incident distributions between them.

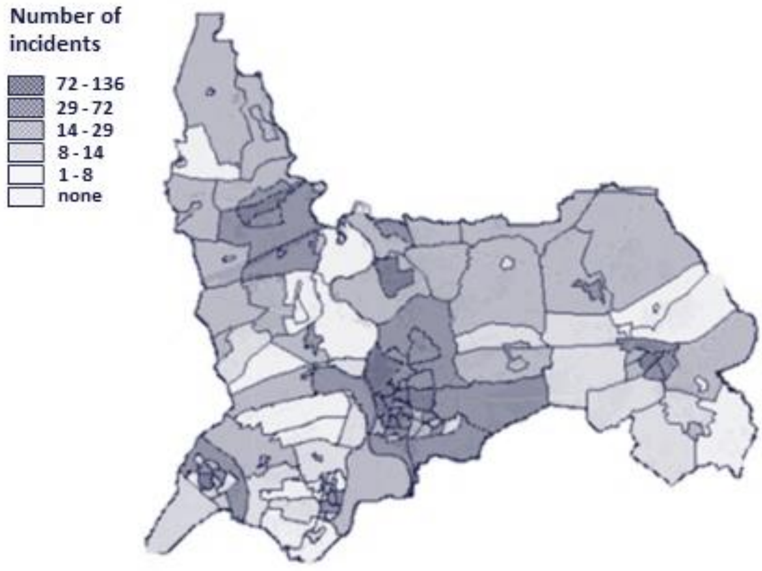


Figure 7: ***Distribution of incidents among the subareas and smaller districts of IJsselstreek.***

For the second hypothesis, i.e., to identify yearly seasonal fluctuations, we plotted the number of weekly incidents from the last three years, as can be seen in Figure 8. Note that we plot weeks (and not months for example) to have the number of weekdays equal for each point. It might happen that the number of incidents on Saturdays is much higher compared to other days of the week, so taking months (with unequal numbers of Saturdays) is not representative.

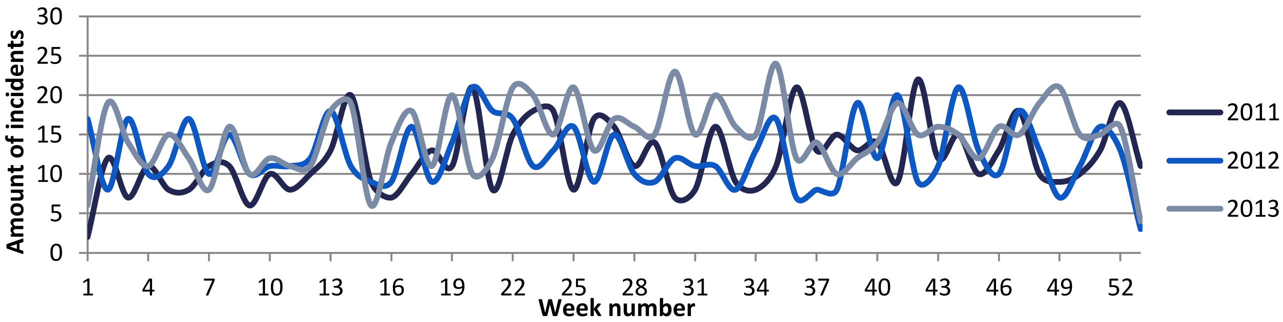


Figure 8: ***Weekly amount of incidents from the last three years (2011 – 2013).***

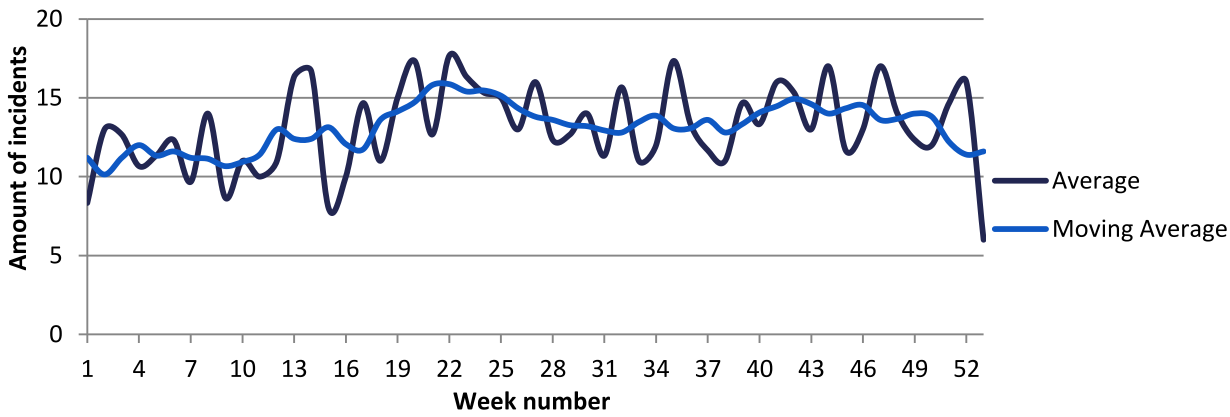


Figure 9: ***Average weekly amount of incidents from the last three years (2011 – 2013), including a moving average of five weeks.***

It is hard to state whether there are yearly patterns in incident distributions, so we also plotted an average of the three years, as shown in Figure 9. It is still hard to see a clear pattern (if one exists), so we added a moving average to it: for each week we take the average of the number of incidents from (i) that week, (ii) one and (iii) two weeks prior to that week and (iv) one and (v) two weeks after that week. So each week consists of an average of five weeks to smooth out the fluctuations. Now we notice small differences, but this does not seem significant. To be sure, we use a *two sample t-test with equal variances*, where we divide the years into quarters to check whether each quarter is significantly different from the other quarters. The results can be found in Table 2. If the *p-value* is smaller than 0.05, we assume that the concerned quarters differ significantly from each other (marked red).

	Q1	Q2	Q3	Q4
Q1	1			
Q2	0.024	1		
Q3	0.124	0.466	1	
Q4	0.006	0.831	0.309	1

Table 2: ***t-test resulting p-values for comparing the amount of prio 1 incidents for each quarter in a year.***

From these results, we conclude that only the first quarter differs significantly from the two adjacent quarters (Q2 and Q4). Furthermore, the other quarters (Q2, Q3 and Q4) do not differ significantly from each other. When we perform the same test again with semesters, where we divide the year in two halves where we expect the highest difference, i.e., the darker days versus the lighter days, we get a *p-value* of **0.341**. This is higher than the critical 0.050 value, so we conclude that there is no significant difference in the amount of weekly incidents at the darker days and the lighter days. We distinguish only the first quarter from the rest of the year. Note that it is however possible that there are more patterns recognizable, for different types of prio 1 incidents. For example, robberies can have different distributions of incidents than traffic accidents. The separation between all different types of incidents is not part of this research, because this data is not accessible, so we consider for this research only the total number of prio 1 incidents.

The third hypothesis concerns the day of the week and the fourth hypothesis concerns the time of the day. Now, it is assumed that there are different patterns for both factors, but also that they depend on each other, i.e., the distribution of incidents for a day depends on the day of the week. Therefore we combine both hypotheses to show these dependencies. Figure 10 shows the incident distribution for each hour of the day (with smooth lines between them) for every day in the week.

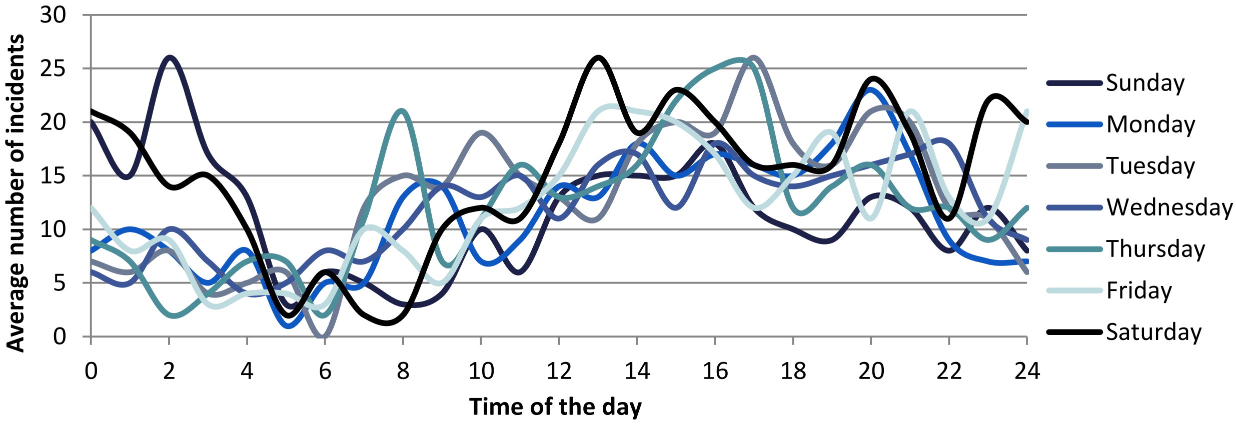


Figure 10: ***Average amount of prio 1 incidents for each hour of the day for each day of the week.***

We can see from this figure that the number of incidents is different during the weekend nights and that the number of incidents between 6:00 and 22:00 seems to follow the same distribution for every day in the week, although with high fluctuations. When we perform a *two sample t-test with equal variances*, where we count for each day only the number of incidents between 22:00 and 06:00, we get the resulting *p-values* as presented in Table 3. The values which are smaller than 0.05 are marked red and they imply significant difference.

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
Sun	1						
Mon	0.000	1					
Tue	0.000	0.210	1				
Wed	0.000	0.186	0.897	1			
Thu	0.000	0.372	0.707	0.627	1		
Fri	0.024	0.001	0.000	0.000	0.000	1	
Sat	0.045	0.000	0.000	0.000	0.000	0.000	1

Table 3: ***t-test resulting p-values for comparing the amount of prio 1 incidents between 22:00 and 06:00 for each day of the week.***

Here we see that the Fridays, Saturdays and Sundays are significantly different from each other, while the others are not. In comparison, we perform the same *t-test*, but now for the incidents registered between 06:00 and 22:00, resulting in the *p-values* as presented in Table 4. Here we see that only the Sundays are significantly different to almost every other day of the week, but the Fridays and Saturdays are not that different here. Therefore we conclude that there are differences between day of the week and hour of the day and that they depend on each other.

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
Sun	1						
Mon	0.000	1					
Tue	0.003	0.518	1				
Wed	0.090	0.080	0.258	1			
Thu	0.034	0.113	0.354	0.785	1		
Fri	0.000	0.523	0.199	0.018	0.025	1	
Sat	0.000	0.899	0.585	0.089	0.126	0.428	1

Table 4: ***t-test resulting p-values for comparing the amount of prio 1 incidents between 06:00 and 22:00 for each day of the week.***

For the last hypothesis, we check whether prio 2 incidents behave in a same way as prio 1 incidents. If so, this is very useful, since on average there happen only about two prio 1 incidents per day, but about eighteen prio 2 incidents a day. Therefore, we have a lot more data that we can use to predict incidents if this prio 2 data behaves in a same way. When we draw the same graph with the amount of incidents per week, as we did before, for both prio 1 and prio 2 incidents, we get the graph as presented in Figure 11. Note that we use for prio 1 incidents the *moving average*, like we did in Figure 9 and we normalized all incidents to fractions of their totals to compare them fairly.

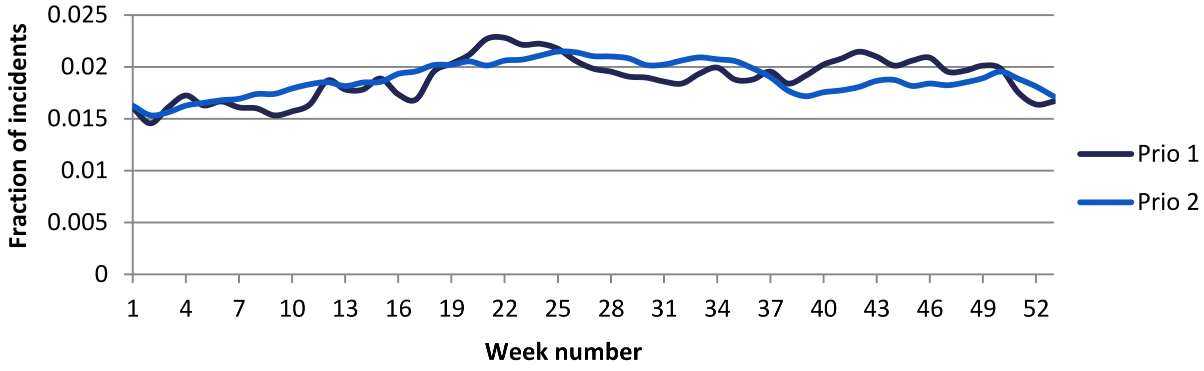


Figure 11: ***Normalized weekly amount of prio 1 (moving average) and prio 2 (normal average) incidents.***

We see that, for the first half year, the amount of prio 2 incidents rise almost every week. It is hard to state whether these prio 2 incidents behave in a same way. We know that for the prio 1 incidents, only the first quarter differs significantly from the other quarters. We can also compare the differences in quarters for prio 2 incidents. If we perform the *t-test with equal variances*, we get the results as presented in Table 5. Here we see significant differences between almost all quarters. Also with a *p-value* of **0.000** when we compare the lighter days with the darker days, i.e., Q1 + Q4 versus Q2 + Q3, we have to acknowledge different patterns during a year. The differences between quarters are higher than for prio 1 incidents, but the patterns are not that different.

	Q1	**Q2**	**Q3**	**Q4**
Q1	1			
Q2	**0.000**	1		
Q3	**0.000**	0.941	1	
Q4	**0.006**	**0.007**	**0.016**	1

Table 5: ***t-test resulting p-values for comparing the amount of prio 2 incidents for each quarter in a year.***

For the days in a week and hours in a day, we can also plot a graph for prio 2 incidents, like we did before for prio 1 incidents. This graph is shown in Figure 12. Here we notice the same kind of behaviour as for the prio 1 incidents: a significant higher amount of incidents during the weekend nights and almost the same patterns from around 6:00 until 20:00. For this comparison we use correlation coefficients, where we check whether the behaviour is the same or not. The results can be found in Table 5. We do not have a fixed critical value, but we interpret the results with common sense. Values near zero indicate no correlation and values around 1 indicate strong positive correlation. We assume that the correlation is strong enough in this case to state that prio 2 data behaves in a same way as prio 1 data. Again, we recommend a better understanding of patterns within the priority categories, but that is not part of this research.

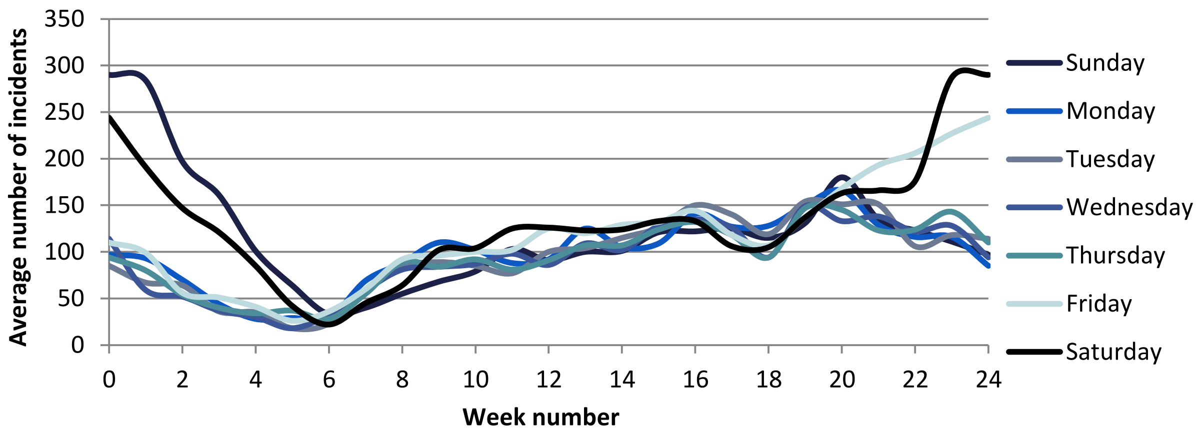


Figure 12: *Average amount of prio 2 incidents for each hour of the day for each day of the week.*

Weekday	Correlation coefficient
Sun	0.69
Mon-Thu	0.86
Fri	0.63
Sat	0.70

Table 6: *Correlation coefficients for prio 1 versus prio 2 data.*

To complete this analysis, we have to take a look at dependencies between different sub areas of IJsselstreek and the other discussed factors, like hour of the day, day of the week and week of the year. For simplicity, we divide the area into two groups, called *urban* and *rural*. Because more than one third of the incidents from the last three years happened around the city Zutphen, we call those incidents *urban* and the other incidents *rural*. Moreover, Zutphen is by far the most crowded municipality (see Section 2.6).

When we normalize the number of incidents (both groups the same average) and have a look at the incidents per week, we get the graph from Figure 13. Here we cannot notice clear differences. With a correlation coefficient of **0.63**, we conclude that both categories do not have significant differences in the amount of incidents per week over a year.

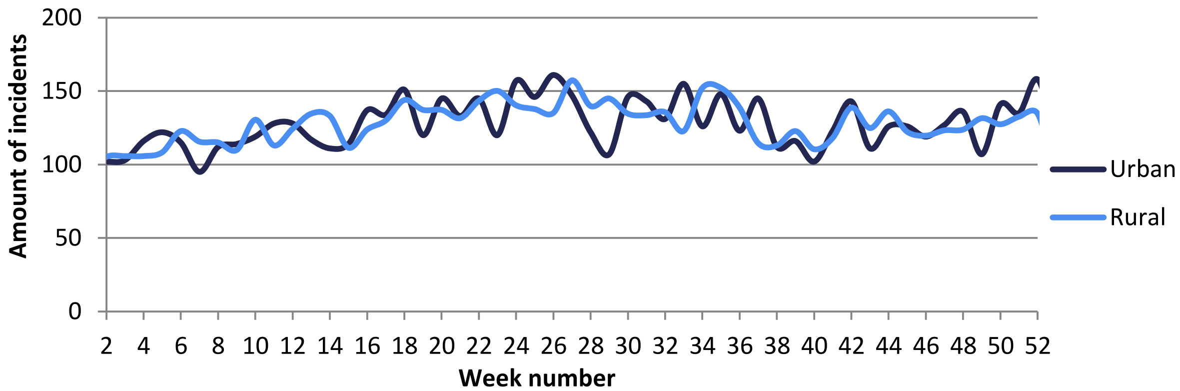


Figure 13: *Normalized weekly amount of incidents in urban and rural areas.*

For the days in a week and hour of a day, we created the same graphs as we did before, but now for both the urban (Figure 14) and the rural (Figure 15) category. The patterns look very similar, so we check this again with correlation coefficients. The results can be found in Table 7. All values are between 0.71 and 0.90, so we conclude that there is no significant difference between urban and rural areas in the amount of daily and weekly incidents.

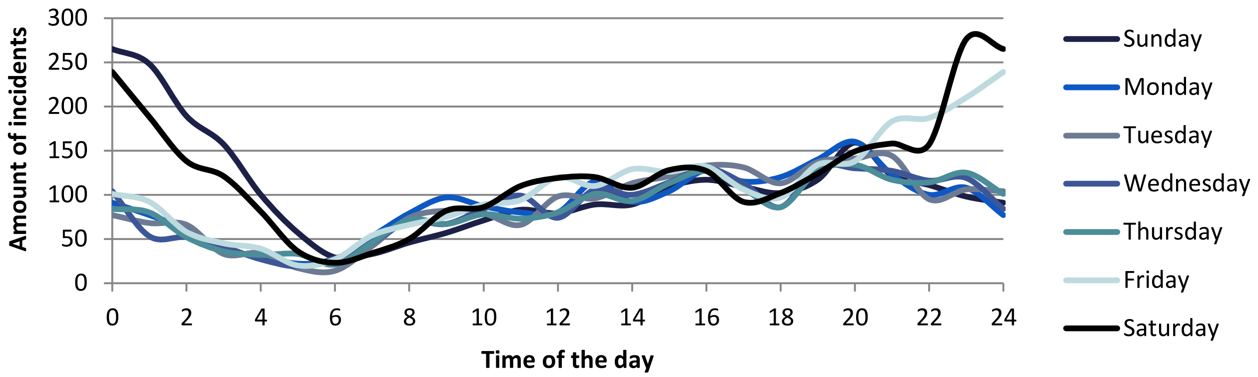


Figure 14: ***Average amount of urban incidents for each hour of the day for each day of the week.***

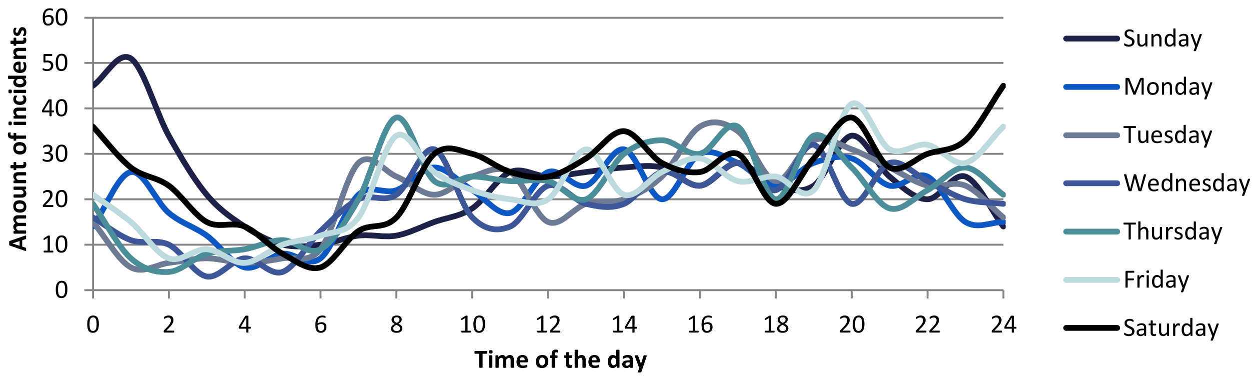


Figure 15: ***Average amount of rural incidents for each hour of the day for each day of the week.***

Weekday	Correlation coefficient
Sunday	0.90
Monday	0.81
Tuesday	0.77
Wednesday	0.75
Thursday	0.71
Friday	0.76
Saturday	0.78

Table 7: ***Correlation coefficients for urban versus rural areas.***

A complete overview of the dependent factors that determine the amount of incidents is shown in Figure 16. Here it can be seen that the amount of incidents is dependent on the week of a year, the area and the priority (independent from each other) and that it is dependent on the combination of day of the week and hour of the day. This information can be used to create a suitable forecast, which is presented in Chapter 5.

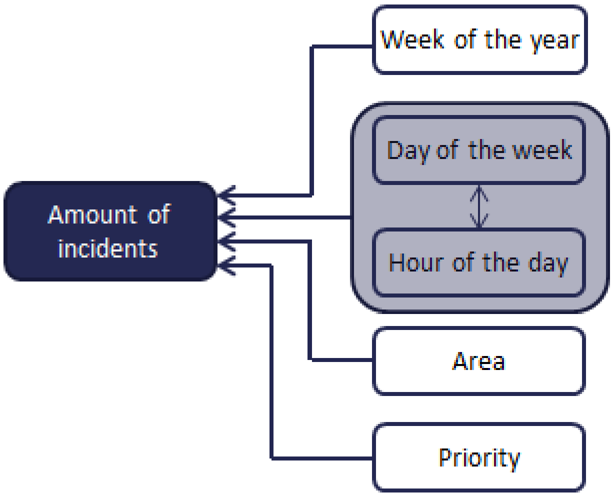


Figure 16: ***Overview of the factors that influence the amount of incidents.***

2.4 DISPATCH DELAY TIME

As defined in Section 2.1, we should use the information that we have available to estimate the dispatch delay time, i.e., the time that the ECC needs to handle the received call for an incident and to give orders to the nearest emergency unit. We use the data about historical incidents of the last three years (2011 – 2013). For each incident we calculate the dispatch delay time by subtracting the date/time that a call is received from the date/time that an emergency vehicle is notified. The data is plotted in a histogram in Figure 17 and we find an average of 3:06 and a median of 2:43.

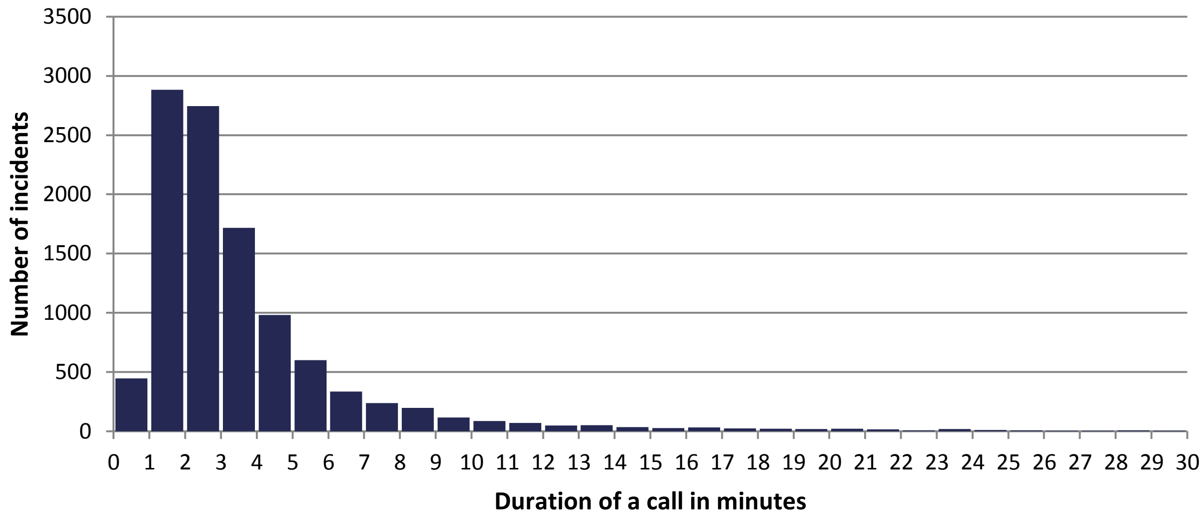


Figure 17: ***Historical data about the dispatch delay time from the ECC.***

From the graph we recognize a typical pattern for applications like this, where the proportion of events is large at the beginning (the left side) of the graph and where the right side forms a long tail. It is obvious that a lot of calls are handled within a few minutes, but there are exceptions where it takes (a lot) more time. The shape of the histogram appears to be of the lognormal form, so we check if this density function fits the data. The lognormal function contains a shape parameter σ and a scale parameter e^{μ} which have to be estimated. Using the frequently used maximum-likelihood estimator calculation, results in the following estimates of the parameters:

$$\hat{\mu} = \frac{\sum_{i=1}^{n} \ln X_i}{n-1} = 1.055$$

$$\hat{\sigma} = \left[\frac{\sum_{i=1}^{n} (\ln X_i - \hat{\mu})^2}{n-1}\right]^{\frac{1}{2}} = 0.681$$

When we plot the density function as a curve, using the two estimators, and combine this with the histogram of the historical data, we get the graph that is shown in Figure 18. We see that the curve fits well in the histogram and, using the chi square test for goodness of fit, we conclude that the dispatch delay time follows a lognormal LN(1.055, 0.681^2) distribution.

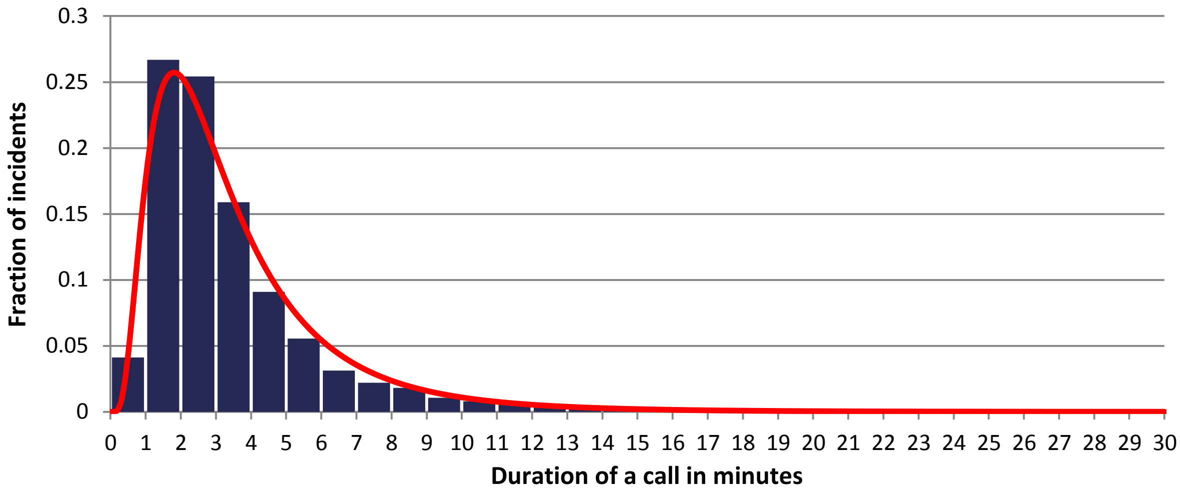


Figure 18: ***Historical data about the dispatch delay time including the well-fitted LN(1.055, 0.681^2) density function.***

2.5 INCIDENT HANDLING TIME

For the incident handling time, we have data available for some of the incidents that include the date/time that a vehicle leaves the spot, but it appears that the registration of those date/times is not done in a systematic way. Therefore, and as can be concluded after conversations with different people from the organization, this data is not reliable enough to use. When analysing the data, the assumption of a high unreliability of this data is confirmed, as can be seen in Figure 19.

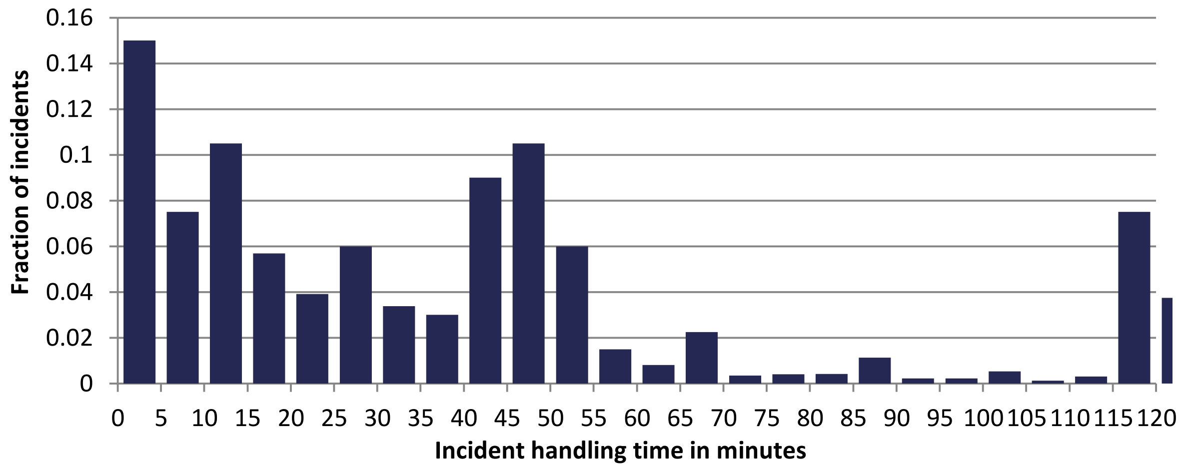


Figure 19: ***Historical data about the incident handling time.***

Unfortunately we cannot draw here a suitable function, so we have to make assumptions. After conducting interviews with emergency crews, it appears that the handling of incidents varies a lot, but we can assume that about half of the incidents take less than half an hour and there is a negligible small chance that incidents take longer than three hours. This information will be used in Chapter 6.

2.6 GEOGRAPHY

When we analyse the area of IJsselstreek, where we perform our research, we notice that we can call IJsselstreek a rural area. On average, the area of IJsselstreek has 267.18 inhabitants/km^2, where the countrywide average is 404.94 inhabitants per km^2 (CBS, 2013). See Table 8 for the distribution among the four municipalities.

	Inhabitants	*Area (km^2)*	*Inhabitants per km^2*
Voorst	23,741	126.52	187.65
Brummen	21,184	85.05	249.08
Lochem	33,333	215.19	154.90
Zutphen	47,233	42.93	1,100.23
Total	***125,491***	***469.69***	***267.18***

Table 8: ***Inhabitants and area per municipality of IJsselstreek.***

When we take a look at the characteristics of the area, we see that the area of IJsselstreek includes two important waterways: the river *the IJssel* and the channel *Twentekanaal*. Both waterways limit the crossing from one side to the other side. This applies especially for the IJssel, since there are only two bridges, both nearby Zutphen. This means that we can conclude that there is no way to drive directly between the municipalities Voorst and Lochem, although the areas are adjacent to each other. On the other hand, the Twentekanaal has eight bridges within the area of IJsselstreek, which makes the crossing from the north side to the south side of Lochem more accessible; see Figure 20 (Google, 2013).

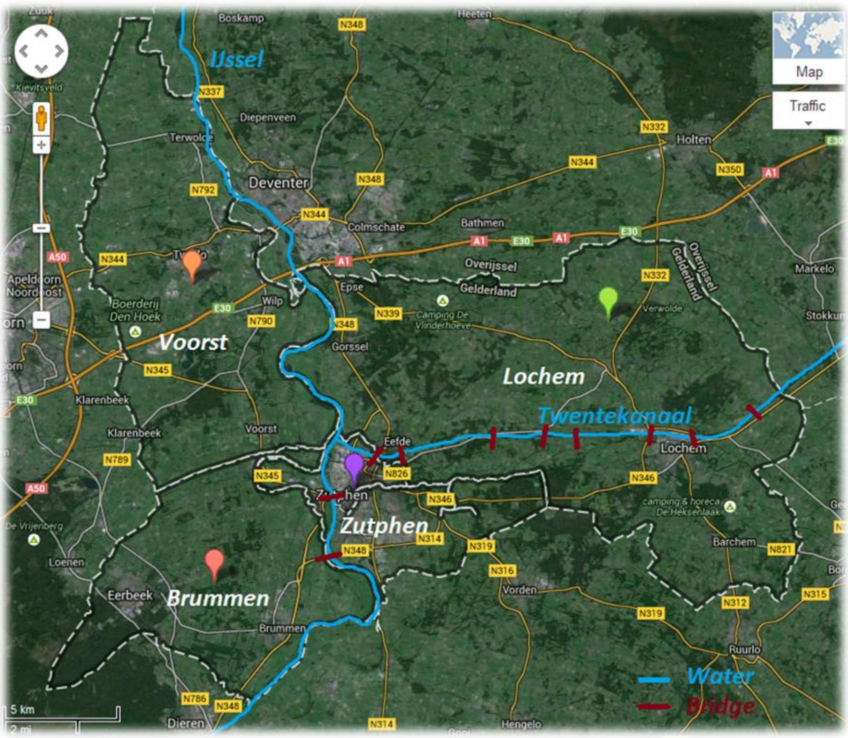


Figure 20: ***The area of IJsselstreek, including their municipalities, waterways and bridges.***

Of course, it is allowed to travel outside of the area of IJsselstreek and this is probably in a significant number of cases a lot faster. For example, travelling from the north side of Voorst to the north side of Lochem using the motorway *A1* is faster than travelling via Zutphen. Also the use of the motorway *A50* of the west side of Voorst and Brummen can make a difference in the travel times. However, the travel times can still be crucial when we have to travel from one point to another. When we travel from the most northern point in Voorst to the most south-eastern point of Lochem, it takes at least one hour (no traffic jams included), although emergency vehicles may travel faster for prio 1 incidents. Moreover, travelling from (the starting location) Zutphen to one of the corners takes more than half an hour, implying the need of a smartly organized deployment of emergency units within the area.

2.7 FUTURE SITUATION

In the ideal situation, we know in advance where and when an incident will happen and there is always (or at least in 90% of the cases) an emergency vehicle nearby. However, this is not possible since we do not know exactly where and when an incident will happen, although we can make use of forecasts to have emergency units nearby places where incidents are likely to happen. For the surveillance options, we pointed out in Section 2.3 that there are two extreme points regarding the coordination level. In the desired situation, it is however not allowed to fully coordinate the emergency vehicles from a higher level, due to the general policy. This means that we can create a model that generates routing schedules for emergency vehicles for each time unit. The other extreme point where we have no control at all over the positioning of emergency vehicles is also not preferable, since we cannot make a difference to the current situation then. Therefore it is desired to come up with recommendations about the organization of emergency units without generating complete routing schedules. With the use of forecasts we can predict in some way where possible incidents will happen. This should give us information on how many emergency units the police needs and where to position them.

2.8 CONCLUSION

In this chapter we aimed to give a clear overview of the current situation and also a brief description of what the results of this research should lead to. We described the routing of a prio 1 incident and the deployment of emergency units. We conclude that the service time of a prio 1 incident can be divided into the response time plus the handling time of such an incident. Where we focus basically on travel times, we see that this is only one part of the ’15 minute response time’. Therefore we made an estimate of the dispatch delay time and we concluded that the reaction time is part of the travel time. Moreover, we have to take the handling time of incidents into account, since during this time the relevant emergency vehicle is not usable for other prio 1 incidents (called congestion, see Chapter 3).

Furthermore we can basically distinguish three surveillance options, where currently the emergency vehicles drive based on experience. Another option is to drive to areas where, according to historical incident data, it is likely that relevant incidents will happen. We concluded that surveillance based on forecasts requires probably less emergency units to cover the area of IJsselstreek. We aim to model the situation in such a way that we give recommendations about how many emergency units are needed and how to organize them.

We also showed some characteristics of the area of IJsselstreek, including the accessibility limitations of going from one point to another, mostly due to the river the IJssel. With calculating the travel times from one point to another we also have to take traffic jams into account, which are discussed in a the next chapter.

Finally, we made an analysis of the available incident data. We conclude that the last three years the target of being for at least 90% of the high-priority incidents on-time is not met. With the reorganization kept in mind, the target might be even harder to achieve when there is no change in the organization of emergency units, because of the decreasing amount of physical police stations. Another conclusion from the incident data is that we can recognize patterns. It is shown that the incident data correlates with the day of the week and the time of the day. When we look at the month of the year, we cannot recognize a clear pattern, although there are some months when more incidents have happened the last three years than other months.

3. LITERATURE RESEARCH

In this chapter we give a review of the available literature that is relevant for this research. We distinguish three topics, where we start in Section 3.1 with the Location Covering Problem (LCP). This is followed by forecasting in Section 3.2 and the estimation of travel times in Section 3.3. We close this chapter with conclusions in Section 3.4.

3.1 LOCATION COVERING MODELS

The core problem of location covering could be seen as follows. A certain area needs a specific service, like villains that potentially need medical service. Furthermore, there are targets, often set by a high level like the government, which requires for the total area that for a certain percentage of incidents, ambulances should arrive within a certain amount of time. Although the targets could differ, this problem is applicable to all emergency services, like police and fire departments (Geroliminis, Kepaptsoglou, & Karlaftis, 2011). When we are faced with such a problem, there are three main decisions to make:

1. *Where to locate the facilities?*
2. *How many servers per facility?*
3. *How to position each server?*

Emergency response service providers are typically concerned with the problem of improving the response time for a particular area with a limited amount of servers or determining the least number of servers needed to meet specified response time targets. Such response time targets include the requiring of a certain percentage of requests be reached within a certain amount of time. In the U.S., e.g., there is a general objective, originated from 1973, which states that ambulances should arrive in 95% of the requests within 10 minutes in urban areas (Ball & Lin, 1993). To reach targets like this, it is crucial to determine where to locate one or more facilities, known as the facility location problem, and how to position the fleet of servers. Note that in most of the models, a demand is set to be covered if at least one server can serve the emergency call within a predefined distance standard.

The problem of covering a certain area such that the required number of emergency vehicles is minimized, is known as the Location Set Covering Problem (LSCP), formulated by Toregas et al. (1971). Three years later the Maximal Coverage Location Problem (MCLP) was introduced (Church & ReVelle, 1974), which tries to maximize the covering of demand points, given a fixed fleet size. These two early problems form the basis of a lot of extensions that are widely used.

The first shortcoming of the LSCP and MCLP that was acknowledged is the problem that when a facility is called for service, the other demand points in that area are not covered anymore. In the literature, there are mainly two directions of solutions to prevent this issue, known as congestion (Beraldi & Bruni, 2009). One of them is the Double Standard Model (DSM), which provides multiple coverage of demand points, proposed by Gendreau et al (1997). The other direction of solutions includes busy probabilities, which are defined as the probability that at a random time period, a random vehicle is busy with the handling of an incident. For a certain time period, the busy probability can be determined as follows:

$$busy\ probability = \frac{expected\ service\ time}{total\ available\ service\ time}$$

Here the total available service time is equal to the length of the time period multiplied by the number of vehicles available in that period. The expected service time is equal to the total expected demand in the period multiplied by the mean service time of the handling of an incident. Among the first proposed models where such busy probabilities are included are the Maximum Expected Covering Location Problem (MEXCLP) by Daskin (1983) and the Maximum Availability Location Problem by ReVelle et al. (1989). These models strongly assume that facilities are independent and all facilities have the same busy probabilities. These assumptions are relaxed in hypercube queuing models, which are more realistic but also more complex models. Larson et al. (1974) were among the first to combine queuing theory with facility location modelling into the hypercube model and there are many extensions that use a queuing framework for modelling all kinds of uncertainty (Rajagopalan, Saydam, & Xiao, 2008). For a detailed overview of these extensions we refer to Galvão and Morabito (2008).

Another extension, known as the TIMEXCLP (Maximum Expected Covering Location Problem with Time Variation), was proposed by Repede et al. (1994), where the objective is to optimize the expected coverage at various points in time. This model was integrated into a decision support system for the emergency medical service in Louisville, Kentucky and results show that response time decreased by 36%. So, besides the spatial demand variation, TIMEXCLP also integrates temporal demand variation. For other extensions of the MEXCLP including time variation we refer to Owen and Daskin (1998).

Another extension concerns the so-called dynamic models. In the previous mentioned probabilistic models, congestion is taken into account using either busy probabilities or covering the demand points multiple times, where the allocation does not change after each incident. Dynamic models do take reallocation into account, resulting in, e.g., new allocations of other vehicles when one vehicle is dispatched. Gendreau et al. (2001) proposed such a dynamic probabilistic model, solving it using Tabu Search and extended it with a limitation on the number of allowed reallocations (Gendreau, Laporte, & Semet, 2006). For a complete overview of dynamic extensions, like penalty costs per reallocation, we refer to Brotcorne et al. (2003), Iannoni et al. (2009) and Pillac et al. (2013).

Besides the LCP concept, there is another class of problems, called Vehicle Routing Problems (VRP). Such a problem is described as follows. Given are the start and end location of a vehicle, the capacity of that vehicle and a group of demand points. The goal is to minimize the total traveling distance, while fulfilling all demand points. This initial VRP concept is proposed by Dantzig and Ramser (1959) and a lot of extensions are developed hereafter. Multiple vehicles with different capacities, different start/end locations and the addition of time windows are examples of extensions of the original VRP (Cordeau, Desaulniers, Desrosiers, Solomon, & Soumis, 2002). The VRP concept is widely used in the world of transportation and logistics. When we apply this concept to our situation, we can define the start/end locations as the physical police stations. There is a fixed number of vehicles that has to serve as much demand as possible within the time window of 15 minutes per demand request, where demand is in our situation defined as high-priority incidents. The demand is considered as fulfilled when the vehicle is at the location within 15 minutes. The demand is however unknown beforehand, but we can use expected demand. The extension of the VRP where demand is (partly) unknown is called the Dynamic VRP (DVRP). We refer to Toth and Vigo (2002) for a complete overview of the DVRP concept and its extensions.

While LCP models were developed, also various solution and evaluation techniques were developed in order to solve and/or evaluate the proposed models. There are mainly three categories of these techniques: exact methods, heuristics and simulation (Law, 2007, pp. 12-14). In a number of cases, LCP models can be solved to optimality within a reasonable amount of time. The case study for emergency vehicles in Louisville, Kentucky solves the formulated TIMEXCLP to optimality within a few minutes on a high-end computer. A high-end computer from 1992 is probably not the same as a high-end computer today, so today it might be even faster to solve. We see that in the most complex LCP models, including the hypercube queuing models, solutions were generated using all kind of heuristics like Genetic Algorithm, Tabu Search, Lagrangian Relaxation, Simulated Annealing, Ant Colony Optimization and Local Search heuristics (Li, Zhao, Zhu, & Wyatt, 2011). Most of the facility location models are formulated mathematically and are either solved to optimality using Branch and Bound (B&B) or solved to near-optimality using one of the above heuristics, depending on the complexity of the model. In a number of cases, simulation is used in combination with heuristics to get near-optimal results or to analyse the system performance (Li, Zhao, Zhu, & Wyatt, 2011).

For the DVRP concept, solution methods are far more complex and therefore time-consuming. Only for relatively small problems, exact algorithms can be used for solving. Heuristics are developed to solve these problems to near-optimality, but compared to the LCP concept, these are still more time-consuming. When we face the scenario where we have only one police station, we might get results that are good enough, e.g., solving the DRVP using the Clark & Wright heuristic. However, we have other scenarios that include two police stations, comparable with two depots in the DRVP, which are harder to solve. Furthermore, we do not solve such a model once and use the same output for a long time period (then the solving time would not be an issue), but we continually solve the model again and again after new incidents happen. This means that, using a model that takes, e.g., two minutes to solve, the time to drive to an incident is then reduced by two minutes. This is not favourable and therefore we give a high priority to the solving time when choosing a suitable model.

In Table 9, the concept matrix is depicted, where the mentioned concepts including their main characteristics are listed. It can be seen that three of the six concepts can be solved within a reasonable amount of time. This is our main driver, since we need quick responses to incident updates. Therefore, we do not want to use one of the three models which are hard to solve, if other models are also useful. This means that we prefer to define our problem as a LCP instead of a VRP. We see that the TIMEXCLP is able to deal with spatial and temporal demand variances and reallocations are possible, which makes this concept the best from the ones that are solvable within a reasonable amount of time. The only disadvantage is that there is no straightforward possibility to include busy probabilities which are vehicle/location dependent. For example, when vehicle A is positioned at a location where few incidents happen and vehicle B is positioned at a location where a lot of incidents happen, the TIMEXCLP concept assumes that both vehicles have the same probability (the average of both) of being busy with the handling of an incident. This is not representative in this case, because vehicle B is more likely to be busy than vehicle A. The Hypercube Queuing Model is the only concept which relaxes this assumption, but the disadvantage is the huge amount of time necessary to solve this. Furthermore, the research about the positioning of emergency medical vehicles in Louisville, Kentucky has shown that the TIMEXCLP concept can result in good solutions (Repede & Bernardo, 1994). Therefore we aim to apply the TIMEXCLP concept to our situation.

	Concept					
Characteristics	*Maximum Availability Location Problem*	*Maximum Expected Covering Location Problem*	*Maximum Expected Covering Location Problem with Time Variation*	*Hypercube Queuing Model*	*Vehicle Routing Problem with Time Windows*	*Dynamic Vehicle Routing Problem with Time Windows*
Abbreviation	**MALP**	**MEXCLP**	**TIMEXCLP**	**HQM**	**VRP**	**DVRP**
Solvable within a reasonable amount of time?	Yes	Yes	Yes	No	No	No
Spatial demand variance possible?	Yes	Yes	Yes	Yes	Yes	Yes
Temporal demand variance possible?	No	No	Yes	Yes	Yes	Yes
Vehicle/location dependent busy probabilities?	No	No	No	Yes	No	No
Reallocation possible?	No	Yes	Yes	Yes	No	Yes

Table 9: ***Concept matrix of the Location Covering Problem and Vehicle Routing Problem concepts.***

3.2 FORECASTING

Time series forecasting is a technique that predicts future events, based on the same sort of events that happened in the past. This means that the outcomes of the future events are not known, but we can make predictions based on historical data about these events. In our case we want to know something about future incidents in the area of IJsselstreek. Forecasts can be based on (1) an extrapolation of what has been observed in the past, called statistical forecasting, and (2) information about future events (Silver, Pyke, & Peterson, 1998). When forecasting time series from historical data, the following components can be distinguished (Brockwell & Davis, 2002):

- *Level*, the scale of a time series
- *Trend*, the growth/decline over time
- *Seasonal variations*, the daily/weekly/monthly patterns
- *Random fluctuations*, the residue due to unpredictability

With the analysis of the historical data, these components can be estimated and a forecast can be made for the next time periods. However, the general forecasting techniques are hard to apply to incident forecasting in small precincts, since there is usually not much data available, a high amount of randomness and a lot of different types of crime (Gorr, Olligschlaeger, & Thompson, 2003). One way of recognizing patterns in crime per location is to make use of hot spots (Sherman, Gartin, & Buerger, 1989), which are areas with a high crime density. Also Willing et al. (1982) and Kelling et al. (1998) recognize the criminality of certain places in their approaches. Another popular model is the Spatial and Temporal Analysis of Crime program (STAC), which clusters high-density crime points within ellipses and convex hulls (Block, 1995). This method allows for spatial and temporal variances, which is necessary as we concluded from Chapter 2. Liu and Brown developed a point-pattern-based density model, which is a prediction model for hot spots that relates characteristics of an area to preferences of criminals (Liu & Brown, 2003). These methods are relatively easy to use and to update and are therefore commonly used. However, when a lot of data is available to forecast, these methods are too general and more specific methods are preferable. So, when not much incident data is available, it is useful to create hot spot ellipses, but when there is more incident data available, a more detailed forecasting method is preferred (Liu & Brown, 2003).

In our situation, we have an area which we will divide into different subareas where we want a forecasting procedure that provides us with forecasted values of the amount of incidents per subarea, for each time period, where we acknowledge the existence of daily, hourly and weekly patterns. The size of a subarea will be determined in Chapter 5, but we know that if we choose the size of the subarea too small, we do not have enough data to make reliable forecasts. On the other hand, if we choose the size of the subarea too large, we cannot make good decisions. Generalization is often used to make the forecasted values more reliable (Sutton & Barto, 1998). Sutton and Barto state that one can use generalization in a lot of ways, in both a spatial and temporal direction. For example, when forecasting incidents for a certain subarea, at a certain time period, one might consider using incident data from a little earlier and a little later for that subarea, or for neighbour subareas around that same time period. Also a strong dependence on population density is often considered as a useful fact to forecast crime. These generalization methods are useful when a limited amount of data is available for certain areas or time intervals, but Sutton and Barto claim also that this is not necessary when there is sufficient data available (Sutton & Barto, 1998).

3.3 TRAVEL TIMES

When we separate the region of IJsselstreek into different subareas, we have to say something about the travel times between and within those sectors. In the early LCP models, travel times are supposed to be deterministic. These travel times are directly related to the distance and do not incorporate different traffic densities. The first models that do incorporate time-dependent travel times, e.g., traffic jams, make use of piecewise linear continuous speed functions (Hill & Benton, 1992) and are used for all kind of routing models (Horn, 2000). However, two other studies showed that including time-dependent travel times has only minor impact on the average response times: Kolesar et al. (1975) analysed the positioning of fire engines in New York and Budge et al. (2010) analysed the positioning of ambulances in Calgary, both resulting in travel times proportional to travel distances.

We see that most implemented location covering models are done on the North American continent, where the streets and traffic in Europe have definitely different characteristics. Schmid and Doerner (2010) defined three important factors that have to be taken into account, regarding the determination of travel times:

- During peak times, the traffic density tends to be high, which affects the travel time of emergency vehicles. The traffic density can depend on the type of area (urban/rural) and the time (hour/day/month/season) and it can be affected due to incidental events like a sports event or a car accident that blocks the road.
- The travel speed of emergency vehicles depends on the type of incident. Where in case of life-threatening incidents an emergency vehicle is allowed to make use of acoustic and light signals, this is not allowed for other incidents. In the Netherlands these life-threatening incidents are categorized as ‘prio 1’, like we mentioned already in Chapter 1.
- However emergency vehicles are allowed to speed up and make use of acoustic and light signals for high-priority incidents, there are still some legal regulations concerning the maximum travel speed, including the requirement not to endanger other road users.

For determining the travel times, taking into account time-dependency, we can start with determining the distances. Nowadays it is not difficult to get exact point-to-point distances, e.g., by making use of Google Maps (Google, 2013). Moreover, there are even reliable tools that incorporate traffic jams in the calculation of travel times, specialized for The Netherlands, see Locatienet (2013), ANWB (2013) and Modelit (2013). We should be able to get realistic travel times of emergency vehicles when we combine these tools with the above three points.

Since the number of travel times grows fast with the number of subareas, we can only calculate these travel times manually for a limited number of subareas. Therefore, we will probably benefit from a system that calculates travel times automatically. Google has an *application programming interface* available for calculating distances, called Google Distance Matrix API (Google, 2014). Given a set of locations, this Distance Matrix API gives travel times and distances between all locations as a result. Another option to get all the travel times is to get a small number of travel times exactly and then make assumptions of the remaining travel times, e.g., by multiplying the Euclidean distance with a factor (Brimberg, Walker, & Love, 2007). When it is possible, we prefer the first option of calculating all travel times automatically, because these are exact and can be calculated for different moments in time (including rush hours).

3.4 CONCLUSION

In this chapter we searched for relevant scientific literature to give us insight in different location covering models. We described six methods to solve the problem, and we concluded that the Maximum Expected Covering Location Problem with Time Variation (TIMEXCLP) is the most suitable one for our case, based on five criteria that we showed in a concept matrix. The goal of the TIMEXCLP is to maximize expected coverage of a certain area, for various time intervals. Where the standard MEXCLP does not allow for time variations with respect to incident distributions and travel times, this extended problem does. Furthermore, it is solvable in a reasonable amount of time and reallocation of vehicles is possible, which are preferable characteristics.

For the forecasting part, we found basic models that imply the distinguishing of level, trend and seasonal factors. However, when looking specifically for *incident* forecasting, most methods from the scientific literature state that it is hard to predict incidents because of the high amount of uncertainty. Generalization can be a solution when there is not much incident data available, but in our case there is sufficient data available (about 20,000 incident records from three years) and there are clearly patterns recognizable, as can be concluded from Chapter 2. Therefore, we conclude that general methods, like the hot spot ellipses from the STAC model are not preferable in our situation. In Chapter 5 we create our own forecasting method, based on level, trend and seasonal factors, including common sense, and we test the generated method with a relatively easy method where we ignore daily/hourly and weekly patterns.

When we split the area of IJsselstreek into different sub areas, we need exact travel times between all areas. We conclude that it is not difficult to get those times exactly, e.g., from Google (2013), although the number of travel times needed grow fast with the number of areas. Therefore we need a method that provides us relatively quickly the needed travel times, instead of calculating all travel times manually. We prefer the most accurate method, which is making use of the Google Distance Matrix API (2014).

4. COVERING MODEL

In this chapter we aim to give a formulation of the covering model. We start with a problem description, including an answer on how to apply this model in our situation in Section 4.1. Then we formulate the basic covering model in Section 4.2, followed by extensions of this model. In Sections 4.3 – 4.6 we add time dimensions, including demand variation over time (4.3), fleet size variation over time (4.4), travel time variation over time (4.5) and busy probabilities over time (4.6). In Section 4.7 we add the personnel schedule extension, after which we present the complete model in Section 4.8 and we end this chapter with conclusions in Section 4.9.

4.1 PROBLEM DESCRIPTION

From the previous chapter, we learned that the most suitable model to use for our situation is the TIMEXCLP, which is an extended Location Covering Problem, where time is included. In the next sections, this concept is explained, including extensions applicable specifically for our situation, but first we are faced with the problem how to apply this concept to our situation. We know from Chapter 3 that the TIMEXCLP optimizes the coverage fraction, which means that when we solve the formulated problem for a given time period, we get as a result the fraction of incidents that is expected to be visited on time. Furthermore, we get the locations of the available vehicles per time period as a result. However, when an incident happens, there is an emergency vehicle busy with the handling of the incident and the (temporary) new situation requires perhaps another positioning of the remaining vehicles, which means that the problem should be solved again, which is called reallocation. When this incident is handled, the according vehicle is again available for the upcoming time periods and the problem should be solved again, etc.

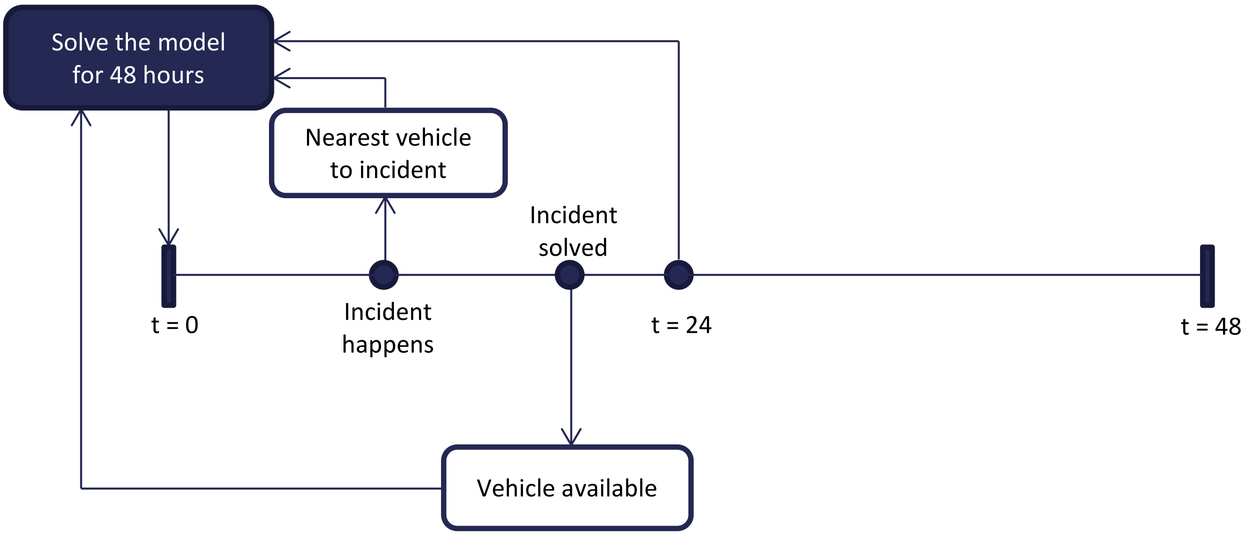


Figure 21: ***The process of positioning vehicles and incident handling.***

This means that, depending on the frequency of incidents, the model might be solved frequently. On average, about two high-priority incidents per day happen at the region of IJsselstreek (see Chapter 2), so this implies that on average the model should be solved about twice a day. However, emergency vehicles also handle the lower priority incidents (prio 2). This means that when such an incident happens, the nearest vehicle handles this incident. This changes the location of that vehicle and influences the coverage fraction of prio 1 incidents. It is assumed that prio 2 incidents are less important and when an emergency unit is busy with the handling of a prio 2 incident and a prio 1 incident happens, it is allowed to abort the current prio 2 incident and drive to the (more important) prio 1 incident. This influences the situation in the following way. When a prio 2 incident happens, the nearest emergency vehicle drives to the location of that incident. Now the position of that vehicle is changed for that time period, just like when a prio 1 incident happens, but now the vehicle is still available for a prio 1 incident. This means that the TIMEXCLP is solved again, but now with a fixed vehicle location for one of the vehicles and with this vehicle still covering part of the area. Since prio 2 incidents happen on average between sixteen and seventeen times a day, we have to solve the TIMEXCLP between eighteen and nineteen times per day on average. For on-time planning reasons, we decided to solve the TIMEXCLP for *48 hours*. This time period is long enough to give insight in where the vehicles are positioned in the nearby future, when the model is used. Then we use an event-driven rolling horizon to position the emergency vehicles to optimality. This means that we solve the positioning model initially for 48 hours, but when an event happens (an incident occurs), we solve the model again for the upcoming 48 hours. To get insight in the results of the model for at least the upcoming 24 hours, we solve the model also after 24 hours when no incident happened in the past 24 hours. The whole process of positioning vehicles and reacting on incidents is shown in Figure 21. Note that after solving the model for the upcoming 48 hours, one of three possible events can occur that triggers a new 48 hour period solving: (i) an incident occurs, (ii) an incident is solved or (iii) 24 hours of time has passed (no incident happened in the past 24 hours).

4.2 BASIC MODEL

In the next sections, the TIMEXCLP concept is explained, where we start with a basic LCP and extend it with useful components to make it suitable for our situation. We start with suitable demand points. This means that we split up the area of IJsselstreek into different nodes $i \in \mathcal{I}$, such that we can calculate in each node the expected demand, i.e., the expected number of prio 1 incidents that occur. Note that we do not take prio 2 incidents into account here, because otherwise the busy probabilities are not representative anymore. This is explained in Section 4.6. Furthermore, we have a set of possible locations $j \in \mathcal{J}$ where the vehicles $k \in \mathcal{K}$ can be positioned. These locations can be (partly) the same as the nodes *i* but this is not necessary. An example of an area divided into nodes *i* including possible vehicle locations *j* can be found in Figure 22. It can be seen that when locations *j1* and *j3* are assigned for vehicles, nodes *i1*, *i2*, *i3*, *i4*, *i6*, *i9*, *i13* and *i14* are covered, while the remaining nodes are not covered. The radius of the drawn circles equals here the time limit that a vehicle has in order to be on time at the incident, which is in our case less than 15 minutes. When the demand is equal for all nodes *i*, we have a coverage of ${}^{8}/_{14} = 57.1\%$.

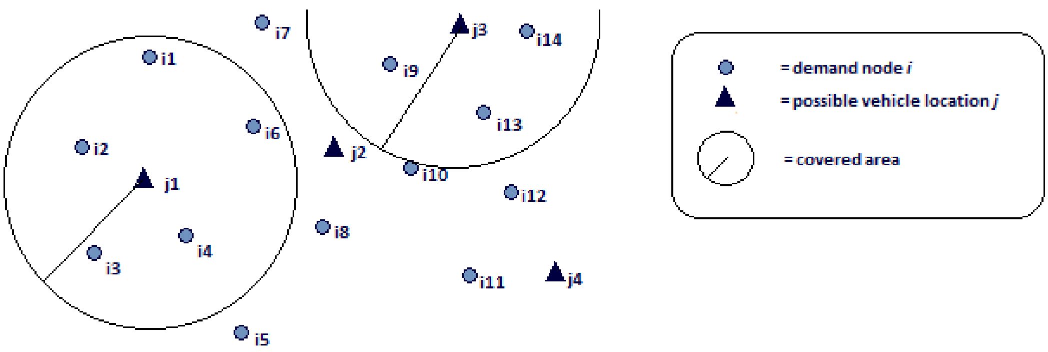


Figure 22: ***Example of covering of a certain area, including demand nodes and possible vehicle locations.***

Since we have expected demands for all nodes *i*, we define the parameter d_i as the demand size of node *i*. Furthermore, we have a fixed fleet size *f* and we define busy probabilities *q* of the vehicles, i.e., the probability that a randomly selected vehicle will be busy. This is an important addition, because we take into account the probability that an incident occurs, while the nearest vehicle is currently busy with another incident. We explain the calculation of the busy probabilities in Section 4.4. Furthermore, we want to maximize the total expected coverage of the area, which is for each node the demand size multiplied by the probability of a vehicle being available. This results in the following basic model:

$$\max \quad \sum_i \sum_k d_i(1-q)q^{k-1}Y_{ik} \tag{1}$$

$$\text{subject to} \quad \sum_k Y_{ik} \leq \sum_j X_j a_{ij} \quad \forall i \tag{2}$$

$$\sum_j X_j \leq f \tag{3}$$

$$X_j \in \mathbb{N} \quad \forall j \tag{4}$$

$$Y_{ik} \in \{0,1\} \quad \forall i,k \tag{5}$$

In this model Y_{ik} represents whether or not vehicle k covers node i. Furthermore, X_j represents the number of vehicles allocated to location j and the parameter a_{ij} represents elements from the $I{\cdot}J$ coverage matrix, being 1 if node i is covered by location j and 0 otherwise. The goal function (1) makes sure that the total coverage is maximized, which is the product of the demand size per node and the availability of the vehicles in that node, summed over all nodes. If there are multiple vehicles that cover the same node, the product of the demand size and availability is multiplied by the busy probability for each extra vehicle, because the demand can only be covered by the second vehicle for the time that the first vehicle is not available: the busy probability. Constraint (2) makes sure that the number of vehicles that cover a given node i cannot be more than the sum of the numbers of vehicles allocated at locations j that cover that node i. Constraint (3) restricts the total number of vehicles allocated to all the possible locations to be at most the fleet size f. Constraint (4) implies that the number of vehicles allocated per possible location should be a nonnegative integer and the Y_{ik} variable should be binary. An overview of all the mathematical notations that are used can be found in Appendix A.

The formulated basic model is known as the MEXCLP in the literature (Daskin, 1983) and it forms a suitable basis for our situation, because we aim to look for the best locations of vehicles in order to maximize the coverage of prio 1 incidents, given expected demand points in the area. The model does take the demand expectations and fleet size into account, as well as the probability that a random vehicle is not available (busy probability). It is, however, a stationary model that does not take, e.g., demand and fleet size variations over time into account. As we concluded already in the previous chapter, we have to extend this problem to the TIMEXCLP where we add the following extensions:

- Demand variation over time
- Fleet size variation over time
- Travel time variation over time
- Busy probability variation over time
- Start and end locations

4.3 DEMAND VARIATION OVER TIME

When we analyse the expected demand, we see that demand is not equal for each time of the day, day of the week and month of the year. This means that we cannot calculate one average demand from the last few years, but we have to make a forecast for different time intervals. The MEXCLP is not able to deal with demand variation over time, but, as can be read in Section 3.1, the MEXCLP can be adjusted so that it can take demand variation in time into account. Therefore we transform the demand size d_i to d_{it}, which is now the expected demand in node i at time t.

Now the new objective function (1) becomes:

$$max \quad \sum_{t}\sum_{i}\sum_{k} d_{it}(1-q)q^{k-1}Y_{ik} \quad , \tag{6}$$

where the summation over the time period is the length of the planning horizon. Furthermore, in order to get a coverage *fraction* from the objective function, we also want the demand d_{it} to be fractional. This means that we define d_{it} as the fraction of expected demand at node *i* during time period *t*, i.e., the expected demand during time period *t* at node *i* divided by the total expected demand during time period *t*. This makes the value of the goal function (6) to be nonnegative and at most equal to 1. Therefore we can easily compare the outcomes with the target values and the current values.

4.4 VEHICLE VARIATION OVER TIME

In the desired situation, it should be possible to locate a different number of vehicles for different time periods. For example, when demand is high at Voorst on Mondays and low on Tuesdays, it should be possible to have a vehicle nearby Voorst on Monday and not on Tuesday. We introduce the time index for the two variables in order to deal with this. Now X_{jt} becomes the number of vehicles allocated at location *j* during time period *t* and Y_{ikt} becomes 1 when the *k*-th vehicle added to the fleet during time period *t* covers node *i*.

Furthermore, when we know in advance that for certain time periods a smaller or larger number of vehicles is available, known as a variable fleet size over time, we can add this in our model. Let us introduce the index *t* to the fleet size *f*, resulting in f_t which is now the fleet size at time period *t*.

Combining the above model extensions, our new objective function (6) becomes:

$$max \quad \sum_{t}\sum_{i}\sum_{k} d_{it}(1-q)q^{k-1}Y_{ikt} \tag{7}$$

Constraints (2), (3), (4) and (5) become respectively:

$$\sum_{k} Y_{ikt} \leq \sum_{j} X_{jt}a_{ij} \quad \forall i,t \tag{8}$$

$$\sum_{j} X_{jt} \leq f_t \quad \forall t \tag{9}$$

$$X_{jt} \in \mathbb{N} \quad \forall j,t \tag{10}$$

$$Y_{ikt} \in \{0,1\} \quad \forall i,k,t \tag{11}$$

The objective function (7) aims to maximize the total coverage for the whole planning horizon, depending on how large we choose the maximum time period *T*. Now, due to the possibility to have a different number of vehicles at a place for different time periods, the model should now consider options where vehicles can actually travel from one surveillance location to another in the next time period, in order to cover other demand nodes. Constraint (8) makes sure that, for each time period *t*, the number of vehicles that cover a given node *i* cannot be more than the sum of the numbers of vehicles allocated at locations *j* that cover that node *i*. Constraint (9) restricts the total number of vehicles allocated per time period *t* to be at most the number of available vehicles for that time period. Constraints (10) and (11) are the same as before, although with an extra index t.

4.5 TRAVEL TIME VARIATION OVER TIME

Depending on the time of the day, day of the week and month of the year, travel times vary. This has a direct impact on the nodes i that can be covered within the specified time limit (15 minutes) from the possible vehicle locations j. In order to deal with this variation, we transform the parameter a_{ij} into a_{ijt}, which is now defined as 1 if location j covers node i at time period t and 0 otherwise. For the model, only constraint (8) changes into the following constraint:

$$\sum_k Y_{ikt} \leq \sum_j X_{jt} a_{ijt} \quad \forall i, t \tag{12}$$

This means that at rush hours for each location the number of covering nodes is possibly smaller than beyond the rush hours. The a_{ijt} values should be determined by analysing the travel times in the area.

4.6 BUSY PROBABILITY VARIATION OVER TIME

Where the demand and fleet size vary over time, also the busy probability should vary over time. The system wide busy probability per time period can be expressed as the ratio of the expected service time to total service time available (Repede & Bernardo, 1994). When we define u as the mean service time and m as the length of period t (both in minutes), we can express the probability that a random vehicle is busy at time period t, q_t, as:

$$q_t = \sum_i d_{it} u / f_t m \tag{13}$$

In order to set a lower bound for the search for an optimal number of vehicles and to make sure that the service capacity exceeds the expected demand, the following constraint on the fleet size f_t can be added (Repede & Bernardo, 1994):

$$f_t \geq \max_i \lceil d_{it} u / m \rceil \tag{14}$$

Although this formulation provides us with a lower bound, it is still possible that the expected demand exceeds the service capacity, resulting in a busy probability greater than 1. Since this is not favourable, we prefer to make sure that $0 < q_t \leq 1$. So, instead of taking the maximum highest integer, we take the sum over all demands per period, resulting in the following new constraint:

$$f_t \geq \left\lceil \sum_i d_{it} u / m \right\rceil \tag{15}$$

Including the busy probabilities per time period, the new objective function becomes:

$$max \quad \sum_t \sum_i \sum_k d_{it} (1 - q_t) q_t^{k-1} Y_{ikt} \tag{16}$$

We see that the new formulation of the problem is equal to the TIMEXCLP, which is a so-called type IV system (Repede & Bernardo, 1994): It allows for variations in both the locations and the active fleet size.

4.7 START AND END LOCATIONS

In our situation, we distinguish a series of scenarios, where the locations and number of the physical police stations differ. For each scenario, we know how much police stations (one or two) are positioned at which location. It is given that the eight hour shift of each crew has fixed start and end times, as shown in Figure 23. Furthermore, the emergency units switch their shifts at the physical police stations. For example, when only one police station, located at Zutphen, is available in the whole area, all emergency units from the morning shift will end their shifts around 15:00 and the units from the afternoon shifts start around that same time. When a unit is on its way during a shift changing time and an incident occurs, of course the relevant unit will first handle the incident and then change the shift. This means that there is no gap that emergency units are unavailable, but there are certain points in time where the emergency units have fixed locations. At all three shift changing points per day, all emergency units are positioned at the locations of the physical police stations. Moreover, the number of emergency vehicles that change shifts is fixed per location, since it is not allowed that an emergency unit starts its shift at one location and ends its shift at another location. For example, when there are two police stations (Zutphen and Twello) and three emergency vehicles, there are two possible options: Two units starting/ending at Zutphen and one at Twello or two starting/ending at Twello and one at Zutphen.

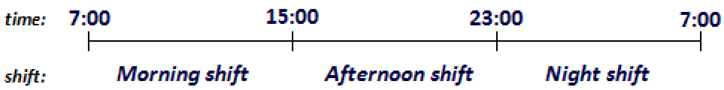


Figure 23: ***The shifts of the emergency vehicle personnel.***

We include these start and end times for each emergency unit as follows. We introduce the set $\mathcal{P}$ which is a subset of $\mathcal{J}$ (possible vehicle locations) and contains the locations of police stations. Furthermore, we introduce the set $\mathcal{S}$, which is a subset of $\mathcal{T}$ (time period) and contains the shift change time periods. Now we want to make sure that at each shift time period, all emergency units are located at the police stations. Therefore we add the following constraint:

$$\sum_{p} X_{ps} \geq f_s \quad \forall\, s \tag{17}$$

This constraint implies that the total number of emergency units positioned at the police stations during the shift change period is equal to the fleet size of that time period. Furthermore, we want to include the restriction that each unit belongs to one police station, i.e., shift change locations are fixed for each emergency unit. Therefore we have to introduce a parameter, n_p, which is the number of available emergency units at police station p. Now we can restrict the number of emergency units located at each police station at the shift change period as follows:

$$X_{ps} \leq n_p \quad \forall\ p, s \tag{18}$$

Constraint 18 indicates that the number of emergency units positioned at each police station during the shift change period cannot exceed the predetermined number of available emergency units per station. This parameter n_p is an input for the model, but we can try different options when we simulate it, because we are free to choose how to divide the number of emergency units among the police stations. When we simulate different options, we can compare the outcomes and find out what the best option is.

4.8 COMPLETE MODEL

In the previous sections of this chapter, we made changes and additions to the basic model. For completeness, we present here the total positioning model:

$$max \quad \sum_t \sum_i \sum_k d_{it}(1 - q_t)q_t^{k-1}Y_{ikt} \tag{19}$$

$$s.t. \quad \sum_k Y_{ikt} \le \sum_j X_{jt}a_{ijt} \quad \forall i, t \tag{20}$$

$$\sum_j X_{jt} \le f_t \quad \forall t \tag{21}$$

$$q_t = \sum_i d_{it}u/f_t m \quad \forall t \tag{22}$$

$$f_t \ge \left\lceil \sum_i d_{it}u/m \right\rceil \quad \forall t \tag{23}$$

$$\sum_p X_{ps} \ge f_s \quad \forall\, s \tag{24}$$

$$X_{ps} \le n_p \quad \forall\; p, s \tag{25}$$

$$X_{jt} \in \mathbb{N} \quad \forall j, t \tag{26}$$

$$Y_{ikt} \in \{0,1\} \quad \forall i, k, t \tag{27}$$

Note that this is only an overview of what we already showed, so no constraints are added or changed here.

4.9 CONCLUSION

In this chapter we started with formulating the problem description for positioning emergency vehicles to optimality. We concluded that vehicles react on both prio 1 and prio 2 incidents, where a prio 2 incident can be aborted when a prio 1 incident happens and no other vehicle is available. The whole process of positioning vehicles and reacting on incidents is showed in Figure 21.

After this, we formulated the most suitable Location Covering Problem for our situation, which is an extended version of the TIMEXCLP concept. This model includes spatial and temporal demand variations, which is necessary for our situation. We also incorporated the variations in fleet size over time, because we are able to position vehicles at different places for different time periods. To make the model more suitable, we defined the ability to have different travel times per time period. This is useful, because we have to deal with rush hours in the area. Furthermore, the model includes the so-called busy probabilities, i.e., the probability that a random vehicle is busy with the handling of a high-priority incident. We showed that this is depending on the demand of the area and the number of vehicles available and that it can differ per time period. To complete the model, we include restrictions that allow for personnel schedules and fixed locations of police stations, including a fixed shift changing location for each emergency unit. We concluded that we can solve our formulated TIMEXCLP model to optimality for each time period within a reasonable amount of time. This is necessary, because on average the model should be able to get solved on average between eighteen and nineteen times per day, which is the average number of prio 1 plus prio 2 incidents.

5. FORECASTING MODEL

In this chapter, we present our method to forecast incidents that happen in the area of IJsselstreek. In Section 5.1 we define what the problem of forecasting looks like, followed by the determination of the demand node dimensions in Section 5.2. Then we describe our actual forecasting method with respect to incidents in Section 5.3 and the forecasting of travel times in Section 5.4. We close this chapter with conclusions in Section 5.5.

5.1 PROBLEM DESCRIPTION

The main problem we are facing here is how to make a suitable forecast for the whole area of IJsselstreek, which is the input of the model. As we stated in Section 3.2, we want to divide the whole area in multiple subareas. These subareas have to be large enough in order to make reliable forecasts (reduce the forecast error), but also small enough in order to be able to make good decisions. In the next section, we focus on the size of the demand nodes. We already saw in Chapter 2 that there are differences between incident distributions over time and space. Therefore, we cannot just aggregate all the incidents to make the same forecast for every region at every time of the day, day of the week and month of the year, but we have to transform the available data into a time and space dependent forecast. Furthermore, we can also use prio 2 incident data, which gives us a more reliable forecast, because of the huge amount of data. However, we have to be careful with the 'simple transformation' of these data since the behaviour of prio 2 data is not completely the same as the behaviour of prio 1 data (see Section 2.3). In Section 5.3 we propose a method to make a forecast, taking all the independent and dependent relationships into account. Like we concluded in Chapter 2, we are not able to distinguish different types of prio 1 incidents (robberies, traffic incidents, etc.). We might expect to recognize different patterns between those different types, but this is beyond the scope of our research. We therefore aggregate all prio 1 incidents and aim to recognize patterns on this higher level. We recommend however to analyse also the different types of prio 1 incidents, which might result in an even more accurate forecast.

Besides the forecast with respect to incident data, we also need to create a travel time forecast, where we face the problem of how to determine all distances for all different time periods. When we have, say, 50 demand nodes and we distinguish two different travel time periods (rush hour and no rush hour), we have to determine $50^2 \cdot 2 = 5000$ different distances. This can be time-consuming, so we propose a quick method in Section 5.4.

5.2 NODE DIMENSIONS

When dividing the areas into multiple subareas, we choose to create the subareas all equally sized. When we use equally sized subareas, we can use easy methods to determine to which subareas incidents belong. We do take geographical constraints into account when we determine the travel distances, but not for the sizes of the subareas. The most representative shape of a subarea is a circle, since traveling from the centre to all points on the circumference is always the same Euclidean distance. The disadvantage of a circle is that it is impossible to cover the whole area with circles without the overlapping of one or more circles. We do not want overlapping subareas or an area that is not covered at all, so we are looking for a so-called regular tessellation, which is a tiling of regular polygons, i.e., a grid of equally sized areas that cover the whole area, without overlapping areas. As Ghyka (1977, p. 76) stated, there are only three regular polygons that fit a regular tessellation, which are the hexagon, the square and the triangle, see Figure 24.

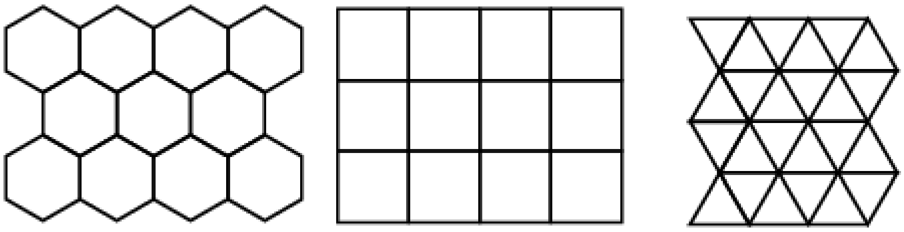


Figure 24: ***The three possible regular tessellations, from left to right: hexagon, square and triangle.***

Since the hexagon is most similar to a circle, we choose for the regular hexagons to fill the tessellation. Also Van Urk (2012) showed that this is a good usable shape, although he uses Euclidean distances in his research about helicopter positioning, which is different from this ground emergency vehicle positioning research.

Unlike the square grid, we cannot use a simple two-dimensional coordinate system in the hexagon grid, since the hexagons are not placed equally left and right to each other, but more down-left/down-right/up-left and up-right to each other. Although we find in the scientific literature coordinate systems that use three-dimensional grids, we cannot translate this easily to the coordinate system from the incident data. Moreover, we know from the previous chapter that we have to define demand points *i*, which implies that the use of one-dimensional coordinates is preferred. So, we are faced with the problem of translating *x* and *y* coordinates from incidents to one-dimensional hexagon shaped demand nodes. The incident data consists of *x* and *y* coordinates from the Rijksdriehoekstelsel, which is a Dutch Cartesian coordinate system for the whole area of the Netherlands. It consists of coordinates within the range [-7.000, 300.000] for the *x* coordinates and within the range [289.000, 629.000] for the Y coordinates, where the coordinates represent meters. So traveling 500 meters to the east is equal to a difference of 500 in *x* coordinates. The origin of this coordinate system is located at the spire of the *Onze Lieve Vrouwetoren* at Amersfoort.

Now, when we want to fit the incident data into the correct hexagons, we first transform the *x* and *y* coordinates into the hexagon coordinates, where we use a two-dimensional coordinate system. Then we transform these hexagons into one-dimensional nodes *i*, which can be used as demand nodes for the TIMEXCLP model.

Step 1: Convert the Rijksdriehoek coordinates (x, y) into hexagon coordinates (u, v)

In this step, we want to place all the incidents with *x* and *y* coordinates into hexagons with *u* and *v* coordinates. As a basis, we want the centre of the (0,0) hexagon to be equal to the physical police station at Zutphen. This location has the Rijksdriehoekstelsel coordinates of (210.500, 461.200). Now, when we move one hexagon up, the *v* coordinate of the hexagon grid increases by 1 and when we move one hexagon down, the *v* coordinate decreases by one. For the *u* coordinate, we can move in two ways. We can either increase the *u* coordinate by 1 when going up right (and decrease by 1 when going down left) or we increase the *u* coordinate by 1 when going up left (and decrease by 1 when going down right). We choose for the first option, which is more intuitive (adding 1 for going up and right). An example of the hexagon coordinate grid is showed in Figure 25.

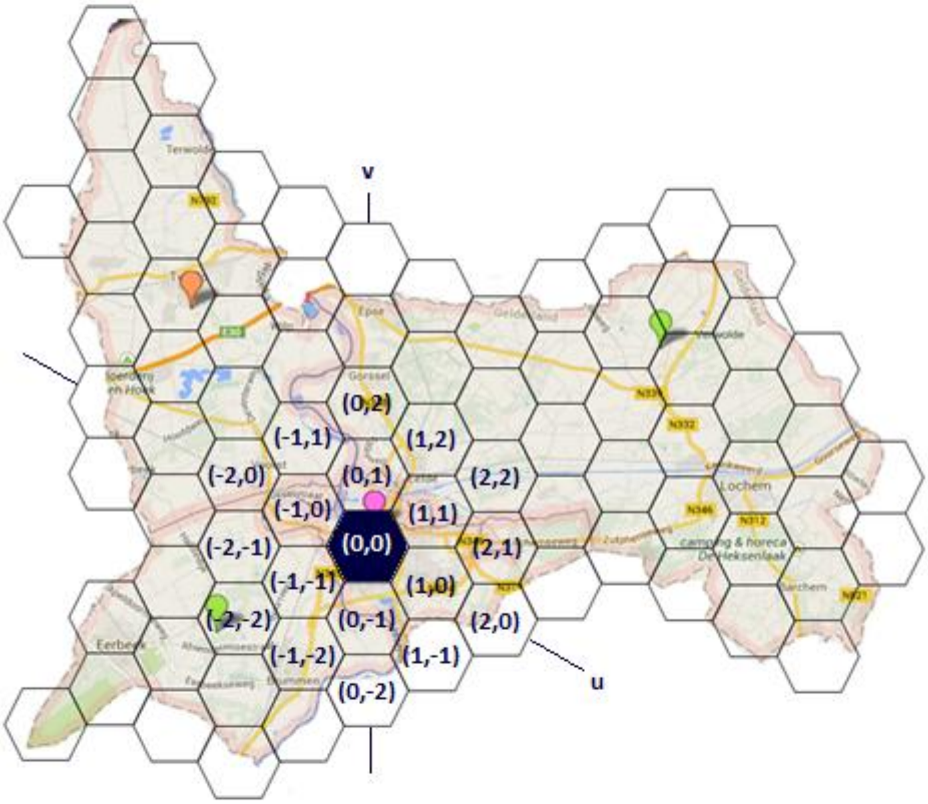


Figure 25: ***Covering of IJsselstreek with regular hexagons, including the two-dimensional coordinate system.***

When we know the height *h* of a hexagon, we can also calculate the radius by $\frac{h}{\sqrt{3}}$, which is a property of a regular hexagon. When we have *x* and *y* coordinates, we can transform this to *u* and *v* coordinates as follows, using the properties of regular hexagons:

$$u^- = \left\lfloor \frac{x - 210{,}500}{\frac{3}{2}r} \right\rfloor$$

$$u^+ = u^- + 1$$

$$v^- = \left\lfloor \frac{y - 461{,}200 + \frac{1}{2}h \cdot u^-}{u^-} \right\rceil$$

$$v^+ = \left\lfloor \frac{y - 461{,}200 + \frac{1}{2}h \cdot u^+}{u^+} \right\rceil$$

The corresponding *x* and *y* coordinates belong either to the hexagon (u^-, v^-) or to the hexagon (u^+, v^+), depending on the shortest distance from the (x, y) point to the centre of these hexagons. Therefore we have to calculate first the *x* and *y* coordinates of the centres of the two hexagons, which can be done as follows:

$$x_{centre\ u^-,v^-} = \frac{3}{2}r \cdot u^- + 210{,}500$$

$$y_{centre\ u^-,v^-} = h \cdot v^- - \frac{1}{2}h \cdot u^- + 461{,}200$$

This can be done in the same way for $x_{centre\ u^+,v^+}$ and $y_{centre\ u^+,v^+}$ and the shortest distance from the incident to these two centres of hexagons determines to which hexagon the incident belongs.

Step 2: Determine the height and radius of the hexagons

In order to make a one-dimensional grid of hexagons, we have to know how much hexagons cover the area of IJsselstreek. We prefer a numbering from 1 to the number of hexagons, without gaps, since this makes the calculations of the model much easier. We can start counting for example from the most left down hexagon to the most left upper one and continue counting with the next row, etc. Therefore, we need the height of a single hexagon in order to count the number of hexagons in each column and each row and make a simple one-dimensional grid.

We choose for a height of 3000 meters, because this area is detailed enough (we distinguish 85 different hexagons that fill the area of IJsselstreek) and we have enough incident data per hexagon. From the height, we can calculate the radius by $\frac{h}{\sqrt{3}}$, resulting in a radius of about 1,732 meters for a height of 3,000 meters. Note that the sides of a regular hexagon are equal to the radius. See Figure 26.

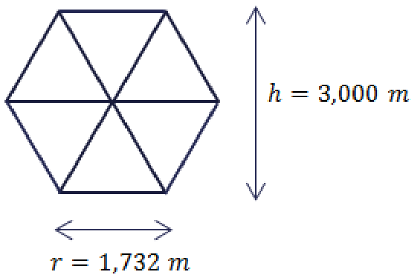


Figure 26: ***The height and radius of a single regular hexagon.***

Step 3: Convert the two-dimensional hexagon coordinates into demand nodes i

For this step, we first look at the dimensions of the *u* and *v* coordinates. We can calculate from the equations given earlier that the lowest possible *u* coordinate in the area of IJsselstreek is -5 and the highest is 8, for *h* = 3,000 meter. For the *v* coordinate, this is -5 and 11 respectively. To transform the hexagon coordinates into demand nodes, we use the following formula:

$$i = 56 + 10 \cdot u + v$$

With this formula, all hexagons in the area of IJsselstreek are numbered, starting with the lower left hexagon (-5, -5) equal to 1 until the upper right hexagon (8, 11) equal to 147. This means that we have a grid of hexagons which form a kind of square grid, implying that not all of the hexagons are positioned in the area of IJsselstreek. After all the calculations for determining to which hexagon the incidents belong, we conclude that only 85 hexagons cover the area of IJsselstreek. For the remaining hexagons, zero incidents happened in the past three years. However, we do take all the 147 demand nodes into account to get a complete numbering from 1 to 147, without gaps. Now we have a complete covering of IJsselstreek with regular hexagons, numbered from 1 to 147, which equals the demand nodes *i*. In Figure 27, a part of the numbering of hexagons is shown.

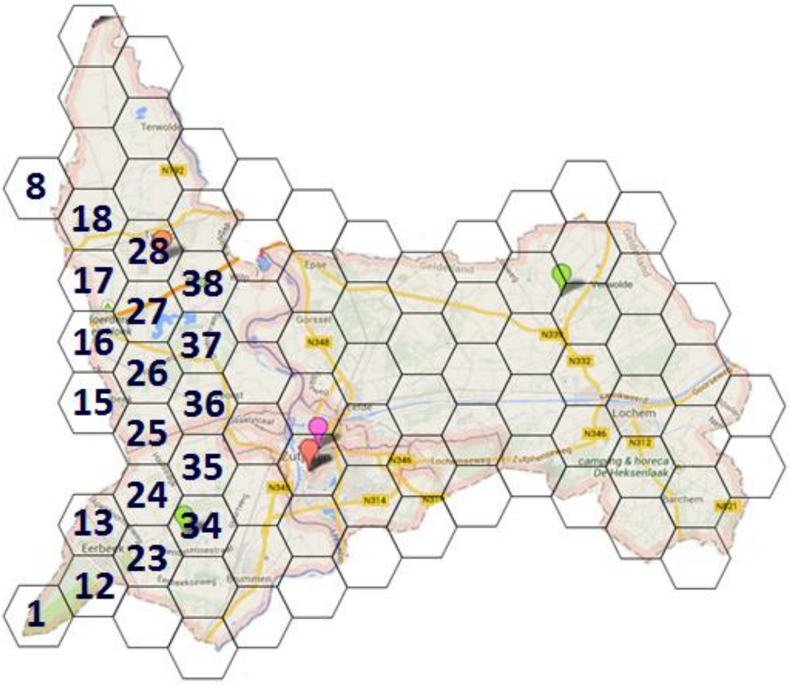


Figure 27: ***Covering of IJsselstreek with the (partly numbered) demand nodes i.***

5.3 INCIDENT FORECAST

In Section 2.3, we recognized all kind of patterns. We gave an overview of the dependent and independent factors that influence the amount of incidents, as presented in Figure 28 (a copy of Figure 16). Here we noticed three independent factors and two dependent factors. With this information, we propose a method to generate a suitable forecast. We are interested in generating a forecast for a given number of days and for each day a forecast for each half hour (each time stamp *t*).

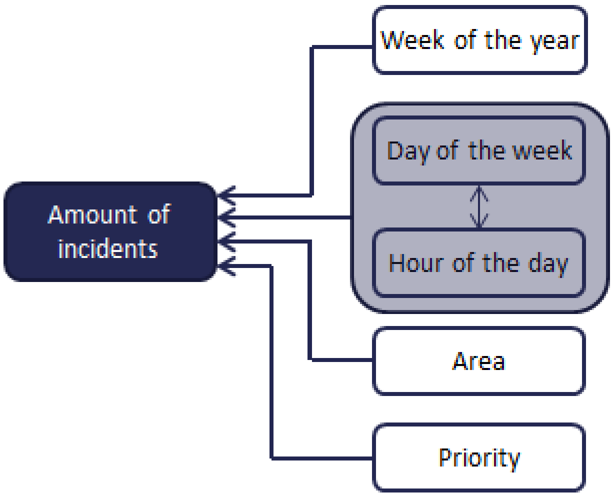


Figure 28: ***Overview of the factors that influence the amount of incidents.***

<u>*Step 1: Weekly forecast*</u>

We want to use as much available information as possible, so we start with an average per week of all incidents, both prio 1 and prio 2, for the total area of IJsselstreek. For example, when we want to forecast a number of days in week 26 of next year, we start with calculating an average of all incidents that happened in the past years in week 26. We then have an expected number of incidents for week 26. However, like we showed in Section 2.3, there are fluctuations between the weeks due to random noise, so we want to use more data to forecast one week. We showed significant differences between quarters of a year, but quarters do not always contain an equal number of weekdays. Therefore we propose to use seven weeks of data to forecast one week, starting with three weeks before the corresponding week until three weeks after that week. For example, when we want to forecast again a number of days in week 26, we take a weekly average of the weeks 23 until 29. The number seven in this case is chosen based on common sense. We showed significant difference for different parts of the year and from Figure 9 we conclude that seven weeks is large enough to cover random noise in certain weeks.

<u>Step 2: Weekly prio 1 forecast</u>

After we generated an expected number of incidents per week, we can proceed with multiplying this by the general prio 1 fraction, which is:

$$prio\ 1\ fraction = \frac{total\ number\ of\ prio\ 1\ incidents}{total\ number\ of\ incidents} \tag{28}$$

Note that we use all incidents and not only the incidents from the corresponding week, because we showed that the priority is independent of the week of a year.

<u>Step 3: Weekly prio 1 forecast for all subareas</u>

The next step is to divide the number of weekly prio 1 incidents over the different subareas. Again, taking independency into account, we can use all incidents to get fractions for all subareas. For each subarea *i*, we multiply the number of weekly prio 1 incidents by the following fraction:

$$subarea\ fraction\ (i) = \frac{total\ number\ of\ incidents\ at\ subarea\ i}{total\ number\ of\ incidents} \tag{29}$$

The result is a weekly forecast for all prio 1 incidents, for all subareas *i*.

<u>Step 4: Daily/half hourly prio 1 forecast for all subareas</u>

Now we want to transform the generated weekly forecast into a forecast for every half hour time interval. This means that we have to take the dependency between day of the week and time of the day into account. Therefore we start with calculating *day/time fractions* for every weekday *w* and time period *t* as follows:

$$daily/time\ fraction\ (w,t) = \frac{total\ number\ of\ incidents\ for\ weekday\ w\ at\ time\ t}{total\ number\ of\ incidents} \tag{30}$$

When we multiply the weekly forecast from the third step with these fractions, we get a forecast for each time period for each day. A clear overview of the total forecasting method is shown in Figure 29. We notice that a lot of information is used, which makes the forecast extremely reliable. For example, when we want to forecast the number of incidents for the 22nd of September in 2014, we use the following information:

- We notice that the corresponding week number is 39. Therefore we use all incidents that happened in weeks 36 until 42 to get a weekly estimate.
- We use all incidents to calculate the general prio 1 fraction.
- We use all incidents to calculate subarea fractions.

- We notice that the corresponding date is a Monday. We now use information from all incidents that happened on a Monday.

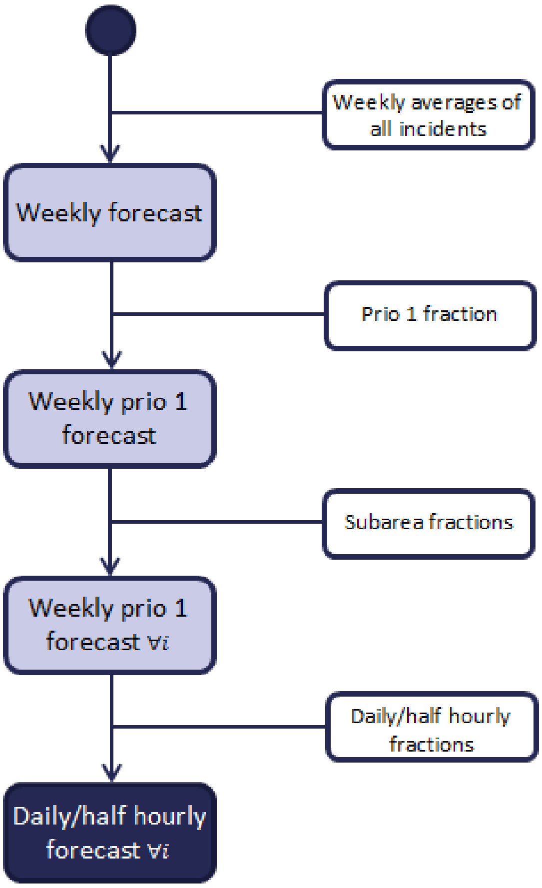


Figure 29: ***Visualized forecasting method.***

When we follow the forecasting procedure with the available data (all prio 1 and prio 2 incidents from 2011 – 2013), we can create forecasts, which we can compare with the known data. We plotted the weekly forecast, like we proposed as a moving average of seven weeks, in Figure 30. Here we also plotted the weekly forecast if we would only take the average of each week. We see that our forecast is smoother than the weekly average (as we expected), but we want to know if that is preferable. Therefore we calculated *Mean Squared Errors* (MSE) for a number of cases if we compare the forecasted data for 2013 (based on the data from 2011 – 2012) with the real data from 2013. The MSE is calculated as follows:

$$Mean\ Squared\ Error = \frac{1}{n}\sum_{i=1}^{n}\left(\hat{X}_i - X_i\right)^2 \tag{31}$$

Here the $\hat{X}_i$ represents the predicted value and X_i the true value. Note that, because we calculate the MSE for a year, *n* is equal to 52 (52 weeks). The lower the MSE, the more accurate the forecast is. We compared four different moving averages, the weekly averages and the total average. The total average means every week the same expected amount of incidents. Note that the weekly average is the least smooth option and the total average is the smoothest option. The moving averages are in between those two options and a higher number of a moving average implies a smoother line. The resulting Mean Square Errors are shown in Table 10. We can conclude that the moving average of seven is the most accurate option, so we made a good decision.

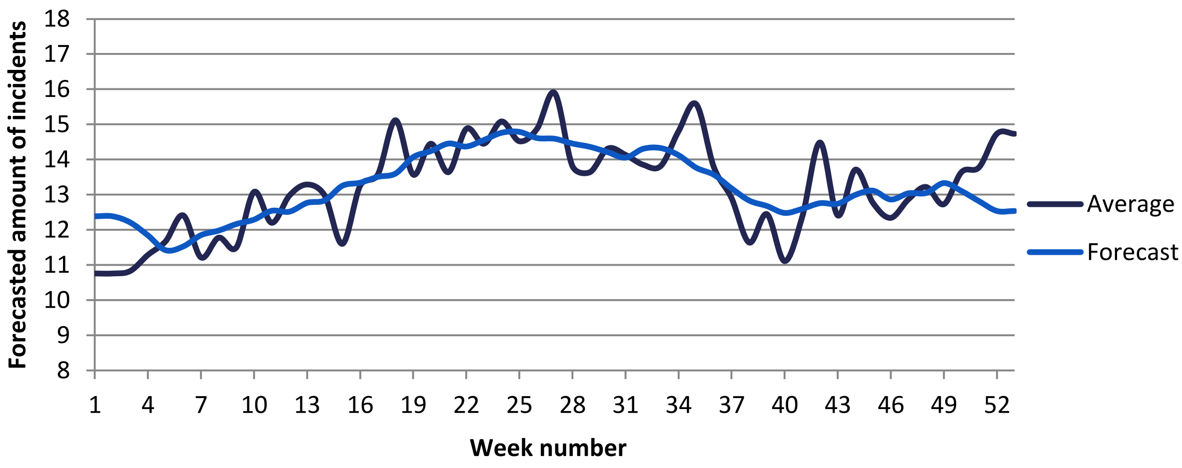


Figure 30: ***Weekly averages and forecasted weekly prio 1 incidents for 2014.***

Option	MSE
Weekly average	22.19203
Moving average (3)	21.13951
Moving average (5)	21.07690
Moving average (7)	21.03218
Moving average (8)	21.07452
Total average	22.31022

Table 10: ***Mean Squared Errors for different smoothing options.***

The result of choosing this option to predict weekly values for 2014 is also plotted in Figure 31, where it is combined with the historical data of incidents from 2011 – 2013. We recognize the typical pattern of a smooth forecast, where the real data fluctuates somewhere around this line.

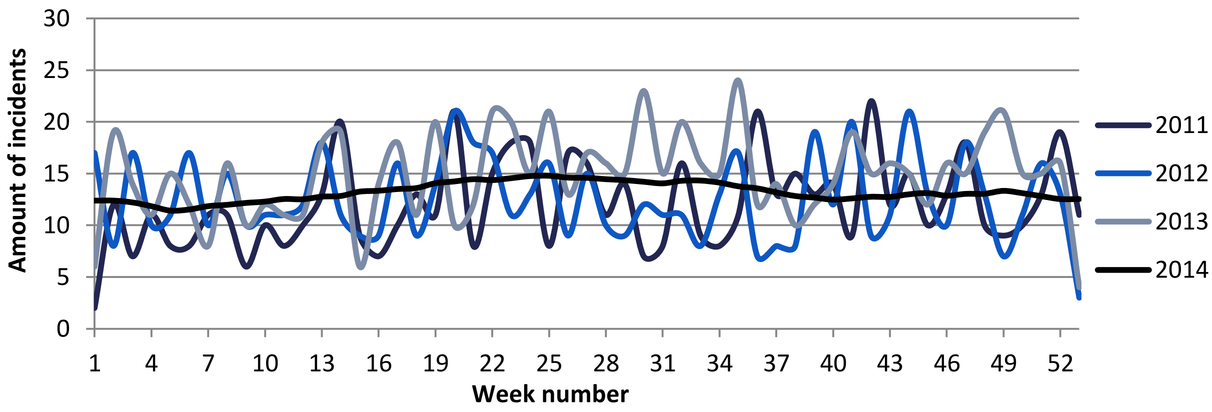


Figure 31: ***Historical incidents from 2011 – 2013 plus a weekly forecast of 2014.***

When we take a look at the weekdays and hours, we can draw a similar graph as we did before in Section 2.3, but now for the forecasted data when we want to forecast one week from 2014. Arbitrarily chosen, we use week 39 and subarea 56 as an example. The forecasted weekdays versus hours of the days are plotted in Figure 32. We recognize again the typical patterns that we saw before (in Section 2.3), where the expected amount of incidents at the weekend nights is high and hourly patterns differ per weekday. We notice that we have extremely low incident predictions. For example, for every day in the week in week 39, we expect about 0.003 prio 1 incidents to happen between 05:30 and 06:00. Of course this is impossible, since only positive integer values are possible. However, this is not a surprising result, since the prediction of the whole week 39 is equal to 12.68 prio 1 incidents, which is less than 2 per day. Then, if we divide this in half hour time periods (48 in a day), we get an average of less than 0.04 incidents per half an hour for the whole region of IJsselstreek. Then, we have about 82 possible subareas within IJsselstreek, resulting in an average of about 0.0005 prio 1 incidents per half an hour per subarea.

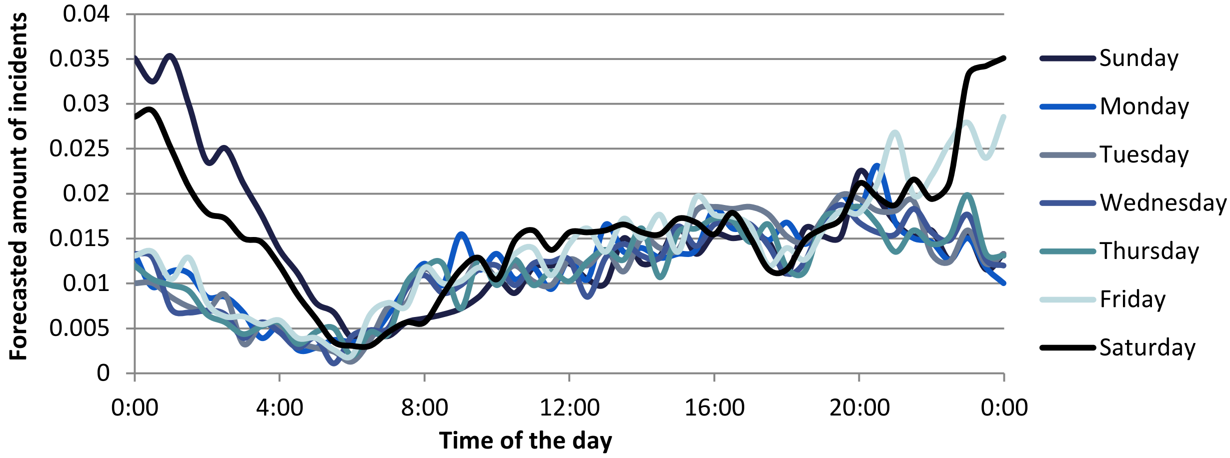
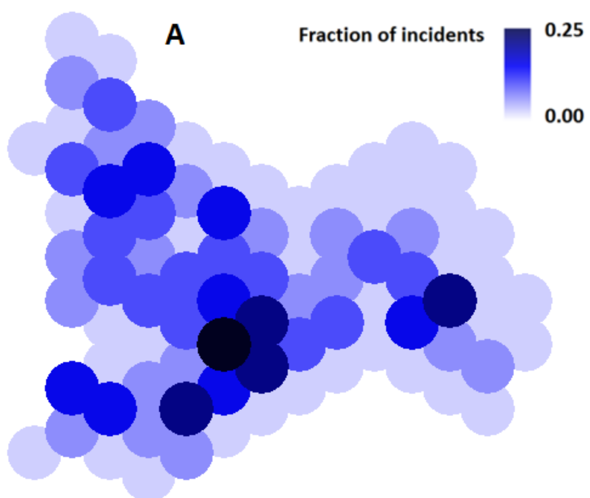


Figure 32: ***Forecasted day and hours of week 39, subarea 56.***

The question is now whether this data is useful, since we know beforehand that the forecasts per half an hour and per subarea are always wrong (except maybe for the areas where we predict zero incidents), because we never predict at least one prio 1 incident to happen, but always small fractions. However, from Chapter 4 we know that we use the forecast as an input for the Location Covering Problem, where we are interested in vehicle locations for a fixed fleet such that the expected coverage is maximized. This means that it does not matter if we use small fractions, because the LCP will look for the total coverage per time period. We visualized the forecasted incident data of week 39 per subarea (A) versus the real data (B) in Figure 33. Here we see that the forecasted data is way more spread than the real data, but when vehicles were placed based on this forecast, a good coverage is likely.



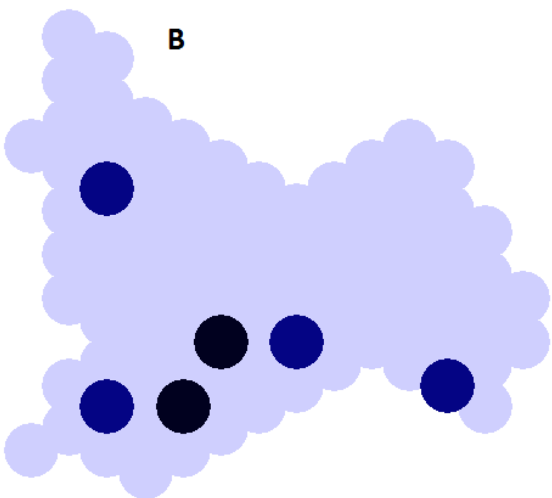


Figure 33A: ***Forecasted incident data of week 39, 2013.***

Figure 33B: ***Real incident data of week 39, 2013.***

5.4 TRAVEL TIME FORECAST

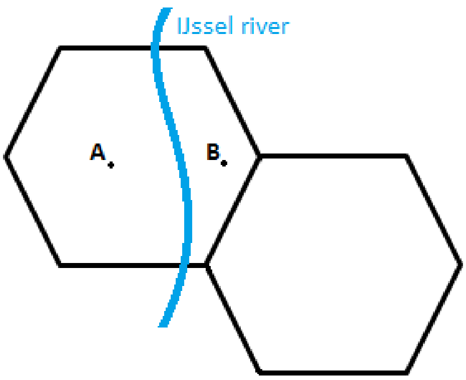
In order to fill in the coverage matrix from Section 4.2, i.e., determine for each parameter a_{ij} if vehicle location j covers demand node i (a_{ij} = 1) or not (a_{ij} = 0), we have different options. First of all, like we already mentioned in Section 3.3, we can determine manually the travel times from one point to another, using applications that can estimate travel times very accurately, but only for a limited number of points. So when we have 147 nodes and the same number of possible vehicle locations, we have to determine $147^2 = 21{,}609$ travel times. Even if we make the number of possible vehicle locations a lot less, say only four locations (and still 147 demand nodes), we still have to determine almost 600 travel times. We learned from Section 3.3 that we can use the Google Distance Matrix API to calculate all the exact travel times. However, Google has a limitation of 2,500 requests per 24 hour period and 100 per 10 seconds. Furthermore, if we have to manually enter all coordinate points every time in the address bar of an internet browser, it will still cost a lot of time, so we programmed a function in Microsoft Excel Visual Basic for Applications, that:

- posts given coordinates to an URL,
- converts the obtained XML file into Excel cells and creates a distance matrix,
- applies this method for at most 100 elements per 10 seconds and 2,500 per 24 hours.

Since we need 21,609 elements, we had to run this procedure for nine days, but eventually we got all exact travel times of the midpoints of all 147 hexagons. Furthermore, we concluded in Section 2.6 that the river the IJssel can have a significant impact on the travel times, since travel times from those areas depend on the side of the IJssel where the vehicle is currently positioned. Therefore, we use the following method, based on common sense, for every hexagon where this river flows. We introduce the so-called *area fractions* for both the left side and the right side of each hexagon where the IJssel flows as follows:

$$areafraction_{side,hexagon} = \frac{surface_{side,hexagon}}{total\ surface_{hexagon}} \tag{32}$$

These area fractions are the fractions of the left side and the right side of the surface of each hexagon. We use those area fractions as probabilities that, given a vehicle that is positioned in such a hexagon, it is positioned on the left side or the right side of that hexagon. For each side, we can draw a midpoint, resulting in two points per hexagon. Then we can calculate for both points (left and right) the travel times to all other hexagons. Then we multiply both travel times with the calculated area fractions and add them, resulting in an expected travel time. See Figure 34 for a graphical explanation of this method.



Area fractions		Travel time to neighbour hexagon	
A (left side):	0.78	From A:	10 min
B (right side):	0.22	From B:	3 min

Expected travel time: $0.78 \cdot 10 + 0.22 \cdot 3 = 8.46$ min

Figure 34: ***Graphical explanation of the travel time determination for the hexagons where the IJssel flows.***

For the differences between time periods (rush hours versus off-peak hours), we use the calculated travel times as a basis, where no traffic density is incorporated, and multiply crowded periods with factors, based on the extra time it takes to get from one location to another. It appears that, when we search for travel times during 'rush hours', we get typical graphs like Figure 35. In this figure, we averaged ten different travel times within the area of IJsselstreek for the weekdays, obtained from the Tripcast route planner website (Modelit, 2013). Here we see that before 07:00 and after 20:00 the shortest travel time can be achieved (low traffic density). Furthermore, there are two typical rush hours around 09:00 and 17:00 and in between those peaks we define the traffic density as normal, where 'normal' is equal to the travel times calculated from the Google Distance Matrix API. For all half hour time periods *t* we can now apply *traffic factors*, which can be found in Table 11.

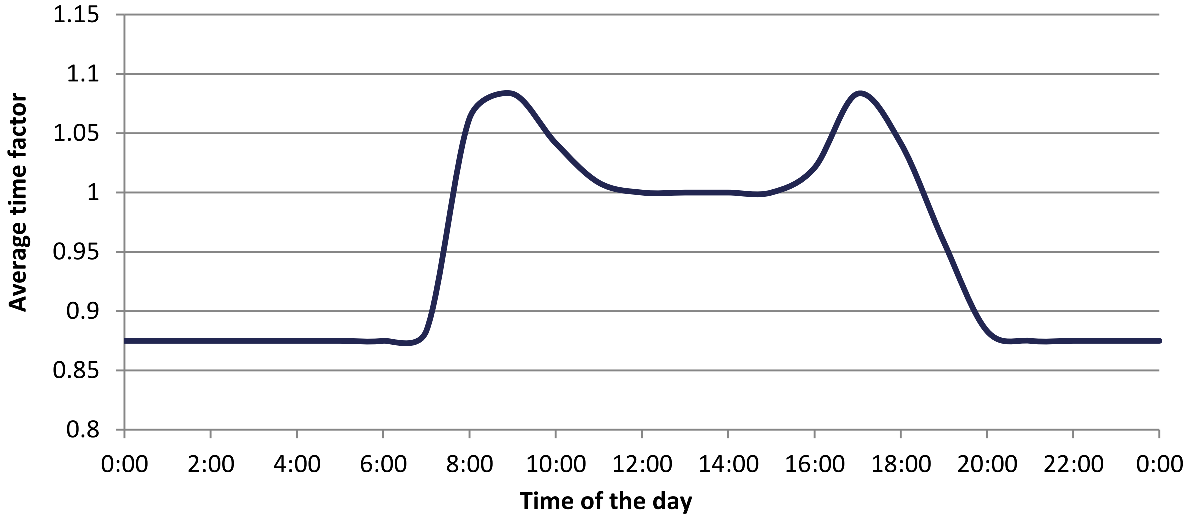


Figure 35: ***Rush hour factors for each time of the day, for weekdays.***

t	*Period*	*Traffic factor*
1-14	00:00 - 07:00	0.88
15	07:00 - 07:30	0.92
16	07:30 - 08:00	1.02
17	08:00 - 08:30	1.07
18	08:30 - 09:00	1.08
19	09:00 - 09:30	1.07
20	09:30 - 10:00	1.05
21	10:30 - 11:00	1.01
22 - 32	11:00 - 16:00	1.00
33	16:00 - 16:30	1.02
34	16:30 - 17:00	1.06
35	17:00 - 17:30	1.07
36	17:30 - 18:00	1.05
37	18:00 - 18:30	1.02
38	18:30 - 19:00	0.98
39	19:00 - 19:30	0.94
40	19:30 - 20:00	0.90
41 - 48	20:00 - 00:00	0.88

Table 11: ***Calculated traffic factors.***

For the weekends, we obtained the travel times as well from the Tripcast route planner website (Modelit, 2013), but we noticed a negligible difference in travel times for each hour of the day. Average travel times are for each hour of the day in the weekends equal to the average weekday travel times from 11:00 to 16:00. For the weekdays, we multiplied the travel times between 11:00 and 16:00 with a factor 1 (see Table 11), so we do the same for each hour of both weekend days. This means that, for the weekends, the travel times calculated from the Google Distance Matrix API are exactly the same as the real travel distances (always *traffic factor* 1).

Finally, we stated in Chapter 2 that police cars are allowed to drive faster when they rush to a high-priority incident. This means that the travel times reduce, compared to the normal driving travel times. After conversations with different people from the organization, we conclude that it is likely that travel times of emergency vehicles that use acoustic and light signals reduce by 20% when compared to normal traffic on an average trip. One may argue whether this is the same for all time intervals (rush hours versus off-peak hours), but there is no data available within the organization that proves this. Therefore we assume that this 20% is valid for all time intervals and we divide all travel times calculated above by the *rush factor* 1.2.

Now we have exact travel times for every time interval, which is a combination of (i) the general car travel time calculated from the Google Distance Matrix API (ii) multiplied by a *traffic factor* and (iii) multiplied by the *rush factor*. We can now fill the covering matrix where node i is said to be covered by vehicle location j if the calculated travel time plus the average dispatch delay time is smaller than 15 minutes.

5.5 CONCLUSION

In this chapter we developed a forecasting method where we combined the incident patterns that we found in Chapter 2. First we split the total area of IJsselstreek into different subareas, with the shape of hexagons and a height of 3,000 meters. Given the x and y coordinates of all available incidents, we can assign all incidents to one of the hexagons, using the properties of regular hexagons.

After the creation of subareas, we proposed a method to produce a forecast for all subareas for each time period, using all available incident data. We started with weekly forecasts where we combine both prio 1 and prio 2 data, where we use a moving average of seven weeks to reduce the forecast error. For the second step we multiply the weekly forecasts with the prio 1 fraction, resulting in weekly prio 1 forecasts. Then we apply subarea fractions (step 3) to create a forecast for each subarea and we complete the forecast by multiplying each subarea forecast with daily/hourly factors.

For the calculation of travel times, we used the Google Distance Matrix API, where we created a Microsoft Excel Visual Basic for Applications function to generate all travel times automatically. We made these travel times more accurate by adding area fractions, where we take the geographical constraint of the river the IJssel into account. Moreover, we incorporate rush hours by multiplying the travel times with traffic factors and we multiply all travel times with a rush factor, which is the factor that an emergency vehicle is allowed to drive faster to get on time at high-priority incidents.

6. EXPERIMENT RESULTS

In this chapter we present the results of simulating the developed forecasting method and positioning model. In Section 6.1 we describe the experimental design, where we present the configurations of the simulation and the key performance indicators. We proceed in Section 6.2 with an overview of the used simulation tool, followed by the results of the simulation experiments in Section 6.3. We close this chapter with conclusions in Section 6.4.

6.1 EXPERIMENTAL DESIGN

In Chapter 4, we described a mathematical optimization model to solve the Location Covering Problem. We are interested in how well this model performs if we would use this model in the future. Instead of implementing the model and see what happens, we use the more cost efficient way of *simulation* to analyse different configurations of the proposed model, as well as alternative methods. From Chapter 2 it is clear that our key performance indicator is the on-time percentage of prio 1 incidents. However, in cooperation with the police, we come up with two extra performance indicators, resulting in the following three key performance indicators:

- On-time percentage of prio 1 incidents
- Average response time of prio 1 incidents
- Fairness of area covering

Note that in Chapter 2 we defined the on-time percentage as the percentage of prio 1 incidents that have a *response time* lower than 15 minutes. The response time (the second performance indicator) is therefore related to the on-time percentage. It is possible that one solution has a better on-time percentage, but also a higher average response time than another solution. For the third performance indicator, fairness of incident covering, we use common sense that does not prefer solutions where some areas can never be visited on time. To make this clear, we come up with the following definition of fairness as a performance indicator:

'The fraction of subareas which are always covered at least once per day'

In this definition, 'covering' refers to a vehicle that is close enough to a subarea that if an incident happens, it can respond within 15 minutes. 'Always covering at least once per day' means that we check for every subarea whether it is covered or not. After conversations with different people from the organization, we conclude that it is interesting to know how many subareas are covered at least once per *day*. Therefore, we check for every day, for every subarea, if this subarea is covered (at least once). If this is for at least one day not the case, then this subarea is said to be '*unfair*'. If we count all the unfair areas and divide this number by the total number of subareas, we get the so-called *fairness factor*. This means that, if we simulate for one year, we count for all 365 days if each subarea is covered at least once within each day, and if a subarea is for at least one day in this year not covered, we mark this subarea as *unfair*. As a result, after one year of simulation, every subarea is marked either as 'fair' or 'unfair'.

For the experimental design of the simulation, we have to determine the simulation length, the data we use and the different configurations where we are interested in. We distinguish two possibilities:

- Use the incident data from the years 2011 – 2012 to forecast and simulate for the year 2013.
- Use the incident data from the years 2011 – 2013 to forecast and simulate for the year 2014.

Option 1: Use the model for 2013 (real demand)

For the first option, we benefit from the fact that we know exactly what happened in 2013 and we are therefore able to compare the simulated results with the actual results. In other words, we can forecast the year 2013 with data from 2011 and 2012, apply our developed positioning model to the real incident data of 2013 and see what would have happened if the police would have used this model in 2013 (model performance). We can also use the incident data of 2013 to compare the actual achieved results (real performance) with the simulated performance. This whole process is shown in Figure 36.

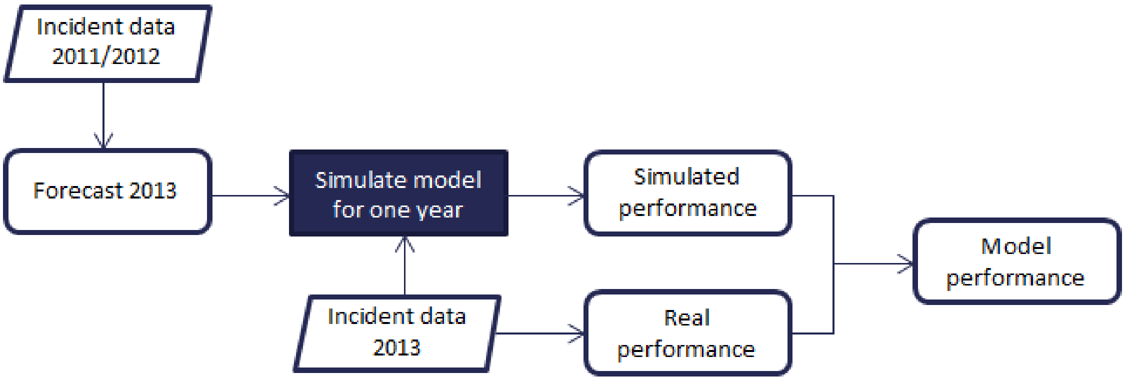


Figure 36: ***The process of simulating for known data.***

Option 2: Use the model for the future (stochastic demand)

The other option is using all available incident data to forecast and apply the positioning model for the *future*, where the arrival of incidents is unknown. The benefit is that we can use more data to forecast and that we can simulate multiple times with different generated incidents. Note that we are able to generate random incidents using statistical functions, based on the forecast data. Assuming that the occurrence of incidents follows a Poisson distribution, we can use the expected number of incidents for each time interval and demand node as the arrival rate. Then we can generate Poisson arrivals for each time interval and each demand node, resulting in a discrete number of incidents happening at each time interval at each demand node. Using this method will create incident data for the future, say for 2014, and we can apply our positioning model to this data, resulting in a simulated performance. This process is shown in Figure 37.

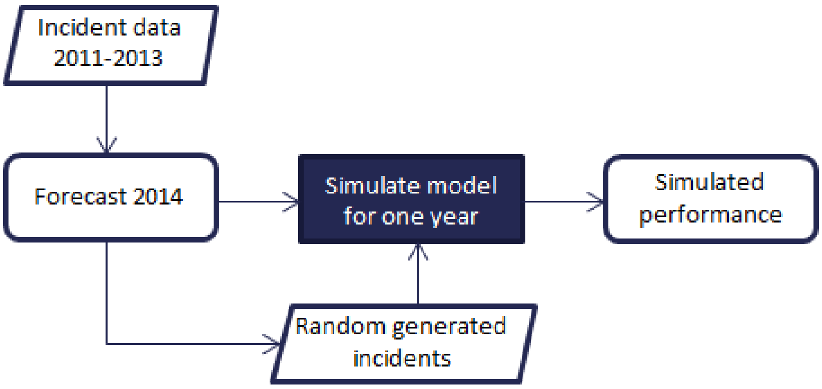


Figure 37: ***The process of simulating for the future.***

We choose to perform both options in the following way. We first simulate for 2013 (option 1) and then perform the second option, where we forecast and simulate in the future (for 2014). If the results are not totally different, we choose to average those experiments. In this way, we believe we get the most reliable results. We can perform the simulation for 2014 multiple times, where we generate every time new incident data. Taking the average running time (about 1.5 hours per experiment) into account, we perform for each configuration:

- One experiment where we simulate for 2013 and use real incident data
- One experiment where we simulate for 2014 and use random generated dataset A
- One experiment where we simulate for 2014 and use random generated dataset B

This can be interpreted as three replications per experiment, where we have three different datasets. The benefit of using both a dataset from 2013 and generated datasets from 2014 is that we can (i) check if the generated datasets from 2014 are kind of the same as the dataset from 2013 and (ii) compare the results of the simulation with the dataset from 2013 with the *actual* results from 2013. These two checks are used to validate (i) the forecast (is the generated dataset representative?) and (ii) the simulation (are the results comparable with the actual results achieved in 2013?). The number of incidents per dataset is presented in Table 12. For the random generation of dispatch delay times and incident handling times, we use the statistical formulas from Section 2.4 and from Section 2.5 respectively.

	Number of prio 1 incidents	Number of prio 2 incidents	Total
Dataset 2013	784	6106	6890
Dataset 2014 A	803	5941	6744
Dataset 2014 B	732	6288	7020

Table 12: ***Number of prio 1 and prio 2 incidents per dataset.***

In Table 12, we do not notice exceptional differences between the dataset from 2013 and both generated datasets from 2014.

For our simulation, we are interested in different experimental factors. First, we want to see how well the optimal positioning model performs. We test this with a basic and alternative way of positioning, namely having all available vehicles standby at the police stations. So our first experimental factor is the *positioning method*, where the range includes the *optimal* method and the *standby* method.

The second and third experimental factors concern the *number and locations of the police stations* and the *number of available vehicles*. The values within the range of these experimental factors are known. Varying from one to four police stations, where Zutphen is included in every value, results in eight different values within the range of the second experimental factor. Furthermore, we want to vary the number of available vehicles from two to five vehicles, resulting in a range of four for this experimental factor.

For the fourth experimental factor, we are interested in different values for the time that it takes for all vehicles to change their shifts. Probably in an optimal way, this time is as small as possible, where we assume that at least one hour is needed to get to the police station and the next crew is available to get positioned. However, in the current situation it appears that emergency crews are about four hours per shift busy with administrative work, indicating that the vehicle is near the police office for about four hours on average. We are interested in the effect of this behaviour. Therefore, the range of this *shift changing time* experimental factor includes four values: 1 hour, 2 hours, 3 hours and 4 hours.

Then we want to incorporate *fairness* as an experimental factor. We already defined fairness as a key performance indicator, but we are also able to control fairness. For example, we can add a restriction to the model that forces the positioning model to cover each area at least once per day, which is considered to be fair according to our definition at the beginning of this section. We call this the *covering fairness*. Note that there is no general definition of fairness, but for this research we agreed on the definition. After conversations with different people of the organization, we also agreed on another interesting fairness value, that we call *visibility fairness*, which we define as follows:

'The fraction of subareas which are always visited at least once per week.'

The choice of the time period length can highly affect the outcomes in terms of on-time percentages and response times. In cooperation with the police, we agreed that it is interesting to know what happens when every subarea has to be visited at least once per *week*. This idea of visiting every subarea at least once per week rises from the problem of being seen as a police unit, which is assumed to be preferable for civilian safety feelings. This means that we have now three different values within the range of this *fairness* experimental factor: No fairness, covering fairness and visibility fairness.

For the sixth experimental factor, we are interested in how much impact our forecast from Chapter 5 has on the outcomes. To compare this, we come up with an alternative method, which works as follows:

1. Calculate the average number of prio 1 + prio 2 incidents per hexagon from all the available data.
2. Multiply these numbers by the overall prio 1 fraction to get average numbers of prio 1 incidents per hexagon.

This simple method does not allow for time variations in hour of the day, day in the week or week of the year, but the execution does not require many calculations and is therefore quicker than the method from Chapter 5, which is assumed to perform better. We have two values for the range of this experimental factor: The (assumed to be) best forecasting method and the quick forecasting method.

The six experimental factors, including their ranges, are presented in Table 13. When we would perform all the possible experiments that can be created from the different combinations of settings for our experimental factors, we get $2 \cdot 8 \cdot 4 \cdot 4 \cdot 3 \cdot 2 = 1{,}536$ different experiments. Such a setup is denoted as a full factorial design. When we perform three replications per experiment (with the three different datasets), we need to perform $4{,}608$ simulation runs. This is too high and not necessary, because not all possible experiments are relevant.

Experimental factors		Range			
1	**Positioning method**	Optimal	Standby		
2	**Police station locations**	Zutphen	Zutphen/Twello	Zutphen/Lochem	Zutphen/Eerbeek
		Zutphen/Twello/Eerbeek	Zutphen/Twello/Lochem	Zutphen/Eerbeek/Lochem	Zutphen/Twello/Eerbeek/Lochem
3	**Number of vehicles**	2	3	4	5
4	**Shift change time**	1 hour	2 hours	3 hours	4 hours
5	**Fairness level**	No fairness	Covering fairness	Visibility fairness	
6	**Forecasting method**	Best	Quick		

Table 13: ***Different configuration options for the experimental design.***

For our experimental design, we start with a base policy, which is shown in Table 14.

Experimental factor		Value
1	Positioning method	Optimal
2	Police station locations	Zutphen
3	Number of vehicles	3
4	Shift change time	1 hour
5	Fairness level	No fairness
6	Forecasting method	Best

Table 14: ***Base policy of the experimental design.***

This is our first experiment and from this base policy we develop the other experiments. The first set of experiments consists of variations in both the locations of police stations and the number of vehicles. This results in a set of $8 \cdot 4 = 32$ configurations. The results from these experiments give us information about the effects on extra vehicles and police station locations with respect to the expected on-time percentage, given that we use our developed positioning model.

To test the developed mathematical positioning model, we have the alternative way of 'positioning' where all vehicles are standby at the available police stations, so we change the first experimental factor from *optimal* to *standby*. For this set, we also vary the number and location of police stations and the number of available vehicles, resulting again in $8 \cdot 4 = 32$ configurations.

For the next set of experiments, we change the shift change time, where we have four options. Instead of performing experiments for every police station location option, we reduce the number to four, where we take the best options for every number of police stations, resulting from the experiments of the first two sets. So we take one location (Zutphen), two locations (Zutphen + the best other one), etc. We also reduce the number of vehicles (otherwise we have too much experiments), where we only consider two and three vehicles. This means that for this set, we have $4 \cdot 2 \cdot 4 = 32$ configurations, which should be sufficient to analyse the effects of changing the length of the shift change time.

To evaluate the fairness, we do want to include all the number of vehicle options, since this will probably affect the fairness drastically. It is much more extensive for two vehicles to visit every hexagon at least once per week than for five vehicles. Taking still the four location options into account, we have $4 \cdot 4 \cdot 3 = 48$ experiments.

To evaluate the forecasting method, we perform a couple of experiments. We test the two forecasting methods where we fix the number of vehicles to three and we still take four location options into account. This results in $2 \cdot 4 = 8$ different experiments. This provides us enough information about the performances of the forecasting methods.

We now have a total number of 128 configurations, which are presented in Figure 38. Taking the number of experiments per configuration into account (three), we need to perform $128 \cdot 3 = 384$ experiments.

Set 1: 32 experiments *1. Positioning method:* **Optimal** *2. Police station locations:* **8 options** *3. Number of vehicles:* **4 options** *4. Shift change time:* **1 hour** *5. Fairness level:* **No fairness** *6. Forecasting method:* **Best**	**Set 2: 32 experiments** *1. Positioning method:* **Standby** *2. Police station locations:* **8 options** *3. Number of vehicles:* **4 options** *4. Shift change time:* **1 hour** *5. Fairness level:* **No fairness** *6. Forecasting method:* **Best**	**Set 3: 32 experiments** *1. Positioning method:* **Optimal** *2. Police station locations:* **4 options** *3. Number of vehicles:* **2 options** *4. Shift change time:* **4 options** *5. Fairness level:* **No fairness** *6. Forecasting method:* **Best**
Set 4: 48 experiments *1. Positioning method:* **Optimal** *2. Police station locations:* **4 options** *3. Number of vehicles:* **4 options** *4. Shift change time:* **1 hour** *5. Fairness level:* **3 options** *6. Forecasting method:* **Best**	**Set 5: 8 experiments** *1. Positioning method:* **Optimal** *2. Police station locations:* **4 options** *3. Number of vehicles:* **3** *4. Shift change time:* **1 hour** *5. Fairness level:* **No fairness** *6. Forecasting method:* **2 options**	**Total** Number of configurations: 152 Overlapping configurations: 24 Total number of configurations: **128**

Figure 38: ***All different sets of experiments for the simulation.***

6.2 SUPPORTING TOOL

To execute the experiments, we programmed the experiments in an optimization software system, called *Advanced Interactive Multidimensional Modeling System* (AIMMS) and we call the system the *Emergency Vehicle Positioning System* (EVPS). A screenshot of this system can be found in Figure 39. First of all, the EVPS allows for the loading of incident data to generate forecasts and it shows the expected incident distributions on a map. Besides the forecasting method, also the Location Covering Model is incorporated to position vehicles on the same map. This means that we can use the EVPS to position the vehicles at any time and see what the expected coverage is.

We can also use the EVPS to simulate the model, where we are able to vary all the experimental factor values from the previous section. We can include different police station locations, which are presented on the map. Furthermore, the simulation requires as input:

- $\lambda_t(P1)$: Arrival rates of prio 1 incidents, for every time period t (derived from the loaded incident data).
- $\lambda_t(P2)$: Arrival rates of prio 2 incidents, for every time period t (derived from the loaded incident data).
- $f(D)$: Probability density functions for the dispatch delay time (from Section 2.4).
- $f(H)$: Probability density functions for the incident handling time (from Section 2.5).

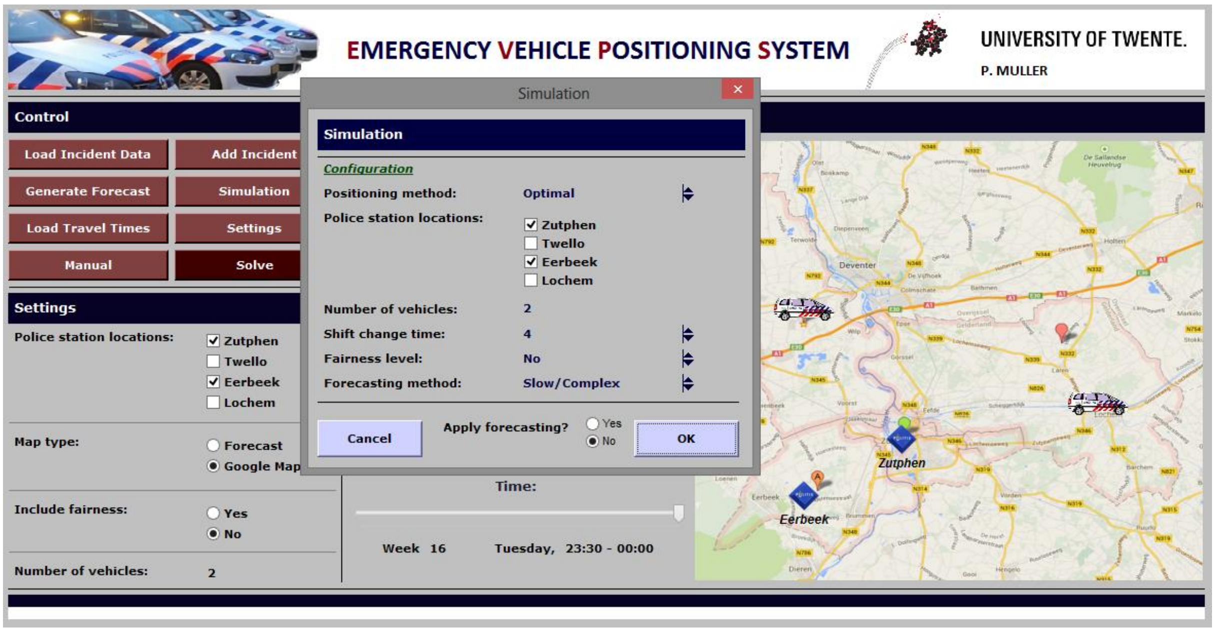


Figure 39: ***Screenshot of the Emergency Vehicle Positioning System, built in AIMMS.***

With this information, we set up the simulation, which follows the following process (shown in the flowchart from Figure 40). Here we start with initializing the simulation, which includes generating the forecast and the input parameters. After this, the positioning model is solved for the first time, resulting in all available vehicles positioned to optimality in the area of IJsselstreek. Then, as we already concluded in Section 4.1, there are three possible events that can occur as time passes by that triggers the simulation to do something:

- An incident happens;
- An incident is solved;
- 24 hours of time elapsed where no incident happened.

When an incident happens, the nearest vehicle is assigned to drive to the location of that incident and the response time can be calculated. For a prio 1 incident, the assigned vehicle is temporarily not available (until the incident is solved) and for a prio 2 incident, we fix the position of the assigned vehicle, where this vehicles is still available for a new prio 1 incident, if no other vehicle is available. Then the positioning model is solved again. When an incident is solved, the assigned vehicle is available again and the positioning model is solved again. Finally, when 24 hours of time elapsed where no incident happened, we solve the positioning model again (for the upcoming 48 hours), to make sure that at least 24 hours of planning in the future is available.

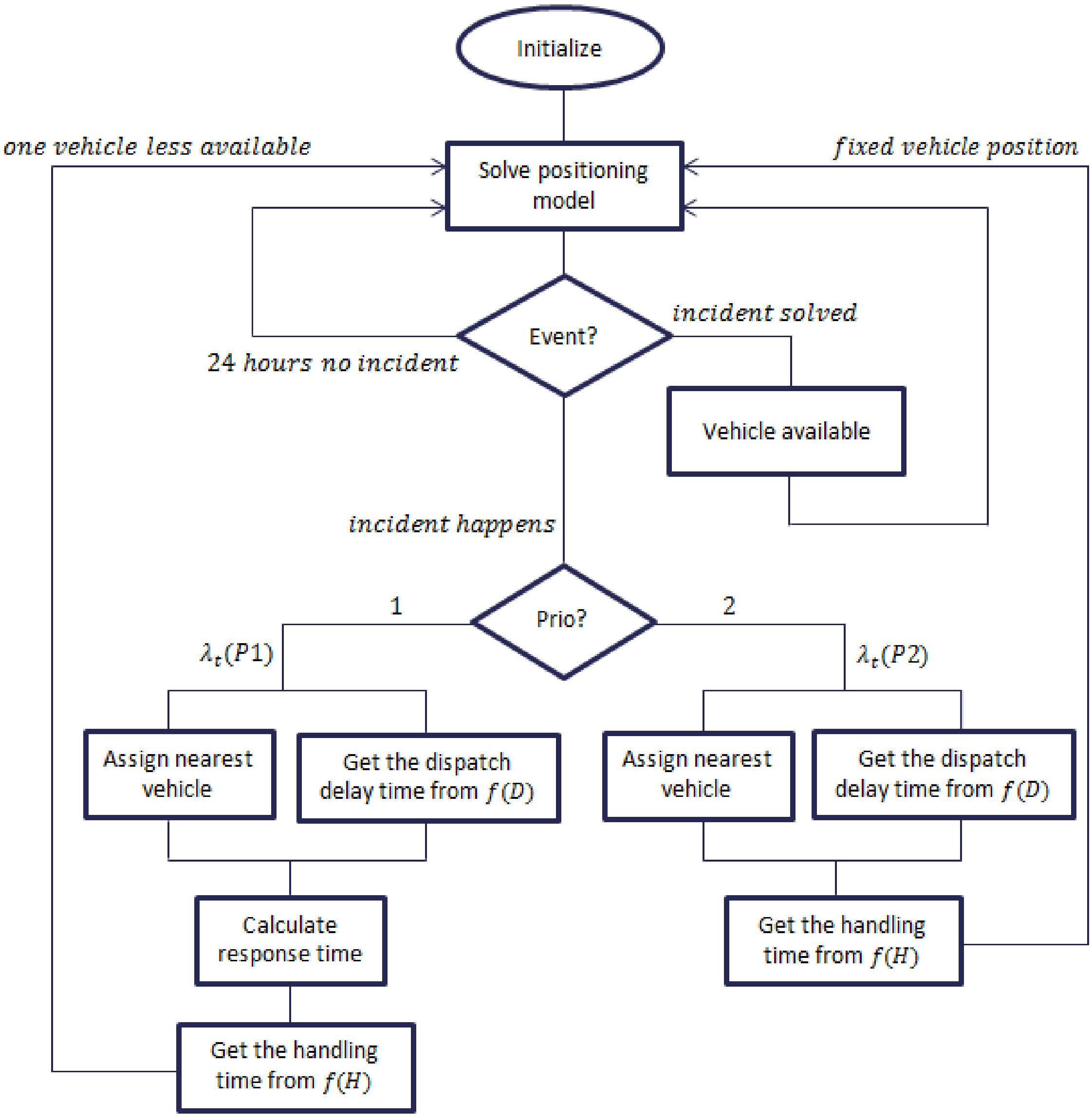


Figure 40: ***Flowchart of the incident handling and positioning process.***

6.3 SIMULATION RESULTS

We performed the 384 experiments as described in Section 6.1, using the software described in Section 6.2. For the validation of the results, we first present the results of applying the base policy to the simulation for all three replications. These results can be found in Table 15. Here we see that the results are not totally different. Furtermore, when we compare the results with the actual results from 2013 (Chapter 2), where the on-time percentage of 2013 was 82.0%, we conclude that the simulation does not produce extraordinary results.

Replication	Dataset	On-time percentage	Response time
1	2013	81.2%	10.6
2	2014 A	81.3%	10.5
3	2014 B	80.8%	10.6
	Average	**81.1%**	**10.6**

Table 15: ***Results from performing three replications of the base-policy simulation.***

Therefore, for all further configurations, we took the average of the three replications, which are presented in the tables in this section. We discuss the results per set, as derived from Figure 38.

Set 1: Optimal positioning model, 32 experiments

See Tables 16, 17 and 18 for the experimental results. Here we applied the optimal positioning method, the best forecasting procedure, a shift change time of only one hour and no fairness restrictions. We see in Table 16 that adding an extra vehicle always improves the on-time percentage, where the greatest benefit can be obtained by going from two to three vehicles, regardless of the chosen police station locations. In terms of response times, we notice that again the greatest benefit can be obtained by going from two to three vehicles, but that, on average, going from four to five vehicles does not change the response time.

On-time percentage	*Number of vehicles*				
Locations	*2*	*3*	*4*	*5*	***Average***
Zutphen	71.6%	81.1%	82.2%	85.4%	**80.1%**
Zutphen/Twello	73.3%	84.5%	86.9%	89.6%	**83.6%**
Zutphen/Eerbeek	71.6%	81.1%	85.6%	87.3%	**81.4%**
Zutphen/Lochem	72.5%	85.2%	87.1%	90.0%	**83.7%**
Zutphen/Twello/Eerbeek	73.3%	84.7%	88.4%	91.7%	**84.5%**
Zutphen/Twello/Lochem	73.7%	87.5%	92.6%	93.8%	**86.9%**
Zutphen/Eerbeek/Lochem	74.2%	85.6%	88.4%	91.7%	**85.0%**
Zutphen/Twello/Eerbeek/Lochem	74.2%	87.5%	92.6%	96.0%	**87.6%**
Average	**73.1%**	**84.6%**	**88.0%**	**90.7%**	**84.1%**

Table 16: ***On-time percentages for set 1.***

Response time	*Number of vehicles*				
Locations	*2*	*3*	*4*	*5*	***Average***
Zutphen	12.1	10.6	10.3	10.3	**10.8**
Zutphen/Twello	12.0	10.1	9.8	9.7	**10.4**
Zutphen/Eerbeek	12.1	10.6	10.1	10.0	**10.7**
Zutphen/Lochem	12.1	10.2	9.7	9.6	**10.4**
Zutphen/Twello/Eerbeek	12.0	10.1	9.5	9.5	**10.3**
Zutphen/Twello/Lochem	12.0	9.9	9.0	9.1	**10.0**
Zutphen/Eerbeek/Lochem	12.0	10.1	9.5	9.4	**10.3**
Zutphen/Twello/Eerbeek/Lochem	12.0	9.9	9.0	9.0	**10.0**
Average	**12.0**	**10.2**	**9.6**	**9.6**	**10.4**

Table 17: ***Response times in minutes for set 1.***

Fairness percentage	Number of vehicles				
Locations	2	3	4	5	**Average**
Zutphen	43.5%	51.8%	62.4%	77.6%	**58.8%**
Zutphen/Twello	43.5%	51.8%	76.5%	78.8%	**62.6%**
Zutphen/Eerbeek	43.5%	51.8%	62.4%	78.8%	**59.1%**
Zutphen/Lochem	43.5%	51.8%	78.8%	94.1%	**67.1%**
Zutphen/Twello/Eerbeek	43.5%	51.8%	78.8%	94.1%	**67.1%**
Zutphen/Twello/Lochem	43.5%	51.8%	78.8%	94.1%	**67.1%**
Zutphen/Eerbeek/Lochem	43.5%	51.8%	78.8%	94.1%	**67.1%**
Zutphen/Twello/Eerbeek/Lochem	43.5%	51.8%	78.8%	94.1%	**67.1%**
Average	**43.5%**	**51.8%**	**74.4%**	**88.2%**	**64.5%**

Table 18: *Fairness percentages for set 1.*

Furthermore, if we compare the different options for two and for three police station locations, we see that Twello and Lochem are the most promising locations. They both outperform Eerbeek in terms of on-time percentages and response times, where there is not much difference between the combinations Zutphen/Twello and Zutphen/Lochem.

Set 2: Alternative positioning model, 32 experiments

For this series of experiments, we did the same as the first set, but we changed the positioning method from optimal to standby. This means that all vehicles are always located at the police stations, until an incident happens. The results of these experiments can be found in Tables 19, 20 and 21.

On-time percentage	Number of vehicles				
Locations	2	3	4	5	**Average**
Zutphen	65.6%	70.7%	72.2%	72.4%	**70.2%**
Zutphen/Twello	69.3%	77.3%	79.3%	80.7%	**76.6%**
Zutphen/Eerbeek	65.6%	76.7%	78.7%	80.1%	**75.3%**
Zutphen/Lochem	66.8%	78.4%	80.1%	81.8%	**76.8%**
Zutphen/Twello/Eerbeek	69.3%	80.4%	86.6%	87.8%	**81.0%**
Zutphen/Twello/Lochem	69.3%	82.7%	90.3%	91.8%	**83.5%**
Zutphen/Eerbeek/Lochem	69.0%	80.4%	86.6%	88.1%	**81.0%**
Zutphen/Twello/Eerbeek/Lochem	69.3%	82.7%	90.9%	96.0%	**84.7%**
Average	**68.0%**	**78.7%**	**83.1%**	**84.8%**	**78.7%**

Table 19: *On-time percentages for set 2.*

Here we see that we can draw the same conclusions as we did for the first set: Going from two to three vehicles has the greatest benefit, compared with adding extra vehicles, and Twello and Lochem outperform Eerbeek again in terms of on-time percentages and response times.

Response time		Number of vehicles			
Locations	2	3	4	5	**Average**
Zutphen	11.5	10.4	10.2	10.2	**10.6**
Zutphen/Twello	11.3	9.7	9.3	9.1	**9.9**
Zutphen/Eerbeek	11.5	9.9	9.6	9.3	**10.1**
Zutphen/Lochem	11.5	9.5	9.2	8.8	**9.7**
Zutphen/Twello/Eerbeek	11.3	9.5	8.4	8.3	**9.4**
Zutphen/Twello/Lochem	11.3	9.3	8.0	7.7	**9.1**
Zutphen/Eerbeek/Lochem	11.3	9.5	8.4	8.0	**9.3**
Zutphen/Twello/Eerbeek/Lochem	11.3	9.3	8.0	7.4	**9.0**
Average	**11.4**	**9.6**	**8.9**	**8.6**	**9.6**

Table 20: ***Response times in minutes for set 2.***

Fairness percentage		Number of vehicles			
Locations	2	3	4	5	**Average**
Zutphen	25.9%	48.0%	60.1%	60.1%	**48.5%**
Zutphen/Twello	37.8%	50.6%	63.1%	63.1%	**53.7%**
Zutphen/Eerbeek	37.8%	50.2%	60.1%	60.1%	**52.1%**
Zutphen/Lochem	37.8%	50.6%	63.1%	63.1%	**53.7%**
Zutphen/Twello/Eerbeek	37.8%	50.6%	67.9%	67.9%	**56.1%**
Zutphen/Twello/Lochem	37.8%	50.6%	67.9%	67.9%	**56.1%**
Zutphen/Eerbeek/Lochem	37.8%	50.6%	67.9%	67.9%	**56.1%**
Zutphen/Twello/Eerbeek/Lochem	37.8%	50.6%	67.9%	67.9%	**56.1%**
Average	**36.3%**	**50.2%**	**64.8%**	**64.8%**	**54.0%**

Table 21: ***Fairness percentages for set 2.***

If we compare the first and the second set, i.e., the difference between an optimal positioning method versus a basic method, we see some surprising results. We see that, like we expected, a significant improvement can be realized when applying the optimal positioning method in terms of on-time percentages. This is not surprising, because the model aims to optimize this on-time percentage all the time. An average of 5.4% improvement can be obtained, where there is not much difference between the numbers of vehicles chosen. For the police station locations, the greatest benefits can be obtained by the smaller amount of police stations, as can be seen in Tables 16 and 19.

However, when we compare the response times (Tables 17 and 20), we notice that the standby method outperforms the optimal way of positioning, on average by 0.8 minutes (= 48 seconds). This is quite surprising, but we can explain this as follows. The optimal positioning model aims to cover as much expected demand areas as possible, where covering indicates potentially reaching the area within 15 minutes. This means that the model prefers solutions where, for example, vehicles can reach three areas in 12 minutes instead of two areas in 4 minutes and one in 16 minutes. In this example, the average response time would be 12 minutes in the preferred case and 8 minutes in the other case. Therefore the optimal positioning model accepts a higher response time while optimizing the on-time percentage.

Set 3: Shift changing times, 32 experiments

For the third set of experiments, we show the impact of having greater shift changing times. Like we mentioned in Section 6.1, if the shift changing time is smaller, there is more time left to position the vehicles to the best locations. We distinguish shift changing times of one, two, three and four hours and we also include the results of the standby method from the second experiment set. We include those results, because the standby method can be seen as a maximal shift changing time (eight hours), where all vehicles are always located at the police offices. The results are shown in Tables 22 and 23.

On-time percentage		Shift changing times				
Locations	***Vehicles***	*1 hour*	*2 hours*	*3 hours*	*4 hours*	*Standby*
Zutphen	*2*	71.6%	68.2%	67.3%	66.2%	65.6%
	3	81.1%	77.8%	76.7%	74.4%	70.7%
Zutphen/Twello	*2*	73.3%	72.2%	71.0%	68.8%	69.3%
	3	84.5%	84.1%	84.1%	83.6%	77.3%
Zutphen/Twello/Lochem	*2*	73.7%	72.2%	71.0%	69.9%	69.3%
	3	87.5%	86.3%	86.0%	85.2%	82.7%
Zutphen/Twello/Eerbeek/Lochem	*2*	74.2%	72.2%	71.6%	70.5%	69.3%
	3	87.5%	87.1%	86.0%	85.2%	82.7%
Average	***2***	**73.2%**	**71.2%**	**70.2%**	**68.8%**	**68.4%**
	3	**85.1%**	**83.8%**	**83.2%**	**82.1%**	**78.3%**

Table 22: ***On-time percentages for set 3.***

From Table 22, we conclude that the duration of the shift changes influences the on-time percentages significantly. Each hour extra results, on average, in a decrease of the on-time percentage for both the two and the three vehicle options. On average, the decrease for going from one hour to four hours is equal to 4.4% for the two vehicle option and 3.0% for the three vehicle option.

Furthermore, we conclude that, although it differs per location option and number of vehicles, the average response times decrease for each extra hour extra. This is quite surprising, although this can be explained again by the fact that the model optimizes the percentage of covering within 15 minutes, instead of the overall response time. It is however interesting to keep in mind that a better on-time percentage results almost always in a worse overall response time.

Response time		Shift changing times				
Locations	***Vehicles***	*1 hour*	*2 hours*	*3 hours*	*4 hours*	*Standby*
Zutphen	*2*	12.1	12.2	12.2	12.1	11.5
	3	10.6	10.5	10.7	10.7	10.4
Zutphen/Twello	*2*	12.0	11.7	11.6	11.6	11.3
	3	10.1	9.8	9.9	9.8	9.7
Zutphen/Twello/Lochem	*2*	12.0	11.7	11.6	11.6	11.3
	3	9.9	10.0	9.8	9.5	9.3
Zutphen/Twello/Eerbeek/Lochem	*2*	12.0	11.7	11.6	11.6	11.3
	3	9.9	10.0	9.8	9.5	9.3
Average	***2***	**12.0**	**11.8**	**11.7**	**11.7**	**11.4**
	3	**10.2**	**10.1**	**10.0**	**9.9**	**9.7**

Table 23: ***Response times in minutes for set 3.***

Set 4: Fairness, 48 experiments

For this set of experiments, we test the effects on the on-time percentages and response times when we force the positioning model to include fairness in two different ways. The results can be found in Tables 24 and 25. We notice that adding coverage constraints has a great impact on the on-time percentages, where visibility fairness has the greatest impact, with, depending on the number of vehicles, an average dropdown between 4.2% and 7.0%, compared to the original (no fairness) method. For the response times (Table 24), there are no large differences between no fairness and covering fairness. However, the response times for visibility fairness are significantly larger, varying on average between 1 and 2 minutes larger than for the original method (no fairness). Apparently, forcing the model to visit every subarea at least once per week results in a far from optimal on-time percentage and response time.

On-time percentage		***Fairness***		
Locations	***Vehicles***	*No fairness*	*Covering fairness*	*Visibility fairness*
Zutphen	*2*	71.6%	67.7%	63.8%
	3	81.1%	80.0%	75.7%
	4	82.2%	81.6%	78.0%
	5	85.4%	84.8%	80.4%
Zutphen/Twello	*2*	73.3%	69.5%	67.0%
	3	84.5%	83.4%	79.9%
	4	86.9%	84.2%	82.1%
	5	89.6%	88.8%	84.9%
Zutphen/Twello/Lochem	*2*	73.7%	69.5%	67.0%
	3	87.5%	83.4%	81.1%
	4	92.6%	91.3%	88.4%
	5	93.8%	92.0%	90.0%
Zutphen/Twello/Eerbeek/Lochem	*2*	74.2%	69.9%	67.0%
	3	87.5%	83.4%	81.1%
	4	92.6%	91.8%	89.2%
	5	96.0%	95.4%	90.7%
Average	***2***	**73.2%**	**69.2%**	**66.2%**
	3	**85.1%**	**82.6%**	**79.5%**
	4	**88.6%**	**87.2%**	**84.4%**
	5	**91.2%**	**90.3%**	**86.5%**

Table 24: ***On-time percentages for set 4.***

Response times		Fairness		
Locations	***Vehicles***	*No fairness*	*Covering fairness*	*Visibility fairness*
Zutphen	*2*	12.1	12.1	13.2
	3	10.6	10.5	12.1
	4	10.3	10.5	11.7
	5	10.3	10.3	11.5
Zutphen/Twello	*2*	12.0	11.9	12.9
	3	10.1	10.4	12.0
	4	9.8	9.9	11.6
	5	9.7	9.5	11.3
Zutphen/Twello/Lochem	*2*	12.0	11.9	12.9
	3	9.9	9.7	12.0
	4	9.0	9.2	11.4
	5	9.1	9.2	11.0
Zutphen/Twello/Eerbeek/Lochem	*2*	12.0	11.9	12.9
	3	9.9	9.7	12.0
	4	9.0	9.3	11.3
	5	9.0	9.2	11.0
Average	***2***	**12.0**	**12.0**	**13.0**
	3	**10.2**	**10.1**	**12.0**
	4	**9.5**	**9.7**	**11.5**
	5	**9.5**	**9.6**	**11.2**

Table 25: ***Response times in minutes for set 4.***

Set 5: Forecasting method, 8 experiments

For the last set of experiments, we test the forecasting method from Section 5 versus the simple one that we explained in Section 6.1. We performed the simulation for the optimal positioning model with three vehicles and we used four different location settings. The results can be found in Tables 26 and 27. Here we see that for each location option, the on-time percentage when applying the best forecasting method is better than the on-time percentage when applying the simple forecasting method. We also notice that on average an improvement of more than 3% can be realized. For the response time, there is on average a slight improvement of 0.1 minute. However, the simple method has a smaller calculation time. The calculation time of the advanced forecasting method is 43.8 seconds and the calculation time of the simple forecasting method is 12.5 seconds, when forecasting for one year ahead (2014) with three years of incident data is available (2011 – 2013; 19,795 records). We conclude that the calculation times are negligible (less than a minute to forecast one year) which makes it beneficial to apply the more advanced forecasting method.

On-time percentage	***Forecasting method***	
Location	*Best*	*Quick*
Zutphen	81.1%	78.0%
Zutphen/Twello	84.5%	82.2%
Zutphen/Twello/Lochem	87.5%	83.8%
Zutphen/Twello/Eerbeek/Lochem	87.5%	83.9%
Average	**85.1%**	**82.0%**

Table 26: ***On-time percentages for set 5.***

Response time		Forecasting method
Location	Best	Quick
Zutphen	10.6	10.8
Zutphen/Twello	10.1	10.2
Zutphen/Twello/Lochem	9.9	10.2
Zutphen/Twello/Eerbeek/Lochem	9.9	10.0
Average	**10.2**	**10.3**

Table 27: ***Response times in minutes for set 5.***

6.4 CONCLUSION

This chapter aims to give an answer on the questions that we recall from Section 3.1:

1. *Where to locate the facilities?*
2. *How many vehicles per facility?*
3. *How to position each vehicle?*

We set up a simulation with 384 different experiments to test the optimal positioning model versus the alternative way of having all vehicles standby at the police offices, compare the impact on larger shift changing times, compare the impacts on adding fairness constraints and testing the forecasting method from Chapter 5. We distinguish five sets of experiments to analyse these configurations and, after programming this in AIMMS, we come to the following conclusions.

Concerning four options regarding the number of police stations, it turns out that having only one location is not enough to guarantee a 90% on-time percentage. If the base team IJsselstreek decides to have two police stations, it appears that the combinations Zutphen/Twello and Zutphen/Lochem outperform the combination Zutphen/Eerbeek. Thus, it would be wise to maintain the police station at Twello or Lochem, where there is no great difference between those two in terms of on-time percentages and response times. Moreover, when three police stations stay open, the best combination turns out to be Zutphen/Twello/Lochem, where Eerbeek is again outperformed.

Taking a look at the minimum of 90% coverage with the emergency vehicles, we conclude that at least four vehicles are needed when there are three or four police stations. When having only one or two police stations, a minimum of 90% is never reached.

Further, we conclude that the optimal positioning model performs significantly better than the alternative standby method, where improvements of almost 7% can be realized, depending on the locations of the police stations and the number of vehicles available, but this is at the cost of a higher overall response time. Since the goal concerns only the on-time percentage, we recommend applying this optimal positioning model.

Furthermore, we conclude that applying a larger shift changing time results in a worse on-time percentage, but also a better response time. Taking only the on-time percentage into account, we recommend reducing the shift changing times as much as possible.

Finally, we recommend to apply the forecasting method as described in Chapter 5, since it turns out to perform better (more than 3% improvement of the on-time percentage) than a simple forecasting method. The calculation time is negligible, since this is still less than one minute to create a forecast for one year.

7. CONCLUSION AND DISCUSSION

In this chapter, we present the conclusions that can be drawn from the report (Section 7.1), the recommendations for the organization (Section 7.2) and we complete this report with suggestions for further research (Section 7.3).

7.1 CONCLUSION

We started this research with a clear goal, which was defined as:

'Give the base team IJsselstreek insight into the consequences of (i) using different police station locations and (ii) the number of emergency vehicles, with respect to the response times of high-priority incidents.'

Currently, the base team IJsselstreek has four police stations and three emergency vehicles that react on high-priority incidents in the area. Taking the closing of two or maybe three police stations in the nearby future into account, the base team IJsselstreek needs a solution to still cover the total area and an answer on how many vehicles are needed in order to be for at least 90% of the high-priority incidents on the spot within 15 minutes. Moreover, it appeared that this 90% is never met in the past three years (2011 – 2013), so they require a solution to cover the area in a more effective way.

Searching for information in the scientific literature, it appeared that this problem can be approached roughly in two different ways, known as a *Dynamic Vehicle Routing Problem* (DVRP) and a *Location Covering Problem* (LCP). We concluded, based on six different characteristics, that an extended variant of the LCP, called *Maximal Expected Coverage Location Model with Time Variation* (TIMEXCLP) is the most suitable approach for our situation. Since this model is developed initially to position ambulances at fixed places, we adapted it to make it useful for the positioning of emergency police vehicles. We included personnel schedules, made it suitable for rush hours, demand variation and fleet size variation and we are still able to solve the problem to optimality within a reasonable amount of time.

The developed optimal positioning model requires demand nodes as input and therefore we divided the area of IJsselstreek into 85 different hexagon-shaped subareas. We developed a function that provides us with accurate travel times between all subareas, where rush hours, geographical constraints (concerning the river IJssel) and faster driving to high-priority incidents are incorporated. To provide the model with expected demand, i.e., the expected number of high-priority incidents, for each subarea and for each time period, we developed a forecasting method. This forecasting method allows for seasonal patterns for the week of the year, the day of the week and hour of the day. When producing a forecast for the upcoming year (2014), we used available incident data from the years 2011 – 2013. Because high-priority (prio 1) incidents occur on average only less than twice a day within the area of IJsselstreek, we also include prio 2 incidents, which happen on average about seventeen times a day, to make the forecast more reliable.

To analyse the results of applying the developed forecast and positioning model, we set up different simulation experiments, which we executed in a software system called *Advanced Interactive Multidimensional Modeling System* (AIMMS). For this simulation, we distinguish five sets, where we first test the optimal positioning model where we include the eight different police station location options and varying the number of vehicles from two to five. From these results we conclude that, when choosing for two or three police stations, it is not wise to have a police station located at Eerbeek in terms of on-time percentages, response times and fairness percentages. The combinations Zutphen/Twello, Zutphen/Lochem and Zutphen/Twello/Lochem outperform all other two and three police station options. To achieve the 90% on-time goal with only emergency vehicles, at least four vehicles and at least two police station locations are needed.

Furthermore, we tested this optimal positioning method with an alternative way of 'positioning', where all vehicles are located at the available police stations and wait until an incident happens. Depending on the locations of the police stations and the number of vehicles, we concluded that a significant improvement can be made when using the optimal positioning method in terms of on-time percentages. However, the realized improvement in on-time percentages is at the cost of slightly higher overall response times. This is due to the fact that the optimal positioning model prefers solutions where the 'within 15 minutes' covering is maximized, instead of a total overall minimized response time.

Then we concluded that on-time percentages decrease slightly when incorporating fairness restrictions, which prevent solutions where some areas are never reachable within 15 minutes. We also concluded that a shift changing time as small as possible is preferred, in order to be positioned in an optimal way as long as possible. Finally we indicated the benefits of a good forecast, where our developed forecasting method performed significantly better than a simple method.

7.2 RECOMMENDATIONS

For the base team IJsselstreek, we give the following recommendations. First of all, when in the nearby future at least one of the police stations needs to close, we recommend closing the current police station located at Eerbeek. Regardless of the positioning method and other variables, this is the least useful location in terms of on-time percentages for the total area of IJsselstreek. When closing two police stations, again closing Eerbeek is the wisest decision and it does not make a significant difference to close Twello or Lochem. Therefore we recommend taking the on-time percentages obtained from the simulation experiments into account when closing on or two police stations, in order to make adequate decisions.

Secondly, we recommend (gradually) using the optimal positioning model to optimize the positions of emergency vehicles and improve the on-time percentages with the same amount of resources. This can be obtained by using the developed positioning model at the *Emergency Control Centre* (ECC), where a real-time overview of the positions of emergency vehicles and locations of currently active incidents are visualized. Using the information from the positioning model as a supporting tool can potentially result in an overall higher on-time percentage. In an ideal situation, this model is combined with the *Geographical Information System* that the police use, so we recommend developing an integrated support tool. Furthermore, although currently there is no higher coordination level above the emergency crews that gives them positioning orders, it might be wise to let emergency crews get insight in the consequences when making non-optimal decisions.

Finally, we recommend applying forecasts based on our developed method. Although there is room for improvement, we showed that a good forecast can make a significant difference in the performance of a positioning method.

7.3 FURTHER RESEARCH

During this research, we made some assumptions and encountered several points for improvement. In this final section, we discuss those points for further research briefly.

First of all, in this research we considered solely the emergency vehicles, which are specially equipped vehicles that react on high-priority incidents. However, there are more units present at the region of IJsselstreek, like motor crews, officers on scooters, bicycles, etc. Although the emergency crews always drive to high-priority incidents eventually, it might happen that other units arrive earlier at the spot. Then the response time of the first unit (not per se an emergency unit) is the leading response time. Therefore, it can be of interest to include other units in the analysis.

Secondly, in this research we created some sort of closed area (IJsselstreek) with clear boundaries. However, cooperation with neighbouring areas might be wise to create an overall optimized model, instead of optimizing for each separate base team. In our research, it might occur that vehicles drive at the borders of IJsselstreek, where they already cover parts of other base teams. This can be included in a holistic view of multiple base teams.

Finally, we showed the importance of a good forecasting method. Since we aggregated all the prio 1 and all the prio 2 incident data, we might miss some important patterns. For example, robberies may show different patterns than traffic incidents. We were not able to obtain those data, but it might be helpful to analyse this and create an even more accurate forecast.

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APPENDIX A: MATHEMATICAL NOTATIONS

Sets

$\mathcal{I}$	Demand nodes
$\mathcal{J}$	Vehicle locations
$\mathcal{K}$	Vehicles
$\mathcal{T}$	Time periods
$\mathcal{P} \subseteq \mathcal{J}$	Police station locations
$\mathcal{S} \subseteq \mathcal{T}$	Shift changing periods

Parameters

a_{ijt}	Parameter that indicates if node $i \in \mathcal{I}$ is covered by vehicle location $j \in \mathcal{J}$ for time period $t \in \mathcal{T}$
d_{it}	Expected demand at node $i \in \mathcal{I}$ for time period $t \in \mathcal{T}$
f_t	Fleet size for time period $t \in \mathcal{T}$
m	Length of each time period $t \in \mathcal{T}$
n_p	Number of available vehicles at police $p \in \mathcal{P}$
q_t	Busy probability for time period $t \in \mathcal{T}$
u	Mean service time of an incident

Variables

X_{jt}	Number of vehicles positioned at vehicle location $j \in \mathcal{J}$ for time period $t \in \mathcal{T}$
Y_{ikt}	Whether vehicle $k \in \mathcal{K}$ covers node $i \in \mathcal{I}$ for time period $t \in \mathcal{T}$