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BACHELOR THESIS

ACOUSTIC WAVE PROPAGATION IN MULTILAYERED MEDIA

AN APPLICATION TO HIP IMPLANTS

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Contents

1 Introduction

This chapter provides an introduction to several subjects which are essential to understanding the context of this paper. Firstly, some information about OCON is given. Secondly, the problem on which this thesis is based is described. Thirdly, the goal, the research question and sub-questions for this paper are formulated. Finally, the structure of this thesis is discussed.

1.1 About OCON

OCON Orthopedic Clinic is a young company, founded in 2010. The clinic is derived from two partnerships within the *Ziekenhuis Groep Twente* (ZGT): orthopedics and anesthesia. The unique part of this company is that they work within the walls of the hospital, but still are independent. Because of this they can specialize en focus optimally on patient care, in order to maintain their goals: high quality and attainable for everyone.[2]

Within OCON scientific research is ongoing. Research contributes directly to the optimization of patient care because it is aimed at investigating and improving the effects of the treatment of orthopedic surgeons. This research is mainly practical. On the basis of the results of studies, orthopedic surgeons can quickly switch their treatments, for example by changing the treatment protocols. Because OCON has a research department it shows that it has a critical and constructive attitude towards its delivered patient care while continuous improvement is pursued. By doing research the orthopedic surgeons are 'forced' to delve into a particular treatment by reading scientific publications and/ or maintain collaboration with orthopedic surgeons who are working somewhere else. In this way they keep abreast of the latest findings and developments in orthopedics.[3]

1.2 Problem description

Our research is aimed at a complications of total hip implants. The components of the hip implant may be cemented into place or may be press-fit into the bone to allow the bone to grow onto the components. The uncemented hip implant is commonly used by OCON, the cemented hip implant is chosen in case the bone is too fragile or in case the shape of the bone is not suitable for a uncemented hip implant.

A hip implant has to migrate with the bone to create a strong connection. In this thesis focuses on the cases where the hip implant gets loose in an early stage due to a failing migration. The migration might fail due several complications, for example due to infections. In any case, a layer of tissue forms between the bone and the implant. The tissue layer might be detected using X-rays or MRI scans, but those methods require the patient to visit a hospital and are quite expensive.

1.2.1 A new method

What if there would be a method that is able to detect a loose hip implant which is faster, cheaper and smaller. Ultrasound might be that method. Ultrasound is already used in diagnostic sonography, but it is never used to see through bone. A small device will be placed upon the hip and sends ultrasonic waves to the bone and can 'hear' whether or not there is a space between the bone and implant. It can not only be used in hospitals but possibly also on the first line in health care, by physiotherapists and general practitioners. With this innovative device loose hip implants will be easier and earlier discovered.

1.3 Goal, research question and sub questions

Based on the problem description in section 1.2, the goal, research question and sub-questions for this thesis have been created, as described in this section.

1.3.1 Goal

The goal of our research is *to define a mathematical model which simulates ultrasonic waves through a hip and to conclude whether it is possible or not to detect a (infection) layer between the bone and hip implant with ultrasound.*

1.3.2 Research question and sub questions

Our goal is translated in the following research question:

Is it possible to detect a layer between bone and the hip implant using ultrasonic waves?

The following sub question must be answered in order to answer our research question.

1. How do ultrasonic waves propagate in different media?
2. Can differences in density be detected with ultrasound?
3. Is it possible to 'see' through bone with ultrasound?

1.4 Structure of the thesis

In this section an overview of the thesis is given.
The following chapter is about the mathematical model. The chapter describes how the model is created and the assumptions and limitations it is based on. In chapter 3 an analysis of the model is made. This analysis contains systematic checks and a validation by an experiment. The discussion in chapter 4 contains the results from the analysis and limitations of this thesis. The result of the thesis is given in chapter 5, the conclusion.

2 Mathematical model

This chapter describes the model which is used to simulate the propagation of ultrasonic waves through a given medium. Firstly, an impression is given of what the model has to look like. Secondly, the concepts that are used to formulate the model are defined, followed by the complete description of the model. Finally, the assumptions and limitations of the model are addressed.

2.1 Design of the model

In reality the propagation of waves in a medium is a very complex problem. The application usually determines which factors are taken into account. Sound propagation near a supersonic jet, for example, behaves very differently than ultrasound in kidneys. This model focuses on the factors which are relevant for ultrasound in soft tissues and bones.

The term ultrasound is given to sound waves with a frequency greater than the upper limit of the human hearing range. These frequencies start at 20 kilohertz. For our purposes, medical imaging, the frequencies range from 1 megahertz to 20 megahertz. To make these ultrasound waves a transducer is used, a device which converts electrical signals to acoustic waves.

2.1.1 Plane waves

Sound waves are inherently a three dimensional phenomenon. Imagine a speaker, for example, producing sound which travels in all directions. The problem is that modeling sound waves in a three dimensional space is generally hard. However for small scales the wave approximately propagates in one direction and is constant along the plane perpendicular to the direction in which the wave travels. These waves are called plane waves, see also figure 1. If A-mode medical imaging is used, this reduction applies. Resulting in a model that only needs to describe the propagation of the wave along one axis, which is called the x-axis.

Figure 1: A plane wave showing surfaces of constant phase (Szabo 2014, p56)

2.1.2 Introduction to the model

In the sections below we build a model to describe the behavior of an acoustic wave. The main problem is predicting what returns at the transducer given a medium and the wave which our transducer emits. The medium is modeled as a series of layers of different materials.

The model describes how the pressure in this medium changes along a line, because the change in pressure is what represents the local amplitude in a acoustic wave. Various physical laws are used to relate these changes in the pressure to the parameters of the medium.

2.2 Concepts within the model

The model contains three fundamental parts: the wave equation, reflection and transmission factors, and the power law. The wave equation describes how waves propagate in a given medium. The reflection and transmission factors describe what happens at the border between two different media and the power law describes the losses in the ultrasound due absorption and refraction.

2.2.1 One dimensional loss-less linear wave equation

The basis of the model is formed by the wave equation, the propagation of ultrasonic waves can be described using this equation. In our one dimensional model, the wave equation is given by the second order partial differential equation, see equation (1) (Szabo 2014, p58). In this equation c is defined as the propagation speed of the wave and $p(x,t)$ measures the pressure at a given point and time.

$$p_{tt}(x,t) - c^2 p_{xx}(x,t) = 0 \tag{1}$$

The initial conditions are

$$\begin{aligned} p(x,0) =& p_0(x) \\ p_t(x,0) =& q_0(x) \end{aligned}$$

In the most general case, the wave equation has no boundary conditions and is therefore defined for the entire x-axis. If that is the case, a analytical solution is given by D'Alembert:

$$p(x,t) = \frac{1}{2}(p_0(x-ct) + p_0(x+ct)) + \frac{1}{2c}\int_{x-ct}^{x+ct} q_0(s)ds. \tag{2}$$

The wave equation is not enough to make a realistic model. The equation is lacking in two areas. Firstly, the equation describes wave propagation on an uniform medium with constant c while we want to describe the propagation in multiple layers with different constants. Besides that, the equation does not describe what happens to a wave on the border between two different media. Thirdly, the equation does not describes losses. According to this equation, a wave continues to infinity without losing amplitude. In the next section we describe what happens at the borders between different media.

Though equation (2) is useful, we define a function $Q_0(x)$:

$$Q_0(x) = \frac{1}{c}\int_0^x q_0(s)ds$$

Equation (2) then reduces to:

$$p(x,t) = \frac{1}{2}(p_0(x-ct) + Q_0(x-ct)) + \frac{1}{2}(p_0(x+ct) + Q_0(x+ct))$$

Physically speaking the first term constitutes to forwards traveling waves and the second term to a backwards traveling waves. Therefore we can define:

$$\begin{aligned} p_f(x,t) &= \frac{1}{2}(p_0(x-ct) + Q_0(x-ct)) \\ p_b(x,t) &= \frac{1}{2}(p_0(x+ct) + Q_0(x+ct)) \\ p(x,t) &= p_f(x,t) + p_b(x,t) \end{aligned}$$

Note that p_f is a function of $x - ct$ only and p_b is a function of $x + ct$ only. This means that we can express a shift in time as a shift in space.

$$p(x,t+\tau) = p_f(x,t+\tau) + p_b(x,t+\tau) = p_f(x-c\tau,t) + p_b(x+c\tau,t)$$

2.2.2 Reflection and transmission

This paragraph describes what happens on the border between two different media. We start with the easiest situation of a layered structure: a two layered system. The wave propagates normally through the first layer until the boundary of the first and second layer. When the wave hits that boundary, there is a reflection and a transmission of the wave. The relative amplitude of the reflection, the reflection factor, is defined as $RF_{1,2}$ and the transmission factor as $TF_{1,2}$. The index $(1,2)$ indicates that these factors belong to the boundary between medium 1 and medium 2. Both factors are dependent on the acoustic impedance's Z of medium 1 and medium 2. The impedance is defined as $Z = c \cdot \rho$, in which c and ρ are the propagation speed and density of the media, respectively. The formulas for the reflection and transmission factors are given by equation (3) and (4) (Szabo 2014, p61).

$$RF_{1,2} = \frac{Z_2 - Z_1}{Z_2 + Z_1} \tag{3}$$

Z_1 and Z_2 are the impedance of the first and second medium respectively. Whenever $Z_1 = Z_2$ the reflection factor is zero, which translates to no reflection. For $Z_2 = 0$ follows that $RF = -1$, in that case the whole wave reflects, but with an negative amplitude.

$$TF_{1,2} = \frac{2Z_2}{Z_2 + Z_1} \tag{4}$$

It is easy to see that if $Z_1 = Z_2$ the transmission factor is $TF_{1,2} = 1$ and there is a full transmission. If $Z_2 = 0$ than $TF_{1,2} = 0$ and there is no transmission at all.

After the boundary between medium 1 and medium 2 the wave propagates with an amplitude of $p \cdot TF_{1,2}$ and the reflected part propagates back with an amplitude of $p \cdot RF_{1,2}$, see figure 2.

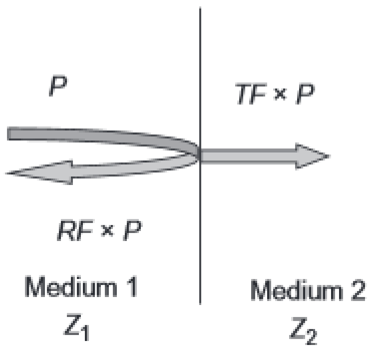


Figure 2: A schematic overview of reflection and transmission on a boundary. (Szabo 2014, p61)

2.2.3 Power law

In this paragraph we model the effects of losses on a wave. We use a model in which losses only affect the amplitude of the wave. In reality a change in phase velocity also occurs. This phenomenon is called dispersion.

Assume a forward traveling wave of a single frequency f with corresponding angular frequency ω and wave number k. The wave starts with an amplitude of A_0 at $(x,t) = (0,0)$. In a loss-less homogeneous medium it can be described as equation (5) (Szabo 2014, p83).

$$p_f(x,t) = A_0 e^{i\omega(t-\frac{x}{c})} = A_0 e^{i(\omega t - kx)} \tag{5}$$

In a real medium small losses, called attenuation, occur. These losses have two effects. They reduce the amplitude and they change the phase velocity. For now we only consider the reduced amplitude by a given factor. Then the wave can be expressed as equation (6).

$$p_f(x,t) = A_0 A(x,t) e^{i(\omega t - kx)} \tag{6}$$

Here $A(x,t)$ is the effect of the attenuation. Note that $A(0,0) = 1$, because the wave still starts at an amplitude of A_0 . We now make some assumptions about $A(x,t)$. We assume that the medium doesn't change with time. Thus that $A(x,t)$ is time-invariant: $A(x,t) = A(x)$. We further assume that we a have homogeneous material. Suppose we know $A(x)$ and $A(h)$ for a certain x, h. Because the material is homogeneous we can translate the effect of the attenuation. Since $A(h)$ is a factor to indicate the loss a wave accumulates over a distance h, we can find $A(x+h)$ by multiplying $A(x)$ and $A(h)$, thus $A(x+h) = A(x)A(h)$. With these assumptions we can find a formula for $A(x)$

$$\begin{aligned} A'(x) &= \lim_{h\to 0} \frac{A(x+h) - A(x)}{h} = -A(x) \lim_{h\to 0} \frac{1 - A(h)}{h} = -\alpha A(x) \\ A(x) &= e^{-\alpha x} \end{aligned}$$

We defined α as $\lim_{h\to 0} \dfrac{1 - A(h)}{h}$. Because $A(x)$ denotes a loss, $A(x) < 1$ for $x > 0$. Therefore α is positive. This α is called the attenuation factor which depends on the frequency f. Empirical findings for α in various fluids and soft tissues have confirmed a power law, see equation (7) (Szabo 2014, p84).

$$\alpha(f) = \hat{\alpha}|f|^y \tag{7}$$

Here the $\hat{\alpha}$ and y are material constants. We stress the fact that this power law is not an analytical result but merely an experimental one.

2.3 The model

In this section all the concepts are combined into one coherent model. Firstly, the geometry is constructed. Then a set of recursive relations is constructed which describes the multi layered problem. Lastly, these relations are solved for a particular case.

2.3.1 Geometry

In this paragraph a geometry for our model is defined.

Suppose we have a structure consisting of N layers with finite length and a transducer is placed at $x = 0$, which produces signals in the positive direction. For convenience, an extra layer is placed behind the transducer, layer 0, and after the structure, layer $N+1$. Thus we have a piecewise homogeneous, infinite domain.

Define the position of the boundary between layer i and $i+1$ as $x_{i,i+1}$, $0 \le i \le N$. Note that $0 = x_{0,1} \le x_{1,2} \le \cdots \le x_{N,N+1}$. And define the length of layer i as $L_i = x_{i,i+1} - x_{i-1,i}$. Each layer has its own speed of sound c_i, acoustic impedance Z_i and attenuation factor $\alpha_i(f)$.

To be sure that a wave does not leave the rightmost boundary of our structure, the impedance $Z_{N+1} = 0$. Then a wave fully reflects on the rightmost boundary. But if we want to be sure that a wave leaves the rightmost boundary of our structure undisturbed, we take $Z_{N+1} = \infty$. The same argument holds for Z_0. We define $c_0 = c_{N+1} = 1$ and $\alpha_0 = \alpha_{N+1} = 0$, as these values are not relevant.

2.3.2 Description of the wave

Paragraph 2.2.1 describes the linear wave equation on an infinite homogeneous domain. The paragraph shows that the pressure $p(x,t)$ can be separated into a forward traveling component $p_f(x,t)$ and a backward traveling component $p_b(x,t)$. For our piecewise homogeneous domain this is still correct in the individual layers.

At the boundary between two layers p_f and p_b are defined as the value of p_f and p_b just before they arrived at the boundary. Thus $\lim_{\tau\to 0^+} p_f(x-c\tau, t-\tau) = p_f(x,t)$ and $\lim_{\tau\to 0^+} p_b(x+c\tau, t-\tau) = p_b(x,t)$.

The wave produced by the transducer, $p_0(t)$, is assumed to be known. We further assume that $p_0(t)$ is harmonic with frequency f, so the power law can be used. We turn the transducer on at $t=0$, thus $p_f(x,t) = p_b(x,t) = p_0(t) = 0, \forall t < 0$. We define that $p_f(0,t) = \dfrac{p_0(t)}{TF_{0,1}}$. Moreover we assume that no wave is entering our structure on the rightmost edge: $p_b(x_{N,N+1}, t) = 0, \forall t \geq 0$. We are interested in the wave which returns our transducer at time t, $p_b(0,t)$.

In equation (3) we derived that the forward and backward traveling wave are functions of $x-ct$ and $x+ct$ respectively in an homogeneous, lossless medium. For a piecewise homogeneous, lossy medium this is not true, but we can amend the equation by restricting x to a single layer and introducing a attenuation factor:

$$\begin{aligned} p_f(x, t+\tau) &= e^{-\alpha_i(f)c_i\tau} p_f(x - c_i\tau, t) \\ p_b(x, t+\tau) &= e^{-\alpha_i(f)c_i\tau} p_b(x + c_i\tau, t) \end{aligned} \tag{8}$$

Here, x, $x - c_i\tau$ and $x + c_i\tau$ are located in layer i.

2.3.3 Recursive equations

In this paragraph we show that the wave in a layer is fully determined by the transmission and the reflection at the boundaries at this layer. This is done by formulating a set of recursive equations which quantifies those effects.

We start with the forward traveling component. As x we choose the boundary between layer i and $i+1$, $x_{i,i+1}$. Within layer i we can use (8) up to $x_{i-1,i}$:

$$\begin{aligned} p_f\left(x_{i,i+1}, t\right) &= e^{-\alpha_i(f)(x_{i,i+1} - x_{i-1,i})} p_f^*\left(x_{i-1,i}, t - \frac{x_{i,i+1} - x_{i-1,i}}{c_i}\right) \\ &= e^{-\alpha_i(f)L_i} p_f^*\left(x_{i-1,i}, t - \frac{L_i}{c_i}\right) \end{aligned} \tag{9}$$

Here p_f^* is the wave just after it reflects on the boundary, since p_f is defined as the wave just before it reflects on the boundary. If we then apply the equation (8):

$$\begin{aligned} p_f\left(x_{i,i+1}, t\right) &= e^{-\alpha_i(f)L_i} p_f^*\left(x_{i-1,i}, t - \frac{L_i}{c_i}\right) \\ &= e^{-\alpha_i(f)L_i}\left(TF_{i-1,i}\, p_f\left(x_{i-1,i}, t - \frac{L_i}{c_i}\right) + RF_{i,i-1}\, p_b\left(x_{i-1,i}, t - \frac{L_i}{c_i}\right)\right) \\ &= e^{-\alpha_i(f)L_i} TF_{i-1,i}\, p_f\left(x_{i-1,i}, t - \frac{L_i}{c_i}\right) \\ &\quad + e^{-\alpha_i(f)L_i} RF_{i,i-1}\, p_b\left(x_{i-1,i}, t - \frac{L_i}{c_i}\right) \end{aligned} \tag{10}$$

The equation above is the result of the combination of the theory of linear wave propagation, reflections and transmissions and attenuation. The same argument can be applied for the backward traveling component and gives a similar equation:

$$
\begin{aligned}
p_b\left(x_{i-1,i},t\right) = e^{-\alpha_i(f)L_i} TF_{i+1,i}\, p_b\left(x_{i,i+1}, t-\frac{L_i}{c_i}\right) \\
+ e^{-\alpha_i(f)L_i} RF_{i,i+1}\, p_f\left(x_{i,i+1}, t-\frac{L_i}{c_i}\right)
\end{aligned} \tag{11}
$$

If equations (10) and (11) are combined for $i = 1, \cdots, N$, i.e. for all internal boundaries, with the equations for the external boundaries, $p_f(0,t) = \dfrac{p_0(t)}{TF_{0,1}}, p_b(x_{N,N+1},t) = 0, \forall t \geq 0$, we get a full set of equations. These equations are recursive in time, because the waves are expressed as a function of the waves in the past. Because we assumed that $p_f(x,t) = p_b(x,t) = 0$ for all negative time, the equations are solvable. The full set of equations is:

$$
\begin{aligned}
p_f\left(x_{i,i+1},t\right) &= e^{-\alpha_i(f)L_i} TF_{i-1,i}\, p_f\left(x_{i-1,i}, t-\frac{L_i}{c_i}\right) \\
&\quad + e^{-\alpha_i(f)L_i} RF_{i,i-1}\, p_b\left(x_{i-1,i}, t-\frac{L_i}{c_i}\right) && \text{for } i = 1, \cdots, N, \forall t \geq 0 && (12) \\
p_b\left(x_{i-1,i},t\right) &= e^{-\alpha_i(f)L_i} TF_{i+1,i}\, p_b\left(x_{i,i+1}, t-\frac{L_i}{c_i}\right) \\
&\quad + e^{-\alpha_i(f)L_i} RF_{i,i+1}\, p_f\left(x_{i,i+1}, t-\frac{L_i}{c_i}\right) && \text{for } i = 1, \cdots, N, \forall t \geq 0 && (13) \\
p_f(0,t) &= \frac{p_0(t)}{TF_{0,1}} && \forall t \geq 0 && (14) \\
p_b(x_{N,N+1},t) &= 0 && \forall t \geq 0 && (15) \\
p_f(x,t) &= p_b(x,t) = 0 && \forall x \in \mathbb{R}, \forall t < 0 && (16)
\end{aligned}
$$

To solve the recursive equations in order to obtain $p_b(0,t)$ at a certain time t, given a function $p_0(t)$ and a geometry. Use equation (13) to write $p_b(0,t)$ as a function of p_f and p_b at some earlier time. Then use equations (12) and (13) to expand those functions and so on. If one of the terms is $p_f(0,\tilde{t})$ or $p_b(x_{N,N+1},\tilde{t})$ then use equations (14) and (15). All other terms eventually get to negative time, which nullifies them.

If we want to know p_f inside the layer i. First, we find p_f at the left boundary of layer i, $x_{i-1,i}$ and then we use the equation (8).

2.3.4 The model evaluated for a specific case

The recursive equations (12) to (16) are generally hard to solve. To demonstrate our model, we apply it to a specific case: a single layer.

Consider a single layer of glass with a transducer placed on it. We can apply the model described above with $N = 1$. Layer 0 and 2 correspond to the air and layer 1 to glass. Z_0, Z_2 are the acoustic impedance of air respectively. c, Z_1 and $\alpha(f)$ are the speed of sound, the acoustic impedance and the attenuation factor of glass respectively. The glass has a thickness of L. The transducer is placed in at $x = 0$, at the border between between the air and glass.

We repeat our boundary conditions: $p_f(0,t) = p_0(t)/TF_{0,1}$ and $p_b(L_1,t) = 0$, since $x_{N,N+1} = x_{1,2} = L_1$. So a harmonic forward wave $p_0(t)$ with frequency f is send from the transducer at $x = 0$. The response we want to measure is $p_b(0,t)$, the backward wave at $x = 0$ for a certain time t.
Now, we can apply the recursive equations (12) to (16). We start with equation (13):

$$
p_b\left(0,t\right) = e^{-\alpha(f)L} TF_{2,1}\, p_b\left(L, t-\frac{L}{c}\right) + e^{-\alpha(f)L} RF_{1,2}\, p_f\left(L, t-\frac{L}{c}\right)
$$

Equation (15) is used to conclude that $p_b\left(L, t-L/c\right) = 0$. We use equation 12 on $p_b\left(L, t-L/c\right)$:

$$p_b\left(0,t\right) = e^{-\alpha(f)2L} RF_{1,2}\; TF_{0,1}\; p_f\left(0, t-2\frac{L}{c}\right) + e^{-\alpha(f)2L} RF_{1,2}\; RF_{1,0}\; p_b\left(0, t-2\frac{L}{c}\right)$$

We can use equation (14) on $p_f\left(0, t-2L/c\right)$:

$$p_b\left(0,t\right) = e^{-\alpha(f)2L} RF_{1,2}\; p_0\left(t-2\frac{L}{c}\right) + e^{-\alpha(f)2L} RF_{1,2}\; RF_{1,0}\; p_b\left(0, t-2\frac{L}{c}\right)$$

We can now repeat these steps on $p_b\left(0, t-2L/c\right)$. But first, we define $n = \lfloor t\frac{c_1}{2L_1}\rfloor$.

$$\begin{aligned} p_b\left(0,t\right) = {}& e^{-\alpha(f)2L} RF_{1,2}\; p_0\left(t-2\frac{L}{c}\right) + e^{-\alpha(f)4L} RF_{1,2}^2\; RF_{1,0}\; p_0\left(t-4\frac{L}{c}\right) + \cdots \\ & + e^{-\alpha(f)2nL} RF_{1,2}^n\; RF_{1,0}^{n-1}\; p_0\left(t-2n\frac{L}{c}\right) + e^{-\alpha(f)2(n+1)L} RF_{1,2}^{n+1}\; RF_{1,0}^n\; p_0\left(t-2(n+1)\frac{L}{c}\right) \\ & + e^{-\alpha(f)2(n+1)L} RF_{1,2}^{n+1}\; RF_{1,0}^{n+1}\; p_b\left(0, t-2(n+1)\frac{L}{c}\right) \end{aligned}$$

We can now apply (16) to conclude that $p_0\left(t-2(n+1)\frac{L}{c}\right) = p_b\left(0, t-2(n+1)\frac{L}{c}\right) = 0$, because $t-2(n+1)\frac{L}{c} < 0$.

$$p_b\left(0,t\right) = \sum_{k=1}^{n} e^{-\alpha(f)2kL} R_{1,2}^k\; R_{1,0}^{k-1}\; p_0\left(t-2k\frac{L}{c}\right)$$

If p_0 is periodic with period $2\frac{L}{c}$, then we can use the geometric series to find an explicit answer:

$$p_b\left(0,t\right) = p_0\left(t\right)\sum_{k=1}^{n} e^{-\alpha(f)2kL} R_{1,2}^k\; R_{1,0}^{k-1} = p_0\left(t\right) e^{-\alpha(f)2L} R_{1,2}\; \frac{1-e^{-\alpha(f)2nL} R_{1,2}^n\; R_{1,0}^n}{1-e^{-\alpha(f)2L} R_{1,2}\; R_{1,0}}$$

2.4 Assumptions and limitations of the model

In this paragraph all assumptions made are recalled and the limitations of the model are shown.

We used a one dimensional model to describe the acoustic waves. This means that we reduced our medium to a few parallel layer and that the waves travel only perpendicular to these layers. Reflection under an angle, for example, isn't taken into account. We further used that these layers are homogeneous and have sharp edges. In reality this is not the case, so we might not get clear reflections. Instead a sequence of interfering waves could show up.

We assumed that the power law holds for the materials we use. Experiments have confirmed this for fluids and soft tissues (Szabo 2014, p 85). We also assumed that the transducer sends out pure harmonic waves of a single frequency. In reality the signal is send for finite time and has some noise incorporated, which distorts our waveform.

The model also didn't take dispersion into account but this is usually small (Szabo 2014, p 85). Further, in real tissues non-linear effects can occur. They can distort the shape of the waveform into a sawtooth, which contains higher harmonics. Because higher frequencies are damped more, this can introduce extra losses.

3 Analysis of the model

3.1 Systematic checks

In this section some systematic checks with our model are executed to determine if the model behaves normally in some basic situations. There are several situations described in which the answers to the problems are trivial. The model should at least behave properly in these easy situations to give proper answers for more complicated situations.

In our first basic situation the model is as follows. There is only one layer with $L = 1$, $c = 1$ and $Z = 1$. Behind that layer, there is a boundary which is fully reflecting, i.e. $Z_{outside} = 0$. Also the starting point at the transducer is fully reflecting the waves. There is no attenuation, diffraction or diffusion. There is one pulse which is a sinusoidal with frequency $f = 1$ and amplitude $A = 1$. We would expect to get a pulse back after 2 seconds, 4 seconds and every two seconds after that. Our model gives indeed these pulses at these times, see figure 3.

In the second basic situation the layer is divided in two equal layers, with a fully transmitting boundary in between. As we physically didn't change the structure, it should yield the same results. As expected, the results are indeed the same. Even if we divide these layers further, the results of our model stay the same. Look at the figures 4 and 5 for the amount of layers, $n = 2$ and $n = 3$.

For the third basic situation we added an attenuation factor to the situation. The attenuation factor was $\alpha = \frac{1}{2}\ln(2)$. This attenuation factor means that the pulse dampens with half for every reflection. The results were as expected and the amplitude halved for each subsequent reflection, see figure 8.

For the fourth situation the boundary is changed. The backward boundary is not fully reflecting but reflects only half of the full amplitude back. The prediction is to see no difference in the waves seen from the previous example. The predictions are correct as shown in figure 7.

All our created situations are checked and the model behaves exactly the way it is supposed to. So we can conclude that it at least checks for common sense in physics and wave propagation theory. In the next section the model is checked with the help of an experiment to further validate the model for the situation.

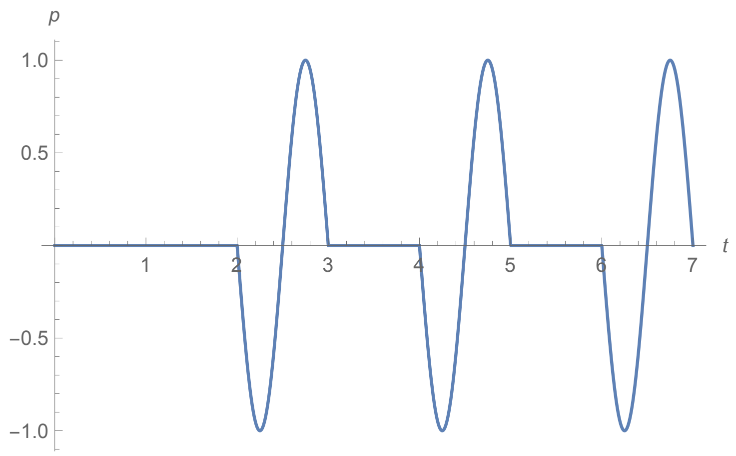


Figure 3: The return signal form a single pulse with unit frequency and unit amplitude in a single layer of unit length with unit speed of sound, full reflection at the boundaries and no attenuation.

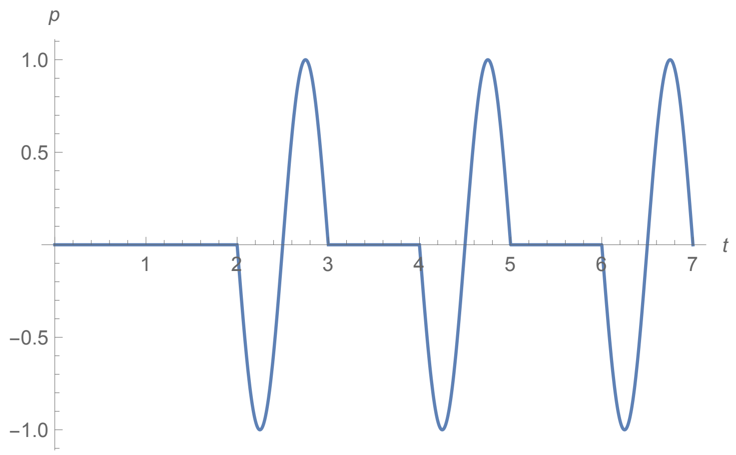


Figure 4: The return signal form a single pulse with unit frequency and unit amplitude in a two layers of half unit length with unit speed of sound, full reflection at the boundaries and no attenuation.

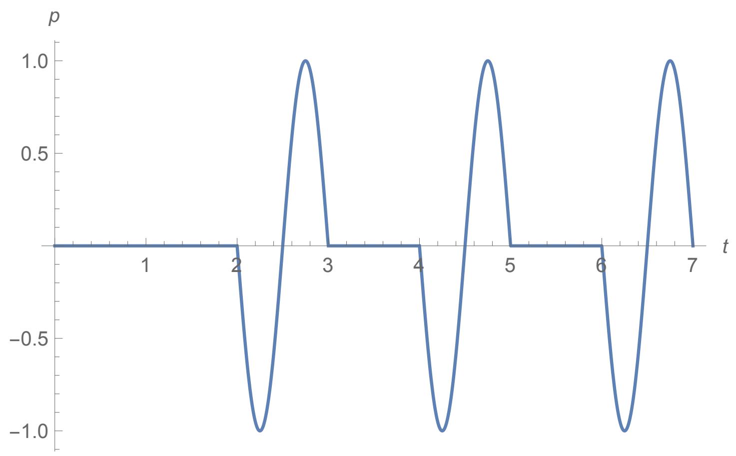


Figure 5: The return signal form a single pulse with unit frequency and unit amplitude in three layers of third unit length with unit speed of sound, full reflection at the boundaries and no attenuation.

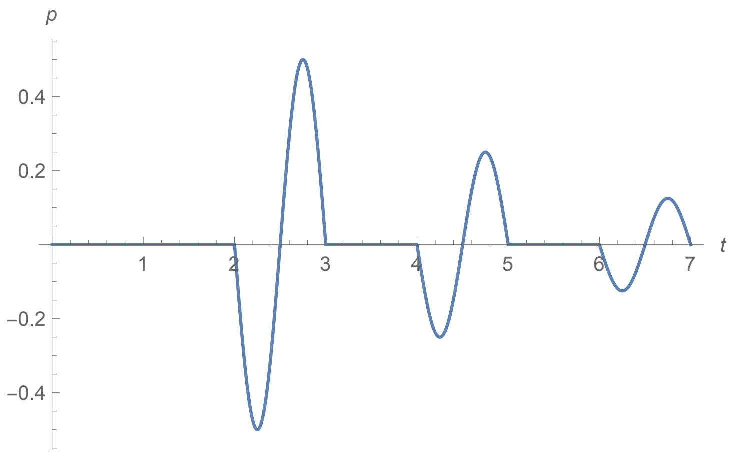


Figure 6: The return signal form a single pulse with unit frequency and unit amplitude in a single layer of unit length with unit speed of sound, full reflection at the boundaries and an attenuation factor of $\frac{1}{2}\ln(2)$.

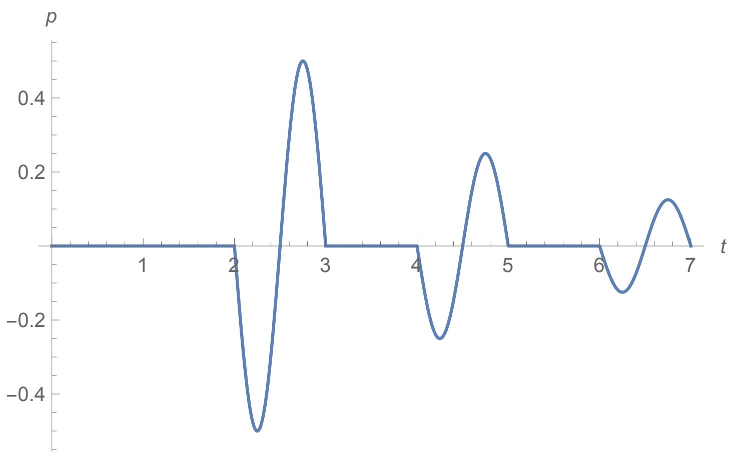


Figure 7: The return signal form a single pulse with unit frequency and unit amplitude in a single layer of unit length with unit speed of sound, full reflection at the left boundary, half reflection at the right boundary and no attenuation.

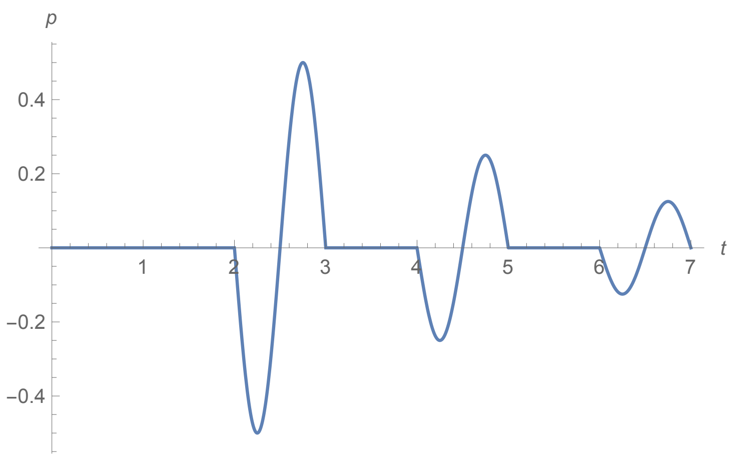


Figure 8: The return signal form a single pulse with unit frequency and unit amplitude in a single layer of unit length with unit speed of sound, full reflection at the left boundary, half reflection at the right boundary and no attenuation.

3.2 Physical experiment

In order to validate the model, we executed a physical experiment. The situation that is created in the experiment is recreated in the model with the corresponding parameters to make a useful comparison between results of the model and the experiment. The set-up of the experiment is given in the next section.

3.2.1 Setup

During the physical test the reflected waves of ultrasound are measured in three different situations. In the first situation there is a simple plastic cup filled with water. In the second situation gelatin is added to the water in the cup to mimic human tissue. In the third situation a layer of epoxy resin is added to the water and gelatin to mimic human bone.

In all cases the transducer send a pulse of ultrasound straight down to the bottom of the cup, the borders between the different layers reflect parts of the pulse back to the transducer. These reflections returning to the transducer are then measured.

Materials
For this experiment we used the measurement equipment of Victor Sluiter. With the proper software we were able to send sinusoidal pulses with a certain frequency and measure the response. The transducer, which sends and receives the signals, was build in a small construction. This construction is placed over the cup and with the screw-thread we are able to adjust the height of the transducer.

For our three situations we need 3 cups and the proper ingredients. These ingredients are water, gelatin and epoxy resin.

Preparation
We prepared 3 different cups like we stated earlier.

Cup 1 is filled with 3 centimeter water.
Cup 2 is filled with a layer of 2 centimeter gelatin beneath a layer of 1 centimeter water.
Cup 3 is filled with 1 centimeter gelatin, a bar of 2 cm wide and 0.2 centimeter thick epoxy resin, again 1 centimeter of gelatin and on top 1 centimeter of water.

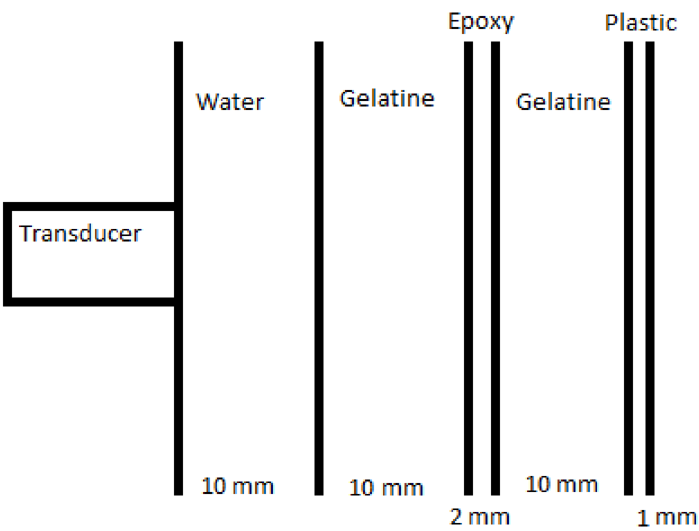


Figure 9: A schematic overview of cup 3.

The construction which holds the transducer is attached to the small plank with the screw-thread and the nuts. The cups are placed upon small pieces of wood such that there is a layer of air under the cup, because ultrasound dies out directly if it contacts air. The transducer is placed in its holder. The measurement equipment is connected to the transducer and adjusted such that it rest just in the water. A schematic overview of the third situation is given in figure 9.

Measurements
For each cup the response is measured for several frequencies. These frequencies are 1 MHz, 5 MHz, 6.5 MHZ, 6.8 MHz, 7 MHz, 10 MHz and 20 MHz and 3 measurements are saved for each combination of cup and frequencies.

3.2.2 Expectations of the experiment

In this section the expectations of the experiment are described. The mathematical model that was described in the section 2.3 is used to get these results. If the model is correct the results of the experiment should be close to the predicted values. Therefore the predictions are sketched first and are compared to the actual results later on. These predictions are based on an frequency of 6.8 MHz, for compactness.

3.2.2.1 Cup 1

In the first experiment there is a cup filled with only water, cup 1. There are 3 centimeters of water and the bottom of the cup is 1 millimeter plastic. With these parameters and the given parameters for the speed of sound in different media found in appendix A.2, the following predictions were done in figure 10:

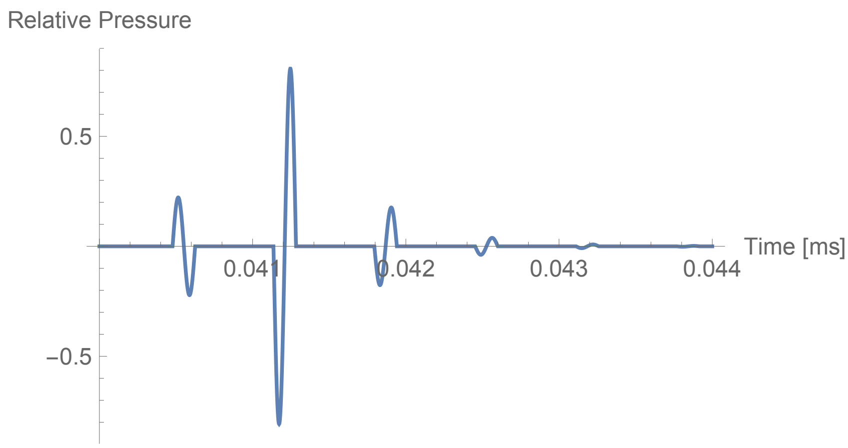


Figure 10: Predictions of the model for cup 1.

There is a first sine wave just before 0.041 milliseconds. This is caused by the reflection of the boundary between water and the bottom of the cup. After the first there is a second bigger wave which is the wave traveled through the boundary between water and the bottom and reflected at the boundary between the bottom and air. Because most of the wave is transmitted through the first border but almost everything is reflected when the boundary with air appears, the amplitude of this wave is larger. After these two primary waves there are only reflections of reflections which are significantly smaller and reduce exponentially.

3.2.2.2 Cup 2

In the second experiment there is a cup filled with 1 centimeter water and 2 centimeter of gelatin and a plastic bottom of 1 millimeter. For the speed of sound in gelatin and water we used the values as given in appendix A.2. These are the predictions, see figure 11:
There is a small sine wave coming back at 0.0135 milliseconds with a relative pressure $6 \cdot 10^{-3}$. This wave is really small because of the small difference in impedance between water and gelatin and because the amplitude is so small it is left out of this figure. The picture is zoomed in at the peaks which correspond to the return from the bottom. There is a reflection around 0.0415 milliseconds. Again the first wave seen is the small wave which is reflected at the boundary

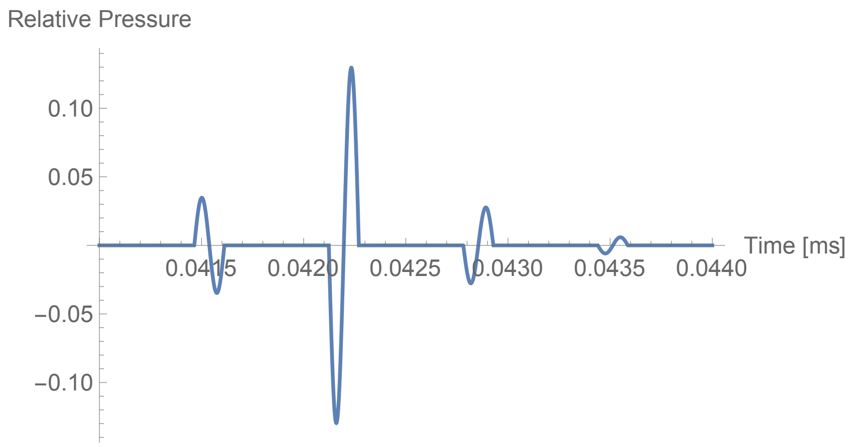


Figure 11: Predictions of the model for cup 2.

gelatin and the bottom. Most of the wave is transmitted here and reflected at the boundary between plastic and air. The bigger wave corresponds to that reflection. The waves after that are again reflected reflections and fade out exponentially.

3.2.2.3 Cup 3

In the third experiment the cup is filled top to bottom with 1 centimeter water, a layer of 1 centimeter gelatin, a layer of 0.2 centimeter epoxy, another layer of 1 centimeter gelatin and at the end the bottom of 1 millimeter plastic. The speed of sound in the different materials can be found in appendix A.2. The prediction is as follows, see figure 12:

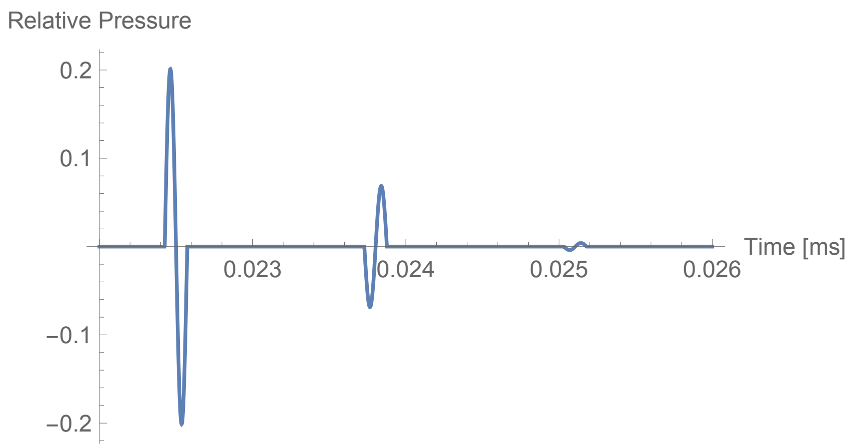


Figure 12: Predictions of the model for cup 3.

There is a wave coming back after 0.022 milliseconds which is fundamentally earlier than the other predictions. This is because the difference in impedance with epoxy and gelatin is big and this is the wave that returns from this reflection. Also there is a reflection of the lower end of the epoxy. Further there are reflections of reflections which fade out exponentially as well. Of course there is a small wave coming back off the boundary water to gelatin, but this small and irrelevant again so we left it out. More interesting are the reflections coming back of the bottom, because these waves would have penetrated the epoxy twice. Once on the way to bottom and once after the reflection when it is returning to the transducer. Some small waves are showed in the following picture, see figure 13:
After about 0.036 milliseconds there is a first small wave which is the reflection from gelatin to the plastic bottom. This reflection is again really small and not relevant. If we look at the scale the amplitude is only around 0.002 and almost undetectable. After this there is the bigger wave which came back from the boundary of the bottom with air which again is a bigger wave. This one spikes up to an amplitude of 0.015 and is easy to detect with the proper equipment. After this wave there are some reflected reflections again which fade out exponentially. In between those small waves some really small waves which can't be detected propagate and these aren't

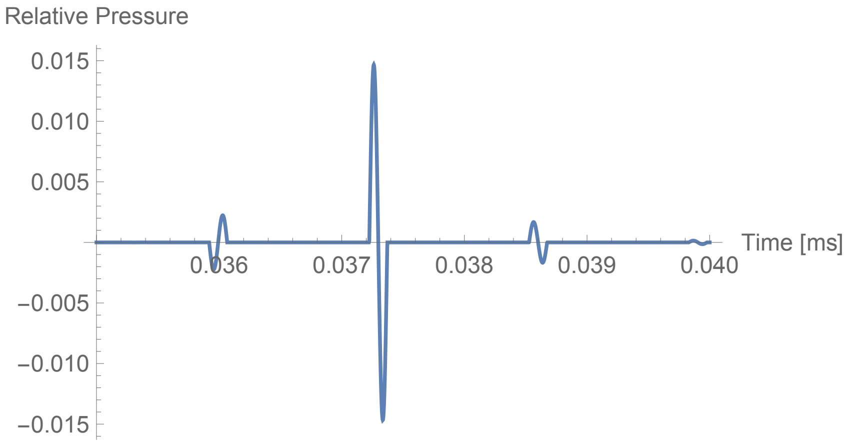


Figure 13: Predictions of the model for cup 3.

important.

3.2.2.4 Expected difference with the results

In the pictures above there was a really smooth signal in a homogeneous media so the signal stays a good-looking sine wave with only reflections at the boundaries. Some differences are expected in the results of the experiment. The following differences are to be expected:

- The signal won't be a smooth sine wave, but will be a bigger signal which is a dispersed sine wave.
- The signal is dispersed so the highest peak will be lower, because the loss of energy through dispersion.
- The boundary between water and gelatin won't be homogeneous in reality and there will be a bigger bump over there.
- There will be some random noise. There is actually not a problem with this because we can average our signal.

These are all some problems we expect to differ from the reality. This does not mean that the experiment is worthless. If our signal comes back around the designated times and if the peaks will be somewhat in the same shape there still is a successful experiment. The shape of the wave changed but this is expected. This is not taken into account in this model because it will make the model too complex. The predictions make the experiment really important because the model can be checked with reality even though it will differ at some points.

3.3 Results

In this paragraph we take a closer look at the results of the experiment. Some question to be asked are: Do the results make any sense? What do we see in the results? Are they similar to the results of our model? Using the speed of sound and the impedance of the materials used in this situation, we are able to explain the results. In appendix A.2 these constants are given. A comparison of the result of the experiment and the model is given in section 3.4. These results are from 6.8 MHz.

Cup 1

In the first situation there is a simple cup of water. With the transmission and reflection factors something can be said about the amplitude of the wave and with the speed of sound and the thickness of the layers an estimation can be made for the returning time for the pulse.

Using equations (3) and (4) the reflection and transmission factors are determined on the border from water to the plastic, from plastic to water and from plastic to air.

	RF	TF
Water $\rightarrow$ polythylene	0.239	1.239
Polythylene $\rightarrow$ water	-0.239	0.761
Polythylene $\rightarrow$ air	-0.999	0.0003

Table 1: Overview of the reflection and transmission factors in cup 1.

The first reflection (`I`) which is measured, is the single reflection from the top of the bottom layer. Therefore the RF is equal to 0.239. the second reflection (`II`) will be the reflection from the bottom of the bottom layer. The pulse transmits from water to plastic, reflects from plastic to air and transmits from plastic to water back. The total reflection factor is $(1.239 \cdot -0.999 \cdot 0.761 =) - 0.942$. The amplitude of the second reflection is significant higher and reversed compared to the first reflection. Another reflection (`III`) within the bottom has a total RF of $(1.239 \cdot -0.999 \cdot -0.239 \cdot -0.999 \cdot 0.761 =) - 0.225$. Yet another reflection (`IV`)has a total RF of -0.05. These results are visible in figure 14 where the start of the reflection is indicated by a vertical dashed line.

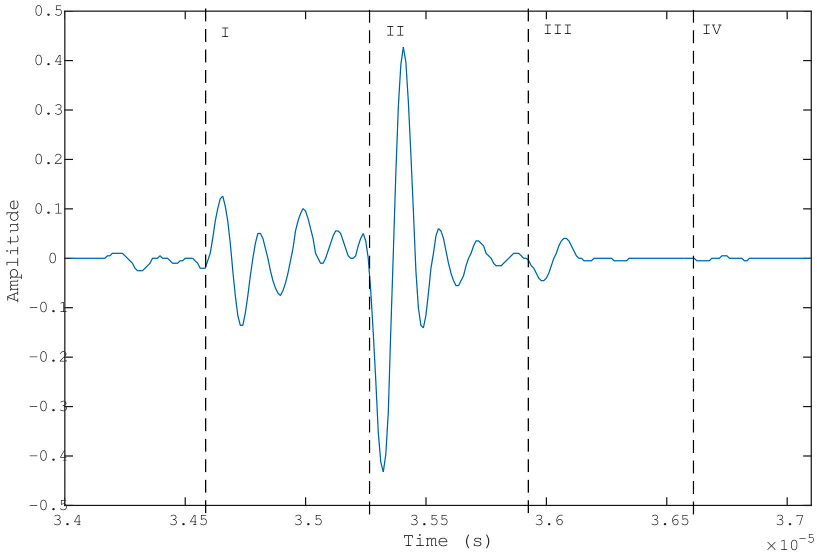


Figure 14: The result of the experiment with cup 1.

With the speed of sound in the materials we calculate the thickness of the layers. Assuming that the speed of sound in water is equal to 1482 m/s, we can say that the first reflection that returned in 3.46×10^{-5} seconds, traveled 5.1 centimeter. The layer of water therefore must have been 2.55 centimeter thick. The difference between the first and second pulse equals the time it took to travel back and forth through the bottom of the cup. This difference is 6.5×10^{-7} seconds in a material where the speed of sound is 2411 m/s. The distance traveled is 1.5 millimeter, the thickness of the bottom is therefore estimated to be 0.75 millimeter.

Cup 2

The results of the second situation are quite similar to those of the first situation. In this situation a layer of gelatin is added to the cup, but this is not evident in the results. If the observer looks closely to each measurement, he could see a small peak around 1.39×10^{-5} seconds. This might be the first reflection back, caused by the border between the water and gelatin. To confirm this we determine the RF on this border.

	RF	TF
Water → gelatin	-0.009	0.991
Gelatin → water	0.009	1.009
Gelatin → polythylene	0.248	1.248
Polythylene → gelatin	-0.248	0.752
Polythylene → air	-0.999	0.0003

Table 2: Overview of the reflection and transmission factors in cup 2.

The properties of gelatin are similar to water, the impedance of gelatin is 1.452 MRayls against 1.482 MRayls for water. The RF from water to gelatin is -0.009, so over 99 percent of the amplitude lost due the reflection. It is therefore likely that the small peak around 1.39×10^{-5} seconds is caused by the border between water and gelatin. The location of the peak suggests that the pulse has traveled a total of 2.06 centimeters through water, so twice through a layer of 1.03 centimeter water. This matches with the setup of the experiment: There is a cup with 2 centimeter gelatin beneath 1 centimeter water.

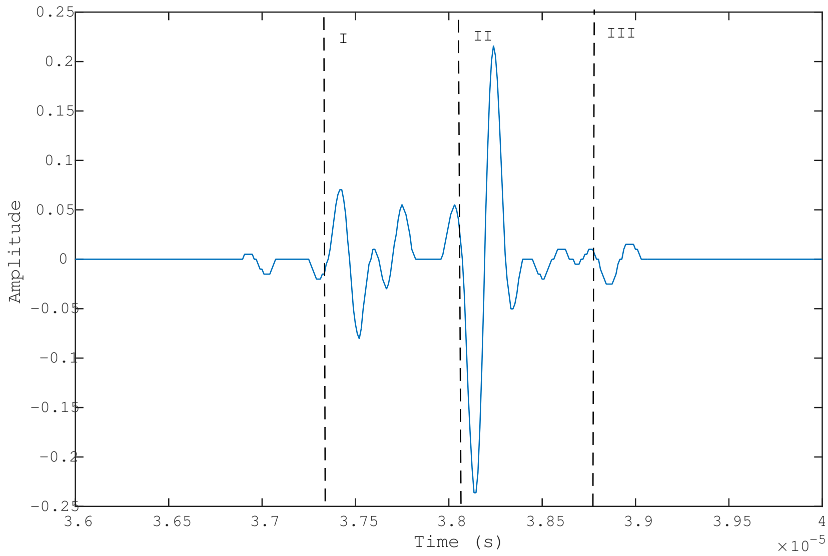


Figure 15: The result of the experiment with cup 2.

Besides this small peak there are similar results compared to the first situation. In figure 15 the first three reflections of the bottom are indicated with dashed lines. The fourth reflection that was visible in the first situation is not visible in this situation. The differences between the two figures are found in the location and the amplitude of the waves. The first reflection occurs at 3.73×10^{-5} seconds, which gives us that the total thickness of the layers of water and gelatin is 2.77 centimeter, therefore the location of the waves is probably changed by a difference in thickness. The different amplitude of the reflection is not caused by difference in the reflection and transmission factor. An overview of all necessary factors is given in table 2. These factors give 0.248, -0.937 and -0.232 as the total reflection factors for reflection `(I)`, `(II)` and `(III)`, respectively. They are almost equal to those in the first situation. The losses are most likely caused by absorption and dispersion in the gelatin.

Cup 3

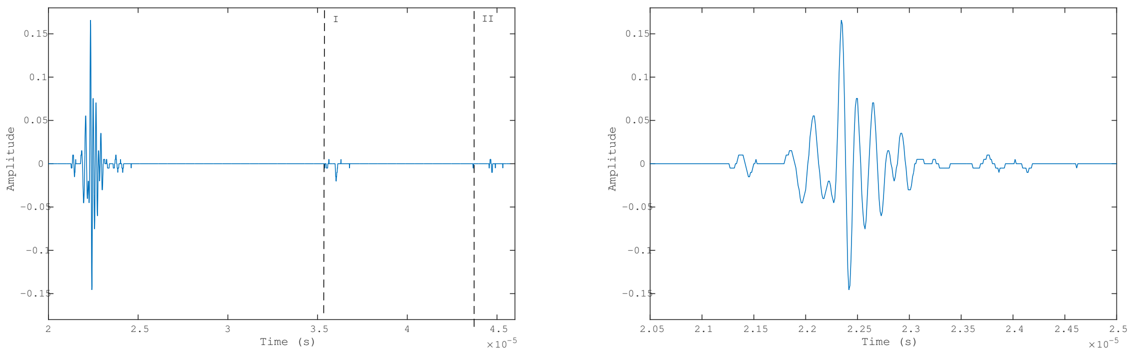
In the results of the third and last situation are separated the first main response and two small responses. These small responses are labeled `(I)` and `(II)` in order of time in figure 16a. The first small response `(I)` is located at 3.5×10^{-5} seconds, this is the reflections of the bottom of the cup. Response `(II)` is the second reflection from the epoxy. The estimation is based on time necessary to travel through the cup. Reflection `(I)` occurs at the same time as the reflection of the bottom in cup 1 and 2. Reflection `(II)` occurs at the doubled time of the main reflection.

In this situation a layer of epoxy resin is used to mimic a layer of bone, in the next calculations and in the model assumptions are made for the properties of bone for the epoxy to determine the RF and TF.

	RF	TF
Water → gelatin	-0.009	0.991
Gelatin → water	0.009	1.009
Gelatin → polythylene	0.248	1.248
Polythylene → gelatin	-0.248	0.752
Gelatin → bone	0.628	1.628
Bone → gelatin	-0.628	0.371
Polythylene → air	-0.999	0.0003

Table 3: Overview of the reflection and transmission factors in cup 3.

It is hard to tell much about the main reflection. As expected a big reflection from the layer of epoxy is seen. But in the results of the experiment as illustrated in figure 16b it is hard to identify different waves apart from each other. The total reflection factors are 0.617 and -0.371 respectively for the reflection on the top and bottom of the layer epoxy. Therefore the reflection of the top would be a large positive sine wave, while the reflection of the bottom of the layer of epoxy would be a negative sine with approximated 2/3 of the amplitude of the reflection of the top. However, these expectations can't be recognized in the results of the experiment. There is a large positive wave in the middle of figure 16b, which might be the reflection of the top, but we can't explain the waves occurring before and after this wave.



(a) Resuls of the experiment.

(b) Close up on the results.

Figure 16: Results of the experiment of cup 3.

3.4 Comparison between expectations and results

After the experiment the results should be compared to the predicted values. In this section these two are compared extensively and there is an attempt to verify our model with the results of the experiment. The comparison is made between all the cups in the following paragraphs. Because of the measurement errors in using certain ways of measuring we had some big errors between the two. We used some physical formulas to correct those errors to make sure the results are more trustworthy. For the analysis of these measurement errors we refer to section 4.1.

Comparison cup 1

The setup of the first cup was relative easy. With a cup only filled with water the results should be really close to the predictions. With the tweaking of the measurement errors, the moment of return of the sine wave was quite accurate. There is however a big error in the amplitude of the wave. The relative pressure dropped considerable, more than predicted. It was around a factor of 1/3. Also the shape of the wave is quite different. The returned signal in reality is wider than the model predicted. This is because the model doesn't take dispersion of the wave into account. The wave sent out by the transducer disperses more over the distance traveled and in this small cup the results of dispersion are already quite visible. This dispersion is probably the cause of the lowered amplitude of the wave. Because the energy of the wave is dispersed over a wider time area the energy coming in at that one moment is reduced.

In figure 17 we see that the wave which returned is quite well predicted. There are some errors in the shape and height of the wave. But the change of shape has influence on the amplitude.

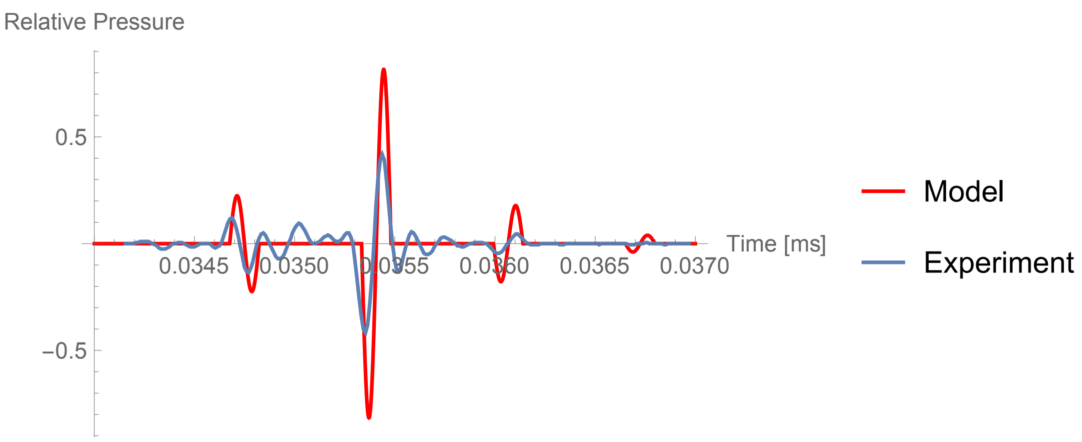


Figure 17: Comparison of result of the experiment and model of cup 1.

Comparison cup 2

The layered structure of cup 2 was a bit more complicated than the first one. With a layer of gelatin after the first layer of water, an extra reflection is expected because of the small difference of acoustic impedance between water and gelatin. This made the prediction for the model already a lot more complicated because of the reflected waves being reflected again.

The amount of time the wave traveled through the different layers before it came back was again accurate after tweaking it with some measurement error and physics. The reflection of the boundary between water en gelatin was hardly noticeable. The difference between the two layers was even smaller than predicted. The shape and amplitude of the sine wave are off again in the result. The dispersion in the different layers probably caused this variation. There was almost no difference in the result for cup 1. This is because the properties of gelatin and water are more alike than predicted.

Comparison cup 3

The layered structure of the third cup was by far the most complicated. There is also is some layer of epoxy in this setup which behaves a lot like human bone. Therefore this experiment is also the most important for the conclusions and in validating the model. The experiment gave nothing back except for some peak which is not identified as the boundary epoxy gelatin for sure. There is too much dispersion to be completely sure that this is indeed the desired layer.

The time needed for the wave to travel through the whole cup is again quite similar to the predictions. This means the model simulates the speed of the traveling waves incredibly well. After the experiments with cup 1 and cup 2 this was foreseen because the sample of epoxy is relative small in relation to the whole cup and does not influence the time needed much. All the waves came back around the predicted time.

The boundary between water and gelatin is not really visible in the experiment in contrast with the values given by our model. That result is not shocking at all since cup 2 showed this result as well. This is because the acoustic impedance of water and gelatin are quite close.

The wave reflecting from the sample of epoxy was quite big as expected, as is illustrated in figure 18. Though there are way more reflections coming back from this piece of epoxy. This is because the structure of epoxy is more complicated than water and gelatin and because it is a solid. This produces more reflections within the material. Most of the energy of the wave is lost with this sample of epoxy in the cup.

We also predicted that there would be a wave coming back from the bottom of the cup all the way through the epoxy twice. Although the wave would be small it should return. In the results of the experiment we can see that there is a wave returning from the bottom of the cup but it is not sure if this is the desired wave. The time to return to the starting point of the wave is the same as predicted. There is a major difference in the amplitude of wave coming back as a result of the dispersion. Unfortunately this means we lost almost all level of detail. The result is still valuable though. The wave coming back and still in some shape of the sine and at the expected is a result that can be valuable for further research.

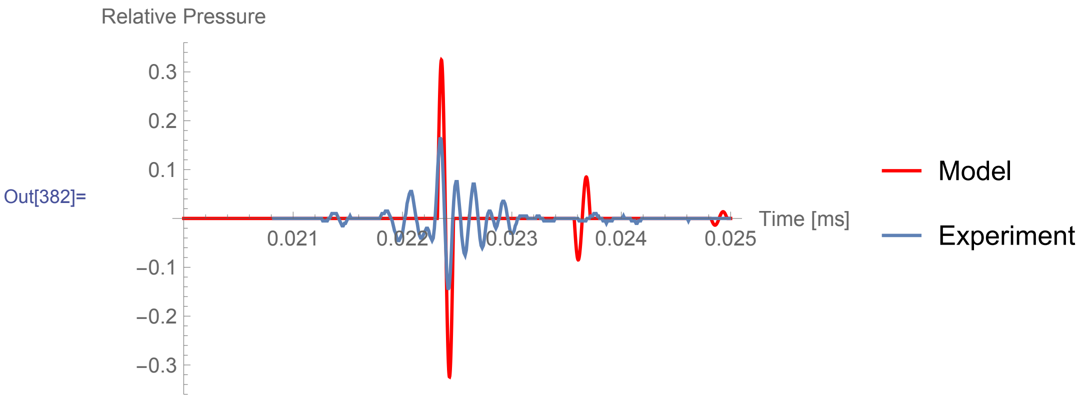


Figure 18: Comparison of result of the experiment and model of cup 3 at 6.8 MHz.

These are the results in case of the 6.8 MHz signal. The experiment was executed of several frequencies, and it is known that signals with a lower frequency are able to penetrate deeper. So in this case it is also interesting to compare the results of the 1 MHz signal. The model suggest we receive reflections with a high amplitude, the results of the experiment are however disappointing. We receive something back at the expected time as figure 19 shows. The shape of the sine is not recognizable. Therefore no verdict can be given based on these results. This could an artifact of the transducer as it was optimized for 6.8 MHz.

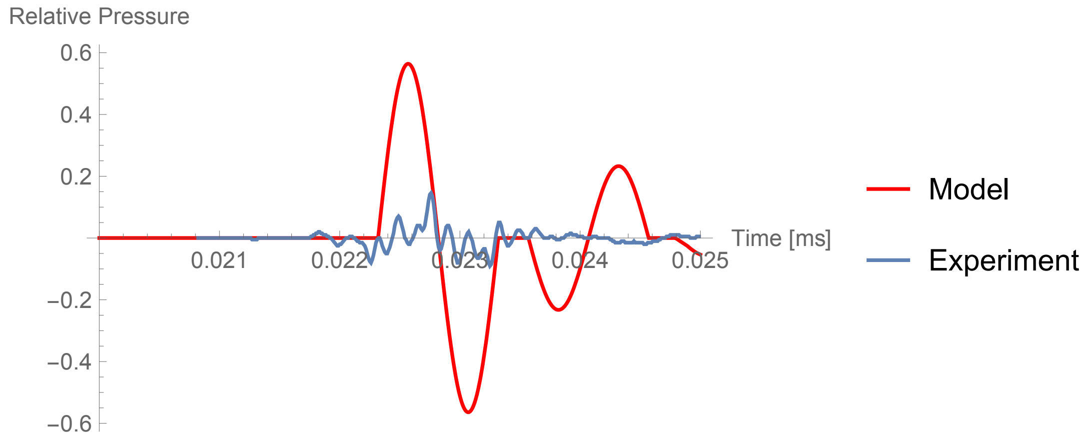


Figure 19: Comparison of result of the experiment and model of cup 3 at 1 MHz.

3.5 Application of the model on a hip

In this section we apply the model to a schematic view of a hip with an hip implant. We will investigate with and without an infection layer.

Schematically a hip consist of a few layers of tissue: Fat, muscle and bone. For our purposes we will add an titanium implant to this. The resulting structure is then given by: fat, muscle, bone, titanium, bone, muscle, fat, air. We will also consider another structure with a water-like infection layer: fat, muscle, bone, titanium, bone, muscle, fat, air.

For clarity we won't show the multiple reflections. For a frequency of 6.8 MHz the result is shown in figure 20. The first peak is a result of the reflection at the boundary between the fat layer and the muscle layer. The second is from the reflection at the boundary between the muscle layer and the bone layer. After this layer no significant reflections return. The next peak turns out to have an amplitude of 0.0003 which is insignificant, as the resolution of our transducer used in the experiment was 0.005. Unsurprisingly, The graphs with and without an infection layer are the same, as the infection layer does not yield any significant reflections. It is safe to say that with a frequency of 6.8 MHz, the infection layer is undetectable.

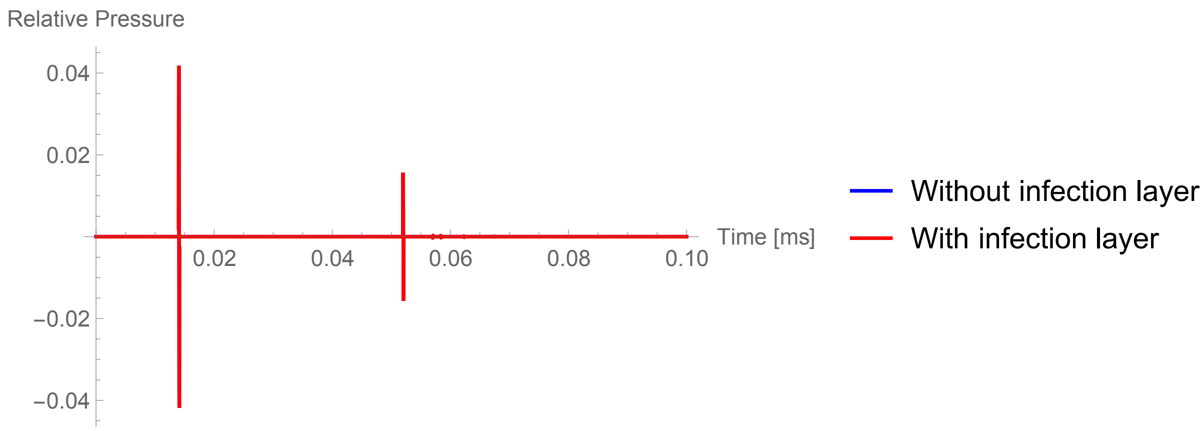


Figure 20: Our model applied to a schematic view of a hip with a frequency of 6.8 MHz.

For a frequency of 1 MHz the results are more useful, see figure 21. First, the scale of the graph is different. The amplitude of the second peak, for example, has increased with a factor of ten. The first two peaks are the same as above, however this time there are more peaks. In the case without the infection layer, there are three more peaks form the reflections of the bone and the titanium layer, the titanium and the second bone layer and the second bone and the second muscle layer respectively. In the case with an infection layer, there are two peaks close together which are the reflections of the bone to the water layer and the water to the titanium layer. After that, there are the same peaks as in the other case but with an lowered amplitude. This could be because the large difference between the impedance of water and bone, and water and titanium. This could be used to distinguish these cases from each other. However our experiment did not validate our model for these frequencies.

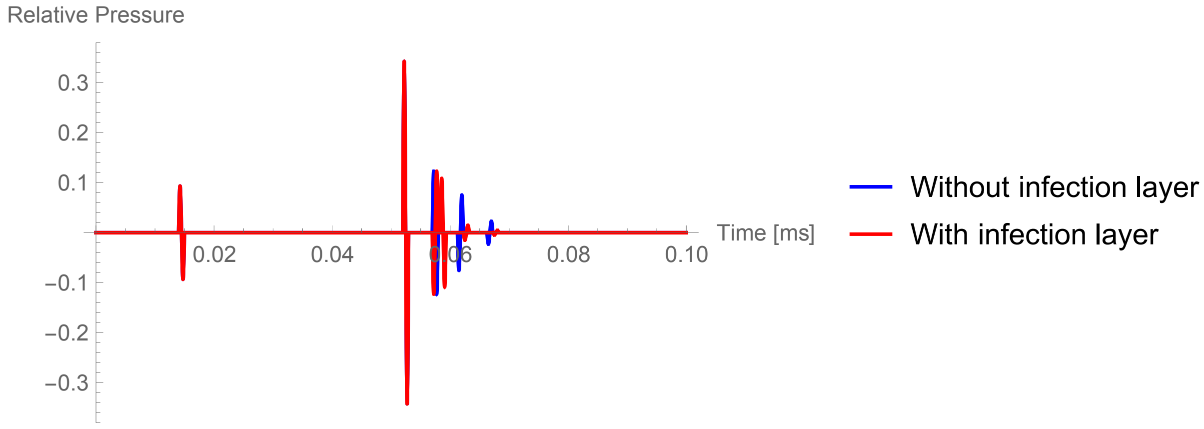


Figure 21: Our model applied to a schematic view of a hip with a frequency of 1 MHz.

4 Discussion

In this section we review our model and experiment. We discuss the relevance and accuracy of our model and experiment.

4.1 Experiment

Experiments always delivers some errors in our results. In this section we are going to discuss the influence and magnitude of these errors to be able to quantify our results. There are a lot of errors possible in our situation: think of an error measuring the depth of the water in the cup, the influence of temperature, the height of the transducer, the thickness of the bottom of the cup. We will sum up these errors here and relate the magnitude of these errors to our experiment.

- For the measurement of the depth of the water in the cup we have used a ruler. This ruler is exact within millimeters. So there will an error of $0,5$ mm because of this.
- The temperature of the room should be around 20 degrees Celsius. It should be at least 18 degrees and at most 22 degrees Celsius. This means that the impedance of the water should be around $1,483$ MRayl and should a vary a maximum of $0,007$ MRayl to both sides with the difference in possible temperatures.
- The construction we have build to support the hanging transducer above the cup has some error as well. Because we could not put the transducer exactly hanging within the first drops of water. There will be only a small error of 1 mm with this construction.
- The bottom of the cup cannot be measured exactly. We measured a value of 1 mm. There will be some small error in this value of around 0.2 mm.
- The composition of the cup is not exactly known. We used polyethylene as a good approximation. In reality the cup is not exactly this kind of plastic and the acoustic impedance will differ some from the value we used.
- The height of the gelatin we used, will not be exactly the real height. During transportation of the cup the gelatin did not stay stable in the cup. Therefore this can differ around 5 mm. Though the problem should not be too big because the acoustic impedance of water is close to that of water.
- The thickness of the epoxy layer is also measured with a ruler. We roughly measured 2 mm of thickness but this can also vary 0,5 mm because of the error that a ruler gives when measuring with your eye.
- When a pulse hits the point of measurement, this point will move a bit. This gives us some small error. By averaging the signal, this error will vanish.
- When using a transducer, you will get some random noise everywhere. By averaging the signal we managed to eliminate most of this noise from our experiment.

4.2 Model

In section 2.4 we showed the assumptions we made when we developed the model. To summarize: We used a one dimensional linear wave equation in a layered structure with power law losses and using a single harmonic signal. We will now discuss the effects of the assumptions and show how the model could be expanded

The experiment shows that our model fails to predict the details of waves. This is partially due to the exclusion of dispersion and non-linear effects. These non-linear effects produce harmonics (Szabo 2014, p504). These harmonics distort the waveform and result in more attenuation, because their frequencies are higher, see figure 22. The dispersion is a result of a change in phase velocity as is required by causality (Szabo 2014, p84). Dispersion is absent in our model, because it doesn't incorporate a change in phase velocity.

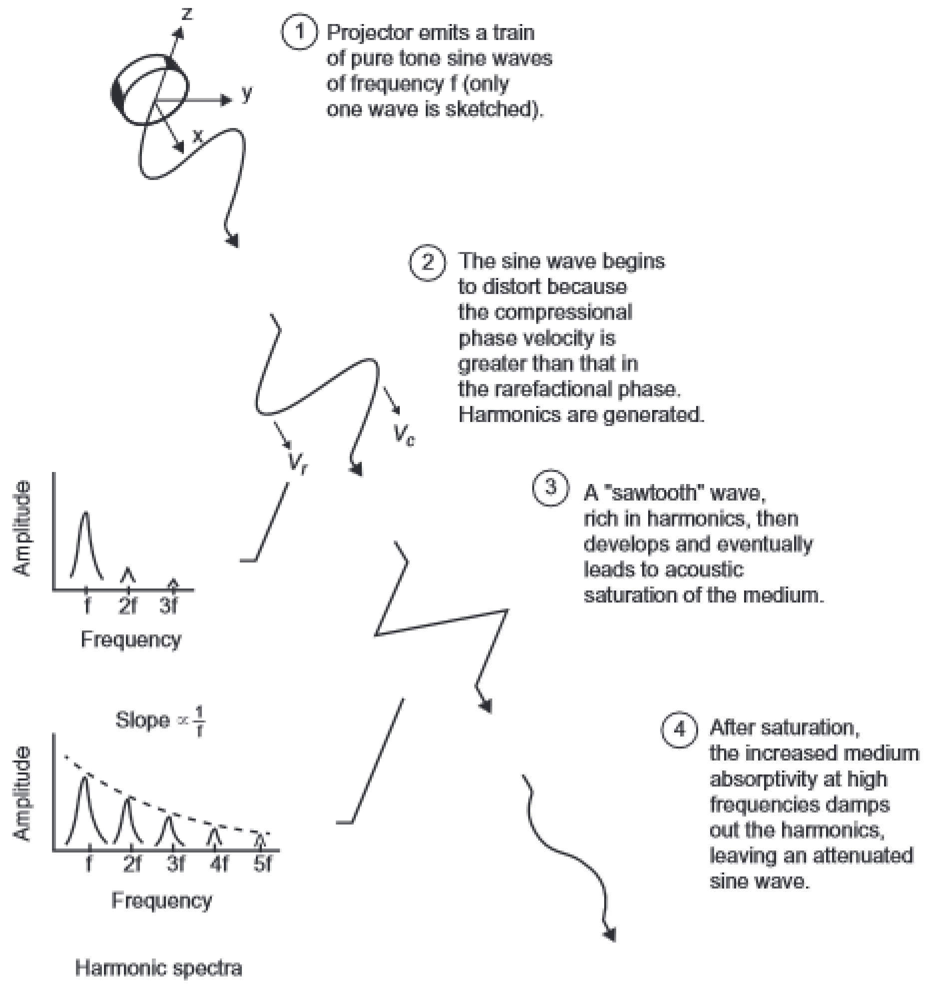


Figure 22: Evolution of a Shock Wave Beginning from a Plane Sinusoidal Wave Source. (Szabo 2014, p504)

The most inclusive model for ultrasound wave propagation is called the heterogeneous Westervelt equation (Treeby et al. 2012), see equation (17). It is a full three dimensional heterogeneous non-linear wave equation with losses according to a power law. This model is too complex to yield any analytic results, so it should be handled numerically. This can be done using a k-space pseudo-spectral method (Treeby et al. 2012).

$$\begin{aligned} &\nabla^2 p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} - \frac{1}{\rho_0}\nabla\rho_0 \cdot \nabla p + \frac{\beta}{\rho_0 c_0^4}\frac{\partial^2 p^2}{\partial t^2} - L\nabla^2 p = 0 \\ &L = -2\alpha_0 c_0^{y-1}\frac{\partial}{\partial t}\left(-\nabla^2\right)^{y/2-1} + 2\alpha_0 c_0^y \tan(\pi y/2)\left(-\nabla^2\right)^{(y+1)/2-1} \end{aligned} \tag{17}$$

We should also consider the inverse problem (Nagayasu 2006). For now we have always assumed we knew the structure of the medium on which we apply our formula's. However, generally speaking, this is not true. In our application, a hip-implant, we have a good idea of the layers inside a hip. So we could compare the model with a layer of bone resorption to a model without it. However it would be better if we could derive the structure of the medium given some output signals.

Another concept that comes up when using a transducer is diffraction (Szabo 2014, p 167). In our model we assumed that the transducer produces pure plane waves. However close to the transducer this is not true. The aperture of the transducer acts as a lens, focusing the acoustic wave, see figure 23. This process is called beam forming. It can be used to focus the acoustic wave on an particular area, to amplify the return form this area.

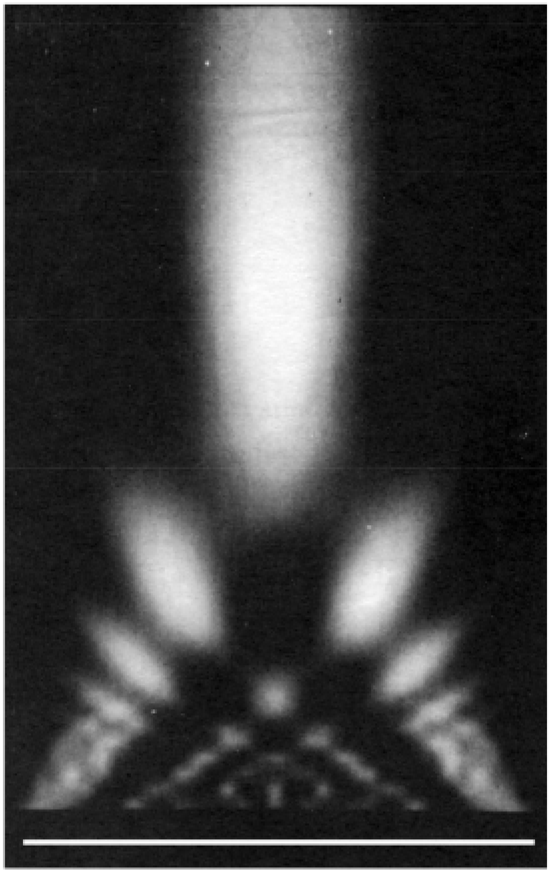


Figure 23: Top View of a Diffracted Field from a 40-wavelength-wide Line Aperture on the bottom. (Szabo 2014, p169)

We could also consider measuring at different frequencies. However this doesn't give us much results, because the speed of propagation and the reflection and transmission factors are independent of the frequency. Only the attenuation is dependent of the frequency. Moreover, we did not find any literature concerning this.

Finally, we could investigate the so-called shear waves (Qiang et al., 2015). It turns out that in solids, like bone, both longitudinal and transverse waves occur. These transverse waves are a product of shear pressures in a material. These waves usually travel at a different speed than the longitudinal waves. Therefore it produces a different wave pattern, not included in our model.

5 Conclusion

5.1 Answering the research questions

In this section we give our conclusion about the possibility to detect hip failures with ultrasound. This was the primary objective but on our way there were some other problems that are tackled. We will make some statements about those secondary questions as well.

Our first sub question is about describing the propagation of acoustic waves in different media. Especially with the waves propagating in a multilayered structure. The linear wave model with absorption turns out to be a good model to use for this problem at normal ultrasound frequencies, according to an experiment. The waves propagate with the expected speed and reflect back as expected. However, the amplitude turns out to be lower than predicted because of wave dispersion. For low ultrasound frequencies the experiment does not validate the model. This may be due to dispersion and non-linear effects, not included in the model, or because of an artifact from the transducer.

Secondly, we tried to measure the difference in density in the different media. It can be done as long as there is a big enough difference between the densities or the speed of sound differs greatly between different media. This will result in reflections which are strong enough to reflect back to the transducer.

Further we wanted to detect hip failure with ultrasound. To do this we need to detect a layer which resembles water between the hip implant and the hip bone. We concluded using our model that for normal frequencies this is impossible, but our model suggests that is might be possible for low ultrasound frequencies. However as mentioned above for these frequencies our model does not have experimental validation. Therefore we can't make a firm statement about the possibility or impossibility to detect hip failure with low frequency ultrasound.

5.2 Future research

Future research should focus on two areas: expanding the model and comparing the model with better data. The model could be expanded with non-linear effects and dispersion. This will yield a more realistic model, especially at low ultrasound frequencies. A broader discussion of expansions to the model is found in section 4.2.

Better data could be obtained if actual tissues are used. An experiment could be arranged with animal tissues. This experiment would yield data with a closer resemblance to a real human hip. On top of that a transducer should be used which is optimized for low ultrasound frequencies. Another option is to take ultrasound measurements on patients with hip implants. This would be especially useful in combination with X-ray measurements to determine if implant failure has occurred.

6 References

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A Appendix

A.1 Index

Name	symbol	unit
Spatial coordinate	x	m
Time	t	s
Pressure	p	Pa
Density	ρ	kg mm^{-3}
Density at rest	ρ_0	kg mm^{-3}
Frequency	f	Hz
Isotropic speed of sound	c	m s^{-1}
Angular frequency	ω	rad s^{-1}
Wavelength	λ	m
Wavenumber	k	m rad^{-1}
Acoustic Impedance	Z	Pa s m^{-1} or $Rayl$
Transmission factor between layer i and j	$TF_{i,j}$	-
Reflection factor between layer i and j	$RF_{i,j}$	-
Attenuation factor	α	m^{-1}
Power law exponent	y	-
Power law prefactor	$\hat{\alpha}$	Hz^{-y}m^{-1}
Nonlinearity parameter	β	-

Table 4: Index of variables

A.2 Constants

Tissue	c (m/s)	α	y	ρ (kg/m^3)	Z $(MRayl)$	B/A
Air @ $20°C$	343	-	-	1.204	$4.13 \cdot 10^{-4}$	-
Blood	1584	0.14	1.21	1060	1.679	6
Bone	3198	3.54	0.9	1990	6.364	-
Brain	1562	0.58	1.3	1035	1.617	6.55
Breast	1510	0.75	1.5	1020	1.540	9.63
Epoxy	3070 [4]	-	-	1200 [5]	3.684	-
Fat	1430	0.6	1	928	1.327	10.3
Gelatin	1482 [4]	-	-	980 [6]	1.452	-
Heart	1554	0.52	1	1060	1.647	5.8
Kidney	1560	10	2	1050	1.638	8.98
Liver	1578	0.45	1.05	1050	1.657	6.75
Muscle	1580	0.57	1	1041	1.645	7.43
Polythylene	2460 [4]	-	-	980 [7]	2.411	-
Spleen	1567	0.4	1.3	1054	1.652	7.8
Water @ $20°C$	1482.3	2.17e-3	2	1000	1.482	4.96

Table 5: Properties of tissues

Note all data in table 5 except lines 6,8 and 13 is from (Szabo 2014, p785).