Analysis of the Super-Regenerative Receiver using Transmit Reference

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Abstract

The Super-Regenerative Receiver (SRR) is analysed regarding the suitability and desirability of the SRR for Transmit Reference (TR) modulation. Due to its simple design and low power dissipation, the SRR is a popular receiver topology in low power design such as Wireless Sensor Networks (WSN).

TR modulation uses two tones for the wireless communication where one tone acts as the reference for the actual modulated tone. The spacing between the two tones is defined as $\Delta \omega$ and the receiver demodulates the data by using the reference tone. The advantage of TR is that the wireless communication is not based on the absolute frequency of the carrier tones but on the frequency spacing. Because the reference is sent as well, there is no local oscillation at the receiver, which reduces the power consumption of the receiver.

The SRR is analysed in the time domain and frequency domain. The frequency spectrum at the output of the SRR was calculated by means of a matrix calculation, the Matrix Model. From the analysis, it appears that the output spectrum consists of infinite side lobes, spaced by the quench frequency, $\omega_q$. Based on the analysis with the Matrix Model, it is concluded that the SRR can be assumed to be linear.

The main trade-off in the design of a SRR is between gain and bandwidth. The gain is set by the negative surface of the total conductance. The bandwidth is set by the slope of the quench signal at the sensitivity period and the Q-Factor of the bandpass filter.

When the output of the SRR is squared, each data side lobe is multiplied with its peer reference side lobe. The multiplication between the data and the reference results in the down conversion of the data to $\Delta \omega$.

The SRR is suitable for TR when $\omega_q$ is at least two times larger than $\Delta \omega$. The best input noise folding performance is achieved when $\omega_q$ is set to two times $\Delta \omega$. This results in a decrease of the NF of 2 $dB$ compared to the lowest NF at other quench frequencies.

This thesis is structured as follows. First, an introduction of the research is given in Chapter 1. The SRR is analysed in the time domain and frequency domain in Chapter 2. Next, the design procedures for the design of the SRR are presented in Chapter 3. In Chapter 4, the TR modulation scheme is applied to the SRR and an analysis is given regarding input noise folding and frequency spacing. The results are discussed in Chapter 5 and this thesis is summarized and concluded in Chapter 6. This thesis is the result of the Master Graduation Assignment of the Electrical Engineering curriculum at the University of Twente.

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Chapter 1

Introduction

1.1 Introduction

Nowadays, in current applications, short- to midrange communication systems such as Wireless Sensor Network (WSN) require Ultra-Low Power (ULP) designs for their receivers [1]. ULP designs have a power dissipations of a few mW. The Super-Regenerative Receiver (SRR) is able to meet these demands and the trend shows that the power consumption of the SRR is moving deeper in the sub-mW region [2][3]. This range of power consumption is called Ultra²Low Power (U²LP). Due to its simplicity and few components, the SRR can achieve a very low power dissipation of a few hundred µW with high voltage gain [3][4][5]. This makes the SRR a good candidate as receiver topology for U²LP designs.

1.2 Super Regeneration

The SRR is based on a bandpass filter with an added negative resistance. This topology was first published by Armstrong in 1922 [6]. The principle of the amplification is based on periodically changing the conductance from positive to negative. In Figure 1.1, a simplified schematic of the SRR is shown.

The SRR shown in Figure 1.1 consists of a parallel RLC filter and a negative resistance. The RLC filter has a bandpass characteristic from input current to output voltage. By changing the total conductance of the RLC filter, the state of the RLC filter is changed. When the total conductance is positive, the SRR is behaving as a normal RLC filter and the incoming signal is filtered. When the total conductance is zero, the SRR behaves as a oscillator that resonates at the resonance frequency of the RLC filter. Each time the input signal adds energy to the RLC filter, the amplitude of the oscillation increases. When the total conductance is negative, the SRR oscillates at the center frequency and the amplitude of the oscillation increases, without addition of energy of the input signal.

By changing the total conductance periodically from positive to negative, the input signal is periodically amplified. The periodic change of the total conductivity is the fundamental concept of the SRR and the oscillation and attenuation of the signal is called quenching. The control signal of the negative resistance is called the quench signal.

This topology has become popular during the second world war for its low power dissipation and simple design. The SRR was implemented in the Identification Friend or Foe (IFF) Mark III pulse transponders of the Royal Air Force in the United Kingdom. The IFF system identified the
aircraft by radio communication. After the war, the SRR lost popularity due to less selectivity and sensitivity compared to the heterodyne receiver. With current high demands on receiver’s power dissipation, the SRR regained popularity.

![Simplified circuit of the Super-Regenerative Receiver.](image)

**Figure 1.1:** Simplified circuit of the Super-Regenerative Receiver.

### 1.3 Transmit-Reference Modulation

In typical wireless communication, the information is modulated on a single sinusoidal tone that is transmitted. Because the receiver knows the absolute frequency of the transmitted tone, the receiver can demodulate the information. Transmit Reference (TR) uses two sinusoidal tones for the communication where one tone functions as the reference for the other tone [7]. In Figure 1.2, a system design is shown of the TR architecture of a transmitter and receiver.

![System architecture of a transmitter and receiver using Transmit Reference modulation](image)

**Figure 1.2:** System architecture of a transmitter and receiver using Transmit Reference modulation

At the transmitter, a carrier tone is modulated with the data and the carrier frequency is shifted in frequency. The frequency shift is defined as $\Delta \omega$. At the antenna, the original unmodulated tone is added to the modulated tone and it acts as a reference for the other tone and both tones are transmitted. The frequency spectrum of the transmitted tones are shown in Figure 1.3a.

At the receiver, the reference tone and the modulated tone are squared. When two tones are squared and filtered by the bandpass filter, it gives the following equation.
(a) Freq. spectrum at the input of the receiver.

(b) Freq. spectrum at the output of the squarer.

(c) Freq. spectrum after the bandpass filter.

(d) Freq. spectrum at the output of the receiver.

Figure 1.3: Freq spectra at the receiver using TR.

\[
\left[ \cos(\omega \lambda t) + m(t)\cos(\omega \gamma t) \right]^2 \cdot H_{BPF}(\omega) \\
= \left[ \cos(\omega \lambda t)^2 + m(t)^2\cos((\omega \gamma)t)^2 + 2 \cdot m(t)\cdot \cos(\omega \lambda t)\cos(\omega \gamma t) \right] \cdot H_{BPF}(\omega) \\
= \left[ \frac{1}{2} + \frac{1}{2}\cos(2\omega \lambda t) + \frac{1}{2}\cos(2\omega \gamma t) + m(t)\cos((\omega \lambda - \omega \gamma) t) + m(t)\cos(2\omega \gamma t) \right] \cdot H_{BPF}(\omega) \\
= m(t)\cos((\omega \lambda - \omega \gamma) t) = m(t)\cos(\Delta \omega t)
\]

(1.1)

Where \( \cos(\omega \gamma t) \) is the modulated tone, \( \cos(\omega \lambda t) \) is the reference tone and \( H_{BPF}(\omega) \) is the bandpass filter transfer function after the squarer. \( H_{BPF}(\omega) \) has bandpass frequency of \( \omega_0 \). The frequency spectrum at the output of the squarer is shown in Figure 1.3b. Due to the squarer, the modulated tone and the reference tone are multiplied with each other and the multiplication of the two tones will down convert the signal. When two sinusoidal tones are multiplied with each other, it results in two tones at the sum frequency and difference frequency of the squared tones. The difference frequency between the two tones is set by the transmitter to \( \Delta \omega \). By bandpass filtering the output of the squarer at \( \Delta \omega \), the tone with the difference frequency tone is selected. The frequency spectrum and the bandpass characteristic is shown in Figure 1.3c. The signal is further down converted to baseband and filtered by a low pass filter. Now the signal can be converted to the digital domain and the information can be extracted. The frequency spectrum after the down conversion to baseband and low pass filtering is shown in Figure 1.3d.

Unlike with conventional wireless communication, the synchronization between the transmitter and receiver with TR is not based on the absolute frequency of the carriers but on the frequency offset \( \Delta \omega \) between the two tones. As long as \( \Delta \omega \) is fixed and known at the receiver, the absolute frequency of the transmitted tones can differ at the transmitter and the receiver can still demodulate the signal correctly without knowing the absolute frequencies of the tones.

Because the reference is sent as well, there is no need to reproduce the high frequency signal at the receiver. This saves power as the reproduction of a clean high frequency signal for demodulation at the receiver is power consuming.
1.4 Thesis

The SRR can amplify the signal at low power consumption which makes it a good candidate for applications that demand U2LP specifications. In this thesis, a research is presented on the suitability and desirability of the Super-Regenerative Receiver (SRR) using Transmit-Reference (TR) modulation.

First in Chapter 2, the operation of the SRR is scrutinized and a time domain analysis and frequency domain analysis of the receiver is presented. Secondly, in Chapter 3, a design procedure is presented to design a SRR. In Chapter 4, the TR modulation is applied to the SRR and the operation is discussed based on frequency spacing and noise folding. The results of this research are discussed and recommendations are given in Chapter 5. In Chapter 6, this thesis is concluded and summarized.
Chapter 2

Super Regeneration

2.1 Introduction

In Section 1.2, the behaviour of the SRR was explained in a nutshell. In this chapter, the SRR is scrutinized and a mathematical analysis of the behaviour of the SRR is given. The analysis gives more insight on how this time varying system works and it determines whether the SRR can be assumed to be linear. The analyses give a good foundation for the further analysis of the SRR with TR.

In Section 2.2, a time domain analysis is given of the SRR. In Section 2.3, a frequency domain analysis is given and it is compared to a Simulink model. Finally, the Chapter is summarized and concluded in Section 2.4.

2.2 Time Domain Analysis

To start the analysis of the SRR in the time domain, first an analysis of a bandpass filter is given. The circuit of this filter is shown in Figure 2.1. The circuit consists of a second order parallel RLC filter and it has a bandpass filter characteristic around the resonance frequency. The input signal is differentially inserted by means of a current at $I_{in}^+$ and $I_{in}^-$. The output signal is present at the same nodes as a voltage ($V_{out}^+$ and $V_{out}^-$). Using the current to voltage relation, the differential equation of a parallel RLC filter is as follows.

$$\frac{v}{R} + \frac{1}{L} \int_0^t v dt + C \frac{dv}{dt} = i_{in},$$

Where $v$ is the output voltage, $i_{in}$ is the input current and $R$, $L$ and $C$ is the resistance, inductance and capacitance of the RLC filter, respectively. $i_{in}$ is a sinusoidal signal, defined as follows.
\[ i_{in} = A \cdot \sin(\omega t) \]  
\[2.2\]

Where \( A \) is the input amplitude and \( \omega \) is the input frequency. By applying Equation 2.2 to Equation 2.1 and differentiating it with respect to \( t \) and dividing it by \( C \), the differential equation is as follows.

\[
\frac{d^2v}{dt^2} + \frac{1}{RC} \frac{dv}{dt} + \frac{v}{LC} = A \omega \cdot \cos(\omega t)
\]
\[2.3\]

To find the homogeneous solution for \( v \), \( i_{in} \) is set to 0 and the assumption is made that \( v \) is of exponential form. This assumption is made because the derivative of an exponential function is also an exponential function and the homogeneous solution can be found. \( v \) is defined as follows [8].

\[
v = V \cdot e^{st}
\]
\[2.4\]

Where \( V \) is the amplitude and \( s \) is complex input frequency. Applying the definition of Equation 2.4 at Equation 2.3, it results in the following equation.

\[
V \cdot e^{st} \left( s^2 + \frac{s}{RC} + \frac{1}{LC} \right) = 0.
\]
\[2.5\]

For Equation 2.5 to be equal to 0, \((s^2 + \frac{s}{RC} + \frac{1}{LC})\) has to be zero. \( e^{st} \) is never zero and \( V \) cannot be zero because this would imply that \( v \) is always zero, even when there is energy stored in the inductor or capacitor [8]. In order to find where the quadratic term is zero, the roots of the quadratic term have to be found. The roots are found by using Quadratic Formula and the roots are as follows.

\[
s_{1,2} = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}},
\]
\[2.6\]

For simplicity, the following definitions are made.

\[
\alpha = \frac{1}{2RC} = \frac{g_{tot}}{2C} \quad \text{(2.7)}
\]

\[
g_{tot} = \frac{1}{R} \quad \text{(2.8)}
\]

\[
\omega_0 = \frac{1}{\sqrt{LC}} \quad \text{(2.9)}
\]

\[
\omega_d = \sqrt{\alpha^2 - \omega_0^2} \quad \text{(2.10)}
\]

\[
s_{1,2} = -\alpha \pm \omega_d \quad \text{(2.11)}
\]

Where \( g_{tot} \) is the total conductance of the RLC filter. Because the quadratic function contains two roots \( (s_1 \text{ and } s_2) \), the voltage response \( (v) \) can be either \( s_1 \) or \( s_2 \). Therefore the solution of the homogeneous solution is as follows.

\[
v(t) = V_1 e^{s_1 t} + V_2 e^{s_2 t}
\]
\[2.12\]
There are three possible outcomes for $s_{1,2}$. The first outcome is that $s_{1,2}$ are real numbers. Then $\omega_0^2 < \alpha^2$ and the RLC filter is overdamped. This means that signal will not oscillate and it is damped. The second outcome is when $\omega_0^2 = \alpha^2$, than $s_{1,2}$ are equal to $\alpha$ and the RLC filter is critically damped. The third outcome is when $\omega_0^2 > \alpha^2$. Then, $s_{1,2}$ are imaginary numbers as the root of Equation 2.10 is negative. The RLC filter is then underdamped. The signal oscillates and eventually it will be damped out when the total conductance is positive. The underdamped voltage response of the RLC filter is at interest for the analysis of the SRR. The underdamped voltage response of the SRR is as follows.

$$v(t) = V_1 \cdot e^{(-\alpha+j\omega_d)t} + V_2 \cdot e^{(-\alpha-j\omega_d)t}$$

$$= V_1 \cdot e^{-\alpha t} e^{j\omega_d t} + V_2 \cdot e^{-\alpha t} e^{-j\omega_d t}$$

$$= e^{-\alpha t} ((V_1 + V_2)\cos(\omega_d t) + j(V_1 - V_2)\sin(\omega_d t))$$

$$= e^{-\alpha t} \left( B_1 \cos(\omega_d t) + B_2 \sin(\omega_d t) \right)$$

(2.13)

Where $B_1 = V_1 + V_2$ and $B_2 = j(V_1 - V_2)$. The values of $B_1$ and $B_2$ are related to the initial values of $v(0^+)$.

To find the values of $B_1$ and $B_2$, the assumption is made that the voltage is built up to the input amplitude ($A$) from previous cycles at $t = 0$ and that the slope of the voltage response is $0$ at $t = 0$. This results in the following equations [8],

$$v(0^+) = A = e^{\alpha 0} (B_1 \cdot 1 + B_2 \cdot 0) = B_1$$

(2.14)

$$\frac{dv(0^+)}{dt} = 0 = \alpha \cdot B_1 + \alpha \cdot B_2$$

(2.15)

$$B_2 = -B_1$$

(2.16)

Applying the assumptions and the values of $B_{1,2}$, Equation 2.13 is as follows.

$$v = e^{-\alpha t} \left[ A \cdot \cos(\omega_d t) - A \cdot \sin(\omega_d t) \right]$$

(2.17)

Using Equation 2.7, Equation 2.17 will be as follows.

$$v = A \cdot e^{-g_{tot} t \frac{t}{2 \epsilon}} \left[ \cos(\omega_d t) - \sin(\omega_d t) \right]$$

(2.18)

Equation 2.17 shows that the amplitude of the voltage response is proportional to $e^{-g_{tot} t}$. By adding negative resistance ($g_m(t)$) to the parallel RLC filter, $-g_{tot}$ is changed. When the total conductance is negative, $e^{-g_{tot} t}$ becomes positive. The principle of the SRR is to control $g_{tot}$ in a periodic way. The value of the negative resistance is controlled by the quench signal. The schematic with added negative resistance is shown in Figure 2.2.

Figure 2.2: Simplified circuit of the Super-Regenerative Receiver.
In order to make the total conductance smaller negative, the conductance of negative resistance should elevate the initial conductance (defined by R) at the RLC filter. The conductance of the negative resistance is defined as follows.

\[ g_m(t) = a_0 + a_1 \cos(\omega_q t) \]  \hspace{1cm} (2.19)

Where \( a_0 \) is the constant term of the negative conductance and \( a_1 \) is the amplitude of the periodic term. The total conductance is defined as follows.

\[ g_{tot}(t) = g - g_m(t) = \frac{1}{R} - g_m(t) \]  \hspace{1cm} (2.20)

At each cycle when \( g_m(t) \) is larger than \( g \), \( g_{tot} \) becomes negative and \( \alpha \) becomes negative. This gives the exponential function a positive power and the output voltage will be increased. In Figure 2.3, a plot of the total conductance and the amplification is shown. The amplification of the input signal starts when the \( g_{tot} \) changes from positive to zero (at time instance \( \pi/2 \) and \( 5\pi/2 \) in Figure 2.3). The time around transition from positive to negative conductance is called the sensitivity period [9]. At the sensitivity period, the SRR is most sensitive and it has an infinite gain at the resonance frequency for an infinite small time instance. This can be observed from the impedance. When the total conductance is 0, the circuit shown in Figure 2.2 will only consist of a parallel \( L \) and \( C \). The impedance of the circuit at \( \omega_0 \) is as follows.

\[ Z(\omega_0) = \frac{1}{j\omega C + \frac{1}{j\omega L}} = \frac{j\omega L}{(j\omega)^2 LC + 1} = \frac{j\omega L}{-1 + 1} \]  \hspace{1cm} (2.21)

With an impedance of infinite, the \( I \) to \( V \) conversion of the bandpass filter is at its highest and thus most sensitive.

Equation 2.18 shows that the amplitude of the output voltage is a function of the amplitude of the input current (A). Based on Equation 2.18, it seems that the SRR is a linear time varying system.

In order to guarantee a stable operation after each quench cycle, the average value of the total conductance (\( \bar{g}_{tot} \)) should be positive [10]. This guarantees that there is no residual energy remaining in the inductance and capacitor of the RLC filter from previous quench cycles and that the oscillation is damped out after each quench cycle. \( \bar{g}_{tot} \) is determined by \( a_0 \) in Equation 2.19.

The periodically change of the conductance of the circuit results in that the input signal is only sensed at the sensitivity period. The sensitivity period is determined by the quench signal and the frequency of the sensitivity period is equal to \( \omega_q \). Therefore, \( \omega_q \) should meet the Nyquist criterion that each bit should be sampled twice to be able to detect it [11]. This means that the quench frequency should be at least two times the baud rate.

### 2.3 Frequency Domain Analysis

Because the system is time variant, the frequency spectrum cannot be determined by taking the Fourier transform of the impulse response of the SRR. Therefore a different approach is used to determine the frequency response [12]. This analysis gives more insight on the frequency spectrum at the output of the SRR that is crucial for the analysis of the SRR using TR.

To start the analysis, the circuit of the SRR is converted to a block diagram, shown in in Figure 2.4. In Figure 2.4, the second order RLC filter is represented as the bandpass filter and \( g_m(t) \) is represented as the transconductance feedback amplifier with an amplification of \( g_m(t) \). The input and output of the SRR are called \( x(t) \) and \( y(t) \), respectively. \( u(t) \) is defined as the feedback result that is added to the input. \( x(t) \) and \( u(t) \) are established in the current...
domain and $y$ is defined in the voltage domain. The signal is received by the antenna and a Transconductance Amplifier (TCA) amplifies and converts the voltage to the current domain. The converted current is defined as the input current of the SRR($x(t)$) and it is defined as follows.

$$x(t) = A \cdot \cos(\omega_0 t) \quad (2.22)$$

Where $A$ is the input amplitude and $\omega_0$ is the input frequency of the input signal. At the first loop iteration, the input signal $x(t)$ is filtered by the bandpass filter $z(t)$. The transfer function of the bandpass filter is defined as $z(t)$ because a conversion is made from the current domain to the voltage domain. Thus, the transfer function is equal to the impedance. Next, the output signal is fed back and added to the input through $g_m(t)$. $g_m(t)$ can be seen as a mixer and an amplifier in parallel due to the periodic term and constant term of $g_m(t)$. $g_m(t)$ mixes the output signal $y(t)$ and the mixing results in the addition of two side lobes in the frequency domain. The side lobes are spaced with $\omega_q$. The output of $g_m(t)$ that is defined as $u(t)$ is then added to the input and the next loop iteration begins. The input signal at the $z(t)$ now contains three tones, two side lobes and the input tone. Each time the signal is mixed by $g_m(t)$, the tones are mixed and more side lobes are added. After infinite loop iterations, the frequency spectrum at $y(t)$ consists of infinite side lobes that are all spaced with $\omega_q$. The bandpass filter attenuates the side lobes that are not at the resonance frequency. Now $y(t)$ can be defined as follows.
The frequency domain can be calculated. The calculation is as follows.

\[ y(t) = \sum_{n=-\infty}^{\infty} y_n \cdot e^{j(\omega_0 + n\omega_q)t} \]  

(2.23)

Where \( y_n \) is the amplitude coefficient of each side lobe. With \( y(t) \) and \( g_m(t) \) defined, \( u(t) \) can be determined.

\[
u(t) = g_m(t) \cdot y(t) \\
= (a_0 + \frac{a_1}{2} \cdot (e^{j\omega_0 t} + e^{-j\omega_0 t})) \sum_{n=-\infty}^{\infty} y_n \cdot e^{j(\omega_0 + n\omega_q)t} \\
= a_0 \sum_{n=-\infty}^{\infty} y_n \cdot e^{j(\omega_0 + n\omega_q)t} + \frac{a_1}{2} \sum_{n=-\infty}^{\infty} y_n \cdot e^{j(\omega_0 + (n+1)\omega_q)t} \\
+ \frac{a_1}{2} \sum_{n=-\infty}^{\infty} y_{n+1} \cdot e^{j(\omega_0 + n\omega_q)t} \\
= \sum_{n=-\infty}^{\infty} (a_0 \cdot y_n + \frac{a_1}{2} \cdot y_{n-1} + \frac{a_1}{2} \cdot y_{n+1}) \cdot e^{j(\omega_0 + n\omega_q)t} \]  

(2.24)

The time domain response of the block diagram of the SRR is as follows.

\[ y(t) = (x(t) + u(t)) \ast z(t) \]  

(2.25)

To avoid the convolution in Equation 2.25 of \( u(t) \) and \( x(t) \) with \( z(t) \), the transfer is made to the frequency domain. A convolution in the time domain is a multiplication in the frequency domain. The Fourier transforms of \( g_m(t) \), \( x(t) \), \( y(t) \) and \( u(t) \) are as follows.

\[ G_m(\omega) = \mathcal{F}\{g_m(t)\} = a_0 \cdot \delta(\omega) + \frac{a_1}{2} (\delta(\omega - \omega_q) + \delta(\omega + \omega_q)) \]  

(2.26)

\[ X(\omega) = \mathcal{F}\{x(t)\} = A \cdot \delta(\omega - \omega_0) \]  

(2.27)

\[ Y(\omega) = \mathcal{F}\{y(t)\} = \sum_{n=-\infty}^{\infty} y_n \cdot \delta(\omega - \omega_0 - n\omega_q) \]  

(2.28)

\[ U(\omega) = \mathcal{F}\{u(t)\} = \sum_{n=-\infty}^{\infty} (a_0 \cdot y_n + \frac{a_1}{2} \cdot y_{n-1} + \frac{a_1}{2} \cdot y_{n+1}) \cdot \delta(\omega - \omega_0 - n\omega_q) \]  

(2.29)

By summing the current and using the current-to-voltage relation, the transfer function in the frequency domain can be calculated. The calculation is as follows.

\[ I(\omega) = V(\omega) \cdot \left(1/R + 1/j\omega L + j\omega C\right) \]  

(2.30)
Rewriting Equation 2.30 to \( Z(\omega) \), gives the following equation and the transfer function.

\[
\frac{V(\omega)}{I(\omega)} = \frac{1}{1/R + 1/j\omega L + j\omega C} = \frac{j\omega/C}{-\omega^2 + j\omega/RC + 1/LC} \tag{2.31}
\]

Now Equation 2.25 can be rewritten in the frequency domain and \( Y(\omega) \) can be calculated.

\[
Y(\omega) = (X(\omega) + U(\omega)) \cdot Z(\omega)
\]

\[
= A \cdot \delta(\omega - \omega_0) \cdot Z(\omega_0) + \sum_{n=-\infty}^{\infty} (a_0 \cdot y_n + \frac{a_1}{2} \cdot y_{n-1} + \frac{a_1}{2} \cdot y_{n+1}) \cdot \delta(\omega - \omega_0 - n\omega_q) \tag{2.32}
\]

Based on Equation 2.32, the values for \( y_n \) can be calculated. The values of \( y_n \) can be generalized for the value of \( n \) as follows.

\[
y_n = \begin{cases} 
(A + a_0 \cdot y_0 + \frac{a_1}{2} (y_{-1} + y_1)) \cdot H(\omega_0) & \text{for } n = 0, \\
(a_0 \cdot y_n + \frac{a_1}{2} (y_{n-1} + y_{n+1})) \cdot H(\omega_0 + n\omega_q) & \text{for } n \neq 0 \end{cases} \tag{2.33}
\]

The summation goes from minus infinity to plus infinity as there are infinite loop iterations. Because of the bandpass filter characteristic of \( Z(\omega) \), side lobes with a frequency that is far away from \( \omega_0 \) are almost fully attenuated by \( Z(\omega) \) and these side lobes can be neglected without introducing a too large error. The upper and lower limits are defined as \( N \) and \( -N \), respectively. Equation 2.32 can now be written in a matrix form.

\[
y = Zx + G_mZy \tag{2.34}
\]

Where \( y, Z, x \) and \( G_m \) are defined as follows.

\[
y = [y_{-N}, \ldots, y_0, \ldots, y_N]^T \tag{2.35}
\]

\[
Z = \begin{bmatrix}
H(\omega_0 - N\omega_q) & 0 & \cdots & 0 \\
0 & \ddots & \ddots & \ddots \\
\vdots & \ddots & H(\omega_0) & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & 0 \\
0 & \ddots & \ddots & 0 & H(\omega_0 + N\omega_q)
\end{bmatrix} \tag{2.36}
\]

\[
x = [0, \ldots, A, \ldots, 0]^T \tag{2.37}
\]

\[
G_m = \begin{bmatrix}
a_0 & \frac{a_1}{2} & 0 & \cdots & 0 \\
\frac{a_1}{2} & a_0 & \frac{a_1}{2} & \ddots & \vdots \\
0 & \ddots & \ddots & \ddots & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots \\
0 & \cdots & 0 & \frac{a_1}{2} & a_0 \frac{a_1}{2} \\
0 & \cdots & 0 & a_0 & \frac{a_1}{2}
\end{bmatrix} \tag{2.38}
\]

Where \( y \) and \( x \) are vectors with a length of \( 2N + 1 \) and \( Z \) and \( G_m \) are square matrices with a size of \( 2N + 1 \). When a non sinusoidal quench waveform is used, \( G_m \) can be changed to the
Fourier coefficients of the desired waveform and so can the calculation of the frequency spectrum be extended to any waveform. Equation 2.34 can be rewritten as follows.

\[(I - G_m Z)y = Zx\] (2.39)

Where \(I\) is the unit matrix with a size of \(2N + 1\). To find the values of \(y_n\), Equation 2.34 needs to be solved. As this matrix solution consists of up to \(2N + 1\) equations and \(N\) can be up to 200, \texttt{Matlab}\’s function \texttt{linsolve} is used to solve the equations for the value of \(y_n\). \texttt{linsolve} returns \(y\), containing the amplitudes for each frequency component. Based on this method, the frequency spectrum at the output of the SRR can be calculated. Because the calculation only consists of multiplications and summations, it is a linear operation. This model for the calculation of the frequency spectrum of SRR is called the \textit{Matrix Model}.

The passband impedance of \(Z(\omega)\), that is shown in Figure 2.2, is defined by the value of \(R\). The open loop gain (\(\zeta\)) can be calculated as follows.

\[\zeta = R \cdot a_0 = \frac{a_0}{g}\] (2.40)

For an initial stable system with positive feedback, the open loop gain should be smaller than 1. This relation shows that \(a_0\) should be smaller than \(g\). This means in the time domain analysis that the average total conductance should be positive and this conclusion is also drawn in Section 2.2.

The presented Matrix Model is validated by comparing the values of \(y_n\) with the frequency spectrum obtained by simulating the \texttt{Simulink} model of the SRR. A screen shot of the \texttt{Simulink} model is shown in Figure 2.5. The parameters of the SRR are as follows.

- \(R = 50 \, \Omega\)
- \(L = 2 \, nH\)
- \(C = 15.64 \, nF\)
- \(\omega_0 = 900 \, MHz\)
- \(Z(s) = \frac{s/C}{s^2 + s/RC + 1/\sqrt{LC}}\)
- \(a_0 = 19.95 \, mS\)
- \(a_1 = 0.2 \, mS\)
- \(\omega_q = 500 \, kHz\)

In Figure 2.6, the calculated frequency spectrum and simulated frequency spectrum of the SRR are plotted.

Because the Matrix Model does follow the frequency spectrum of the \texttt{Simulink} model, it can be concluded that Matrix Model gives a good representation of the frequency spectrum of the SRR and it seems that, based on the time domain analysis and the Matrix Model, the SRR is linear. The simulated frequency spectrum does not meet the matrix model for frequencies that are located further away from the center frequency. The power of these side lobes is lower than the quantization noise of the \texttt{FFT} calculation and therefore these side lobes cannot be detected.

In order verify whether the SRR can be assumed to be linear, the frequency spectrum is calculated by the Matrix Model for two tones. The calculation of the frequency spectrum is done for each tone individually and the outcomes are then added. Next, the two tones are applied
simultaneously to the Simulink model and the frequency spectrum is plotted. In Figure 2.7, the calculated frequency spectrum and simulated frequency spectrum of the two tones is presented. When Figure 2.7 is studied, it can be concluded that the response of the SRR for two tones individually and added is the same as the response of the SRR when the two tones are applied simultaneously. This shows that the SRR can be assumed to be linear.

### 2.4 Conclusion

In this Chapter, the time domain analysis and frequency domain analysis are presented. The time domain analysis is derived from an underdamped parallel RLC filter with a periodic changing conductance. The frequency domain analysis is based on a Matrix Model. Because the system is time variant, the frequency domain analysis cannot be calculated by taking the Fourier transform of the impulse response of the SRR. The Matrix Model allows to calculate the frequency spectrum and frequency response of the SRR faster than other simulation programs such as Simulink. Based on the voltage response shown in Equation 2.18 and the validation of the Matrix Model, it is assumed that the SRR is linear. The response of the SRR for two tones individually is equal to the response of the SRR when the two tones are applied simultaneously.
Figure 2.6: Plot of the frequency spectrum of the SRR, calculated by means of the Matrix Model (black) and simulated by Simulink (red).

Figure 2.7: Plot of the frequency spectrum of the SRR for two tones of 900 MHz and 905 MHz. The stem plot (black) is calculated by the Matrix Model and the line plot (red) is simulated by means of the Simulink model.
Chapter 3

Design Procedures

3.1 Introduction

In this Chapter, the analysis that is discussed in Chapter 2, is applied to design a SRR. This chapter is structured in such a way that it gives a design procedure for the design of a SRR and it gives insight on the trade-offs of the SRR design. The design procedure is discussed in Section 3.2 to 3.4 and the conclusion is drawn in Section 3.5.

3.2 Resonance Frequency & Open Loop Gain

The first step of the design of the SRR, is to define the bandpass filter. The bandpass filter is the main building block that defines the center frequency of the receiver. For the optimum selectivity and gain, the resonance frequency of the bandpass filter ($\omega_0$) is set to be equal to the carrier frequency of the wireless communication.

The next step is to define the passband impedance of the bandpass filter. The passband impedance is defined as the current to voltage conversion gain of the bandpass filter at the resonance frequency. With the passband impedance determined, $a_0$ can be defined. In order to have a stable system, the average open loop gain should be smaller than 1. This guarantees that the oscillation that is built up by previous quench cycles is fully attenuated after each quench cycle. $a_0$ is calculated as follows.

$$a_0 < \frac{1}{Z(\omega_0)} \quad (3.1)$$

The closer the average open loop gain is to 1, the more sensitive it is. The sensitivity comes with the price of stability. When the passband impedance is due to mismatch slightly higher than expected, $\zeta$ is higher than 1 and the system is unstable.

To achieve amplification, $a_1$ has to be set. For amplification and enable quenching, the open loop gain should be periodically larger than 1. Thus $Z(\omega_0) \cdot (a_0 + a_1) > 1$, assuming that $a_1$ is the maximum value of the periodic term. Higher values of $a_1$ gives a higher open loop gain and therefore more amplification. For now, $a_1$ should be set to a value so that the open loop gain is larger than 1 and it enables quenching. The gain of the receiver will be set later on.
3.3 Quench Frequency

In wireless communication, the maximum baud rate is defined. The maximum baud rate defines the channel width and frequency bandwidth. For the design of the SRR, the maximum baud rate determines the minimum quench frequency ($\omega_q$). Due to the sampling like behaviour of the amplification, the Nyquist criterion should be met. This criterion states that the minimum sample frequency should be twice the maximum baud rate [11]. To be able to detect any modulation and coding scheme (i.e. Binary Phase Shift Keying (BPSK) and Manchester coding), two samples of each bit are needed to determine the incoming bit. A higher $\omega_q$ will give more information of the amplitude as the signal is sampled more often per bit.

$\omega_q$ does not only affect the maximum baud rate, but is also influences the gain. To find the relation of gain and $\omega_q$, the voltage response is analysed.

Based on Equation 2.18, when $g_{tot}$ is negative, the input signal is amplified. The amplification stops when $g_{tot}$ changes from negative to positive again. This transition point is the end of the quench cycle. The longer $g_{tot}$ is negative, the more the signal is amplified. When the quench frequency is larger, $g_{tot}$ is for a shorter period negative and the oscillation has less time to build up. This results in a smaller amplification.

![Figure 3.1: Frequency response at the output of the SRR where $\omega_q$ is 100 kHz (red) and 1 MHz (black). The frequency response is calculated by means of the Matrix Model](image)

The change of $\omega_q$ only affects the gain around the center frequency (the center frequency is 900 MHz in Figure 3.1) whereas the gain out of band is unchanged. By setting $\omega_q$ at twice the baud rate, the Nyquist criterion is met and the highest gain is achieved.

3.4 Bandwidth

For the best performance of the receiver, the bandwidth of the receiver should be equal to the bandwidth of the communication channel. Any other undesired signals will be attenuated and the quality of the incoming signal is higher. The baud rate and standards of the communication protocol determine the bandwidth of the channel. The bandwidth of the SRR is determined by...
the slope of the quench signal at the sensitivity period and the Quality factor (Q-factor) of the bandpass filter.

### 3.4.1 Slope

At the sensitivity period, the average open loop gain is for an infinite small time instance perfectly 1 and the SRR will oscillate at $\omega_0$. Do not confuse the open loop gain with the transfer function of the SRR that is infinite at the sensitivity period. The duration of the sensitivity period is determined by the slope of the quench signal. The smaller the slope at the sensitivity period, the longer the sensitivity period is and the longer it will oscillate at $\omega_0$. When the receiver oscillates perfectly at $\omega_0$, all other frequencies will be attenuated and the resonance frequency is infinitely amplified. The slope of a sinusoidal quench signal is defined as follows.

$$\frac{dg_{\text{m}}(t)}{dt} = \frac{d}{dt}\left[a_0 + a_1 \cdot \sin(\omega_q t)\right] = a_1 \cdot \omega_q \cdot \cos(\omega_q t) \quad (3.2)$$

Equation 3.2 shows that the slope of the quench signal is proportional to $a_1$. By increasing $a_1$, the gain will be increased (as discussed in Section 3.2) but the bandwidth of the receiver will decrease as well. In Figure 3.2, the frequency response of the SRR for different values of $a_1$ is shown. From Figure 3.2 can be seen that the gain and bandwidth of the SRR is increased. It is desired to have the highest possible gain and maintain the desired bandwidth to receive the full communication channel. Therefore, the value of $a_1$ consists of a trade-off between the bandwidth and the highest possible gain.

![Figure 3.2: Frequency response of the SRR where $a_1$ is 150 $\mu$S and 300 $\mu$S.](image)

A different approach for changing the slope of the quench signal at the sensitivity period to get the desired bandwidth, is changing the waveform from a sinusoidal signal to a sawtooth. A sawtooth has a smaller slope at the transition point and this translates to a smaller bandwidth. The smaller bandwidth of a sawtooth quench waveform comes with the price of less gain as the negative surface of the sawtooth is less than of a sine. When $a_0$ and $g$ are neglected, the negative surface of a sinusoidal waveform and the sawtooth waveform are as follows.
A−sine = \int_{\pi/\omega_q}^{2\pi/\omega_q} a_1 \cdot \sin(\omega_q t) dt = \frac{-2 \cdot a_1}{\omega_q} \quad (3.3)

A−saw = \int_{\pi/\omega_q}^{2\pi/\omega_q} a_1 - \frac{2a_1 \cdot \omega_q t}{2\pi} dt = \frac{-\pi \cdot a_1}{2\omega_q} \quad (3.4)

Equation 3.3 and 3.4 shows that the negative surface of a sinusoidal waveform is $\frac{4}{\pi}$ larger than the negative surface of a sawtooth waveform. This increases the gain of the SRR. In Figure 3.3, the waveforms of both signals are plotted. When $\phi_{off}$ is close to 0, i.e. $\zeta$ is close to 1, it can be observed from in Figure 3.3 that the slope at the sensitivity period of the sinusoidal waveform is larger than the slope of the sawtooth waveform.

In Figure 3.4, the frequency response is plotted for a sinusoidal quench waveform and a sawtooth waveform. The plots in Figure 3.4 are generated by using a Simulink model instead of the Matrix Model to save time. To adapt the Matrix Model for a sawtooth is time consuming.

Figure 3.3: Time domain plot of the waveform of sine wave (black) and a sawtooth (red).

Figure 3.4: Frequency response of the SRR where $\kappa(t)$ is a sinusoid (red) and a sawtooth (black). The frequency responses are simulated by Simulink using the model shown in Figure 2.5.
3.4.2 Q-Factor of the Bandpass Filter

The bandwidth of the SRR can also be adjusted by changing the Q-factor of the bandpass filter. The Q-factor quantifies the bandwidth of the bandpass filter at the resonance frequency. The Q-factor is defined as follows.

\[ Q = \frac{\omega_0}{BW_{3\,dB}} \]  

(3.5)

Where \( \omega_0 \) is the resonance frequency of the bandpass filter and \( BW_{3\,dB} \) is the width of the frequency band where the gain is less than 3 dB of the gain at the resonance frequency. Keep in mind that this is not the Q-factor of the SRR but the Q-factor of the bandpass filter itself.

A higher initial value of the Q-factor means that the passband of the bandpass filter is narrower and that the frequency response of the bandpass filter has a larger slope away from the pass band. Thus, a signal with a frequency that is close to the resonance frequency but out of the \( BW_{3\,dB} \), will be attenuated more when the Q-factor is higher. A bandpass filter with a higher Q-factor is more frequency selective and due to this selectivity, bandwidth of the SRR becomes narrower.

As discussed in Section 2.3, the output spectrum of the SRR consists of infinite side lobes around the center frequency. When the initial Q-factor is increased, the side lobes are attenuated more by the bandpass filter. Because the gain of the SRR is equal to the sum of the power of all side lobes, the gain of the SRR decreases when the Q-factor is increased.

The bandwidth of the SRR can be adjusted by changing the slope of the quench signal at the sensitivity period or by changing the Q-factor of the bandpass filter. The slope at the sensitivity period can changed by changing the amplitude of the periodic term or changing the quench waveform. The Q-factor of the bandpass filter can be changed by changing the initial Q-factor of the bandpass filter.

The influence of the parameters discussed in this Chapter is summarized and displayed in Figure 3.5.

![Figure 3.5](image_url)

Figure 3.5: Sketched influence of the different parameters on the frequency response of the SRR. The black line is the original frequency response that acts as reference. At the green line, the Q-factor of the bandpass filter is increased. At the red line, \( \omega_0 \) is decreased. At the blue line, \( a_1 \) is increased and at the purple line, \( a_0 \) is increased.
3.5 Conclusion

The frequency response of the SRR is determined by the quench signal and the bandpass filter. The main trade-off in the design of a SRR is between gain and bandwidth. The gain and bandwidth are changed by the amplitude of the periodic term of the quench signal and the Q-factor of the RLC filter. By changing these parameters, the frequency response can be set to the desired specifications.
Chapter 4

Transmit Reference

4.1 Introduction

In this Chapter, the Transmit Reference (TR) modulation scheme is applied to the SRR. Traditional amplifiers will amplify the signal and only a few distortion components will be added to the signal that are spaced with the input frequency. As discussed in Chapter 2, the SRR generates infinite side lobes at the output that are narrowly spaced around the center frequency. The reference tones and the side lobes should not interfere with the demodulation of the signal. This makes the analysis of the SRR in combination with SRR not trivial compared to the analysis with conventional Low Noise Amplifier (LNA)s.

First, in Section 4.2 a system overview is given of the SRR. Secondly, the frequency spacing of the SRR and the TR is discussed in Section 4.3. Next, the noise folding of the system is discussed in Section 4.4. This Chapter is concluded and summarized in Section 4.5.

4.2 System Overview

The block diagram of the system design is shown in Figure 4.1. The system consists of a balun, transconductance amplifier, a SRR, a squarer and a bandpass filter. First, the signal is received by the antenna and the signal is converted from single ended to differential by the balun. The transconductance amplifier (TCA) converts the received voltage to a current that is inserted in the SRR. The TCA functions as a matching filter and it isolates the SRR from the antenna as well [4].

The converted current is amplified by the SRR and next, it is squared by the squarer to mix down the data to $\Delta \omega$. As mentioned in Chapter 1, TR uses two tones for the down conversion where one tone functions as the reference for the other tone that is containing the information. The down mixing by the squarer is as follows. Two sinusoidal tones with a frequency of $\omega_\lambda$ and $\omega_\gamma$ are received and amplified by the SRR. $\omega_\lambda$ represents the modulated tone and $\omega_\gamma$ represents the unmodulated reference tone. The frequency spacing between the tones is $\Delta \omega$.

Because the frequency difference between the reference and the data should be as small as possible to increase the robustness against fading [7], $\Delta \omega$ is small compared to the carrier frequency and the data is available at a low frequency. Fading can result in a change of the propagation time that is frequency dependent. When $\Delta \omega$ is large, the possibility that the two signal will encounter a different propagation time is larger than when $\Delta \omega$ is smaller [13]. A difference in propagation time will deteriorate the down conversion of the data.
With current Analogue-to-Digital Converter (ADC)s, the output of the bandpass filter after the squarer can be directly connected to an ADC for further processing. With this system, the bandpass filter functions as an anti-aliasing filter as well for the ADC filter out higher frequency components that are too high for the ADC.

Figure 4.1: System Overview of a Transmit Reference system using the Super Regenerative Receiver.

4.3 Frequency Spacing

As discussed in Section 2.3, the output spectrum of the SRR consists of multiple side lobes that each contain information. The modulated tone is defined as $\omega_{\lambda}$ and the reference tone is defined as $\omega_{\gamma}$. The side lobes, defined as $\omega_{\lambda 1}, \omega_{\lambda 2}, \omega_{\lambda n}, \ldots$, are each spaced with the quench frequency ($\omega_q$). The reference tone will also produce its side lobes and when the signal is squared, each side lobe multiplied with each other. This results in self mixing ($\omega_{\lambda n}^2$, $\omega_{\gamma n}^2$) and intermodulation of all side lobes ($\omega_{\lambda n} \cdot \omega_{\gamma n}$).

In Figure 4.2 and 4.3 is the frequency spectrum shown at the input and at the output of the squarer, respectively. The red arrows represent the side lobes of the reference and the black arrows represent the side lobes of the modulated data. In Figure 4.3, red and black dotted arrows represent a side lobe due to the intermodulation of a reference side lobe and modulated side lobe where the black arrow represent a self-modulation of a reference side lobe or a modulated side lobe.

The multiplication of $\omega_{\lambda n} \cdot \omega_{\gamma n}$ is the fundamental principle for the TR modulation. Each side lobe is multiplied with its peer side lobe of the reference tone and the result of the multiplication is the down mixing of the signal to $\Delta \omega$. All other intermodulation products ($\omega_{\lambda n} \cdot \omega_{\gamma m}$) are multiplications of side lobes with other side lobes that will not mix down the signal to $\Delta \omega$.

In order to avoid that other intermodulation products will fold back in the band of interest, close to $\Delta \omega$, the spacing between each side lobe should be at least 2 times larger than $\Delta \omega$. The spacing between each side lobe is determined by $\omega_q$, so therefore $\omega_q$ should be at least twice as large as $\Delta \omega$. When $\omega_q$ is smaller than two times $\Delta \omega$, the intermodulation of $\omega_{\lambda n}$ and $\omega_{\gamma n+1}$ will fold close the $\Delta \omega$. This would give a interference that deteriorate the quality of the signal.

When $\omega_q$ is for example 4 times $\Delta \omega$, only $\omega_{\lambda n} \cdot \omega_{\gamma n}$ will fold to $\Delta \omega$. All other intermodulation products will fold back in the band of interest, close to $\Delta \omega$, the spacing between each side lobe should be at least 2 times larger than $\Delta \omega$. The spacing between each side lobe is determined by $\omega_q$, so therefore $\omega_q$ should be at least twice as large as $\Delta \omega$. When $\omega_q$ is smaller than two times $\Delta \omega$, the intermodulation of $\omega_{\lambda n}$ and $\omega_{\gamma n+1}$ will fold close the $\Delta \omega$. This would give a interference that deteriorate the quality of the signal.

When $\omega_q$ is exactly 2 times $\Delta \omega$, only $\omega_{\lambda n} \cdot \omega_{\gamma n}$ will fold to $\Delta \omega$. All other intermodulation products will fold to $3 \cdot \Delta \omega$ ($\omega_{\lambda n} \cdot \omega_{\gamma n-1}$), $4 \cdot \Delta \omega$ ($\omega_{\lambda n} \cdot \omega_{\lambda n+1}$) and $5 \cdot \Delta \omega$ ($\omega_{\lambda n} \cdot \omega_{\gamma n+1}$).

When $\omega_q$ is exactly 2 times $\Delta \omega$, not only $\omega_{\lambda n} \cdot \omega_{\gamma n}$ will fold to $\Delta \omega$ but the intermodulation of $\omega_{\lambda n+1} \cdot \omega_{\gamma n}$ is also mixed down to $\Delta \omega$. Because it is a multiplication of the reference tone and the information, the information is demodulated and the signal is added to the other down conversion products. This results in extra gain of the SRR as more the signal power at $\Delta \omega$ will be higher. The frequency spectrum at the input and at the output of the squarer when $\omega_q$ is two times $\Delta \omega$ is shown in Figure 4.4 and 4.5, respectively.

With $\omega_q$ equal to $2 \cdot \Delta \omega$, only the self-mixing of the tones will produce other side lobes in the spectrum that are spaced $\omega_q$ from $\Delta \omega$. When $\Delta \omega$ is small, the bandpass filter should be steeper.
in order to suppress the other tones sufficient. A smaller $\Delta \omega$ and a smaller $\omega_q$ will make the requirements of the bandpass filter more tough. The advantage of a smaller $\Delta \omega$ is that the TR will be more robust against fading [13].

\subsection{4.4 Noise Folding}

When the output of the SRR is mixed down by means of squaring, noise is also be mixed down and it folds back to the band around $\Delta \omega$ when the noise is wide band. Each multiplication of the noise and a side lobe will result in noise folding to $\Delta \omega$. The noise power at $\Delta \omega$ will be increased and it deteriorates the Signal-to-Noise Ratio (SNR). In Figure 4.6, the Noise Figure (NF) is plotted as function of $\omega_q$ for different values of $\Delta \omega$. The NF quantizes the deterioration of the SNR and it is defined as follows.

$$NF = \frac{SNR_{in}}{SNR_{out}}$$ (4.1)

At the input of the SRR, a noise source with a bandwidth of 200 MHz is connected where all other blocks are noise free. This bandwidth is chosen in order to simulate a low Q-factor matching circuit at the input of the receiver. The noise bandwidth at the output of the receiver is 1 MHz around $\Delta \omega$. The noise analysis is done by means of a Simulink simulation. The NF is calculated by taking the frequency spectrum of the input and output and calculating the input SNR and output SNR. The model of the Simulink simulation is shown in Figure 4.7.
CHAPTER 4. TRANSMIT REFERENCE

Figure 4.4: Frequency spectrum where $\omega_q$ is two times $\Delta\omega$ at the input of the squarer. The side lobes of the modulated tone are displayed in black and the reference side lobes are displayed in red.

Figure 4.5: Frequency spectrum where $\omega_q$ is two times $\Delta\omega$ at the output of the squarer. The intermodulation of the modulated side lobes and the reference side lobes are displayed in red and black. The self mixing of the side lobes are displayed in black.

The NF is the result of noise folding of the squarer. Due to the large bandwidth of the noise, the NF is not affected when $\omega_q \gg \Delta\omega$. A decrease of NF is shown where $\omega_q = 2 \cdot \Delta\omega$. At this quench frequency, the signal power is increased due to the additional signal folding, where the noise power remains the same and this increases the SNR at the output. Here, the NF is decreased to 3.51 dB, 5.76 dB and 5.07 dB for $\Delta\omega$ of 1 MHz, 2 MHz and 3 MHz respectively.

The NF for higher frequencies of $\Delta\omega$ is a few dB higher. When the tones are spaced further apart and the reference tone is further away from the center frequency, the reference tone is amplified less and this gives less signal power at the output. The noise has a different transfer function than the signal due to the non-linear squarer.

4.5 Conclusion

In this Chapter, the Transmit Reference modulation is applied to the SRR. For the down conversion of the data, the signal at the output of the SRR is squared and then filtered by a bandpass filter. $\Delta\omega$ should be at least two times larger than $\omega_q$ for the TR modulation to function. For the lowest NF due to noise folding, $\omega_q$ should be 2 times larger than $\Delta\omega$. 
Figure 4.6: Plot of the Noise Figure as function of the quench frequency for different frequency spacings.

Figure 4.7: Screen shot of the Simulink model used for the calculation of the NF.
Chapter 5

Discussion

5.1 Non-Idealities

This research is based on the analysis of the SRR in combinations with the TR modulation scheme. The analyses and simulations are done on a very high level of abstraction of the system. The system is built up from building blocks that are assumed to be ideal. Ideal building blocks are free from noise and non-idealities and have an infinite headroom. The question rises whether the same conclusions are drawn when the building blocks are not ideal.

5.1.1 Non-Linearity

The frequency spectrum at the output of the SRR consists of an infinite number of side lobes, produced by the mixing with $g_m(t)$. When $g_m(t)$ is not fully linear, it would have odd distortion (even distortion can be cancelled due by using a differential signal), i.e. the 3rd order distortion component would generate a side lobe after one loop iteration at a spacing of 3 times $\omega_q$. After three loop iterations, a side lobe is added at three times $\omega_q$ due to the mixing product of the fundamental frequency component. It can be assumed that the fundamental component is larger than the 3rd order component, so the power introduced by the fundamental frequency at the side lobe at 3 times $\omega_q$ is larger than the side lobe introduced by the 3rd order distortion component. The expectation is therefore that non-linearity in $g_m(t)$ would not have a large effect on the performance of the SRR.

This affect is also shown when the sinusoidal waveform is replaced by a sawtooth waveform. The sawtooth consists of many harmonics and the demodulation by squaring seems not to be affected by the additional side lobes that are introduced by the harmonics of the quench waveform.

The influence of the non-linearity at the feedback amplifier on the performance of the SRR can be calculated by means of the Matrix Model. 3rd order distortion can be added by changing the matrixes of the Matrix Model.

By comparing the frequency spectrum and frequency response of the SRR with and without 3rd order distortion, a conclusion can be drawn what the influence is of distortion in the feedback amplifier at the performance of the SRR. This comparison is not included to this research and the non-linearity in $g_m(t)$ should be investigated.

In the analysis of the SRR with TR, the squarer is assumed to have a perfect second order transfer function. When the squarer has to be realized in IC technology, the transfer function
CHAPTER 5. DISCUSSION

will not be perfectly second order. i.e. the transfer function will have an exponential behaviour. The result of this non-ideality is not included in this research.

When the squarer is not perfectly second order, the side lobes will not only be squared but also a power of three. The expectation is that this will manifest itself by having a lower gain at $\Delta \omega$ and a higher gain at other intermodulations that will fold to other frequencies. With the less signal power at $\Delta \omega$, the NF will increase.

The different transfer function of the squarer can be calculated by means of a series expansion of the transfer function or simulated with the Simulink model by replacing the ideal squarer for a different transfer function.

5.1.2 Noise

Noise introduced by $g_m(t)$ and the squarer is not added in the analysis of the NF that is presented in this thesis. Noise introduced by $g_m(t)$ is filtered by the bandpass filter. The fastest way to find the influence of the noise is to simulate with Simulink by adding broadband noise source to the quench signal and the squarer and calculate the NF. By adding no input noise to the simulation, the NF is only caused by the noise of $g_m(t)$ and the squarer.

5.1.3 Interference

The analysis of TR is done on the basis of two tones present at the input. In a radio harsh environment, the frequency band is filled with other signals that are received as well. Because the SRR is linear, the unwanted signals produce extra side lobes that are added and presented to the input of the squarer. When the side lobes of the interferer are multiplied with the other side lobes at the squarer, the possibility that any combination of the intermodulation of the side lobes will fold back to $\Delta \omega$ is insurmountable.

How large an interferer can be and how far it should be from the center frequency until the information is deteriorated can be simulated by means of the Simulink model, presented in Figure 4.7. Large signals that are far away from the center frequency of the SRR can, due to the SRR, produce side lobes that eventually fold to the center frequency and interfere with the desired signal. This makes the interference simulation less trivial as with conventional receivers. Before the simulation, first the minimum Signal-to-Interference-plus-Noise Ratio (SINR) should be calculated to still be able to demodulate the signal correctly. When the minimum SINR is determined, the interferer can be swept in frequency and power so the parameters of the interference performance can be determined.

5.2 Noise Figure

The NF seems to decrease when the quench frequency is a power of 2 the frequency spacing. It can be observed at $\omega_q$ is 2 MHz, 4 MHz and 8 MHz for $\Delta \omega$ is 1 MHz in Figure 4.6. The calculations show that the SNR at the output is increased. This is contrary to the expectation. At the frequencies that are not two times $\Delta \omega$, the SNR at the output should not be increased as intermodulations of the reference side lobes and its non peer side lobes do not fold to $\Delta \omega$ and therefore do not increase the SNR.

The power of two suggests that it is due to the calculation of the FFT. The FFT is used to calculate the frequency spectrum of the time-domain signal and the SNR at the input and output are derived from this calculated frequency spectrum. Because it is a discrete and digital operation, leakage and errors can be made when $\omega_q$ is a power of two of $\Delta \omega$. 

A.L. van Uem
5.3 Recommendations

The introduction of the non-idealities of the building blocks will result in a simulation that is closer to the reality of when the system is realized in a circuit. Therefore it is recommended to include the non-idealities of the building blocks in the simulation to come to results to be able to make a better prediction of how the system would operate when it is realized in a circuit. Based on these simulations, the conclusion have to be redrawn whether the TR is suitable and desirable for TR modulation.

When the conclusion is redrawn whether the SRR is suitable for TR modulation with all non-idealities included in the simulation, the SRR should be realized in a circuit. With the circuit designed, the power consumption can be calculated and determine whether the combination of the SRR and TR is a good alternative for $U^2LP$ designs.

An mathematical description of the bandwidth of the SRR is not presented in this thesis. In Section 3.4 it is discussed that the bandwidth of the SRR is influenced by the slope of the quench waveform at the sensitivity period. How this bandwidth is determined is not known from a mathematical point of view. It is recommended that this phenomenon is studied more to find more insight on how the bandwidth of the SRR is achieved.
Conclusion

In this thesis, the SRR is scrutinized and the suitability and desirability of the SRR in combination with TR is studied. Due to the simple design and low power dissipation, the SRR is a popular receiver topology for U2LP design.

TR modulation uses two tones where one tone acts as the reference for the modulated tone. The communication is not based on the absolute frequency but on the frequency offset between the two tones, defined as $\Delta \omega$. Due to the presence of the reference, there is no local oscillator used for demodulation at the receiver and this reduces the power dissipation further.

The frequency spectrum at the output of the SRR is calculated by a matrix solution that is solved in Matlab. The matrix solution is defined as the Matrix Model. The Matrix Model calculates the frequency spectrum in a much faster way compared to Simulink simulations and it gives an mathematical description of the frequency spectrum. The frequency spectrum of the SRR contains of an infinite set of discrete side lobes, each spaced at the frequency of the quench signal $\omega_q$. Based on the analysis with the Matrix Model, the SRR is assumed to be linear.

The main trade-off in the design of a SRR is between gain and bandwidth. The gain is set by the negative surface of the total conductance. The bandwidth is set by the slope of the quench signal at the sensitivity period and the Q-factor of the bandpass filter.

By squaring the output of the SRR, each data side lobe is multiplied with its peer reference side lobe. Each multiplication of the modulated side lobe and the reference results in the down conversion of the data to $\Delta \omega$. The SRR is suitable for TR when $\omega_q$ is at least two times larger than $\Delta \omega$. When $\omega_q$ is two times larger than $\Delta \omega$, this results in a decrease of the NF of 2 dB compared to the lowest NF at other values of $\omega_q$. 
References


