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FINAL REPORT

The analysis of a mixed steam-gas aero derivative turbine

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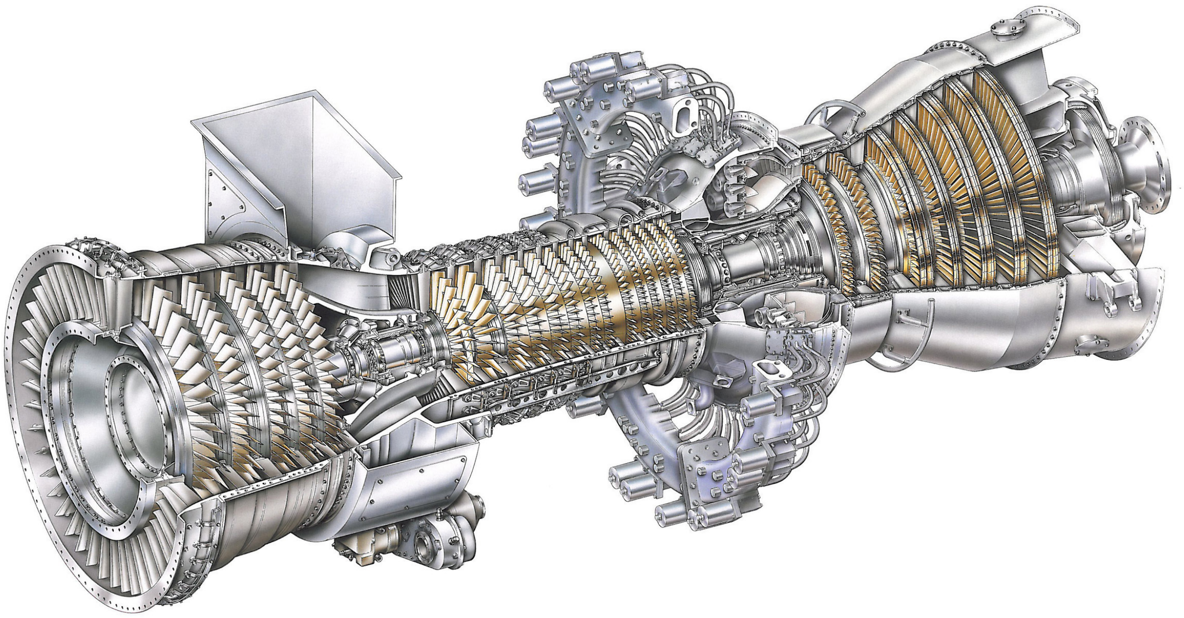

Abstract

An analysis is done by the use of thermodynamics for an aero derivative gas turbine which utilizes steam injection to increase its efficiency. This type of cycle is known as the steam injected gas turbine cycle. The main purpose of this research was to develop a better understanding of how optimal cycle efficiency is reached, for which the steam of injection is generated by use of the turbine exhaust heat.

A model has been developed using the software Engineering Equation Solver to simulate the simple gas turbine, steam generation and effects after steam injection. Input parameters for the model are taken for the GE LM6000 turbine as provided by General Electric. Turbine property results are compared with literature for validation and show the same characteristic behaviour.

Several parameter influences are visualized and explained. It has been found that this type of cycle shows a very specific point where the efficiency is the highest. By using steam injection for the chosen turbine and parameters an efficiency gains of around 11% and power augmentation of 45% is possible to be achieved.

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Nomenclature

Abbreviations	
CC	combustion chamber
EES	engineering equations solver
GT	gas turbine
PR	pressure ratio
SF	steam fraction
STIG	steam injected gas turbine
TIT	turbine inlet temperature

Symbols		Unit
c	specific heat capacity	kJ/kgK
η	efficiency	-
h	enthalpy	kJ/kg
LHV	lower heating value	kJ/kg
$\dot{m}$	mas-flow	kg/s
s	entropy	kJ/kgK
$\dot{W}$	work	kJ/s
w	specific work	kJ/kg
$\dot{Q}$	heat	kJ/s
q	specific heat	kJ/kg
T	temperature	K or °C

Subscripts	
a	air
amb	ambient condition
eco	economizer
c	compressor
cc	combustion chamber
f	fuel
g	combustion gas
is	isentropic
p	pressure
s	steam
sh	superheated
t	turbine
vap	evaporation
w	water

1. Introduction

1.1. Background

In the past power-plants could be used round the clock in baseload to produce the energy needed to meet demand. Nowadays a part of this energy is also generated by sustainable energy resources like solar and wind, however they are dependent on environmental conditions and therefore supply energy in a highly variable manner. This variable power results in destabilization of the electrical grid, for which in order to counter this the produced power by plants has to be adjusted accordingly [1]. This is problematic, since steam power-plants using the combined cycle of gas/steam-turbines have a high thermal inertia; this cycle takes a few hours to start-up and is not suitable to be frequently interrupted. A gas turbine (GT) is however much more flexible and therefore very suitable for this purpose, were it not for the fact that a GT has a much lower thermal efficiency if compared to the combined cycle.

The efficiency can be boosted however. One method is to use the waste heat of the GT to create steam, where after this is injected into the turbine itself and allowed to mix with the internal gasses. This cycle set-up is also know as the STeam Injected Gas (STIG) cycle. It has the flexibility of a simple GT cycle, while its efficiency approaches the initial mentioned combined cycle. Implementation could be achieved without minor modifications to the current device, which results in a low costs of investment to provide a solution to increase the GT efficiency and have a device that can easily adjust to the demand of power generation.

1.2. Previous studies and perspectives

Water/steam injection in gas turbines has been used for a long time. Firstly it was used as way to reduce pollutants like NOx emission, but it was also noticed that with certain set-ups it could have additional performance benefits [2]. In 1976 Prof. D. Y. Cheng proposed that the heat of the GT exhaust gas could be used to generate steam in a heat recovery steam generator (HRSG). He showed that after injection in the GT this method could proof useful to increase the efficiency and augment power output.

General Electric later used this technique on their aero-derivative engines and named them STIG turbines [3]. Efficiency gains and power augmentation of 10% and 50-70%, respectively, were shown to be achieved. STIG turbines were however not able to achieve the same maximum efficiencies as combined cycles, which reached up to 10-15% higher values. As a result the latter was preferred for base load power generation [4].

Nevertheless, the STIG cycle is a very interesting and simple way to increase efficiency and augment power production. For moderate plant sizes in the mid-power range of 1-50 MW STIG performance can be better than available combined cycles [3] and they have a higher specific work compared to combined cycles under certain conditions. Furthermore, for the workings of the STIG it consume a lot of water, which means that in areas where this is a scarce resource the recovery of water is necessary [5].

1.3. Objectives

The effects of STIG turbines in terms of efficiency increase, work augmentation and NOx reduction are well documented in literature. However an explanation about how the most efficient injection point for STIG can be found is not well described.

In order to provide more insight in the workings of the STIG an aero derivative GT will be selected for which the cycle will be analysed by means of thermodynamics. A simple model is created for which the parameters of design and operation will be identified. Afterwards the model result are compared to literature for validation and a performance comparison will be made with simple cycle performance.

The main objective of this research is to make the cycle as efficient as possible by optimizing the water consumption with the exchanged heat of the exhaust gasses and to get a detailed explanation how the most efficient point is reached.

2. Theory

In this chapter basic theory, maths and knowledge is explained, serving as reference for the rest of the report.

2.1. The ideal gas turbine

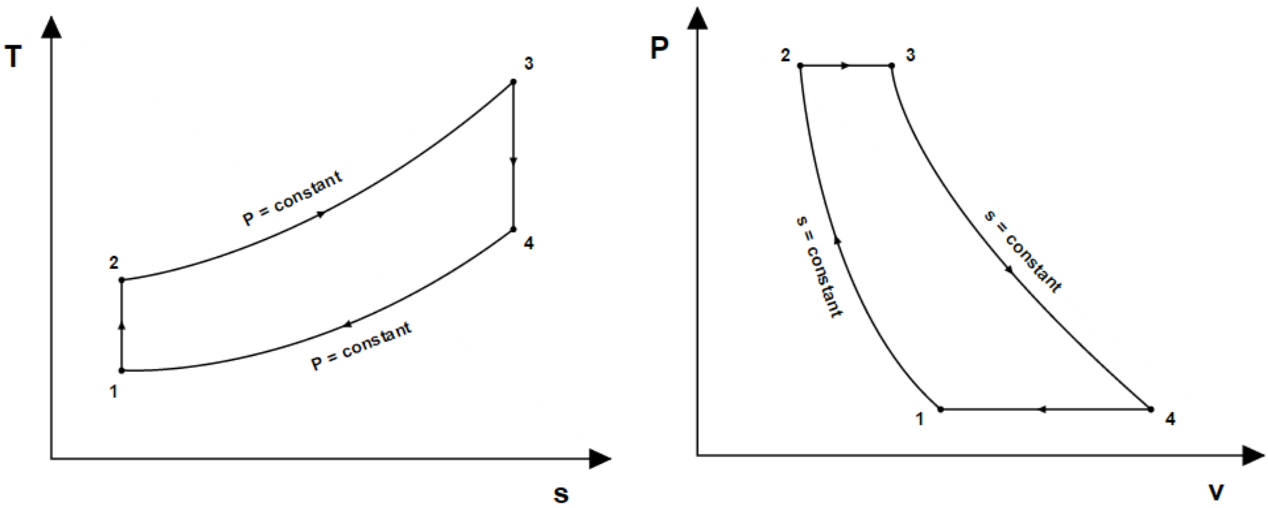
The general working of a GT is as following. Fresh air from the surroundings is sucked into the compressor, which increases its temperature and pressure. The high pressure air is mixed with some kind of fuel and ignited in the combustion chamber (CC); this results in a higher temperature gas. This gas is then led into the turbine, where it is expanded to ambient pressure. This allows the turbine to produce power, where after the gasses are ejected into the atmosphere. Some of this power is then used to drive the compressor, while the remainder can be used to produce useful work.

A property of these gas power cycles is that the fluid throughout the cycle remains a gas. Energy is provided by means of internal combustion by burning some type of fuel, which means the composition of the fluid will change from air and fuel to some kind of combustion mixture. Since the fluid does not undergo an entire thermodynamic cycle but exits the engine at some point as exhaust gas, we are dealing with an open cycle. The analysis of such cycles can be simplified greatly by using approximations which are known as the air-standard assumptions [6]:

1. The working fluid is air, which continuously circulates in a closed loop and always behaves as an ideal gas.
2. All the processes that make up the cycle are internally reversible.
3. The combustion process is modelled as a heat-addition process from an external source.
4. The exhaust process is replaced by a heat-rejection process that restores the working fluid to its initial state.

If additionally is assumed that air has constant specifics heats determined at room temperature, it is called cold-air-standard.

The open-cycle GT can now be modelled as a closed ideal cycle also known as the Brayton cycle. The processes can be visualized for better understanding in T-s and P-v diagrams as seen in figure 2.1. A schematic of the cycle can be seen in figure 2.2.



(a) T-s diagram (b) P-v diagram

Figure 2.1.: Brayton cycle visualized.

The following four internally reversible processes can be seen:

- 1-2 Isentropic compression in the compressor
- 2-3 Constant pressure heat addition
- 3-4 Isentropic expansion in the turbine
- 4-1 Constant pressure heat rejection

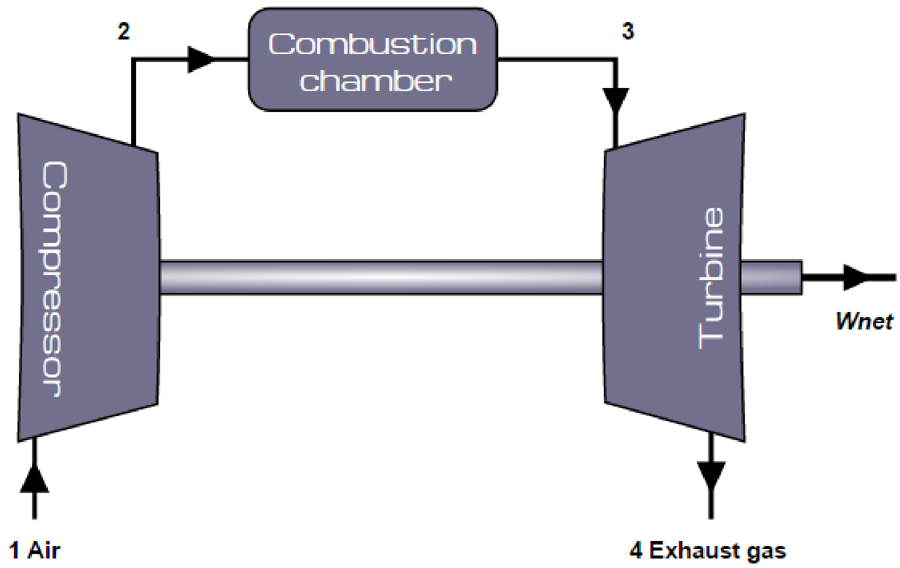


Figure 2.2.: Schematic overview of a simple gas turbine.

By utilizing the first law of thermodynamics and assuming steady flow we get the following general energy balance:

$$\begin{aligned} E_{in} - E_{out} &= \Delta E_{system} = 0 \\ E_{in} &= E_{out} \\ \dot{Q}_{in} + \dot{W}_{in} + \Sigma_{in}\dot{m}\left(h + \frac{V^2}{2} + gz\right) &= \dot{Q}_{out} + \dot{W}_{out} + \Sigma_{out}\dot{m}\left(h + \frac{V^2}{2} + gz\right) \end{aligned} \tag{2.1}$$

For the Brayton cycle with air-standard assumptions, assuming no change in kinetic and potential energies, the following relationships can be deduced for the compressor and the turbine work.

$$\dot{W}_c = \dot{m}_a(h_2 - h_1) \tag{2.2}$$

$$\dot{W}_t = (\dot{m}_a + \dot{m}_f)(h_3 - h_4) \tag{2.3}$$

The difference between the compressor and turbine work gives us the net produced power.

$$\dot{W}_{net} = \dot{W}_t - \dot{W}_c \tag{2.4}$$

The heat added in the CC can be expressed as:

$$\dot{Q}_{in} = \dot{m}_f \cdot LHV_f = (\dot{m}_f + \dot{m}_a) \cdot h_3 - \dot{m}_a \cdot h_2 \tag{2.5}$$

Hence, the efficiency can be determined using (2.4) and (2.5):

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}_{in}} \tag{2.6}$$

Increasing the pressure ratio (PR) of the compressor will also increases the adiabatic efficiency of the Brayton cycle, so it would be useful to express this in a simple relationship. Again the air-standard assumptions are used, cp, cv and γ

remain constant throughout the cycle, therefore cold-air-standard; the PR of the turbine and compressor are equal and all components have an efficiency of 100 %. The output of specific work with the required input of specific heat is then compared:

$$\eta_{Brayton} = \frac{w_{net}}{q_{in}} = \frac{q_{in} - q_{out}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} \tag{2.7}$$

where

$$q_{in} = h_3 - h_2 = c_p(T_3 - T_2) \tag{2.8a}$$
$$q_{out} = h_4 - h_1 = c_p(T_4 - T_1) \tag{2.8b}$$

Since process 1-2 and 3-4 are isentropic, hence adiabatic and reversible, the second isentropic relation for ideal gasses can be used.

$$\frac{T_B}{T_A} = \left(\frac{P_B}{P_A}\right)^{\frac{(k-1)}{k}} \text{ where k = 1.4 (heat capacity ratio air)} \tag{2.9}$$

Furthermore $P_2 = P_3$ and $P_1 = P_4$, therefore:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{(k-1)}{k}} = \left(\frac{P_3}{P_4}\right)^{\frac{(k-1)}{k}} = \frac{T_3}{T_4} \Longrightarrow \frac{T_4}{T_1} = \frac{T_3}{T_2}$$

Substituting equations (2.8a) and (2.8b) in (2.7) and simplifying gives:

$$\eta_{th,Brayton} = 1 - \frac{c_p(T_4 - T_1)}{c_p(T_3 - T_2)} = 1 - \frac{T_1(\frac{T_4}{T_1} - 1)}{T_2(\frac{T_3}{T_2} - 1)} = 1 - \frac{T_1}{T_2}$$

Therefore the theoretic adiabatic efficiency of an ideal Brayton cycle can be expressed as:

$$\eta_{Brayton} = 1 - P_r^{-\frac{(k-1)}{k}} \tag{2.10}$$

where the pressure ratio is defined as:

$$P_r = \frac{P_2}{P_1} \tag{2.11}$$

Equation 2.10 and 2.11 show us that the higher the ratio, the higher the efficiency becomes.

2.2. Ideal and actual cycle

Comparing the ideal Brayton cycle to an actual gas-turbine cycle one would notice some important differences. During the addition of heat and the rejection constant pressure was assumed. In reality one would notice a drop of pressure. Furthermore, the actual work needed by the compressor and the work produced by the turbine would be higher and lower, respectively. This is caused by irreversibilities like friction in the components, which lower the total adiabatic efficiency.

In order to understand the effect of irreversibilities on the performance, reversible processes need to be explained. A reversible process is defined as *"a process that can be reversed without leaving any trace on the surroundings"* [6]. A system and its environment can therefore be reversed between two states without entropy generation. In reality this is never possible, but as idealization it makes systems easier to analyse. For work-producing devices a system will produce the most work, while for work-consuming devices it will need the least work [6]. Actual device performance can therefore be compared to these theoretical ideal devices.

To indicate this loss in total efficiency, the difference between the ideal isentropic work and actual work is given by parameters called the isentropic efficiencies, defined for the compressor and turbine as:

$$\eta_c = \frac{h_{is;2} - h_1}{h_2 - h_1} \tag{2.12}$$

and

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{is;4}} \tag{2.13}$$

Because of these isentropic efficiencies an actual device can never reach efficiencies as stated in equation 2.9. Instead it is only able to reach a lesser maximum value that is limited by the turbine inlet temperature (TIT).

2.3. STIG

The steam injected gas turbine (STIG) is based on the idea that the high temperature exhaust gasses can be used to turn water into steam and when injected into the turbine it increases the cycle efficiency. A schematic can be seen in figure 2.3. Literature tells us that the injected steam can amount to 10-20% of the air mass flow [5] and generation is limited by the amount of usable thermal energy available in the exhaust gas [7]. Both points will be further analysed in chapter 4.

While maintaining the same total turbine design mass flow the entering compressor air-flow can be lowered, hence decreasing the compressor work. This means the efficiency as stated in equation 2.6 is increased [8]. On the other hand, since now a mass-portion of evaporated water has to be heated to the TIT - and water having a higher specific heat capacity than air - the injected fuel will have to increase in order to ensure the same TIT; more about this can be seen in chapter 3.2

GTs have undesirable combustion products like NOx and CO formed due to high primary zone temperatures in the combustion chamber. Steam has a higher heat capacity than air and therefore functions as a heat sink when injected, which reduces the overall primary temperature. In order to prevent local high temperature spots which could still produce these undesirable emissions the steam should be injecting far up-stream of the CC in order to allow for proper mixing [7] [8] [9].

The water used for the production of the injected steam has to be of ”high quality”, meaning it is de-mineralized. This makes the process more complex, however it is necessary to protect against corrosion on internal parts like the turbine blades at high temperatures [3]. This is especially important for aero-derivative engines, which need higher quality water compared to ”heavy-duty” machines [5]. In order to prevent excessive high-quality water consumption, the injected steam can be condensed and recovered from the exhaust gas in order to recycle it for steam generation. Literature shows that practical solutions are achievable, however more research in this area for actual implementation could be possible [3] [5] [7].

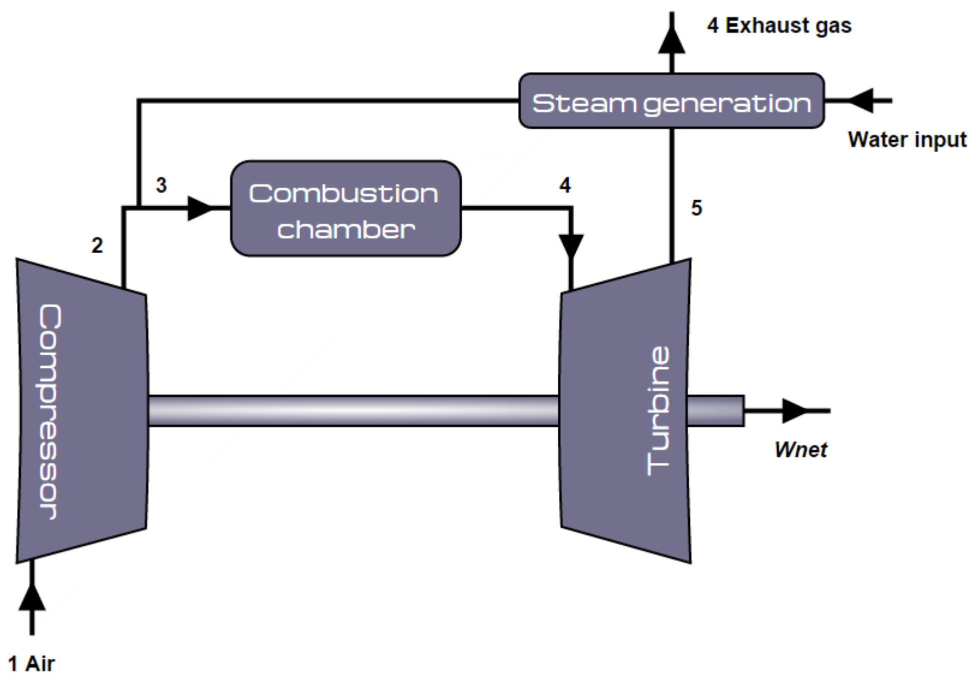


Figure 2.3.: Schematic overview of the STIG setup.

2.4. OTSG

For the generation of steam HRSGs are normally used. It has fixed sections to pre-heat/economize, evaporate and superheat the water. They do have several drawbacks, the most important being that a cold start-up can take up to 2 hours or more. This is caused amongst other because it consists of a steam drum through which the water has to loop some multiple times.

Remember that the idea of utilizing STIG on a GT was because it could proof to be a good solution for quick power generation. A very attractive alternative is therefore the Once-Through Steam Generator (OTSG). The OTSG configuration is much simpler. It has an inlet for heated air at the bottom and an outlet at the top, while uninterrupted tubes run internally. The water is heated by the GT exhaust heat to steam in one run though the system. It therefore does not have fixed sections like the HRSG, which allows for a much smaller and flexible construction, while having less maintenance. Start-up times to generate usable superheated steam can be achieved in 15 minutes, which is much more desirable [10] [11].

Since the process of economization, evaporation and super heating do not have fixed sections, the mass-flow of water through the OTSG will therefore determine where these three specific processes will occur. Since the temperature of the exhaust gas at the inlet remains more or less constant, this system is ideally designed to work at a certain water mass-flow rate. To create a model for the OTSG the exchanged heat between the exhaust gas and the input water is analysed; figure 2.4 gives a schematic representation. An important design parameter is to make sure there is always a minimum temperature difference between point 2 and 7, also known as the pinch point, to make sure that during the entire processes heat is being transferred from the gas to the water [8]. More about this in chapter 3.3.

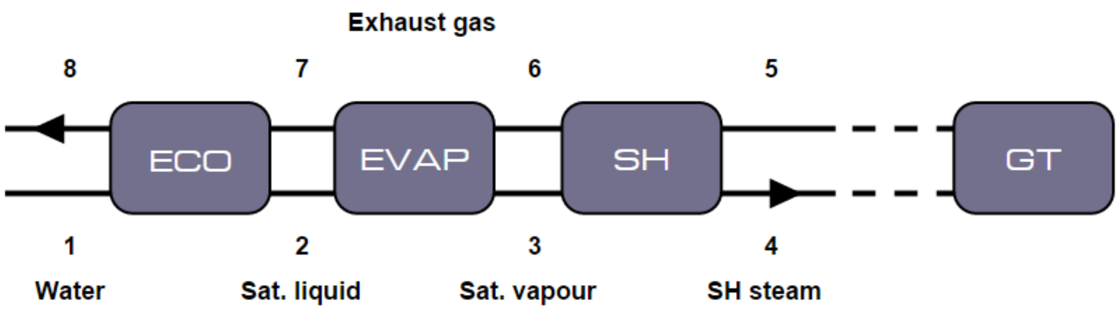


Figure 2.4.: Schematic overview of the OTSG.

2.5. Engine choice and parameters

For this assignment an aero-derivative GT is considered, since this type of engine is attractive for several reasons. First of all if compared to industrial gas turbines they have a quick start-up capability and can do this frequently without causing significant maintenance costs [12], which for quick power generation therefore is an ideal property.

Another property of aero-derivative engines is that the compressor has variable inlet guide vanes with which the sucked in air can be reduced. With other GTs a portion of the compressed air has to be blown off in order for the GT to accept the injected water and maintain the designed mass-flow, which reduces the efficiency gain [2]. Furthermore as can be seen in chapter 2.1 the higher the PR of a GT, the higher its adiabatic efficiency is. Aero-derivative engines have very high PRs - around 30, therefore reach high efficiency values of around 40% [5].

It could also be considered to create a new type of GT specifically created for mixed gas-steam usage. While this could certainly be achieved, considering the R&D costs related to creating a new engine and that the employment of it is much more specific this option is unattractive. Moreover an aero-derivative engine only needs minor adjustments to implement STIG, which makes this economically a more feasible option [13].

General Electric is a multinational corporation which produces several types of these GTs. Their latest models are the LM6000 series, which generate power between 40 and 50 MW. They have PRs that reach over 30 and efficiencies of around 42% [14]. The LM6000-PG was chosen, for it has one of the highest PRs that is in use today. Input parameters for the model are taken from data sheets and brochures as provided by General Electric [14] [15], which are as following:

$T_{1,amb}$	15	$^\circ$C
W_{net}	52,4	MW
P_{ratio}	33,2	
Exhaust flow	141	$\frac{kg}{s}$
Exhaust T	499	$^\circ$C

Table 2.1.: Manufacturer data LM6000-PG

3. EES model

In this chapter the most important parts of the model will be shown and explained. The code of the complete model containing all equations including detailed comments can be viewed as separate parts in Appendix A for the simple GT, Appendix B for the STIG addition and Appendix C for the OTSG. Appendix D shows the calculated results of the system as a whole.

The thermodynamic analysis of the STIG is done with help of Engineering Equation Solver (EES). EES is a software package used to find solutions for system of equations. In addition it contains many material properties, specialized thermodynamic functions and can solve iterative problems, making it a powerful tool for solving thermodynamic and heat-transfer problems. The schematic workings of the model can be viewed in figure 3.1. It consists of the three parts that iteratively work together as a whole.

Values for variables can be looked up if certain other parameters are known. For example if the temperature of ideal air is known, one could look up the enthalpy at this point. EES has excellent in-program functions and databases for this purpose, which are used many times. To make this process therefore easier to interpret a distinction is made between looked up variables and solutions of equations. The former are called 'determined' while the latter are 'calculated'. Note that variable sub-numbers coincidence with positions as given by figure 2.2, 2.3 and 2.4.

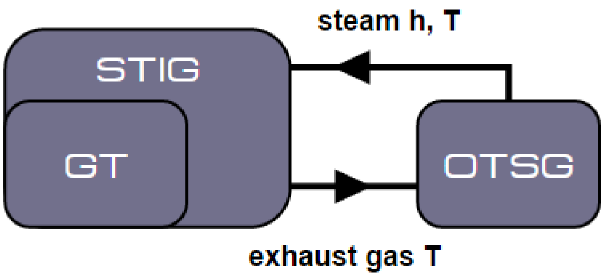


Figure 3.1.: Schematic overview of the EES model.

3.1. Simple GT model

In addition to the manufacturer data as seen in 2.1, some parameter assumptions for simple GT calculations are made.

TIT	1260	°C
η_c	93	%
η_t	86	%
$LHV_{methane}$	50000	kJ/kg
η_{cc}	100	%

Table 3.1.: Model parameter assumptions

Using these parameter, the simple GT model is able to approach the power output and turbine outlet temperature, while having a slightly higher efficiency compared to the chosen turbine. See table 3.2.

Compressor

The main assumptions and manufacturer data are used as input parameters. Given this, the enthalpy h_1 and entropy s_1 of the compressor inlet air can be determined from the temperature T_1 and the pressure P_1. Since the flow through the compressor is isentropic and the PR is known, the entropy $s_{is;2}$ and pressure P_2 can be determined; and hence $T_{is;2}$ and

	Manufacturer	Model	Unit
P_{ratio}	33,2	33,2	-
m_{tot}	141	141	$\frac{kg}{s}$
$T_{1;amb}$	15	15	°C
W_{net}	52,4	51,8	MW
η_{cycle}	41,6	42,2	%
Heat rate	8660	8537	$\frac{kJ}{kWh}$
Exhaust T	499	495	°C

Table 3.2.: Manufacturer data compared with simple GT model results

$h_{is;2}$. Using equation (2.12) to correct for the isentropic compressor efficiency, h_2 can be calculated. Now the enthalpies before and after the compressor are known, therefore using equation (2.2) the compressor work can be calculated:

$$\dot{W}_c = \dot{m}_{a;1}(h_2 - h_1) \tag{3.1}$$

Combustion chamber

The TIT T_3 is known, which allows to determine h_3. Using the first law as seen in equation (2.1), the energy balance over the CC can be solved for the fuel mass-flow:

$$\dot{m}_{a;1} * h_2 + \dot{Q}_{in} = (\dot{m}_{a;1} + \dot{m}_{f;1}) * h_3 \tag{3.2}$$

where as seen in (2.5):

$$\dot{Q}_{in} = \dot{m}_{f;1} \cdot LHV_f \tag{3.3}$$

Notice that the fuel needed is therefore directly depended on the TIT.

Turbine

The addition of heat is isobaric, therefore $P_3 = P_2$. With T_3, the entropy s_3 can be determined and P_4 is calculated from the PR. From the latter two $T_{is;4}$ and hence $h_{is;4}$ can be determined. Using (2.13) to correct for the isentropic turbine efficiency, h_4 can be calculated. Now the enthalpies before and after the turbine are known. Using equation (2.3) gives us the turbine work:

$$\dot{W}_t = (\dot{m}_{a;1} + \dot{m}_{f;1})(h_3 - h_4) \tag{3.4}$$

3.2. STIG model

The simple GT model is expanded in order to simulate the workings of the STIG. The total mass-flow $\dot{m}_{tot}$ of the system now consists of the air-, steam- and fuel-flow, which is kept at the same value as the simple GT. The input of steam x will be expressed as mass fraction of $\dot{m}_{tot}$.

Compressor

In order to maintain the same $\dot{m}_{tot}$ the air mass-flow $\dot{m}_{a;2}$ has to be reduced. Since h_1 and h_2 do not change, the compressor work $W_{c;stig}$ will be lower if compared to the simple cycle and therefore needs to be recalculated using (2.2):

$$\dot{W}_{c;stig} = \dot{m}_{a;2}(h_2 - h_1) \tag{3.5}$$

Injection

It is assumed the steam to be isenthalpically throttled to the same pressure as P_2 when injected into the compressed air. The energy balance of the injection then looks like:

$$\dot{m}_{a;2} * h_2 + \dot{m}_s * h_{s;3} = (\dot{m}_{a;2} + \dot{m}_s) * h_{stig;3} \tag{3.6}$$

which serves as inlet for the combustion chamber. The enthalpy of the injected steam $h_{s;3}$ is calculated by the last part of the whole model as seen in chapter 3.3.

Combustion chamber

In the CC fuel is added, after which it is ignited. The gas- and steam flow can be expressed separately:

$$(\dot{m}_{a;2} + \dot{m}_s) * h_{stig;3} + LHV_f * \dot{m}_{f;2} = (\dot{m}_{a;2} + \dot{m}_{f;2}) * h_{g;4} + \dot{m}_s * h_{s;4} \tag{3.7}$$

where the outlet of the CC looks like:

$$(\dot{m}_{a;2} + \dot{m}_{f;2}) * h_{g;4} + \dot{m}_s * h_{s;4} = (\dot{m}_{a;2} + \dot{m}_{f;2} + \dot{m}_s) * h_{stig;4} \tag{3.8}$$

Equations (3.6), (3.7) and (3.8) can be expressed as a single equation:

$$\dot{m}_{a;2} * h_2 + \dot{m}_s * h_{s;3} + LHV_f * \dot{m}_{fuel} = (\dot{m}_{a;2} + \dot{m}_{f;2}) * h_{g;4} + \dot{m}_s * h_{s;4} \tag{3.9}$$

Assuming the gas-mixture to be ideal air, $h_g = h_a$, it can be rewritten to the following:

$$\dot{m}_{f;2} * (LHV_f - h_{a;4}) = \dot{m}_{a;2} * (h_{a;4} - h_2) + \dot{m}_s * (h_{s;4} - h_{s;3}) \tag{3.10}$$

Knowing the pressure and the TIT, now numbered $T_{stig;4}$ and $P_{stig;4}$, we can determine the enthalpies $h_{a;4}$ and $h_{s;4}$. Since all other variables are known, the new fuel mass-flow $\dot{m}_{f;2}$ of the STIG can be calculated. It can therefore again be seen that the amount of this flow is directly dependent on the energy required to bring the air and steam to the TIT.

Furthermore, if compared to the rewritten equation (3.2):

$$\dot{m}_{f;1} * (LHV_f - h_3) = \dot{m}_{a;1} * (h_3 - h_2) \tag{3.11}$$

it is seen that the injection of steam has added an extra term defining the fuel mass-flow. This indicates that the flow will be higher compared to the simple GT, which can be explained by the fact that now a portion of steam is heated instead of air. Since steam has a higher heat capacity, more energy is needed to reach the same TIT.

Turbine

Again, the heat addition happens isobaric, therefore the entropies of air and steam can be determined from $P_{stig;4}$ and $T_{stig;4}$. After isentropic expansion the isentropic temperatures and enthalpies for the steam and water can be determined. However, we can not use equation (2.13), since that would yield different results for the outlet temperatures $T_{stig;air;5}$ and $T_{stig;steam;5}$. The following equation is therefore used:

$$\eta_t = \frac{x_a * h_{a;4} + x * h_{s;4} - x_a * h_{a;5} - x * h_{s;5}}{x_a * h_{a;4} + x * h_{s;4} - x_a * h_{a;is;5} - x * h_{s;is;5}} \tag{3.12}$$

where $x_a = (1 - x)$ and x the injected steam mass-fraction, with which the condition $T_{stig;air;5} = T_{stig;steam;5}$ is set. This can now be solved for $h_{a;5}$ and $h_{s;5}$ from which the turbine outlet temperature $T_{stig;5}$ can be determined. For no steam injection $(x = 0)$ equation (3.12) would have the same format as (2.13).

Now it is possible to calculate the turbine work as given in the energy balance:

$$(\dot{m}_{a;2} + \dot{m}_{f;2}) * h_{a;4} + \dot{m}_s * h_{s;4} = \dot{W}_{t;stig} + (\dot{m}_{a;2} + \dot{m}_{f;2}) * h_{a;5} + \dot{m}_s * h_{s;5} \tag{3.13}$$

3.3. OTSG model

The OTSG is viewed at three specific stages. The economizing of input water to saturated liquid, the heating of saturated liquid to a saturated vapour and the superheating of steam. Values for the latter serves as input for the STIG.

The balance equations as from the water side:

$$\dot{Q}_{eco} = \dot{m}_s * (h_{w;2} - h_{w;1}) \tag{3.14}$$
$$\dot{Q}_{vap} = \dot{m}_s * (h_{w;3} - h_{w;2}) \tag{3.15}$$
$$\dot{Q}_{sh} = \dot{m}_s * (h_{w;4} - h_{w;3}) \tag{3.16}$$

For the exhaust gas side:

$$\dot{Q}_{eco} = (\dot{m}_{a;2} + \dot{m}_f; 2) * (h_{a;7} - h_{a;8}) + \dot{m}_s * (h_{s;7} - h_{s;8})$$
$$\dot{Q}_{vap} = (\dot{m}_{a;2} + \dot{m}_f; 2) * (h_{a;6} - h_{a;7}) + \dot{m}_s * (h_{s;6} - h_{s;7})$$
$$\dot{Q}_{sh} = (\dot{m}_{a;2} + \dot{m}_f; 2) * (h_{a;5} - h_{a;6}) + \dot{m}_s * (h_{s;5} - h_{s;6})$$

which can be rewritten to:

$$\dot{Q}_{eco} = ((\dot{m}_{a;2} + \dot{m}_f; 2) * c_{a;78} + \dot{m}_s * c_{w;78}) * (T_{stig;7} - T_{stig;8}) \tag{3.17}$$
$$\dot{Q}_{vap} = ((\dot{m}_{a;2} + \dot{m}_f; 2) * c_{a;67} + \dot{m}_s * c_{w;67}) * (T_{stig;6} - T_{stig;7}) \tag{3.18}$$
$$\dot{Q}_{sh} = ((\dot{m}_{a;2} + \dot{m}_f; 2) * c_{a;56} + \dot{m}_s * c_{s;56}) * (T_{stig;5} - T_{stig;6}) \tag{3.19}$$

4. Analysis

In this chapter the EES model will be analysed for its behaviour and is compared to what is described in literature. Furthermore it is determined under what conditions the STIG will function at its most efficient point.

4.1. Verification with literature

Starting with the simple GT the specific work and related efficiency at different PRs can be plotted. This is done for several TIT values in figure 4.1. Note that the PR-range goes from 10 to 60, the former value being quite low, while the latter very high if compared to current standards, which is simply done to present a clearer view of the behaviour.

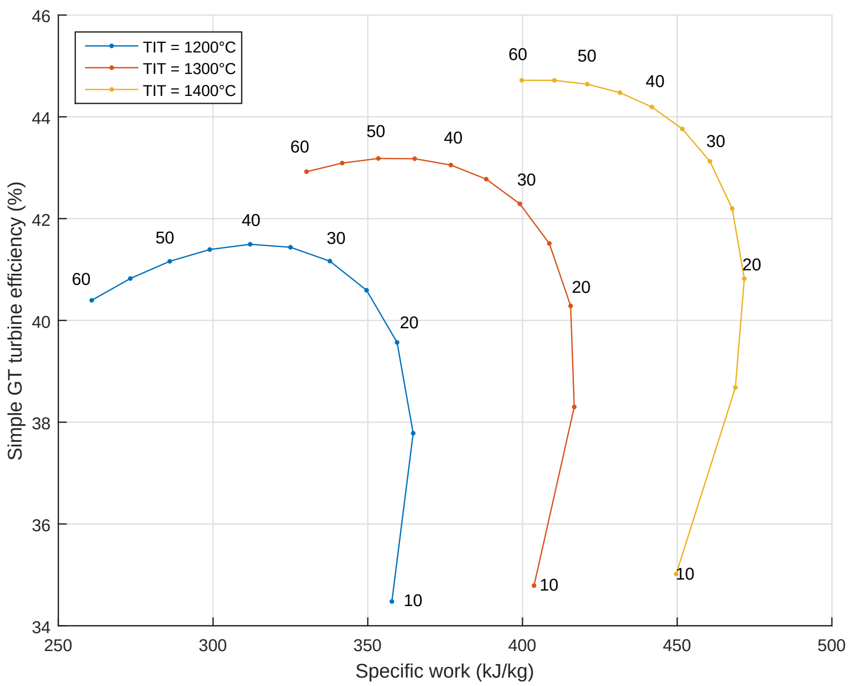


Figure 4.1.: Simple GT specific work and efficiency for given P-ratios at $TIT = 1200, 1300, 1400\,^{\circ}\mathrm{C}$.

Moving to higher values of the TIT we see an 'unfolding' pattern, while it shifts to higher specific work and efficiency values. This pattern is characteristic for GTs and is reflected in literature [5] [8] [16]. One interesting results is that for a given TIT there is a certain PR where the efficiency is at its highest. Increasing the PR is known to have positive effects, however going further than this maximum value is actually ineffective. Current turbines operate at TIT and PR values of around $1200\,^{\circ}\mathrm{C}$ and 30, respectively [8], which means that benefits from PR increase are still possible for the assumed parameters.

The same behaviour can be presented for the STIG, see figure 4.2. Now a certain steam fraction (SF) defined as fraction of the total mass flow is injected. For every PR value, the system utilizes the most efficient injection point; more about this in section 4.2. Again the same basic unfolding behaviour is present, also as seen in literature [16]. One of the most distinctive differences with the simple GT is that the region consists of much higher specific work and efficiency values. Furthermore, if compared to the same TIT of the simple GT, the most efficient point is positioned a much lower PR. For the assumed parameters, current turbines would therefore already be near the most efficient area if STIG would be applied.

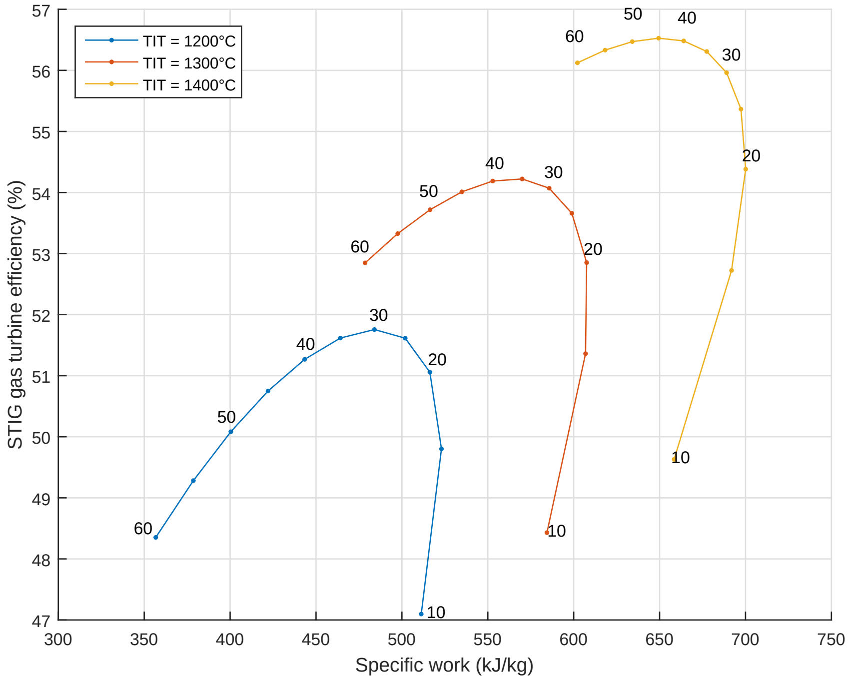


Figure 4.2.: STIG specific work and efficiency for given P-ratios at $TIT = 1200, 1300, 1400\,°\mathrm{C}$.

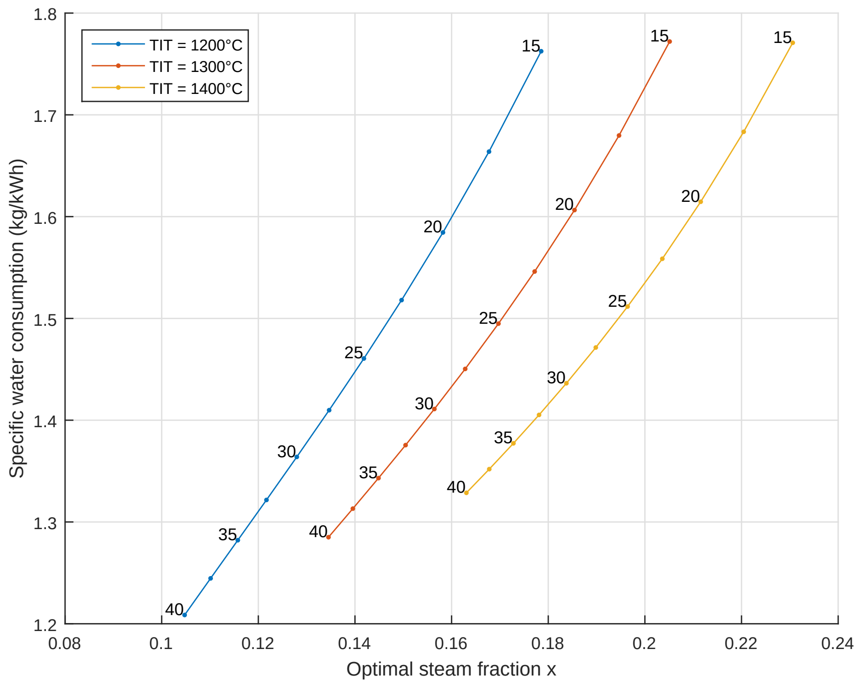


Figure 4.3.: STIG specific water consumption and optimal steam fraction at P-ratios for $TIT = 1200, 1300, 1400\,°\mathrm{C}$.

Another important factor is the water consumption, see figure 4.3. For a given TIT and PR, the STIG will have a certain optimal SF x and specific water consumption $\left(\frac{\dot{m}_{water}[\frac{kg}{h}]}{\dot{W}_{net}[kW]}\right)$. If pressure is increased at constant TIT, the water consumption will decrease. If the TIT is increased, so does the water usage at constant pressure. This behaviour is also present in literature [16] [17] . Notice that with figure 4.2 and 4.3 one could determine the optimal PR and SF for a given TIT, and then find the resulting efficiency, specific water consumption and work output.

4.2. STIG efficiency

The STIG has a very distinctive point at which it operates at the highest efficiency. In order to explain the reason behind this, some additional behaviours have to be explained first. To illustrate this, the following plots are generated by using the model-values as shown in table 3.1 and 3.2.

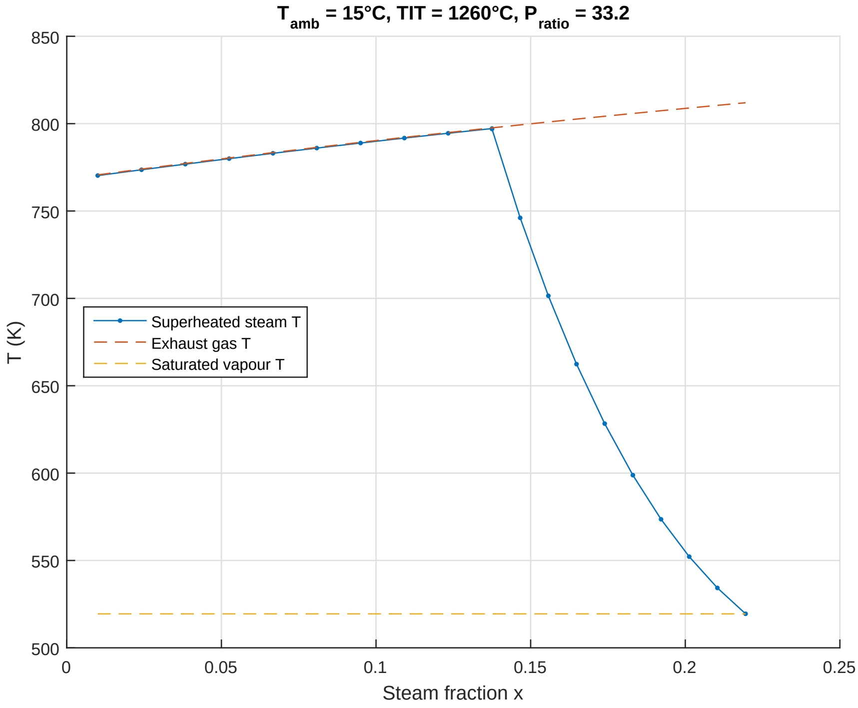


Figure 4.4.: Highest possible steam temperature for certain mass-fraction x

Figure 4.4 shows the last step in the creation of steam, the part where it is superheated prior to injection. There are two important boundaries, the temperature of the steam when it is a saturated vapour, since from here on it will be superheated, and the turbine outlet temperature, for this is the gas used for heating. For a given mass fraction x the blue line shows to which temperature the steam is able to be superheated.

For example, at $x = 0.17$ the generated steam would be around $T = 650K$. If the SF would be decreased, the result would be that the exhaust heat has to be transferred to a smaller mass-flow, resulting in a higher overall temperature. The opposite also happens. If the steam mass-flow is increased the exhaust heat is now transferred to larger mass-flow, resulting in a lower overall temperature.

Notice that around $x = 0.22$ it hits a limit. The mass-flow has become so large that there is just enough heat to produce saturated steam. SFs higher than this specific point can therefore not be achieved by solely using the turbine exhaust gasses. Furthermore, the left side of the graph consists of relatively low SFs. As a result of this small steam mass-flow, there is more than enough heat in the turbine exhaust gas to bring it to maximum temperature; its limit being assumed to be the exhaust gas temperature.

Three important regions can now distinguished. Region I, where the SF is so low the steam can be brought up to a maximum temperature, region II, where the steam can be heated to a certain temperature, but lower than maximum, and region III, which consists of SFs which can not be generated by using the turbine outlet gasses.

If one would look at the efficiency as shown in figure 4.5, where region I and II meet the most efficient point can be found. Here lays the SF which utilizes the most transfer heat while still reaching the maximum temperature, hence having the highest enthalpy. On both sides the efficiency drops rapidly. The reason for lower SFs being twofold: lowering the steam mass-flow goes at the expense of the gained work reduction in the compressor. Additionally the turbine outlet temperature decreases, likewise does the enthalpy of the injected steam which is accompanied by an increase of fuel flow. The reason for higher SFs being that despite the efficiency increase because of more steam, the lower temperature/enthalpy of the injected steam decreases the efficiency at a faster rate.

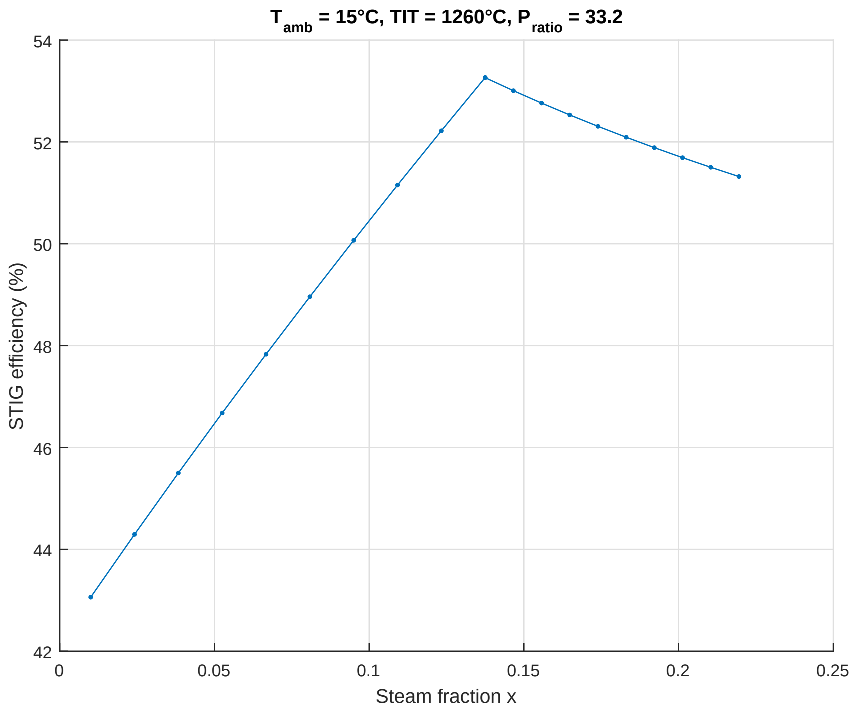


Figure 4.5.: STIG efficiency for certain amount of injected steam x.

Lastly, in figure 4.6 we see a similar behaviour as in figure 4.3, only now for the turbine outlet exhaust gas temperature. For a given TIT it is seen that the outlet temperature will decrease with higher PRs. This is because of the isentropic relation between the temperature and PR of the turbine as seen in equation 2.9. It shows that increasing the PR while maintaining the same TIT results in a decrease of the outlet temperature. As a cause this decreases the amount of heat that can be transferred to generate steam, thus lowering x_{opt}

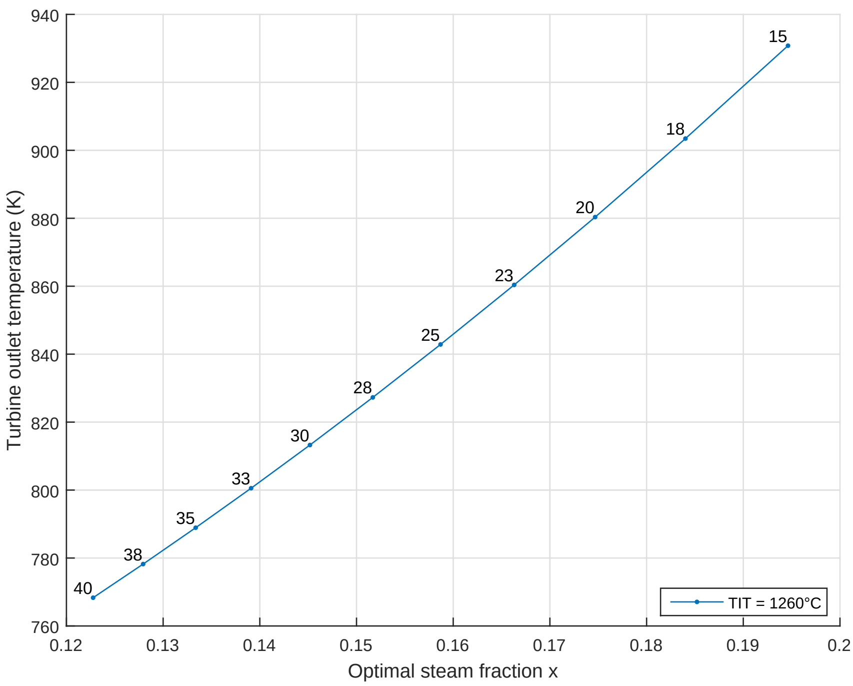


Figure 4.6.: Turbine outlet temperature and optimal steam fraction x for certain pressure ratio at given TIT.

5. Discussion

A model has been created and is a simple representation; this means there is still room to expand upon. For example, pressure drop in the CC could be introduced, a separate power turbine could be added and instead of one segment for the compressor and turbine they could be split in (several) high and lower pressure segments like present in actual turbines. Furthermore, average heat capacities are assumed as approximation for the OTSG model; using variable specific heats could provide a more exact analysis. These changes could improve the model without making it too complex.

One of the prime reasons the STIG and other water injection methods are so limited used is because of the large amount of high-quality water being used. To put the 21 kg/s from the model into perspective: if the Horst-tower was transformed into a reservoir and filled from the bottom to the top with water, there would be enough for approximately 2 weeks before everything would be evaporated into the atmosphere. Winning back the injected water is therefore very important to decrease the associated treatment costs or if water is a scarce resource. Thereupon it was considered to take a look at condensation of the exhaust gas, however later it was decided that not sufficient time was available for this addition. Efforts would better be put in getting the EES models up and running and validating the results.

Lastly, while creating the model and reviewing literature it became apparent that numerical results are very susceptible to changes of values for the input parameters. Therefore as general remark, one should be cautious when numerical results are compared between different turbines if these parameters do not match, since one could end up drawing the wrong conclusions. Therefore to show the workings, this report tried to put focus on showing what the characteristic behaviours are of the system.

6. Conclusions and recommendations

From the analysed data a few conclusions can be made:

- The model shows expected behaviour as seen in literature for the simple GT as well as the STIG
- Simple GTs at a certain TIT value have a maximum PR at which the cycle efficiency is the highest. Further increase of the ratio is therefore not useful.
- The STIG shows similar behaviour, albeit for lower maximum PRs and achieves higher values for cycle efficiencies and specific work.
- Increasing the TIT at constant pressure ratio increases specific water consumption, while increasing the pressure ratio at constant TIT reduces it.
- The most efficient point for the STIG can be found for the SF where the generated steam utilizes the most heat from the exhaust gasses while still reaching the maximum temperature, hence having the highest enthalpy before injection.
- By utilizing the STIG method for the chosen turbine, an efficiency increase and work augmentation of around 11% and 45% respectively can be achieved.

Additional research is still suggested. The most important recommendation is to look at the possibilities and implementation for condensation of the exhaust gas of a STIG turbine, since a lot of room for research is still possible in the area of recovery of the injected steam. What also could be interesting is to compare the usage of water of STIG with the water consumption of rankine cycles using cooling towers, which also evaporate water into the atmosphere. Furthermore, the EES model could be improved upon, amongst others as stated in the discussion.

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A. Simple gas turbine

Input data

$$P_{ratio} = 33,2 \tag{A.1}$$

$$T_1 = 288,15 \;\; [\text{K}] \tag{A.2}$$

$$P_1 = 100,325 \;\; [\text{kPa}] \tag{A.3}$$

$$T_3 = (1260 + 273,15) \;\; [\text{K}] \tag{A.4}$$

$$\dot{m}_{tot} = 141 \;\; [\text{kg/s}] \tag{A.5}$$

$$\dot{m}_{tot} = \dot{m}_{a;1} + \dot{m}_{f;1} \tag{A.6}$$

$$\eta_c = 93/100 \tag{A.7}$$

$$\eta_t = 86/100 \tag{A.8}$$

$$LHV = 50000 \;\; [\text{kJ/kg}] \qquad \text{Methane} \tag{A.9}$$

Compressor inlet conditions

$$h_1 = \text{h}\left(Air; \; \text{T} = T_1\right) \tag{A.10}$$

$$s_1 = \text{s}\left(Air; \; \text{T} = T_1; \; \text{P} = P_1\right) \tag{A.11}$$

Compressor analysis

$$s_{is;2} = s_1 \qquad \text{In ideal case, process 1-2 is isentropic} \tag{A.12}$$

$$P_{ratio} = \frac{P_2}{P_1} \qquad \text{Definition of the ratio, determines P2} \tag{A.13}$$

$$T_{is;2} = \text{T}\left(Air; \; \text{s} = s_{is;2}; \; \text{P} = P_2\right) \qquad \text{T}_{is;2} \text{ is the isentropic value of T[2] at the compressor exit} \tag{A.14}$$

$$h_{is;2} = \text{h}\left(Air; \; \text{T} = T_{is;2}\right) \qquad \text{Knowing T}_s\text{, we can determine h}_s \tag{A.15}$$

$$\eta_c = \frac{h_{is;2} - h_1}{(h_2 - h_1)} \qquad \text{Adiabatic efficiency Eta}_c = \text{w}_{dot;c;ideal}/\text{w}_{dot;c;actual} \tag{A.16}$$

Compressor adiabatic efficiency; This allows to determine the actual h[2]

$$\dot{m}_{a;1} \cdot h_1 + \dot{W}_c = \dot{m}_{a;1} \cdot h_2 \qquad \text{Derivative of the energy balance, assuming: adiabatic, ke=pe=0, gives W}_{dot;c} \tag{A.17}$$

External heat exchanger analysis

$$h_3 = \text{h}\left(Air; \; \text{T} = T_3\right) \qquad \text{Known temperature, so we can deduce the enthalpy} \tag{A.18}$$

$$\dot{m}_{a;1} \cdot h_2 + \dot{Q}_{in} = (\dot{m}_{a;1} + \dot{m}_{f;1}) \cdot h_3 \qquad \text{Energy balance, assuming W=0 and ke=pe=0, to determine Q}_{dot} \tag{A.19}$$

$$\dot{Q}_{in} = LHV \cdot \dot{m}_{f;1} \qquad \text{Used to determine the fuel flow in order to get T3} \tag{A.20}$$

Turbine inlet conditions

$$P_3 = P_2 \qquad \text{process 2-3 is ideally at constant pressure} \tag{A.21}$$

$$s_3 = \text{s}\left(Air; \; \text{T} = T_3; \; \text{P} = P_3\right) \tag{A.22}$$

Turbine analysis

$$s_{is;4} = s_3 \qquad \text{Ideal case, process 3-4 is isentropic} \tag{A.23}$$

$$P_{ratio} = \frac{P_3}{P_4} \qquad \text{Definition of the ratio, determines P4} \tag{A.24}$$

$$T_{is;4} = \mathrm{T}\left(Air;\ \mathrm{s} = s_{is;4};\ \mathrm{P} = P_4\right) \qquad \text{T}_{is;4} \text{ is the isentropic value of T[4] at the turbine exit} \tag{A.25}$$

$$h_{is;4} = \mathrm{h}\left(Air;\ \mathrm{T} = T_{is;4}\right) \qquad \text{And therefore gives the isentropic enthalpy at that point} \tag{A.26}$$

Turbine adiabatic efficiency Eta$_t$ = W$_{dot;t}$ /Wts$_{dot}$ turbine is known+ allows calculation of h[4]

$$\eta_t = \frac{h_3 - h_4}{\left(h_3 - h_{is;4}\right)} \tag{A.27}$$

Energy balance, assuming: adiabatic, ke=pe=0. Gives W$_{dot;t}$

$$\left(\dot{m}_{a;1} + \dot{m}_{f;1}\right) \cdot h_3 = \dot{W}_t + \left(\dot{m}_{a;1} + \dot{m}_{f;1}\right) \cdot h_4 \tag{A.28}$$

Analysis of the cycle

$$\dot{W}_{net} = \dot{W}_t - \dot{W}_c \qquad \text{Definition of the net cycle work, in kW} \tag{A.29}$$

$$\eta_{cycle} = \dot{W}_{net}/\dot{Q}_{in} \qquad \text{Cycle thermal efficiency} \tag{A.30}$$

$$Bwr = \frac{\dot{W}_c}{\dot{W}_t} \qquad \text{Back/work ratio} \tag{A.31}$$

The following points are determined only to produce a T-s and P-v plot

$$T_2 = \mathrm{T}\left(Air;\ \mathrm{h} = h_2\right) \tag{A.32}$$

$$T_4 = \mathrm{T}\left(Air;\ \mathrm{h} = h_4\right) \tag{A.33}$$

$$s_2 = \mathrm{s}\left(Air;\ \mathrm{T} = T_2;\ \mathrm{P} = P_2\right) \tag{A.34}$$

$$s_4 = \mathrm{s}\left(Air;\ \mathrm{T} = T_4;\ \mathrm{P} = P_4\right) \tag{A.35}$$

$$s_{is;1} = s_1 \tag{A.36}$$

$$s_{is;3} = s_3 \tag{A.37}$$

$$T_{is;1} = T_1 \tag{A.38}$$

$$T_{is;3} = T_3 \tag{A.39}$$

$$v_1 = \mathrm{v}\left(Air;\ \mathrm{T} = T_1;\ \mathrm{P} = P_1\right) \tag{A.40}$$

$$v_2 = \mathrm{v}\left(Air;\ \mathrm{T} = T_2;\ \mathrm{P} = P_2\right) \tag{A.41}$$

$$v_3 = \mathrm{v}\left(Air;\ \mathrm{T} = T_3;\ \mathrm{P} = P_3\right) \tag{A.42}$$

$$v_4 = \mathrm{v}\left(Air;\ \mathrm{T} = T_4;\ \mathrm{P} = P_4\right) \tag{A.43}$$

$$v_{is;1} = \mathrm{v}\left(Air;\ \mathrm{T} = T_{is;1};\ \mathrm{P} = P_1\right) \tag{A.44}$$

$$v_{is;2} = \mathrm{v}\left(Air;\ \mathrm{T} = T_{is;2};\ \mathrm{P} = P_2\right) \tag{A.45}$$

$$v_{is;3} = \mathrm{v}\left(Air;\ \mathrm{T} = T_{is;3};\ \mathrm{P} = P_3\right) \tag{A.46}$$

$$v_{is;4} = \mathrm{v}\left(Air;\ \mathrm{T} = T_{is;4};\ \mathrm{P} = P_4\right) \tag{A.47}$$

GT solutions

$Bwr = 0,587$	$\eta_c = 0,93$ [-]	$\eta_{cycle} = 0,4217$	$\eta_t = 0,86$ [-]
$LHV = 50000$ [kJ/kg]	$\dot{m}_{tot} = 141$ [kg/s]	$P_{ratio} = 33,2$ [-]	$\dot{Q}_{in} = 122785$ [kJ/s]
$\dot{W}_c = 73600$ [kJ/s]	$\dot{W}_{net} = 51779$ [kJ/s]	$\dot{W}_t = 125379$ [kJ/s]	

Row	h_i [kJ/kg]	$h_{is;i}$ [kJ/kg]	$\dot{m}_{a;i}$ [kg/s]	$\dot{m}_{f;i}$ [kg/s]	P_i [kPa]	s_i [kJ/kg-K]
1	288,5		138,5	2,456	100,3	5,664
2	819,8	782,6			3331	5,712
3	1676				3331	6,468
4	787,1	642,3			100,3	6,675

Row	$s_{is;i}$ [kJ/kg-K]	T_i [K]	$T_{is;i}$ [K]	v_i [m^3/kg]	$v_{is;i}$ [m^3/kg]
1	5,664	288,2	288,2	0,8244	0,8244
2	5,664	797,7	763,8	0,06875	0,06582
3	6,468	1533	1533	0,1321	0,1321
4	6,468	767,9	633,2	2,197	1,812

B. STIG

Input values

$\dot{m}_{tot} = \dot{m}_{a;2} + \dot{m}_{f;2} + \dot{m}_s$ We want to keep the total designed mass flow the same (B.1)

$x = \dot{m}_s / \dot{m}_{tot}$ Express steam input as total mass fraction (B.2)

$T_{stig;4} = T_3$ $T_{stig;4}$ is now the turbine inlet temperature, which was T_3 in the GT model, same value (B.3)

SWITCH CASE A:

A1: Calculate for chosen x-value

$x = 0.15$ Mass percentage of steam (B.4)

A2: In order to calculate at x_{opt}:

$Tw_4 = T_{stig;5}$ Condition for most efficient energy transfer of exhaust gas and water (B.5)

A3: In order to calculate at x_{max}:

$Tw_4 = Tw_5$ Condition for max steam generation (B.6)

Compressor analysis

$\dot{m}_{a;2} \cdot h_1 + \dot{W}_{c;stig} = \dot{m}_{a;2} \cdot h_2$ Derivative of the energy balance, assuming: adiabatic, ke=pe=0, gives $W_{dot;c;stig}$ (B.7)

Mass flow of air will be lower, hence compressor work needs to be recalculated

Mix of steam and air

$P_{stig;3} = P_2$ Steam enters at the same pressure (B.8)

$hd = hw_4$ Enthalpy of injected steam (B.9)

Combustion chamber analysis

General energy balance:

Inlet: $\dot{m}_a$ * h[2] + $\dot{m}_s$ * $h_{steam;3}$ = ($\dot{m}_a$ + $\dot{m}_s$) * $h_{stig;3}$

Combustion: ($\dot{m}_a$ + $\dot{m}_s$) * $h_{stig;3}$ + LHV * $dot_{m;f}$ = ($\dot{m}_a$ + $\dot{m}_f$) * h4g + $\dot{m}_s$ * h4s

Outlet: ($\dot{m}_a$ + $\dot{m}_f$) * h4g + $\dot{m}_s$ * h4s = ($\dot{m}_a$ + $\dot{m}_f$+$\dot{m}_s$) * $h_{stig;4}$

As a single main equation:

$\dot{m}_a$ * h[2] + $\dot{m}_s$ * $h_{steam;3}$ + LHV * $dot_{m;f}$ = ($\dot{m}_a$ + $\dot{m}_f$) * h4g + $\dot{m}_s$ * h4s

Rearange to get:

$\dot{m}_{f;2}$* (LHV-h4g) = $\dot{m}_{a;2}$* (h4g-h[2]) + $\dot{m}_s$*(h4s-h$_{steam;3}$)

$$\dot{m}_{f;2}\cdot(LHV - hg_4) = \dot{m}_{a;2}\cdot(hg_4 - h_2)+\dot{m}_s\cdot(hs_4 - hd) \tag{B.10}$$
Solve for the needed m$_{dot;f}$ in order to get the desired TIT / T$_{stig;4}$

$$hs_4 = \mathrm{h}\left(Steam;\ \mathrm{T} = T_{stig;4};\ \mathrm{P} = P_1 \cdot P_{ratio}\right) \tag{B.11}$$
Enthalpy of steam at TIT

$$hg_4 = ha_4 \tag{B.12}$$
We assume the enthalpy of the combustion gas to be the same of air, only small difference, $<5\%$

$$ha_4 = Enthalpy\left(Air;\ \mathrm{T} = T_{stig;4}\right) \tag{B.13}$$
Gives the enthalpy of air at the TIT

$$\dot{Q}_{in;stig} = \dot{m}_{f;2} \cdot LHV \tag{B.14}$$
Used to determine the needed Q$_{dot;in;stig}$ in order to get the desired TIT / T$_{stig;4}$

Turbine inlet conditions

$$P_{stig;4} = P_{stig;3} \tag{B.15}$$
Isobaric combustion throughout the combustion chamber

$$s_{stig;air;4} = \mathrm{s}\left(Air;\ \mathrm{T} = T_{stig;4};\ \mathrm{P} = P_{stig;4}\right) \tag{B.16}$$
s$_{stig;4}$ Can now be determined

$$s_{stig;steam;4} = \mathrm{s}\left(Steam;\ \mathrm{T} = T_{stig;4};\ \mathrm{P} = P_{stig;4}\right) \tag{B.17}$$
s$_{stig;4}$ Can now be determined

Turbine analysis

$$s_{is;stig;air;5} = s_{stig;air;4} \tag{B.18}$$
Ideal case, process 4-5 is isentropic

$$s_{is;stig;steam;5} = s_{stig;steam;4} \tag{B.19}$$
Ideal case, process 4-5 is isentropic

$$P_{ratio} = \frac{P_{stig;4}}{P_{stig;5}} \tag{B.20}$$
Known ratio, determines P$_{stig;5}$

$$T_{is;stig;air;5} = \mathrm{T}\left(Air;\ \mathrm{s} = s_{is;stig;air;5};\ \mathrm{P} = P_{stig;5}\right) \tag{B.21}$$
T$_{is;stig;5}$ is the isentropic value of T[5] at the turbine exit

$$ha_{is;5} = \mathrm{h}\left(Air;\ \mathrm{T} = T_{is;stig;air;5}\right) \tag{B.22}$$
Which can be used to determine the isentropic enthalpy at that point

$$T_{is;stig;steam;5} = \mathrm{T}\left(Steam;\ \mathrm{s} = s_{is;stig;steam;5};\ \mathrm{P} = P_{stig;5}\right) \tag{B.23}$$
T$_{is;stig;5}$ is the isentropic value of T[5] at the turbine exit

$$hs_{is;5} = \mathrm{h}\left(Steam;\ \mathrm{T} = T_{is;stig;steam;5};\ \mathrm{P} = P_{stig;5}\right) \tag{B.24}$$
Which can be used to determine the isentropic enthalpy at that point

$$xs = x \tag{B.25}$$

$$xa = (1 - x) \tag{B.26}$$

$$\eta_t = \frac{xa \cdot ha_4 + xs \cdot hs_4 - xa \cdot ha_5 - xs \cdot hs_5}{(xa \cdot ha_4 + xs \cdot hs_4 - xa \cdot ha_{is;5} - xs \cdot hs_{is;5})} \tag{B.27}$$
Solves for ha[5] and hs[5]

Boundary condition to calculate ha[5] and hs[5]

$$T_{stig;steam;5} = T_{stig;air;5} \tag{B.28}$$

$$T_{stig;air;5} = \mathrm{T}\left(Air;\ \mathrm{h} = ha_5\right) \tag{B.29}$$

$$T_{stig;steam;5} = \mathrm{T}\left(Steam;\ \mathrm{h} = hs_5;\ \mathrm{P}\ = P_1\right) \tag{B.30}$$

Energy balance, assuming: adiabatic, ke=pe=0. Gives $\dot{W}_{t;stig}$

$$(\dot{m}_{a;2} + \dot{m}_{f;2}) \cdot ha_4 + \dot{m}_s \cdot hs_4 = \dot{W}_{t;stig} + (\dot{m}_{a;2} + \dot{m}_{f;2}) \cdot ha_5 + \dot{m}_s \cdot hs_5 \tag{B.31}$$

Analysis of the cycle

$$\dot{W}_{net;stig} = \dot{W}_{t;stig} - \dot{W}_{c;stig} \quad \text{Definition of the net cycle work, in kW} \tag{B.32}$$

$$\eta_{cycle;stig} = \dot{W}_{net;stig}/\dot{Q}_{in;stig} \quad \text{Cycle thermal efficiency} \tag{B.33}$$

$$Bwr_{stig} = \dot{W}_{c;stig}/\dot{W}_{t;stig} \quad \text{Back/work ratio} \tag{B.34}$$

The following points are determined only to produce a T-s and P-v plot

$$sw_1 = \mathrm{s}\left(Water;\ \mathrm{T} = Tw_1;\ \mathrm{P} = Pw\right) \tag{B.35}$$

$$sw_2 = \mathrm{s}\left(Water;\ \mathrm{T} = Tw_2;\ \mathrm{x} = 0\right) \tag{B.36}$$

$$sw_3 = \mathrm{s}\left(Water;\ \mathrm{T} = Tw_3;\ \mathrm{x} = 1\right) \tag{B.37}$$

$$sw_4 = \mathrm{s}\left(Water;\ \mathrm{T} = Tw_4;\ \mathrm{P} = P_{stig;3}\right) \tag{B.38}$$

$$s_{stig;5} = \mathrm{s}\left(Air;\ \mathrm{T} = T_{stig;5};\ \mathrm{P} = P_{stig;5}\right) \tag{B.39}$$

C. OTSG

Input values OTSG

$$T_{stig;5} = T_{stig;air;5} \tag{C.1}$$

$Pw = P_1 \cdot P_{ratio} + 400$ [kPa] Workpressure of water is a bit higher for injection, this if for practical reasons concerning less exergy (C.2)

SWITCH CASE B: between rich ($x < x_{opt}$) and normal region ($x_{opt} =< x =< x_{max}$)

B1: These are to calculate (parametric table) values for $x_{opt} =< x =< x_{max.}$ Works with A1, A2 and A3

$T_{stig;7} - Tw_2 = 10$ [K] Pinch point Tw[1], taken at 10 degrees difference (C.3)

$$Tw_4 = \mathrm{T}\left(Steam;\ \mathrm{P} = Pw;\ \mathrm{h} = hw_4\right) \tag{C.4}$$

B2: These are to calculate (parametric table) values for $x < x_{opt.}$ Works only with A1

$Tw_4 = Tw_{stig;5}$ Max temperature the steam can get (C.5)

$$hw_4 = Enthalpy\left(Steam;\ \mathrm{T} = Tw_4;\ \mathrm{P} = Pw\right) \tag{C.6}$$

Balance equations

Balance equations input water side

$$\dot{Q}_{eco} = \dot{m}_s \cdot (hw_2 - hw_1) \tag{C.7}$$

$$\dot{Q}_{vap} = \dot{m}_s \cdot (hw_3 - hw_2) \tag{C.8}$$

$$\dot{Q}_{sh} = \dot{m}_s \cdot (hw_4 - hw_3) \tag{C.9}$$

Balance equations output exhaust gasses side

$\dot{Q}_{eco}$ = ($\dot{m}_a$ + $\dot{m}_f$)*(h7g-h8g) + $\dot{m}_s$ * (h7s-h8s)

$\dot{Q}_{vap}$ = ($\dot{m}_a$ + $\dot{m}_f$)*(h6g-h7g) + $\dot{m}_s$ * (h6s-h7s)

$\dot{Q}_{sh}$ = ($\dot{m}_a$ + $\dot{m}_f$)*(h5g-h6g) + $\dot{m}_s$ * (h5s-h6s)

Rewritten:

$$\dot{Q}_{eco} = \left((\dot{m}_{a;2} + \dot{m}_{f;2}) \cdot c78a + \dot{m}_s \cdot c78s\right) \cdot (T_{stig;7} - T_{stig;8}) \tag{C.10}$$

$$\dot{Q}_{vap} = \left((\dot{m}_{a;2} + \dot{m}_{f;2}) \cdot c67a + \dot{m}_s \cdot c67s\right) \cdot (T_{stig;6} - T_{stig;7}) \tag{C.11}$$

$$\dot{Q}_{sh} = \left((\dot{m}_{a;2} + \dot{m}_{f;2}) \cdot c56a + \dot{m}_s \cdot c56s\right) \cdot (T_{stig;5} - T_{stig;6}) \tag{C.12}$$

$$\dot{Q}_{rec} = \dot{Q}_{eco} + \dot{Q}_{vap} + \dot{Q}_{sh} \tag{C.13}$$

Additional variables definition

Average specific heat capcitites on several points

$$c78s = \mathrm{c_p}\left(Water;\ \mathrm{T} = \frac{T_{stig;7} + T_{stig;8}}{2};\ \mathrm{P} = Pw\right) \tag{C.14}$$

$$c67s = \mathrm{c_p}\left(Water;\ \mathrm{T} = \frac{T_{stig;6} + T_{stig;7}}{2};\ \mathrm{P} = Pw\right) \tag{C.15}$$

$$c56s = \mathrm{c_p}\left(Steam;\ \mathrm{T} = \frac{T_{stig;5} + T_{stig;6}}{2};\ \mathrm{P} = Pw\right) \tag{C.16}$$

$$c78a = \mathrm{c_p}\left(Air;\ \mathrm{T} = \frac{T_{stig;7} + T_{stig;8}}{2}\right) \tag{C.17}$$

$$c67a = \mathrm{c_p}\left(Air;\ \mathrm{T} = \frac{T_{stig;6} + T_{stig;7}}{2}\right) \tag{C.18}$$

$$c56a = \mathrm{c_p}\left(Air;\ \mathrm{T} = \frac{T_{stig;5} + T_{stig;6}}{2}\right) \tag{C.19}$$

Known enthalpies and temperatures of water

$$hw_1 = \mathrm{h}\left(Water;\ \mathrm{T} = Tw_1;\ \mathrm{P} = Pw\right) \quad \text{Entry h} \tag{C.20}$$

$$hw_2 = \mathrm{h}\left(Water;\ \mathrm{x} = 0;\ \mathrm{P} = Pw\right) \quad \text{hf} \tag{C.21}$$

$$hw_3 = \mathrm{h}\left(Water;\ \mathrm{x} = 1;\ \mathrm{P} = Pw\right) \quad \text{hg} \tag{C.22}$$

$$Tw_1 = (273,15 + 35)\ [\mathrm{K}] \quad \text{Input T water} \tag{C.23}$$

$$Tw_2 = \mathrm{T_{sat}}\left(Water;\ \mathrm{P} = Pw\right) \quad \text{T at hf} \tag{C.24}$$

$$Tw_3 = \mathrm{T_{sat}}\left(Steam;\ \mathrm{P} = Pw\right) \quad \text{T at hg} \tag{C.25}$$

Extra analysys entire cycle

$$\dot{W}_{spec} = \dot{W}_{net}/\dot{m}_{tot} \tag{C.26}$$

$$\dot{W}_{spec;stig} = \dot{W}_{net;stig}/\dot{m}_{tot} \tag{C.27}$$

$$WC_{spec} = \dot{m}_s \cdot 3600/\dot{W}_{net} \tag{C.28}$$

D. Complete model solutions

$Bwr = 0,587$	$Bwr_{stig} = 0,456$	$c56a = 1,089$ [kJ/kg-K]	$c56s = 2,282$ [kJ/kg-K]
$c67a = 1,057$ [kJ/kg-K]	$c67s = 2,461$ [kJ/kg-K]	$c78a = 1,027$ [kJ/kg-K]	$c78s = 4,557$ [kJ/kg-K]
$\eta_c = 0,93$ [-]	$\eta_{cycle} = 0,4217$	$\eta_{cycle;stig} = 0,5326$	$\eta_t = 0,86$ [-]
$hd = 3503$ [kJ/kg]	$LHV = 50000$ [kJ/kg]	$\dot{m}_s = 19,38$ [kg/s]	$\dot{m}_{tot} = 141$ [kg/s]
$Pw = 3731$ [kPa]	$P_{ratio} = 33,2$ [-]	$\dot{Q}_{eco} = 17782$ [kJ/s]	$\dot{Q}_{in} = 122785$ [kJ/s]
$\dot{Q}_{in;stig} = 141336$ [kJ/s]	$\dot{Q}_{rec} = 64991$ [kJ/s]	$\dot{Q}_{sh} = 13593$ [kJ/s]	$\dot{Q}_{vap} = 33616$ [kJ/s]
$WC_{spec} = 1,348$ [kg/kJ]	$\dot{W}_c = 73600$ [kJ/s]	$\dot{W}_{c;stig} = 63106$ [kJ/s]	$\dot{W}_{net} = 51779$ [kJ/s]
$\dot{W}_{net;stig} = 75279$ [kJ/s]	$\dot{W}_{spec} = 367,2$ [kJ/kg]	$\dot{W}_{spec;stig} = 533,9$ [kJ/kg]	$\dot{W}_t = 125379$ [kJ/s]
$\dot{W}_{t;stig} = 138385$ [kJ/s]	$x = 0,1375$	$xa = 0,8625$	$xs = 0,1375$

Row	h_i [kJ/kg]	ha_i [kJ/kg]	$ha_{is;i}$ [kJ/kg]	hg_i [kJ/kg]	hs_i [kJ/kg]	$hs_{is;i}$ [kJ/kg]	hw_i [kJ/kg]	$h_{is;i}$ [kJ/kg]
1	288,5						149,9	
2	819,8						1067	782,6
3	1676						2802	
4	787,1	1676		1676	5301		3503	642,3
5		819,2	642,3		3540	3487		
6								
7								
8								

Row	$\dot{m}_{a;i}$ [kg/s]	$\dot{m}_{f;i}$ [kg/s]	P_i [kPa]	$P_{stig;i}$ [kPa]	s_i [kJ/kg-K]	sw_i [kJ/kg-K]	$s_{is;i}$ [kJ/kg-K]	$s_{is;stig;air;i}$ [kJ/kg-K]
1	138,5	2,456	100,3		5,664	0,5037	5,664	
2	118,8	2,827	3331		5,712	2,759	5,664	
3			3331	3331	6,468	6,098	6,468	
4			100,3	3331	6,675	7,251	6,468	
5				100,3				6,468
6								
7								
8								

Row	$s_{is;stig;steam;i}$ [kJ/kg-K]	$s_{stig;i}$ [kJ/kg-K]	$s_{stig;air;i}$ [kJ/kg-K]	$s_{stig;steam;i}$ [kJ/kg-K]	T_i [K]	Tw_i [K]	$T_{is;i}$ [K]	$T_{is;stig;air;i}$ [K]
1					288,2	308,2	288,2	
2					797,7	519,4	763,8	
3					1533	519,4	1533	
4			6,468	8,831	767,9	797,2	633,2	
5	8,831	6,716						633,2
6								
7								
8								

Row	$T_{is;stig;steam;i}$ [K]	$T_{stig;i}$ [K]	$T_{stig;air;i}$ [K]	$T_{stig;steam;i}$ [K]	v_i $[\mathrm{m}^3/\mathrm{kg}]$	$v_{is;i}$ $[\mathrm{m}^3/\mathrm{kg}]$
1					0,8244	0,8244
2					0,06875	0,06582
3					0,1321	0,1321
4		1533			2,197	1,812
5	772,5	797,2	797,2	797,2		
6		720,2				
7		529,4				
8		446,1				