Internship Report

Assessment of influence of play in joints on the end effector accuracy in a novel 3DOF (1T-2R) parallel manipulator

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Preface

This internship assignment was carried out at TU Delft and was a compulsory module as a part of my Master studies in Mechanical Engineering at the University of Twente. This internship was carried out for a period of 14 weeks, from 16th of November 2015 to 19th of February 2016, and during this period I was exposed to different research projects being carried out at the Delft Haptics lab, apart from in-depth knowledge of the project I was working on.

The main objectives of this internship assignment were to get acquainted with Parallel Kinematics, which included deep understanding of Screw theory and its application to parallel manipulator analysis, and to get acquainted with various methods to assess the effects of play at the joints of parallel mechanisms on the end effector accuracy.

To achieve the same, the influence of play on the static performance (end effector accuracy) of a novel 3-DOF (1T-2R) parallel manipulator, which is intended to act as a haptic master device for steerable needles, was tried to analyze. The device being a limited-DOF parallel manipulator has some constrained directions and the play at the ball bearings affects the performance and quantifying this was the main aim of this internship.

Contents

1. Introduction

Parallel manipulators have had an indelible impact on industry applications in the past few years due to its inherent advantages in terms of accuracy as compared to serial manipulators, although some [1] have pointed out that this question of superiority is debatable. However, the consideration of accuracy is always crucial in terms of demand that different applications have and there are various factors that can hamper this. One of the main factors affecting the end effector accuracy are the clearances in joints. The effect of clearance on the positioning error and accuracy of parallel manipulators has been looked into by many authors over decades and brief overview of different approaches is summarized here. The problem of modelling joint clearance in mechanisms and assessing its effect on the pose (position and orientation) error has been approached by different methods, however, most of them were limited to planar or single loop mechanisms and their extension to analysis of effects in parallel manipulators was not addressed (in some cases) or was not demonstrated.

The current study aims at analyzing the effects of clearance on a novel 3-DOF parallel manipulator intended to act as a haptic master device. The aforementioned device has universal and revolute joints and the play in the constrained directions are mainly due to clearance in the revolute joints. This report discusses screw theory in brief and then goes on to provide a brief literature survey of various relevant methods of modelling the play at joints and then describes the implementation of the selected method of modelling on the system under study. During the implementation of the selected method, certain problems were encountered which are discussed later in this report and given the timeframe of this assignment, a detailed analysis of the same was not possible and possible explanations are summarized at the end.

2. A Brief Summary of Screw Theory

Screw theory is one of the strongest tools for analysis of robotic manipulators and it uses a combination of algebra and calculus of pair of vectors such as forces, moments, angular and linear velocities to do so. This section provides a brief introduction to Screw theory and a comprehensive overview of important concepts used in parallel kinematics is given. [2] is used as a reference for all the figures and references for detailed information.

2.1. Description of screw motion

Screw motion is described as rigid body motion consisting of a rotation about an axis through an angle, followed by translation along the same axis. Figure 2.1 shows the description of a general screw motion.

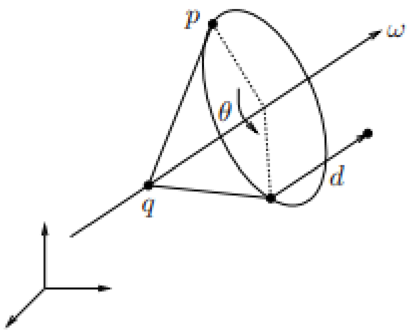

Figure 2.1: General screw [2]

Since this motion resembles the motion of a screw, it is called screw motion. Further, the pitch of a screw motion is defined to be the ratio of translation to rotation, $h = d/\theta$, where d is the translation and θ is the rotation about the axis in radians.

Thus the net translational motion after θ radians of rotation about the axis is hθ. The axis is then represented as a directed line through a point; choosing $q \in R^3$ as the point on the axis and $\omega \in R^3$ as the unit vector specifying the direction. The axis is then a set of points defined by

$$l = (q + \lambda\omega : \lambda \in R) \qquad (2.1)$$

The above definitions are valid when the screw motion consists of a nonzero rotation followed by translation. In case of zero rotation, we consider the axis as a line through the origin in the direction *v* and the magnitude of this vector is 1. The pitch of this screw is ∞ and the amount of translation along the direction *v* is given by its magnitude.

2.2. Twists and Wrenches

Now that we know the description of screw, we can go ahead to describe velocities and forces using this description of screw. **Twists** are generalization of velocities of rigid bodies and any rigid body motion can be expressed using a twist. Geometrically, these are elements of the Lie algebra se(3) associated to the Lie group SE(3). For a more mathematical understanding of the theory of twists, the reader is referred to section 3.2 to section 3.3 of chapter 1 of [2]. To understand the geometric interpretation of twist, consider the following figure.

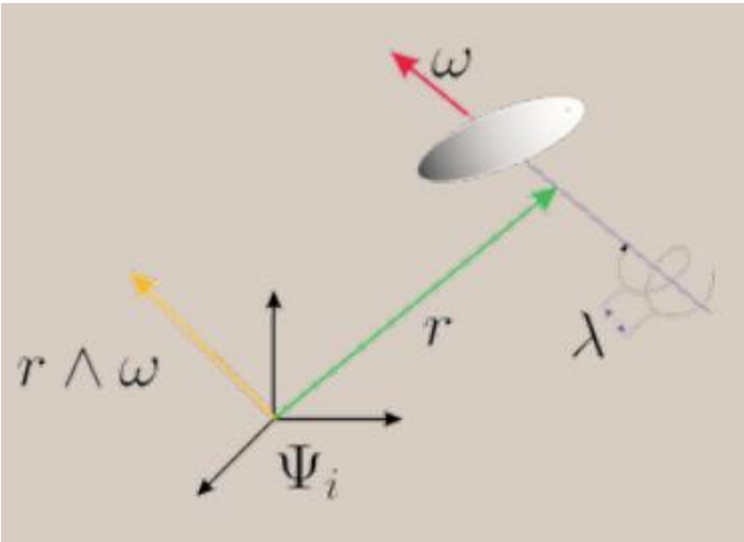

Figure 2.2: geometric interpretation of a twist [Lecture sheets-Modern Robotics-Stefano Stramigioli, University of Twente]

The figure above shows the geometric interpretation of a twist. A twist can be written as shown below

$$\begin{pmatrix} \omega \\ v \end{pmatrix} = \underbrace{\begin{pmatrix} \omega \\ r \wedge \omega \end{pmatrix}}_{\text{rotation}} + \lambda \underbrace{\begin{pmatrix} 0 \\ \omega \end{pmatrix}}_{\text{translation}}$$

In the above expression, ω represents the axis around which rotation takes place and the magnitude of ω gives the magnitude of rotation and "r" is the arm connecting the origin of the reference frame and the axis "ω". The scalar λ is the pitch that relates the rotation about the axis to the translation along the axis. For a twist, the pitch is defined as the ratio of translation along the axis to the rotation about the axis. Hence, in the representation of twist, it can be seen that when ω≠0, the v component along ω is λω and r∧ω is the one orthogonal to ω. The ∧ operator implies cross product .Hence, in a twist, the upper part (ω) represents the rotation and "v" represents the translation.

If we have ω and v, we can find the arm "r" and pitch λ suing expressions

$$r = \frac{\omega \wedge v}{||\omega||^2}$$

$$\lambda = \frac{\omega^T v}{||\omega||^2}$$

The reader is referred to [2] for detailed derivation of the above expressions.

Wrenches are dual to twists. These are linear operators that relate twists to power. According to Poinsot's theorem, any system of forces can be expressed as a pure linear force along a line plus a pure moment around it. Hence, a wrench is defined as shown below

$$\begin{pmatrix} \tau \\ F \end{pmatrix} = \underbrace{\begin{pmatrix} r \wedge F \\ F \end{pmatrix}}_{\text{force}} + \lambda \underbrace{\begin{pmatrix} F \\ 0 \end{pmatrix}}_{\text{moment}}$$

The geometrical representation of a wrench is shown in figure 2.3. As shown in the figure, the linear force is defined and the pitch λ relates the linear force F to the moment about the axis. The pitch in a wrench is defined as the ratio of angular rotation about the axis to the linear force. The reader is referred to section 5 of chapter 1 in [2] for more detailed mathematical explanation of wrenches.

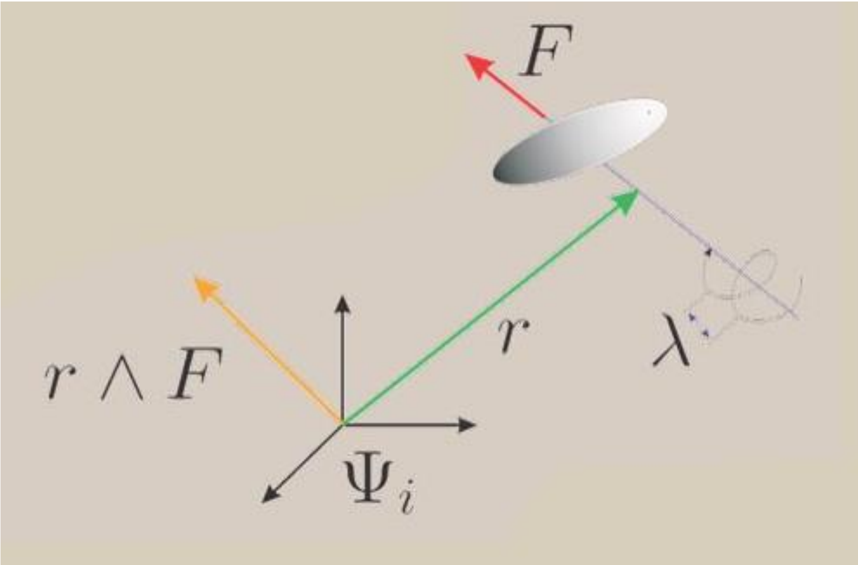

Figure 2.3: geometric interpretation of a wrench [Lecture sheets-Modern Robotics-Stefano Stramigioli, University of Twente]

2.3. Reciprocity and Applications

The dot product of wrenches and twist gives the instantaneous power associated with the moving rigid body under the influence of applied force. A wrench F is then said to be reciprocal to a twist V if the instantaneous power is zero, i.e., F . V = 0. Let V be the twist about screw S_1 and let F be the wrench along the screw S_2. The reciprocal product between a twist and a wrench, expressed in screw coordinates, is then defined as follows.

For a twist

$$V = M_1 \begin{bmatrix} q_1 \times \omega_1 + h_1\omega_1 \\ \omega_1 \end{bmatrix}$$

And a wrench

$$F = M_2 \begin{bmatrix} \omega_2 \\ q_2 \times \omega_2 + h_2\omega_2 \end{bmatrix}$$

Where, M_1 and M_2 are the magnitudes of the screws.

We can assume that the axes are closest at points q_1 and q_2 and hence q_2 can be written as $q_2 = q_1 + dn$, where n is the unit normal vector connecting the two axes. Figure 2.4 shows the above description.

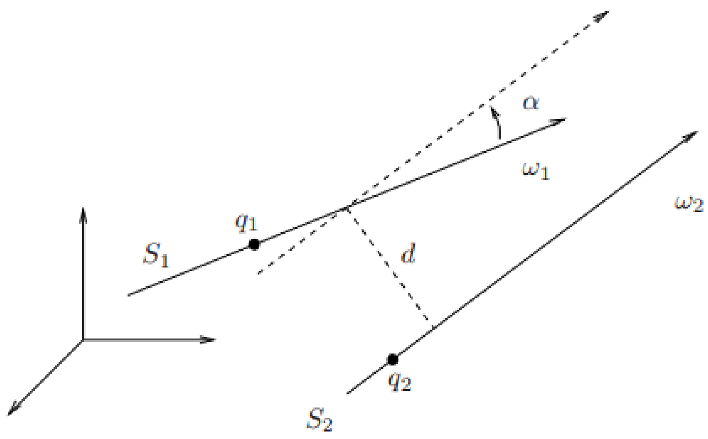

Figure 2.4: Notation for reciprocal screws [2]

The reciprocal product is then given by

$$\begin{aligned} V \cdot F &= M_1M_2\big(\omega_2 \cdot (q_1 \times \omega_1 + h_1\omega_1) + \omega_1 \cdot (q_2 \times \omega_2 + h_2\omega_2)\big) \\ &= M_1M_2\big(\omega_2 \cdot q_1 \times \omega_1 + h_1\omega_1 \cdot \omega_2 + \omega_1 \cdot (q_1 + dn) \times \omega_2 + h_2\omega_1 \cdot \omega_2\big) \\ &= M_1M_2\big((h_1 + h_2)\cos\alpha - d\sin\alpha\big), \end{aligned}$$

In the above expression, the basic idea lies behind multiplying the linear components of both, the twist and wrench, and the rotational components of both, which gives the respective instantaneous work done. Thus, the screws are reciprocal only if the reciprocal product is zero. The reciprocal product is generally represented by the symbol $\otimes$.

Reciprocal screws are widely used in analyzing the kinematic properties of mechanisms. As an example, we can consider the context of grasping an infinitely rigid object. The wrenches applied to the object can be viewed as a set of constraining screws. Now, if there are any instantaneous motions (twists), then these correspond to motions that are not constrained by the applied wrench and hence the reciprocal product of these twists with the applied wrenches are then 0, implying that the applied wrenches have no contribution to generating/restricting these twists. A noteworthy example to mention here would be [3] in which the authors have shown that the Jacobian of a limited DOF parallel manipulator can be derived making use of the theory of reciprocal screws.

2.4. Coordinate transformation

Now that we know about how velocities and forces can be represented using screw coordinates, it's handy to know how the coordinate transformation can be performed for them. This is a very useful tool for kinematic analysis of mechanisms. The reader is referred to section 4.4 of chapter 2 of [2] for more detailed information.
Twists and wrenches transform depending on their coordinate frame of reference. The expressions below gives the coordinate transformation for twists

$$T_{\bullet}^{j,\bullet} = Ad_{H_i^j} T_{\bullet}^{i,\bullet}$$

Where,

$$Ad_{H_i^j} := \begin{pmatrix} R_i^j & 0 \\ \tilde{p}_i^j R_i^j & R_i^j \end{pmatrix}$$

Here, R_i^j represents the rotation matrix for transformation from reference frame "i" to reference frame "j" and R_j^i represents the rotation matrix for transformation from reference frame "j" to reference frame "i". The $\tilde{p}_i^j$ represents the following: P_i^j is the vector connecting the origin of reference frame “i” to that of reference frame “j”.

For, $P_i^j = [p_1\ p_2\ p_3]^T$

$\tilde{p}_i^j$ is given by,

$$\begin{bmatrix} 0 & -p_3 & p_2 \\ p_3 & 0 & -p_1 \\ -p_2 & p_1 & 0 \end{bmatrix}$$

Similarly, the coordinate transformation of wrenches is given by the following expression

$$(W^i)^T = Ad^T_{H_i^j}(W^j)^T$$

Where

$$Ad^T_{H_i^j} := \begin{pmatrix} R_j^i & -R_j^i\tilde{p}_i^j \\ 0 & R_j^i \end{pmatrix}$$

3. Literature Survey

In this section, a summary of the literature survey performed to find the different modelling techniques to model the effect of play in joints on end effector performance is summarized. Some authors concentrated on the dynamic effects of clearance (Flores, Ambrósio [4]), such as impacts in pairs and vibrations, whereas some concentrated on kinematic modelling Parenti-Castelli and Venanzi [5], Venanzi and Parenti-Castelli [6], which is more helpful for preliminary analysis of mechanism performance. Most approaches of analyzing the effects of clearances can be broadly classified into stochastic and deterministic methods. Stochastic methods (Dhande and Chakraborty [7], Wei-Liang and Qi-Xian [8], Chaker, Mlika [9]) describe the clearance due to displacement through probability distribution function and Deterministic methods (Venanzi and Parenti-Castelli [6], Innocenti [10], Parenti-Castelli and Venanzi [5]) try to exactly determine the displacement of the mechanism links. Wu and Rao [11] used the method of intervals to model clearance and manufacturing errors.

A screw theory method was presented by Tsai and Lai [12] to analyze the transmission performance of linkages that have joint clearance by treating the joint clearances as virtual links in this study, but this method was valid only for planar mechanisms and the effectiveness was demonstrated only for single loop linkages. An extension of this method to multi loop linkages was presented in Tsai and Lai [13], but again was valid only for planar mechanisms. The main drawback of this method, i.e., it’s limitation to planar mechanism analysis, was overcome in methods proposed by Parenti-Castelli and Venanzi [5]. They showed a method to evaluate the clearance influence in spatial parallel mechanisms with focus on kinematic modelling and by using the principle of virtual work. Venanzi and Parenti-Castelli [6] used a deterministic technique to assess the effect of clearance for both planar and spatial, open-, and closed-chain mechanisms (not for over constrained mechanisms). The method uses a kinematic approach to do so. In this method, the clearances are modeled as virtual generalized displacement and local models are defined for different joint pairs and maximization of pose error function is carried out to determine the largest possible error. Chaker, Mlika [9] analyzed a spherical parallel manipulator with clearance and manufacturing errors by modelling the clearance and manufacturing errors as small displacements and by using screw theory and stochastic results were presented. They proved that the method of superposition doesn’t work when both manufacturing error and clearance are considered. Frisoli, Solazzi [14] used a method based on screw theory to estimate the pose accuracy in spatial parallel manipulators with revolute joint clearance. This method performs a 2 step maximization (the first step gives a suboptimal estimation of the pose error function and the second step is an iterative numerical procedure) pose error function and the pose error is a quadratic function of the end-effector displacement, and can converge to exact maximum pose error in a limited number of iterations. The effectiveness of this method has been demonstrated with application examples where worst case angular and linear position accuracy in translating fully parallel manipulators is determined.

Since this investigation is very close (in terms of study on clearance in revolute joints) to the study performed by Frisoli, Solazzi [14], the method used by them will be used for analyzing the clearance effect on end effector performance and look into ways to redesign the haptic master device to reduce the play below human thresholds.

4. Method to model the effect of play in joints on the end effector accuracy

The following method was described in Frisoli, Solazzi [14]. This is a method based on screw theory for the analysis of position accuracy in spatial parallel manipulators with revolute joints clearances and since our system of interest also has revolute joints (only), this is a very good reference study.

In this method, the displacement due to clearances are modelled as additional degrees of freedom in each kinematic pair, whose effect is equivalent to removing the kinematic constraints made ineffective by clearances. As an example, consider the case of a rotational joint, we can introduce two rotations perpendicular to the revolute joint axis, and three independent translations, as shown in figure 4.1, for joint 11, represented by twist $\$_{11}$.

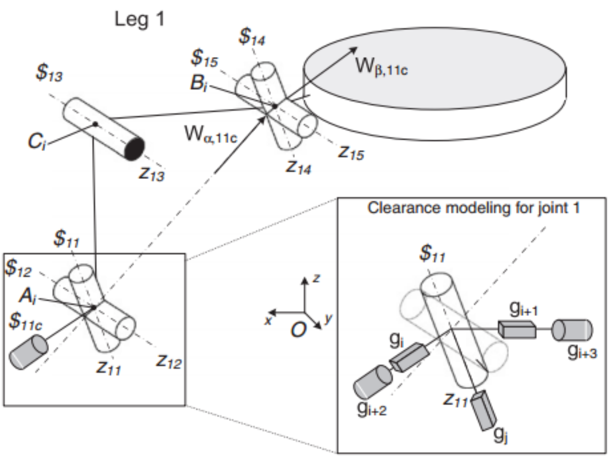

Figure 4.1: Figure showing one of the legs of a parallel manipulator with associated clearances indicated for the joint $\$_{11}$ [Frisoli, Solazzi [14]]

The hypothesis of small displacements holds strong because in most mechanisms, the joint's clearance is upper bounded and is of at least one order lower than the mechanism's dimensions. Based on this this hypothesis, the following assumptions are made:

- The contribution of different joints, due to joint clearances, at the end effector, are independent of each other.
- The end-effector pose error is a linear function of joint clearance contributions.

- Velocity analysis can be used to study the effect of clearances and screw theory can be used to represent the induced infinitesimal displacements. The pose of the mechanism affects the influence of each clearance on the overall displacement of the end effector.

4.1. Obtaining the associated twists

Consider a parallel manipulator, whose generic leg is represented in figure 4.1. Assume all the active DOFs locked by the actuators (represented as darkened) so that the mechanism is statically determined and has a mobility of zero according to Grubler's criterion. Assume an isostatic distribution of reaction forces, which are a system of six linearly independent constraint wrenches [W_{im}], with subscript i=1,….,n_i indicating the leg number and m=1,….,n_m the wrench number. They represent the active and passive constraints of the kinematic chains of the mechanism, according to [3]. Figure 4.2 shows a 3URU (universal-revolute-universal, indicating the types of joints in each leg) parallel manipulator, with W_{i1}, i=1,2,3 are the actuation wrenches of zero pitch and W_{i2}, i=1,2,3 are the passive constraint wrenches of infinite pitch for leg i.

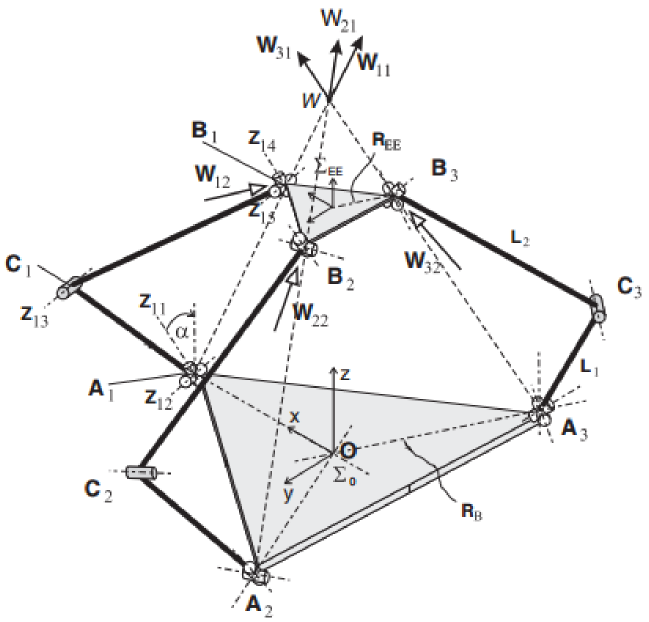

Figure 4.2: Kinematics of 3URU parallel manipulator. Manipulator actuated at joints C_1, C_2 and C_3. [14]

Now, each clearance displacement can be modelled by a twist $\$_{ijc}$, with $i=1,\ldots.,n_i$ indicating the leg number, $j=1,\ldots.,n_j$ indicating the joint number and $c=1,\ldots.,n_c$ indicating the clearance number. For each kinematic pair of g DOFs, upto 6-g clearance displacements can be associated. For example, in the detailed view in figure 4.1, the joint $\$_{11}$ has 3 translational and 2 rotational additional degrees of freedom defined (Universal joint can be modelled as a combination of two revolute joints and $\$_{11}$ is one of the revolute kinematic pairs constituting the universal joint).

The resultant generic twist at the end effector, $\mathbf{T}_{\underline{\mathbf{ijc}}}$, induced by the clearance $\$_{\underline{ijc}}$(clearance displacement of clearance $\underline{c}$ at joint $\underline{j}$ of leg $\underline{i}$) – the underline is used to specify that the particular leg/joint/clearance, for example, $\$_{\underline{i}jc}$ points at all the joints and clearances of the particular specified leg $\underline{i}$ and $\$_{\underline{ijc}}$ specifies the particular clearance of the specified joint of specified leg. The resultant generic twist of the end effector, $\mathbf{T}_{\underline{\mathbf{ijc}}}$, is given by

$$\mathbf{T}_{\underline{\mathbf{ijc}}} = \sum_{j=1, j\neq j_a}^{n_j} (\delta\theta_{\underline{i}j} \$_{\underline{i}j}) + g_{\underline{ijc}} \$_{\underline{ijc}} \tag{4.1}$$

Where, $\delta\theta_{ij}$ indicates small displacements along the Lagrangian coordinates of the joints j, except the actuated ones which are considered locked and $g_{\underline{ijc}}$ is the additional clearance displacement.

Now, with each twist ijc, we can define a set of reciprocal wrenches W_{ijc}, that are of two types, namely, wrench of type α, $W_{\alpha,\underline{ijc}}$, and wrench of type β, $W_{\beta,\underline{ijc}}$. The wrenches of type α are reciprocal to the additional clearance DOF $_{\underline{ijc}}$ and to all the twists $_{\underline{i}j}$, $j=1,\ldots.,n_j$, $j\neq j_a$, of the considered leg $\underline{i}$. The following equations hold for these wrenches ($\otimes$ represents reciprocal product)

$$\begin{cases} \mathbf{W}_{\alpha.\underline{\mathbf{ijc}}} \otimes \$_{\underline{ijc}} = 0 \\ \mathbf{W}_{\alpha.\underline{\mathbf{ijc}}} \otimes \$_{\underline{i}j} = 0 \quad \text{for } j=1\ldots n_j, j\neq j_a \end{cases} \tag{4.2}$$

Similarly, the wrenches of type β are defined reciprocal to all the twists $_{ij}$, j=1,….,n_j, $j \neq j_a$, but not to the considered clearance DOF $\$_{\underline{ijc}}$. The following equations define this type of wrench

$$\begin{cases} \mathbf{W}_{\beta,\underline{\mathbf{ijc}}} \otimes \$_{\underline{ijc}} \neq 0 \\ \mathbf{W}_{\beta,\underline{\mathbf{ijc}}} \otimes \$_{\underline{ij}} = 0 \quad \text{for } j = 1 \ldots n_j, j \neq j_a \end{cases} \tag{4.3}$$

To consider the effect of multiple clearances, linear superimposition is employed. The virtual work done by the α and β wrenches can be computed using the following expressions

$$\begin{cases} \mathbf{W}_{\alpha,\underline{\mathbf{ijc}}} \otimes \mathbf{T}_{\underline{\mathbf{ijc}}} = 0 \\ \mathbf{W}_{\beta,\underline{\mathbf{ijc}}} \otimes \mathbf{T}_{\underline{\mathbf{ijc}}} = g_{\underline{ijc}} \left(\mathbf{W}_{\beta,\underline{\mathbf{ijc}}} \otimes \$_{\underline{ijc}} \right) \end{cases} \tag{4.4}$$

Also, the work done by the constraint wrenches, W_{im}, laong the displacement $\mathbf{T}_{\underline{\mathbf{ijc}}}$ must be zero and so the following equation holds

$$\mathbf{W}_{\mathbf{im}} \otimes \mathbf{T}_{\underline{\mathbf{ijc}}} = 0 \text{ , for legs } i \neq \underline{i}, \forall m \tag{4.5}$$

In an isostatic system, the wrenches of type α, type β and W_{im}, $i \neq \underline{i}$ define a system of 6 wrenches that can be arranged in a matrix W:

$$W = (W_{\beta,\underline{ijc}}\ W_{\alpha,\underline{ijc}}\ \ldots\ W_{im}),\ i \neq \underline{i} \tag{4.6}$$

Equations 4.4 and 4.5 can be put in matricial form as:

$$W^T I^* \mathbf{T}_{\underline{\mathbf{ijc}}} = \begin{pmatrix} g_{\underline{ijc}} \left(\mathbf{W}_{\beta,\underline{\mathbf{ijc}}} \otimes \$_{\underline{ijc}} \right) \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{4.7}$$

Here, the reciprocity product is represented in matrix form through the matrix I^*, which is composed of 3x3 identity and zero matrices.

$$I^* = \begin{pmatrix} 0_3 & I_3 \\ I_3 & 0_3 \end{pmatrix} \tag{4.8}$$

I^* and W^T are always invertible and hence the contribution can be obtained by using the relation

$$\mathbf{T}_{\underline{\mathbf{ijc}}} = \left(W^T I^*\right)^{-1} \begin{pmatrix} g_{\underline{ijc}}\left(\mathbf{W}_{\beta,\underline{\mathbf{ijc}}} \otimes \$_{\underline{ijc}}\right) \\ 0 \\ \vdots \\ 0 \end{pmatrix} = \hat{\mathbf{T}}_{\underline{\mathbf{ijc}}} g_{\underline{ijc}} \tag{4.9}$$

Where the "T" term on the right hand side of the above equation indicates the screw associated with the motion associated by the unitary clearance $\$_{\underline{ijc}}$.

The overall displacement at the end effector must be the linear composition of contributions by individual clearances-multiplied by the associated clearance values g_{ijc}.

$$\$ = \sum_{(i=1,j=1,c=1)}^{(i=n_i,j=n_j,c=n_c)} \hat{\mathbf{T}}_{\mathbf{ijc}} g_{ijc} = T\mathbf{g} \tag{4.10}$$

Here, T and **g** are given by

$$T = \left(\hat{\mathbf{T}}_{111} \cdots \hat{\mathbf{T}}_{11\mathbf{n_c}} \cdots \hat{\mathbf{T}}_{1\mathbf{n_j}\mathbf{n_c}} \cdots \hat{\mathbf{T}}_{\mathbf{n_i}\mathbf{n_j}\mathbf{n_c}}\right)$$
$$\mathbf{g} = \begin{pmatrix} g_{111} \\ \vdots \\ g_{11n_c} \\ \vdots \\ g_{1n_jn_c} \\ \vdots \\ g_{n_in_jn_c} \end{pmatrix}. \tag{4.11}$$

4.2. Determination of clearance

To determine the maximum pose error, the following expression can be formulated.

$$f(\mathbf{g}) = \$^T M \$ = \mathbf{g}^T T^T M T \mathbf{g} = \mathbf{g}^T A \mathbf{g}. \tag{4.12}$$

Where, M is a metrics matrix and is positive definite and symmetric and f(g) is a cost function that has to be maximized to obtain the worst possible error.

Since this analysis is limited to rotational joints, the clearances in the joints are modelled as small displacements and based on this two types of constraints are defined. These two types of constraints define the maximization clearance problem to find a suboptimal estimate of the maximum clearance. Taking this suboptimal solution as the starting point, the optimal value of maximum clearance is obtained through gradient descent maximization procedure. The reader can refer section 3 of [14] for detailed derivation and explanation in this regard.

4.3. Final effect quantified

Once the maximum clearance value matrix is obtained, the matrix of individual twist contributions, T, and the matrix of maximum clearance values, g, are substituted in equation 4.10 to obtain the final twist at of the end effector.

5. Application of play modelling to the 3-DOF Haptic Master device

5.1. Description of the haptic master device and the problem due to play

The parallel manipulator under consideration here is intended to act as a haptic master device for steerable needles. It's a 3 DOF (1T-2R) system with translation allowed along Z-axis and rotations allowed about the X and Y axes. The figure below shows the haptic master device

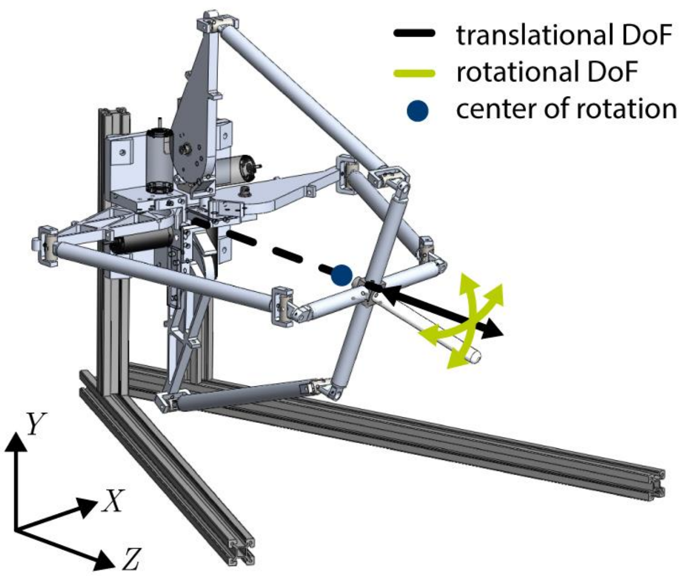

Figure 5.1: 3DOF parallel haptic master device

The links are made of hollow aluminum tubes and each leg is connected to the base by bolts and bearings such that they allow free rotation at each of those connections and the rest of the kinematic pairs are either revolute or universal or spherical joints.

It was observed that there were small motions along DOFs intended to be constrained at the end effector and this was mainly due to the play at the joints. It was also observed that the play was mainly in the joints connecting the legs to the base, i.e., the revolute joints with bearings. The analysis was therefore restricted to analyzing the contribution of this play to the end effector accuracy. In this study, the influence of

play in the joint connecting the right leg (as in figure 5.1) to the base is studied.
The procedure as outlined in section 4 of this report was employed.

5.2. Defining twists

First, the twists representing the joints and virtual joints were defined. The origin of frame of reference was assumed to be at the center of the base. The fig below shows the right leg with its joint representations.

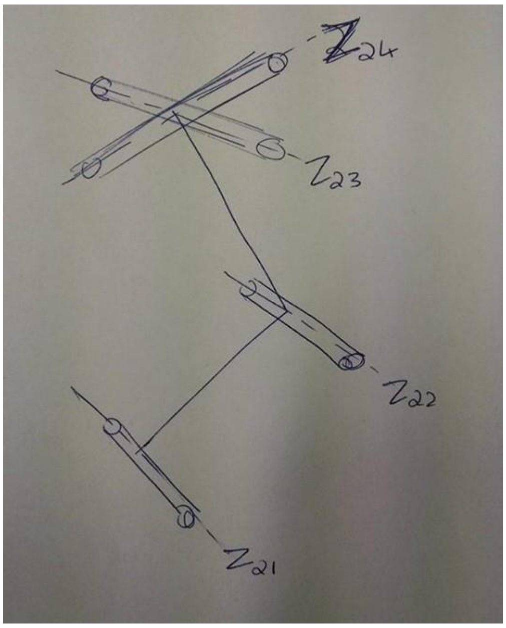

Figure 5.2: Right leg of the haptic master device

Z_{21}.....Z_{24} represent the versors aligned along the joints of the leg. The unit twists representing these joints are denoted as $\$_{21}$....$\$_{24}$. The figure below shows joint $\$_{21}$ along with the associated virtual degrees of freedom, which is also our joint of interest.

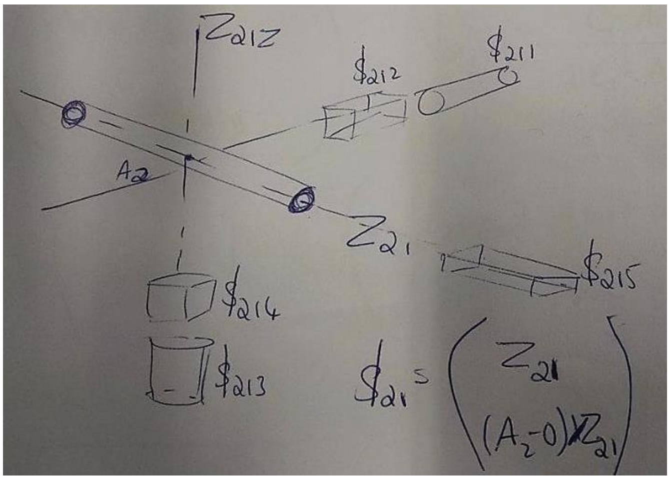

Figure 5.3: Joint 1 and its associated virtual DOFs

The figure above shows the associated virtual degrees of freedom, which are 2 rotational and 3 translational DOFs. Z_{21} is the versor along the revolute joint considered and Z_{21z} is the versor perpendicular to Z_{21} and oriented along Z-axis. The unit twist associated with joint 1 is given by

$$\$_{21} = \begin{pmatrix} Z_{21} \\ (A_2 - O) \times Z_{21} \end{pmatrix} \tag{5.1}$$

The unit twists representing the virtual DOFs are given by

$$\$_{211} = \begin{pmatrix} \omega_{21} \\ (A_2 - O) \times \omega_{21} \end{pmatrix} \tag{5.2}$$

Where, $\omega_{211} = Z_{21} \times Z_{21z} / || Z_{21} \times Z_{21z} ||$

$$\$_{212} = \begin{pmatrix} 0 \\ \omega_{212} \end{pmatrix} \tag{5.3}$$

Where, $\omega_{212} = Z_{21} \times Z_{21z} / || Z_{21} \times Z_{21z} ||$

$$\$_{213} = \begin{pmatrix} Z_{21z} \\ (A_2 - O) \times Z_{21z} \end{pmatrix} \tag{5.4}$$

$$\$_{214} = \begin{pmatrix} 0 \\ Z_{21z} \end{pmatrix} \tag{5.5}$$

$$\$_{215} = \begin{pmatrix} 0 \\ Z_{21} \end{pmatrix} \tag{5.6}$$

5.3. Determination of wrench system

Now the alpha and beta wrenches as described in section 4 were identified. The wrenches were identified by mere observation and they are given below

$W_{\alpha,\cancel{211}}$ = (does not exist)

$$W_{\beta,\cancel{211}} = \begin{pmatrix} Z_{23} \\ (B_2 - O) \times Z_{23} \end{pmatrix} \tag{5.7}$$

$$W_{\beta,\cancel{212}} = \begin{pmatrix} Z_{23} \\ (B_2 - O) \times Z_{23} \end{pmatrix} \tag{5.8}$$

$W_{\beta,\cancel{211}}$ = (does not exist)

$W_{\alpha,\cancel{213}}$ = (does not exist)

$$W_{\beta,\cancel{213}} = \begin{pmatrix} Z_{23} \\ (B_2 - O) \times Z_{23} \end{pmatrix} \tag{5.9}$$

$$W_{\beta,\cancel{214}} = \begin{pmatrix} Z_{23} \\ (B_2 - O) \times Z_{23} \end{pmatrix} \tag{5.10}$$

$W_{\beta,\cancel{214}}$ = (does not exist)

$W_{\alpha,\cancel{215}}$ = (does not exist)

$$W_{\beta,\cancel{215}} = \begin{pmatrix} Z_{23} \\ (B_2 - O) \times Z_{23} \end{pmatrix} \tag{5.11}$$

The constraint wrenches were defined for the other legs that constitute the W matrix as in equation 4.10.

5.4. Solving the system of equations

Once the wrenches and twists are defined, the system of equations as shown in section 4 are solved. Equation 4.9 is solved to obtain the individual twists and then the maximum clearance matrix is obtained using the procedure outlined in [14] and the final twist of the end effector is calculated using equation 4.10. This was formulated in MATLAB and the code is given in Appendix A1.

6. Results and discussion

Appendix A2 shows the result after running the code. The following problems were encountered in the implementation of the procedure outlined in [14]:

- The "$W^T I^*$" matrix in equation 4.9 was a rank deficient matrix since one of our constraint wrenches from other legs was zero and also in some cases, the alpha and/or beta wrench was zero. Hence, the inverse of the matrix didn't exist. This led us to taking the pseudoinverse of the matrix for further analysis.
- The obtained twist matrix "T" was found to be a rank deficient matrix implying that the twist contributions from the individual virtual DOFs were dependent/related (since they were same vectors were different scaling).
- The implementation of the method to obtain maximum clearance was unsuccessful as the results obtained were very unrealistic.

Although the physical interpretation of the obtained twists for the end effector look acceptable(the directions of translation are expected) , the inability to obtain the exact numerical values for the exact influence of the joint on the end effector pose error remains a major setback of this research. Also, the reason for dependent twist matrix is unknown and hence the results obtained (twists of the end effector) can be doubted for their correctness.

This research had to be terminated at this point due to timeframe of the assignment. However, to obtain acceptable results, the optimization process can be looked into more carefully as the obtained clearance values are unrealistic. Also, the reason for the twist matrix being rank deficient can be looked into for possible relevant interpretations. It is highly probable that solving the above mentioned issues can lead to actual pose error at the end effector.

References:

1. Briot, S. and I. Bonev, *Are parallel robots more accurate than serial robots?* CSME Transactions, 2007. **31**(4): p. 445-456.
2. Murray, R.M., et al., *A mathematical introduction to robotic manipulation*. 1994: CRC press.
3. Joshi, S.A. and L.-W. Tsai. *Jacobian analysis of limited-DOF parallel manipulators*. in *ASME 2002 International design engineering technical conferences and computers and information in engineering conference*. 2002. American Society of Mechanical Engineers.
4. Flores, P., et al., *Dynamic behaviour of planar rigid multi-body systems including revolute joints with clearance.* Proceedings of the Institution of Mechanical Engineers, Part K: Journal of Multi-body Dynamics, 2007. **221**(2): p. 161-174.
5. Parenti-Castelli, V. and S. Venanzi, *Clearance influence analysis on mechanisms.* Mechanism and Machine Theory, 2005. **40**(12): p. 1316-1329.
6. Venanzi, S. and V. Parenti-Castelli, *A new technique for clearance influence analysis in spatial mechanisms.* Journal of Mechanical Design, 2005. **127**(3): p. 446-455.
7. Dhande, S. and J. Chakraborty, *Mechanical error analysis of spatial linkages.* Journal of Mechanical Design, 1978. **100**(4): p. 732-738.
8. Wei-Liang, X. and Z. Qi-Xian, *Probabilistic analysis and Monte Carlo simulation of the kinematic error in a spatial linkage.* Mechanism and Machine Theory, 1989. **24**(1): p. 19-27.
9. Chaker, A., et al., *Clearance and manufacturing errors' effects on the accuracy of the 3-RCC Spherical Parallel Manipulator.* European Journal of Mechanics-A/Solids, 2013. **37**: p. 86-95.
10. Innocenti, C., *Kinematic Clearance Sensitivity Analysis of Spatial Structures With Revolute Joints (*).* Journal of mechanical design, 2002. **124**(1): p. 52-57.
11. Wu, W. and S. Rao, *Interval approach for the modeling of tolerances and clearances in mechanism analysis.* Journal of Mechanical Design, 2004. **126**(4): p. 581-592.
12. Tsai, M.-J. and T.-H. Lai, *Kinematic sensitivity analysis of linkage with joint clearance based on transmission quality.* Mechanism and Machine Theory, 2004. **39**(11): p. 1189-1206.
13. Tsai, M.-J. and T.-H. Lai, *Accuracy analysis of a multi-loop linkage with joint clearances.* Mechanism and machine theory, 2008. **43**(9): p. 1141-1157.
14. Frisoli, A., et al., *A new screw theory method for the estimation of position accuracy in spatial parallel manipulators with revolute joint clearances.* Mechanism and Machine Theory, 2011. **46**(12): p. 1929-1949.

Appendix

A1. The MATLAB code for the analysis carried out to obtain the twist at the end effector due to play at joint 1 of the right leg of the haptic master device.

```
clear all
close all
clc

%% to get the constraint wrenches

Rbc_x= [ 1 0 0
    0 cosd(0) -sind(0)
    0 sind(0) cosd(0)];

Rbc_y= [ cosd(0) 0 -sind(0)
    0 1 0
    -sind(0) 0 cosd(0)];

Rbc_z= [ cosd(90) -sind(90) 0
    sind(90) cosd(90) 0
   0 0 1];
Rbc= Rbc_x *Rbc_y *Rbc_z;

Pbc =[0 0 -0.500]'; % platform RF origin fixed at 500mm along Z axis from
the base RF origin

Pbc_cap= [ 0 0.500 0
    -0.500 0 0
    0 0 0 ];

%% to transform wrench from platform RF to base RF

%% for leg2(teun) or leg1(me)
Wc1_all_PRF = Needle_3DOF_Wcleg2(0,0.5,0,0.1350);
Wc1_1_PRF_teun= Wc1_all_PRF(:,1);
Wc1_1_PRF(1:3,:)=Wc1_1_PRF_teun(4:6,:);Wc1_1_PRF(4:6,:)=Wc1_1_PRF_teun(1:3,
:); %since teun has defined torque first and then force


Wc1_2_PRF_teun= Wc1_all_PRF(:,2);
Wc1_2_PRF(1:3,:)=Wc1_2_PRF_teun(4:6,:);Wc1_2_PRF(4:6,:)=Wc1_2_PRF_teun(1:3,
:);

Wc1_1(1:3,:)= Rbc'* Wc1_1_PRF(1:3,:);
Wc1_1(4:6,:)= (-Rbc'*Pbc_cap)*Wc1_1_PRF(1:3,:)+ Rbc'*Wc1_1_PRF(4:6,:);
% Wc1_1= [0 0 0 0 0 0 ]';
Wc1_2(1:3,:)= Rbc'* Wc1_2_PRF(1:3,:);
Wc1_2(4:6,:)= (-Rbc'*Pbc_cap)*Wc1_2_PRF(1:3,:)+ Rbc'*Wc1_2_PRF(4:6,:);
%% leg1 (teun)

Wc2_all_PRF=Needle_3DOF_Wcleg1(0);

Wc2_1_PRF_teun= Wc2_all_PRF(:,1);
```

```
Wc2_1_PRF(1:3,:)=Wc2_1_PRF_teun(4:6,:);Wc2_1_PRF(4:6,:)=Wc2_1_PRF_teun(1:3,
:); %since teun has defined torque first and then force

Wc2_1(1:3,:)= Rbc'* Wc2_1_PRF(1:3,:);
Wc2_1(4:6,:)= (-Rbc'*Pbc_cap)*Wc2_1_PRF(1:3,:)+ Rbc'*Wc2_1_PRF(4:6,:);


Wc2_2_PRF_teun= Wc2_all_PRF(:,2);
Wc2_2_PRF(1:3,:)=Wc2_2_PRF_teun(4:6,:);Wc2_2_PRF(4:6,:)=Wc2_2_PRF_teun(1:3,
:);

Wc2_2(1:3,:)= Rbc'* Wc2_2_PRF(1:3,:);
Wc2_2(4:6,:)= (-Rbc'*Pbc_cap)*Wc2_2_PRF(1:3,:)+ Rbc'*Wc2_2_PRF(4:6,:);




%% for leg3(teun) or leg4(me)

Wc4_all_PRF= Needle_3DOF_Wcleg3(0);

Wc4_1_PRF_teun= Wc4_all_PRF(:,1);
Wc4_1_PRF(1:3,:)=Wc4_1_PRF_teun(4:6,:);Wc4_1_PRF(4:6,:)=Wc4_1_PRF_teun(1:3,
:); %since teun has defined torque first and then force
Wc4_1_PRF=[0 0 0 0 0 0 ]'; % to not consider the constraint along y axis in
leg 4
Wc4_2_PRF_teun= Wc4_all_PRF(:,2);
Wc4_2_PRF(1:3,:)=Wc4_2_PRF_teun(4:6,:);Wc4_2_PRF(4:6,:)=Wc4_2_PRF_teun(1:3,
:);


Wc4_1(1:3,:)= Rbc'* Wc4_1_PRF(1:3,:);
Wc4_1(4:6,:)= (-Rbc'*Pbc_cap)*Wc4_1_PRF(1:3,:)+ Rbc'*Wc4_1_PRF(4:6,:);

Wc4_2(1:3,:)= Rbc'* Wc4_2_PRF(1:3,:);
Wc4_2(4:6,:)= (-Rbc'*Pbc_cap)*Wc4_2_PRF(1:3,:)+ Rbc'*Wc4_2_PRF(4:6,:);

%% to calculate the alpha and beta wrenches

A2 =[ 0.070 0 0]';
B2 = [ 0.135 0 0.500]';
Oc= [0 0 0 ]';
Ob = [ 0 0 0.500]';

Z21= [0 1 0 ]';
Z21z = [ 0 0 1]';
Z23= [0 1 0 ]';
Z24=[1 0 0 ]';

%% $211 wrenches

W_alpha_211= [0 0 0 0 0 0 ]';

omega_211= cross(Z21,Z21z)/sqrt(sum((cross(Z21,Z21z).*(cross(Z21,Z21z)))));
W_beta_211(1:3,:) = Z23;

W_beta_211(4:6,:) = cross((B2-Oc),Z23);
```

```
%% $212 wrenches


omega_212= cross(Z21,Z21z)/sqrt(sum((cross(Z21,Z21z).*(cross(Z21,Z21z)))));

W_alpha_212(1:3,:)  = Z23;
W_alpha_212(4:6,:)= cross((B2-Oc),Z23);

W_beta_212 = [0 0 0 0 0 0 ]';


%% $213 wrenches

W_alpha_213= [ 0 0 0 0 0 0 ]';

W_beta_213(1:3,:)  = Z23;
W_beta_213(4:6,:)= cross((B2-Oc),Z23);


%% $214 wrenches

W_alpha_214(1:3,:)= Z23;
W_alpha_214(4:6,:)= cross(B2-Oc,Z23);


W_beta_214= [0 0 0 0 0 0 ]';


%% $215 wrenches

omega_215= cross(Z23,Z24)/sqrt(sum((cross(Z23,Z24).*(cross(Z23,Z24)))));

W_alpha_215(1:3,:)= 0;
W_alpha_215(4:6,:)= 0;

W_beta_215(1:3,:)= Z23;
W_beta_215(4:6,:)= cross(B2-Oc,Z23);

%% individual virtual joint T matrices

%% $211 --> The complete Wrench matrix(W) and get the T matrix


Dollar_211(1:3,:)  = omega_211;
Dollar_211(4:6,:)  = cross(A2-Oc,omega_211);

Rec_pro_211=
W_beta_211(1)*Dollar_211(4)+W_beta_211(2)*Dollar_211(5)+W_beta_211(3)*Dolla
r_211(6)+W_beta_211(4)*Dollar_211(1)+W_beta_211(5)*Dollar_211(2)+W_beta_211
(6)*Dollar_211(3);


W_211(:,1)= W_beta_211;
W_211(:,2)= W_alpha_211;
```

```
W_211(:,3)= Wc1_1;
W_211(:,4)= Wc1_2;
W_211(:,5)= Wc4_1;
W_211(:,6)= Wc4_2;

I_star= [zeros(3) eye(3)
    eye(3) zeros(3)];

Product_matrix_211 = [ Rec_pro_211 0 0 0 0 0 ]'
T_ijc_211 = pinv(W_211'*I_star)*Product_matrix_211

%% $212 --> The complete Wrench matrix(W) and get the T matrix

Dollar_212(1:3,:) = [0 0 0 ]';
Dollar_212(4:6,:) = omega_212;

Rec_pro_212= 0;


W_212(:,1)= W_beta_212;
W_212(:,2)= W_alpha_212;
W_212(:,3)= Wc1_1;
W_212(:,4)= Wc1_2;
W_212(:,5)= Wc4_1;
W_212(:,6)= Wc4_2;


I_star= [zeros(3) eye(3)
    eye(3) zeros(3)];

Product_matrix_212 = [ Rec_pro_212 0 0 0 0 0 ]';


T_ijc_212 = pinv(W_212'*I_star)*Product_matrix_212


%% $213 --> The complete Wrench matrix(W) and get the T matrix

Dollar_213(1:3,:) = Z21z;
Dollar_213(4:6,:) = cross(A2-Oc,Z21z);


Rec_pro_213= W_beta_213(1)*Dollar_213(4)+    W_beta_213(2)*Dollar_213(5)+
W_beta_213(3)*Dollar_213(6)+    W_beta_213(4)*Dollar_213(1)+
W_beta_213(5)*Dollar_213(2)+    W_beta_213(6)*Dollar_213(3);

W_213(:,1)= W_beta_213;
W_213(:,2)= W_alpha_213;
W_213(:,3)= Wc1_1;
W_213(:,4)= Wc1_2;
W_213(:,5)= Wc4_1;
W_213(:,6)= Wc4_2;


I_star= [zeros(3) eye(3)
    eye(3) zeros(3)];
```

```
Product_matrix_213 = [ Rec_pro_213 0 0 0 0 0]' ;

T_ijc_213 = pinv(W_213'*I_star)*Product_matrix_213


%% $214 --> The complete Wrench matrix(W) and get the T matrix

Dollar_214(1:3,:) = [0 0 0 ]';
Dollar_214(4:6,:) = Z21z;


Rec_pro_214= W_beta_214(1)*Dollar_214(4)+    W_beta_214(2)*Dollar_214(5)+
W_beta_214(3)*Dollar_214(6)+    W_beta_214(4)*Dollar_214(1)+
W_beta_214(5)*Dollar_214(2)+    W_beta_214(6)*Dollar_214(3);

W_214(:,1)= W_beta_214;
W_214(:,2)= W_alpha_214;
W_214(:,3)= Wc1_1;
W_214(:,4)= Wc1_2;
W_214(:,5)= Wc4_1;
W_214(:,6)= Wc4_2;


I_star= [zeros(3) eye(3)
    eye(3) zeros(3)];

Product_matrix_214 = [ Rec_pro_214 0 0 0 0 0]';

T_ijc_214 = pinv(W_214'*I_star)*Product_matrix_214

%% $215 --> The complete Wrench matrix(W) and get the T matrix

Dollar_215(1:3,:) = [0 0 0 ]';
Dollar_215(4:6,:) = Z21;

Rec_pro_215= W_beta_215(1)*Dollar_215(4)+    W_beta_215(2)*Dollar_215(5)+
W_beta_215(3)*Dollar_215(6)+    W_beta_215(4)*Dollar_215(1)+
W_beta_215(5)*Dollar_215(2)+    W_beta_215(6)*Dollar_215(3);

W_215(:,1)= W_beta_215;
W_215(:,2)= W_alpha_215;
W_215(:,3)= Wc1_1;
W_215(:,4)= Wc1_2;
W_215(:,5)= Wc4_1;
W_215(:,6)= Wc4_2;

I_star= [zeros(3) eye(3)
    eye(3) zeros(3)];

Product_matrix_215 = [ Rec_pro_215 0 0 0 0 0]';

T_ijc_215 = pinv(W_215'*I_star)*Product_matrix_215

%% total T_cap_ijc matrix

T_ijc_21= T_ijc_211+T_ijc_212+T_ijc_213+T_ijc_214+T_ijc_215
```

```
%% T matrix - with all T_cap_ijcs arranged in a matrix

T(:,1)= T_ijc_211;
T(:,2)= T_ijc_212;
T(:,3)= T_ijc_213;
T(:,4)= T_ijc_214;
T(:,5)= T_ijc_215

%% "A" matrix

A= T'*eye(6)*T;

%% find gmax

ri=0.000013;
[V,D]=eig(A)

maxeigenvalue= max(max(D))
[num idx] = max(D(:));
[maxeigenidx maxeigenidy] = ind2sub(size(D),idx)

Vmax=V(:,maxeigenidy)
gmax= ri.* Vmax

%% to find gopt

thetai= atan2(gmax(2),gmax(1));
thetaiplus1= atan2(gmax(4),gmax(3));
phii= atan2(sqrt(gmax(4)^2+gmax(3)^2),sqrt(gmax(2)^2+gmax(1)^2));

gopt(1)= ri*(cos(thetai)*cos(phii));
gopt(2)= ri*(sin(thetai)*cos(phii));
gopt(3)= ri*(cos(thetaiplus1)*sin(phii));
gopt(4)= ri*(sin(thetaiplus1)*sin(phii));
gopt(5)= sign(gmax(5));
gopt=gopt'

%% calculating optimal g


g=[0 0 0 0 0 ]';
for i= 1:size(diag(D))

    g=g+ 0.000013*V(:,i);


end

g

%% calculating the final associated twist

finaltwist_ijc_211= g(3).*T_ijc_211;
finaltwist_ijc_212= g(1).*T_ijc_212;
finaltwist_ijc_213= g(4).*T_ijc_213;
finaltwist_ijc_214= g(2).*T_ijc_214;
finaltwist_ijc_215= g(5).*T_ijc_215;
```

```
finaltwist_ijc=
finaltwist_ijc_211+finaltwist_ijc_212+finaltwist_ijc_213+finaltwist_ijc_214
+finaltwist_ijc_215

T
```

A2. The result obtained after running the code:

Product_matrix_211 =

```
  -0.5000
        0
        0
        0
        0
        0
```

T_ijc_211 =

```
   0.2000
  -0.0000
  -0.0000
  -0.0000
  -0.4000
        0
```

T_ijc_212 =

0

0

0

0

0

0

T_ijc_213 =

-0.0260

0.0000

0.0000

0.0000

0.0520

0

T_ijc_214 =

0

0

0

0

0

0

```
T_ijc_215 =

  -0.4000
   0.0000
   0.0000
   0.0000
   0.8000
        0


T_ijc_21 =

  -0.2260
   0.0000
   0.0000
   0.0000
   0.4520
        0


T =

   0.2000         0   -0.0260         0   -0.4000
  -0.0000         0    0.0000         0    0.0000
```

```
  -0.0000         0    0.0000         0    0.0000
  -0.0000         0    0.0000         0    0.0000
  -0.4000         0    0.0520         0    0.8000
        0         0         0         0         0


V =

  -0.8308   -0.1123         0    0.3128   -0.4465
        0         0   -1.0000         0         0
   0.1065   -0.9926         0    0.0093    0.0580
   0.3466    0.0459         0    0.9369         0
  -0.4223    0.0084         0    0.1558    0.8929


D =

  -0.0000         0         0         0         0
        0   -0.0000         0         0         0
        0         0         0         0         0
        0         0         0    0.0000         0
        0         0         0         0    1.0034


maxeigenvalue =
```

```
    1.0034


maxeigenidx =

     5


maxeigenidy =

     5


Vmax =

   -0.4465
         0
    0.0580
         0
    0.8929


gmax =

   1.0e-04 *
```

```
   -0.0580
         0
    0.0075
         0
    0.1161


gopt =

   -0.0000
    0.0000
    0.0000
         0
    1.0000


g =

   1.0e-04 *

   -0.1400
   -0.1300
   -0.1064
    0.1728
    0.0825
```

```
finaltwist_ijc =

   1.0e-04 *

  -0.0588
   0.0000
   0.0000
   0.0000
   0.1176
        0


T =

   0.2000         0   -0.0260         0   -0.4000
  -0.0000         0    0.0000         0    0.0000
  -0.0000         0    0.0000         0    0.0000
  -0.0000         0    0.0000         0    0.0000
  -0.4000         0    0.0520         0    0.8000
        0         0         0         0         0

>>
```