UNIVERSITY OF TWENTE

MASTER THESIS

On the Risk-Free Rate

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A thesis submitted in fulfillment of the requirements for the degree of Master of Science

in

Financial Engineering and Management
Department Industrial Engineering and Business Information Systems

May 25, 2016

Abstract

Currently several governments receive money for borrowing up to seven years. This fundamentally contradicts with time preference implying that investors always require a positive return. Furthermore, it evokes questions about the functioning of the bond market, and about government bonds' adequacy as proxy for the risk-free rate. Although government bonds were historically almost unquestionably used as risk-free rates, we formally disentangle the two concepts by defining a risk-free asset as theoretical concept and government bonds as estimators. This strict separation enables us to select the best estimators for the risk-free rate for valuation purposes.

Firstly, we distinguish two methods to estimate the risk-free rate: proxies and models. Proxies are observable variables and estimate an unobservable variable by closely resembling it. Models consist of several variables together related by theory to the unobservable variable. We evaluate two risk-free proxies: German government bonds and Overnight Indexed Swaps. We discard two other proxies: General Collateralized Repurchase Agreements and our developed 'Market-Implied Risk-Free Rate', because they are not available for longterm maturities needed for valuations. Besides the proxies, we construct a 'Macro Model' by regressing macro-variables on the German government bond in a period it closely resembled a risk-free asset. We discard other models, because the Macro Model has historically the best explanatory power for risk-free proxies.

Subsequently, we define three evaluation criteria for risk-free estimators: Consistency, Intelligibility and Availability. Firstly, we measure Consistency by comparing the risk-free estimators with a Market-Implied Risk-Free Rate, which we define as the market's view of a risk-free portfolio. Secondly, we evaluate Intelligibility by comparing the resemblance of the estimators with a risk-free asset. Finally, we measure Availability as the publishing frequency of the estimators. Our aggregated analysis shows that the German government bond is the best risk-free estimator by performing as good as or better than the alternatives on all criteria.

Afterwards, we analyze deficiencies of the German government bond as risk-free estimator and their causes. We show that credit risk, flights-to-liquidity, and Quantitative Easing programs all deviate the Bund from the risk-free rate. We propose adjustments for the former two and not for the latter, because our estimation of that deficiency is inaccurate. We propose using CDS to eliminate partially the credit risk exposure and using German agency bonds to partially eliminate the liquidity premium. Although our adjustments improve the Bund as risk-free estimator conceptually, the adjustments worsen the Consistency of the risk-free estimator. We attribute this to the neutralizing effect of the positive credit risk premium and the negative liquidity premium of the Bund. Concluding, we recommend to use German government bonds to estimate the risk-free rate.

Finally, we reflect on theory stating that investors require a positive return and show it to be incorrect. We propose three reasons for the acceptance of negative returns: (i) speculation about bond price increases or currency appreciation, (ii) regulatory requirements for financial institutions, and (iii) lack of alternative assets. We think that all three factors push the lower bound of interest below zero. However, the question remains; "What is the lower bound?"

Acknowledgments

Firstly, I would like to express my sincere gratitude to my first supervisor Reinoud Joosten for his continuous support during my research. He triggered me to think further by discussing concepts elaborately and he helped me in improving the quality of my thesis. Besides my first supervisor, I would like to thank Berend Roorda, for his insightful comments sharpening my view.

My sincere thanks also goes to my PwC colleagues and especially Maurice Koerse, who provided me the opportunity to join their team as intern. They made my six months at PwC great, because of the professional experiences, received advice and new friendships.

Last but not least, I would like to thank Micheline, my family, and friends for supporting me throughout writing this thesis and in my life in general.

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List of Abbreviations

AHP	Analytic Hierarchy Process
BLUE	Best Linear Unbiased Estimator
BoE	Bank of England
BoJ	Bank of Japan
Bund	German Government Bond
CAPM	Capital Asset Pricing Model
CDS	Credit Default Swap
CRRA	Constant Relative Risk Aversion
CV	Continuing Value
DU	Discounted Utility
EAPP	Extended Asset Purchase Program
ECB	European Central Bank
EONIA	Euro Overnight Index Average
EV	Enterprise Value
FCF	Free Cash Flow
FED	Federal Reserve
GC Repo	Generalized Collateral Repurchase Agreements
HICP	Harmonized Index of Consumer Prices
KfW	Kreditanstalt für Wiederaufbau
LIBOR	London Interbank Offered Rate
MAE	Mean Absolute Error
OIS	Overnight Indexed Swap
PSPP	Public Securities Purchase Program
QE	Quantitative Easing
RMSE	Root Mean Squared Error
WACC	Weighted Average Cost of Capital

Chapter 1

Research Design

We have divided the research design into a conceptual and a technical one, as proposed by Verschuren & Doorewaard (2010). The conceptual part describes the goal of this research. It covers the Problem Context in Section 1.1, our Research Objective in Section 1.2, and the Research Questions in Section 1.3. Subsequently, we describe the plan to realize this study in our technical design. It covers our Research Strategy and Thesis Outline in Section 1.4.

1.1 Problem Context

The risk-free rate is the required return on a risk-free asset and is a fundamental concept in finance. A risk-free asset is a theoretical concept and is an asset without any exposure to financial risks. It pays a specified unit in a currency at a certain date in the future in every possible state of the world. The concept has attractive characteristics making it a building block for many finance theories. Fisher (1930) was probably the first to introduce formally the concept to describe the time-value of money. Subsequently, Markowitz (1952) used a risk-free asset's uncorrelatedness with other assets in Modern Portfolio Theory to construct a portfolio with an optimal risk-return combination. Later, Sharpe (1964) continued on this theory to develop the Capital Asset Pricing Model (CAPM). Finally, Black & Scholes (1973) used the certain return of a risk-free asset in option pricing. We study the risk-free rate in the context of business valuations. Within this context the risk-free rate is used to construct the Weighted Average Cost of Capital (WACC), which is finally used to discount all expected cash flows of a company to determine its value (Koller *et al.*, 2010).

In reality no asset fully satisfies all characteristics of a risk-free asset, because it is impossible to eliminate all financial risks (Damodaran, 2010). Therefore the risk-free rate can only be estimated. After a preliminary literature survey we distinguish two risk-free estimator types: proxies and models. Proxies are observable variables and estimate an unobservable variable by closely resembling the theoretical concept (Bai & Ng, 2005). Models consist of several observable variables and are constructed based on theory of a risk-free asset. Currently the most used risk-free estimators are government bonds, resembling the concept due to their assumed absence of credit risk. Governments are regarded as creditworthy, because they can raise taxes and in theory can use monetary policy, i.e., print additional money, to meet their outstanding obligations. However, recently questions have been raised about the adequacy of government bonds as risk-free proxy. This is due to two deficiencies that have been recognized:

- *Negative long-term real yields*. Currently real yields of several Euro-Area government bonds are negative, when we subtract the inflation expectation from nominal yields (CapitalIQ, 2016). This implies that investors are losing purchasing power over time, contradicting time preference stating that humans prefer direct over delayed consumption (Frederick *et al.*, 2002). Nominal bond yields require an inflationary and real compensation for borrowing money. These components compensate investors for the decrease of purchasing power of the currency and the time-value of money, respectively (Fisher, 1930).

- *Increased default risk of governments*. Two factors make the negligibility of the credit risk of European government bonds questionable (ECB, 2014). Firstly, Euro-Area governments do not longer have the control of the money supply, because this is transferred to the European Central Bank (ECB). Secondly, the credit risk of European governments has risen significantly after the financial crisis.

1.2 Research Objective

These two deficiencies evoked questions about the adequacy of government bond yields as risk-free proxy. Therefore our objective is to provide recommendations for estimating the risk-free rate for valuation purposes. Firstly, we analyze the most common used proxies, by conducting a literature survey. Secondly, we develop a model as an alternative to the current existing proxies. Subsequently, we evaluate all risk-free estimators on evaluation criteria derived from literature and expert interviews. Finally, we analyze the deficiencies of the best risk-free estimator to further improve its adequacy. The results of this theory-oriented research project contribute to a more adequate estimation of the risk-free rate for valuation purposes.

1.3 Research Questions

To achieve the research objective, we have formulated research questions for structuring our research. Our main research question is:

What is the best estimator of the risk-free rate for valuation purposes?

We have broken down the main research question into four research questions, which are all divided into sub-questions. In this way we are able to research concepts independently and to incrementally answer the main research question.

1. What is the risk-free rate for valuation purposes?
   (a) What is the risk-free rate from a theoretical perspective?
   (b) What are components of the risk-free rate?
   (c) What function does the risk-free rate have in valuations?
2. How do proxies estimate the risk-free rate for valuation purposes?
   (a) Which proxies are used to estimate the risk-free rate for valuation purposes?
   (b) Why are these proxies assumed to resemble the risk-free rate?
   (c) What are the deficiencies of these proxies?
3. How do models estimate the risk-free rate for valuation purposes?
   (a) What are approaches to model the risk-free rate?
   (b) What are weaknesses of these models?
4. How well do the selected methods estimate the risk-free rate for valuation purposes?
   (a) What are criteria to evaluate estimators from an academic perspective?
   (b) What are criteria to evaluate estimators from a practical perspective?
   (c) What are improvements to the best estimator of the risk-free rate for valuation purposes?

The first research question focuses on the definition of the risk-free rate. It is the foundation for the rest of the thesis by providing a theoretical definition of the risk-free rate. Furthermore, it reviews the concept specifically for valuation purposes, so we can clearly define this research's scope. The second and third Research Question cover the two risk-free estimator approaches; proxies and models, respectively. Finally, the last Research Question evaluates all estimators for the risk-free rate. As stated previously, in reality no actual risk-free asset exists, so it is impossible to test the adequacy of the proxies directly. To overcome this problem we will formulate evaluation criteria. We will determine these from a theoretical and a business perspective. After the evaluation, we will further analyze the deficiencies of the best method and propose improvements.

1.4 Thesis Outline

In our thesis we will use a combination of desk research and interviews for our data gathering. Desk research enables us to conduct an extensive analysis of the theoretical concepts, making it well suited for the theory-oriented objective of this thesis. Furthermore, this research strategy is also very well suited to conduct many similar tests on the proxies. We also will conduct interviews conducted to provide practical insights for the research.

Figure 1.1 shows the outline of this thesis. Chapter 2 conducts a literature analysis to the the theoretical concept of the risk-free rate. We conduct a literature search to define the concept. Furthermore, we conduct interviews to provide practical insights to the specific use of the concept in valuations. Subsequently, we describe the estimators of the risk-fee rate in Chapter 3 and 4. Thirdly, we evaluate the selected estimators in Chapter 5 and propose improvements for the best method in Chapter 6. Finally, we conclude in Chapter 7 by answering the main research question and providing suggestions for further research.

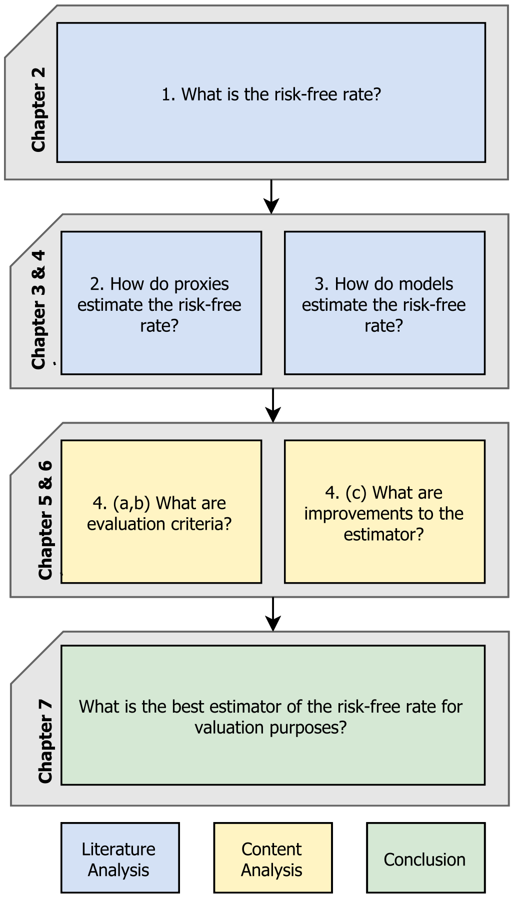

FIGURE 1.1: Thesis Outline

Chapter 2

The Risk-Free Rate

In this Chapter we answer the first Research Question by defining and describing the function of the risk-free rate for valuation purposes. Firstly, we deduce from the theoretical concept of a risk-free asset a practical definition of a risk-free proxy in Section 2.1. Subsequently, we describe in Section 2.2 the components for which an investor requires return from a risk-free asset. Afterwards, we describe in Section 2.3 the function of the risk-free rate in valuations. Defining the specific usage in valuations enables us to focus the scope of this thesis. Finally, we synthesize our findings in Section 2.4 by answering the first Research Question.

2.1 Definition

Investors consider investments on a risk-return trade-off. Figure 2.1 shows the components of the required return of an asset by an investor. The required return consists of the return on a risk-free asset, i.e., the risk-free rate, and additional risk premia for financial risks. Hull (2015) defines financial risk as the possibility of financial loss, implying implicitly that the return of a risk-free asset has a standard deviation of zero.

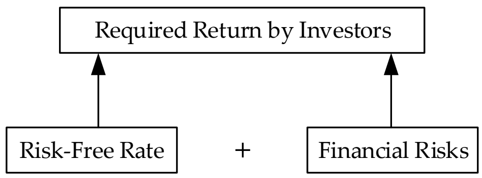

FIGURE 2.1: Components of Required Return by Investors

Thus a risk-free asset always pays a predetermined cash flow. However, the certainty of a specified cash flow does not eliminate all financial risks. In reality investors are bound to a return in a certain currency, inducing the inclusion of inflation and exchange-rate risk. The former is the risk of a greater depreciation of the currency's purchasing power than initially expected (Bekaert & Wang, 2010). Exchange-rate risk is the risk of the depreciation of the foreign currency in which the return is denominated (Berk *et al.*, 2012). Furthermore, this definition also includes liquidity risk, because a predetermined cash flow at a certain date sets no requirements to the ease of trading before maturity. Liquidity risk is the ease with which assets can be sold (Hull, 2015). Concluding, we define a risk-free asset as follows:

An asset paying a specified unit in a currency at a certain date in the future in any possible state of the world.

Figure 2.2 shows the cash flows of a risk-free asset with a payoff at year T. It has a single positive cash flow at maturity. This cash flow is certain in every state of the world and is a specified unit, 100+r, in the currency Euro. The present value of the asset is 100€, implying that the risk-free rate for T years is $r/T\%$ annually.

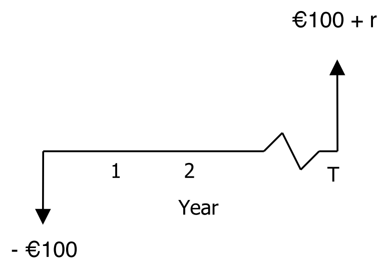

FIGURE 2.2: Example of Cash Flows from Risk-Free Asset

In reality no asset guarantees a specified cash flow in any possible state of the world and thus no 'true' risk-free asset exists. Therefore, we have to use proxies and models to estimate the risk-free rate. We transform the theoretical definition of a risk-free asset, into a practical definition for a risk-free proxy. We alter the definition to be able to quantify the exposure to financial risks. A practical definition of a risk-free proxy enables us to select risk-free proxies and finally evaluate them. Thus, defining a risk-free proxy is a trade-off between the inclusion of financial risks to make the concept more practical and staying close to the theoretical concept. To understand the considerations in this trade-off, we describe the most common financial risks affecting the cash flow at maturity in Table 2.1. The exposure to and probability of the financial risks differs per asset type, therefore we randomly list the risks. We have excluded interest-rate risk from the overview, because we define this as the resultant of all financial risks.

TABLE 2.1: Definition of Financial Risks

Risk type	Description	Examples
Credit Risk	Credit risk arises from the possibility that counterparties may default. Default is the failure to promptly pay financial obligations when due (Hull, 2015).	Bankruptcy or a late interest payment.
Reinvestment Risk	The risk that future cash flows from an asset cannot not be reinvested at the prevailing interest rate when the asset was initially purchased (Damodaran, 2008).	Market interest rate drops during the duration of the loan, so coupons cannot be reinvested at the prevailing rate.
Prepayment Risk	The risk of the early unscheduled return of principal of loans (Damodaran, 2008).	When principal is returned early, future interest payments will not be paid.

Scholars differ over the inclusion of financial risks in defining a risk-free proxy. One group stays true to the strict definition of a risk-free asset, while another only excludes credit risk. We think that controllability is key in deciding in the trade-off for the risk-free proxy. We classify financial risks as either controllable or uncontrollable. Controllable financial risks can be excluded by agreements and uncontrollable financial risks cannot. We classified the three major financial risks in these categories in Figure 2.3. We classify prepayment and reinvestment risk as controllable, because they can be excluded by specific agreements, e.g., a bond can be structured to only include a single payment and no right to early redemption. On the other hand we classify credit risk as uncontrollable, implying the impossibility to completely exclude it. Credit risk cannot be entirely removed, because of the existence of low probability high impact events, e.g., natural disasters.

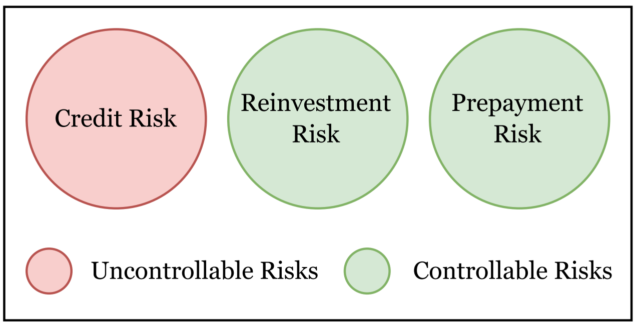

FIGURE 2.3: Included Financial Risks in Risk-Free Proxy Definition

Concluding, we think it is most accurate for a risk-free proxy definition to exclude all controllable financial risks and minimize all uncontrollable risks. Hereby we stay as closely possible to the concept of a risk-free asset, but acknowledge the impossibility to exclude uncontrollable financial risk in reality. Finally, we also stress that our definition of a risk-free asset includes exchange-rate, inflation and liquidity risk. We classify the first two as controllable financial risks, because both can be included by paying in a certain currency. However, we cannot fully include liquidity risk, because we need a certain level of liquidity for efficient market pricing. Therefore, we require a risk-free proxy to have a sufficient level of liquidity. We regard liquidity above this minimum as an excessive liquidity premium, inflating the price of a risk-free asset. Synthesizing, this leads to the following definition of a risk-free proxy.

An asset paying a specified unit in a currency at a certain date in the future with minimal credit risk and sufficient liquidity.

2.2 Components of Required Return

The risk-free rate is the required return of a risk-free asset by investors. They require compensation for postponing consumption and this can be broken down into two elements. Firstly, investors require compensation for the time value of money. Time preference is the economic term describing the preference of humans for direct over delayed consumption (Frederick *et al.*, 2002). This component of the required return is called the real rate. Secondly, investors require compensation for inflation. Investors are not interested in money itself, but in the purchasing power it represents in goods. In order to maintain their purchasing power at the same level, investors require compensation for the expected inflation. This component of the risk-free rate is called the inflation rate. Together both components form the nominal risk-free rate. Fisher (1930) firstly described these components and formalized their relationship at time t below. The latter approximation holds when the real and inflation rate are small.

$$\text{Nominal Rate}_t = (1 + \text{Real Rate}_t) * (1 + \text{Inflation Rate}_t) - 1 \approx \text{Real Rate}_t + \text{Inflation Rate}_t$$

The equation has become known as the Fisher equation and can be used to decompose the required return in a real and inflationary component ex-post and ex-ante. The ex-post decomposition in hindsight determines the components by using the realized inflation to determine the real rate. Decomposing the required return ex-ante needs forecasts of future inflation and real rates. An inseparable aspect of forecasting returns is uncertainty. Investors dislike uncertainty about future cash flows and therefore require additional compensation. This additional compensation is called a risk premium, which is the difference between the expected return from a risky asset and a certain return. In valuing the future real and inflation rate the market sums the expected rate with the risk premium. According to Fisher (1930) no interest risk premia exist on the long-term, however scholars have shown that they exist on the short-term (Bekaert & Wang, 2010; García & Werner, 2010). Figure 2.4 displays schematically the decomposition of the nominal risk-free rate ex-ante.

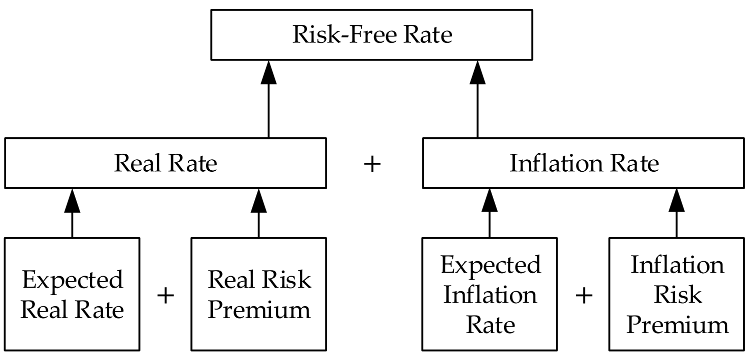

FIGURE 2.4: Components of the Risk-Free Rate

The risk-free rate components determine the required return for a certain period. Of course the length of the period also influences the required return. The relationship between the investment term and the interest rate is called the term structure. The visualization of this relationship is called a yield curve. Generally a longer maturity is accompanied with a higher required return, because a longer maturity is often accompanied with a larger uncertainty, resulting in a higher required risk premia. However, short-term uncertainties sometimes outweigh this effect (Bernoth *et al.*, 2012).

2.2.1 Inflation Rate

Inflation is the rate at which the general level of prices for consumer goods and service has risen over a certain period. In the Euro-Area the realized inflation is measured by the ECB with the harmonized index of consumer prices (HICP). This is a price basket of everyday items, durable goods and services weighted by the importance in the household budget (Berk *et al.*, 2012). The inflation rate is an important macro-economic variable and therefore is also forecasted. It is forecasted by surveys and mathematical models (Guimaraes, 2012). Important inflation drivers are macro-economic developments and the money supply (Gordon, 1975; Mankiw, 2008). An overview of the main drivers of inflation is listed in Table 2.2.

TABLE 2.2: Drivers of Inflation

Driver	Description	Effect
Demand-pull inflation	Aggregate demand due to increased private and government spending.	Demand grows faster than supply, thereby rising the general price level.
Cost-push inflation	Drop in aggregate supply.	Supply decreases faster than demand, thereby increasing inflation.
Built-in inflation	Vicious circle of inflation induced by adaptive expectations.	When everyone expects a certain inflation, the expectation will be incorporated in the price setting and results in the realization of the expectation.
Increase of money supply	Increased by the government or central bank.	By increasing the money supply, the general price level increases.

After central banks adapted inflation targeting as policy in the 1990s, the inflation in the Euro-Area has become rather stable. The ECB quantified her objective of price stability in an inflation target of 2%. It uses several monetary instruments to achieve this target. A stable and positive inflation realized that the inflation component of the risk-free was positive and rather stable over the latest years. However, the inflationary compensation does not need to be positive, in case of expected deflation investors can accept a negative inflationary compensation. They accept this, because the purchasing power is expected to grow over time. Since October 2013, the HICP has dropped below 1% and the ECB has taken significant measures to increase inflation. It has set overnight interest rates negative and started the Quantative Easing (QE) program in which it buys government bonds with the goal to stimulate investments. However, until today the measures have not succeeded and the Euro-Area has been exposed to a near zero to negative inflation.

2.2.2 Real Rate

The real rate compensates investors to lend money instead of immediately consuming it (Grabowski, 2010). Due to this positive time preference, it is assumed that the real rate is always positive. The real rate is determined by the neutral real rate and cyclical factors. The former is determined by the equilibrium at which supply and demand for capital meet. This equilibrium changes over time and is the result of the time preference of investors at a time. The latter determinant, cyclical factors, is the influence of the short-term interest rates, set by the central banks (Archibald & Hunter, 2013; Cour-Thimann *et al.*, 2006). Table 2.3 summarizes the drivers of the real rate.

TABLE 2.3: Drivers of the Real Rate

Driver	Sub-driver	Description	Effect
Neutral Real Rate	Savings and investment equilibrium	The demand and supply equilibrium of the capital markets, determined by the time preference of consumers and the investment options.	When the savings demand increases, the equilibrium rate shifts downward.
	Impediments to international capital flows	Impediments to capital markets between countries affect the equilibrium.	Impediments prevent capital to flow freely between low interest rate countries to high interest rate countries.
Cyclical factors		Adjustments of central banks to lean against inflationary pressure, also known as the real interest rate gap.	Central banks determine the short nominal interest rate and thereby influence the long-term real rates.

The real rate is an unobservable variable and is can be calculated ex-post with the Fisher equation. Ex-ante the real rate can be estimated by using expectations of the inflation rate or mathematical models to decompose the nominal interest rate. Most of these models are based on the assumption that the high-rated government bonds reflect the nominal risk-free rate. Furthermore the rates of inflation-linked bonds issued by governments are used (Guimaraes, 2012; Canova, 2002).

2.3 Business Valuations

The most used business valuation method is the discounted cash flow method. This method defines the enterprise value (EV) as the net present value of all future free cash flows (FCF). The FCF are the available cash flows to investors and represents the cash that a company is able to generate after setting apart the money required to maintain or to extend its assets base. It is computed by subtracting the capital expenditures from the operating cash flow. The discount rate used in the valuation is the WACC. We show the EV calculation below where FCF_t represents the FCF at time t and r_{wacc} the WACC.

$$EV = \sum_{t=1}^{T} \frac{FCF_t}{(1 + r_{wacc})^t}$$

Accurate projections of the FCF are only possible to a certain period. The projections of FCF after the projection period are based on the simplifying assumption that a company growths indefinitely at a fixed rate. For companies with a finite lifetime an adjustment to this assumption is made. The cap of the perpetuity growth-rate is the risk-free rate, because companies cannot outgrow the economy until perpetuity. The net present value of this growing perpetuity is called the continuing value (CV) (Koller *et al.*, 2010). When the WACC is greater than the growth rate, the CV is defined as below where g represents the growth rate. The total EV is the sum of the EV in the forecast period with the discounted CV. We show the complete derivation of the CV in Appendix A.

$$CV = \frac{FCF_T}{r_{wacc} - g}$$
$$EV = \sum_{t=1}^{T} \frac{FCF_t}{(1 + r_{wacc})^t} + \frac{CV}{(1 + r_{wacc})^T}$$

The valuation uses a fixed discount rate instead of a yearly rate. Theoretically a yearly discount rate is more accurate, however practitioners use a fixed rate for two reasons. Firstly, the usage of a fixed rate simplifies the valuation, and secondly it causes negligible difference in respect to the yearly method. The maturity of the fixed discount rate is based on duration matching, i.e., matching the duration of the expected cash flows with the duration of a risk-free asset. Generally practitioners use a 10-year risk-free rate when valuing business with an indefinite horizon (Damodaran, 2008). The WACC represents the minimum return on an existing asset base to satisfy all capital providers. When a company is unable to generate the WACC as the return, capital providers switch to better risk-return investment opportunities. Therefore, the WACC is used as a discount rate. When a company finances itself through equity and debt, then the WACC is calculated as the weighted average of both costs of capital as is shown below. Interest payment on debt can be deducted from taxes and therefore the cost of debt is adjusted for the tax rate (Berk *et al.*, 2012). Where E represents the total equity, D the total debt, K_e the cost of equity, K_d the cost of debt, and t the tax rate.

$$\text{WACC} = \frac{E}{(E + D)} K_e + \frac{D}{(E + D)} K_d (1 - t)$$

The first component of the WACC, the cost of equity is usually calculated with CAPM introduced by Sharpe (1964). It is based on the Modern Portfolio Theory introduced by Markowitz (1952) stating that investors make a trade-off between risk and expected return. Risk in this context is defined as the standard deviation of the possible returns. Investors analyze all investment opportunities and only consider investments yielding an optimal risk-return trade-off. All investments yielding such a trade-off together form the efficient frontier. This is shown in Figure 2.5 with the reference to 'previous efficient frontier'.

Modern Portfolio Theory states that inclusion of a risk-free asset changes the efficient frontier. A risk-free asset is in this context characterized as an asset with zero risk, implying it has a return without any standard deviation. When the simplifying assumption is made that an investor can borrow at the risk-free rate, a tangent through point M, the maximum of the efficient frontier, is created. This line is called the 'new efficient frontier' in Figure 2.5 (Hull, 2015). Consider for example when an investor has an investable amount of 1 and forms an investment i by putting β_i in the risky portfolio m and investing the remaining part $1 - \beta_i$ in a risk-free asset. This results in the following expected return and risk equations.

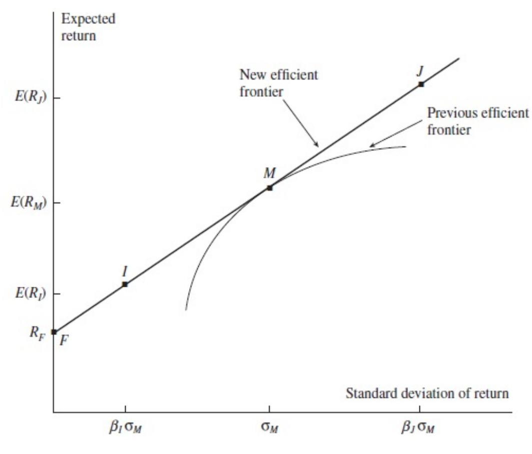

FIGURE 2.5: Efficient Frontier

$$E(r_i) = (1 - \beta_i) r_f + \beta_i E(r_m)$$
$$\sigma_i = (1 - \beta_i) \sigma_f + \beta_i \sigma_m = \beta_i \sigma_m$$

Furthermore, Markowitz (1952) reasons that the portfolio of risky assets M, consists of all risky assets, because the risk-return of a specific asset balances with the optimal market portfolio. Thereofre the portfolio M is referred to as the market portfolio. Sharpe (1964) has built on the knowledge of the market portfolio in determining the required return for individual investments. His CAPM states that the required return on an investment should reflect the extent to which the investment contributes to the risks of the market portfolio. The common procedure to determine this contribution, is to regress the return of an individual investment to the market portfolio.

$$r_{i,t} = a + \beta r_{m,t} + \epsilon_{i,t}$$

Where a and β are constants and ϵ is a random variable equal to the regression error. The equation shows that an investment is exposed two uncertain components. Firstly, the component βr_m which is the multiple of the market return and referred to as systematic risk. This risk cannot be diversified away by investing in other investments. Therefore, an investor requires additional compensation for this risk. The second uncertain component, ϵ, is non-systematic risk. When we assume that non-systematic risks of assets are independent of each other, then they can be diversified away in a large portfolio. Therefore, an investor does not require additional compensation for non-systematic risk. Concluding, investors thus only require compensation for their exposure to systematic and this determines the required return of an individual investment. This asset pricing model is known as the CAPM and shown below.

$$E(K_e) = r_f + \beta(E(r_m) - r_f)$$

The second component of the WACC, the cost of debt, is calculated by summing the risk-free rate with an additional credit risk premium, as shown below. A risk premium is required for the credit risk of a specific company. Credit risk is determined based on publicly known bond rates, credit ratings or a comparison with a peer group. The risk-free rate functions in the cost of debt as a benchmark asset without any credit risk.

$$K_d = r_f + \text{Credit Risk Premium}$$

Figure 2.6 shows the decomposition of the WACC schematically. The risk-free rate has an increasing effect on the WACC, when the asset has a postive beta. Theoretically assets with a negative beta exist, but they are very uncommon. Such companies should profit from negative market returns and vice versa, e.g., restructuring consulting firms and gold. Thus generally an increase of the risk-free rate, when holding everything else constant, increases the discount rate and reduces the enterprise value.

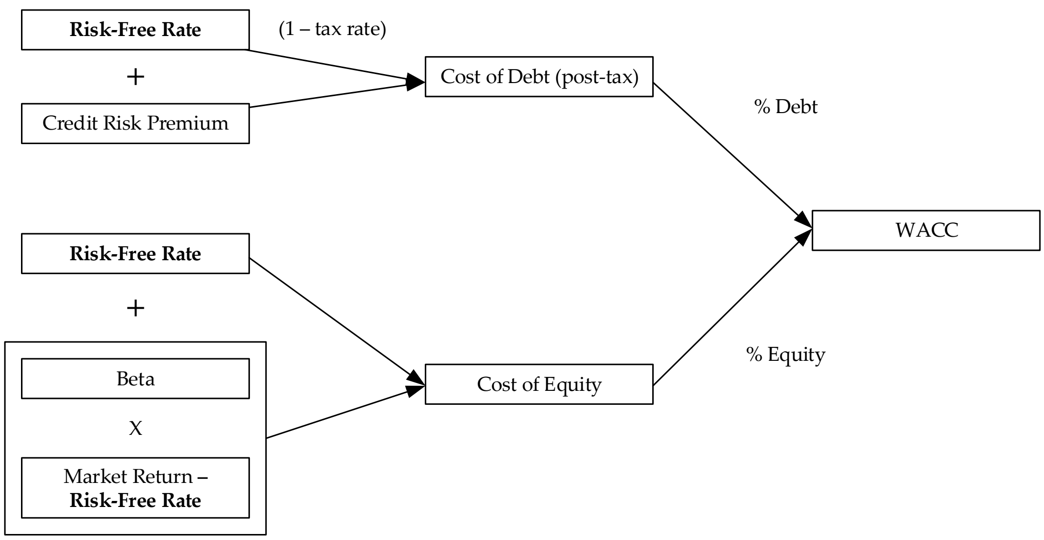

FIGURE 2.6: WACC Decomposition

2.4 Risk-Free Rate for Valuation Purposes

In this Chapter we have reviewed the theoretical concept of the risk-free rate. According to the strict theoretical definition a risk-free asset is not exposed to any financial risk. However, we acknowledge that in reality no true risk-free asset exists and therefore propose a practical definition of a risk-free proxy; *"An asset paying a specified unit in a currency at a certain date with minimal credit and sufficient liquidity."* Our definition excludes all controllable financial risks and minimizes all uncontrollable risks. We use our definition of the risk-free proxy to evaluate estimators in Chapter 5.

Subsequently, we showed that the risk-free rate consists of an inflation and real compensation. The first component compensates for the expected decreased purchasing power of the currency and the second component for the delayed consumption. Humans have time preference and therefore it is assumed that the real component of the risk-free rate should always be positive. Furthermore, we have shown that several methods decompose nominal risk-free rate into these two components, of which using inflation-indexed bonds is the most used approach.

Finally, we explained that the most used approach to value businesses is the discounted cash flow method, discounting all expected FCF with the WACC to determine the value of a business. In determining the WACC the risk-free rate is used for two functions. Firstly, it is regarded as an asset with a return with zero standard deviation and this characteristic is used to determine the required return on equity via CAPM. Secondly, the risk-free rate is used as a benchmark to determine the required return on risky debt. Furthermore, we explained that cash flows should be discounted with a matched maturity of the risk-free rate and that practitioners use a 10-year maturity. This maturity is used, because it matches with the duration of the expected cash flows.

Chapter 3

Risk-Free Proxies

Risk-free proxies closely resemble a risk-free asset and therefore are suitable estimators. In this Chapter, we answer the second Research Question by describing four risk-free proxies. The first and most used risk-free proxies are government bonds, which we describe in Section 3.1. Secondly, we describe Overnight Indexed swaps (OIS) in Section 3.2. Thirdly, we describe generalized collateral repurchase agreements (GC repo) in Section 3.3. Fourthly, we describe our own developed 'Market Implied Risk-Free Rate' in Section 3.4. For all proxies we describe their resemblance to a risk-free asset by evaluating their financial risk exposure. Finally, we conclude this Chapter in Section 3.5 by selecting suitable risk-free proxies for valuation purposes.

3.1 Government Bonds

Government bonds are loans issued by national governments with the obligation of periodic coupon payments and the repayment of the face value at maturity. Generally the bonds are denominated in the country's own currency, but some governments issue also foreign debt for strategic reasons. The government bond yields of the Euro-Area countries differ due to differences in liquidity and perceived credit risk. The German government bond (Bund) has in general had the lowest yield and therefore we use it as proxy for the Euro risk-free rate. The Bund has a low yield, because of its high liquidity and low credit risk (Ejsing *et al.*, 2015).

Government bonds of developed countries have been used traditionally as risk-free proxy, because of their low credit risk. Governments of developed countries are regarded as creditworthy, because of their long-term vision and ability to raise taxes. A conformation of the low credit risk is the high credit ratings of governments of developed countries (CapitalIQ, 2016). Furthermore, governments can in theory use their control over the money supply to meet their financial obligations, i.e., print additional money to pay their creditors (Damodaran, 2008; Dacorogna & Coulon, 2013). However, since the financial crisis in 2008 the negligibility of credit risk of developed countries has been questioned. Credit agencies downgraded several countries and also Credit Default Swaps (CDS) spreads have risen (CapitalIQ, 2016). A CDS provides insurance against the default of a reference entity, and thus indicates the probability of default (Hull & White, 2013). Concluding, the credit risk of developed countries is still low, but certainly not absent.

The government bond market is large with high trading volumes and low transaction costs. Liquidity can be measured in many methods and all indicate a high liquidity of the government bond market. Fleming (2003) is in favor of using the relative bid-ask spread on assets to measure market's liquidity and showed that spreads on government bonds are very small. The high liquidity inflates the value of the Bund, distorting it from the price of a risk-free asset requiring only a sufficient liquidity for efficient market pricing.

Finally, government bonds resemble a risk-free asset because of the absence of prepayment and reinvestment risk. Regular government bonds are not callable and include coupon payments (Bloomberg L.P., 2015). The first characteristic means that governments cannot repay their loan earlier than agreed and thus excludes prepayment risk. Coupon payments exposes government bonds to reinvestment risk. However, coupon-bearing bonds can be stripped to zero-coupons bonds, which are bonds with only a single payment. The removal of intermediate payments eliminates reinvestment risk, because the bond has no exposure to fluctuating interest rates until maturity. Synthesizing, we conclude that zero-coupon government bonds have no exposure to prepayment and reinvestment risk and thus resemble a risk-free asset.

3.2 Overnight-Indexed Swaps

An OIS is an interest rate swap based on unsecured overnight interbank borrowing, i.e., all overnight loans between banks without collateral. The Euro OverNight Index Average (EONIA) is the average overnight rate in the Euro-Area and is the floating component of the interest rate swap. All major currencies have an overnight market, however they slightly differ in construction (Edu-Risk International, 2015). We use the EONIA as example, because it is the overnight rate for the Euro market. To better grasp the concept of an OIS, Subsection 3.2.1 firstly explains an interest rate swap. Subsequently, Subsection 3.2.2 explains the EONIA. Finally, Subsection 3.2.3 describes OIS as risk-free proxy.

3.2.1 Interest Rate Swaps

In a plain vanilla interest rate swap a company exchanges a variable for a fixed cash flow stream or vice versa. It agrees to pay interest at a predetermined fixed rate on a notional principal for a number of periods. In exchange it receives interest of a floating rate on the same notional principal for the same period. A swap is structured in a way that its net present value at initiation is zero, this means that no cash flows are exchanged at initiation (Hull, 2008).

$$PV\left[\Big(\prod_{t=1}^{T}(1+\text{Floating}_t)\Big)-1\right] = PV\left[(1+\text{Fixed})^T-1\right]$$

This equation is realized by agreeing to a fixed rate equating the above equilibrium. Thus issuers of an interest rate swap have an expectation of the development of the variable rate. In theory two companies can enter directly into an interest rate swap agreement, but in reality each deals with a financial intermediary. Financial institutions act as a market maker for interest rate swaps and provide bid and offer quotes for which they are prepared to exchange floating rates (Hull, 2008). The most common floating rates like the London Interbank Offered Rate (LIBOR) and the EONIA are quoted for maturities up to thirty years.

The structure of interest rate swaps induce a lower credit risk relatively to ordinary loans with the same notional and maturity. Firstly, the creditworthiness of the variable component of an interest rate swap always remain stable. For example the LIBOR rate is always based on A-rated banks and has no risk of declining credit quality. On the other side an ordinary loan to a A-rated bank is exposed to declining credit quality. Secondly, the exchanged cash flows are smaller, because no notional is exchanged and cash flows are netted. The notional itself is only used to calculate interest and not exchanged itself, because it would be meaningless to exchange the same notational at maturity. Furthermore, the cash flows are netted, meaning only the difference between the floating and fixed rate is exchanged. This means that the exchanged cash flows are smaller, reducing credit risk. The netting occurs annually and at maturity (Collin-Dufresne & Solnik, 2001). We show a numerical example of an interest rate swap in Appendix B.

3.2.2 Euro Overnight Index Average

Overnight rates are the rates in the government-organized interbank market where banks with excess reserves lend to banks that need to borrow to meet their reserve requirements (ECB, 2014). EONIA is the overnight rate in the Euro-Area and is daily calculated by the ECB. The rate is calculated as the weighted average of the actual transactions of panel banks. Currently this panel consists of 35 creditworthy banks (European Banking Federation, 2013). The ECB directly influences the overnight rate by setting a rate corridor. Its *marginal lending facility* is the rate at which banks can borrow from the ECB and forms the upper bound of the corridor. The lower bound is the *deposit facility*, the compensation banks receive for their overnight deposits of the ECB. The EONIA is always between this corridor, because all panel banks have access to the facilities of the ECB. The corridor is only changed between meeting dates, making EONIA relatively constant between these periods. Figure 3.1 shows the development of the overnight rates (ECB, 2015). The EONIA has sharply decreased after the financial crisis in 2008. Although not entirely clear from the figure, the deposit facility is also below the EONIA in 2014 and 2015.

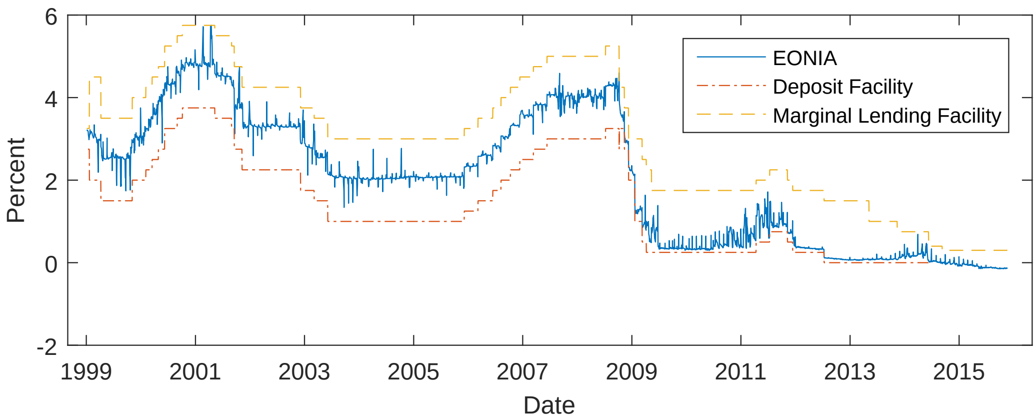

FIGURE 3.1: Development of Overnight Rates

3.2.3 Euro Overnight Index Average Swap

The Euro OIS is an interest rate swap based on the EONIA. The floating rate during an interest period is calculated by compounding the daily EONIA rates. For non-business days the EONIA is not quoted, so it is assumed to remain constant for the non-quoted days. Below we show the calculation of the compounded EONIA for an interest period with T number of business days.

$$\text{Floating Rate} = \left(\prod_{t=1}^{T} \left(1 + \frac{n_t \text{EONIA}_t}{360} \right) \right) - 1$$

where n_t is the number of calendar days between business day t and the next business day. According to market European market convention, the interest is calculated based on a year with 360 days, a simplification of reality (Edu-Risk International, 2015). As earlier stated in Subsection 3.2.2, EONIA depends strongly on the corridor set by the ECB, which is only changed at ECB meeting dates. Therefore, EONIA swaps are quoted by market makers based on standard tenors and in short-term also with maturities corresponding to ECB meeting dates. The short-end of the curve is constructed with the assumption that the EONIA remains constant between meeting dates. The longer-end of the curve is constructed by interpolating between quoted maturities.

OIS are regarded to be risk-free proxies especially due to their superior credit qualities. OIS have the same credit quality as a continually refreshed overnight loan between highly-rated banks. Overnight borrowing is regarded to be credit risk free, thus also its derivative the OIS. Secondly, the prepayment risk of OIS is absent, due to the fact that no notional is exchanged. Thirdly, conventional OIS are exposed to reinvestment risk, because swaps are settled annually and at maturity. Thus they have intermediate payments inducing reinvestment risk, however these intermediate payments can be eliminated by bootstrapping. This is the same process as used in the construction of zero-coupon government bonds. The OIS market is still developing and although its liquidity is concentrated on the short-term maturities (ECB, 2014), also the long-term maturities are relatively liquid. This is shown by the relative small relative bid-ask spread for long-term maturities for OIS compared to corporate bonds (CapitalIQ, 2016). This indicates that the OIS yields incorporate a liquidity premium.

3.3 Generalized Collateral Repurchase Agreements

A repurchase agreement is a sale of a security with the agreement to repurchase the security at a specified price. Repos are a type of a short-term collateralized loan. We show the calculation of the repo rate r below. The rate is calculated by determining the difference between the sale price, P_s, and the repurchase price, P_r, for a period with T days (Hull, 2015).

$$
\begin{aligned}
P_s(1 + r\frac{T}{360}) &= P_r \\
r\frac{T}{360} &= \frac{P_r}{P_s} - 1 = \frac{P_r - P_s}{P_s} \\
r &= \frac{P_r - P_s}{P_s}\frac{360}{T}
\end{aligned}
$$

A GC repo is a repo type using a predefined basket of safe securities as collateral. The basket primarily contains government and agency securities with the highest credit quality. Agency securities are issued by government agencies and are guaranteed by the central government. Therefore, these securities have the same credit risk as government bonds. GC repos are regarded to be a risk-free proxy, because they have a low exposure to financial risks. Firstly, credit risk is virtually absent due to the posted collateral. A small credit risk exposure exists, due to the market movements of the collateral, i.e., the collateral can decrease in value. However, this is usually resolved by overcollateralization. Furthermore, GC repos are unexposed to prepayment and reinvestment risk by construction. Their agreement does not include any coupon payments and specifies a fixed repayment date. Lastly, the liquidity of GC repos is very small. The GC repo market is split in overnight and term repos, where the former is highly liquid and the latter illiquid especially for maturities longer than a week (Ejsing *et al.*, 2015).

3.4 Market Implied Risk-Free Rate

Hull *et al.* (2004) and Blanco *et al.* (2005) construct a risk-free portfolio by combining a corporate bond and a CDS. We construct a risk-free estimator by averaging many of such portfolios and call it the 'Market Implied Risk-Free Rate'. We use the term 'Market Implied', because it is indirectly implied by the market. Firstly, we elaborately explain the concept of the Market Implied Risk-Free Rate in Subsection 3.4.1. Subsequently, we construct the proxy in Subsection 3.4.2. Finally, we review the function of Market Implied Risk-Free Rate as proxy in Subsection 3.4.3.

3.4.1 Concept of Market Implied Risk-Free Rate

A portfolio of a corporate bond and a corresponding CDS is risk-free, because it has no credit risk. The credit risk of the corporate bond is eliminated by a CDS, which we assume to be accurately priced for large corporations. The relationship below should hold, where a holder receives interest from the corporate bond, y, and has to pay the CDS spread, s.

$$r_f = y - s$$

When the above relation would not hold, then arbitrage possibilities would exist. When for example s is greater than the difference between y and r_f, an arbitrageur could make a risk-less profit by purchasing a risk-free bond, and selling a corporate bond and the CDS. However, in this arbitrage argument we made two assumptions. Firstly, we assumed that market participants can short corporate bonds and borrow at the risk-free rate (Hull *et al.*, 2004). According to us a reasonable assumption, because it is often made in asset pricing theory. Secondly, we assumed that a CDS also pays the accrued interest of a bond, while it only pays the bond's face value. Duffie (1999) proposed an adjustment for this incorrect assumption. When A is the expected accrued interest at the time of default, then the expected payoff from a CDS equating the face value of a bond and its accrued interest is $1 + A$ times the CDS spread. Therefore, Duffie

(1999) proposed the following improved estimation of the risk-free rate:

$$r_f = y - s(1 + A)$$

Most European corporate bonds pay annual interest (CapitalIQ, 2016), so we can determine the expected accrued interest by assuming a certain probability of default of corporates. When we assume consistently with Hull *et al.* (2004) that the probability of default is uniformly distributed over a year, the expected accrued interest A becomes $\frac{y}{2}$. In Appendix B we show that the Market Implied Risk-Free variant without the adjustment for accrued interest more accurately reflects the risk free rate. We show this by comparing the spreads of the Market Implied Risk-Free variants to the average of the Bund and OIS. We use the latter as benchmark, because they are the only available risk-free proxies for a 5-year maturity. Therefore, in the rest of this thesis we use the variant without the accrued interest adjustment as the Market Implied Risk-Free Rate. Furthermore, our analysis shows that market participants do not value the loss of accrued interest.

3.4.2 Construction of Market Implied Risk-Free Rate

We construct the Market Implied Risk-Free Rate for a 5-year maturity, because of the low liquidity of the longer CDS tenors and the low availability of longterm corporate bonds. Annual CDS tenors up to ten years are available, however they are much less liquid for longer maturities. This is shown by the relatively high bid-ask spreads (CapitalIQ, 2016). The low liquidity disturbs accurate market pricing. Furthermore, only a few companies issue 10 year bonds: in our sample period on average 3 companies had 10 year bonds monthly available (Bloomberg L.P., 2015). This forces the Market Implied Risk-Free Rate to be based on only a small group of companies and exposes it to company specific risks. These reasons combined, force us to use only the Market Implied Risk-Free Rate for maturities up to 5 years.

Firstly, we analyzed the Europe CDS index of Markit Group (2015) containing the 125 most actively traded CDSs. We use this index due to its high liquidity. Subsequently, we filtered our selection by two criteria. Firstly, companies must have at least an A S&P rating. Houweling & Vorst (2002) have shown that bond and CDS portfolios of highly-rated companies better reflect the risk-free rate. Secondly, companies must be headquartered in the Euro-Area. This ensures that the majority of their bonds are issued in Euros. This filter resulted in a selection of 29 companies. Subsequently, we obtained for these companies the 5-year CDS spreads and all outstanding bonds from 2008 to 2015 from (Bloomberg L.P., 2015). We only incorporated bonds meeting the following criteria in our sample set:

1. Bonds have to be Euro denominated with fixed coupons. The first component of the criterion makes the loans suitable for determining the Euro risk-free rate. Furthermore, fixed coupons enable us to forecast cash flows.

2. Bonds must not be puttable, callable, convertible or reverse convertible. This ensures that bonds have a fixed maturity and do not incorporate additional risk premiums.

3. Bonds must not be subordinated or structured. These bonds are exposed to different risks than regular bonds.

4. Bonds must be issued publicly, otherwise the market is unable to function efficiently.

5. The amount issued must be at least €100 million, because large issues ensure sufficient liquidity.

To perform an accurate comparison we use bonds with a maturity of exactly five years. Blanco *et al.* (2005) constructed five year bonds by linearly interpolating between the two bonds with the closest maturity to five years. Hull *et al.* (2004) constructed them by regressing all available bonds between two and ten years. They use these two thresholds, because the 'normal' yield curve has a different slope outside of them. We constructed five year bonds with both methods for our data. Both methods show similar outcomes, however the results of the regression method have a lower standard deviation. Therefore, we prefer the regression method, because a lower standard deviation indicates more robust results. Three companies had too few bond issues meeting the requirements, so for in total 26 companies 5-year bond yields have been calculated. The Market Implied Risk-Free Rate is determined as the simple average of all companies in the sample. We have used the simple average, because we consider the assumptions of a risk-free portfolio equally holds for all companies. The descriptive statistics of the companies per sector are shown in Table 3.1. The companies are grouped by sector classification of CapitalIQ (2016), because it enables a sector specific review. The most remarkable observation is that the Financials have the largest CDS spread, but their bond yield is not proportionally higher in comparison to the other sectors. A possible explanation is that the Financials conduct the largest bond issues and so their bonds are traded more actively. Investors prefer this liquidity and therefore require lower bond yields.

TABLE 3.1: Descriptive Statistics of Corporates in our Sample Set from 2008:01 to 2015:12

Sector	No. Companies	Bond Yield	CDS Spread
Consumer Discretionary	3	197.31	91.36
Consumer Staples	3	228.14	54.91
Energy	1	250.35	63.61
Financials	11	263.59	119.21
Healthcare	2	213.68	56.65
Industrials	3	216.28	85.45
Materials	3	228.86	57.85
Utilities	3	234.02	73.76
Mean		229.03	76.56

The bond yield and CDS spread are denominated in basis points and are simple averages of the sector. The mean is the simple average of all companies and is used to determine the Market Implied Risk-Free Rate.

3.4.3 Market Implied Risk-Free Rate as Proxy

The Market Implied Risk-Free Rate is a risk-free proxy, because of its low exposure to financial risks and its relative low liquidity. The proxy has virtually no exposure to credit risk, and only a small exposure to reinvestment and prepayment risk. Credit risk is eliminated, because of the CDS in the portfolio. Furthermore, the proxy has a small exposure to reinvestment and prepayment risk. The former is caused by the annual payments of the CDS premia and bond coupons, inducing intermediate payments. The size of this risk is small, because the payments are in opposite directions and thereby neutralize each other partially. Prepayment risk is caused by the probability of a default, where the face value of the loan is immediately repaid. This forces an investors to reinvest the face value at the then prevailing risk-free rate. The size of this risk is very small, because we only selected A-rated companies having a very low probability of default. Finally, the Market Implied Risk-Free Rate is suitable as proxy, because of its relative low liquidity. The relative bid-ask spread, a common measure of liquidity, is on average four times larger for corporate than for government bonds. However, the spread is still much smaller than non-investment grade bonds (Chen *et al.*, 2007). All these characteristics let the Market Implied Risk-Free Rate closely resemble a risk-free asset.

3.5 Selected Risk-Free Proxies

In this Chapter we have described four risk-free proxies. We have shown that all closely resemble the risk-free asset, but have different deficiencies. Two proxies are not suitable as risk-free estimators for valuation purposes, because of the absence of long-term maturities. GC repo is only available for maturities up to one year and the Market Implied Risk-Free Rate up to five years. Therefore, we will only evaluate the Bund and the OIS as risk-free estimators.

Chapter 4

Risk-Free Models

We answer the third Research Question in this Chapter by providing an overview of the existing methods to model the risk-free rate. We explain the different methods in Section 4.1 and select the most suitable model within the context of this thesis. Subsequently, we construct our own model in Section 4.2.

4.1 Modeling Methods

After a literature review we have selected three risk-free models. The first is based on a consumption based asset pricing model and defines the risk-free rate as a function of intertemporal choice. Subsection 4.1.1 describes this 'Consumption Model'. The second method defines the risk-free rate as a common factor of several market interest rates. The differences between these interest rates are related to specific factors and by calculating these factors we can determine the risk-free rate. Subsection 4.1.2 explains this so-called 'Co-Movements Model'. The last approach uses macro-economic variables to explain the risk-free rate and we refer to it as the 'Macro Model'. This method regresses macro variables on a historical accurate risk-free proxy to model the risk-free rate. Subsection 4.1.3 explains this approach in more detail. We conclude this section by selecting the most suitable model for construction in Subsection 4.1.4.

4.1.1 Consumption Model

Campbell (2003) constructed the Consumption Model by defining the risk-free rate as a result of intertemporal choice. Intertemporal choice are decisions involving trade-offs between costs and benefits occurring at different times. Multiple psychological factors have a role in determining a person's intertemporal choices. Economists have simplified intertemporal choice problems by comparing utility instead of real goods. Utility in this context represents the satisfaction experienced by the consumption of a good. This concept is used, because it is a more universal measure than real goods. The foundation of intertemporal choice is that humans prefer immediate over delayed utility. This preference has been translated into a discount function. The intertemporal utility function for the present ($t = 0$) is given below. In the function $U(C_t)$ represents the utility of consumption C at time t, and $D(t)$ represents the discount function at time t (Frederick *et al.*, 2002). With positive time preference $D(t)$ is a decreasing function and always smaller than 1. We reflect on several discounting models in Appendix C. This reflection has led us to use the discounted utility model in this thesis, as it is the most widely used model.

$$U = \sum_{t=0}^{T} D(t)U(C_t)$$

The Consumption Model of Campbell (2003) assumes that humans make choices in order to maximize utility over their lifetime. Thus humans equate the cost of consuming one real dollar less at time t and the expected benefit of investing one real dollar in asset i at time t and consuming the dollar and its proceeds in $t+1$. He finds this optimum by solving the first-order condition listed below, where the left-hand-side represents the benefit of utility now and the right-hand-side represents the discounted expected benefit of delayed utility. In this function $E_t\Big[(1 + r_{i,t+1})U'(C_{t+1})\Big]$ represents the expected value of the return of asset i, $1 + r_{i,t+1}$, and the derivative of the utility of consumption at time $t+1$, $U'(C_{t+1})$.

$$U'(C_t) = D(t)E_t\Big[(1 + r_{i,t+1})U'(C_{t+1})\Big]$$

Campbell (2003) further specifies the risk-free rate by assuming a certain utility function. He uses the power utility function, which is the most used utility function. Its widespread usage is related to its simplicity and the disagreement on better models. It is simple, because it is the only utility function with a constant relative risk aversion (CRRA) coefficient γ. Risk aversion is the reluctance of a person to accept a uncertain higher expected payoff over a certain lower expected payoff. Besides its simplicity, the power utility has the advantage of being scale-invariant, meaning that is constant over time as wealth increases. The power utility function Campbell (2003) uses, is specified below.

$$U(C_t) = \frac{C_t^{1-\gamma} - 1}{1-\gamma}$$

Finally, Campbell (2003) defines the risk-free rate, by assuming that the joint conditional distribution of asset returns and the stochastic discount factor is lognormally distributed and homoskedastic. The Consumption model defines the risk-free rate as an asset with a standard deviation of zero, resulting in the specification of the risk-free rate as below. We show the full derivation of Campbell (2003) in Appendix A. This equation states that the risk-free real rate is linear in expected consumption growth, with a slope equal to the CRRA. Furthermore, the variance of the consumption growth has a negative effect on the risk-free rate. Campbell (2003) interprets the latter as a precautionary savings effect, implying that a higher volatility of consumption growth induces more saving.

$$log(1 + r_f) = -log(D(t)) + \gamma E_t[\Delta log(C_{t+1})] - \frac{1}{2}\gamma^2\sigma_m^2$$

Weil (1989) was the first to empirically test the Consumption Model with historical data. In his analysis he used the time period ranging from 1889 to 1978. In this period the growth per capita real consumption was 1.83% and the real return on a relatively risk-free security was 0.80%. With the knowledge of these variables only the discount factor and the CRRA are unknown. Weil subsequently implied both variables, by making assumptions about the maximum value of the variables. He capped CCRA at a value of 10, the maximum experimentally observed by Mehra & Prescott (1985), and limited the discount factor to $.98$, implying almost no discounting. The results of Weil (1989) are listed in Table 4.1 and show that both maximum values result in strange implied factors. Firstly, a CCRA of 10 implies a discount factor greater than 1, implying a negative time preference. Secondly, a discount factor of $.98$ implies a CRRA of 29, which is far greater than experimentally observed values (Arrow, 1971). The implied results are very far off from the experimentally observed discount factor and CRRA and thus seem incorrect. Weil (1989) calls this discrepancy between theory and reality the 'Risk-Free Rate Puzzle'. It poses the question: 'Why can the risk-free rate be so low with the observed discount factors and CRRA?' This puzzle is linked to the equity premium puzzle discovered by Mehra & Prescott (1985), questioning the high observed risk equity risk premia with the observed discount factors and risk aversion. After the research of Weil (1989), scholars have tried to resolve the Risk-Free Rate Puzzle by altering the utility functions of humans (Kandel & Stambaugh, 1991; Mello & Guimaraes-Filho, 2004), however this has not yet resolved in a solution.

TABLE 4.1: Historically Implied Discount Factor and CRRA

r_f	ΔC_{t+1}	σ_c	D	γ
0.80%	1.83%	3.57%	1.12	10.00
0.80%	1.83%	3.57%	0.98	29.37

Concluding, we appreciate the approach of the Consumption Model to model the risk-free rate as a function of human intertemporal choice. It has great explanatory power by linking psychological motives to the risk-free rate. However, the model with the current assumptions has proven to be a bad explainer of the risk-free rate historically. This is as a big disadvantage of the approach.

4.1.2 Co-Movements Model

The Co-Movements Model defines the risk-free rate as the common factor underlying all market interest rates. It interprets market interest rates r_i at time t as the sum of the risk-free rate, credit and liquidity premia and a disturbance term.

$$r_{i,t} = r_{f,t} + \text{Credit Premium}_{i,t} + \text{Liquidity Premium}_{i,t} + \epsilon_{i,t}$$

Reinhart & Sack (2002) and Feldhutter & Lando (2008) propose that besides these four factors market interest rates are also influenced by idiosyncratic shocks, i.e., shocks specific to an asset. For example, they state that government bonds are exposed to these shocks, because they have advantages not shared by other assets, e.g., their widespread use as collateral in repo transactions. The number of unique characteristics in comparison to other bonds determines the exposure to idiosyncratic shocks. Therefore they include an idiosyncratic shock term in the definition of several market interest rates.

The influence of the factors to the market interest rates is quantified with factor loadings. The risk-free rate is assumed to influence all market interest rates equally, i.e., its factor loading is one for all market interest rates. The other factors all influence the interest rates differently. The risk-free rate is solved from the Co-Movements Model by quantifying the factor loadings of market interest rates. This can be done in two approaches, either by determining anchor points or by maximum likelihood estimation. The first method determines specific interest rates as anchor points without any exposure to specific risks. For example Ejsing *et al.* (2015) determines the OIS as an interest rate with a zero factor loading to credit risk. When anchor points for all factor loadings have been determined the model can be solved recursively or by regression. The advantage of this method is its ease and intelligibility. However, it has the disadvantage that it relies on arbitrary anchor points. As stated in Section 2.1, in reality no assets without credit or liquidity risk exposure exist, so by definition these anchor points are only approximations. The second approach in solving the factor loadings uses maximum likelihood estimation. This approach determines the factor loadings by calculating the maximum likelihood of the factor loadings within a selected set of interest rates, which can be done with various techniques. The advantage of this method is that it does not rely on subjective methods, however it does require complex maximum likelihood estimators.

The main advantage of the Co-Movements Model's approach to model the risk-free rate is its economic interpretation. It isolates risk premia in existing market interest rates to determine the risk-free rate. Thereby it let risk-free proxies more closely resemble a risk-free asset. However, the biggest disadvantage of the model lies within the determination of the factor loadings. The first approach relies on subjective anchor points decreasing the accuracy of the model. This is caused by the fact that these anchor points are per definition approximations. Furthermore, the second other approach uses complex maximum likelihood estimation methods decreasing the economic interpretation and are outside the scope of this thesis.

4.1.3 Macro Model

The Macro Model defines the risk-free rate as a function of macro-economic variables. Firstly, Taylor (1993) has shown that movements in the short risk-free rate are related to movements in macro variables. Furthermore, Ang & Piazzesi (2003) continued on this observation by analyzing the term structure based on macro-variables and latent variables. The latter are variables which are not directly observable. We discard the option to model the risk-free rate with latent variables, because they lack economic interpretation. In the Macro Model the risk-free rate is a dependent variable and is explained by several independent macro-economic variables. The model is described by k independent variables for time t below. In the model α represents the regression constant and β_i the coefficient of a macro-economic variable x_i. The coefficients of the model are determined by regression macro-variables on a risk-free proxy.

$$r_{f,t} = \alpha + \beta_1 x_{1,t} + ... + \beta_k x_{k,t}$$

Scholars roughly agree on the determination of the explanatory variables for the risk-free rate. Three groups can be classified: Policy Instruments, Real Activity and Inflation. Firstly, Policy Instruments are variables influenced by the central banks to steer monetary policy, e.g., overnight interest rates. Secondly, Real Activity variables describe the economy's activity, e.g., industrial production and employment percentages. Finally, the Inflation group contains variables regarding past or future inflation, e.g., realized HICP and inflation forecasts (Diebold *et al.*, 2006; Taylor, 1993; Rudebusch *et al.*, 2006; Ang & Piazzesi, 2003). Besides these macro-variables types, other variables representing liquidity and credit risk are also frequently used by scholars, e.g., bid-ask and CDS spreads (Reinhart & Sack, 2002; Pericoli & Taboga, 2015).

The advantage of the Macro Model is that it is constructed easily, it has high explanatory power, and its coefficients have economic interpretation. However, it has the disadvantage that it has only a limited link to the risk-free rate theory. Furthermore, the model has to be regressed to an risk-free estimator at construction, making it inevitably prone to estimation errors.

4.1.4 Selected Risk-Free Model

We have discussed three approaches to model the risk-free rate. Firstly, we have shown that the Consumption Model defines the risk-free rate as a function of intertemporal choice, linking it strongly to risk-free theory. However, the model has shown to have only limited explanatory power of risk-free proxies. Secondly, the Co-Movements Model defines the risk-free rate as the common factor underlying all market interest rates. It isolates the risk-free rate by quantifying other factors influencing the market interest rates and thus improves existing risk-free proxies. The downside of this method lies with the quantification of the factor loadings, they either have to be determined arbitrarily by setting anchor points or by using complex maximum likelihood estimation methods. Finally, the Macro Model defines the risk-free rate as a function of several macro-variables. The model can be constructed easily, it has high explanatory power and it can be economically interpreted. However, it lacks underlying theory linking macro-variables to risk-free rate theory. Furthermore, it has to be constructed with a risk-free proxy, making it inevitably exposed to estimation errors of the risk-free proxy.

Considering all pros en cons of the different methods and the limited time of this research, we select the Macro Model as the best option to model the risk-free rate. Firstly, we are confident that we can minimize the estimation errors by selecting a accurate risk-free proxy. Furthermore, we are capable to construct this model within the time constraint set by this thesis.

4.2 Macro Model Construction

To construct the Macro Model, we firstly select an accurate historical risk-free proxy. Subsequently, we select macro-economic variables optimizing the explanatory power of the Macro Model. We describe this process in Subsection 4.2.1. Finally, we describe the regression and the constructed model in Subsection 4.2.2.

4.2.1 Data Description

The first step in constructing the Macro Model is the choice of an accurate historical proxy. Secondly, we also have to choose a period in which it accurately reflected the risk-free rate. As described in Chapter 3, the Bund has traditionally been regarded as the best risk-free proxy and therefore we use it as proxy. Subsequently, we minimize the estimation errors by selecting a period in which the Bund most accurately reflected the risk-free rate. We choose the period between 2000 and 2008. We leave the pre-Euro period out of the consideration, since it may have currency specific issues and we have enough observations. Furthermore, we discard the period after 2008, because Ejsing *et al.* (2015) have shown that since then the Bund has been impacted by financial risks. They show that liquidity and credit risk factors have had a large impact on the Bund as a result of the banking crisis and its impact on public finances. The sample period contains 96 observations; a sufficient number to conduct a regression. Furthermore, this period includes the Dot-com financial crisis, from mid of 2000 to end of 2001. We evaluated the effect of this crisis by comparing its in- and exclusion. Finally, we decided to include the complete sample period, because it realized lower standard errors and a larger explanatory power of the model. Concluding, we construct the Macro-Model by regressing macro-variables on the Bund in the period between 2000 and 2008, because it is the best available risk-free proxy.

We considered several macro-economic variables proposed by scholars for constructing the Macro Model. We only compared macro-variables published with at least a monthly frequency, because a lower frequency would significantly limit the use of the Macro Model. Furthermore, we considered the change of macro-variables when it contains more information than its absolute level, e.g., indices. We also considered volatilities of the variables. Finally, we transformed all variables to an annual basis, because the Bund yields are also quoted annually. Finally, we selected five variables optimizing the explanatory power of the model. The following model at month t is constructed:

$$\text{Bund}_t = \alpha + \beta_1 \text{PPI}\Delta_t + \beta_2 \text{UE}\Delta_t + \beta_3 \text{GER}\Delta_t + \beta_4 \text{EONIA}_t + \beta_5 \text{Bid-Ask}\sigma_t + \epsilon_t$$

Table 4.2 describes the used variables and shows the summary statistics. Appendix C describes in more detail the selected and excluded variables. Firstly, it shows the development of the selected variables in the sample period. Secondly, it describes the reason for excluding variables. Reasons for exclusion were that the variables proved to be insignificant, lacked historical data, or were highly correlated with other dependent variables.

TABLE 4.2: Variable Description and Summary Statistics from 2000:01 to 2007:12

Group	Variable	Description	Source	Mean	Std. Dev.
Risk-Free Proxy	Bund	10Y German annual zero-coupon bond yield.	Bloomberg	4.36	0.64
Inflation	PPIΔ	Annual change of European PPI.	ECB	2.81	1.94
Real Activity	UEΔ	Annual change of European unemployment rate.	ECB	-3.07	5.68
Real Activity	GERΔ	Annual change of MSCI German stock index.	MSCI	2.54	28.74
Policy Instrument	EONIA	Euro-Area average of unsecured overnight interbank borrowing.	ECB	3.19	0.95
Liquidity Risk	Bid-Askσ	Annual historical volatility of the relative bid-ask spread of the most recent issued 10Y German government bond.	Bloomberg	0.96	0.31

All variables are expressed in percentages. The variables representing annual change, denoted with a Δ, are calculated at time t as $(I_t/I_{t-12}) - 1$ where I_t is the respective index at time t. The Bid-Askσ at time t is calculated as the standard deviation of the 12 latest monthly returns where a monthly return, Bid-AskΔ_m, at time t is calculated as $(\text{Bid-Ask}_t/\text{Bid-Ask}_{t-1}) - 1$. The monthly standard deviation at time t is calculated as $\sqrt{\frac{1}{12}\sum_{i=t-12}^{t}(\text{Bid-Ask}\Delta_{m,i} - \mu)^2}$ where μ is the average of the 12 latest monthly returns. The standard deviation is annualized by multiplying by $\sqrt{12}$.

The German annual zero-coupon bond yields have been retrieved from Bloomberg L.P. (2015). They have been calculated as the average of the stripped bonds traded in the market. As a comparison the yields of three different maturities have been obtained: 5-, 10- and 20-years. The standard deviation is relative small in the sample period.

The Producer Price Index (PPI) is a price index measuring the average sales prices received by domestic producers of goods and services over time. The annual growth of the index measures the price inflation from the perspective of the seller. The HICPΔ, the annual change of the European HICP, was insignificant and therefore the PPIΔ is included. A possible explanation for this is the different focus of both indices. The HICP only constitutes products and goods for consumers and the PPI includes a broader range. This broader range perhaps better reflects the total economy and therefore has a significant relation with the Bund yield. In the sample period two periods of deflation occurred; one from the end of 2001 to the mid of 2002 and one in February 2004. The index is obtained from the ECB (2015) and includes all industries except the construction and the energy sector. These industries are frequently excluded from inflation indices, because these are the most volatile sectors (ECB, 2015). We prefer a stable index, because we are interested in the long-term inflation trend and not in short term jumps.

The model incorporates two variables measuring Real Activity: the annual change in the unemployment rate (UEΔ) and the annual change of the German MSCI stock index (GERΔ). UE measures the number of unemployed Euro-Area citizens as a percentage of the total labour force and is obtained from the ECB (2015). The unemployment rate on average decreased in the sample period, but between mid of 2001 and mid of 2004 it increased. GERΔ is calculated as the annual change of the German MSCI stock index and is obtained from MSCI (2015). The sample period covers the burst of the Dot-com bubble and this is clearly shown by large losses in 2001. The MSCI country stock indices are adjusted from cash dividends and capital repayments, which are reinvested in the index. These adjustments enable the index to show the total return earned by investors. Furthermore, the indices contain all large and mid cap companies. These characteristics make this index preferable over the DAX, the German stock index.

The EONIA is the European overnight index average of overnight unsecured borrowing between banks and is obtained from ECB (2015). As explained in Subsection 3.2.2 the rate strongly depends on the policy rates set by the ECB. The EONIA decreased between 2000 and 2003, and it remained stable until the end of 2005 afterwards it increased.

The bid-ask spread is the difference between the bid and offer prices of a finacial product. Fleming (2003) proposes the relative bid-ask spread as the best measure of market liquidity. The relative bid-ask spread is calculated as: $\frac{P_a - P_b}{P_m}$ where P_a, P_b and P_m are the ask, bid and mid prices of the bond, respectively. The annual historical volatility is calculated as the standard deviation of the twelve latest monthly returns. The figure is annualized by multiplying it with the square root of twelve (Hull, 2015). The volatility of the bid-ask spread remained very stable in the sample period. The bond prices are obtained from Bloomberg L.P. (2015).

4.2.2 Regression

To obtain accurate results from a regression several assumptions must hold concerning the disturbance terms ϵ_t and the independent variables. When all assumptions hold, then the estimators are known as Best Linear Unbiased Estimator (BLUE) and have several desirable properties (Brooks, 2008). Table 4.3 shows the assumptions and the tested results on our data.

Our regression satisfies the first, second and fourth BLUE assumption. However, the third assumption is not satisfied, meaning that the estimators are no longer efficient. The first assumption is satisfied in our analysis, because it includes a constant. The second assumption also holds, because the general test for heteroscedasticity of White (1980) shows that our data is homoscedastic. Error terms are called homoscedastic when they have a constant variance, and are called heteroscedastic otherwise. The fourth assumption is also satisfied, because we

TABLE 4.3: BLUE Assumptions

Assumption	Interpretation	Result
$E(\epsilon_t) = 0$	The errors have zero mean.	$\surd$
$var(\epsilon_t) = \sigma^2 < \infty$	The errors have constant and finite variance.	$\surd$
$cov(\epsilon_i, \epsilon_j) = 0$	The errors are uncorrelated between observations.	$\times$
$cov(\epsilon_t, x_t) = 0$	The errors and independent variables are uncorrelated.	$\surd$

have strong indication that we have no omitted variables problem. We have this indication by checking the impact of 8 control variables (Brooks, 2008). The third assumption is not satisfied for our analysis, because the error term is autocorrelated. Our error term is positively autocorrelated according to the autocorrelation test of Durbin & Watson (1951). The partially satisfied BLUE assumptions induce that the estimation of the standard errors is invalid due to the bias resulting from autocorrelation. However, the estimates of the coefficients do remain unbiased. In the case of positive autocorrelation the estimated variances of the OLS estimators underestimate the true variances and standard errors, and thereby inflate t-values. As a result the F- and t-tests are unreliable. Newey & West (1987) have developed a variance-covariance estimator that is consistent in the presence of both heteroscedasticity and autocorrelation. The estimator uses lagged residuals to evaluate the autocorrelation. By estimating consistent standard errors we realize reliable F- and t-tests. Our applied Newey-West modification uses four lags, a number based on the rule of thumb on our number of 96 observations (Brooks, 2008).

To conduct hypothesis testing we have to assume that the errors terms are standard normally distributed; $\epsilon_t \sim N(0, \sigma^2)$. The normality test developed by Bera & Jarcque (1981) shows that the error terms of our data are not normally distributed. A quantile-quantile plot of the error terms shows two outliers; the observations of January and February 2000. We correct for these outliers by creating a new dummy variable called Outlier Dummy. This variable equals one for the January and February 2000 observations and is zero otherwise. Thereby it knocks out the outliers. After incorporating the dummy variable the Jarcque-Bera test shows that the error term is standard normally distributed. A final implicit assumption is that independent variables are uncorrelated. When this not the case there is multicollinearity. We checked for multicollinearity by calculating the VIF values and the Pearson correlation coefficients (Brooks, 2008). Both measures show no proof of multicollinearity for our data.

Table 4.4 shows the regression results of the 5-, 10- and 20-year Bund yields. The coefficients of the macro-variables differ per maturity. UE is the only insignificant variable for the regression on the 5-year Bund yield, all the other variables are significant. The goodness of fit statistic, the R^2, describes how well a model explains the variations in the dependent variable. The model has a high explanatory power for all three maturities. Consistent with Ang & Piazzesi (2003), the explanatory power decreases with the maturity. When we compare the importance of variables with different maturities, we see that UEΔ has an increasing influence when the maturity is extended. A possible explanation is that change in unemployment has a delayed effect on the economy and therefore has a bigger impact on longer maturity bonds. All the other macro-variables do not significantly change with different maturities.

The last column shows our a priori expected signs of the coefficients. The largest mismatch between expectation and realization is for the PPIΔ variable. We expected a positive PPIΔ, indicating inflation over a year, to have an increasing effect on the Bund yield. However, the actual coefficient states differently. A possible explanation can be that the PPIΔ is backward looking, while the risk-free rate incorporates a forward-looking inflation. Besides PPIΔ, also the UEΔ coefficient for the 5-year Bund is different then expected, however this coefficient is not significantly negative.

TABLE 4.4: Determinants of 10Y Bund Yield from 2000:01 to 2007:12

Variables	5-year Bund	10-year Bund	20-year Bund	A priori sign expected
PPIΔ	-0.0982**	-0.0815**	-0.0656*	+
	(0.0395)	(0.0330)	(0.0345)	
UEΔ	-0.0059	0.0443**	0.0753***	+
	(0.0189)	(0.0204)	(0.0246)	
GERΔ	0.0068***	0.0063***	0.0060**	+
	(0.0024)	(0.0021)	(0.0024)	
EONIA	0.5042***	0.5609***	0.6414***	+
	(0.0913)	(0.1034)	(0.1272)	
Bid-Askσ	-0.0067***	-0.0065***	-0.0058**	-
	(0.0015)	(0.0019)	(0.0022)	
Outlier Dummy	0.0169***	0.0176***	0.0176***	n.a.
	(0.0016)	(0.0018)	(0.0022)	
Constant	0.0350***	0.0341***	0.0360***	n.a.
	(0.0042)	(0.0047)	(0.0057)	
Observations	96	96	96	
F-Statistic	80.77	64.24	55.41	
R^2	0.8488	0.8201	0.7995	

Coefficients significance levels are denoted: *** p<0.01, ** p<0.05, * p<0.1.
Newey-West corrected standard errors are reported in parentheses.

As stated the Macro Model is based on the 10-year Bund and can be defined as below. The model describes the relation of the macro-variables to the risk-free rate from 2000 to 2007. In estimating the risk-free rate in future periods, we assume the relation of the macro-variables to the risk-free rate remains fixed. We do understand that this relation changes over time, but we think this only occurs slowly. Therefore we think that our fixed assumed model provides an adequate estimation of the risk-free rate in the future. We removed the dummy variable in the model, because its only purpose was knocking out two outliers in the sample period.

$$r_{f,t} = 0.0341 - 0.0815\text{PPI}\Delta_t + 0.0443\text{UE}\Delta_t + .0063\text{GER}\Delta_t + 0.5609\text{EONIA}_t - 0.0065\text{Bid-Ask}\sigma_t$$

Chapter 5

Evaluation of Risk-Free Estimators

The pre-selection of the risk-free estimators has resulted in three candidates for evaluation; the Bund, the OIS and the Macro Model. We answer the first two sub-question of the fourth Research Question by formulating evaluation criteria in Section 5.1. Subsequently, we evaluate the estimators on these criteria in Section 5.2. Finally, we conclude in Section 5.3 by describing the results of our analysis and selecting the best risk-free estimator.

5.1 Criteria

We formulate several evaluation criteria, because we acknowledge that no perfect single measure of the risk-free rate exists. We conducted a literature review to requirements of estimators, but this preliminary review did not result in many relevant criteria. Therefore, we refined the search to financial proxies and found four interesting articles. BIS (2013) and Financial Stability Board (2014), both financial regulators, reviewed criteria for financial benchmarks in the aftermath of the LIBOR-scandal. IOSCO (2013), the international regulator for the securities and futures market, set criteria for financial benchmarks for oil prices. Fleming (2003) used several criteria for the evaluation of liquidity measures. Besides a literature review, we used PwC experts as a second source of information for the formulation of evaluation criteria. By interviewing PwC employees we determined specific criteria regarding the use of the risk-free rate for valuations purposes.

These analyses lead to the formulation of three criteria: Availability, Consistency and Intelligibility. Firstly, Availability evaluates the publication frequency of an estimator. The risk-free rate changes over time, so it is important for an estimator to incorporate the latest market developments. Consistency is the second criterion and measures to which extent the estimators are in line with the Market Implied Risk-Free Rate, which we define as the most accurate mid-term risk-free proxy. When estimators are not in line with this estimator, they are affected by other factors and thus less accurate. The last criterion, Intelligibility, is the level to which estimators are linked to risk-free rate theory. The criterion tests the link between theory and practice. Scholars also proposed Reliability and Robustness as indicators. The former measures the resistance of an estimator to market manipulation and obviously has received much attention in the LIBOR reform studies. However, we are confident that manipulation risk of our estimators is negligible, due to the fact that all estimators are either transaction based or are determined by governmental institutions. A transaction based variables is based on actual market transactions, which are very difficult to manipulate for large markets. Furthermore, we discarded Robustness as criterion. This criterion measures the stability of the estimators in periods of financial stress. We discarded this criterion, because we assume that our risk-free estimators function adequately during periods of market stress. We based this assumption on the fact that all our risk-free estimators are considered to be safe-havens and thus have remained liquid in historical periods of market stress.

To realize a consistent evaluation, we structure the evaluation via the Analytic Hierarchy Process (AHP) of Saaty (1980). AHP assigns weights to the criteria by pair-wise evaluation. When one criterion is preferred, a score ranging from 1 to 9 is assigned. Otherwise, a score ranging from $\frac{1}{3}$ to $\frac{1}{9}$ is assigned. Figure 5.1 shows the AHP scale (Saaty, 1980).

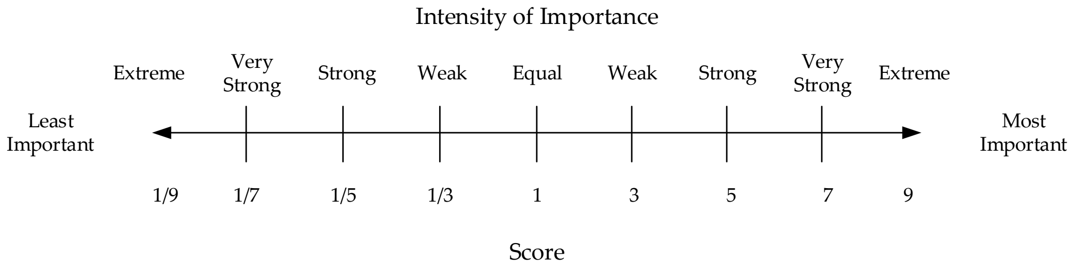

FIGURE 5.1: AHP Scale of Intensity of Importance

We determined the pairwise evaluation of the criteria after interviews with PwC employees and show it in Table 5.1. Consistency is chosen as the most important criterion, because accurate pricing of the risk-free rate is seen as the boundary condition of an estimator. The second ranked criterion is Intelligibility, which is due to its emphasis on the theory regarded to be less important than Consistency. However, we do value its importance, because an intelligible estimator creates market trust. We ranked Availability as the lowest priority, because we assume that the drivers of the risk-free rate are rather stable over time.

TABLE 5.1: Pairwise Comparison Matrix of Evaluation Criteria

Criteria	Availability	Consistency	Intelligibility	Priorities
Availability	1	$\frac{1}{5}$	$\frac{1}{4}$	0.10
Consistency	5	1	2	0.57
Intelligibility	4	$\frac{1}{2}$	1	0.33

Consistency Ratio = 0.01 < 0.10

The last column shows the priority given to a criterion. These are calculated by firstly normalizing the relative weights and subsequently averaging the rows. The normalization is done by dividing each entry by the sum of its column. The sum of the priorities is by definition equal to 1 and the relative weights can be interpreted as a ratio scale (Saaty, 1980). The pairwise comparisons are consistent according to the consistency ratio shown below Table 5.1. This ratio developed by Saaty (1980) is a measure of the consistency of the judgments. Ten percent is proposed by him as upper limit of an acceptable consistency level. We omitted the computation of the consistency ratio and all the full pairwise comparison matrices for the criteria for brevity, but we show them in Appendix D. Synthesizing, Figure 5.2 shows the three evaluation criteria with their corresponding priority. Furthermore, it shows whether we evaluate the estimator quantitatively or qualitatively.

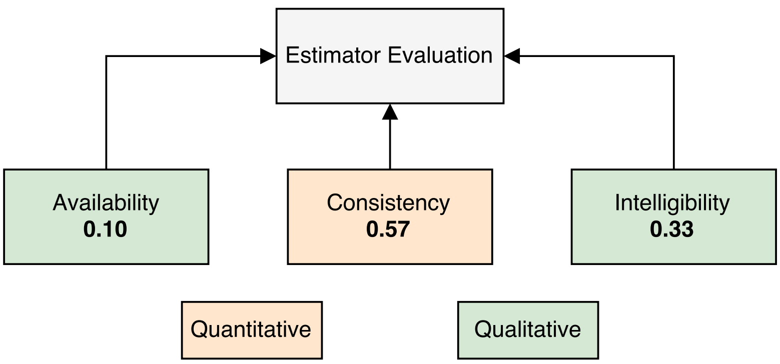

FIGURE 5.2: Criteria for Estimator Evaluation with Weights

5.2 Evaluation of Criteria

We evaluate the three risk-free estimators from 2008 until 2015, because Ejsing *et al.* (2015) have shown that from 2008 credit risk has significantly impacted the Bund yields. Figure 5.3 shows the development of the three estimators. The Bund and OIS co-move through the complete sample period and the Macro Model diverges between May 2011 and May 2015. The drop of the Macro Model in May 2015 is caused by a sharp increase of the historical volatility of the relative bid-ask spreads. The divergence between the three estimators from 2013 on it self does not solely imply that one path is better than another.

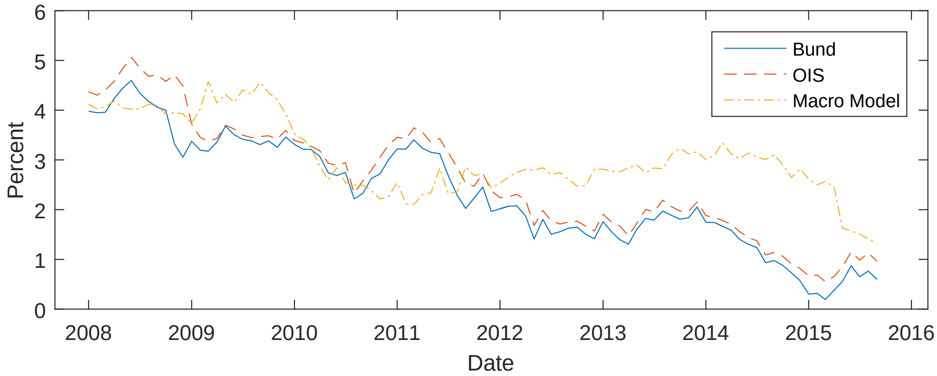

FIGURE 5.3: Development of Risk-Free Estimators

5.2.1 Consistency

We define Consistency as the accuracy at which the estimators resemble the Market Implied Risk-Free Rate for the 5-year maturity. The Market Implied Risk-Free Rate is a risk-free estimator and we discarded it as risk-free proxy for valuation purposes, because it is not available for longer maturities. However, for the five year maturity the Market Implied Risk-Free Rate is conceptually superior to the Bund and OIS as risk-free proxy, because it has a low liquidity and no credit risk exposure. Furthermore, the comparison to the 5-year maturity is a good indication for the 10-year maturity performance, because these maturities of the risk-free estimators are highly comparable, confirmed by the high correlation coefficient: 0.98.

Figure 5.4 shows the difference between the risk-free estimators and the Market Implied Risk-Free Rate. The Macro Model deviates the most, especially from 2012. The Bund and OIS perform better and only deviate during the financial crisis from 2007 to 2009 from the Market Implied Risk-Free Rate. Their largest spread occurred in October 2008 just after Lehman Brothers filed for bankruptcy (Brittanica, 2016). The average deviation of the Bund and OIS during the financial crisis is nine times larger than the average deviation in the period afterwards. A possible explanation is a flight-to-liquidity, in which investors fled from illiquid corporate bonds to liquid assets like the Bund and OIS. This flight was not driven by credit risk, because the CDS spreads of corporates did not increase in the same proportion. We exclude this period from the evaluation, because liquidity disturbs the analysis. Another interesting observation is that the Bund and OIS spreads are positive during the complete sample, except between May 2012 and September 2013. This was the period of the European sovereign debt crisis, in which in May 2012 the first rumors of a 'Grexit' originated and in which in September 2013 the Euro-Area officially emerged from a recession (Brittanica, 2016). A possible explanation for the negative spread can be a flight from the Bund and OIS to corporate bonds. This can be regarded as a flight-to-quality with an opposite direction than usual, where investors favored corporate bonds over the Bund and OIS proportionally more than their difference in credit risk. Consistent with this explanation is that in March 2012, the first month in which the Market Implied Risk-Free Rate was smaller than the estimators, the average CDS spread increased and the average bond yield decreased. This is an counterintuitive observation and can explained by the flight-to-quality from risk-free proxies to corporate bonds.

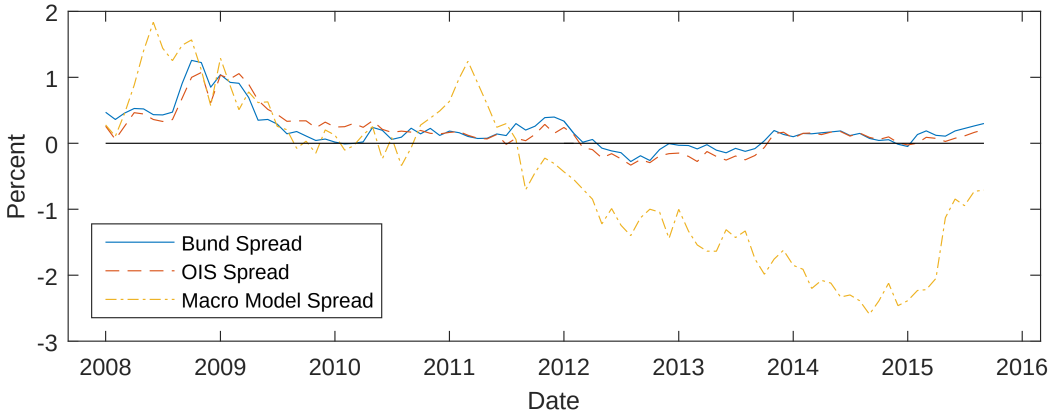

FIGURE 5.4: Development of Estimators to Implied Risk-Free Rate Spreads

To formally evaluate the accuracy, we analyze the error of the estimators. Below we defined the error as the difference between the estimators and the average Market Implied Risk-Free Rate. Where y represents the observation of the Market Implied Risk-Free Rate and $\hat{y}_i$ represents the prediction of risk-free estimator i at time t. We are interested in positive and negative deviations of the predictor and therefore analyze the squared and the absolute error. Furthermore, we define accuracy as a predictor that over time results in the lowest error, so we analyze the average. This provides us with two forecast error measures: the root mean squared error (RMSE) and the mean absolute error (MAE). The RMSE is the root of the average of the squared errors. Taking the root ensures that the RMSE has the same units as the quantity being estimated. The MAE is the average of the absolute errors. Both measures are scale-dependent and are negatively oriented scores, meaning the smaller the score the better performance. The difference between the measures is the weighting of outliers, which are defined as errors larger than one percentage point. The RMSE assigns more weights to outliers due to the squaring of the errors before averaging them. Evaluating the estimators on both criteria provides us with a robust indication of the accuracy.

$$\epsilon_{i,t} = y_t - \hat{y}_{i,t} \qquad \text{RMSE}_i = \sqrt{\frac{\sum_{t=1}^{n}(\epsilon_{i,t})^2}{n}} \qquad \text{MAE}_i = \frac{\sum_{t=1}^{n}|\epsilon_{i,t}|}{n}$$

Table 5.2 shows the RMSE and the MAE from 2009:07 to 2015:09. Both accuracy measures indicate that the Macro Model performs worse than the other estimators. The RMSE of the other estimators is rather similar and the MAE of the Bund is lower than that of OIS. The difference between the accuracy measures, indicates that the Bund has relative more extreme errors than OIS. However, we still can conclude that the Bund is the most accurate estimators of the Market Implied Risk-Free Rate. The corresponding relative priorities are shown in the last column.

TABLE 5.2: Consistency of Risk-Free Estimators

	RMSE	MAE	Priorities
Bund	0.165	0.137	0.65
OIS	0.193	0.170	0.28
Macro Model	1.325	1.072	0.07

5.2.2 Intelligibility

The criterion Intelligibility measures to which extent estimators are in line with the theoretical concept of the risk-free rate. We asses the criterion based on two subcriteria; Financial Risks and Physical Asset. The first evaluates the proxies on the exposure to the financial risks defined in our definition of a risk-free proxy. The latter subcriterion evaluates the link of an estimator to a physical asset. We think that a strong link of an estimator to a physical asset enhances its understandability and transparency. Deviations in the estimator can then easily be traced back to physical changes in the market. Table 5.3 shows the scores of the three estimators on all criteria. The Bund and the OIS have a similar total score. The OIS performs slightly better on the Financial Risk subcriterion, but the Bund on the Physical Asset subcriterion. The Macro Model scores the worst, because it has no underlying physical asset.

TABLE 5.3: Intelligibility of Risk-Free Estimators

Criteria	Bund		OIS		Macro Model	
Credit Risk	◕	Low Credit Risk	●	Virtually absent		n.a.
Reinvestment Risk	●	Absent	●	Absent		n.a.
Prepayment Risk	●	Absent	●	Absent		n.a.
Sufficient Liquidity	◑	Very Liquid market	◕	Liquid market		n.a.
Financial Risks	◕	Good	●	Excellent		n.a.
Physical Asset	●	German Government bonds	◑	Interest rate swaps	○	No link
Intelligibility	◕	**Good**	◕	**Good**	○	**Very Poor**

The estimators are evaluated on the criteria on a 5-point scale with harvey balls, where ● represents excellent, ◕ good, ◑ satisfactory, ◔ poor and ○ very poor. Financial risks is an aggregate score of the separate financial risks. Intelligibility is the aggregate score on the criteria Financial Risks and Physical Asset.

Financial Risks

The Bund and OIS are proxies and are thus based on an underlying asset. Therefore we only compare compare asset characteristics of these estimators with our definition of a risk-free proxy. The Macro Model is based on five underlying macro-economic variables and thus is not based on a physical asset. Due to the absence of the link with a physical asset it is impossible to evaluate the Macro Model on the exposure to financial risks.

Firstly, the credit risk of the Bund and the OIS are both very low, but we regard the Bund to have a higher credit risk. Indications of the credit risk of the Bund are credit ratings and CDS spreads. The former is the highest for the Bund, but the latter shows a certain credit risk exposure. The average 5-Year CDS spread on Bunds was 36.8 basis points from 2008 until 2015 indicating an annual default probability of .6%, when we assume an recovery rate of 40% (CapitalIQ, 2016). Therefore, we conclude that the Bund has a low credit risk exposure. The OIS has virtually no credit risk exposure. It has the same credit quality as an continually refreshed overnight loan between European banks, which are assumed to be virtually free of credit risk (Ejsing *et al.*, 2015). Secondly, both the Bund and the OIS are unexposed to reinvestment risk. This only holds for zero-coupon variants of both financial assets, because these only include one payment at maturity. Thirdly, the Bund and OIS are both unexposed to prepayment risk, because both assets are non-callable and thus have a specified maturity. Finally, all liquidity indicators show that the both markets are highly liquid in comparison to corporate bonds (CapitalIQ, 2016). However, ECB (2014) showed that the market for the Bunds is more liquid than that of the OIS, especially for longer maturities. Concluding, we regard the OIS market to be more liquid than minimal necessary, because it has more liquidity than the corporate bonds market. Furthermore, we regard the Bund market to be much more liquid than necessary.

Physical Asset

This subcriterion evaluates the link of the estimators to a physical asset. Firstly, the Bund has a strong link to a physical asset which is understandable for market participants. The proxy is based on outstanding zero-coupon bonds in the market. Furthermore, its yield curve is created by interpolating between the trading bonds and so closely resembles actual traded bonds.

Secondly, OIS are linked to an interest rate swap making it less understandable for market participants due to the complexity of the instrument. The interest-rate swap itself is based on EONIA, which is an asset with a maturity of one day. The OIS for long maturities is thus indirectly linked to physical asset, because it is a financial derivative based on EONIA. Thirdly, the Macro Model is not linked to a physical asset making its understandability low. Instead the model is based on macro-economic variables. Taylor (1993) has theoretically described the effect of macro-variables on the risk-free rate. However, the absence of a direct link to a physical asset leads to the lack of economic interpretation of the estimator.

5.2.3 Availability

We define Availability as the publication frequency of an estimator. The risk-free rate is the market's required return on a risk-free asset and is composed of the required real return and the market's expected inflation. These components change over time due to changing investor preferences and macro-economic conditions. Market participants require a risk-free rate estimator incorporating current market conditions, therefore a daily available risk-free estimator is important.

The Bund and OIS trade on financial markets and therefore market participants can obtain quotes at all trading days. The Macro Model is per definition only calculated monthly, because it is based on monthly macro-economic data. Additionally, this data is published with approximately one month lag by the ECB, so the rate is already outdated when it is published. Therefore, we conclude that the Bund and OIS have a better Availability.

5.3 Results

We calculated the total priorities by taking the weighted sum of all criteria's priorities for the estimators and show them in Table 5.4. We conclude that the Bund is the best estimator for the risk-free rate for valuation purposes. The Bund is approximately twice as good as OIS and eight times as good as the Macro Model. The main reason for its high score is that the Bund scores is at least as good or better than all other estimators on all criteria. The Bund outperforms the other two estimators on Consistency and scores as good as OIS on Intelligibility and Availability.

TABLE 5.4: Overall Priorities of Risk-Free Estimators

Criteria	Availability	Consistency	Intelligibility	Overall Priority
Weights	0.10	0.57	0.33	
Bund	0.45	0.65	0.47	0.57
OIS	0.45	0.28	0.47	0.36
Macro Model	0.09	0.07	0.05	0.07

Consistency Ratio = 0.03 < 0.10

Aspects of the analysis are judgment-based and therefore it is of added value to verify the robustness of the results. The scores of the criteria Availability and Intelligibility are judgment-based. However, we are confident that this is supported by an adequate literature analysis. Furthermore, the criteria weights are judgment-based. For this component we think it is good to assess the robustness of the model by conducting a sensitivity analysis. However, it is unnecessary with the current scores, because no other estimator outperforms the Bund at a criterion. Therefore, we conclude that the Bund is robustly the best risk-free estimator.

Chapter 6

Improvements to the German Government Bond

In the preceding Chapter we showed that the Bund is the best available risk-free estimator for valuation purposes, however it still has some deficiencies. In this Chapter, we answer the last sub-question of the fourth Research Question by analyzing the causes and sizes of potential deficiencies and propose improvements. Figure 6.1 shows the potential deficiencies of the Bund as a risk-free estimator. We split the deficiencies into conceptual and market ones. Conceptual deficiencies are risk exposures of the Bund and deviate it from a theoretical risk-free asset. As described in Subsection 5.2.2 the Bund is exposed to credit risk and although it is uncontrollable, we aim to quantify its impact and propose improvements in Section 6.1. Besides the conceptual deficiencies, we distinguish two deficiencies induced by the market. Firstly, asset flights significantly decrease Bund yields during market stress periods. We analyze the causes of asset flights and describe whether they deviate the Bund yields from the risk-free rate in Section 6.2. Furthermore, we expect QE programs to distort the capital markets. In this program central banks purchase at large volume government bonds to increase inflation. Considering the size of the program, we assume it deviates Bund yields from the risk-free rate. In Section 6.3 we firstly describe the program and then analyze the impact of QE. Finally, we conclude this Chapter in Section 6.4 by reviewing the proposed adjustments to the Bund.

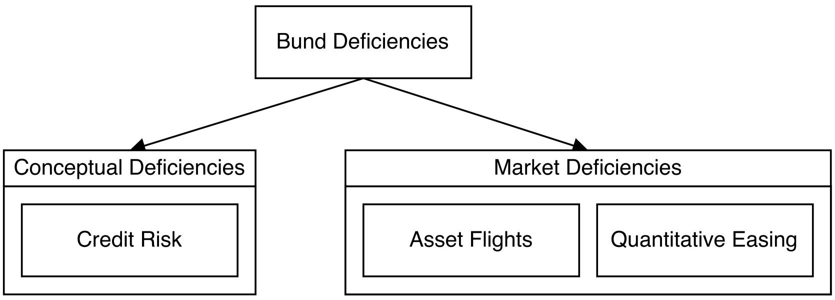

FIGURE 6.1: Bund's Potential Deficiencies as Risk-Free Proxy

6.1 Credit Risk

CDS spreads indicate that Bunds are exposed to credit risk; the average 10-year CDS spread was 37 basis points in 2015 (CapitalIQ, 2016). Theoretically we can combine Bunds with a CDS to obtain a risk-free portfolio in the same manner as we have constructed the Market Implied Risk-Free Rate portfolio. However, Arora *et al.* (2012) showed that CDS spreads for large governments are not accurately priced due to counterparty risk. Counterparty risk is the risk of financial losses due to the default of the insurance issuer. Counterparty risk is highly correlated with defaults of systemically important governments like Germany. Thereby this counterparty risk decreases the expected payoff of a CDS and thus results into a lower spread. Despite this underestimation, we propose to subtract the CDS spread from the Bund yields, because it still partially removes the credit risk of Germany.

6.2 Asset Flights

Asset flights occur in times of market stress when investors flee to safe assets. Flights-to-quality and -liquidity describe investor's preference for creditworthy and liquid assets, respectively. A well-known example of an asset flight was after the default of Russia in August 1998, leading to the default of the hedge fund Long-Term Capital Management (Hull, 2015). In this Section we firstly describe the causes of asset flights, subsequently show their occurrences and impact, and finally propose adjustments to the Bund.

Asset flights are caused by two reasons; increased risk premia and risk aversion. Firstly, risk premia increase during times of market stress. Credit risk is increased, due to the worsened economic circumstances, and liquidity risk is increased, because of the larger probability of the need to liquidate a portfolio. Secondly, risk aversion of investors increases due to the increased uncertainty (Vayanos, 2004). Usually flights-to-quality and -liquidity coincide in market stress periods, related by Beber *et al.* (2009) to the generally positive correlation between the creditworthiness and liquidity of assets. Furthermore, they showed that credit quality is the main determinant in bond spreads, but that in times of market stress participants prefer liquidity. A consequence from the increased importance of short-term transaction costs in respect to long-term default costs, when investors defensively rebalance their portfolio. Vayanos (2004) showed this especially holds for fund managers with the need to liquidate their portfolio below a certain threshold. Furthermore, Hull (2015) reasons this explanation also holds for leveraged hedge funds needing to meet collateral obligations, e.g., Long-Term Capital Management. Our definition of a risk-free asset does exclude credit risk, but does not exclude liquidity risk. Therefore flights-to-quality have a similar impact on the Bund and the risk-free rate, because investors flee to credit-risk free assets. However, flights-to-liquidity distort the Bund from the risk-free rate, because investors flee to liquidity which is not a characteristic of a risk-free asset.

Longstaff (2004) proposed to measure the impact of flights-to-liquidity by looking to the agency to government bond spread. He showed that the spread between bonds issued by the central government and government agencies is completely attributed to differences in perceived liquidity. Agency bonds are guaranteed by the central government and therefore have the exactly same credit quality as government bonds. Therefore, he concluded that the agency to government bond spread shows the flight-to-liquidity premia in bond prices. We use the bonds of the development bank Kreditanstalt für Wiederaufbau (KfW), the largest German agency, to quantify flights-to-liquidity in the German bond market. KfW bonds are explicitly guaranteed by the German government, meaning that they bear the same credit risk as German government bonds. Figure 6.2 shows the 10-year KfW-Bund spreads from April 2011 to October 2015 obtained from Bloomberg L.P. (2015). We computed the spread by subtracting zero-coupon Bunds from zero-coupon KfW bonds. We only show this limited period, because before April 2011 no constant maturity KfW bonds were available on Bloomberg. As expected the spread is the largest at the end of 2011, a market stress period due to the European sovereign debt crisis (Brittanica, 2016). After this period the flight-to-liquidity premia have decreased, and it even become slightly negative in the second half of 2015.

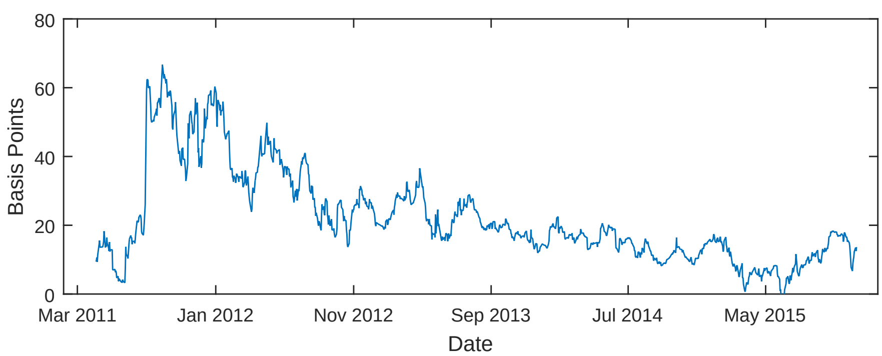

FIGURE 6.2: Development of KfW to Bund Spread

We showed that flights-to-liquidity significantly decrease the Bund yields and thereby distort the Bund from the risk-free rate. A solution for this distortion requires a liquidity anchor point, from which we measure deviation caused by liquidity. The difficulty in deciding on such anchor point is that no asset has a 'fixed' liquidity level and that all other asset characteristics should be the same. We could use the KfW as liquidity anchor point, however this asset could also be influenced by flights-to-liquidity and potentially underestimate the true impact. Nevertheless, using KfW bonds instead of Bunds still eliminates partially liquidity premia. Therefore, we propose to use KfW bonds to improve the risk-free estimator.

6.3 Quantitative Easing

QE is a form of monetary policy implemented by central banks to fight deflation by purchasing assets on a large scale. We expect these programs distort capital markets and decrease Bund yields because of their large size. This expectation is confirmed by studies of Gagnon *et al.* (2011) and Joyce *et al.* (2011) to the impact of the US and UK QE programs, respectively. Furthermore, Neely (2011) has showed that the US QE program also globally decreased government bond yields, implying the spill-over effects of the national programs. Therefore, we estimate the total impact of all QE programs on the 10-year Bund yield. Firstly, we describe the concept of QE in Subsection 6.3.1. Subsequently, we estimate the impact of the ECB QE program in Subsection 6.3.2 and the impact of the foreign programs in Subsection 6.3.3. Finally, we propose adjustments for the capital market distortion as a result of the QE programs in Subsection 6.3.4.

6.3.1 Concept of Quantitative Easing

The goal of most central banks is to achieve price stability and they have quantified this in a 2% inflation target. To achieve this goal, central banks implement monetary operations. In normal times they steer inflation by influencing the short-term interest rates through open market transactions of short-term assets. In conventional monetary policy these transactions are a means to achieve a desired interest rate. However, when the interest rate is at or near zero, also known as the 'Zero-Lower-Bound', central banks can no longer decrease the short-term interest rate and have to resort to unconventional monetary measures. QE is such an unconventional monetary policy. It is an asset purchase program transforming market transactions of central banks into a goal rather than a means. The Bank of Japan (BoJ) firstly implemented QE in the 1990s to fight deflationary pressures after the real estate bubble. Subsequently, the Federal Reserve (FED), the Bank of England (BoE) and the ECB have adopted similar policies, although differing in size and targeted securities (Joyce *et al.*, 2012).

The ECB implemented four different asset purchase programs of which three are still currently active; the Covered Bond Program, the Asset-Backed Securities Purchase Program and the Public Securities Purchase Program (PSPP). Combined these programs are called the Expanded Asset Purchase Program (EAPP). Although all these programs can be classified as QE, the media only used this term for PSPP. This is due to its relatively large size and its purchases of government bonds (Financial Times, 2015b). We mean, consistently with the media, only PSPP when referring to ECB's QE. Furthermore, we only analyze this program, because PSPP is the only ECB program directly impacting Bund yields. We explain the concept of PSPP and its desired purpose more elaborately in Appendix E. Besides the ECB, three others central banks also implemented QE. We show the main characteristics of the programs in Table 6.1 (FED, 2016; BoE, 2016; BoJ, 2016; ECB, 2016). Although the programs differ in size, they all share the goal to increase inflation. We computed the size of the programs by only including the components purchasing government bonds. Furthermore, we only reviewed QE programs after the financial crisis, and thus discarded the BoJ's QE programs from the 1990s. We have used this threshold, because the current impact of older programs on Bund yields is highly uncertain. Table 6.1 shows that the FED and the BoE implemented their programs much earlier than the other two central banks. Furthermore, the size of the programs differs significantly. The BoJ program is by far the largest, approximately ten times the size of the ECB program.

TABLE 6.1: Overview of International QE Programs

Central Bank	Start Date	End Date	Description	Size
FED	November 2008	October 2014	Occurred in three stages, QE1 in March 2009, QE2 in March 2010 and QE3 in September 2012.	€2,120 bn.
BoE	March 2009	June 2012	Incrementally increased at quarterly BoE meetings.	€468 bn.
BoJ	April 2013	Open-Ended	Firstly introduced in April 2013, expanded in size October 2014.	€18,185 bn.
ECB	April 2014	March 2017	The program is prolonged in March 2015 and expanded in size in March 2016.	€1,544 bn.

The size of the programs are calculated by summing all announced agency and government bonds purchases. We based the BoJ program size until April 2016, because it is open-ended. All size estimates are exchanged to Euros at the spot rate of 6/4/2016 (CapitalIQ, 2016)

Gagnon *et al.* (2011) and Joyce *et al.* (2011) analyzed the impact of the QE programs in the US and the UK. They used two approaches to quantify the impact: an event-study and a portfolio balance model. The former analyzes changes in interest rates around official events of the programs and assumes the impact is the cumulative change. It is based on the Efficient Market Hypothesis assuming that asset prices immediately reflect new publicly information. Thereby asset prices adjust to the news about a transaction, instead of the actual transaction itself. The second approach uses time-series analyses to estimate the impact of a decreased supply of government bonds. It assumes that changes in asset supply affect the required return of investors. We conduct an event study to quantify the QE impact on Bund yields, because it requires less assumptions than the other one, e.g., Gagnon *et al.* (2011) and Joyce *et al.* (2011) already use three different time-series analyses for their portfolio balance model.

6.3.2 Impact of ECB's Quantitative Easing Program

Event studies are used in economics to measure the impact of specific events. Generally the quantify the impact by subtracting the actual change with the expected change, which is based on an time window prior to the event (MacKinlay, 1997). However, we look directly to the actual change, consistent with Gagnon *et al.* (2011) and Joyce *et al.* (2011). Thereby they implicitly assume no expected change, because of the mean reverting characteristics of interest rates. In our event study we examine changes of Bund yields around official PSPP communications. In quantifying QE's impact with this approach, we make the following assumptions: (i) markets are efficient, (ii) our event set includes all announcements affecting the expectations about PSPP, (iii) time windows are wide enough to capture the full impact of the announcement, and (iv) non-PSPP news on the event days has a negligible impact on Bund yields.

Table 6.2 shows the selected event set. We constructed this set after reviewing publicly available data of the ECB (2016). All events, except one, are press conferences after the ECB's General Council meetings. During these meetings the ECB decides on monetary policies. The event on 2 July 2015 was an update on the ECB's website and only a minor addition to PSPP, so a separate General Council Meeting was not needed. We excluded unofficial announcements, because firstly it is hard to find all announcements speculating about PSPP and secondly during these probably also non-PSPP information is stated. The first two selected events occurred before the official announcement of PSPP. However, these events strongly indicated the likeliness of PSPP and we therefore included them in the event set. On 22 January 2015, the ECB officially announced PSPP. The sequential two announcements were minor additions to the program. On 3 December 2015, the ECB prolonged PSPP and announced further additions. In the one but last event the ECB stated "that there are no limits in its mandate", strongly indicating a near-future increase of the program. The last announcement was on 10 March 2016 and is a large expansion of the EAPP program. This extension is regarded as the 'last toolkit' for the ECB.

TABLE 6.2: PPSP Announcements

Date	Event Type	PSPP Actions	Other Actions
3/4/2014	G.C. Meeting	Stated that the Governing Council is unanimous in its commitment to using unconventional measures including QE.	
7/8/2014	G.C. Meeting	Indicated that QE including government bonds is still on the table.	Preparations for Asset-Backed Securities purchases
22/1/2015	G.C. Meeting	Announced PSPP as part of the EAPP.	
2/7/2015	Website Update	Expanded the eligible public securities for PSPP.	
3/9/2015	G.C. Meeting	Increased the issue share limit from 25 to 33%.	
3/12/2015	G.C. Meeting	Extended the EAPP; (i) prolonged program until March 2017, (ii) reinvest the principal payments on the securities purchased under the EAPP to stimulate liquidity, and (iii) make debt issued by regional and local governments eligible for PSPP.	Lowered interest rates.
21/1/2016	G.C. Meeting	Indicated that "there are no limits in the ECB's mandate".	
9/3/2016	G.C. Meeting	Expanded the EAPP; (i) increased monthly purchases to €80 billion, (ii) increased issue share limit to 50%, (iii) included non-bank corporate bonds eligible for EAPP.	Lowered interest rates, and launched TLTROII

The determination of the window size is crucial to an event study. Too short and the full impact of an event is not included, too long and other factors can disturb the results. Due to the novelty of the QE programs Gagnon *et al.* (2011) and Joyce *et al.* (2011) assume that the market at least needed one day to incorporate the effect of the announcement. Therefore, we computed the cumulative impact for one-, two- and three-day time-windows in Figure 6.3 CapitalIQ (2016). We show the most important maturities eligible for PSPP to analyze possible different impacts along the term structure. PSPP's impact estimates are very sensitive to changes in the windows size. Surprisingly, the PSPP's impact using a one-day window is positive. A possible explanation for this fact is that the market did not fully incorporate PSPP's impact. A positive impact is inconsistent with our expectations and other research, and therefore we discard the option for a one-day time window. Using a longer window size results in a larger negative impact, a finding consistent with Gagnon *et al.* (2011) and Joyce *et al.* (2011). Furthermore, it inevitably increases the disturbance of non-PSPP announcements, and therefore we decide to use the two-day time window for our analysis. Independently from the time window size, the PSPP's negative impact increases for longer maturities for almost all cases. The large relative change of longer-term yields in comparison to short-term yields, suggests that the announcements reduced longer-term rates by reducing the term premium, as opposed to signaling a commitment to keep policy rates low for an extended period of time. The latter would decrease the shorter-term rates proportionally with the the longer-term rates.

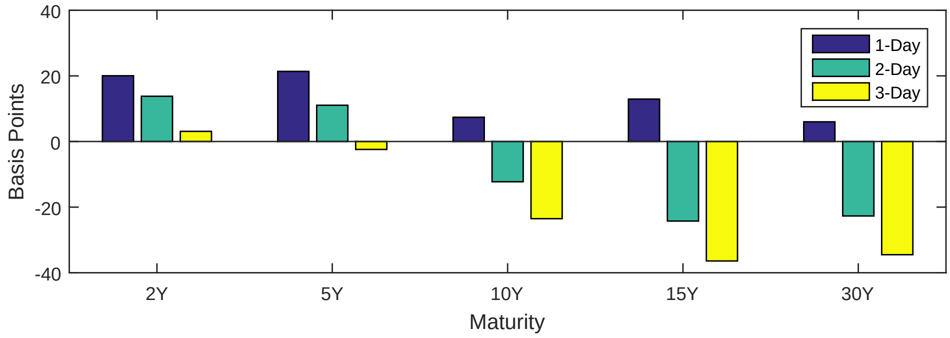

FIGURE 6.3: PSPP's Cumulative Impact on Bund Yields for Three Time Windows

Figure 6.4 shows the impact of each event on the 10-year Bund yield. We only show this maturity, because it is mostly used for business valuations. A complete overview of the impact on each maturity is shown in Appendix E. Our analysis shows a cumulative negative impact of 12.3 basis points on the 10-year Bund yield. The direction of the impact is consistent with our expectation and foreign QE impact studies. When we look at specific events, the positive impact on 3 December 2015 stands out. This can be explained by the market expecting larger stimulating measures from the ECB (Het Financieele Dagblad, 2015). Also the sign of the effect of the last event differs from expectations. In this event the EAPP is largely extended, so we expected a negative impact. The positive impact can be explained by the announcement of the ECB that it does not anticipate to increase the simulating monetary policies in the future (ECB, 2016). Thus the market could interpreted this extension as the last toolkit of the ECB, inducing increased bond yields. Furthermore, the announcement of the program, on 22 January 2015, and its first extension, on 3 September 2015, had as expected the largest negative impact.

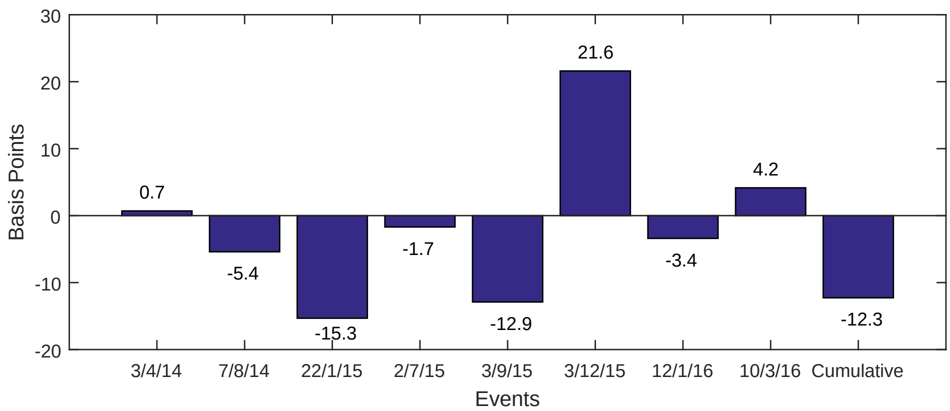

FIGURE 6.4: PSPP's Impact on the 10Y Bund yield

6.3.3 Impact of Foreign Quantitative Easing Programs

We expect that the impact of a foreign program is the product of its size and its bond market integration with Germany. We think the latter factor is important, because it shows the willingness to substitute national with German debt of investors. We use the correlation of the 10-year Bund returns to the 10-year foreign bond returns as proxy for the foreign bond market integration with Germany. We compute the correlation between the returns from November 2008, the start of the FED program, until March 2016. Table 6.3 shows our expected relative impact of the programs. All bond returns are positively correlated as expected, because the economies of the four countries are heavily integrated. However, the correlation differs tremendously. The UK bond returns are the most correlated as expected, due to the large economic integration between the UK and Germany. On the other side the Japanse bond returns are only slightly correlated with the German bond returns, implying a not highly integrated bond market. Based on our hypothesized relation of QE size and bond market integration, we expect the ECB program to have the largest impact, and the BoE program to have the smallest.

TABLE 6.3: Expected Relative Impact of QE Programs

Central Bank	Size	Correlation	Expected Order
FED	€2,120 bn.	0.182	3
BoE	€468 bn.	0.499	4
BoJ	€18,185 bn.	0.045	2
ECB	€1,544 bn.	1	1

Government Bond yields are obtained from CapitalIQ (2016)

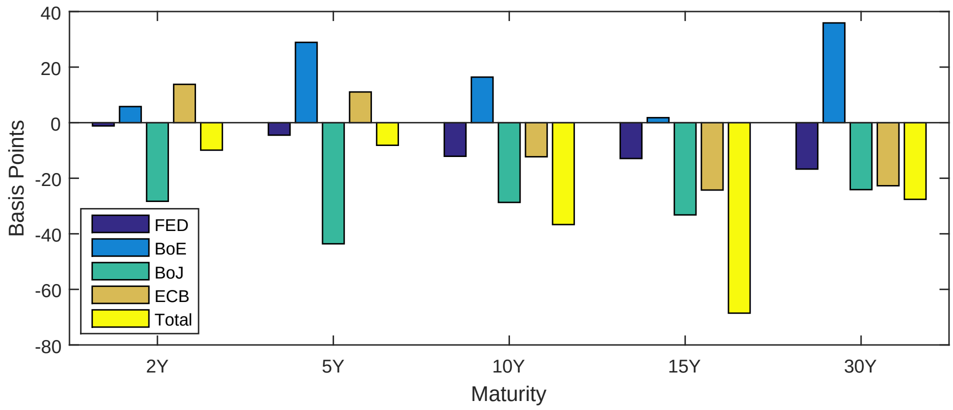

FIGURE 6.5: QE's Cumulative Impact on the Bund Yield

To compare the impact of the QE programs, we compared the Bund yield change relatively to the yield at the announcement of the program. We think this measure is more informative than the absolute impact, because of the importance of the prevailing Bund yield level. Table 6.4 shows that the BoJ program has the largest negative relative impact. This is not in line with our expectation, but the underestimation of Japan's bond market integration with Germany can be a possible explanation. The other findings are in line with our expectation. On an absolute level the FED program has the largest impact, but this can be explained by the relatively high Bund yields at the announcement of the program. As stated earlier, we expect our BoE impact underestimates the true impact, because the market could anticipate program increases.

TABLE 6.4: Impact of QE Programs

Central Bank	Impact	Relative Impact	Actual Order	Expected Order
FED	-15.1	-5.5%	3	3
BoE	16.4	5.2%	4	4
BoJ	-12.1	-9.4%	1	2
ECB	-12.3	-7.6%	2	1

Impact is denominated in basis points, and relative impact in percentages. Government Bond yields are obtained from CapitalIQ (2016)

6.3.4 Adjustment for Quantitative Easing

Before we adjust for the impact of the QE programs on the Bund yields, we firstly review to which extent our event study assumptions have held. Firstly, we assumed Efficient Markets where asset prices immediately reflect publicly available information. Our use of two-day window is not in line with this assumption. However, considering the novelty of the program and a similar effect in the US and the UK, we think it is reasonable to assume that the market needed some time to react to the news. Therefore, we think the assumption still holds when using two-day windows. Secondly, we assumed that our event set included all events affecting QE expectations. We think we incorporated all official announcements, however we have the impression that we did not include all rumors. This is indicated by the unexpected positive impact of the BoE program and several separate events of other programs. We can only explain the positive impact by the market's anticipation of even larger stimulating measures. Due to these missed rumors we potentially underestimated the QE programs' impact. Therefore, we conclude that this assumption does not hold. Thirdly, we assumed that our event windows were wide enough to capture the QE's full impact. We think this assumption holds, because we used relatively long time windows and showed that the direction of the impact was the same for longer time windows. Finally, we assumed that non-QE events had a minor effect on the Bund prices. We think this assumption also holds, because the events did not incorporated major non-QE announcements. Concluding, we showed that QE negatively impacted Bund yields. However, we have strong indications that our estimate underestimates QE's impact, based on the unexpected positive impact of some announcements. Considering this likely underestimation, we think its conceptually better to not adjust the Bund yields for QE.

6.4 Proposed Improvements to German Government Bond

Initially, we expected three factors to be a deficiency of the Bund as risk-free estimator. Firstly, we showed that the Bund's exposure to credit risk conceptually deviates the proxy from the theoretical concept by adding a risk premium. This risk exposure is partially estimated by CDS, and therefore we propose to subtract CDS spreads from Bund yields. Secondly, we showed that only asset flights where investors chase liquidity are a deficiency. Flights-to-quality where investors flee to safe assets, impact the Bund yields in the same manner as a risk-free rate asset. We propose to use KfW bonds instead of Bunds to adjust for flights-to-liquidity. Finally, we showed that QE decreased Bund yields. However, we were have strong indications that our result underestimates the true impact, because we missed rumors with market expectations. Therefore, we propose not to make any adjustments to the Bund yields for QE. Synthesizing, we propose to adjust the Bund for credit risk and flights-to-liquidity. We eliminate partially the credit risk exposure by subtracting the CDS spread from the Bund. Furthermore, we obtain a smaller liquidity premium by using KfW bonds instead of Bunds. Conceptually, a portfolio of a KfW bond and a German CDS has a closer resemblance to a risk-free asset.

We test formally whether these adjustments are an improvement by analyzing the Consistency of the adjusted estimator with the Market Implied Risk-Free Rate, the same approach as used in Subsection 5.2.1. Figure 6.6 shows the difference between the Market Implied Risk-Free Rate to the Bund and the KfW-CDS portfolio. The period starts only at 2011, because the KfW constant maturity bonds are unavailable before this period. The Bund and the KfW-CDS portfolio show a similar deviation throughout the sample period, only before 2012 the KfW-CDS portfolio shows a larger deviation. We classified this period as a flight-to-liquidity period in Section 6.2. When we assume that the KfW-CDS portfolio is completely credit risk-free, this deviation can be entirely attributed to a higher liquidity of KfW bonds. Thus flights-to-liquidity also decrease KfW bond yields and thus the KfW to Bund spread underestimates the total flight-to-liquidity premium.

Table 6.5 shows the accuracy measures RMSE and MAE for the Bund and the KfW-CDS portfolio in respect to the Market Implied Risk-Free Rate. Both measures indicate that the Bund performs much better than the KfW-CDS portfolio. Furthermore, we also evaluated the adjustments independently as risk-free estimators, i.e., Bund-CDS and KfW, and both show an even worse performance than the KfW-CDS portfolio. Therefore, the Bund without any adjustments most accurately reflects the risk-free rate.

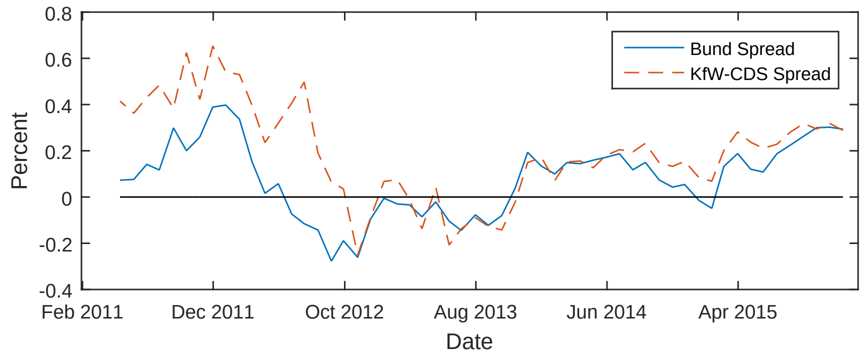

FIGURE 6.6: Bund and KfW-CDS to Market Implied Risk-Free Rate Spreads

TABLE 6.5: Consistency of Bund and KfW-CDS

	RMSE	MAE
Bund	0.1421	0.1700
KfW - CDS	0.2347	0.2828
Difference	65%	66%

We can explain the better performance of the Bund by decomposing the three risk-free proxies into three components: the risk-free rate, a credit risk premium and a liquidity premium, consistent with Reinhart & Sack (2002). We assume that the Market Implied Risk-Free Rate is conceptually the most accurate risk-free estimator for the 5-year maturity and therefore set it equal to the risk-free rate. Furthermore, we assume that the Bund incorporates a credit risk premium (C_b) and a liquidity risk premium (L_b). When we assume that the CDS entirely eliminates the credit risk of the KfW-CDS portfolio, this portfolio only deviates from the risk-free rate by a liquidity premium (L_k). We expect that the credit risk premium has an increasing impact and the liquidity premia have a decreasing impact on the bond yields.

$$\begin{aligned}\text{Market Implied Risk-Free Rate} &= r_f \\ \text{Bund} &= r_f + C_b - L_b \\ \text{KfW-CDS} &= r_f - L_k\end{aligned}$$

The accuracy measures show that on average the absolute error of the Bund is smaller than that of the KfW-CDS portfolio. Furthermore, we know that the liquidity of the Bund is larger than that of the KfW, because the KfW to Bund spread is almost always positive as shown in Figure 6.2. Furthermore, we can induce that the neutralizing effect of the liquidity premium on the credit risk premium makes the Bund the best risk-free estimator. Concluding, we recommend to make no adjustments to the Bund, because it does not improve its estimation accuracy.

$$\begin{aligned}L_b &= L_k + \text{KfW} - \text{Bund} \\ \text{KfW} - \text{Bund} &> 0 \\ L_b &> L_k \\ |C_b - L_b| &< |L_k|\end{aligned}$$

Chapter 7

Conclusion

The current low government bond yields evoked questions about the adequacy of government bonds as a risk-free estimator. Mechanically applying these yields as risk-free rates implies low discount rates and high business valuations. Furthermore, German government bonds currently have negative real returns for the complete term structure and even negative nominal returns for maturities up to seven years (Deutsche Finanzagentur, 2016; CapitalIQ, 2016). We studied the risk-free rate concept and reviewed multiple estimators to answer our main research question: "*What is the best estimator of the risk-free rate for valuation purposes?*" We answer this question in Section 7.1 by recapitulating our study's approach and its outcomes. Subsequently, we propose topics for further research in Section 7.2. Finally, we conclude our thesis in Section 7.3 by reviewing potential causes of the current low interest rates.

7.1 Conclusions

The risk-free rate is a fundamental part of finance, but the theoretical concept only received little attention. An indication of this lack of attention is that governments bonds are often mistakenly interpreted as the concept itself (Financial Times, 2015a). However, recent market circumstances with historical low government bond yields, evoked questions about the concept and the adequacy of German government bonds as risk-free estimator. Therefore, we conducted research to the best estimator for the risk-free rate. Theoretically, the risk-free rate is the required return on an asset without any financial risks. However, in reality it is impossible to eliminate all financial risks and therefore we proposed a practical definition for a risk-free proxy: "*An asset paying a specified unit in a currency at a certain date with minimal credit risk and sufficient liquidity.*" We acknowledge that in reality credit risk cannot be removed entirely and thus aim to minimize its exposure. All the other financial risks are controllable, so a risk-free proxy should not be exposed to them. Furthermore, we require an asset to have only a sufficient liquidity to guarantee efficient market pricing and minimize the liquidity premium.

We distinguished two methods to estimate the risk-free rate: via proxies and models. Proxies closely resemble a risk-free asset and therefore accurately estimate it, on the other side models are theoretically linked to the risk-free rate. We concluded that the German government Bond and the Overnight Indexed Swaps are suitable as risk-free proxies for valuation purposes. Furthermore, we discarded the Generalized Collateral Repurchase Agreements and our developed 'Market-Implied Risk-Free Rate' as proxies, because both are unavailable for long-term maturities required for business valuations. Besides the proxies, we constructed the 'Macro Model' by regressing macro-variables on the German government bond in a period when it closely resembled a risk-free asset. We reviewed these three estimators on three evaluation criteria; Consistency, Intelligibility, and Availability. We showed that the German government bond performs as good as or better than the other estimators on all criteria and is thus the best risk-free estimator for valuation purposes. Our own developed Macro Model showed to be the worst estimator, especially due to lack of Intelligibility. Overnight Indexed Swaps are a reasonable alternative estimator and only show a lower score on Consistency.

By definition a risk-free estimator only approximates the risk-free rate, so even the German government bond has deficiencies. We distinguished three deficiencies of the German government bond as risk-free estimator: credit risk exposure, flights-to-liquidity, and Quantitative Easing. Subsequently, we proposed adjustments for the former two and not for the latter, because our impact estimation for Quantitative Easing is inaccurate. We proposed to use CDS spread to eliminate partially the credit risk exposure and to use KfW bonds to eliminate partially the liquidity premium. Although our adjustments improve the Bund conceptually as risk-free estimator, the adjusted Bund is less consistent with the Market Implied Risk-Free Rate. We regard the latter as best estimate of the risk-free rate for the 5-year maturity. We relate the lower consistency to the neutralizing effect of the positive credit risk premium and negative liquidity premium of the Bund. Therefore, we recommend valuators to use the German government bond as risk-free estimator.

7.2 Further Research

We distinguish three areas for further research to the best risk-free estimator for valuation purposes. Firstly, the concept of the risk-free rate can be defined more strictly by determining a minimal liquidity level. Secondly, our methodology can be improved by determining more evaluation criteria and increasing the measurement frequency. Thirdly, the Bund as risk-free estimator can be improved by better quantifying its deficiencies.

Firstly, the definition of a risk-free asset can be improved by stricter defining sufficient liquidity. We defined sufficient liquidity as the liquidity level needed for the market to efficiently price the asset. Later on, we implicitly used the liquidity of A-rated corporate bonds as sufficient liquidity by using them as basis for the Market Implied Risk-Free Rate. Further research can more elaborately focus on this choice. Perhaps lower-rated corporate bonds have a lower but also sufficient liquidity? Another approach would be to analyze values of assets without a primary market. However, it remains questionable whether assets are then priced efficiently.

The second area of further research focuses on the evaluation of the estimators. Due to time constraints we evaluated the estimators on only three criteria. More extensive research to other financial benchmarks and expert interviews can improve the evaluation criteria. This enables a broader evaluation of the characteristics of a risk-free estimator. Furthermore, we recommend to increase the frequency used in the evaluation to improve the level of detail. We were constrained to a monthly frequency, because of the limited availability of the Macro Model.

Finally, further research can be conducted to the identified deficiencies of the German government bond as risk-free estimator. Our proposed CDS adjustment only partially removes credit risk and its accuracy can be improved by reducing counterparty risk. Firstly, CDS can use collateralization, a common method to reduce counterparty risk of financial instruments. Collateral has to be posted when the mark-to-market value significantly increases. The posted collateral can be paid to the buyer when a CDS issuer defaults. A second strategy to reduce counterparty risk would be to use central clearing of CDS contracts, as is already the case for several other derivatives. Central clearing mitigates the credit risk of the buyer and seller of a derivative to the central clearing house, which has lower credit risk due its diverse risk exposure and netting opportunities. Downside of both proposed measures is that they require a change of standard CDS contracts, making it not likely to be implemented soon. Furthermore, we acknowledge that counterparty risk can never be entirely eliminated, because of the enormous global consequences when Germany defaults. The second deficiency of the German government bond as a risk-free estimator is the capital market distortion of Quantitative Easing. The program has been a novelty in its size and structure, so until today only little research has been conducted to its impact. We think the alternative measuring approach also used by Gagnon *et al.* (2011) and Joyce *et al.* (2011) can improve the estimate of Quantitative Easing's impact. This alternative approach analyzes the impact of bond supply reduction in 'normal' times and multiplies this with the reduced bond supply by Quantitative Easing.

7.3 Discussion

Currently German government bonds yield negative real returns. This contradicts with human time preference stating that investors require a compensation for lending money, implying that the required real return should always be positive. What is then the reason for the negative real yields observed? We formulated three potential explanations of the paradoxically negative real yields in Figure 7.1. Of these three factors, we regard the last two as remaining potential causes. We discard the first factor stating that the inflation expectation is incorrect, because the complete term structure of inflation-linked German government bonds is also negative (Deutsche Finanzagentur, 2016). These bonds are indexed to inflation, so they are not exposed to potential incorrect inflation expectations.

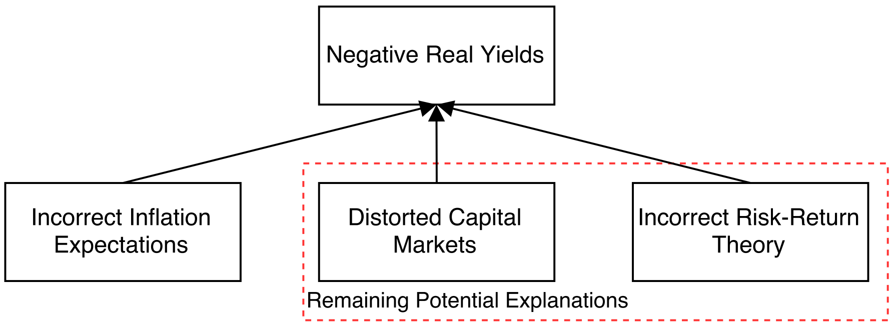

FIGURE 7.1: Potential Explanations of Negative Real Rates

Thus only two potential factors explaining the negative real German government bond yields remain. We think that both factors cause the current observed negative real yields. Firstly, we already indicated that the capital market is distorted by showing the increased bond prices by the Quantitative Easing programs and flights-to-liquidity. Both factors reduced globally government bond yields, causing the risk-free estimators to deviate from the risk-free rate. We recommend the German government bond as best risk-free estimator, however this remains a relative judgment compared to the other available estimators.

Secondly, we regard insufficient risk-free theory as an explanation for the current negative real risk-free rate. Traditionally, economic theory states that zero is the effective lower bound of interest. This assumption showed not to hold in the current environment. We propose three explanations for investors to accept a negative real and even a negative nominal return: (i) speculation about bond price increases or currency appreciation (ii) regulatory requirements for financial institutions to purchase government bonds, and (iii) lack of alternative assets. The first explanation assumes that investors offset negative returns by increasing asset prices. The second describes the obligatory purchases of safe assets by financial institutions. The third and last relates the acceptance of negative returns to the difficulty of finding alternative methods to safely store their assets. This lack of alternative assets forces investors to accept a negative return as a 'holding cost'. Concluding, we think that all factors pushed the lower bound of interest below zero. However, the question remains: "What is the new lower bound?" We can only philosophize about it and think that it is up to a few percentage points below zero, especially considering the holding cost accompanied with cash storage. We consider this an excellent topic for further research.

Chapter 8

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Appendix A

Mathematical Derivations

This Appendix shows the mathematical derivations for the used formula in this thesis. Firstly, Section A.1 shows the derivation for the continuing value used in discounted cash flow valuations. Secondly, Section A.2 shows the derivation of the Consumption Model.

A.1 Derivation of the Continuing Value

The CV is the value of the cash flows generated by the company after the explicit forecast period. The cash flows in this period are assumed to grow indefinitely at a fixed rate. The CV is defined below where FCF_t represents the FCF at time t, r_{wacc} the company's WACC used for discounting and g the company's growth rate.

$$CV = \frac{\text{FCF}_T}{1+r_{wacc}} + \frac{\text{FCF}_T(1+g)}{(1+r_{wacc})^2} + ... + \frac{\text{FCF}_T(1+g)^{\infty-1}}{(1+r_{wacc})^{\infty}}$$
$$CV = \frac{\text{FCF}_T}{1+r_{wacc}} + \frac{\text{FCF}_T}{(1+r_{wacc})}\Big(\frac{1+g}{1+r_{wacc}}\Big) + ... + \frac{\text{FCF}_T}{(1+r_{wacc})}\Big(\frac{1+g}{1+r_{wacc}}\Big)^{\infty-1}$$

The CV has the same structure as an infinite geometric series. From calculus the sum of such a series is known when the absolute value of the common ratio is smaller than 1. The common ratio in the continuing value is $\frac{1+g}{1+r_{wacc}}$, thus for the condition to hold r_{wacc} should always be greater than g. This means that the discount rate should be greater than the growth rate, a logical assumption considering a perpetuity. Thus calculus enables a further simplification of the CV.

$$\begin{aligned}
CV &= \sum_{k=0}^{\infty} \frac{\text{FCF}_T}{1+r_{wacc}}\Big(\frac{1+g}{1+r_{wacc}}\Big)^k = \frac{\frac{\text{FCF}_T}{1+r_{wacc}}}{1-\frac{1+g}{1+r_{wacc}}} \\
&= \frac{\frac{\text{FCF}_T}{1+r_{wacc}}}{1-\frac{1+g}{1+r_{wacc}}}\frac{1+r_{wacc}}{1+r_{wacc}} = \frac{\text{FCF}_T}{(1+r_{wacc})-(1+g)} \\
&= \frac{\text{FCF}_T}{r_{wacc}-g}
\end{aligned}$$

A.2 Derivation of the Consumption Model

The Consumption Model defines the risk-free rate as an resultant of intertemporal choice models. These models resemble human behavior over time and their preferences define the risk-free rate. We listed the derivation of the Consumption Model by Campbell (2003) below. The total utility in a person's lifetime in discrete time is given by

$$U = \sum_{t=0}^{T} D(t)U(C(t))$$

where $D(t)$ represents the discount factor and $U(C_t)$ the utility of consumption C at time t. The Consumption Model assumes that humans make choices in order to maximize utility over their lifetime. This means that between periods a human equates the cost of consuming one real dollar less at time t and the expected benefit of investing one real dollar in asset i at time t and consuming the dollar and its proceeds in $t+1$. This optimum can be found by solving the first-order condition listed below, where the left-hand-side represents the benefit of consuming now and the right-hand-side represents the discounted expected benefit of delayed consumption.

$$
\begin{aligned}
U'(C_t) &= D(t)E_t\Big[(1+r_{i,t+1})U'(C_{t+1})\Big] \\
1 &= E_t\left[(1+r_{i,t+1})D(t)\frac{U'(C_{t+1})}{U'(C_t)}\right] \\
1 &= E_t[(1+r_{i,t+1}M_{t+1})]
\end{aligned}
$$

Where $M_{t+1} = D(t)U'(C_{t+1})/U'(C_t)$ represents the intertemporal marginal rate of substitution of the investor, also known as the 'stochastic discount factor'. When we assume that the joint conditional distribution of asset returns and the stochastic discount factor is lognormally distributed and homoskedastic, a further simplification can be deduced. From statistics it is known that a lognormally distributed random variable X has the following property

$$log(E_t[X]) = E_t[log(X)] + \frac{1}{2}Var(log(X))$$

Thus when we take logs of the first-order condition of the maximization equation, we obtain

$$0 = E_t[log(1+r_i)] + E_t[M_{t+1}] + \frac{1}{2}(\sigma_i^2 + \sigma_m^2 + 2\sigma_{im})$$

In this formula the variances denote the variance of log returns of the random variables and the covariance denotes the covariance of the log returns. Campbell (2003) defines a risk-free asset as an asset with a standard deviation of zero over its future returns. This characteristic further simplifies the equation.

$$
\begin{aligned}
0 &= log(1+r_f) + E_t[log(M_{t+1})] + \frac{1}{2}\sigma_m^2 \\
log(1+r_f) &= -E_t[log(M_{t+1})] - \frac{1}{2}\sigma_m^2
\end{aligned}
$$

The above equation holds for all kind of utility functions. When we assume that humans have a power utility function, we can specify the slog stochastic discount factor as below. The advantage of the power utility function is that it has a CRRA.

$$
\begin{aligned}
U(C_t) &= \frac{C_t^{1-\gamma} - 1}{1-\gamma} \\
U'(C_t) &= C_t^{-\gamma} \\
M_{t+1} &= D(t)U'(C_{t+1})/U'(C_t) \\
&= D(t)(C_{t+1}/C_t)^{-\gamma} \\
log(M_{t+1}) &= log(D(t)(C_{t+1}/C_t)^{-\gamma}) \\
&= log(D(t)) - \gamma(log(C_{t+1}) - log(C_t))) \\
&= log(D(t)) - \gamma\Delta log(C_{t+1})
\end{aligned}
$$

Since we now have specified the log stochastic discount factor, we can further specify the risk-free rate equation.

$$
\begin{aligned}
log(1+r_f) &= -E_t[log(D(t)) - \gamma\Delta log(C_{t+1})] - \frac{1}{2}\gamma^2\sigma_m^2 \\
&= -log(D(t)) + \gamma E_t[\Delta log(C_{t+1})] - \frac{1}{2}\gamma^2\sigma_m^2
\end{aligned}
$$

This equation states that the risk-free real rate is linear in expected consumption growth, with slope equal to the CRRA. The variance of the consumption growth has a negative effect on the risk-free rate. The latter can be interpreted as a precautionary savings effect, implying that a higher volatility of consumption growth induces more saving (Campbell, 2003).

Appendix B

Background for Risk-Free Proxies

In this Appendix we provide more background detail about two risk-free proxies. Firstly, we explain the interest rate swaps more elaborately with an example in Section B.1. Secondly, we show that the Market Implied Risk-Free Rate without the accrued interest adjustment better resembles the risk-free rate in Section B.2.

B.1 Interest Rate Swaps

As an illustration for the functioning of a swap a numerical example is provided. Suppose Company A agrees to swap an interest rate of 3% per annum on a principal of € 100 million and in return receives 6-month LIBOR + .5%. The agreement specifies that payments are exchanged every 6 months and the interest rate is quoted with semiannual compounding. The swap agreement is schematically displayed in Figure B.1.

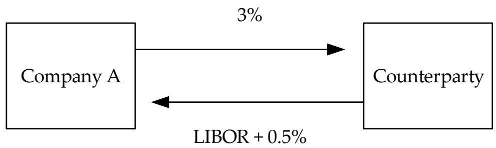


FIGURE B.1: LIBOR Swap

The first exchange is due on the July 1, 2016. Company A would receive 1.5 $(0.5 * .05 * 100)$ and pay 1.1 $(0.5 * 2.2 * 100)$. This thus results in a net cash flow of -1.4. In total six exchanges would occur, which are all shown in Table B.1.

TABLE B.1: Cash Flows in € millions from LIBOR Swap

Date	6-Month LIBOR + .5%	Floating Cash Flow Received	Fixed Cash Flow Paid	Net Cash Flow
1-1-2016	2.2%			
1-7-2016	2.8%	1.10	−1.5	−0.40
1-1-2017	3.3%	1.40	−1.5	−0.10
1-7-2017	3.5%	1.65	−1.5	0.15
1-1-2018	3.6%	1.75	−1.5	0.25
1-7-2018	3.9%	1.80	−1.5	0.30
1-1-2019	4.4%	1.95	−1.5	0.45

Usually companies use a swap agreement to transform a liability into a more desirable cash flow. In the Figure B.2 Company A has borrowed at a floating rate of LIBOR $+1\%$ and the Counterparty has borrowed at the fixed rate of 3.3%. The swap enables Company A to exchange a floating for a fixed rate, as can be seen in Table B.2. Company A has transformed its floating cash flow into a fixed cash flow of 3.5%. The same principle of cash flow transformation can be applied to transform the cash flows from an asset. Companies enter into such contracts because of multiple reasons, e.g., hedging and speculation (Hull, 2015).

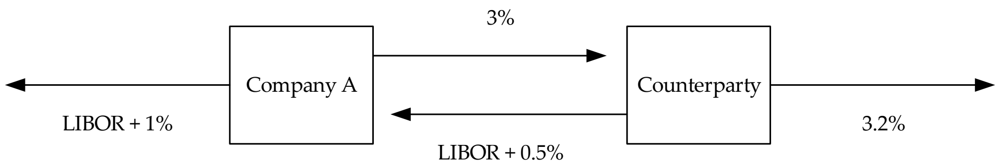


FIGURE B.2: Transformation of Cash Flows through LIBOR Swap

TABLE B.2: Transformation of Cash Flows through LIBOR Swap

	Company A		**Counterparty**
+	LIBOR +1%	+	3.20%
-	(LIBOR + 0.5%)	-	3%
+	3%	+	LIBOR + 0.5%
+	3.50%	+	LIBOR + 0.7%

B.2 Adjustment to Market Implied Risk-Free Rate

In Subsection 3.4.1 we described two variants of the Market Implied Risk-Free Rate. Hull *et al.* (2004) proposed a corporate bond and a CDS as a risk-free portfolio. However, Duffie (1999) argued this does not incorporate the accrued interest and proposed an adjustment. The difference between the two variants is the adjustment for the accrued interest; $s\frac{y}{2}$, which is equally affected by the bond yield and the CDS spread. In this Section we will evaluate which variant better reflects the risk-free rate by comparing their accuracy with the average of the Bund and the OIS. We use the average of these risk-free proxies, because they are highly correlated, indicated by correlation coefficient of .99. Furthermore, we compare it only to the Bund and OIS, because these are the only risk-free proxies available for the five-year maturity.

Figure B.3 shows the difference between the two variants of the Market Implied Risk-Free Rate and the average risk-free proxy. The variant with the adjustment for the accrued interest is always smaller than the average the average risk-free proxy, and the the variant without the adjustment almost always larger.

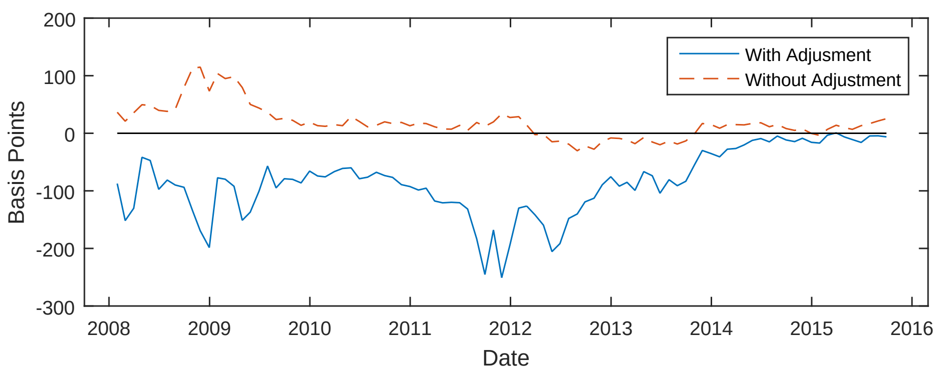


FIGURE B.3: Market Implied Risk-Free Rate Variants to the Average Risk-Free Proxy Spreads

To formally evaluate the accuracy, we analyze the error of the variants. Below we defined the error as the difference between the two variants of the Market Implied Risk-Free Rate and the average risk-free proxy. Where y represents the observation of the average risk-free proxy and $\hat{y}_i$ represents the prediction of the Market Implied Risk-Free Rate variant i at time t. We are interested in positive and negative deviations of the predictor and therefore analyze the squared and the absolute error. Furthermore, we define accuracy as a predictor that over time results in the lowest error, so we analyze the average. This provides us with two forecast error measures: the root mean squared error (RMSE) and the mean absolute error (MAE).

$$\epsilon_{i,t} = y_t - \hat{y}_{i,t} \qquad \text{RMSE}_i = \sqrt{\frac{\sum_{t=1}^{n}(\epsilon_{i,t})^2}{n}} \qquad \text{MAE}_i = \frac{\sum_{t=1}^{n}|\epsilon_{i,t}|}{n}$$

The RMSE is the root of the average of the squared errors. Taking the root ensures that the RMSE has the same units as the quantity being estimated. The MAE is the average of the absolute errors. Both measures are scale-dependent and are negatively oriented scores, meaning the smaller the score the better performance. The difference between the measures is the weighting of outliers, which are defined as errors larger than one percentage point. The RMSE assigns more weights to outliers due to the squaring of the errors before averaging them. Evaluating the estimators on both criteria provides us with a robust indication of the accuracy.

To evaluate the accuracy of the estimators we calculated the RMSE and the MAE from 2009:07 to 2015:09 and show them in Table B.3. Both measures indicate that the Market Implied Risk-Free Rate variant without the accrued interest adjustment better represents the risk-free rate.

TABLE B.3: Accuracy of Market Implied Risk-Free Rate Variants

	RMSE	MAE
With Adjustment	1.0272	0.8574
Without Adjustment	0.3468	0.2474
Difference	202%	251%

Appendix C

Background for Risk-Free Models

We explain more in depth two risk-free models. Firstly, we explain several intertemporal choice models in Section C.1. Secondly, we describe additional data used in the construction of the Macro Model in Section C.2.

C.1 Intertemporal Choice Models

This Section reviews the most important discounting models. The discounting model we use in this thesis is the DU model, because this is the mostly used. The main criticism on this model is the observed decrease of the discount factor which is inconsistent with the by DU model assumed constant factor. Firstly, we describe the DU model in more detail in Subsection C.1.1. Subsequently, Subsection C.1.2 describes alternative discounting models.

C.1.1 Discounted Utility Model

Samuelson (1937) has introduced the DU Model in which all underlying intertemporal human motives are condensed into a single parameter; the discount rate. The DU model is the standard for evaluating economic human behavior in time, mainly due to its ease of use. The model specifies a decision maker's intertemporal preferences over consumption profiles with the discount function shown below. $D(t)$ is the person's discount function and is interpreted as the relative weight a person attaches to his utility in a certain period. ρ represents the individual's pure rate of time preference and reflects the collective effects of his psychological motives. The discount function is decreasing for persons with a positive time preference (Samuelson, 1937).

$$D(t) = \left(\frac{1}{1+\rho}\right)^t$$

A main underlying assumption of the DU model is that humans use a constant discount factor. Constant discounting entails that a person evaluates time in a consistent manner. This means that delaying two dated outcomes by a common amount, should have no impact. However, intuition and empirical research have indicated that the impact of a constant time difference between two outcomes becomes less significant as both outcomes are made more remote (Prelec & Loewenstein, 1991). The classic example is that a person who is indifferent between €20 today and €25 in a month will most likely prefer €25 in eleven months to €20 in ten. The finding that the discount rate decreases as the delay increases is called declining impatience. This and other anomalies have created the room for alternative discounting models.

C.1.2 Alternative Intertemporal Choice Models

Multiple scholars relate the observation of non-constant discounting to declining impatience of humans. To properly model this aspect of human behavior a hyperbolic discount function is introduced, because this model incorporates a declining discount factor. Laibson (1997) was the first to formalize a functional form of a discount function which captures the essence of hyperbolic discounting. His Quasi-Hyperbolic discounting function has the following functional form. In this discount function β and δ are constants between 0 and 1. As can be seen the first condition states that rewards taken at the present time are not discounted. Afterwards the discount factor decreases gradually.

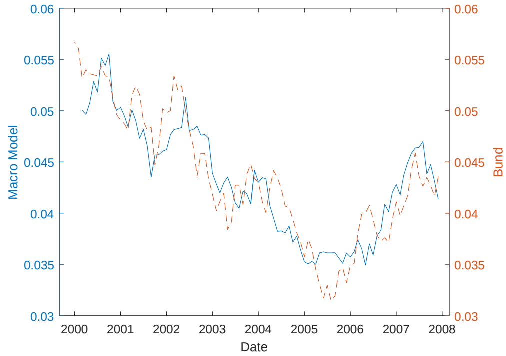
$$D(t) = \begin{cases} 1 & \text{if } t = 0 \\ \beta\delta^t & \text{if } t > 0 \end{cases}$$

However, also scholars have conducted experiments contradicting hyperbolic discounting (Rubinstein, 2003). Read (2001) has taken another path in explaining the observed declining discount factor by relating it to sub-additivity. He bases his findings on studies of Kahneman & Knetsch (1992) describing sub-additivity in expected utility studies. Sub-additivity means that the price put on a good is greater if the good is first divided into parts which are evaluated individually and summed afterwards, than if it is evaluated in its entirety. The first reason for this behavior is that people pay more attention to an object if it is divided than if it is a part of a larger whole. Secondly, the regression effect is another factor. This effect describes that subjective estimates are typically biased in the direction of the midpoint of whatever range is estimated, leading to overestimates of small quantities and underestimates of larger ones. Read (2001) shows with experiments that when a delay is divided into more subintervals, the larger the discount factor becomes. He argues that sub-additivity, and not an actual declining discount function, explains the empirical evidence of non-constant discounting. His proposition is summarized below.

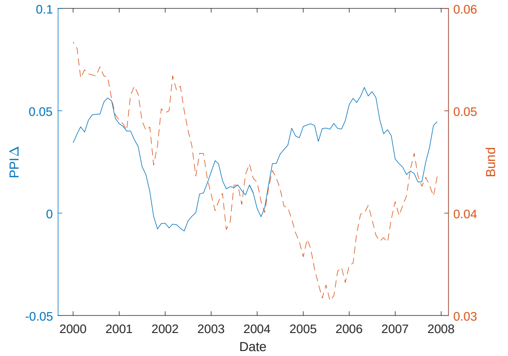
$$D_{0\to T} < D_{0\to T'\to T} \quad \text{in which } T' < T$$

C.2 Macro Model's Additional Data

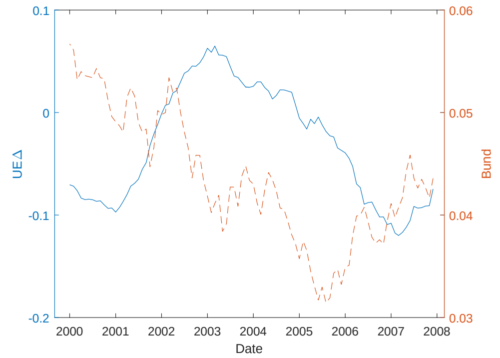
Figure C.1 shows the historical development of the variables compared to the Bund. Also the Macro-Model itself is plotted against the Bund. Besides the selected variables, we also evaluated other macro-variables. Table C.1 shows the variables not incorporated in the Macro Model. They have been selected, because they were used by scholars modeling the risk-free rate (Diebold *et al.*, 2006; Taylor, 1993; Rudebusch *et al.*, 2006; Ang & Piazzesi, 2003; Reinhart & Sack, 2002; Pericoli & Taboga, 2015). The description of the variables and the reason for excluding are described. For all the variables the annual volatility is tested, but all proved to be insignificant. For the sake of brevity these variables are not described separately.



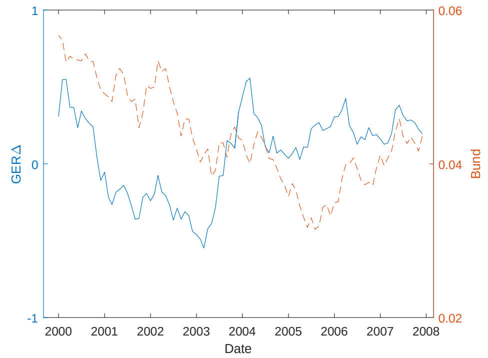
(A) Macro Model



(B) PPIΔ

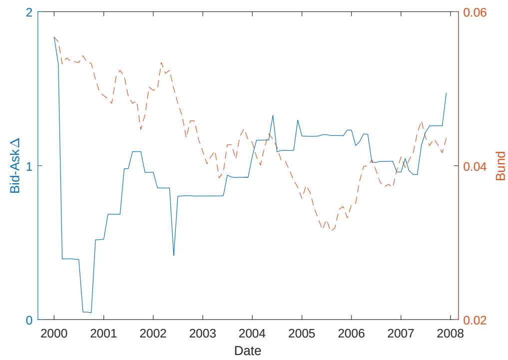


(C) UEΔ



(D) GERΔ

(E) EONIA



(F) Bid-Askσ

FIGURE C.1: Development of Macro-Economic Variables and 10Y Bund Yields

TABLE C.1: Excluded Variables for Macro Model

Group	Variable	Source	Description	Reason
Inflation	HICPΔ	ECB (2015)	Annual change of European HICP, measuring the price development of goods and services for consumers.	Insignificant.
Inflation	PCOMΔ	ECB (2015)	Annual change of the European Commodity Price Index, measuring the price development of spot commodity prices.	Insignificant.
Inflation	Inflation Swap	Bloomberg L.P. (2015)	The 5 year inflation swap exchanges the future European HICP with a fixed rate. Thus it is an indication of the market's expected inflation.	Only data up to June 2004.
Real Activity	IPΔ	ECB (2015)	Annual change of the European Industrial Production Index, measuring the production output. It is commonly used as a proxy for the GDP, because both variables are highly correlated and the latter is only published quarterly.	Insignificant.
Real Activity	WORLDΔ	MSCI (2015)	Annual change of MSCI World Index. The index measures all worldwide stocks.	Lower explanatory power than the highly correlated GERΔ.
Real Activity	VSTOXXΔ	Bloomberg L.P. (2015)	The implied volatility of all the outstanding EURO STOXX 50 options. Thus it is an indication for the expected European stock market volatility	Insignificant.
Credit Risk	CDS	Bloomberg L.P. (2015)	5-year CDS spread of Germany. This is the premium paid to an insurer for a default of Germany. Thus it is an indication for the German Credit Risk.	Only data up to March 2003.
Liquidity Risk	Kfw-Bund Δ	Bloomberg L.P. (2015)	Annual change of the 10-year Bund to KfW spread. The KfW is a German development bank and its obligations are guaranteed by the German Government. This spread is used by Ejsing *et al.* (2015) as an indicator of liquidity risk.	Only data up to April 2011.

Appendix D

Detailed Evaluation of Proxies

This Appendix describes the evaluation of the proxies in more detail. All AHP's pairwise comparison matrices of the criteria are shown below. Furthermore, the Consistency Ratio of all comparison matrices are denoted below the table. The Consistency Ratio is developed by Saaty (1980) as a measurement of the degree of consistency of the subjective judgments. He has shown that a matrix is fully consistent if the size of the matrix n is equal to the matrix' eigen value λ. The Consistency Index is a measure of relative consistency and is defined below:

$$\text{CI} = \frac{\lambda_{max} - n}{n - 1}$$

where λ_{max} is the maximal eigenvalue and n is the matrix' size. The Random Consistency Index is the average Consistency Index of 500 randomly filled matrices. The Random Consistency Index computed by Saaty (1980) for matrices up to a size of 10 judgments is listed in Table D.1.

TABLE D.1: Random Consistency Index for Multiple Matrix Sizes

n	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

Saaty (1980) proposes that a judgment matrix is of an acceptable consistency level if the Consistency Index is not greater than 10% of the Random Consistency Index. The Consistency Ratio is calculated by dividing the Consistency Index by the Random Consistency Index: CR = CI/RI and if smaller than 10% is assumed to be consistent.

TABLE D.2: Pairwise Comparison Matrix of Evaluation Criteria

Criteria	Availability	Consistency	Intelligibility	Priorities
Availability	1	$\frac{1}{5}$	$\frac{1}{4}$	0.10
Consistency	5	1	2	0.57
Intelligibility	4	$\frac{1}{2}$	1	0.33

CI = 0.01, RI = 0.58, CR = 0.01 < 0.10

TABLE D.3: Pairwise Comparison Matrix of Consistency

Consistency	Bund	OIS	Macro Model	Priorities
Bund	1	3	7	0.65
OIS	$\frac{1}{3}$	1	5	0.28
Macro Model	$\frac{1}{7}$	$\frac{1}{5}$	1.00	0.07

CI = 0.02, RI = 0.58, CR = 0.03 < 0.10

TABLE D.4: Pairwise Comparison matrix of Intelligibility

Intelligibility	Bund	OIS	Macro Model	Priorities
Bund	1.00	3.00	9.00	0.47
OIS	0.33	1.00	9.00	0.47
Macro Model	0.11	0.11	1.00	0.05

CI = 0.00, RI = 0.58, CR = 0.00 < 0.10

TABLE D.5: Pairwise Comparison Matrix of Availability

Availablility	Bund	OIS	Macro Model	Priority Vector
Bund	1	1	5	0.45
OIS	1	1	5	0.45
Macro Model	$\frac{1}{5}$	$\frac{1}{5}$	1	0.09

CI = 0, RI = 0.58, CR = 0.00 < 0.10

Appendix E

Public Securities Purchase Program

This appendix more elaborately describes PSPP. Firstly, Section E.1 describes the concept of PSPP and its desired effect. Subsequently, Section E.2 shows the impact of all event dates on all eligible maturities. Finally, we show a more detailed impact analysis of the foreign QE programs in Section E.3.

E.1 Concept of Public Securities Purchase Program

On 22 January 2015 the ECB announced PSPP. In this program the ECB purchases bonds issued by Euro-Area central governments, agencies and European institutions. The program is part of the EAPP which combined purchases asset up to an amount of €60 billion per month. €50 billion will directed towards PSPP of which 88 percent will be used to purchase government and agency debt. Agency debt is guaranteed by central governments and thus has the same credit quality. An example of a Dutch agency is the Bank Nederlandse Gemeenten, financing publicly owned organizations. The remaining 12 percent is allocated to purchase supranational debt, e.g., European Union. Figure E.1 shows the allocation of the monthly asset purchases by the ECB (2016`

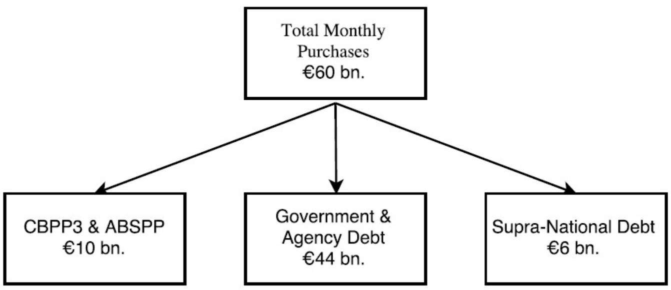


FIGURE E.1: Monthly EAPP Asset Purchases

PSPP has some specific boundaries set for the duration of the program and the eligible assets. The program was initially to be carried until September 2016, but was prolonged until March 2017. The ECB is allowed to purchase public securities from countries with a high credit rating or which are benefiting from an EU financial assistance program. It can only acquire debt maturing in 2 to 30 years from the secondary market, meaning that the ECB cannot directly finance governments. Finally the ECB is only allowed to purchase bonds with a yield equal or greater than the deposit rate. On top the of the eligibility constraints, the ECB is limited to purchase a 33% issue limit and a 33% issuer limit. These criteria are implemented to preserve market functioning (ECB, 2016).

The goal of the asset purchasing program is to achieve the price stability target, which is defined as 2% annual inflation. Joyce *et al.* (2012) explains that PSPP impacts inflation with two channels; the portfolio substitution channel and the bank-funding channel. Figure E.2 shows the relation of the two channels with inflation. Firstly, PSPP impacts inflation via the so-called portfolio substitution channel meaning changing the quantity and mix of the financial assets held by the public. Market participants obtain cash from the purchased assets by the ECB. As an result the duration of a market participant's portfolio changes. Most market participants want match the duration of their asset portfolio with its liabilities portfolio. Thus to again obtain a similar duration, the market participants purchases other long-dated assets, e.g., corporate bonds. This additional demand will enable easing credit conditions for corporates and result in a higher capital gain. Finally households, the ultimate owners of risky corporate assets, can consume part of this increased wealth. Another impact of the portfolio substitution channel is that due to the central bank's purchases the average duration of the stock of bonds held by the private sector decreases. This may decrease the term premium; the premium required to hold longer-term bonds. The decrease of the term premium also eases credit conditions for corporates. Secondly, PSPP impacts inflation due to the bank funding channel meaning changes in the availability of bank funding. PSPP increases the bank deposits of market participants, because they do not directly purchase replacing assets. Increased bank deposits stimulate banks to ease credit conditions, finally increasing public demand.

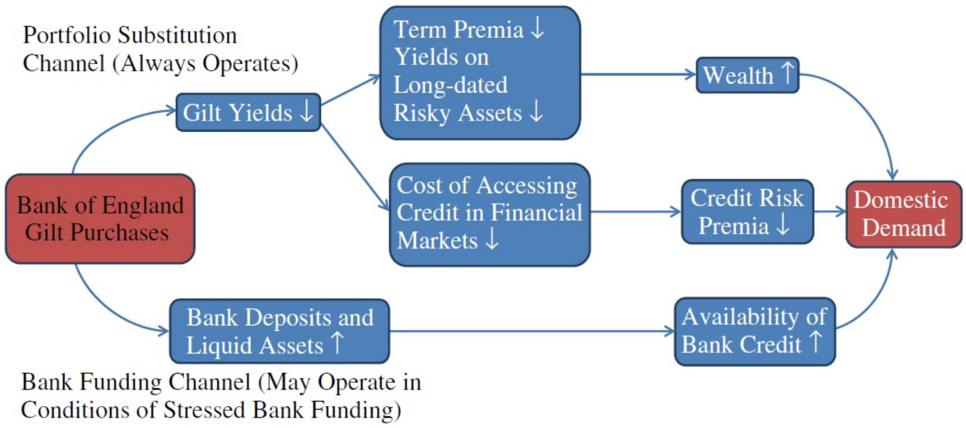
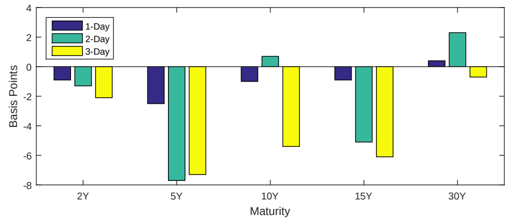


FIGURE E.2: QE's Impact Channels

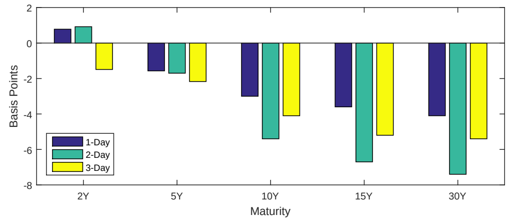
The actual impact of the QE program is a highly debated topic. As explained the program only indirectly impacts the domestic demand, so it is highly depended on the transmission of the market participants. When market participants do not reinvest the acquired cash, the impact is much smaller. Market participants can be hesitant to reinvest cash, due to multiple reasons, e.g., regulation for financial institutions prescribing certain levels of government bond holdings and cash, and absence of other investment opportunities. Studies to PSPP's and other QE programs' impact are growing, but encounter difficulty in isolating its impact. Given the obvious lags involved before the effects get fully passed through output and inflation, researchers have to rely on constructing model-based policy and no-policy counterfactuals. The construction of these counterfactuals is difficult to the the current unusual circumstances. Despite all the difficulties, multiple researchers have stated that the policy causes positive inflationary pressures (Joyce *et al.*, 2012).

E.2 Impact of Public Securities Purchase Program

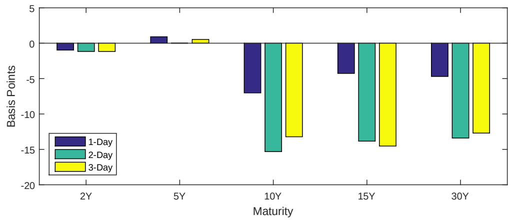
Figure E.3 shows PSPP's impact on all event dates (Bloomberg L.P., 2015). The PSPP had a positive impact on the Bund yields on two events; 3 December 2015 and 10 March 2016. We explain this positive impact, because the market expected a larger stimulus program. With almost all events PSPP had a larger negative for longer maturities.



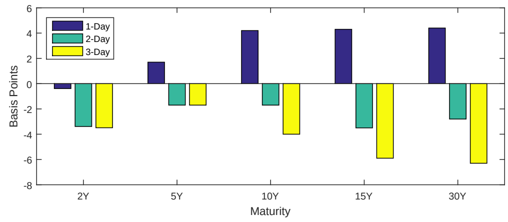
(A) 3 April 2014



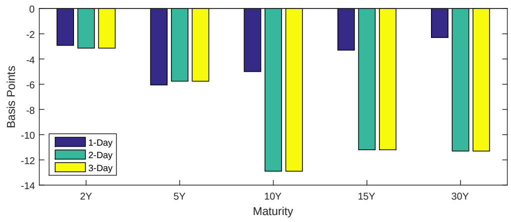
(B) 7 August 2014



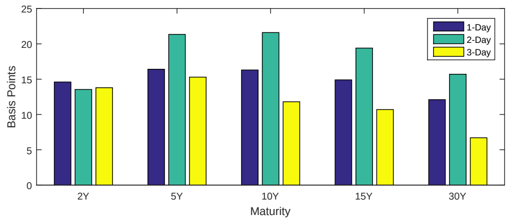
(C) 22 January 2015



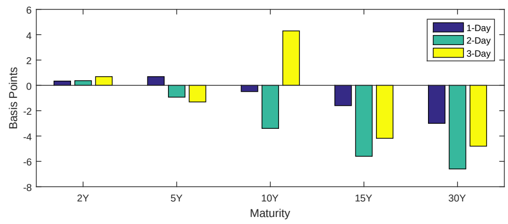
(D) 2 July 2015



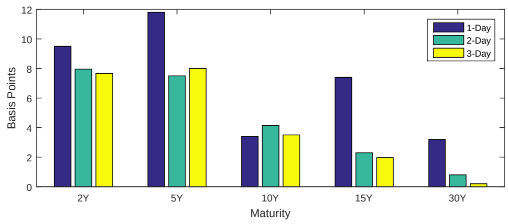
(E) 3 September 2015



(F) 3 December 2015



(G) 12 January 2016



(H) 10 March 2016

FIGURE E.3: PSPP's Impact of all Events on 10Y Bund Yield

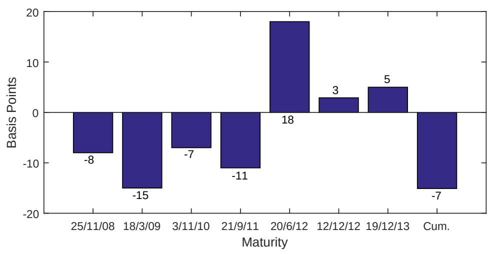
E.3 Foreign Quantitative Easing Programs

For the event study, we selected all official announcements concerning government bond purchases of the foreign QE programs in Table E.1 (FED, 2016; BoE, 2016; BoJ, 2016). For each event the date, the central bank, and the announcement are described. The FED implemented three QE programs every time with an increased size. The BoE incrementally increased the size of its program. The BoJ implemented by far the largest program and increased its size once.

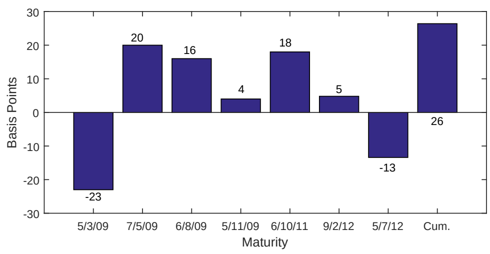
Figure E.4 shows the impact of all events on the 10-year Bund yield. The last three events of the FED program have a positive impact on the Bund yield. A possible explanation for the largest positive impact can be that the market expected an increase of the asset purchasing program, instead of an extension. Furthermore, the last event of the FED announced the tapering of the QE program, making a positive impact logical. The impact of BoE programs fluctuate per event. Only two events had a negative impact on the Bund yields; the announcement of the QE program, and the last increase of the QE program. We assume that the positive impact of the other events can be attributed the market's overestimation of the incremental program size increase. The BoJ program only is slightly positive for the second event, in which it expanded the annual purchases. A possibility of the almost absence of impact can be that the market already expected the increase of the program.

TABLE E.1: Foreign QE Announcements

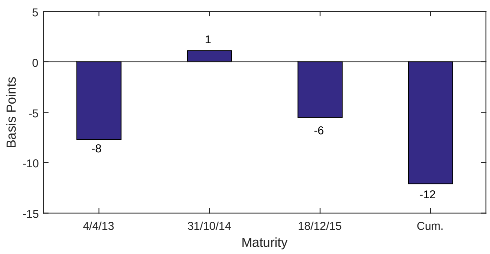
Date	Central Bank	Announcement
25/11/2008	FED	Announced QE1 with purchases of $100 billion of agency debt.
5/3/2009	BoE	Announced QE with purchases of £75 billion of government bonds.
18/3/2009	FED	QE1 expanded with additional $300 billion of government bonds
7/5/2009	BoE	QE expanded by increasing purchases up to £125 billion.
6/8/2009	BoE	QE expanded by increasing purchases up to £175 billion.
5/11/2009	BoE	QE expanded by increasing purchases up to £200 billion.
3/11/2010	FED	QE2 announced with purchases of $600 billion in government bonds.
21/9/2011	FED	Operation Twist announced with purchase of $400 billion of long-term government bonds and sale of equal amount of short-term bonds.
6/10/2011	BoE	QE expanded by increasing purchases up to £275 billion.
9/2/2012	BoE	QE expanded by increasing purchases up to £325 billions.
20/6/2012	FED	Operation Twist extended with $45 billion monthly purchases.
5/7/2012	BoE	QE expanded by increasing purchases up to £375 billion.
12/12/2012	FED	QE3 expanded by monthly purchasing $85 billion of government bonds.
4/4/2013	BoJ	QE announced with annual purchasing ¥50 trillion of government bonds.
19/12/2013	FED	Announced tapering of QE program with $10 billion every month.
31/10/2014	BoJ	QE expanded by increasing annual purchases up to ¥80 trillion.
18/12/2015	BoJ	QE altered by purchasing more longterm bonds.



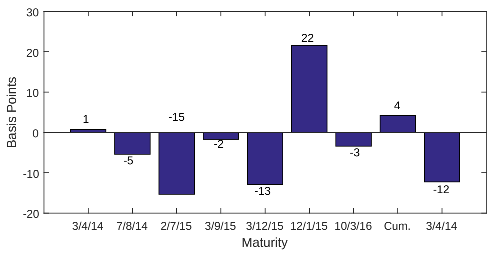
(A) FED



(B) BoE



(C) BoJ



(D) ECB

FIGURE E.4: QE's impact of all Events on 10Y Bund Yield