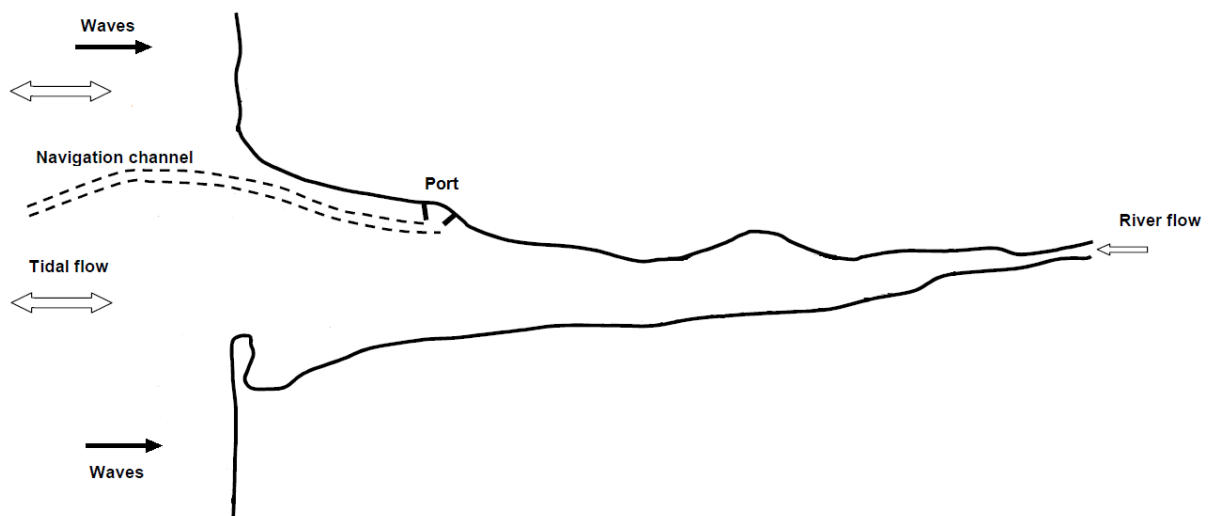

PORT AND CHANNEL SEDIMENTATION: QUANTIFYING UNCERTAINTIES USING AN EMULATOR

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MSC Thesis

in Civil Engineering and Management



UNIVERSITY OF TWENTE.

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Abstract

All around the world, expensive computationally models have been used in feasibility studies in order to predict the amount of port siltation in a certain area. Depending on how many millions of cubic meters of fine sediments brought into the navigation channel (by the tides, waves and rivers) need to be dredged, the construction of such a port can become impractical due to high maintenance costs. But if the uncertainties associated with this natural process are taken into account, for a better prediction, then those models cannot be used due to the many runs required to compute these uncertainties. The goal of this study is to quantify these uncertainties in port siltation using an emulator. An emulator is a simpler model which aims to generate an output as close as possible from the original model but with a lower computational time.

In order to reach the goal of this study, a model and two emulators with a lower horizontal resolution will be used. The model and the two emulators are termed as sixteen grid cells model, eight grid cells model and four grid cells model due to the number of grid cells along their respective navigation channel width. Moreover, four steps will be applied as follows. Firstly, the uncertain input parameters are selected with help of expert judgement and literature review. Secondly, we choose the one-at-a-time method (OAT) to determine among the uncertain parameters which contributed most to the uncertain in the model output, named as qualitative sensitivity analysis. Thirdly, we apply the Latin Hypercube Sampling method (LHS) to sample the previously selected input parameters which are used in the model runs for uncertainty quantification. Finally, we investigate mapping functions that, along with the emulator, are capable to translate the amount of uncertainty in port siltation computed by the model.

Regarding the sensitivity analysis, the uncertain input parameters to which the siltation is the most sensitive are: bed roughness, settling velocity and significant wave height, according to the model and the two emulators. However, the four grid cell model presented very low siltation values when compared to the sixteen and eight grid cells models. Because of this, it is not used in uncertainty quantification. With respect to the uncertainty quantification, the eight grid cells model present a smaller amount of uncertainty in the model output when compared to the sixteen grid cells model. This is an unrealistic result proving that the eight grid cells emulator cannot be used for uncertainty quantification. As for the identification of mapping functions, four mapping functions are identified and verified through box plots.

From this study, three important conclusions can be highlighted. First, the eight grid cells model is a good replacement for the sixteen grid cells model with respect to siltation computation in the navigation channel. Second, the four grid cells model can be used for a 'parameter screening' in qualitative sensitivity analysis. Third, the eight grid cell model can be used for uncertainty quantification if the specified mapping functions are applied. Summarizing, it is possible to quantify the uncertainty in a navigation channel using an emulator as long as a mapping function is applied. With attention to the fact that the channel width should be around 10 grid cells, otherwise the hydrodynamics and morphodynamics are not well represented by the emulator.

Keywords: uncertainty analysis, sensitivity analysis, emulator, Delft3D, mapping function.

Preface & acknowledgments

*“Valeu a pena? Tudo vale a pena
Se a alma não é pequena.”*

Fernando Pessoa, Mensagem (1934)

These verses are from *Mar Português*, one of the most famous poems of Fernando Pessoa, in which the poet describes the story of the Portuguese navigators who left all behind in search of new possibilities. In these verses he wonders “Was it worth it? All is worthwhile, if the soul is not small”. This is exactly the feeling of gratitude that I have after going through this journey called master thesis. Throughout these six months there were moments of rough seas, but also of calm waters. Everything started two years ago in a land far away from here known as Brazil. Despite of the sadness of leaving my family and friends behind, the challenging of a new adventure in The Netherlands, a country different in so many aspects from mine, spoke louder. Here, every day has been a constant lesson contributing to make me a better professional and human being. This master thesis marks the end of this adventure and the start of a new one. But I cannot end this journey without thanking from all my heart the people and institutions which made this dream come true.

First of all, I would like to thank my family specially my mother, father and sister who in spite of their huge affection for me let me go to achieve my dream. Thanks for being always so supportive.

I acknowledge the University of Twente for supporting my dream with the University of Twente scholarship, without it nothing of this would have ever been possible.

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Special thanks to Carles and Redmar for always been there for me and for their immense help, support and patience during my thesis. I will be forever grateful. Also thanks to my friends in Enschede and Delft for the relaxing and chilling moments of everyday life which made my stay in The Netherlands much nicer.

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1 Introduction

1.1 Background

Many ports around the world suffer from siltation on their berthing and manoeuvring areas, requiring maintenance dredging activities to restore navigable depths (see Figure 1). Sedimentation of harbour basins and navigation channels is a serious problem. Maintenance costs are a critical element in the economic feasibility of a port, particularly when a relatively long access channel requires frequent dredging. The maintenance dredging requirement for large sea ports and navigation channels may exceed tens of millions m³/year of sediment removal (Deltares, 2011).

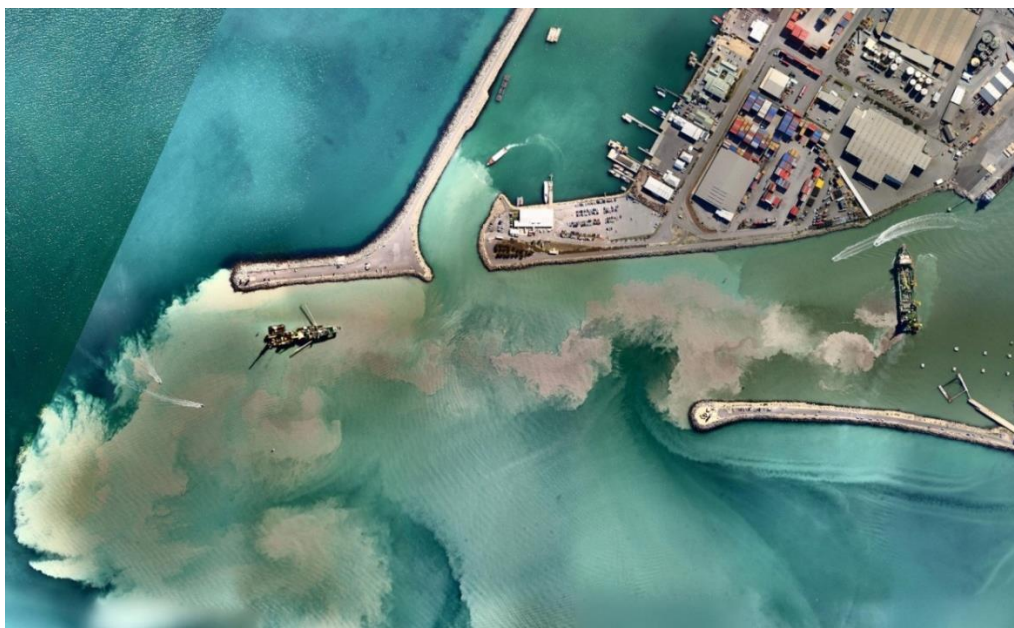


Figure 1: Dredging of the Port of Fremantle's Shipping Channels, 2010, Perth, Australia. (Becker, et al., 2013)

Due to the expensive maintenance of ports, predictions of port sedimentation are an important factor for the feasibility study of building a port in a certain area. Many studies have been conducted using empirical, numerical and physical models to study sedimentation processes in ports and channels both qualitatively and quantitatively. However, modelling such processes is not an easy task, partly due to the several known and unknown uncertainties in both the model input and within the model itself, which later are translated into the model output.

Scientific decision support can provide assistance to policymakers in order to make decisions, while taking into account uncertainties surrounding that decision (Walker, et al., 2003). The identification and quantification of these uncertainties (known as uncertainty analysis) usually requires many model

runs. Also, a sensitivity analysis is important to determine the most important elements of a model and to 'screen out' the irrelevant part. According to Saltelli (2000), uncertainty quantification should always come together with a sensitivity analysis (see Figure 2).

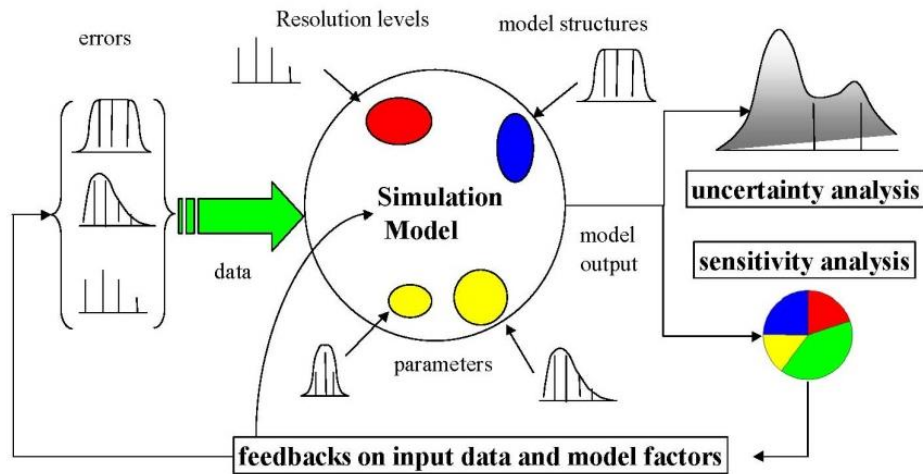


Figure 2: Uncertainty analysis scheme (Wikipedia, 2016).

For complex numerical models such as coastal morphodynamic models, implementing those two analyses become infeasible due to the performance of a large number of model evaluations. Hence, using an emulator can present as a better option. An emulator is a low-order, computationally efficient model identified from the original large model and then used to replace it for computationally intensive applications (Castelleti, et al., 2012). The complex numerical models known as “fine” models have been replaced by models with a coarser mesh known as “coarse” models (a type of an emulator) which, although are less accurate, are less computationally expensive than the fine models (Koziel & Bandler, 2008). In case the coarse model is a good representation of the fine model to be optimized, a mapping function between both can be identified. A mapping function aims to generate the output of the fine model from the output of the coarse model. Under those circumstances, a coarse model together with a mapping function would give accurate results with less computational time. Therefore, the emulator could induce to faster reliable results being a better option than the model for the demands of the engineering market.

1.2 Research objectives

1.2.1 Knowledge gap

The knowledge gap to be filled in this study is to quantify the uncertainties encountered in port sedimentation using an emulator. Although, many studies as Kuang et al. (2014), Plüß et al. (2014), Lund (1990) and Marlowe (1979) have been done in port sedimentation using predicted models, none of them quantified the existent uncertainty in the amount of siltation calculated by these models using an emulator.

1.2.2 Study case

In the present study, a port feasibility study is proposed by a client of Deltares, an independent institute for applied research in the field of water and subsurface. The client is interested in building a port in a certain area, which can be represented as seen in Figure 3. The area is mainly composed by mud and the transport of sediments is affected by the river discharge, waves and tidal flow. Due to client's request the location of the port cannot be revealed. This is irrelevant to the aim of this study.

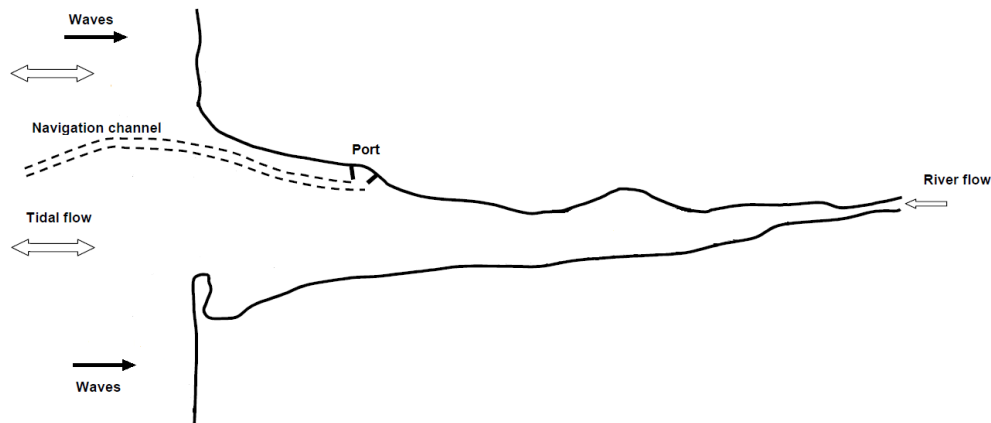


Figure 3: Scheme of the study area.

Prior to building this port, the client wants to know the expected amount of annual siltation that is likely to take place in the navigation channel, in relation to the future costs of maintenance that it would require. The client is confronted with a range of possible amounts of future siltation. A distribution representing this uncertainty range of siltation, such as (cumulative) distribution function, provides this information (see Figure 4 as an example of cumulative distribution of annual amount of siltation).

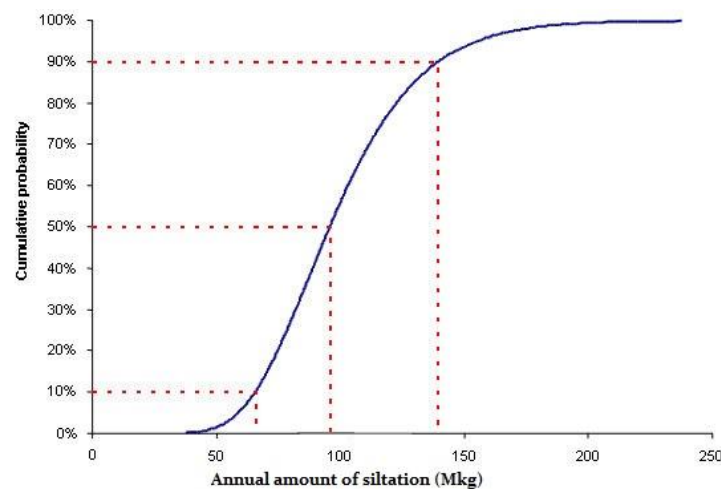


Figure 4: Example of a cumulative probability function of siltation.

A cumulative distribution function (CDF) of a distribution function X , evaluated at x , is the probability that X will take a value less than or equal to x . Thus, this example shows that the probability that the annual amount of siltation will be less than or equal to 100×10^6 kg is of the 50%.

1.2.3 Model study

In this study, we simulate port sedimentation using Delft3D. Delft3D is an open source, flexible, integrated modelling framework, developed by Deltares, simulating two and three-dimensional flow, waves, sediment transport and morphology, water quality and ecology and is capable of handling many interactions between these processes. Among the several modules of the software (see Figure 5) the used module is Delft3D-FLOW which is a multidimensional (2D or 3D) hydrodynamic simulation program which calculates non steady flow and transport phenomena that result from tidal and meteorological forcing on a rectilinear or curvilinear, boundary fitted grid.

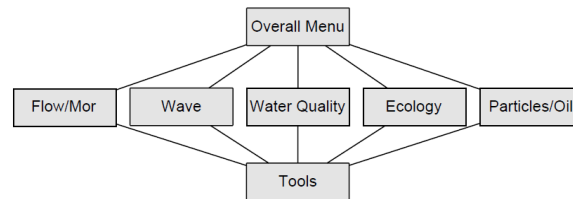


Figure 5: Delft3D Modules. (Deltares, 2014)

The original model can be found in Figure 6. The navigation channel has a depth of 20m MSL. The Boundary Conditions (BC's) consist of offshore tidal elevation on the left and the river discharge flowing into the system towards the navigation channel. The Initial Conditions (IC's) are: 0.25 m for water level, 35 ppt for salinity and 0 kg/m³ for concentration of mud.

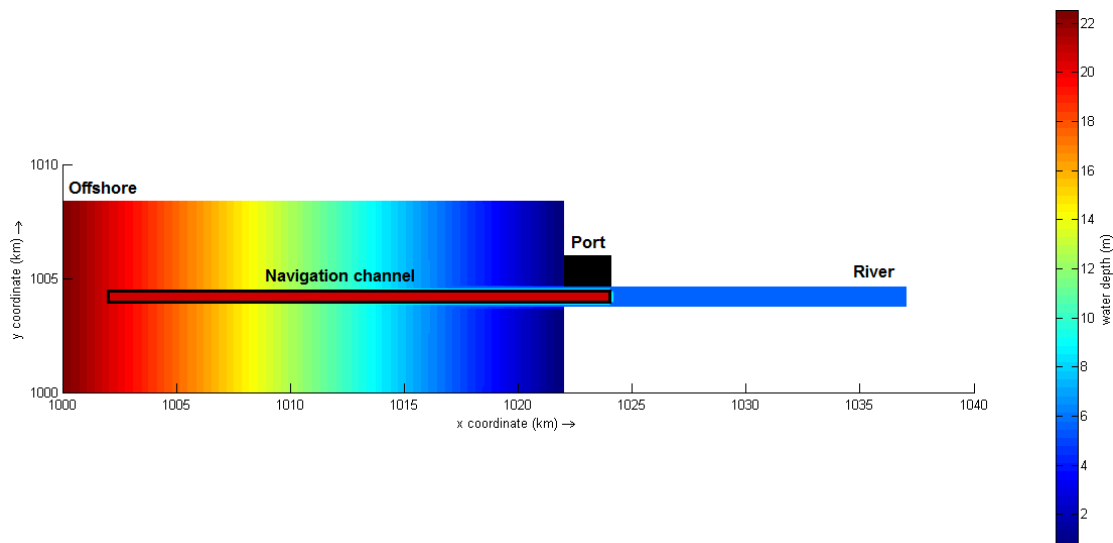


Figure 6: Schematization of the model.

In this study, three Delft3D models will be used, varying in number of horizontal grid cells. All models have eight layers over the vertical. The original model has sixteen grid cells over the width of the navigable channel, while the emulators feature eight and four grid cells. In order to differentiate from the two emulators, one will be called eight grid cells model and the other four grid cells model. The original model will be called sixteen grid cells model. Figure 7 shows a plan view of the three models where the navigable channel lies within the depicted red rectangle: it consists of sixteen, eight and four grid cells in the y-direction for the sixteen grid cells model, eight grid cells model and the four grid cells model, respectively.

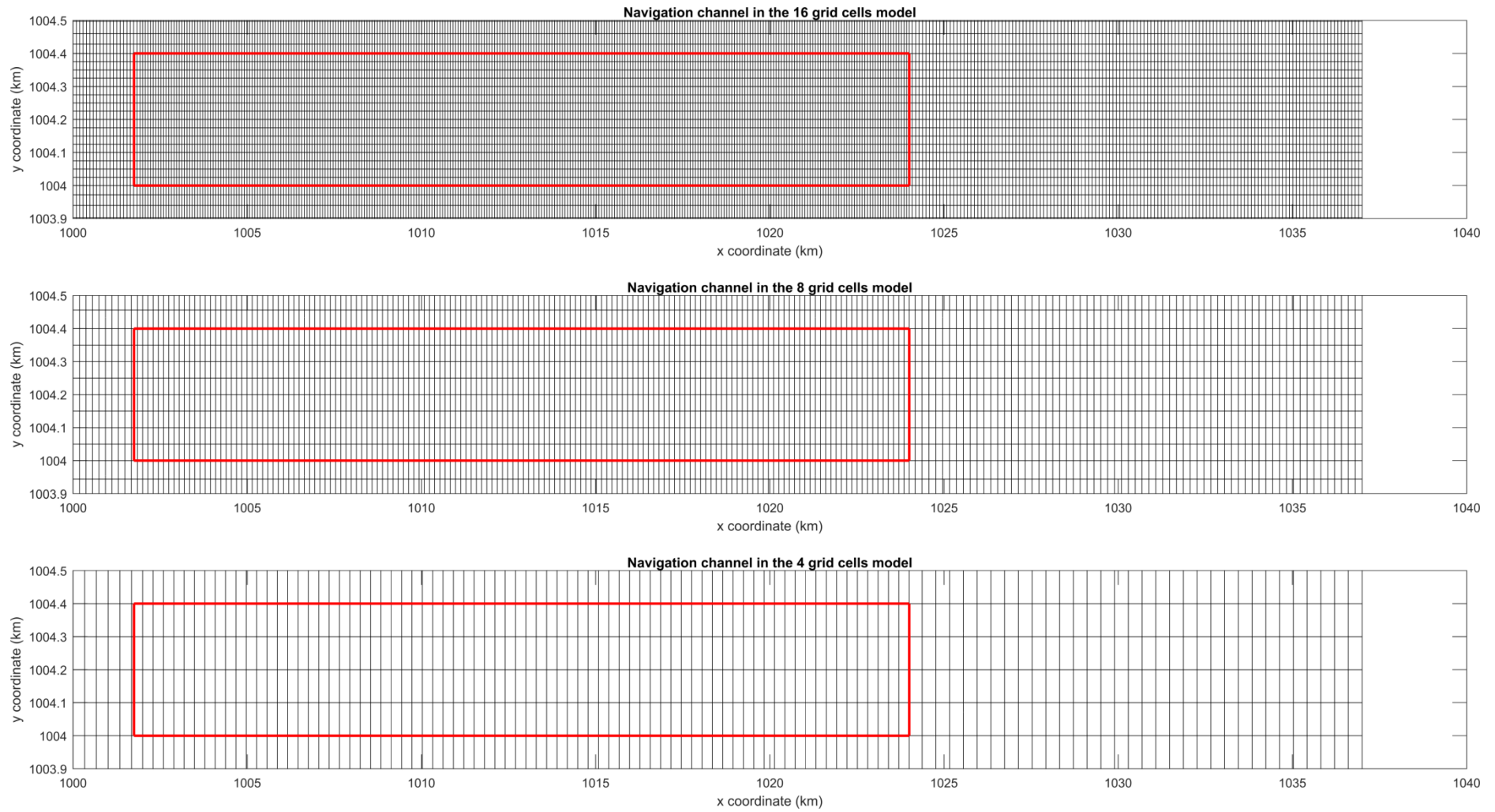


Figure 7: Difference in the number of grid cells in the navigation channel between the three models, not that the scaling is not 1 on 1 within this picture.

In the particular case analysed, sedimentation processes are mainly driven by currents (flow) and wind waves. These processes will be modelled in Delft3D by using the 'Flow' and 'Wave' modules (see Figure 8). These modules can be run simultaneously in the so-called 'online-coupling' mode, which allows interaction between the modules with a coupling interval that was set to one hour. The updated bottom, water level and velocity information are passed to the wave model and wave-induced forces, wave heights, periods and directions are returned to the flow module. For the purpose of this study, it was decided not to update the bathymetry because it is assumed that the port will be dredged continuously for maintenance purposes.

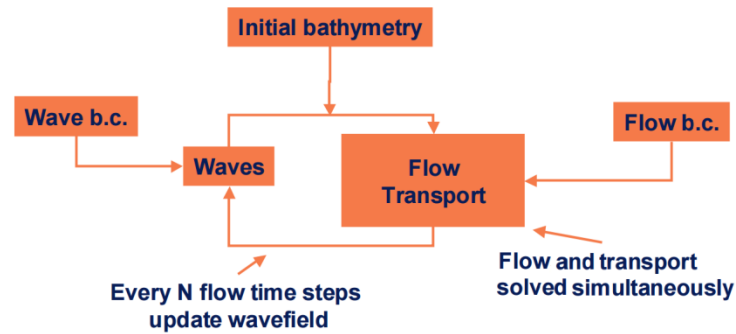


Figure 8: General chart of Delft3D system. Adapted from (Roelvink & Walstra, 2005)

Besides the hydrodynamic grid, there is also a wave grid defined in the Delft3D-WAVE for each of the three models. Figure 9 shows the wave and hydrodynamic grids defined in Delft3D-WAVE and Delft3D-FLOW, respectively. The wave grid is bigger than the hydrodynamic grid in order to guarantee that the wave information will be passed to the flow grid without problems.

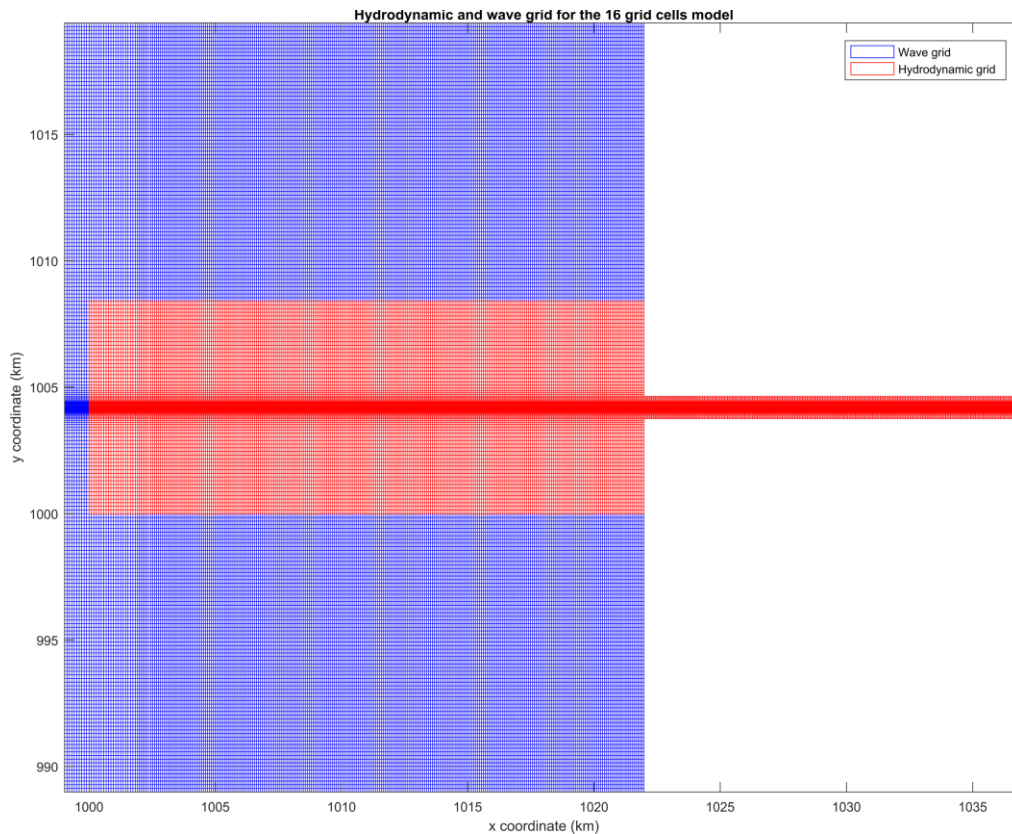


Figure 9: Wave grid (blue) and hydrodynamic grid (red) for the sixteen grid cells model.

As the bathymetry is not updated, the quantification of the amount of siltation in the channel is computed through the available mass within the uniform bed. The type of bed adopted is a uniformly well-mixed bed consisting of one layer of non-depleting muddy sediment. All the sediment is available for erosion. When sediments are deposited, they are added to the layer and when sediments are eroded, the layer exports sediments to the water column.

1.3 Research objective

The objective of this study is:

Can an emulator help estimating the uncertainty of navigation channel siltation in a guide and robust way?

Naturally there are several limitations and assumptions. First of all, the following study is applicable for a Delft3D model, which will be considered sufficient to represent the morphodynamic and hydrodynamics processes in port siltation. Second, the model and the emulators differ in the number of gridcells, while the underlying relations for hydrodynamic and morphodynamic processes are the same in both original and emulators models. At last, the chosen intervals for the parameters also restrict the applicability of this study.

1.4 Research questions

The goal is to answer the following research questions:

1. What are the uncertain input parameters with respect to a coastal morphodynamic model?
2. What are the most sensitive uncertain input parameters for a Delft3D model?
3. In what way does the amount of uncertainty in siltation differ between model and emulator?
4. Can a mapping function be applied to the emulators in order to map the amount of uncertainty in the siltation distribution computed by the model?

2 Coastal morphodynamic modelling

When modelling a coastal environment, several difficulties need to be taken into account. First, the interaction between hydrodynamics and sediment is very complex and poorly understood (Vollmers, 1989). Besides that, several of the parameters cannot have their values determined from the site due to factors like low budget for data acquisition, difficulty in measuring or its random behaviour. Despite all these difficulties it is still possible to make reasonable predictions of the expected amount of siltation in navigable channels, for example. A brief explanation about the hydrodynamic processes, sediment transport, siltation and uncertainties will be given in this chapter in order to make clear for the reader the steps taken towards the goal of this study.

2.1 Hydrodynamic processes

The nearshore hydrodynamics are important for sediment transport due to their capacity of stirring up sediments from bed and transport them along the surf zone. When waves come from deep to shallow waters, their characteristics as wave height and direction change. Shallow water processes include shoaling, wave breaking, wave orbital velocities and bed shear stress, which are the most relevant for the present work.

When waves come from deep water propagate perpendicularly towards the coast and reach a water depth less than about half the wavelength, their speed slows down due to the bed shear stress. Considering energy conservation, as the velocity goes down, the wave height increases: the waves experience shoaling. The ratio between wave height and wave length is called wave steepness. However, this increase in height is controlled by the dissipation due to wave breaking. The wave breaking occurs when the particle velocity exceeds the velocity of the wave crest causing the wave crest to become unstable.

Under wave surface, fluid particles describe an orbital path. Ideally, from the water surface to the bottom their trajectory changes from circular to ellipsoidal. At the same time, orbital diameter becomes smaller further down from the surface. Above the bed there is a turbulent layer called wave boundary layer which is approximately 5 to 10 cm. This turbulence is enhanced by bed roughness in combination with high flow orbital velocities. In this layer, the orbital velocity is responsible for varying the bed shear stress during the time which can set sediments on the bed into motion. The orbital velocity is zero at the bed and is equal to non-zero wave-averaged horizontal velocity (free-stream velocity) at the top of the layer. This velocity which has the same direction as wave propagation is extremely important for the transport of sediment onshore.

Accordingly, bed shear stress occurs due to currents and to waves. In order to determine the bed shear stress due to waves, Jonsson (1966) came up with the concept of wave friction factor which is a similar concept used to calculate the bed shear stress due to currents. However, while current friction factor is related to depth averaged current velocity, wave friction factor is related to free-stream velocity. The bed shear stress due to currents can be calculated as follows:

$$\tau_b = c_f \rho |\vec{U}| \vec{U} \quad (1)$$

$$c_f = \frac{g}{c^2} \quad (2)$$

$$|\vec{U}| = \frac{|\vec{u}_*|}{k} \ln \left(1 + \frac{H}{e z_o} \right)_0 \quad (3)$$

where τ_b is the bed shear stress due to currents, c_f is the current friction factor, ρ is the density of water, $\vec{U}$ is the depth-averaged flow velocity, g is the gravity and C is the Chézy friction coefficient, $\vec{u}_*$ is friction velocity due to currents and waves and z_o is bed roughness length.

In this study, the bottom roughness will be computed using the Manning's formulation. For a constant Manning value the bottom roughness increases rapidly with decreasing depth. As a result, the flow will be shifted to deeper water and the velocities nearshore will be reduced. This will greatly affect the longshore sediment transport.

$$C = \frac{\sqrt[6]{H}}{n} \quad (4)$$

where n is the Manning coefficient in N/m^2 , H is the water depth in m . The magnitude of the wave-averaged bed shear-stress due only to waves is

$$|\vec{\tau}_w| = \frac{1}{2} \rho_0 f_w u_{orb}^2 \quad (5)$$

where $\vec{\tau}_w$ is the magnitude of the bed stress for waves alone in N/m^2 , ρ_0 is the reference density of water in kg/m^3 , f_w is the total friction wave factor and the u_{orb} is the orbital velocity in m/s . For the orbital velocity the following formulation is used:

$$u_{orb} = \frac{1}{4} \sqrt{\pi} \frac{H_{rms} \omega}{\sinh(kH)} \quad (6)$$

and H_{rms} is the square root of the wave height in $m^{1/2}$, ω is the wave angular frequency in s^{-1} , k is the wave number. The bed shear stress due to waves and currents can be determined by several formulations: Feddersen et al., Fredsøe, Bijker, Grant&Madsen (see Figure 10). Among the shear stress formulations the Fredsøe formulation presented the most robust when compared to observation data. (Roelvink D., 2010) This formulation will be adopted in this study.

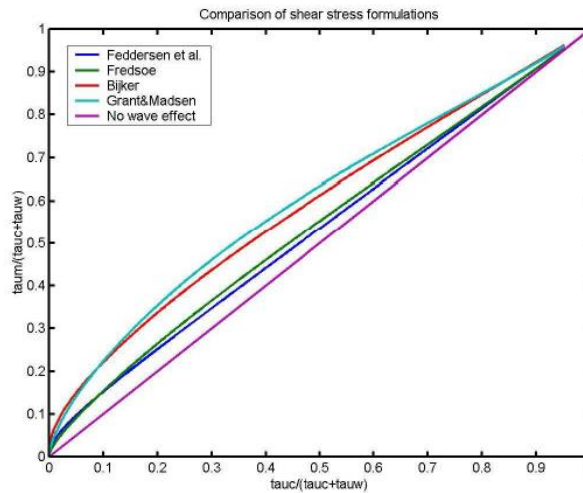


Figure 10: Comparison of mean shear stress due to waves and currents. (Roelvink D., 2010)

Soulsby et al. (1993) fitted one standard formula for these wave-currents models which each model has its own coefficients. The parameterization of Soulsby for the maximum bed shear-stress for combined waves and currents is

$$|\vec{\tau}_{max}| = Z(|\vec{\tau}_b| + |\vec{\tau}_w|) \quad (7)$$

2.2 Sediment transport

Sediment transport is a crucial bond between waves, currents and morphological changes. It is related with current, orbital velocities, sediment properties, bed forms and bed roughness. The sediment transport can be divided into bed load transport and suspended load transport. The bed load transport takes place through sediments which roll, shift or make small jumps over the seabed, staying in contact with the bed. The suspended load transport occurs via sediments which are lifted up above the critical flow velocity and are transported in suspension by the flow. If the flow velocity decreases until a certain value, then the suspended sediments settle down.

As previously mentioned, the sediment transport is related to the sediment properties. In fact, the sediments can be divided into two main groups: cohesive and non-cohesive. This classification is important to sediment transport because the upward sediment flux from the bed into the water column strongly depends on the bed cohesiveness. This cohesiveness is related to the capacity of certain materials to stick together due to chemical constitution which will increase greatly the resistance against erosion. Sand is a non-cohesive material while silt, clay and organic materials are cohesive materials. The fluid-sediment mixture of (salt) water, silt, clay and organic materials known as mud is a cohesive material.

The upward sediment flux from the bed into the water column is commonly accepted to be controlled by the critical bed shear stress of the bed material and the actual bed shear stress. If the actual bed shear stress is bigger than the critical bed shear stress, erosion will take place. On the other hand, if the actual bed shear stress is smaller than the critical bed shear stress, deposition will take place. In case of non-cohesive sediments, the critical bed shear stress for erosion and sedimentation are the same. As a result, the transport of sand can be well predicted by empirical sediment transport formulas as Van Rijn, Engelund-Hansen and Meyer-Peter-Muller. In case of cohesive materials, the critical bed shear stress for erosion is larger than the critical bed shear stress for sedimentation. In case the actual bed shear stress is between the critical bed shear stress for erosion and for sedimentation there will be no exchange with the bottom layer. In fact, it is not commonly accepted the existence of a critical shear stress for deposition (Winterwerp, 2004).

According to Eq. 8 it can be observed that the change of the bed level is controlled by the rate of deposition and erosion:

$$(1 - \varepsilon) \frac{\partial z_b}{\partial t} + \frac{\partial S_x}{\partial x} + \frac{\partial S_y}{\partial y} = D - E \quad (8)$$

$$E = M \left[\frac{\tau}{\tau_{cr,e}} - 1 \right] \text{ for } \tau > \tau_{cr,e} \quad (9)$$

$$D = w_s C \left[1 - \frac{\tau}{\tau_{cr,d}} \right] \text{ for } \tau < \tau_{cr,d} \quad (10)$$

where ε is the porosity, S_x and S_y are the sediment transport in the x- and y-direction, D is the deposition rate of suspended sediment, E is the erosion rate of suspended sediment, M is the erosion parameter, τ is the actual critical bed shear stress, $\tau_{cr,e}$ is the critical bed shear stress for erosion, $\tau_{cr,d}$ is the critical bed shear stress for sedimentation, w_s is the settling velocity and C is the concentration of sediments in the water column.

In a coastal environment the sediment transport on cross-shore and alongshore direction has an important role on the morphodynamics system. Wave orbital motion is very important for the sediment transport on cross-shore direction. This type of sediment transport is essential to stir up the sediments and keep them into suspension in the water column. But it does not contribute to actually transport these sediments along the coast. That is done by the alongshore sediment transport. Regarding it, the

wave-induced surf zone longshore flow is important because it produces a longshore current along the coast. The wave-induced flow is determined by wave characteristics as wave height, period and direction.

2.3 Channel sedimentation

In ports the navigable channels unfortunately work as ‘sediments trap’ which increases the maintenance costs due to the need of maintaining the required channel depth for the ships to enter in the port. In the channel the velocities become smaller due to the higher water depth, causing a reduction in the sediment transport capacity. Consequently, the bed load particles and part of the suspended load particles will be deposited in the channel. In addition, the gravitational effect induces a downward force on bed-load particles on the side slopes of a channel. On the other hand, the dynamic nature of the channels produces shifting shoals and banks (Van Rijn, 2013). Figure 11 shows the main factors responsible for the sedimentation in a navigation channel. For the studied navigation channel, the currents caused by tides and waves also approach alongside the channel as showed in Figure 11.

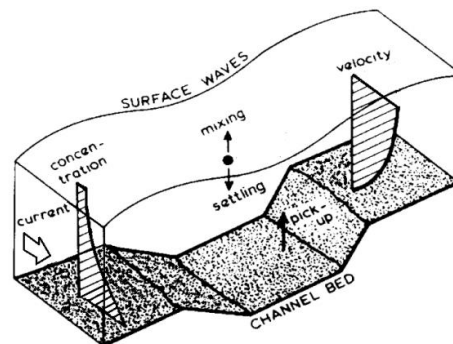


Figure 11: Channel sedimentation. Adapted from (Van Rijn, 2005).

If the waves are included then more sediment is expected to be deposited in the channel due to the capacity of the waves to stir up sediments which will be transported by the flow. Therefore, the amount of sedimentation that may take place in a navigable channel depends on the rate of sediment transport, which is controlled by the flow, the waves, the sediments approaching to the channel, and the trapping efficiency of the channel influenced by the geometry, dimensions, channel orientation and the sediment characteristics.

2.4 Uncertainties

Although the idea of uncertainty is quite familiar to most of the people its sources are not always easy to identify. A general definition of uncertainty would be: “a situation in which something is not known, or something that is not known or certain” (Cambridge, 2016). But in environmental modelling, in his first chapter brings two classical definitions of uncertainty: one derived from set theory and one from statistics. Conforming to the first definition, uncertainty appears when there is a set of possible alternatives when only one is required. And conforming to the second definition, there is a universal set of alternatives which can be expressed in terms of some measure, in the range from 0 to 1. For Beven (2009) both definitions have their limitations. After an overview of the various definition of uncertainty, he observes that having a different definition will lead to different ways of representing

sources of uncertainty in the modelling process, which in turn will result in different uncertainty estimation of the model output. He conclude that in real applications the most important aspect of uncertainty estimation is to make better decisions, and that different methods of uncertainty estimation are necessary for different types of problems.

According to Scheel et al. (2014), the imperfect description of the physics in the model, the inability to accurately define model input parameters and the natural variations in the model forcing have introduced uncertainty in morphological model predictions. Therefore, identification and quantification of those uncertainties is not only important to better understand the studied morphological system, but also to be able the decision maker to take risks informed decisions. Fortunato et al (2009) also realized the increase importance of uncertainty in determining the limits of predictability of morphodynamics models due to the increase owed to the growing of computational resources. For Scheel et. al. (2014) the sources of these uncertainties are (see Figure 12):

- Forcing uncertainty: it is an inherent uncertainty in the forcing condition due to its natural variation in space and time. For example: storms, disasters, climate changes, etc;
- Parameter uncertainty: it is an uncertainty in (physical) model parameters. It occurs due to the absence of enough data and knowledge about the empirical formulas, parameters resemblance with reality, measurement techniques, initial conditions, etc;
- Model uncertainty: it is an epistemological uncertainty due to the incompleteness of conceptualizing a real system into a model. For example, errors in numerical methods, underlying assumptions, limited processes, etc;
- Unknown uncertainty sources: it is inherent uncertainty which comes from unknown and unpredictable sources. For example, human errors, wrong assumptions, etc.

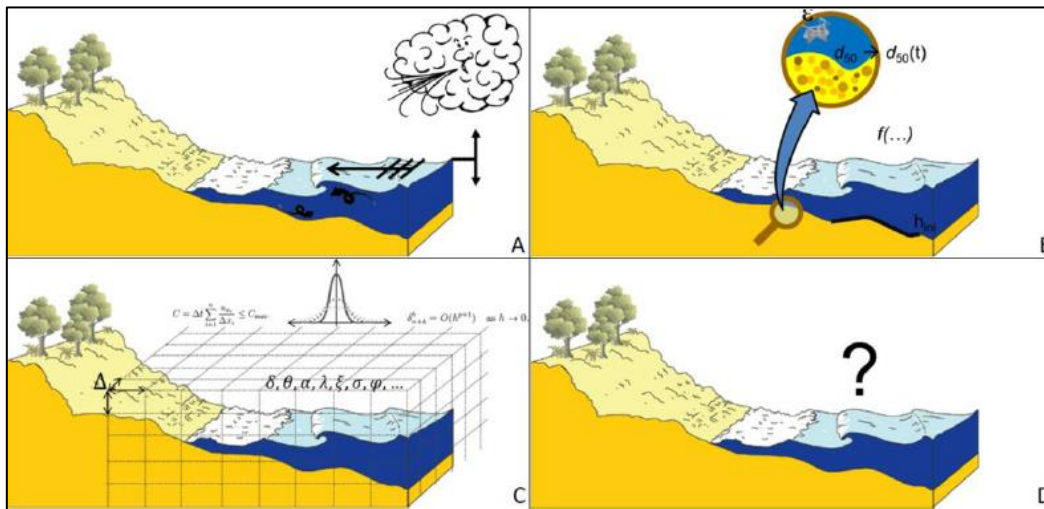


Figure 12: Sources of uncertainty in morphodynamics models. A) Forcing Uncertainty. B) Parameter uncertainty. C) Model uncertainty D) Unknown uncertainty sources (Scheel, et al., 2014).

As it can be noted, most of these uncertainties cannot be eliminated due to its inherent aspect of the model or the system. However, the quantification of these uncertainties will contribute to make clearer the limits of predictability of morphodynamics models for both decision makers and scientists.

3 Methodology

The methodology applied is based on the workflow suggested by Scheel et. al. (2014) is shown in Figure 13. This workflow was expanded with the final step “Identifying mapping function” in order to take into account the last step of this study.

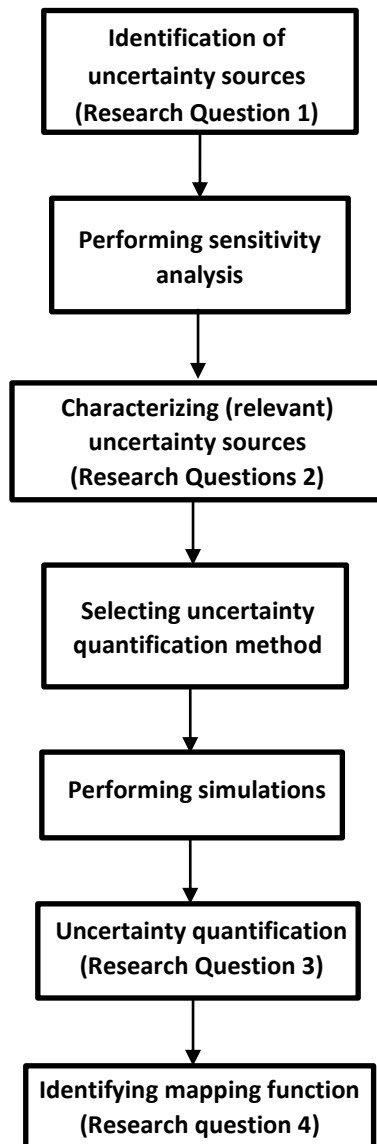


Figure 13: Adopted uncertainty analysis workflow.

A brief overview of each step can be found in this chapter.

3.1 Identification of uncertainty sources

Numerical models need input parameters in order to give the output of interest. The number and type of input parameters depend on the assumptions made by the modeller. The user is interested in including all the input parameters which are relevant to describe the phenomenon analysed.

As it was mentioned before, modelling is not an easy task due to the uncertainties in many of its aspects, including in the input. In order to deal with modelling uncertainties, a previous sensitivity analysis and uncertainty analysis are recommended in several disciplines. While sensitivity analysis focus on how uncertainty in the output of model can be apportioned to different sources of uncertainty in the model input, the uncertainty analysis focus on quantifying these uncertainties in model output (Baveye, et al., 2007) . The importance of sensitivity analysis in disciplines as applied econometrics is illustrated by Peter Kennedy, who stated “Thou shall confess in the presence of sensitivity” which can be translated into “Look at uncertainties before going public with findings”.

Among the several sources of uncertainty mentioned by Scheel et al. (2004), the parameter uncertainty will be the focus of this study. In that case, it is necessary to take a look into the model and identify the uncertain input variables. In Appendix 1, most of the input variables of the Delft3D-FLOW and Online Delft3D-WAVES for the model are listed. Several input variables are constant (e.g. gravity) and others were assumed to be constant (e.g. water density) for simplicity.

The first filter to eliminate non important parameters was the knowledge from an expertise who has been modelling coastal areas during years and have gained insight on which parameters and processes can be discarded due to their small influence on model results. In different areas the subjective analysis has been showed to be a valuable tool for uncertainty analysis including Cooke, et al. (2002), Morgan, et al. (1995) and Evans et al. (1994). In this study, this method was used in order to reduce the number of input variables to a feasible size. In view of that, an interview with Freek Scheel, a consultant and researcher in coastal engineer at Deltares, was carried out in order to discuss the uncertain input parameters in a Delft3D coastal morphodynamics model. In addition, but also a research in literature was done.

3.2 Performing sensitivity analysis

Here the goal is to analyse how much the model output is affected while changing the input variables. For that, a sensitivity method is applied. From all existing sensitivity methods it was chosen to use the one-at-a-time method (OAT). This method consists of varying one parameter at time while keeping the others fixed (Hamby, 1994). The way in which the number and values of the parameters have to be chosen is dependent on the type of model output of interest. For this study, the extreme values of the variables (e.g 1st and 99th percentiles) are not important. On the contrary, an overview of the whole output distribution is more interesting. With this intention, while one variable is set to the 10th or 90th percentile of its distribution, the others will be left constant, by that, to the 50th percentile of their respective distribution as schematized in Figure 14.

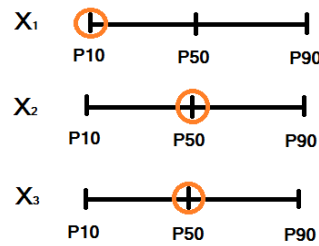


Figure 14: One-at-a-time method (OAT) for the studied case. Here, X_1 , X_2 and X_3 are model input parameters which values are changed one at a time in order to investigate how the model's output reacts to those changes. The values which these parameters will be set are the 10th, 50th and 90th percentile of their respective distribution.

The advantages of this method are: simplicity in implementation, computationally inexpensive and obtention of a rough idea of model's behaviour (Saltelli, 1999). The disadvantages are: assumptions of no correlation between the variables and linearity of the model. These two assumptions are discussed below. Firstly, it is assumed that the input parameter can be independently determined. Although the required critical shear stress to set non-cohesive sediments into motion decreases with the particle size, for cohesive sediments this is not a straightforward linear relation (Brown & Bearman, 1999). Under those circumstances, it can be assumed no correlation between settling velocity and critical shear stress. The other variables do not present correlation among them. Second, the model is clearly not linear due to the complexity between the inputs, for example the interaction of waves and current bed shear stress (Deltares, 2014). Due to that, the implementation of a 'full sensitivity analysis' cannot be considered, but instead a 'parameter screening' is implemented. The last consists on determining which parameters are more important, but not quantifying the sensitivity exactly.

3.3 Characterizing (relevant) uncertainty sources

Here the results of last step are analysed. Firstly, the variables which the model output is more sensitive to are chosen. This sensitivity of the model output is expressed by its distribution range. Thus, the models which the output present big ranges are more sensitive to a certain parameter variation than the models which output remained constant for the same parameter variation. Afterwards, the same samples are used as model input in the sensitivity analysis for the three models and as a consequence a comparison between the amounts of siltation calculated by them is made. Due to coarsening of the grid, it is expected a different amount of siltation between the model and the two emulators.

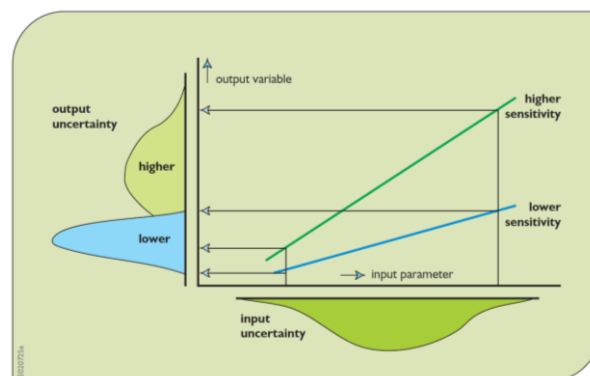


Figure 15: Schematic diagram relating input parameter uncertainty to model output uncertainty sensitivity. (Loucks & van Beek, 2005)

3.4 Selecting uncertainty quantification method

The uncertain quantification method chosen is the Latin Hypercube Sampling (LHS) which was described by McKay et al. (1979). The LHS method is a type of Monte Carlo, a sampling method that generates random samples from the probability distributions. The main advantage of LHS when compared to crude Monte Carlo is the more equally division of the sample probability distribution in order to guarantee that the whole probability distribution is sampled. First, the range of each variable is divided into n non-overlapping intervals of equal probability called strata. After, the values of these variables are randomly chosen in a way that each range is sampled only once. Later, these values are randomly combined to n sets of Latin Hypercube samples. In the end, each of the n samples contains only one value for each variable (see Figure 16).

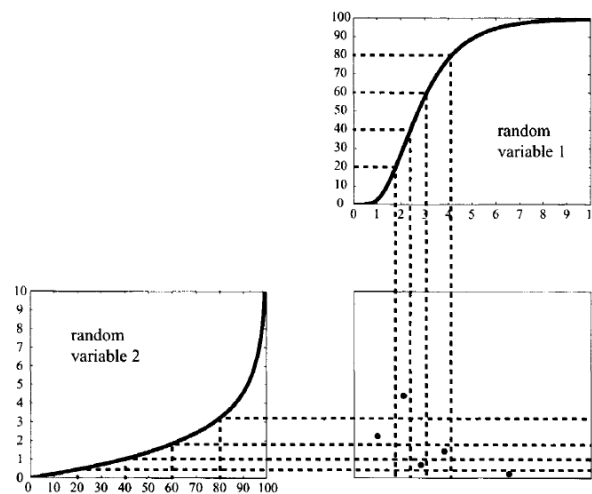


Figure 16: Latin Hypercube Sampling scheme. (Van der Klis, 2003)

These samples work as different input scenarios for the model which will generate outputs equi-distributed over the desired output distribution. Among the many advantages of LHS are :

1. Unbiased estimates for means and distribution functions;
2. Dense stratification across the range of each sampled variable;
3. Uncertainty and sensitivity analysis results obtained with Latin Hypercube sampling have been observed to be quite robust even when using relatively small samples (i.e $n = 50 - 200$);
4. The means and distribution functions estimated tend to be more stable than in the Crude Monte Carlo. In other words, the amount of variation between results obtained with different samples generated by the particular sampling technique under consideration is smaller.

The disadvantage is the fact that its accuracy cannot be measured differently than in the crude Monte Carlo, which different formulations can be found in literature (Morgan & Henrion, 1990). For that, it would be necessary to repeat the LHS several times and therefore, this would attenuate its advantage of using a smaller sample size (Van der Klis, 2003).

It is necessary to choose n intervals which the probability distribution of the variables will be divided into equal probability. Initially the probability distribution of the most sensitive uncertain input parameters was divided by $n = 80$ and as a consequence that is also the number of the sample size.

Although there is no formulation which recommends the sample size for LHS, it can be found in several studies as in Chang, et al. (1993), Yeah, et al. (1993), Christiaens, et al. (2001) and Van der Klis (2003) that a sample size of 80 is a reasonable number.

Due to the different computational time of the models, different sample sizes were adopted. For a single run, the sixteen grid cells model takes approximately three days, the eight grid cells model takes around one day and the four grid cells model takes approximately five hours. Consequently, it is not feasible to use a sample size of $n = 80$ for the sixteen and eight grid cells model.

Thereupon the four grid cells model used the original sample size of 80 samples, while the eight grid cells model and the sixteen grid cells model used subsamples of the original sample size of 40 and 20 samples. An example of LHS subsamples $n = 4$ and $n = 2$ from an original LHS sample $n = 8$ can be found in Figure 17.

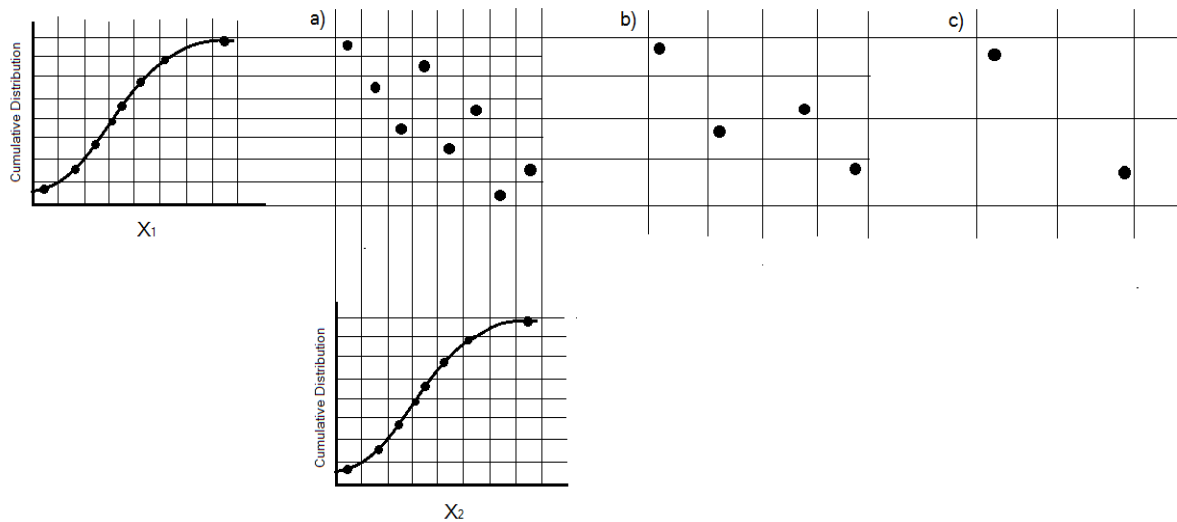


Figure 17: Example of subsampling from LHS. A) It is the original LHS ($n=8$). B) It is a subsample of the original LHS ($n=4$). C) It is subsample of the last one ($n=2$).

Despite the different subsample sizes, both come from the original one. As a result, it is possible to compare the cumulative distribution function of the siltation of the three models. The final LHS runs can be found in the scheme in Figure 18. In this scheme, the columns represent the three sample sizes initially adopted (80, 40 and 20 samples), the rows are the three models (sixteen grid cells model, eight grid cells model and four grid cells model), the symbol X represents the known siltation data and $?$ the unknown siltation data.

16	X	X	?
8	X	X	?
4	X	X	X
	20	40	80

Figure 18: Scheme for the LHS runs from the sensitivity analysis.

3.5 Uncertainty quantification

For the uncertainty quantification, the results from the LHS runs are used to fit a probability distribution function and to build the cumulative distribution functions and the box plots of the amount of siltation in the navigation channel for the three models. Afterwards, these uncertainty quantifications are compared as illustrated in Figure 19a. The verification is done by comparing these amounts of uncertainty between the samples sizes, $n=40$ and $n=20$, showed in Figure 19b.

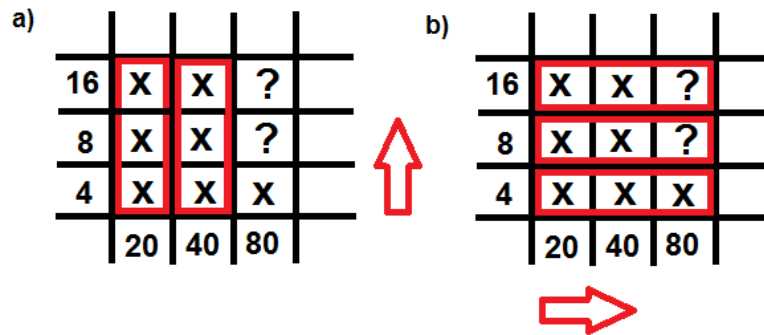


Figure 19: Scheme adopted in uncertainty quantification. a) Comparison among the different models. b) Verification between different samples sizes.

Because it was only possible to run the 80 samples in the four grid cells model, this sample will not be included in the further comparisons.

3.6 Identifying mapping function

For each of the three sample sizes the amounts of siltation calculated by the three models are plotted two by two. A polynomial fit is applied in order to build a mapping function between the two respective models. These mapping functions are compared and verified using the same scheme from the uncertainty quantification. Additionally, the parameters from the particular probability distribution function found in the previous step are also compared and verified using the same scheme.

4 Results

4.1 Identification of uncertainty sources

After the interview and the literature review, five uncertain input variables are chosen from the thirty six input variables found in the two tables in Appendix 1. The uncertain input parameters can be found in the following table:

Table 1: Uncertain input parameters.

Uncertain Parameter	Unit	P10	P50	P90	Data
Settling Velocity	ms^{-1}	0.1	0.5	1.0	Normal
Bed roughness	Nm^{-2}	0.018	0.020	0.22	Normal
Wave height	m	0.71	1.20	1.30	Data-based
Discharge	m^3s^{-1}	0.24	1.24	110.60	Data-based
Critical Shear Stress for Erosion	Nm^{-2}	0.4	0.5	0.6	Normal

Table 1 also shows the type of data and the 10th, 50th and 90th percentiles of the probability distribution or data series for each parameter. It was assumed that the settling velocity, bed roughness and critical shear stress have a normal distribution. In relation to the wave height and the discharge, time-series are provided by Deltares. Figure 20 and Figure 21 show the distribution and the 10th, 50th and 90th percentiles for the wave height and the discharge, respectively. As previously mentioned in 'Study Case' the location of the port cannot be revealed, and as consequence the same applies to the locations where both time-series were measured.

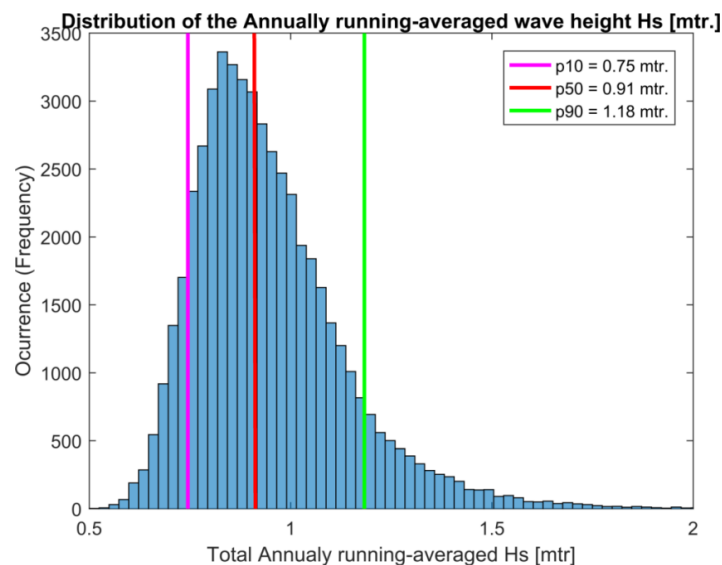


Figure 20: Distribution of the wave height at port's location.

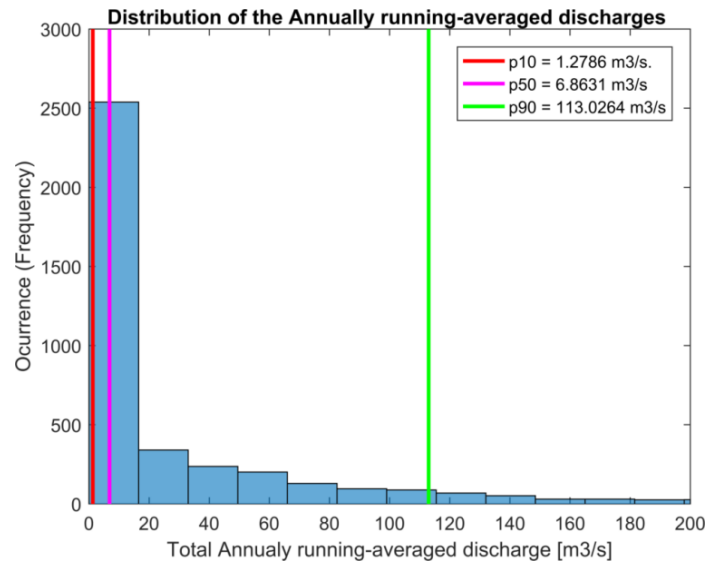


Figure 21: Distribution of the discharge at port's location.

4.2 Performing sensitivity analysis

Within the OAT method, 11 runs for each model were performed resulting in a total of 33 runs. For the three model, a reference case was simulated in which all the uncertain input variables are set to their median value of their respective distribution. The values used in the reference cases can be found in Table 1. The run period is set from 01-07-2014 to 01-08-2014.

4.3 Characterizing (relevant) uncertainty sources

An error bar plot is generated in order to show the results from the sensitivity analysis. Figure 22 shows the different values of siltation inside of the channel when varying the bed roughness for the four grid cells model. For example, if the value of bed roughness is set to its 10th percentile while the other four uncertain input parameters are set to their median values, the siltation will be approximately 10.0×10^6 kg (represented by the magenta line in Figure 22).



Figure 22: Error bar of the bed roughness for the four grid cells model.

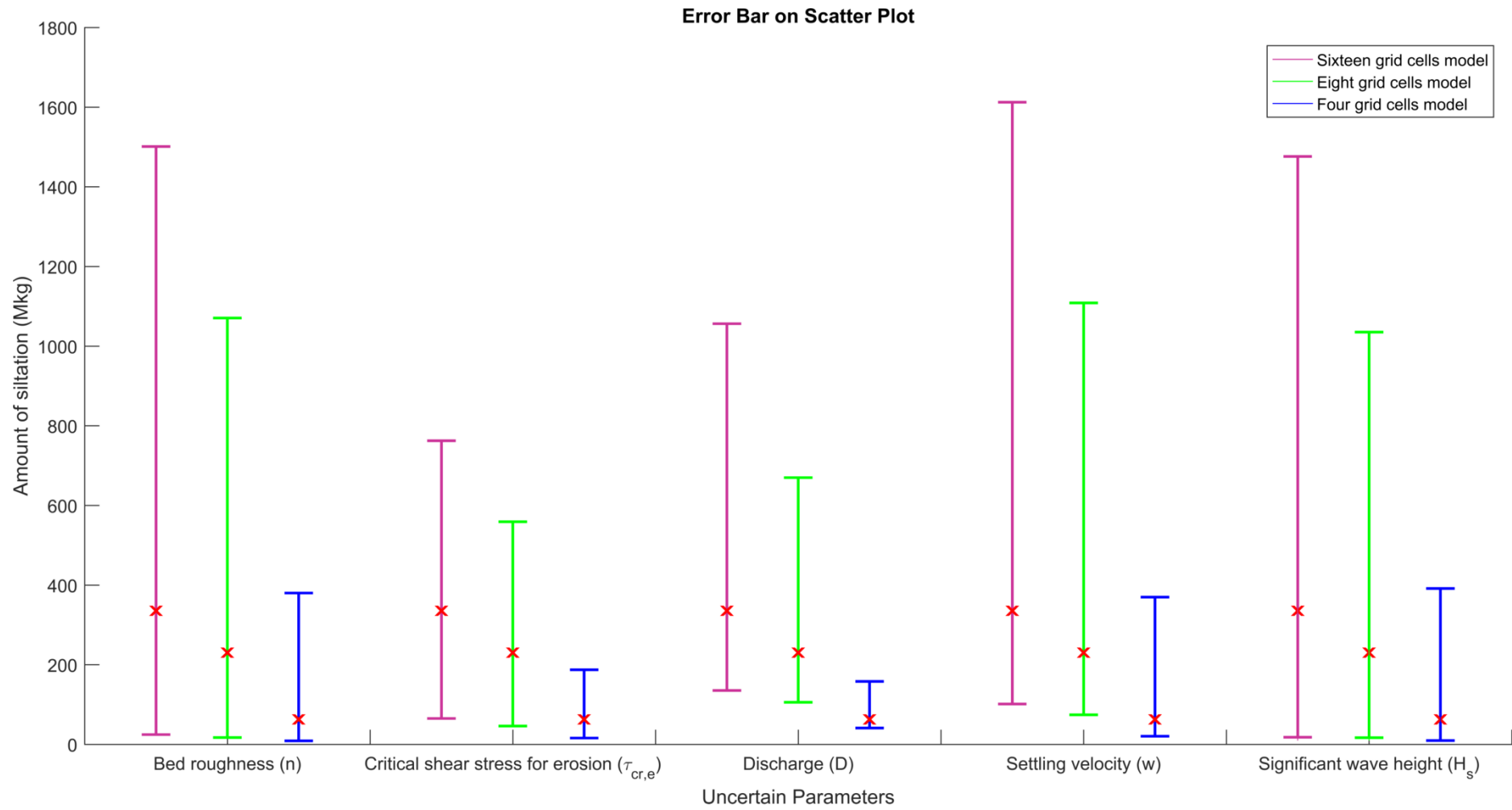


Figure 23: Results from the sensitivity analysis using the OAT method.

From the sensitivity analysis results showed in Figure 23, three important conclusions can be made:

1. **The three most sensitive parameters are the bed roughness, the settling velocity and significant wave height.** As it can be observed in Figure 23, the siltation is more sensitive to the variation of the bed roughness than the critical shear stress. Given that, when the bed roughness varies the amount of siltation becomes more distant from its reference value. As a result, the siltation is more sensitive to the bed roughness than to the critical shear stress. The same happens to the settling velocity and the significant wave height. The three models showed the same results.
2. **It can be observed that two factors seem to dominate the output sensitivity: the resolution (among the models) and the parameter range (among the parameters).** Table 7 found in Appendix 2 brings the exact values of the total amount of siltation for the three models obtained in the sensitivity analysis. The resolution factor is related to the difference in the amount of siltation from the model to the four grid cells model. The resolution factors are represented by $fr_{16,8}$, $fr_{8,4}$ and $fr_{16,4}$ and are found in Table 7. In the table below, Table 2, the resolution factors for the reference case are presented.

Table 2: Resolution factors for the reference cases.

Resolution factor ($fr_{i,j}$)	Value [-]
$fr_{16,8}$	1.5
$fr_{8,4}$	3.7
$fr_{16,4}$	5.3

These factors are computed using the following formulation:

$$fr_{i,j} = \frac{S_i}{S_j}$$

where S_i is the total amount of siltation calculated by the model i and S_j by the model j , in which i and j are the sixteen, eight and four grid cells models. The second line of Table 7 shows that most variables obey a factor of 1.5 from the sixteen grid cells model to eight grid cells model for 50th, 10th and 90th percentiles. However, the same does not happen from the eight grid cells model to the four grid cells model and for the sixteen grid cells model to the four grid cells model. Table 8 in Appendix 2 brings information about the parameter range factors which can be defined as:

$$fp_{I,m} = \frac{S_{I,P90}}{S_{I,P10}}$$

where I is the uncertain input parameters (see Table 3), m represents the three models, S is the amount of siltation when varying one of the uncertain input variables I for a respective model m and $P10$ and $P90$ are the 10th and 90th percentiles of the distribution for the uncertain input parameters.

Table 3: Variable I from the parameter range factor.

Variable I	Symbol
bed roughness	n
critical shear stress for erosion	$\tau_{cr,e}$
discharge	D
settling velocity	w
significant wave height	H_s

Table 4 shows that the uncertainties in model input are translated to the model output in a similar way to the sixteen grid cells model and eight grid model. For example, the ratio between the siltation amounts when the bed roughness set to its 90th distribution percentile and its 10th distribution percentile is similar two models. By that, both models present the same magnitude of sensibility, or range, when varying the input parameters. Nonetheless, the four grid cells model is less sensitive than the other two, except for the settling velocity.

Table 4: The range parameter factors for the three models and five uncertain input parameters.

Parameter range factor ($fu_{I,m}$)	Value [-]
$fp_{n,16}$	60.2
$fp_{n,8}$	60.8
$fp_{n,4}$	41.2
$fp_{\tau_{cr,e},16}$	11.7
$fp_{\tau_{cr,e},8}$	12.0
$fp_{\tau_{cr,e},4}$	11.5
$fp_{D,16}$	7.8
$fp_{D,8}$	6.3
$fp_{D,4}$	3.8
$fp_{w,16}$	15.9
$fp_{w,8}$	14.9
$fp_{w,4}$	17.8
$fp_{H_s,16}$	81.0
$fp_{H_s,8}$	60.7
$fp_{H_s,4}$	39.5

In conclusion, both sixteen grid cells model and eight grid cells model present similar resolution and parameter range factors.

3. **There is a decrease in the total amount of siltation calculated from the sixteen to the four grid cells model.** In fact, if all the uncertain parameters are set to their median value, the total amount of siltation for the four grid cells model is around 60×10^6 kg, for the eight grid cells model will be 230×10^6 kg and for the sixteen grid cells model will be 330×10^6 kg. Thus, some of the morphodynamics and/or hydrodynamics processes seem to have not been well calculated with coarsening of the grid in the x- and y- directions. However the difference between the sixteen grid cells model and the eight grid cells is an order of 1.5 while from the sixteen to the four grid cells is an order of 5. For the purpose of understanding this difference in the amount of siltation calculated by the three models, a deeper look into the

morphodynamics and hydrodynamic processes for siltation in muddy areas is done in the following topic.

In relation to the last two aspects, the four grid cells model presented results considerable poor and because of that this model will not be used in the uncertainty quantification and consequently, in the identification of mapping functions.

4.3.1 Investigation

According to Van Maren (2009) the transport of mud depends on the fluid dynamics mainly driven by bed shear stress, flow velocity profile and sediment concentration depending on the erosion rate, availability of sediments and age of the sediments. The initial sediment layer and running period are the same for all the models, consequently the availability and the age of the sediments are also the same. The process of siltation in muddy areas consists of a series of important morphodynamic and hydrodynamic processes. First, due to the bed shear stress the bed is eroded, second the eroded sediments are lifted up by the orbital velocity and brought into suspension, and then these sediments will settle down caused by the settling velocity

The average and maximum values of previously mentioned important parameters for siltation in a navigation channel for the three models are analysed. These parameters are the maximum bed shear stress due to currents and waves, the bed shear stress due to current, the orbital velocity and the horizontal velocity at the bed which plots are in Appendix 4. It is important to quantify this difference between the models within the intent of finding a link between the variables and the difference in the amount of siltation calculated by the models.

However, the models have different numbers of grid cells in which turns the use of ratio between the models prohibitive. With this in mind, an interpolation method was used with the purpose of having the three models with the same number of grid cells and then comparing the results between them. Nevertheless the interpolated values in the right and below boundaries in the eight and four grid cells models present errors caused by the difference when defining the values and the locations of these parameters in the grid cells (see Figure 24).

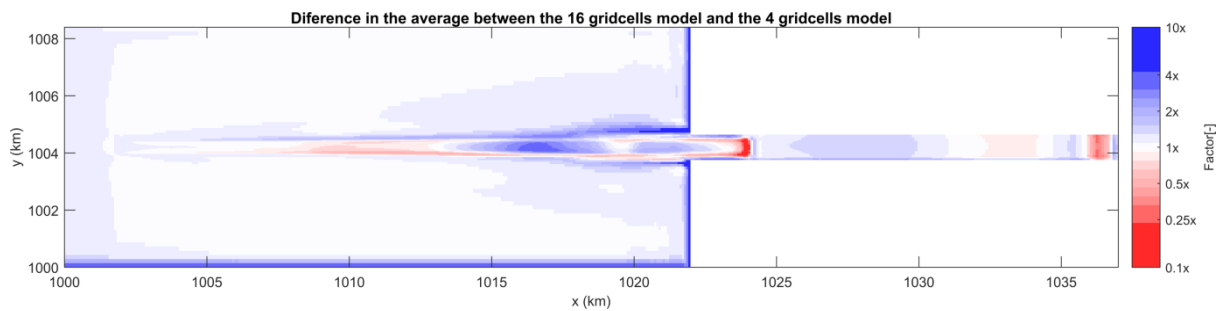


Figure 24: Error in the above and right boundaries of the studied area due to the linear interpolation.

The comparison is given by the ratio between the average values of parameter P calculated by the sixteen grid cells model and the model m , represented as $mean\left\{\frac{P_{16}}{P_m}\right\}$, and by the ratio of the maximum of the same values, represented as $max\left\{\frac{P_{16}}{P_m}\right\}$, both averaged in time. The parameter P represents the

maximum bed shear stress due to currents and waves, the bed shear stress due to currents, orbital velocity or horizontal velocity nearby the bed. The plots comparing the average and maximum values of these four parameters between the models are in Appendix 5. In these plots a special attention is given to the values locate nearby and inside the channel and in the surf zone because at those locations the important processes responsible for siltation occurs as wave breaking and cross shore and along shore sediment transport.

In summary, the average and maximum values calculated for these four parameters by the eight grid cells model and sixteen grid cells model are relatively similar. In contrast, the four grid cells model presented much smaller values for these parameters in those locations. In the four grid cells, both maximum and average values for the maximum bed shear stress due to currents and waves and the orbital velocity are greatly underestimated. These two parameters are ten times smaller in some of those locations in the four grid cells model in relation to the values in the six grid cells model. This might have been caused by coarsening of the wave grid. Due to the wave forcing being poorly computed, the erosion of the bed was underestimated and that explains the considerable small amount of sediments computed by the four grid cells model observed in the sensitivity analysis.

4.4 Selecting uncertainty quantification method

The LHS is chosen to quantify the uncertainty in the output. The idea is to use LHS subsamples from the original LHS sample due to the expensive computational time required by the eight grid cells model and the sixteen grid cells model. The original sample has 80 samples in which 40 samples are chosen, and from this subsample, 20 samples are chosen to the end that the three models have a common sample space.

The original sample is represented by a matrix with 80 rows and 3 columns. Each row consists of three different cumulative probabilities values for the three parameters sampled by the LHS method. The first, second and third columns correspond to the sampled cumulative probabilities of the bed roughness, settling velocity and significant wave height distributions, respectively. For the bed roughness and settling velocity a normal probabilistic distribution are assumed and for the wave height a generalized extreme value probabilistic distribution is chosen to be valid (see Figure 25).

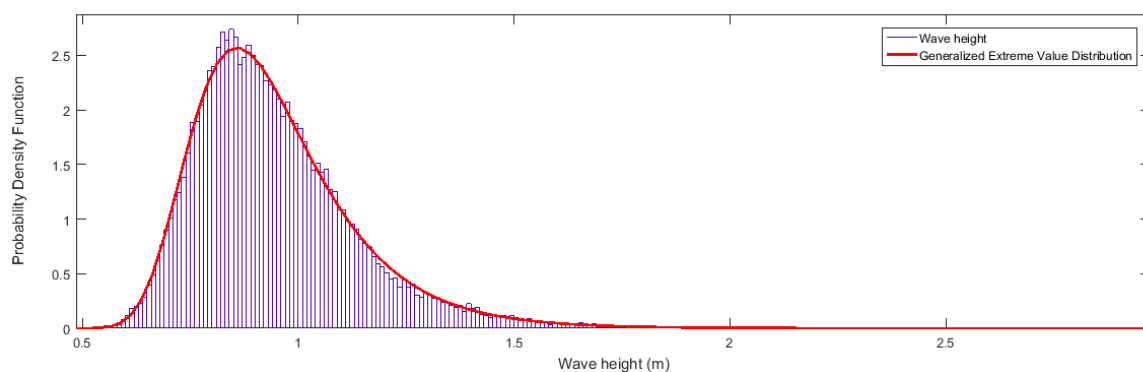


Figure 25: Generalized Extreme Value distribution fit.

Looking carefully at Figure 26, it can be noticed that each column and each line have one value. That is to say, that both settling velocity and bed roughness distributions before divided into 80 parts with equally probability are sampled. This plot is a 2D-LHS sampling arrangement for two of the three parameters. The other two 2D plots and a 3D plot can be found in Appendix 3. The next step after generating the 80 samples is to generate the 40 subsamples from the 80 samples and at last the 20 subsamples from the 40 samples.

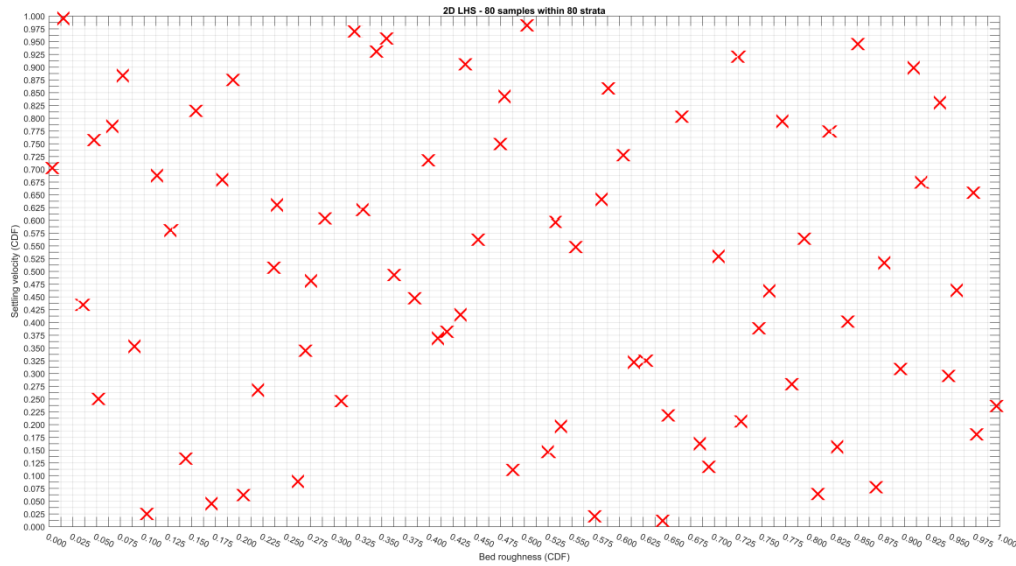


Figure 26: The Latin hypercube arrangement of the original sampling points.

However, the computation of the 40 subsamples requires many days running in Matlab due to the several possibilities of combination during the simulation for a 3D LHS. The number of combinations are $80!/(40!(80-40)!)$ which is approximately 1.07×10^{23} possible combinations. Due to the short time available for this study, the best guess given by Matlab is adopted. Therefore, after 1.000.000 iterations the best guess was 37 samples within 40 strata. For each of the three dimensions three strata are missing. In order to make sure that all the strata are sampled, eight samples from the 80 samples are added. In total there are 45 samples within 40 strata. Given that, there are strata which are sampled more than once. The same problem happens when subsampling 20 samples from the 40 samples. In the end the best guess is 25 samples within 20 strata.

4.5 Uncertainty quantification

Among all the probability distribution functions (pdf's), the lognormal was chosen to fit the distribution of siltation in the navigation channel calculated by the three models using different sample sizes. A random variable X is said to be log-normally distributed if $\log(X)$ is normally distributed. In this case, only positive values are possible and the distribution is skewed to the left. Skewed distributions are characterized by low mean values, large variances and non-negative values. These characteristics can be observed in Figure 27 and Figure 28. In addition, a chi-square goodness-of-fit test is applied and it confirms that the data samples come from a lognormal distribution.

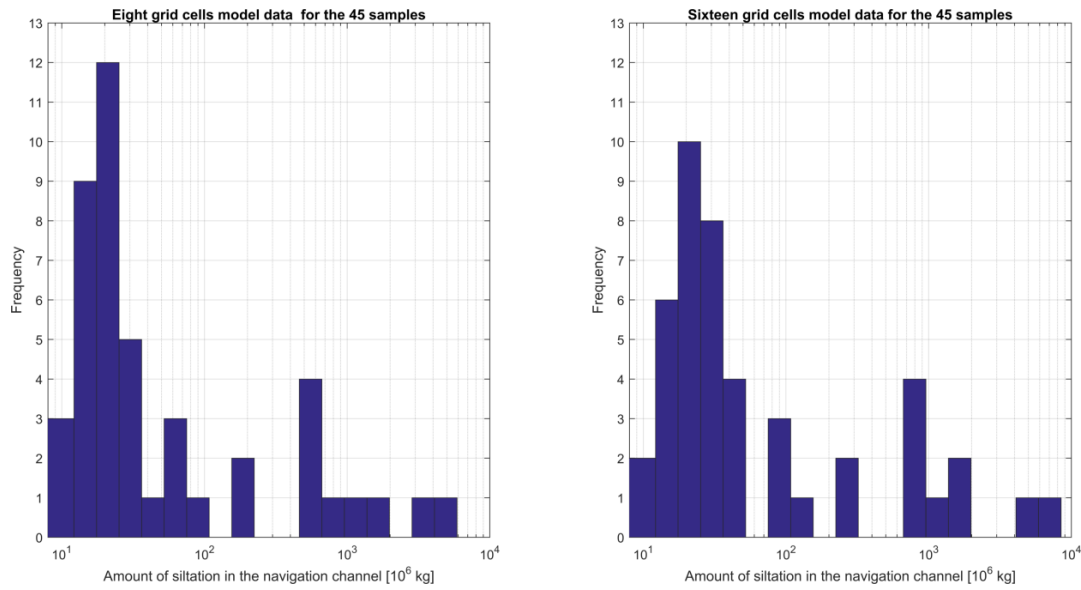


Figure 27: Histogram of the siltation for the eight and sixteen grid cells model using the 45 samples for the uncertainty quantification.

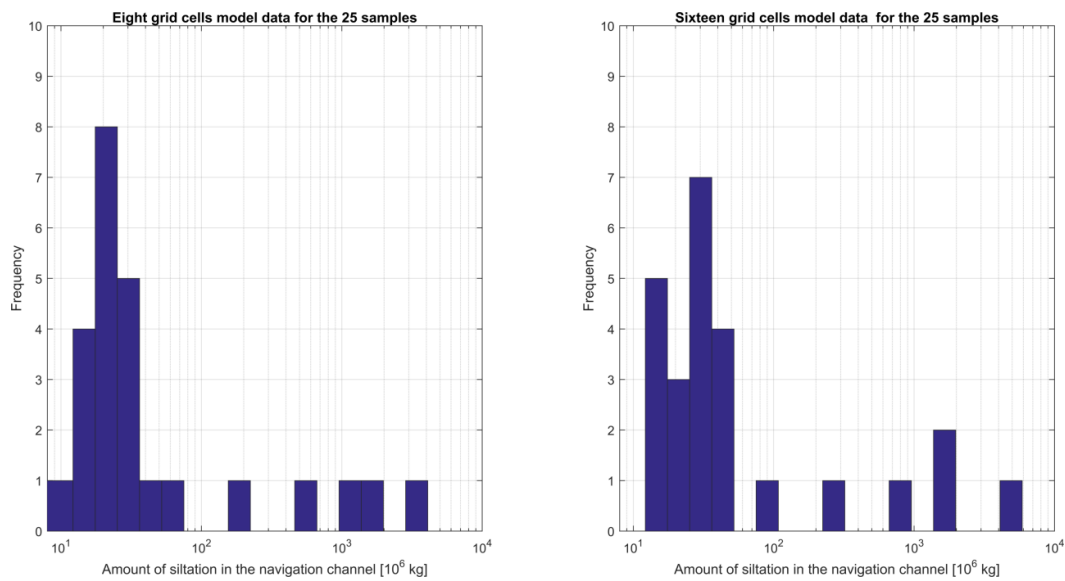


Figure 28: Histogram of the siltation for the eight and sixteen grid cells model using the 25 samples for the uncertainty quantification.

From the two histograms above, an important aspect can be noticed. A sample size of 45 can represent quite well the siltation distribution for both models. Despite the decrease in the amount of siltation calculated by the eight grid cells model, both distributions are quite similar as Figure 27 shows. In contrast, a sample size of 25 does not seem sufficient big to represent the siltation distribution of both models as showed Figure 28 in which the distributions look less alike. Additionally, for the 25 samples the amount of siltation for both models is underestimated when compared to the 45 samples. This low performance when using 25 samples is a consequence of sampling the input parameter distributions by an insufficient number of strata. Thus, the siltation distribution and its extremes are not well represented.

A lognormal distribution is characterized for the median (μ) and standard deviation (σ) of $\log(X)$. Most commonly, the uncertainty on a quantity is quantified in terms of the standard deviation, σ which is the

positive square root of variance, σ^2 . In order to determine these values in terms of X , the measure data, the two formulations $\mu^* = e^\mu$ and $\sigma^* = e^\sigma$ are used, where μ^* and σ^* are called the 'back-transformed' values. Lognormal cumulative distribution functions (cdf's) for the siltation computed by the sixteen grid cells model and eight grid cells model using the 45 and 25 samples are found in Figure 29 and Figure 30.

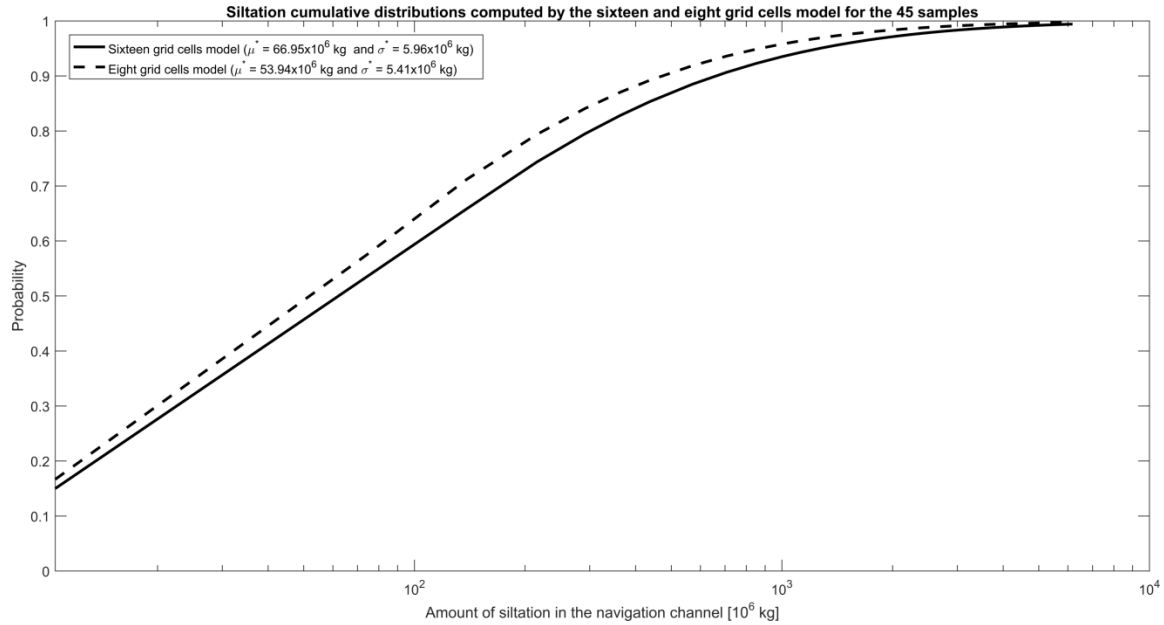


Figure 29: According to these cdf's, for the same probability the values of siltation computed by the eight grid cells model are smaller than by the sixteen grid cells model using the same 45 sample size. It can be observed that the difference in the amount of siltation between both models is small for the lower and higher percentiles.

However it increases considerably from the 60th to the 90th percentiles. Thus, the extremes values are well represented by the eight grid cells model.

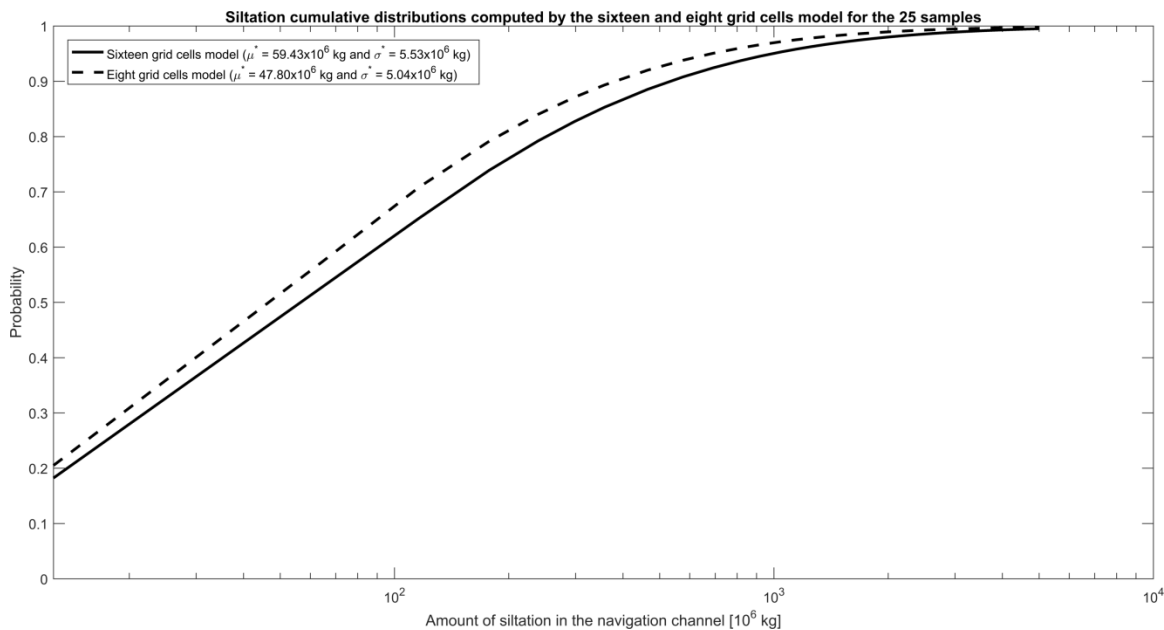


Figure 30: According to these cdf's, for the same probability the values of siltation computed by the eight grid cells model are smaller than the sixteen grid cells model for the same 25 sample size. The difference in the amount of siltation between both models is small for the lower percentiles, it increases considerably from the 60th to the 90th percentiles and then it decreases again.

The differences in the cdf's between both models showed in Figure 29 and Figure 30 are bigger for the 25 samples than for the 45 samples. The legends in these figures show the 'back-transformed' median and standard deviations for both models. For the 45 samples, $\mu = 3.99$ and $\sigma = 1.69$ for the eight grid cells model and $\mu = 4.20$ and $\sigma = 1.78$ for the sixteen grid cells model. For the 25 samples, $\mu = 3.87$ and $\sigma = 1.62$ for the eight grid cells model and $\mu = 4.08$ and $\sigma = 1.71$ for the sixteen grid cells model.

Not only cumulative distribution are created with the purpose of uncertainty analysis, but also box plots. In descriptive statistics, a boxplot is a convenient way of graphically illustrate groups of data along their quartiles. On each box, the central mark indicates the median, and the bottom and top edges of the box indicate the 25th and 75th percentiles, respectively. The lines extending vertically from the boxes (whiskers) indicate variability outside the upper and lower quartiles. The outliers are plotted as individual points. Box plots are non-parametric in which the variation in samples of a statistical population is displayed without making any assumptions of the underlying statistical distribution. The spacing between the different parts of the box indicates the degree of dispersion (spread) and skewness in the data, and shows outliers.

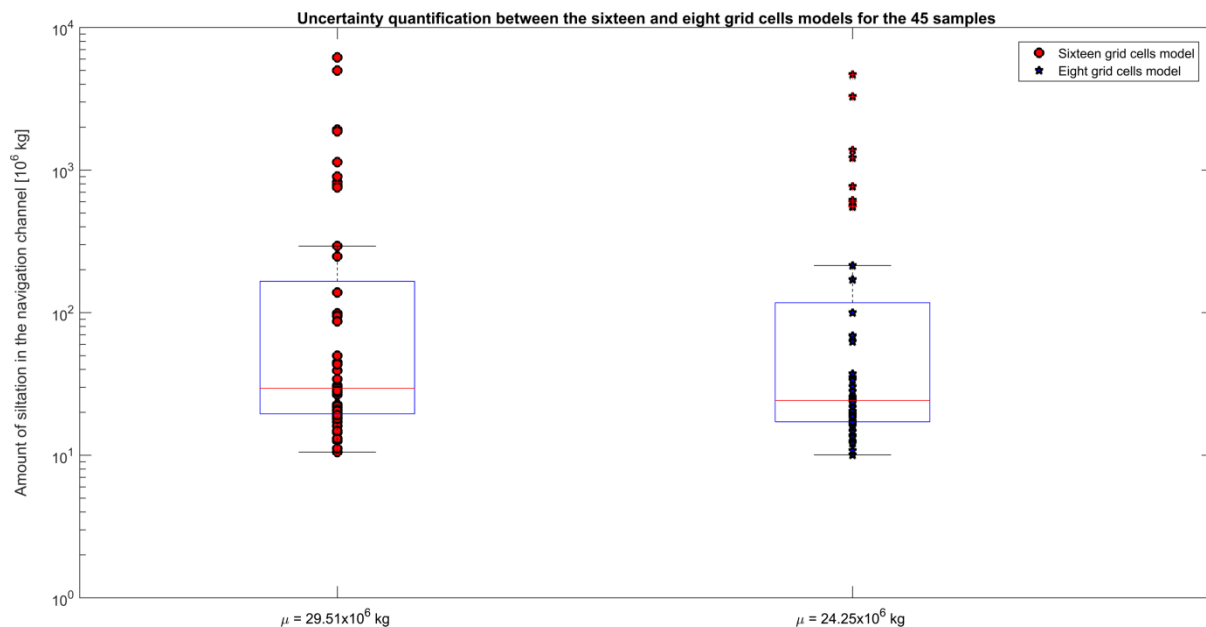


Figure 31: New scatter plot and box plot combined for uncertainty analysis. From this plot, the box plot corresponding to the siltation data computed by the eight grid cells models is smaller than the box plot of the sixteen grid cells model. The size of the box plot corresponds to the range of the measured data. The results of the eight grid cells model are not reliable.

The most important aspect of Figure 31 is the smallest amount of uncertainty in the siltation data computed by the eight grid cells model when compared to the sixteen grid cells model. The eight grid cells model gives a false idea of uncertainty in the siltation distribution. Therefore, the eight grid cells model is not reliable to be implemented in uncertainty quantification. It is also important to realize that the median calculated by the eight grid cells model is underestimated if compared to the median of the sixteen grid cells model for the same sample.

4.6 Identifying mapping function

In order to identify a mapping function from the eight grid cells model to the sixteen grid cells model, both distributions were plotted against each other and a polyline fit was applied as Figure 32 and Figure 33 present. The aim of these polyline fits is to map the amount of siltation inside of the channel from one model to the other. A fitting curve in a logarithm scale is chosen in order to better fit entire the siltation data.

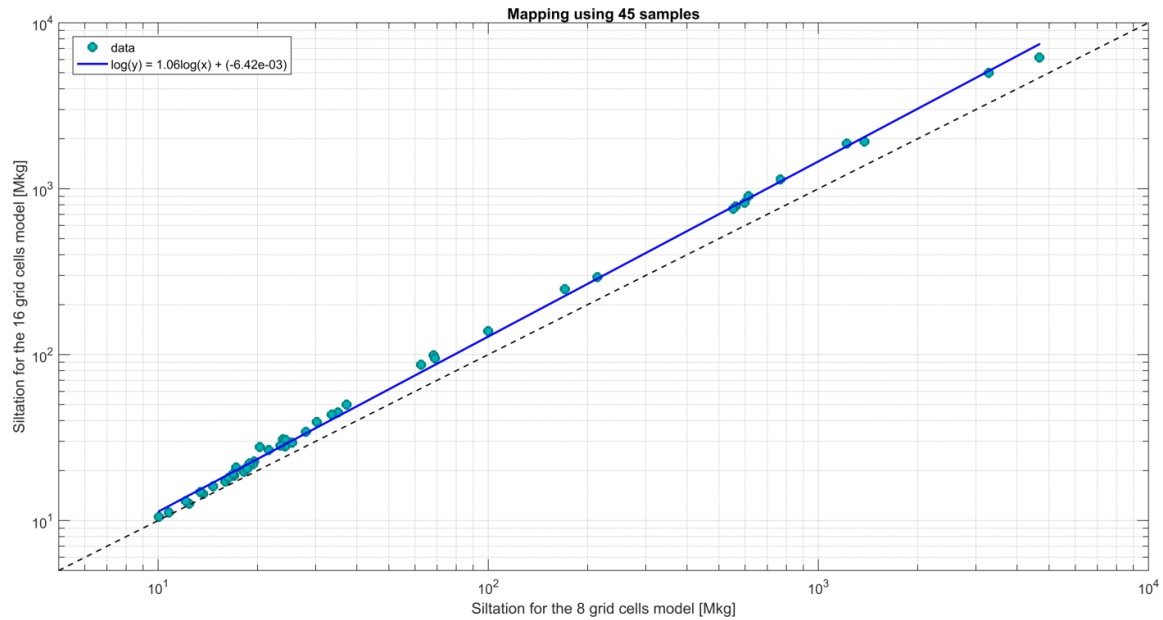


Figure 32: Experimental mapping function relating the amount of siltation calculated by the sixteen grid cells model and eight grid cells model with 45 samples. The plot shows that the logarithms of both siltation distributions are related by a linear fit.

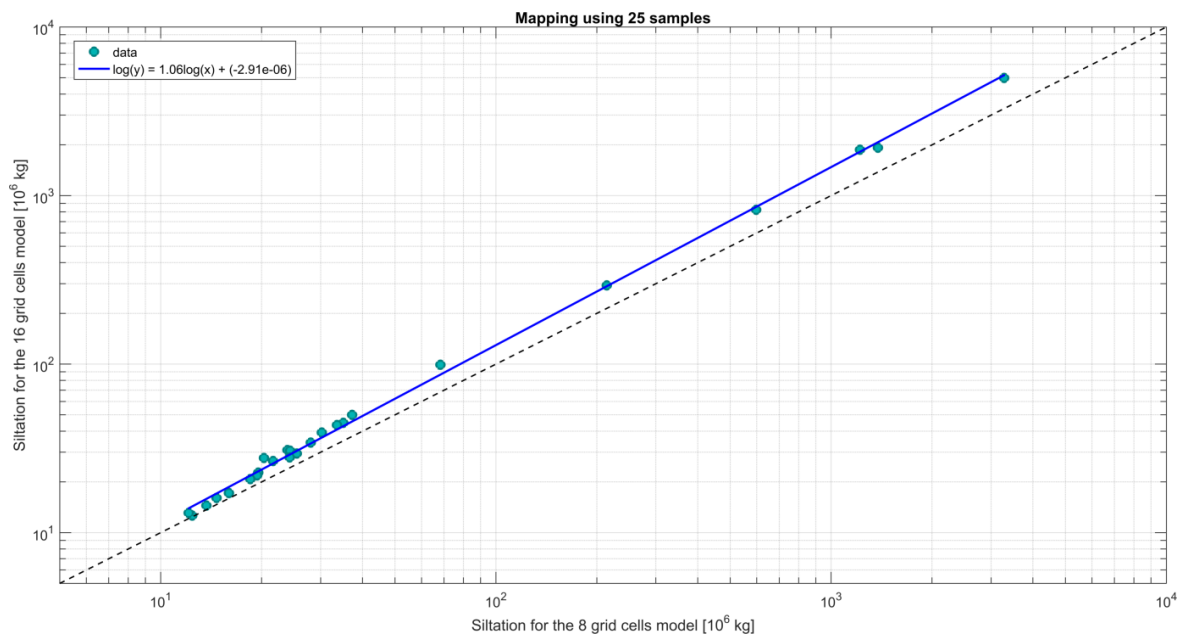


Figure 33: Experimental mapping function relating the amount of siltation calculated by the sixteen grid cell model and eight grid cells model with 25 samples. The plot shows that the logarithms of both siltation distributions are related by a linear fit.

These two experimental mapping functions termed as $f_{exp,25}$ and $f_{exp,45}$ for the 25 samples and 45 samples, respectively, have a slope-intercept form $y = mx + c$ where m is the slope and c is the intercept. If the slope and the intercept are known, a linear relation between two variables can be found.

Looking at the experimental mapping functions in Figure 32 and Figure 33, it can be observed that both mapping functions have the same slope, but different intercepts. The logarithms of the siltation of the two models have the same linear relationship for both samples as $m = 1.06$. While the intercept of the mapping function f_{25} is very small (close to zero), the intercept of the mapping function f_{45} is bigger which reduces the difference between the logarithms of both siltations. Therefore, when using 45 samples the values of siltation between the sixteen and eight grid cells models are closer than when using 25 samples.

Regarding the lognormal mapping functions, two diagrams were created to show the results for the two samples. The diagram for the 25 samples and the 45 samples can be found in Figure 34 and Figure 35, respectively.

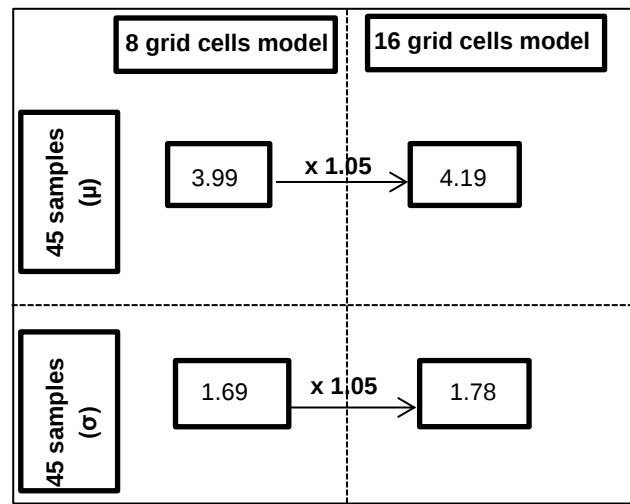


Figure 34: Diagram presenting the results for the 45 samples.

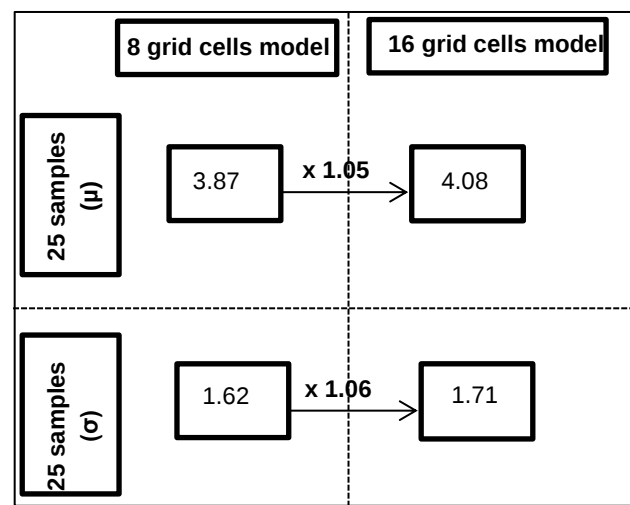


Figure 35: Diagram presenting the results for the 25 samples.

The values of the medians and standard deviations for the lognormal siltation distributions are inside of the rectangles and the lognormal mapping functions $f_{logn,25}$ and $f_{logn,45}$ are the ratios above the arrows. The $f_{logn,25}$ consists of the ratios related to the median and standard deviation in Figure 34 and $f_{logn,45}$ consists of the ratios in Figure 35. It can be noticed that these mapping functions are almost the same for the two samples. For both models, the medians and standard deviations are underestimated when using only 25 samples.

From the *Uncertainty Quantification* chapter, it was concluded that the eight grid cells model cannot be used in uncertainty quantification because it is not a good representation of the expected range of siltation that takes place in a navigation channel. It is worth investigating if the eight grid cells model together with a mapping function can be used in uncertainty quantification. In addition, it is also investigated the results when using a mapping function constructed from a 25 samples but using 45 samples. In that case, the advantages are determining a mapping function with part of the data and using the other part for validation.

Firstly, the two experimental mapping functions found in Figure 32 and Figure 33 were applied to the siltation data computed by the eight grid cells model for the 45 samples. The comparison between these mapped siltation distributions and the siltation distribution calculated by the sixteen grid cell model is showed in Figure 36.

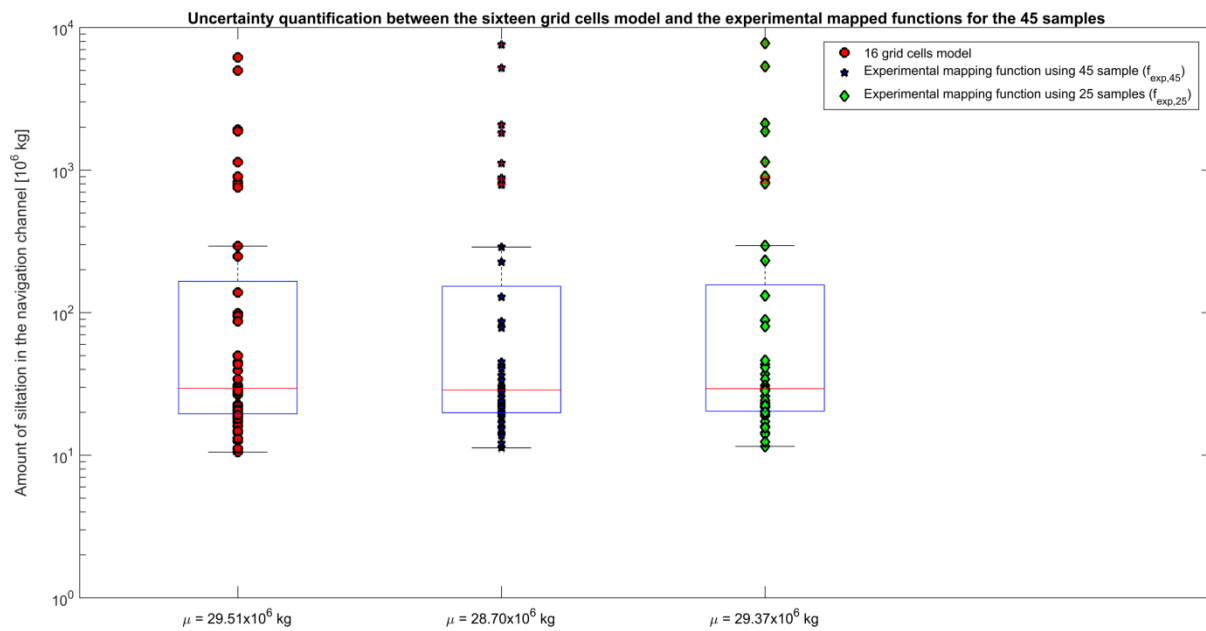


Figure 36: Visual uncertainty quantification for the experimental mapped siltation distributions and the siltation distribution from the sixteen grid cells model using box plots.

In Figure 36, the first box plot and the red circles represent the siltation distribution computed by the sixteen grid cells model, the second box plot and the blue pentagrams represent the experimental mapped siltation for the $f_{exp,45}$ and the third box plot and green diamonds represent the experimental mapped siltation for the $f_{exp,25}$. After applying these mapping functions, the siltation distributions look very alike. Besides the similar values of medians, the range represented by the size of the box plots (including the whiskers) is approximately the same. Therefore, the eight grid cells model when combined with one of the experimental mapping functions present results very similar to the six grid cells model for uncertainty quantification.

Regarding the lognormal mapping functions, they are applied to 45 random numbers generated from a lognormal distribution with $\mu=3.99$ and $\sigma=1.69$ which are the parameters values for the eight grid cells model siltation distribution, see Figure 34. Next, a comparison of uncertainty quantification between these lognormal mapped siltation distributions and the siltation distribution computed by the sixteen grid cells model is showed in Figure 37.

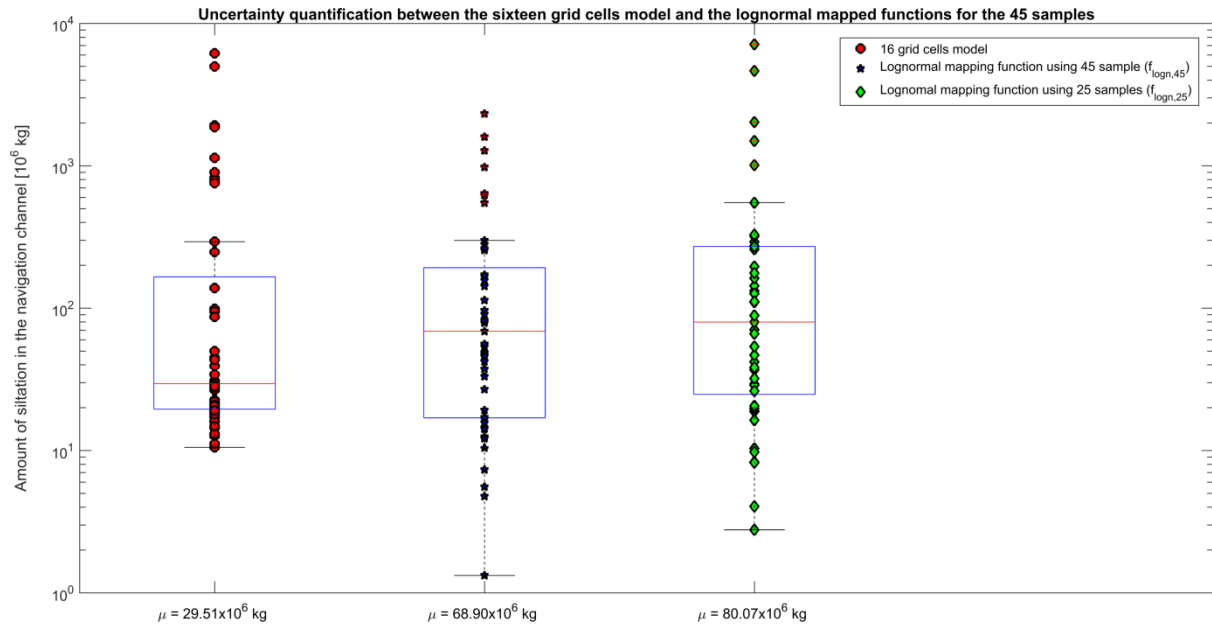


Figure 37: Visual uncertainty quantification for the lognormal mapped siltation distributions and the siltation distribution from the sixteen grid cells model using box plots.

The first box plot and the red circles represent the siltation distribution computed by the sixteen grid cells model, the second box plot and the blue pentagrams represent the mapped siltation for $f_{exp,45}$ and the third box plot and green diamonds represent the mapped siltation for $f_{exp,25}$. Within this plot, two important aspects need to be highlighted. Firstly, the medians of the mapped siltation distributions are greatly overestimated as a consequence of the lognormal distribution fit. Secondly, mapped siltation distributions have a larger amount of uncertainty than in the siltation from the sixteen grid cell models. Therefore, fitting lognormal distribution to siltation data do not give correct results that can be used for uncertainty quantification.

5 Discussion

A Delft3D model and two emulators were used to quantify the amount of uncertainty in the siltation distribution. The results are compared and mapping functions are applied in order to translate the amount of uncertainty in the model output from the emulator output. Here some points about the limitations and assumptions of this study are discussed:

1. In this study, some of the assumptions made as no wind effect, perpendicular approach of the waves to the coast and low discharge had a big impact on the sensitivity analysis in which the most sensitive uncertain parameters presented to be the settling velocity, bed roughness and significant wave height. In case these assumptions are not made, the outcome of the sensitivity analysis might be different;
2. This study is very case specific due to the assumptions and parameters intervals adopted. Due to that, the results of this study are quite limited. However, the methodology here applied in uncertainty quantification using an emulator can also be implemented in other studies;
3. The LHS method shows to be a good sampling method as long as the sample size is not very small. A sample size of 25 showed to give results less accurate than the 45 samples.
4. The lognormal probability density function fits the siltation distribution from the sixteen and eight grid cells model, nonetheless it has the disadvantage of increasing its median value;

6 Conclusion

Let's highlight three important conclusions from this study. First, the eight grid cells model is a good replacement for the sixteen grid cells model with respect to siltation computation in the navigation channel. Second, the four grid cells model can be used for a 'parameter screening' in sensitivity analysis. Third, the eight grid cell model can be used for uncertainty quantification if the specified mapping functions are applied. Summarizing, it is possible to quantify the uncertainty in a navigation channel using an emulator as long as a mapping function is applied. With attention to the fact that the channel width should be around 10 grid cells, otherwise the hydrodynamics and morphodynamics are not well represented by the emulator. Equally important, the research questions presented in *Introduction* can now be answered.

1. What are the uncertain input parameters with respect to a coastal morphodynamic model?

For a coastal morphodynamic model which the study area is characterized by low wind velocities, waves coming from deep sea and a river discharge, the uncertain input parameters will be river discharge, critical shear stress for erosion, settling velocity, bed roughness and significant wave height.

2. What are the most sensitive uncertain input parameters for a Delft3D model?

For both model and emulators, the most sensitivity parameters are the settling velocity, wave height and the bed roughness. Although the discharge is an important parameter, because it brings sediments into the system, it was found to not be relevant in this study due to its low values.

3. In what way do the uncertain in the siltation between model and emulators differ between them?

The siltation distribution computed by the eight grid cells model presented much less uncertainty when compared to the siltation distribution computed by sixteen grid cells model. However, this is not a realistic result. Given that, the eight grid cells underestimate the real range of the siltation distribution. The four grid cells model does not correctly compute the hydrodynamics and morphodynamics due to its coarse grid, thus it was not applied in uncertainty quantification.

4. Can a mapping function be applied to the emulators in order to map the amount of uncertainty in the siltation distribution computed by the model?

For this study, the relation between the siltation computed by the sixteen grid cells model and eight grid cells models is log linear. The experimental mapping functions are $\log(Y) = 1.06\log(X) - 6.42 \times 10^{-3}$ and $\log(Y) = 1.06\log(X) - 2.91 \times 10^{-6}$ where X is the siltation data calculated by the eight grid cells model and Y is the mapped siltation. If these mapping functions are applied to the eight grid cells model data, then this model can be used in uncertainty quantification. The lognormal mapping functions do not present good results in uncertainty quantification.

7 Recommendation

This study used a Delft3D model and two emulators in order to calculate the amount of uncertainty in port siltation. Within the known assumptions and conditions of this study, some recommendations can be done:

- For 'screening parameter', or qualitative sensitivity analysis, is recommended to use a coarse emulator instead of a model, because it will give the same result in a much short period of time.
- For uncertainty quantification, an emulator can be used as long as it has around 10 grid cells along the channel width and whether there is a mapping function capable to map the siltation distribution of this emulator to its model. If that is not the case, the model should be used in uncertainty quantification. Because the emulator without a mapping function gives a siltation distribution in which the amount of uncertainty is underestimated may causing future losses to the client;
- In order to build mapping functions, few data points are necessary. In future works in which is desired to determine the amount of uncertainty in port siltation using an emulator, part of the data calculated by the model can be reserved to build a mapping function while the other part to validate it. For a emulator which has around 10 grid cells along the navigation channel and the most sensitivite parameters are the settling velocity, wave height and bed roughness, the experimental mapping function $\log(Y) = 1,06\log(X)$ can be applied for uncertainty analysis using 45 LHS samples;
- For future works, it may be worth to investigate until what sample size the mapping function can be improved. Here, the mapping function from the 45 samples fits better the data than the 25 samples. It is expected that this improvement of the mapping function reaches its limits due to the own limitation of the emulator;
- The possibility of using an emulator with a mapping function brings the possibility of making this case specific into a more general case. Given that, other uncertain input parameters and/or wider parameter ranges can be adopted in order to verify the existence of a mapping function which together with an emulator could be applied in several projects. Thus, a applied research institute as Deltares would be able to give in a short period of time the feasibility of building a port in a certain area based on the uncertainty quantification done by an emulator together with this more general mapping function.

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Appendix 1: Subjective sensitivity

Table 5: Delft3D-FLOW input parameters.

Delft3D-FLOW				
Input parameter	Variable	Uncertain	Comment	Unit
Initial conditions	Water level	N	Do not have a big influence on the system analysed.	m
	Salinity	N	Determined in the model set-up.	ppt
	Sediment concentration	N	Determined in the model set-up.	Kgm ⁻³
Boundaries	Offshore	N	Constant	m
	River discharge	Y	Due to prediction uncertainty in precipitation, run-off and evaporation in the river watershed.	m ³ s ⁻¹
	Neumann-north	N	Constant	m
	Neumann-south	N	Constant	m
Physical Parameters	Gravity	N	Constant	ms ⁻²
	Water density	N	Constant	kgm ⁻³
	Temperature	N	Constant	°C
	Horizontal eddy viscosity	N	Calibration parameters.	m ² s ⁻¹
	Horizontal eddy diffusivity	N	Calibration parameters.	m ² s ⁻¹
	Vertical eddy viscosity	X	Not considered in the model.	m ² s ⁻¹
	Vertical eddy diffusivity	X	Not considered in the model.	m ² s ⁻¹
	Ozmidov length scale	X	Not considered in the model.	m
	Specific density	Y	Depends on the mineral composition	kgm ⁻³
	Dry bed density	Y	It will not change due to short run period.	kgm ⁻³
	Fresh settling velocity	Y	It will affect the cohesive sediment settling velocity.	mms ⁻¹
	Saline settling velocity	Y	It will affect the cohesive sediment settling velocity.	mms ⁻¹
	Hindered settling velocity	N	Significant for multiple suspended sediment fractions.	mms ⁻¹
	Critical bed shear stress for erosion	Y	Uncertain parameter.	Nm ⁻²
	Erosion Parameter	N	It is assumed to be constant.	
	Salinity for saline velocity	N	It is assumed to be constant.	ppt

Table 6: Delft3D-WAVES input parameters.

Delft3D-WAVES				
Input parameter	Variable	Uncertain	Comment	Unit
Physical Parameters	Gravity	N	Constant	ms^{-2}
	Water density	N	Constant	kgm^{-3}
	North w.r.t. x-axis	N	Constant for the model.	deg
	Minimum depth	N	Constant (=0.05)	m
	Wind speed	x	Not considered in the model.	ms^{-2}
	Wind direction	x	Not considered in the model.	deg
	Depth-induced breaking Alpha	N	Constant	(-)
	Depth-induced breaking Gamma	N	Constant	(-)
	Bottom friction	N	Constant	m^2s^{-3}
Boundaries	Significant wave height	Y	Due to prediction uncertainty.	m
	Peak period T_p	N	Constant	s
	Direction (nautical)	N	Constant	deg
	Directional spreading	N	Constant	-

Appendix 2: Sensitivity analysis

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Table 7: Resolution factors and amount of siltation in the navigation channel.

Models	Reference (P50)	Bed roughness		Critical S. S. for Erosion		Discharge		Settling Velocity		Significant Wave Height	
		P10 (kg)	P90 (kg)	P10 (kg)	P90 (kg)	P10 (kg)	P90 (kg)	P10 (kg)	P90 (kg)	P10 (kg)	P90 (kg)
16GRID	335.713.900,0	24.932.176	1.501.509.900	65.402.409	762.833.910	135.702.090	1.056.635.000	101.580.570	1.612.810.000	18.235.234	1.476.567.400
$Fr_{16,8}$	1,5	1.4	1.4	1.4	1.4	1.3	1.6	1.4	1.5	1.1	1.4
8GRID	230.746.620,0	17.620.665	1.070.783.500	46.513.312	559.353.780	106.119.760	670.044.680	74.616.003	1.108.763.000	17.050.050	1.035.352.900
$Fr_{8,4}$	3.7	1.9	2.8	2.9	3.0	2.6	4.2	3.6	3.0	1.7	2.6
4GRID	62.921.560,0	9.227.299,5	380.390.730	16.246.230	187.632.560	41.309.384	158.449.450	20.773.887	370.027.630	9.919.559	391.620.060
$Fr_{16,4}$	5.3	2.7	4.0	4.0	4.1	3.3	6.7	4.9	4.4	1.8	3.8

Table 8: Parameter factors and amount of siltation in the navigation channel.

odels	Bed roughness (n)		Critical S. S. for Erosion ($\tau_{cr,e}$)		Discharge		Settling Velocity		Significant Wave Height	
	P10 (kg)	P90 (kg)	P10 (kg)	P90 (kg)	P10 (kg)	P90 (kg)	P10 (kg)	P90 (kg)	P10 (kg)	P90 (kg)
16GRID	24.932.176	1.501.509.900	65.402.409	762.833.910	135.702.090	1.056.635.000	101.580.570	1.612.810.000	18.235.234	1.476.567.400
$fp_{I,16}$	60.2		11,7		7,8		15,9		81.0	
8GRID	17.620.665	1.070.783.500	46.513.312	559.353.780	106.119.760	670.044.680	74.616.003	1.108.763.000	17.050.050	1.035.352.900
$fp_{I,8}$	60.8		12,0		6,3		14.9		60.7	
4GRID	9.227.299,50	380.390.730	16.246.230	187.632.560	41.309.384	158.449.450	20.773.887	370.027.630	9.919.559	391.620.060
$fp_{I,4}$	41.2		11.5		3.8		17.8		39.5	

Appendix 3: Latin Hypercube samples

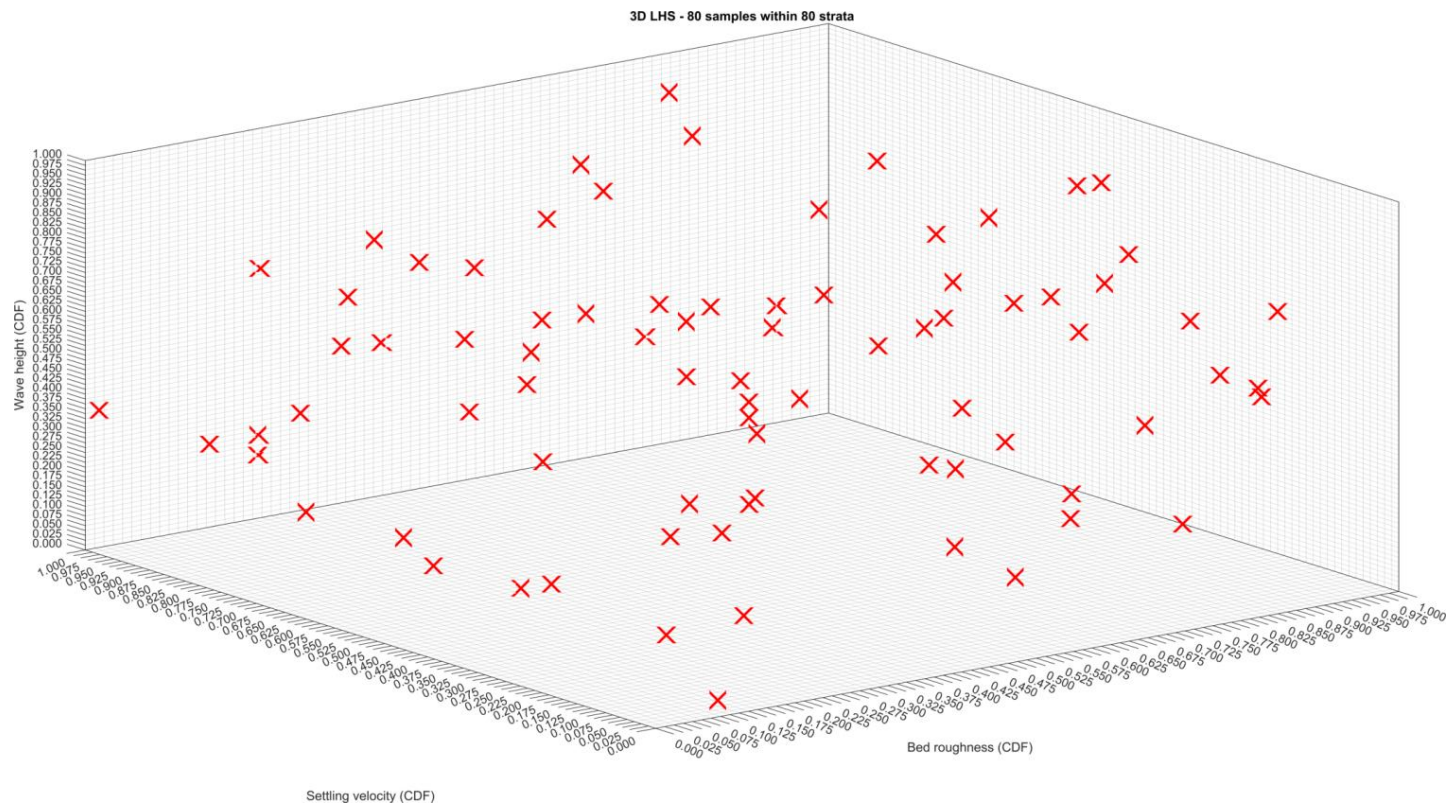


Figure 38: 3-Dimensional LHS's (80 strata).

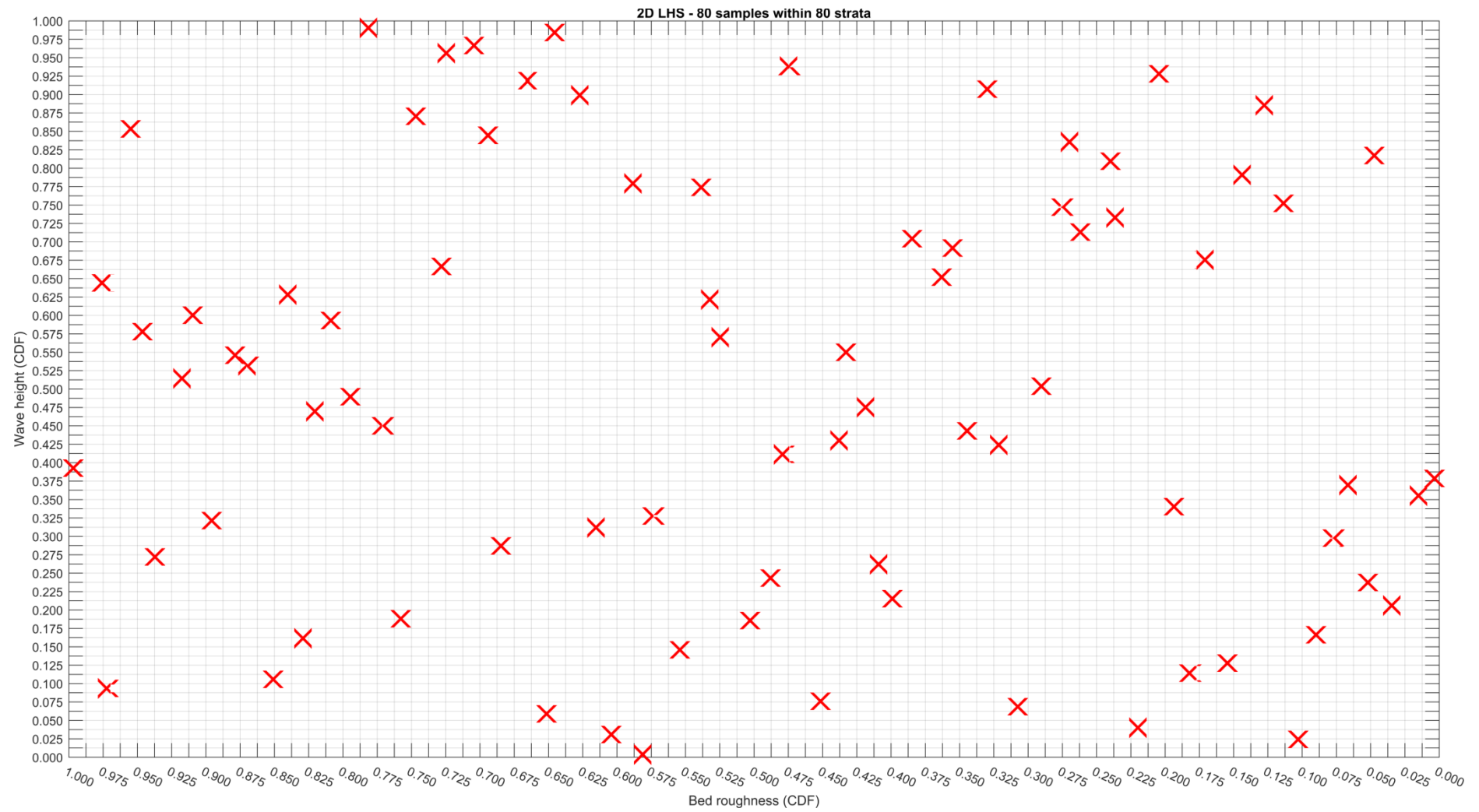


Figure 39: 2-Dimensional LHS's (80 strata).

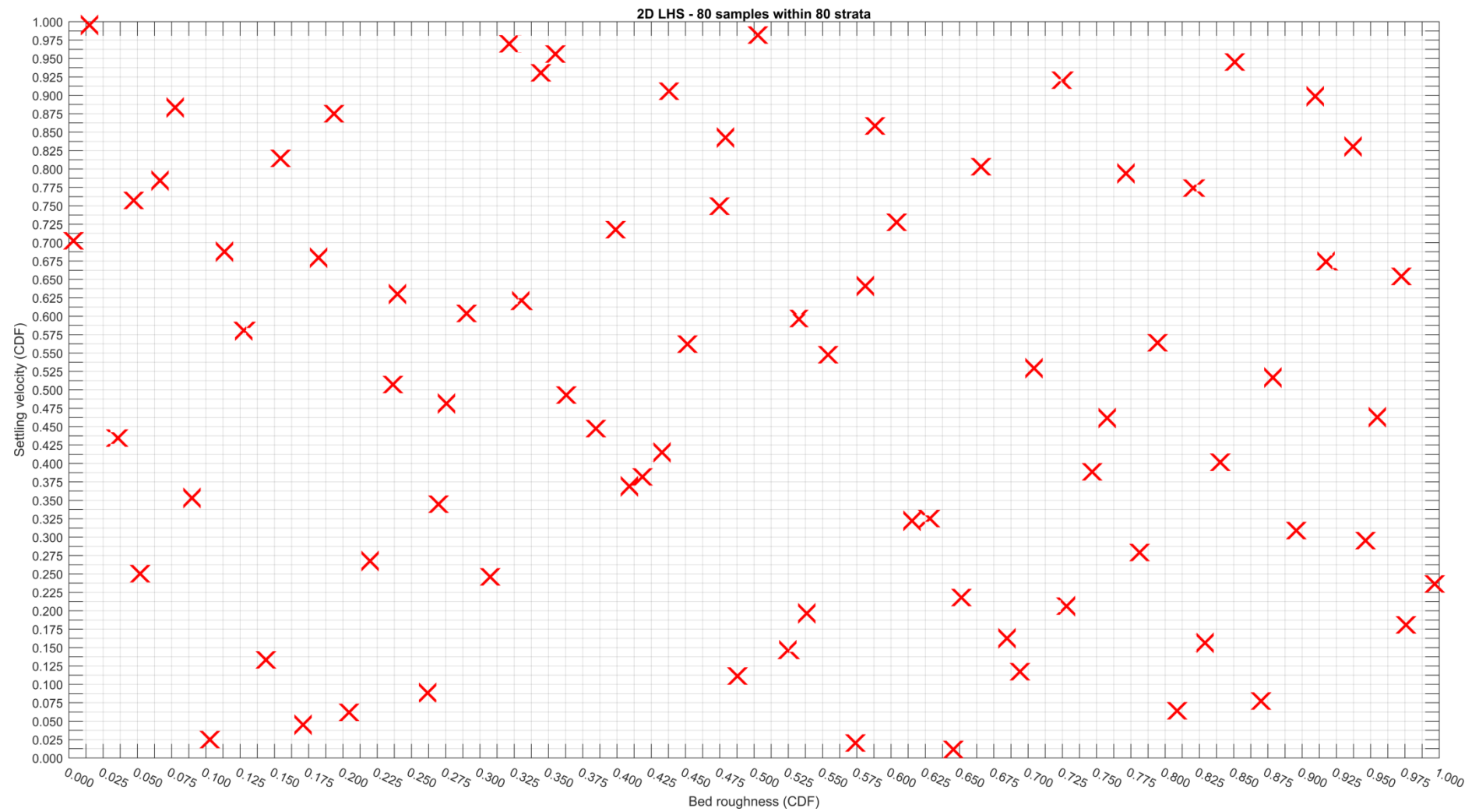


Figure 40: 2-Dimensional LHS' sample (80 strata).

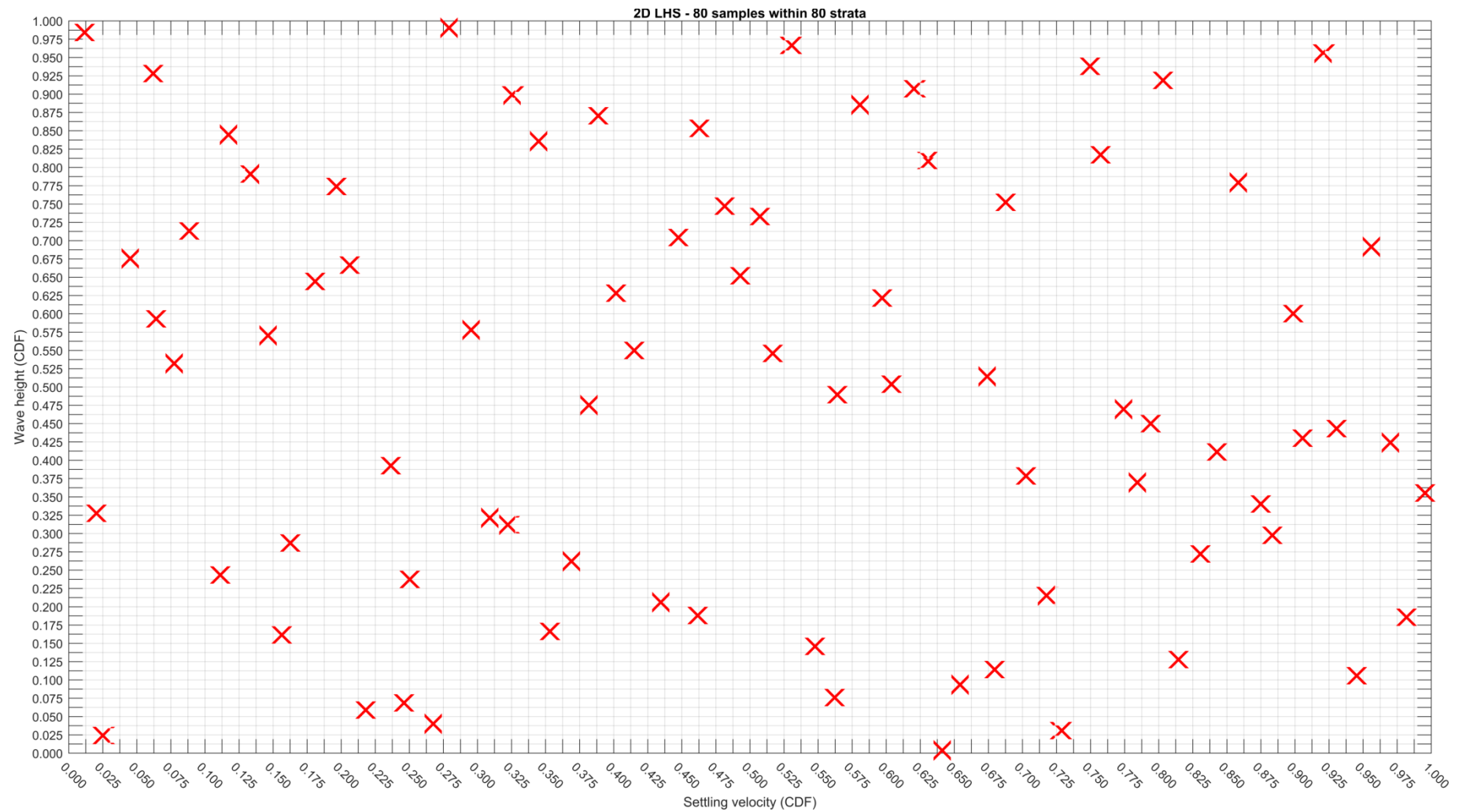


Figure 41: 2-Dimensional LHS' sample (80 strata).

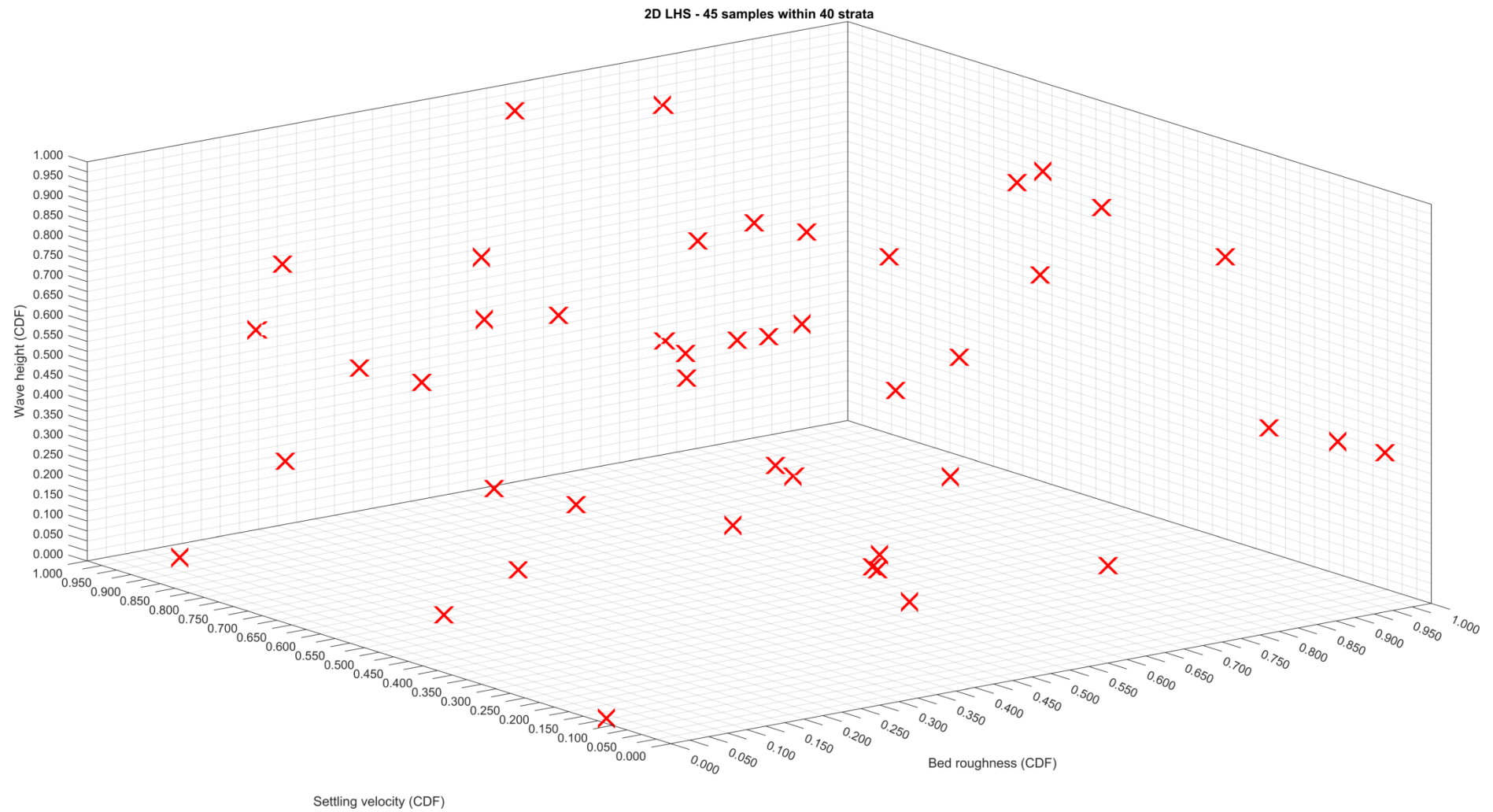


Figure 42: 3-Dimensional LHS's (40 strata)

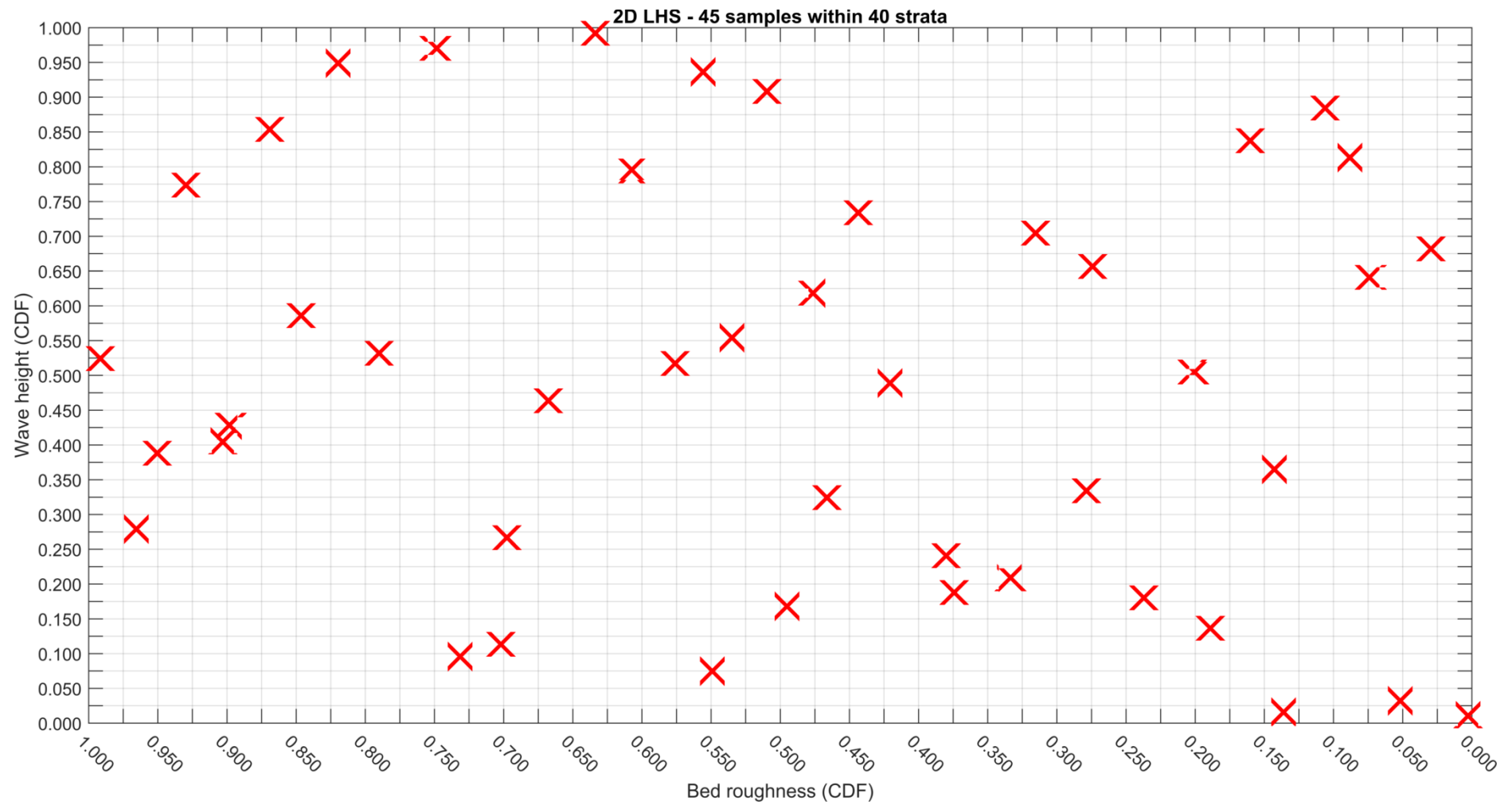


Figure 43: 2-Dimensional LHS' sample (40 strata).

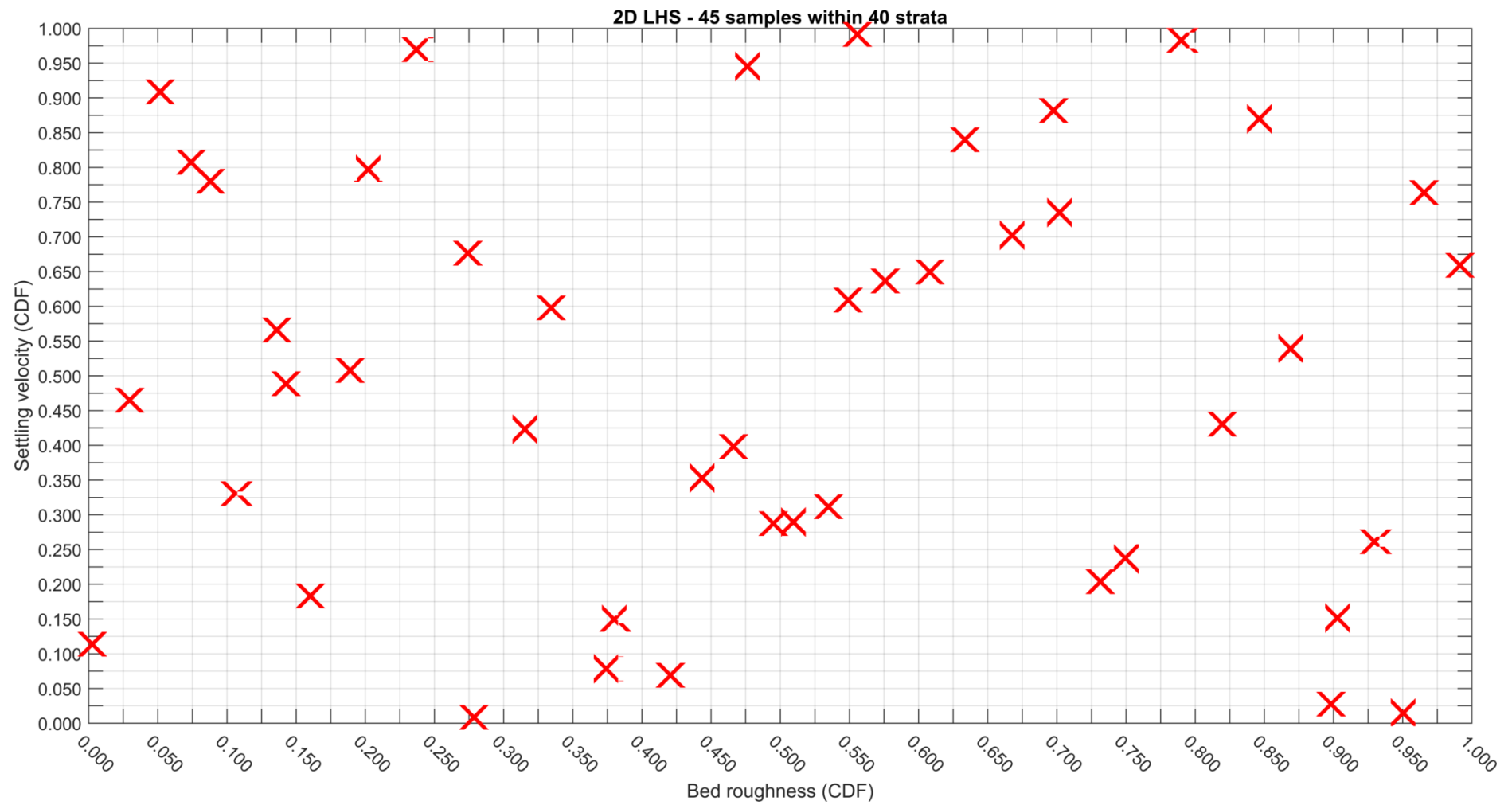


Figure 44: 2-Dimensional LHS' sample (40 strata).

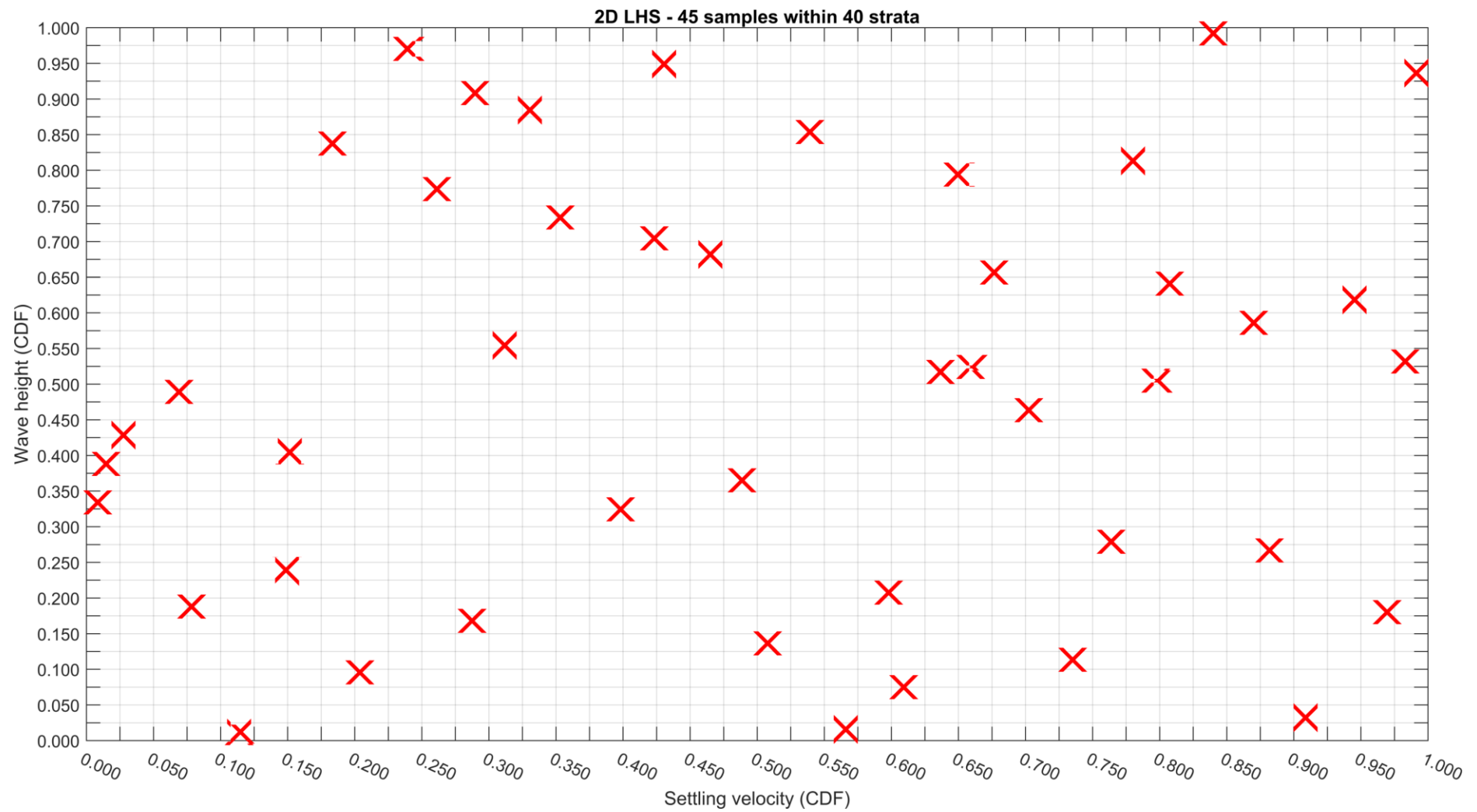


Figure 45: 2-Dimensional LHS' sample (40 strata).

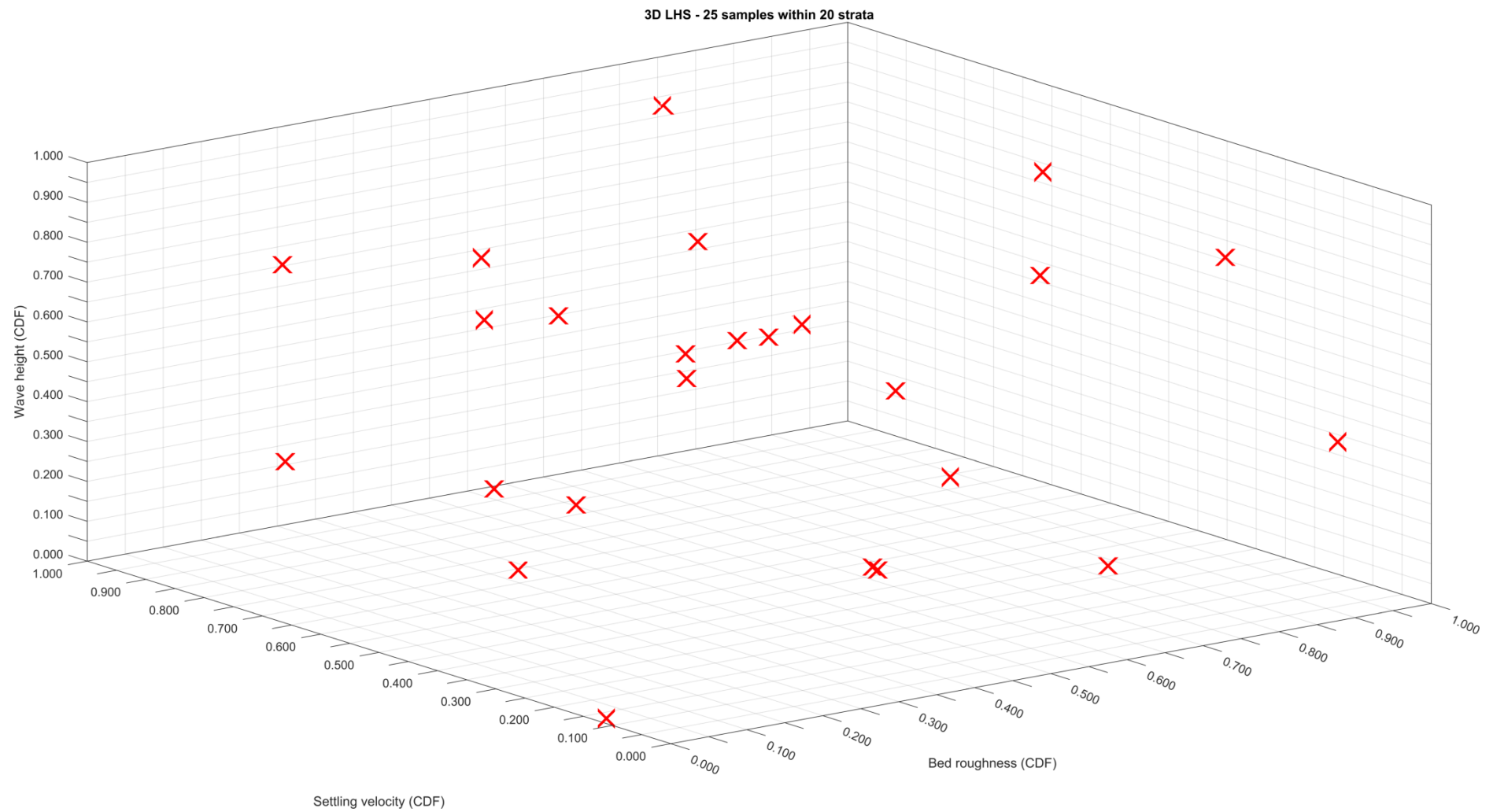


Figure 46: 3-Dimensional LHS' sample (20 strata).

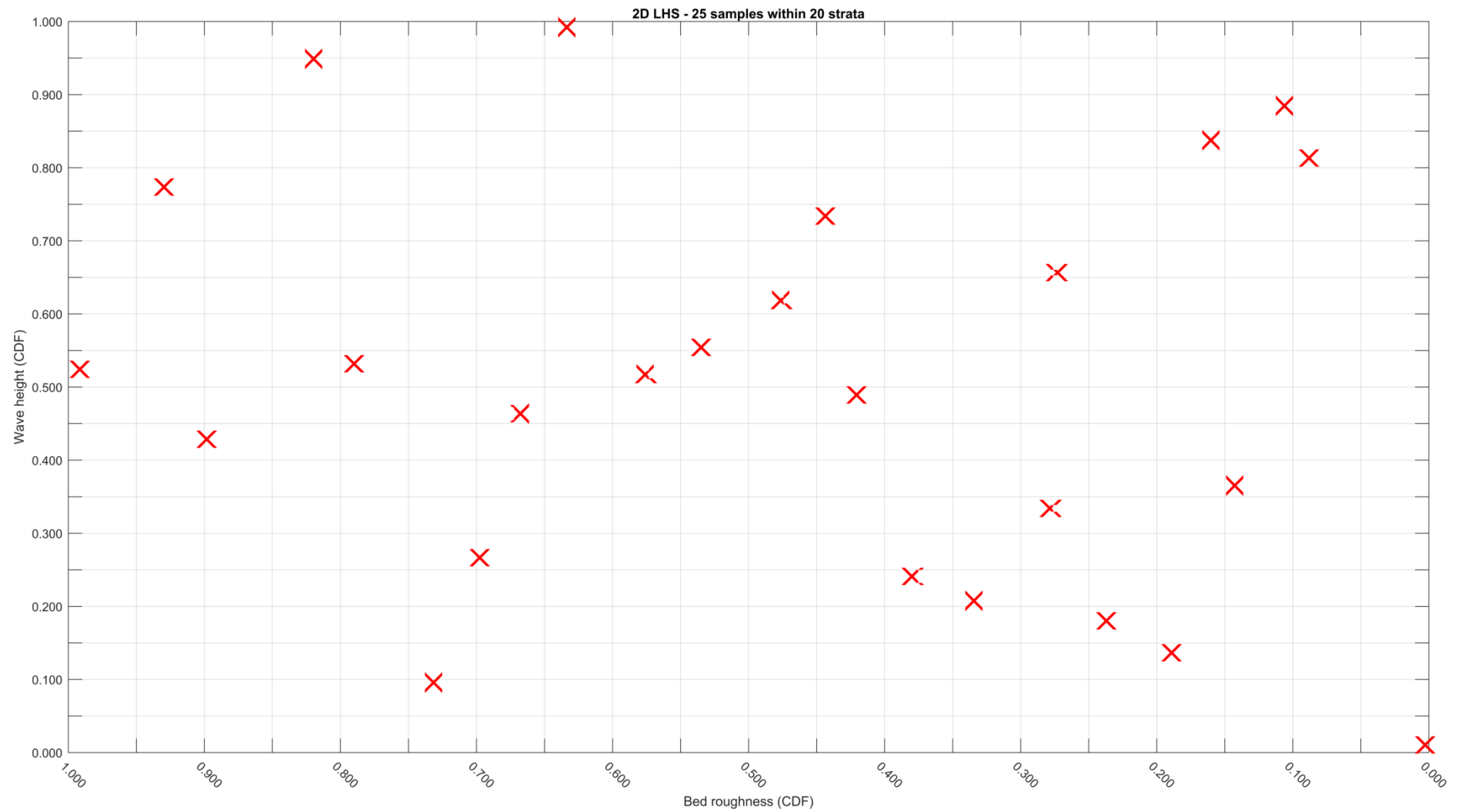


Figure 47: 2-Dimensional LHS' sample (20 strata).

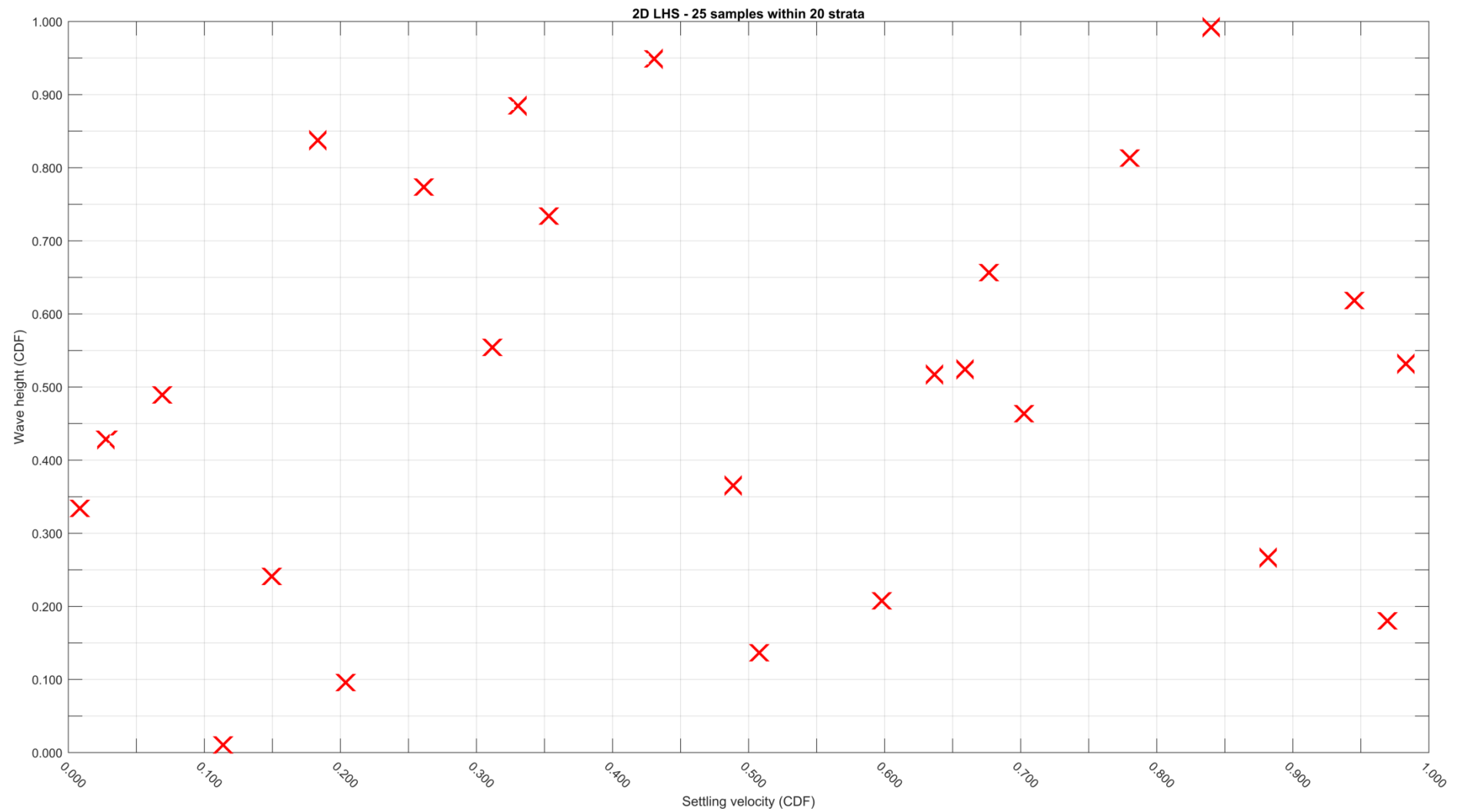


Figure 48: 2-Dimensional LHS' sample (20 strata).

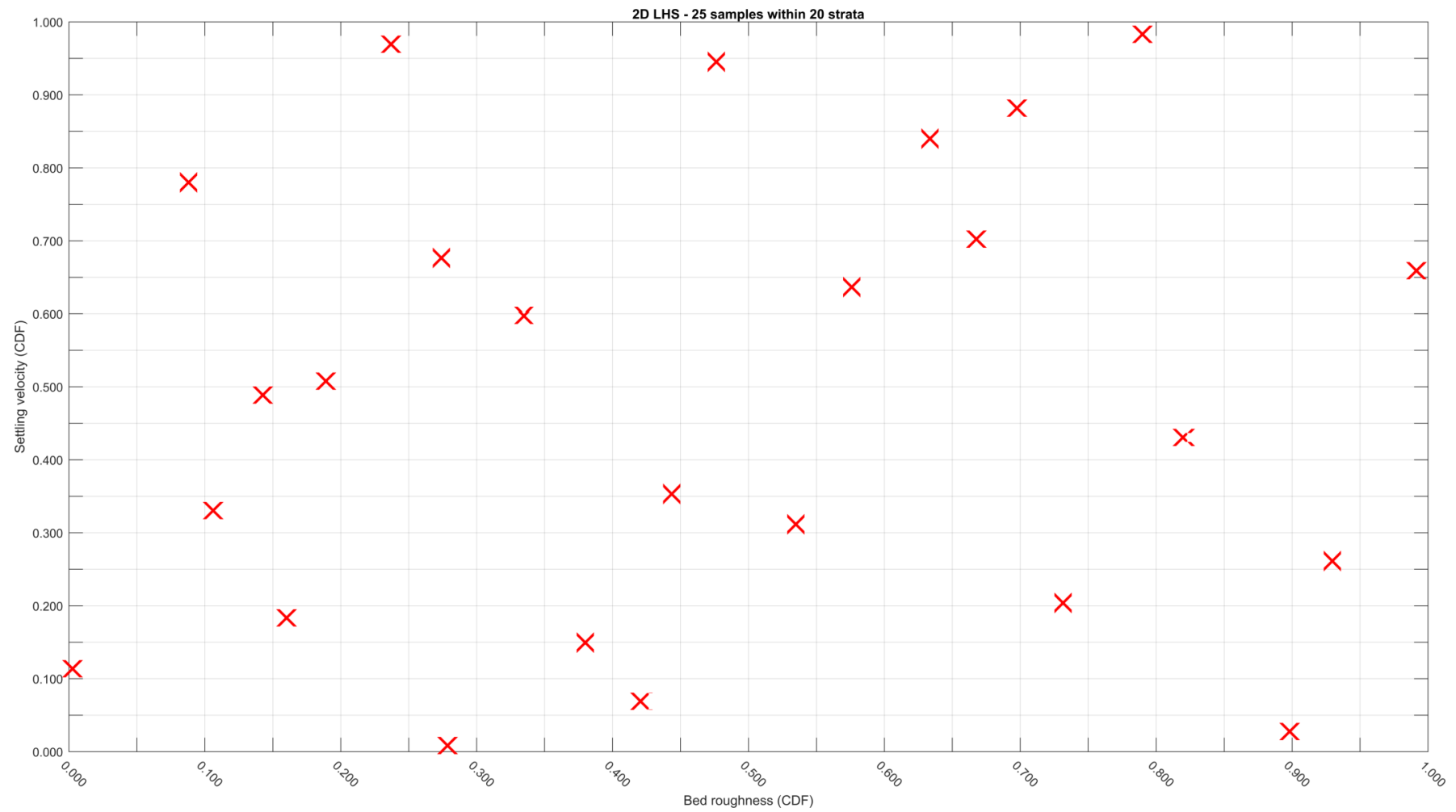


Figure 49: 2-Dimensional LHS' sample (20 strata).

Appendix 4: Investigation of the maximum bed shear stress due to currents and waves, the bed shear stress due to currents, the orbital velocity and the horizontal velocity at the bed

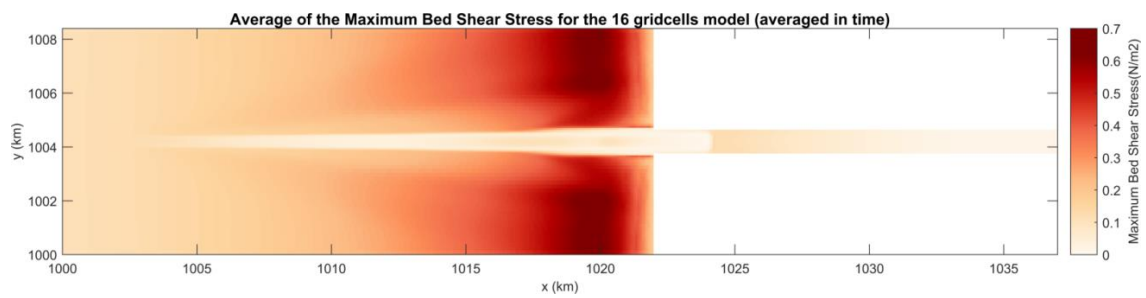


Figure 50: Median values of the maximum bed shear stress averaged in time computed by the 16 grid cells model.

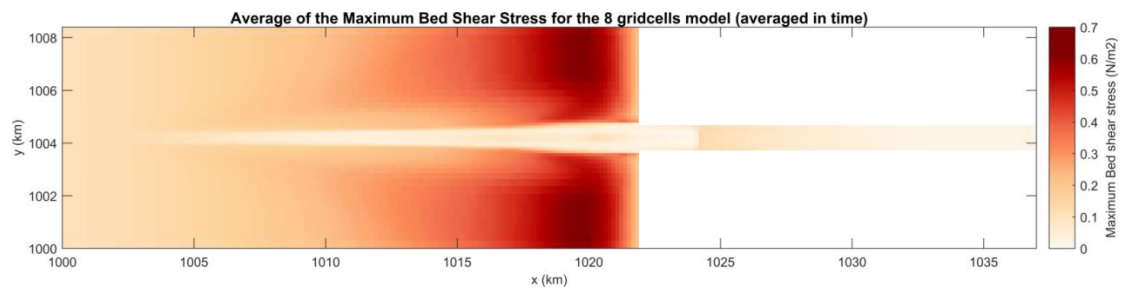


Figure 51: Median values of the maximum bed shear stress averaged computed by the 8 grid cells models.

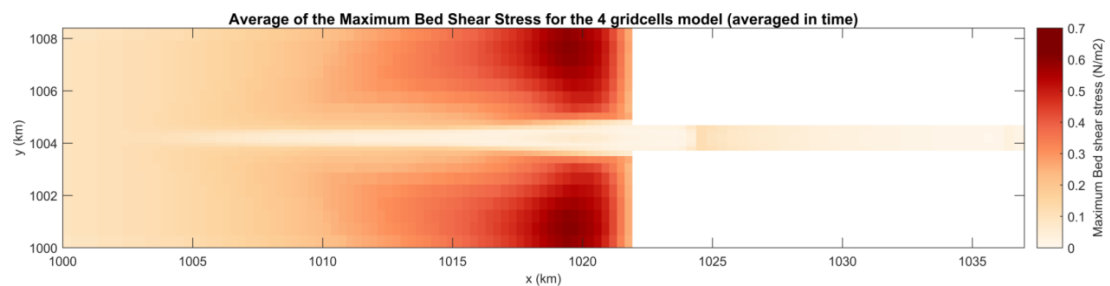


Figure 52: Median values of the maximum bed shear stress averaged in time computed by the 4 grid cells model.

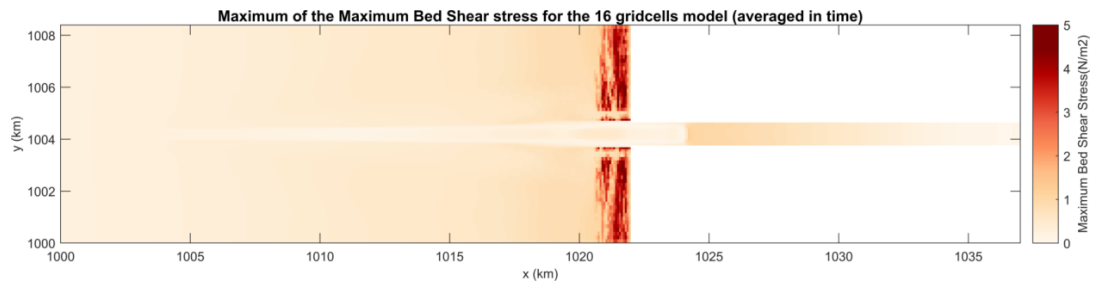


Figure 53: Highest values for the maximum bed shear stress averaged in time computed by the 16 grid cells model

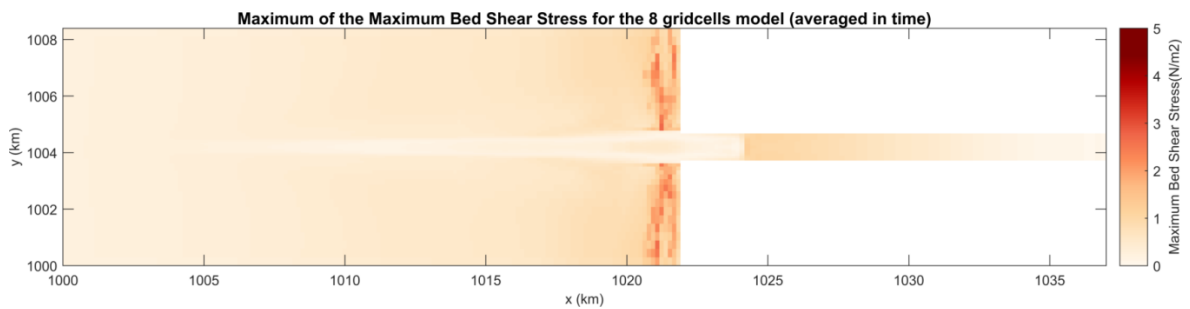


Figure 54: Highest values for the maximum bed shear stress averaged in time computed by the 8 grid cells model.

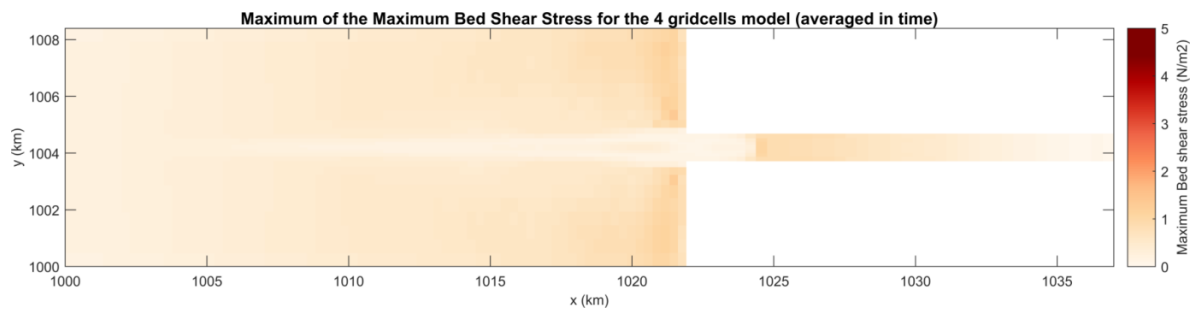


Figure 55: Highest values for the maximum bed shear stress averaged in time computed by the 4 grid cells model.

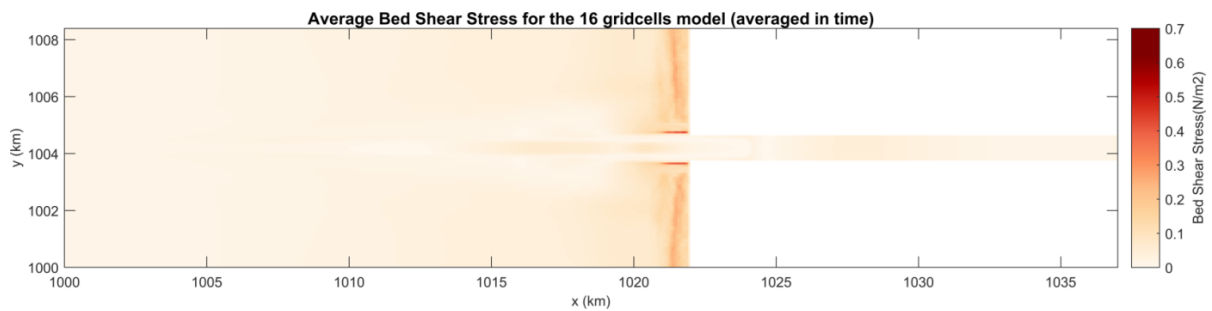


Figure 56: Median values for the bed shear stress averaged in time computed by the 16 grid cells model.

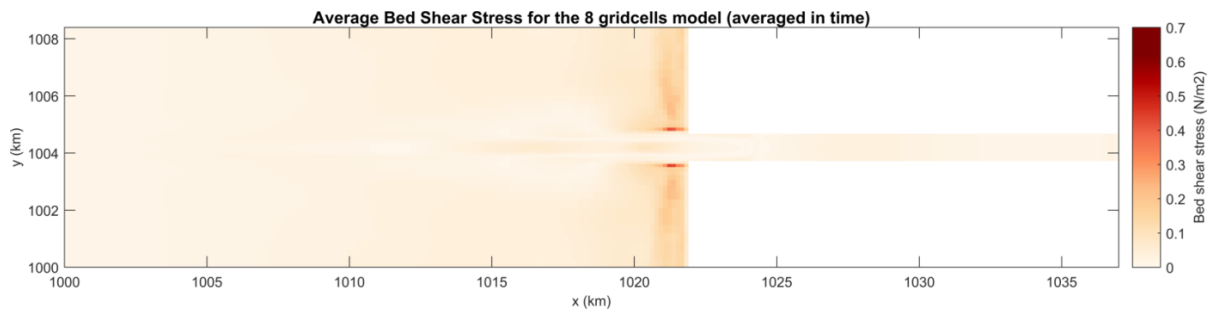


Figure 57: Median values for the bed shear stress averaged in time computed by the 8 grid cells model.

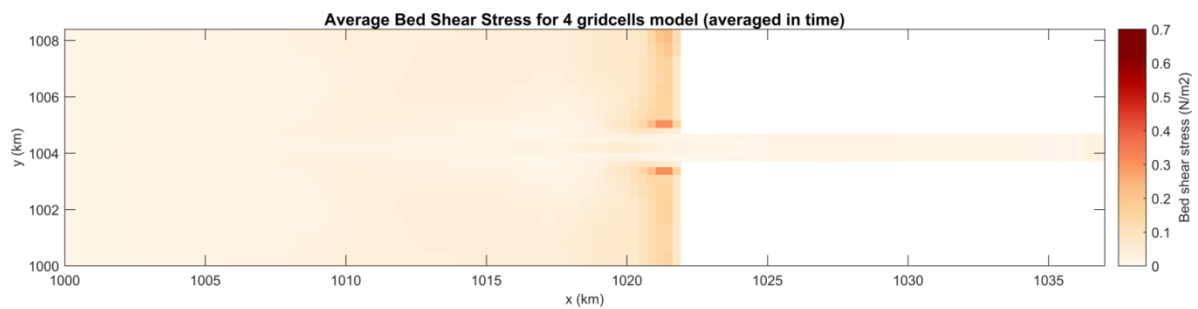


Figure 58: Median values for the bed shear stress averaged in time computed by the 4 grid cells model.

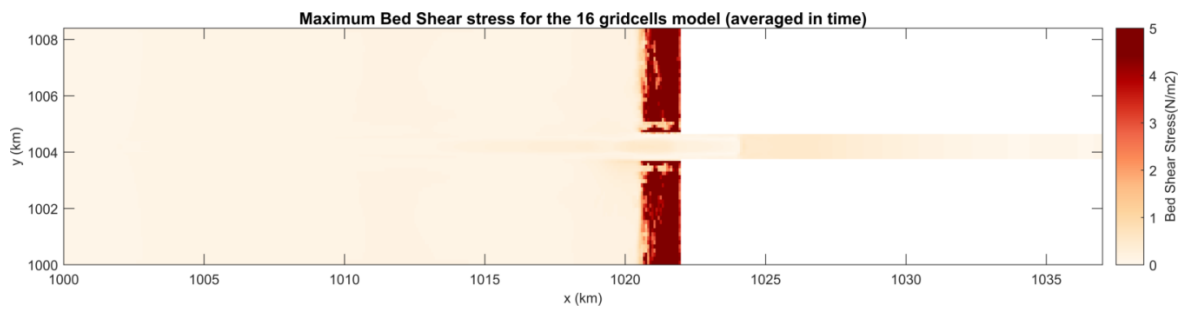


Figure 59: Highest values for the bed shear stress averaged in time computed by the 16 grid cells model.

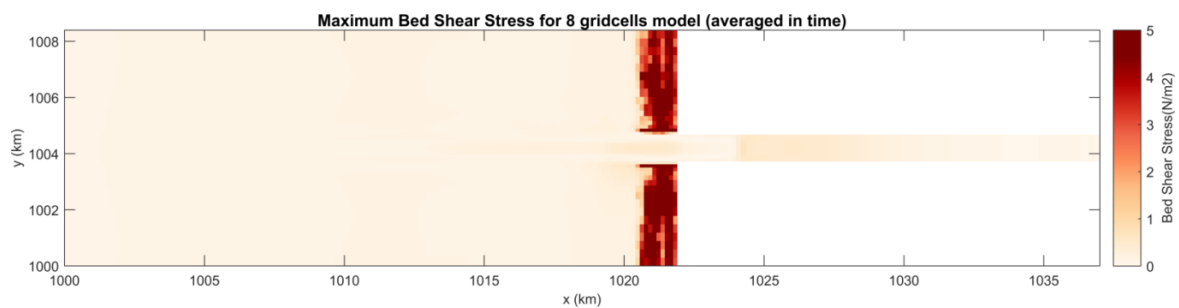


Figure 60: Highest values for the bed shear stress averaged in time computed by the 4 grid cells model

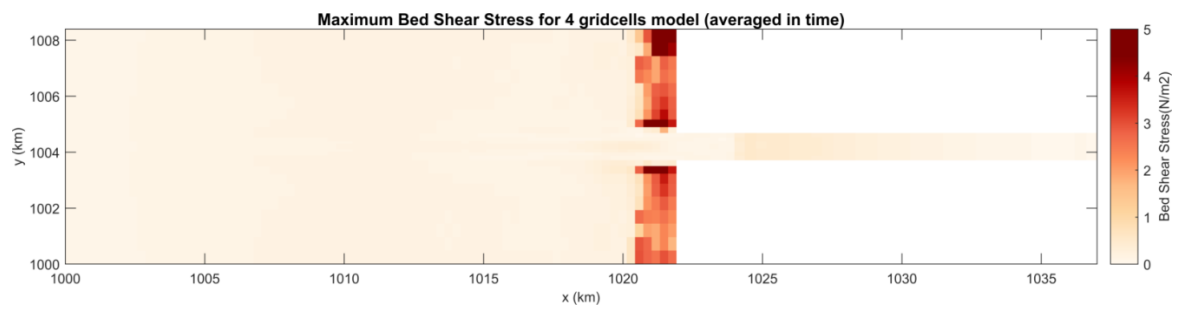


Figure 61: Median values for the bed shear stress averaged in time computed by the 4 grid cells model.

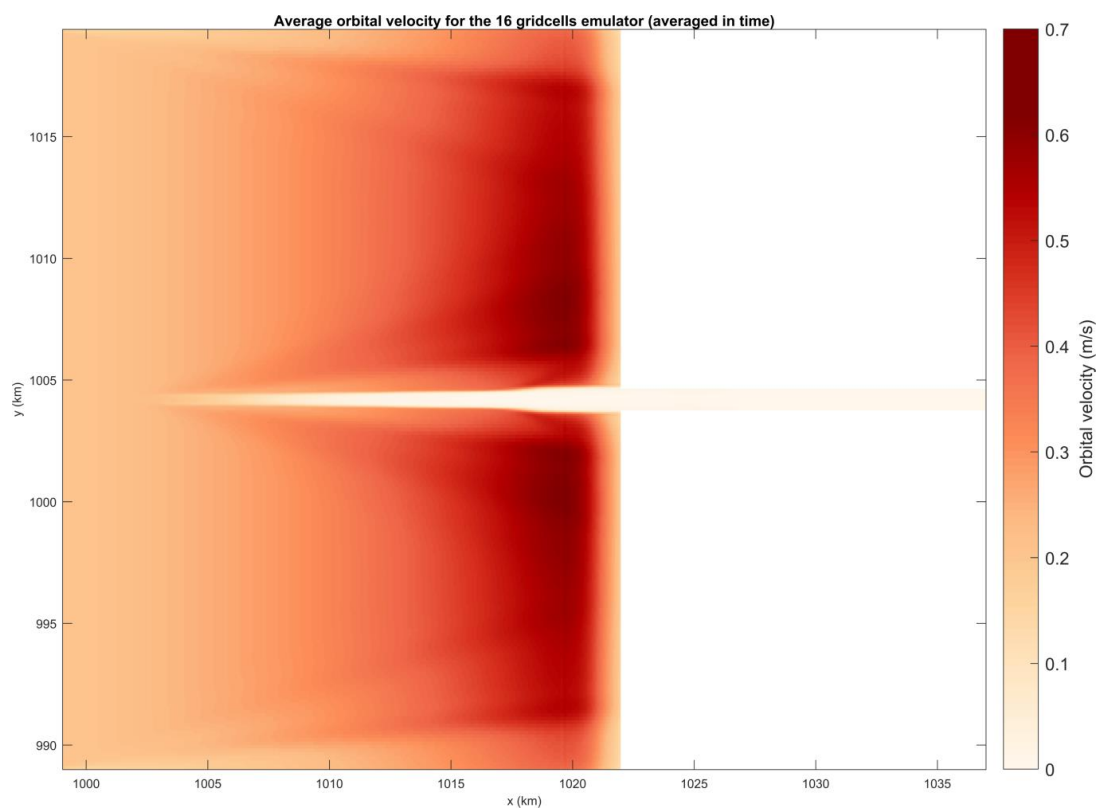


Figure 62: Median values for the orbital velocity averaged in time computed by the 16 grid cells model.

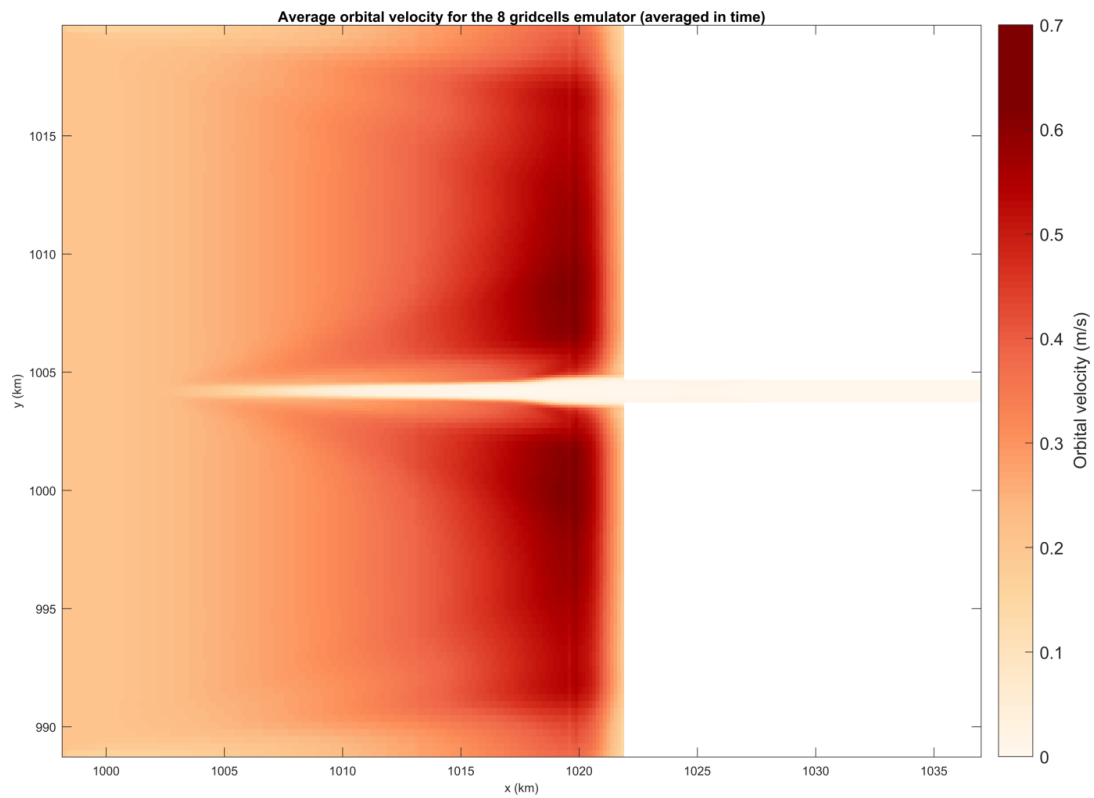


Figure 63: Median values for the orbital velocity averaged in time computed by the 8 grid cells model.

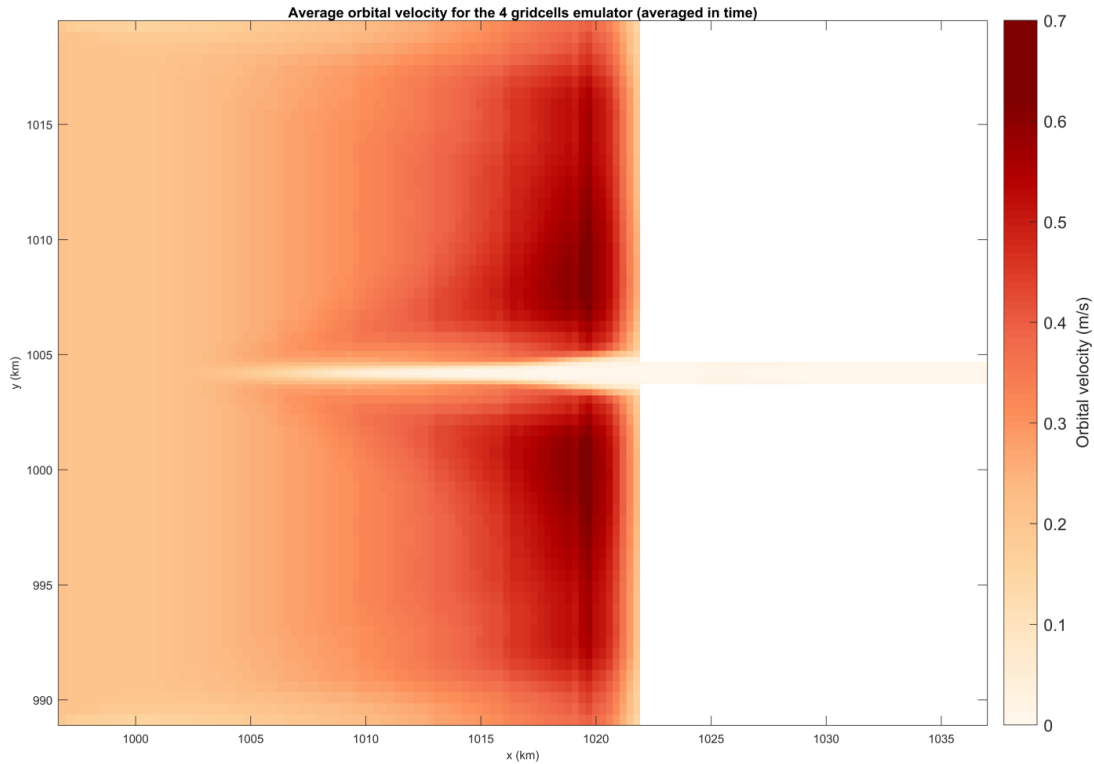


Figure 64: Median values for the orbital velocity averaged in time computed by the 4 grid cells model.

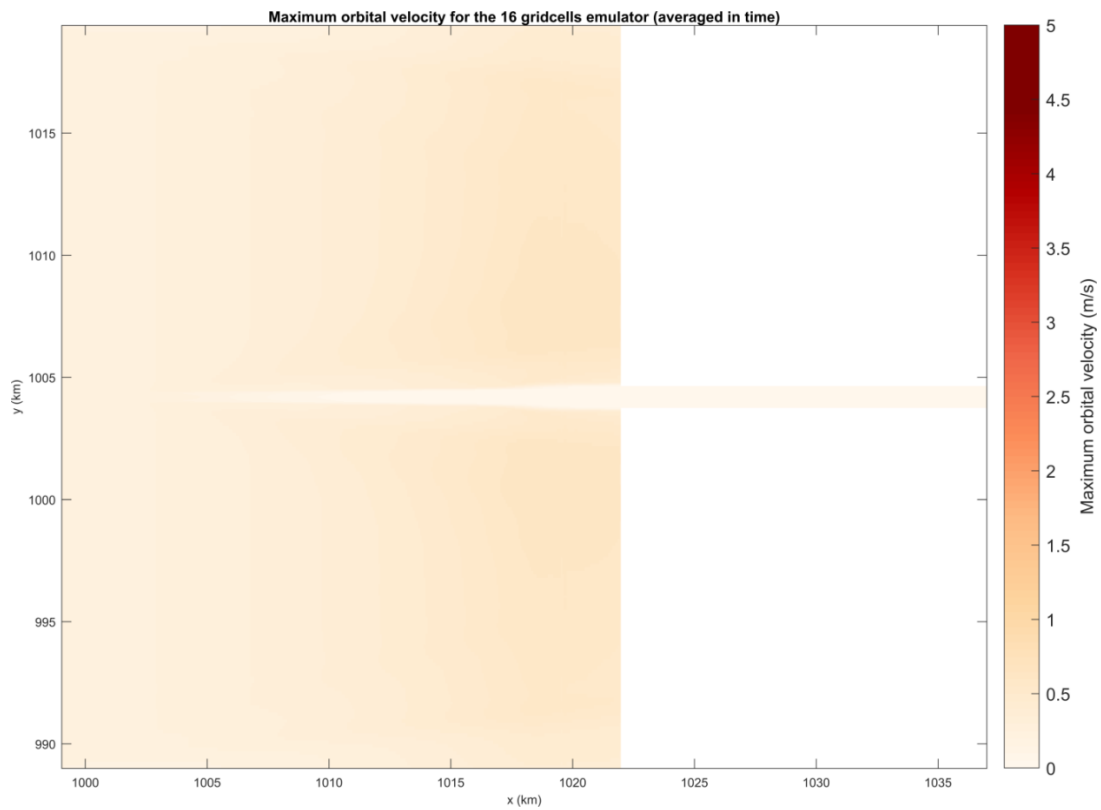


Figure 65: Highest values for the orbital velocity averaged in time computed by the 16 grid cells model.

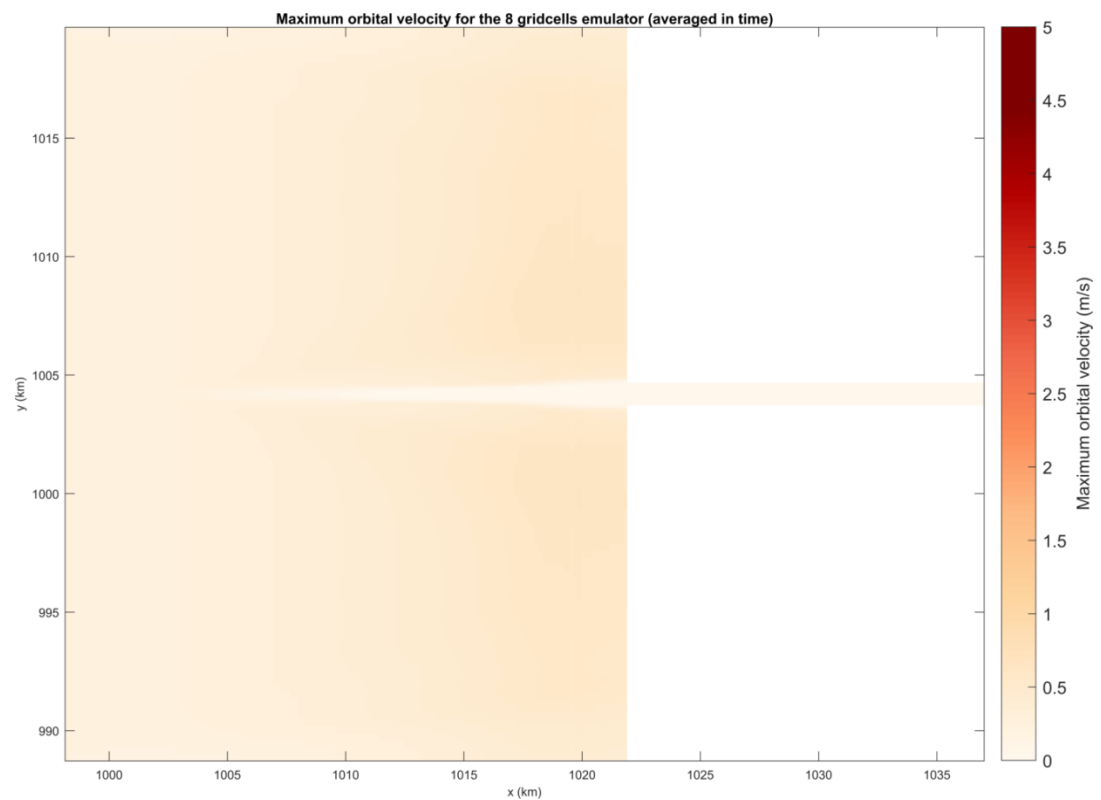


Figure 66: Highest values for the orbital velocity averaged in time computed by the 8 grid cells model.

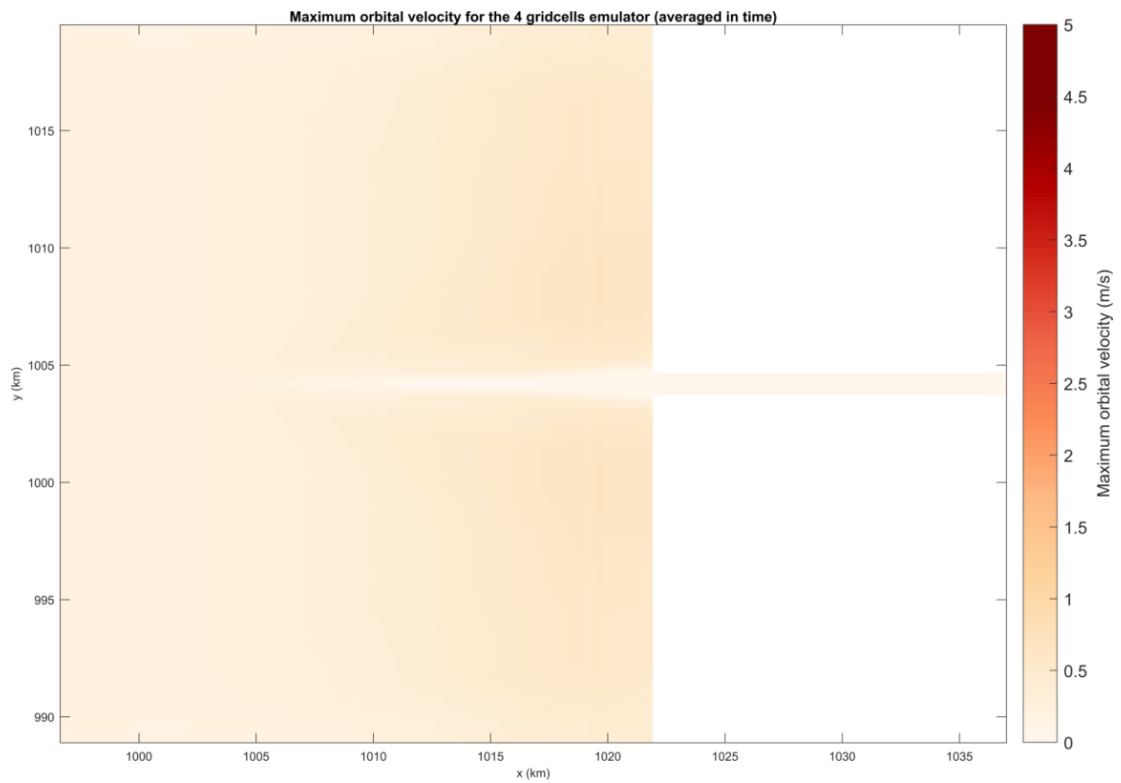


Figure 67: Highest values for the orbital velocity averaged in time computed by the 4 grid cells model.

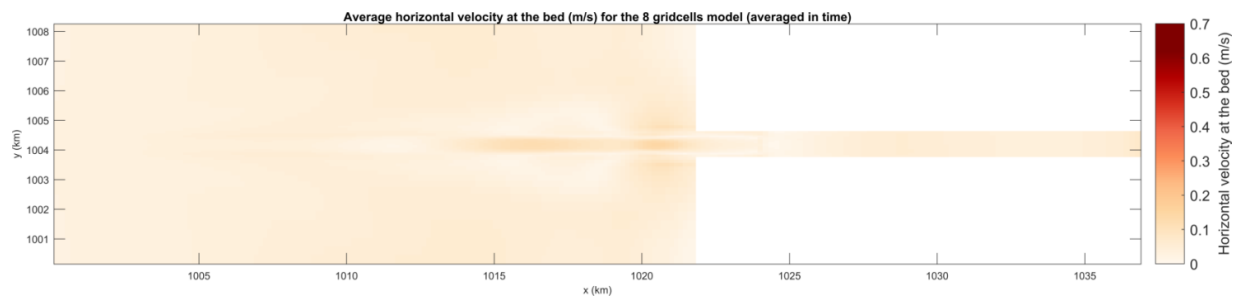


Figure 68: Median values for the horizontal velocity at the bed averaged in time computed by the 16 grid cells model.

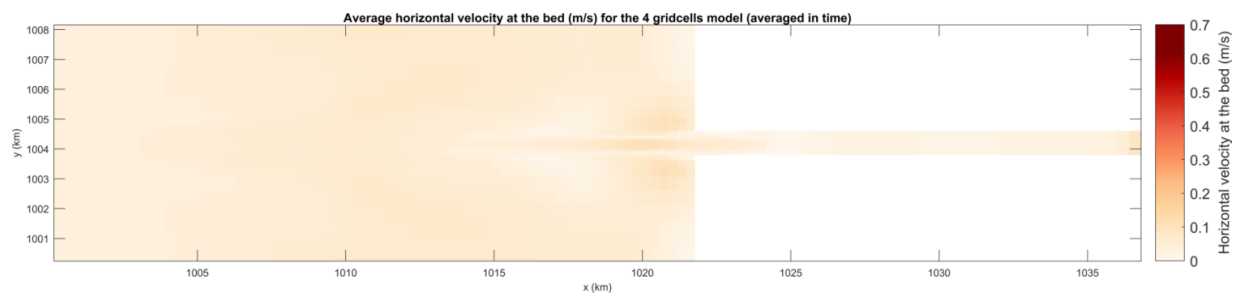


Figure 69: Median values for the horizontal velocity at the bed averaged in time computed by the 8 grid cells model.

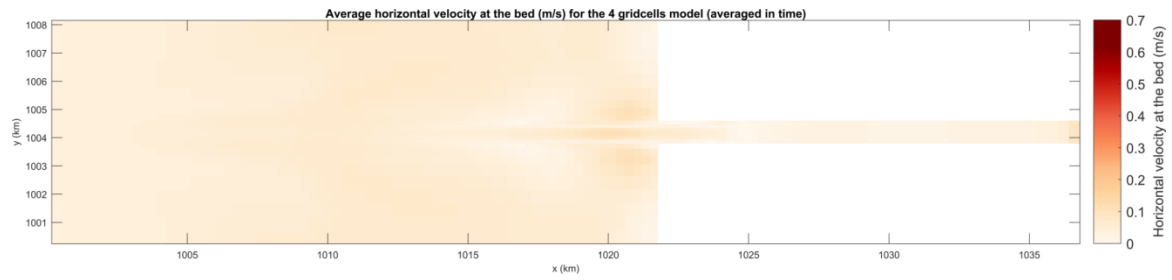


Figure 70: Median values for the horizontal velocity at the bed averaged in time computed by the 8 grid cells model.

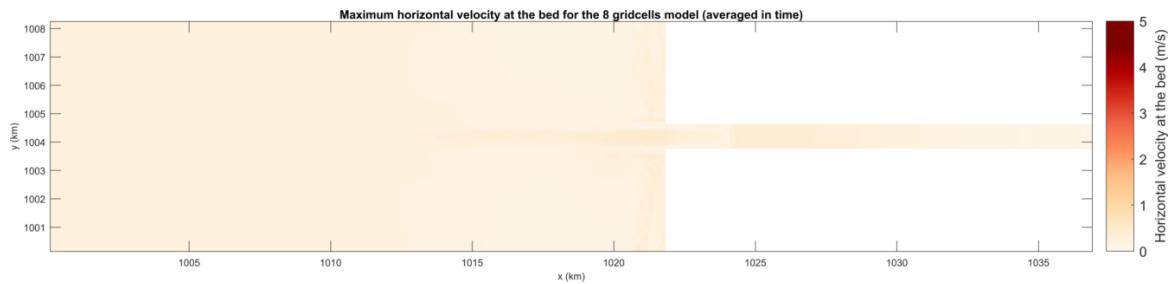


Figure 71: Highest values for the horizontal velocity at the bed averaged in time computed by the 4 grid cells model.

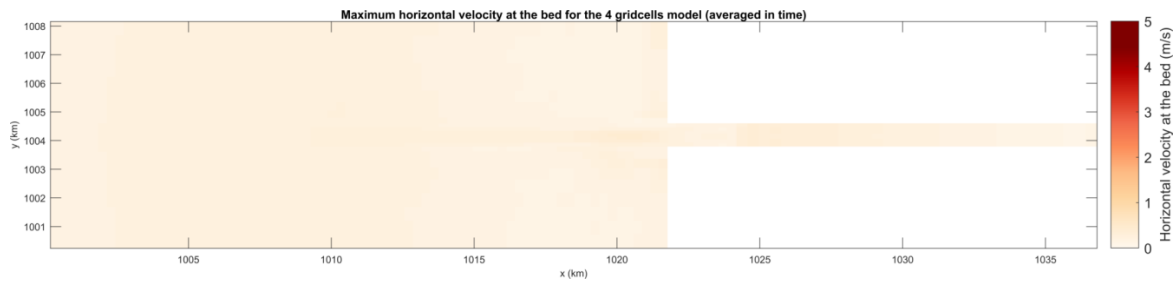


Figure 72: Highest values for the horizontal velocity at the bed averaged in time computed by the 4 grid cells model.

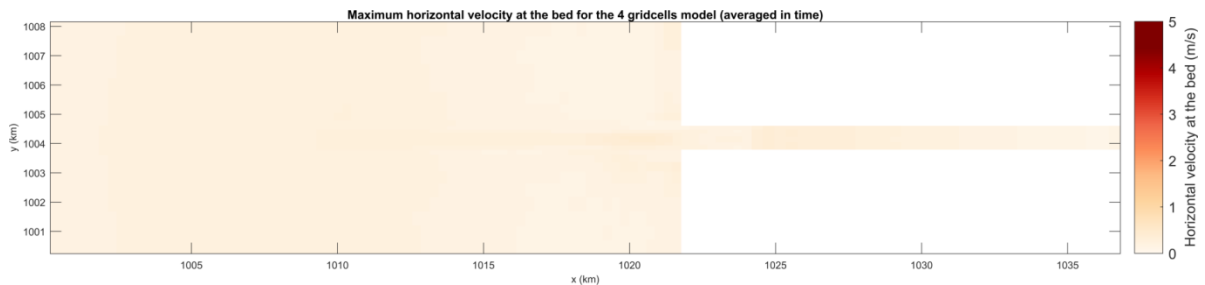


Figure 73: Highest values for the horizontal velocity at the bed averaged in time computed by the 4 grid cells model.

Appendix 5: Comparison between maximum bed shear stress due to currents and waves, bed shear stress due to currents, orbital velocity and horizontal velocity at the bed

- $\text{mean} \left\{ \frac{\tau_{max,16}}{\tau_{max,8}} \right\}$

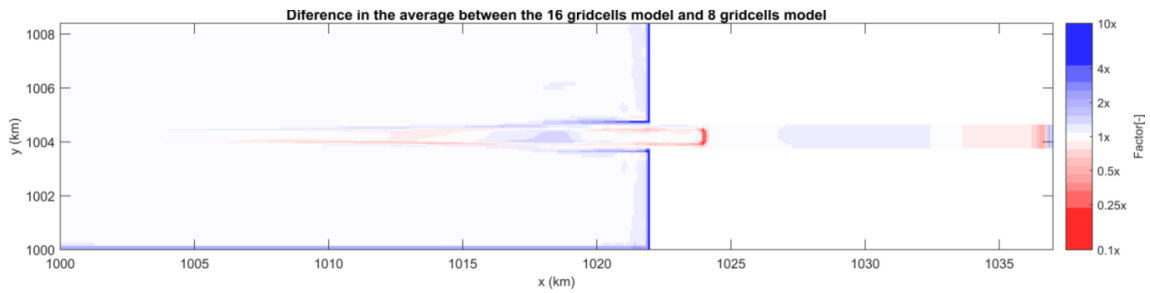


Figure 74: Ratio of the average values maximum bed shear stress between the 16 and 8 grid cells model.

- $\text{mean} \left\{ \frac{\tau_{max,16}}{\tau_{max,4}} \right\}$

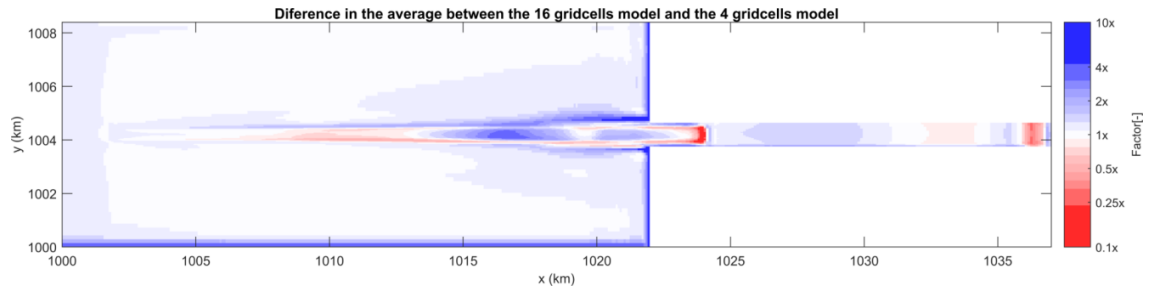


Figure 75: Ratio of average values of the maximum bed shear stress between the 16 and 4 grid cells model.

- $\text{max} \left\{ \frac{\tau_{max,16}}{\tau_{max,8}} \right\}$

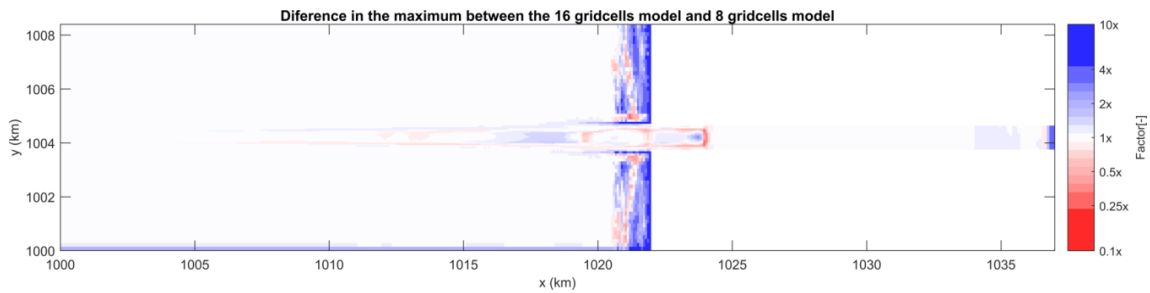


Figure 76: Ratio of maximum values of the maximum bed shear stress between the 16 and 8 grid cells model.

- $\max \left\{ \frac{\tau_{c,16}}{\tau_{c,4}} \right\}$

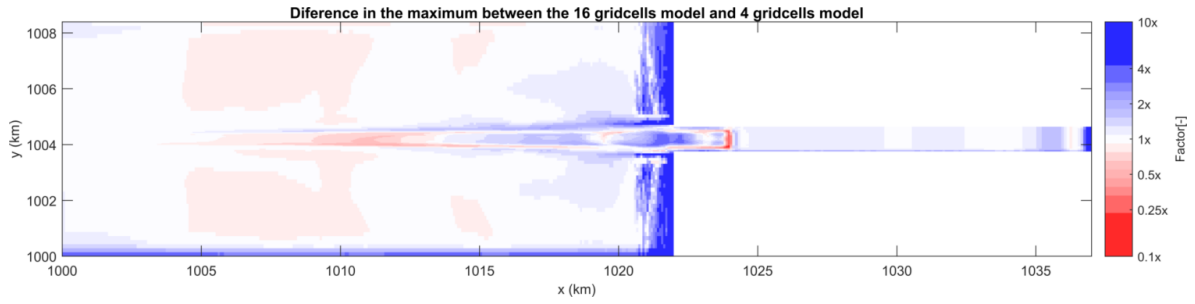


Figure 77: Ratio of maximum values of the maximum bed shear stress between the 16 and 4 grid cells model.

- $\text{mean} \left\{ \frac{\tau_{c,16}}{\tau_{c,8}} \right\}$

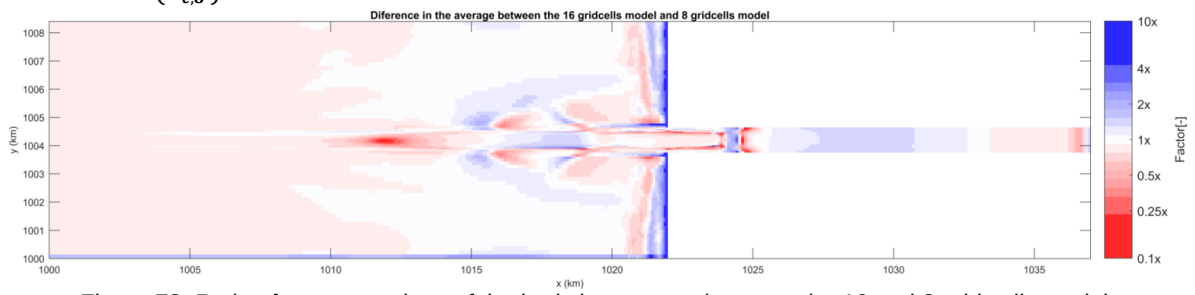


Figure 78: Ratio of average values of the bed shear stress between the 16 and 8 grid cells model.

- $\text{mean} \left\{ \frac{\tau_{c,16}}{\tau_{c,4}} \right\}$

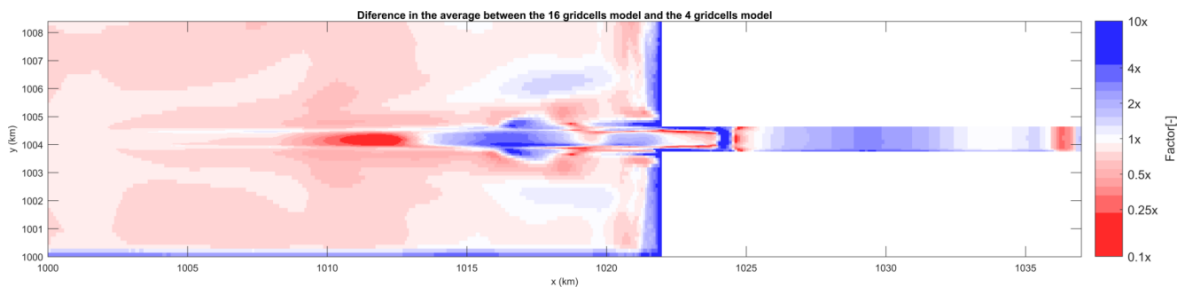


Figure 79: Ratio of average values of the bed shear stress between the 16 and 4 grid cells mode

- $\max \left\{ \frac{\tau_{c,16}}{\tau_{c,8}} \right\}$

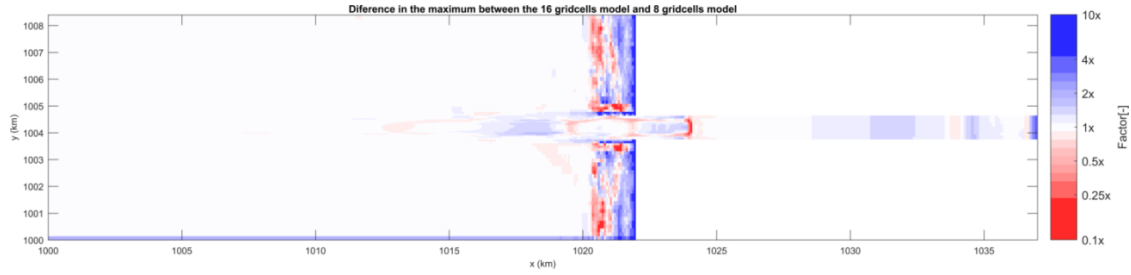


Figure 80: Ratio of maximum values of the bed shear stress between the 16 and 8 grid cells model.

- $\max \left\{ \frac{\tau_{c,16}}{\tau_{c,4}} \right\}$

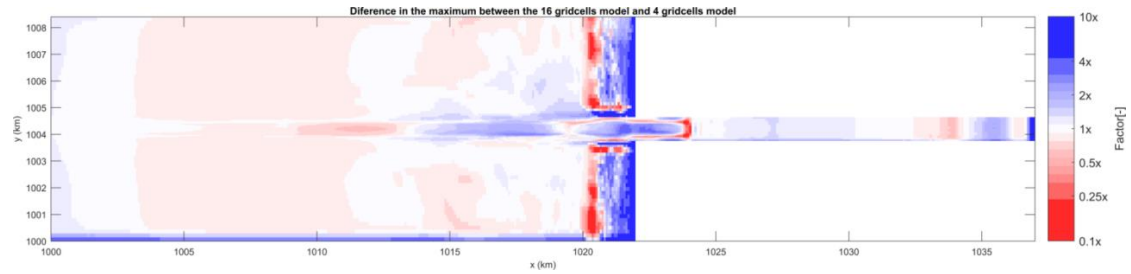


Figure 81: Ratio of maximum values of the bed shear stress between the 16 and 4 grid cells model.

- $\text{mean} \left\{ \frac{u_{orb,16}}{u_{orb,8}} \right\}$

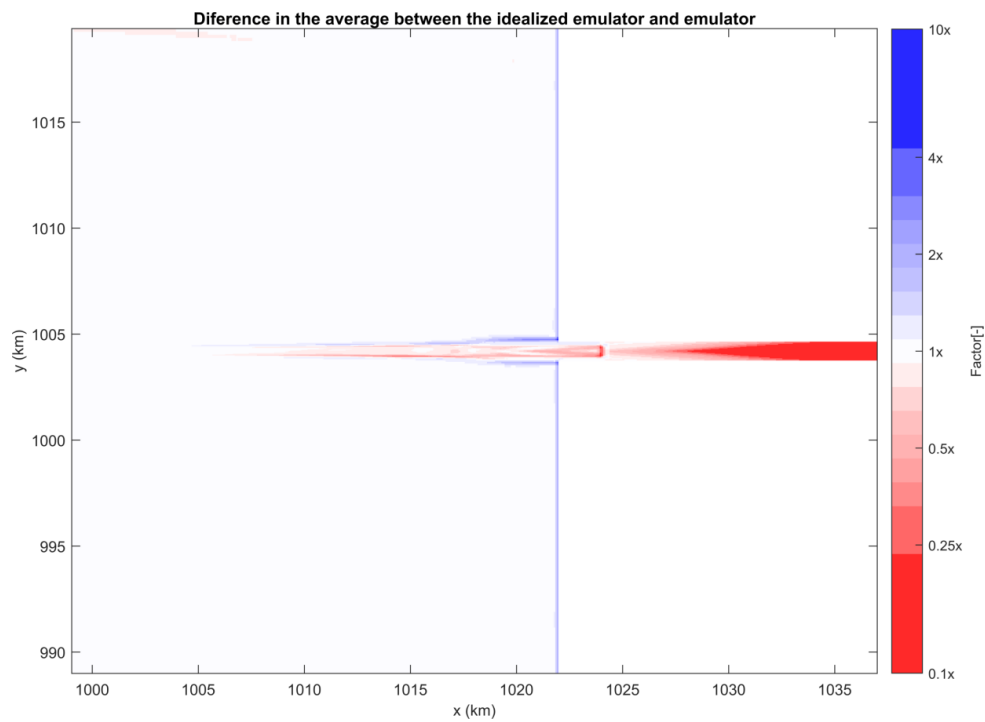


Figure 82: Ratio of the average values of the orbital velocity of the 16 and 8 grid cells model.

- $\text{mean} \left\{ \frac{u_{orb,16}}{u_{orb,4}} \right\}$

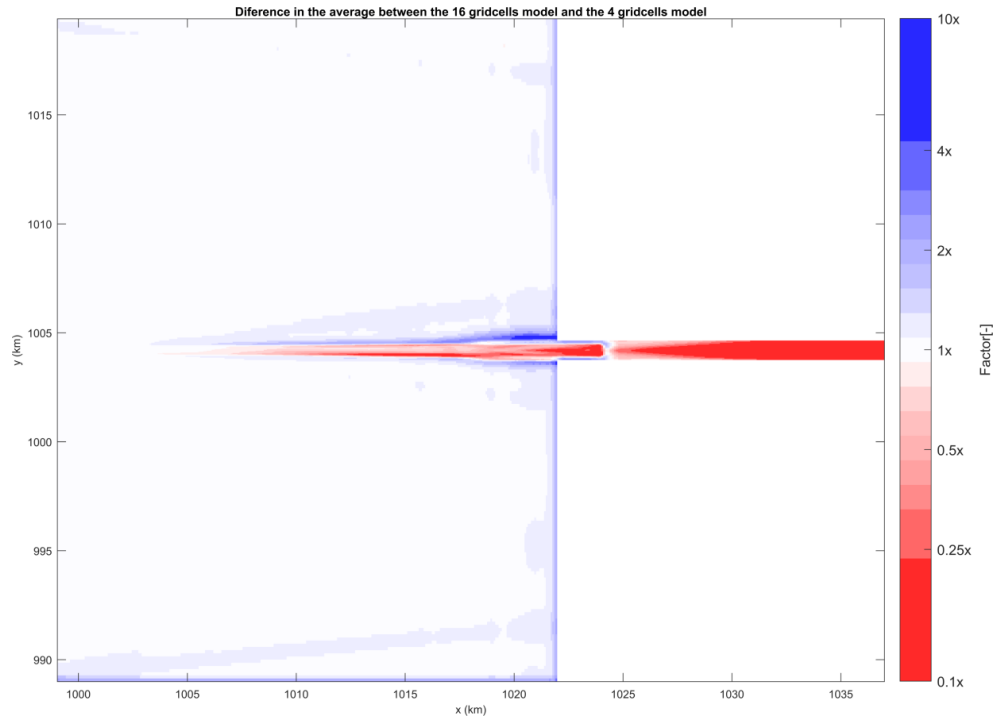


Figure 83: Ratio of the average values of the orbital velocity of the 16 and 4 grid cells model.

- $\text{max} \left\{ \frac{u_{orb,16}}{u_{orb,8}} \right\}$

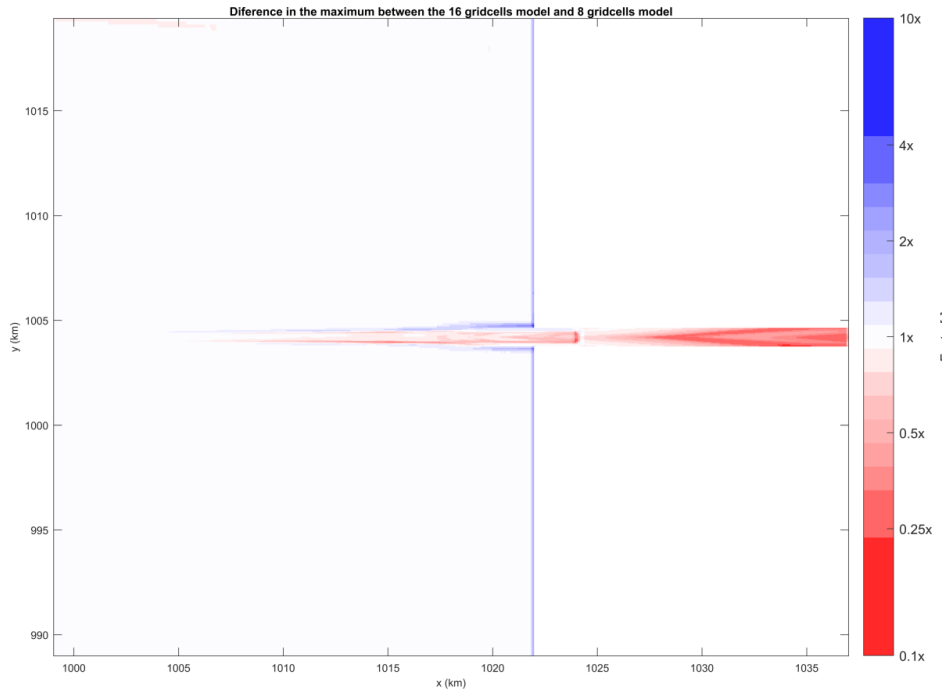


Figure 84: Ratio of the maximum values of the orbital velocity of the 16 and 8 grid cells model.

- $\max \left\{ \frac{u_{orb,16}}{u_{orb,4}} \right\}$

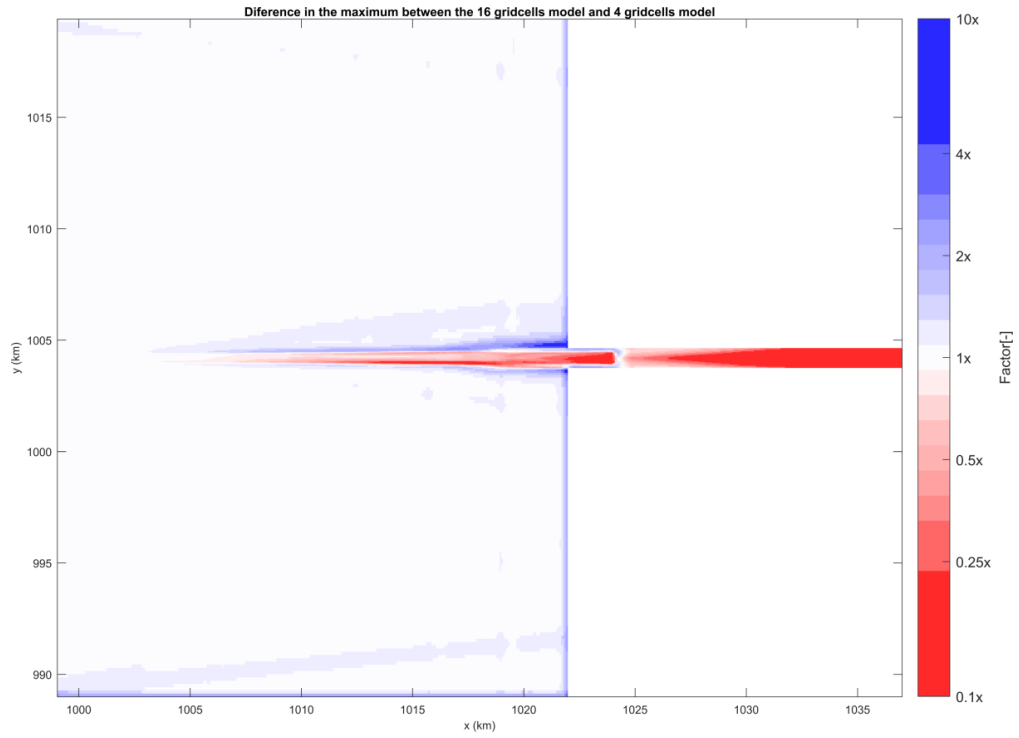


Figure 85: Ratio of the maximum values of the orbital velocity of the 16 and 4 grid cells model.

- $\text{mean} \left\{ \frac{u_{b,16}}{u_{b,4}} \right\}$

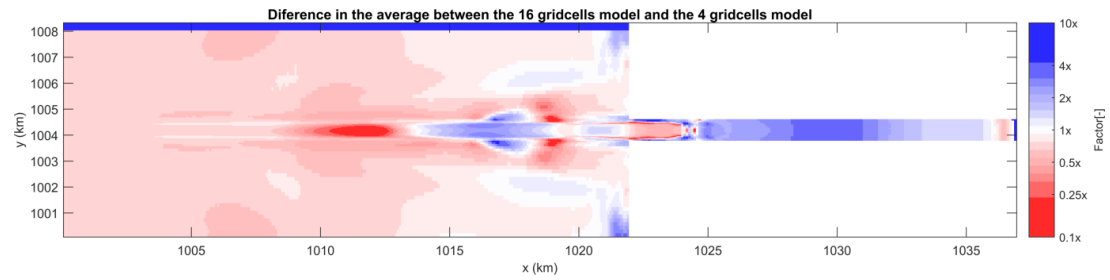


Figure 86: Ratio of the average values of the horizontal velocity at the bed of the 16 and 8 grid cells model.

- $\text{mean} \left\{ \frac{u_{b,16}}{u_{b,8}} \right\}$

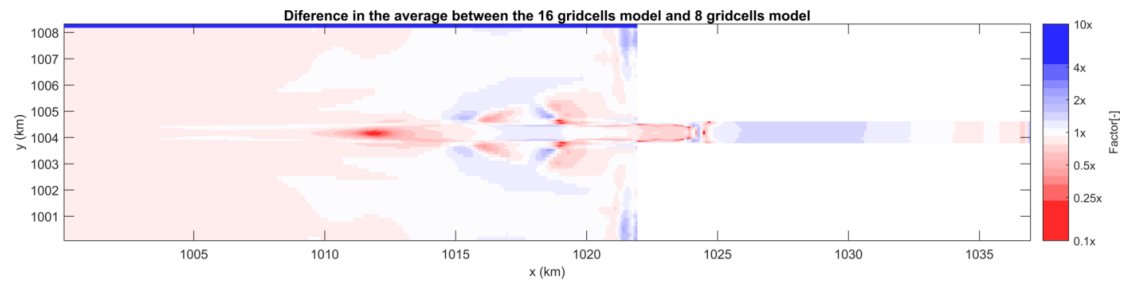


Figure 87: Ratio of the average values of the horizontal velocity at the bed of the 16 and 4 grid cells model.

