Modelling the Output Spectrum of a Direct Digital Synthesizer exploiting a Digital to Time Converter

Pepijn Ekelmans, s1442988, Bachelor Student Electrical Engineering, University of Twente, Enschede, Netherlands

***Abstract*—A DDS (Direct Digital Synthesizer) is a highly configurable system that is capable of outputting square waves with a large range of different frequencies. The system described in this paper uses the MSB of the output of an accumulator as a square wave signal. This edges in this signal suffer from large delays, which are corrected by using logic and a DTC (Digital to Time Converter). The delay of the edges are increased in such a way that a clean signal is created with equal periods and the desired frequency. Unfortunately, this signal is a victim of quantization caused by the digital nature of the input of the DTC and the output of the accumulator. This causes quantization spurs in the frequency spectrum, which is undesirable. By modelling these quantization spurs, the system can be analyzed and optimized. Due to the dependence of the output on a total of four parameters (the resolution, tuning number and clock frequency of the accumulator and the resolution of the DTC) an extensive model is needed to predict the energy of these spurs. This model is created by considering the difference between the delayed edge and the desired edge as error pulses subtracted or added from the ideal output signal. These error pulse can be modelled as square waves with a certain duty cycle and time-shift, which are also based on the four parameters defined previously. The duty cycle is determined by the duration of the pulse, while the time-shift is caused by the location of the pulse in the output signal. By defining all these components separately and using the law of superposition, a complete equation has been created. This equation is capable of predicting the exact energy for the carrier signal and every existing quantisation spur in the frequency spectrum of the output signal. A simulation shows how the equation matches the output of a simulated DTC-based DDS created in Matlab. Slight inaccuracies have been observed based on the sample size of the simulated output signal. The equation will be able to help in determining the quality of an existing DTC-based DDS and could provide information about ways to improve the system.**

***Keywords*—DDS, DTC, Quantisation Spurs, FFT, Model**

I. INTRODUCTION

A DDS (Direct Digital Synthesizer) is a system capable of generating square waves with any frequency defined by the user and is therefore very reconfigurable. In this paper a model will be presented that describes a DDS that uses the properties of a DTC (Digital to Time Converter) to create signals with significantly accurate periods. A Digital to Time Converter is a component that takes a digital input and delays an edge accordingly. The necessity of this component will be made clear later in this introduction. Another option that has the same functionality as the DTC would be a DLL (Delay Locked Loop). Even though it is a much less complex system than the DTC, the DLL has the disadvantage that its precision decreases when the resolution of its input increases. High resolution is very important in this system, so a DTC is used instead. The exact details of the DTC used in the DDS analysed by this paper are described in [2].

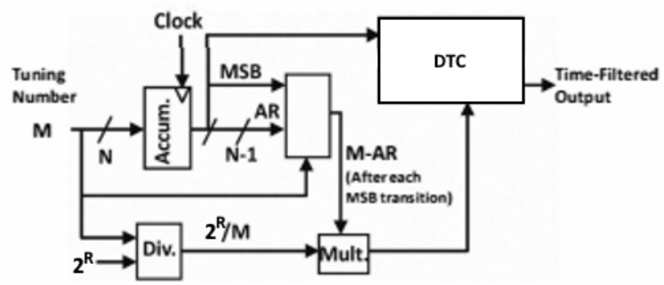

Fig. 1. A block schematic of the DDS exploiting the workings of a DTC [1]

The system that will be modelled in this paper is shown in figure 1. This block schematic is based on another DDS that uses a DLL instead of a DTC[1]. The schematic will be explained in section II.

In section II, the time and frequency spectrum of the square wave output of a DTC-based DDS will be observed and explained. Based on this behaviour, an equation will be constructed in section III. In section IV, a simple example will show that this equation correctly describes the quantisation spurs in the frequency spectrum of the output signal. The paper is concluded in section V.

II. OUTPUT CHARACTERISTICS OF DTC-BASED DDS

The first part of the system is the accumulator. An accumulator has a binary output with N bits resolution of which the value gets increased by tuning number M after every rising edge of a clock signal with frequency f_{clk}. After the output reaches its highest binary value, it reverts back to zero, as the carrier is discarded. By reading just the MSB (Most Significant Bit) of the binary output, a square wave can be constructed. This concept is visualised in figure 2.

As is shown in figure 3 and 2 however, there is one problem with the system; the edges do not always occur on the right point in time. Due to the larger steps of M, the actual falling and rising edges often occur after the ideal falling or rising edge. This overflow in time can be fixed by the rest of the system. Due to causality the delay can not be removed, but will instead be pushed further. By pushing all edges further in time to one common total delay, all rising and falling edges will be at the same distance from each other (figure 4

The added delay will be created by the last part of the system: the DTC. This DTC has an internal DAC that steers it,

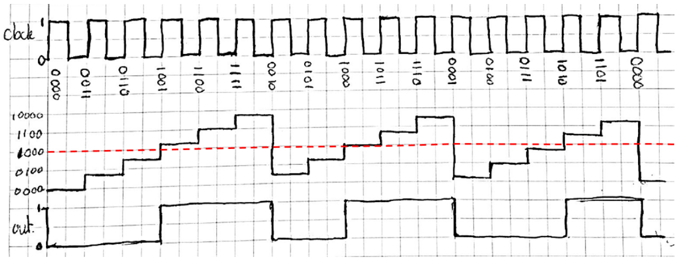

Fig. 2. An example of how the accumulator MSB creates an irregular square wave with frequency $f_{clk}\frac{M}{2^N}$. In this picture, N = 4 and M = 3. M periods are drawn, which allows the accumulator output to end exactly where it started.

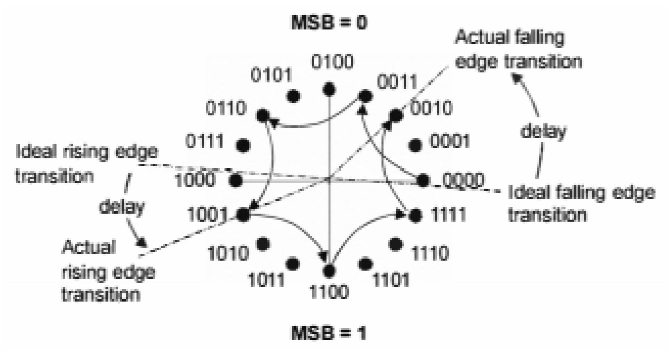

Fig. 3. A cycle showing the the output of the accumulator. In this case the output of the accumulator (N) has 4 bits and the tuning number (M) is 3 [1]

which means that the DTC needs a digital input. That input is called the delay word (DW). The highest possible delay word adds a delay of one clock period to an edge. This means that if a delay needs to be half a clock period, the delay word will be half of its maximum value. The maximum delay word is depended on the resolution R of the DTC. If a DTC has a resolution of R, then its highest delay word can be 2^R. Calculating the delay word is done by defining with which fraction of a clock period an edge needs to be delayed. This can be expressed as $\frac{M-AR}{M}T_{clk}$ in seconds in which AR is the amount of "overflow bits" between the MSB edge transition and the ideal edge transition, as defined in figure 3. This fraction needs to be multiplied with the maximum delay word. However, since the input of the DTC is digital, this value needs to floored to an integer value. This causes a quantization effect. The function that describes the delay word that is submitted to the DTC is equation 1 [1].

$$DW = floor\Big(2^R\frac{M-AR}{M}\Big) \quad (1)$$

Due to the digital nature of the input of the DTC, the delay is quantized. This means that some edges occur earlier than they should. The higher the resolution of the DTC, the smaller these errors become. This need for high resolutions is why a DLL would not be a good choice for a system like this. In contrast to the DLL, the DTC would remain much more stable for higher resolutions, which is important in this system. Figure 5 gives a clear indication of the consequences of quantisation in the delay of the edges.

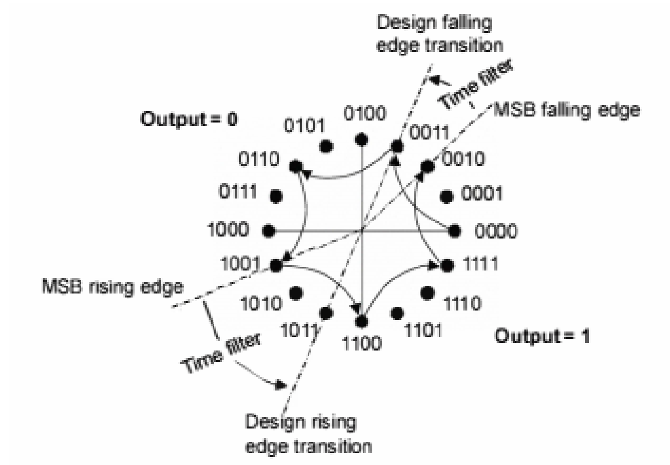

Fig. 4. The idea behind the DTC-based DDS [1]

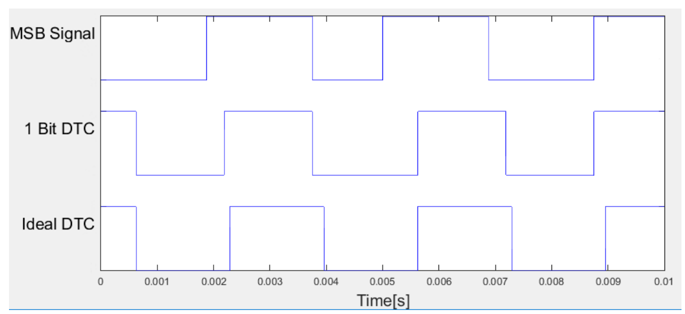

Fig. 5. Some delay words do not completely correct the signal due to quantization

In the frequency spectrum, the quantization of the delays is visible as quantisation spurs. In this paper, an equation will be derived that models the exact power of those quantisation spurs as a function of the system parameters N, M, R and T_{clk}. This can help to determine the effectiveness of increasing or decreasing certain parameters. It is also useful for determining the quality of DTC-based DDS systems.

In this paper, it is assumed that the clock signal has a constant period, no jitter and a rising and falling time of zero. Furthermore, the DAC inside the DTC has an INL and DNL of 0 and the comparator in the DTC does not produce delay by itself. None of the parts in the system produce any random noise.

For the cases when M and 2^N are not coprimes, M and N can be made smaller for the efficiency of calculating the equation later on. For example, a system with N = 5 and M = 6 produces the same output as a system with N = 4 and M = 3. The first system takes twice as many steps per clock cycle on an accumulator cycle that is also twice as long. Therefore, one MSB period will be just as long, as the amount of steps are the same. A smaller value of M will simplify the equation, so for ease of use, the parameters M' and N' will be created. These parameters are defined by $M' = \frac{M}{gcd(M,2^N)}$ and $N' =$

$N - \log_2(gcd(M, 2^N))$.

III. CONSTRUCTION OF THE EQUATION

A. Constructing parts of the equation

The difference between the quantized signal and the "ideal" signal (in this case an ideal signal is a signal without and delayed edges due to quantization) could be seen as a small square "error" pulse. If these error pulses are correctly subtracted from or added to the ideal signal, a signal will be created that exactly represents the output of a DTC-based DDS. This idea is visualised in figure 6. A signal that is the sum of two signals has a frequency spectrum that is the sum of the individual frequency spectra of the original two signals [4] (Appendix A, equation 8). This means that if the frequency spectra of the ideal signal and the error pulses are known, the frequency spectrum of the output of the DTC-based DDS can be described.

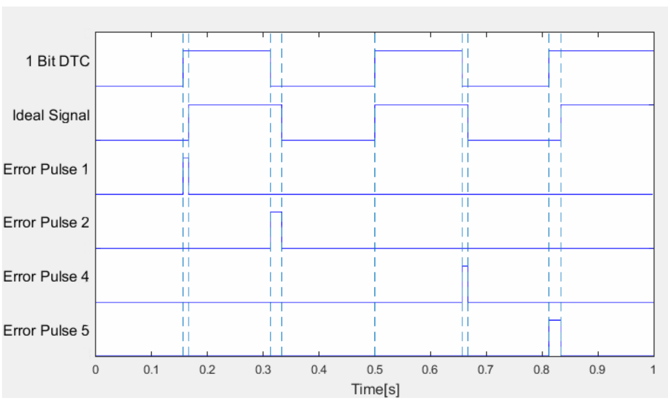

Fig. 6. Visualisation of the error pulses created in a DTC-based DDS with an accumulator resolution N' of 4, a tuning number M' of 3, a clock frequency f_{clk} of 16 Hz and a DTC resolution R of 1. The 16 Hz clock signal creates an error period T_{error} of 1 second, as the accumulator takes $2^{N'} = 16$ steps to return to its starting point. By adding error pulse 1 and 5 and subtracting pulse 2 and 4 from the ideal signal, the output of a DTC-based DDS with a 1 bit DTC can be mimicked.

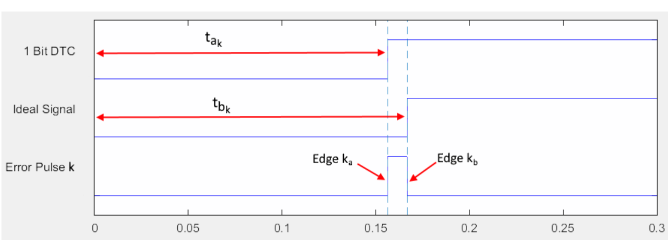

Fig. 7. Visualisation of the error pulses created in a DTC-based DDS with an accumulator resolution N' of 4, a tuning number M' of 3, a clock frequency f_{clk} of 16 Hz and a DTC resolution R of 1. By adding error pulse 1 and 5 and subtracting pulse 2 and 4 from the ideal signal, the output of a DTC-based DDS with a 1 bit DTC can be mimicked.

An error pulse k (in which k corresponds to the k^{th} edge in one period of the ideal signal) is defined as a periodic pulse that is only nonzero between an edge k_a in the quantized signal and its corresponding edge k_b in the non-quantized ideal signal. Edge k_a always represents the left edge of the error pulse, since it comes from the quantized signal with floored delays. A floored value is always either lower than or equal to its original value. The time periods between the start of the period and edge k_a and k_b respectively are called t_{a_k} and t_{b_k}. This is visualised in figure 7. An error pulse occurs every $2^{N'}$ clock periods, defining $T_{error} = T_{clk} * 2^{N'}$. This is because for tuning number M', there are M' unique periods. This is clearly visible in figure 2. These M' unique periods always have a total duration of $T_{clk} * 2^{N'}$. For every edge, a unique error pulse has to be created. Since the tuning number merely defines the amount of periods inside the given time period of $T_{clk} * 2^{N'}$, every edge always occurs only once in this period of time. This gives the error pulses a period T_{error} equal to $T_{clk} * 2^{N'}$.

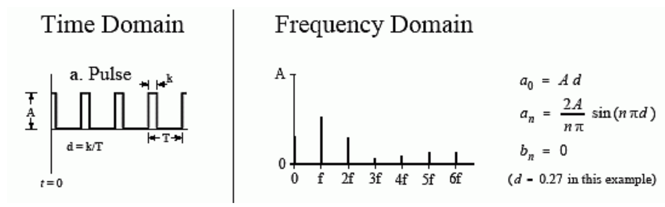

Fig. 8. Fourier Series of a square wave with a variable duty cycle [3]

To describe the frequency spectrum of the error pulses and the ideal signal, the Fourier Series described in figure 8 is used. This figure describes the Fourier Series of a square wave with a variable duty cycle. The difference between error pulses and the square wave described in figure 8, is that the error pulses occur on a different point in the period. By using the time-shifting theorem of the Fourier Transform [4] (Appendix A, equation 9), this can also be accounted for. This is unfortunately only applicable for the Fourier Transform, meaning that the Fourier series will have to be converted into a Fourier transform. This will be done by using equation 2. By combining these three equations, equation 3 is derived, where T is the period of the signal. This period is $T_{error} = T_{clk} * 2^{N'}$ for error pulses and $T_{ideal} = \frac{T_{clk} * 2^{N'}}{M'}$ for the ideal signal. The complete derivation of this function will be displayed in Appendix B.

$$X_k(f) = \sum_{n=-\infty}^{\infty} X_k[n]\delta\left(f - \frac{n}{T}\right) \quad (2)$$

$$X_k(f) = \frac{2A}{\pi}\exp(-j2\pi f t_{0_k}) \sum_{n=-\infty}^{\infty} \frac{\sin(n\pi d_k)}{n}\delta\left(f - \frac{n}{T}\right) \quad (3)$$

For the ideal signal, the duty cycle and time-shift are always the same. The ideal signal has a duty cycle of 0.5 and is time-shifted by $\frac{3}{4}T$, in which T is the period of the error signal $T_{error} = T_{clk} * 2^{N'}$. Since the period in figure 8 starts three quarters of a period after the falling edge (at which the ideal signal starts in this model) it should be time-shifted by that amount. T_{error} is used, because the equation for the ideal signal will be constructed by creating it with the frequency of the error signal, after which it will be dilated to the correct frequency.

To describe the error pulses with equation 3, a duty cycle and a time-shift have to be defined for each pulse individually. As has been explained before, the error pulse signal is only nonzero between edge k_a and k_b. The duty cycle of the error pulse is the fraction of the period in which the value of the signal is nonzero. The duty cycle of error pulse k can be derived by subtracting t_{a_k} from t_{b_k} (as defined in figure 7) and dividing that sum by the duration of one period of the error pulse, $T_{error} = T_{clk} * 2^{N'}$. The resulting equation is defined in 4. In Appendix C, the equation is derived in detail.

$$d_k = \frac{k}{2M'} - \frac{floor(\frac{k2^{N'+R}}{2M'})}{2^{N'+R}} \quad (4)$$

It is assumed that the time-shift of the error pulse is 0 when it is depicted as in figure 8; nonzero and symmetrical in t = 0. This means that, to create an error pulse from the time signal in figure 8, the time signal first has to shift half the duration of the pulse and then has to shift over the duration of t_{a_k}. The duration of the pulse is defined by the duty cycle of the pulse multiplied by the period of the error pulse T_{error}. This brings the total time-shift to $t_{0_k} = 0.5 * d_k * T_{error} + t_{a_k}$. The complete derivation of this is equation 5. In Appendix C, the derivation is explained in detail.

$$t_{0_k} = \frac{T_{error}}{2}\left(\frac{floor(\frac{k2^{N'+R}}{2M'})}{2^{N'+R}} + \frac{k}{2M'}\right) \quad (5)$$

B. Construction of the total equation

The ideal signal can be constructed using equation 3. As was defined in the previous section, the ideal signal has a duty cycle of 0.5 and is time-shifted by $\frac{3}{4}T_{error} = \frac{3*T_{clk}*2^{N'}}{4}$. The ideal signal has a time signal that, relative to the error pulses, is dilated by a factor M'. According to the Fourier Transform Dilation Theorem[4] (Appendix A, equation 10), this means that the ideal signal can be described by using equation 3 and dividing that function and its variable f by M'. The function then becomes equation 6. The derivation of this function is described in Appendix D.

$$X_{ideal}(f) = \frac{2A}{M'\pi}\exp(-j\frac{\pi f}{M'}\frac{3 * T_{clk} * 2^{N'}}{2})* \sum_{n=-\infty}^{\infty}\frac{\sin(\frac{n\pi}{2})}{n}\delta\Big(f - \frac{nM'}{T_{clk} * 2^{N'}}\Big) \quad (6)$$

The error pulses are more complex and require a more general definition. Some pulses are subtracted from the ideal signal, while others are added. As a point of reference to define which pulses do what, it is assumed that the output signal of the DTC-based DDS starts exactly after a falling edge where the accumulator output is 0, as is done in figure 6 and figure 2. Since the edges k_a of the quantized signal are always earlier than the edges k_b, an error pulse that is created for a rising edge always needs to be added to the ideal signal while an error pulse that is created for a falling edge always needs to be subtracted from the ideal signal. By giving the edges of the ideal signal a value k ranging from 1 to 2M'-1, it is clear that rising edges occur on odd and falling edges occur on even edges. This means that odd numbered error pulses need to be added, while even numbered error pulses should be subtracted. Edge 0, M' and 2M' all have a pulse with duty cycle 0, as the edge of the quantized signal and the ideal signal occur at the exact same time. This means that these values can be excluded from the equation. By combining the remaining pulses, a total equation 7 can be defined, with d_k defined in equation 4 and t_{0_k} defined in equation 5.

$$\begin{aligned} X(f) = \frac{2A}{\pi}* \\ \Bigg(\Bigg(\frac{1}{M'}\exp(-j\frac{\pi f}{M'}\frac{3 * T_{clk} * 2^{N'}}{2})* \\ \sum_{n=-\infty}^{\infty}\frac{\sin(\frac{n\pi}{2})}{n}\delta\Big(f - \frac{nM'}{T_{clk} * 2^{N'}}\Big)\Bigg) \\ +\Bigg(\sum_{p=0,(2p+1)\neq M'}^{M'-1}\exp(-j2\pi f t_{0_{(2p+1)}})* \\ \sum_{n=-\infty}^{\infty}\frac{\sin(n\pi d_{(2p+1)})}{n}\delta\Big(f - \frac{n}{T_{clk} * 2^{N'}}\Big)\Bigg) \\ -\Bigg(\sum_{p=1,(2p)\neq M'}^{M'-1}\exp(-j2\pi f t_{0_{(2p)}})* \\ \sum_{n=-\infty}^{\infty}\frac{\sin(n\pi d_{(2p)})}{n}\delta\Big(f - \frac{n}{T_{clk} * 2^{N'}}\Big)\Bigg)\Bigg) \end{aligned} \quad (7)$$

As was explained previously, N' is the resolution of the accumulator, M' is the tuning number, T_{clk} is the clock period and R is the resolution of the DTC. Furthermore, A is the amplitude of the signal in the time domain.

IV. COMPARISON WITH FFT SIMULATIONS

To prove that this equation is really capable of describing the behaviour of the frequency spectrum of the output of a DTC-based DDS, the output of the system and the output predicted by the equation can be compared via a simulation. This simulation was created in Matlab. As a test and for ease of visualisation, the scenario of accumulator resolution N' = 4, tuning number M' = 3 and DTC resolution R = 1 is analyzed in this case. An ideal square wave signal with the desired frequency was created along with four error pulses such that adding and subtracting said error pulses from the ideal signal created the real output of the DDS based DTC. This has been done in figure 6 and works as intended, meaning that the original five components combine into the actual output of the system. This should mean that adding and subtracting the frequency spectra of the error pulses correctly from the frequency spectrum of the ideal signal should indeed produce a correct representation of the output of the system. After calculating the frequency spectrum from the simulated output of the DTC-based DDS, the equation is constructed. By constructing parts of the function and adding and subtracting them accordingly, an exact prediction of the function should be created. The result can be seen in figure 9.

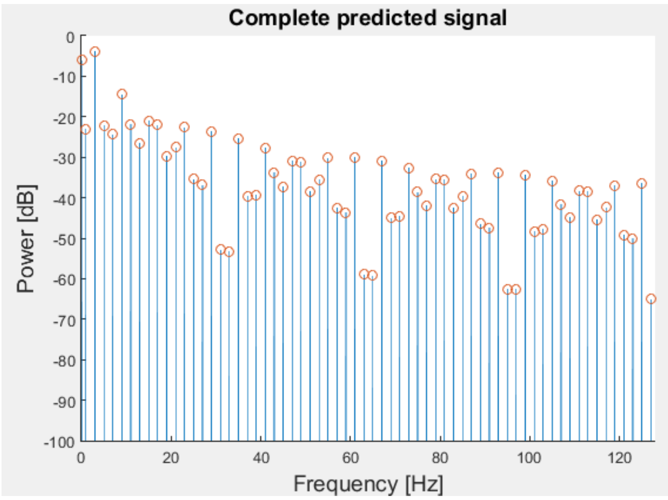

Fig. 9. The frequency spectrum of the output of a DTC-based DDS with accumulator resolution N' = 4, tuning number M' = 3, DTC resolution R = 1 and f_{clk} = 16Hz. The blue signal is the simulated output, while the red circles indicate the prediction done by the equation. This figure was created using 65536 samples for every T_{error} seconds.

In this figure, small inaccuracies can be found. To determine whether these are caused by a finite amount of samples in the simulation signal or an inaccuracy regarding the prediction equation itself, a test was done. The results of this test are displayed in figure 10.

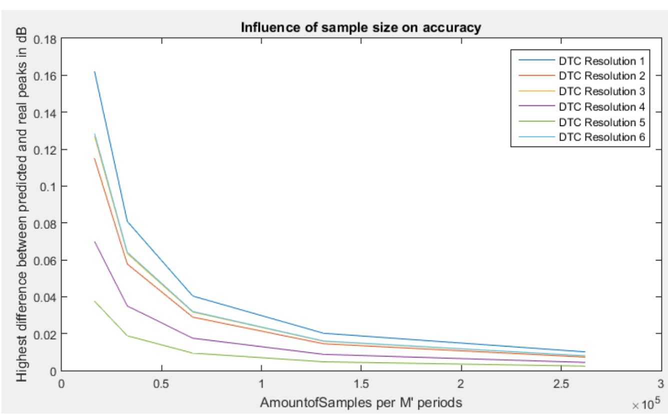

Fig. 10. The highest difference between a predicted point and its simulated output in dB. As can be seen, a doubling of the sample rate halves the highest difference.

Figure 10 shows that the difference between a predicted peak and a simulated peak is halved when the amount of samples in the simulation is doubled. Since a signal in the real world could be compared to a signal with an infinite sample size, this equation is very likely to be a good model for the output of a DTC-based DDS.

V. Conclusion

An equation was found that was capable of describing the energy of important harmonic peaks in the frequency spectrum of a DTC-based DDS with a variable accumulator resolution N, tuning number M, DTC resolution R and clock period T_{clk}. According to measurements, this equation corresponds to a real world frequency spectrum with an infinite amount of samples. This makes the equation a good tool for determining whether it is worth to upgrade the quality of certain components within the DTC-based DDS. It can also help in determining the quality of an already existing DTC-based DDS. The Spurious-Free Dynamic Range (SFDR) can also be found with the help of the equation by finding the highest quantisation spur between 0 and $2f_0$ for any frequency multiple of f_0/M' and determining the difference between the power of the carrier signal and the power of the highest quantisation spur in dBc.

Appendix A
Used Fourier Transform theorems

A. Linearity

$$ag_1(t) + bg_2(t) \leftrightharpoons aG_1(f) + bG_2(f) \tag{8}$$

B. time-shifting

$$g(t - t_0) \leftrightharpoons G(f)exp(-j2\pi f t_0) \tag{9}$$

C. Dilation

$$g(at) \leftrightharpoons \frac{1}{|a|}G\Big(\frac{f}{a}\Big) \tag{10}$$

Appendix B
Constructing the base equation for error pulse k

As defined in figure 8, the Fourier Series of a square wave with a variable duty cycle is defined by equation 11.

$$X_k[n] = \frac{2A}{n\pi}\sin(n\pi d_k) \tag{11}$$

To use the time-shifting theorem, which is a Fourier Transform theorem, equation 11 needs to be transformed into a Fourier Transform. This is done with the help of equation 2 and results in equation 12

$$X_k(f) = \frac{2A}{\pi}\sum_{n=-\infty}^{\infty}\frac{\sin(n\pi d_k)}{n}\delta\Big(f - \frac{n}{T}\Big) \tag{12}$$

After this transformation, the function can be time-shifted according to equation 9.

$$X_k(f) = \frac{2A}{\pi}\exp(-j2\pi f t_{0_k})\sum_{n=-\infty}^{\infty}\frac{\sin(n\pi d_k)}{n}\delta\left(f - \frac{n}{T}\right) \tag{13}$$

Finally, if this equation is used for error pulses, $T = T_{error} = T_{clk} * 2^{N'}$, while if the equation is used for the ideal signal, $T = T_{ideal} = \frac{T_{clk} * 2^{N'}}{M'}$

APPENDIX C
CONSTRUCTING THE EQUATION FOR THE DUTY CYCLE AND TIME-SHIFT OF AN ERROR PULSE

To define the duty cycle and the time-shift, the position of the right and left edge need to be determined. These are defined as t_{a_k} and t_{b_k}, as depicted in figure 7. For easy of explanation, three time periods are defined: T_{clk}, T_{ideal}, T_{error}. These periods are shown in figure 11.

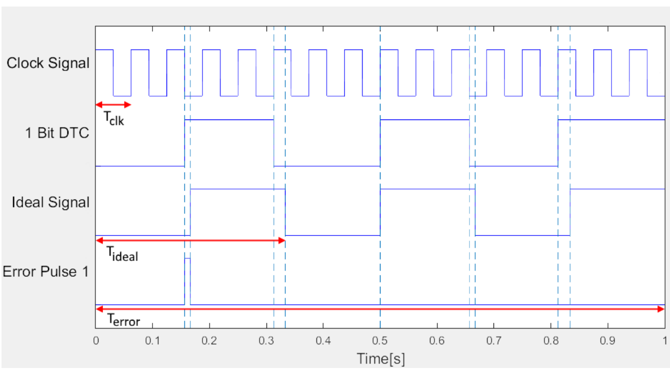

Fig. 11. The highest difference between a predicted point and its simulated output in dB. As can be seen, a doubling of the sample rate halves the highest difference.

Edge k_b (figure 7) of every error pulse corresponds to one of the edges of the ideal signal. This means that these edges are evenly spaced over the complete length of T_{error}. To find t_{b_k}, the function simply needs to be multiply the value of k with the length between two of these edges, which is equal to $\frac{T_{ideal}}{2}$. This is defined by equation 14

$$t_{b_k} = \frac{k * T_{error}}{2M'} \tag{14}$$

The left edge corresponds to edges k_a of the quantized signal. The accumulator that generates the MSB signal only changes its output value when a rising edge occurs. This means that during one period of T_error, $2^{N'}$ possible edges can occur. When the DTC is added, the amount of possible edges doubles for every bit of resolution of the DTC. This is because a DTC with resolution R can make time steps of $\frac{1}{2^R}T_{clk}$. This brings the total amount of steps to $2^{N'} * 2^R$. These steps have a stepsize of $\frac{T_{clk}}{2^R}$. The edges of the ideal signal are approximated with the step that is closest to the ideal edge k_b, but also lower than said edge. The step that represents this definition best can be located by calculating how many integer steps fit between the start of the period and the edge k_a in question. This is expressed in equation 15

$$Amountofsteps = floor(\frac{k * 2^{N'} * 2^R}{2M'}) \tag{15}$$

Dividing the amount of steps between the start of the period and edge k_a by the total amount of steps in the system gives the fraction of the period T_{error} that is equal to the amount of time between the start of the period and edge k_a, which is the basic definition of t_{a_k}. This means that multiplying the fraction with period T_{error} gives t_{a_k}. Equation 16 describes this.

$$t_{a_k} = \frac{floor(\frac{k * 2^{N'} 2^R}{2M'})}{2^{N'} 2^R} T_{error} \tag{16}$$

The duty cycle d_k can be defined by subtracting the t_{a_k} from t_{b_k} and dividing the total by T_{error}, as is done in equation 17.

$$d_k = \frac{k}{2M'} - \frac{floor(\frac{k * 2^{N'+R}}{2M'})}{2^{N'+R}} \tag{17}$$

The time-shift of the error pulse is zero when the error pulse is nonzero and symmetrical at t = 0, as is defined in figure 8. If this is assumed, the time-shift of an error pulse can be defined as the sum of half of the pulse duration plus t_{a_k}. The pulse duration is defined by the duty cycle of the error pulse multiplied by the total duration of the period of the pulse, T_{error}. The total time-shift can be defined as $t_{0_k} = \frac{t_{period}}{2} d_k + t_{left}$, which is the basis for equation 18.

$$t_{0_k} = \frac{t_{period}}{2}\left(\frac{k}{2M'} + \frac{floor(\frac{k * 2^{N'+R}}{2M'})}{2^{N'+R}}\right) \tag{18}$$

APPENDIX D
CONSTRUCTING THE EQUATION FOR THE IDEAL SIGNAL

The function for a normal repeating square wave with a variable duty cycle and time-shift is defined as equation 3. Dilating this function with M' creates equation 19.

$$X_{ideal}(f) = \frac{2A}{M'\pi}\exp(-j\frac{\pi f}{M'}t_{0_k}) * \sum_{n=-\infty}^{\infty}\frac{\sin(n\pi d_k)}{n}\delta\left(\frac{f}{M'} - \frac{n}{T_{error}}\right) \tag{19}$$

Since the ideal function has a fixed duty cycle and time-shift these can be filled into equation 19 resulting in equation 20. This equation can be simplified to equation 21. The time-shift value can still be used because the part of the function responsible for the time-shift is dilated by M'.

$$X_{ideal}(f) = \frac{2A}{M'\pi}\exp(-j\frac{2\pi f}{M'}\frac{3 * T_{clk} * 2^{N'}}{4M'}) * \sum_{n=-\infty}^{\infty}\frac{\sin(n\pi * 0.5)}{n}\delta\left(\frac{f}{M'} - \frac{n}{T_{error}}\right) \tag{20}$$

$$X_{ideal}(f) = \frac{2A}{M'\pi}\exp(-j\frac{\pi f}{M'}\frac{3 * T_{clk} * 2^{N'}}{2M'}) * \sum_{n=-\infty}^{\infty}\frac{\sin(\frac{n\pi}{2})}{n}\delta\left(f - \frac{nM'}{T_{clk} * 2^{N'}}\right) \tag{21}$$

REFERENCES

[1] Earl McCune, *"Time-filtered squarewave output from direct digital synthesis"* (2010), http://ieeexplore.ieee.org/xpl/login.jsp?tp=&arnumber=5517929&url=http%3A%2F%2Fieeexplore.ieee.org%2Fxpls%2Fabs_all.jsp%3Farnumber%3D5517929

[2] Jiayoon Zhiyu Ru, Claudia Palattella, Paul Geraedts, Eric Klumperink, Bram Nauta, *"A High-Linearity Digital-to-Time Converter Technique: Constant-Slope Charging"* (22-05-2015) http://ieeexplore.ieee.org/xpl/login.jsp?tp=&arnumber=7108057&url=http%3A%2F%2Fieeexplore.ieee.org%2Fiel7%2F4%2F4359912%2F07108057.pdf%3Farnumber%3D7108057

[3] Steven W. Smith, Ph.D., *"The Scientist and Engineer's Guide to Digital Signal Processing, Chapter 13: Continuous Signal Processing, The Fourier Series"* (1998), http://www.dspguide.com/ch13/4.htm

[4] S. Haykin, M. Moher, *"An Introduction to Analog and Digital Communications, 2nd edition"* (2006), page 275