Internship Report

Design of droplet impingement measurement set-up

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Preface

From the beginning of September until halfway December I did my internship at the University of Brunswick in Brunswick. I was stated at the Multiphase and Icing Flow research group located at the Institute for Fluid Mechanics. In this time I helped several PhD students with their research. Calibration of the icing wind tunnel, building and designing measurement set ups were few of the many tasks. Since describing everything is unnecessary I chose to discuss the design of the new droplet impingement wheel in this report in detail.

I would like to thank Arne Baumert and David Burzynski in particular for their guidance through this internship. Arne for letting me help calibrate the multiphase wind tunnel and David for helping me design parts of the droplet impingement wheel. I have learned in these few weeks a lot about how it is to work at a research institute and which things I can expect when I would apply at a research facility. I would like to ask David to send me a video of the system when it has actually been build. I am very excited to see it in action and hear if everything works as it is supposed to.

Summary

This report covers the design of parts of the droplet impingement wheel. The goal is to design the shaft, frame and safety screens and all the steps taken in this process are covered in this report. Design criteria of the shaft, static strength analysis, dynamic strength analysis, final CAD design and mounting order are presented.

Design criteria

The flowing media should be methanol, ethanol and water in this watertight system. The wheel should rotate at velocities between 250 rpm and 1000 rpm with a maximum deviation of ± 0.01 at 1000 rpm. A confocal chromatic sensor should be used in combination with an optic rotary joint in order to measure the film thickness during the rotation. Steady film requirement during the whole rotation should be achieved by making sure that eigenfrequencies of the system are at least five times higher than the actuating frequency.

Static strength analysis

In the static strength analysis minimum shaft diameter as well as reaction forces on the bearings are calculated. The shear force and moment are determined as function of the position in the parts are presented and used to calculate the maximum shear force and maximum internal moment. Also the endurance life of the bearings is estimated.

Dynamic strength analysis

The eigenfrequencies of the shaft and medium and optical signal supply system are estimated using the theory of the dynamics of continuous systems. Bending, torsional and longitudinal vibration eigenfrequencies are estimated for the shaft. For the medium and optical signal supply system only bending vibrations can occur and eigenfrequencies for this case are estimated.

Final CAD design and mounting order

The final design is presented with a rendered photograph and all the features covered discussed. Mounting order of all the parts is shown with guiding pictures and comments.

Frame and safety screens

The design of the complete frame including safety screens is presented with rendered pictures. Since the frame is built out of standard components the components as well as their functionality is explained.

Table of Contents

Preface ii

Summary iii

Actual experiment 1

Improving the design 2

Subsystems of the design 3

Designed parts 5

- Power system 5
- Medium and optical signal supply system 5
- Supporting frame 6

Designing the power system and medium and optical signal supply system 6

- Design specifications 7
- Static strength analysis 7
- Dynamical strength analysis 10
- Power system keys analysis 14
- Seal design 14
- CAD Designs 16
- Mounting order and rendered photo 17

Designing the frame 18

- Frame 18
- Building blocks 18

Designing the safety screens 20

- Used screens 20
- Glass thickness 20

Appendix A: Measurement equipment configuration I

Appendix B: Results from static strength analysis II

Appendix B: Results from static strength analysis (continued) III

Appendix C: Dynamic strength analysis results IV

Appendix D: Power system keys analysis results V

Appendix E: O-ring and groove dimensions VI

Actual experiment

Since David Burzynski has to continue the research done by its predecessor B.W. Faßmann first the abstract of part of his research will be given:

"The vertical impact of single, mono dispersed water droplets on a dry smooth surface was studied experimentally by means of shadowgraphy. A glass substrate was mounted on a rotating wheel to obtain high impact velocities. The droplets were generated on demand. While the Ohnesorge number was kept constant, Weber number and Reynolds number were varied by adjusting the impact velocity. In all performed experiments, splashing was observed. The distinction of the different measurement series was done by the use of the Weber number. The different Weber numbers were, 3,500, 5,000 and 10,000. Phase-locked images were taken and the temporal evolution of the impact was reconstructed by means of the nondimensional impingement time. The outcome of the measurement was analyzed by digital image processing to quantify the distribution of the diameter of the resulting secondary droplets in size and time as well as their velocity, and the total deposited mass fraction remaining on the surface after the impingement. In all cases, the greater part of the impinging primary droplet remained on the substrate." [1]

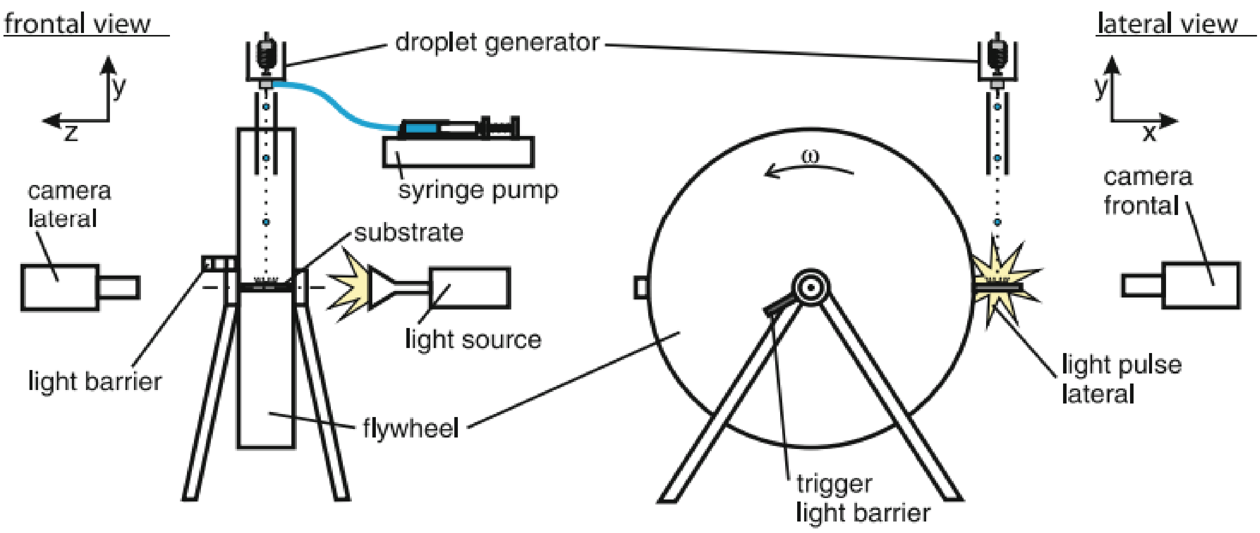

Figure 1: Experimental set-up by W.B. Faßmann (2013)

If one then would take a look at the set-up (**Figure 1**) one could see the functions this design had:

- To create the high velocity impact of the droplets the wheel was rotated at high rotational velocities.
- The droplet generator made the droplets and released them with a trigger. This meant that the diameter of the droplets could be chosen as well as the moment the droplet would hit the surface.
- The syringe pump transports the medium to the droplet generator and controls the droplet volume.
- The trigger light barrier sends a signal to the system and triggers the camera and light source making sure that every picture that is taken contains a droplet.

[1] W.B. Faßmann (2013) High velocity impingement of single droplets on a dry smooth surface Exp Fluids (2013) 54:1516

Improving the design

In order to improve and extend the research of Faßmann David Burzynski has the following requirements:

- The experiments should also be done on wet surfaces so a steady film should be introduced on the rotating surface.
- The range of the Weber number should be increased. Since the Weber number is proportional to the density the media used for the droplets should be varied between water ethanol and methanol. These properties give the opportunity to change the Reynolds number while keeping the Weber number constant. Also the Weber number is quadratically proportional to the velocity so the maximum rotation velocity of the wheel should be increased.
- The accuracy of the wheel's rotation should be improved. Especially when the system runs at low velocities the actual velocity of the wheel deviates too much from the set velocity. This leads to difficulties in synchronizing the system.
- The impact angle should be varied. The previous experiments were all done with a 90 degree angle between the surface and droplet. Change in this angle could be very interesting and should be examined.
- An optic sensor should be used in combination with an optic rotary joint to measure the film thickness during the whole rotation of the system.
- It should be possible to implement a heating element in the future at the position of the film. Heating the film could be interesting as an extension of the research. In order to be able to implement this heating system power should be supplied to the rotating surface. This means that either the optic rotary joint should also include a power connection or two separate slip rings should be implemented.
- Because of the high rotational velocities of the wheel in the future a breaking system and also a safety screen frame should be added to the system. If there are some problems with the measurement or the system David wants to be able to break the system in order to make it as safe as possible. Also the breaking system should possibly be triggered with an external acceleration sensor on the wheel. This means that if some vibrations occur that are not visible the system should detect these and break the wheel. The safety screens make sure that if for instance the rotating surface releases nobody gets injured. Medium that drips down the surface can be collected at the bottom and reused.

Since these are the new requirements the measurement system should have it is also very important to learn from the first experiments made by Faßmann. Since he was the first one to address the problem several problems occurred during the experiments which should be solved in the new design. Faßmann tried to introduce the water film but has not succeeded because he was not able to create a steady film at high velocities. The problems that need to be solved are:

- Close attention should be payed to making the system watertight. Because of the many leaks the system had the created water film was never steady. Since this is the requirement to publish the results all the measurements were not published.
- All the vibrations in the rotating system should be as low as possible. Due to these vibrations the water film was disturbed with waves.

Subsystems of the design

In order to efficiently develop the new design the choice was made to divide the whole design into subsystems. Every subsystem had its specifications and problems and was solved separately. If one would look at the complete design the following parts could be recognized (see **Figure 2**):

- *The rotating surface ("Paddle"):* This surface would convert the medium supply into a steady film and has a collection device to suck away the medium and air mixture. Also the optical sensor is used to measure the film thickness during the rotation of the Paddle.
- *Rotating wheel*: Should rotate at a fixed velocity without vibrations in any way so that the film of the Paddle is uniform and the experiments can be made as consistently as possible to make the experiments reproducible. One way to do this there should be a system included in the wheel that enables the change of the moment of inertia. Also there should be a possibility to connect the necessary tubings that provides the Paddle with medium and extracts the medium and air mixture. The system trigger is on a fixed position on the wheel making sure that every component needed for a good picture is activated at the right time.
- *Medium and optic signal supply system*: Inserts the medium and extracts the medium and air mixture. The optical cable should enter the system here since at the motor no room is available due to the direct contact between the shaft and the motor. Since this is the connection of the static part to the rotating part a rotational joint must be implemented. The optical cable is connected to a control box.
- *Power system:* Is the connection of the rotating shaft of the wheel to the motor, the breaking system and the motor itself. This system is connected to the supporting frame and controlled with the measurement equipment.
- *Medium pump and mixture suction pump:* inserts the medium into the system and extracts the medium and air mixture.
- *Supporting Frame:* All the important parts are connected to a very sturdy frame. This to make sure that there are as few vibrations as possible. Once the set-up is mounted all the distances to all the components are fixed and aligned.
- *Droplet generator:* contains the droplet trigger as well as the syringe pump. This system makes sure that all the droplets that hit the Paddle are as uniform as possible and hit the Paddle at the moment when the photo is taken.
- *Measurement equipment:* Contains all the equipment that is needed to make the pictures at the right moment and as consistent as possible. These components contain the camera(s), laser, computer, Programmable Timing Unit (PTU), sensor control box and all the connections between these components. The measurement set-up of the old experiment can be found in Appendix A.

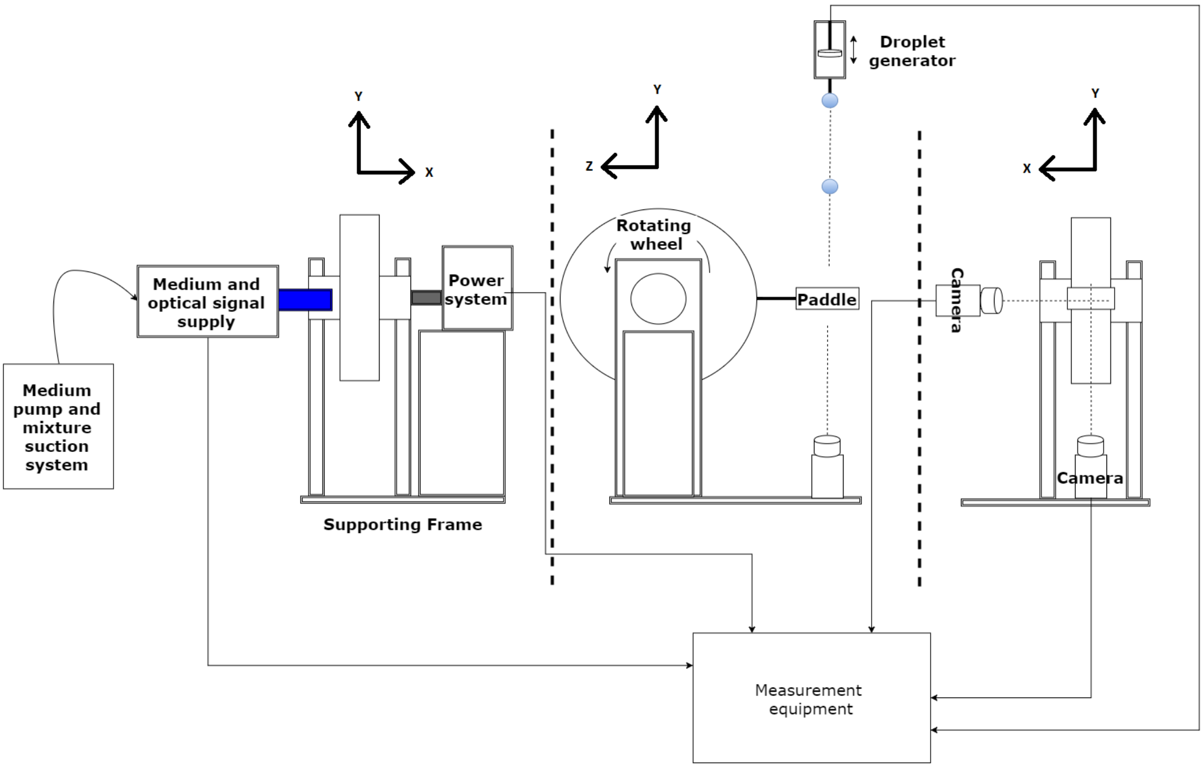

Figure 2: Complete design divided into subsystems

Designed parts

In this chapter I would like to discuss the parts of the design that I designed. The total design consists also out of parts that I consulted in but not designed myself and thus will not be presented here. The parts that I actually designed myself are the *power system, medium and optical signal supply system,* the *supporting frame* and the *safety screens.* Since the designs were supposed to be improvements of the old design I first discuss the old designed parts and state the problems with these components. When all the improvements that are needed are known a model is made of the future design and several calculations are made to proof that the future design will not fail during its use.

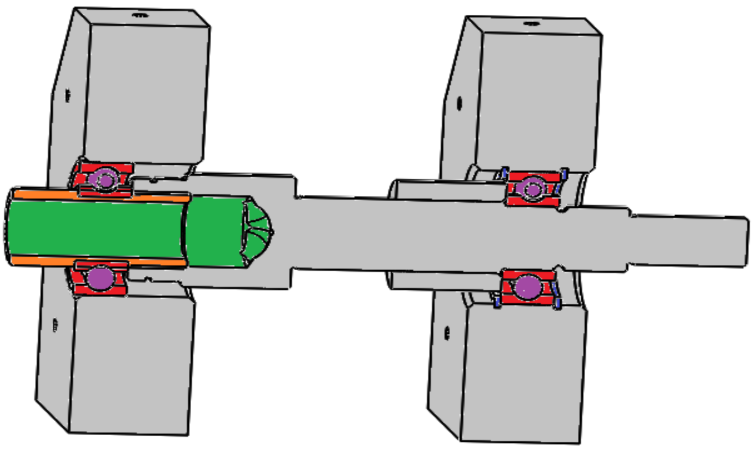

Figure 3: Schematic representation of the old power system. Bearings (red) ,supporting frame (gray), mix of in- and outflow (green) and closing rings (blue).

Power system

Looking at the old power system **(Figure 3)** one can see that the wrong bearings are chosen. Simple ball bearings constrain axial movement and since this is done twice the system is overdetermined. Extra care needs to be taken into choosing the right bearings for the future design. Mounting the tubes to the shaft was very difficult due to the limited space and needs to be made easier in the new design.

Medium and optical signal supply system

Even though the old design **(Figure 4**) did not contain the optical signal support taking a close look at the old medium supply system was very useful to exactly find where the problems were and how they could be solved.

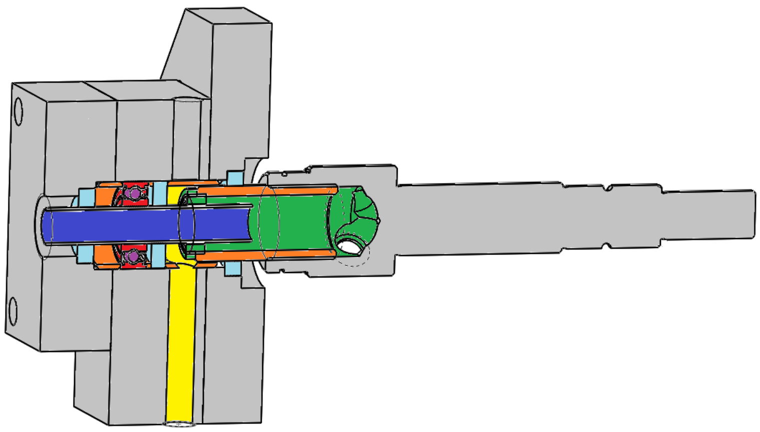

Figure 4: Schematic representation of old water supply system. inflow (dark blue), mix of in- and outflow (green) and outflow (yellow)

The first problem is that the flows are not separated at the location highlighted in green. This means that the inflow to the Paddle can also contain air which disturbs the steady flow. Secondly the seals highlighted in light blue were not properly designed and caused leakage. Third problem was the choice of bearings. The bearing that is shown in red is a simple ball bearing that constrains axial displacement and in combination with the two ball bearings on the power system meant that the system was overdetermined. This caused unnecessary stresses on the bearings. Final and most important problem was the poor combination of materials. Some steel parts were connected to aluminum parts and caused corrosion.

Supporting frame

As can be seen in (**Figure 5**) the old supporting frame consists out of two equal parts that are connected with the floor connection flanges. Due to the welded connections the tolerances of the frame were too high making the connections to the floor very difficult. Also the space between the bars was too small causing mounting difficulties. Mounting the motor was done externally using X-95 profiles and created alignment issues. The overall problem was that the old frame took too much space compared to the functions it had. The new frame should be smaller and have more functions besides holding the wheel.

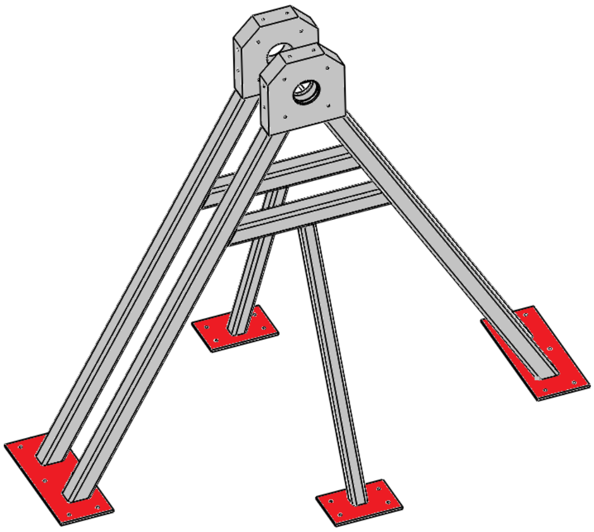

Figure 5: Schematic representation of the old frame. Frame (gray) and connection to the ground (red)

Designing the power system and medium and optical signal supply system

At the shaft medium supply, optical signal supply and also the rotational velocity of the wheel are achieved. Since there are two positions on the shaft where these features can be inserted a combination of two components was needed. The choice to combine the medium and optical signal at one side and the power supply on the other side was made. Due to the rigid connection of the shaft and engine there was not enough space to fit in another feature. The other option was to use a belt between the shaft and the engine but this could introduce alignment problems and thus unnecessary vibrations. Since I was responsible for the design of these components I would like to describe here in detail the steps I took to make the original ideas into the final design. Since this process can be described in chronological order I present these as the following phases:

- As in every design I started listing the **design specifications** that we made.
- **Static strength analysis** and **dynamical strength analysis:** A full *static strength analysis* is made and determines the reaction forces and important stresses so that the right bearings can be chosen as well as the right geometry. Also a *dynamical strength analysis* is done to estimate the eigenfrequencies of the two systems. This chapter gives only the derivation of the formulas and the formulas itself. *For the static strength analysis results see Appendix B and for the dynamical strength analysis results see Appendix C.*
- **Power system keys analysis:** The torque of the motor needs to be converted into rotational velocity of the wheel. The connection of the motor to the shaft and wheel is by the usage of keys. In order to make sure that the keys are strong enough the maximum stress as function of the applied torque needs to be determined. This chapter gives only the derivation of the formulas and the formulas itself. *For the power system keys analysis results see Appendix D.*
- **Seal design:** Making the system watertight had proven to be difficult in the past so extra time and effort was spent on the design of the O-rings and also choosing the right material for this

application. This chapter gives only the derivation of the formulas and the formulas itself. *For the O-ring and groove dimensions see Appendix E.*

- **CAD Design:** Since at this stage all the requirements of the system are known the design is made and presented.
- **Mounting order and rendered photo:** Special care was taken to make the system simple to mount and the way to do is explained here. In the last step the complete system is visualized with a rendered photograph.

Design specifications

In order to make sure that all the requirements can be achieved and checked at the end of the design some quantifiable numbers need to be addressed to the system requirements **(Table 1)**:

System requirement	Specified range
Rotational velocity of the wheel	250 rpm – 1000 rpm
Maximum deviation of the rotational velocity	$\pm$0.01 rpm at 1000 rpm
Media useable in system	Water, ethanol and methanol
Optic sensor to measure the film thickness during the whole rotation of the wheel	Confocal Chromatic Sensor
Optic rotary joint + electrical slip ring	Single channel-multimode optic fiber
Breaking system (should be triggered by accelerator sensor on the wheel)	Implemented in electric motor or separate system
System watertight	No leakage of medium anywhere
No disturbing vibrations	Eigen frequencies of rotating system 5 times higher than actuating frequency (>100 Hz)
Film quality	Steady film over the whole rotation with uniform thickness and no disturbance waves.

Table 1: Design specifications

Static strength analysis

Reaction forces and maximum stresses on the power system

The bearings that support the shaft are chosen such that the system is simply supported and thus statically determined. This means that there is one bearing that can be represented by a roll support and one bearing that can be represented by a normal not moving support. If one then would take a look at the Free Body Diagram (see **Figure 6**) the reaction forces on the bearings can be easily calculated and are:

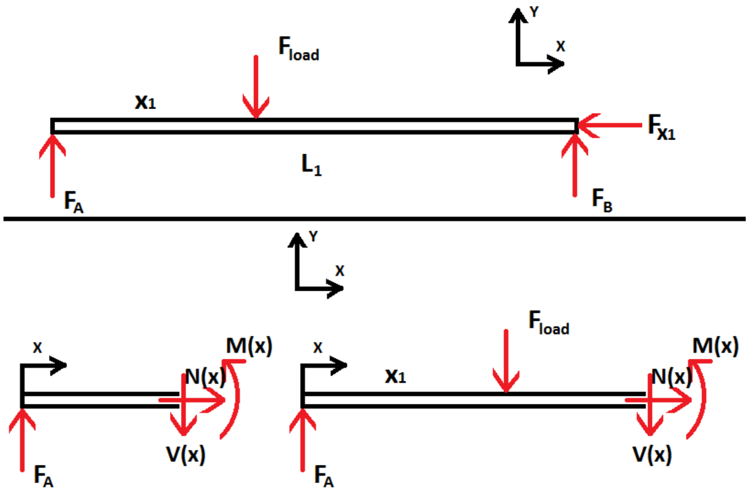

Figure 6: Free Body Diagram shaft

$$\sum F_x = 0 \Rightarrow F_{x1} = 0$$

$$\sum F_y = 0 \ and \ \sum M_z = 0 \Rightarrow$$

$F_A = F_{load} * \left(\frac{L_1 - x_1}{L_1}\right)$, $F_B = F_{load} * \frac{x_1}{L_1}$ $\boldsymbol{with}\ \frac{\boldsymbol{x_1}}{\boldsymbol{L_1}} > \frac{\boldsymbol{1}}{\boldsymbol{2}}$

Now the forces on the bearings are known thus an estimate can be made of the lifespan of the bearings[2]:

$$L_{10} = \left(\frac{C}{F}\right)^p, \begin{cases} C: dynamic\ load\ factor\ [N] & p: livelife\ exponent\ [-] \\ F: force\ on\ bearing\ [N] & L_{10}: Life\ expectancy\ [no. rotations] \end{cases}$$

The internal shear force and internal moment as function of the position are:

$$V_1(x) = \begin{cases} F_A & for \quad 0 \le x < x_1 \\ F_A - F_{load} & \quad\ \ x_1 < x \le L_1 \end{cases} \ and\ M_1(x) = \begin{cases} F_A x & for \quad 0 \le x < x_1 \\ F_A x - F_{load}(x - x_1) & \quad\ \ x_1 < x \le L_1 \end{cases}$$

The maximum internal shear stress of the shaft and also the maximum bending stress can then be determined:

$$\tau_{shear,max} = \frac{V_{1,max}}{A_{solid\ shaft}} = \frac{|F_A - F_{load}|}{\pi {r_{shaft}}^2}, \qquad \sigma_{bending,max} = \frac{M_1(x_1) * r_{shaft}}{I_{solid\ shaft}} = \frac{F_A * x_1 * r_{shaft}}{\frac{\pi}{4} {r_{shaft}}^4}$$

Now these two formulas can be used to calculate the minimum solid shaft diameter for the shear stress and bending stress. The larger of the two is the minimum solid shaft diameter that is needed so that the shaft will not fail due to static forces.

$$d_{min,shear} = 2\sqrt{\frac{|F_A - F_{load}|}{\pi * \tau_{max}}}$$

$$d_{min,bending} = 2 * \sqrt[3]{\frac{4 * F_A * x_1}{\pi * \sigma_{max}}}$$

Reaction forces and maximum stresses on the medium and optical signal supply system

The same procedure can be done to calculate the important values for the water and optical signal system (for Free Body Diagram see **Figure 7**):

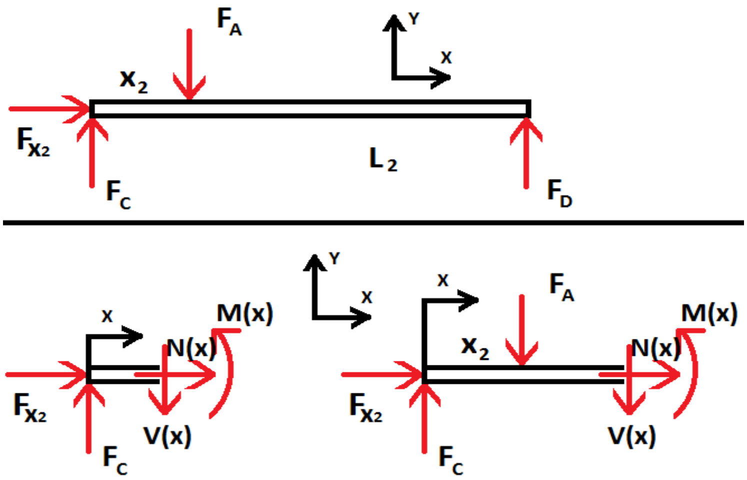

Figure 7: Free Body Diagram water and optical signal system

$\sum F_x = 0 \Rightarrow F_{x2} = 0$

$\sum F_y = 0\ and\ \sum M_z = 0 \Rightarrow$

$F_C = F_A * \left(\frac{L_2 - x_2}{L_2}\right)$, $F_D = F_A * \frac{x_2}{L_2}$, $\boldsymbol{with}\ \frac{\boldsymbol{x_2}}{\boldsymbol{L_2}} < \frac{\boldsymbol{1}}{\boldsymbol{2}}$

[2] Catalogue Schaeffler KG, (2008) "Wälzlager"

Now the forces on the bearings are known thus an estimate can be made of the lifespan of the bearings[3]:

$$L_{10} = \left(\frac{C}{F}\right)^p, \begin{cases} C: dynamic\ load\ factor\ [N] & p: livelife\ exponent\ [-] \\ F: force\ on\ bearing\ [N] & L_{10}: Life\ expectancy\ [no. rotations] \end{cases}$$

The internal shear force and internal moment as function of the position are:

$$V_2(x) = \begin{cases} F_C \\ F_C - F_A \end{cases} for \begin{matrix} 0 \le x < x_2 \\ x_2 < x \le L_2 \end{matrix}\ and\ M_2(x) = \begin{cases} F_C x \\ F_C x - F_A(x - x_2) \end{cases} for \begin{matrix} 0 \le x \le x_2 \\ x_2 < x \le L_2 \end{matrix}$$

The maximum internal shear stress of the medium system and also the maximum bending stress can then be determined:

$$\tau_{shear,max} = \frac{V_{2,max}}{A_{solid\ shaft}} = \frac{F_C}{\pi r_{shaft}{}^2}, \qquad \sigma_{bending,max} = \frac{M_2(x_2) * r_{shaft}}{I_{solid\ shaft}} = \frac{F_C * x_2 * r_{shaft}}{\frac{\pi}{4} r_{shaft}{}^4}$$

Now these two formulas can be used to calculate the minimum solid shaft diameter for the shear stress and bending stress. The larger of the two is the minimum solid shaft diameter that is needed so that the shaft will not fail due to static forces.

$$d_{min,shear} = 2r_{min,shear} = 2\sqrt{\frac{F_C}{\pi * \tau_{max}}},\ d_{min,bending} = 2r_{min,bending} = 2 * \sqrt[3]{\frac{4 * F_C * x_2}{\pi * \sigma_{max}}}$$

[3] Catalogue Schaeffler KG, (2008) “Wälzlager”

Dynamical strength analysis

Eigenfrequencies of the power system

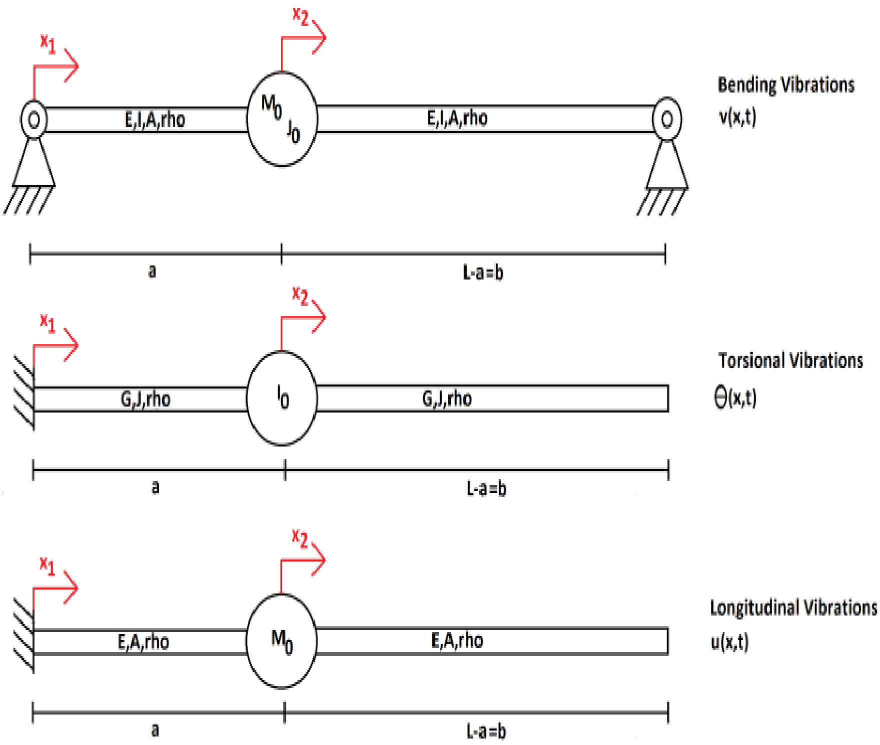

Figure 8: Vibrational model for the power system

In order to calculate the estimated eigenfrequencies of the power system one has to look first at the theory of the dynamics of continuous systems. In this case three load cases can be found and also vibrations in these directions are very important. In order to be able to calculate analytic solutions to these equations a simplified model of the system is introduced (see Figure 8). The mass and inertia are introduced as representation of the wheel and due to this discontinuity the domain needs to be split into two pieces. Also the distinction is made between the different load cases. As one can see the boundary conditions slightly differ for the different configurations.

Bending vibrations

The equation of motion that needs to be solved for bending vibrations of continuous systems (homogenous and prismatic shaft):

$$\rho A \frac{\partial^2 v(x,t)}{\partial t^2} + EI \frac{\partial^4 v(x,t)}{\partial x^4} = F(x,t)$$

In order to calculate the eigenfrequencies of the unforced system the solution is found by the separation of variables method (only for standing waves):

$$v(x,t) = X(x)T(t), \ \Rightarrow \frac{T''}{T} = -\frac{EI}{\rho A} * \frac{X''''}{X} = -\omega^2$$

So the equations that need to be solved are:

$$T''(t) + \omega^2 T(t) = 0, \ X''''(x) - \beta^4 X(x) = 0, \ \beta^4 = \frac{\rho A}{EI}\omega^2$$

And the solutions to these differential equations are:

$$T(t) = A_t cos(\omega t) + B_t sin(\omega t)$$

$$X(x) = A_x \cos(\beta x) + B_x \sin(\beta x) + C_x \cosh(\beta x) + D_x \sinh(\beta x)$$

Now the general solution of the differential equation is known. In order to make the solution realistic for the chosen model the constants and frequencies need to be determined. Due to the concentrated mass one equation to describe the whole system is not enough. In this case two functions are used that are valid in the two parts of the domain.

$$v_1(x_1,t) = X_1(x_1)T_1(t) \;\; for\; 0 \leq x_1 \leq a \quad v_2(x_2,t) = X_2(x_2)T_2(t) \;\; for\; 0 \leq x_2 \leq L-a$$

The easy boundary conditions that come with this configuration are:

$$v_1(0,t) = 0,\; EI\left.\frac{\partial^2 v_1}{\partial x_1^2}\right|_{x_1=0} = 0,\; \Rightarrow A_{x1} = C_{x1} = 0$$

$$v_2(L-a,t) = 0,\; EI\left.\frac{\partial^2 v_2}{\partial x_2^2}\right|_{x_2=L-a} = 0$$

$$\left.\frac{\partial v_1}{\partial x_1}\right|_{x_1=a} = \left.\frac{\partial v_2}{\partial x_2}\right|_{x_2=0}, v_1(a) = v_2(0)$$

At the position of the mass the equations need to be connected to satisfy continuity conditions and also the acceleration of the mass needs to be accounted for. To determine odd and even functions the boundary conditions slightly differ:

$$\sum F_y = F_2 \mp F_1 = m_0 a_0 = m_0 \ddot{v}_1 \Rightarrow EI\left(\left.\frac{\partial^3 v_2}{\partial x_2^3}\right|_{x_2=0} \mp \left.\frac{\partial^3 v_1}{\partial x_1^3}\right|_{x_1=a}\right) = \left.\frac{m_0 \partial^2 v_2}{\partial t^2}\right|_{x_2=0}$$

$$\sum M = M_2 \mp M_1 = J_0 \alpha \Rightarrow EI\left(\left.\frac{\partial^2 v_2}{\partial x_2^2}\right|_{x_2=0} \mp \left.\frac{\partial^2 v_1}{\partial x_1^2}\right|_{x_1=a}\right) = J_0 \left.\frac{\partial^3 v_2}{\partial x \partial t^2}\right|_{x_2=0}$$

Where the minus sign determine the odd functions and the plus sign the even functions. All these conditions can be written in matrix vector form. Since this multiplication is equal to zero the trivial solution can be found which means a static system or if the determinant is zero the eigenfrequencies are found (b=L-a).

$$\begin{bmatrix} 0 & 0 & cos(\beta b) & sin(\beta b) & cosh(\beta b) & sinh(\beta b) \\ 0 & 0 & -\beta^2 cos(\beta b) & -\beta^2 sin(\beta b) & \beta^2 cosh(\beta b) & \beta^2 sinh(\beta b) \\ \beta cos(\beta a) & \beta cosh(\beta a) & 0 & -\beta & 0 & -\beta \\ sin(\beta a) & sinh(\beta a) & 1 & 0 & 1 & 0 \\ \beta^3 cos(\beta a) & -\beta^3 cosh(\beta a) & \frac{m_0\omega^2}{EI} & \mp\beta^3 & \frac{m_0\omega^2}{EI} & \pm\beta^3 \\ \beta^2 sin(\beta a) & -\beta^2 sinh(\beta a) & \mp\beta^2 & \frac{J_0\omega^2}{EI}\beta & \pm\beta^2 & \frac{J_0\omega^2}{EI}\beta \end{bmatrix} \begin{Bmatrix} B_{x1} \\ D_{x1} \\ A_{x2} \\ B_{x2} \\ C_{x2} \\ D_{x2} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

So now the eigenfrequencies of the system can be calculated simply by substituting all values for the system and finding for which ω the determinant of the matrix is equal to zero. To check if the solution is valid one can check a special case which is known. Since for a simply supported beam without mass the eigenfrequencies are known:

$$\omega_n = n^2\pi^2\sqrt{\frac{EI}{\rho A L^4}}\; for\; n = 1,2,\ldots\infty$$

This is validated and the eigenfrequencies of the system are equal to the theoretical values.

Torsional vibrations

The equation of motion that needs to be solved for torsional vibrations of continuous systems (homogenous and prismatic shaft):

$$\rho J \frac{\partial^2 \Theta(x,t)}{\partial t^2} - GJ \frac{\partial^2 \Theta(x,t)}{\partial x^2} = T(x,t)$$

The same process as shown in *bending vibrations* can be used to determine the solutions to the unforced equation:

$$T(t) = A_t cos(\omega t) + B_t sin(\omega t)$$

$$X(x) = A_x \cos(\beta x) + B_x \sin(\beta x)$$

$$with\ \beta^2 = \frac{\rho}{G}\omega^2$$

The simple boundary conditions for the clamped-free shaft and the connection of the functions:

$$\Theta_1(x_1 = 0) = 0,\ \ GJ \frac{\partial \Theta_2}{\partial x_2}\bigg|_{x_2 = L-a} = 0,\ \ \Theta_1(x_1 = a) = \Theta_2(x_2 = 0)$$

Due to the concentrated mass in the middle and thus polar moment of inertia (note that due to the unsymmetrical boundary conditions even functions are not possible):

$$GJ\left(\frac{\partial \Theta_2}{\partial x_2}\bigg|_{x_2=0} - \frac{\partial \Theta_1}{\partial x_1}\bigg|_{x_1=a}\right) = I_0 \frac{\partial^2 \Theta_2}{\partial t^2}\bigg|_{x_2=0} = -\omega^2 I_0 \Theta_2(x_2 = 0)$$

The system of equations that needs to be solved is (b=L-a):

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -\beta sin(\beta b) & \beta cos(\beta b) \\ cos(\beta a) & sin(\beta a) & -1 & 0 \\ \beta sin(\beta a) & -\beta cos(\beta a) & \frac{\omega^2 I_0}{GJ} & \beta \end{bmatrix} \begin{Bmatrix} A_{x1} \\ B_{x1} \\ A_{x2} \\ B_{x2} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

If one then would check the special case when the polar moment is equal to zero and a=L/2 one can validate the model to the theoretical eigenvalues the system should have:

$$\omega_n = \frac{n\pi}{2L}\sqrt{\frac{G}{\rho}},\ for\ n = 1,3,5, \dots, \infty$$

This is validated and the eigenfrequencies of the system are equal to the theoretical values.

Longitudinal vibrations

The equation of motion that needs to be solved for longitudinal vibrations of continuous systems (homogenous and prismatic shaft):

$$\rho A \frac{\partial^2 u(x,t)}{\partial t^2} - EA \frac{\partial^2 u(x,t)}{\partial x^2} = N(x,t)$$

The same process as shown in *bending vibrations* can be used to determine the solutions to the unforced equation:

$T(t) = A_t cos(\omega t) + B_t sin(\omega t),\ X(x) = A_x \cos(\beta x) + B_x \sin(\beta x)\,,\ \beta^2 = \frac{\rho}{E}\omega^2$

The simple boundary conditions are:

$$u_1(x_1 = 0) = 0,\ \frac{\partial u_2}{\partial x_2}\bigg|_{x_2 = L-a} = 0,\ u_1(x_1 = a) = u_2(x_2 = 0)$$

Force equilibrium at the concentrated mass gives:

$$EA\left(\frac{\partial u_2}{\partial x_2}\bigg|_{x_2=0} - \frac{\partial u_1}{\partial x_1}\bigg|_{x_1=a}\right) = m_0 \frac{\partial^2 u_2}{\partial t^2}\bigg|_{x_2=0}$$

The matrix vector equation will then be:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -\beta sin(\beta b) & \beta cos(\beta b) \\ cos(\beta a) & sin(\beta a) & -1 & 0 \\ \beta sin(\beta a) & -\beta cos(\beta a) & \frac{\omega^2 m_0}{EA} & \beta \end{bmatrix} \begin{Bmatrix} A_{x1} \\ B_{x1} \\ A_{x2} \\ B_{x2} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

If one then would check the special case when the polar moment is equal to zero and a=L/2 one can validate the model to the theoretical eigenvalues the system should have:

$$\omega_n = \frac{(2n-1)\pi}{2L} * \sqrt{\frac{E}{\rho}}\ for\ n = 1,2,3, \dots, \infty$$

This is validated and the eigenfrequencies of the system are equal to the theoretical values.

Eigenfrequencies of the water and optical signal supply system

For the water and optical signal supply system only *bending vibrations* are important since torsion and longitudinal directions are not exited.

The theory of solving the equations is equal as presented for the power system only the boundary conditions are different.

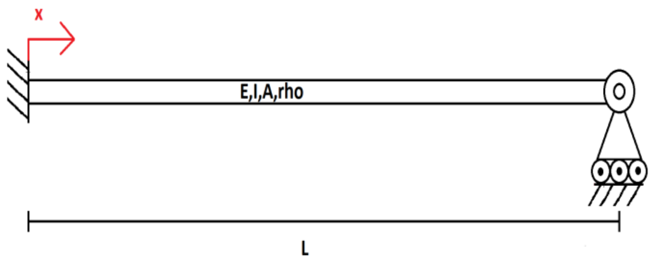

Figure 9: Vibrational model medium and optical signal supply system

The boundary conditions that come with this configuration are:

$$v(0,t) = 0, \frac{\partial v}{\partial x}\bigg|_{x=0} = 0$$

$$v(L,t) = 0,\ EI\,\frac{\partial^2 v}{\partial x^2}\bigg|_{x=L} = 0$$

The system of equations that needs to be solved is:

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & \beta & 0 & \beta \\ cos(\beta L) & sin(\beta L) & cosh(\beta L) & sinh(\beta L) \\ -\beta^2 cos(\beta L) & -\beta^2 sin(\beta L) & \beta^2 cosh(\beta L) & \beta^2 sinh(\beta L) \end{bmatrix} \begin{Bmatrix} A_x \\ B_x \\ C_x \\ D_x \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}\ with\ \beta^4 = \frac{\rho A}{EI}\omega^2$$

And the eigenfrequencies are found when the determinant is equal to zero.

Power system keys analysis

In order to guide the power delivered by the motor to the shaft and onto the wheel keys are used. Since these keys should sustain shear stress and compression stress to guide the torsional moment formulas for these are derived (for free body diagram see **Figure 10**):

$$T = F * \frac{D}{2} \Rightarrow \tau_{shear} = \frac{F}{A_{shear\ plane}} = \frac{2T}{DWL}$$

$$\sigma_{compression} = \frac{F}{0.5 * A_{compression\ plane}} = \frac{4T}{DHL}$$

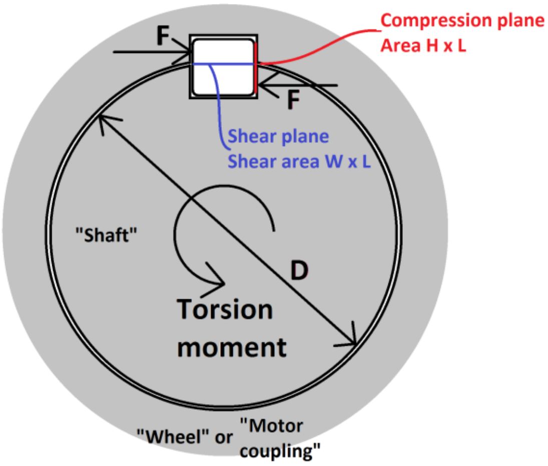

Figure 10: Free body diagram of keys

Since the two load cases appear at the same time the maximum stresses that appear in the key are the principal stresses according to:

$$\sigma_1 = \frac{\sigma_{compression}}{2} + \sqrt{\left(\frac{\sigma_{compression}}{2}\right)^2 + \tau_{shear}^2},\ \sigma_2 = \frac{\sigma_{compression}}{2} - \sqrt{\left(\frac{\sigma_{compression}}{2}\right)^2 + \tau_{shear}^2}$$

To ensure that the key not fails during use the maximum principal stress should not exceed the maximum allowed stress of the key. For a simple case the maximum allowed stress can be the yield stress of the material but in order to account for fatigue and other failure mechanisms it is common to use a safety factor. This means that the maximum allowed stress of the key is the yield stress multiplied with a safety factor.

$$if \begin{cases} \sigma_1 \geq \sigma_2 \\ \sigma_2 \geq \sigma_1 \end{cases} then \begin{cases} \sigma_1 \leq \sigma_{max} \\ \sigma_2 \leq \sigma_{max} \end{cases}, \quad where\ \ \sigma_{max} = N_{safety} * \sigma_{yield}\ where\ [1 \leq N_{safety} \leq 4]$$

Seal design

Dimensions

To ensure a watertight system as well as to make sure that the in- and outflow cannot mix O-rings need to be designed.

To fix the O-ring in the groove the initial O-ring diameter should be smaller than the groove diameter so that the O-ring is slightly stretched and snugly fitting.

$$O_D = \frac{G_D}{1 + \%_{stretch}}\ where\ 1 \leq \%_{stretch} \leq 5$$

The cross section diameter of the O-ring should be such that for every possible tolerance combination the hole is completely closed. Since the system is rotating a percentage of compression is

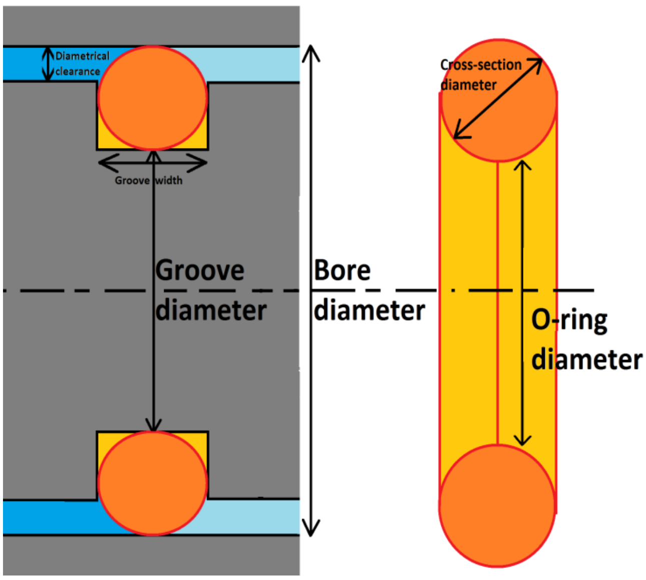

Figure 11: O-ring sketch

introduced to make sure that the O-ring is exerting force onto the surface and seals perfectly:

The maximum cross section diameter of the O-ring is:

$$max(Ring_{C.S.D}) = \frac{\left(\frac{min\ Bore_D - max\ Groove_D}{2}\right)}{1 - max(\%_{compression})} - Ring_{C.S.D\ tolerance}\ , C.S.D. = Cross\ Section\ Diameter$$

In the same way the minimum cross section diameter can be determined:

$$min\left(Ring_{C.S.D)}\right) = \frac{\left(\frac{max\ Bore_D - min\ Groove_D}{2}\right)}{1 - min(\%_{compression})} + Ring_{C.S.D\ tolerance}$$

Due to the rotational velocity the percentage of compression influences the amount of friction generated by the O-rings. For dynamical purposes a compression percentage between 10% and 30% is commonly used.

Used material

Since the O-ring should endure water, ethanol and methanol not all materials can be used. Also low friction should be introduced so a hard material should be selected. Combining these requirements with the fact that the medium is used at room temperature gives the best material for this purpose to be ethylene propylene diene terpolymer (commonly called EPDM).

Surface roughness

To reduce friction and wear the rotating surface should be as smooth as possible. Also the groove should have some surface roughness to keep the O-ring from rotating. The recommended[4] surfaces roughness's are:

For $\begin{cases} Contact\ surface: & R_{mean} = 1.6\mu m \\ Groove: & R_{mean} = 3.2\ \mu m \end{cases}$ and $\begin{matrix} R_{max} = 6.3\mu m \\ R_{max} = 12.5\mu m \end{matrix}$ and should be implemented.

[4] https://www.parker.com/literature/ORD%205700%20Parker_O-Ring_Handbook.pdf

CAD Designs

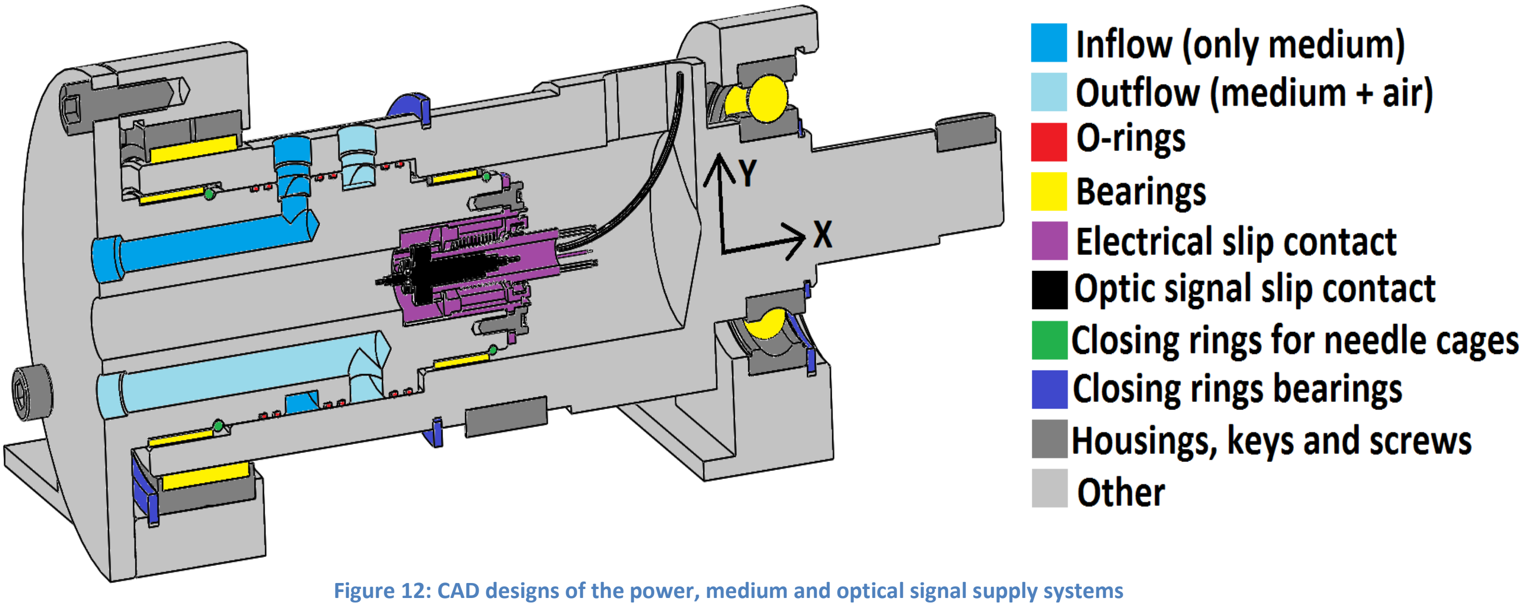

Figure 12: CAD designs of the power, medium and optical signal supply systems

Here the final CAD-designs (see **Figure 12**) of the power, medium and optical signal supply systems are presented. The subsystems that can be recognized are:

- **In- and outflow:** From the static part on the left the medium flows through the top channel to the rotating chamber. This chamber is always filled with medium and secured from leakage with O-rings. From the rotating chamber it flows through the shaft and into a tube that is connected to the paddle. On the way back it enters another rotating chamber which is secured with O-rings and leaves the static part on the lower channel.
- **Bearings:** The shaft is simply supported using one simple ball bearing (on the right) and one needle bearing without inner ring (on the left). The simple ball bearing constrains movement in x- and y-direction whereas the needle bearing enables x-direction movement. The medium and optical signal supply system has two needle cage bearings and these are used to direct the forces from the shaft to the frame and relieve possible forces on the O-rings. X-direction movement is enabled and is used to insert the system into the shaft.
- **Closing rings:** All the closing rings are used to make sure that after mounting the bearings and wheel are only able to move in the directions they are supposed to. On the other side of the bearings solid edges are used to center the bearings.
- **Hybrid slip contact:** The hybrid slip contact consists out of two slip rings that are combined to one product. The optical signal slip ring is placed inside an electrical slip contact and fixed there. The total product can be bought and is fixed on the static part using M4 screws. The optical cable leaves the slip contact and is directed to the wheel at a big radius to prevent signal loss.
- **Housings:** The bearing housings are used to fix the whole shaft system to the frame. On the left there can be seen that the medium and optical supply system is fixed to the housing using M8 screws.
- **Keys:** As mentioned in “Power system keys analysis” keys are used to guide the torque of the motor to the wheel.

Mounting order and rendered photo

The right way to mount the whole shaft system is presented here. Several subsystem mounting orders are explained and the rendered photograph of the total system will be given.

1. Medium and optical signal supply system

Every part that should be fixed on the system should be inserted from the right and slide into negative x-direction (see **Figure 13**). First the bigger needle cage should be pressed onto the solid surface and secured with the closing ring. Then the six O-rings can be fixed in their grooves. After the O-rings the second needle cage plus closing ring can be fixed. Last step is securing the hybrid slip contact with four M4 screws making sure that the cables are fitted through the gaps.

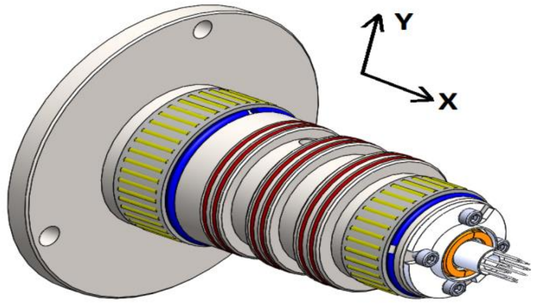

Figure 13: Mounting medium and optical signal supply system

2. Fixed bearing and wheel

First the simple ball bearing should be fixed on the shaft and secured with its closing ring. Second the bearing should be fixed in the casing and closed with its corresponding closing ring. Final step is sliding the wheel on the shaft from the left (see **Figure 14**) and fixed by its closing ring and key.

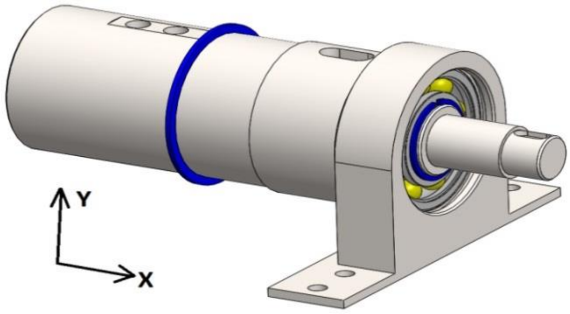

Figure 14: Mounting the simple ball bearing and wheel (wheel not displayed)

3. Floating bearing

Fixing the needle bearing (see **Figure 15**) in its casing is the same process as done with the simple ball bearing. After the bearing is fixed the shaft can be mounted in the bearing. NOTE: no closing ring necessary here due to axial freedom of the shaft at this position.

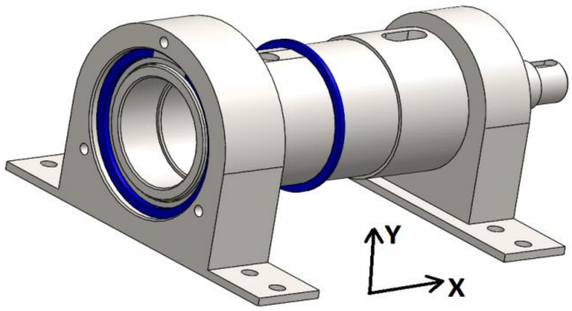

Figure 15: Mounting needle bearing in its casing and shaft

4. Securing the medium and optical signal supply system (rendered photograph)

The final step for the whole assembly is sliding the whole medium and optical signal supply system in the shaft and fixing it with three M8 screws. In **Figure 16** the result of the last step can be seen. The whole assembly including the wheel can now be mounted on the frame.

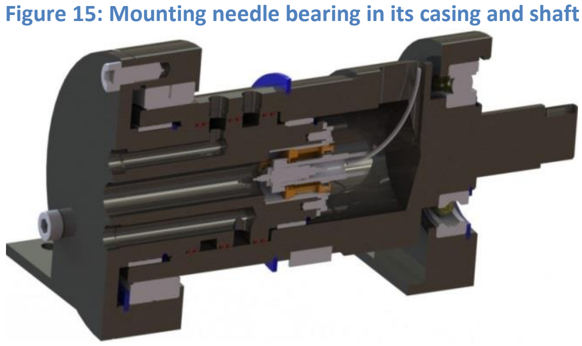

Figure 16: Rendered photo of cut of final assembly

Designing the frame

In this chapter I would like to present the new frame which I designed. After taking a close look at the old frame the choice has been made to make the frame out of standard elements provided by a well-known company in Germany[5]. The aluminum profiles that are used can be easily connected to make a light and stiff construction. Also the possibility to change things in the future or adding new features can be easily achieved. This was very difficult with the old frame due to the welded connections. All the connections in the new frame will be made by using standard building blocks which use M8 screws.

Figure 17: Rendered photo of frame

Frame

As can be seen in **Figure 17** the frame consists out of standard aluminum profiles which are connected in a way to create trusses. The two parts that can be recognized are the motor support and the medium supply support which are connected with four profiles. On the right the part that supports the motor can be seen. The plate on which the motor is mounted is connected to the frame and separated with rubber foam to reduce vibrations. The two bearing cases are also protected from vibrations using rubber foam.

Building blocks

Profiles

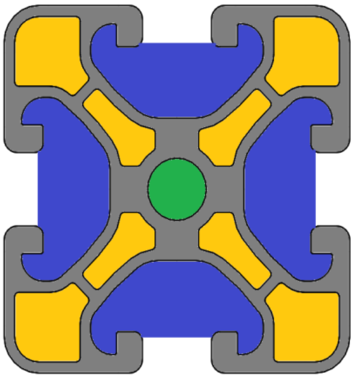

Figure 19: Cross section of aluminum profile “Profil 8 40x40 leicht natur”

Figure 18: "Nutenstein" and screw

The profiles[6] that are used have a particular shape (see **Figure 19**). Indicated in grey is the actual geometry whereas the other colors are to indicate other functions. The blue parts are cavities where the so called “Nutenstein[7]” can fit (see **Figure 18**). These are parts that can slide over the whole length of the profile and can be used to fix other parts to these profiles. The big hole in the middle which is indicated in green can be used once there is a threading tapped in there to connect parts on the ends of the profiles. The orange cavities are weight reductions.

5 http://www.item24.de/produkte/item-baukastensysteme/mbsystembaukasten.html
6 http://product.item24.de/produkte/produktkatalog/products/profile-und-zubehoer-1.html
7 http://product.item24.de/produkte/produktkatalog/products/nutensteine.html

Corner connections

In the design four types of corner connections are used. These are:

Verbindungssatz 8 40x40x40 [8]

This connection can be used once in the middle of the profile a threading is tapped and connects up to three profiles. In the design these connectors are used to create the cube that supports the motor and create the frame for the medium support. In **Figure 17** these connections can be seen as black cubes.

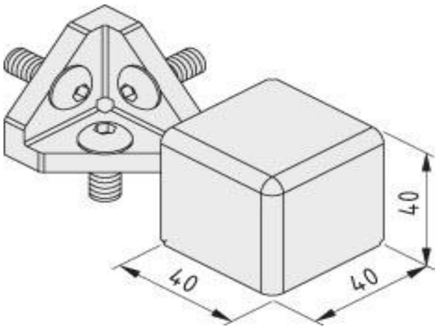

Figure 20: "Verbindungssatz 8 40x40x40"

Figure 21: "Winkelelement 8 T1-40"

Figure 22: "Winkelelement 8 T2-40"

Figure 23: "Gehrungs-Verbindungssatz 8"

Winkelelement 8 T1-40 [9]

This connector block is used to fix a 45 degree angled profile in a corner (see **Figure 21**). The T1 connector as well as the T2 connector are used for creating the trusses and can be found in **Figure 17** as black triangles.

Winkelelement 8 T2-40 [10]

This connector block is used to fix two 45 degree angled profiles to each other and to a 90 degree profile as can be seen in **Figure 22**. The T2 connector as well as the T1 connector are used for creating the trusses and can be found in **Figure 17** as black triangles.

Gehrungs-Verbindungssatz 8 [11]

The last connector (see **Figure 23**) is used for fixing parts that are at strange angles. Since the frames are not twice as high as wide the angle of the profile is not 45 degrees and another connector is needed. In **Figure 17** these connectors are not explicitly drawn since they are fixed inside the profiles. Six of these are used to connect the three angled profiles.

---

[8] http://product.item24.de/produkte/produktkatalog/produktdetails/products/eck-verbindungssaetze/verbindungssatz-8-40x40x40-schwarz-41608.html

[9] http://product.item24.de/produkte/produktkatalog/produktdetails/products/winkelelemente/winkelelement-8-t1-40-natur-38800.html

[10] http://product.item24.de/produkte/produktkatalog/produktdetails/products/winkelelemente/winkelelement-8-t2-40-natur-38802.html

[11] http://product.item24.de/produkte/produktkatalog/produktdetails/products/gehrungs-verbindungssaetze/gehrungs-verbindungssatz-8-49230.html

Designing the safety screens

The safety screens that have been designed are presented in **Figure 24.** Also these screens are made out of the standard profiles and assembled using M8 screws. Two parts can be distinguished and are both supported by wheels to make the system mobile. At the left the more complex safety screen can be seen which features a hole for the droplets (on the top) and holes in the other parts for cameras and lasers. Since the screens are made out of safety glass images can be distorted and holes are necessary. One can see that at the very left the hole in the screen is as small as possible. In order to protect the camera from a possible breaking paddle the slit is smaller than the paddle.

Figure 24: Rendered photograph of safety screens

Used screens

Since the material of the screens endures water, ethanol and methanol not all materials will suffice. In fact all possible clear plastics were not resistant to the three combinations of materials. Since normal glass is resistant to all these media the choice has been made to make the safety screens out of safety glass. Safety glass is stronger than normal glass and can endure the high impact of the paddle. To make sure that the glass does not break at impact the minimum thickness needs to be calculated.

Glass thickness

The impact energy of the paddle needs to be calculated in order to choose the right glass thickness:

$$E_{impact} = \frac{1}{2} m_{paddle} * v_{paddle}^2 = \frac{1}{2} * \frac{1}{2} * (30)^2 = 225\, Joules$$

So with making the choice for the glass one must take care that the glass can endure 225 Joules. The glass that is chosen[12] has a thickness of 6 mm thick and meets the specifications necessary.

[12] http://www.glas-behrens.de/catalog/sicherheitsglaeser/7ab17f67-a039-4dad-9ca9-49f8dfddf64c.aspx

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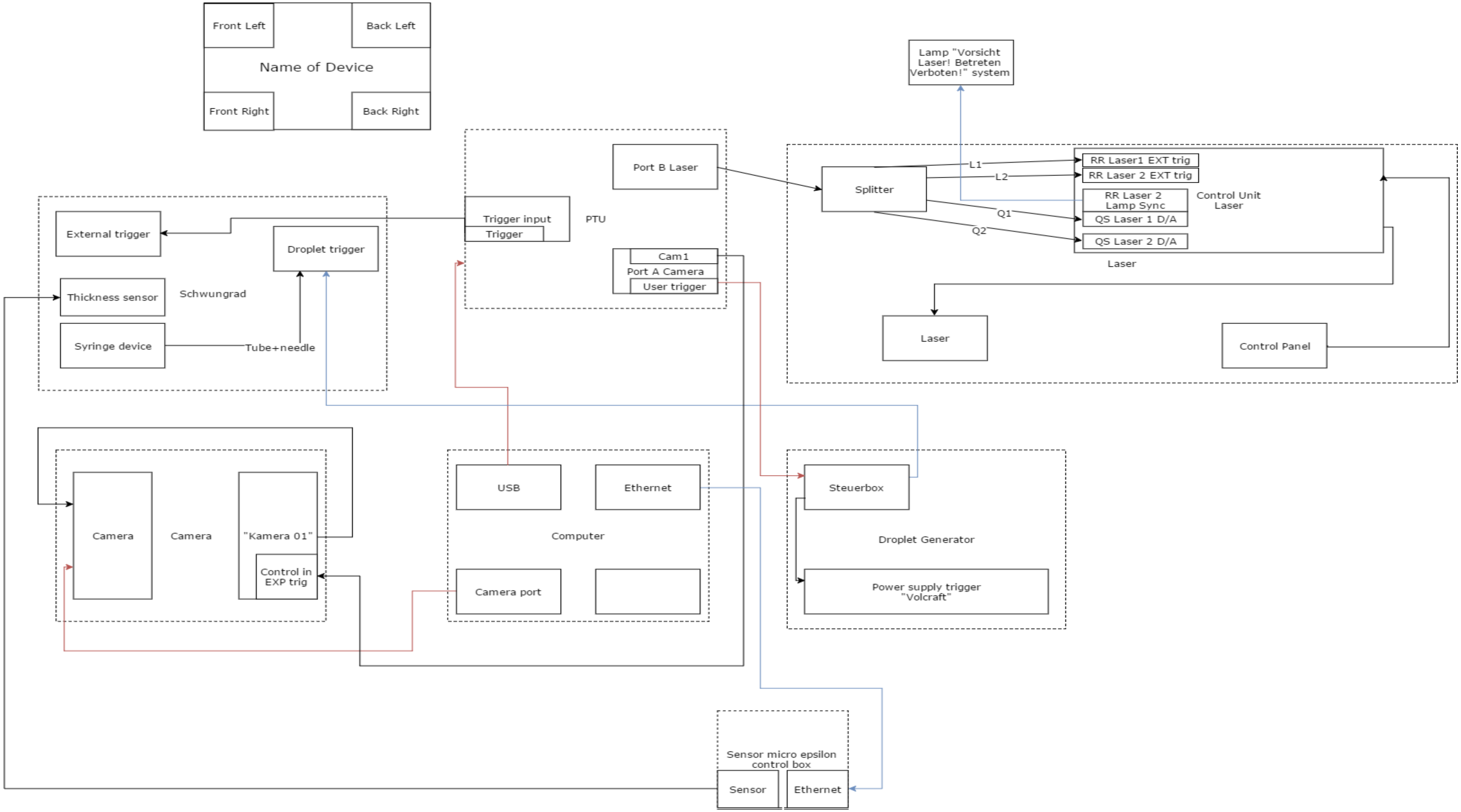
Appendix A: Measurement equipment configuration



Appendix B: Results from static strength analysis

Shaft system					
System parameter	Value	*Dimension*	**Important property**	Value	*Dimension*
Length shaft (L_1)	0.185	*m*	**Cross section area (A_solidshaft)**	0.005	*m^2*
Offset wheel (X_1)	0.094	*m*	**Moment of inertia (I_solidshaft)**	3.2*10^(-5)	*m^4*
Loading force (F_load)	100	*N*	**Force on bearing A (F_A)**	46.89	*N*
Radius shaft (r_shaft)	40	*Mm*	**Force on bearing B (F_B)**	51.18	*N*
Elasticity Module steel (E)	210	*GPa*	**Maximum shear force (V1_max)**	-51.18	*N*
Maximum stress steel (σ_max)	500	*MPa*	**Maximum internal moment (M1_max)**	4.41	*Nm*
Maximum shear stress steel (τ_max)	300	*MPa*	**Maximum bending stress (σ_bendingmax)**	0.005	*MPa*
			Maximum shear stress (τ_shearmax)	-0.01	*kPa*
Maximum dynamic load bearing A (C_A)	80000	*N*	**Minimum diameter necessary due to bending stress (d_minbending)**	4	*mm*
Maximum dynamic load bearing B (C_B)	18000	*N*	**Minimum diameter necessary due to shear stress (d_minshear)**	0.47	*mm*
Maximum rotational velocity wheel (n)	1000	*rpm*	**Approximate endurance life bearing A (L_10A)**	4.97*10^15	*Rotations*
Livelife exponent (p)	3	**-**	**Approximate endurance life bearing B (L_10B)**	4.35*10^13	*Rotations*

Appendix B: Results from static strength analysis (continued)

Medium and optical signal supply system					
System parameter	Value	*Dimension*	**Important property**	Value	*Dimension*
Length shaft (L_2)	0.087	*m*	**Cross section area (A_solidshaft)**	0.002	*m^2*
Offset wheel (X_2)	0.0073	*m*	**Moment of inertia (I_solidshaft)**	3.2*10^(-6)	*m^4*
Loading force (F_A)	46.89	*N*	**Force on bearing C (F_C)**	42.98	*N*
Radius shaft (r_shaft)	22.5	*Mm*	**Force on bearing D (F_D)**	3.9	*N*
Elasticity Module steel (E)	210	*GPa*	**Maximum shear force (V2_max)**	42.98	*N*
Maximum stress steel (σ_max)	500	*MPa*	**Maximum internal moment (M2_max)**	0.312	*Nm*
Maximum shear stress steel (τ_max)	300	*MPa*	**Maximum bending stress (σ_bendingmax)**	0.002	*MPa*
			Maximum shear stress (τ_shearmax)	27.02	*kPa*
Maximum dynamic load bearing C (C_C)	29500	*N*	**Minimum diameter necessary due to bending stress (d_minbending)**	2	*mm*
Maximum dynamic load bearing D (C_D)	22500	*N*	**Minimum diameter necessary due to shear stress (d_minshear)**	0.43	*mm*
Maximum rotational velocity wheel (n)	1000	*rpm*	**Approximate endurance life bearing C (L_10C)**	3.23*10^14	*Rotations*
Livelife exponent (p)	3	**-**	**Approximate endurance life bearing D (L_10D)**	1.92*10^17	*Rotations*

Appendix C: Dynamic strength analysis results

Shaft system					
System parameter	Value	*Dimension*	**Important property**	Value	*Dimension*
Length shaft (L_1)	0.185	*m*	**Cross section area (A_hollowshaft)**	0.0017	*m^2*
Offset wheel (X_1)	0.094	*m*	**Moment of inertia (I_hollowshaft)**	1.82*10^(-5)	*m^4*
Mass wheel (m_0)	10	*kg*			
Polar moment of inertia wheel for bending (J_0)	0.42	*kg*m^2*	**First bending eigenfrequency**	6333	*Rad/s*
Polar moment of inertia wheel for torsion (I_0)	0.84	*kg*m^2*	**Second bending eigenfrequency**	26250	*Rad/s*
Outer radius shaft (r_out)	40	*mm*			
Inner radius shaft (r_in)	32.5	*mm*	**First torsional eigenfrequency**	6013	*Rad/s*
Elasticity Module steel (E)	210	*GPa*	**Second torsional eigenfrequency**	54300	*Rad/s*
Shear Module steel (G)	79.3	*GPa*			
Density steel (ρ)	7700	*kg/m^3*	**First longitudinal eigenfrequency**	18370	*Rad/s*
Maximum rotational velocity	1000	*Rpm*	**Second longitudinal eigenfrequency**	91430	*Rad/s*
Maximum rotational velocity	104.7	*Rad/s*			

Medium and optical signal supply system					
System parameter	Value	*Dimension*	**Important property**	Value	*Dimension*
Length shaft (L_2)	0.125	*m*	**Cross section area (A_hollow)**	0.0015	*m^2*
Outer radius shaft (r_out)	25	*mm*	**Moment of inertia (I_hollow)**	4.60*10^(-6)	*m^4*
Inner radius shaft (r_in)	12.5	*mm*			
Elasticity Module steel (E)	210	*GPa*	**First bending eigenfrequency**	288100	*Rad/s*
Density steel (ρ)	7700	*kg/m^3*	**Second bending eigenfrequency**	933600	*Rad/s*
Maximum rotational velocity	1000	*Rpm*			
Maximum rotational velocity	104.7	*Rad/s*			

Appendix D: Power system keys analysis results

Wheel key					
System parameter	Value	*Dimension*	**Important property**	Value	*Dimension*
Torsional moment on shaft (T)	30	*Nm*	**Shear stress (τ_shear)**	5.05	*MPa*
Diameter shaft (d_shaft)	80	*mm*	**Compression stress (σ_comp)**	20	*MPa*
Length key (L)	15	*mm*	**Principal stress 1 (σ_1)**	21.2	*MPa*
Width key (W)	9.9	*mm*	**Principal stress 2 (σ_2)**	-1.2	*MPa*
Height key (H)	5	*mm*	**Safety factor (N_safety) in this case**	23.6	-
Maximum stress steel (σ_max)	500	*MPa*			

Coupling key					
System parameter	Value	*Dimension*	**Important property**	Value	*Dimension*
Torsional moment on shaft (T)	30	*Nm*	**Shear stress (τ_shear)**	30	*MPa*
Diameter coupling shaft (d_shaft)	25	*mm*	**Compression stress (σ_comp)**	68.6	*MPa*
Length key (L)	10	*mm*	**Principal stress 1 (σ_1)**	79.84	*MPa*
Width key (W)	8	*mm*	**Principal stress 2 (σ_2)**	-11.3	*MPa*
Height key (H)	7	*mm*	**Safety factor (N_safety) in this case**	6.3	-
Maximum stress steel (σ_max)	500	*MPa*			

Appendix E: O-ring and groove dimensions

O-ring and Groove dimensions (see Figure 11)		
Important property	Value	*Dimension*
O-ring diameter	56.88	*mm*
O-ring cross section diameter	1.78	*mm*
O-ring cross section diameter tolerances	± 0.08	*mm*
O-ring material	EPDM	
Groove diameter	57.4	*mm*
Groove width	2.4	*mm*
Bore diameter	60	*mm*
Maximum diametrical clearance	0.068	*mm*
Minimum diametrical clearance	0.030	*mm*
Groove and surface material	Stainless steel 1.4571	
Mean contact surface roughness	1.6	μm
Maximum contact surface roughness	6.3	μm
Mean groove surface roughness	3.2	μm
Maximum contact surface roughness	12.5	μm