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MASTER THESIS

The CVA trade-off: Capital or P&L?

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Abstract

Credit valuation adjustment (CVA) has become an important aspect of accounting and regulatory standards. On the one hand, the regulatory standards (Basel accords) demand a capital risk charge for CVA volatility. Basel allows to hedge the CVA, which results in a reduction of the risk charge. On the other hand, the accounting standards (IFRS) require that the financial instrument is valued at fair value, which is achieved by including CVA. Consequently, changes in CVA have an effect on P&L, since fluctuations of the instrument's value affect the balance sheet equity. However, there is a mismatch between hedging the regulatory risk charge and accounting P&L volatility. The hedge instruments reducing the risk charge cause additional P&L volatility, due to the fact that the regulatory view on CVA is more conservative than the accounting one (Berns, 2015; Pykhtin, 2012). There is a trade-off between achieving risk charge reduction and creating additional P&L volatility.

We present a methodology to define the optimal hedge amounts, which leads to maximal CVA charge reduction while minimizing additional P&L volatility. The Hull-White model is selected to simulate the risk factors determining the value of the interest rate swap of time. By applying the Monte Carlo method, the expected exposure path is found. Furthermore, the CDS spreads are simulated, which are used in the CVA calculation and CDS calculation. The combination of results are implemented in the regulatory and accounting regimes, yielding in the CVA risk charge and CVA P&L. Using the optimization criteria, we found the optimal hedge amount for each risk appetite. In the implementation case we use an interest rate swap, due to the notional size of interest rate derivatives to the OTC market. Here, we present a step-by-step guidance from implementing the risk factor model to finding the optimal hedging amount. For a bank more focused on capital, we see that the hedge amount should be set closer to Basel's EAD level. For a bank focused on reducing additional P&L volatility, we find that the hedge amount should be set closer to expected exposure level.

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Chapter 1

Introduction

1.1 Background

The recent financial crisis showed that *counterparty credit risk* (CCR) is to be considered significant. According to Gregory (2009), CCR is the risk that a counterparty in a derivatives transaction will default prior to expiration of a trade and will not make current and future payments required by the contract. High-profile bankruptcies and bailouts of Lehman Brothers, Bears Stearns, Merril Lynch, AIG and more, erased the misconception that certain counterparties would never fail.

CCR is a specific form of credit risk. Traditionally, credit risk is thought of as lending risk, where a party borrows money from another party and fails to pay some or the whole amount due to insolvency. However, CCR differentiates on two aspects from traditional credit risk (Gregory, 2009):

1. The future credit exposure of a derivative contract is uncertain, since the exposure is determined by the market value of the contract.
2. Counterparty credit risk is normally bilateral, since the value of a derivatives contract could have a negative or positive value.

CCR exposure is significant in the *over-the-counter* (OTC) market. In an OTC transaction two parties do a trade without the supervision of an exchange. Exchange-traded contracts are standardized, follow the terms of the exchange and eliminate the CCR due to third party agreements to guarantee the contract payments as agreed on. In an OTC trade there are no third party agreements, which means the CCR will not be eliminated and there is a risk that the contract will not be honored (Hull, 2012). The advantage of OTC trades on the other hand, lies in the possibility to create highly specified derivative contracts adapted to the needs of the two parties.

During the financial crisis banks suffered large CCR losses on their OTC derivatives portfolios. Most of these losses were not caused by the counterparty default but due to the change in value of the derivative contracts. The *Basel Committee on Banking Supervision* (BCBS) states that (BCBS, 2011):

"*During the financial crisis, roughly two-thirds of losses attributed to Counterparty Credit Risk were due to Credit Valuation Adjustments losses and only about one-third were due to actual defaults*".

The downgrade of a counterparty's creditworthiness, or the fact the counterparty is less likely than expected to meet their obligations, caused the value of the derivative contracts to be written down (BCBS, 2015). The risky price of a derivative can be thought of as the risk-free price (the price assuming no counterparty risk) minus a component to correct for counterparty risk. This latter component is called *credit valuation adjustment* (CVA) (Gregory, 2009).

Accounting standards (IFRS) require that the value of a financial instrument includes counterparty credit risk, which leads to the fair value of the instrument. The fair value is achieved by a valuation adjustment referred to as CVA. The CVA has an effect on the *profit and loss* (P&L), since losses (gains) caused by fluctuations of the counterparty's credit quality reduce (increase) the balance sheet equity. Regulatory standards (Basel Accords) demand a capital charge for future changes in credit quality of the counterparty, i.e. CVA volatility (Berns, 2015). Basel II/III define the capital charge in two parts, namely: a charge for the current CVA (CCR capital requirements) and a charge for future CVA changes (CVA risk charge). Since banks actively manage CVA position by their CVA desks, regulatory standards allow to reduce the CVA risk charge by entering in eligible hedges such as CDS hedges. Hedging future CVA volatility leads to a decrease of CVA risks, which implies that less capital is necessary.

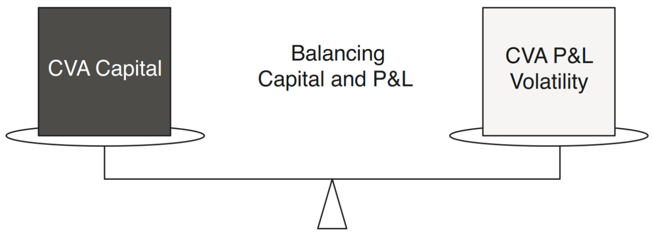

FIGURE 1.1: Balancing between CVA P&L hedging and capital optimization (Lu, 2015).

As illustrated, CVA can be interpreted from the accounting perspective or the regulatory perspective. However, there is a mismatch between the regulatory capital reduction and accounting CVA P&L hedging (Lu, 2015). Between the two regimes, different valuation methods are applied for CVA. In general, the regulatory view on CVA is more conservative than the accounting one (Pykhtin, 2012). Intuitively, regulatory capitals are set to protect from stressed situations based on events with small probability, while accounting CVA P&L looks at the expected exposure at present time (Lu, 2015). This difference leads to the following problem of hedging the CVA risk charge: eligible hedge instruments reduce the regulatory CVA risk charge, while under IFRS the hedge instrument is recognized as a derivative and accounted for at fair value through P&L introducing P&L volatility. In other words, hedging CVA exposures from the regulatory perspective leads to overhedging from the accounting perspective, meaning a part of the hedge would be naked from the accounting point of view because of the mismatch in exposure profiles, and therefore a mismatch in spread sensitivity. This overhedging creates additional P&L volatility (Berns, 2015). Figure 1.1 shows the trade-off between choosing CVA P&L volatility and regulatory CVA capital. If the firm is more focused on reducing capital, it exposes itself to more CVA P&L volatility, and vice versa (Lu, 2015).

A real-world example of the mismatch between the regulatory and accounting treatments of CVA can be found in the recent history of Deutsche Bank. The bank used a hedging strategy to achieve regulatory capital relief in the first half of 2013. The reduction in the CVA risk charge led to large losses due to P&L volatility. The mismatch forces banks to decide which regime is more significant. As J. Kruger, Lloyds Banking Group, puts it well: *"It's a trade-off. How much volatility is it worth to halve your CVA capital? That's the million-dollar question"* (Carver, 2013).

On the first of July 2015, the BCBS released a consultative document discussing a review of the credit valuation adjustment risk framework. [1] The document presents a proposed revision of the current CVA risk framework set out in Basel III. The proposal sets forth two different frameworks to accommodate different types of banks, namely the "FRTB-CVA framework" and the "Basic-CVA framework". The aim of the proposition is to capture all CVA risks, better alignment with industry practices for accounting purposes, and better alignment with the market risk framework. Unfortunately, the mismatch still exists within the new CVA risk framework, due to recent developments on the proposed framework leaving only the punitive "Basic-CVA framework".

1.2 Research design

1.2.1 Research objective and questions

The current accounting and regulatory regimes lead to the situation where it is hard to reduce the CVA risk charge and lower P&L volatility. The mismatch between the two regimes demands for a trade-off. Therefore, the research objective of this thesis is defined as follows:

Propose a methodology to define the optimal hedge amounts, which leads to maximal CVA risk charge reduction while minimizing additional P&L volatility.

Research questions are set up to achieve our research objective. The research questions are formulated as follows:

1. What is credit valuation adjustment?
   (a) What are the components of the Basel CVA approach?
   (b) What are the components of the IFRS CVA approach?
2. How do regulatory CVA and accounting CVA have an influence on P&L?
   (a) How does CVA have an impact on P&L volatility?
   (b) How do CVA hedges have an impact on P&L volatility?

[1] Basel Committee on Banking Supervison, *Review of the Credit Valuation Adjustment Risk Framework*, July 2015, www.bis.org/bcbs/publ/d325.htm.

3. How to define the optimal hedge amount leading to maximal CVA charge reduction and minimal additional P&L volatility?

The first question targets the definition of CVA by describing the underlying mathematical foundation and implementation of regulatory CVA and accounting CVA. The second question focuses on the interplay of the hedges on P&L. Finally, the last question aims at defining the optimal hedge amount.

1.2.2 Scope

The concept of CVA is broad and complex. Therefore it is critical to have a distinct demarcation to make it clear which components of CVA are in- and excluded within this research.

1. **Basel CVA risk charge:** Three approaches are proposed by the Basel CVA framework, namely the *Internal Models approach* (IMA-CVA), the *Standardized approach* (SA-CVA) and the *Basic approach* (BA-CVA). We consider the BA-CVA only, since IMA-CVA and SA-CVA are too complex to model by ourselves. Furthermore, recent developments show that IMA-CVA is canceled by the BCBS [2] and SA-CVA is under consideration to be axed as well [3].

2. **Risk mitigation:** Reducing CVA risk is possible in multiple ways, such as hedging, netting agreements, credit support annex, special purpose vehicles and more. Since the research objective is to define the optimal hedge ratio, we focus on the risk mitigation via hedging. Adding more risk mitigation tools would make the results indistinct.

3. **Accounting framework methodology:** In accounting literature there is no specific method prescribed to calculate CVA. Various approaches are available to compute CVA. We use the most commonly used approach in practice by derivative dealers and end users.

4. **xVA:** Different valuation adjustments are generalized by the term XVA. It quantifies the values of components such as counterparty risk, collateral, funding or margin. Examples of xVA are CVA, DVA, FVA, ColVA, KVA and MVA (Gregory, 2015). We include CVA only since we are focusing on counterparty risk.

1.2.3 Outline

The thesis outline is set up as follows:

Chapter 2. CVA Frameworks: The concept of CVA is introduced by defining its components. The components combine into a standardized formula to compute CVA. Next, the regulatory framework with regard to the CVA risk charge is described. Furthermore, the accounting framework with the CVA approach used in this research is explained. We choose the most common CVA approach, since multiple accounting alternatives for CVA computations are available.

[2] Risk.net, *Dealers fret over Basel CVA revisions*, October 2015, http://www.risk.net/2427911

[3] Risk.net, *Basel considered axing standardised approach to CVA calculation*, November 2016, http://www.risk.net/2477114

Chapter 3. Optimization methodology: A step-by-step methodology is introduced to combine the accounting framework and the regulatory framework to define the optimal hedge amount. We use the methodology on a pre-specified interest rate derivative contract, namely an interest rate swap.

Chapter 4. Model foundation: Here, the necessary models and valuation techniques are explained. The methodology suggests to use an interest rate model to generate input to compute CVA for the interest rate swap. The procedure behind the calibration of the interest rate model is explained. Furthermore, pricing of the interest rate derivatives and credit hedges are presented.

Chapter 5. Implementation case: The results of the implementation of the pre-specified financial contract within the optimization methodology are presented. The model is calibrated to represent the current time. It shows the interplay between the regulatory and accounting frameworks effected by the hedge amount.

Chapter 6. Conclusion: Finally, we come to a conclusion based on the findings of the implementation case. We discuss the limitations of our research and give suggestions for further research.

Chapter 2

The CVA frameworks

In this chapter we introduce the regulatory CVA framework and the accounting CVA framework. First, we start by explaining credit valuation adjustment and present a generalized formula by Gregory (2009) to compute CVA. Next, we examine the proposed regulatory CVA framework and in particular the basic approach. Then, the accounting CVA framework is discussed, which is based on IFRS13. Lastly, the difference in hedging is explained with the use of an example.

2.1 Credit valuation adjustment

2.1.1 Definitions and notations

Asymmetry of potential losses with respect to the value of the underlying transaction is one of the characterizing features of counterparty credit risk (Gregory, 2009). A contract is considered to be an asset to the firm if the *mark-to-market* (MtM) is positive and a liability if the MtM value is negative. If the counterparty defaults and the contract is considered an asset then the loss would be the value of the contract at that specific time since the counterparty is unable to undertake future contract commitments. [1] We define exposure for uncollateralized trades as follows:

Definition 2.1. Exposure. *Let $V(t)$ be the default-free MtM value of a contract at time t. The Exposure at time t for the non-negative part of position $V(t)$ is defined as:*

$$E(t) = max(V(t); 0)\,. \tag{2.1}$$

The contract value can be expressed as the expectation of the future contract values in all future scenarios. In other words, different future scenarios have different future exposures. When these future exposures are combined we get an exposure distribution at a future point in time. The expected exposure is the average of this exposure distributions (Lu, 2015). We define the expected exposure as follows:

Definition 2.2. Expected Exposure. *Expected Exposure at time t is defined as:*

$$EE(t) = \mathbb{E}\big[E(t)\big]\,. \tag{2.2}$$

[1] assuming no collateral, netting agreements or other risk mitigators.

The combination of future scenarios gives an exposure distribution (i.e., probability distribution). For risk management practices this probability distribution may be used to find the worst exposure at a certain time in the future with a certain confidence level. For example, with a confidence level of 99% implies the potential exposure that is exceeded with less than 1% probability (Lu, 2015). We define the potential future exposure with a high confidence level quantile as follows:

Definition 2.3. Potential Future Exposure. *Potential Future Exposure at time t is defined as:*

$$\mathrm{PFE}_{\alpha}(t) = inf\{x : \mathbb{P}(E(t) \leq x) \geq \alpha\} \tag{2.3}$$

where α is the confidence level and $\mathbb{P}$ the real-world measure.

The percentage of the outstanding claim recovered when a counterparty defaults is represented by the *recovery rate* (RR). The outstanding claim recovered can also be expressed alternatively by the *loss given default* (LGD), which is the percentage of the outstanding claim lost (Gregory, 2015). These percentages depend on, among other things, the default time, valuation of the derivatives at the default time, the remaining assets of the defaulting party and the seniority of the derivative trade (Lu, 2015). We define the loss given default as follows:

Definition 2.4. Loss Given Default. *The Loss Given Default is defined as:*

$$\mathrm{LGD} = 1 - \mathrm{RR} \tag{2.4}$$

where RR is the Recovery Rate.

The probability of default describes the likelihood of a default over a particular time horizon. The probability of default may be defined as real-world, where the actual probability of default is estimated via historical data, or as risk neutral, where the probability of default is estimated via market-implied probabilities (Gregory, 2015). We define the default probability as follows:

Definition 2.5. Default probability. *The incremental Default Probability of a given time frame is defined as:*

$$PD(t, t + dt) = H(t)dt \tag{2.5}$$

and the total default probability from time 0 to T is:

$$PD(0, T) = \int_0^T H(t)dt \tag{2.6}$$

2.1.2 Introduction of CVA

Credit valuation adjustment (CVA) is defined as the adjustment to the value of derivatives due to expected loss from future counterparty default. Intuitively, one can view CVA as the difference between the risk free value of a derivative and the risky value of a derivative, where the counterparty's default is allowed (Lu, 2015):

$$\text{CVA} = \Pi(t) - \Pi^{risky} \tag{2.7}$$

where Π is risk-free value without counterparty risk at time t and Π^{risky} is the market value of the portfolio accounting counterparty risk at time t. We define the CVA term as follows (Gregory, 2009):

$$\text{CVA} = \text{LGD} \int_0^T D(t)EE(t)H(t)dt \tag{2.8}$$

where $D(t)$ is the relevant risk-free discount factor. An important note is that the risk neutral measure should be taken is we are interest in the expectation the market price of credit risk, while the real world measure should be taken if interested in exposures in risk management perspective (Timmer, 2014). Equation (2.8) shows that CVA is built up by three components: loss-given-default, exposure and default probability. Here we assume no wrong-way risk, i.e. dependency between default probability and market risk exposure, since CVA pricing is already a complex process excluding wrong-way risk. Including wrong-way risk is out of scope for this thesis. Furthermore wrong-way risk is a broad subject, which can give birth to multiple potential research subjects for future theses [2].

Until this point we considered the pricing of CVA from the perspective that the institution was risk-free themselves and could not default, which is referred to *unilateral CVA* (CVA). This seems like a straightforward assumption, since the accountancy concepts are based on the assumption that a business is a "going concern" and will remain in existence for an indefinite period. However, credit exposure has a liability component and can be included in the pricing of the counterparty risk, which is known as *debt value adjustment* (DVA), where own creditworthiness is taken into account (Gregory, 2009). We define the DVA term as follows (Gregory, 2009):

$$\text{DVA} = \text{LGD} \int_0^T D(t)ENE(t)H(t)dt\,. \tag{2.9}$$

where $ENE(t)$ is defined as the Expected Negative Exposure. Here the Negative Exposure is the negative part of the default-free MtM value. Including CVA and DVA is referred to as *bilateral CVA* (BCVA). We define BCVA as follows:

$$\text{BCVA} = \text{CVA} - \text{DVA}\,. \tag{2.10}$$

[2]For the interested, see Hull and White (2012) and Delsing (2015) to learn more about CVA and wrong-way risk.

2.1.3 Hedging

By definition is CVA a complex but an important concept. Managing CVA positions is an essential part of risk management. CVA risks involve two types of factors, namely market risk factors and credit risk factors. Market risk factors influence the derivative valuations and market risk exposure by changes in underlying factors such as interest rate, FX rates and equities. Credit risk factors include counterparties' default risk by underlying CDS spreads.

To make hedging decision, it must be clear to know the purpose of hedging. The most common credit risk hedging goals are (Lu, 2015):

- Reduction of CVA P&L fluctuations
- Reduction of counterparty credit default risk
- Reduction of the regulatory capital requirements

CVA P&L fluctuations show the influence on a daily basis, while counterparty default risk and regulatory capital requirements are more of a tail risk, due to the very small probability.

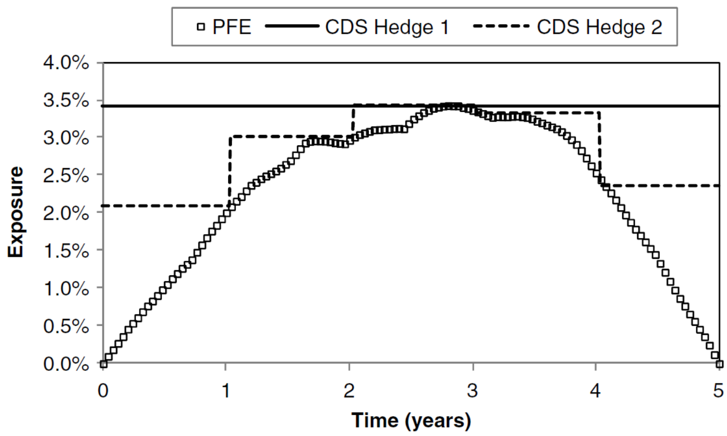

FIGURE 2.1: Two types of CDS hedging strategies: strategy CDS Hedge 1 involves a single 5 year CDS and strategy CDS Hedge 2 involves a 1-,2-,3-,4- and 5-year CDSs (Gregory, 2009)

Let us consider an interest rate swap, where we want to hedge CVA movements caused by credit spread changes. This concept is illustrated using the following simple example. In this case we assume an upwards-sloping credit curve,[3] which results in a total CVA of 1.5 bps based on the exposure profile in Figure 2.1. Two types of hedging strategies using CDSs are shown. CDS hedge 1 uses one 5-year CDS protection with the initial cost of 10.3 bps.[4] CDS hedge 2 uses multiple CDSs, a term structure hedge, to match the exposure profile better with the initial cost of cost 8.1 bps.[5] (Gregory, 2009) If the CVA changes due to movements in credit spread, the CDS

[3]1year = 100 bps, 2year = 150 bps, 3year = 200 bps, 4year = 250 bps, 5year = 300 bps.
[4]300*3.42%
[5](100*-0.92%)+(150*-0.42%) + (200*0.11%) + (250*0.97%) + (300*2.35%)

will compensate these movements.

Over time the exposure profile might be different due to changes in the underlying market factors. If the credit hedge is static (i.e., not adjusted from the time of initiation), the CDS hedge is less effective to compensate for the CVA movements, due to differences in sensitivities. Alternatively, hedging could be done dynamically (i.e., adjust the hedge to the new exposure profile at specific points in time). Then, the CVA sensitivities are compensated by CDS movements. The more points in time that the hedge is adjusted to the exposure profile, the better the hedge will offset the CVA movements. However, increasing the number of times adjusting the hedge will lead to higher hedging costs.

A solution to the problem that the CDS might deviate from the exposure profile is making use of a *contingent CDS* (CCDS). A CCDS works the same as a standard CDS, except that the notional amount of protection is based on the value of the derivative contract at the default time. For example, if a derivative contract has an exposure of $10m at the counterparty default time, then the CCDS will pay a protection amount of $10m. In other words, the CCDS follows the exposure profile of the derivative (Gregory, 2009).

Until this point we only considered hedges against credit spread movements. As noted before, CVA is also driven by an exposure component influenced by market risk factors. Dependent on the type of derivative contract, FX, interest rate and so on, different types of hedges may be used to offset the CVA caused by exposure movements. However, exposure hedges are not included within the scope of this thesis, because the regulatory CVA framework does not allow exposure hedges, as we will see later on.

2.2 Regulatory CVA framework

2.2.1 CVA under the Basel Accords

Basel I & II

The first Basel Accord was the start of international standards for banking regulation. The Basel I Accord was set up in 1988 by the BCBS to define international risk-based standards for capital adequacy. From 1988 it was gradually accepted by the members of the G-10 countries and many other countries around the world. The Accord mainly focused on credit risk with the introduction of *risk-weighed assets* (RWA) to reflect the bank's total credit exposure accordingly. The capital a bank has to hold is calculated based on the exposure the RWA generate (Hull, 2012).

The Basel I Accord set the foundation of capital requirements, however it showed some weaknesses (it lacked risk sensitivities). Therefore, the BCBS introduced a new framework with a set of rules supplementing and improving the Basel I Accord, known as Basel II. The Accord includes market risk and operational risk next to credit risk. The Basel II framework consists of three pillars: (I) minimal capital requirements, (II) supervisory review, and (III) market discipline (Gregory, 2009).

As stated before, CCR is the risk that the counterparty does not fulfill its obligations on a derivative contract. Introduced as part of the Pillar I was the CCR charge, which is a capital charge to mitigate the losses on an OTC contract caused by the default of the counterparty. Two approaches are available under Basel II to compute the CCR charge, namely the Standardised approach and the Internal Ratings-Basel approach.

Basel III

The financial crisis showed that the Basel II framework had shortcomings, including insufficient capital requirements, excessive leverage, procyclicality and systematic risk (Gregory, 2009). The BCBS proposed a new set of changes to the previous regulatory framework, which are set up in Basel III. Basel III largely focuses on counterparty credit risk and CVA. As stated by the BCBS most of the losses did not arise by defaults of the counterparty but from credit deterioration of the counterparty affecting the fair value of the derivative contracts. The CCR charge under Basel II was focused on the actual default of the counterparty rather than the potential accounting losses that can arise from CVA . To close this gap in the framework, the BCBS introduced the CVA risk charge to capitalise against variability in CVA (BCBS, 2015).

The current CVA framework consists of two approaches for computing this CVA risk charge, namely the "Advanced Approach" and the "Standardised Approach". Changes in the credit spreads are the drivers of CVA variability within the two approaches. Both approaches do not take exposure variability driven by daily changes of market risk factors into account. The current advanced approach is only available for banks who meet the criteria to use the *internal model method* (IMM) for computing the *exposure at default* (EAD). The current standardised approach is a pre-defined regulatory formula using rating-based risk weights to compute the CVA risk charge (BCBS, 2015).

Review of the CVA risk framework

On the first of July 2015, the BCBS released the consultative document named 'Review of the Credit Valuation Adjustment Risk Framework'. This consultative document presents a proposed revision on the current CVA framework set out in Basel III capital standards for the treatment of counterparty credit risk. The reasons for revising the current CVA framework are in threefold: capturing all CVA risks and better recognition of CVA hedges, alignment with industry practices for accounting purposes, and alignment with proposed revision to the market risk framework (BCBS, 2015).

The proposal sets forth two different frameworks to accommodate different types of banks. Firstly, the "FRTB-CVA framework" is available to banks which meet pre-specified conditions set out in the *fundamental review of the trading book* (FRTB). This framework consists of a proposed *standardised approach* (SA-CVA) and a proposed *internal models approach* (IMA-CVA). Secondly, the "Basic CVA framework" is available for banks which do not meet the pre-specified FRTB conditions. This framework consists of a proposed *basic approach* (BA-CVA).

Since the release of the consultative document several developments regarding the proposed CVA risk framework have taken place. In October 2015 the BCBS revised the proposed framework by eliminating the IMA-CVA approach (Sherif, 2015b) and in November 2015 to axe the SA-CVA approach (Sherif, 2015a), leaving the BA-CVA approach.

2.2.2 Proposed Basic Framework

The definitions and notations of the Basic CVA approach are adopted from BCBS (2015).

Basic CVA approach formula

The basic CVA capital charge K is calculated according to

$$K = K_{spread} + K_{EE} \tag{2.11}$$

where K_{spread} is the contribution of credit spread variability and K_{EE} is the contribution of EE variability to CVA capital. The EE variability component consists of a simple scaling of $K_{spread}^{unhedged}$ by β. Here β is set to 0.5, which assigns one-third of the capital requirement to EE variability.

$$K_{EE} = \beta K_{spread}^{unhedged} \quad . \tag{2.12}$$

Combining Equation (2.11) and Equation (2.12) results in the following basic CVA capital charge

$$K = K_{spread} + \beta K_{spread}^{unhedged} \quad . \tag{2.13}$$

The component of the basic CVA capital charge, $K_{spread}^{unhedged}$, is calculated via

$$K_{spread}^{unhedged} = \sqrt{(\rho \cdot \sum_c S_c)^2 + (1-\rho^2) \cdot \sum_c S_c^2} \tag{2.14}$$

where

- $S_c = RW_{b(c)} \cdot \sum_{NS \in c} M_{NS} \cdot EAD_{NS}$ is the supervisory ES of CVA of counterparty c, where the summation is performed over all netting sets with the counterparty
- $b(c)$ is the supervisory risk bucket of counterparty c
- RW_h is the supervisory weight for risk bucket b
- EAD_{NS} is the EAD of netting set NS calculated according to the Annex 4 of the Basel framework and used for default capital calculations for counterparty risk
- M_{NS} is the effective maturity for netting set Ns
- ρ is the supervisory correlation between the credit spread of a and the systematic factor

The composition of $K_{spread}^{unhedged}$ is similar to the K_{spread}, however the hedging components are absent. Hence, EE variability cannot be hedged, which implies that the basic CVA framework does not recognize exposure hedges.

The other component of the basic CVA capital charge, K_{spread}, is calculated via

$$
\begin{aligned}
K_{spread}^2 = (\rho \cdot \sum_c (S_c - \sum_{h \in c} r_{hc} S_h^{SN}) - \sum_i S_i^{ind})^2 \\
+ (1-\rho^2) \cdot \sum_c (S_c - \sum_{h \in c} r_{hc} S_h^{SN})^2 \\
+ \sum_c \sum_{h \in c} (1 - r_{hc}^2)(S_h^{SN})^2
\end{aligned}
\tag{2.15}
$$

where

- $S_h^{SN} = RW_{b(h)} M_h^{SN} B_h^{SN}$ is the supervisory ES of the price of single-name hedge h
- $S_t^{ind} = RW_{b(i)} M_i^{ind} B_i^{ind}$ is the supervisory ES of price index of hedge i
- $b(e)$ is the supervisory risk bucket of entity e (single-name or index)
- B_h^{SN} is the discounted notional of single-name hedge h
- M_h^{SN} is the remaining maturity of single-name hedge h
- B_i^{ind} is discounted notional of index hedge i
- M_i^{ind} is remaining maturity of index hedge i
- r_{hc} is the correlation between the credit spread of counterparty c and the credit spread of a single-name hedge h of counterparty c.

The K_{spread} component consists of three major terms under the square root partitioned by the plus symbols. The first term accumulates for the systematic components of CVA in combination with the systematic components of the single-name and index hedges. The second term accumulates for the unsystematic components of CVA in combination with the unsystematic components of the single-name hedges. The last term accumulates for the components of indirect hedges, which are not aligned with counterparties' credit spreads.

Eligible hedges

Eligible hedges under the Basic CVA framework are single-name CDS, single-name contingent CDS and index CDS. An additional requirement is set up for eligible single-name hedges, which states that the single-name hedges must (i) reference the counterparty directly, (ii) reference an entity legally related to the counterparty or (iii) reference an entity that belongs to the same sector and region as the counterparty.

Comparison to the current Standardised Approach

The Basic CVA approach is based on the current Standardised Approach. The most important changes between the approaches are (BCBS, 2015):

- The 99% VaR of the standard normal distribution is replaced by the 97.5% Expected Shortfall of the standard normal distribution. The factor is integrated into the risk weights.
- EAD is divided by the alpha multiplier to approximate the discounted EE curve better.
- The risk of non-perfect hedges between the credit spread of the counterparty and the credit spread of the hedge is introduced into the formula.
- Multiple netting sets and multiple single-name hedges related to the same counterparty are explicitly treated within the formula.
- Risk weights are defined for the SA-TB single-name credit spread buckets plut two extra bucket for credit indices. Therefore, risk weights based on ratings are discarded.

EAD frameworks

In March 2014 the BCBS released a document named 'The standardised approach for measuring counterparty credit risk exposures' (BCBS, 2014). This document presents a formulation for its *standardised approach* (SA-CCR) for measuring EAD for counterparty credit risk. The new approach replaces the current non-internal model, the *current exposure method* (CEM) and the *standardised method* (SM). A brief summary of the SA-CCR for measuring EAD is given in Appendix A.

2.3 Accounting CVA framework

2.3.1 CVA under IFRS

IFRS13

Similar to the regulatory framework, the accounting framework acknowledged the fact that major bank default and losses due to credit deterioration during the financial crisis highlighted the urgency to implement CCR adjustment to the valuation process of derivatives. On the first of January 2013 IFRS 13 Fair Value Measurement became effective. IFRS 13 requires that derivative contracts are valuated at fair value, which includes the counterparty credit risk into derivative valuations. IFRS 13 defines fair value as (IASB, 2011):

"The price that would be received to sell an asset or paid to transfer a liability in an orderly transaction between market participants at the measurement date."

The fair value is measured based on market participants' assumptions. Furthermore, IFRS 13 states explicitly that the fair value of a liability should reflect the effect of non-performance risk, including an entity's own credit risk. This results in considering the effects of credit risk when determining the fair value, by including DVA and CVA on derivatives. However, many entities cited a number of reasons for neglecting DVA in their derivative valuations, including: the counter-intuitive impact of recognising a gain or loss due to own credit deterioration, the difficulty to monetise from own credit gain, the increase of systematic risk due to hedging DVA, and anomalies in accounting standards (EY, 2014).

2.3.2 IFRS13 calculation approach

The accounting literature does not describe a specific method to estimate the effect of credit risk on the fair value of derivatives. Several credit adjustment valuation methods are available. The degree of sophistication in the credit adjustment valuation methods differ significantly dependent on the several factors, such as cost and availability of modeling, availability of data, derivative instruments, and more (EY, 2014).

EY (2014) states that the 'expected exposure approach' is the most advanced approach used and common practice within the financial sector to calculate credit adjustments. Therefore, we choose this approach to calculate the accounting CVA. The approach is considered to be the most theoretically pure approach, includes the bilateral nature of derivatives, and can be applied at transaction level. Unfortunately, the approach is costly to implement at large scale, involves highly complex modeling and advanced technical skills, and an excellent IT infrastructure (EY, 2014).

The approach simulates market variables to compute the price of the derivative over time, resulting in an exposure path. By simulating multiple exposure paths, the average results in the expected exposure path. For CVA only positive exposure paths (EE) are used, while for DVA the negative exposure paths (ENE) are used. Furthermore, default probabilities of the counterparty are inserted for CVA, and own default probabilities are inserted for DVA. The approach is defined as follows:

$$\text{CVA} = \text{LGD} \int_0^T D(t)EE(t)H(t)dt \tag{2.16}$$

$$\text{DVA} = \text{LGD} \int_0^T D(t)ENE(t)H(t)dt\,. \tag{2.17}$$

Eligible hedges

The accounting framework has a broader understanding of hedging CVA risk in comparison to the regulatory framework. Next to credit risk hedges, such as CDS, contigent CDS and index CDS, market risk hedges are allowed too. Since credit risk hedges only allow for capital relief seen from the regulatory framework, we focus on credit risk hedges only.

2.4 Hedging anomaly: regulatory vs accounting

In the previous sections we saw that hedging with eligible instruments lowers regulatory CVA capital and reduces accounting P&L fluctuations. Let us consider a derivative contract with one single counterparty. The goal is to hedge against CVA credit spread sensitivity due to changes in counterparty's credit spread. The hedges we choose are eligible hedges according to both frameworks, such as a single-name CDS. We would like the CDS to compensate for credit spread movements in CVA, such that the hedge creates delta neutrality. The delta neutral condition is defined as follows:

$$\Delta CVA = \Delta CDS \tag{2.18}$$

where Δ describes the first order derivative of CVA and CDS. The value of a CDS consists of a premium leg and a default leg. The default leg of the CDS should compensate for the CVA fluctuations. The cashflow of the default leg is given by $D(0,\tau)B\mathbf{1}_{\tau<T}$, where D is the discount factor, B the payment amount (or notional) at default time τ if $\tau < T$. The value of the default leg is given by (Berns, 2015):

$$PV_{defaultleg} = \text{LGD } B \int_0^T D(t)H(t)dt\,. \tag{2.19}$$

By comparing Equation (2.16) and Equation (2.19), the two equations coincide, if the notional B is set equal to EE^*. This results in the optimal hedge amount B from IFRS perspective:

$$B = EE^* \tag{2.20}$$

where EE^* is the average expected exposure of $EE(t)$ defined as:

$$EE^* = \frac{1}{T}\int_0^T EE(t)dt\,. \tag{2.21}$$

The CVA credit spread sensitivities can be written as:

$$\Delta_{CVA} = EE^*\Delta_{CDS} \tag{2.22}$$

where Δ_{CDS} is the credit spread sensitivity of the default leg of the CDS with notional amount $B = 1$.

The risk charge is given by Equation (2.11), where only K_{spread} can be hedged. Let us consider K_{spread} without index hedges and only direct single-name CDS hedge, which simplifies Equation (2.15) into:

$$K_{spread} = \sqrt{(\rho \cdot \sum_c (S_c - S_h))^2 + (1-\rho^2) \cdot \sum_c (S_c - S_h)^2}\,. \tag{2.23}$$

If we assume $S_c = S_h$, the K_{spread} term is cancelled out. In other words, the hedge is optimal from the regulatory perspective to compensate for credit spread movements. By setting $S_c = S_h$ we find that the optimal hedge amount B from the regulatory perspective is:

$$B = EAD\,. \tag{2.24}$$

The CVA credit spread sensitivities can be written as:

$$\Delta_{CVA} = EAD\Delta_{CDS} \tag{2.25}$$

where Δ_{CDS} is the credit spread sensitivity of the default leg of the CDS with notional amount $B = 1$.

By comparing Equation (2.20) and Equation (2.24) we see that the hedge notional for hedging on the accounting side is EE^*, while the notional amount to reduce the CVA risk charge is EAD. Normally, $EAD > EE^*$ holds due to the conservative regulatory assumptions. Equation (2.22) and Equation (2.25) implicate that the CVA sensitivities differ due to hedging at different notional amounts, and hence creating more P&L volatility if a firm is focusing on capital reduction.

We use a simple example to illustrate this concept. Let us consider one interest rate swap with an single counterparty with a maturity of 5 years. We show the realized P&L each quarter over the last two and a half years of the swap. We assume a flat exposure profile, i.e., EE^*, so that the CDS with notional $B = EE^*$ should cancel all P&L fluctuations. The hedge is set on two levels, namely EE^* and EAD. The left plot of Figure 2.2 shows the individual P&L effects of CVA, the EE^* hedge and the EAD hedge. The individual P&L of the EAD hedge is considerably larger then the P&L of the EE^* hedge. The right plot shows the residual P&L, i.e., CVA P&L minus the P&L of the hedges. No P&L volatility is observed at EE^* level, whilst there is substantially P&L volatility at EAD level.

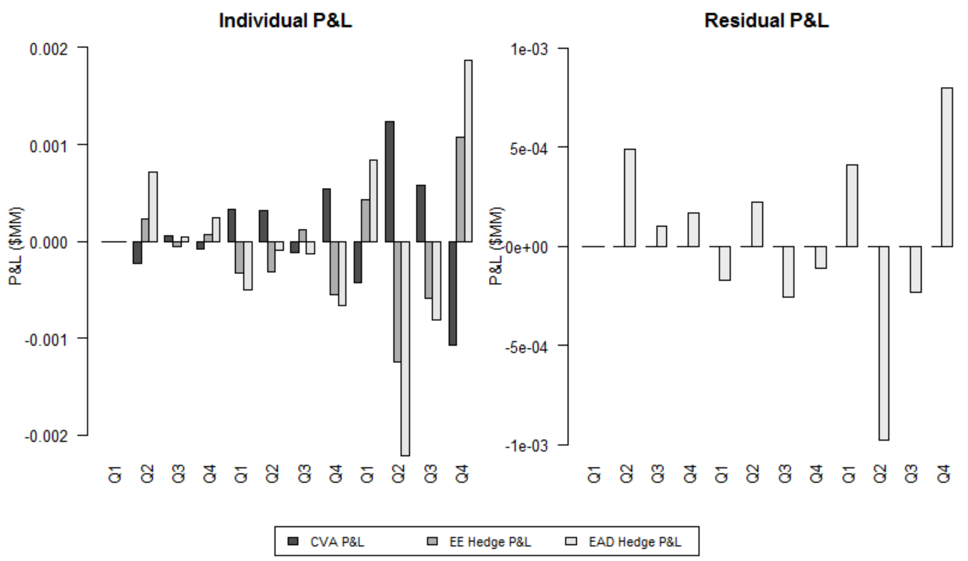

FIGURE 2.2: On the left the individual P&L of the CVA, the EE hedge and the EAD hedge. On the right the residual P&L due to the two hedges, computed by extracting the individual hedge P&L from the individual CVA P&L.

Chapter 3

Optimization methodology

CVA applies to bilateral OTC traded derivatives. Even after the financial crisis, the OTC market is still large in size (543 Trillion USD) (BIS, 2016). Several types of financial contracts are traded in the OTC market, such as interest rate contracts, equity-linked contracts, commodity contracts, foreign exchange contracts and more. As presented in Figure 3.1 interest rate contracts make up the largest part of the OTC market. Not only the performance of financial firms is affected by interest rate fluctuations but also the performance of non-financial firms. Both types of firms search to protect themselves against interest rate fluctuations by entering in interest rate linked contracts.

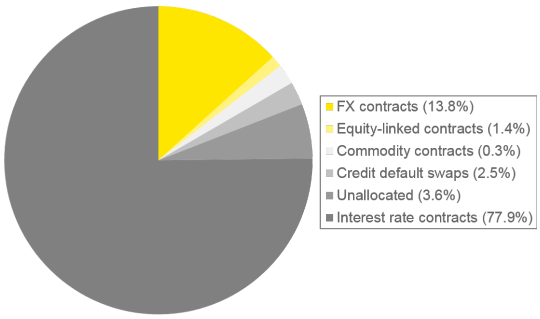

FIGURE 3.1: Notional outstanding of different OTC derivatives expressed in percentages of the OTC market in 2016 (BIS, 2016).

We focus on a specific interest rate derivative, namely an interest rate swap, due to OTC notional size and the impact on the OTC market. The methodology presented is linked to the interest rate swap. The methodology could be used for other derivative types, such as FX, equity and commodity contracts. However, the underlying models should be adjusted to the type of product.

3.1 Methodology

The presented methodology aims to define the optimal hedge amount of the hedge. We assume that a bank either wants to hedge P&L and/or reduce the risk charge, which implies that the hedge amount B is always set at EE level, EAD level or in between. Equation (3.1) shows that the notional B is adjusted by a scaling factor α on the interval of [0,1].

$$B = \alpha \; EE + (1-\alpha) \; EAD \,. \tag{3.1}$$

Following Berns (2015), we introduce a synthetic volatility (σ_{syn}), which consists of the additional P&L volatility ($\sigma_{cva,pnl}$) caused by changes of the hedge instruments and regulatory risk charge volatility ($\sigma_{cva,reg}$) [1]. Equation (3.2) shows that the additional P&L volatility and risk charge volatility depend on the hedge notional B.

$$\sigma^2_{syn}(B) = \omega \; \sigma^2_{cva,pnl}(B) + (1-\omega) \; \sigma^2_{cva,reg}(B) \,. \tag{3.2}$$

Due to the mismatch in sensitivities of CVA and hedges, there is no complete offsetting. Therefore, both volatilities move in opposite directions. A higher hedge notional decreases $\sigma_{cva,reg}$, while increasing $\sigma_{cva,hed}$. The goal is to minimize σ_{syn} by adjusting B by changing the scaling factor α. This leads to the optimal allocation between P&L volatility and the CVA risk charge following this optimization criteria (Berns, 2015). The ω is the weight factor on the interval [0,1] and represents the risk appetite of the bank between P&L volatility and the CVA risk charge. With ω close to 0, the bank is more focused on reducing the risk charge, while a ω close to 1, the bank is more focused on lowering additional P&L volatility. Section 3.2 provides a more in depth description of the risk appetite ω.

To define the optimal hedge ratio we follow the following steps:

Step 1. The set up of the interest rate model: We start by setting up the interest rate model. The Hull-White model is selected to model the evolution of the short rates of interest. The model is chosen because it can fit the initial term structure of interest rates and is used for risk-management purposes within the financial industry (Brigo, 2007). More complex multifactor extensions of single factor models are available. However, the Hull-White model satisfies to model interest rates sufficiently in order to reach the objective of this thesis.

The model parameters are calibrated to reflect the initial term-structure using implied market data. The Hull-White model is given by:

$$dr(t) = [\vartheta(t) - ar(t)] \, dt + \sigma dW(t) \tag{3.3}$$

where a, $\vartheta(t)$ and σ represent the mean reversion rate, long term short rate and volatility of the short rate, respectively. The diffusion term $W(t)$ denotes the Wiener process under the risk neutral measure $\mathbb{Q}$, which makes it an Ornstein-Uhlenbeck process.

Step 2. The modeling of short rates: The idea of the calibration is to set $\vartheta(t)$ to the initial term-structure using implied market data and calibrate a and σ, so that the model fits observed market prices. Based on

[1] Berns (2015) and Pykhtin (2012) show we can interpret the risk charge as a value at risk measure with a portfolio of positions subject to normally distributed CVA changes

real market data,[2] the calibrated Hull-White model is used to model short rates over the time of the interest rate derivative.

Step 3. Determine exposures over time: The simulated short rates path is used to calculate the zero-coupon bond prices. These zero-coupon bond prices are used as input to price the interest rate swap. The interest rate swap prices are calculated over time, which gives the price path of the swap. Based on the price of the swap at time t we can determine the (expected) exposure of the swap at time t using Equation (2.1) and Equation (2.2).

Step 4. Determine accounting CVA and CVA risk charge: The different exposure paths of the interest rate swap lead to an exposure distribution. From this distribution we extract different properties, which are implemented in the accounting framework and the regulatory framework. The accounting framework yields the CVA using Equation (2.16), while the regulatory framework provides the CVA risk charge using Equation (2.11).

Step 5. Define the optimal hedge ratio: The notional amounts of eligible hedge instruments for the accounting CVA and the regulatory CVA charge show the interplay between the two frameworks using Equation (3.1) and Equation (3.2). For every risk appetite ω we set out the optimal hedge amount to see the interplay between the trade-off of regulatory capital and P&L volatility.

3.1.1 Monte Carlo method

Monte Carlo simulation is based on the risk-neutral approach. By generating a large number of random price paths via simulation a distribution function of possible outcomes is generated. From this distribution function different properties (e.g. expected price path, potential future exposure) can be determined. In our case we repeat Step 1 to 3 N times to find the expected values using the Monte Carlo method and the probability theory of the law of large numbers.[3] More specifically, different zero-coupon bond prices are generated from the short rate paths, resulting in multiple paths of interest rate swap prices and a different exposure path.

3.2 The Bank's risk appetite ω

The banking business has become a broad landscape due to wide variety of banking activities. Each bank has its own specific capital structure, i.e., the financing of assets by the combination of equity and debt. The most important factors driving the capital structure are size, activity diversity and market risk (ECB, 2014; ECB, 2016). Large banks usually have a high degree of leverage, because it could lead to greater profitability and return on equity. These banks are usually better diversified geographically or across product lines and can liquidate positions more easily, while smaller banks do not have advantage and search for financial stability (ECB, 2016). Within the

[2] extracted from a Bloomberg terminal

[3] following (Sen and Singer, 1993) on the theory of large numbers

CVA trade-off, each bank has to decide individually between the amount of capital reduction it would like to achieve and the lowering of additional P&L fluctuations. We define this as the risk appetite of the bank, which is the choice between P&L volatility and the CVA risk charge. For example, a smaller bank is in search of financial stability and constant earnings (i.e., less P&L volatility) would focus on lowering P&L volatility and concentrate less on reducing the CVA risk charge. While another bank could be more focused on capital reduction to increase profitability and return on equity and would take P&L volatility for granted. A bank's risk appetite could also be somewhere in between, where the focus lays on reducing the CVA risk charge and lowering P&L volatility.

Chapter 4

Model foundation

In this chapter we lay the foundation of the model to define the optimal hedge amount. We start by introducing essential definitions and notations to understand interest rate characteristics. We introduce interest rate derivatives, credit default swaps and the Hull-White interest rate model. Then, we elaborate on the valuation models of the interest rate derivatives using the Black model and the Hull-White model. Lastly, we describe the steps to calibrate the Hull-White model to current market data.

4.1 Definitions and notations

The definitions and notations are adopted from Brigo (2007) and Brigo (2012).

Definition 4.1. Bank Account (Money-market account). *We define $M(t)$ to be the value of a bank account at time $t \geq 0$. We assume $M(0) = 1$ and that the bank account evolves according to the following differential equation:*

$$M(t) = M(t)r(t)dt \tag{4.1}$$

where r_t is a positive function of time. As a consequence,

$$M(t) = M_0 \cdot \exp \int_0^t r(s)ds\,. \tag{4.2}$$

Definition 4.2. Stochastic discount factor. *The (stochastic) discount factor $D(t,T)$ between two time instants t and T is the amount at time t that is "equivalent" to one unit of currency payable at time T, and is give by*

$$D(t,T) = \frac{M(t)}{M(T)} = \exp\Big(-\int_t^T r(s)ds\Big)\,. \tag{4.3}$$

Definition 4.3. Zero-coupon bond. *A T-maturity zero-coupon bond (pure discount bond) is a contract that guarantees its holder the payment of one unit of currency at time T, with no intermediate payments. The contract value at time $t < T$ is denoted by $P(t,T)$. Clearly, $P(T,T) = 1$ for all T.*

Definition 4.4. Time to maturity *The time to maturity $T - t$ is the amount of time (in years) from the present time t to the maturity $T > t$.*

Definition 4.5. Year fraction, Day-count convention *We denote by $\tau(t,T)$ the chosen time measure between t and T, which is usually referred t oas year fraction between the dates t and T. When t and T are less than one-day distant (typically when dealing with limit quantities involving time to maturities tending to zero), $\tau(t,T)$ is to be interpreted as the time difference $T-t$ (in years). The particular choice that is made to measure the time between two dates reflects what is knows as the day count convention.*

Actual/360. A year is assumed to be 360 days long and the year fraction between two dates is the actual number of days between them divided by 360. If we denote by $D_2 - D_1$ the actual number of days between the two dates, $D_1 = (d_1, m_1, y_1)$ included and $D_2 = (d_2, m_2, y_2)$ excluded, we have that the year fraction in this case is

$$\frac{D_2 - D_1}{360}\,. \tag{4.4}$$

Definition 4.6. Continuously-compounded spot interest rate *The continuously-compounded spot interest rate prevailing at time t for the maturity T is denoted by $R(t,T)$ and is that constant rate at which an investment of $P(t,T)$ units of currency at time t accrues continuously to yield a unit amount of currency at maturity T. In formulas:*

$$R(t,T) := -\frac{ln(P(t,T))}{\tau(t,T)}\,. \tag{4.5}$$

The continuously-compounded interest rate is therefore a constant rate that is consistent with the zero-coupon-bond prices in that

$$\exp(R(t,T)\tau(t,T))P(t,T) = 1 \tag{4.6}$$

from which we can express the bond price in terms of the continuously-compounded rate R:

$$P(t,T) = \exp\{-R(t,T)\tau(t,T)\}\,. \tag{4.7}$$

The simple-compounded spot interest rate is defined as:

$$L(t,T) := \frac{1-P(t,T)}{\tau(t,T)P(t,T)}\,. \tag{4.8}$$

The annually-compounded spot interest rate is defined as:

$$Y(t,T) := \frac{1}{(P(t,T))^{1/\tau(t,T)}} - 1\,. \tag{4.9}$$

Definition 4.7. Zero-coupon curve *The zero-coupon curve (sometimes also referred to as "yield curve") at time t is the graph of the function*

$$T \mapsto \begin{cases} L(t,T) & t < T \le t+1 \quad \textit{(years)} \\ Y(t,T) & T > t+1 \quad \textit{(years)} \end{cases} \tag{4.10}$$

such a zero-coupon curve is also called the *term structure of interest rates* at time t.

Definition 4.8. Zero-bond curve *The zero-bond curve at time t is the graph of the function*

$$T \mapsto P(t,T) \quad T > t \tag{4.11}$$

which, because of the positivity of interest rates, is a T-decreasing function starting from $P(t,t) = 1$. Such a curve is also referred to as the term structure of discount factors.

Definition 4.9. Simply-compounded forward interest rate *The simply-compounded forward interest rate prevailing at time t for the expiry $T > t$ and maturity $S > T$ is denoted by $F(t;T,S)$ and is defined by*

$$F(t;T,S) := \frac{1}{\tau(T,S)}\Big(\frac{P(t,T)}{P(t,S)} - 1\Big). \tag{4.12}$$

Definition 4.10. Instantaneous forward interest rate *The instantaneous forward interest rate prevailing at time t for the maturity $T > t$ is denoted by $f(t,T)$ and is defined as*

$$f(t,T) := \lim_{S \downarrow T^+} F(t;T,S) = -\frac{\partial \log P^M(t,T)}{\partial T} \tag{4.13}$$

so that we also have

$$P(t,T) = \exp\Big(-\int_t^T f(t,u)du\Big) \tag{4.14}$$

(Brigo, 2007; Brigo, 2012).

4.2 Interest rate swap, swaptions and CDSs

4.2.1 Interest rate swap

An *interest rate swap* (IRS) is an OTC agreement between two counterparties where a fixed rate of interest payments is exchanged for a floating rate of interest payments. A receiver (payer) IRS receives (pays) the fixed rate and pays (receives) the floating rate.

Let $\{T_0, \ldots, T_n\}$ be a set of future times of which $T_0, \ldots, T_{n-1}$ correspond to the dates on which the floating rate is determined, or reset dates, and $T_1, \ldots, T_n$ correspond to the dates on which payments are exchanges, or payments dates. Let $\tau_i = T_i - T_{i-1}$ be the year fractions between two consectutive dates for $i = 1, \ldots, n$ and we assume constant year fractions, so $\tau_i = \tau$. Let K be the fixed rate, or swap rate . We define the value of an IRS as follows (Brigo, 2007):

$$
\begin{aligned}
IRS^{payer}(t,T,N,K) &= \sum_{i=1}^{n} P(t,T_i)N_\tau(F(t,T_{i-1},T_i)-K) \\
&= N\sum_{i=1}^{n} P(t,T_i)\Big(\frac{P(t,T_{i-1})}{P(t,T_i)}-1\Big) - N\tau\sum_{i=1}^{n} P(t,T_i)K \\
&= N(P(t,T_0)-P(t,T_n)) - N\tau\sum_{i=1}^{n} P(t,T_i)K \\
&= B^{float}(t) - B^{fixed}(t,K)
\end{aligned}
\tag{4.15}
$$

$$
IRS^{receiver}(t,T,N,K) = B^{fixed}(t,K) - B^{float}(t) \tag{4.16}
$$

where $B^{float}(t)$ and $B^{fixed}(t)$ represent the value of the floating-rate bond with notional N and the value of the fixed-rate bond with notional N, respectively. We see that a payer (receiver) IRS can be interpreted as a long (short) position in a floating-rate bond and a short (long) position in a fixed-rate bond. Usually the swap rate is chosen such that the sum of the values of the fixed-rate bond and floating-rate bond (thus the IRS) is zero at initiation. The swap rate at time t for period T_0 to T_n is given by:

$$
S_{0,n}(t) = \frac{P(t,T_0)-P(t,T_n)}{\sum_{i=1}^{n}\tau P(t,T_i)}\,. \tag{4.17}
$$

4.2.2 Swaption

A swaption is an option on an IRS. The holder of an European swaption has the right, not the obligation, to enter an IRS at a given future time and a predetermined strike K. The swaption maturity T_m coincides with the first reset date of the underlying IRS. The length of the IRS ($T_n - T_m$) is referred as the tenor of the swaption. Market practice is to valuate swaptions using Black's formula, given by Brigo (2007):

$$
Swaption(t_0,T,\tau,N,K) = N\sum_{i=m+1}^{n} \tau_i P(t_0,T_i)[\omega S_{m,n}(t_0)\Phi(\omega d_1) - \omega K\Phi(\omega d_2)] \tag{4.18}
$$

with $\omega \in \{-1,1\}$, where $\omega = 1$ results in a payer swaption and $\omega = -1$ results in a receiver swaption and

$$
d_1 = \frac{log(\frac{S_{m,n}(t_0)}{K}) + \frac{\sigma_{m,n}^2}{2}T_m}{\sigma_{m,n}\sqrt{T_m}} = d_2 + \sigma_{m,n}^2\sqrt{T_m}\,. \tag{4.19}
$$

4.2.3 Credit default swap

A *credit default swap* (CDS) provides protection against default and can be thought of as an insurance contract on default. Here party A (the protection buyer) agrees with party B (the protection seller) to protect against the default at time τ of party C (the reference entity). Party A pays premiums at fixed intervals until either maturity T, or if party C defaults at time $\tau \leq T$.

The premiums at fixed intervals is referred to the premium leg of the CDS. If $\tau > T$, party B pays nothing to party A. If $\tau \leq T$, party B pays a certain cash amount to party A. The cash amount, or notional, is typically set to the LGD of reference party C. The cash amount payment is referred to the default leg of the CDS.

Let R be the CDS spread at a set of times $\{T_{a+1},,T_b\}$, let $\alpha_i = T_i - T_{i-1}$, let $T_0 = 0$ and R is fixed in advance at time 0. Premium payments continue up to default time τ if this occurs before maturity T_b or until maturity T_b if no default occurs. The price at time t of the CDS is denoted by $CDS_{a,b}(t, R, \text{LGD})$. The CDS price is computed via risk-neutral valuation. The CDS pricing formula, given by Brigo (2012):

$$
\begin{aligned}
CDS_{a,b}(t, R, \text{LGD}) &= -R\,\mathbb{E}_t[P(t,\tau)(\tau - T_{\beta(\tau)-1})\mathbf{1}_{\{T_a<\tau\leq T_b\}}] \\
&- \sum_{i=a+1}^{b} P(t,T_i)\alpha_i R\,\mathbb{E}_t[\mathbf{1}_{\{\tau\geq T_i\}}] + \text{LGD}\,\mathbb{E}_t[\mathbf{1}_{\{T_a<\tau\leq T_b\}}P(t,\tau)] \\
&= -R\Big[P(t,T_i)\alpha_i\mathbb{Q}_{\tau\geq T_i} + \int_{T_{i-1}}^{T_i}(u - T_{i-1})P(t,u)d\mathbb{Q}(\tau\leq u)\Big] \\
&+ \text{LGD}\int_{T_a}^{T_b} P(t,u)d\mathbb{Q}(\tau\leq u) \\
&= PV^{defaultleg}(t) - PV^{premiumleg}(t)\,.
\end{aligned}
\tag{4.20}
$$

Default probabilities can be estimated using credit spreads from CDSs. The hazard rate λ between time zero and time T is given by:

$$
\lambda = \frac{R(T)}{LGD} \tag{4.21}
$$

where $R(T)$ is the credit spread for a maturity T. If CDS spreads of different maturities are known, it is possible to bootstrap the term structures of hazard rates (Hull, 2012) [1].

4.3 Hull-White One Factor

4.3.1 Short rate process

The dynamics of the short rate of a mean-reverting stochastic process is described by the Vašiček model , which is expressed in the following stochastic differential equation (Vašiček, 1977):

$$
dr(t) = [\theta - ar(t)]\,dt + \sigma dW(t) \tag{4.22}
$$

where a, θ and σ represent the mean reversion rate, long term short rate and volatility of the short rate, respectively. The diffusion term W_t denotes the Wiener process under the risk neutral measure $\mathbb{Q}$, which makes it an Ornstein-Uhlenbeck process (Hull, 2011). The model captures mean reversion, which is an essential characteristic of interest rates. Unfortunately, the

[1] We follow the methodology behind the credit curve bootstrapping of Nezet (2015).

model cannot satisfactorily reproduce the initial yield curve $T \to P(0,T)$, due to lack of flexibility of the parameters of the models. Hull and White (1990) introduced the time-varying parameter of θ (denoted as $\vartheta(t)$) within the Vasicek model, named the Hull-White One Factor model. The function $\vartheta(t)$ can be defined such that it fits the current term structure of interest rate. The Hull-White One Factor model is given by the dynamics:

$$dr(t) = [\vartheta(t) - ar(t)]\,dt + \sigma dW(t)\,. \tag{4.23}$$

According to Brigo (2007), more complex multifactor extensions of single factor models are available. However, the Hull-White model satisfies to model interest rates sufficiently to reach the objective of this thesis. Furthermore, the Hull-White model is a commonly used approach within the banking industry (Delsing, 2015).

4.3.2 Pricing of zero-coupon bond and option

We focus on the Hull-White One Factor model given by the dynamics:

$$dr(t) = [\vartheta(t) - ar(t)]dt + \sigma dW(t) \tag{4.24}$$

where a and σ are positive constants and $\vartheta(t)$ is chosen to fit the term structure of interest rates observed in the market. Let us assume that the term structure of discount factors observed in the market is a smooth function $T \to P^M(0,T)$. We denote by $f^M(0,T)$ the instantaneous forward rates by the market term structure at time 0 for maturity T as associated with the bond prices, i.e. (Brigo, 2007),

$$f^M(0,T) = -\frac{\partial \log P^M(0,T)}{\partial T}\,. \tag{4.25}$$

The function $\vartheta(t)$ is defined as (Brigo, 2007),

$$\vartheta(t) = \frac{\partial f^M(0,t)}{\partial T} + af^M(0,t) + \frac{\sigma^2}{2a}(1-e^{-2at}) \tag{4.26}$$

where $\frac{\partial f^M}{\partial T}$ denotes the partial derivative of f^M, which we define as,

$$\lim_{t\to 0} \frac{\partial f^M(0,T)}{\partial T} = \frac{f^M(0,T+\Delta t) - (f^M(0,T-\Delta t)}{2\Delta t}\,. \tag{4.27}$$

The price of a pure discount bond at time t at time T is given by (Brigo, 2007)

$$P(t,T) = A(t,T)e^{-B(t,T)r(t)} \tag{4.28}$$

where

$$A(t,T) = \frac{P^M(0,T)}{P^M(0,t)} \exp\left[B(t,T)f^M(0,t) - \frac{\sigma^2}{4a}(1-e^{-2at})B(t,T)^2\right] \tag{4.29}$$

$$B(t,T) = \frac{1}{a}\left[1 - e^{-a(T-t)}\right]. \tag{4.30}$$

The price of an European call option $ZBC(t,T,S,X)$ at time t with strike X, maturity T and written on a pure discount bond maturing at time S is given by (Brigo, 2007)

$$ZBC(t,T,S,X) = P(t,S)\Phi(h) - XP(t,T)\Phi(h - \sigma_p) \tag{4.31}$$

where

$$\sigma_p = \sigma\sqrt{\frac{1 - e^{-2a(T-t)}}{2a}}B(T,S) \tag{4.32}$$

$$h = \frac{1}{\sigma_p}\log(\frac{P(t,S)}{P(t,T)X} + \frac{\sigma_p}{2}\,. \tag{4.33}$$

The price of an European put option $ZBP(t,T,S,X)$ is given by (Brigo, 2007)

$$ZBP(t,T,S,X) = XP(t,T)\Phi(-h + \sigma_p) - P(t,S)\Phi(-h)\,. \tag{4.34}$$

4.3.3 Calibration

The pricing of bonds and options within the Hull-White model has been discussed. The Hull-White model needs to be calibrated to real-market data. The function $\vartheta(t)$ is chosen to make the model consistent to the initial term structure. The goal of the calibration is to estimate the remaining set of parameters P.

$$P = \{a, \sigma\}\,. \tag{4.35}$$

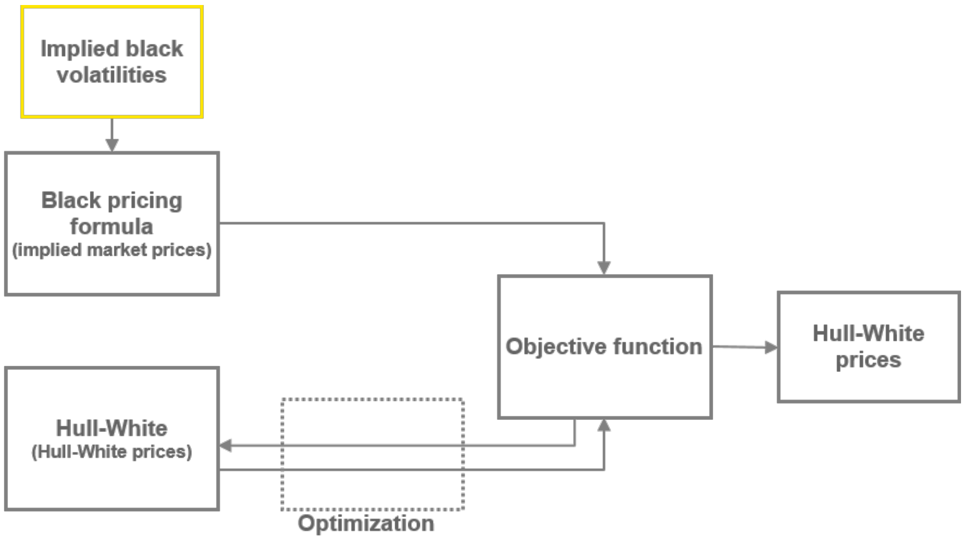

FIGURE 4.1: The Hull-White optimization process to fit observed market prices.

Since we are working under the risk-neutral measure we would like to set the model parameters such that the model is consistent with current market prices. We use the inverse problem approach for the calibration, where we calculate the causal factors from a set of observations. The goal is to find model parameters to fit observed market prices. Since our aim is to value an interest rate swap, we calibrate the model to swaption prices. The problem is ill-posed, because it is possible that multiple parameter sets may be consistent with the market prices. The calibration is an optimization problem, where we aim to minimize the difference between the swaption market prices and swaption model prices. Figure 4.1 illustrates the calibration procedure. We optimize using the least squares approach.[2] Assuming we have M different observed swaption prices, the formulation of the optimization problem is given by

$$\min_P \sum_{i=1}^{M} \left(Swaption_i^{market} - Swaption_i^{model}(P) \right)^2 \tag{4.36}$$

where $Swaption_i^{market}$ and $Swaption_i^{model}$ represent the i^{th} swaption prices for the market and model, respectively (Delsing, 2015).

The swaption market quotes implied Black volatilities and these implied volatilities of swaptions are used as input in the Black pricing formula, which in return give the market prices of the swaptions.

According to Delsing (2015), Equation (4.36) the optimization problem is a non-convex function without a particular structure. In this case, most gradient-based optimization methods cannot guarantee a global minimum and only a local one. However, by picking initial parameters close enough to the global minimum will converge towards the global minimum. Therefore, we apply the *Genetic Algorithms* (GA) optimization method to find good initial parameters (Delsing, 2015). Other optimization methods were considered, such as *Branch and Bound* (BB), and *Stimulated Annealing* (SA), however Delsing (2015) states that GA is faster than BB and seen as a generalization of SA. The Genetic Algorithms are used to find a good initial parameter set, based on the implementation, see Scrucca (2016), and is included through the GA package for R.

[2] Obviously many other optimization functions are available.

Chapter 5

Implementation case

In this chapter we use a stylized derivative contract to show the implementation of the methodology presented in Chapter 3. We start off by setting up and calibrating the Hull-White model. Next, the short rates are modeled using the calibrated Hull-White model. Then, the exposures are computed and implemented within both frameworks to find the P&L results and the CVA risk charges. Finally, the optimal hedge amounts are predicted for each risk appetite.

The derivative contract is a fix-float IRS, with the details displayed in Table 5.1. The IRS is valued at each fiscal quarter, where 1 January (Q1), 1 April (Q2), 1 July (Q3) and 1 October (Q4) are defined as the start of the four quarters. For each quarter the P&L and risk charge are determined. The results are split up in two time frames: *Time frame I* (01 Jan 2014 - 01 Oct 2016) and *Time frame II* (01 Jan 2017 - 01 Jan 2019). Time frame I considers results from the past. At each fiscal quarter, the data at that time is used to compute results by repeating methodology steps 1 to 4. Time frame II considers simulation results in the future, which implies the necessity of predicting variables in the future. We assume that the calibrated parameter set P on 01 Oct 2016 has the predictive power to estimate variables in future points in time. Furthermore, future CDS spreads are necessary to generate results in the future. We generate these CDS spreads using a mean-reverting model based on data from the past. This procedure is described in Appendix B.[1]

TABLE 5.1: Details of the interest rate swap contract used for the implementation case.

Interest rate swap contract		
	Fixed leg	*Floating leg*
Effective date	01-01-14	01-01-14
Maturity date	01-01-19	01-01-19
Currency	USD	USD
Rate	1.507%	6M USD LIBOR
Notional	1MM	1MM
Payment frequency	Semiannual	Semiannual
Day count	ACT/360	ACT/360
Counterparty	Deutsche Bank A.G.	

[1] To predict real-case scenarios, it is necessary to include a PD model to estimate CDS spreads.

5.1 Implementation of steps

We follow the steps of the methodology presented in Chapter 3.

Step 1. The set up of the interest rate model

We calibrate the Hull-White model to the relevant underlying rate, the 6 month USD LIBOR. As explained in Chapter 4, to calibrate the Hull-White model to a specific point in time, we need the data of the term structure of the underlying rate and swaption volatility surface. We use only ATM swaptions volatilities, where the swap rate is equal to the strike of the swaption, i.e., $K = S_{m,n}(0)$, where K represents the strike of the swaptiopn and $S_{m,n}(0)$ represents the swap rate at time 0. The underlying term structure and the swaption volatilities are presented in Appendix C.

The calibration procedure of the Hull-White model to the swaption volatility surface at time zero delivers the outcome of the parameter set $P = \{a, \sigma\}$, where $a = 0.04518101$ and $\sigma = 0.011371370$. The outcomes of the parameter set P of other points in time are presented in Appendix D.

Step 2. The modeling of short rates

With the calibrated Hull-White model we can construct the model implied term structure. Figure 5.1 shows that the model implied term structure follows the market term structure well. In Appendix E, we show by using a Chi-square goodness of fit test there is no significant difference between the two term structures.

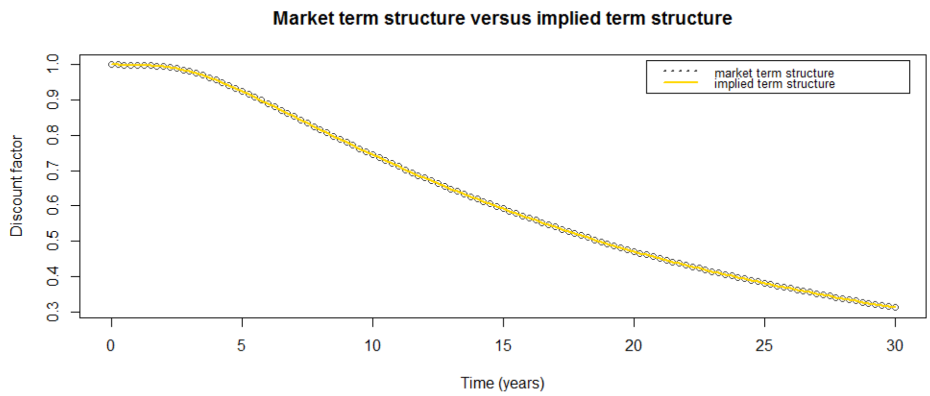

FIGURE 5.1: Comparison of the observed market term structure and the modeled implied term structure on 01-01-2014.

The Hull-White model simulates interest rate paths of the 6M USD LIBOR over the time of maturity of the IRS. One simulated interest path is shown in figure 5.2. Multiple paths are simulated with the use of the Monte Carlo method leading to different interest rate paths due to randomness under the Ornstein-Uhlenbeck process. Figure 5.3 presents 100 simulated interest rate paths [2]. Over time the short rate reverts to the long-term short rate $\vartheta(t)$ following from (4.26). The paths show an upward trend since the long-term short rate lies above the initial starting point. The mean of the simulated 6M USD LIBOR paths after 5 years is 0.0432.

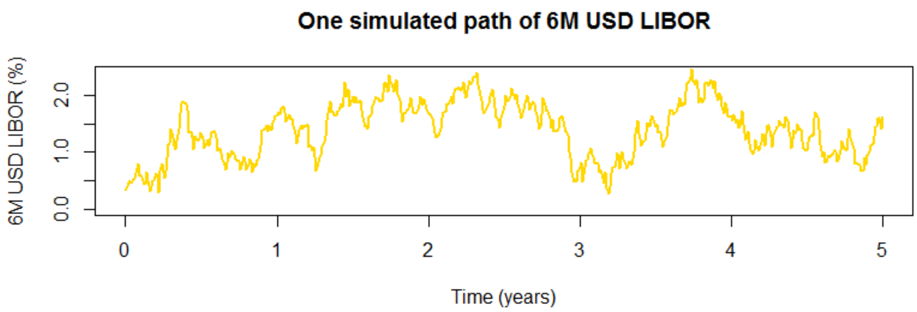

FIGURE 5.2: One simulated path of 6M USD LIBOR on 01-01-14 with the use of the calibrated Hull-White model.

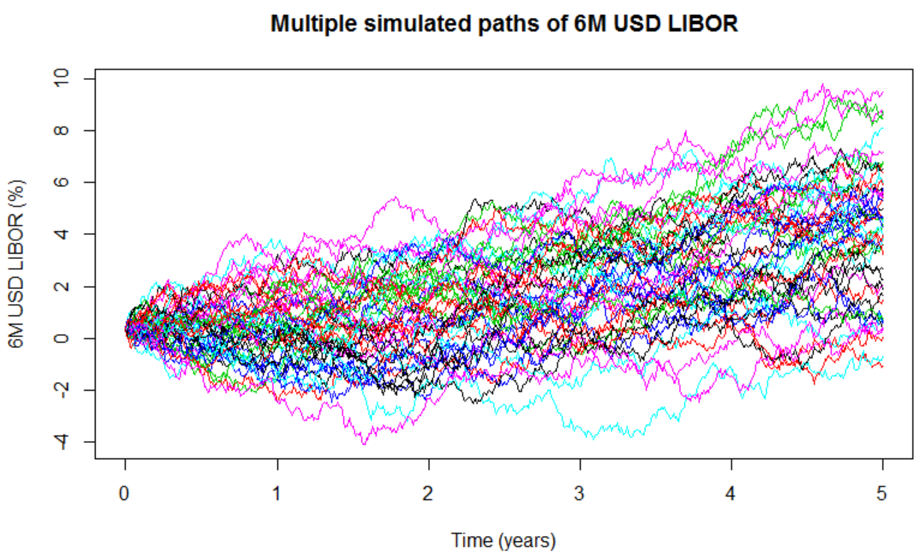

FIGURE 5.3: Multiple simulated paths of 6M USD LIBOR on 01-01-14 with the use of the calibrated Hull-White model. It shows an upward trend because the long-term short rate lies above the initial starting point.

[2]For the sake of example, N is set to 100. For computations N is set to 10000.

Step 3. Determine exposures over time

The simulated interest rate paths are used to compute the value of the IRS over time for every individual path using Equation (4.15) and Equation (4.16). When these exposures are combined we get an exposure distribution at a future point in time. Figure 5.4 shows several characteristics, which are extracted from this exposure distribution.

The exposure profiles of payer and receiver swaps differ due to the shape of the yield curve and its volatility. An upward sloping yield curve creates higher (positive) exposure on a payer swap compared to the (positive) exposure on a receiver swap as a results of that the value of early fixed payments is high compared to floating payments. Contrarily, for a downward sloping yield curve it is vice versa. In our case the yield curve is upward sloping, since the payer swap shows higher exposures than the receiver swap.

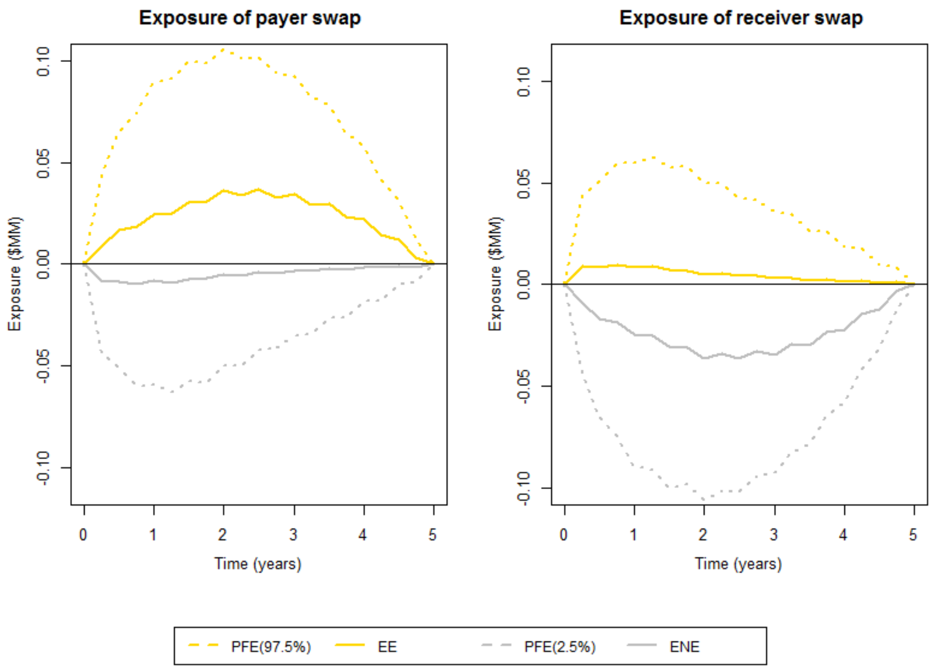

FIGURE 5.4: Positive and negative exposure and PFE profiles of receiver and payer swap. The payer swap shows higher positive exposures than the receiver swap due to an upward sloping yield curve.

As Figure 5.5 shows the hazard rate curve is upward sloping. This means that the market expects default on the long-term instead on the short-term. A downward sloping curve is usually seen for a firm in distress, where short-term default is expected. The firm's credit curve changes over time by experiencing less or more financial distress. As mentioned before, the default probabilities of time frame I are extracted from CDS spreads observed in the market, while the default probabilities of time frame II are simulated following a mean-reverting model.

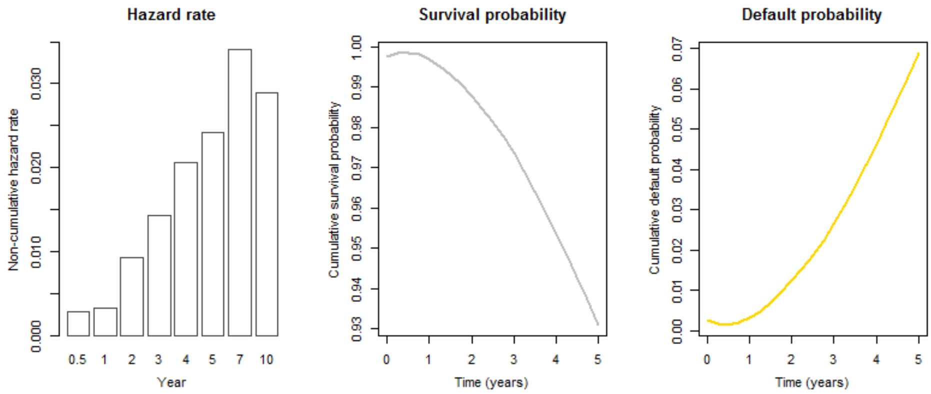

FIGURE 5.5: Hazard rate, survival probability and default probability of counterparty on 01-01-14.

Step 4. Determine Accounting CVA and Basel Capital Charge

The combination of steps 1-3 leads to the computation of the accounting CVA and the CVA risk charge. Figure 5.6 shows CVA over time. The "CVA realized" considers time frame I, which implies actual realized CVA observed from this point in time. The peak is the highest at the half of time to maturity, due to the peak in exposure profile and high CDS spreads. The "CVA projected" looks at time frame II, where we see that CVA is leveling off. The exposure reduces since we approach time to maturity, which leads to lower impact of CDS spread sensitivities.

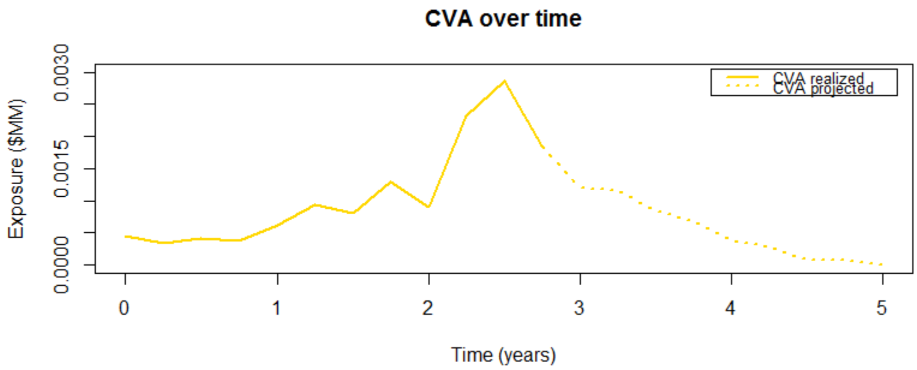

FIGURE 5.6: The modeled CVA over time of the contract. The solid line shows the realized CVA, while the dotted line show the project CVA in future points in time.

As discussed in Chapter 2, $EAD > EE^*$ holds due to the conservative regulatory assumptions. This implicates that the CVA sensitivities differ due to hedging at different notional amounts. We use the actual expected exposure profile $EE(t)$ and not the average expected exposure profile EE^*. This means the hedges are not set to EE^* but to the maximum of EE, identical as the CDS hedge I in Figure 2.1. Figure 5.7 shows how EAD and EE

change over time. The EAD is larger than EE at every point in time, which cause additional P&L and differences in the risk charge amount.

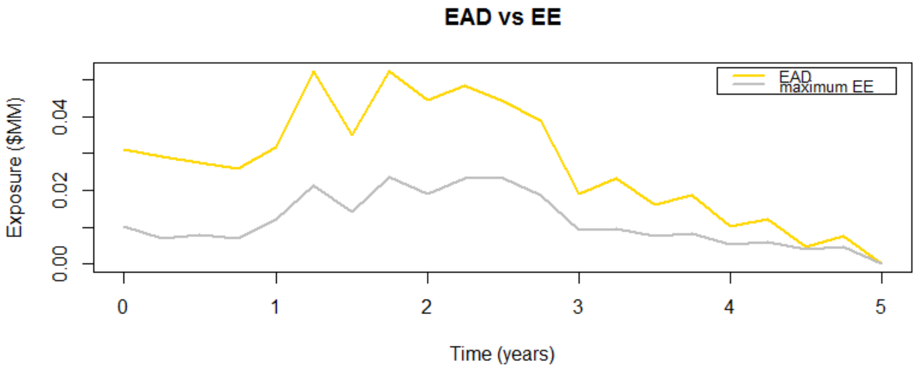

FIGURE 5.7: The modeled EAD and EE over time of the contract. The EAD is at any point in time larger than EE due to conservative regulatory assumptions.

As shown in Figure 5.6, we see that CVA fluctuates over time. The changes in the value of CVA have an effect on the P&L. Figure 5.8 displays the CVA fluctuations on P&L considering time frame I. Furthermore, three type of hedges are used to compensate for the CVA movements. The hedges are set at α's of 0 (EE level), 0.5 ($\frac{1}{2}[EE + EAD]$ level) and 1 (EAD level) following Equation (3.1). Hedging at EE level does not always lower P&L compared to not hedging. The hedge does not fully mitigate credit spread sensitivity, due to the a misalignment in exposure profile and CDS profile. Also the CVA risk charge is given in Figure 5.8. We see that the a higher hedge notional leads to more reduction on the CVA risk charge. When we hedge at EAD level, we do not get full capital relief. This is due to the K_{EE} term in Equation (2.12), because the exposure component cannot be hedged. Similarly, Figure 5.9 displays the P&L fluctuations and risk charge considering time frame II. We see that P&L and the risk charge both reduce closer to the maturity of the contract.

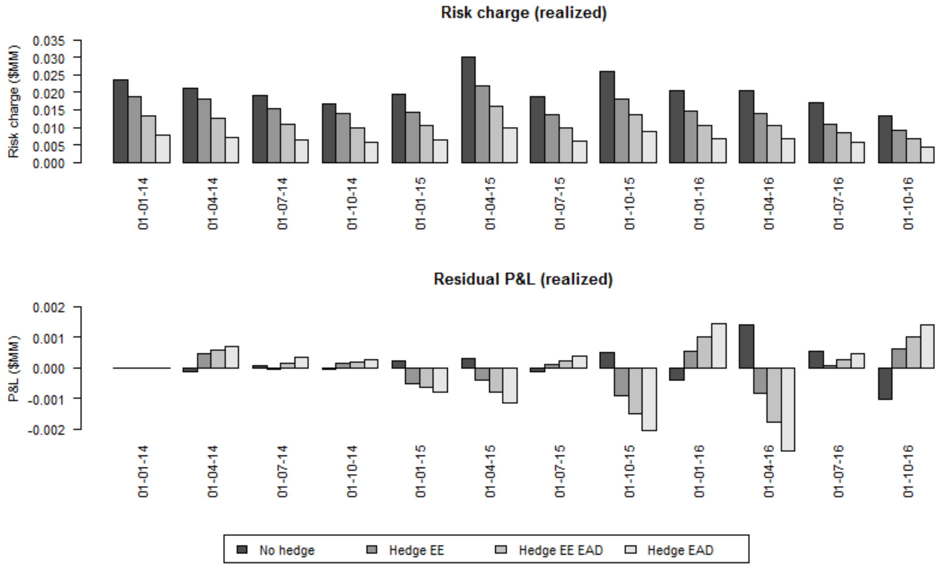

FIGURE 5.8: The realized risk charge and P&L over time frame I.

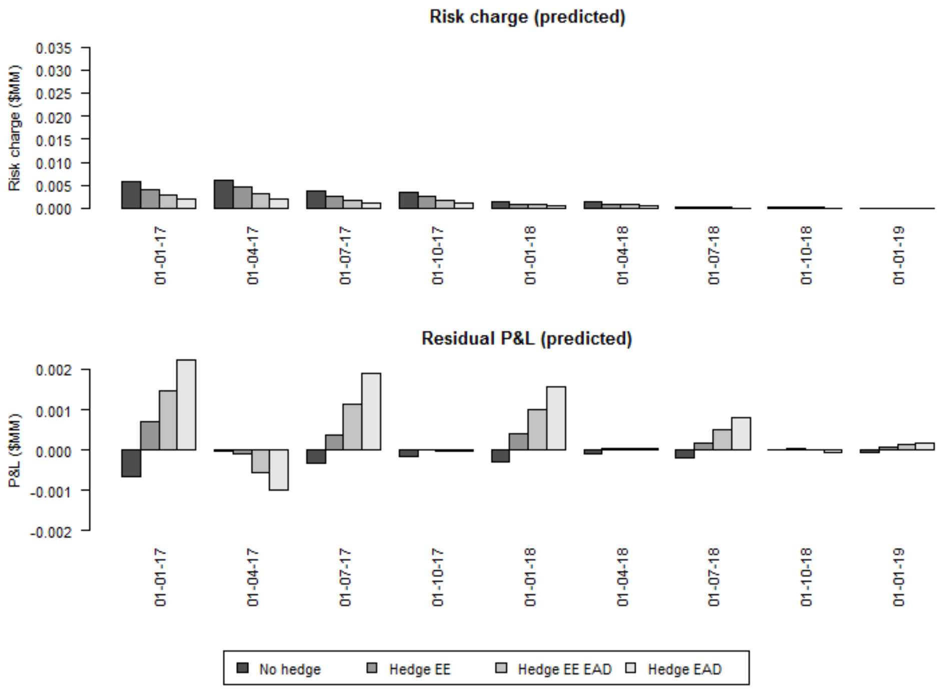

FIGURE 5.9: The predicted risk charge and P&L over time frame II.

Step 5. Define the optimal hedge ratio

To define the optimal hedge ratio, we use Equation (3.1) and Equation (3.2)[3]. By minimizing σ^2_{syn} of Equation (3.2), we can find the optimal hedge notional B by adjusting α in Equation (3.1). However, Equation (3.2) is dependent on ω, which defines the risk appetite of the bank between P&L volatility and the CVA risk charge. The risk appetite ω is different for each individual bank as explained in Section 3.2. Therefore we do not set a specific risk appetite, but find the optimal α for each risk appetite ω. As mentioned in Section 3.1, α lays on the interval of [0,1], where $\alpha = 0$ the hedge notional is set on EAD, while $\alpha = 1$ the hedge notional is set on EE. Likewise, ω lays on the interval of [0,1], where $\omega = 0$ the focus lies on reducing the risk charge, while $\omega = 1$ the focus lies on lowering additional P&L volatility.

The power of the model is to find the optimal hedge amounts by simulating future results. Here, the forecasted P&L results and CVA risk charges in future points in time are predicted using our simulation model. Figure 5.10 shows the optimal scaling factor α for each ω. Here the optimal hedge amounts are chosen such that maximal CVA risk charge reduction is achieved and additional P&L volatility is minimized. For a ω lower than 0.28, we see that the α should be set to 0. For a ω between the range of 0.28-0.71, we find that the α grows linearly. For a ω larger than 0.71, we see that the α should be set to 1. A bank more focused on reducing capital ($\omega = 0$) should set the hedge notional closer to EAD level. A bank more focused on reducing additional P&L volatility ($\omega = 1$) should set the hedge notional closer to EE level.

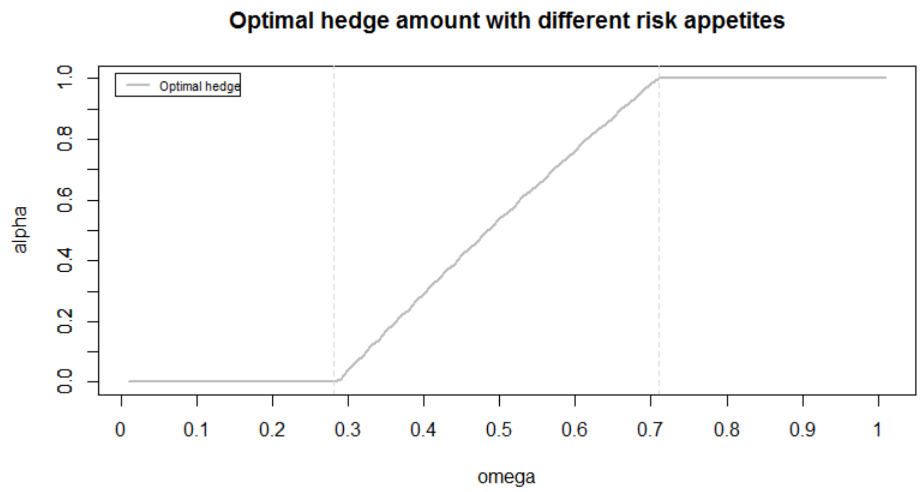

FIGURE 5.10: Scaling factor α set out against risk appetite ω, which represents the optimal hedge amount for each type of risk appetite.

[3]To recall Equation (3.1) "$B = \alpha EE + (1-\alpha)\ EAD$" and Equation (3.2) "$\sigma^2_{syn}(B) = \omega\ \sigma^2_{cva,pnl}(B) + (1-\omega)\ \sigma^2_{cva,reg}(B)$"

Optimal hedge amounts sensitivity to model parameters

The optimal hedge amounts are found by following the steps described in the methodology. Here, the model is calibrated to the market data to predict future results. The outcome presented in Figure 5.10 seems to be clearly and non-contentious. However, the result is based on specific input parameters, while other input parameters could lead to different results. Therefore, we look at the effect of Hull-White input parameters on the optimal hedge amounts.

The first parameter in consideration is the σ within the Hull-White model given by Equation (4.23). The parameter describes the volatility of the short rate. Figure 5.11 shows the optimal hedge amounts by changing parameter σ. We find that larger σ's have a greater effect on the change of the optimal hedge amounts than smaller σ's. This means that for a higher volatility, i.e., a larger σ, we need to pick a higher α, i.e., an optimal hedging amount closer to EE level, and vice versa for a lower volatility. Furthermore, we see more non-linearity at 1.5σ and 2σ between the scaling factor α and the risk appetite ω, where α is not 0 or 1.

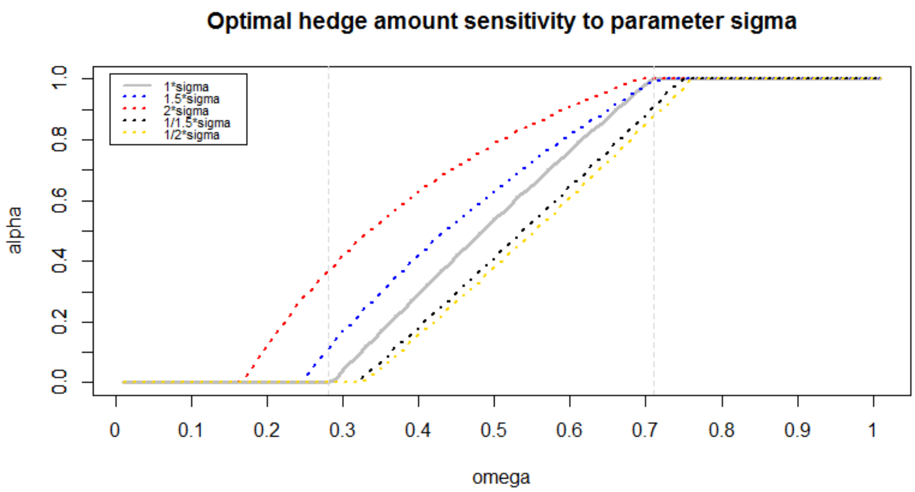

FIGURE 5.11: Scaling factor α set out against risk appetite ω with different volatility parameters sigma. The parameter sigma varies from 1/2 times sigma to 2 times sigma.

The second parameter in consideration is the k within the Hull-White model given by Equation (4.23). The parameter describes the speed of the reversion to the mean. Figure 5.12 shows the optimal hedge amounts by changing parameter k. The effects of different k's are not that great in comparison to the effect of different σ's. A change of a factor 10 in k gives minimal changes in the optimal hedge amounts. There is a small shift horizontally and a slight shift in gradient in the optimal hedge amounts, where α is not 0 or 1.

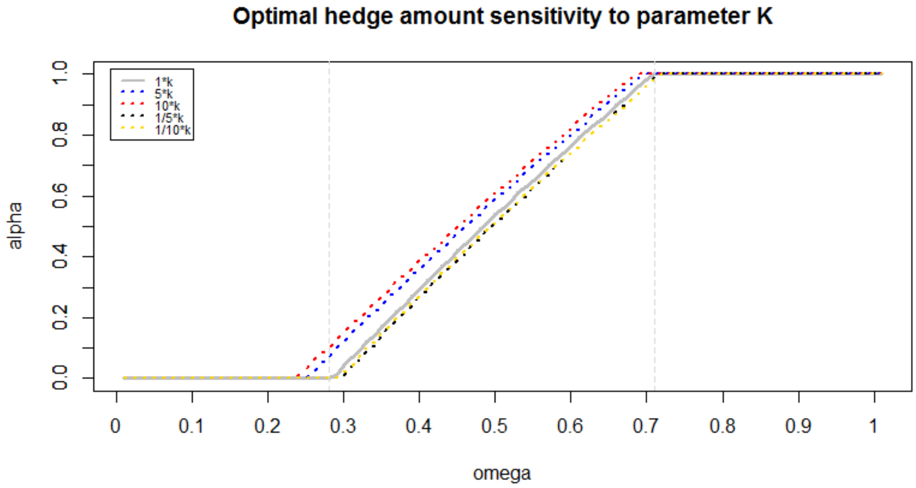

FIGURE 5.12: Scaling factor α set out against risk appetite ω with different volatility parameters k. The parameter k varies from 1/10 times sigma to 10 times k.

Chapter 6

Conclusion

In this chapter we conclude based on the findings. We also discuss the limitations of our research and present recommendations to focus on for further research.

6.1 Conclusion

In this thesis, we looked at the mismatch between the accounting CVA framework and the regulatory CVA framework. The accounting and regulatory regimes lead to the situation where it is hard to reduce the CVA risk charge and lower P&L volatility. This demands for a trade-off between the two regimes. Therefore, the goal of the research was to propose and implement a methodology to define the optimal hedge amount, which leads to maximal CVA risk charge reduction while minimizing additional P&L volatility.

We proposed a methodology composed of a step-by-step guide to find the optimal hedge amounts. In our case, we focused on interest rate swaps particularly. The interest rate model we introduced is the Hull-White model, which is calibrated to market swaption prices by using the least squares approach. We find that this interest rate model is capable of simulating future implied term structures. A large part of this thesis is dedicated to the pricing of the interest rate swap using the predicted implied term structures. With the implementation of the accounting and regulatory frameworks, we find, as expected, that the regulatory framework is more conservative in comparison to the accounting framework with regards to exposure profiles. Using the historical time frame I, we demonstrated the trade-off between the two regimes. Using time frame II, we predicted the future CVA risk charges and P&L volatility. We used historical CDS spreads for the first time frame, while predicting CDS spreads in the future for the second time frame using a mean-reverting model.

We demonstrated that our methodology is capable of estimating the optimal hedge amounts B by adjusting scaling factor α using a single-name CDS hedge instrument. This is shown for each type of risk appetite ω of the bank. The power of the model lies in the possibility to simulate future results to find the optimal hedge amount. For a bank more focused on capital, we see that the hedge amount should be set closer to EAD level. For a bank focused on P&L volatility, we find that the hedge amount should be set closer to EE level. Furthermore, the effect of changes in Hull-White input parameter on the optimal hedge amounts is shown, where the volatility has a large impact on the outcome in comparison to the speed of reversion.

One of the advantages is that our methodology offers guidance to finding the optimal hedge amount. Furthermore, by following this methodology a lot of freedom is provided, because the user can implement different types of risk factor models and valuation models to his liking.

6.2 Discussion

In this section we discuss limitations of our research.

- **Probability default model:** In our case we compute historical probability of default using market CDS spreads and future probability of default spreads using a mean-reverting model. However, this mean-reverting PD model has been arbitrarily chosen to compute CVA over time and therefore does not reflect reality. We propose to implement a PD model calibrated to real market conditions to compute future default probabilities.
- **Optimization criteria:** We follow an adjusted version of the optimization criteria of Berns (2015), where we minimize σ^2_{syn} to find the optimal hedge amount B. Other types of optimization criteria formulas could be derived to meet the research goal.
- **Risk neutral measure:** In our case we use the Hull-White model calibrated to market swaption prices, using the risk neutral measure $\mathbb{Q}$ for the accounting and regulatory frameworks. Under the accounting regime this is the correct approach. However, under the regulatory regime the real world measure $\mathbb{P}$ should be used instead of using historical market data to fit parameters.
- **Hedge instrument:** We use a 5-year CDS as the hedge instrument. Since we do not assume an average exposure path EE^*, but the actual exposure path EE, the hedge 5-year CDS does not hedge perfectly (see Chapter 2.1.3). The hedge always leads to a reduction in the risk charge, but due to the misalignment in exposure not always in a lower P&L volatility. By implementing different hedging strategies, such as CCDS or using multiple CDSs (see Figure 2.1), the exposure path could be matched significantly better.

6.3 Further research

In this section we present multiple recommendations for further research.

- **Portfolio level and derivative types:** This methodology is applied to a single interest rate derivative. Other OTC derivative types can be included as well, such as FX, equity-linked and commodity-linked. Furthermore, we believe that the model brings more benefits if it is extended to portfolio level, i.e., including various derivative products, counterparties and other conditions.
- **Hedge instruments and strategies:** We use one type of eligible hedge instrument, namely the 5-year CDS. The strategy is to use dynamic

hedging over each quarter. Other hedge instruments, eligible under both regimes, can be implemented within this methodology. An important factor of considering hedge instruments is the liquidity of these instruments. Also, the effect of a hedge strategy is interesting to investigate, since the number of times adjusting the hedge could have an effect on the optimal hedge amount.

- **Model choice:** We use the Hull-White model to simulate interest rates. As we see, the effect of input parameters has an effect on the optimal hedge amounts. The next step should be to do an in-depth sensitivity analysis for more parameters. Also, the effect of other interest rate models could be examined. Furthermore, we focus on the regulatory BA-CVA approach and the accounting expected exposure approach. Other regulatory and accounting approaches could be implemented in the model to extend the scope of the model.
- **Including XVa, WWR and risk mitigators:** In our case we excluded other valuation adjustments, wrong-way risk and risk mitigators. Recently, discussion of other valuation adjustments and wrong-way risk is getting more attention. The effect on the optimal hedge amount could be investigated by including other valuation adjustments, wrong way risk and risk mitigators.

Appendix A

Regulatory EAD framework

This appendix gives a brief summary of the regulatory EAD framework for interest rate derivatives. For a detailed description we refer to 'The standardised approach for measuring counterparty credit risk exposures' (BCBS, 2014).

EAD calculation:

The calculation of exposure at default (EAD) under the SA-CCR is calculated via

$$EAD = \alpha * (RC + PFE) \tag{A.1}$$

where α is 1.4, RC is the replacement cost, and PFE is the potential future exposure.

Replacement costs calculation:

The replacement costs (RC) is defined as the greater of (i) the current market value of the derivative contract and (ii) zero.

$$RC = \max[V; 0] \tag{A.2}$$

where V is the value of the derivative transaction.

PFE calculation:

The PFE add-on consists of (i) an aggregate add-on component and (ii) a multiplier.

$$PFE = multiplier * AddOn\,. \tag{A.3}$$

Multiplier calculation

The multiplier is calculated via

$$multiplier = \min\left[1; Floor + (1 - Floor) * \exp\left(\frac{V}{2 * (1 - Floor) * AddOn}\right)\right] \tag{A.4}$$

where $\exp(\cdots)$ equals to the exponential function, $floor$ is 5%, V is the value of the derivative transaction.

AddOn calculation

The AddOn component is calculated via

$$AddOn_j^{IR} = SF_j^{(IR)} \cdot EffectiveNotional_j^{(IR)} . \tag{A.5}$$

The SF_j^{IR} represents the interest rate supervisory factor, which is set on 0.50%. The effective notional D_{jk}^{IR} is calculated for time bucket k of hedging set j according to:

$$D_{jk}^{(IR)} = \sum_{i \in Ccr_j, MB_k} \delta_i * d_i^{(IR)} * MF_i^{(type)} \tag{A.6}$$

where notional $i \in Ccr_j, MB_k$ refers to trades of currency j that belon to maturiy bucket k. The parameter δ_i is defined as the supervisory delta adjustment, which is $+1$ is long in the primary risk factor and -1 is short in the primary risk factor. The parameter MF_i represents a maturity factor. The aggregation across maturity buckets for each hedging set is calculated according to the following formula

$$\begin{aligned} EffectiveNotional_j^{(IR)} = {} & [(D_{j1}^{(IR)})^2 + (D_{j1}^{(IR)})^2 + (D_{j1}^{(IR)})^2 \\ & + 1.4 * D_{j1}^{(IR)} * D_{j2}^{(IR)} + 1.4 * D_{j2}^{(IR)} * D_{j3}^{(IR)} \\ & + 0.6 \cdot D_{j1}^{(IR)} \cdot D_{j3}^{(IR)}]^{1/2} \end{aligned} \tag{A.7}$$

$$d_i^{IR} = TradeNotional \cdot \frac{\exp(-0.05 \cdot S_i) - \exp(-0.05 \cdot E_i)}{0.05} . \tag{A.8}$$

Appendix B

CDS spreads

Future CDS spreads, in specific time frame II, are modeled using a mean-reverting model. The parameters of the model are based on the available CDS spread data of time frame I. We model the 6m CDS spread over time.

We make use of an Ornstein-Uhlenbeck process to simulate the 6m CDS spread over time. It has the property that the process tends to drift towards its long-term mean, i.e. it is mean-reverting. The generalized equation of an Ornstein-Uhlenbeck process is given by:

$$dx(t) = \theta(\mu - x(t))dt + \sigma dW(t) \tag{B.1}$$

where $\theta > 0$, μ and σ are parameters and $W(t)$ denotes the Wiener process.

Based on the available 6m CDS data we set $\theta = 1$, $\mu = 64$ and $\sigma = 75$. However, the CDS spreads could go negative using these parameters. Therefore we floor the 6m CDS spreads at 10bps, since we assume that a counterparty always has a default probability. Also, historical spreads of the counterparty do not show lower values than 10bps. Now, the 6m CDS spread is simulated by implementing the parameters into equation (B.1).

We assume that the other CDS tenors (1y, 2y, 3y, 4y, 5y, 7y, 10y) move in parallel to the 6m tenor. Therefore, the other CDS tenors are calculated by adding a specific spread add-on to the 6m spread. The add-on is based on data of time frame I and is the difference between the average CDS spread of a specific tenor, other than the 6m CDS, and the average CDS spread of the 6m tenor. Table B.1 shows the add-on in bps. For example, if the 6m CDS spread is 10bps, then the 1y CDS spread results in 15bps (10+5) and the 5y CDS spread results in 65bps (10+55).

TABLE B.1: The add-on per CDS tenor seen from the 6m CDS tenor

CDS tenor	**Add-on (bps)**
6m	-
1y	5
2y	15
3y	30
4y	45
5y	55
7y	75
10y	90

Figure B.1 shows the CDS spreads of the tenors over time. The actual CDS spreads are used until $t_{2.75}$ and the simulated CDS spreads are used from $t_{2.75}$. The actual CDS spread cross each other, due to the changing slope of the hazard rate over time (as mentioned in Section 5.1). The simulated spreads do not have this characteristic, since we always use a pre-specified add-on.

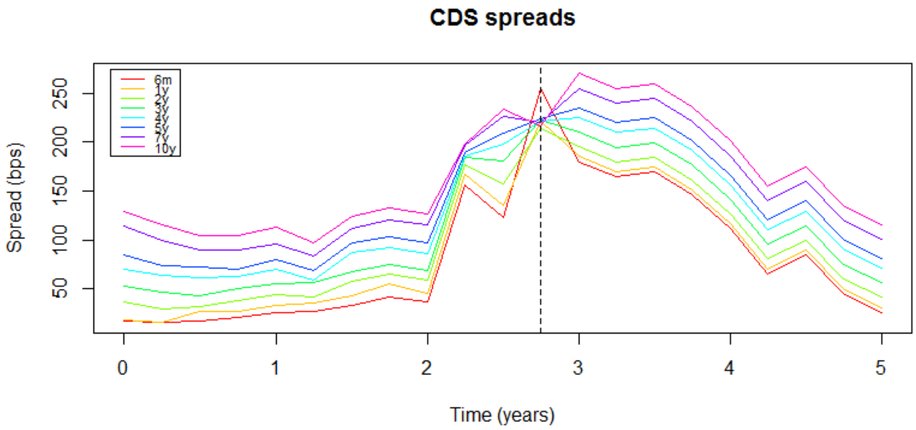

FIGURE B.1: The left side of $t_{2.75}$ shows actual CDS speads, while the right side of $t_{2.75}$ shows the simulated CDS spreads.

Appendix C

Market data

Data extracted from Bloomberg Terminal. In this appendix the data of 01-01-14 is shown. Data of other dates are available upon request.

TABLE C.1: Swaption volatilities on 01-01-14.

Expiry	**1Yr**	**2Yr**	**3Yr**	**4Yr**	**5Yr**	**6Yr**	**7Yr**	**8Yr**	**9Yr**
1Mo	44.38	48.48	47.18	42.93	37.73	32.19	28.67	28.02	25.95
3Mo	53.46	53.58	49.8	44.77	38.69	33.51	30.1	29.55	27.54
6Mo	60.27	58.74	52.24	44.1	38.41	33.68	30.54	30.05	28.24
9Mo	61.96	57.72	50.86	43.71	37.27	33.14	30.52	29.61	27.96
1Yr	63.83	57.29	48.34	40.8	35.59	32.07	29.77	29.48	28.02
2Yr	53.64	44.63	38.22	33.61	30.67	28.58	26.98	26.96	25.91
3Yr	39.65	34.74	31.27	28.71	26.79	25.46	24.43	24.62	23.83
4Yr	31.37	28.82	26.85	25.31	24.02	23.2	22.5	22.78	22.28
5Yr	26.97	25.27	24.09	23.15	22.31	21.7	21.14	21.51	21.12
6Yr	24.54	23.36	22.46	21.69	21.05	20.54	20.14	20.54	20.21
7Yr	22.4	21.61	20.91	20.37	19.88	19.52	19.17	19.61	19.38
8Yr	20.84	20.19	19.71	19.3	18.99	18.67	18.36	18.87	18.65
9Yr	19.25	18.9	18.55	18.37	18.1	17.82	17.61	18.14	17.95
10Yr	17.96	17.69	17.59	17.44	17.22	17.06	16.89	17.45	17.27
12Yr	17.35	17.06	16.82	16.71	16.57	16.36	16.17	16.74	16.66
15Yr	15.64	15.44	15.46	15.41	15.36	15.22	15.13	15.79	15.76

Expiry	**10Yr**	**12Yr**	**15Yr**
1Mo	22.69	20.81	18.87
3Mo	24.22	22.25	20.18
6Mo	25.16	23.24	21.16
9Mo	25.57	23.64	21.47
1Yr	25.31	23.58	21.57
2Yr	23.77	22.39	20.58
3Yr	22.21	21.13	19.7
4Yr	20.99	20.14	18.95
5Yr	20.06	19.35	18.31
6Yr	19.22	18.57	17.63
7Yr	18.42	17.82	17.01
8Yr	17.75	17.17	16.41
9Yr	17.1	16.58	15.84
10Yr	16.49	16.02	15.27
12Yr	15.9	15.43	14.67
15Yr	14.95	14.46	13.72

TABLE C.2: Discount rates on 01-01-14.

Payment date	**Discount**
5-1-2015	0.995998
3-7-2015	0.993078
4-1-2016	0.98858
3-1-2017	0.971354
3-1-2018	0.944616
3-1-2019	0.909728
4-1-2021	0.832141
3-1-2024	0.719884
5-1-2026	0.651659
3-1-2029	0.563621
3-1-2034	0.443509
4-1-2039	0.355144
4-1-2044	0.287128

Appendix D

Calibration parameters

TABLE D.1: Starting parameters for each quarterly time step.

time frame	date	k	sigma	r0	swap time
I	1-1-2014	0.04518101	0.01137137	0.3464	0
I	1-4-2014	0.033748474	0.009647178	0.3279	0.25
I	1-7-2014	0.029416889	0.009187273	0.3279	0.5
I	1-10-2014	0.0341414	0.00906414	0.3247	0.75
I	1-1-2015	0.033459392	0.009036356	0.3648	1
I	1-4-2015	0.031870835	0.009354411	0.4034	1.25
I	1-7-2015	0.038000235	0.009698421	0.44835	1.5
I	1-10-2015	0.031450729	0.009098801	0.531	1.75
I	1-1-2016	0.026990294	0.008315626	0.84225	2
I	1-4-2016	0.024140119	0.00761476	0.9011	2.25
I	1-7-2016	0.012935267	0.006415365	0.92365	2.5
I	1-10-2016	0.014321835	0.006096298	1.24778	2.75
II	1-1-2017	0.014321835	0.006096298	1.26266	3
II	1-4-2017	0.014321835	0.006096298	1.279485	3.25
II	1-7-2017	0.014321835	0.006096298	1.295196	3.5
II	1-10-2017	0.014321835	0.006096298	1.307183	3.75
II	1-1-2018	0.014321835	0.006096298	1.33598	4
II	1-4-2018	0.014321835	0.006096298	1.374019	4.25
II	1-7-2018	0.014321835	0.006096298	1.39161	4.5
II	1-10-2018	0.014321835	0.006096298	1.384896	4.75
II	1-1-2019	0.014321835	0.006096298	1.401037	5

Appendix E

Goodness of fit

We use the Chi-squared test as a goodness of fit test to see if the implied term structure differs statistically from the market term structure.

Let O_i be the observed frequencies in category i and E_i be the expected frequencies in each category, for each of the k categories $i = 1, 2, 3, \cdots, k$, into which the data has been grouped. The test statistic is given by:

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} \,. \tag{E.1}$$

The hypothesis are:

- $H_0 : O_i = E_i$
- $H_1 : O_i \neq E_i$

with the summation over all k categories, and $k - 1$ degrees of freedom (Gingrich, 2004).

The goodness of fit test is based on an α of 1%. By applying the test statistic on the data we find the error of the data $\chi^2 = 0.1376$, whereas the critical value is $\chi^2 = 158.9502$. Since 0.1376 < 158.9502, we do not reject H_0.

The model's term structure follows the initial can be ascribed to the initial term structure.

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