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# Dynamic pricing for camping and bungalow parks: integer linear programming models for revenue maximization


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MSc Thesis
May 2017

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## Abstract

More and more airlines, hoteliers, webshops and other companies apply dynamic pricing strategies to increase their revenue. This project presents a deterministic integer linear program to find a dynamic pricing strategy to increase revenue of camping and bungalow parks. A simulation framework is proposed to evaluate the performance of a dynamic pricing strategy. The computational experiments are used to validate and test the performance of this linear program. The results show an increase of the revenue relative to the pricing strategy that is currently used by most camping and bungalow parks.



**Keywords:** Dynamic pricing, deterministic integer linear programming, camping, bungalow, revenue maximization.

# Preface

This thesis is the final project of my study Applied Mathematics at the University of Twente. I performed my research for the group Discrete Mathematics and Mathematical Programming in the master's program Operations Research.

This project is done in cooperation with Stratech, which develops software for niche markets. Stratech puts industry-specific issues into innovative solutions for (international) organizations which are active in different sectors. One of these sectors is the recreation sector, which is the sector of the camping and bungalow parks.

Stratech observes that dynamic pricing is an upcoming research field. Competitors of Stratech already started small studies to develop tools to support camping and bungalow parks with pricing decisions. Stratech suspect that dynamic pricing will be integrated in the recreation sector soon. Therefore, Stratech also want to start with some studies in this research field.

Stratech attempt to provide their software with a tool which support camping and bungalow parks with pricing decisions to increase their revenue. At the beginning of September I was asked to perform a study on dynamic pricing models and here is where my project started. The research that I performed on this topic in the last 7 months is worked out in this report.

## Acknowledgements

I have enjoyed working on this project and I want to thank some people who helped and support me during this project.

I would like to thank Niek Beimer who gave me the opportunity to perform my final assignment at Stratech and being one of my supervisors. He was very helpful on the practical sight of this project. He had the practical knowledge that was needed to come up with an appropriate model. He was always prepared to give me the right data and taking time for brainstorm meetings.

Next I would like to thank Marc Uetz for being my supervisor of this project. He pushed me into the right mathematical direction and our conversations were useful to get positive progress during this project.

I also would like to thank Nelly Litvak who was prepared to take place into the assessment committee and for the time spending on reading and judging my master thesis

I thank Yoeri Boink and Stefan Klootwijk to give up some time for me to listen en help me with some mathematical problems. In special Yoeri, who also was prepared to read my thesis and give me feedback.

Finally, I am very grateful to my parents and my family for supporting me. Especially I want to thank Marieke for her extra support and always being there for me.

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# 1 Introduction

The leisure industry is a worldwide active industry. One big part of the leisure industry is the industry that rents *campgrounds and bungalows*. The companies of this part of the industry can outsource tasks like administration, marketing and the management of bookings. Stratech is a company that develops software for these tasks. Since 1989 Stratech develops innovative software solutions for niche markets and one of these software solutions is Stratech-RCS and around 300 companies, domestically and abroad, use this software package. Currently, the seasonal prices (low, mid and high season) for each accommodation are set at the beginning of the year. Except for discounts, these prices are hold till the end of the year. Stratech observes that more and more airlines, hoteliers, web shops and other companies successfully apply *dynamic pricing strategies* to increase their revenue. Therefore, Stratech aims to augment Stratech-RCS with a dynamic pricing tool to support camping and bungalow parks with pricing decisions.

In general, a dynamic pricing strategy is a pricing strategy in which businesses set flexible prices for products or service based on current market demands. The goal of dynamic pricing is to adjust the price of a product or service on the situation and/or customer to maximize profits. Dynamic pricing has become a more popular research field since the Airline Deregulation Act of 1978 [20]. The Airline Deregulation Act is the law that deregulated the airline industry in the United States, removing U.S. Federal Government control over such things as fares, routes and market entry of new airlines. The resulting free market has led to an increased number of flights and a decrease of fares. The development of dynamic pricing models has been upcoming since then. Later, computers became faster and big data turned into a popular research field. All of this led to more sophisticated dynamic pricing algorithms and thereby more companies integrated dynamic pricing strategies [15, chapter 1]. It is most likely that dynamic pricing strategies will soon be applied by camping and bungalow parks.

Dynamic pricing is a form of *revenue management*, which is the science of managing a limited amount of supply to maximize revenue, by dynamically controlling the price/quantity offered [1, 3, 4, 15]. In terms of business practice, varying prices is often the most natural mechanism for revenue management. Many industries use various forms of dynamic pricing to respond to market fluctuations and uncertain demand [15, chapter 9] and pricing is one of the most effective variables that managers can manipulate to encourage or discourage demand. Pricing is not only important from a financial point of view, but also from an operational point of view. Prices influence the decision of customers and thereby help to regulate inventory [4]. Furthermore, *demand forecasting* plays an important role in the revenue management [12,19]. Accurate demand forecasts are crucial to a valuable revenue management, because prices are adjusted using the demand forecast [4,17,19].

A big challenge in dynamically setting prices is the uncertainty in predicting customer behavior. How do customers respond on price changes? What is the customer willing to pay? But even when the answers to these questions are known, it is not directly clear how prices should be adjusted to obtain a higher revenue taking the limited amount of accommodations into account.

Stratech asks for an algorithm which periodically changes the offered prices to support camping and bungalow parks to increase their revenue. Stratech requires that the prices will not changed real time or high frequently, but at most once a day. In addition, prices should not be deviated to much from the static prices and the algorithm will be used to support camping and bungalow parks to make price decisions. Also, the algorithm needs to be generic in the sense that all users of Stratech-RCS could use this algorithm.

In this project we developed an algorithm that computes a dynamic pricing strategy for camping and bungalow parks, which periodically assigns a price class to each possible reservation to maximize the total expected revenue. The performance of the obtained pricing strategy is evaluated in a developed simulation framework. What this exactly means becomes clear in the remainder of the report.

The remainder of the thesis is organized as follows: In Chapter 2 we give a literature review on dynamic pricing models and solutions approaches and also a brief review on demand forecasting models. In Chapter 3 a network revenue management model of the dynamic pricing problem is proposed together with a model for the demand elasticity. Further, some practical modeling problems are discussed at the end of this chapter. In Chapter 4 we propose a deterministic integer linear program (ILP) which finds the optimal expected revenue of the network revenue management model and some methods are proposed to tackle some practical modeling problems. In Chapter 5 a demand forecasting model is proposed where customer requests are generated with a Bernouilli process. Also, a Monte Carlo simulation is proposed which is used to generate sample paths for the computational experiments. Finally, a ILP is proposed to maximize revenue of a single a sample path. In Chapter 6 we proposed the computational experiments to test and validate the performance of the proposed ILP and analyze the results of these experiments. In Chapter 7 we give a critical review on proposed model. In Chapter 8 we give a summary of this project and recommendations for future research. Throughout the paper we use examples to support the reader for understanding.

# 2 Literature review

A large volume of the literature on dynamic pricing models is focused on the airline and hotel industry. Literature specifically on dynamic pricing models for camping and bungalow parks are very scarce. Fortunately, the hotel industry requires to handle the same kind of problem. In particular, the hotel industry faces similar problems in revenue management concerning demand forecast, customer behavior, occupancy, variable lengths of stay and limited and variable accommodation types. Camping and bungalow parks deviates from the hotel industry on average length of stay, average time between booking date and arrival date, cancellation rate and demand elasticity. For example, the average length of stay is close to 11 days in summer, and reservations for summer can occur more than 9 months in advance for early bookers, while the average length of stay in the hotel industry is close to 4 days and most reservation occur less than 3 months in advanced. Some companies allow unsynchronised arrivals (Wednesday, Saturday, Sunday) when others impose Saturday to Saturday stays. Further, camping and bungalow parks have different ancillary costs, inventory size, inventory heterogeneity and customer segments. We have focused on the literature of dynamic pricing models concerning the hotel industry and keep in mind these differences to come up with an appropriate model and solution strategy.

## 2.1 Dynamic pricing

A clear literature review on dynamic pricing in the hotel industry can be found in [2]. The works [4] and [15] present an overview of dynamic pricing models for revenue management. From these articles we obtained that the dynamic pricing problem is modeled and solved in many different ways. The dynamic pricing problem in the hotel industry is usually formulated as a network revenue management problem. In [15, chapter 3] and section 3.1 a description is given of this network. The network revenue management is widely used in the airline industry. However, in contrast to the airline industry, the end of the horizon in not clear in the hotel industry. In the airline industry each flight has a certain departure day, called end of horizon. After this day no seats can be sold. In the hotel industry we do not have such derparture day after no rooms can be sold anymore, there is no clear end of horizon. Rolling horizon procedures are used to solve the problem at a given *cut-off date* (i.e. end of horizon). To point this out, customers usually can not make a booking before a certain time. For example, if customers can not book longer than 1 year in advance, then the cut-off date is set on one year ahead and after each day the horizon 'rolls' one day forward. The work of [10] discusses rolling horizon models and techniques for the hotel revenue management.

Two main solution approaches for the network revenue management problem are deterministic linear programming (LP) and dynamic programming (DP). Generally, LP's generate, if frequently resolved, good pricing strategies. However, the deterministic approach ignores demand uncertainty, which is the main weakness of LP's. A stochastic DP formulation can overcome this weakness, but its state space can easily suffer from Bellman's main curse of dimensionality.

In both main approaches a distinction is made between a choice based (or dependent) demand model and an independent demand model. In the choice-based demand model, customers are assumed to choose among all available possible reservations according to prespecified choice probabilities. An independent demand model assumes that demand for each possible reservation comes from different customers and that the demand for a product is lost when the product is not available or the price is too high. The work of [13] shows that choice-based availability control can improve revenue, compared to models that are based on the independent demand model.

A commonly used approach in the DP's is *DCOMP*, which is a decomposition of the network problem into a set of smaller problems where each concerns only one resource (i.e. a single night stay). A clear introduction to this concept can be found in [15, chapter 4] and examples of the DCOMP approach can be found in [7,13,22,23]. The works [13,15,18] propose strong heuristics for the network revenue management in practice. The works [8] and [22] consider variants of DCOMP and [13] studied both deterministic and stochastic LP in a simulating setting. The DP approach is popular in recent research and shows to generate strong heuristics [24].

The deterministic linear programming approach is one of the traditional approaches for making pricing decisions in network revenue management. This deterministic linear program assumes that the arrivals of customers are given by deterministic functions of the prices. The LP dates back to the work of [9] and it has been widely used by practitioners [8]. The work of [16] proposed a deterministic choice based linear model (CDLP) to solve the network revenue management problem with column generation techniques. In [14] two new method are proposed to solve the CDLP efficiently.

Besides the LP and DP approach, another solution approach is proposed in [3]. The authors developed a price optimization framework based on price multipliers. The price is the product of four optimised multipliers (time, capacity, length of stay and group size). Each multiplier varies around one and provides a varying discount or surcharge over some seasonal reference price set by the company. A Monte Carlo simulation from [21] is used for simulating the demand.

Several different modeling approaches and different solution strategies were found in the literature. Stratech asks for a model in which price changes occurs at most once a day. In all models that use a DP approach prices may change every time period, where a time period correspond to a small enough interval of time that there is at most one booking at each time period. As a consequence, the time intervals are too small and a DP formulation is not suitable in our context. A LP generates (if frequently resolved) good pricing strategies [24]. Therefore, we chose a LP approach in this project. Furthermore, a choice based demand model shows to obtain better performance, but reliable choice probabilities are necessary. Unfortunately, such reliable choice probabilities can not be conducted from the available data of Stratech. Therefore, an independent demand model is used in this project. The LP approach with independent demand that is used in this project is a modification of the LP proposed in [8].

## 2.2 Demand forecasting

A demand forecast in the hotel industry (and camping and bungalow parks) has three dimensions: time of booking, time of arrival and the length of stay. The work of [21] composed two competing philosophies in the forecasting theory. One approach is based on using the historical data to develop an empirical formula for the forecasting variable (number of future arrivals). The other approach focuses on simulating a predefined model forward in time to obtain the forecast.

The work of [19] compared several forecasting methods and distingished three forecasting methods: historical booking methods, advanced booking methods and combined booking methods. Methods like moving average, exponential smoothing and other autoregressive models are often used in practice [11]. All these methods are used to develop an empirical formula for the forecasting. The authors of [19] argued that demand forecasting is quite company specific and one needs to be careful by a general usage of a demand forecasting model.

A simulating predefined model is proposed in the work of [21]. The authors give a clear description of a Monte Carlo simulation to forecast demand. We decided to use this simulation in this project, because of its good performance in the hotel industry. Furthermore, this approach is also used in other works to simulate future demand, see for example [3, 5–7].

# 3 Modeling

A *dynamic pricing strategy* is a pricing strategy in which businesses set flexible prices for products or service based on current market demands. The dynamic pricing problem is modeled as a revenue management network. Furthermore, we propose how we model the effect of price changes on demand, called *demand elasticity*. This demand elasticity is a part of the demand forecasting and, hence, also plays an important role in the dynamic pricing problem. Finally, we present an performance evaluation framework for a dynamic pricing strategy.

## 3.1 Network revenue management

Camping and bungalow parks typically rent multiple *object types* (e.g. campground, luxury campground, bungalow, camper ground etc.). We assume that the demand model differ along the object types and that customers do not choose along objects. For example, a customer that intends to book for a bungalow would not search for a campground even when the price of the bungalow is higher than his willing of pay. We model the dynamic pricing problem for a *single object type*. Furthermore, we model the dynamic pricing problem as a *Revenue Management network*.

We first define a time horizon which denotes the begin and the end of the considered time period.

**Definition 1** *We define $T = \{d_1, d_2, ..., d_e\}$ as the* ***time horizon*** *(in days). The first and last day of the arrival horizon are denoted by $d_1$ and $d_e$ respectively.*

To remark, $d_e$ denotes the cut-off date of the time horizon.

Customers arrive on a certain day in the time horizon. However, some camping and bungalow parks allow unsynchronised arrivals (Wednesday, Saturday, Sunday) when others impose Saturday to Saturday stays.

**Definition 2** *We define $\mathcal{H} = \{a_1, a_2, ..., a_e\}$ as the* ***arrival horizon*** *(in days), where arrivals may occur. The first and last day of the arrival horizon are denoted by $a_1$ and $a_e$ respectively.*

We also define a set of *night stays* that can be consumed by the customer. A night stay is the night between two consecutive days. For example, the night of '27-May-2017 on 28-May-2017' and night of '28-May-2017 on 29-May-2017' etc. are night stays.

**Definition 3** *The set $\mathcal{I}$ denotes the set of offered night stays. A single* ***night stay*** *$i \in \mathcal{I}$ is denoted by the pair $(d{,}d+1)$ with $d{,}d+1 \in T$, where $d+1$ denotes the day after $d$.*

Fig 1 illustrates how $T$, $\mathcal{H}$ and $\mathcal{I}$ relate to each other. A *reservation* $r$ is a sequence of successive night stays. For example, if we receive a customer reservation request for the weekend 27-May-2017 till 29-May-2017, then the two night stays '27-May-2017 on 28-May-2017' and '28-May-2017 on 29-May-2017' are consumed by this customer. A reservation typically consists of an *arrival day* and a number of nights reserved, called *length of stay* (LoS).

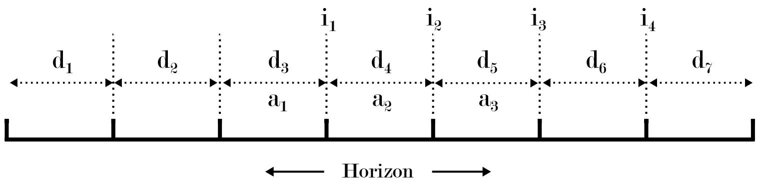


Figure **1**: Time horizon $d_1$ to $d_7$ of one week, with allowed arrivals $a_1$ to $a_3$ and offered night stays $i_1$ to $i_4$

**Definition 4** *We define $\mathcal{L}_a$ as the set of possible **lengths of stay** from some arrival day $a \in \mathcal{H}$.*

**Definition 5** *A **reservation** $r$ is denoted by the pair $(a, l)$, with $a \in \mathcal{H}$ and $l \in \mathcal{L}_a$. The set $\mathcal{R} = \{(a, l) \mid a \in \mathcal{H}, l \in \mathcal{L}_a\}$ denotes the set of all possible reservations.*

Moreover, consider some reservation $r$ with arrival day $a \in \mathcal{H}$ and $l \in \mathcal{L}_a$, then $\{(a, a+1); (a+1, a+2); ...; (a+l-1, a+l)\} \subseteq \mathcal{I}$. We use $a_r$ to indicate that this is the arrival day of reservation $r$ and $l_r$ to indicate the length of stay of reservation $r$.

Furthermore, each company has a certain amount of available objects to rent. The number of available objects is called the *capacity*.

**Definition 6** *The **capacity** is defined by $c = \{c_1, ..., c_m\}$, where $c_i$ denotes the capacity of night stay $i$ at the beginning of the decision horizon.*

Each reservation consumes a number of night stays. The *night stay consumption* for all reservations is denoted by the $m \times n$ matrix $A$, with $m$ the total number of considered night stays and $n$ the total number of reservations.

**Definition 7** *The $m \times n$ matrix $A$ represents the **night stay consumption** of all reservations, where the $(i, r)^{th}$ element, $a_{i,r}$, denotes the quantity of night stay $i$ consumed by a reservation $r$; $a_{i,r} = 1$ if night stay $i$ is used by reservation $r$ and $a_{i,r} = 0$ otherwise.*

Let $A^i$ be the $i^{th}$ row of $A$ and $A_r$ be the $r^{th}$ column of $A$, respectively. To simplify the notation, we use $r \in A^i$ to indicate that reservation $r$ uses night stay $i$ and $i \in A_r$ to indicate that night stay $i$ is used by reservation $r$.

We are allowed to charge different prices of each possible reservation separately. From now on, each possible reservation can be placed in a certain *price class* and $p_r$ denotes the *reference price* of reservation $r$. The reference price is the price established by the company at the beginning of the year or, in other words, the price that is charged if we did not use dynamic prices.

**Definition 8** *The set $\mathcal{K}$ is a non-empty set of integers, called **price classes**. We define $\{p_r^k \mid k \in \mathcal{K}\}$ as the set of **prices** for reservation $r$. The price that is charged for reservation $r$ has to take a value in this set.*

We require that $p_r \in \{p_r^k \mid k \in \mathcal{K}\}$, which indicates that there is a price class for the reference price. To give an example, consider three prices classes, i.e. $\mathcal{K} = \{1, 2, 3\}$. Price class 1 indicates a 10% discount, price class 2 follows the reference price and price class 3 indicates a 10% surcharge of the price. Then $p_r^1 = 0.9 \cdot p_r$, $p_r^2 = 1 \cdot p_r$ (= reference price) and $p_r^3 = 1.1 \cdot p_r$.

Stratech asks for periodically change of prices. Hence, price changes may occur at different time points and can hold for a certain time period. Therefore, we define the *decision periods*. A decision period is a certain time period of at least one day (e.g. day or week) and at the beginning of each decision period prices can be changed which are hold till the end of the decision period.

**Definition 9** *We define $\mathcal{D} = \{w_1, w_2, ..., w_e\}$ as the set of* ***decision periods*** *with $w_1$ the first decision period and $w_e$ the last decision period. We define $d_w^1 \in T$ and $d_w^e \in T$ as the first and last day of decision period $w$ respectively. Prices are charged at the beginning of $d_w^1$ and holds till the end of $d_w^e$.*

To clarify, consider some decision period $w$, then prices of all reservation $r$ with $a_r \geq d_w^1$ can be changed, i.e. arrival day $a_r$ is later (or on the same day) in the time horizon than the first day of decision period $w$. A *request* can be made on a certain day before arrival. Moreover, a request for reservation $r$ can be received in each decision period $w$ if $a_r \leq d_w^e$. Each request that is received in decision period $w$ for a reservation $r$ with $a_r \geq d_w^1$ is offered for the price $p_r^k$ if price class $k$ is charged for reservation $r$ in decision period $w$. Fig 2 illustrates the relation between $\mathcal{H}$ and $\mathcal{D}$.

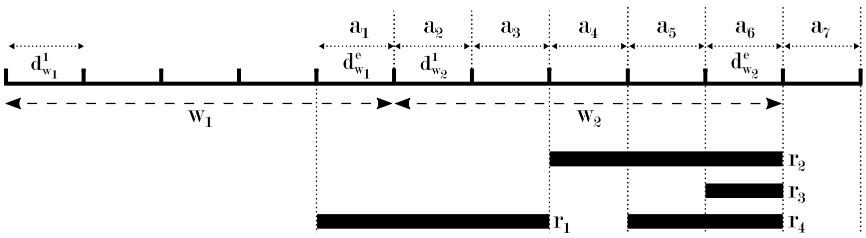


Figure **2**: *Decision period $w_1$ and $w_2$ are periods of five days. At the beginning of day $d_{w_1}^1$ we can charge new prices for all reservation $a_r \geq d_{w_1}^1$, i.e. $r_1$, $r_2$, $r_3$, $r_4$. These prices holds till the end of day $d_{w_1}^e$. At the beginning of $d_{w_2}^1$ we can charge new prices for all reservation $a_r \geq d_{w_2}^1$, i.e. $r_2$, $r_3$, $r_4$. These prices hold till the end of day $d_{w_2}^e$.*

Throughout the paper, we reserve $d \in T$, $a \in \mathcal{H}$, $i \in \mathcal{I}$, $l \in \mathcal{L}$, $r \in \mathcal{R}$, $k \in \mathcal{K}$, $w \in \mathcal{D}$ as the indices for days, arrival days, night stays, length of stay, reservations, price classes and decision period respectively. The following example is used to illustrate the network revenue management.

**Example 1** *Consider a small bungalow park which decide to start changing prices for his accommodations on Monday $8^{th}$ of May, 2017. The week Monday May $22^{th}$ to Sunday May $28^{th}$ is the last week of the time horizon and contains the ascension weekend. Thus, the considered time horizon is given by $T = \{$ May $8^{th}$, May $9^{th}$,...,May $28^{th}\}$. The park requires that arrivals only occur in the ascension weekend except for the Sunday, so $\mathcal{H} = \{$Friday May $26^{th}$, Saturday May $27^{th}$ $\}$. For shorter notation, $\mathcal{H} = \{a_1, a_2\} = \{Fr, Sa\}$. Further, a reservation needs to be made for three night stays if Friday is the arrival day and for at most two night stays if Saturday is the arrival day. Hence, $\mathcal{L}_{a_1} = \{3\}$ and $\mathcal{L}_{a_2} = \{1, 2\}$. The park requires weekly price changes, so $\mathcal{D} = \{w_1, w_2, w_3\} = \{$May $8^{th}$ - May $14^{th}$),(May $15^{th}$ - May $21^{th}$),(May $22^{th}$ - May $28^{th}$)$\}$. Thus, price changes in decision period $w_1$ occur at the beginning of day $d^1_{w_1}$ (= May $8^{th}$) and are hold till the end of day $d^e_{w_1}$ (= May $14^{th}$). The same principle holds for periods $w_2$, $w_3$. The set of offered night stays that can be consumed by the customer is given by $\mathcal{I} = \{(Fr, Sa), (Sa, Su), (Su, Mo)\}$. Assume that the park has only 3 accommodations available for the ascension weekend, i.e. $c = \{3, 3, 3\}$. Now, the night stay consumption matrix $A$ is given by*

$$A = \begin{array}{c} \\ (Fr,Sa) \\ (Sa,Su) \\ (Su,Mo) \end{array} \begin{array}{c} \begin{array}{ccc} r_1 & r_2 & r_3 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix} \end{array}$$

*and the set of considered reservation becomes $\mathcal{R} = \{(Fr, 3), (Sa, 1), (Sa, 2)\}$. See fig 3 for a visualization of $\mathcal{H}$, $\mathcal{D}$ and $\mathcal{R}$ and consider $r_1 = (Fr, 3)$, $r_2 = (Sa, 1)$, $r_3 = (Sa, 2)$. Assume price $p^k_{r_3}$ is charged for reservation $r_3$ in decision period $w_1$ and one request for $r_3$ is received in decision period $w_1$. There is enough capacity, so the request can be accepted and night stays $\{i | i \in A^{r_3}\}$ are consumed. The revenue increases with $p^k_{r_3}$ and the capacity becomes $c = \{3, 2, 2\}$.*

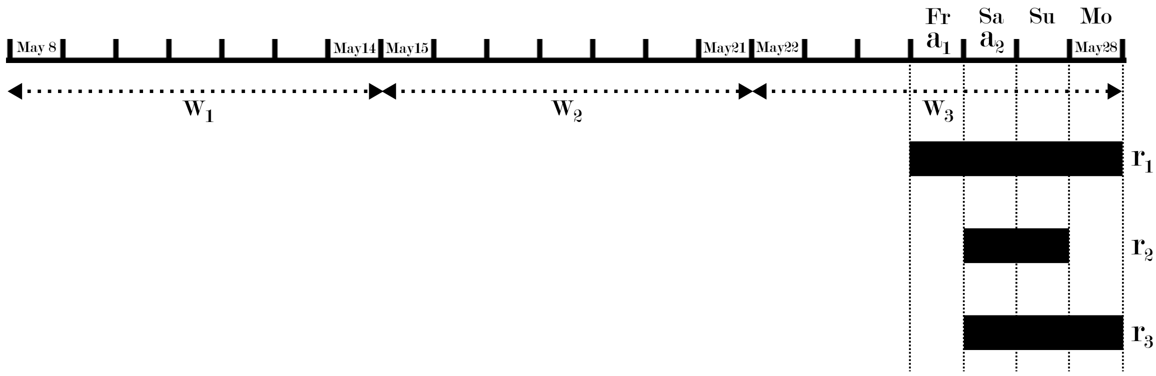


Figure **3**: Visualisation of $\mathcal{H}$, $\mathcal{D}$ and $\mathcal{R}$ of example 1

The developed ILP uses the expected number of requests for some reservation at a certain price class in a certain decision period. This is the most important input parameter of the ILP proposed in section 4.1.

**Definition 10** *We define $b^k_{r,w}$ as the* ***expected number of requests*** *when price class $k$ is charged in decision period $w$ for reservation $r$. We define $b_{r,w}$ as the the expected number of requests for reservation $r$ in decision period $w$ at the reference price. The expected number of request $b_{r,w}$ is represented by a matrix $b$ with the reservations $r$ on the rows and decision periods $w$ on the columns.*

Note that $b^k_{r,w} = 0$ for all $k$ if $a_r < d^1_w$. We further note that $b^k_{r,w}$ also could be zero if $a_r \geq d^1_w$, but that it is possible in practice that reservation $r$ in decision period $w$ is actually received, even if it was not expected. This leads to some practical problems, but this is discussed later in more detail in.
We now employ a few assumptions for the expected number of requests. First, we assume that there exist some $\phi \in \mathcal{K}$ such that $b^{\phi}_{r,w} = 0$. In this case, if there is not enough capacity to serve a request for reservation $r$, then we charge the (large) price $p^{\phi}_r$ to ensure that we do not receive a request for reservation $r$ in decision period $w$. Second, because we use an independent demand model, we assume that $b^k_{r,w}$ depends only on the price for reservation $r$ in decision period $w$, but not on the prices for the other reservations in decision period $w$.

Our goal is to find a pricing strategy which maximize the revenue of camping and bungalow parks.

**Definition 11** *A* ***pricing strategy*** *is denoted by the binary vector $u^k_{r,w}$, with $\sum_{k \in \mathcal{K}} u^k_{r,w} = 1$ for all $r \in \mathcal{R}$ and $w \in \mathcal{D}$. If $u^k_{r,w} = 1$ then price class $k$ is charged for reservation $r$ in decision period $w$.*

In words, a dynamic pricing strategy assigns a single price class in each decision period to each possible reservation.

**Definition 12** *If $k = \{k | p_r = p^k_r\}$ and $u^k_{r,w} = 1$ for all $r$ and $w$, then $u^k_{r,w}$ is called a* ***static pricing strategy****. If $u^k_{r,w}$ is not a static pricing strategy, then $u^k_{r,w}$ is called a* ***dynamic pricing strategy****.*

In words, a static pricing strategy is the pricing strategy which assigns the price class of the reference price in each decision period to each possible reservation. Then the objective of the network revenue management problem is to find pricing vector $u^k_{r,w}$ which maximize revenue.

## 3.2 Demand elasticity

The demand elasticity is the degree to which demand varies with its price. Camping and bungalow parks that currently use Stratech-RCS never used dynamic pricing strategies before. As a consequence that there is an insufficient amount of data concerning the effect of price changes on demand. Therefore, an assumption needs to be made. We assume that the demand is a linear function of the price. In Chapter 5 we propose a demand model to find the expected values $b_{r,w}$ and the values $b^k_{r,w}$ are computed by a linear *demand elasticity function*. Usually, the demand decreases when the price increases and visa versa. Within this project, the demand elasticity function is modeled as a linear price class dependent function, denoted by $DE(k)$. Note that a more sophisticated demand elasticity might be more appropriate in practice. In practice, the demand elasticity probably depends on the seasonality, time of booking and perhaps also on the reservation. The values $b^k_{r,w}$ are obtained by multiplying $b_{r,w}$ with the value of $DE(k)$ that varies around one. The demand elasticity gives the expected amount of increase or decrease. Hence, a value of $DE(k)$ that is bigger than one indicates an increase of demand and a value smaller than one indicates a decrease of demand. The demand elasticity function outputs the value 1 if the price class of the reference price is charged. In general,

$$b^k_{r,w} = b_{r,w} \cdot DE(k) \qquad k \in \mathcal{K}.$$

In short, $b_{r,w}$ is estimated with a demand forecasting model and $b^k_{r,w}$ is estimated by the demand elasticity function. We use the following example to illustrate the demand elasticity model.

**Example 2** *Consider the setting of example 1. In addition, consider the set of price classes $\mathcal{K} = \{1, 2, 3, 4\}$, where price class 1 indicates a 10% discount, 2 indicates the reference price, 3 indicates a 10% surcharge and 4 indicates price class $\phi$. At the beginning of each decision period some price class $k \in \mathcal{K}$ is assigned for each reservation $r \in \mathcal{R}$. Assume that the demand forecast model outputs*

$$b = \begin{array}{c} \\ r_1 \\ r_2 \\ r_3 \end{array} \begin{array}{c} \begin{array}{ccc} w_1 & w_2 & w_3 \end{array} \\ \begin{pmatrix} 2.2 & 0 & 1 \\ 0 & 1.8 & 0 \\ 0 & 0 & 0.7 \end{pmatrix} \end{array}.$$

*As an illustration, the expected number of requests in decision period $w_2$ for reservation $r_2$ (i.e. $b_{2,2}$) equals 1.8 and equals zero for reservations $r_1$ and $r_3$. Note that the values are allowed to be continuous because it is an expected value. Let $k \in \mathcal{K}$ and consider the array $\alpha = [0.9, 1, 1.1, \infty]$. $\alpha(k)$ indicates the $k^{th}$ element of $\alpha$ and consider the demand elasticity function*

$$DE(k) = \begin{cases} -2\alpha(k) + 3 & \textit{if } \alpha(k) \le 1.1 \\ 0 & \textit{if } \alpha(k) > 1.1 \end{cases} \tag{1}$$

*for all reservations $r$ and decision period $w$ (see fig 4). This demand elasticity indicates that if the price increases with 10%, then we expect that the demand decreases with 20%. On the opposite, if the price decreases with 10%, then we expect that the demand increases with 20%. Moreover, if price class 4 is charged, then we expect no requests. Thus, if the price is increased with 10% in decision period 2 for reservation $r_2$, then $b^3_{2,2} = 1.8 \cdot 0.8 = 1.44$. Similarly, the value $b^k_{r,w}$ can be found for all $k$, $r$ and $w$.*

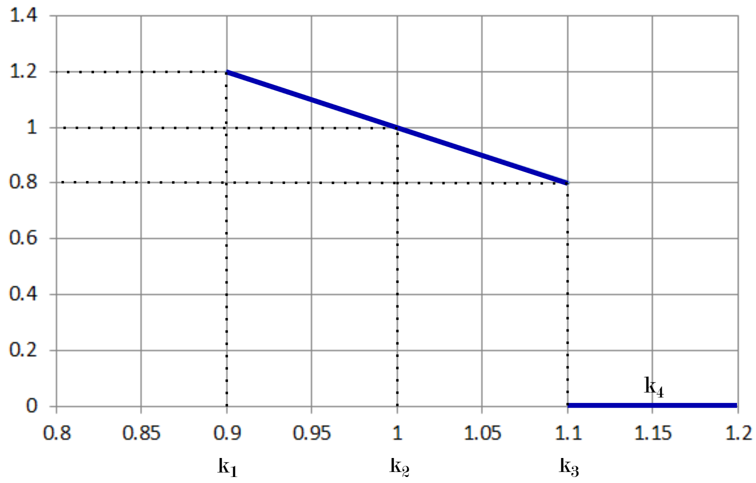


Figure **4**: Demand elasticity function DE(k) of example 2

## 3.3 Expected revenue

If the price for some reservation increases or decreases, then the demand elasticity function outputs an expected decrease or increase of demand. With these expectations the expected revenue can be calculated. The expected revenue is obtained by multiplying the expected number of requests by the price that is charged for the reservation of the request. To point forward, in section 4.1 we propose a ILP to find the *maximum expected revenue.* The following example illustrates how $DE(k)$ and the values $b_{r,w}$ are used to obtain the expected revenue.

**Example 3** *Consider the setting of example 2 and assume that the static pricing strategy is applied and the bungalow park now has an infinite capacity. The reference price ($p_r$) for reservations $r_1$, $r_2$ and $r_3$ are €150,-, €50,- and €100,- respectively. With matrix b from example 2 and prices $p_r$ the expected revenue ($E[Rev]$) is computed by*

$$\begin{aligned} E[Rev] &= \sum_{w \in \mathcal{D}} \sum_{r \in \mathcal{R}} b_{r,w} p_r \\ &= b_{1,1} p_{r_1} + b_{2,2} p_{r_2} + b_{1,3} p_{r_1} + b_{3,3} p_{r_3} \\ &= (2.2 \cdot 150) + (1.8 \cdot 75) + (1 \cdot 150) + (0.7 \cdot 100) = €640. \end{aligned} \tag{2}$$

*In addition, consider the following two pricing strategies. In the first strategy the price is decreased with 10% for each reservation at the beginning of each period. In the second strategy the price is increased with 10% for each reservation at the beginning of each period. The expected revenue of the first strategy is computed by*

$$E[Rev] = \sum_{w \in \mathcal{D}} \sum_{r \in \mathcal{R}} DE(1) \cdot b_{r,w} \cdot 0.9 p_r = \sum_{w \in \mathcal{D}} \sum_{r \in \mathcal{R}} 1.2 \cdot b_{r,w} \cdot 0.9 p_r = €720.$$

*The expected revenue of the second strategy is computed by*

$$E[Rev] = \sum_{w \in \mathcal{D}} \sum_{r \in \mathcal{R}} DE(3) \cdot b_{r,w} \cdot 1.1 p_r = \sum_{w \in \mathcal{D}} \sum_{r \in \mathcal{R}} 0.8 \cdot b_{r,w} \cdot 1.1 p_r = €480.$$

*It is obvious that the first strategy gives the optimal expected revenue, because* $1.2 \cdot 0.9 > 1 > 0.8 \cdot 1.1$.

previous example shows that the optimal expected revenue is easily found with an infinite capacity, but in practice there is a capacity constraint. In section 4.1 we proposed a ILP which takes into account this capacity constraint and solves these kind of instances in general. The ILP finds the right trade off between price surcharge and change in demand, given the available capacity. The ILP find the optimal expected revenue and the output variables are used as a dynamic pricing strategy.

## 3.4 Performance evaluation framework

Eventually we want to evaluate the performance of the pricing strategy of this algorithm relative to the static pricing strategy that is currently used by most of the companies. In this section we discuss the practical problems that arise in the evaluation of the performance of a pricing strategy and how we overcome these problems.

### 3.4.1 Performance of a pricing strategy

In practice, the performance of a pricing strategy could be obtained with a *practical experiment*. For example, we could test the performance of two pricing strategies on two comparable bungalow parks in the same time period. One bungalow park applies a price strategy and the other bungalow park applies the other pricing strategy in the same time period. At the end both revenues can be compared to evaluate the performance of both pricing strategies. Such practical experiment was not suitable for this project and we developed a method to evaluate the performance computationally.

It would be easy if the expected revenue of several pricing strategies can be evaluated as in example 3 and that we can claim that the pricing strategy with the highest expected revenue performs the best. But note that the capacity is infinite in this example and that the expected revenue of a pricing strategy is meaningless if the capacity constraint in not taken into account. To explain, consider the setting of example 3, but now with a finite capacity $c = \{3, 3, 3\}$. With equation (2) we computed the expected revenue of €640 when the static pricing strategy will be applied. But note that $r_1$, $r_2$ and $r_3$ all consumes night stay $(Sa, Su)$ and in the coming three weeks we expect a total of 5.7 (= 2.2 + 1.8 + 1 + 0.7) requests which makes use of night stay $(Sa, Su)$. So, the expected number of reservations that consumes night stay $(Sa, Su)$ violates the capacity of that night ($5.7 > 3$). An expected revenue of €640 is obtained, but the capacity constraint is not taken into account. Hence, it would be incorrect to claim that €640 represent the expected revenue of the static pricing strategy correctly. In contrast, the (maximum) expected revenue of ILP (6) - (9) proposed in section 4.1 does take the capacity constraint into account. Hence, we set up a *simulation framework* in which the performance of both pricing strategies can be evaluated.

### 3.4.2 Simulation approach

The developed simulation framework finds a, so called *simulated expected revenue*, for a pricing strategy. This revenue is computed in a simulated way and the global idea of the simulation is described below. We assume that the pricing strategy with the highest simulated expected revenue has the best performance.

**Global idea simulation approach** Simulate future requests (also called *demand*) for some time and arrival horizon. Manage each request in chronological order and check if the reservation of this request does not violate the capacity. If the request fits and can be accepted, then the total simulated expected revenue increases with the price suggested by the pricing strategy for this request and the available capacity is updated. If the request can not be accepted, then the request is denied and the total simulated expected revenue does not increase.

The simulation experiment sounds accessible, but if price changes are made we also need to predict the behavior of the customers on these changes and some practical problems arise. Moreover, the demand should change somehow due to the price changes and we need to predict the expected future requests that would have occurred if we had used a certain pricing strategy. For example, assume that we generated a future request for some reservation $r$ in decision period $w$. The customer of this request will behave differently on different prices for reservation $r$. What would this customer do if we charge the reference price or increase the price of the reservation? We modeled the customers behavior on prices with the same demand elasticity model as used for the expected number of requests. The expected increase or decrease due to the demand elasticity function is now applied on single request. For example, if we receive a request and price class $k$ is charged and $DE(k) = 1.2$ then we expect an amount of 1.2 of this request. Similarly, if $DE(k) = 0.8$, then we expect an amount of 0.8 of this request. If the static pricing strategy is used, then $DE(k) = 1$ and we do not expect an increase or decrease of any requests. Therefore, the simulated expected revenue is easily found by above simulation for the static pricing strategy.

Now we only need to find the simulated expected revenue of a dynamic pricing strategy. But, if we use a dynamic pricing strategy it becomes more complex, because a practical problem arise here. As we already saw above, if we use the demand elasticity model of section 3.2 we could end up with fractional expected future requests, called *continuous demand* (also illustrated in example 4). With this continuous demand we can compute the simulated expected revenue in a similar way as described in above simulation approach, but a continuous demand is not realistic. In practice customers just accept or deny the price of a reservation. In section 4.2.1 we propose how we obtain a demand in which each customer just accepted or denies the price of a reservation. We employ a few more definition before we illustrate a continuous demand with an example.

**Definition 13** *A* ***demand stream*** $Q$ *is a set of chronological ordered simulated requests. We define* $q(n) = (r, w)$ *as the* $n^{th}$ *request of demand stream* $Q$ *which denotes the request for reservation* $r$ *and is received in decision period* $w$.

**Definition 14** *We define* $q_{r,w} \in Q$ *as the total* ***number of realized requests*** *for reservation* $r$ *in decision period* $w$ *in demand stream* $Q$*. The values* $q_{r,w}$ *are represented by the matrix* $q$ *with the reservations on the row and decision periods on the columns.*

**Definition 15** *We define* $Q^u$ *as the* ***continuous demand stream****, which is the obtained expected demand stream by applying some pricing vector* $u^k_{r,w}$ *on demand stream* $Q$.

**Example 4** *Consider the setting of example 3 and assume we have a demand stream* $Q = \{q(1), ..., q(5)\} = \{(r_1, w_1), (r_1, w_1), (r_2, w_2), (r_3, w_3), (r_1, w_3)\}$ *(see fig 5). Thus, we consider*

$$\begin{array}{cc} & \begin{array}{ccc} w_1 & w_2 & w_3 \end{array} \\ b = \begin{array}{c} r_1 \\ r_2 \\ r_3 \end{array} & \begin{bmatrix} 2.2 & 0 & 1 \\ 0 & 1.8 & 0 \\ 0 & 0 & 0.7 \end{bmatrix} \end{array} \quad \textit{and} \quad \begin{array}{cc} & \begin{array}{ccc} w_1 & w_2 & w_3 \end{array} \\ q = \begin{array}{c} r_1 \\ r_2 \\ r_3 \end{array} & \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}.$$

*As an illustration, we expected* 1.8 *requests for reservation* $r_2$ *in decision period* $w_2$ *and we actually received 1 request for reservation* $r_2$ *in decision period* $w_2$*, which is denoted by* $q(3)$ *(or* $q_{2,2}$*). If the static pricing strategy is applied on this demand stream, then the simulated expected revenue (E[SimRev]) is computed by*

$$E_{sim}[Rev] = \sum_{w \in \mathcal{D}} \sum_{r \in \mathcal{R}} b_{r,w} p_r = 2 \cdot 150 + 1 \cdot 50 + 1 \cdot 100 + 1 \cdot 150 = €600.$$

*When the price is decreased with 10% for each reservation at the beginning of each period, then the simulated expected revenue is computed by*

$$E_{sim}[Rev] = \sum_{w \in \mathcal{D}} \sum_{r \in \mathcal{R}} DE(1) \cdot q_{r,w} \cdot 0.9 p_r = \sum_{w \in \mathcal{D}} \sum_{r \in \mathcal{R}} 1.2 \cdot q_{r,w} \cdot 0.9 p_r = €768 \qquad (3)$$

*And we obatin a continuous demand stream* $Q^u = \{1.2 \cdot q(1), ..., 1.2 \cdot q(5)\} = \{1.2 \cdot (r_1, w_1), 1.2 \cdot (r_1, w_1), 1.2 \cdot (r_2, w_2), 1.2 \cdot (r_3, w_3), 1.2 \cdot (r_1, w_3)\}$*. This means that* $q^u(n) \in Q^u$ *now consumes 1.2 of the capacity instead of 1 on the night stays* $i \in A_r$ *with* $r$ *the reservation of request* $q(n)$*. Thus, if we apply a pricing strategy on demand stream* $Q$*, then we could end up with a continuous demand stream* $Q^u$*.*

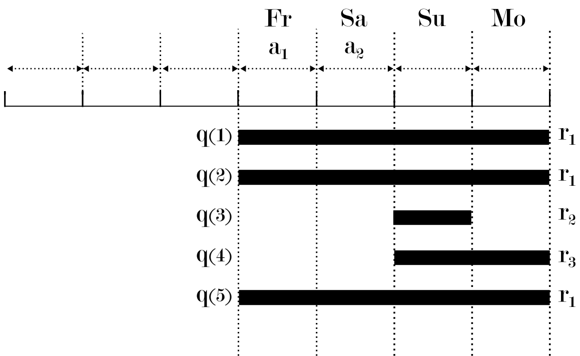


Figure **5**: Visualization of demand stream $Q$ from example 4

Note that we can perform the computation in equation (3), because we still have an infinite capacity. In section 4.2.3 we propose how $E_{sim}[Rev]$ is computed with a finite capacity.

Unfortunately, a continuous demand stream is not a realistic demand stream and the simulated expected revenue is calculated with this continuous demand stream. The simulated expected revenue of an expected *discrete demand* due to the price changes would be more realistic. We come back to this later.

#### 3.4.3 Distinction of expected revenues

The intended purpose of the simulated expected revenue is that it represents a realistic performance of a dynamic pricing strategy.

**Definition 16** *We define $\tilde{Q}^u$ as the* **discrete demand stream** *computed from the (possibly) continuous demand stream $Q^u$.*

In section 4.2.1 we propose a method to obtain a discrete demand stream $\tilde{Q}^u$ from a continuous demand stream $Q^u$.

We now formulate three different expected revenues, which are used in the remainder of this report. We first obtain an expected revenue using pricing vector $u^k_{r,w}$ and values $b^k_{r,w}$ and $p^k_r$ as proposed in example 3. Second, we obtain a simulated expected revenue using pricing vector $u^k_{r,w}$ and a possibly continuous demand stream $Q^u$ and prices $p^k_r$ as proposed in example 4. Third, we obtain a, so called, realistic expected revenue using pricing vector $u^k_{r,w}$ and a discrete demand stream $\tilde{Q}^u$ and prices $p^k_r$.

**Definition 17** *The* **expected revenue** *$E[Rev]$ is the revenue obtained using some pricing vector $u^k_{r,w}$, expected requests $b^k_{r,w}$ and prices $p^k_r$.*

**Definition 18** *The* **simulated expected revenue** *$E_{sim}[Rev]$ is the revenue obtained using using some pricing vector $u^k_{r,w}$, demand stream $Q^u$ and prices $p^k_r$.*

**Definition 19** *The* **realistic expected revenue** *$E_{real}[Rev]$ is the revenue obtained using some pricing vector $u^k_{r,w}$, demand stream $\tilde{Q}^u$ and prices $p^k_r$.*

In section 4.2.3 we describe how $E[Rev]$, $E_{sim}[Rev]$ and $E_{real}[Rev]$ are actually computed.

Eventually we want to validate that the simulated expected revenue is a good approximation of the realistic expected revenue, because we assume that this realistic expected revenue gives a better representation of the practice. Therefore, we define the following.

**Definition 20** *The* **integrality gap** *($G_{int}$) is the relative difference between the simulated expected revenue of demand stream $Q$ and the average realistic expected revenue ($\overline{E}_{real}[Rev]$) over all discrete demand stream $\tilde{Q}^u$ obtained from $Q^u$.*

The integrality gap of a pricing strategy is computed by

$$G_{int} = \frac{|\ \overline{E}_{real}[Rev] - E_{sim}[Rev]\ |}{E_{sim}[Rev]} \cdot 100\% \tag{4}$$

Note that there is no integrality gap of the static pricing strategy, because $E_{sim}[Rev] = E_{real}[Rev]$ in this case. The integrality gap is used to validate that the simulated expected revenue gives a good approximation of the simulated expected revenue.

Now it is also important to know how the expected revenue of the ILP and the simulated expected revenue relates. Therefore, we define the following.

**Definition 21** *The* ***simulation gap*** *($G_{sim}$) is the relative difference between the expected revenue and the average simulated expected revenue $\overline{E}_{sim}[Rev]$ over all (possibly) continuous demand streams $Q^u$.*

The simulation gap is computed by

$$G_{sim} = \frac{|\overline{E}_{sim}[Rev] - E[Rev]|}{E[Rev]} \tag{5}$$

The simulation gap is used to validate that the proposed ILP gives a good approximation of the simulated expected revenue.

# 4 Deterministic integer linear programming

In this section we propose a deterministic integer linear program (ILP) to find a dynamic pricing strategy which periodically assigns a single price class to all possible reservations to maximize the expected revenue. Further, a method is proposed to compute a discrete demand stream from a continuous demand stream. Next, we propose and tackle some practical problems that occur and propose the method that is used to compute the simulated and realistic expected revenue. Finally we propose two additional solution approaches which make use of the ILP solution.

## 4.1 Aggregate expected reservation pricing model

The expected number of requests $b^k_{r,w}$ is used to find the optimal price class for each reservation $r$ for every decision period $w$. The expected revenue is maximized with ILP (6) - (9) which finds the right trade off between price surcharge and change in demand, given the available capacity. The maximum expected revenue is found by assigning a single price class to each reservation $r$ at the beginning of decision period $w$ taking into account that the capacity $c_i$ is not violated for all night stays $i \in \mathcal{I}$. This problem is formulated as a deterministic integer linear program, which we call *aggregated expected reservation integer linear program* (AER-ILP). It is called the aggregated expected reservation ILP because a price class is assigned to each possible reservation separately and is based on the expected values $b^k_{r,w}$. The dynamic pricing strategy that maximize the expected revenue is defined by $\hat{u}^k_{r,w}$ which are the output variables of the AER-ILP.

$$\max \sum_{w \in \mathcal{D}} \sum_{\substack{r \in \mathcal{R} \\ a_r \geq d^1_w}} \sum_{k \in \mathcal{K}} u^k_{r,w} p^k_r b^k_{r,w} \tag{6}$$

$$s.t. \sum_{w \in \mathcal{D}} \sum_{\substack{r \in \mathcal{R} \\ a_r \geq d^1_w}} \sum_{k \in \mathcal{K}} a_{i,r} u^k_{r,w} b^k_{r,w} \leq c_i \qquad \forall i \in \mathcal{I} \tag{7}$$

$$\sum_{k \in \mathcal{K}} u^k_{r,w} = 1 \qquad \forall r \in \mathcal{R}, \forall w \in \mathcal{D} \tag{8}$$

$$u^k_{r,w} \in \{0,1\} \qquad \forall r \in \mathcal{R}, \forall k \in \mathcal{K}, \forall w \in \mathcal{D} \tag{9}$$

The objective function (6) accounts for the maximum expected revenue over the time horizon $T$. The constraints of (7) ensure that the capacity of all night stays $i$ do not violate the capacity of night stay $i$. Constraints (8) ensure that each reservation is offered at a single price each decision period. Constraints (9) ensures that $u^k_{r,w}$ is one or zero for all $k$, $r$ and $w$.

## 4.2 Validation and simulation of the expected revenue

In section 3.4.3 we argued about the integrality gap due to the continuous demand. In this section we propose the method that is used to compute a (more realistic) discrete demand from a continuous demand and is used to find this integrality gap. Further, we give an extension of the demand elasticity function, which is needed to overcame some practical problems. Finally, we propose how the simulated and realistic expected revenue are actually computed.

### 4.2.1 Discrete demand simulation

In section 3.4.2 we illustrated that the demand elasticity function gives a certain expected increase or decrease of requests and we showed that we could end up with a continuous demand stream in the simulation framework. Now we explain how we obtain a discrete demand stream $\tilde{Q}^u$ from a fractional demand stream $Q^u$.

We computed a discrete demand stream $\tilde{Q}^u$ from a continuous demand stream $Q^u$ in a simulated way. For each $q(n) \in Q^u$ the following simulation is performed to obtain $\tilde{Q}^u$. If price class $k$ is charged for the reservation of request $q(n)$ and $DE(k) < 1$, then with probability $DE(k)$ one request $q(n)$ is added to $\tilde{Q}^u$. If $DE(k) < 1$, then one request $q(n)$ is added to $\tilde{Q}^u$ and with probability $1-DE(k)$ an extra request $q(n)$ is added to $\tilde{Q}^u$. If $DE(k) = 1$, then one request $q(n)$ is added to $\tilde{Q}^u$. We use the following example to illustrate this simulation of a single request $q(n) \in Q^u$ discrete demand.

**Example 5** *Consider request $q(2) = 1.2 \cdot (r_1, w_1)$ from demand stream $Q^u$ of example 4. Also consider the demand elasticity function $DE(k)$ and the set of price classes as in example 2. If we charge price class 3 for reservation $r_1$ in decision period 1, then $DE(3) = 0.8$ and the customer of request $q(2)$ accepts the price $1.1 \cdot$ €150 = €165 with probability $DE(k) = 0.8$ and deny this price with probability $(1 - DE(k)) = 0.2$. If we charge price class 1 for reservation $r_1$ in decision period 1, then $DE(1) = 1.2$ and we receive an extra identical request with probability $1 - DE(k) = 0.2$, otherwise we only receive $q(2)$. If we charge the reference price we just receive request $q(2)$.*

With above simulation we can generate a large number of different discrete demand streams $\tilde{Q}^u$. The simulated expected revenue for each simulated discrete demand stream can be found by algorithm 1 of section 4.2.3 and we can compute $\overline{E}_{real}[Rev]$ by averaging over all simulated continuous demand streams. This average simulated expected revenue is used to find the integrality gap. These discrete demand are reasonable to use to find the integrality gap, because if we perform enough discrete demand streams and average the number of requests over all these demand streams then we end up with the (original) continuous demand stream.

### 4.2.2 Extension of the demand elasticity function

Before we propose the method that is used to find the simulated and realistic expected revenue of a certain pricing strategy, an annotation need to be made. The demand elasticity is used to find the expected number of increase or decrease of demand. Now, it could be the case that demand stream $Q$ does not contain a request for reservation $r$ in decision period $w$ while $b_{r,w} > 0$. The AER-ILP assigned a price class for this request, namely price class $k$ for which $\hat{u}^k_{r,w} = 1$. In the simulation experiment we want to know if we could have received a request for reservation $r$ if a certain price class is assigned for this reservation. As an illustration of this case, take matrix $b$ in example 4, but now we also expected 2 request for reservation $r_2$ in decision period $w_1$ and matrix $q$ is still the same. In this case, $q_{r_2,w_1} = 0$, while we expected 2 request in this week. With current demand elasticity $DE(k)q_{r_2,w_1} = 0$ for all $k$, but if we use a price class which decrease the price for reservation $r$ in decision period $w_1$, then we assume that there should be some expectation that we receive a request $q_{r,w}$ under this new price. Moreover, we assume that if we did not expect any request for reservation $r$ in decision period $w$ and $q_{r,w} = 0$, then we do not expect that we receive a request for reservation $r$ in decision period $w$ at any price class.

We extend our demand elasticity function to tackle this problem. We first propose the following.

**Proposition 1** *If $\hat{u}^k_{r,w} = 1$, $p^k_r > p_r$ and $q_{r,w} = 0$ for some $k$, $r$ and $w$, then $q^u_{r,w} = 0$.*

In words, if we did not receive a request for reservation $r$ in decision period $w$ and we increased the price of $r$ in decision period $w$, then we expect no request for reservation $r$ in decision period $w$. Now, if a pricing strategy assigns a price class which decreases the price for $r$ in decision period $w$, then their should be some probability that we received a request for $r$ in decision period $w$.

**Definition 22** *If $b_{r,w} > 0$ and $q_{r,w} = 0$ for some reservation $r$ and decision period $w$, then we call this an* ***unrealized request*** *for reservation $r$ in decision period $w$.*

If a request is an unrealized request then the demand elasticity becomes

$$DE^0_{r,w}(k) = \begin{cases} (DE(k) - 1)b_{r,w} & \text{if } DE(k) \geq 1 \\ 0 & \text{if } DE(k) < 1 \end{cases} \tag{10}$$

Function (10) states that if we decrease the price of reservation $r$ in decision period $w$, then we have an expected increase of a certain percentage, namely $DE(k) - 1$, of the number of request we did expected ($b_{r,w}$). Using definition 22 and function (10) we propose the following.

**Proposition 2** *If $b_{r,w} > 0$ and $q_{r,w} = 0$ for some reservation $r$ in decision period $w$, then $DE(k) = DE^0_{r,w}(k)$.*

Hence, an *unrealized request* has the potential to become a request if we charge a lower price for the corresponding reservation and decision period. From now on, we state that a demand stream $Q$ also contains *unrealized request*. We illustrate this using example 4, if we indeed expected two request for reservation $r_2$ in decision period $w_1$, then we have $Q = \{q(1), ..., q(6)\} = \{(r_1, w_1), (r_1, w_1), (r_2, w_1), (r_2, w_2), (r_3, w_3), (r_1, w_3)\}$, where $(r_2, w_1)$ is an unrealized request.

### 4.2.3 Computation of the revenue

The proposed AER-ILP computes a maximum expected revenue and outputs a pricing strategy vector $\hat{u}^k_{r,w}$. In this section we describe how the simulated and realistic expected revenues are computed for both static and dynamic pricing strategies.

#### Simulated and realistic expected revenue

The demand elasticity function gives the value one for all reservations $r$ and decision periods $w$ if the static pricing strategy is used. Therefore, with the simulation approach we always end up with a (realistic) discrete demand stream. The following algorithm is used to compute $E_{sim}[Rev]$ (or $E_{real}[Rev]$) of a discrete demand stream $Q$ with pricing strategy $u^k_{r,w}$.

**Algorithm 1** *For $n = 1$ to $|Q|$ (or $|\tilde{Q}^u|$) the following is performed. If request $q(n) \in Q$ (or $|\tilde{Q}^u|$) is a realized request and reservation $r$ of this request does not violate the capacity (i.e. if $c_i - 1 \geq 0 \quad \forall i \in A_r$), then $E_{sim}[Rev]$ (or $E_{real}[Rev]$) increases with $p^k_r$ with $k = \{k | u^k_{r,w} = 1\}$ and the capacity is updated with $c_i = c_i - 1$ for all $i \in A_r$. If request $q(n) \in Q$ (or $|\tilde{Q}^u|$) is an unrealized request or the reservation of this request does violate the capacity, then this request is denied and $E_{sim}[Rev]$ (or $E_{real}[Rev]$) does not increase*

The following example illustrates algorithm 1.

**Example 6** *Consider the setting of example 4, but now with a slightly different expected request matrix b. We now also expected two requests for reservation $r_2$ in decision period $w_1$. Thus,*

$$b = \begin{array}{c} \\ r_1 \\ r_2 \\ r_3 \end{array} \begin{array}{c} \begin{array}{ccc} w_1 & w_2 & w_3 \end{array} \\ \begin{pmatrix} 2.2 & 0 & 1 \\ 2 & 1.8 & 0 \\ 0 & 0 & 0.7 \end{pmatrix} \end{array} \quad \text{and} \quad q = \begin{array}{c} \\ r_1 \\ r_2 \\ r_3 \end{array} \begin{array}{c} \begin{array}{ccc} w_1 & w_2 & w_3 \end{array} \\ \begin{pmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{array}.$$

*and demand stream $Q$ contains also an unrealized request, so $Q = \{q(1), ..., q(6)\} = \{(r_1, w_1), (r_1, w_1), (r_2, w_1), (r_2, w_2), (r_3, w_3), (r_1, w_3)\}$, where $(r_2, w_1)$ is the unrealized request. Consider a (finite) capacity $c = \{3, 3, 3\}$ for the bungalow park and a static pricing strategy is applied. Then $E_{sim}[Rev]$ is computed with algorithm 1 as follows. Start at $n = 1$, so $q(1) = (r_1, w_1)$ is received at first. We obtain that request $q(1)$ fits (i.e. $c_i - 1 > 0 \quad \forall i \in A_{r_1}$), so the total simulated expected revenue increases with $p_{r_1} =$ €150 and the capacity is updated ($c = \{2, 2, 2\}$). Similarly, $q(2) = (r_1, w_1)$ fits, so the total simulated expected revenue increases with $p_{r_1} =$ €150 and the capacity is updated ($c = \{1, 1, 1\}$). Next, $q(3) = (r_2, w_1)$ is 'received' which is an unrealized request, but the total simulated expected revenue does not increases, because $DE(k) = 1$ and $DE^0_{r_2,w_1}(k) = 0$. Then, $q(4) = (r_2, w_2)$ is received and this request fits, so the total simulated expected revenue increases with $p_{r_2} =$ €50 and the capacity is updated ($c = \{1, 0, 1\}$). Now we obtain that request $q(5) = (r_2, w_2)$ does not fit, because $c_i - 1 < 0$ for $i = (Sa, Su)$ and this request is denied. Also $q(6)$ does not fit anymore. Thus, if we have a finite capacity $c = \{3, 3, 3\}$ and use the static pricing strategy we obtain*

$$E_{sim}[Rev] = 2 \cdot 150 + 1 \cdot 50 = €350.$$

It becomes a little bit more complex if $E_{sim}[Rev]$ is computed for a dynamic pricing strategy. In this case, we also have to deal with the extended demand elasticity function as described in section 4.2.2. The following algorithm is used to find $E_{sim}[Rev]$ when a dynamic pricing strategy is applied.

**Algorithm 2** *For $n = 1$ to $|Q|$ the following is performed. If $q(n) = (r,w) \in Q$ is a realized request, then $DE = DE(k)$ with $k = \{k|\hat{u}^k_{r,w} = 1\}$. Now, If value $DE$ of does not violate the capacity (i.e. if $c_i - DE \geq 0 \quad \forall i \in A_r$), then $E_{sim}[Rev]$ increases with $DE \cdot p^k_r$ and the capacity is updated with $c_i = c_i - DE$ for all $i \in A_r$. If request $q(n)$ is an unrealized request, then $DE = DE^0_{r,w}$. Now, if this $DE$ does not violate the capacity, then $E_{sim}[Rev]$ increases with $DE \cdot p^k_r$ for $k = \{k|u^k_{r,w} = 1\}$ and the capacity is updated.*

The following example is used to illustrate algorithm 2.

**Example 7** *Consider expected request matrix $b$, demand stream $Q$ and capacity $c$ from example 6. Now a dynamic pricing strategy is applied which assigns price class 1 (decrease price with 10%) for each reservation in each decision period. $E_{sim}[Rev]$ is computed with algorithm 2 as follows. Start at $n = 1$, so $q(1) = (r_1, w_1)$ is received at first. This is a realized request, so $DE = DE(1) = 1.2$. The value $DE$ fits in the capacity (because $c_i - DE > 0 \quad \forall i \in A_r$), so $E_{sim}[Rev]$ increases with $1.2 \cdot 0.9 \cdot p_{r_1} =$ €$162$ and the capacity is updated ($c = \{1.8, 1.8, 1.8\}$). Similarly, for $q(2) = (r_1, w_1)$ we have enough capacity, so $E_{sim}[Rev]$ increases with €162 and the updated capacity becomes $c = \{0.6, 0.6, 0.6\}$. Next, $q(3) = (r_2, w_1)$ is 'received' which is an unrealized request, so $DE = DE^0_{r,w}(1) = (DE(k)-1)b_{r,w} = (1.2-1)\cdot 2 = 0.2$. Value $DE = 0.2$ fits in the capacity, so $E_{sim}[Rev]$ increases with $0.2 \cdot 0.9 \cdot p_{r_2} =$ €$9$ and the updated capacity becomes $c = \{0.6, 0.4, 0.6\}$. Then, $q(4) = (r_2, w_2)$ is received, which is a realized request with $DE = 1.2$. This request does not fit anymore, because $c_i - 1.2 < 0 \quad \forall i \in A_r$, so $E_{sim}[Rev]$ does not increase and request $q(4)$ is denied. Also, request $q(5)$ and $q(6)$ do not fit and are denied. Thus, if we have a finite capacity $c = \{3, 3, 3\}$ and use the above dynamic pricing strategy, then the simulated expected revenue is denoted by*

$$E_{sim}[Rev] = 2 \cdot 162 + 1 \cdot 9 = €333.$$

In this case, the dynamic pricing strategy which decrease the price with 10% gives a lower simulated expected revenue than the static pricing strategy. Hence, we assume that this it is not the optimal pricing strategy. The simulated expected revenue is used to quantify the performance of a pricing strategy. We assume that the pricing strategy with the highest simulated expected revenue has the best performance.

## 4.3 Additional approaches

In this section we propose two additional solution approaches which come up during this project. The first solution approach is to use the dynamic pricing strategy obtained from the AER-ILP to dynamically change prices per night stays instead of prices per reservations. The second solution approach is an extension of the dynamic pricing strategy that is obtained from the AER-ILP, which includes price decisions for unexpected requests.

### 4.3.1 Night stay pricing

In this section we discuss an additional model to find a pricing strategy which use prices per night stay instead of prices per possible reservation. We developed such model because in the Stratech-RCS software the calculation of the prices for reservations are often based on prices per night stay. This means that the price for some reservation $r$ is computed by summing up the prices of all night stays $i \in A_r$.

But, pricing night stays instead of pricing each reservation separately seems a kind of sub-optimal. We argue as follows, a price for night stay $i$ has an effect on the prices of all reservations $r \in A^i$. It is harder to manage the occupancy with a *night stay pricing strategy*, because if we use night stay prices, then we can not deny single requests by charging a large price like price class $\phi$. To explain, if we use price class $\phi$ for a certain night stay, then we have no request for all reservations $r \in A^i$, which is probably undesirable. We attempt to find an appropriate night stay pricing strategy and in the computational experiments we test this strategy. We now propose two methods to compute a pricing strategy which periodically change night stay prices.

We computed two pricing strategies using night stay prices which are obtained from the output variables of the AER-ILP. We first divide the price for each reservation by the number of nights that is consumed by this reservation to obtain a night stay price for all night stays $\{i|A_r\}$. Next, we average all night stay prices $i$ of reservations $r \in A^i$ to obtain the price of night stay $i$.

**Definition 23** *$p_{i,w}$ is a **night stay pricing strategy**, where price $p_{i,w}$ is charged for night stay $i$ in decision period $w$.*

The night stay pricing strategy $p_{i,w}$ is found by

$$p_{i,w} = \frac{\left(\sum_{r \in A^i} \frac{\sum_{k \in K} \hat{u}^k_{r,w} p^k_r}{l_r}\right)}{|\{r|r \in A^i\}|}. \tag{11}$$

Where $|\{r|r \in A^i\}|$ denotes the number of reservation that consumes night stay $i$ and $\hat{u}^k_{r,w}$ denotes the output variables of the AER-ILP. Further, we define $p^*_{r,w}$ as the price for reservation $r$ in decision period $w$ that is obtained from the night stay pricing strategy and is computed by

$$p^*_{r,w} = \sum_{i \in A_r} p_{i,w}. \tag{12}$$

Now, $E_{sim}[Rev]$ is computed in a quite similar way as algorithm 2 of section 4.2.3, but we first need to mention the following. There are no price classes for reservation anymore, but just prices $p^*_{r,w}$. Therefore, the price class dependent demand elasticity function can not be used here. To overcome this, a linear demand elasticity function is used, which has the same linear slope as the price class dependent elasticity function. This demand elasticity function is based on function (1) of section 3.2. We consider

$$DE(p^*_{r,w}) = -2\frac{p^*_{r,w}}{p_r} + 3 \tag{13}$$

with extended demand elasticity function

$$DE^0_{r,w}(p^*_{r,w}) = \begin{cases} (DE(p^*_{r,w}) - 1)b_{r,w} & \text{if } DE(p^*_{r,w}) \geq 1 \\ 0 & \text{if } DE(p^*_{r,w}) < 1 \end{cases} \tag{14}$$

Where $\frac{p^*_{r,w}}{p_r}$ denotes increase (or decrease) of the price relative to the reference price. We developed two different methods to compute the simulated expected revenue of a night stay pricing strategy. In the first method night stay prices are used for all request and $E_{sim}[Rev]$ is computed in a similar way as algorithm 2, but now with demand elasticity functions (13) and (14) and prices $p^*_{r,w}$ computed with equations (11)and (12). In the second method is quite similar to the first one, but a request for reservation $r$ in decision period $w$ is now denied if $\hat{u}^{\phi}_{r,w} = 1$. The first method is called the *night stay pricing method* and the second method is called *AER-ILP night stay pricing method*

#### 4.3.2 Unexpected requests

An extra annotation need to be made on the proposed AER-ILP. Notice that the AER-ILP only gives a meaningful price class for reservation $r$ in decision period $w$ if $b_{r,w} > 0$, because if $b_{r,w} = 0$, then $u^k_{r,w}p^k_r b^k_{r,w} = 0$ for all price classes. In practice it is possible that we receive a request for reservation $r$ in decision period $w$ while we did not expected this request. We need to make a decision which price we charge for a request that we did not expected, i.e. the price for a request for reservation $r$ in decision period $w$ while $b_{r,w} = 0$

We used the following two methods to decide which price is charged for a reservation of an unexpected request. The first method is called the *reference method.* In this method the reference price is charged for the reservation of an unexpected request. If the request is not an unexpected request, then price class $k = \{k|\hat{u}^k_{r,w} = 1\}$ is charged, with $\hat{u}^k_{r,w}$ the output variables of the AER-ILP. The second method is called the *night stay method.* In this method the price for the reservation of an unexpected request is computed using prices $p^*_{r,w}$ as described in section 4.3.1. If the request is not an unexpected request, then the price class $k = \{k|\hat{u}^k_{r,w} = 1\}$ is charged. The simulated expected revenue of the reference method and the night stay method can be found with algorithm 3 and 4 respectively, which are quite similar to algorithm 2.

**algorithm 3** *For $n = 1$ to $|Q|$ perform the following. If $q(n)$ is an unexpected request and $\hat{u}^{\phi}_{r,w} = 0$, then perform algorithm 1 for this single request. If $q(n)$ is not an unexpected request and $\hat{u}^{\phi}_{r,w} = 0$, then perform algorithm 2 for this single request.*

**algorithm 4** *For $n = 1$ to $|Q|$ The following is performed. If $q(n) \in Q$ is an unexpected request and $\hat{u}^{\phi}_{r,w} = 0$, then perform algorithm 2 for this single request, but now with demand elasticity function (13) and (14) and prices $p^*_{r,w}$ computed with equation (12). If $q(n)$ is not an unexpected request and $\hat{u}^{\phi}_{r,w} = 0$, then perform algorithm 2 for this single request.*

# 5 Demand forecasting

Before we discuss the computational experiments we end up with a chapter about the demand forecasting, which also played an important role in this project. In this section a model is proposed to forecast the *request process*. The request process is the process of incoming customer reservation requests. The model is based on the work of [21], which showed to have a good performance in the hotel industry. The proposed demand model is used to generate sample paths and to find appropriate values $b_{r,w}$ for the computational experiments. The values $b^k_{r,w}$ are determined by the demand elasticity function as described in section 3.2. At the end of this chapter we propose a Monte Carlo simulator to generate sample paths.

## 5.1 Demand model

A request is typically made in a certain time period before the intended arrival day. The number of periods between an arrival day and the day of the request is called the *lead time*. A lead time of zero represents the so called *walk-in* customers who request in decision period $w$ with $d^1_w \leq a_r \leq d^e_w$. The *occupancy* is the number of objects that are occupied at a particular night stay. We employ the following definition.

**Definition 24** *The* ***total requests*** *at any period $\tau$ before arrival day $a$ is the total number of requests made exactly $\tau$ periods before the particular arrival day $a$. The* ***request curve*** *is the graph of total requests as a function of the lead time.*

In fig 6 we illustrate the number of periods $\tau$ before arrival day $a$. As illustration, decision week $w_1$ is two period before $a$ and so $\tau = 2$. Fig 7 shows an example of the request curve for the first

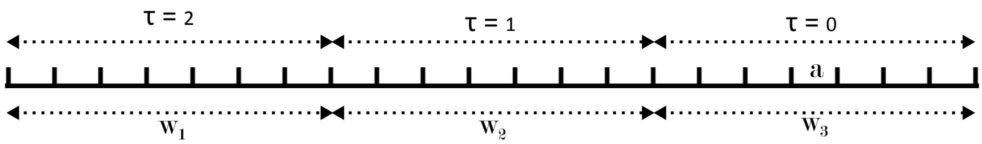


Figure **6**: Number of periods before arrival day $a$

arrival day of the ascension weekend in 2014 of a certain bungalow park in the Netherlands with periods of one day.

Clearly, requests appear over time and a certain *seasonality* is involved here. For different periods in the year there is a high occupancy and periods with a low occupancy. Also, for different periods in the year there is a higher number of requests then other periods. Seasonality is a major factor that considerably effects the number of requests. Camping and bungalows parks usually divide their arrival horizon into different seasons and set prices along these seasons. Typically, a season with a high number of expected arrivals contains higher prices than a season with a low number of expected arrivals. We assume that we have $S$ different seasons during the year and $S_a$ indicates the season in which arrival day $a$ appears.

The request process is modeled as a Bernoulli process and we employ the following definition.

**Definition 25** *We define $B(\tau, a)$ as the expected number of request for arrival day $a$ that are made $\tau$ periods before arrival.*

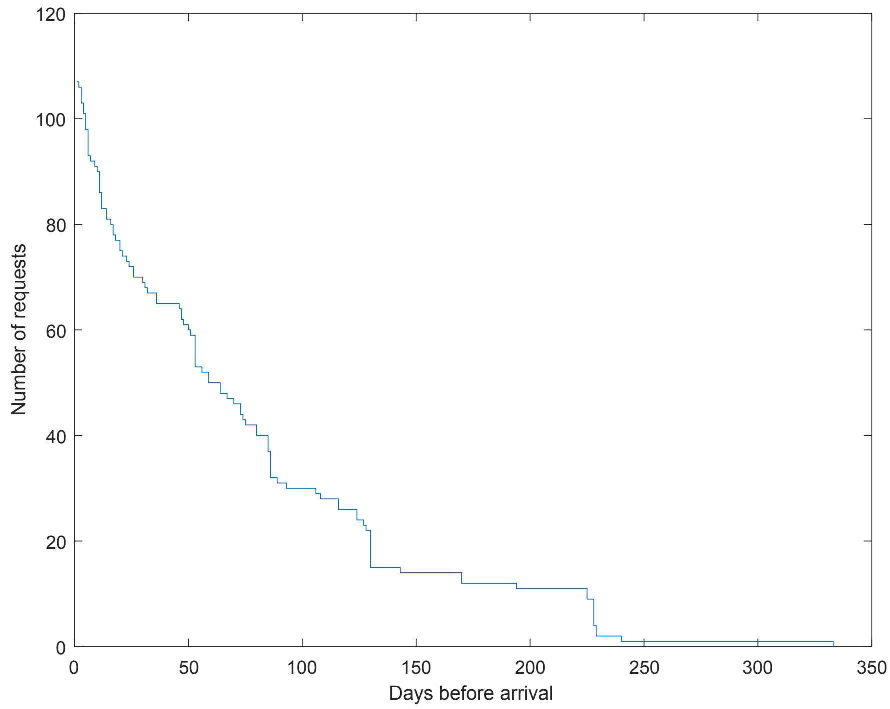


Figure **7**: Request curve

We use $B(0, a)$ to denote the expected number of walk-in customers at arrival day $a$. We assume that reservations obey a binomial distribution with probability $\rho$. Thus,

$$B(\tau, a) = N\rho$$

with $N$ the size of the 'potential' population of requests and $\rho$ the probability that a request will occur. We need to find an estimate of $B(\tau, a)$ to generate future requests. However, from the historical data we only have one realization of $B(\cdot, a)$. Of course, it will be inaccurate to base the estimate of $B(\tau, a)$ on this single observation. We analyzed the historical data of several camping and bungalow parks in the Netherlands. This data includes the date of received a request, the reservation of this request and the object type chosen by the customer of the request. From this data we observed that the shape of the request curve is quite the same within each season. As consequence, we can make our estimate more accurate by separating the effect of the request behavior $B(\tau, \cdot)$ from the arrival process $B(\cdot, a)$. In particular, if we take period $M$ as an upper bound beyond no requests occur, then we assume that

$$B(\tau, a) = B'(\tau)s(a)$$

with $\sum_{\tau=1}^{M} B'(\tau) = 1$ and $s(a)$ the number of arrivals on arrival day $a$. Now, our goal is to find a good estimate $\hat{B}(\tau, a)$ for $B(\tau, a)$ using

$$\hat{B}(\tau, a) = \hat{B}'(\tau)\hat{s}(a).$$

Where $\hat{B}'(\tau)$ and $\hat{s}(a)$ are estimates of $B'(\tau)$ and $s(a)$ respectively.

When this expected number of request for each arrival day is computed, the LoS needs to be computing of each request. The LoS plays an important role, because the LoS can actually impact the occupancy. We consider a distribution of the LoS obtained from the historical data. From the data of several bungalow parks in the Netherlands we observed that the lead time does not typically impact the LoS. The major influence factor on the LoS comes from the seasonality and day of the week (i.e. Su,Mo,..,Sa) of the arrival day $a$, defined by $a_d$. Let $l \in \mathcal{L}_a$, then we need to find the probabilities $Pr(l \mid S_a, a_d)$ for all $S$ and $a$, which is the probability that an arrival on arrival day $a$ has a length of stay of $l$.

If the expectations $\hat{B}(\tau, a)$ and probabilities $Pr(l \mid S_a)$ are known, then the request progress can be simulated over time. We developed a *Monte Carlo simulator* (MC-sim) which perform Bernoulli trails to find the number of arrivals of a certain arrival day and add a LoS to each request with the corresponding probability distribution. This simulator is described in section 5.2 in more detail. Using the stochastic model we compute $b_{r,w}$ by

$$b_{r,w} = \hat{B}(w_{a_r} - w, a_r) \cdot Pr(l_r \mid S_{a_r}, a_d). \tag{15}$$

Where $w_{a_r}$ is the decision period for which $d_w^1 \leq a_r \leq d_w^e$ and $w_{a_r} - w$ denotes the number of periods before arrival day $a$. In the work of [21] some methods are given to find the $\hat{B}'(\tau)$, $\hat{s}(a)$ and probabilities $Pr(l_r|S_{a_r}, a_d)$. In this project some easy methods are applied to find these estimates and are only used to get an indication of the values $b_{r,w}$ to come up with appropriate expected number of request in the computational experiments.

## 5.2 Monte Carlo simulator

For purpose of simulating the request process, a binomial distribution is considered together with the expectations $B(\tau, a)$ for all $\tau$ and $a$ and probabilities $Pr(l_r|S_{a_r})$ for all $r$. The two quantities $N$ and $\rho$ are set such that the following two equations are satisfied:

$$B(\tau, a) = N\rho \tag{16}$$

$$Var = N\rho(1-\rho). \tag{17}$$

To complete the simulation, each generated reservation needs to be provided with a length of stay. The MC-sim is build up in the following steps:

1. Initialize $Q = \emptyset$.
2. For $w = w_1, ...w_e$ perform the following:
   (a) For all $a \in \{a|d_w^e \geq a \geq d_w^1\}$ perform the following:
   If $B(\tau, a) \neq 0$ (with $\tau = w_a - w$) then Solve (16) and (17) for $N$ and $\rho$ with values $B(\tau, a)$ and $Var$. Generate the total number of arrivals (denoted by $X$) for arrival day $a$ with the binomial distribution and parameters $N$ and $\rho$.
   (b) If X > 0, then pick for each generated arrival a length of stay using the LoS probabilities $Pr(l|S_a, a_d)$.
   (c) If X = 0, then generate an *unrealized request* for all reservations with arrival day $a$ and length of stay $l \in \{l|Pr(l|S_a, a_d) > 0\}$.
   (d) Randomly add the requests generated in step (2b) and (2c) to $Q$ to obtain all (*realized* and *unrealized*) request of decision period $w$
3. Output demand stream $Q$

After the Monte Carlo simulation, we end up with a demand stream $Q$, which is called a sample path. So, a sample path is a chronological list of requests and such a path is used as a sample path for our numerical experiments.

**Definition 26** *A **sample path** is a demand stream $Q$ generated by the MC-sim.*

## 5.3 Maximize revenue of Monte Carlo path

In this section we proposed the computation of the maximum simulated expected revenue of a sample path. The maximum simulated expected revenue for a certain sample path is used as a benchmark in the computational experiments.

**Definition 27** *We define $q^k_{r,w}$ as the expected number of realized (and unrealized) requests at price class $k$ for reservation $r$ in decision period $w$.*

The values $q^k_{r,w}$ are computed from $q_{r,w}$ with the same demand elasticity model is used in section 3.2. Hence,

$$q^k_{r,w} = q_{r,w} \cdot DE(k) \qquad \forall k \in \mathcal{K}$$

with $DE(k)$ the same linear price class dependent function as used for computation of the values $b^k_{r,w}$. Moreover, if $q_{r,w} = 0$ and $b_{r,w} > 0$, then we use the extended demand elasticity function as described in section 4.2.2 to compute $q^k_{r,w}$. The maximum simulated expected revenue of a sample path is found by solving the AER-ILP with values $q^k_{r,w}$ instead of the values $b^k_{r,w}$ and is found in ILP (18) - (21). The output variables $\hat{v}^k_{r,w}$ denotes the optimal dynamic pricing strategy for sample path $Q$. ILP (18) - (21) is called the *optimal aggregated simulated expected reservation ILP* (OPT-ILP) and the maximum simulated expected revenue and pricing strategy $\hat{v}^k_{r,w}$ is found by solving

$$\max \sum_{w \in \mathcal{D}} \sum_{\substack{r \in \mathcal{R} \\ a_r \geq d^1_w}} \sum_{k \in \mathcal{K}} v^k_{r,w} p^k_r q^k_{r,w} \tag{18}$$

$$s.t. \sum_{w \in \mathcal{D}} \sum_{\substack{r \in \mathcal{R} \\ a_r \geq d^1_w}} \sum_{k \in \mathcal{K}} a_{i,r} v^k_{r,w} q^k_{r,w} \leq c_i \qquad \forall i \in \mathcal{I} \tag{19}$$

$$\sum_{k \in \mathcal{K}} v^k_{r,w} = 1 \qquad \forall r \in \mathcal{R}, \forall w \in \mathcal{D} \tag{20}$$

$$v^k_{r,w} \in \{0, 1\} \qquad \forall r \in \mathcal{R}, \forall k \in \mathcal{K}, \forall w \in \mathcal{D}. \tag{21}$$

The objective function (18) accounts for the maximum *simulated expected revenue* over the time horizon. The constraints of (7) ensure that the capacity of all night stays $i$ do not violate the capacity of night stay $i$. Constraints (8) ensure that each reservation is offered at a single price each decision period. Constraints (9) ensures that $v^k_{r,w}$ is one or zero for all $k$, $r$ and $w$.

# 6 Computational experiments and analysis

In this chapter we describe and discuss the computational experiments that are performed to test analyze the proposed pricing model. The experiments are divided into four subjects: validation, performance, sensitivity and practice.

The *validation experiments* are performed to test how well the proposed pricing model serves its intended purpose. We validate that the optimal expected revenue is a reasonable quantity in the sense that this value gives a good approximation of the simulated expected revenue. Furthermore, we validate that the simulated expected revenue is a reasonable quantity in the sense that this value gives a good approximation of the realistic expected revenue.

The *performance experiments* are performed to quantify the performance of the AER-ILP. We computed the simulated expected revenue of the static pricing strategy and the dynamic pricing strategy from the output variables of the AER-ILP. We also compared both pricing strategies with the benchmark obtained from the OPT-ILP.

The *sensitivity experiments* are performed to evaluate the sensitivity of the AER-ILP. We solved the AER-ILP using input values $b^k_{r,w}$ which are computed with different linear slopes of the demand elasticity function. We also solved the AER-ILP using modified input values $b^k_{r,w}$. With the output variables of both experiments we computed the simulated expected revenue to evaluate the sensitivity of the AER-ILP.

The *practical experiments* are performed to evaluate the performance of the AER-ILP in a more practical setting. We computed the simulated expected revenue in the case that unexpected request occur. Furthermore, we computed the simulated expected revenue in the case that reservations are priced using prices per night stay.

We first give an experiment setup in which we initialize all model parameters. Next, we described per subject (validation, performance, sensitivity and practice) the performed experiments and analyze the results directly after each experiment. Finally, we give a critical view on the developed model.

## 6.1 Experimental setup

The chosen model parameters attempt to represents a practical simulation setting for the high season period of a camping or bungalow park. The AER-ILP and the OPT-ILP are implemented in Matlab and all experiments are performed on an Intel Core processor 2.4 GHz.

### Time horizon and decision periods

The time horizon $T_\beta$ indicates the time horizon where $\beta \in \{4, 8, 16, 32\}$ denotes the number of weeks till the end of the horizon. For example, $T_4 = \{d_1, d_2, ..., d_{28}\}$ and $T_{32} = \{d_1, d_2, ..., d_{224}\}$. The decision horizon $\mathcal{D}$ and the arrival horizon $\mathcal{H}$ are set as follows.

- If time horizon $T_\beta$ is considered, then $\mathcal{D} = \{w_1, w_2, ..., w_e\}$, where $w_j \in \mathcal{D}$ denotes the $j^{th}$ week of $T_\beta$ and $d^e_{w_e} = d_e$.
- If time horizon $T_4$ or $T_8$ is considered, then $\mathcal{H} = \{a_1, a_2, ..., a_e\}$, with $a_e = d_e$ and $a_1$ denotes the arrival day exactly 4 weeks before $a_e$.
- If time horizon $T_{16}$ or $T_{32}$ is considered, then $\mathcal{H} = \{a_1, a_2, ..., a_e\}$, with $a_e = d_e$ and $a_1$ denotes the arrival day exactly 8 weeks before $a_e$.

Thus, a request can be made during the whole time horizon $T_\beta$, but the arrival day of the reservation of a request is in the last month or in the last two months of the horizon. We choose these time series because in practice this last month(s) could be seen as the month(s) in the summer holidays. We remark that we slightly abuse definition 3 in the experiments, because we allowed requests for reservations which have its arrival day within time period $T_\beta$ and its departure outside this horizon. For example, in time period $T_4$ a request can be made for the reservation with arrival day $d_28$ and length of stay of 14 days.

### Demand model parameters

We designed a matrix $B(\tau, a)$ for each time horizon $T_\beta$. Further, to reduce complexity, the maximum LoS is 14 days and the LoS probabilities of table 1 holds for all arrival days.

| LoS | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Pr | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.02 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.05 | 0.2 |

Table **1**: LOS probabilities

So, it is most likely that a reservation of a request has a LoS of 7 and 14 days. With $B(\tau, a)$ of each $T_\beta$ and LoS probabilities from table 1 the values $b_{r,w}$ are computed by equation (15) of section 5.1 for each $T_\beta$.

### Sample paths

Sample paths are generated with the MC-sim of section 5.2 using the matrix $B(\tau, a)$, Var = 0.3 and the LoS probabilities from table 1. We obtained that it is time consuming to find the optimal simulated expected revenue for large sample paths. The larger the time horizon and number of reservations, the more request are generated by the MC-sim. Therefore, a different amount of sample paths is used for each time horizon. The number of sample paths per time horizon $T_\beta$ are proposed in second column of table 2. The sample paths for each time horizon $T_\beta$ are generated on

| | # samples | # size Q |
|---|---|---|
| $T_4$ | 1000 | 250 |
| $T_8$ | 500 | 850 |
| $T_{16}$ | 250 | 2500 |
| $T_{32}$ | 100 | 3200 |

Table **2**: The number and size of the sample paths per time horizon

forehand with the MC-sim of section 5.2. The set $Q_\beta$ includes all sample paths for time horizon $T_\beta$. For example, $Q_4$ includes 1000 sample paths for time horizon $T_4$. The last column of table 2 gives the rounded average the number of request per sample path for each time horizon.

### Capacity

Different capacities are used for each time horizon $T_\beta$ and each night stay in the arrival horizon contains the same capacity. The capacities are divided into three categories: low (L), medium (M) and high (H). A low capacity means that the expected occupancy on each night stay is significant higher than actually fits in the capacity. A medium capacity means that the expected occupancy is not significant or slightly higher than could fits in the capacity. A large capacity means that the expected occupancy is slightly higher than fits in the capacity. The three capacity categories for time horizon $T_{16}$ are illustrated in fig 8. According to fig 8, we established that the capacity of 200 is called a low capacity, because the expected occupancy is significantly higher than the capacity. The capacity of 300 is called a medium capacity, because the expected occupancy is not significantly or slightly higher than the capacity. The capacity of 400 is called a high capacity, because the expected occupancy is slightly higher than the capacity. The used capacities for each time horizon can be found in table 3.

From now on, an *instance* is defined by $\{\beta, C\}$, with $\beta$ the considered time horizon $T_\beta$ ($\beta = 4$, 8, 16 or 32) and $C = \{L, M, H\}$ the capacity category. For example, with instance $\{16, M\}$ we consider a time horizon of 16 weeks with a capacity of 300 of each night stay.

| | | Capacity category | | |
|---|---|---|---|---|
| | | Low | Medium | High |
| Time horizon | $T_4$ | 50 | 75 | 100 |
| | $T_8$ | 100 | 150 | 200 |
| | $T_{16}$ | 200 | 300 | 400 |
| | $T_{32}$ | 300 | 450 | 600 |

Table **3**: Capacity per category and time horizon

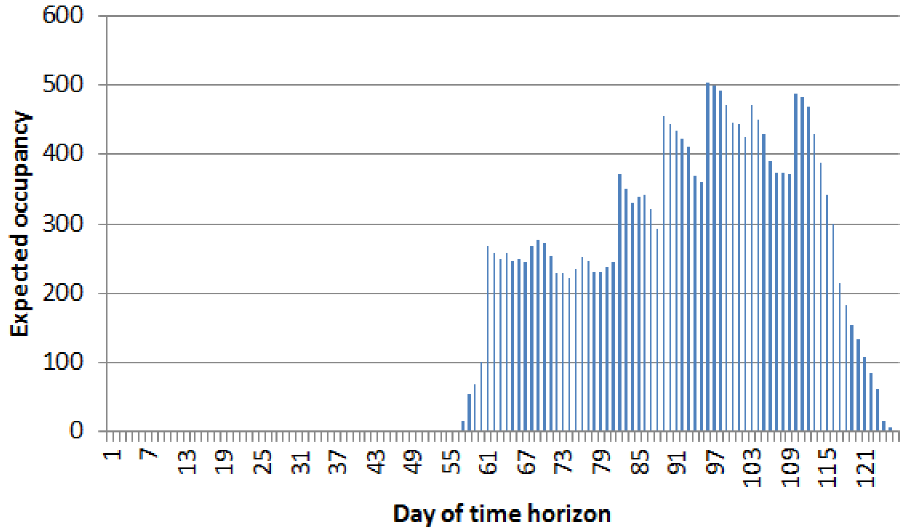


Figure 8: The expected occupancy in time horizon $T_{16}$

### Prices and demand elasticity

The price $p_r$ for each reservation $r$ only depends on the LoS of reservation $r$ and are proposed in table 4.

| LoS | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Price (€) | 40 | 80 | 117 | 155 | 193 | 230 | 270 | 305 | 343 | 381 | 418 | 456 | 494 | 531 |

Table 4: Prices in per length of stay

Furthermore, twelve price classes are considered (i.e. $\mathcal{K} = \{k_1, k_2, ..., k_{12} = \{1, 2, ..., 12\}\}$) and

$$\alpha = [0.9, 0.92, 0.94, 0.96, 0.98, 1, 1.02, 1.04, 1.06, 1.08, 1.1, \infty].$$

The linear price class dependent function $DE(k)$ is denoted by

$$DE(k) = \begin{cases} -2\alpha(k) + 3 & \alpha(k) \leq 1.1 \\ 0 & \text{if } \alpha(k) > 1.1 \end{cases} \tag{22}$$

This is the same linear function as in example 2.
Note that $k_{12}$ correspond to price class $\phi$ and $k_6$ correspond to the price class of the reference price.

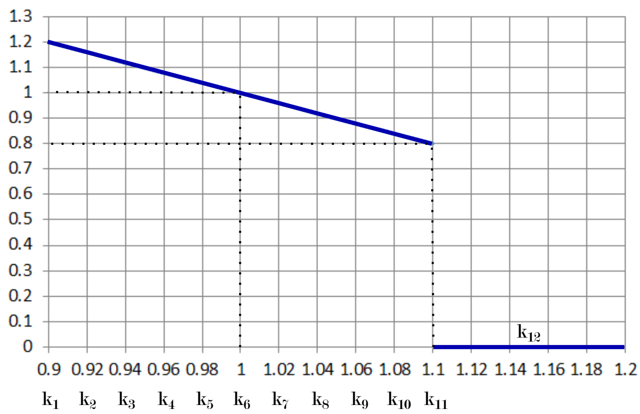


Figure **9**: Demand elasticity function DE(k) with twelve price classes

## 6.2 Validation analysis

In section 3.4.3 we defined the quantities $E[Rev]$, $E_{sim}[Rev]$, $\overline{E}_{sim}[Rev]$, $E_{real}[Rev]$ and $\overline{E}_{real}[Rev]$. With these quantities we can find the integrality gap with equation (4) and the simulation gap with equation (5) which can also be found in section 3.4.2. We set up an experiment to find the integrality gap for the pricing strategies obtained from the AER-ILP (6) - (9) and the OPT-ILP (18) - (21). Next, we set up an experiment to find the simulation gap for the AER-ILP.

### 6.2.1 Integrality gap

The integrality gap is used to validate that the simulated expected revenue gives a good approximation of the simulated expected revenue.

#### Experiment

The experiment to find the integrality gap is build up is described below and this experiment is performed 100 times for each instance $\{\beta, C\}$ with input parameters $c$ obtained from table 3 and the generated $b^k_{r,w}$ for time horizon $T_\beta$.

Experiment in steps:

1. Randomly pick one sample path from $Q_\beta$.
2. Solve the AER-ILP.
3. Compute $E_{sim}[Rev]$ for the picked sample path with dynamic pricing strategy $\hat{u}^k_{r,w}$ and algorithm 2 of section 4.2.3.
4. Generate 1000 discrete demand stream as described in section 4.2.1.
5. Compute $E_{real}[Rev]$ of each discrete demand stream with algorithm 1 of section 4.2.3.
6. Compute $\overline{E}_{real}[Rev]$ over all 1000 discrete demand streams.
7. Compute $G_{int}$ with equation (4) in section 3.4.3.

The final integrality gap of the pricing strategy of the AER-ILP is found by averaging the computed integrality gap over the 100 experiments. In a similar way we computed the integrality gap of the pricing strategy of the OPT-ILP by solving the OPT-ILP instead of the AER-ILP in step (2) and use input parameter $q^k_{r,w}$ (from the sample path picked in step (1)) instead of the values $b^k_{r,w}$.

#### Results and discussion

Fig 10 presents the integrality gap of the pricing strategies obtained from both AER-ILP and OPT-ILP for all instances $\{\beta, C\}$. We observe that the integrality gap obtained from the AER-

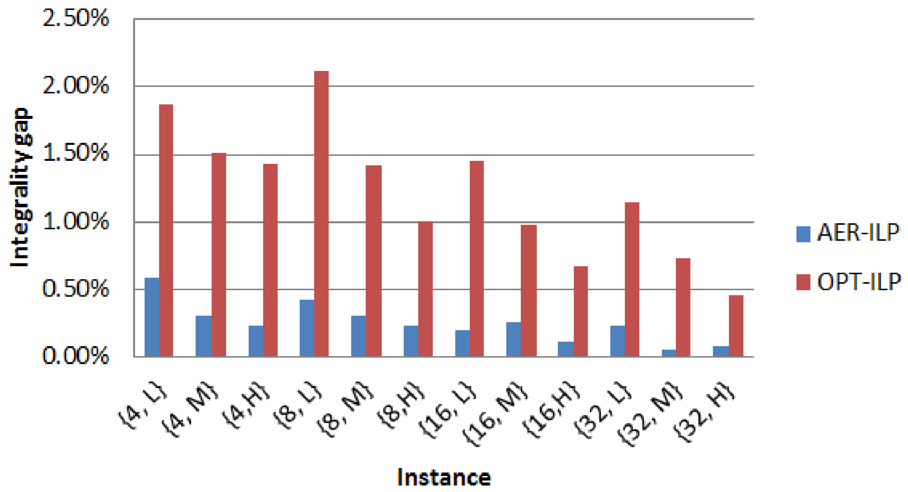


Figure **10**: Integrality Gap of the AER-ILP and the OPT-ILP

ILP and the OPT-ILP are smaller than 0.6% and 2.12% respectively. It seems that these values are not significantly high in both cases. But we need to realize that the value of the simulated expected revenues increases with the size of the time horizon, because more requests can be accepted. Therefore, the absolute difference of the simulated expected revenue may significant. In fig 11 we present the average simulated expected revenues obtained from the sample paths. We observe that an integrality gap of 1% in instance $\{32, H\}$ means that the realistic expected revenue deviates approximately €10.000 from the simulated expected revenue (± €1.000.000). But for instance $\{4, L\}$ this means that the realistic expected revenue deviates approximately €500 from the simulated expected revenue (± €50.000). Computing these absolute values in a similar way for the integrality gaps 0.6% and 2.12% still results in not significant high values although the absolute difference between $E_{real}[Rev]$ and $E_{sim}[Rev]$ is quite larger for instances with a longer horizon. Fortunately, we also observe that the integrality gap decreases the longer the time horizon. To explain this latter, the larger the amount of request the more variation is possible. The simulated extra or denied reservations do not make a significant impact on the total occupancy if there is a large amount of capacity.

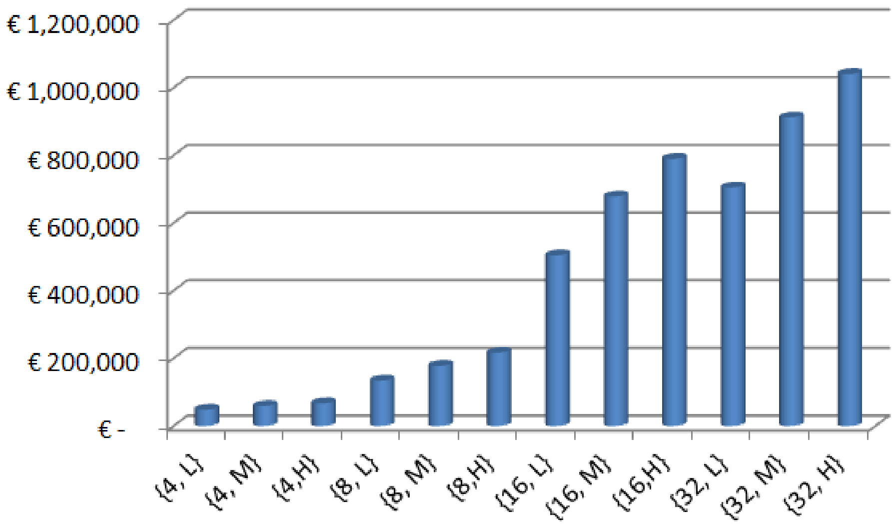


Figure **11**: The simulated expected revenue per instance

We also inspected the variance of the realistic expected revenue. The variance of each instance is computed by

$$Var_{\beta,C} = \sqrt{\sum_{j=1}^{1000}\left(E_{real}[Rev]_{j,\beta,C} - \overline{E}_{real}[Rev]\right)^2}$$

with $Var_{b,C}$ the variance of instance $\{b, C\}$ and $E_{real}[Rev]_{j,\beta,C}$ the realistic expected revenue of the $j^{th}$ computed discrete demand for instance $\{b, C\}$. For the instances with short time horizon ($\beta = 4, 8$) the variance is approximately $2\% - 3\%$ of $\overline{E}_{real}[Rev]$ and is quite similar for all capacity categories. For the instances with a long time horizon ($\beta = 16, 32$) the variance is approximately $0.5\% - 1\%$ of $\overline{E}_{real}[Rev]$ and is also quite similar for all capacity categories. The variance is small in all cases, which means that the integrality gap is quite consistent over all discrete demand streams. Taking this all into account, we assume that the simulated expected revenue gives a good approximation of the realistic expected revenue.

We further observe that the integrality gap of the AER-ILP is smaller than the integrality gap of the OPT-ILP. To explain, the OPT-ILP gives an optimal dynamic pricing policy and capacities are quite tight in this solution and continuous demand is allowed. If we simulate the discrete demand from the continuous demand, then the realistic expected revenue is always lower than the simulated expected revenue. On the other hand, the solution of the AER-ILP is not optimal for a sample path and capacities may not tight. If we simulate the discrete demand from this continuous demand, we observed that the realistic expected revenue could be higher than the simulated expected revenue. Therefore, on average we have a lower integrality gap.

Another observation is that the integrality gap decreases if the capacity of the instance increases. We can argue this similar as above, the bigger the capacity the more request can be accepted and the more variation is possible in the discrete demand.

### 6.2.2 Simulation gap

The simulation gap is used to validate that the AER-ILP gives a good approximation of the simulated expected revenue.

#### Experiment

The experiment to find the simulation gap is build up as described below and this experiment is performed for all instances $\{\beta, C\}$ with input parameters $c$ obtained from table 3 and the generated $b^k_{r,w}$ for time horizon $T_\beta$.

Experiment in steps:

1. Compute $E[Rev]$ by solving the AER-ILP (6) - (9).
2. Compute $E_{sim}[Rev]$ for each sample path of $Q_\beta$ with pricing strategy $\hat{u}^k_{r,w}$ and algorithm 2 of section 4.2.3.
3. Compute $\overline{E}_{sim}[Rev]$ over all sample paths $Q_\beta$.
4. Compute $G_{sim}$ with equation (4) of section 3.4.3.

#### Results and discussion

Fig 12 presents the simulation gap of the AER-ILP for all instances $\{T_\beta, c\}$. We noticed that the simulation gap is lower than 2.87% for each instance and the simulation gap is decreasing by the size of the capacity for instances with time horizon $\beta$ = 8 and 16. For the instances with time horizon $\beta$ = 4 and 32 we see the same effect, except that the instance with medium capacity shows a lower simulation gap than the instance with high capacity. Besides, the effect of the

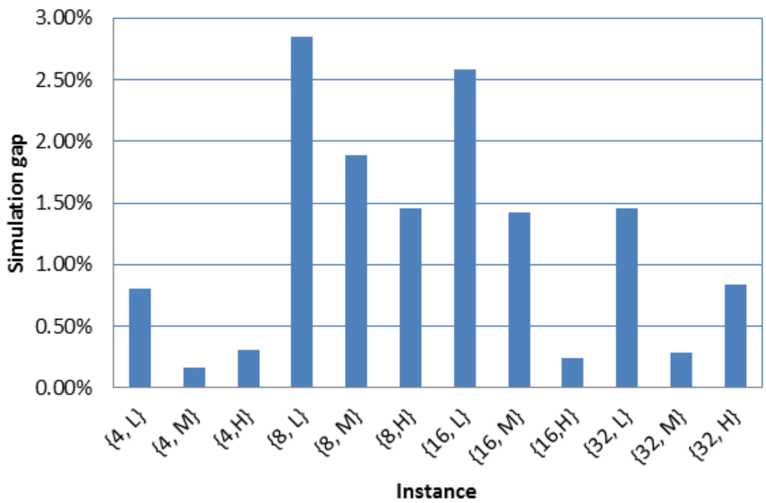


Figure **12**: Simulation Gap

simulation gap on the absolute difference of simulated expected revenue is also not significantly high as already argued in section 6.2.1. Further, we can not observe some consistencies in the results of the simulation gap, which could be caused by the randomly generated $B(\tau, a)$ for each time horizon. It could be the case that the AER-ILP computes a better approximation of the simulated expected revenue for different expected requests matrices. Fortunately, the simulation gap is quite small along all instances.

We also inspected the variance of the simulated expected revenue for each instance, which is computed by

$$Var_{\beta,C} = \sqrt{\sum_{j=1}^{|Q_\beta|} \left( E_{sim}[Rev]_{j,\beta,C} - \overline{E}_{sim}[Rev] \right)^2}$$

with $Var_{b,C}$ the variance of instance $\{b, C\}$ and $E_{real}[Rev]_{j,\beta,C}$ the simulated expected revenue of the $j^{th}$ sample path of instance $\{b, C\}$. For the instances with shortest time horizon ($\beta = 4$) the variance is approximately $3.5 - 4\%$ of the mean absolute difference, for the instances with time horizon of 8 weeks ahead ($\beta = 8$) the variance is approximately $1.5\% - 2\%$ of the mean absolute difference, and for the instances with a long horizon ($\beta = 16, 32$) the variance is approximately $0.5\% - 1\%$ of the mean absolute difference. In all cases the variance was quite similar along the capacity categories. The simulation gap of the instance with the shortest time horizon shows the smallest simulation gap, but the highest variance. Overall, the variance is quite small, especially for the instance with a long time horizon, which means that the simulation gap is also quite consistent over all sample paths.

Taking this all into account, we assume that the expected revenue gives a good approximation of the simulated expected revenue.

## 6.3 Performance analysis

In this section we describe two experiments to quantify the performance of the AER-ILP. We assumed that the pricing strategy which generates the highest simulated expected revenue has the best performance. The first experiment is used to find the performance of the dynamic pricing strategy from the AER-ILP relative to the static pricing strategy. The second experiment is used to find the performance of the dynamic pricing strategy from the AER-ILP relative to the benchmark obtained from the OPT-ILP.

### 6.3.1 Revenue improvement

We first evaluate if the dynamic pricing strategy of the AER-ILP leads to a revenue improvement relative to the static pricing strategy.

#### Experiment

Below experiment finds $E_{sim}[Rev]$ for the dynamic pricing strategy obtained from the AER-ILP and the static pricing strategy. This experiment is performed for each instances $\{\beta, C\}$ with parameters $c$ and the generated $b^k_{r,w}$ for time horizon $T_\beta$.

Experiment in steps:

1. Solve the AER-ILP.
2. Compute $E_{sim}[Rev]$ for each sample path of $Q_\beta$ using pricing strategy $\hat{u}^k_{r,w}$ and algorithm 2 of section 4.2.3.
3. Compute $E_{sim}[Rev]$ for each sample path using the static pricing strategy and algorithm 2 of section 4.2.3.
4. Compute $\overline{E}_{sim}[Rev]$ over all $E_{sim}[Rev]$ computed in step (2) and step (3) separately.

#### Results and discussion

Fig 13 presents the performance of the AER-ILP relative to the static pricing strategy for all instances $\{T_\beta, c\}$. This figure shows that the dynamic pricing strategy of the AER-ILP has a better performance than the static pricing strategy in each instance. The figure also shows that the AER-ILP performs better on instances with a small time horizon relative to the static pricing strategy. Thereby, the AER-ILP performs better on instances with a low capacity. To explain these occurrences, we need to take a look at the distribution of the used price classes from the output variables of the AER-ILP. See fig 14 for a price class distribution of instances $\{16, 200\}$, $\{16, 300\}$ and $\{16, 400\}$. This figure shows that price class 11 and 12 are frequently used in the instances with low capacity. In contrast, price class 1 is frequently used in instances with high capacity. This is not surprising, because for an instance with a low capacity we expect a significantly higher occupancy than fits in the capacity. A lot of request need to be denied (i.e. use price class 12) or need to be given a higher price (i.e. price class 11) to meet the capacity constraints. On the other hand, if we have a high capacity, then the number of ('desirable') requests can be increased by using price class 1 and not many request need to be denied to meet the capacity constraint. So,

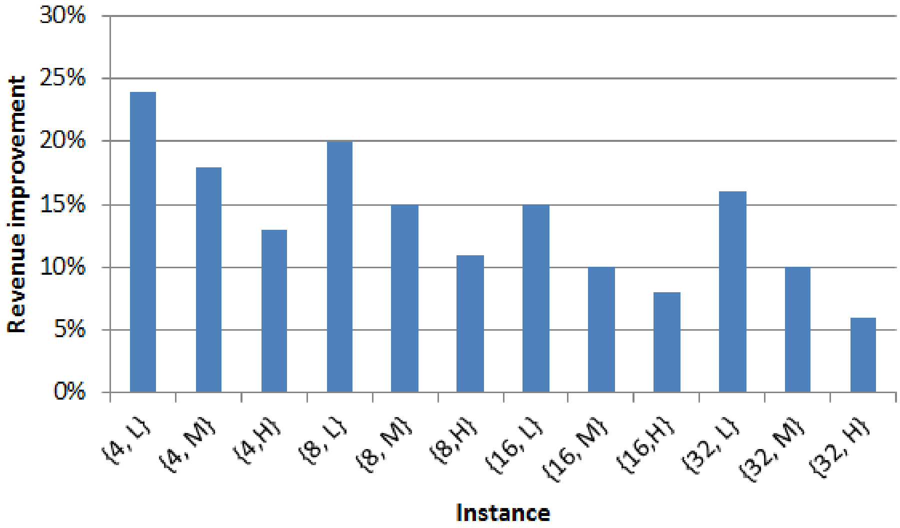


Figure **13**: Percentage improvement of the simulated expected revenue of AER-ILP relative to the static pricing strategy

with the AER-ILP we get a solution where we deny 'undesirable' request and accept 'desirable' requests. With the static pricing strategy we need to accept all request by its reference price and this latter (apparently) leads to a sub optimal result. This effect is the strongest on instances with a short horizon and low capacity.

### 6.3.2 Benchmark

The benchmark is found by solving the OPT-ILP and this value is compared to the simulated expected revenues of the static pricing strategy and dynamic pricing strategy obtained from the AER-ILP.

#### Experiments

The experiment to find the benchmark is build up as described below and this experiment is performed for all instances $\{\beta, C\}$ with input parameters $c$ from table 3 and the generated $b^k_{r,w}$ for time horizon $T_\beta$.

Experiment in steps:

1. Compute the maximum $E_{sim}[Rev]$ of each sample path $Q_\beta$ by solving the OPT-ILP for each sample path.
2. Compute the maximum $\overline{E}_{sim}[Rev]$ over all sample paths of $Q_\beta$.

We also use the $\overline{E}_{sim}[Rev]$ of the AER-ILP of previous experiment.

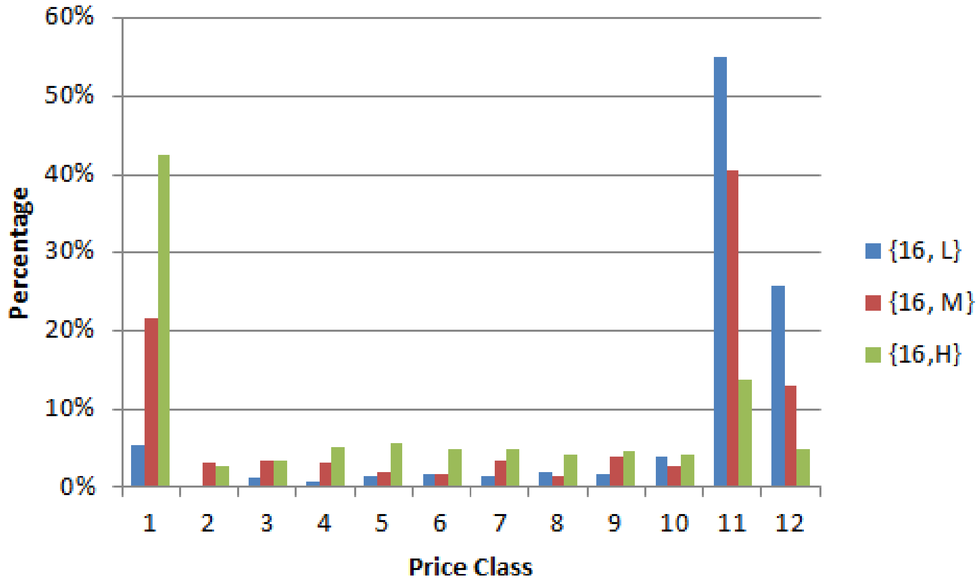


Figure **14**: Price class distribution of instances $\{16, 200\}$, $\{16, 300\}$ and $\{16, 400\}$

**Result and discussion**

Fig 15 presents the performance of the AER-ILP and the static pricing strategy relative to the benchmark for all instances $\{\beta, C\}$. A performance of 0.95 indicates that the simulated expected revenue is 95% of the maximum simulated expected revenue. First observe that the performance of the AER-ILP is close to the OPT-ILP for all instances. If we take a closer look to the distribution of the price classes of both AER-ILP and OPT-ILP, then we can see that the distributions of the AER-ILP and OPT-ILP are quite the same (see fig 16). The exact price class assignment is to complex to analyze, but is seems that the AER-ILP uses a quite same price class assignment as the OPT-ILP, because it computes a simulated expected revenue of around 96% of the optimal simulated expected revenue for each instance.

Additionally, in fig 13 we observed that the AER-ILP obtains the highest revenue improvement on the instance with the lowest capacity and shortest time horizon. Thereby, we noticed that the static pricing strategy has the worst performance relative to the benchmark on this instance (see also fig 15). Apparently, the chosen model parameters of this instance are unfavorable for the static pricing strategy, because the performance of the AER-ILP is consistent over all instances.

We remark that we also wanted to find the performance of the relaxation of the AER-ILP, but due to a Matlab error these results were not reliable and could (unfortunately) not be analyzed in this section.

### 6.3.3 Computational time

The CPU times of the AER-ILP for all instances $\{T_\beta, c\}$ can be found in fig 17. The larges CPU time is observed at the instance with the longest time horizon and the lowest capacity. This is not surprising, because this is the 'hardest' problem to solve, because this instance does have to most optimization variables and the capacities are very tight. Overall, the CPU times are quite good,

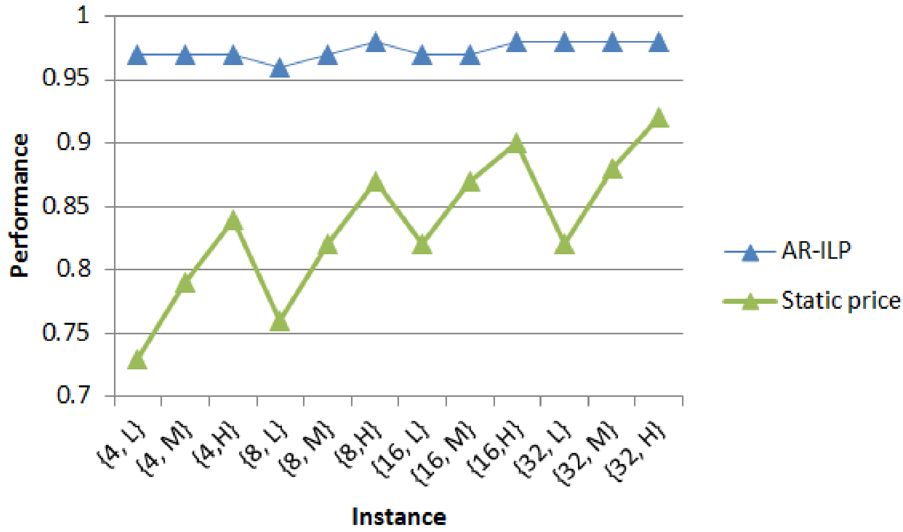


Figure **15**: AER-ILP and AER-ILP Relaxation performance relative to the benchmark

in the sense that it can be used in practice. Instances $\{T_{32}, L\}$, $\{T_{32}, M\}$ and $\{T_{32}, H\}$ are instances that we could have in practice and price changes occur weekly, so there is enough time to run the ILP.


Figure **16**: the price class distribution for the AER-ILP and OPT-ILP for instance $\{16, M\}$

## 6.4 Sensitivity analysis

In this section we describe two experiments to evaluate the sensitivity of the AER-ILP. In the first experiment we use different linear slopes of the demand elasticity function to compute the values $b^k_{r,w}$, which are used as input of the AER-ILP. In the second experiment we modified the values $b_{r,w}$ and use these as input of the AER-ILP. Moreover, in this latter experiment the values $b^k_{r,w}$ are computed by the demand elasticity function as described in the experimental set up.

### 6.4.1 Modified demand elasticity function

In this experiment different linear slopes are used. The demand elasticity function (22) has a linear slope of -0.2. The linear slopes -0.3, -0.4 and -0.5 are used to evaluate the sensitivity. For example, a linear slope of -0.4 indicates that a price increase of 10% leads to a 40% decrease of demand and a price decrease of 10% leads to a 40% increase of demand.

#### Experiment

The experiment to evaluate the performance of the AER-ILP with different linear slopes of the demand elasticity function is build up as described below. This experiment is performed for each instances $\{\beta, C\}$ and each modified linear slope with input parameters $c$ and the $b^k_{r,w}$ generated with the modified demand elasticity function for time horizon $T_\beta$.

Experiment in steps:

1. Solve the AER-ILP.
2. Compute $E_{sim}[Rev]$ of each sample of $Q_\beta$ with dynamic pricing strategy $\hat{u}^k_{r,w}$ from (1) and algorithm 2 of section 4.2.3.

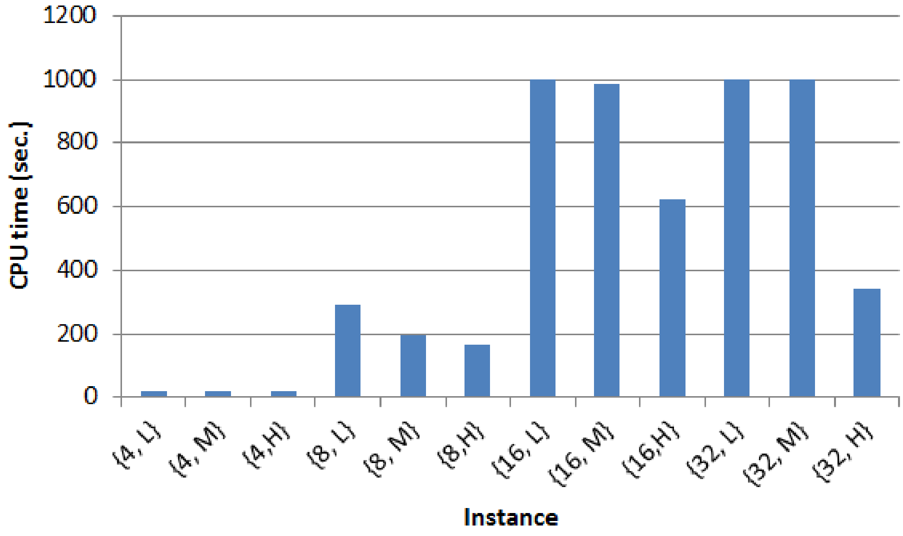


Figure **17**: CPU times in seconds of the AER-ILP and the OPT-ILP

The same benchmark is used as found in the experiment of section 6.3.2 to quantify the performance of the AER-ILP with different slopes of the demand elasticity function.

#### Results and discussion

Fig 18 presents the performance of the AER-ILP for different linear slopes of the demand elasticity function on instance $\{8, M\}$. The results of the other instances are quite the same and can be found in Appendix B. Fig 18 shows that if the linear slope decreases with 0.1, then the performance

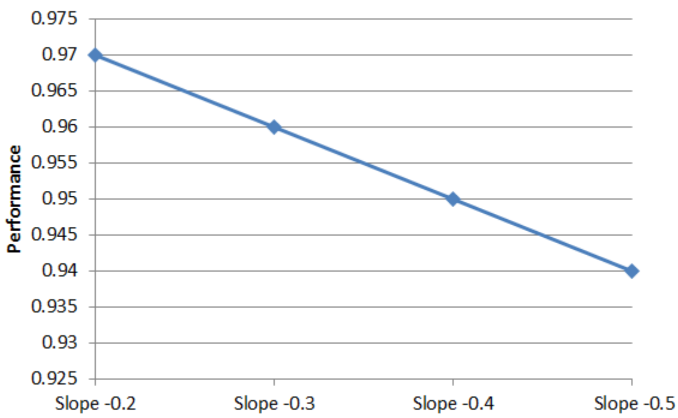


Figure **18**: Performance AER-ILP on instance $\{8, M\}$ for different linear slopes of the demand elasticity function

decreases with 1%. So, a different linear slope of the demand elasticity function does not have a significant influence on the performance of the AER-ILP. Even when a slope of -0.5 is used, the performance is still way better than the static pricing strategy. Intuitively, this is not what we expected, because the demand elasticity has plays an important role in the AER-ILP. We noticed

| | $b_{r,w} < 5$ | $b_{r,w} \geq 5$ |
|---|---|---|
| Modification 1 | 0.1 | 0.5 |
| Modification 2 | 0.2 | 1 |
| Modification 3 | 0.4 | 1.5 |
| Modification 4 | 0.75 | 2 |
| Modification 5 | 1 | 3 |

Table **5**: The values that are randomly add or subtract

in the experiments that the values $b_{r,w}$ are often quite very small, because the reservations are highly aggregated. The demand elasticity add or subtract a certain percentage of this values. A certain percentage of a small number, is still a small number. Hence, the overall effect of the modified linear slopes is not significant as we see in the results.

In this setting the AER-ILP is not very sensitive for a change in the demand elasticity function. Unfortunately, this experiment does not give us that much information about the sensitivity of the AER-ILP in general, because we made the assumption that the demand elasticity function is always linear with a slope of -0.2 and the reservation were highly aggregated. In practice, we will possibly have an different reservation aggregation and a different demand elasticity model. This could lead to other sensitivities of the AER-ILP.

### 6.4.2 Modified expected number of requests

Because the values $b^k_{r,w}$ are often quite small, we used another approach in this experiment. Now we randomly add or subtract a certain value to every $b_{r,w}$ as follows. If $b^k_{r,w} < 5$, then we add or subtract a 'small' value. If $b^k_{r,w} \geq 5$, then we add or subtract a 'large' value. Five experiments were performed and the values that were randomly added of subtracted can be found in table 5. To remark, if a value is subtracted while $b_{r,w} = 0$, then $b_{r,w}$ remains zero. And, if a value is subtracted from $b_{r,w}$ and $b_{r,w}$ becomes $< 0$, then $b_{r,w} = 0$.

#### Experiment

Experiment in steps:

1. Solve the AER-ILP with the modified expected request matrix as input.
2. Compute $E_{sim}[Rev]$ of each sample of $Q_\beta$ with dynamic pricing strategy $\hat{u}^k_{r,w}$ from (1) and algorithm 2 of section 4.2.3.

We use the same benchmark as the experiment in section 6.3.2 to quantify the performance of the AER-ILP with a modified values $b_{r,w}$ as input.

**Results and discussion**

Fig 19 shows the results of above experiment for all instances $\{\beta, M\}$. The results of the other instances are quite similar and can be found in Appendix B. The figure shows that the performance

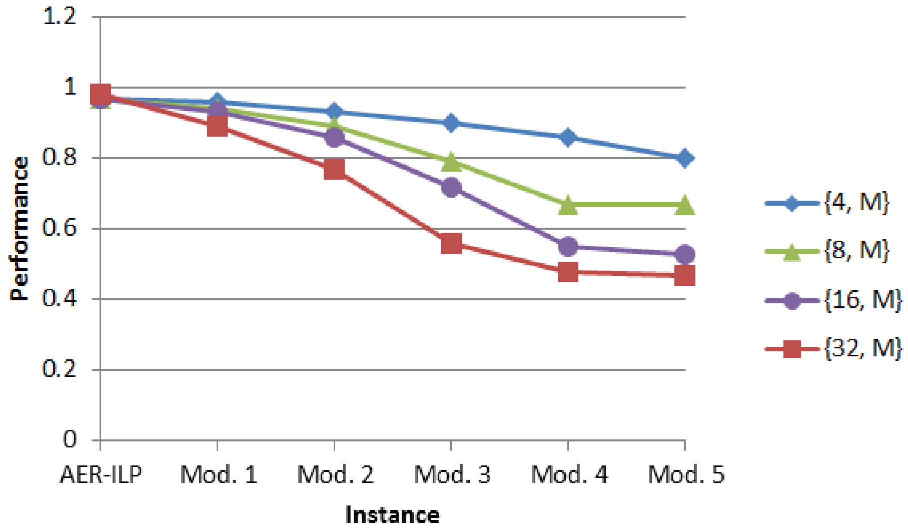


Figure **19**: Performance of the AER-ILP for all modifications on instances $\{\beta, M\}$

of the AER-ILP is more sensitive on instances with a long time horizon than on instances with a short time horizon. To explain, afterwards we noticed that a long time horizon has a larger matrix $b$ and contains more zeros than the matrix $b$ of the shorter time horizon. For the instances with a long time horizon more zeros become a positive number and values close to zero could become zero. The method to modify matrix $b$ is quite 'rigorous' on the larger matrices $b$. Therefore, it is not surprising that the instances with a larger horizon are more sensitive in this experiment. In this setting the AER-ILP is quite sensitive because the modified input leads to a (approximately) 40% decrease of performance if modification 5 is used as.

Also in this experiment the sensitivity of the AER-ILP does not become totally clear, because the modification had a different effect along the instances. So, here we mention that additional experiments are needed to find the sensitivity of the performance of the AER-ILP on a modified expected request matrix.

## 6.5 Practical analysis

In this section we propose two experiments. On one hand, an experiment is propose to measure the performance of the AER-ILP on sample paths including unexpected requests. On the other hand, an experiment is proposed to quantify the performance of the AER-ILP using night stay prices as described in section 4.3.1.

### 6.5.1 Unexpected requests

The MC-sim of section 5.2 does not generate unexpected request, but in practice they could appear. To generate a demand stream $Q$ including unexpected request, we added an extra step after step 2d of the MC-sim. We added the following step:

2e) consider every arrival day $a \in \{a|d_w^1 \geq a \geq d_w^e\}$. If $B(\tau, a) = 0$ then we randomly add a request to $Q$ with arrival day $a$ and a LoS taken from the LoS distribution. If this reservation is already made in one of the other periods $w$ of $Q$, then randomly remove one of these request, otherwise do nothing.

In this way we do not add to much extra request to the sample path, but the sample path now contains unexpected requests. For each instance $\{\beta, C\}$ we generated the same number of sample paths as proposed in table 2. The set $Q_\beta^m$ includes all sample paths for time horizon $T_\beta$ using above modified MC-sim.

#### Experiment

The experiment to evaluate the performance of the AER-ILP using the reference method and the night stay method as described in section 4.3.2 is build up as described below. This experiment is performed for each instances $\{\beta, C\}$ with input parameters $c$ and the generated $b_{r,w}^k$ for time horizon $T_\beta$.

Experiment in steps:

1. Solve the AER-ILP
2. Compute $E_{sim}[Rev]$ of each sample of $Q_\beta^m$ with the *reference method* and algorithm 3 of section 4.3.2.
3. Compute $E_{sim}[Rev]$ of each sample of $Q_\beta^m$ with the *night stay method* and algorithm 4 of section 4.3.2.

#### Results and discussion

Fig 20 proposes the results of above experiment for instance $\{8, 150\}$. We analyze only this instance, because we noticed that the results for the other instances were quite similar. Fig 20 shows that the night stay pricing method has the best performance. Furthermore, the strategy to assign the reference price to each unexpected request is still better than the static pricing strategy. From this experiment we conclude that it is better to use the night stay method than the reference method to price unexpected request.

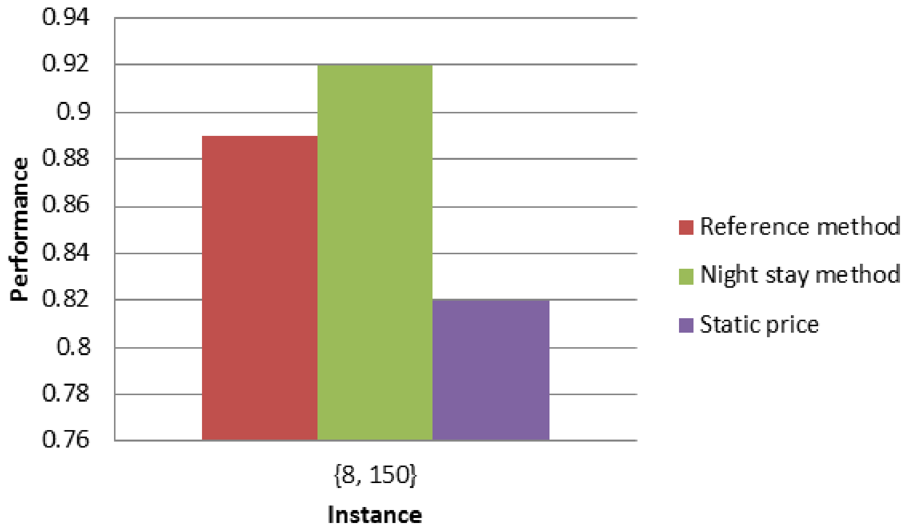


Figure **20**: Performance of reference method, night stay method and the static pricing strategy for instance $\{8, 150\}$

We also observe that the performance of the AER-ILP is decreased with 4%. The unexpected request does have influence on the performance, but is not significant. It is not surprisingly that the performance is decreased, because now the same effect occurs as we argued in section 6.3.1. That is that in both proposed method the 'undesirable' request are not denied, but are offered by the price suggested by the methods. Apparently, this lead to weaker performance.

### 6.5.2 Night stay prices

In section 4.3.1 we proposed a method to compute the prices per night stay $p_{i,w}$ by equation (11) of section 4.3.1. Furthermore, we proposed two different night stay pricing method to obtain the simulated expected revenue using night stay prices.

#### Experiment

The experiment to evaluate the performance of the AER-ILP using the *night stay pricing method* and the *AER-ILP night stay pricing method* as described in section 4.2.3 is build up as described below. This experiment is performed for each instances $\{\beta, C\}$ with input parameters $c$ and the generated $b^k_{r,w}$ for time horizon $T_\beta$.

Experiment in steps:

1. Solve the AER-ILP
2. Compute $E_{sim}[Rev]$ of each sample of $Q_\beta$ with the 'night stay pricing method' as described in section 4.2.3.
3. Compute $E_{sim}[Rev]$ of each sample of $Q_\beta$ with the 'AER-ILP night stay pricing method' as described in section 4.2.3.

#### Results and discussion

Fig 21 shows the results of above experiment. The results show that the 'AER-ILP night stay

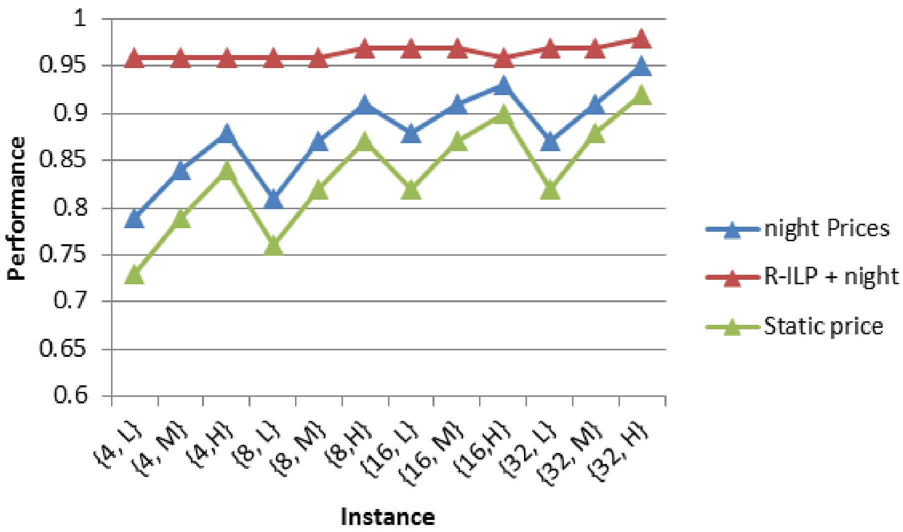


Figure **21**: performance of using night stay prices

method' obtains a better performance than the 'night stay pricing method'. We also observe that both methods still perform better than the static pricing strategy. It is remarkable to see that the 'AER-ILP night stay pricing method' almost has the same performance as the AER-ILP. It seems that the most important task is to price 'desirable' requests and deny 'undesirable' requests. This experiment supports the argumentation in section 4.3.1, where we argued that prices per night stay leads to sub-optimal results. With prices per night stay it is harder to manage the occupancy, because the price of night stay has influence in the price of all reservation $r \in A^i$.

# 7 Critical review

In this section we give a critical review on the proposed pricing model and experiments. We discuss the assumptions and model choices that are made during this project which makes the model less realistic.

In the proposed pricing model we assumed that the demand for each object comes from different customers and that the demand for an object is lost when the object is not available or the price is higher than the customers willing to pay. Additionally, the model is build for a single object type, assuming that customers do not choose along different object types. In practice, customers probably choose along different reservations and along different object types. For example, consider a customer that wants to book for a campground for the last two weeks of the summer holidays. We assumed that this customer is only interested in a campground for this specific two weeks in the summer holiday and if there is no campground available for these two weeks (or the price is high), then we lose this customer. But in practice, this customer will probably look for alternatives (other week or other object type) and we probably not lose this customer. But we do not know the impact of this in. In the hotel industry it is already showed that choice based demand models show better performance than the independent demand model, but for camping and bungalow parks it could be possible that the independent demand model gives a good approximation of this effect.

We also noticed that the proposed pricing model is only suitable in the high season. To explain, we noticed that a camping or bungalow park is fully occupied in the high season, while this is not the case for the low and mid season. The values $b^k_{r,w}$ are small for the low and mid season and

$$\sum_{w \in \mathcal{D}} \sum_{r \in \mathcal{R}} \sum_{k \in \mathcal{K}} a_{i,j} u^k_{r,w} b^k_{r,w} \leq c_i$$

always holds $\forall i \in \mathcal{I}$ even if we decrease the price for all $r$ and $w$. In this case we do not have this capacity constraint and we just use the price class for which we have the highest value of $p_r \cdot b^k_{r,w}$. This gives us a trivial solution as we already obtained in example 3 in section 3.3. Therefore, another pricing model is needed to find appropriate pricing strategies for the low and mid season.

Next, we modeled the demand elasticity with a price class dependent function, which holds for all reservations in every decision period. In practice, the demand elasticity probably depends on the seasonality, time of booking and perhaps also on the reservation. So, a more sophisticated demand elasticity model might be more appropriate in practice. Moreover, we suspect that the performance of the AER-ILP would be similar for other kind of demand models, but we did not test this in this project.

Further, we developed a simulation framework to evaluate the performance of a pricing strategy. We attempted to model the customers behavior on price changes with the demand elasticity model of section 3.2 and the extended demand elasticity model of section 4.2.2. With these models we computed the simulated expected revenue as described in section 4.2.3 to quantify the performance of a pricing strategy. It is very likely that customer behavior on price changes can be better modeled with other (more sophisticated) models to obtain a more realistic quantification of the performance of a pricing strategy. This may lead to other performance results of the proposed pricing model.

We also developed some methods to make pricing decisions for unexpected request and some methods to price reservations with night stay prices. Also in this case, there are probably other (more sophisticated) methods to make pricing decisions for unexpected request and find appropriate night stay prices to maximize revenue. These method may shows better results than those proposed in this project.

We also proposed a demand forecasting model which performs well in the hotel industry. But most reservations in the hotel industry occur less then 3 months in advanced, while the lead time for camping and bungalows could be more than 9 months. The influence of this aspect is not tested in this project. Therefore, the proposed demand model may be inaccurate or not suitable for camping and bungalow parks.

Furthermore, an annotation need to be made on the capacity constraint of the AER-ILP. This constraint does not take into account the planning board that is used by the users of Stratech-RCS. In practice, every accepted request is assigned to a specific object in the Stratech-RCS software. Some of them are fixed in the planning board, while others can be shifted. It could be the case that there is enough capacity, but that some request can not be accepted. An example of this is given in fig 22. It is possible to reschedule the planning board to overcome this, but some camping and bungalow parks allow customers to choose a specific object type and the reservation of this request is then fixed in the planning board. This aspect is not taken into account in the proposed pricing model.

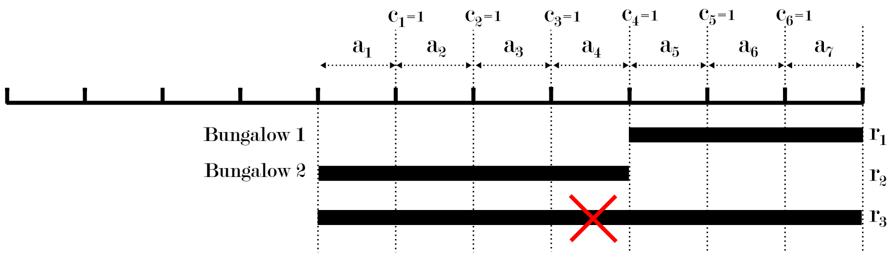


Figure **22**: Because $r_1$ and $r_2$ are planned on bungalow 1 and bungalow 2 respectively, $r_3$ does not fit in the planning board, while there is enough capacity available.

Finally we want to mention that the performed experiments take into account only a single experimental set up. All users of Stratech-RSC have there own 'characteristics' and manage their camping or bungalow parks in many different ways. For example, a part of the customers of Stratech impose Saterday to Saterday arrivals, which lead to a less highly aggregation of reservation than customer who impose daily arrivals. It would be desirable to perform experiments with different experimental settings to analyze the performance of the proposed pricing model. For example, different levels of reservation aggregation, other demand elasticity functions, other (or more) price classes, different expected request matrices or length of stay probabilities. Also, other (more sophisticated) sensitivity experiments could be performed to quantify the sensitivity of the AER-ILP. Additionally, it is also interesting to compare the performance of a daily decision period or monthly price decision periods.

# 8 Summary and recommendations

In this last chapter we give a summary of the work that is done within this project and end up with recommendations for future research and give some recommendations for Stratech on future work that has to be done to develop an appropriate dynamic pricing tool to support camping and bungalow parks to increase their revenue.

## 8.1 Summary

In this project we developed an algorithm for camping and bungalow parks that computes a dynamic pricing strategy which periodically assigns a price class to each possible reservation to maximize the total expected revenue. In a simulation framework we evaluated the performance of the obtained pricing strategy.

We started by modeling the dynamic pricing problem as a network revenue management problem and derived a model for the demand elasticity with a linear price class dependent function. Next, we developed an integer linear program, called aggregated expected reservation ILP (AER-ILP), which maximize the total expected revenue using the expected number of request at each price class as main input parameter. The AER-ILP finds the right trade off between price surcharge and expected change in demand, given the available capacity. The output variables of the AER-ILP are used as a pricing strategy which assigns a certain price class to each possible reservation for each period in which price changes are allowed. This price class decreases, increases or holds the initial price of the reservation.

We set up a simulation framework for the validation and analysis of the performance of the AER-ILP. In this framework we defined the so called simulated expected revenue, which quantifies the performance of the AER-ILP.

The computational experiments are used to validate the model, analyze the performance and sensitivity of the AER-ILP and to analyze the performance of of the AER-ILP including unexpected request and use a night stay pricing strategy. We validated that the proposed pricing model serves its intended purpose and gives a reasonable output. Furthermore, the AR-ILP propose strong performance relative to the static pricing strategy and the benchmark. We also found that the CPU time of the AER-ILP is suitable for practice purpose. The sensitivity of the AER-ILP did not become very clear in these experiments, because the reservations were highly aggregated in the simulation setting and one of the experiments was quite 'rigorous'.

We also did performed some additional experiment. We noticed that a big part of the users of Stratech-RSC make use of night stay pricing and derive the price for a reservation on the price per night stay. We argued that it might be sub-optimal to depend reservation prices on prices per night stay. In the computational experiments we found that this is indeed the case. Furthermore, the AER-ILP is based on the expected number of request which are determined at forehand. We noticed that in practice we can receive requests that we did not expected. Therefore, we performed an experiments to evaluate the performance of the AER-ILP by simulating unexpected requests. This effect was not significant and the AER-ILP still performed well.

From this project we conclude that the developed AER-ILP could perform well in practice provided that an independent demand model gives a good approximation of the demand of camping and bungalow park customers and that we have a reliable demand elasticity model and expected demand forecast. From the experimental result we suspect that the AER-ILP is valuable for the high season weekends (ascension, Pentecost, Easter, Fronleichnam etc.), because customers want to book for such specific weekend and there are no alternatives. The independent demand model probably gives a good representation of the customer demand.

## 8.2 Recommendations future research

In the critical review on the model we already give some aspects which could improve the proposed pricing model. From a practical point of view the dynamic pricing problem is a very complex research area, because there are a lot of aspect in the customers booking behavior. A task for future research is to (accurately) analyze customer behavior. This can, for example, be done with web clicks analysis or performing (extensive) data analysis on the historical data. Moreover, it is an important tasks to come up with a reliable demand forecasting model. In this project we only proposed the demand model, but did not test its performance in the setting of camping and bungalow parks.

As already mentioned in the literature review, LP's generates good pricing strategies when they are frequently resolved. For future research is would be valuable to evaluate the performance of the AER-ILP in practice (and on real date) and frequently resolve the algorithm with updated capacities and expected requests.

Futhermore, due to the lack of time in this project we could not perform and analyze all valuable experiments. In the critical review we already mentioned that it would be desirable to perform more (sophisticated) experiments, especially for the analysis of the sensitivity. Also, experiments in different experimental set up settings are not performed yet and are valuable to found out for which users this pricing model is suitable.

Another aspect for future research is to include planning board constraints, as we already mentioned in the critical review. This makes the problem more complex, but makes it more realistic. Every user of Stratech-RCS uses this planning board and so it is an important aspect to take into account.

## 8.3 Recommendations for Stratech

We end up with some recommendation for Stratech and discuss the work that has to be done to develop an appropriate pricing tool for their customers.

First of all, Stratech needs to realize that integrating dynamic pricing models requires time and investment. Time is needed to gather data of customer behavior on price changes. This data is necessary to obtain a sophisticated demand elasticity model. A camping or bungalow park needs to start with small price changes to collect this data. The work of [3] proposed a model which uses price multiplier to obtain the price for each reservation. This model is an accessible model to make some (small) price changes and gather data of the customer booking behavior. Furthermore, Stratech needs to invest in a reliable demand forecasting model. We mention that a demand forecasting is often company specific and needs to be careful managed. Stratech needs to keep in mind that it is possible that there is not a generic demand forecasting model that can be used for each company. Another valuable investment would be to analyze web clicks on the sites of the camping and bungalow parks. This data will give much more information than the historical booking data.

Furthermore, Stratech can think about a pricing model which uses a choice based demand model, which is (probably) a more realistic model. Also in this case, additional data is needed to find customer choice probabilities. Besides, this project proposed a model which is only suitable for the high season periods, so Stratech need to come up with another pricing model to determine appropriate prices in the low and mid season.

Also, it was an important result that it is essential to offer the 'right' reservations. Apparently, if the reservation are highly aggregated, then there are a lot of 'undesirable' reservation, which cause lower revenues. Stratech could do some extra research on this topic.

As a final note, there is still much research to do and one of the most important task is to gather data on customers booking behavior and customers responses on price changes, because these are the most important input parameters for pricing models.

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# 1 Appendices

## A Notation and definitions

All definition and notions are summarized in this appendix.

| | |
|---|---|
| $T = \{d_1, ..., d_e\}$ | The time horizon with $d_1$ and $d_e$ the first and last day respectively. |
| $\mathcal{I} = \{1, ..., m\}$ | The set of offered night stays, with night stay $i \in \mathcal{I}$ |
| $\mathcal{H} = \{a_1, ..., a_e\}$ | The arrival horizon with $a_1$ and $a_e$ the first and last day respectively |
| $\mathcal{D} = \{w_1, ..., w_e\}$ | The decision horizon with $w_1$ and $w_e$ the first and last period respectively |
| $d_w^1, d_w^e$ | The first and last day of decision period $w$. |
| $\mathcal{L}_a = \{1, ...L\}$ | The set of possible LoS at day $a \in \mathcal{H}$ and $L$ the maximum length of stay. |
| $\mathcal{R} = \{(a, l) \| a \in \mathcal{H}, l \in \mathcal{L}_a\}$ | The set of considered reservations. $m$ is the size of set $\mathcal{R}$. |
| $r = (a, l)$ | Reservation $r$ consist of an arrival day $a$ and LoS $l$ |
| $a_r$ | Arrival day of reservation $r$ |
| $l_r$ | Length of stay of reservation $r$ |
| $c = \{c_1, ..., c_m\}$ | The network capacity with $c_i$ the capacity for night stay $i$ |
| $A$ | The $m \times n$ night stay consumption matrix. $A^i$ is the reservation incidence vector. $A_r$ is the night stay incidence vector. |
| $\mathcal{K}$ | The set of price classes |
| $p_r^k$ | The price of reservation $r$ at price class $k \in \mathcal{K}$ |
| $b_{r,w}^k$ | The expected number of requests for reservation $r$ at price class $k$ in period $w$. $b_{r,w}$ is the expected number of requests for reservation $r$ in period $w$ following the reference price |
| $DE(k)$ | linear price dependent demand elasticity function |
| $\hat{u}_{j,w}^k$ | The optimal solution variables of AER-ILP (6) - (9) |
| $\hat{v}_{j,w}^k$ | The optimal solution variables of OPT-ILP (18) - (21) |
| $Q$ | Demand stream with chronological ordered requests |
| $Q^u$ | Possibly continuous demand stream obtained from pricing strategy |
| $u_{r,w}^k$ | |
| $\tilde{Q}^u$ | Discrete demand stream obtained from $Q^u$ |
| $p_{i,w}$ | The price charged for night stay $r$ in period $w$ |
| $p_{i,w}^*$ | The price charged for night stay $i$ in period $w$ obtained from $\hat{u}_{j,w}^k$ |

| | |
|---|---|
| $B(\tau, a)$ | The expected number of arrivals on day $a$ that are made exactly $w$ days before arrival |
| $N$ | The number of 'potential' population |
| $\rho$ | The probability that a request will occur |
| $S_a$ | Indicates the season in which day $a$ appears |
| $M$ | Upper bound, the period beyond no requests occur |
| $B'_{S_a}(w)$ | The shape of the request process with $\sum_{w=1}^{M} B'_S(w) = 1$ |
| $s(a)$ | The level of the request process, i.e. the (expected) number of arrivals on day $a$ |
| $R(\tau, a)$ | The actual number of arrivals at day $a$ that are made exactly $\tau$ periods before arrival |
| $a_d$ | Weekday of arrival day $a$ |
| $Pr(l \mid S_a, d_a)$ | The probability that a reservation for arrival day $a$ has a LoS of exactly $l$ days. |
| AER-ILP | Aggregated expected reservation integer linear program |
| OPT-ILP | Simulated Aggregated expected reservation integer linear program |

# B Computational results

| **arrival day** \| $\tau$ | **1** | **2** | **3** | 4 |
|---|---|---|---|---|
| **1** | 0.00 | 3.26 | 5.97 | 0.00 |
| **2** | 0.00 | 4.37 | 4.15 | 2.20 |
| **3** | 6.11 | 2.59 | 9.21 | 0.00 |
| **4** | 3.06 | 0.00 | 9.06 | 5.72 |
| **5** | 0.00 | 0.00 | 0.00 | 46.88 |
| **6** | 0.00 | 0.00 | 4.70 | 2.06 |
| **7** | 7.58 | 2.79 | 9.65 | 0.00 |
| **8** | 0.00 | 0.00 | 0.00 | 0.00 |
| **9** | 0.00 | 2.32 | 0.00 | 0.00 |
| **10** | 7.07 | 3.24 | 3.40 | 0.00 |
| **11** | 0.00 | 0.00 | 0.00 | 6.51 |
| **12** | 0.00 | 0.00 | 0.00 | 44.21 |
| **13** | 4.58 | 3.90 | 0.00 | 8.87 |
| **14** | 0.00 | 4.21 | 0.00 | 0.00 |
| **15** | 0.00 | 2.47 | 0.00 | 3.24 |
| **16** | 0.00 | 8.10 | 0.00 | 4.35 |
| **17** | 0.00 | 0.00 | 0.00 | 5.73 |
| **18** | 3.63 | 0.00 | 0.00 | 8.00 |
| **19** | 0.00 | 0.00 | 0.00 | 0.00 |
| **20** | 4.58 | 5.63 | 0.00 | 8.41 |
| **21** | 0.00 | 5.32 | 0.00 | 0.00 |
| **22** | 0.00 | 0.00 | 0.00 | 0.00 |
| **23** | 0.00 | 0.00 | 0.00 | 0.00 |
| **24** | 8.85 | 4.94 | 4.32 | 0.00 |
| **25** | 0.00 | 8.62 | 6.48 | 0.00 |
| **26** | 32.72 | 32.36 | 0.00 | 42.68 |
| **27** | 0.00 | 2.31 | 5.91 | 2.53 |
| **28** | 6.42 | 2.73 | 2.41 | 0.00 |

Table **6**: Expected request matrix for time horizon of 4 weeks ahead

| **Instance** | **AR-ILP** | **Static price** | **Revenue** |
|---|---|---|---|
| {4, L} | 0.97 | 0.73 | €48.096,- |
| {4, M} | 0.97 | 0.79 | €58.836,- |
| {4,H} | 0.97 | 0.84 | €66.910,- |
| {8, L} | 0.96 | 0.76 | €133.994,- |
| {8, M} | 0.97 | 0.82 | €177.453,- |
| {8,H} | 0.98 | 0.87 | €216.258,- |
| {16, L} | 0.97 | 0.82 | €504.289,- |
| {16, M} | 0.97 | 0.87 | €679.187,- |
| {16,H} | 0.98 | 0.9 | €789.800,- |
| {32, L} | 0.98 | 0.82 | €704.681,- |
| {32, M} | 0.98 | 0.88 | €911.867,- |
| {32, H} | 0.98 | 0.92 | €1.040.340,- |

Table **7**: Performance AER-ILP and static pricing strategy relative to the benchmark

| **Instance** | **k = 1** | **k = 2** | **k = 3** | **k = 4** | **k = 5** | **k = 6** | **k = 7** | **k = 8** | **k = 9** | **k = 10** | **k = 11** | **k = 12** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| {4, L} | 45.81% | 0.25% | 0.25% | 0.00% | 0.74% | 0% | 0.74% | 0.99% | 0.99% | 1.23% | 17.98% | 31.03% |
| {4, M} | 47.78% | 0.25% | 0% | 0.00% | 0.74% | 1.48% | 1.48% | 1.48% | 2.22% | 4.19% | 21.43% | 18.97% |
| {4,H} | 47.29% | 0.25% | 0.99% | 0.99% | 1.48% | 1.48% | 5.17% | 1.72% | 3.94% | 3.69% | 19.21% | 13.79% |
| {8, L} | 12.99% | 0.28% | 1.02% | 0.56% | 0.56% | 1.11% | 0.83% | 1.39% | 2.32% | 2.23% | 40.72% | 35.99% |
| {8, M} | 13.64% | 1.02% | 0.74% | 1.02% | 1.21% | 1.02% | 2.60% | 0.56% | 2.13% | 1.95% | 46.94% | 27.18% |
| {8,H} | 16.88% | 1.30% | 2.04% | 2.41% | 2.69% | 1.58% | 3.43% | 1.95% | 2.50% | 6.03% | 43.69% | 15.49% |
| {16, L} | 5.33% | 0.18% | 1.10% | 0.74% | 1.53% | 1.56% | 1.44% | 1.96% | 1.56% | 3.83% | 54.90% | 25.84% |
| {16, M} | 21.55% | 3.10% | 3.40% | 3.16% | 2.02% | 1.78% | 3.49% | 1.35% | 3.95% | 2.76% | 40.40% | 13.03% |
| {16,H} | 42.43% | 2.67% | 3.37% | 5.03% | 5.55% | 4.81% | 4.84% | 4.08% | 4.63% | 4.14% | 13.70% | 4.75% |
| {32, L} | 19.79% | 0.64% | 0.66% | 1.05% | 1.52% | 2.18% | 1.74% | 1.05% | 2.20% | 2.10% | 46.50% | 20.57% |
| {32, M} | 32.75% | 3.06% | 2.50% | 2.86% | 4.82% | 4.13% | 5.09% | 2.13% | 4.06% | 3.33% | 25.83% | 9.44% |
| {32, H} | 62.21% | 2.50% | 2.15% | 3.42% | 1.71% | 2.15% | 3.33% | 2.20% | 3.74% | 3.11% | 10.08% | 3.40% |

Table **8**: Price Class distribution

| **Instance** | **AER-ILP** | **OPT-ILP** |
|---|---|---|
| {4, L} | 20.27 | 1.97 |
| {4, M} | 20.17 | 1.62 |
| {4,H} | 20.97 | 1.52 |
| {8, L} | 291.32 | 68.06 |
| {8, M} | 194.31 | 95.35 |
| {8,H} | 164.67 | 62.63 |
| {16, L} | 999.87 | 367.72 |
| {16, M} | 988.44 | 323.37 |
| {16,H} | 622.83 | 185.29 |
| {32, L} | 999.72 | 417.31 |
| {32, M} | 999.89 | 335.29 |
| {32, H} | 343.15 | 126.35 |

Table **9**: CPU times AER-ILP and OPT-ILP

| Instance | Slope -0.2 | Slope -0.3 | Slope -0.4 | Slope -0.5 | Static price |
|---|---|---|---|---|---|
| {4, M} | 0.97 | 0.96 | 0.95 | 0.95 | 0.79 |
| {8, M} | 0.97 | 0.96 | 0.95 | 0.94 | 0.82 |
| {16,M} | 0.97 | 0.96 | 0.95 | 0.95 | 0.87 |
| {32,M} | 0.98 | 0.97 | 0.96 | 0.96 | 0.88 |

Table **10**: Performance of AER-ILP with different demand elasticity functions

| Instance | AER-ILP | Mod. 1 | Mod. 2 | Mod. 3 | Mod. 4 | Mod. 5 | Static price |
|---|---|---|---|---|---|---|---|
| {4, L} | 0.97 | 0.94 | 0.93 | 0.88 | 0.8 | 0.8 | 0.73 |
| {4, M} | 0.97 | 0.96 | 0.93 | 0.9 | 0.86 | 0.8 | 0.79 |
| {4,H} | 0.97 | 0.97 | 0.95 | 0.89 | 0.83 | 0.8 | 0.84 |
| {8, L} | 0.96 | 0.93 | 0.86 | 0.79 | 0.73 | 0.71 | 0.76 |
| {8, M} | 0.97 | 0.94 | 0.89 | 0.79 | 0.67 | 0.67 | 0.82 |
| {8,H} | 0.98 | 0.95 | 0.91 | 0.82 | 0.67 | 0.61 | 0.87 |
| {16, L} | 0.97 | 0.9 | 0.8 | 0.69 | 0.62 | 0.63 | 0.82 |
| {16, M} | 0.97 | 0.93 | 0.86 | 0.72 | 0.55 | 0.53 | 0.87 |
| {16,H} | 0.98 | 0.96 | 0.91 | 0.8 | 0.59 | 0.52 | 0.9 |
| {32, L} | 0.98 | 0.82 | 0.66 | 0.56 | 0.55 | 0.53 | 0.82 |
| {32, M} | 0.98 | 0.89 | 0.77 | 0.56 | 0.48 | 0.47 | 0.88 |
| {32, H} | 0.98 | 0.92 | 0.84 | 0.66 | 0.47 | 0.45 | 0.92 |

Table **11**: Performance AER-ILP with modified expected request matrix