UNIVERSITY OF TWENTE.

Bachelor Thesis

The impact of price elasticity in the sales and operations planning

PUBLIC VERSION

In this public version, the names of the company and products are replaced by fictive names. Some sections and figures are adjusted or completely removed. Due to the anonymisation, there may be inconsistencies in the data and calculations.

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Industrial Engineering & Management

Enschede
July, 2017

Company X

Document title The impact of price elasticity in the sales and operations planning

Bachelor thesis for the bachelor program Industrial Engineering and Management at the University of Twente

Status Public report

Date 20-07-2017

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Management summary

The monthly allocation of the raw material at Company X (further referred to as CX) is planned with a sales and operations planning (S&OP) model. The S&OP model allocates the raw material to aggregated product categories, so-called plan products. Around 20% of the total raw material supply is processed into the plan product 'product 1'.

CX has a market share of more than 50% in product 1 and their production quantities influence the market price. The effect between quantity and price is usually expressed in the term price elasticity. Currently, CX does not consider the effect of price elasticity in their S&OP model. Previous research expresses the relation between quantity and price mathematically and shows positive results in a fictional case study with piecewise linear approximations. Piecewise linear approximations are necessary to implement price elasticity in the S&OP model of CX. This research investigates the impact of implementing price elasticity in the actual S&OP model with real data of CX. Therefore, the main research question of this thesis is:

What is the impact of the implementation of price elasticity in the sales and operations model of Company X?

At first, we investigate the current situation. Processing raw material into product 1 results in the by-production of product 3 and product 4. The same applies for the processes other products, like product 2, which results in the by-production of product 4. To measure the valorisation, or added value, for processing raw material into a certain products and corresponding by-products, CX categorised its products and by-products in certain 'baskets'. We will analyse the financial effects of the products with the change of the valorisation of these baskets due to price elasticity.

The analysis of the current situation is followed by a literature review about price elasticity in sales and operations planning. The literature shows positive results. However, it does not provide a direct answer to our main research question. In additional to the literature review, we find a general approach for piecewise linear approximations. We adopt the general approach from the literature to construct piecewise linear approximations of the revenue curve of product 1.

Then, we specify the details of the piecewise linear approximation of product 1 and adjust the S&OP model to implement the new prices and quantities. The main assumption and therefore the main limitation of this research is that we only consider the effect price elasticity at product 1 and assume that all other factors remain constant. We assume that competitors do not respond to the decisions of CX and that the demand and prices of other products do not change. We assume that prices of product 1 only respond to the quantity of the same month.

After we made the assumptions and implemented price elasticity, we investigate the impact of price elasticity in the S&OP model. By means of experiments, we find that a model with 10 segments provides adequate accuracy for the implementation of price elasticity. The model with price elasticity suggests to reduce the annual product 1 quantity with 4.1% and to increase the product 2 quantity with 11.8% compared to the current model. This change results in an annual improvement of the valorisation of 68 thousand euros, which is 0.52% of the revenue of product 1, product 2 and corresponding by-products.

The impact of price elasticity is significantly higher in months in which the valorisation of product 1 is low relative to the valorisation of product 2. Therefore, the impact of price elasticity can significantly improve when the valorisation of product 1 and product 2 become closer to each other than in the current market situation.

The sensitivity analysis shows that the financial improvement is nullified when the forecasted prices of product 1 and product 2 deviate with 6 to 10% over the whole year. Because these situations are highly unlikely, the improvements in valorisation have a low risk regarding the input data. However, we discover larger risks from the worst-case scenario. Then, a loss of 220 thousand euros can be made due to change in quantity of product 1 to product 2. The worst-case scenario shows a lower risk in months in which the valorisation of product 1 is low relative to the valorisation of product 2 and confirms the increased potential of these months.

Although the annual financial improvement of price elasticity is 68 thousand in our research, we do not recommend starting to use price elasticity as it is used in this research. We do not recommend this, because we assume in this study that competitors do not response and that the prices only react on the quantity of the same month. However, these assumptions are not consistent with reality and thus limit the practical feasibility of this research. In the worst-case scenario, CX makes an expected loss of 220 thousand euros. Nevertheless, the results of price elasticity are more promising when the valorisation of product 1 becomes low relative to the product 2 valorisation. Therefore, we recommend combining the concepts of this study with more knowledge about the product 1 market to benefit from the effects of price elasticity when the valorisation of product 1 becomes close to the product 2 valorisation.

As future research, we suggest to extend the S&OP model with more intelligent market behaviour. We recommend investigating how to implement the response of competitors, enhanced price behaviour and price elasticity of other products into the S&OP model, while keeping the human effort for the S&OP within bounds.

Preface

To complete the bachelor Industrial Engineering & Management at the University of Twente, I performed at in-depth research assignment at Company X about the impact of price elasticity on their sales and operations planning. The findings of this study are reported in this bachelor thesis.

For the past three month I worked at Company X at Department X. This gave me the opportunity to experience the complexity and possibilities of the supply chain and planning of Company X. I gained a lot of knowledge about these processes during my time at Department X. I would like to thank my colleagues for the interesting conversations and their help during my research.

This report would not be here without the support of my supervisors. At first, I would like to thank my supervisor at Company X. He guided me through the research and we had valuable discussions about the ideas and insights of this study. I would also like to thank Matthieu van der Heijden, my supervisor from the University of Twente, for his valuable feedback and scientific insights.

Finally, I would like to thank my friends and family for their support during this research.

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Glossary

Basket	A group of products consisting of a main product and corresponding by-products to which raw material can be allocated and completely processed.
Disturbance capacity	Spare capacity planned to anticipate on disturbances
ET	Equivalence table
CX	Company X
Hub	S&OP division of Company X
Raw material flow	The amount of raw material allocated to a certain basket
P-effect	The financial effect in revenue caused by a change in price
Plan product	Aggregated product category consisting of products with comparable production processes and market behaviour
Price elasticity	Change in price caused by the change in quantity
Q-effect	The financial effect in revenue casued by a change in quantity
Raw material-product 1 ratio	The ratio of additional raw material sales which ends up in product 1
S&OP	Sales and Operations Planning
Valorisation	The process of creating value or adding value to a product
VBA	Visual Basic for Applications, the programming language within Microsoft® Excel

1. Introduction

In this chapter, we introduce the research conducted at Department X of Company X. The research investigates the effects of implementing price elasticity in the Sales and operations Planning model. At first, we give a brief description of Company X, Department X and the Sales and operations Planning process. We describe previous research at Company X about price elasticity. Afterwards we describe the aim of this research with the problem statement, relevance, scope and research questions.

1.1. Company X

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Department X

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Sales & Operations Planning

With the monthly Sales & Operations Planning (S&OP) process, CX determines for the coming 18 months how to allocate the raw material supply in a product mix that adds the most value, given the capacity, the raw material supply and the market. CX has to process all raw material supplied. Therefore, the S&OP is an important process to balance supply and demand. To make optimal decisions, the S&OP planners use an S&OP tool in which the supply chain of CX is modelled on an aggregated level. The S&OP tool financially optimizes the planning.

The process is a close collaboration between Department X and 10 S&OP divisions of CX, called 'hubs'. Those hubs have their own product category (e.g. product 1 or product 2). Hubs are not location specific and multiple hubs can serve one factory. For example, a factory can make product 1, product 2 and product 3, all planned by another hub.

The S&OP process brings the information of those hubs together to distribute the raw material in the most valorising way and therefore prevents the negative effects of local optimization of the hubs. The hubs provide Department X with their aggregated demand volumes, available production hours and sales prices for the coming 18 months. Besides the information of the hubs, the forecast of the raw material supply and the external raw material sales are used. The output of the S&OP process is a plan which allocates the raw material to the hubs and their aggregated product categories, called plan products.

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Figure 1.1 - The S&OP process divides the raw material over the S&OP hubs

Previous research

Previously, Company X worked together with Wageningen University to test and implement a new S&OP tool and investigate the practical and financial feasibility of including price elasticity.

The research of Van Haperen (2016) concluded that all necessary functionalities are present in the new tool to implement price elasticity. A fictional case study was performed and shows a significant financial improvement. Van Haperen (2016) concluded that further research is needed with actual data of Company X. He also stated that piecewise linear approximations are necessary to implement price elasticity in the S&OP model. Van Haperen (2016) expressed the relationship between the change in price and the change in quantity as follows.

For every decrease in quantity (Q), the sales price (P) increases with a constant value (C):

$$\Delta P = -\Delta Q * C \quad (1)$$

In which:
ΔP = Change in price
ΔQ = Change in quantity $(Q - Q_0)$
C = Price elasticity constant

With this value for the extent to which the price changes caused by changes in quantity, Van Haperen (2016) expressed the relationship between the produced quantity and the sales price mathematically.

$$R = -C * Q^2 + (Q_0 * C + P_0) * Q \quad (2)$$

In which:
R = Revenue
C = Price elasticity constant
Q = Quantity
Q_0 = Initial quantity
P_0 = Initial sales price

With (2) Van Haperen (2016) included the effect of price elasticity in the revenue function. Appendix A clarifies the complete derivation of this expression. This equation is very useful, because the initial quantity and the initial sales price for every month are reported by CX.

1.2. Problem statement

The Market Intelligence department of CX investigates the markets in which CX operates. They observe that the sales volumes of CX of product 1, product 7 and product 11 have influence on their price in the markets, especially within the product 1 market. When additional volumes are pushed on the market, a downwards price effect is observed over the whole volume. The relation between quantity and price is usually expressed in term price elasticity. Price elasticity is defined as the ratio of the percentage change in quantity to the percentage change in price (Case, Fair, & Oster, 2011). Throughout this report, the term 'price elasticity' is used to refer to the change in price caused by the change in quantity. This effect is not yet considered in the S&OP process, while the research of Van Haperen (2016) shows a significant financial improvement in a fictional case study. Besides the potential, required data and technical possibilities are available.

Currently, the hubs provide an average sales price and sales prices when the quantity changes with -10% and +10%. According to the business controller of Department X who responsible for gathering the sales prices for the S&OP, the average sales prices are accurate, but the ±10% prices are based on intuition rather than facts or calculations. To improve the price and revenue in the S&OP process, Department X wants to investigate a new approach which considers the price elasticity of a product.

Currently, the hubs provide an average sales price and sales prices when the quantity changes with -5% and +5%. According to the controlling analyst of the MVA who responsible for gathering the sales prices for the E S&OP, the average sales prices are accurate, but the ±5% prices are based on intuition rather than facts or calculations. To improve the price and revenue estimations in the E S&OP process, the MVA wants to investigate a new approach which considers the price elasticity of a product.

The main problem is that CX has no insight in the impact of the implementation of price elasticity within the actual S&OP model. They believe that the more accurate approach of price elasticity will result in a better valorisation, which means that CX can add more value to the supplied raw material. Therefore, the price elasticity approach should be implemented and the effects of the implementation should be investigated to solve the problem.

1.3. Relevance

The S&OP model plans every month how nearly 1 million kilos raw material will be processed and sold. From this amount, around 20% will be processed into product 1. Therefore, it is expected that a small improvement in the S&OP model changes the allocation of thousands of kilos raw material. Because improving the model can have a large impact, it is important to investigate the effects with this study.

With the current model, prices regarding the additional volumes in the product 1 market are inaccurate. Especially too high price estimations by the model can result in selling product 1 for lower prices than expected. This situation is undesirable for CX, because they lose money due to the lower prices. They may even sell their product 1 below the cost price. The raw material used for the production of product 1 could have been used for better purposes with a higher valorisation. The implementation of price elasticity can prevent such situations.

1.4. Scope

The S&OP model considers a lot of plan products. It is impossible for this project to implement the price elasticity for all products for which price elasticity is observed. Therefore, the scope of the study is the price elasticity of product 1. The Market Intelligence department knows the value of price elasticity in the product 1 market and product 1 covers a large share of the production volume of CX. We exclude price elasticity of other products in this research.

The focus of the project is to implement price elasticity into the S&OP model for the product 1 market and analyse the effects of this implementation. It is important to investigate the potential financial improvements and to find out how decision-making changes within the S&OP model. All required data, such as the value for price elasticity, demand data and sales prices, is available and we take these data as given. Therefore, market research on the price elasticity is outside the scope of this research. Organizational implications due to the implementation of price elasticity, such as changing information flows, are out of scope.

1.5. Research questions

The main problem of Department X is that they do not know which impact the implementation of price elasticity will have on the sales and operations planning and financial results. The objective of this research project is to implement the effect price elasticity at product 1 in the S&OP model and to investigate the effects of the planning and valorisation. The described objective leads to the following main research question:

What is the impact of the implementation of price elasticity in the sales and operations model of Company X?

To provide insight into the impact of the implementation of price elasticity, we answer four sub questions.

1. *How does Company X currently manage demand in the S&OP process?*

Chapter 2 describes the current situation. To adjust and improve the S&OP model with price elasticity, we first provide insight into the current situation. We investigate the relevant markets and the current revenue curve. We also elaborate about the way of managing demand and the current model of CX. Furthermore, we elaborate about the methods CX uses to analyse the valorisation of their products.

2. *What literature is available related to the use and implementation of price elasticity in sales and operations planning of Company X?*

We answer this question in Chapter 3. We investigate the concepts of sales and operations planning, oligopoly and price elasticity. We use a systematic literature review protocol to answer the research question and provide insight into the impact of price elasticity according to literature. We also study literature about piecewise linear approximations to support the implementation of price elasticity in the S&OP model.

3. *How should we adjust the S&OP model to implement the price elasticity of product 1?*

In Chapter 5 we elaborate about the adjustments of the model necessary for this research. At first, we investigate the influences in the market to implement price elasticity correctly. We determine the characteristics of the piecewise linear approximation of product 1. We will also list our simplifications and assumptions and validate the model. We also describe the actual adjustments and implementation of the model and explain our key performance indicators.

4. *What is the impact of price elasticity on decision-making and the valorisation of the S&OP model?*

In Chapter 6 we describe the numerical results of the impact of the implementation of price elasticity. Due to price elasticity, both decisions and the valorisation change. To gain broad understanding of the model and the effects of price elasticity, we perform experiments with a different number of segments, perform a sensitivity analysis and a worst-case scenario. We also investigate several scenarios.

After we answered all research questions, we draw conclusions, mention the limitations of this research and provide recommendations and future research. At the end of the report are appendices which provide additional information about this research.

2. Current situation

In this chapter, we describe which aspects are involved in managing the demand in the S&OP process. We answer the first research question:

How does Company X currently manage demand in the S&OP process?

In this section, we investigate the relevant markets, the current revenue curve and describe the demand and supply chain modelling of the S&OP model. We also discuss the concept of equivalence tables, which is used to evaluate the valorisation of different products.

2.1. Relevant products and markets

We focus for this research on two main products, product 1 and product 2. Product 2 is a common alternative of product 1 regarding the processing of raw material. Both products are important to balance the supply and demand of CX. They are produced in high volumes with low margins and making a loss on these products is not uncommon. Therefore, small improvements have a significant impact on the profit. Although product 1 and product 2 are used for the same purpose, the market characteristics of the two products are totally different.

The product 1 market is a small market in which CX competes with mainly smaller national companies. Company X has a market share of over 50%. Due to the large market share, CX can significantly influence the market price with its own volumes. Decreasing the product 1 production leads to a shortage in the market and increases prices. Because relatively few companies produce product 1, which are large enough to influence the market price, we characterise the product 1 market as an oligopolistic market.

On the other hand, the product 2 market is a large global market in which CX competes with a lot of companies around the world. Although CX is large company in the Netherlands, in the world they only have a market share of 6%. The quantities of CX do not significantly influence the market price. We can characterise the product 2 market a market with almost perfect competition, because it is a market with a lot of suppliers which cannot individually influence the market price.

2.2. Current revenue curve

As described in Section 1.2, the hubs currently provide an average sales price of the current quantity and sales prices when the quantity changes with -10% or +10%. When we analyse the planned sales prices of product 1, we see that the average price, the -10% and +10% prices are equal in each month. Therefore, the downwards price effect is not considered at this moment by the product 1 hub. When we consider the more accurate method with price elasticity, the revenue has a downward slope. Figure 2.1 shows the revenue curve for one month of the current method and the revenue curve when price elasticity is included. The revenues are quite close to each other near the current production quantity of ±2,000 tons. However, at a quantity of 2,500 tons is the deviation between the two methods already 10 thousand euros. This deviation leads to sub optimal decisions in the S&OP plan.

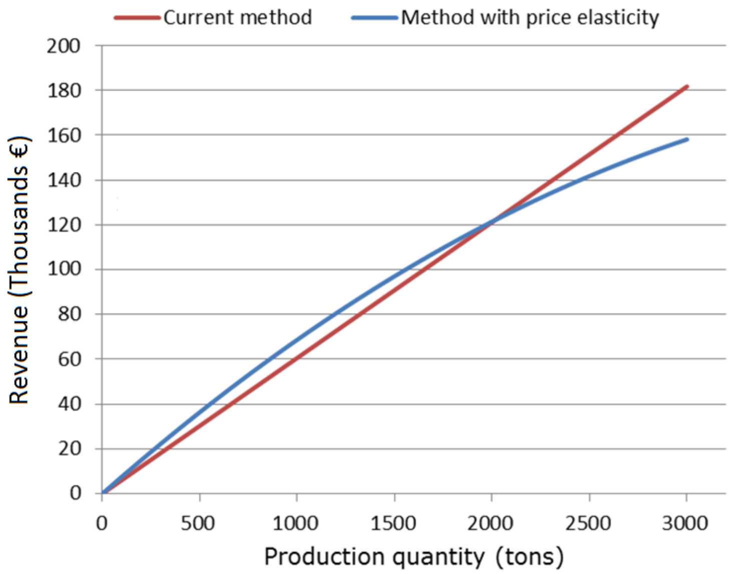

Figure 2.1 - Revenue curves for one month of methods without and with price elasticity

We do not plot historic data in Figure 2.1 to analyse the accuracy of the revenue curve with price elasticity. The price and revenue are dependent on a lot of more factors than the production quantity of CX. Due to constantly changing market conditions (e.g. governmental regulations and the world raw material supply), the historic data cannot provide an unbiased result. On top of that, market analysis is out of the scope of this research. We take the price elasticity as given and reliable, provided by the Market Intelligence department of CX. Their insight into the markets and price elasticity is far beyond the reach of this research project. Nevertheless, in this project we manage the potential inaccuracy with the sensitivity analysis in Chapter 5.

2.3. S&OP tool

CX is currently developing and implementing a new S&OP tool to support the S&OP process. The S&OP tool is a deterministic tool which consists of multiple coherent tables to model a supply chain. Currently, there are tables present for all main characteristics of the supply chain, among others the raw material supply, factories, products, machines and customers. At the background of the tool runs a linear programming solver to find the financially optimal solution. Not all supply chains and products are modelled at the moment of this research.

Because of a linear solver and tables in which we need to fill prices and quantities, we cannot implement mathematical functions like the revenue curve. Therefore, Van Haperen (2016) stated that CX has to apply piecewise linear approximations to implement the revenue curve in the S&OP tool. Instead of one price for a product, we can divide the demand in several segments. By using multiple tables, we can provide different prices at certain volumes and approximate the revenue curve.

2.4. Demand modelling in the S&OP tool

At this moment is the demand of the products in the S&OP tool is divided into three segments, with corresponding quantities and sales prices. The segments are called the fixed, the reducible and the additional segment. The S&OP process cannot adjust the quantity of the fixed segment, but it can change the quantities of the reducible and additional demand.

The S&OP model is obliged to fulfil the fixed demand, due to a high valorisation of the products or contractual agreements with customers. The reducible demand and the additional demand give the model flexibility to balance supply and demand to obtain a high level of valorisation of the raw material. The hubs provide the demand-driven quantity to Department X. This is the quantity demanded by the customers of CX, which is equal to the fixed and reducible quantity. Ideally, this is the quantity which CX will produce. However, CX has the obligation to process all supplied raw material. Unfortunately, the supply of raw material usually does not match the raw material needed to fulfil the demand. With a raw material shortage, Department X should cut into the production quantities. The amount which can be cut is the reducible demand. With a raw material surplus, Department X should plan extra production to process all raw materials, which are called supply-driven quantities. The amount which is produced extra is the additional segment. The system with the three segments is visualised in Figure 2.2.

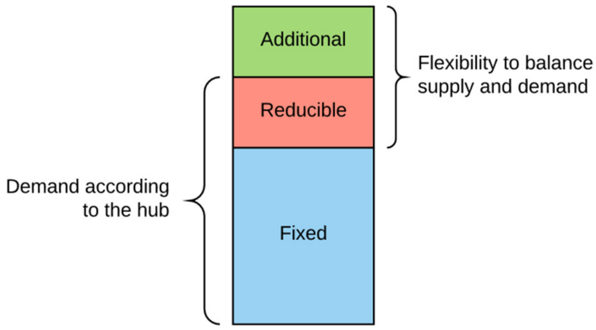

Figure 2.2 - The demand system with three segments

We must note that not all products have three segments. Profitable products only have a fixed demand which is usually equal to the production capacity. These products should not be reduced, because they add a lot of value. An additional demand is not possible, because the production is already limited by its production capacity.

2.5. Supply chain in the S&OP tool

The S&OP model is an aggregated model and therefore a simplified version of the actual supply chain of CX. All main characteristics are present in the model, among others the raw material supply, factories, products, machines, transport costs, inventory costs, operational costs, capacity constraints and customers. Considering interdependence of the factories and the various products, the S&OP model can be characterized as a multi-echelon, multi-product supply chain model.

In the model, the supply chains of all 'hubs' are modelled. Hubs are the S&OP divisions of CX, which manage their own product category (e.g. product 1 or product 2) and corresponding supply chain. The supply chains show some overlap, because factories often produce multiple products of different categories. For example, a factory produces product 1 and product 2. Therefore, hubs are not location specific and multiple hubs can manage one factory. The supply chain of each hub is modelled in the S&OP model.

Figure 2.3 is a visualisation of the modelled supply chain of a hub. Each hub is modelled similarly with its own products. The beginning of the supply chain is the supply of raw material. The raw material is allocated to different machines at different factories. Each machine can have one or more operations which process raw material into main products and by-products. There is an operation for each machine-main product combination. The product lines from each machine to the main products illustrate which products a machine can make. Most of the machines have one operation, but the machines in factory 1 have two operations; they can make both product 1 and product 6. The product lines of each machine to the by-products are not visualised in Figure 2.3 to maintain overview. All operations in the supply chain produce the by-products product 3 and product 4. The operations are modelled as one aggregated version of the actual processes, which actually consists of multiple successive operations. By-products of each hub are exchanged with other hubs. Other hubs use these by-products as input in their production processes. For example, product 4 is further processed into product 7 and product 9. The main products go through a fictional distribution centre to distribute the products from different factories to the customers. The model is not allowed to hold any stock throughout the months.

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Figure 2.3 - Structure of the modelled supply chain

The actual customers of CX are not modelled in the S&OP model. What is modelled, are hub specific customers. In the model, the hub has a 'customer' for each segment, i.e. a fixed, a reducible and an additional customer. This is necessary in order to assign different sales prices at different production levels to a product. The sales prices and quantities of products are linked to a specific customer with a forecast item. Each customer has a different priority regarding the fulfilment of the demand of its products. Fixed demand has an extremely high penalty when their demand is not fulfilled. The reducible demand has a low penalty when their demand is not fulfilled and the additional demand does not involve a penalty. This is consistent with the situation of the three segments. Because of the extremely high penalty, the fixed demand should always be fulfilled. The reducible demand can be reduced and due to the low penalty, it is prevented that when de reducible and additional sales prices are equal, the additional demand is fulfilled instead of the reducible.

2.6. Valorisation and equivalence tables

When including price elasticity in the model, it affects the valorisation, or added value, of product 1. Increasing the production quantity decreases the valorisation per ton product 1 due to the decrease in sales price. Because the S&OP model aims for the best financial performance and thus for the highest valorisation, it is important to have insight into the valorisation of products to understand the decision-making of the model.

To measure the valorisation of a product, not only the main product should be considered, but also by-products like product 3 and product 4. Therefore, Department X has divided its products into so-called 'baskets', such as a product 1 basket with product 3 and product 4 and a product 2 basket with product 4 as by-product. With these baskets, Department X can compare the valorisation of its products.

For the calculation of the valorisation, Department X considers the revenue, recipe costs and production costs for the main and the by-products. Recipe costs are the costs of added components to produce the end products. The cost of supplied raw material is the only important component which is left out of the calculation of the valorisation. These costs are not considered, because CX determines the raw material price and therefore, the cost of supplied raw material. CX uses the valorisation of its products is input for determining the raw material price. Figure 2.4 represents the basic calculation of the valorisation of a basket.

$$\begin{array}{ll} \left.\begin{array}{l} Output\ quantity * sales\ price \\ \quad -recipe\ costs \\ \quad -production\ costs \end{array}\right\} & \begin{array}{l} For\ the \\ main\ and \\ by-products \end{array} \\ \hline & \hfill + \\ Valorisation\ of\ a\ basket & \end{array}$$

Figure 2.4 - Basic calculation for the valorisation

To determine the output quantity and recipe costs, CX uses a dynamic bill of materials (BOM). This BOM is dynamic, because the composition of raw material has a seasonal pattern. The composition of raw material influences the output quantity. To compare the valorisation of products, Department X uses an equal unit of measure. They calculate the valorisation of a basket based on 1 ton of raw material intake. The forecast of the valorisation of the baskets for the coming months are reported in equivalence tables (ETs). Figure 2.5 shows an example of an ET with the five baskets. We see that the valorisation of each basket changes throughout the months. There are two main causes for these changes: market developments and the composition of raw material. Changing market conditions influence the sales price of the main and by-products.

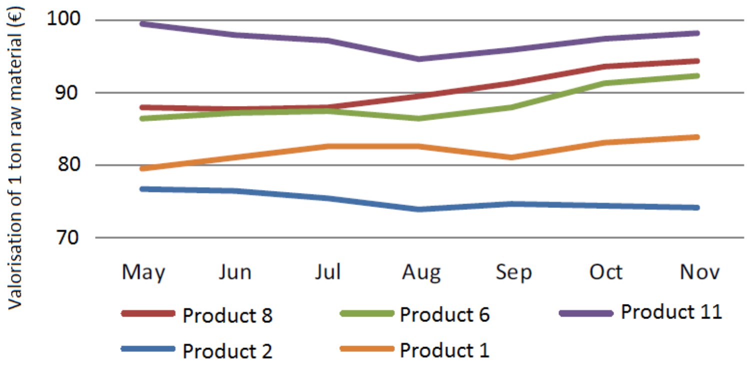

Figure 2.5 - Example of an equivalence table

The valorisation of the baskets is important for balancing supply and demand. With a shortage in the raw material supply, the production of the basket with the lowest valorisation should be reduced. With a raw material surplus, the production of the best valorising basket should be increased. In the ET we see that product 11 is the best valorising basket, while product 2 is the worst valorising basket.

2.7. Conclusion

In this chapter, we investigated the current situation at CX. With this chapter, we answer our first research question:

How does Company X currently manage demand in the S&OP process?

Our investigation concludes that:

- The currently used revenue curve is linear, while the actual revenue curve is concave. This can lead to sub optimal decisions in the S&OP plan.
- The concave revenue curve results from the characteristics of the oligopolistic market of product 1 and the market share of CX. This effect is not observed in the product 2 market.
- The S&OP tool of CX consists of multiple coherent tables and a linear solver. Therefore, piecewise linear approximations are necessary to implement price elasticity.
- The demand in the S&OP model is divided into three segments, i.e. the fixed, reducible and additional segment.
- The supply chain in the S&OP model is modelled according to the aggregated supply chain of each hub and with a 'customer' for each demand segment.
- The forecast of the added value of the products of CX are reported in equivalence tables, which show the valorisation of 1 ton raw material into a 'basket', consisting of one main product and corresponding by-products.

In the following chapters, we further investigate the impact of using the actual concave revenue curve. We adjust the current demand and supply chain model and apply piecewise linear approximations. We use the calculations of the valorisation in the equivalence tables in this research to financially analyse the effects of price elasticity.

Before we investigate the influence of price elasticity within CX, we review scientific literature about the effects of price elasticity in sales and operations planning. We also describe the literature about piecewise linear approximations, because this is necessary to implement price elasticity in the S&OP model.

3. Literature research

In this chapter, we study scientific literature relevant to the research project. The literature study is twofold. First, we perform a literature review to investigate the status of research about the use of price elasticity within sales and operations planning. Second, we perform an explorative study about piecewise linear approximations. Ultimately, we provide an answer to the second research question:

> *What literature is available related to the use and implementation of price elasticity in sales and operations planning of Company X?*

3.1. Price elasticity in sales and operations planning

To provide a comprehensive answer to the question, we use a systematic literature review protocol to select relevant literature. Appendix A shows the review protocol. The concept matrix with the key findings is reported in Appendix B. We combine sales and operations planning with price elasticity and oligopolistic markets. We focus on the oligopolistic markets, because the product 1 market is considered as an oligopolistic market. Figure 3.1 visualises the overlap between the topics. Not a lot of literature all three exists that combines concepts together. To overcome this issue, we look for literature that combines sales and operations planning with the other topics (combination A and B). Because we are only interested in the impact on the sales and operations planning, we exclude the combination of oligopolistic markets and price elasticity (combination C) from our review. We first give a description of the concepts of the literature review. Then we discuss the impact of price elasticity on the sales and operations planning.

Figure 3.1 - Overlap between the topics

Sales and operations planning

Sales and operations planning (S&OP) is a planning process that combines different business plans into one integrated set of plans (Thomé, Scavarda, Fernandez, & Scavarda, 2012). The purpose of S&OP is to balance or integrate demand and supply plans at an aggregate level, usually on a monthly basis (Feng, D'Amours, & Beauregard, 2008; Wallace & Stahl, 2008; Thomé et al., 2012). S&OP uses aggregate data, such as clusters of resources and product families, rather than individual stock keeping units (Grimson & Pike, 2007; Noroozi & Wikner, 2016). S&OP should be a cross-functional process in which different functions, such as sales and marketing, production, purchasing, finance, human resources and product development should cooperate and agree on the final plan (Wallace & Stahl, 2008). Due to the alignment of plans and performances of different functions, S&OP supports the business strategic plan and can lead to improvements in profit customer satisfaction and organizational atmosphere (Chen & Chen, 2008; Feng et al., 2008). The participation of people is very important for a successful execution of the S&OP process (Grimson & Pike, 2007). The suggested planning horizon of an S&OP process is between three months and three years (Grimson & Pyke, 2007). Nevertheless, most researchers emphasize on the horizon of between 12 and 18 months (Wallace & Stahl, 2008), especially for companies with a seasonal profile (Grimson & Pike, 2007). S&OP can increase the profitability of the supply chain, because supply and demand are matched in a coordinated process. Supply is managed by using capacity, inventory, subcontracting and backlogs. Demand is managed by using short-term price discounts and promotions (Chopra & Meindl, 2013). To summarizS&OP, Thomé et al. (2012, p. 2) described the main features of S&OP as follows:

- It is a cross-functional and integrated tactical planning process within the firm
- It integrates all the plans of the business in a unified plan
- It has a planning horizon from less than three months to over 18 months
- It bridges strategy and operations
- It creates value and is linked with the performance of the firm

Oligopoly

An oligopoly is an industry characterized by a few dominant firms, each large enough to influence the market price. Products in an oligopolistic market may be homogenous or differentiated. Companies in an oligopolistic market do not only compete in price, but also in marketing and new products (Case et al., 2011).

A lot of different models from the fields of game theory and competitive strategy are developed to increase profitability in an oligopoly. A simple and common model is the Cournot model, introduced by the mathematician Antoine Augustin Cournot in the 19th century (Case et al., 2011). In the Cournot model, all firms produce a homogeneous product and compete in production quantities. Product prices are variable functions of the collective market supply. Therefore, companies do not control prices, but they influence them with their production decisions (Tominac & Mahalec, 2017). Companies set optimal production quantities to maximize their profit, given their competitors production quantities (Ma, Zhu, & Wang, 2013).

Price elasticity

The law of demand states that a decrease in product price leads to increase in product demand and vice versa. Price elasticity is the concept which expresses the degree of responsiveness between the sales price and the demand of a product (Lui, Shah, & Papageorgiou, 2012). Price elasticity is defined as the ratio of the percentage change in quantity to the percentage change in price (Case et al., 2011).

Impact of price elasticity in S&OP

Now we have identified the concepts within the research topic, we try to investigate the impact when these concepts come together. Table 3.1 presents the effects of including price elasticity on the profit, revenue, production quantity and sales price according to the articles which are reviewed in the systematic literature review.

Aspect of the operations	Increased	Decreased	Stable	Not considered
Profit	Algarni et al. (2007), Calfa & Grossmann (2015), Chen & Chen (2008), Hjaila et al. (2014), Kaplan et al. (2011), Lui et al. (2012), Ma et al. (2013), Tang et al. (2015), Tominac & Mahalec (2017)	Karmarkar & Rajaram (2012)		Farris & Darley (1964)
Revenue	Calfa & Grossmann (2015), Kaplan et al. (2011), Tang et al. (2015), Tominac & Mahalec (2017)	Karmarkar & Rajaram (2012), Lui et al. (2012)	Hjaila et al. (2014)	Algarni et al. (2007), Chen & Chen (2008), Farris & Darley (1964), Ma et al. (2013)
Quantity	Calfa & Grossmann (2015), Kaplan et al. (2011), Tang et al. (2015), Tominac & Mahalec (2017)	Karmarkar & Rajaram (2012), Lui et al. (2012)	Algarni et al. (2007), Hjaila et al. (2014)	Chen & Chen (2008), Farris & Darley (1964), Ma et al. (2013)
Sales price	Lui et al. (2012), Ma et al. (2013)	Calfa & Grossmann (2015), Kaplan et al. (2011), Karmarkar & Rajaram (2012), Tominac & Mahalec (2017)	Farris & Darley (1964), Hjaila et al. (2014)	Algarni et al. (2007), Chen & Chen (2008), Tang et al. (2015)

Table 3.1 - Classification of the effects of price elasticity

In Table 3.1, we see that in most cases the profitability of a supply chain increased when price elasticity is included in the operations planning, especially with a very elastic market (Chen & Chen, 2008; Kaplan, Türkay, Karasözen, Biegler, 2011; Lui et al., 2012). Besides that, in most articles the revenue increased because of higher production quantities and lower prices. In the case of Lui et al. (2012), the revenue and quantity decreased, but due to higher prices, the profit increased. Production planning in an oligopolistic market can benefit from the analysis of the behaviour of competitors (Tominac and Mahalec, 2017).

Nevertheless, we cannot translate the results towards our research, because a complex supply chain network makes it difficult to understand the system response to price elasticity (Kaplan et al., 2011). Besides that, the approach used to approximate the revenue curve can significantly affect the decision-making and the economic behaviour of the supply chain (Hjaila, Zamarripa, Shokry, & Espuña, 2014). The research of Hjaila et al (2014) and Calfa and Grossmann (2015) both observed that more complex models which are closer to the real price behaviour provide better solutions and economic advantages relative to simpler and less accurate models, especially when the demand is price sensitive. Therefore, an accurate model is important to make financially optimal decisions. However, more complex models required larger computational effort and resources information flows should be integrated in the process (Hjaila et al., 2014).

Besides the effects in Table 3.1, authors also considered costs (Algarni et al., 2007; Hjaila et al., 2014) which decreased in those cases. Inventory was also considered, which decreased in the study of Calfa and Grossmann (2015) and with the study of Lui et al. (2012), the inventory deviation decreased.

3.2. Piecewise linear approximations

The S&OP tool of Company X can only process linear data. Unfortunately, the revenue function determined by Van Haperen (2016) is a quadratic function (Section 1.1). To implement price elasticity in the S&OP model, Van Haperen suggested to linearize the function by using piecewise linear approximations. In this section, we investigate available literature about computational methods for piecewise linear approximation to support our research.

Piecewise linear approximations are widely used to approximate non-linear functions, also in the field of planning and scheduling, supply and demand curves, and in the allocation of resources in general (Keha, de Farias Jr., & Nemhauser, 2004; Kontogiorgis, 2000). In general, a piecewise linear function consists of multiple linear line segments K over the interval [l, u] (Yamamura & Tamura, 2012). The two end-points of each segment are called breakpoints. The slope of the function changes at each breakpoint (Winston, 2004). The most common approach to compute a piecewise linear function is through linear interpolation between sample coordinates, because it preserves the concavity and continuity of the nonlinear function (Kontogiorgis, 2000; D'Ambrosio, Lodi, & Martello, 2010). Figure 4.2 illustrates an example of a nonlinear function and its piecewise linear approximation with three segments.

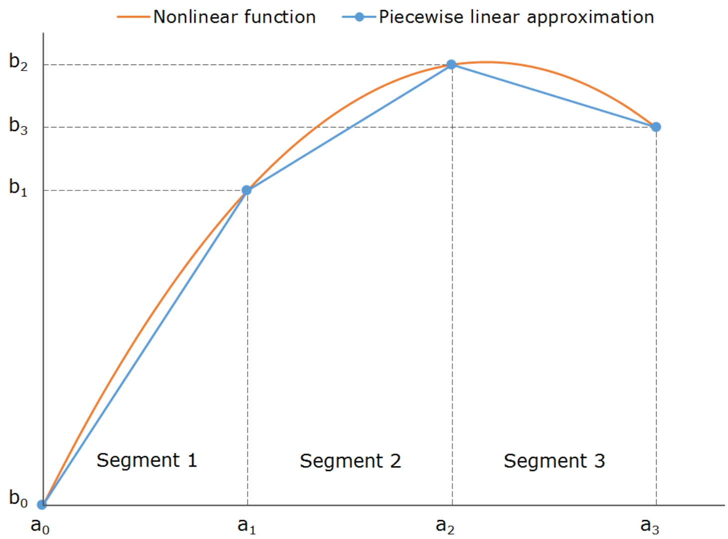

Figure 3.2 - Example of a piecewise linear approximation

The accuracy of an approximation is mainly based on the number of linear segments. The accuracy increases when increasing the number of segments in an interval. However, more segments also require more computational effort (Keha et al., 2004). Barros and Weintraub (1986) report 14 uniform segments provide the adequate accuracy for the approximation of supply and demand functions of sugar and wheat. Experimentation shows that 5 to 10 segments is a proper approximation for airline revenue curves (Curry, 1990).

Besides the number of segments, also the placement of the breakpoints has influence on the accuracy of the approximation. The placement of the breakpoints is usually left to the domain expertise and skill of the modeller (Kontogiorgis, 2000). However, the optimal placement can be calculated objectively. Where the magnitude of the curvature is greater, an approximation needs more breakpoints for the same accuracy (Kontogiorgis, 2000). The curvature can be expressed with the second derivative. In the case of a quadratic function is the second derivative constant. Therefore, an equal distribution of the breakpoints is optimal (Kontogiorgis, 2000).

Now, we elaborate about the general appraoch of linear interpolation to compute the piecewise linear function $g(X)$ of the quadratic function $f(X)$ over the interval [l, u]. The approach is based on to the method used by Yamamura and Tamura (2012) and similar to the methods of Kontogiorgis (2000), Keha, de Farias Jr. and Nemhauser (2004) and D'Ambrosio, Lodi and Martello (2010).

For the number of breakpoints K we take $l = a_0 < a_1 < \cdots < a_K = u$. The piecewise linear function $g(X)$ is linear for the segments [a_{j-1}, a_j] for $j = 1, 2, \ldots, K$. When we determined the segments, we take samples of b_j at the breakpoints a_j:

$$b_j = f(a_j) \qquad for\ j = 0,1, \ldots, K \tag{3}$$

Now, we introduce the auxiliary variables δ_j to formulate X and $g(X)$.

The value of δ_j satisfies:

- $0 \leq \delta_j \leq a_j - a_{j-1}$
- If $\delta_j = 0\ (1 \leq k \leq K-1)$, then $\delta_{j+1} = 0$
- If $0 < \delta_k < a_k - a_{k-1}\ (1 \leq k \leq K-1)$, then $\delta_j = a_j - a_{j-1}$ for $1 \leq j \leq k-1$ and $\delta_j = 0$ for $k+1 \leq j \leq K$

Then, every value of X in [l, u] can be written as:

$$X = a_0 + \delta_1 + \delta_2 + \cdots + \delta_K \tag{4}$$

With the linear interpolation between the samples we can finish the general computational approach for the piecewise linear functions. $g(X)$ can be written as:

$$g(X) = b_0 + \frac{b_1 - b_0}{a_1 - a_0}\delta_1 + \frac{b_2 - b_1}{a_2 - a_1}\delta_2 + \cdots + \frac{b_K - b_{K-1}}{a_K - a_{K-1}}\delta_K \tag{5}$$

3.3. Conclusion on the literature study

In this literature study, we reviewed literature about the impact of using price elasticity in sales and operations planning and studied literature about piecewise linear approximations. Therefore, this chapter provides the answer to our second question:

> *What literature is available related to the use and implementation of price elasticity in sales and operations planning of Company X?*

According to literature, we found the following about price elasticity in sales and operations planning:

- The profitability of a supply chain improves in most cases by using the concept of price elasticity, especially in markets with a high price elasticity.
- Production planning in an oligopolistic market benefits from the analysis of the behaviour of competitors.
- A complex supply chain makes it difficult to understand the response to price elasticity.
- Models which are closer to the real price behaviour provide better solutions and economic advantages relative to less accurate models.

We also studied literature about piecewise linear approximations. We conclude that:

- We have a general approach for piecewise linear approximations, which we adopt for the implementation of price elasticity in the S&OP model.
- An accurate approach to approximate the revenue curve is important to make financially optimal decisions for the production quantity.
- The main factor for the accuracy of the approximation is the number of segments; more segments provide a higher accuracy.
- Approximations with 5 to 14 segments provide adequate accuracy for the approximation of supply, demand and revenue functions.

Literature is promising about the implementation of price elasticity. However, it does not offer a direct solution to our research. Therefore, we further investigate the implementation of price elasticity in the S&OP process of CX. Several articles confirm that the approach to model price elasticity is an important factor. In the next chapter, we further analyse how we approach price elasticity and construct our model with piecewise linear approximations. Furthermore, we perform experiments in Chapter 5 with the number of segments, because this is an important factor for the accuracy of the approximation.

4. Model adjustment

In this chapter, we investigate how we can approach price elasticity and construct the S&OP model in a way it considers the price elasticity of product 1. We also define key performance indicators to analyse the results. We answer our third research question:

How should we adjust the S&OP model to implement the price elasticity of product 1?

4.1. Influences in the market

At first, we elaborate about the influences in the product 1 market which we consider in this research. The main influence is the production quantity of product 1 of CX. We also consider two other factors, the sales of raw material to competitors and the production of product 5.

As mentioned in Section 1.2, CX observes that the production quantity of CX has influence on the price in the product 1 market. The quantity and price form together the revenue of CX. The production of product 1 increases the quantity. The increase in quantity increases the revenue of product 1. An increase in quantity also leads to a decrease in price, which decrease the revenue.

We also find another effect on the revenue, the additional sales of raw material. Not all raw material supplied is processed by CX; some raw material is sold to competitors. CX sells a fixed quantity of raw material to competitors, because they have a contractual agreement. On top of that, CX sells additional quantities when the factories do not have the capacity to process the raw material supply in its own factories. These additional quantities are mainly processed into product 1 and increase the total volume in the market. According to the market analyst of FC, 60 to 100 percent of the additional raw material sales end up in product 1. Therefore, the additional sales of raw material affect the market price of product 1.

Before selling raw material, CX first tries to allocate the raw material to its own factories. One of these products is product 5. The product 5 market has a fixed demand, but CX can produce additional volumes. In that case, CX drives other competitors of the market. Instead of product 5, competitors process 60 to 100 percent of the raw material into product 1. Therefore, each unit of raw material used to produce additional volumes of product 5, leads to the same amount of raw material available at competitors. This behaviour affects the product 1 market in the same way as additional raw material sales. Because the effect of product 5 is equal to the raw material sales, we include the effect of product 5 within the sales of raw material. The effects on the product 1 market are visualised in Figure 4.1.

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Figure 4.1 - Influence on revenue of product 1

4.2. Including raw material sales

In this section, we elaborate further about the effect of raw material sales and product 5. As mentioned in Section 4.1, 60 to 100 percent of the additional raw material sale is processed into product 1 by competitors. We define this ratio as the 'raw material-product 1 ratio'. The exact ratio is unknown and varies per month. The additional production of product 5 has a similar effect as raw material sales and is included in the additional raw material sales.

The quantity of product 1 which is produced by competitors depends on a few variables. Because we deal with additional raw material sales, the initial volume $QR_0 = 0$. The change in raw material sales (ΔQR) is equal to the total quantity of additional raw material sales (QR). Then, we can express the relation between additional raw material sales (QR) and the change in quantity of the product 1 market (ΔQM) through the following equation:

$$\Delta QM = MC * \mathrm{BOM} * \mathrm{QR} \tag{6}$$

In which:
ΔQM = Change in quantity of the product 1 market by competitors
MC = Raw material-product 1 ratio
BOM = Amount of product 1 which can be produced from one unit of raw material
QR = Quantity of the raw material which is sold additionally to competitors

Because the change in the price of product 1, $\Delta P = -\Delta QM * C$, the change in price of product 1 due to the raw material sales is:

$$\Delta P = -MC * \mathrm{BOM} * \mathrm{QR} * C \quad (7)$$

Because a change in revenue by a change in price can be expressed with $\Delta R = Q * \Delta P$, the effect of raw material sales on the revenue of product 1 of CX is:

$$\Delta \mathrm{R} = -\mathrm{Q} * MC * \mathrm{BOM} * \mathrm{QR} * C \quad (8)$$

From (8), we see that both the quantity of additional raw material sales (QR) and the quantity of product 1 (Q) are factors which determine the influence of raw material sales on product 1. Because of this interdependence is the relationship between raw material sales and product 1 complex.

To calculate the influence, both the quantity of raw material sales and product 1 should be known. Unfortunately, both are decision variables determined by the S&OP model. It is impossible to implement this complexity into the S&OP tool, because we cannot model a price dependent on a variable quantity of another product. To implement the influence in the S&OP tool, we need to make the revenue dependent on one of the quantities and assume the other.

When we assume the quantity of product 1, we can include the effect in the price of raw material sales. When we assume the quantity of raw material sales, we can include the effect in the price of product 1. From the experience of the S&OP planners, the production of product 1 is rather stable, while the quantity of additional raw material sales is far more volatile. This is the case because the sale of raw material is used as the final option when the raw material supply exceeds the capacity of the factories of CX. Due to the volatility, it is better to assume the quantity of product 1 than the quantity of raw material sales.

For the assumption of the product 1 quantity, we can take Q_0. From (8) we can derive the effect on the product 1 revenue when QR is increased with one unit, which should be the offset in the price of raw material sales (ΔPR):

$$\Delta PR = \frac{\Delta R}{\Delta QR} = -\mathrm{Q}_0 * MC * \mathrm{BOM} * C \quad (9)$$

Because $PR = PR_0 + \Delta PR$, we can write the price of raw material sales (PR):

$$PR = \mathrm{PR}_0 - MC * \mathrm{BOM} * \mathrm{Q}_0 * C \quad (10)$$

Now we have included the influence of raw material sales and product 5 on the revenue of product 1 in the sales price of the raw material. With (10), we can determine the adjusted price of raw material sales. The raw material price (PR) is not dependent on the quantity of the raw material sales, so the price remains constant for all quantities. Because this effect is included in the price of raw material, the revenue function of Section 1.1 stays unchanged.

The exact value of the raw material-product 1 ratio is unknown, but it is somewhere between the 60% and 100% according to the market analyst of CX. Therefore, we propose to experiment with multiple ratios to provide insight into the influence of raw material sales.

4.3. Computation of the approximation

In this section, we investigate the characteristics of the piecewise linear approximation. First, we determine the placement of the breakpoints. Then, we adjust the approximation and establish bounds of the approximation. At last, we describe the tool which automates the calculations.

Placement of breakpoints

The placement of the breakpoints has influence on the accuracy of the approximation. The placement of the breakpoints is usually left to the domain expertise and skill of the modeller (Kontogiorgis, 2000). However, we want to determine the placement objectively. Where the magnitude of the curvature is greater, an approximation needs more breakpoints for the same accuracy (Kontogiorgis, 2000). We can express the curvature of a function with the second derivative. The second derivative of the revenue function (Section 1.1: $R = -C * Q^2 + (P_0 + Q_0 * C) * Q)$ is:

$$R'' = -2C \tag{11}$$

This means that the curvature is only dependent on the elasticity value, which is constant. Because the curvature is constant, the optimum placement of the breakpoints is uniform. Therefore, we get an equal deviation and accuracy at all segments of a piecewise linear approximation.

Underestimation and overestimation

Since the revenue curve is concave, the piecewise linear approximation always underestimates the revenue, except from the breakpoints. Ideally, piecewise linear approximation should underestimate as much as it overestimates the revenue. In other words, the piecewise linear approximation should estimate the revenue with an average deviation of zero.

Because all segments have the same absolute deviation, we can obtain an average deviation of zero by increasing the initial value of the piecewise linear function (i.e. b_0). When the surface of deviation above the linear approximation is equal to the surface below the linear approximation, the average deviation is zero (see Figure 4.2a).

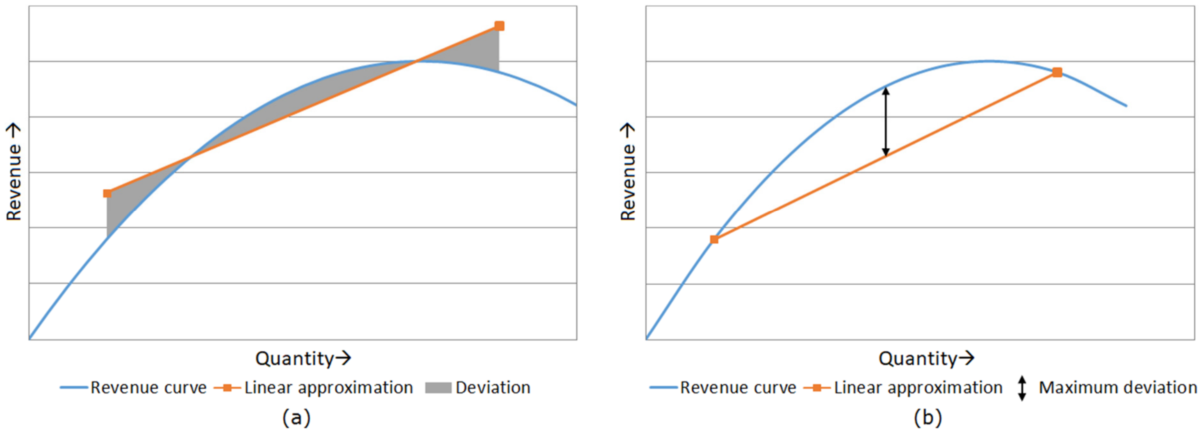


Figure 4.2 - The underestimation equal to the overestimation (a) and the maximum deviation in the middle of a segment (b)

To adjust all piecewise linear curves without loss of generality, we want to use a ratio relative to the maximum deviation. The maximum deviation of an unadjusted piecewise linear approximation of quadratic function is exactly in the middle of each segment, which is illustrated in Figure 4.2b. The ratio to adjust b_0 to obtain an average deviation of zero is $^2/_3$. The proof of this ratio is given in Appendix C. The value of b_0 is as follows:

$$b_0 = \frac{2}{3} * maximum\ deviation \tag{12}$$

Bounds of the approximation

In this section, we establish a lower and an upper bound. We determine these limits because it is not necessary to compute the piecewise linear approximation over an extremely large range. We only approximate the relevant area, because this provides the information we need and prevents unnecessary computational or human effort. Figure 4.3 shows two approximations of the revenue curve with three segments. We see that the approximation with bounds provides a higher accuracy in the relevant area than the approximation without bounds.

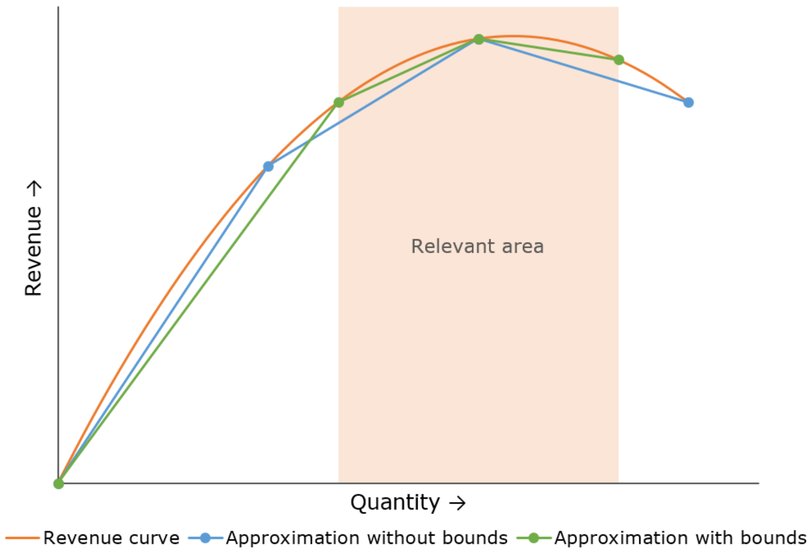


Figure 4.3 - Approximations in the relevant area

We choose a lower bound, because CX will certainly produce a minimum amount of product 1. According to historic data, CX not produced less than 3,000 tons a month in the past years. A lower production is not desirable, because it can possibly disappoint customers and CX can lose market share. Therefore, we take 3,000 tons per month the lower bound.

For the upper bound we consider the production capacity. CX has a certain production capacity for product 1 which cannot be easily increased in the tactic horizon of the S&OP process. The production capacity of product 1 varies per month due to a varying amount of available production hours in the factories. Another factor is that the production lines of factory 1 produce both product 1 and product 6. The production of the more valorising product 6 limits the production capacity of product 1. Therefore, the maximum capacity for a month cannot exceed 4,800 tons and we take that as upper bound.

Now we have the bounds, we model a large segment for the quantity of 0 to 3,000 tons and multiple segments in the range from 3,000 tons to 4,800 tons. In this case, the quantity and revenue can be implemented correctly in the S&OP model.

Computation of piecewise linear approximations

For this research, we need to compute piecewise linear approximations of product 1 for all 18 months of the planning horizon of the S&OP tool. When experimenting with the number of segments, we need to compute 18 piecewise linear approximations for each experiment. To prevent lots of human effort for the calculation, we automated the process into a tool. The tool is coded as a macro written in Visual Basic for Applications, for use within Microsoft® Excel (Office 2010). The outputs of the tool are the sizes of the segments and corresponding sales prices for each month. The inputs for the calculations are:

- Number of segments
- Lower bound

- Upper bound
- Price elasticity constant
- Forecasted initial quantity for each month
- Forecasted initial price for each month

Appendix E shows the macro 'LinearApproximation' which executes the calculations.

4.4. Simplifications and assumptions of the model

To make experimental analysis possible, we make several simplifications and assumptions. Below we list the simplifications and assumptions.

Simplifications

During this research, the S&OP model is in development. Therefore, we make some implications to the model to perform experiments. We make the following simplifications:

- We exclude the hubs Ambient, Chilled and Condensed and their raw material supply from the model. The hubs have only fixed volumes and a fixed raw material supply. Therefore, this does not influence the decisions or financial improvements of the model.
- We simplify the hub Product 7. Instead of processing product 4 into product 7, product 8, product 9 or product 10, we model a customer to which all product 4 can be sold without further processing. The model can sell unlimited product 4 without capacity or demand constraints. The sales price of product 4 is based on the valorisation of processing product 4 into product 8 and product 9. This is the most common method to process product 4. Therefore, this does not have significant influence the decisions or financial improvements of the model.

Assumptions

Besides simplifications, we make assumptions to make this research possible within the scope and time limit of this project. We make the following assumptions:

- We only consider price elasticity of product 1 due to the scope of this research. Prices of other products and by-products are equal to the current method with a fixed, reducible and additional price and demand.
- The demand of product 1 is completely flexible in each month. The model can decrease and increase the production volumes as much as possible. This holds for the long term, but for the coming months, around 70% of the sales is contractually determined.
- The initial quantity in a month is biased because of the number of days in the month. For example, in February is the initial quantity lower than in March. We correct the initial quantities and assume that each month has 30 days to prevent biased results.
- The price of product 1 is forecasted for 6 months, while the S&OP plans 18 months ahead. Therefore, CX assumes that product 1 has the same initial price after 6 months. To prevent biased results from the change in planned quantities after 6 months, we assume that the initial quantity after 6 months is equal to the sixth month, just as the price.
- Competitors do not change their production quantities in response to the changing volumes of CX. Therefore, their only influence on the sales price of product 1 is due to processing raw material sales of CX.
- Sales prices in a month only respond to the quantity of CX in the same month. Therefore, sales prices immediately respond to the decisions of CX and are mutually independent of prices and quantities in other months.
- The price of additional raw material sales is based on the valorisation of product 2, which is the lowest valorising product at the moment of this study. CX does not forecast the price of raw material sales to competitors. Therefore, we assume that the price of additional raw material sales without the product 1 effect is equal to the valorisation of product 2.

- To anticipate on disturbances, CX plans each month a spare capacity equal to 30 thousand kilos of raw material. The 'disturbance capacity' is planned at product 2 or product 1.

4.5. Implement the piecewise linear function in the S&OP tool

In this section, we describe how we adjusted the model to implement the piecewise linear approximation of the revenue function of product 1.

We described the supply chain model in the S&OP tool in section 2.5. The plan products have three segments, i.e. fixed, reducible and additional. In order to implement the piecewise linear approximation with price elasticity, this part of the model needs to be extended. Instead of the fixed, reducible and additional segment, we need a segment for each segment of the approximation. Each segment is modelled with a ´customer´ in the S&OP model. Therefore, the number of customers is increased to the number of segments. Figure 4.4 visualises the change in the structure of the E S&OP model.

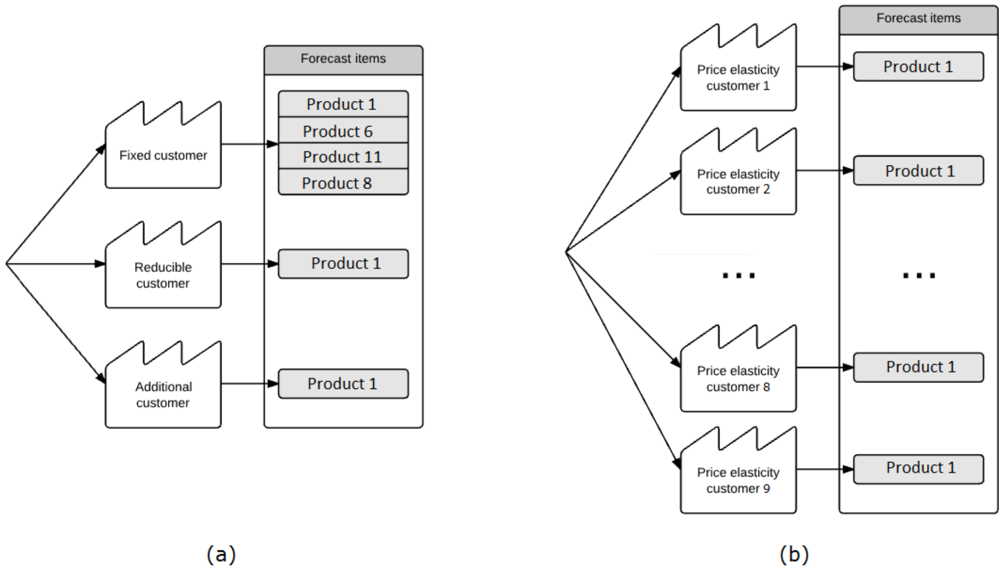


Figure 4.4 - The current model (a) and the model extended with price elasticity (b)

We described the piecewise linear function in Section 3.2 with the equation (5):

$$g(Q) = b_0 + \frac{b_1 - b_0}{a_1 - a_0}\delta_1 + \frac{b_2 - b_1}{a_2 - a_1}\delta_2 + \cdots + \frac{b_K - b_{K-1}}{a_K - a_{K-1}}\delta_K \qquad (5)$$

The S&OP tool is not a tool in which we can copy this piecewise linear function. As described in Section 2.3, the S&OP tool consists of multiple coherent tables. For product 1, the production quantity and the sales price of at each month need to be filled in a segment specific table. The production quantity and the sales price can be extracted from the piecewise linear function. The production quantity is the segment size and the corresponding sales price is the slope of the segment. From the piecewise linear function $g(Q)$ we consider the part of the equation before δ_j, $b_i - b_{i-1}/a_i - a_{i-1}$, as the slope of segment i and thus the sales price for that segment.

Besides determining the slopes, we have also defined an adjusted value b_0 in section 4.3. A base value like b_0 cannot be implemented into the S&OP tool. It is possible to add a new segment of a quantity of 1 and a sales price with the value of b_0, but this is not desired. Instead, we incorporate this value in the price of the 'zeroth' segment with the quantity from 0 to 3,000. We determine the revenue as follows:

$$\text{Sales price adjusted with } b_0 = \frac{sales\ price\ of\ first\ segment * 3{,}000 + b_0}{3{,}000} \qquad (13)$$

Note: This calculation is similar to the calculation of the slopes in the piecewise linear approximation. Therefore, we actually included the adjustment into b_1 and keep b_0 equal to zero. Nevertheless, in this case can we implement the piecewise linear approximation correctly. These adjustments are considered in the macro used to compute the piecewise linear approximations (Appendix E).

4.6. Validation of the model

The S&OP model is in development during this study. All required functionalities are present in the model. However, the results of the model are not yet validated. Validation is required before performing our research to provide correct and reliable results. Validation checks if the model is an accurate representation of reality for the particular objectives of the study (Law, 2007).

To validate the model, we performed a black-box validation with the S&OP plan of May 2017. This plan was made with the old S&OP tool. A black-box validation compares the outputs of the model with the real system (figure 4.5). The idea is that similar input should result in similar output. The real system is in our case the old S&OP tool. The input of both models is the data of CX of May 2017. We analyse the quantities of the solution of the model with the quantities in the S&OP plan. In consultation with the S&OP planner, the only differences we noticed where caused due to changes in the design between the old and the new S&OP tool. Therefore, we conclude that the model works correctly and provides reliable results.

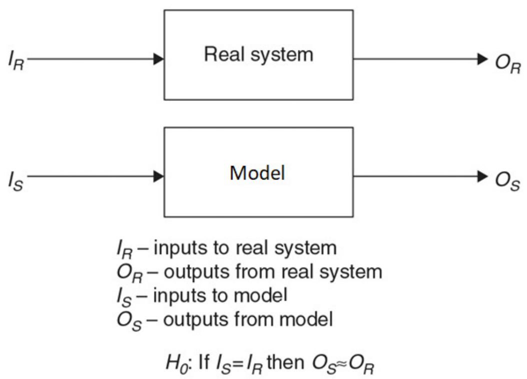


Figure 4.5 - Black-box validation (Robinson, 2004)

4.7. Key performance indicators

To analyse the outcomes of the model, we define key performance indicators (KPI's). In general, we analyse the planned production quantities for the decision-logic of the model and we investigate the valorisation for the financial analysis. At last, to determine the accuracy of the number of segments, we define the percent error of the quantity.

The KPI's described in this section are calculated per month for the first 12 months (June 2017 to May 2018). For the annual values of the KPI's, we take the sum of each month.

Change in quantity

The main task of the S&OP model is to plan the production quantities for the plan products of CX in a financial optimal way. When we change the financial input (e.g. the price of product 1), it is likely that some of the planned production quantities change. Besides the main products, the quantity of corresponding by-products also change. We determine the change in the quantities of the products in outcomes of the current model and the model with price elasticity.

To determine the quantity of a product, we need to take the sum of the quantity in each segment of the product. The number of segments differs per product. Some products have one segment (i.e. the fixed segment), some have three segments (i.e. the fixed, reducible and additional segment) and foil cheese has as many segments as the piecewise linear approximation. To measure the change in quantity of product j, we subtract the quantity of product j in the model with price elasticity from the quantity of product j in the current model. This leads to the following calculation of the change in the quantity of each product j.

$$\Delta Q_j = \sum_{i \in I_{j,PE}} Q_{i,j,PE} - \sum_{i \in I_{j,Curr}} Q_{i,j,Curr} \qquad \forall j \tag{14}$$

In which:
ΔQ_j = Change in the quantity of product j
$Q_{i,j,PE}$ = Quantity in segment i of product j in the model with price elasticity
$Q_{i,j,Curr}$ = Quantity in segment i of product j in the current model

We also determine the percent change in quantity of each product. The percent change is calculated as follows.

$$Percent\ change\ Q_j = \frac{\Delta Q_j}{\sum_{i \in I_{j,Curr}} Q_{i,j,Curr}} * 100\% \qquad \forall j \tag{15}$$

Change in valorisation

To express the improvement of implementing price elasticity financially, we calculate the change in the valorisation. The calculation is similar to the ET calculation in Section 2.6. In this case, we measure the valorisation over the total basket instead of over 1 ton of raw material intake. The valorisation of a basket is the revenue minus the production costs and costs for added components for the main products and by-products. As mentioned in Section 2.6, the only important component left out of the calculation for the valorisation is the cost of supplied raw material. Because the raw material supply remains equal in all situations, the cost of supplied raw material is as equal in each model. Therefore, we can neglect these costs when determining the change in the valorisation. Because the change of the valorisation covers the revenue and all changing costs, we can consider the change of the valorisation as the change of the gross profit.

Both the current model and the model with price elasticity calculate a financially optimal plan. The current model bases the decisions on the current prices, while the model with price elasticity bases the decisions on the prices with price elasticity. The prices in the current model are inaccurate and therefore, the current model makes suboptimal decisions. To measure the improvement of the new model, we base the valorisation of the current model on the prices of the model with price elasticity. In that case, we can determine the change of the valorisation due to the implementation of price elasticity. Figure 4.6 illustrates this method.

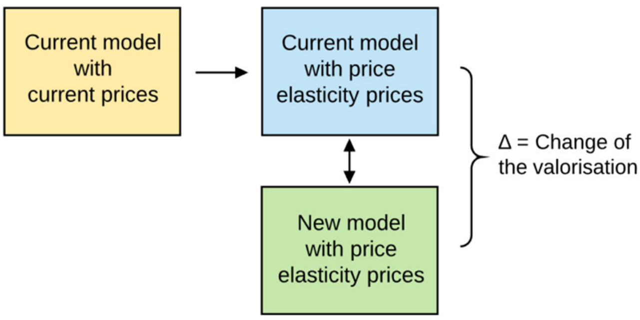


Figure 4.6 - Comparison between the models

We measure the change of the valorisation for each basket. Each basket consists of a different main product and corresponding by-products. Each product consists of one or more segments. Figure 4.7 presents this hierarchy within the foil cheese basket.

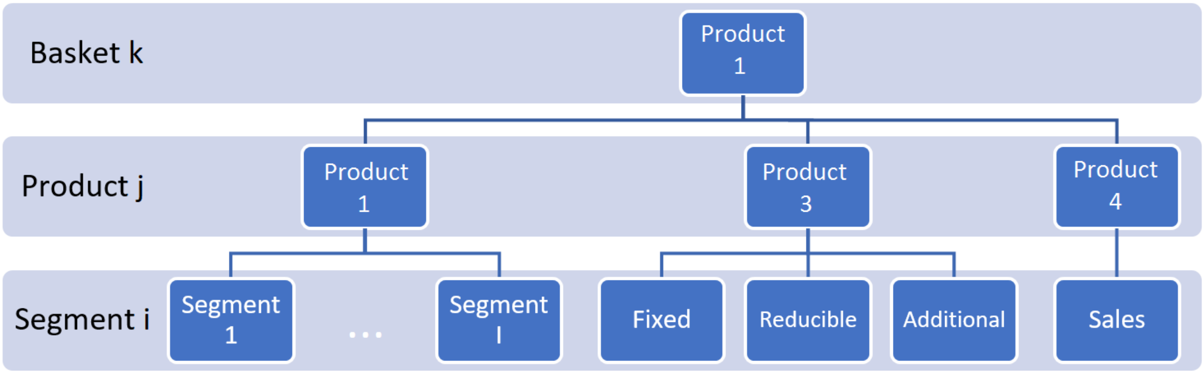


Figure 4.7 - Hierarchy of the product 1 basket

To determine the valorisation of a basket, we use the hierarchy illustrated above. At first, we establish the equation for the revenue. Secondly, we construct the calculation for the production costs. Thirdly, we determine the recipe costs. At last, we combine these equations to form the calculation of the valorisation. To remain overview, the subscript of the used model is left out of the equations (16), (17), (18) and (19).

The revenue is the product of the quantity Q and sales price P. The revenue in basket k is the sum of the revenues in all segments i of all products j in basket k. This leads to the following calculation:

$$Revenue_k = \sum_{j \in J_k} \sum_{i \in I_{j,k}} (Q_{i,j,k} * P_{i,j,k}) \qquad \forall k \tag{16}$$

In which:

$Revenue_k$ = Revenue of basket k

$Q_{i,j,k}$ = Quantity in segment i of product j in basket k

$P_{i,j,k}$ = Price in segment i of product j in basket k

The production costs are equal for each segment i and therefore calculated on the product level of a basket. The production costs of product j are calculated with the total quantity of product j and the production costs per unit (PC_j). The total production costs of basket k is the sum of the production costs of each product j:

$$Production\ costs_k = \sum_{j \in J_k} \left(PC_j * \sum_{i \in I_{j,k}} Q_{i,j,k} \right) \qquad \forall k \tag{17}$$

In which:

$Production\ costs_k$ = Production costs of basket k

$Q_{i,j,k}$ = Quantity in segment i of product j in basket k

PC_j = Production costs per unit of product j

From the outcomes of the model, we can deduce the purchased quantity of recipe products r used to produce product j in basket k. Together with the purchase price of each recipe product, the total recipe costs in basket k are determined by:

$$Recipe\ costs_{\ k} = \sum_{r \in R_{j,k}} (PQ_{r,j,k} * PP_{r,j,k}) \qquad \forall k \tag{18}$$

In which:

$Recipe\ costs_k$ = Recipe costs of basket k

$PQ_{r,j,k}$ = Purchase quantity of recipe product r used for product j in basket k

$PP_{r,j,k}$ = Purchase price of recipe product r used for product j in basket k

The valorisation in basket k is the revenue minus the production costs and recipe costs. This leads to the following equation for the valorisation in basket k:

$$Valorisation_k =$$

$$\sum_{j \in J_k} \sum_{i \in I_{j,k}} (Q_{i,j,k} * P_{i,j,k}) \quad - \sum_{j \in J_k} \left(PC_j * \sum_{i \in I_{j,k}} Q_{i,j,k} \right) - \sum_{r \in R_{j,k}} (PQ_{r,j,k} * PP_{r,j,k}) \qquad \forall k \tag{19}$$

To determine the change in the valorisation in basket k, we subtract the valorisation of the current model from the valorisation in the model with price elasticity:

$$\Delta Valorisation_k = Valorisation_{k,PE} - Valorisation_{k,Curr} \qquad \forall k \tag{20}$$

In which:

$\Delta Valorisation_k$ = Change in the valorisation of basket k
$Valorisation_{k,PE}$ = Valorisation of basket j in the model with price elasticity
$Valorisation_{k,Curr}$ = Valorisation of basket k in the current model

To measure the percent change of the valorisation relative to the revenue, we use the following equation:

$$Percent\ change\ valorisation_k = \frac{\Delta Valorisation_k}{Revenue_k} * 100\% \qquad \forall k \tag{21}$$

Quantity and price effect

We distinguish two effects in the change of the revenue of a product. The quantity effect (Q-effect) is caused by the change of the quantity. The price effect (P-effect) is caused by the change in price. Figure 4.8 visualizes the effects.

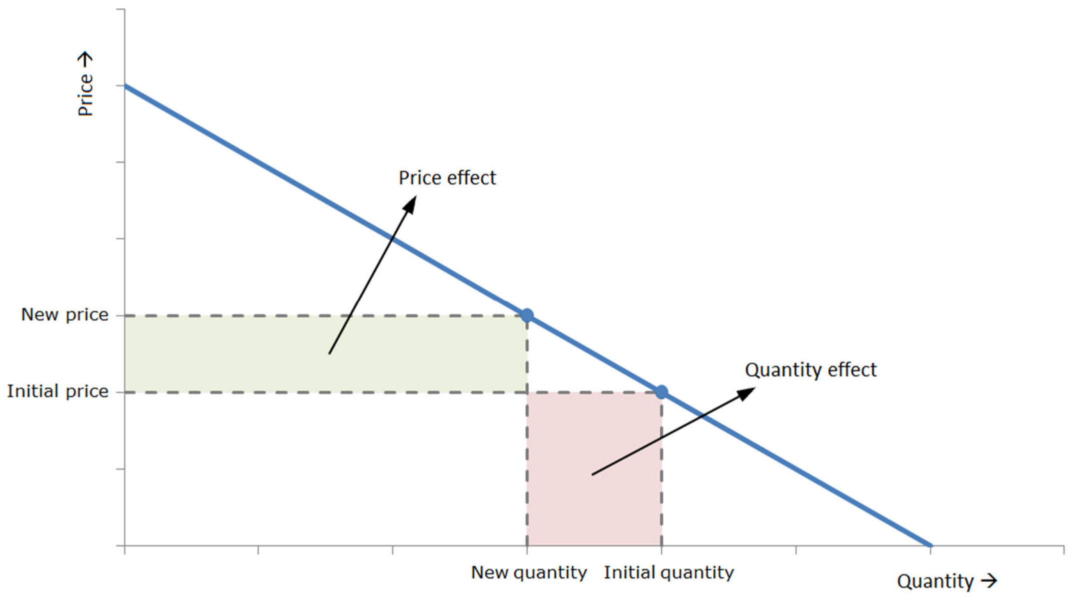


Figure 4.8 - Quantity and price effect

In the figure is the price (y-axis) plotted against the quantity (x-axis). When CX reduces the quantity from the initial quantity to the new quantity, the price increases from the initial price to the new price. The revenue (the area within the dashed lines), decreases due to the change in quantity (red area) and increases due to the change in price (green area).

We can calculate the Q- and P-effect by the following equations.

$$Q - effect = (New\ quantity - \ Initial\ quantity) * Initial\ price \tag{22}$$

$$P - effect = (New\ price - Initial\ price) * New\ quantity \tag{23}$$

In our case, we do not consider the revenue, but the valorisation. Besides that, we do not consider the effect of one product, but the effect of a basket. Nevertheless, the Q- and P-effect can be distinguished from the valorisation of a basket. However, we need to redefine the Q- and P-effect to apply this for our analysis.

We consider the main product as the basis for the initial and new quantity. The by-products change accordingly. To determine the initial and new quantity, we need to take the sum of the quantity within each segment of the main product of basket k. For the initial quantity, we perform this calculation with the outcomes of the current model. For the new quantity, we perform this calculation with the outcomes of the model with price elasticity. This leads to the following calculations for the initial and new quantity of basket k.

$$Initial\ quantity_k = \sum_{i \in I_{main\ product,k,Curr}} Q_{i,main\ product,k,Curr} \tag{24}$$

$$New\ quantity_k = \sum_{i \in I_{main\ product,k,PE}} Q_{i,main\ product,k,PE} \tag{25}$$

In which:

$Initial\ quantity_k$	=	Total quantity of the main product of basket k in the current model
$New\ quantity_k$	=	Total quantity of the main product of basket k in the model with price elasticity
$Q_{i,mainproduct,k,Curr}$	=	Quantity in segment j of the main product of basket k in the current model
$Q_{i,mainproduct,k,PE}$	=	Quantity in segment i of the main product of basket k in the model with price elasticity

In order to consider the valorisation instead of the revenue, we have to consider the valorisation per ton of the main product instead of the price of the main product. Since we determined the total valorisation of a basket in the previous section, we can measure the initial and new valorisation per ton with the following equations.

$$Initial\ valorisation/ton_k = \frac{Valorisation_{k,Curr}}{Initial\ quantity_k} \tag{26}$$

$$New\ valorisation/ton_k = \frac{Valorisation_{k,PE}}{New\ quantity_k} \tag{27}$$

In which:

$Initial\ valorisation/ton_k$	=	Valorisation per ton of basket k in the current model
$New\ valorisation/ton_k$	=	Valorisation per ton of basket k in the model with price elasticity
$Valorisation_{k,Curr}$	=	Valorisation of basket k in the current model
$Valorisation_{k,PE}$	=	Valorisation of basket k in the model with price elasticity

$Initial\ quantity_k$	=	Total quantity of the main product of basket k in the current model
$New\ quantity_k$	=	Total quantity of the main product of basket k in the model with price elasticity

Then, we can determine the Q-effect and the P-effect of the valorisation of basket k with the following equations, which are based on the equations (19) and (20).

$$Q - effect_k = (New\ quantity_k - Initial\ quantity_k) * Initial\ valorisation/ton_k \tag{28}$$

$$P - effect_k = \left(New\ valorisation/ton_k - Initial\ valorisation/ton_k\right) * New\ quantity_k \tag{29}$$

The Q- and P-effect together form the change in valorisation, similar to the revenue of the example in Figure 4.8. The overall change of the valorisation is equal to the change of the valorisation in each basket, which is equal to the Q- and P-effects in each basket. Therefore, the following equation holds:

$$Overall\ change\ of\ the\ valorisation = \sum_{k \in K} \Delta V_k = \sum_{k \in K} (Q - effect_k + P - effect_k) \tag{30}$$

Percent error

To analyse the accuracy of a piecewise linear approximation relative to the actual revenue curve of product 1, we calculate the percent error of the maximum error of the approximation relative to the actual revenue. Due to the adjustment in Section 4.3, the maximum error is obtained at each breakpoint. This leads to the following equation:

$$Percent\ error\ of\ the\ approximation = \frac{|Approximated\ revenue - Actual\ revenue|}{Actual\ revenue} * 100\% \quad (at\ 3{,}000\ tons) \tag{31}$$

We also perform experiments with the number of segments, to analyse the effect of the accuracy on the output of the model. Therefore, we need a KPI to compare the experiments. As KPI, we choose the quantity of product 1, because the quantity is directly influenced by the prices of product 1, which vary due to the number of segments. Besides that, the quantities of product 1 form the basis of the change in valorisation. To measure the accuracy of the quantities of the solutions, we need to know the optimal production quantities. Since we do not know the optimal quantities, we perform a 'benchmark experiment' with a model with a piecewise linear approximation with a low percent error of the approximation. We assume that the solution of this model is optimal and may be used as benchmark for the other experiments. Then, we can determine the level of accuracy of our other experiments. To analyse the results between our benchmark and our experiments, we use the percent error of the quantity of product 1. The calculation is as follows.

$$Percent\ error\ of\ the\ quantity = \frac{\left|Q_{experiment} - Q_{benchmark}\right|}{Q_{benchmark}} * 100\% \tag{32}$$

In which:

$Q_{experiment}$	=	Quantity of product 1 of the experiment
$Q_{benchmark}$	=	Quantity of product 1 of the benchmark experiment

4.8. Conclusion

In this chapter, we investigate how to adjust the S&OP model to implement price elasticity and we answer our third research question:

How should we adjust the S&OP model to implement the price elasticity of product 1?

We described the influences on product 1 in Section 4.1 and expressed the influence of raw material sales and product 5 in Section 4.2 within the price of the additional raw material sales. We use equation (10):

$$PR = \mathrm{PR}_0 - MC * \mathrm{BOM} * \mathrm{Q}_0 * C \tag{10}$$

In Section 4.3 we determined the characteristics of the piecewise linear approximation. Table 4.1 describes the characteristics which should be used for the piecewise linear approximation of the revenue function.

Characteristic	**Description**
Breakpoints	Uniform placement
Adjustment b_0	$^2/_3$ of the initial maximum deviation
Area of approximation	3,000 to 4,800 tons

Table 4.1 - Characteristics of the piecewise linear approximation

In Section 4.4 we described the simplifications and assumptions of the model. Our main assumption is that besides the price elasticity of product 1, everything remains the same. We assume that the demand and prices of other products and the quantities of competitors do not change. Afterwards, we described how we validated the S&OP model and adjusted the model to implement the new prices of product 1. At last, we defined our key performance indicators to analyse the experiments. These are the change in quantity, change in valorisation and the quantity and price effect. In the next chapter, we perform experiments with the adjusted model and analyse them according to the KPI´s.

5. Model evaluation: Numerical results

In this chapter, we analyse the numerical results of the model. For all our experiments, we used the S&OP model with the data of the S&OP cycle from May 2017 of CX. With the data of product 1, we compute several piecewise linear functions according to the method of Section 3.2. We use the characteristics and assumptions as described in Chapter 4. We start to determine the necessary number of segments of our piecewise linear approximation. Afterwards, we analyse the numerical results of the model with price elasticity relative to the current model. In Section 5.3 we provide a sensitivity analysis and Section 5.4 investigates the worst-case scenario. Section 5.5 compares different scenarios to provide more insight into the behaviour and impact of the model. With this chapter, we provide an answer to our fourth research question:

> *What is the impact of price elasticity on decision-making and the valorisation of the S&OP model?*

5.1. Number of segments

From the literature in section 3.2, we find that the number of segments is an important factor for the accuracy of the approximation. An accurate approach is important to make financially optimal decisions for the production quantity. More segments provide a higher accuracy. Literature stated that 5 to 14 uniform segments can provide adequate approximations of revenue functions. In this section, we investigate how many segments an approximation needs to provide adequate accuracy.

Although more segments provide a higher accuracy, the implementation of a piecewise linear approximation of more than 10 segments is not desired due to practical implications. More segments lead to too much human effort, for this study, but mainly when this approach will be implemented within the monthly S&OP cycle of CX. It leads to too much human effort, because a separate manual operation is required for the implementation of each segment in the S&OP model.

Using the macro in Microsoft® Excel, we compute piecewise linear approximations with several number of segments. We compute approximations with 2 to 10 segments and with 20, 30, 40 and 50 segments. We determine the maximum error of the approximation from the revenue curve for each approximation. We calculate the percentage error relative to the revenue in June 2017 at the lower bound of 3,000 tons, which is 940 thousand euros. Figure 5.1 shows the error per number of segments.

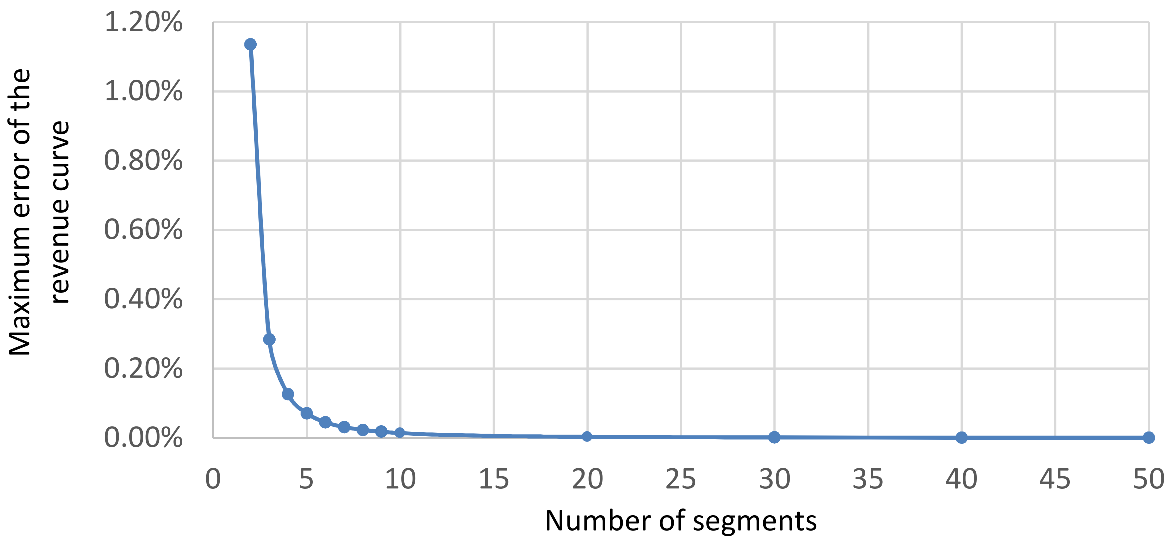


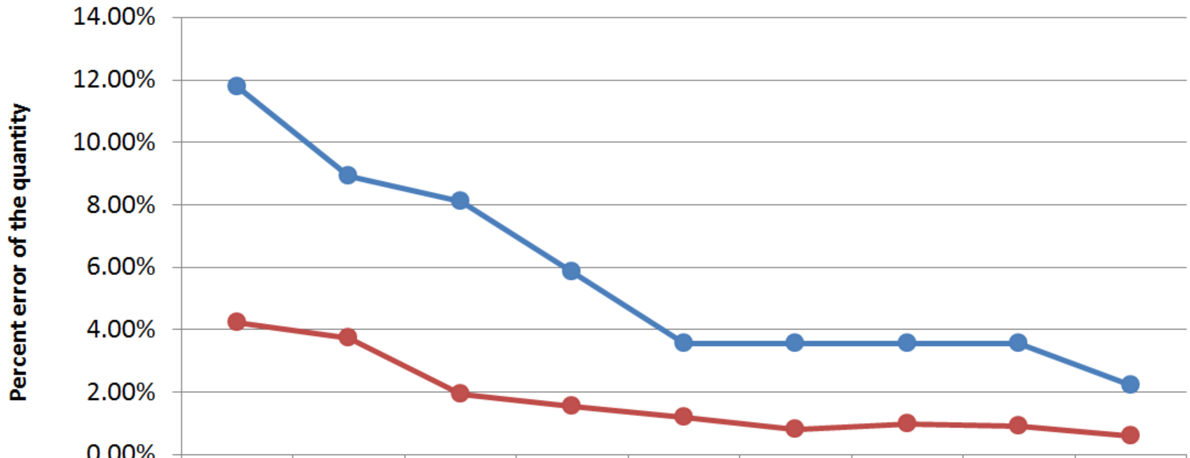
Figure 5.1 - Accuracy of the approximations

We find that the deviation at 2 segments is between the 1.1% and 1.2% and it seems to decrease exponentially with more segments. At 10 segments, the error is 0.014% and at 50 segments, it is only 0.00047%.

The approximation with 2 segments already provides a quite high accuracy. A model with more segments is likely to provide better results. However, we do not know how the S&OP model responds to the number of segments. Therefore, we analyse the decisions of the model by measuring the accuracy of the quantities of product 1 with a varying number of segments.

To measure the accuracy of the optimal quantity of product 1, we perform one experiment with 50 segments as benchmark. Considering the high accuracy of the approximation, we assume that this experiment is optimal and may be used as a benchmark for the other experiments. Besides the experiment with 50 segments, we perform experiments with 2 to 10 segments.

After each experiment, we determine the percent error per month for the first 12 months. Figure 5.2 shows the average percent error and the maximum percent error of each model relative to the benchmark solution.



	2 segments	3 segments	4 segments	5 segments	6 segments	7 segments	8 segments	9 segments	10 segments
Average	4.24%	3.74%	1.93%	1.55%	1.18%	0.82%	0.99%	0.92%	0.57%
Maximum	11.78%	8.91%	8.09%	5.84%	3.54%	3.54%	3.54%	3.54%	2.21%

Figure 5.2 - Accuracy of the number of segments

Figure 5.2 brings two insights:

- More segments lead in general to a lower percent error, but not as significant as with error of the revenue in Figure 5.1.
- Due to the complexity of the model, more segments do not necessarily provide better results.
    - The maximum accuracy is equal with 6 to 9 segments. This is the case because these models have the same quantity in the month of the maximum error.
    - The experiment with 7 segments provides a higher average accuracy than 8 and 9 segments. By chance, the experiment with 7 segments provides a better outcome, because the quantities revolve around the optimum.

The experiment with 10 segments provides the highest accuracy, on average 0.57% error. Therefore, we chose to use 10 segments for further analysis.

5.2. Numerical results of price elasticity

In this section, we describe the differences between the outcomes of the current model and the model with price elasticity. Again, we use the data of CX from May 2017. At first, perform an experiment with the current model. We compare this with the model with 10 segments. Only the product 1 prices and number of segments changed. The current model has the fixed, reducible and additional segment with equal market prices and the model with price elasticity has 10 segments with prices determined by a piecewise linear approximation.

We compare the outcomes of the two models and the quantities of product 1, product 2 and corresponding by-products change. Table 5.1 presents the changes for the coming year from June 2017 to May 2018.

Description	Change	Change (%)
Product 1 quantity	-9,924 tons	-4.1%
Product 2 quantity	+7,622 tons	+11.8%
Product 4 quantity	+3,970 tons	+1.8%
Product 3 quantity	-5,160 tons	-1.7%

Table 5.1 - Annual changes in the S&OP due to price elasticity

In the scenario with price elasticity, 4.1% less product 1 is made and 11.8% more product 2 is made instead. As the decrease in product 1, less product 3 and product 4 is produced. However, because the production of product 2 produces more product 4 than the product 1 production, the total product 4 production increases. The changes in quantities only provide insight into the practical changes. Less product 1 is not necessary better for CX. Ultimately, we want to financially improve the S&OP model. Therefore, we analyse the valorisation in the next section.

Analysis of the valorisation

In this section, we describe the financial effects of price elasticity based on the Q-effect and the P-effect. These effects are explained in Section 4.7. The effect on the revenue caused by a change in quantity is the Q-effect. The effect on the revenue caused by a change in price is the P-effect. Table 5.2 shows the expected financial effects and the effects relative to the revenue for the coming year from June 2017 to May 2018. product 2 has P-effect, because the price stays on the same level for each quantity.

Description	Financial effect		% of the revenue of the product 1 and product 2 basket
Product 2 Q-effect	€	1,325,858	10.02%
Product 1 Q-effect	€	-1,546,275	-11.68%
Product 1 P-effect	€	284,615	2.18%
Overall effect	€	68,521	0.52%

Table 5.2 - Annual effects of the valorisation

Overall, we achieve an increase of the annual valorisation of € 68.5 thousand euros. Relative to the revenue of product 1, product 2 and corresponding by-products, this is an improvement of 0.52%. This seems a minor improvement, but we must bear in mind that product 1 and product 2 are made in high volumes with low margins. The improvement is achieved with significant underlying Q-effects. The valorisation changes with tens of thousands due to the change from product 1 to product 2, the Q-effects. The Q-effect of product 2 is lower, because the valorisation product 2 is lower than product 1. The decrease of the valorisation resulting from the Q-effects is compensated by the increase in the price of product 1, the P-effect.

Besides the overall annual effect and underlying Q- and P-effects, we can also break the annual effect down into the effect per month. Because the quantities and prices of product 1 and product 2 change throughout the months, it is likely that the effect in the valorisation also changes per month.

Valorisation per month

Figure 5.3 represents the improvement of the valorisation per month. We see that the major of the annual improvement is obtained in the first four months (e.g. June 2017 to September 2017). In the first four months, 78% of the annual improvement is obtained, while the last eight months are responsible for 22% of the improvement.

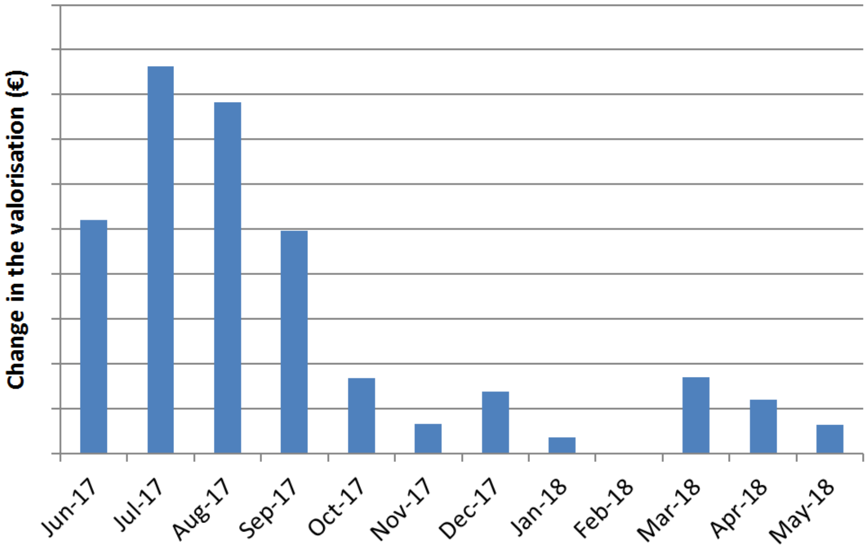


Figure 5.3 – Expected improvement of the valorisation per month

This figure clearly illustrates that there are certain months in with it is significantly more profitable consider the effect of price elasticity than in other months. February 2018 does not show any improvement, because the quantities in this month remain equal. We analyse the underlying changes in the Q- and P-effects with waterfall diagrams of the effects in July 2017 (Large improvement) with December 2018 (Small improvement) in Figure 5.4.

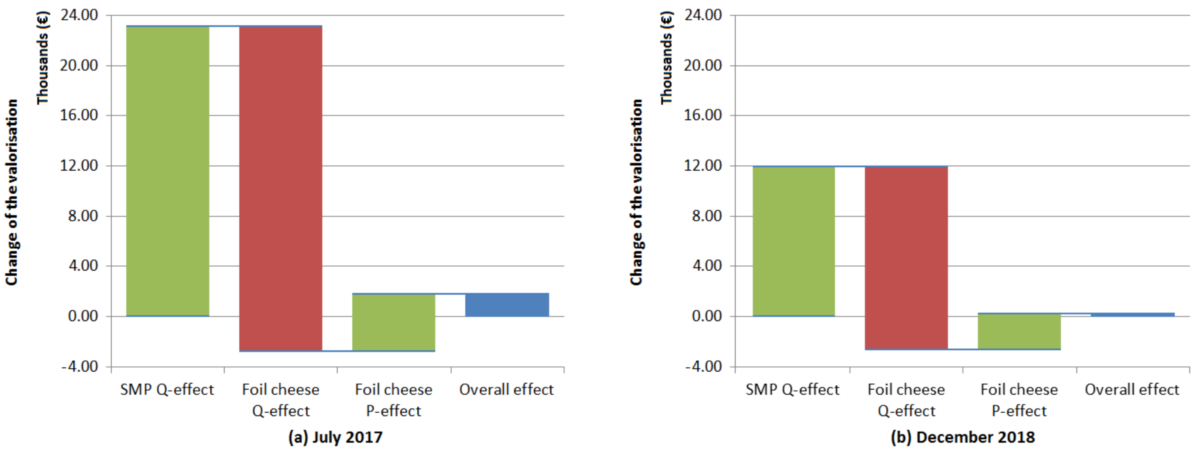


Figure 5.4 - Effects in July 2017 (a) and December 2018 (b)

In the figures, we see that the effects in December 2018 are around 2 times smaller than the effects in July 2017. However, the overall effect in December is 6 times smaller than the overall effect in July. The P-effect in December is almost nullified by the negative result of the Q-effects, while the result of the Q-effects in January is around 60% of the P-effect.

We find major factor for the significant differences in the difference between the valorisation of product 1 and product 2. Figure 5.5 presents an equivalence table with the valorisation of 1 ton raw material processed into product 2 and additional quantities of product 1 for the coming 12 months. The ET valorisation of additional product 1 indicates the added value of processing 100 kg of raw material into product 1, when CX already makes a certain volume. The values are calculated with the ET calculation of Section 2.6 and with the prices of product 1 at certain volumes, obtained from the piecewise linear approximation.

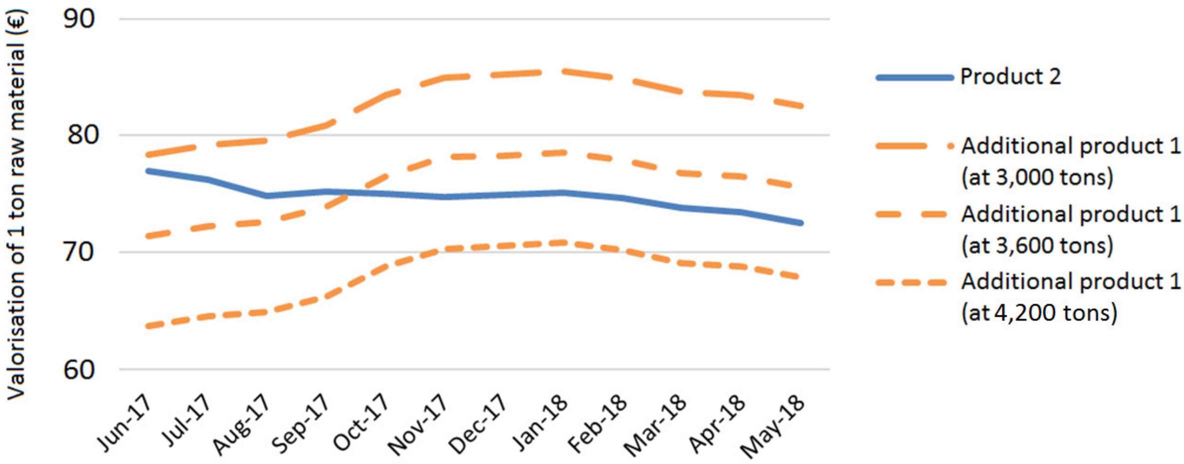


Figure 5.5 - Equivalence table of product 2 and additional product 1

From the ET, we obtain one major insight. In the months with significant improvements (e.g. June 2017 to September 2017) is the ET valorisation of product 1 low, relative to the ET valorisation of product 2. In the months with a small improvement is the ET valorisation of product 1 higher, relative to the ET valorisation of product 2.

We can explain this insight. With a relatively low valorisation of product 1:

- The quantity of product 1 decreases more to obtain a valorisation equal to the product 2 valorisation. This leads to
    - Larger Q-effects
    - A higher product 1 price, so a larger P-effect
- The difference between the Q-effect of product 1 and product 2 is relatively small.

The combination of the relatively smaller difference between the Q-effects and a larger P-effect results in a higher improvement of the valorisation in the first four months. The other months show the opposite effects.

The model provides significant changes in the volumes and establishes Q- and P-effects of thousands of euros. Nevertheless, all results are based on forecasts which can change throughout the time. A small inaccuracy of the forecasts may have a large influence on the valorisation. To investigate these risks, we perform a sensitivity analysis in the next section.

5.3. Sensitivity analysis

With a sensitivity analysis, we want to investigate what the consequences are of incorrect input data of S&OP model. It may be possible that the estimation of the price elasticity differs from the actual price elasticity. In that case, the actual prices of product 1 are different from the prices on which the solution is based. Besides the price elasticity constant, we also analyse the sensitivity of the initial quantities and initial prices of product 1 and the price of product 2.

For the sensitivity analysis, we analyse the financial improvement of the price elasticity model. We use the outcomes of the model with price elasticity and analyse the valorisation by changing different parameters. The changes of the parameters lead to different prices and another valorisation. The sensitivity analysis is ranging from -15% to +15%. -15% means that the value in the actual value is 15% lower than the value used in the model to optimize the S&OP. +15% means that the actual value is 15% higher. We must note that we assume for this analysis that the deviations of the parameters are inaccurate for the whole year. In reality, this is very unlikely, because Department X will probably adjust its forecast when a systematic error occurs. Figure 5.6 shows the results.

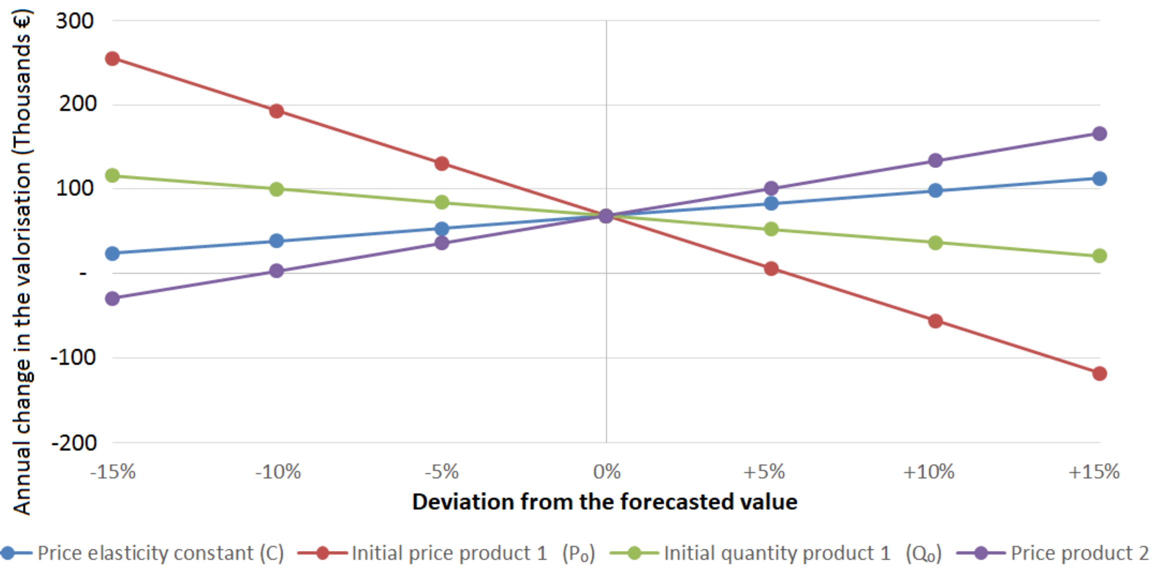


Figure 5.6 - Sensitivity analysis

The sensitivity analysis provides three insights:

- All parameters show a linear trend.
- The price elasticity model becomes more profitable with an actual higher price elasticity constant, a lower initial price of product 1, a lower initial quantity of product 1 and a higher price of product 2 (and vice versa). We can confirm these effects, because they all result in the fact that the valorisation of product 1 becomes lower relative to product 2.
- The initial price of product 1 has the largest impact. At a deviation of -6%, the financial effect is diminished to zero. The price of product 2 has also a significant impact. A deviation of -10% is the financial improvement neglectable. Nevertheless, forecast inaccuracies of product 1 and product 2 over a whole year, which are worse than 6%, will hardly ever occur. Based on these figures, the implementation of price elasticity seems to have a low risk.

The sensitivity analysis provides promising results. However, these numerical results are based on the assumption that competitors do not react. In reality, increasing prices will lead to a response of competitors. They probably increase their production, because product 1 becomes more profitable. In the worst-case, the competitors increase their production quantity in a month with the same amount as CX decreases its quantity. In that case, the prices do not increase but stay on the same level. We analyse this potential risk in the next section.

5.4. Worst-case scenario

Although the sensitivity analysis does not provide significant risks, we want to investigate the risks more extensively. In this section, we analyse the effect when competitors react on the production quantities of CX.

The numerical results and the sensitivity analysis are based on the assumption that competitors do not react. In reality, increasing prices will lead to a response of competitors. They probably increase their production, because product 1 becomes more profitable. In the worst-case, the competitors increase their production quantity in a month with the same amount as CX decreases its quantity. Then, the prices do not increase and the price effect is nullified. Table 5.3 presents an overall negative effect of 220 thousand euros, which is a lot worse than the expected financial improvement of 68 thousand euros.

Description	Annual financial effect		% of the revenue of product 1 and product 2 basket
Product 2 Q-effect	€	1,325,858	10.02%
Product 1 Q-effect	€	-1,546,275	-11.68%
~~Product 1 P-effect~~	~~€~~	~~284,615~~	~~2.18%~~
Overall effect	€	-220,417	-1.66%

Table 5.3 – Worst-case annual financial effects

When we analyse the overall impact per month, we see in Figure 5.7 that the loss per month is not similar to the potential improvement of Section 5.2. The worst-case loss is more than in the potential improvement in every month, except February 2018.

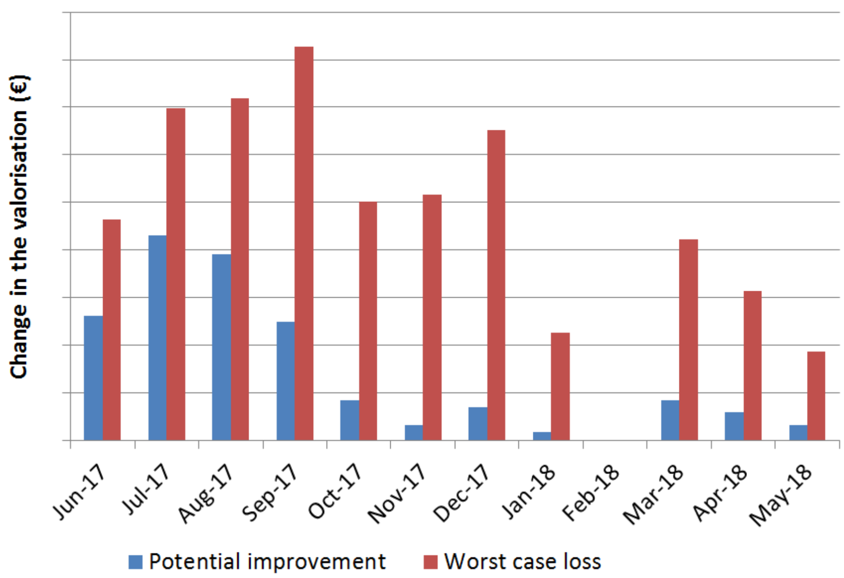


Figure 5.7 - Potential improvement and worst-case loss

Again, the explanation lies in the fact that the valorisation of product 1 is low relative to the valorisation of product 2 in the first four months. The insight about the profitable months in Section 5.2 still holds. However, the P-effect, which was larger in the first four months, is nullified in the worst-case scenario. This means that in months with a relatively low valorisation of product 1 (e.g. June 2017 to September 2017):

- The quantity of product 1 decreases more to obtain a valorisation equal to the product 2 valorisation. This leads to larger Q-effects.
- The difference between the Q-effect of product 1 and product 2 is relatively small

Larger Q-effects and a smaller difference between the Q-effects compensate each other. Consequently, the worst-case losses in the first four months are relatively close to the losses in last months. Therefore, the worst-case scenario and the potential improvement of Section 5.2 both confirm that the impact of price elasticity is more promising when the product 1 valorisation is low relative to the product 2 valorisation. In that case, the expected benefit is higher and the risks are lower.

We have investigated the impact of price elasticity on annual and monthly level and investigated why some months provide more promising results than other months. In the next section, we perform experiments in which the capacity of product 2 and the raw material-product 1 ratio changes. We investigate the impact on the outcomes due to these changes and we determine in which cases this can lead to more promising results.

5.5. Scenarios

This section describes the results of potential scenarios to get a better understanding of the model and determine which cases provide more promising results. We perform experiments and vary two parameters of the model, the capacity of product 2 and the changing raw material-product 1 ratio.

The capacity of product 2 can be an important factor. Less capacity limits the production of product 2. We expect that instead of product 2, more product 1 will be made and more raw material will be sold. On the other hand, the production of product 2 is not limited with more product 2 capacity. We expect in that case that the product 1 production is decreased more than in the current situation and that selling raw material is not necessary. We perform experiments with less capacity by excluding Machine 1 in Factory 1 and experiments with more capacity by adding extra machine in Factory 2 similar to Machine 5.

In Section 4.2, we propose to experiment with the raw material-product 1 ratio, because the exact ratio is unknown. According to the market analyst, it is somewhere between 60% and 100%. Therefore, we choose to experiment with 60%, 80% and 100%. In this case, we expect to get a proper overview of the influence of the sales of raw material.

To analyse the effects, we perform the experiments with the current model and with the model with price elasticity. We perform a full factorial analysis. This means that we perform an experiment for each possible scenario. We have three factors in the capacity, three in the raw material-product 1 ratio and two for each model. This results in a total of 18 experiments. Again, we perform the experiments with the characteristics as we describe in Chapter 4 and the data of CX from May 2017.

Analysis of the scenarios

When we analyse the outcomes of the experiments, we find that the changes in quantity between the current model and the price elasticity model are equal in each scenario, similar to the changes of Table 5.4 in Section 5.2. This means that the decision logic of the model with price elasticity does not change because of a varying capacity or due to a varying raw material-product 1 ratio. Instead of analysing the change between the current model and the model with price elasticity, we analyse the change between the scenarios with a different capacity and a varying raw material-product 1 ratio.

Raw material sales to product 1

We find that the quantities do not change between the scenarios with different raw material-product 1 ratios. The only explanation for this result is that the sale of raw material never becomes more attractive than product 1. To confirm this hypothesis, we analyse the ET with product 2, additional product 1 and the valorisation of raw material sales with the raw material-product 1 ratios of 60%, 80% and 100%, see Figure 5.8.

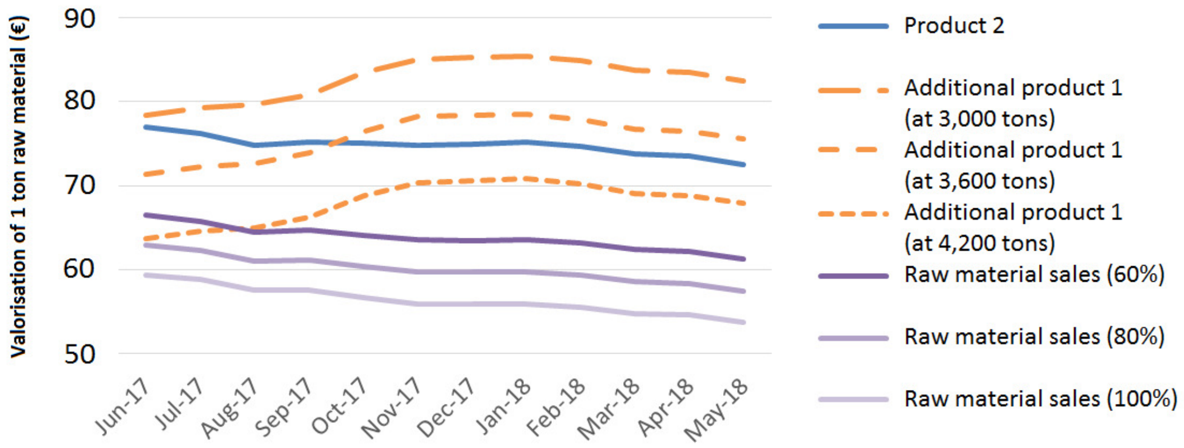


Figure 5.8 - Equivalence table with product 2, product 1 and raw material sales

In the equivalence table, we clearly see that the valorisation of raw material sales is based on product 2. Therefore, the valorisation of raw material sales is most attractive in the first four months. To sell raw material, the product 1 production should increase until the valorisation is equal to raw material. However, in the first four months, the quantity decreases because the valorisation of product 1 is low relative to product 2 (and raw material sales). The relatively high valorisation of product 2 results in a high valorisation of raw material sales, but also in a decrease of product 1. Therefore, it is highly unlikely that the sale of raw material becomes more attractive than product 1. The model only sells raw material out of necessity.

Changing capacity

On the other hand, in the scenarios with a varying capacity, the total quantity of product 2 varies because of capacity constraints. Due to the varying capacity, more or less raw material is sold. The total quantity of product 1 remains equal. We calculate the percent change between the scenario with the current capacity and the scenario with less capacity and between the current capacity and the scenario with more capacity. Table 5.4 presents the percent changes.

Description	Less capacity	More capacity
Product 2 quantity	-20.5%	+1.8%
Product 4 quantity	-13.9%	+0.4%
Additional raw material sales	+2158.7%	-100.0%

Table 5.4 – Annual changes in the S&OP due to a varying capacity

The scenario with less capacity results in a significant decrease of the product 2 quantity of 20.5%. The product 4 quantity changes accordingly, but the percentages show otherwise because the total product 4 quantity larger than quantity that is produced with product 2. In the scenario with more capacity, the quantity of raw material sales is nullified, because all raw material can be processed into product 2. In only leads to a minor increase, because the additional raw material sale in the scenario with standard capacity is already relatively low. This low quantity in the standard scenario leads to a major increase of 2158.7% of the additional raw material sales in the scenario with less capacity.

The main question is why the product 1 quantity remains the same. The quantity is decreased due to price elasticity, so there should be capacity left to produce product 1 instead of selling raw material. After all, the valorisation of product 1 is higher than the additional raw material sales.

We find the explanation in the planned disturbance capacity of 30 thousand raw material equivalents which is planned at product 1 and product 2. The decrease of product 1 production results in disturbance capacity being planned at the production lines of product 1. The capacity left at the production lines of product 1 is in neither of the months more than 30 thousand raw material equivalents. Therefore, in the scenario with less, the product 1 production cannot be increased more because of the necessary anticipation of disturbance.

Changing the capacity of product 2 or varying the raw material-product 1 ratio does not change the situation that all remaining product 1 capacity is planned with disturbance. Changing the capacity only affects the product 2 production and leads to less or more raw material sales, which we see in in the scenarios (Table 5.4). Because the valorisation of raw material sales is in each month lower than product 2 and the valorisation of product 1 does not drop below product 2, raw material sales is always the last option. The changes of product 2 and raw material sales were only caused by the capacity and not by the price elasticity of product 1. Therefore, the decision logic stays the same in all scenarios.

5.6. Conclusions on the results

We investigated the impact of price elasticity on the S&OP model in this chapter. In this section, we conclude our fourth and last research question:

> *What is the impact of price elasticity on decision-making and the valorisation of the S&OP model?*

We find that a model with 10 segments provides adequate accuracy. From the comparison of the current model with the model with price elasticity, we conclude that:

- The product 1 quantity decreases with 4.1%
- The product 2 quantity increases with 11.8%
- The valorisation improves annually 68 thousand euros
- The impact of price elasticity is higher in months in which the valorisation of product 1 is low relative to the valorisation of product 2

The sensitivity analysis (Figure 5.6) shows that a forecast inaccuracy of 6% in the product 1 price or 10% in the product 2 price over the whole year can diminish the financial improvement to zero. Such deviations are not likely for a whole year and therefore, the implementation of price elasticity seems to have a low risk. Nevertheless, we investigate the worst-case scenario in which competitors do react. In that case, the P-effect is nullified and a loss of 220 thousand euros is made. Therefore, it is important to have a comprehensive view of the responses of the competitors.

Experiments with different scenarios provide similar results in the decisions and valorisation. The sale of raw material only becomes attractive with a significant increase of the product 1 production, while the product 1 production decreases due to price elasticity. Therefore, raw material should only be sold when there is no alternative to process it within the company. The scenarios with a changing capacity of product 2 only show a change in product 2 and the raw material sales because of capacity constraints and the planned disturbance capacity and not affect the changes regarding price elasticity.

6. Conclusion and recommendations

In this chapter, we conclude our research and answer our main research question. We draw conclusions in Section 6.1 and elaborate about the limitations of this research in Section 6.2. In Section 6.3 we provide recommendations and in Section 6.4 we propose future research.

6.1. Conclusion

Before this research, CX had the technical possibilities and the required data to implement price elasticity in their sales and operations planning. However, they did not have a specified approach and did not have insight into the impact on the planning and valorisation of their raw material. This research investigates both. We establish an approach to implement price elasticity in the S&OP model and provide insight into the changes of the decision-making of the model and the potential financial improvements. This research answers our main research question:

What is the impact of the implementation of price elasticity in the sales and operations model of Company X?

We reviewed literature about price elasticity in sales and operations planning. The literature shows positive results, but does not provide a direct answer to our research question. In additional to the literature review, we find a general approach for piecewise linear approximations. We adopt the general approach from the literature to construct piecewise linear approximations of the revenue curve of product 1.

We conclude that a model with 10 segments provides adequate accuracy. The model with price elasticity suggest to reduce the annual product 1 quantity with 4.1% and to increase the product 2 quantity with 11.8% compared to the current model. These changes result in an annual improvement of the valorisation of 68 thousand euros, which is 0.52% of the revenue of product 1, product 2 and corresponding by-products.

The decrease of product 1 quantity leads to a decrease of the valorisation of 1.5 million euros, while the valorisation of product 2 increases with 1.3 million euros due to the increase of the product 2 production. The decrease of the quantity of product 1 leads to increased sales prices which improve the valorisation of product 1 with 285 thousand euros.

The impact of price elasticity is significantly higher in months in which the valorisation of product 1 is low relative to the valorisation of product 2. Therefore, the impact of price elasticity can significantly improve when the valorisation of product 1 and product 2 become close to each other.

The sensitivity analysis shows that the financial improvement is nullified when the forecasted prices of product 1 and product 2 deviate with 6 to 10% over the whole year. Because these situations are highly unlikely, the improvements in valorisation have a low risk regarding the input data.

However, we discover larger risks from the worst-case scenario. The P-effect can be nullified by the response of the competition. Then, a loss of 220 thousand euros can be made because of change in quantity of product 1 to product 2. The worst-case scenario shows a lower risk in months in which the valorisation of product 1 is low relative to the valorisation of product 2.

Experiments with different raw material-product 1 ratios and with a changing product 2 capacity show similar results. In all cases, raw material should only be sold when there is no alternative to process it within the company. The decision logic and valorisation due to the price elasticity of product 1 do not change.

6.2. Limitations

Before our recommendations to CX, we mention the limitations of this research. The main limitation of this research is that the real world is far more complex than how we modelled it during this study. This limits the practical feasibility of this research. We divided this main limitation into four limitations:

- We assume that competitors do not change their production quantities. However, in reality competitors probably response to the decision of CX to decrease its quantity of product 1 to increase the price. In that case, decreasing the production does not lead to the expected price increase.
- We assume that prices only react on the quantity of the same month and that the price and quantity in a month are mutually independent of prices and quantities in other months. In practice, prices are not dependent on the quantities of the same months, but are also influenced by the prices and quantities of the months before. Besides that, prices continuously develop and are not stable within a time period of a month.
- We assume a completely flexible demand of product 1 and the results show a decrease in production of 4.1%. In reality, this reduction might not be possible or desirable. Especially on the short-term, when most sales are contractually determined.
- Our research only considers the price elasticity of product 1. However, CX observes price elasticity in other markets. Because we did not include the price elasticity of other products, we did not get a comprehensive overview.

6.3. Recommendations

Although the annual financial improvement of price elasticity is 68 thousand in our research, we do not recommend starting to use price elasticity as it is used in this research. We do not recommend this, because of the limitations of Section 6.2 on which the research is based and because of the worst-case scenario, which results in a loss of 220 thousand euros. Nevertheless, the results of price elasticity will be more promising when the valorisation of product 1 becomes closer to the product 2 valorisation. Therefore, we recommend combining the concepts of this study with more knowledge about the product 1 market to benefit from the effects of price elasticity when the valorisation of product 1 becomes close to the product 2 valorisation.

We also recommend starting the discussion about price elasticity within CX. Especially the product 1 and product 2 hub can benefit from the integral insights of this research, while the product 1 hub and the Market Intelligence department can provide Department X with more knowledge and insights about the product 1 markets.

6.4. Future research

The results of this study show large risks regarding the assumptions of this study. Therefore, future research is necessary before using the methods of this research in practice. CX should investigate to which extend it is possible to enhance the S&OP model with more intelligent market behaviour to provide accurate results, while keeping the human effort for the S&OP within bounds. Suggestions for future research are:

- Analyse how competitors can react on the decisions of CX and investigate how to implement this into the S&OP model.
- Analyse the price elasticity of other products and investigate the impact of the model when including these price elasticities.
- Investigate how prices of product 1 and other products react on the quantities of CX in the current month and previous months and find a way to implement this knowledge into the S&OP model.

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Appendices

Appendix A: Derivation of the revenue function

In this appendix, we describe the complete derivation of the revenue function with price elasticity as described in Chapter 1, based on the research of Van Haperen (2016).

Currently, the revenue (R) is the product of the quantity (Q) and the price (P):

$$R = Q * P \tag{A1}$$

Van Haperen (2016) expressed the relationship between the change in price and the change in quantity as follows. For every decrease in quantity (Q), the sales price (P) increases with a constant value (C):

$$\Delta P = -\Delta Q * C \tag{A2}$$

With the current S&OP process, the initial production quantity and corresponding market price are determined every month, based on the new market conditions. To use this data to construct the new revenue function, Van Haperen introduced the initial sales price (P_0) and the initial quantity (Q_0). The price (P) can be calculated from the initial price (P_0) and the change in price (ΔP):

$$P = P_0 + \Delta P \tag{A3}$$

By combining (A2) and (A3), P can be written as:

$$P = P_0 - \Delta Q * C \tag{A4}$$

Since $\Delta \mathrm{Q} = \mathrm{Q} - \mathrm{Q}_0$:

$$P = P_0 - (Q - Q_0) * C \tag{A5}$$

By substituting (A5) into (A1):

$$R = Q * (P_0 - (Q - Q_0) * C) \tag{A6}$$

Rewriting (A6) to the standard form of $y = ax^2 + bx + c$:

$$R = -C * Q^2 + (P_0 + Q_0 * C) * Q \tag{A7}$$

With (A7) Van Haperen (2016) included the effect of price elasticity in the revenue function. The revenue is expressed as a function of the quantity with the constant values of the initial quantity, initial price and the elasticity constant. Table A.1 gives a brief description of the variables used.

Variable	Description
P	Sales price
P_0	Initial sales price
ΔP	Change in price
Q	Quantity
Q_0	Initial quantity
ΔQ	Change in quantity
C	Price elasticity constant
R	Revenue

Table A.1 - Description of the variables

Appendix B: Systematic literature review protocol

In this section, we describe the systematic literature review protocol used for the literature review of Chapter 3.

The first step is to identify the key words and search strings. Due to the narrow topic, we use broad search strings. Instead of "sales and operations planning", we search for "operations planning" and "production planning". We use them in combination with "oligopoly", "elasticity" and "Cournot" in the title, abstract, keywords or topic in the database of Scopus. An overview of the review protocol is provided in Table B.1. The initial search resulted in 29 articles. After removing duplicates and the articles which are not available for reading, we kept 20 articles for further investigation.

Search string	Scope	Date of search	Nr. of entries
Search protocol for Scopus			
"operations planning"			
AND "oligopoly"	Title, keywords and abstract	May 4, 2017	1
AND "elasticity"	Title, keywords and abstract	May 4, 2017	8
AND "Cournot"	Title, keywords and abstract	May 4, 2017	0
"production planning"			
AND "oligopoly"	Title, keywords and abstract	May 4, 2017	3
AND "elasticity"	Title, keywords and abstract	May 4, 2017	13
AND "Cournot"	Title, keywords and abstract	May 4, 2017	4
Total			**29**
Removing duplicates			-2
Not available for reading			-7
Selecting based on exclusion criteria			-9
Removed after complete reading			-2
Included after complete reading			+2
Total selected for review			**11**

Table B.1 - Overview of the literature review process

Within the search for articles about planning, also other articles came forward. These articles are excluded for further research, because they are irrelevant. Table B.2 shows the exclusion and inclusion criteria.

Nr.	Criteria	Reason for exclusion	Nr. excluded
1	Medical topics	This does not refer to planning	4
2	Topics about risk management	This does not refer to planning	1
3	Topics about mechanics	This does not refer to planning	2
4	Topics about optical networks	This does not refer to planning	1
5	Topics about thermal analysis	This does not refer to planning	1
Nr.	**Criteria**	**Reason for inclusion**	**Nr. included**
1	Articles considering price elasticity in operations planning	Relevant for the aim of the review	2

Table B.2 – Exclusion and inclusion criteria

The selected articles are fully read. Two articles did not fulfil the aim of the review and were removed. Further, during complete reading two articles were included to the selection, because they fulfilled the inclusion criteria, but had been missed in the review selection process (Lui et al., 2012; Hjaila et al., 2014).

Appendix C: Findings literature review

Table C.1 show the articles, keywords, methodology and key findings used for the systematic literature review. The articles are sorted on alphabetical order of the first author.

Article	Keywords	Methodology	Key findings
Algarni, A.A.S., Bhattacharya, K., El-Shatshat, R.A. (2007). Optimal operation of a disco in competitive electricity market with elasticity effects. *2007 IEEE Power Engineering Society General Meeting.* PES.	Distribution system, distribution power flow, Price elasticity, Demand response	Case study with a non-linear programming model	Elasticity functions are necessary to examine the impact of price changes on operations. The impact is highly dependent on the market situation.
Calfa, B.A., & Grossmann, I.E. (2015). Optimal procurement contract selection with price optimization under uncertainty for process networks. *Computers and Chemical Engineering 82,* 330-343.	Optimal contract selection, Price optimization, Uncertainty, Process network production planning	Case study with a stochastic programming model	More complex models, e.g. nonlinear or stochastic models, provide economic advantages in the solutions relative to less complex models, e.g. linear or deterministic models.
Chen, L.T., & Chen, J.M. (2008). Collaborative marketing and production planning with IFS and SFI production styles in an ERP system. *Journal of the Chinese Institute of Industrial Engineers 25,* 337-346.	Collaborative planning, Pricing, Production, Dynamic programming, ERP, Deteriorating item	Case study with a dynamic programming model	Cross-functional coordination between marketing and production planning can lead to substantial improvements in profit, customer satisfaction and organizational atmosphere. The price-elasticity coefficient of the demand function has a significant impact on the profit.
Farris, P.L., & Darley, R.D. (1964). Monthly Price-Quantity Relations for Broilers at the Farm Level. *American Journal of Agricultural Economics 46,* 849-856	Seasonal patterns, Price-quantity relations, Regression	Regression analysis	The price elasticity for broilers at the farm level in the period 1953-1963 shows a seasonal pattern. Seasonal patterns of production would have been necessary to stabilize prices during seasonal changing price elasticity.

Article	Keywords	Methodology	Key findings
Hjaila, K., Zamarripa, M., Shokry, A., Espuña, A. (2014). Application of Pricing Policies for Coordinated Management of Supply Chains. *Computer Aided Chemical Engineering 33*, 475-480	Coordinated management, SC planning, Pricing, Demand elasticity	Case study with a non-linear programming model	Since demand is price sensitive, an adequate pricing policy is important for proper decision-making. The approach used to approximate the revenue curve can significantly affect the planning decisions and the economic behaviour of the whole system. A more accurate approach to the real price behaviour leads to better solutions, leading to significant improvements, although its use may require larger computational effort. Integrating information flows should be used to identify proper pricing behaviour.
Kaplan, U., Türkay, M., Karasözen, B., Biegler, L.T. (2011). Optimization of Supply Chain Systems with Price Elasticity of Demand. *INFORMS Journal on Computing 23,* 557-568.	Mixed-integer nonlinear programming, Supply chain management, Smoothing, Price elasticity of demand	Case study with a non-linear programming model	The price elasticity values for most commodity products remain stable during its lifetime. A complex network makes it difficult to understand system response to price elasticity. An increase of the price elasticity leads to an increase in the production and profitability of the system.
Karmarkar, U.S., & Rajaram, K. (2012). Aggregate production planning for process industries under oligopolistic competition. *European Journal of Operational Research 223,* 680-689.	Aggregate production planning, Competition, Process industry, Nonlinear programming	Case study with a non-linear programming model	An increase in production efficiency, an increase of the market size or a decrease of customer price sensitivity results in increased sales prices, production quantities and profits for producers. The converse also holds.
Lui, S., Shah, N., Papageorgiou, L.G. (2012) Multiechelon Supply Chain Planning with Sequence-Dependent Changeovers and Price Elasticity of Demand under Uncertainty. *AICHE Journal 58,* 3390-3403	Supply chain management, Price elasticity of demand, Inventory control, Pricing, Model predictive control, Sequence-dependent changeovers	Case study with a mixed-integer linear programming model.	The pricing strategy is an important issue to the supply chain, especially when the price elasticity of demand is high. Including price elasticity in the model resulted in lower production quantities and higher sales prices relative to the model without price elasticity. A pricing strategy with price elasticity has a higher flexibility on price and demand management, which results in more profit and less inventory deviations.

Article	Keywords	Methodology	Key findings
Ma, W., Zhu, X., Wang, M. (2013). Production planning for static Cournot duopoly competition under random yield. *Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture 227,* 1888-1900	Production planning, Cournot competition, Random yield, Asymmetric information	Case study with a non-linear programming model	Oligopoly manufacturers compete in their production quantities and their aims are to decide their best target production quantities to maximize their profit. With few powerful companies in an oligopoly, consumers suffer from higher market prices.
Tang, C.S., Wang, Y., Zhao, M. (2015). The Implications of Utilizing Market Information and Adopting Agricultural Advice for Farmers in Developing Economies. *Production and Operations Management 24,* 1197-1215	Emerging markets, Social responsibility, Operational improvements, Competitive production strategies	Case study with a non-linear programming model	The provision of market information improves the farmers´ total welfare in Cournot competition.
Tominac, P., & Mahalec, V. (2017). A Game Theoretic Framework for Petroleum Refinery Strategic Production Planning. *AICHE Journal*.	Refinery planning, Strategic planning, Game theory, Potential game, Nash equilibrium	Case study with a non-linear programming model	In a Cournot oligopoly, product prices are variable functions of the collective market supply. Companies do not control prices, but do influence them with their production decisions. Profitable products are driven by capacity; less profitable products are driven by prices. Production planning in an oligopolistic market benefits from game theoretic analysis.

Table C.1 – Findings of the systematic literature review

Appendix D: Proof for optimal adjustment b_0

In this section, we explain that the ratio to adjust b_0 is ${}^2/_3$ to obtain a revenue function with an average deviation of zero. We begin with the revenue function $f(Q)$:

$$f(Q) = -C * Q^2 - (P^0 + P^0 * C) * Q \tag{8}$$

From equation (D1) we distinguish:
$a = -C$
$b = -(P^0 + Q^0 * C)$
$c = 0$

For simplicity, we can write the revenue function $f(Q)$ as:

$$f(Q) = aQ^2 + bQ \tag{9}$$

Then we take the piecewise linear function $g(Q)$:

$$g(Q) = b_0 + \frac{b_1 - b_0}{a_1 - a_0}\delta_1 + \frac{b_2 - b_1}{a_2 - a_1}\delta_2 + \cdots + \frac{b_K - b_{K-1}}{a_K - a_{K-1}}\delta_K \tag{10}$$

Because b_0 equally translates all segments of the piecewise linear function, we take for simplicity only the first segment of (D3). Then, we can write $g(Q)$ as:

$$g(Q) = \frac{b_1 - b_0}{a_1 - a_0}Q + b_0 \tag{D411}$$

When we take the interval of the first segment [a_0, a_1] = [0, x] and $b_j = f(a_j)$, we can write $g(Q)$ as:

$$g(Q) = \frac{(ax^2 + bx) - (a0^2 + b0)}{x - 0}Q + b_0 \tag{12}$$

We can simplify $g(Q)$ to:

$$g(Q) = (ax + b)Q + b_0 \tag{13}$$

When we subtract (D6) from (D2), we get the function $h(Q)$ as the difference between $f(Q)$ and $g(Q)$ in the interval [0, x]:

$$h(Q) = aQ^2 - axQ - b_0 \tag{14}$$

The average deviation is zero when the surface of overestimation is equal to the surface of underestimation. This situation holds when integral of the difference between the revenue curve and the linear approximation is zero over the interval of the segment:

$$\int_0^x h(Q)\,dQ = 0 \tag{15}$$

This is equal to:

$$\int_0^x aQ^2 - axQ - b_0 \; dQ = 0 \tag{16}$$

When we integrate (D9) we get:

$$\left[\frac{1}{3}aQ^3 - \frac{1}{2}axQ^2 - b_0Q\right]_0^x = 0 \tag{17}$$

We can write (D10) as:

$$-\frac{1}{6}ax^3 - b_0x = 0 \tag{18}$$

Because we want to adjust b_0 with a general ratio relative to the maximum deviation, we express the maximum deviation of the unadjusted $h(Q)$, when $b_0 = 0$, at $h\left(\frac{1}{2}x\right)$:

$$Maximum\ deviation = a\left(\frac{1}{2}x\right)^2 - ax\frac{1}{2}x \tag{19}$$

When we simplify (D12), we get:

$$Maximum\ deviation = -\frac{1}{4}ax^2 \tag{20}$$

With $b_0 = ratio * maximum\ deviation$ and (C13), we can substitute b_0 in (D11) to get:

$$-\frac{1}{6}ax^3 - \left(ratio * -\frac{1}{4}ax^2\right)i = 0 \tag{21}$$

When we simplify (D14), we obtain the ratio:

$$ratio = \frac{2}{3} \tag{22}$$

In short, when we want to adjust the piecewise linear function $g(Q)$ with b_0 to obtain an average deviation of zero, we should take two-thirds of the maximum deviation of the initial approximation as value for b_0.

Appendix E: Macro for the computation of piecewise linear approximations

This section shows the code we use to compute the piecewise linear approximations.

```
Sub LinearApproximation()

Dim ActualRevenue, ApproximatedRevenue, C, Deviation, DeviationQuantity, _
P(), Q(), R(), SegmentSize As Single
Dim i, j, Segments As Integer
Dim W As Worksheet

'Set workbook shortcut
Set W = ThisWorkbook.Sheets("Approximation")

'Remove potential old approximations
W.Range("A9:Z1000").Clear

'Perform the linear approximation for all 18 months
For j = 1 To 18
    'Set number of segments
    Segments = W.Cells(2, 2)

    'Dim arrays of quantity, price and revenue at each segment
    ReDim Q(Segments)
    ReDim P(Segments)
    ReDim R(Segments)

    'Set remaining initial variables
    'Set lower bound
    Q(1) = W.Cells(3, 2)

    'Upper bound
    Q(Segments) = W.Cells(4, 2)

    'Price elasticity constant
    C = W.Cells(5, 2)

    'Initial price
    P(0) = W.Cells(6, j + 1)

    'Initial quantity
    Q(0) = W.Cells(7, j + 1)

    'Set segment size within the bounds
    SegmentSize = (Q(Segments) - Q(1)) / (Segments - 1)

    'Determine quantity and revenue at the end of each segment
    For i = 1 To Segments
        'Quantity at the end of segment i
        Q(i) = Q(1) + (i - 1) * SegmentSize

        'Revenue at the end of segment i
        R(i) = -C * Q(i) ^ 2 + (Q(0) * C + P(0)) * Q(i)
    Next i

    'Calculate corresponding piecewise prices
    For i = 1 To Segments
        P(i) = (R(i) - R(i - 1)) / (Q(i) - Q(i - 1))
    Next i
```

```
    'Calculate the maximum deviation of approximation from the revenue
    'before adjustment
    'The deviation is the largest at the middle of a segment
    'Determine the quantity at the middle of the first segment
    DeviationQuantity = Q(1) + SegmentSize / 2

    'Calculate the approximated revenue
    ApproximatedRevenue = R(1) + (DeviationQuantity - Q(1)) * P(2)

    'Calculate the actual revenue
    ActualRevenue = -C * DeviationQuantity ^ 2 + (Q(0) * C + P(0)) _
    * DeviationQuantity

    'Determine the maximum deviation
    Deviation = ActualRevenue - ApproximatedRevenue

    'Adjust the approximated revenue of the first segment with the maximum
    'deviation
    R(1) = R(1) + 2 / 3 * Deviation

    'Set adjusted price for first segment P(0)
    P(1) = R(1) / Q(1)

    'Plot P and Q for each month
    For i = 1 To Segments
        W.Cells(i * 3 + 6, 1) = "Quantity segment " & i
        W.Cells(i * 3 + 7, 1) = "Price segment " & i
        W.Cells(i * 3 + 7, j + 1) = P(i)

        If i = 1 Then
            W.Cells(i * 3 + 6, j + 1) = Q(i)
        Else
            W.Cells(i * 3 + 6, j + 1) = SegmentSize
        End If
    Next i

    'New maximum deviation is 2 / 3 of the original deviation.
    'Plot new maximum deviation
    W.Cells(2, 5) = 2 / 3 * Deviation

    'Plot new maximum deviation relative to the revenue at the lower bound
    W.Cells(3, 5) = 2 / 3 * Deviation / R(1)
Next j

End Sub
```