Minimising road infrastructure maintenance costs by managing the traffic

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Abstract

This work is a master thesis performed at the company DAT.Mobility. Currently the traffic and maintenance management of a road network are two separated worlds. In this research we have combined these two worlds to reduce the maintenance costs by managing the traffic through the network. Therefore we varied the speed limits in the network by using a local search algorithm. In this thesis have set up models of road deterioration and the traffic movement. From this we used local search determine the minimal costs whereby none of the road authorities has to pay more than they do nowadays. It turns out that the maintenance costs can be influenced by the traffic flows and we found speed limits in the cities Enschede and Hengelo which result in a profit. The actual solution found in this thesis is case specific, but the model can easily be adapted to other road networks.

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Chapter 1

Introduction

The maintenance costs for the Dutch road network increases every year and they are a substantial part of the total budget of municipalities for road infrastructure. At the moment the maintenance occurs when needed. The moment that maintenance is needed depends on many factors like the amount of traffic, the weather conditions, the pavement type and thickness. The department of traffic management is totally separated from the department of road maintenance and it may be a good challenge to combine these two departments to bring a reduction in the maintenance costs.

In this thesis we will research if it is possible to bring a reduction in the maintenance costs by managing the traffic in a certain way and we will make an estimation of the fall in costs. Traffic management can be done in several ways, like bypasses, closing roads, changing speed limits, the building of new roads, less or more lanes on roads etc. In this research we will not consider changes in the infrastructure, which means that we will not look at changes concerning new roads or lanes. We will focus in this research on speed limits.

With this information, we define our research question as follows:

> How can we minimise the maintenance costs for a road network over time, where we will manage the traffic with speed limits?

We divide our research question in several sub questions.

1. Which factors have an influence on the deterioration of the road and how big are these influences? Hereby we can think of the weather, the number of cars and other vehicles, accidents, type of asphalt, speed of the traffic etc.

2. Which static and variable factors will influence the maintenance costs and what is the status of the pavement after maintenance?

3. How will the traffic distribute itself over the road network when the travellers want to reach their destination?

The research question will lead to a bi-level problem, which means that we have to solve two problems separately where the outcome of one problem will influence the other and the other way around. The upper level is the minimisation of the maintenance costs and the lower level is the distribution of the traffic over the road network. Because of this bi-level structure this thesis can be roughly split into four parts: the upper level optimisation problem, the lower level optimisation problem, the solving method and a case study. At first (chapter 2) we will give an explanation of bi-level problems. Then, in the first part (chapter 3) we will answer the first and second sub question, in the second part (chapter 4) we will answer the third sub question. In the third part (chapter 5 and chapter 6) we will look for a method to solve this bi-level problem. The last part (chapter 7) describes a case on which we will apply our model.

For the case study we will use an existing model of the cities Enschede and Hengelo and some surrounding area, in the east of the Netherlands.

Our solving method will be local search, with this method we will compute the costs for each road authority in this part of the Netherlands separately and we will minimise them without letting an authority pay more than in the original situation.

CHAPTER 2

Bi-level optimisation problem

The problem we have in this research is a so called bi-level optimisation problem. This means that the problem is split into two different parts which should be solved separately, but cannot be seen separately because they need information from each other. The two levels of a bi-level optimisation problem are called the lower level and the upper level. The upper level is the actual problem we want to solve, wherefore we need the information from lower level optimisation. A bi-level optimisation problem is in general NP-hard [32].

In our research the upper level is the optimisation of the maintenance costs, in this level we need the traffic flows. The travellers will optimise their route, which results in the lower level optimisation problem.

We want to minimise the maintenance costs by changing speed limits. The speed limits will influence the routes taken by the travellers and the travel flow will influence the maintenance costs. This will be an iterative process, which is shown in Figure 2.1. The stop criterion is case specific, like maximum number of iterations or a certain amount of time.

We will focus in our research on the computation of the maintenance costs, because DAT.Mobility already has models which computes the traffic flow. Also our research is about minimising the maintenance costs, not about assigning the traffic in a right way to the network. Therefore we will use the tools of DAT.Mobility for the lower level.

The total optimisation problem is shown below

$$\begin{aligned} \min_{x} \quad & \sum_{r \in R} C_r(f_r) \\ \text{s.t.} \quad & f(x) = \text{ user equilibrium given speed limits } x, \end{aligned}$$

where R is the set of roads, C_r is the cost function for road r, $f = (f_{r_1}, \ldots, f_{r_{|R|}})$ the flow on the roads and $x = (x_{r_1}, \ldots, x_{r_{|R|}})$ the speed limits on the roads. The drivers on the network will distribute themselves according to an user equilibrium which depends on x, more explanation of this is in section 4.4.

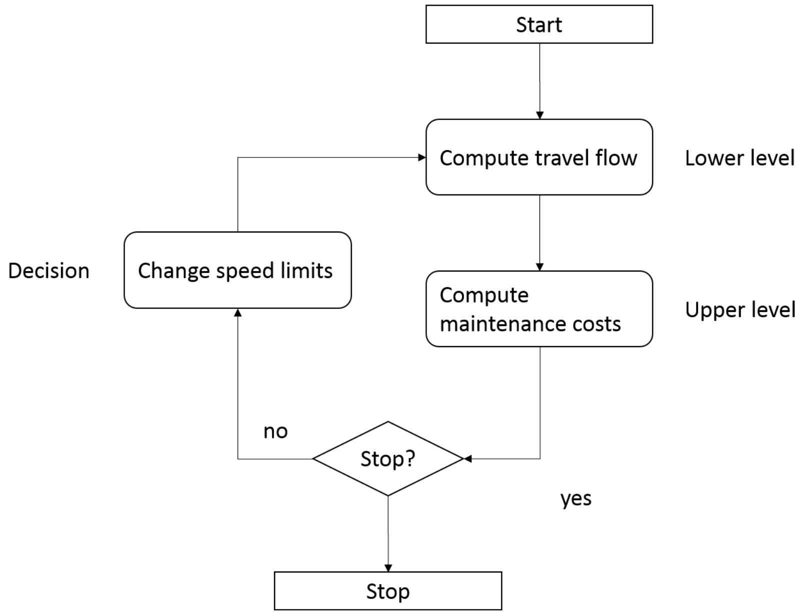


Figure 2.1: The bi-level optimisation problem.

We will start with the upper optimisation problem, afterwards we will look which information we need from the lower level optimisation problem, so we can describe this in an appropriate way.

Chapter 3

Minimising the maintenance costs

Every road will deteriorate. The deterioration depends on different factors. Of course the weather and the number of vehicles which will drive over it, influences the deterioration, but also an accident may cause maintenance needs. There are two different types of maintenance: acute and periodic. The acute maintenance is caused by for example an accident and the periodic maintenance is needed because of deterioration by weather and traffic. The maintenance costs depend on the pavement type. For example, do you have to replace asphalt or rigid pavement.

Municipalities use the instruction manuals of the CROW (knowledge partner for municipalities). Which is a global inspection every one or two years, the inspection card is in Appendix C. After this a budget will be made for the next one or two years [17], [6]. This strategy may not be optimal, because nothing is planned in this strategy, therefore we will look if we can minimise the maintenance costs by another strategy with which money can be saved. A new strategy can be to maintain the road after a fixed time or when the road has a certain state. Another possibility is to set up and solve an optimisation problem to get the optimal strategy given the state of the road over the years.

In this chapter we assume that the the traffic flows are known, so we can set up an optimisation problem for the maintenance costs. For this optimisation problem, we need to know the costs of the different maintenance types and the costs of replacing the pavement. We also have to know the deterioration influence of the traffic flow on the pavement.

The optimisation of the maintenance costs will be done in two steps. First we will formulate some models of the pavement status we found in the literature. From this we will formulate the computation of the maintenance costs, according to the literature. Afterwards we will set up a model according to the models found in the literature. With this model we will compute the maintenance costs in a road network, given the amount of traffic on every road.

3.1 Definitions

Before we will discuss different models, we need some definitions. Also some formulations will be clarified.

Some models use vehicle types. A vehicle type is a group of vehicles with approximately the same characteristics and the same deterioration influences on the pavement.

Different types of pavement are used in the road network. Most used pavements are element pavement, flexible pavement and rigid pavement.

Definition 3.1 (Element pavement)**.** This pavement consists of separate (small) elements, like clinker bricks.

Definition 3.2 (Flexible pavement)**.** Flexible pavement consists of asphalt.

Definition 3.3 (Rigid pavement)**.** Rigid pavement consists of cement concrete.

There are also different types of flexible pavement, because different types of asphalt are used in the Dutch road network. On the motorways most of the time Porous Asphalt (NL: ZOAB, Zeer Open Asfalt Beton) is used. In the municipality networks Asphalt Concrete (NL: DAB, Dicht Asfalt Beton) is used [27].

Because the pavement types differ in structure, the pavement status models can also differ. In the next section we will formulate the pavement status.

3.2 Formulation of the pavement status

One way to formulate the road status is the Present Serviceability Index (PSI) [2]. This index represents the service the road offers to the passengers in the vehicles.

CE Delft has made a model which relates difference in axle loads to the maintenance costs. This model uses the so called Axle Damage Factor (ADF) [9], [10]. It skips the step of the roadstatus and links directly the ADF to the maintenance costs, by giving a rise or fall in the initial costs. We will discuss both models in this section.

3.2.1 Present Serviceability Index

The PSI represents the service the road has for a driver, as a number. Before the PSI was used, people used the PSR (Present Serviceability Rating). This value was obtained by a panel which drove on the road and rated it to its serviceability [2], this is like the procedure of CROW. Because the PSR is not a very precise method, the PSI was introduced: An index rating from approximately five (excellent) to zero (awful) which depends on multiple deterioration types on the pavement. The formula for the PSI differs for flexible pavements and rigid pavements.

The PSI depends on the following factors:

1. Slope Variance (SV): *The variance in the slope in length direction (*Rad^2*).*
2. Rut Depth (RD): *The mean of the ruts in the pavement (*mm*).*
3. Cracking (CR): *The length of the cracks in the pavement (*m/1000m^2*).*
4. Patching (PA): *The area of the patches placed on the pavement (*m^2/1000m^2*).*

The PSI for flexible pavements is given by [18]

$$PSI_{\text{flexible}} = 5.03 - 1.9^{\dagger} \log(1 + SV) - 2.14 \cdot 10^{-3} RD^2 - 0.01\sqrt{0.3048CR + PA}.^{\ddagger} \tag{3.1}$$

The PSI for rigid pavements is given by [18]

$$PSI_{\text{rigid}} = 5.41 - 1.8 \log(1 + SV) - 0.09\sqrt{0.3048CR + PA}.^{\S} \tag{3.2}$$

The variables in the PSI-formula will be explained in the following sub-subsections.

3.2.1.1 Slope Variance

For the slope variance we need multiple measurements, say $y_1, \ldots, y_n$. Every measurement is the slope on an interval in Radians. The slope variance is therefore given by [20]

$$SV = \frac{\sum_{i=1}^{n} y_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} y_i\right)^2}{n-1}.$$

The length of the interval depends on the measuring instrument. We have not found a good model, depending on the axle load or not, of the appearance of slope in a road. Therefore we cannot use the slope variance.

3.2.1.2 Rut Depth

A road with deep rutting is shown in Figure 3.1. The rut depth can be measured on the road itself. But in *"Estimation of Rutting Models by Combining Data From Different Sources"* [3], two models are described to compute the rut depth in millimetres from axle loads and environment factors.

The first model is the WesTrack model. This model is limited to layer thicknesses and axle loads. The other model is the AASHO model, this model is limited to the materials used. Because this

†This value differs in the articles. The values of $1, 1.9$ and 1.91 are found. The reason therefore we could not find. We will use the value 1.9 because that value is found most often.

‡In the literature, this formula is found slightly different, namely $PSI = 5.03 - 1.9\log(1 + SV) - 1.38RD^2 - 0.01\sqrt{CR + PA}$ [19]. The reason is that the original formula works with foot and inch [2]. In the formula used in this report the unit is metre for SV, CR and PA and millimetre for RD.

§Again this formula is found slightly different than the formula found, because the original formula works with foot and inch and in this formula the unit is metre.


Figure 3.1: Rutting. [15]

research is about managing the traffic to minimise the maintenance costs, a model which depends on the axle loads is most suitable. Therefore we use the AASHO model. This model can only be used for flexible pavements.

On road section r, we have for time t:

$$\begin{aligned} RD_{r,t} &= RD_{r,0} + \sum_{s=1}^{t} \Delta RD_{r,s} \\ &= RD_{r,t-1} + \Delta RD_{r,t} \\ &= RD_{r,t-1} + \left[\Delta RD_{r,t}^{AC} + \Delta RD_{r,t}^{U} \right]. \end{aligned}$$

where $RD_{r,0}$ is the initial rut depth of the pavement and $\Delta RD_{r,t}$ the difference in rut depth between time $t-1$ and t. $\Delta RD_{r,t}$ is the sum of $\Delta RD_{r,t}^{AC}$, the difference in rut depth of the asphalt concrete layer, and $\Delta RD_{r,t}^{U}$, the difference in rut depth of the under layer.

According to [3] we take

$$\Delta RD_{r,t}^{AC} = \mu_{r,t} e^{b' N'_{r,t}} \Delta N'_{r,t}, \tag{3.3}$$

$$\Delta RD_{r,t}^{U} = \alpha_{r,t} e^{b N_{r,t}} \Delta N_{r,t}, \tag{3.4}$$

hereby is $\mu_{r,t}$ a function of high temperatures and $\alpha_{r,t}$ a function of the pavement thickness and low temperatures. The functions $\Delta N'_{r,t}$ and $\Delta N_{r,t}$ give the effect of the traffic on the pavement in period t. Further $N'_{r,t} = \sum_{s=1}^{t} \Delta N'_{r,s}$ and $N_{r,t} = \sum_{s=1}^{t} \Delta N_{r,s}$.

To define $\Delta N'_{r,t}$ and $\Delta N_{r,t}$, we introduce the Load Equivalence Factor (LEF). This is defined by

$$LEF = \left(\frac{S}{S_{std}} \right)^{\beta}. \tag{3.5}$$

The LEF gives the ratio of load S to the standard axle load (S_{std}). The power β is called the empirical factor or the load equivalence coefficient.

We have

$$\Delta N'_{r,t} = \sum_{j=1}^{R_S} n_{r,t,S_j} \left(\frac{S_j}{S_{std}}\right)^{\beta_4} + \sum_{j=1}^{R_T} n_{r,t,T_j} 2 \left(\frac{T_j}{2S_{std}}\right)^{\beta_4},$$

$$\Delta N_{r,t} = \sum_{j=1}^{R_S} n_{r,t,S_j} \left(\frac{S_j}{S_{std}}\right)^{\beta_1} + \sum_{j=1}^{R_T} n_{r,t,T_j} \left(\frac{T_j}{\beta_3 S_{std}}\right)^{\beta_2},$$

where

n_{r,t,S_j} = # single axle loads on section r at time t with magnitude S_j.

n_{r,t,T_j} = # tandem axle loads on section r at time t with magnitude T_j.

R_S = # different load magnitudes for single axles.

R_T = # different load magnitudes for tandem axles.

$\beta_1, \ldots, \beta_4$ = parameters.

The difference between $\Delta N'_{r,t}$ and $\Delta N_{r,t}$ can be explained as follows. For the difference in rut depth of the asphalt concrete layer we need $\Delta N'_{r,t}$, on the surface of the pavement. The tandem axle acts like two equal single axles. Therefore we first divide the load by two and after we compute the LEF we multiply it by two. Also the empirical factor is equal because of this reason. In the under layer a tandem axle does not behave like two single axles any more.

Now we just have to define $\mu_{r,t}$ and $\alpha_{r,t}$. As said before, $\mu_{r,t}$ is a function of the high temperatures. We have:

$$\mu_{r,t} = \beta_5 + \beta_6 \cdot TH_t,$$

where

$$TH_t = \begin{cases} 1 & TM_{max,t} > 28.6°C \\ 0 & \text{otherwise} \end{cases}$$

$TM_{max,t}$ = mean maximum temperature in period t ($°C$)

β_5, β_6 = parameters.

The function $\alpha_{r,t}$ depends on the layer thickness and low temperatures. We have

$$\alpha_{r,t} = \beta_7 e^{\beta_8 TL_t - RN_r},$$

where

$$TL_t = TF_t \max\left(TM_{max,t}, 0\right)$$

$$TF_t = \begin{cases} 0 & t = 1 \\ \max\left(0, TF_{t-1} - TM_{min,t}\right) & t = 2, \dots, T \end{cases}$$

$$TM_{min,t} = \text{mean minimum temperature in period } t\ (°C)$$

$$RN_r = \beta_9(LT_{r,1} + OT_r) + \beta_{10}LT_{r,2} + \beta_{11}LT_{r,3}$$

$$LT_{r,1} = \text{Thickness of upper layer (m)}$$

$$OT_r = \text{Thickness of overlay (m)}$$

$$LT_{r,2} = \text{Thickness of base layer (m)}$$

$$LT_{r,3} = \text{Thickness of subbase layer (m).}$$

$$\beta_7, \dots, \beta_{11} = \text{parameters.}$$

As we can see, the rut depth strongly depends on the axle loads. And the combination of axle loads with extreme weather will influence it even more (if β_6 and β_8 are positive).

3.2.1.3 Cracking

An example of cracks in the pavement is in Figure 3.2. A good model for cracking, depending or not on the traffic, is not found. This means that we cannot use cracking in our model.

Figure 3.2: Cracking. [14]

3.2.1.4 Patching

Patching is a repair of the pavement. If there is a crack or a hole in the pavement, it will be filled up (patched) with new pavement [7]. An example of patched areas in the pavement is in Figure 3.3. The patched area is not that good as the original pavement, because the original structure of the pavement is broken. Therefore this repair is in the PSI. The need of patching is caused by axle loads or the weather, but the patches are placed by humans. Therefore, no formula for patching exists.

Figure 3.3: Patching. [22]

3.2.2 Axle Damage Factor

Another model for the pavement status is the Axle Damage Factor, ADF.

In subsubsection 3.2.1.2 we defined the LEF. The ADF uses the LEF too, it is an extended version of the Load Equivalence Factor. The ADF is described in various Dutch papers ([10], [13], [1]).

The Axle Damage Factor depends on the following factors [9]:

1. Tire Factor (TF): *Depends on the tire type, tire inflation pressure, etc.*
2. Suspension Factor (SF): *Can be pneumatic or a leaf spring.*
3. Axle Configuration Factor (ACF): *Single, tandem or tridem.*
4. Load Equivalence Factor (LEF): *Gives the number of standard axles.*

The Axle Damage Factor for an axle is given by

$$ADF = TF \cdot SF \cdot ACF \cdot LEF.$$

As can be seen, the ADF is just an extension of the LEF. The Axle Damage Factor is how many standard axles one given axle is. So the ADF is, similar to the LEF, unitless.

The more axles a vehicle has, the more its weight will be distributed on the road. This means that the more axles, the less the Axle Damage Factor will be. This is expressed in the Axle Configuration Factor.

The Load Equivalence Factor is as defined in Equation (3.5).

The ADF is for one axle of the vehicle, so the total average Axle Damage Factor of a vehicle of type v is the sum of the ADF of all its axles. This means

$$ADF_v = n_{\text{single},v} ADF_{\text{single},v} + n_{\text{tandem},v} ADF_{\text{tandem},v} + n_{\text{tridem},v} ADF_{\text{tridem},v},$$

where $n_{\text{single},v}$ is the average number of single axles on vehicle type v and $ADF_{\text{single},v}$ the average Axle Damage Factor for a single axle for one vehicle of type v. The parameters $n_{\text{tandem},v}$, $n_{\text{tridem axle},v}$, $ADF_{\text{tandem},v}$ and $ADF_{\text{tridem},v}$ are defined similarly.

3.2.2.1 Axle load on the road

In [13], a model is made to define the additional maintenance costs caused by overcharged trucks on motorways. We will describe this model and look if we can use it in general.

Now we know the total average Axle Damage Factor for every vehicle type v. We need a standardised form to express this influence. We will call this $A_{v,r}$ [25].

$$A_{v,r} = F_{\text{lanes}}{}_{v,r} F_{\text{width}}{}_{r} F_{\text{super single}}{}_{v,r} F_{\text{speed}}{}_{v,r} ADF_v \tag{3.6}$$

The $A_{v,r}$ is a factor of how many times a vehicle of type v damages road r in comparison to a standard axle when the vehicle drives the maximum speed. Like LEF and ADF, $A_{v,r}$ is just a measure of standard axles, so also the factor $A_{v,r}$ is unitless. The factors $A_{v,r}$ depends on are

1. Lanes Factor ($F_{\text{lanes}}{}_{v,r}$): *Vehicles will distribute over the number of lanes. Therefore the more lanes, the less damage on one lane.*

2. Width of Lanes Factor ($F_{\text{width}}{}_{r}$): *If the lanes are wide, the vehicles will distribute over the width. This means that if a lane is wide, the damage will be distributed along the width, so there will be less damage at one place.*

3. Super Single Factor ($F_{\text{super single}}{}_{v,r}$): *Some trucks have Super Single tires, this replaces two normal tires, but is smaller. So Super Singles cause more damage to the road.*

4. Speed Factor ($F_{\text{speed}}{}_{r}$): *If vehicles cannot drive the maximum allowed speed it means there is congestion. In case of congestion, vehicles will stop and start more often than if they only drive. The starting and stopping causes more damage to the pavement. So the slower the speed in comparison to the maximal speed limit, the higher the deterioration of the pavement.*

As said before, this model is made for trucks on a motorway. If we want to use this formula also for cars, we can neglect the Super Single Factor, because cars do not have these kind of tires. So for cars we have $F_{\text{super single}}{}_{r} = 1$ for all r. Also the $F_{\text{speed}}{}_{r}$ will differ for cars, because they are allowed to drive faster on a motorway than trucks. Also if we want to use this formula for other roads than motorways we have to take other values for the speed factor than given in [13].

3.3 Maintenance costs

From the previous section we know two ways to formulate the pavement status, we now want to say something about the maintenance costs. In this section we use the pavement status to formulate the maintenance costs.

3.3.1 Maintenance costs using the Present Serviceability Index

The PSI is an index of the pavement status, on which basis we can make decisions when we want to maintain the pavement. Say if $PSI < B$ we want to replace the pavement, for a certain value of B, and maintenance can be done every time slot. With this we will set up an optimisation problem in subsection 3.4.2.

Another option is to look to the factors the PSI depends of, say we replace road r on time t if $RD^{AC}_{r,t} \geq B_1$ and $RD^{U}_{r,t} \geq B_2$.

3.3.2 Maintenance costs using the Axle Damage Factor

We will estimate the maintenance costs of the motorways according to [13], which claims that if the yearly standardised traffic flow f doubles, the yearly repairing costs will increase by 60%. So the cost C_r on road r with flow f_r is given by:

$$C_{\text{motorway},r}(f_r) = \tilde{c}_r 1.6^{\log_2\left(\frac{f_r}{\tilde{f}_r}\right)} = \tilde{c}_r \left(\frac{f_r}{\tilde{f}_r}\right)^{\log_2(1.6)} = \frac{\tilde{c}_r}{\tilde{f}_r^{\log_2(1.6)}} \cdot f_r^{\log_2(1.6)} = \tilde{C}_r \cdot f_r^{\log_2(1.6)}$$

where

$$\begin{aligned} f_r &= \text{traffic flow on road } r \\ \tilde{f}_r &= \text{initial flow on road } r \\ \tilde{c}_r &= \text{initial costs (when intensity is } \tilde{f}_r) \\ \tilde{C}_r &= \frac{\tilde{c}_r}{\tilde{f}_r^{\log_2(1.6)}} \end{aligned}$$

This means we need to have one case in which we know the intensity and the repairing costs for one year. We can just compute the increase or decrease in the maintenance cost for that motorway road section. It is very unlikely that we know for every motorway the present maintenance costs.

We also cannot simply use this for other roads than motorways, because we know nothing about the raise in cost when the standardised intensity doubles for a road which is not a motorway.

3.4 The maintenance model

We discussed two different methods to formulate the maintenance costs and we decided to use the PSI. We will not use the Axle Damage Factor, because this model is made for motorways and the municipalities do not maintain motorways.

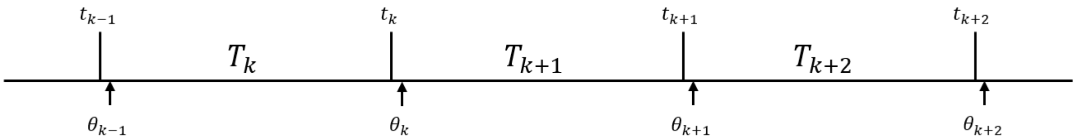


Figure 3.4: Illustration of the time slots. The measure moments are t_i, the period belonging to that measure moment is T_i and the maintenance or replacement is θ_i.

With the PSI we can set-up an optimisation problem. In this optimisation, we want to minimise the total maintenance costs in a city.

3.4.1 Model assumptions

In our research we found nothing about the influence of the traffic on rigid pavements. We know a formulation of the PSI for rigid pavements, but it is unknown how traffic affects the damage factors in the PSI. For flexible pavements we also have a formulation of the PSI. From the municipality of Enschede we know that on open asphalt ravelling is the most common deterioration and on closed asphalt it is rutting. Because in the formulation of the PSI no ravelling is included, we just look to closed asphalt in our model. This assumption seems extreme, but most pavements in municipalities are closed asphalt.

If we just take out the roads with no closed asphalt of our model, this will influence the network and travellers will choose routes which they will not choose in real life. We also cannot say that the maintenance costs for this roads will be zero, because this will influence the measures taken. Therefore we act like every road in the network has a closed asphalt pavement.

We want to make a model for multiple years, say we model T time slots of equal length. We assume that the flow grows with a growth factor which is equal in every time slot.

The PSI differs for every time slot (unless no one uses the road). Every time slot we have to look if the PSI is below the replace level B, if so, we have to replace the pavement. Every time slot we can choose if we want to do nothing, maintain or replace the pavement. This is our decision variable θ. If we do nothing, the pavement will deteriorate further. If we replace the pavement, the pavement is new and therefore the PSI is at its maximum. If we maintain the road, we replace the top asphalt layer. This means that after the maintenance, the pavement is not as good as new, but better than before the maintenance. The under layer will deteriorate further and we take $RD^{AC} = 0$. The maintenance and the replacement will take place at the end of the time slot, directly after the measure moment. An illustration of this is shown in Figure 3.4.

We also make some assumptions about the PSI, Equation (3.1). Because the maintenance is replacing the top layer of asphalt, we never have patched areas on our roads. This means that $PA = 0$. We also did not find any useful model for cracking, therefore we neglect this by taking $CR = 0$. The slope variance can just be measured by special devices, we also do not know the influence of the vehicles on the slope variance. Therefore we will neglect this by taking $SV = 0$.

All this assumptions seems extreme, but the most common deterioration on closed asphalt is rutting and this is still in the PSI.

With this assumptions we have

$$\begin{aligned} PSI &= 5.03 - 1.9\log(1+SV) - 2.14\cdot 10^{-3}RD^2 - 0.01\sqrt{0.3048CR+PA} \\ &= 5.03 - 1.9\log(1+0) - 2.14\cdot 10^{-3}RD^2 - 0.01\sqrt{0.3048\cdot 0+0} \\ &= 5.03 - 2.14\cdot 10^{-3}RD^2 \end{aligned}$$

The costs of maintaining and replacing the pavement will be equal for every road and in every time slot. The influence of inflation is explained later on.

3.4.2 The optimisation problem

We define the state of the pavement on road r at time t with $q_{r,t}$ which depends on the PSI. Say the asphalt concrete layer is replaced in time slot $\tilde{t}_r^{AC}$ and the under layer in time slot $\tilde{t}_r^{U}$. Then $q_{r,t}$ is the PSI from $\tilde{t}_r^{AC}$ up until t for the asphalt layer and $\tilde{t}_r^{U}$ up until t for the under layer. We notate this with $PSI_{r,\tilde{t}_r^{AC},\tilde{t}_r^{U},t}$.

At the first time slot, the roads have an initial pavement status. This is the given information $RD_{r,0}^{AC}$ and $RD_{r,0}^{U}$.

Our decision variables are $\theta_{r,t}$ $\forall r,t$, we have

$$\theta_{r,t} = \begin{cases} 0 & \text{do nothing with road } r \text{ on time } t \\ 1 & \text{maintain road } r \text{ on time } t \\ 2 & \text{replace road } r \text{ on time } t. \end{cases} \tag{3.7}$$

From the decision of $\theta_{r,t}$, we know the state of the pavement at the end of next time slot ($q_{r,t+1}$).

$$q_{r,t+1} = \begin{cases} PSI_{r,\tilde{t}_r^{AC},\tilde{t}_r^{U},t+1} & \theta_{r,t}=0 \\ PSI_{r,t,\tilde{t}_r^{U},t+1} & \theta_{r,t}=1 \\ PSI_{r,t,t,t+1} & \theta_{r,t}=2 \end{cases} \qquad \forall r\in R, 0\le t<T$$

If the state of the pavement, $q_{r,t}$ is below level B, we have to replace the pavement. Therefore we have the constraint

$$q_{r,t} < B \Rightarrow \theta_{r,t} = 2.$$

With this we can define our optimisation problem. The optimisation problem below is not in the standard form, but it can be rewritten in such a way it is. This would be a very big optimisation problem from which it is not clear what it does, for clarity the optimisation problem is not in standard form.

$$
\begin{aligned}
\min_{\theta} \quad & \sum_{t=1}^{T} \delta^{t-1} \sum_{r \in R} L_r W_r C(\theta_{r,t}) && && (3.8) \\
\text{s.t.} \quad & q_{r,t+1} = \begin{cases} PSI_{r,\tilde{t}_r^{AC},\tilde{t}_r^{U},t+1} & \theta_{r,t} = 0 \\ PSI_{r,t,\tilde{t}_r^{U},t+1} & \theta_{r,t} = 1 \\ PSI_{r,t,t,t+1} & \theta_{r,t} = 2 \end{cases} && \forall r, t < T \\
& q_{r,t} < B \Rightarrow \theta_{r,t} = 2 && \forall r, t \\
& n_{v,r} = \text{ \# vehicles of type } v \text{ on road section } r && \forall r
\end{aligned}
$$

Where δ is the discount factor for inflation and deflation and L_r and W_r the length and width for road r respectively.

In the next section the values of the used parameters are given. With this we can define our optimisation problem for our specific situation.

3.5 Values of the parameters

In this section we will define the parameters used in our optimisation model.

3.5.1 Objective function

We start with the parameters in the objective function. The costs of the maintenance and the replacements are as in Appendix B. We have

$$
C(\theta_{r,t}) = \begin{cases} €0.00 & \theta_{r,t} = 0 \\ €14.50 & \theta_{r,t} = 1 \\ €41.00 & \theta_{r,t} = 2. \end{cases}
$$

The values of L_r and W_r are case specific. The inflation rate in 2016 was 0.32%, therefore we take the discount factor $\delta = \left(\frac{1}{1-0.0032}\right)^{\frac{1}{365}} = 1.0000088$.

We take the horizon of our optimisation problem $T = 30$ years and one time slot is one day.

3.5.2 Constraints

In the constraints, there are just parameters in the PSI and B.

In [2], an overview is given what the values of the PSI mean. If the PSI is between 2.0 and 2.9, the pavement status is 'fair', which means that the pavement is barely tolerable for high speed traffic [2], therefore we will choose $B = 2.9$.

variable	value	variable	value
b	$-2.28 \cdot 10^{-7}$	β_6	0.00
b'	$-2.11 \cdot 10^{-6}$	β_7	$1.71 \cdot 10^{-6}$
β_1	2.44	β_8	$4.26 \cdot 10^{-6}$ *
β_2	2.86	β_9	9.28
β_3	1.68	β_{10}	4.77
β_4	0.56	β_{11}	4.29
β_5	$1.70 \cdot 10^{-5}$		

Table 3.1: The values of the parameters used in subsubsection 3.2.1.2 according to [3].

According to [3] we take $S_{std} = 8.0 \cdot 10^4$ Newton.

In Table 3.1, the values for the parameters in the rut depth, according to [3], are shown.

The axle load of every vehicle is unknown to us. This is the reason we have vehicle types. We distinguish two different types: cars and lorries. We call our vehicle set $V = \{v_1, v_2\} = \{v_{\text{car}}, v_{\text{lorry}}\}$. The reason for this division is that in OmniTRANS, the program we will use in the next section, this division has already been made.

The total average weight and other characteristics of vehicles are given in Table 3.2. These values are for Dutch vehicles. We will use the average of these values in our research.

	v_{car}		v_{lorry}		
	car	**mini bus**	**truck**	**lorry**	**bus**
# on the road (1000)	6539	756	83	60	11
weight (kg) †	1022 + 150	1485 + 338	25000	25000	10868 + 1500
growth factor one year	1.018	1.064	0.999	1.053	0.995
# single axles	2	2	2	2	1
# tandem axles	0	0	0	0	1
# tridem axles	0	0	0	1	0

Table 3.2: Characteristics of the vehicles. [16] [23]

We assume that the number of vehicles grows every time slot with the same factor $g_{v_{\text{car}}}$ and $g_{v_{\text{lorry}}}$. Therefore we have $n_{r,t+1,v} = n_{r,t,v} \cdot g_v$, so $n_{r,t,v} = n_{r,v} \cdot g_v^{t-1}$. Where $n_{r,v}$ is the number of vehicles of type v in the basis day.

The average growth factor for the vehicle types are

$$g_{v_{\text{car}}} = \left(\frac{6539 \cdot 1.018 + 756 \cdot 1.064}{6539 + 756} \right)^{\frac{1}{365}} = 1.00006$$
$$g_{v_{\text{lorry}}} = \left(\frac{83 \cdot 0.999 + 60 \cdot 1.053 + 11 \cdot 0.995}{83 + 60 + 11} \right)^{\frac{1}{365}} = 1.00005.$$

*[3] gives this parameter the value $4.26 \cdot 10^{-3}$, but divides it later on by 10^3. For clarity, we define $\beta_8 = 4.26 \cdot 10^{-6}$.

†The weight is combined from [16] and [23]. If a weight is a sum, it is a sum of the empty weight and the weight of the people inside the vehicle.

From the municipality of Enschede (Appendix B), we know that the top layer of the asphalt is 3 cm and the total thickness of the asphalt is 13 cm. Therefore we take

$$LT_{r,1} = 0.03\,\text{m} \qquad \forall r \tag{3.9}$$

$$LT_{r,2} = 0.05\,\text{m} \qquad \forall r \tag{3.10}$$

$$LT_{r,3} = 0.05\,\text{m} \qquad \forall r \tag{3.11}$$

In our assumptions we never have an overlay, so we take

$$OT_r = 0\,\text{m} \qquad \forall r$$

This means that every road has an equal thickness of pavement, so $RN_r = RN\ \forall r$.

3.6 Analysis of the optimisation problem

First we will analyse what the influence is of our assumptions and parameters we have.

3.6.1 The Rut depth

We take one average day in the year, so we have to know the average minimal and average maximal temperature of a year. In [30] the average minimum and maximum temperatures of the months from 1706 till 2014 in the Netherlands are listed. We take these values to compute the expected values of the average minimum and maximum for all the next years.

We have

$$\begin{aligned}
\mathbb{E}[TM_{max,t}] &= \frac{31(7.1+8.8+16.0+22.3+20.5+14.2+7.3)+30(13.1+18.8+17.9+10.2)}{365.25}°C \\
&\quad + \frac{28.25 \cdot 7.6}{365.25}°C \\
&\approx 13.68°C \\
\mathbb{E}[TM_{min,t}] &= \frac{31(-7.0-2.3+7.5+11.2+13.9+13.5+6.0-5.7)}{365.25}°C \\
&\quad + \frac{30(4.3+11.2+10.7+0.6)+28.25 \cdot -6.7}{365.25}°C \\
&\approx 4.83°C.
\end{aligned}$$

Because $\mathbb{E}[TM_{max,t}] \le 28.6°C$ and $\mathbb{E}[TM_{min,t}] \ge 0$ we have $\mathbb{E}[TH_t] = 0$, $\mathbb{E}[TF_t] = 0$ and $\mathbb{E}[TL_t] = 0$. Therefore we take $TH_t = TL_t = 0$. If we fill in these values in the formulas for $\mu_{r,t}$ and $a_{r,t}$ we get

$$\begin{aligned}
\mu_{r,t} &= \beta_5 + \beta_6 \cdot TH_t = \beta_5 \\
a_{r,t} &= \beta_7 e^{\beta_8 TL_t - RN} = \beta_7 e^{-RN}.
\end{aligned}$$

With our assumption of the vehicle types, we can define $\Delta N'_{r,t}$ and $\Delta N_{r,t}$ different. We will use

$$\Delta N'_{r,t} = \sum_{v\in V} n_{r,t,v}\Delta\tilde{N}'_v = \sum_{v\in V} n_{r,v} g_v^{t-1}\Delta\tilde{N}'_v,$$
$$\Delta N_{r,t} = \sum_{v\in V} n_{r,t,v}\Delta\tilde{N}_v = \sum_{v\in V} n_{r,v} g_v^{t-1}\Delta\tilde{N}_v,$$

where $\Delta\tilde{N}'_v$ is the average number of axles in a vehicle of type v times the average axle load of vehicle type v corrected if the vehicles have tandem or tridem axles. $\Delta\tilde{N}_v$ is defined similarly. This definition makes $\Delta\tilde{N}'_v$ and $\Delta\tilde{N}_v$ constants. We define these constants more clearly later on.

Therefore we have

$$\begin{aligned}
N'_{r,\tilde{t}_r^{AC},t} &= \sum_{s=\tilde{t}_r^{AC}}^{t} \Delta N'_{r,s} \\
&= \sum_{s=\tilde{t}_r^{AC}}^{t} \sum_{v\in V} n_{r,v} g_v^{s-1}\Delta\tilde{N}'_v \\
&= \sum_{v\in V} n_{r,v}\Delta\tilde{N}'_v \sum_{s=\tilde{t}_r^{AC}}^{t} g_v^{s-1} \\
&= \sum_{v\in V} n_{r,v}\Delta\tilde{N}'_v \frac{g_v^t - g_v^{\tilde{t}_r^{AC}-1}}{g-1}, \\
N_{r,\tilde{t}_r^{U},t} &= \sum_{v\in V} n_{r,v}\Delta\tilde{N}_v \frac{g_v^t - g_v^{\tilde{t}_r^{U}-1}}{g_v-1}.
\end{aligned}$$

We take

$$\Delta\tilde{N}'_v = n_{\text{single}v}\left(\frac{S_{\text{single}}}{S_{std}}\right)^{\beta_4} + n_{\text{tandem}v}2\left(\frac{S_{\text{tandem}}}{2S_{std}}\right)^{\beta_4} + n_{\text{tridem}v}3\left(\frac{S_{\text{tridem}}}{3S_{std}}\right)^{\beta_4}.$$
$$\Delta\tilde{N}_v = n_{\text{single}v}\left(\frac{S_{\text{single}}}{S_{std}}\right)^{\beta_1} + n_{\text{tandem}v}\left(\frac{S_{\text{tandem}}}{\beta_3 S_{std}}\right)^{\beta_2} + n_{\text{tridem}v}\left(\frac{S_{\text{tridem}}}{\beta_{12} S_{std}}\right)^{\beta_{13}}$$

Where $n_{\text{single}v}$ is the number of single axles of vehicle type v. The parameters $n_{\text{tandem}v}$ and $n_{\text{tridem}v}$ are defined similarly. We expand the formulas used in [3] and given earlier in this report by adding tridem axles.

If we use average vehicles and we fill in these formulas, we get constant values of the axle damage of one average vehicle of type v.

In the formula of $\Delta\tilde{N}'_v$ we filled in the variables added for tridem axles according to the reason that in the top layer the axles in a multi-axle behave like single axles. The values of β_{12} and β_{13} must be estimated. We are not able to validate these values.

For clearness we introduce another constant $\beta_0 = 1$ in $\Delta\tilde{N}_v$, this does not influence the formula:

$$\Delta\tilde{N}_v = n_{\text{single}v}\left(\frac{S_{\text{single}}}{\beta_0 S_{std}}\right)^{\beta_1} + n_{\text{tandem}v}\left(\frac{S_{\text{tandem}}}{\beta_3 S_{std}}\right)^{\beta_2} + n_{\text{tridem}v}\left(\frac{S_{\text{tridem}}}{\beta_{12} S_{std}}\right)^{\beta_{13}}.$$

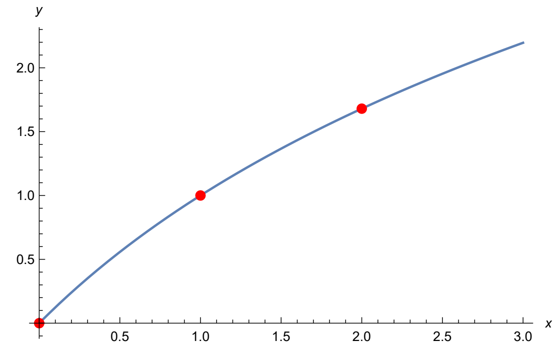


Figure 3.5: Graph of $(0,0)$, $(1,\beta_0)$ and $(2,\beta_3)$ to determine β_{12}.

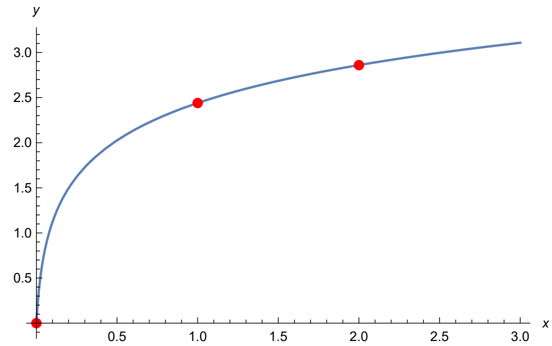


Figure 3.6: Function through $(0,0)$, $(1,\beta_1)$ and $(2,\beta_2)$ to determine β_{13}.

So we have $\beta_0 = 1$ and $\beta_3 = 1.68$, from this we can conclude that the less wheels spread the pressure, the higher is the pressure at one place. So if the pressure is spread by an area which goes to zero, the pressure on one place will go to infinity. We will compute a logarithmic function $y(x)$ through the points $(x_0, y_0) = (0,0)$, $(x_1, y_1) = (1,1)$ and $(x_2, y_2) = (2,1.68)$ and compute $y(3)$ afterwards The formula which fits these points is $y(x) = 2.1511 \ln\left(\frac{x+1.6897}{1.6897}\right)$, see Figure 3.5. So $\beta_{12} = y(3) = 2.20$.

We also use a logarithmic function to estimate β_{13}, we use the points $(0,0)$, $(1,2.44)$ and $(2,2.86)$. The function through these points is $y(x) = 0.6144 \ln\left(\frac{x+0.0192}{0.0192}\right)$, see Figure 3.6. We take $\beta_{13} = y(3) = 3.11$.

Now we can compute the values of the constants $\Delta\tilde{N}_v$ and the $\Delta\tilde{N}'_v$. We assume that the load is divided equally over all the axles.

To keep the computations clear, we will compute $\Delta\tilde{N}'_{v_{\text{car}_i}}$ separately for every vehicle in vehicle type v_{car} and compute the average afterwards. So for $\Delta\tilde{N}'_v$ we have:

$$\begin{aligned}
\Delta\tilde{N}'_{v_{\text{car}}} &= n_{\text{single}\,v_{\text{car}}}\left(\frac{S_{\text{single}}}{S_{std}}\right)^{\beta_4} + n_{\text{tandem}\,v_{\text{car}}} 2\left(\frac{S_{\text{tandem}}}{2S_{std}}\right)^{\beta_4} + n_{\text{tridem}\,v_{\text{car}}} 3\left(\frac{S_{\text{tridem}}}{3S_{std}}\right)^{\beta_4} \\
\Delta\tilde{N}'_{v_{\text{car}\,\text{car}}} &= 2\left(\frac{9.81(1.022+0.150)/2}{80}\right)^{0.56} \approx 0.457780 \\
\Delta\tilde{N}'_{v_{\text{car}_{\text{mini bus}}}} &= 2\left(\frac{9.81(1.485+0.338)/2}{80}\right)^{0.56} \approx 0.586270 \\
\Delta\tilde{N}'_{v_{\text{car}}} &= \frac{6539\cdot 0.457780 + 756\cdot 0.586270}{6539+756} \approx 0.4711.
\end{aligned} \tag{3.12}$$

To keep the computations clear, we will compute $\Delta\tilde{N}'_{v_{\text{lorry}_i}}$ separately for every vehicle in vehicle type v_{lorry} and compute the average afterwards.

$$
\begin{aligned}
\Delta\tilde{N}'_{v_{\text{lorry}_i}} &= n_{\text{single}}v\left(\frac{S_{\text{single}}}{S_{std}}\right)^{\beta_4} + n_{\text{tandem}}v 2\left(\frac{S_{\text{tandem}}}{2S_{std}}\right)^{\beta_4} + n_{\text{tridem}}v 3\left(\frac{S_{\text{tridem}}}{3S_{std}}\right)^{\beta_4} \\
\Delta\tilde{N}'_{v_{\text{lorry}_{\text{truck}}}} &= 2\left(\frac{9.81\cdot 25/2}{80}\right)^{0.56} \approx 2.5404 \\
\Delta\tilde{N}'_{v_{\text{lorry}_{\text{lorry}}}} &= 2\left(\frac{9.81\cdot 25/3}{80}\right)^{0.56} + 3\left(\frac{9.81\cdot 25/3}{3\cdot 80}\right)^{0.56} \approx 3.6657 \\
\Delta\tilde{N}'_{v_{\text{lorry}_{\text{bus}}}} &= \left(\frac{9.81(10.868+1.5)/2}{80}\right)^{0.56} + 2\left(\frac{9.81(10.868+1.5)/2}{2\cdot 80}\right)^{0.56} \approx 2.0184 \\
\Delta\tilde{N}'_{v_{\text{lorry}}} &= \frac{83\Delta\tilde{N}_{v_{\text{lorry}_{\text{truck}}}} + 60\Delta\tilde{N}_{v_{\text{lorry}_{\text{lorry}}}} + 11\Delta\tilde{N}_{v_{\text{lorry}_{\text{bus}}}}}{83+60+11} \\
&= \frac{83\cdot 2.5404 + 60\cdot 3.6657 + 11\cdot 2.0184}{83+60+11} \approx 2.9415
\end{aligned}
\tag{3.13}
$$

And for the $\Delta\tilde{N}_v$ values we have:

$$
\begin{aligned}
\Delta\tilde{N}_{v_{\text{car}}} &= n_{\text{single}}v_{\text{car}}\left(\frac{S_{\text{single}}}{S_{std}}\right)^{\beta_1} + n_{\text{tandem}}v_{\text{car}}\left(\frac{S_{\text{tandem}}}{\beta_3 S_{std}}\right)^{\beta_2} + n_{\text{tridem}}v_{\text{car}}\left(\frac{S_{\text{tridem}}}{\beta_{12} S_{std}}\right)^{\beta_{13}} \\
\Delta\tilde{N}_{v_{\text{car}_{\text{car}}}} &= 2\left(\frac{9.81(1.022+0.150)/2}{80}\right)^{2.44} \approx 0.0032 \\
\Delta\tilde{N}_{v_{\text{car}_{\text{mini bus}}}} &= 2\left(\frac{9.81(1.485+0.338)/2}{80}\right)^{2.44} \approx 0.0095 \\
\Delta\tilde{N}_{v_{\text{car}}} &= \frac{6539\cdot 0.0032 + 756\cdot 0.0095}{6539+756} \approx 0.0040.
\end{aligned}
\tag{3.14}
$$

Again we will compute $\Delta\tilde{N}_{v_{\text{lorry}_i}}$ separately for every vehicle in vehicle type v_{lorry} and compute the average afterwards, to keep the computations clear.

$$
\begin{aligned}
\Delta\tilde{N}_{v_{\text{lorry}_i}} &= n_{\text{single}}v\left(\frac{S_{\text{single}}}{S_{std}}\right)^{\beta_1} + n_{\text{tandem}}v\left(\frac{S_{\text{tandem}}}{\beta_3 S_{std}}\right)^{\beta_2} + n_{\text{tridem}}v\left(\frac{S_{\text{tridem}}}{\beta_{12} S_{std}}\right)^{\beta_{13}} \\
\Delta\tilde{N}_{v_{\text{lorry}_{\text{truck}}}} &= 2\left(\frac{9.81\cdot 25/2}{80}\right)^{2.44} \approx 5.6705 \\
\Delta\tilde{N}_{v_{\text{lorry}_{\text{lorry}}}} &= 2\left(\frac{9.81\cdot 25/3}{80}\right)^{2.44} + \left(\frac{9.81\cdot 25/3}{2.20\cdot 80}\right)^{3.11} \approx 2.2005 \\
\Delta\tilde{N}_{v_{\text{lorry}_{\text{bus}}}} &= \left(\frac{9.81(10.868+1.5)/2}{80}\right)^{2.44} + \left(\frac{9.81(10.868+1.5)/2}{1.68\cdot 80}\right)^{2.86} \approx 0.6119 \\
\Delta\tilde{N}_{v_{\text{lorry}}} &= \frac{83\Delta\tilde{N}_{v_{\text{lorry}_{\text{truck}}}} + 60\Delta\tilde{N}_{v_{\text{lorry}_{\text{lorry}}}} + 11\Delta\tilde{N}_{v_{\text{lorry}_{\text{bus}}}}}{83+60+11} \\
&= \frac{83\cdot 5.6705 + 60\cdot 2.2005 + 11\cdot 0.6119}{83+60+11} \approx 3.9572
\end{aligned}
\tag{3.15}
$$

The thickness of the pavement is also defined and is equal for every road. Therefore we have

$$\begin{aligned}
RN &= \beta_9(LT_{r,1} + OT_r) + \beta_{10}LT_{r,2} + \beta_{11}LT_{r,3} \\
&= 9.28 \cdot 0.03 + 4.77 \cdot 0.05 + 4.29 \cdot 0.05 \\
&= 0.7314
\end{aligned}$$

Therefore the rut depth is given by the following formula

$$RD_{r,\tilde{t}_r^{AC},\tilde{t}_r^{U},t} = RD^{AC}_{r,\tilde{t}_r^{AC},t} + RD^{U}_{r,\tilde{t}_r^{U},t}$$

where

$$\begin{aligned}
RD^{AC}_{r,\tilde{t}_r^{AC},t} &= RD^{AC}_{r,\tilde{t}_r^{AC},t-1} + \Delta RD^{AC}_{r,\tilde{t}_r^{AC},t} \\
&= RD^{AC}_{r,\tilde{t}_r^{AC},t-1} + \mu_{r,t} e^{b' N'_{r,\tilde{t}_r^{AC},t}} \Delta N'_{r,t} \\
&= RD^{AC}_{r,\tilde{t}_r^{AC},t-1} + \beta_5 e^{b' N'_{r,\tilde{t}_r^{AC},t}} \sum_{v \in V} n_{r,v} g_v^{t-1} \Delta \tilde{N}'_v \\
RD^{U}_{r,\tilde{t}_r^{U},t} &= RD^{U}_{r,\tilde{t}_r^{U},t-1} + \Delta RD^{U}_{r,\tilde{t}_r^{U},t} \\
&= RD^{U}_{r,\tilde{t}_r^{U},t-1} + \alpha_{r,t} e^{b N_{r,\tilde{t}_r^{U},t}} \Delta N_{r,t} \\
&= RD^{U}_{r,\tilde{t}_r^{U},t-1} + \beta_7 e^{-RN} e^{b N_{r,\tilde{t}_r^{U},t}} \sum_{v \in V} n_{r,v} g_v^{t-1} \Delta \tilde{N}_v
\end{aligned}$$

with $RD^{AC}_{r,0,0}$ and $RD^{U}_{r,0,0}$ are the initial state of respectively the asphalt layer and the under layer. We also have $RD^{AC}_{r,s,s} = RD^{U}_{r,s,s} = 0 \ \forall r,s$, because this is the difference of rut depth between two the same time slots.

3.7 Example

To make the last section more clear we will give an example of how the PSI on a road can be computed. In Algorithm 3.1 an iterative guideline of how to compute the PSI of one road section is given, if we take $n_{\text{car}} = 0$. We assume in the algorithm that $\alpha, \mu, b, b', \Delta N$ and $\Delta N'$ are given.

According to the algorithm, we will give an example of computing the PSI in three time steps. We take $n_{\text{lorry}} = 5000$ for every time interval and the values of the given variables as computed and given before. This is shown in Figure 3.7.

It can be seen that in three intervals the PSI does not become very low. In the next section we will plot some graphs with more time intervals.

$PSI = 5.03$
$T = 3$
$n = 5000$
$N' = 0$
$N = 0$
$RD^{AC} = 0$
$RD^{U} = 0$
$\alpha = \beta_7 \cdot e^{-RN} = 1.71 \cdot 10^{-6} \cdot e^{-0.7314} \approx 8.2291 \cdot 10^{-7}$
$\mu = \beta_5 = 1.70 \cdot 10^{-5}$
$t = 1$
- $N = 0 + 5000 \cdot 3.9572 = 19786$
- $N' = 0 + 5000 \cdot 2.9415 = 14707.5$
- $\Delta RD^{U} = 8.2291 \cdot 10^{-7} e^{-2.28 \cdot 10^{-7} \cdot 19786} \cdot 5000 \cdot 3.9572 = 0.0162$
- $\Delta RD^{AC} = 1.70 \cdot 10^{-5} e^{-2.11 \cdot 10^{-6} \cdot 14707.5} \cdot 5000 \cdot 2.9415 = 0.2424$
- $RD^{U} = 0 + 0.0162 = 0.0162$
- $RD^{AC} = 0 + 0.2424 = 0.2424$
- $PSI = 5.03 - 2.14 \cdot 10^{-3} (0.2424 + 0.0162)^2 = 5.0299$

$t = 2$
- $N = 19786 + 5000 \cdot 3.9572 = 39572$
- $N' = 14707.5 + 5000 \cdot 2.9415 = 29415$
- $\Delta RD^{U} = 8.2291 \cdot 10^{-7} e^{-2.28 \cdot 10^{-7} \cdot 39572} \cdot 5000 \cdot 3.9572 = 0.0161$
- $\Delta RD^{AC} = 1.70 \cdot 10^{-5} e^{-2.11 \cdot 10^{-6} \cdot 29415} \cdot 5000 \cdot 2.9415 = 0.2350$
- $RD^{U} = 0.0162 + 0.0161 = 0.0321$
- $RD^{AC} = 0.2424 + 0.2350 = 0.4774$
- $PSI = 5.0299 - 2.14 \cdot 10^{-3} (0.4774 + 0.0321)^2 = 5.0293$

$t = 3$
- $N = 39572 + 5000 \cdot 3.9572 = 59358$
- $N' = 29415 + 5000 \cdot 2.9415 = 44123$
- $\Delta RD^{U} = 8.2291 \cdot 10^{-7} e^{-2.28 \cdot 10^{-7} \cdot 39572} \cdot 5000 \cdot 3.9572 = 0.0161$
- $\Delta RD^{AC} = 1.70 \cdot 10^{-5} e^{-2.11 \cdot 10^{-6} \cdot 29415} \cdot 5000 \cdot 2.9415 = 0.2278$
- $RD^{U} = 0.0321 + 0.0161 = 0.0482$
- $RD^{AC} = 0.4774 + 0.2278 = 0.7052$
- $PSI = 5.093 - 2.14 \cdot 10^{-3} (0.7052 + 0.0482)^2 = 5.0281$

Figure 3.7: Computation example with three time intervals.

double $PSI = 5.03$
int T = # time steps
int n = # lorries $= n_{v_{\text{lorry}}}$
double $N' = 0 = \tilde{N}'_{v_{\text{lorry}}}$
double $N = 0 = \tilde{N}_{v_{\text{lorry}}}$
double $RD^{AC} = 0$
double $RD^{U} = 0$
for int $t = 1 : T$

$N = N + n \cdot \Delta N$

$N' = N' + n \cdot \Delta N'$

$\Delta RD^{U} = \alpha e^{b \cdot N} n \Delta N$

$\Delta RD^{AC} = \mu e^{b' \cdot N'} n \Delta N'$

$RD^{U} = RD^{U} + \Delta RD^{U}$

$RD^{AC} = RD^{AC} + \Delta RD^{AC}$

$PSI = PSI - 2.14 \cdot 10^{-3} \left(RD^{U} + RD^{AC}\right)^{2}$

end

Algorithm 3.1: Iterative algorithm to compute the PSI on one road section if $n_{\text{car}} = 0$.

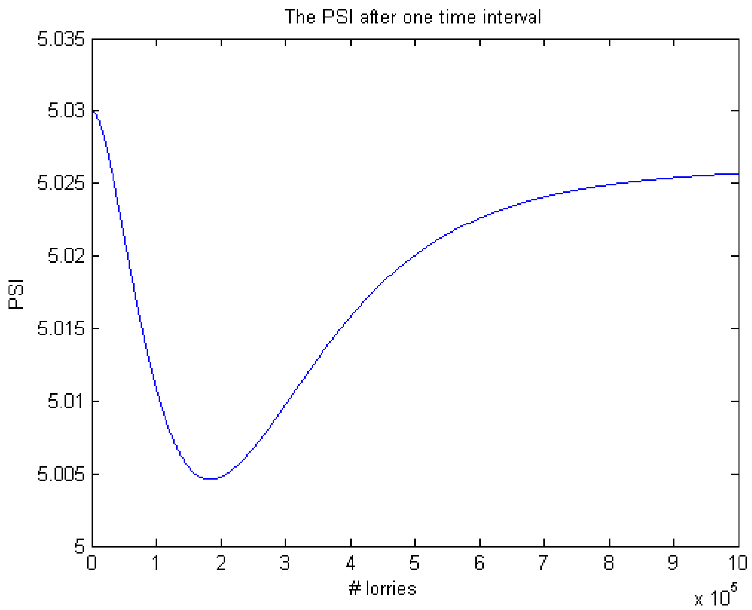


Figure 3.8: The PSI after the first time interval with $RN = 0.7314$.

3.8 Graphs of the Present Serviceability Index

If we plot the PSI in the first interval depending on the number of lorries we get Figure 3.8. In this graph we see that till approximately $1.8 \cdot 10^5$ lorries in one time interval the graph is decreasing. If more lorries would use the road, the PSI would be higher, that is strange because in practice, more axles will cause more damage to the road. We can make the same graph for only cars on the road, see Figure 3.9. The maximum number of cars is approximately $1.0071 \cdot 10^6$.

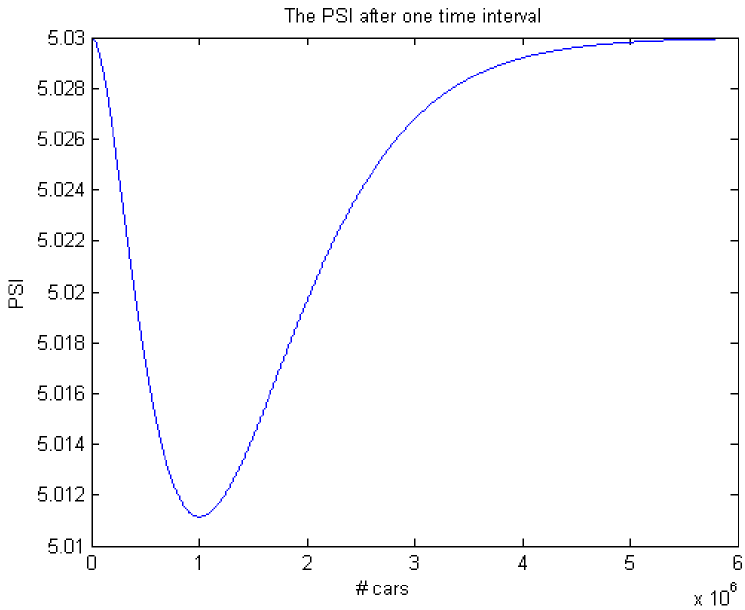


Figure 3.9: The PSI after the first time interval with $RN = 0.7314$.

Therefore we can only use the formula for the PSI if the number of vehicles on the road is low enough. Note that this is only the first time interval, if the growth factor is bigger than one, the number of vehicles increases over time and therefore the start number of vehicles must be lower.

The reason that the formula can just be used for a low enough number of vehicles is because of the shape of the ΔRD function, which is $x \cdot e^{-x}$, see Equation (3.3) and (3.4). For big enough x the part e^{-x}, which converges to zero, determines the behaviour of the function, which means that we have $\lim_{x\to\infty} x \cdot e^{-x} = 0$. This means that the delta rut depth converge to zero if the number of vehicles is to big, which is actually not the case in real life. This means that the PSI cannot be used any more for a large number of vehicles.

The PSI after one time interval if a combination of lorries and cars drive over the road is shown in Figure 3.10. The area where the PSI is usable is below the black line. If this number of vehicles is reached in reality depends on the time interval. In one day, on the busiest roads in the east of the Netherlands this number of vehicles is not reached.

We also made a plot about the PSI over time if we take 1000 lorries. After some trial and error, this is the lowest PSI we can get, see Figure 3.11. Also the rut depth is computed with the same information, this is in Figure 3.12. The reason why we cannot get the PSI lower than approximately 4.75 can be that we just use the rut depth to compute the PSI or that we do not take weather influences into account. In [5] we can see a graph witch has a PSI under the 1.5. This picture is also in Figure 3.13. In [5] not enough information is given to reproduce this graph, the missing information is the number of vehicles on the road. In their discussion the authors indicate the presence of difficulties by calculating the rut depth. Maybe this is the same problem

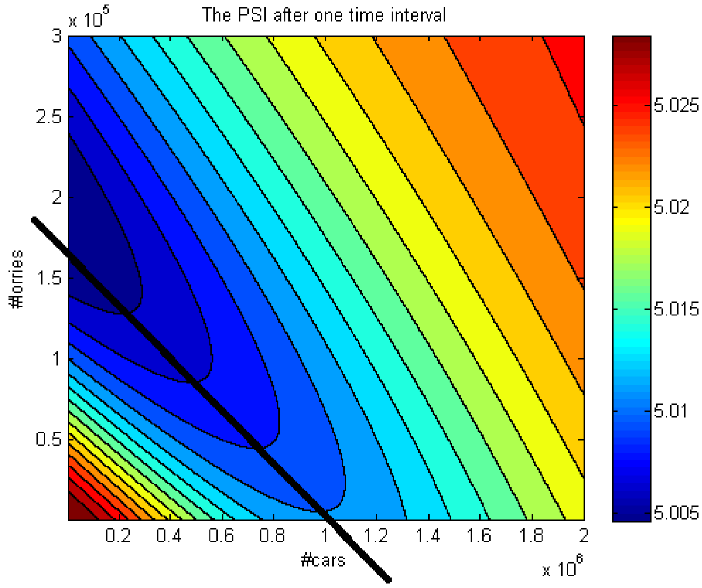


Figure 3.10: The PSI after the first time interval with $RN = 0.7314$.

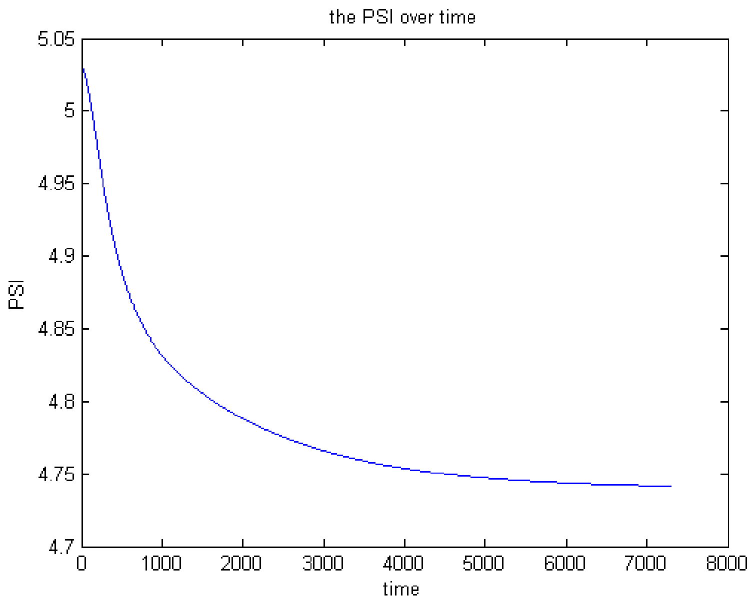


Figure 3.11: The PSI with 1000 lorries over time, with $RN = 0.7314$.

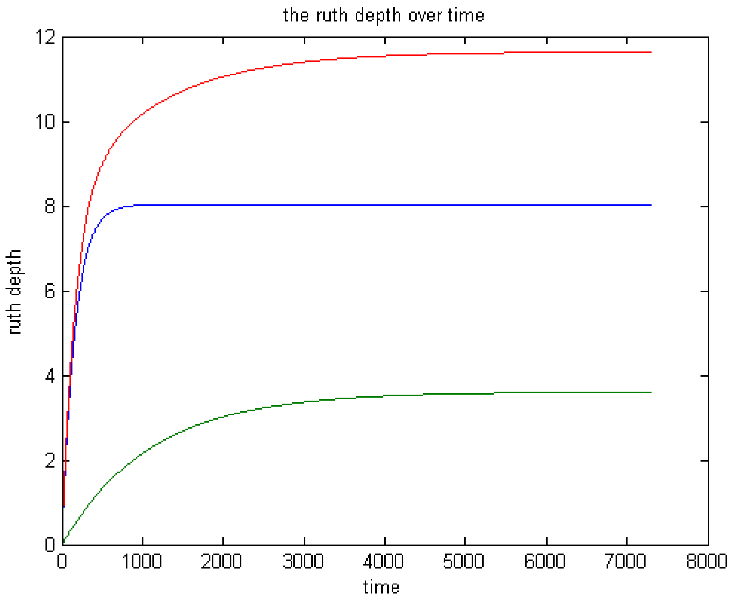


Figure 3.12: The rut depth with 1000 lorries over time, with $RN = 0.7314$. Red is the total rut depth, blue is the rut depth in the asphalt concrete layer and green is the rut depth in the underlayer.

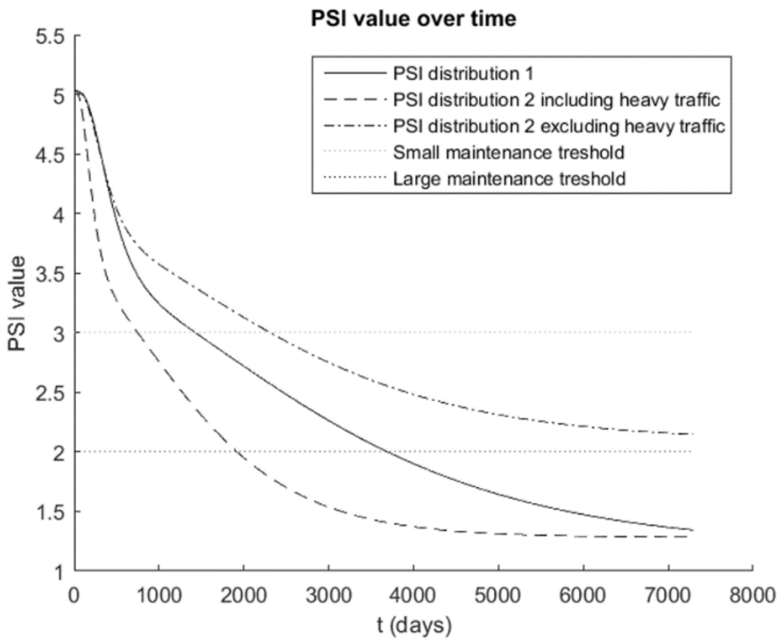


Figure 3.13: The PSI-graph according to [5].

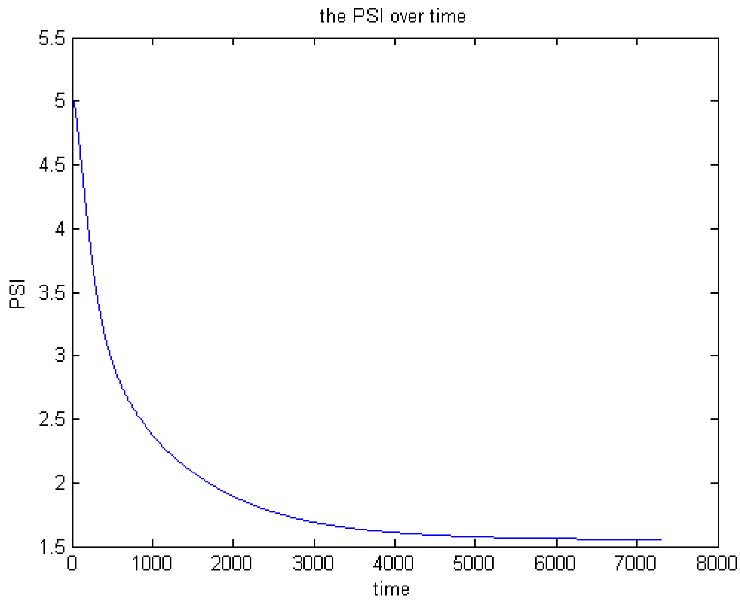


Figure 3.14: The corrected PSI with 1000 lorries over time, with $RN = 0.7314$.

as we can see in our graphs.

To make the PSI still usable in this paper, we will multiply the rut depth by a factor to get approximately the same graph as in [5]. It turns out that a multiplication of the rut depth with 3.46, the PSI-graph looks usable. Therefore we take this factor in our formula for the PSI. This factor can be the average of weather influences and the cracking and slope variance neglected in this paper. If we use the formula

$$\begin{aligned} PSI_{\text{corr}} &= 5.03 - 2.14 \cdot 10^{-3} \left(3.46RD\right)^2 \\ &= 5.03 - 2.57 \cdot 10^{-2} RD^2, \end{aligned} \tag{3.16}$$

the PSI_{corr}-graph looks like Figure 3.14. Therefore we calculate from now on with Equation (3.16) as formula for the PSI.

CHAPTER 4

THE USAGE OF THE ROAD NETWORK

Given a network of roads, people will plan their trip optimally. This means that the travellers want to minimise their 'costs', which may be for example the travel time or the travel distance.

First we will explain the structure of a road network and the Classic Transport Model, afterwards we will look how the usage of the road network fits in these models. Further we will describe which different ways there are for assigning trips to the network and we will give the optimisation problem which is used to get the distribution of the traffic over a network in such a way that no traveller wants to change his or her route, this is called the user equilibrium and is according to Wardrop's first principle. Finally we will describe how OmniTRANS (a software product of DAT.Mobility) assigns the traffic to a network.

4.1 The road network

In this chapter we will model a road network with a graph. The network is a directed graph $G(U,E)$ with a set of nodes U and a set of edges E. The nodes represent the junctions and the edges represent the links or roads. The junctions and the roads have multiple properties, which are called the labels of the nodes and links.

4.2 Classic Transport Model

The classic transport model describes the four phases to model the movements of vehicles over a network (Figure 4.1). The first three phases are just for personal traffic. The data of these phases are given from other models.

Database The model starts with a database. This consists of the data about the network, like labels of the nodes and links and the zones the network is divided in. Also data of possible transport modes (public transport, car, lorry) is in the database.

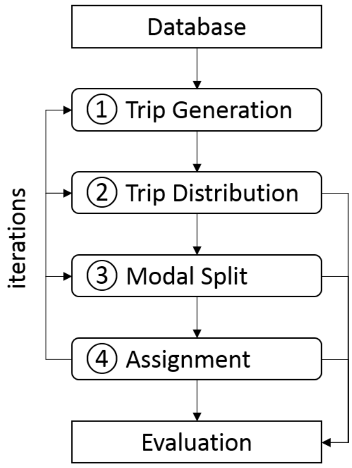


Figure 4.1: The Classic Transport Model. [11]

Trip Generation In this phase, the total number of trips will be generated and the number of trips attracted and produced by each zone of the network.

Trip Distribution Now the trips will be allocated to their particular origin and destination, this means the Origin-Destination-Matrix (OD-Matrix) will be defined.

Modal Split The OD-matrix will be split by different modes of transport, like a car or public transport.

Assignment In this phase the trips for each mode will be assigned to their best route.

Evaluation After every phase the results should be evaluated, which is important because you want to know if the results are plausible.

In our problem we only have to do phase 4, the assignment, and the evaluation. The other phases are outside this research and we assume they are fixed. This means that in our research we assume a fixed demand, therefore the OD-matrix is known.

4.3 Data

If a traveller wants to go to his destination, he needs a mode of transport. Different modes are bikes, motorcycles, cars, taxis, tractors, trucks and walking. Because there are a lot of different vehicles, we will group them in vehicle types v. We will neglect public transport and pedestrians, because we only consider trips made by vehicles on the road which can change their route. We also neglect the cyclists, because they have most of the time separate lanes. Our types are as described in chapter 3:

1. v_{car}: cars, mini buses
2. v_{lorry}: trucks, lorries, buses.

The data of the vehicle types are the influence on the asphalt concrete and the under layer of the pavement which are $\Delta\tilde{N}'_v$ and $\Delta\tilde{N}_v$ respectively. They also have a growth factor g_v.

The road network in which the travellers go to their destination is given, this is the directed graph $G(U,E)$. A link has multiple properties, the properties which are important for this research are

1. L_r: length of road r (m).
2. W_r: width of road r (m).
3. c_r: capacity of road r.
4. $x_{r,v}$: maximum speed of road r for vehicle type v (km/h).

An OD-pair can have $w_i \in U$ as origin and $w_j \in U$ as destination, this is shown as $[w_i, w_j]$. For every OD-pair multiple routes ρ are possible. This routes can depend on the vehicle type, because trucks are not allowed on every road where cars are. Also cars are more likely to take shortcuts. The nodes which can be in an OD-pair are special nodes, these nodes do not represent a junction in our network, but so called zones. This is done because we do not know exactly the origin and destination of travellers, but we know it approximately. These nodes are called nutrition nodes.

4.4 Assignment of the trips to the network

Two types of assignment are possible, static and dynamic assignment. Both assignments are for a certain time interval. With static assignment every person has an origin and a destination between which he wants to travel, the traveller chooses his route to optimise his travel costs. This means that all the traffic from the time interval is placed at once on its route and they are everywhere in the network at the same time. With dynamic assignment, everyone also has a departing time, for every second it will be computed where the traffic is at that moment. This will cost a lot of time, but it is more realistic because the congestion is simulated better. If the person wants to travel from his origin to his destination, he can optimise the travel costs by choosing a route and choosing the departure time.

Another important decision can be made about the capacity constraint. Every road has a capacity, if the flow in that road is bigger than the capacity, there will be congestion and the maximal speed the vehicles are able to drive will decrease. This means that if all the vehicles go via the same route, this may not be optimal, even if the road is part of the initial shortest path from A to B.

For these four different types of assignment, different algorithms are used. A scheme for this is in Table 4.1.

		Stochastic effects included?	
		No	Yes
Capacity constraints	No	All-Or-Nothing	Burrell's equilibrium
included?	Yes	Wardrop's equilibrium	Stochastic user equilibrium

Table 4.1: Classification scheme for traffic assignment. [11]

In this thesis we use static assignment for an average working day, divided in three time slots: morning rush hour (7 till 9 o'clock), afternoon rush hour (16 till 18 o'clock) and the rest of the day. We use static assignment because the available model of our case (Enschede and Hengelo) is static. Also, in our research the actual travel time does not matter for the maintenance costs, only the axle loads of vehicles does matter. The axle load does not depend on departure time, so the static assignment is good enough for our research.

The assignment of v_{lorry} will just be the shortest path, because the lorries will drive on big roads and they all follow the same route if they have the same OD-pair. This type of assignment is a state of practice for lorries. For the assignment of v_{car} we will look for the Wardrop's equilibrium with a pre-load of the lorries on the network.

The Wardrop's equilibrium is the equilibrium the users of the network make if they choose their optimal path. This is stated in Wardrops's first principle.

Theorem 4.1 (Wardrop's first principle [26])**.** *Under equilibrium conditions traffic arranges itself in such a way that no individual trip maker can reduce his path costs by switching routes.*

This means that if all trip makers perceive costs in the same way we can rewrite Wardrop's first principle.

Theorem 4.2 (Wardrop's first principle with equal trip makers [26])**.** *Under equilibrium conditions traffic arranges itself in congested networks such that all used routes between an OD-pair have equal and minimum costs while all unused routes have greater or equal costs.*

Note that the Wardrop's equilibrium is not necessarily the total cost optimal solution!

We take identical travellers with a cost function of time. Every traveller wants to minimise his travel time, in other words, he wants to minimise his delay. The delay on a link will be computed with the Bureau of Public Roads curve, the BPR-function [31].

$$\tau_r(f_r) = \tau_{0,r}\left(1 + A_r\left(\frac{f_r}{c_r}\right)^{B_r}\right)$$

where

$$\begin{aligned}
\tau_r &= \text{final travel time of a vehicle on road } r\\
\tau_{0,r} &= \text{free flow travel time on road } r\\
A_r, B_r &= \text{parameters}\\
f_r &= \text{flow on road } r\\
c_r &= \text{capacity of road } r
\end{aligned}$$

Note that because the travellers are all the same, their free flow travel time must also be the same.

The flow f is a function of the numbers of the different vehicle types using the road. This is expressed in pae, persoon auto equivalent (personal car equivalent). From this OmniTRANS model we have

$$f_r = n_{v_{\mathrm{car}},r} + 1.8 n_{v_{\mathrm{lorry}},r}. \tag{4.1}$$

The reason that lorries have a factor 1.8 is that lorries are longer vehicles than cars and therefore take more place on the road which leads quicker to congestion.

The free flow travel time can be computed from the maximum speed by $\tau_{0,r} = \frac{L_r}{x_r}$. The parameters A_r and B_r depend on the characteristics of the road. The model in OmniTRANS has

$$A_r = \begin{cases} 0.5 & r \text{ has the structure of a motorway} \\ 0.8 & r \text{ has the structure of a superhighway or a 80 km/h road} \\ 1.0 & \text{otherwise} \end{cases}$$

$$B_r = 4.$$

Which means that the higher A_r is, the earlier delay will occur and choosing another route will be interesting.

Note that the value of A_r depends on the structure of the road and not on the maximum speed vehicles are allowed to drive. A road with the structure of a motorway, with the maximum speed 100 km/h has $A_r = 0.5$ instead of the A_r-value of the superhighway.

4.4.1 The optimisation problem

Based on the descriptions given, we can set up an optimisation problem. Every traveller wants to minimise his costs, the travel time. If every traveller acts in this way, every route between nutrition node w_i and w_j must have the same travel time if it is used (Wardrop).

Say the number of trips between OD-pair $[w_i, w_j]$ for vehicle type v via route ρ is given by $\eta_{i,j,\rho,v}$. It is clear that

$$\eta_{i,j,\rho,v} \geq 0 \qquad \forall i,j,\rho,v.$$

We also have

$$\sum_{\rho} \eta_{i,j,\rho,v} = \eta_{i,j,v} \qquad \forall i,j,v,$$

where $\eta_{i,j,v}$ is the total vehicle v supply of node w_i which goes to w_j. This constraint means that it does not matter which route will be taken, the travellers will all reach their destination.

We have the intensity on link r as given in Equation (4.1). The number of vehicle type v on link r is given by $n_{v,r} = \sum_{i,j,\rho} \eta_{i,j,\rho,v} \xi_{i,j,\rho,v,r}$ where

$$\xi_{i,j,\rho,r} = \begin{cases} 1 & \text{link } r \text{ is part of route } \rho \text{ for OD-pair } [w_i, w_j] \\ 0 & \text{otherwise} \end{cases}$$

Therefore we have

$$\begin{aligned} f_r &= n_{v_{\text{car}},r} + 1.8 n_{v_{\text{lorry}},r} \\ &= \sum_{i,j,\rho} \eta_{i,j,\rho,v_{\text{car}}} \xi_{i,j,\rho,v_{\text{car}},r} + 1.8 \sum_{i,j,\rho} \eta_{i,j,\rho,v_{\text{lorry}}} \xi_{i,j,\rho,v_{\text{lorry}},r} \\ &= \sum_{i,j,\rho} \left[\eta_{i,j,\rho,v_{\text{car}}} \xi_{i,j,\rho,v_{\text{car}},r} + 1.8 \eta_{i,j,\rho,v_{\text{lorry}}} \xi_{i,j,\rho,v_{\text{lorry}},r} \right] \\ &= \sum_{i,j,\rho,v} \omega_v \eta_{i,j,\rho,v} \xi_{i,j,\rho,v,r}, \end{aligned}$$

with $\omega_{v_{\text{car}}} = 1$ and $\omega_{v_{\text{lorry}}} = 1.8$.

The objective function in our optimisation problem, which is commonly used in the literature [11], is

$$\sum_r \int_0^{f_r} \tau_r(\zeta)\ \mathrm{d}\zeta.$$

To keep the explanation of this objective function clear, we look at the situation with cars only. The objective function $\sum_r \sum_{\zeta=0}^{f_r} \tau_r(\zeta)$ gives the users equilibrium.

Theorem 4.3. *The minimisation of* $\sum_r \sum_{\zeta=0}^{f_r} \tau_r(\zeta)$ *gives the users equilibrium.*

Proof. The users equilibrium means that no one can choose another route in such a way that it will decrease its travel time. Suppose we are in a global optimum with objective value k and someone can change his route to decrease his travel time with amount $\kappa > 0$.

The new solution is a feasible traffic flow. The objective value of this solution is exactly the previous value decreased with the same amount as the travel time of that specific driver has decreased. The new objective value is therefore $k - \kappa < k$. Which means that we found a new solution with a smaller objective value than the global minimum, which is a contradiction.

Therefore gives the minimisation of $\sum_r \sum_{\zeta=0}^{f_r} \tau_r(\zeta)$ the users equilibrium. ■

Because calculating with an integral is often easier than a summation, the summation is replaced by an integral, this is only allowed if the integral approaches the summation good enough. Because the integral is used everywhere in literature, we assume it is. Also an integral provides the possibility of continuous flow functions. In our research we have cars and lorries and the flow is given by Equation (4.1), which is not always a whole number. Which is no problem with an integral.

This brings us to the standard model for the user equilibrium used in the literature.

$$\begin{aligned}
\text{min.} \quad & \sum_r \int_0^{f_r} \tau_r(\zeta)\ \mathrm{d}\zeta && (4.2)\\
\text{s.t.} \quad & \sum_\rho \eta_{i,j,\rho,v} = \eta_{i,j,v} \qquad \forall i,j,v \\
& \eta_{i,j,\rho,v} \geq 0 \qquad \forall i,j,\rho,v \\
& \sum_{i,j,\rho,v} \omega_v \eta_{i,j,\rho,v} \xi_{i,j,\rho,v,r} = f_r
\end{aligned}$$

The solution space is convex, which means that it can be solved with a steepest descent method. The solution of this optimisation problem gives us the Wardrop Equilibrium [11].

4.4.2 Junction delay

At junctions the travellers have delay too, which has to be taken into account by computing the route costs. The delay on junctions is a lot more complex to compute. The idea is to make a

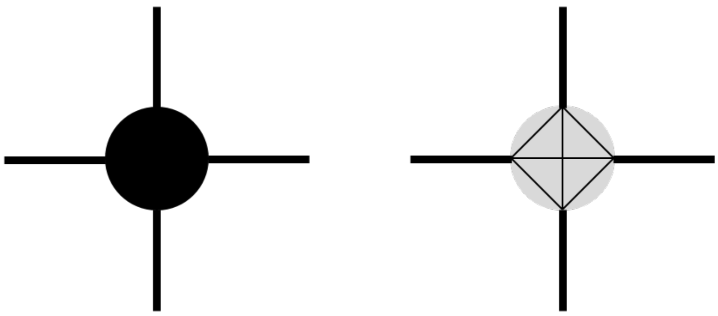


Figure 4.2: The underlying structure of a junction. Instead of one big node, the junction consists of turns.

network of roads in the junction. These roads are called turns. Now the individual turns have a delay, but this delay also depends on the intensity on the other turns. How this depends of all the intensities on the turns in the junction depends on the type of the junction (roundabout, non regulated, traffic lights or priority junction). In *"Static Traffic Assignment with Junction Modelling"* [21] the junction modelling in OmniTRANS is extensively described. The functions used need too much explanation for this paper, therefore we say that the delay for vehicle type v on the junction of road r on a junction is given by $\tilde{\tau}_{r,v}(f)$, with $\tilde{\tau}_{r,v}(f)$ not further specified.

With time delay functions which depend on the intensity on multiple links, the Wardrop's Equilibrium cannot be found any more with the optimisation problem in Equation (4.2). Therefore we will use the junction modelling from OmniTRANS, which is an iterative process.

4.4.3 Iterative process for travel flow determination

OmniTRANS uses an iterative process to compute the User Equilibrium. In the first step it computes the shortest path for the vehicles and assigns the vehicles to this path. If you stop the algorithm here, it is called the AON (All-Or-Nothing) assignment. The iterative process now looks for another shortest path using the BPR-function and the previous assignment as pre-load. Afterwards its averages the traffic on all the routes found till then. This will be done a certain number of iterations. This method is called the Volume-Averaging assignment (VA).

As said before, it is state of practice to use the All-Or-Nothing assignment for lorries. Therefore OmniTRANS uses this assignment for lorries, afterwards, it will assign cars determined by a Volume-Averaging assignment with the lorries as preload.

In the AON assignment the junction modelling is not used, because it is based on free flow conditions, which is the shortest path if the network is empty. This means that the junctions have no delay. Therefore the AON assignment is just a shortest path algorithm which is solvable in polynomial time.

The VA assignment has junction modelling included, which means that this problem is not easy and quick to solve anymore.

CHAPTER 5

The optimal travel flow

We know how to compute the minimal maintenance costs given the flow on the road network (chapter 3). We also know how travellers will distribute themself over the road network (chapter 4). So now we have to determine which routes the vehicles have to drive such that the travel flow is optimal for the maintenance. We can reduce this problem to a kind of min-cost flow problem, of which an efficient solving method is known. Therefore we write our problem in a min-cost flow optimisation problem. Afterwards we have to set up a cost function needed in the min-cost flow problem and we will analyse it. Finally we will find out how we will solve our special min-cost flow optimisation problem.

5.1 Formulation of the min-cost flow problem

The min-cost flow problem is an optimisation problem which decides how the flow has to go through a graph from the source to the target node in such a way that the total costs will be minimal. All the flow is exactly the same, which means that if a link has a flow in one direction, flow in the opposite direction will neutralise this flow.

In our problem we have different flows, because traffic is not neutralising each other. We really have to move flow from w_i to w_j and if there is another flow in the opposite direction on that link, it will not neutralise each other. Therefore we take different flow types.

The flow on link (w_i, w_j) with OD-pair $[w_k, w_l]$ is called $f_{(w_i,w_j)}([w_k, w_l])$. The capacity on link (w_i, w_j) is given by $c_{(w_i,w_j)}$. Note that the capacity of a link is not specific for every OD-pair, because every vehicle of one type has the same influence as another vehicle of the same type. Some nodes have a demand or supply, these nodes are the nutrition nodes in our road network. The supply of node w_i which goes to w_j is given by $d_{i,j}$. The demand of node w_j with origin w_i is therefore also $d_{i,j}$.

The optimisation problem to get the optimal flow is given by

$$\begin{aligned}
\text{min.} \quad & \sum_{(w_i,w_j)\in E} C_{(w_i,w_j)}\left(\sum_{w_k,w_l} f_{(w_i,w_j)}\left([w_k,w_l]\right)\right) & \\
\text{s.t.} \quad & \sum_{w_k,w_l} f_{(w_i,w_j)}\left([w_k,w_l]\right) \le c_{(w_i,w_j)} & \forall i,j \\
& f_{(w_i,w_j)}\left([w_k,w_l]\right) = -f_{(w_j,w_i)}\left([w_k,w_l]\right) & \forall i,j,k,l \\
& \sum f_{(w_i,w_j)}\left([w_k,w_l]\right) = 0 & \forall i \neq k,l \\
& \sum_{w_j} f_{(w_i,w_j)}\left([w_k,w_l]\right) = d_{k,l} & \forall k,l \\
& \sum_{w_i} f_{(w_i,w_j)}\left([w_k,w_l]\right) = d_{k,l} & \forall k,l
\end{aligned}$$

which is an extension of the normal min-cost flow problem.

The capacities c are known from the data in our OmniTRANS model. Also the demands d are known from the data in our OmniTRANS model. The cost function we have to make ourselves.

5.2 The cost function

The flow on a link consists of the number of cars, $n_{v_{\text{car}}}$, and the number of lorries, $n_{v_{\text{lorry}}}$. In this section we talk in terms of n_v instead of flow. We can formulate the time until maintenance or replacement which depends on the number of vehicles on the road. We know the costs of the maintenance and replacement, so we can compute the costs depending on the flow.

The formula for the costs is not easy. If the flow is known for a road, then the costs can be computed according to Algorithm 5.1. The value of δ is the discount factor and is the same as in chapter 3.

if $n_{r,l} == 0$ **and** $n_{r,c} == 0$
    $C_r = 0$
else
    determine times for maintenance (= $T_{r,\text{main}}$) and replacement (= $T_{r,\text{rep}}$)
    $C_r = L_r W_r \left(\sum_{t\in T_{r,\text{main}}} C_{\text{main}}\delta^t + \sum_{t\in T_{r,\text{rep}}} C_{\text{rep}}\delta^t\right)$
end

Algorithm 5.1: Compute the maintenance costs for one road section.

The sets $T_{r,\text{main}}$ and $T_{r,\text{rep}}$ can be computed according to an algorithm of your choice. For getting the optimal costs, Equation (3.8) has to be solved, however this will cost a lot of computation time because it is an MIP (Mixed Integer Program), which, in general, can not be solved in linear time. Therefore an alternative algorithm is given later on, this algorithm gives a fixed strategy to maintain or repair the road. First we give some explanation to understand this algorithm. In the following computations we just use the rut depth of the asphalt concrete layer. The computations with the rut depth of the under layer are similar. For the difference of the rut depth of the asphalt concrete layer between $t_2 - 1$ and t_2 when the layer is placed new at t_1 we have

$$\begin{aligned}
\Delta RD^{AC}_{t_1,t_2} &= \mu \exp\left(b' \sum_{v\in V} n_v \Delta\tilde{N}'_v \sum_{s=t_1}^{t_2} g_v^{s-1} \right) \sum_{v\in V} n_v \Delta\tilde{N}'_v g_v^{t_2-1} \\
&= \mu \exp\left(b' \sum_{v\in V} n_v \Delta\tilde{N}'_v \frac{g_v^{t_2} - g_v^{t_1-1}}{g_v - 1} \right) \sum_{v\in V} n_v \Delta\tilde{N}'_v g_v^{t_2-1} \\
&= \mu \prod_{v\in V} \exp\left(b' n_v \Delta\tilde{N}'_v \frac{g_v^{t_2} - g_v^{t_1-1}}{g_v - 1} \right) \cdot \sum_{v\in V} n_v \Delta\tilde{N}'_v g_v^{t_2-1} \\
&= \mu \prod_{v\in V} \underbrace{\exp\left(\frac{b' n_v \Delta\tilde{N}'_v}{g_v - 1} \right)}_{A'_v}{}^{\left(g_v^{t_2} - g_v^{t_1-1}\right)} \cdot \sum_{v\in V} n_v \Delta\tilde{N}'_v g_v^{t_2-1} \\
&= \mu \prod_{v\in V} A'^{\left(g_v^{t_2} - g_v^{t_1-1}\right)}_v \cdot \sum_{v\in V} n_v \Delta\tilde{N}'_v g_v^{t_2-1}.
\end{aligned}$$

And therefore we have

$$\begin{aligned}
RD^{AC}_{t_1,t_2} &= \sum_{s=t_1}^{t_2} \mu \prod_{v\in V} A'^{\left(g_v^{s} - g_v^{t_1-1}\right)}_v \cdot \sum_{v\in V} n_v \Delta\tilde{N}'_v g_v^{s-1} \\
&= \mu \sum_{s=t_1}^{t_2} \left[\prod_{v\in V} A'^{\left(g_v^{s}\right)}_v A'^{\left(-g_v^{t_1-1}\right)}_v \cdot \sum_{v\in V} n_v \Delta\tilde{N}'_v g_v^{s-1} \right]
\end{aligned}$$

We maintain the road if $RD^{AC} \geq B_1^{AC}$ and we repair the road if $RD^{AC} \geq B_2^{AC}$ and $RD^U \geq B_2^U$. Therefore we want to know t_2 from

$$\begin{aligned}
B_1^{AC} &\leq RD^{AC} \\
B_1^{AC} &\leq \mu \prod_{v\in V} A'^{\left(-g_v^{t_1-1}\right)}_v \cdot \sum_{s=t_1}^{t_2} \left[\prod_{v\in V} A'^{\left(g_v^{s}\right)}_v \cdot \sum_{v\in V} n_v \Delta\tilde{N}'_v g_v^{s-1} \right] \\
\frac{B_1^{AC} \prod_{v\in V} A'^{\left(g_v^{t_1-1}\right)}_v}{\mu} &\leq \sum_{s=t_1}^{t_2} \left[\prod_{v\in V} A'^{\left(g_v^{s}\right)}_v \cdot \sum_{v\in V} n_v \Delta\tilde{N}'_v g_v^{s-1} \right]
\end{aligned}$$

Note that you have to choose B_1^{AC}, B_1^{AC} and B_1^{AC} with the factor described in section 3.8, Equation (3.16), taken into account! We take $B_1^{AC} = B_2^{AC} = 7.22$ and $B_2^U = 1.88$, which means that we will repair the road if the PSI is lower or equal than 2.90.

With this we can define an algorithm to compute the sets $T_{r,\text{main}}$ and $T_{r,\text{rep}}$, which is done in Algorithm 5.2.

So the function for C_r is clearly not continuous and therefore also not differentiable. This means that the algorithms which are used usually to solve an optimisation problem, cannot be used. Therefore we will look to iterative algorithms to solve our optimisation problem. With an iterative algorithm we can also skip the computation of the optimal flow an directly solve the whole problem by taking speed measures to optimise the maintenance costs.

A visualisation of the maintenance costs can be seen in Figure 5.1. Again we see that the formula of the PSI cannot be used for every number of lorries and cars, only the left under corner under the black line, can be used. Further we can see that the number of cars does hardly influence the maintenance costs. Therefore we will compute the maintenance costs just depending on the number of lorries.

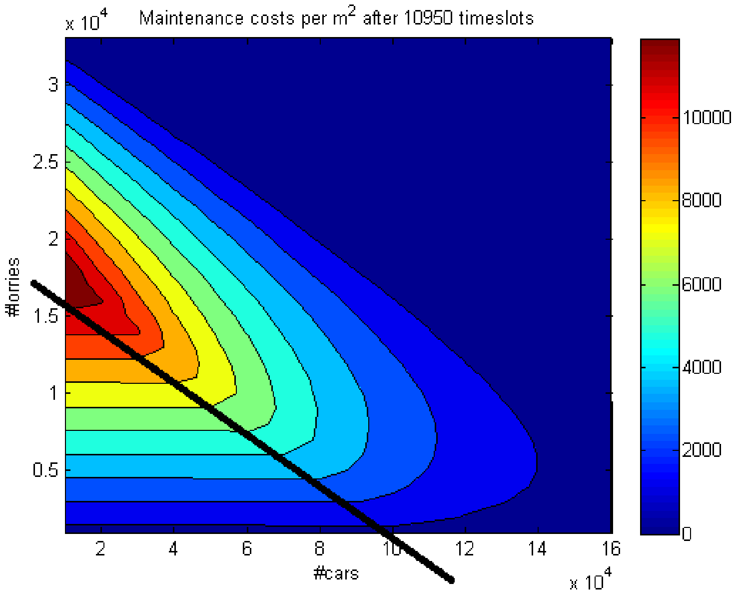


Figure 5.1: The maintenance costs depending on the number of lorries and cars. With $d = 1.000009$ and $RN = 0.7314$, after 30 years.

In Figure 5.2, the costs per square meter are plotted, depending on the number of lorries on the road. The blue line represents the real costs, which is not continuous. The function consists of a lot of step-functions after each other. The red line represents the approximated costs, which is continuous. As we can see, the first part of the blue line is a linear function, afterwards it could be a polynomial or logarithmic function. For the last part, the PSI cannot be used any more, this is around 18000 lorries. From then we approximate it with a horizontal straight line. We choose for the second part a second order polynomial function.

With interpolation on the original function, the function for the costs per square meter is given by

$$C(n_{v_{\text{lorry}}}) = \begin{cases} 0.927 n_{v_{\text{lorry}}} & n_{v_{\text{lorry}}} \leq 4375 \\ -4.050 \cdot 10^{-5} n_{v_{\text{lorry}}}^2 + 1.586 n_{v_{\text{lorry}}} - 2.108 \cdot 10^3 & 4375 < n_{v_{\text{lorry}}} \leq 18260 \\ 13353 & \text{otherwise} \end{cases} \tag{5.1}$$

The total costs for the road is the costs per square meter multiplied with the length and width of the road. The length is known in the OmniTRANS network, but the width is not. The width of a road depends on the road order, which are listed in Table 5.1. We first give some definitions.

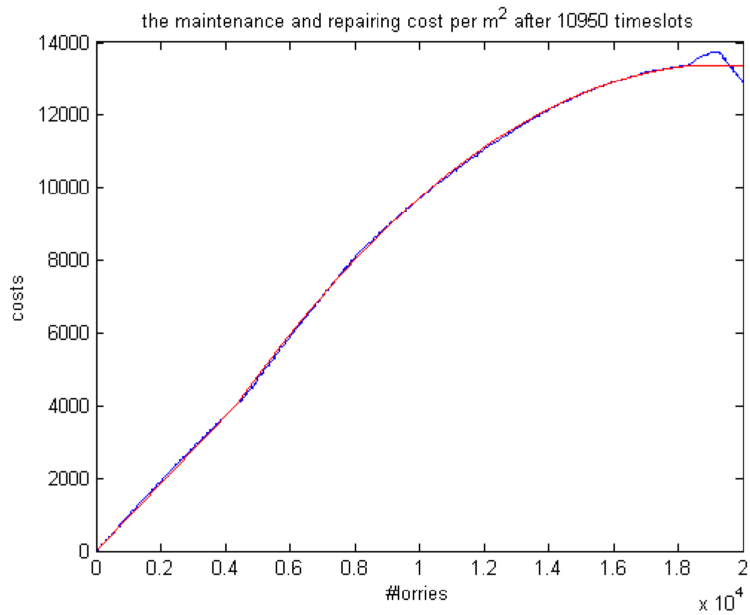


Figure 5.2: The maintenance costs depending on the number of lorries. With $d = 1.000009$ and $RN = 0.7314$, after 30 years. The blue line represents the real costs and the red line indicates the approximated costs

Definition 5.1 (Flow road (NL: stroomweg) [28])**.** Road for a lot of flow with as less as possible delay. For example the motorways (NL: autosnelweg) or the superhighway (NL: autoweg). This road type has most often grade separation.

Definition 5.2 (Flow access road (NL: gebiedsontsluitingsweg) [28])**.** This road group connects flow roads with access roads, therefore it has priority intersections. Also these roads are made for big flows of traffic. Outside the urban area, examples of flow access roads are 80 km/h roads. In the urban area, examples of flow access roads are 50 km/h roads.

Definition 5.3 (Access road (NL: erftoegansweg) [28])**.** These are the local roads, which are meant to exchange traffic. This means that there are intersections. Outside the urban area, access roads are 60 km/h roads and in the urban area they are the 30 km/h roads. These roads does not have a division line for the different directions on the road.

road group	**width of lane**
Access road	3.50 m
Flow access road (2×1)	2.75 m
Flow access road (2×2)	3.10 m
Flow road (2×1)	3.00 m
Flow road (2×2)	3.25 m
Highway	3.50 m

Table 5.1: The grid points of possible velocities for the first step of grid search. (2×2 means that there are two carriageways with both 2 lanes, 2×1 is defined similarly.

It sounds strange that access roads are wider than flow access roads, but this is because flow roads do not have lanes. All the vehicles drive on the same area, the width is therefore the width of the whole road.

It is possible that a road has multiple lanes, if so the lorries will distribute themselves over these lanes. The most right lane has the most lorries on it, so this lane is determining for the maintenance. Most roads with multiple lanes are motorways, in chapter 3 we already introduced F_{lanes}, a correction factor for the number of lanes. These correction factors are shown in Table 5.2 [25]. Even though F_{lanes} is not in the model of the PSI, we will use it, because it gives the distribution of lorries on motorways in the Netherlands and it will make our model more complete.

# lanes	F_{lanes}
One or target lane	1
Two	0.95
Three or more	0.90

Table 5.2: The correction factor for the lanes, according to [25].

5.2.1 Analysis of the cost function

If we analyse Equation (5.1) and look to the costs per lorry, depending on the number of lorries, we get Figure 5.3. The last part of the graph, where the number of lorries is bigger than $1.8 \cdot 10^5$ is grey, because from there we cannot use the formula for the PSI any more.

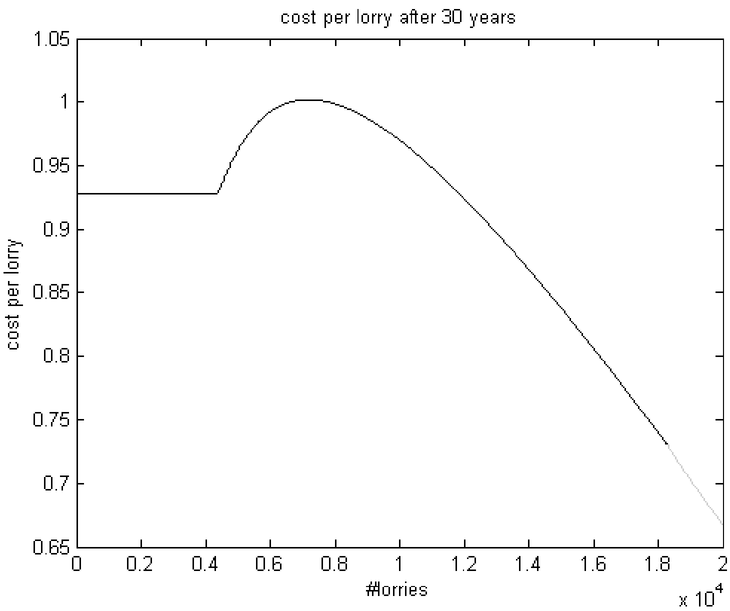


Figure 5.3: The maintenance costs per lorry depending on the number of lorries. With $d = 1.000009$ and $RN = 0.7314$, after 30 years.

It can be seen that when $4375 < n_{v_{\text{lorry}}} < 11896$ per timeslot the costs per lorry are higher than elsewhere, this means that if a certain number of lorries is on the road, the maintenance costs are relatively high. It is likely that you want to avoid this number of lorries on roads. We can mark a green, orange and red zone. In the red zone, the function for the PSI cannot be used, so you do not want that number of lorries on the road. In the orange zone a lorry is relatively expensive. This is shown in Figure 5.4.

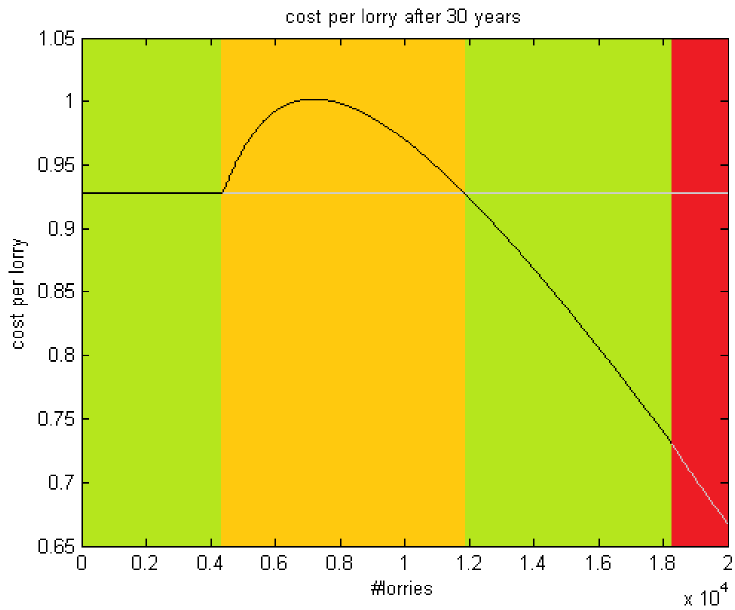


Figure 5.4: The green, orange and red areas for the number of lorries. With $d = 1.000009$ and $RN = 0.7314$, after 30 years.

This does not mean that the maintenance costs are minimal if no (red and) orange roads are in the network, we formulate this in a theorem.

Theorem 5.1. *A network with no orange roads does not necessarily give the minimal maintenance costs.*

Proof. We will prove this by a counterexample.

Take a network with two nodes A and B and three roads r_1, r_2 and r_3. All the roads have the same width, say one meter. The length of road r_1 and r_3 is 15 meter and the length of r_2 is 10 meter. See Figure 5.5.

At first, we put all the traffic (5000 lorries) on road r_2. This means that r_2 is in the orange area. We can compute the maintenance costs

$$C = €\left(-4.050 \cdot 10^{-5} \cdot 5000 + 1.586 \cdot 5000 - 2.108 \cdot 10^{3}\right) \cdot 10 \approx €4.811 \cdot 10^{3} \cdot 10 \approx €4.811 \cdot 10^{4}.$$

Second, we distribute the the traffic of r_2 on r_1 and r_3 equally. This means that on both roads are

2500 lorries, so the roads are in the green area. The total costs therefore are

$$C = €2 \cdot (0.927 \cdot 2500) \cdot 15 \approx €30 \cdot 2.318 \cdot 10^3 \approx €6.95 \cdot 10^4,$$

which is higher than in the first network. ■

So, we will not use this method.

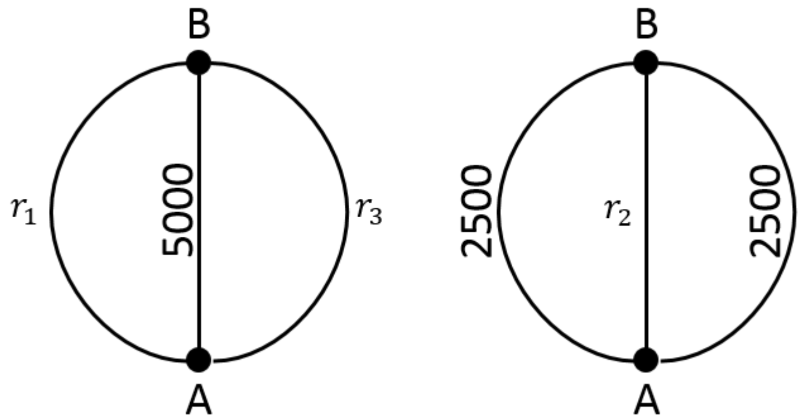


Figure 5.5: A counterexample that only having green roads does not make the maintenance costs as low as possible. The length of r_1 and r_3 is 15m and the length of r_2 is 10m. The costs in the first network are $€4.81 \cdot 10^4$ and in the second network $€6.95 \cdot 10^4$.

5.3 Solving the min-cost flow problem

Now we have the cost function only dependent on the number of lorries. OmniTRANS first assigns the lorries and afterwards the cars, this means that the assigning of cars does not matter any more, which means that we can simplify the min-cost flow problem by taking out the cars from the flow and OD-pairs.

The cost function is not linear, which means that we cannot set up an LP to solve this optimisation problem. We can for example use piecewise linear functions for the costs and compute the flow which minimises the costs. Afterwards we can let the number of lorries on the road network converge (iteratively) to the optimal flows, for example with speed limits. Or we can use an iterative method which assigns the lorries to the network and computes the corresponding costs every iteration.

The first option sounds like a good idea, but the problem with piecewise linear functions is that it makes the optimisation problem only solvable in an ILP (Integer Linear Program), which means that this problem cannot be solved in linear time. It is very likely that the outcome of this method gives a distribution which cannot be reached with measures taken in a network. Or for example, the optimal speed limit would be a decimals number, which is not the practice on roads. It will not always be the case that the speeds rounded on a multiple of ten gives the optimal speed limits. Also in this method we have two long optimisation problems: the min-cost flow problem and the iterative process to come to this distribution. The pro of this method is that you know which direction you have to go with assigning the lorries to the network and it may be possible to reach this distribution with other measures than speed limits.

Iterative methods can be a random walk, local search, grid search or a genetic algorithm. This means that with this method you act like you do not know anything about he solution space and therefore you do not know what the best solution would be. Such a method would be grid search and after a certain number of iterations you can go further with local search. A pro of iterative methods is that we skip the problems which are difficult to solve, we can compute the costs according to Equation (5.1), which will cost little time.

We will use the second idea, the iterative methods. In the next section we analyse some and decide which one we will use further in this research.

int $t'_1 = 1$, **int** $t_1 = 1$, **int** $T_{\text{main}} = \emptyset$, **int** $T_{\text{rep}} = \emptyset$
double $A'_{v_i} = \exp\left(\frac{b' n_{v_i} \Delta\tilde{N}'_{v_i}}{g_{v_i} - 1}\right)$, **double** $A_{v_i} = \exp\left(\frac{b n_{v_i} \Delta\tilde{N}_{v_i}}{g_{v_i} - 1}\right)$
int $t'_{2,1}$, $t'_{2,2}$, t_2, **double** LHS, LHS'_1, LHS'_2
while $t_1 < T$
  $LHS = B_2^U \alpha^{-1} \prod_{v\in V} A_v^{\left(g_v^{t_1-1}\right)}$
  Solve t_2 from $LHS \le \sum_{s=t_1}^{t_2} \left[\prod_{v\in V} A_v^{(g_v^s)} \cdot \sum_{v\in V} n_v \Delta\tilde{N}_v g_v^{s-1}\right]$
  while $t'_1 < t_2$
    $LHS'_1 = B_1^{AC} \mu^{-1} \prod_{v\in V} {A'}_v^{\left(g_v^{t'_1-1}\right)}$
    Solve $t'_{2,1}$ from $LHS'_1 \le \sum_{s=t'_1}^{t'_{2,1}} \left[\prod_{v\in V} {A'}_v^{(g_v^s)} \cdot \sum_{v\in V} n_v \Delta\tilde{N}_v g_v^{s-1}\right]$
    if $t'_{2,1} < t_2$
      $T_{\text{main}} = T_{\text{main}} \cup \{t'_{2,1}\}$
      $t'_1 = t'_{2,1}$
    else
      $LHS'_2 = B_2^{AC} \mu^{-1} \prod_{v\in V} {A'}_v^{\left(g_v^{t'_1-1}\right)}$
      Solve $t'_{2,2}$ from $LHS'_2 \le \sum_{s=t'_1}^{t'_{2,2}} \left[\prod_{v\in V} {A'}_v^{(g_v^s)} \cdot \sum_{v\in V} n_v \Delta\tilde{N}_v g_v^{s-1}\right]$
      if $t'_{2,2} < t_2$
        $T_{\text{rep}} = T_{\text{rep}} \cup \{t'_{2,1}\}$
        $t'_1 = t'_{2,1}$
        $t_1 = t'_{2,1}$
      else
        $T_{\text{rep}} = T_{\text{rep}} \cup \{t'_{2,2}\}$
        $t'_1 = t'_{2,2}$
        $t_1 = t'_{2,2}$
      end
    end
  end
end
while $t'_1 < T$
  $LHS'_1 = B_1^{AC} \mu^{-1} \prod_{v\in V} {A'}_v^{\left(g_v^{t'_1-1}\right)}$
  Solve $t'_{2,1}$ from $LHS'_1 \le \sum_{s=t'_1}^{t'_{2,1}} \left[\prod_{v\in V} {A'}_v^{(g_v^s)} \cdot \sum_{v\in V} n_v \Delta\tilde{N}_v g_v^{s-1}\right]$
  if $t'_{2,1} < T$
    $T_{\text{main}} = T_{\text{main}} \cup \{t'_{2,1}\}$
  end
  $t'_1 = t'_{2,1}$
end

Algorithm 5.2: Compute the maintenance and repair times.

Chapter 6

Solving methods

We are going to solve the optimisation with an iterative algorithm. Each iterative algorithm has some similar computation steps of which some are specific for our problem, therefore we will describe this first. Afterwards we will discuss some iterative algorithms found in literature and look into their applicability. We will discuss genetic algorithms, grid search and local search. We want an algorithm which can give a 'good enough' solution in maximal one night running time. It turns out that genetic algorithms and grid search will cost too much computation time, therefore we will use local search.

6.1 Computation steps

The following computation steps are used in all the iterative algorithms which we will discuss in this chapter:

1. Updating the database
2. Assigning the lorries to the network
3. Get the data needed from the database
4. Decide speed limits for next iteration

The first three steps are nearly the same for the algorithms, the last step is very case specific. Therefore we will only discuss the first three steps in the next subsections.

We will use the program OmniTRANS for the network computations, this means that the first three steps will happen in OmniTRANS. The last step is made in Matlab. Therefore we need some communication between these two programs. The computations OmniTRANS makes are described in so called 'jobs', see **??**, which is a piece of code in OmniTRANS. The execution of the jobs will take place in OmniTRANS, but will be called from Matlab.

6.1.1 Updating the database

When we update the database, we use an UPDATE SQL-query. It is not possible to change multiple speed limits with the same SQL-query if the new value is not the same for the links. Therefore we need a lot of SQL queries to update the database. This means that a good time indication is difficult. The worst case scenario is to change all the speed limits in our case in different queries, this costs approximately 38 s. Note that grouping the roads and directions decreases this computation time a lot.

6.1.2 Assigning the lorries to the network

The assigning will be with an AON strategy, therefore we do not use different daytimes. In OmniTRANS three different daytimes are commonly used: morning rush hour, afternoon rush hour and the rest of the day. It does not influence the assignation if add up the lorries in these daytimes and assign them together. It will cost approximately 1.33 s to run a job of just assigning the lorries, if we call this job from Matlab.

6.1.3 Get the data needed from the database

This part consists of two jobs, the first job is for the database needs of the access roads and the other job selects the database needs for the remaining roads. Both jobs write the data in a text file so that Matlab can work with it. The data needed are

1. type number
2. link number
3. direction
4. # lanes
5. load
6. length

The first one is needed to get the width of the road, the second and third are for the identification of the link. The number of lanes is needed to scale the load on the link to all the lanes as described in Table 5.2. The load is the flow on the road, this value and the length are obviously needed.

6.2 Genetic algorithms

Genetic Algorithms are directly distracted from the evolution theory of Charles Darwin. Given a begin population, it will evaluate over time to a generation with other individuals by survival of the fittest. We will explain this evolution process and how it can be used by solving optimisation problems.

6.2.1 Definitions

The names used in GAs are the same as in the evolution theory. We start with a population, the population of one iteration is a generation. Every generation consists of individuals, which are possible solutions in our optimisation problem. Every individual has a chromosome, the abstract representation of the solution. On the chromosomes lay the genes, every gene is a parameter which has to be optimised. The different values of a gene are the alleles. [29]

6.2.2 The phases of a genetic algorithm

A genetic algorithm has, like evolution, different phases, these are shown in Figure 6.1.

The initialisation is a start population, on the population a selection will occur. This selection is made on the 'fitness' of a solution. The fitness is a measure of how good the solution is for our optimisation problem. After the selection the reproduction starts, this happens with crossing over and mutations. Now we have a new population. If in this population there is a good enough individual, we stop the algorithm. Otherwise we will make a new generation.

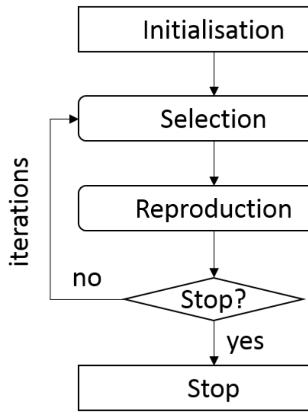


Figure 6.1: The phases of a genetic algorithm.

6.2.3 Genetic algorithms in maintenance costs optimisation

In a genetic algorithm a big population and a lot of iterations are needed to get to an optimal solution if the solution space is not nice shaped. With a big population, the probability to get stuck in a local optimum decreases, however, this will take a lot of computation time. For every individual we have to assign the speeds to the network, assign the lorries to the network and compute the costs. This will cost a lot of time per individual, namely a few seconds. With a population of a few hundred of individuals it will take too much computation time to find an optimal solution. Therefore we will not use this method.

6.3 Grid search

To get an idea of where the good solutions are in the solution space, Grid search (GS) can be used.

To keep the explanation and the pictures clear, we will describe grid search in a two-dimensional solution space. In the two dimensional case we have two variables x, y. The function we want to minimise is $f(x,y)$. In Figure 6.2 the principle is made clear. A grid will be laid down on the solution space. On every point (x_i, y_j) in the grid $f(x_i, y_j)$ will be computed. At four low points which form a box, the grid will be made finer. The whole process of looking for an optimum can be started over again on this grid, until a stopping criterium will be reached. A stopping criterium can for example be the number of iterations or the value of the function $f(x,y)$.

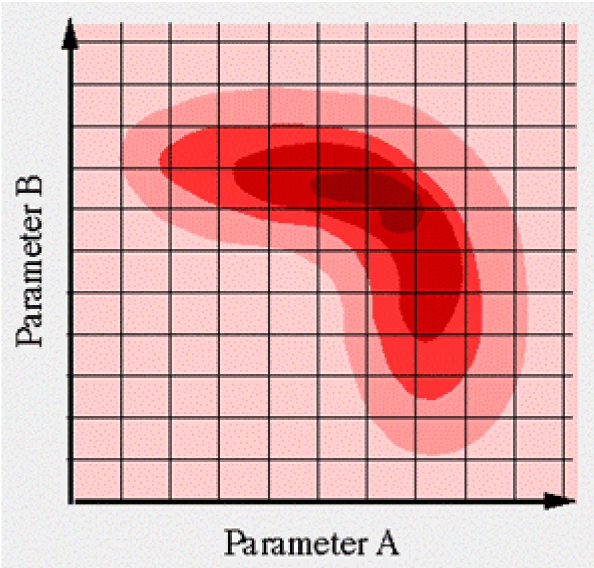


Figure 6.2: Grid search, the darker the color, the lower the function. [8]

The grid can be built up in several ways, like regular, random or according to an algorithm. At the same time, this brings a disadvantage, namely if the grid points are distributed in a wrong way, it can give a distorted view on how the function behaves, an example is given in Figure 6.3.

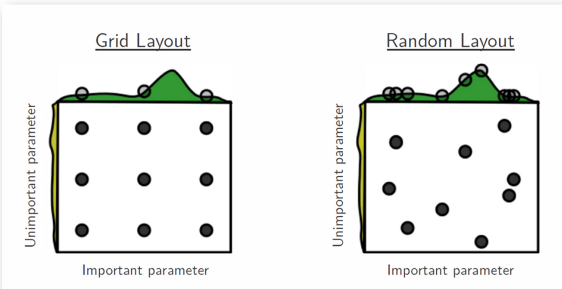


Figure 6.3: Regular grid search vs random search. [4]

It will be clear that you have to know the behaviour of the function before you can choose the grid points distributions well. This is in contrast with the reason grid search is used.

6.3.1 Grid search in maintenance costs optimisation

A good grid is needed before grid search is useful. This means that for every variable multiple values have to be tested. For example, we group all our road variables in nine variables with all five values in our grid, we have $5^9 = 1953125$ solutions which have to be tested. The computation of the costs of one solution depends on the number start-ups of OmniTRANS, the program which assigns the traffic to the network. It costs approximately three seconds to compute the costs for one solution, for 15 solutions it costs approximately six seconds. This means that a formula for the computation time in seconds is $2.79 + 0.21 \cdot \#\text{solutions}$. This means that this grid computation costs approximately $2.79 + 0.21 \cdot 1953125$ seconds, which is almost 5 days, which is too long.

This means that we can take less groups or less possible values in our grid, but this results in a grid from which you cannot conclude, because it is too rough. Therefore we will not use grid search in our research.

6.4 Local search

The local search algorithm is a steepest descent method. It is an iterative method to find an optimal solution. Local search starts with an initial solution in the solution space, then it will look at the neighbours and computes their 'fitness', how well do they apply to be optimal. From these neighbours it will choose one and it will look to the neighbours of this chosen one. This will repeat until the stop criterion is met, this is schematic shown in Figure 6.4.

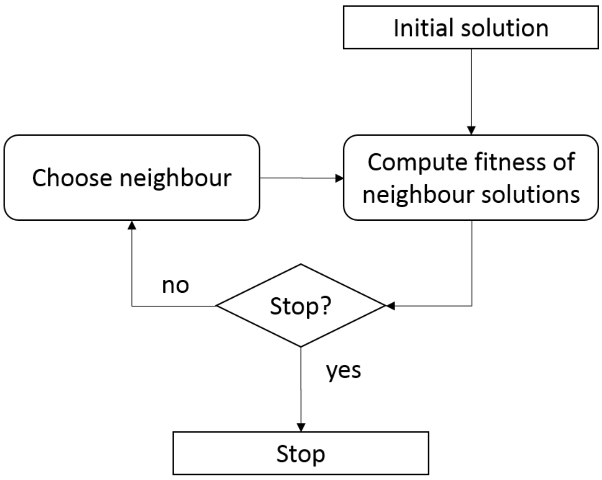


Figure 6.4: The local search principle.

In the following subsections the different parts of local search will be explained in more detail.

6.4.1 Fitness of a solution

The fitness of a solution is how optimal the solution is. Sometimes the optimal value is known and you search for the solution belonging to this optimal value, then the outcome of the objective function can directly be linked to a measure of how optimal the solution is. If the optimal value is not known the fitness can be the value of the objective function of that solution. It is even possible to set up a fitness function which is totally different from the objective function. Normally the higher the fitness, the better the solution, but in case the fitness is the value of the objective function a lower fitness could be better too.

6.4.2 Neighbours

A solution is given in the form $x = (x^1, \ldots, x^m)$, with for x^i the possible values $x^{i,1}, \ldots, x^{i,m_i}$, which have a certain order. A neighbour of x is $\tilde{x}$ with exactly one i for which $x^i \neq \tilde{x}^i$. It depends on the algorithm which value $\tilde{x}^i$ can take. In some algorithms $\tilde{x}^i$ can be every possible value of x^i, in other algorithms $\tilde{x}^i$ has to be $x^{i,j-1}$ or $x^{i,j+1}$ if $x^i = x^{i,j}$.

6.4.3 Stop criterion

Local search is an iterative algorithm, which means that a stop criterion is needed. A logical stop criterion is to stop when the fitness of all the neighbours is worse than the current solution. Other simple stop criteria are a maximal number of steps, a maximal running time or the fitness is above (or under) a certain level. The last one is useful if the optimal value is known.

6.4.4 Local search in maintenance costs optimisation

Local search can be very useful if exact algorithms cannot be used. A disadvantage is that the algorithm can get stuck in a local algorithm. To avoid this, the algorithm can be run multiple times with different starting points. This way you get more local optima. Another way not to converge to the closest optimum is to choose a neighbour with a certain probability. The computation of for example 18 neighbours will cost approximately seven seconds, in one night (eight hours) we can do 4114 iterations, which is a big enough number.

Therefore we will use local search in a specific case, see chapter 7.

Chapter 7

Test case: Enschede and Hengelo

Now we will use the model which is described in the previous chapters on a case. The case we use is about Enschede and Hengelo, two cities in the east of the Netherlands, and some surrounding area. A model of these two cities is available in OmniTRANS inside DAT.Mobility, see the network in Figure 7.1. The data in this model are a forecast for the year 2020. First we will look at how the situation is on the road nowadays and afterwards to some examples to get an idea of the traffic flow influences.

After we formed an idea of the influence of the traffic flow, we will do local search to find an

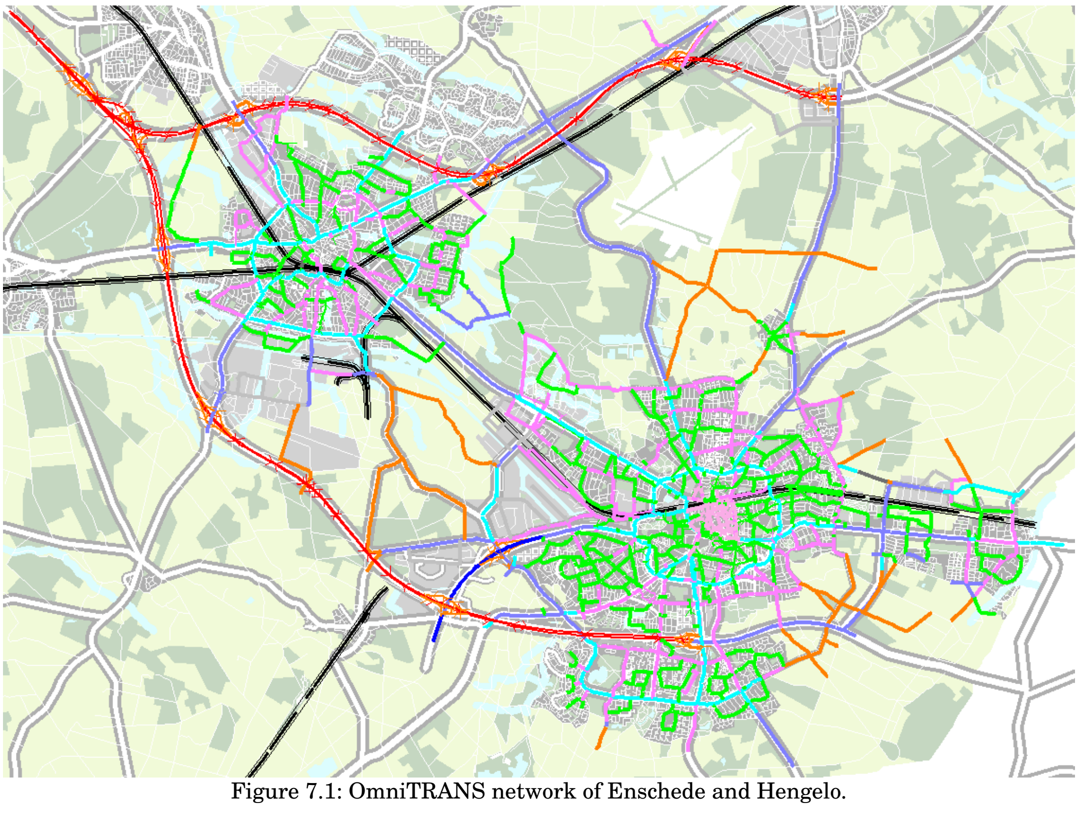


Figure 7.1: OmniTRANS network of Enschede and Hengelo.

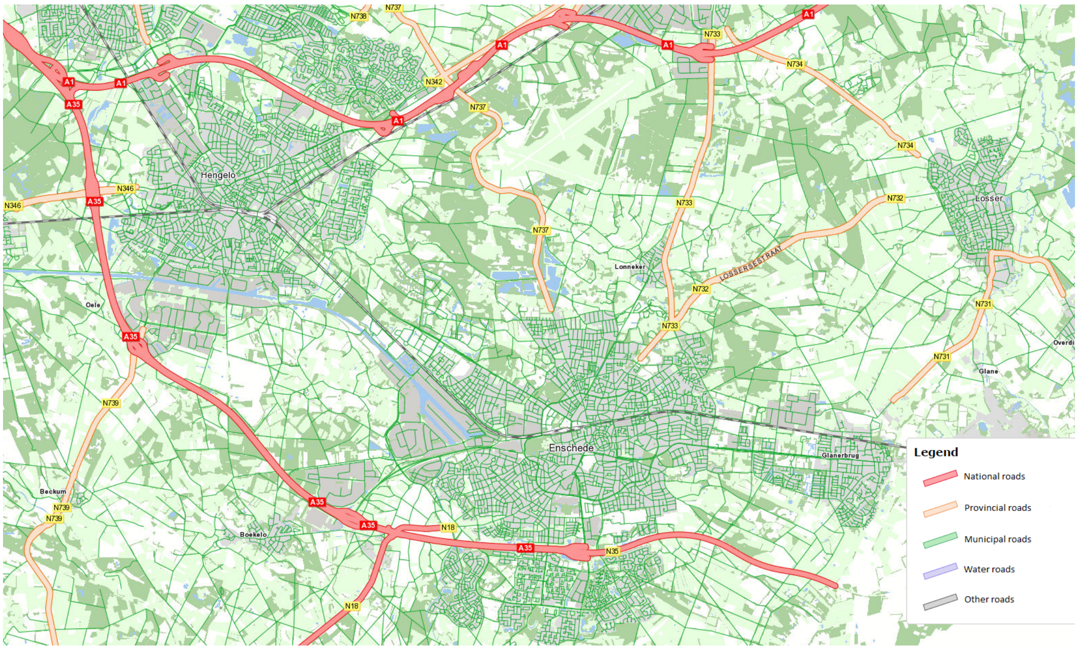


Figure 7.2: The road administrators in Enschede and Hengelo. [24]

optimal solution. Our first implementation has nine road groups (subsection 7.3.1). Hereby, we will look at the outcomes of the local search algorithm with three different fitness functions. First we want to minimise the average costs for all the paying parties, second we want to minimise the costs of the party who has to pay the most in comparison with the original costs and finally we will minimise the total costs when no party has more maintenance costs than in the original situation.

Afterwards we will further decrease the costs by dividing the roads into more than nine groups (subsection 7.3.4). This second implementation has 39 road groups. With this implementation we will minimise the total costs under the condition that none of the parties has more maintenance costs than in the original situation.

7.1 Original situation

First we will look at the original situation with which we can compare some examples of other speed limits. The original speed limits are given. With Equation (5.1) we can compute the total costs for this network, which is given in Table 7.1. In this table we make the division of costs per road authority: the municipalities (Enschede, Hengelo and a part of Oldenzaal and Dinkelland), the province and Rijkswaterstaat (RWS). The last one is the nationwide organisation which maintains motorways and most of the superhighways. The distribution of roads between this three parties is shown in Figure 7.2.

Part of the network	Costs	Total vehicle km
Municipality	€$2.2378 \cdot 10^8$	$7.3413 \cdot 10^4$
Province	€$8.4109 \cdot 10^7$	$2.7550 \cdot 10^4$
Nation	€$2.6880 \cdot 10^9$	$4.0864 \cdot 10^5$
Total	€$2.9960 \cdot 10^9$	$5.0960 \cdot 10^5$

Table 7.1: The costs for 30 years and vehicle kilometres made per day in the original network.

Part of the network	Costs	Total km
Municipality	€$3.8418 \cdot 10^8$	$1.3067 \cdot 10^5$
Province	€$8.6950 \cdot 10^7$	$2.9385 \cdot 10^4$
Nation	€$2.1638 \cdot 10^9$	$3.2091 \cdot 10^5$
Total	€$2.6350 \cdot 10^9$	$4.8762 \cdot 10^5$

Table 7.2: The costs in 30 years and vehicle kilometres made per day in the network if the speed limit is the same everywhere.

7.2 Examples

We can also look at some other speed limits on the network. We want to look if it is even possible to control the costs made for the three parties and how much we influence it with changing the speed limits. We will look at three examples. The first one is on every street the same speed. The second one is high speed limits (120 km/h) on roads outside the urban area and low speed limits (15 km/h) on roads in the urban area. The third one is the opposite of the second one. Afterwards we will discuss the results of the examples.

7.2.1 Example 1

In this example we take the same speed limits on every road. We take a speed limit of 80 km/h. The exact speed limit does not matter, because the distribution of the traffic depends on the ratio of speed limits in the network. The total costs and kilometres made for this network are given in Table 7.2. We can also make a visualisation in OmniTRANS which shows the differences in the traffic flows. This is shown in Figure 7.3.

If on every road the same speed limit applies, lorries will drive their shortest route by distance. This means that the total number of kilometres made is lower. This can be seen by comparing Table 7.1 and 7.2. The shortest routes taken place more in the urban areas, therefore we see the increase in kilometres made in the municipality and the province, while the number of kilometres on the motorways decreases. Likewise the costs for the municipality and the province increase, while the costs for the nation decrease. Also the total costs decrease. Although a decrease of the costs is what we want, we can wonder if it is acceptable to increase the costs for the municipality by 172%.

We can also wonder if the city is liveable and safe if every day for example 300 lorries will drive through a street in a residential area.

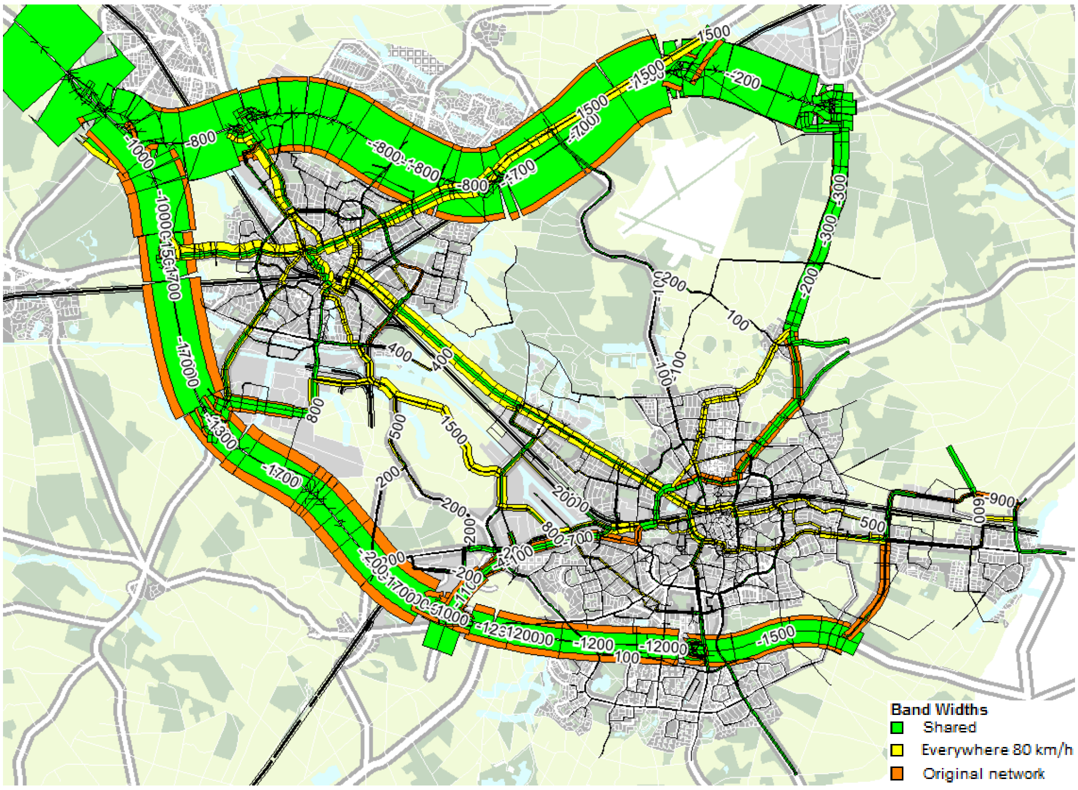


Figure 7.3: The differences in the traffic flow of the original flow and everywhere 80 km/h.

Part of the network	Costs	Total km
Municipality	€$2.1994 \cdot 10^8$	$6.8648 \cdot 10^4$
Province	€$7.1827 \cdot 10^7$	$2.2921 \cdot 10^4$
Nation	€$3.0230 \cdot 10^9$	$4.5613 \cdot 10^5$
Total	€$3.3147 \cdot 10^9$	$5.4770 \cdot 10^5$

Table 7.3: The costs in 30 years and vehicle kilometres made per day in the network when push the traffic outside the cities.

7.2.2 Example 2

In this example we will look what happens if we centre all the traffic on the roads outside the urban area. Therefore we set speed limits outside the urban area to 120 km/h and the other speed limits to 15 km/h. The costs are shown in Table 7.3 and the differences of the traffic flows are shown in Figure 7.4.

As we could expect, the costs for the municipality are lower and the costs for the nation are higher. This is because all the traffic is repressed from the city centre. The total costs are also higher, because the width of the carriageway is a larger than the width of an urban road.

7.2.3 Example 3

Because the motorways are that expensive, it would be likely that managing all the traffic on small streets causes the lowest maintenance costs. Therefore we will look at the opposite of

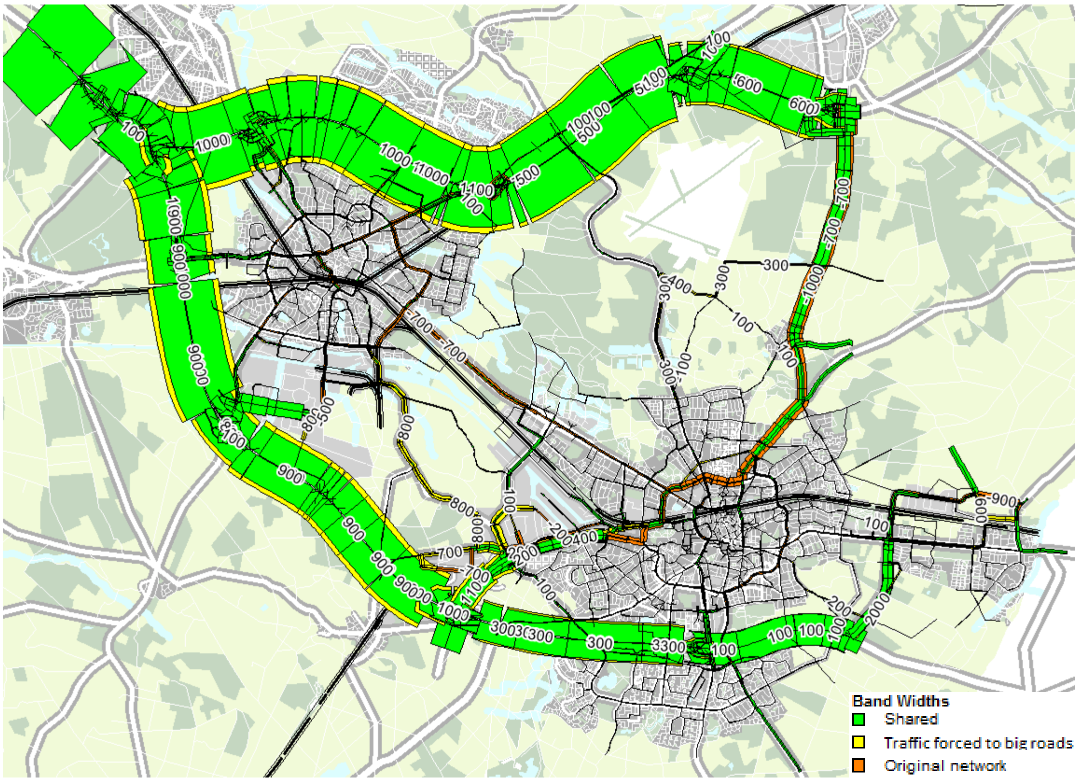


Figure 7.4: The differences in the traffic flow of the original flow and when we centre the traffic to big roads.

Part of the network	Costs	Total km
Municipality	€$1.0951 \cdot 10^9$	$3.8101 \cdot 10^5$
Province	€$2.2878 \cdot 10^8$	$7.7409 \cdot 10^4$
Nation	€$8.6247 \cdot 10^8$	$1.1713 \cdot 10^5$
Total	€$2.1863 \cdot 10^9$	$5.7555 \cdot 10^5$

Table 7.4: The costs per 30 years and vehicle kilometres made per day in the network if we centre the traffic to cities.

subsection 7.2.2, namely the roads in the urban area will be 120 km/h and the other roads will be 15 km/h. With this measures, we centre the traffic on urban roads. The outcome of this is shown in Table 7.4 and Figure 7.5.

Indeed the total costs are lower than in the original network and even lower than in subsection 7.2.1, but the costs for the municipality are approximately five times as high as in the original network.

7.2.4 Discussion of the examples

As we can see in the examples, the traffic and maintenance costs can be influenced by adjusting the speed limits. It costs a lot of effort to prevent the lorries from driving on the motorways. This may be caused by the lorries which just have to go through the network in stead of having their origin or destination somewhere in the network.

From a maintenance perspective, it is better to put the traffic on urban roads than on motorways.

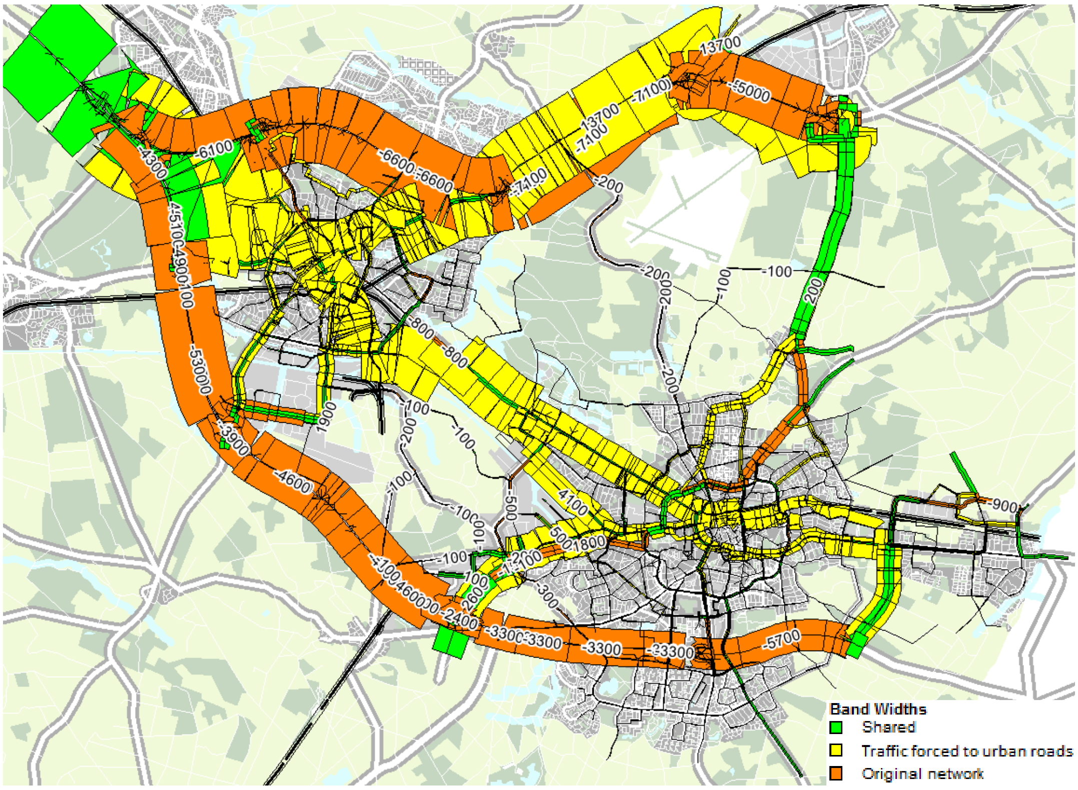


Figure 7.5: The differences in the traffic flow of the original flow and when we centre the traffic to urban roads.

This is caused by high costs for motorways because of their larger width than urban roads. This is not what we want from a viability perspective, therefore it may be a good idea to put limitations of the maximum speed limits in our network, this causes a solution which is executable in real live. Therefore we set the maximum speed in the network on 90 km/h, because lorries are limited on this speed. Outside the urban area, the minimal speed limit is 60 km/h and in the urban area the maximum speed limit is 60 km/h.

We will use local search to look for an optimal solution.

7.3 Local search

We will choose our 'best' measures by local search. In this context best is between quotes because local search will converge to an optimum which is not necessarily a global optimum. In this chapter we will describe the local search algorithms we will use.

7.3.1 First implementation

For the implementation of an algorithm we need to know several things, like the solution space and a stop criterion. In this subsection we will describe the parts needed for the implementation

7.3.1.1 Solution space

In our research the solution x is a vector with the maximum allowed speed on the roads. Because in this research there are two different vehicle types from which we will just look at the lorries, we have

$$x = \begin{bmatrix} x_{v_{\text{lorry}},r_1} \\ \vdots \\ x_{v_{\text{lorry}},r_{|R|}} \end{bmatrix}.$$

This solution space is too big to take the computation time low enough to run algorithms, therefore we will group roads according to their function (motorway, urbanised road, e.d.). In these cases, the x matrix has as many rows as groups. As said in subsection 7.2.4 we will have limited possible speed limits, to get usable results.

OmniTRANS divides the roads in types. In the model we have, these types are:

1. motorway 2 × 4 (130 km/h)
2. motorway 2 × 3 (130 km/h)
3. motorway 2 × 2 (130 km/h)
4. slip road
5. superhighway 2 × 2 (100 km/h)
6. superhighway 2 × 1 (100 km/h)
7. 80 km/h 2 × 2
8. 80 km/h 2 × 1 closed for bikes
9. 80 km/h 2 × 1 cycle path
10. 80 km/h 2 × 1 cycle lane
11. 60 km/h outside urban areas
12. arterial road 2 × 2 (50 km/h)
13. arterial road 2 × 1 (50 km/h)
14. collector road
15. local street (30 km/h)
16. industrial road
17. closed for lorries
18. ferry
19. rush hour lane
20. parking system
21. nutrition link
22. lorry nutrition link
23. cycle path
24. parking path
25. pedestrian path
26. public transport
27. public transport and bike
28. bus lane
29. undefined

In which the 2 × 4 means that there are 2 carriageways with both 4 lanes, 2 × 3, 2 × 2 and 2 × 1 are defined similarly. The speed indication behind the type is the current situation in the Netherlands for cars. The nutrition link is to put the vehicles on the network. The parking system and paths are for another project done with this model.

We will not use the road types ferry, parking system, (lorry) nutrition link, cycle path parking path and pedestrian path. We also neglect the public transport, because we can not influence their route. The other road types we group in nine groups:

1. motorway, rush hour lane, slip road
2. superhighway
3. 80 km/h
4. 60 km/h
5. arterial road
6. collector road
7. local street
8. closed for lorries
9. other (=industrial road and undefined)

We do not take all the possible velocities on every road type, because municipalities do not want vehicles driving 130 km/h in the urban area. Therefore we make the solution space smaller, which decreases the computation time. If the speed of a vehicle type is one, it means that the road is closed for that vehicle type, we do not make it zero because then it can happen that lorries cannot reach their destination. Lorries are limited on 90 km/h, therefore we take the maximum speed limit of 90 km/h. The possible speed limits for the road types are given in Table 7.5.

road group number	**road group name**	**possible speeds of** v_{lorry}
1	motorway	{90,80,70,60}
2	superhighway	{90,80,70,60}
3	80 km/h	{90,80,70,60}
4	60 km/h	{80,70,60,1}
5	arterial road	{60,50,40,30,20,1}
6	collector road	{60,50,40,30,20,1}
7	local street	{60,50,40,30,20,1}
8	closed for lorries	{80,70,60,50,40,30,20,1}
9	other	{80,70,60,50,40,30,20,1}

Table 7.5: The grid points of possible velocities for the first step of grid search.

7.3.1.2 Neighbours

A solution is a vector of speed limits, x, where $x^i \in \{90,80,70,60,50,40,30,20,1\}$. The speed limit of one means that the street is closed, it is not zero, because that may cause unreachable destinations. The maximum speed limit is 90 km/h because the lorries are limited on that speed. A neighbour of solution x is a solution $\tilde{x}$ with exactly on i for which $x^i \neq \tilde{x}^i$ and if $x^i = x^{i,j}$ then $\tilde{x}^i = x^{i,j-1} \vee \tilde{x}^i = x^{i,j+1}$.

7.3.1.3 Fitness

For the fitness function, we analyse the costs for nation, the province and the municipality separately, so we will use the following input:

1. costs for the nation (C_{RWS}),
2. costs for the province (C_{prov}),
3. costs for the municipality (C_{muni}).

The fitness is a function which compares the solution of the current situation with the solution computed. Because the nation, the province and the municipality are independent parties who have to pay, we assume that we can split the fitness function into independent sub fitness functions for the separate parties and sum this up with a certain weight. We take the nation, the province and the municipality as equal parties. This means that we take an average of their fitnesses as the total fitness function. Therefore the fitness of a solution is given by

$$f_{\text{av}}(C_{\text{muni}}, C_{\text{prov}}, C_{\text{RWS}}) = \frac{1}{3} f_{\text{muni}}(C_{\text{muni}}) + \frac{1}{3} f_{\text{prov}}(C_{\text{prov}}) + \frac{1}{3} f_{\text{RWS}}(C_{\text{RWS}}). \tag{7.1}$$

The road administrators want to make lower costs, equal costs are okay and they definitely do not want higher costs. So we make their fitness function positive if they save money and strong negative if they have to spend more money. We will use an exponential function for this.

Say that the current costs for a party are C_{cur}. If they make C_{cur} costs, it is okay and the fitness is zero. If they make zero costs their fitness is one. The shape of the function is a negative exponential function. This means we have for $i \in \{\text{muni}, \text{prov}, \text{RWS}\}$

$$f_i(C_i) = -a \cdot e^{\frac{C_i}{C_{\text{cur},i}} b} + c.$$

We now have three unknowns and we know two instances. Therefore we choose another one, if the current costs are halved, we set the fitness on $\frac{3}{4}$, because the parties are very happy. Now we have three costs-fitness pairs and we can determine the function. We have $(0,1)$, $(\frac{1}{2}C_{\text{cur}}, \frac{3}{4})$ and $(C_{\text{cur}}, 0)$, which makes $a = \frac{1}{8}$, $b = 2\ln(3)$ and $c = \frac{9}{8}$. This makes the fitness functions

$$f_i(C_i) = -\frac{1}{8} \cdot e^{\frac{2\ln(3)}{C_{\text{cur},i}} C_i} + \frac{9}{8} = -\frac{1}{8} \cdot 3^{\frac{2C_i}{C_{\text{cur},i}}} + \frac{9}{8} \qquad i \in \{\text{muni}, \text{prov}, \text{RWS}\}.$$

We want the speed limits to be a little bit according to the design of the type. We can fix this by adding this constraint to the fitness. We will not add these constraints, because we already will restrict the possible speed limits in such a way that this it is not possible to drive for example 130 km/h on an urban road.

We can also add the kilometres made in the fitness-function, but if we do this we optimise the experience of the drivers, which is not the intention of our research.

In Equation (7.1) we made a weighted average fitness function. With this function it is possible that the positive fitness of a party neutralises the negative fitness of another party. Therefore we introduce another fitness function

$$f_{\text{mm}}(C_{\text{muni}}, C_{\text{prov}}, C_{\text{RWS}}) = \min\Big(f_{\text{muni}}(C_{\text{muni}}), f_{\text{prov}}(C_{\text{prov}}), f_{\text{RWS}}(C_{\text{RWS}})\Big), \tag{7.2}$$

which just looks to the lowest fitness.

Another possible fitness is one with constraints. We want to minimise the costs, but under the constraint that neither the municipality nor the province nor the nation has more maintenance costs. We call this fitness function f_{con}, we take

$$f_{\text{con}}(C_{\text{muni}}, C_{\text{prov}}, C_{\text{RWS}}) = C_{\text{muni}} + C_{\text{prov}} + C_{\text{RWS}} + \tilde{C}_{\text{muni}} + \tilde{C}_{\text{prov}} + \tilde{C}_{\text{RWS}}, \tag{7.3}$$

where

$$\tilde{C}_i = \begin{cases} 0 & C_{\text{curr},i} - C_i \leq 0 \\ \frac{C_i - C_{\text{curr},i}}{C_{\text{curr},i}} \cdot a & \text{otherwise.} \end{cases}$$

The value of a should be high enough to make sure that this constraint is not violated. Because the order of magnitude of the total costs is 10^9 and we really do not want the parties to have more maintenance costs, we take $a = 10^{11}$.

Other than ordinary fitness functions, for this fitness applies that the lower the better.

We will look at all these three fitnesses in our research.

7.3.1.4 Next solution

Now we know the neighbours and the fitness function, we can decide how we will choose a neighbour. We use two methods to determine this.

The first method is a decision made by a probability distribution. We take a linear one. Say there are N neighbours, with fitnesses $f_1 \leq \cdots \leq f_N$ We choose them respectively with probability $\delta, 2\delta, \ldots, N\delta$, with a total sum of one. This means that

$$\begin{aligned} 1 &= \delta + 2\delta + \cdots + N\delta \\ &= \delta \sum_{i=1}^{N} i \\ &= \delta \frac{1}{2} N(N+1) \\ \Rightarrow \delta &= \frac{2}{N(N+1)}. \end{aligned}$$

With this method we do not get stuck in a local optimum, because there is always a probability to go to a less optimal solution and go from there to a more optimal local extreme value. The disadvantage of this method is that it is possible to never reach an optimum.

With the second method we will choose the neighbour with the best fitness, this is called 'hill climbing'. An advantage of this algorithm is that you will reach an optimum, the disadvantage is that it you will get stuck in a local optimum, whether you do not know if it is also a global optimum or not.

We will use a combination of both methods, first we use the method with probabilities and after some time, we use the maximum found as a start point for the second method. For the first method we will use different start points.

Also other decision methods are possible, a good suggestion is a decision method to make a probability distribution which depends on the value of the fitness and not only on the order of which solution has a better fitness.

7.3.1.5 Start points and stop criterion

As start points we take the current situation, which is approximately

$$x_1 = (80,80,80,60,50,40,30,1,50).$$

We said approximately because on some roads the speed limits are a little bit different than their type prescribes. Also the speed limit on the roads of type ‘other’ will in practice not be the same everywhere. The reason we set the speed limit of the collector roads on 40 km/h is because in practice it is sometimes 50 km/h and sometimes 30 km/h, and therefore we take the average. Our second start solution is when we take all the speed limits the same, we have

$$x_2 = (60,60,60,60,60,60,60,60,60).$$

Then we have two randomly chosen start solutions which are

$$x_3 = (90,60,60,60,1,40,20,80,60)$$
$$x_4 = (80,90,90,1,50,1,60,60,80).$$

We will stop the algorithm if the maximum number of iterations is met or, with the hill climbing, if we are stuck in a local optimum.

7.3.2 Solutions

In this sections we will look what the different types of fitnesses give for optimal solutions and we will discuss if this is what we want in practice.

7.3.2.1 Average fitness

For the first solution, x_1, we get after some iterations of the linear neighbours and afterwards hill climbing, the maximum solution is $\tilde{x}_{1,\text{av}} = (90,60,60,60,30,20,20,30,40)$. The fitness of $\tilde{x}_{1,\text{av}}$ is

$$f_{\text{av}}(\tilde{x}_{1,\text{av}}) = \frac{f_{\text{muni}}(\tilde{x}_{1,\text{av}}) + f_{\text{prov}}(\tilde{x}_{1,\text{av}}) + f_{\text{RWS}}(\tilde{x}_{1,\text{av}})}{3} = \frac{0.0544 + 0.2940 - 0.0978}{3} = 0.08353.$$

The solutions x_2, x_3 and x_4 stuck after iterations of linear neighbours and hill climbing afterwards to the same local optimum, namely $\tilde{x}_{2,\text{av}} = (90,60,60,70,30,30,20,\{50,60,80\},40)$. Hereby if x^i

is a set in stead of a number, x^i can take all these values to reach the same optimal costs. The fitness of this solution is

$$f_{\text{av}}(\tilde{x}_{2,\text{av}}) = \frac{f_{\text{muni}}(\tilde{x}_{2,\text{av}}) + f_{\text{prov}}(\tilde{x}_{2,\text{av}}) + f_{\text{RWS}}(\tilde{x}_{2,\text{av}})}{3} = \frac{0.0201 + 0.3051 - 0.0643}{3} = 0.08697. \tag{7.4}$$

To compute the costs from the fitness (given by the algorithm), we use

$$\begin{aligned} f_i &= -\frac{1}{8} \cdot 3^{\frac{2C_i}{C_{\text{cur},i}}} + 1\frac{1}{8} & i &\in \{\text{muni}, \text{prov}, \text{RWS}\} \\ \Rightarrow\ C_i &= \frac{C_{\text{cur},i}}{2} \log_3(9 - 8f_i) & i &\in \{\text{muni}, \text{prov}, \text{RWS}\} \end{aligned}$$

Thus the cost vector, consisting of the costs made by the municipality, province and nation respectively, is

$$C(\tilde{x}_{2,\text{av}}) = (€2.21944 \cdot 10^8, €7.1999 \cdot 10^7, €2.756 \cdot 10^9),$$

therefore the profit for $\tilde{x}_{2,\text{av}}$ is given by

$$\begin{aligned} C_{\text{original}} - C(\tilde{x}_{2,\text{mm}}) &= (€2.2378 \cdot 10^8, €8.4109 \cdot 10^7, €2.6880 \cdot 10^9) \\ &\quad - (€2.21944 \cdot 10^8, €7.1999 \cdot 10^7, €2.756 \cdot 10^9) \\ &= (€1.836 \cdot 10^6, €1.211 \cdot 10^7, -€6.8 \cdot 10^7). \end{aligned} \tag{7.5}$$

The best solutions from this is Equation (7.4). But we see that the fitness for the nation is negative, which means that it has more maintenance costs than in the original solution. In the profit vector we see that the nation has $€6.8 \cdot 10^7$ more maintenance costs than before. The total profits are $€1.836 \cdot 10^6 + €1.211 \cdot 10^7 - €6.8 \cdot 10^7 = -€5.405 \cdot 10^7$ which is less than zero and therefore a loss.

We do not want to lose money, so the average strategy is not a good strategy to minimise the maintenance costs. We can also see that the averaging of the fitness for the municipality, the province and RWS turns out to be bad for the RWS. Therefore we will look at maximising the minimal fitness, in hope that this will bring a saving of costs for all the three parties. If we get a positive fitness with this method, we know that all the three parties have less maintenance costs.

7.3.2.2 Max-min fitness

Now we will use a max-min-strategy. If we get the minimal fitness positive, we know that all the road administrators have a positive fitness and thus profit from the new situation.

We take the same start points as with the average strategy and again we first use the linear neighbour strategy and afterwards hill climbing.

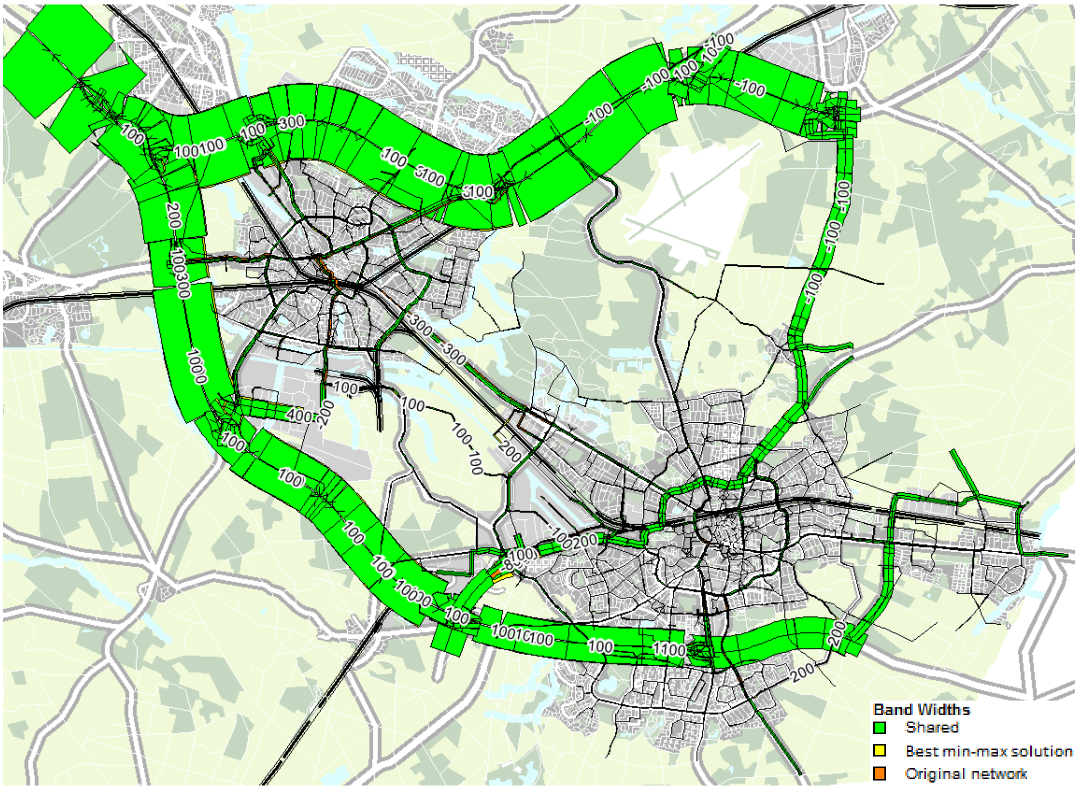


Figure 7.6: The distribution of the traffic of Equation (7.6) in comparison with the current situation.

From x_1 we get the solution $\tilde{x}_{1,\mathrm{mm}} = (90, 80, 70, 60, 40, 30, 20, 1, 50)$ with fitness $f_{\mathrm{mm}}(\tilde{x}_{1,\mathrm{mm}}) = 0.0010$. This means that all parties have a profit, the average fitness is

$$
\begin{aligned}
f_{\mathrm{av}}(\tilde{x}_{1,\mathrm{mm}}) &= \frac{f_{\mathrm{muni}}(\tilde{x}_{1,\mathrm{mm}}) + f_{\mathrm{prov}}(\tilde{x}_{1,\mathrm{mm}}) + f_{\mathrm{RWS}}(\tilde{x}_{1,\mathrm{mm}})}{3} \\
&= \frac{0.0062 + 0.0063 + 0.0010}{3} = 0.0045.
\end{aligned}
$$

From x_2 we get the solution $\tilde{x}_{2,\mathrm{mm}} = (90, 60, 60, 60, 40, 40, 30, 70, 50)$ with fitness $f_{\mathrm{mm}}(\tilde{x}_{2,\mathrm{mm}}) = 0.0380$. This means that all parties have a profit, the average fitness is

$$
\begin{aligned}
f_{\mathrm{av}}(\tilde{x}_{2,\mathrm{mm}}) &= \frac{f_{\mathrm{RWS}}(\tilde{x}_{2,\mathrm{mm}}) + f_{\mathrm{prov}}(\tilde{x}_{2,\mathrm{mm}}) + f_{\mathrm{muni}}(\tilde{x}_{2,\mathrm{mm}})}{3} \\
&= \frac{0.0310 + 0.0559 + 0.0270}{3} = 0.0380. \qquad (7.6)
\end{aligned}
$$

From x_3 and x_4 we get solutions with negative min-max-fitness. Respectively -0.0282 and -0.0374, which means that not all parties have profit and this solution is not useful.

The solution with the highest minimal fitness is also the one with the highest average fitness with the max-min strategy. This is $\tilde{x}_{2,\mathrm{mm}}$. The distribution of the traffic is shown in Figure 7.6.

The cost vector, consisting of the costs made by the municipality, province and nation respectively, is

$$C(\tilde{x}_{2,\text{mm}}) = (€2.20934 \cdot 10^8, €8.2158 \cdot 10^7, €2.65828 \cdot 10^9), \tag{7.7}$$

therefore the profit for $\tilde{x}_{2,\text{mm}}$ is given by

$$\begin{aligned} C_{\text{original}} - C(\tilde{x}_{2,\text{mm}}) = (€2.2378 \cdot 10^8, €8.4109 \cdot 10^7, €2.6880 \cdot 10^9) \\ - (€2.20934 \cdot 10^8, €8.2158 \cdot 10^7, €2.65828 \cdot 10^9) \\ = (€2.846 \cdot 10^6, €1.951 \cdot 10^6, €2.972 \cdot 10^7). \end{aligned}$$

This means that the total profit made is $€2.846 \cdot 10^6 + €1.951 \cdot 10^6 + €2.972 \cdot 10^7 = €3.451 \cdot 10^7$.

We want to minimise the costs made in the network, but under the condition that the paying parties do not have a lot more maintenance costs than they do right now. Therefore we will use a fitness function with constraints.

7.3.2.3 Fitness with constraints

Because we want to minimise the total costs, but no loss for all the parties, we use a fitness with constraints. The disadvantage of this method is that it is so unattractive to let some party pay more than in the original situation that it is more difficult to escape a local minimum.

We take the same start points as with the average and max-min strategy and again we first use the linear neighbour strategy and afterwards hill climbing.

From x_1 we get the solution $\tilde{x}_{1,\text{con}} = (90, 60, 60, 60, 40, 50, 30, 40, 50)$ with fitness $f_{\text{con}}(\tilde{x}_{1,\text{con}}) = 2.9481 \cdot 10^9$. The cost vector is

$$C(\tilde{x}_{1,\text{con}}) = \left(€2.2363 \cdot 10^8, €8.2098 \cdot 10^7, €2.6423 \cdot 10^9\right).$$

The profit per party is

$$\begin{aligned} C_{\text{original}} - C(\tilde{x}_{1,\text{con}}) = \left(€2.2378 \cdot 10^8, €8.4109 \cdot 10^7, €2.6880 \cdot 10^9\right) \\ - \left(€2.2363 \cdot 10^8, €8.2098 \cdot 10^7, €2.6423 \cdot 10^9\right) \\ = \left(€1.5000 \cdot 10^5, €2.011 \cdot 10^6, €4.57 \cdot 10^7\right). \end{aligned}$$

All the indices of the profit vector are positive, which means that all the parties have profit, the total profit made is $€1.5000 \cdot 10^5 + €2.011 \cdot 10^6 + €4.57 \cdot 10^7 = €4.7861 \cdot 10^7$.

From x_2 we get the solution $\tilde{x}_{2,\text{con}} = (90, 80, 70, 60, 40, 30, 20, 70, 50)$ with fitness $f_{\text{con}}(\tilde{x}_{2,\text{con}}) = 2.9941 \cdot 10^9$. The costs for each party separately are

$$C(\tilde{x}_{2,\text{con}}) = \left(€2.2322 \cdot 10^8, €8.3894 \cdot 10^7, €2.6870 \cdot 10^9\right).$$

This means that the profit per party is

$$\begin{aligned} C_{\text{original}} - C(\tilde{x}_{2,\text{con}}) &= \left(€2.2378\cdot 10^8, €8.4109\cdot 10^7, €2.6880\cdot 10^9\right) \\ &\quad - \left(€2.2322\cdot 10^8, €8.3894\cdot 10^7, €2.6870\cdot 10^9\right) \\ &= \left(€5.6000\cdot 10^5, €2.1500\cdot 10^5, €1.0000\cdot 10^6\right). \end{aligned}$$

All the indices of the profit vector are positive, which means that all the parties have profit, the total profit made is $€5.6000\cdot 10^5 + €2.1500\cdot 10^5 + €1.0000\cdot 10^6 = €1.1661\cdot 10^6$.

From x_3 and x_4 we get the same solution $\tilde{x}_{3,\text{con}} = (90, 60, 70, 60, 40, 40, 30, \{20, 60\}, 50)$ with fitness $f_{\text{con}}(\tilde{x}_{3,\text{con}}) = 2.9542\cdot 10^9$.The costs for each party separately are

$$C(\tilde{x}_{3,\text{con}}) = \left(€2.2326\cdot 10^8, €8.3228\cdot 10^7, €2.6477\cdot 10^9\right).$$

This means that the profit per party is

$$\begin{aligned} C_{\text{original}} - C(\tilde{x}_{3,\text{con}}) &= \left(€2.2378\cdot 10^8, €8.4109\cdot 10^7, €2.6880\cdot 10^9\right) \\ &\quad - \left(€2.2326\cdot 10^8, €8.3228\cdot 10^7, €2.6477\cdot 10^9\right) \\ &= \left(€5.2000\cdot 10^5, €8.8100\cdot 10^5, €4.0300\cdot 10^7\right). \end{aligned}$$

All the indices of the profit vector are positive, which means that all the parties have profit, the total profit made is $€5.2000\cdot 10^5 + €8.8100\cdot 10^5 + €4.0300\cdot 10^7 = €4.1701\cdot 10^7$.

The best solution is $\tilde{x}_{1,\text{con}}$, in Figure 7.7 we see the difference between the original traffic and that of $\tilde{x}_{1,\text{con}}$. It may be interesting to look separately at the assigning of internal and external traffic. Internal traffic has its origin and destination inside the network, while external traffic has at least one of them outside the network. The comparison between the internal traffic of the original and the traffic of $\tilde{x}_{1,\text{con}}$ is shown in Figure 7.8. For clearness, the bandwidth in this picture is 10 times as high as in other pictures. The biggest difference we can see is that less internal traffic is going over the Hengelose-/Enschedesestraat, it takes an outside route instead.

The comparison of external traffic is shown in Figure 7.9. We can see that this traffic is more or less the same. Only on small parts in cities it takes another route for a short time.

7.3.3 Discussion

As we can see, we can reach a profit of €41.7 million for all the parties together, this was done by a fitness function with constraints. The best result we get is

$$\tilde{x}_{1,\text{con}} = (90, 60, 60, 60, 40, 50, 30, 40, 50). \tag{7.8}$$

We saw that the internal traffic in particular drives different routes, the external traffic still uses the motorway and other big roads.

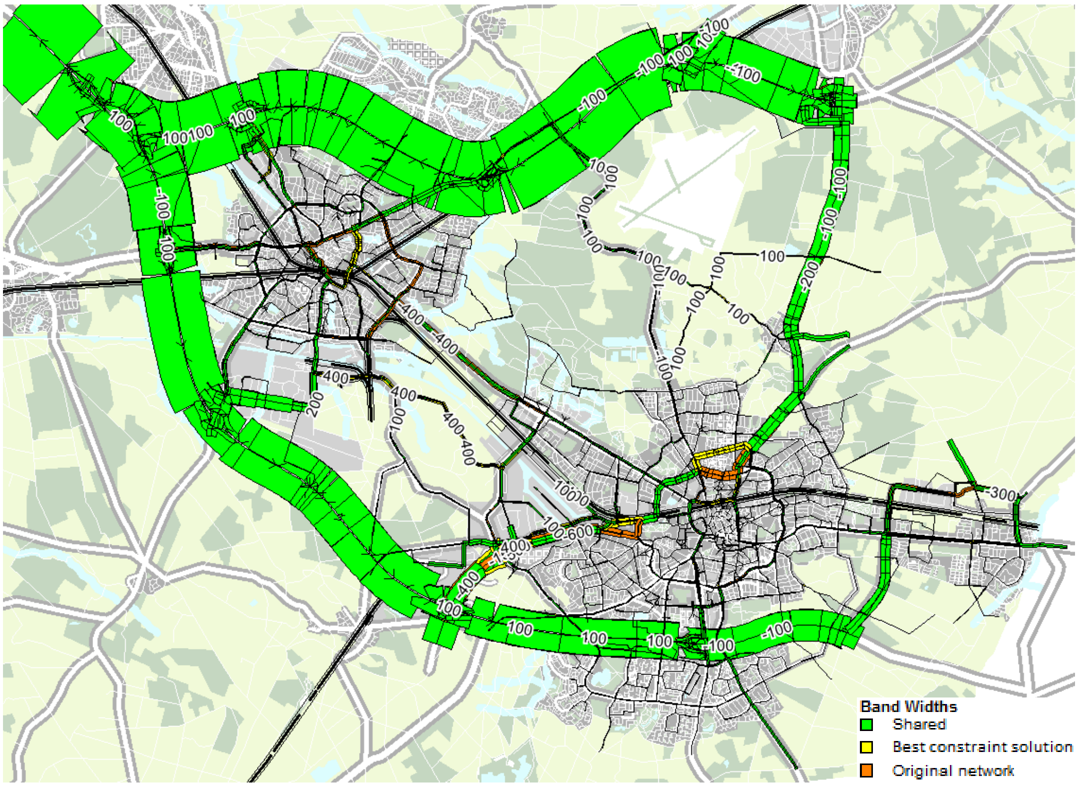


Figure 7.7: The distribution of the traffic of $\tilde{x}_{1,\text{con}}$ in comparison with the current situation.

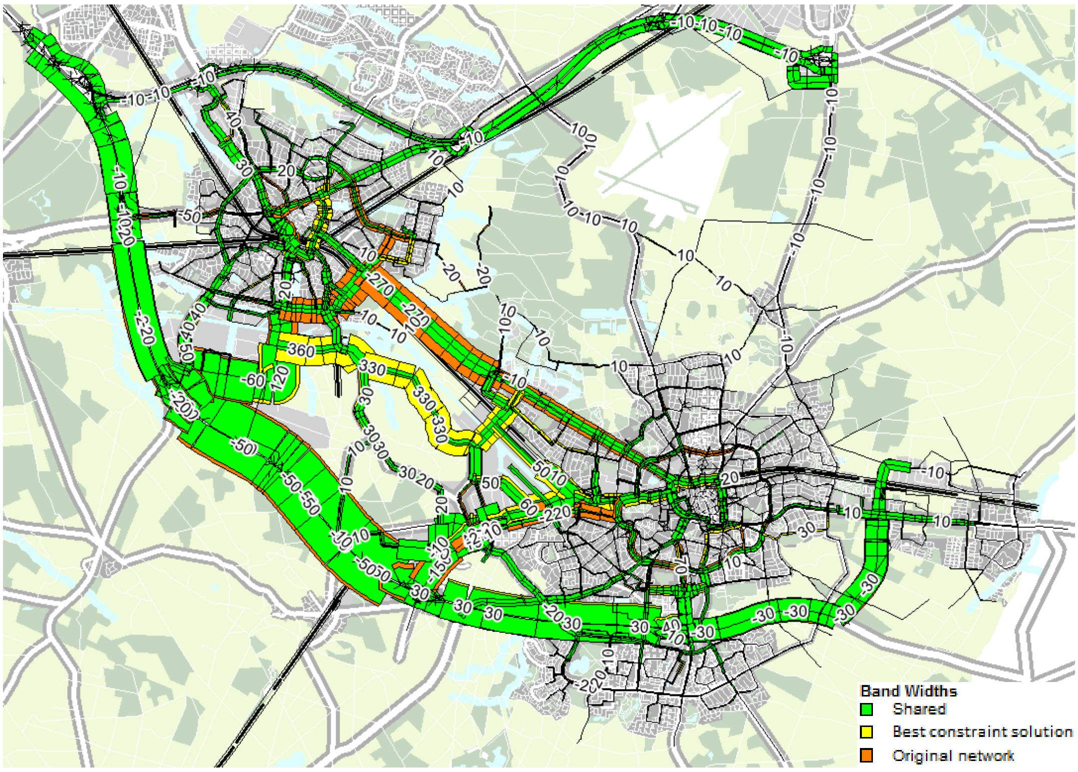


Figure 7.8: The distribution of the internal traffic of $\tilde{x}_{1,\text{con}}$ in comparison with the current situation. With the bandwidth 10 times as high as in other pictures.

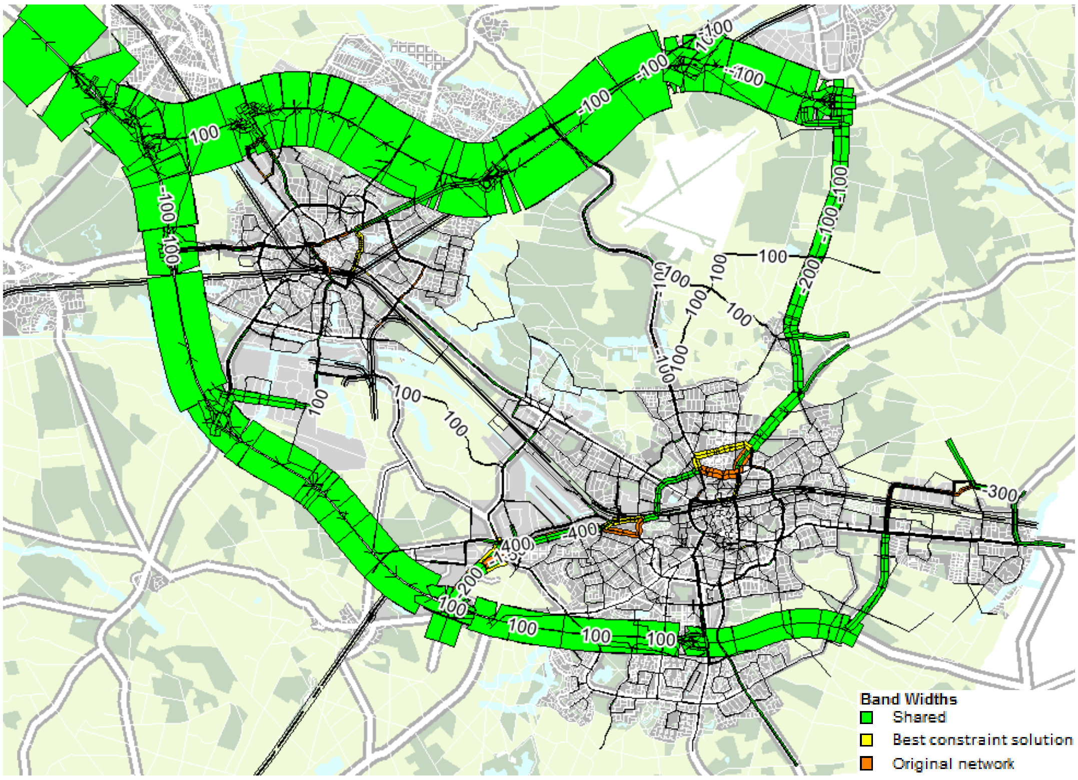


Figure 7.9: The distribution of the external traffic of $\tilde{x}_{1,\mathrm{con}}$ in comparison with the current situation.

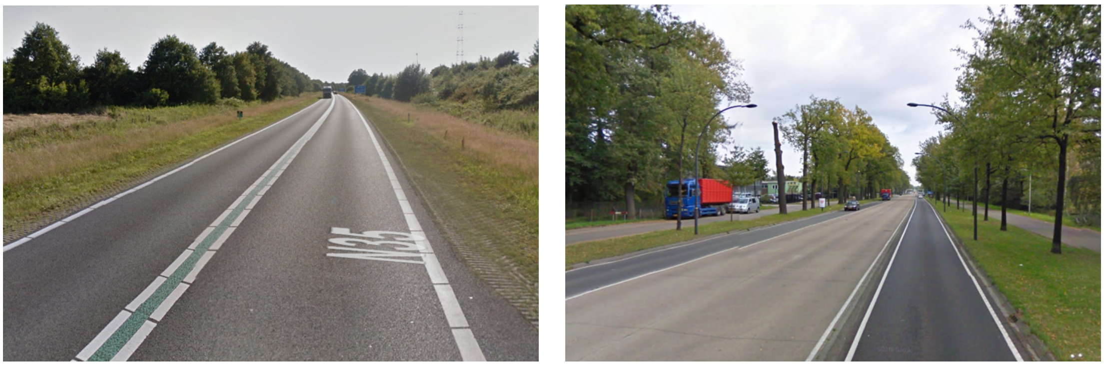


Figure 7.10: The N35 and the Gronausestraat, both are modelled with type 80 km/h road but they are respectively a superhighway and an arterial road. [12]

The result $\tilde{x}_{1,\mathrm{con}}$ is not usable for every road network in the Netherlands, for other cities another solution may result in more profit than this one. Also should be noted that for example the speed limit 90 km/h for the motorway is for roads that have type motorway in the OmniTRANS model. Not every road has in OmniTRANS the type of its function. For example the N35 and a part of the Gronausestraat are modelled with type 80 km/h road, while they are respectively a superhighway and an arterial road. More profit can maybe reached by taking a less restrictive division of road types, such that some arterial roads and superhighways are not grouped together any more. Also the probability of choosing a certain neighbour can be improved. We will make new road groups and a improved decision rule in the next section.

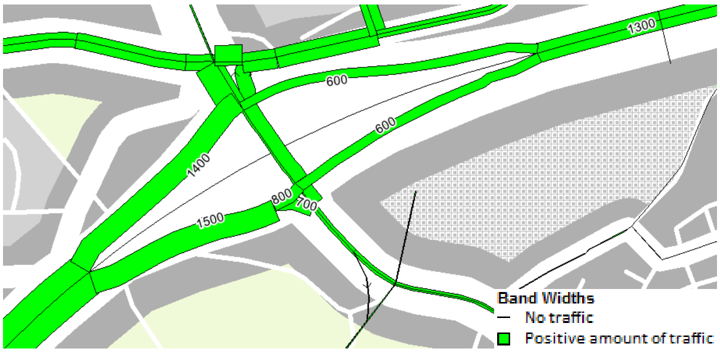


Figure 7.11: The assigning of traffic on a superhighway and his slip roads with a higher speed limit.

Also with another (good) grouping rare assignments like in Figure 7.11, also will not occur. This situation happens because the speed limit on the superhighway is 60 km/h and the speed limit on the slip road is 90 km/h, therefore all the lorries go off and on the superhighway because that is quicker. If we group the slip road together with the motorway or superhighway with which it is connected, this situations will not occur.

7.3.4 Second implementation

With the linear probability distribution of choosing a new neighbour, we do not actually use the value of the fitness-function. Therefore we will define another method to select the new neighbour and use both methods. We will also make our solution space bigger by making more road groups. The fitness will be computed according to the constraint fitness, because we still want to minimise the costs without letting a party pay more than in the current situation.

7.3.4.1 Solution space

The solution space in this implementation is bigger than in the first one. Here we have in total 39 groups, these groups are the motorways, the highways, the ring roads, the urban veins and some rest groups. Note that every urban arterial and such is an apart group. The groups are shown in Table 7.6.

In practice we have 38 groups because the group of remaining 30 km/h roads is empty. This means that a solution vector has size 38. The order of the solution is given in Table 7.6.

7.3.4.2 Next solution

We will choose the new solution according to Algorithm 7.1 or the linear distribution. After this algorithm we do hill climbing. As said before an algorithm according to the fitness value could be better, but in our case we have fitnesses which lie very close to each other. The simple inverse decision rule might not converge to the optimal solution. Nevertheless we will use an inverse decision rule wich is given in Algorithm 7.1.

motorway	50 km/h roads Enschede	50 km/h roads Hengelo
A1	E ring road	H ring road
A35	E inner ring road	H inner ring road
	E outer ring road	H Bornsestraat
superhighway	E Boulevard and Gronausestraat	H Deldenersestraat
N35	E Deurningerstraat	H Deurningerstraat
N342	E Haaksbergerstraat	H Enschedesestraat
Westerval	E Hengelosestraat	H Oelerweg
	E Kuipersdijk and Knalhutteweg	H Oldenzaalsestraat
80 km/h	E Oldenzaalsestraat	H remaining 50 km/h roads
E Oldenzaalsestraat	E Oostweg	
E Vliegveldstraat and Weerseloseweg	E Westerval	**30 km/h**
H Bornsestraat	E Zuiderval	E 30 km/h roads
H Haaksbergerstraat	E remaining 50 km/h roads	H 30 km/h roads
Enschedese-/Hengelosestraat		remaining 30 km/h roads
remaining 80 km/h roads		
		other
60 km/h		remaining 50 km/h roads
remaining 60 km/h roads		remaining roads

Table 7.6: The 39 groups for local search. The E in front of a name means that this street lies in Enschede and an H means that the street lies in Hengelo.

$ns = \{\text{neighbours}\}$
$N = |ns| = \#\text{neighbours}$
$C = (C_1, \ldots, C_N) = \text{cost vector neighbours}$
$iC = \left(\frac{1}{C_1}, \ldots, \frac{1}{C_N}\right)$
$ciC = \text{cumulativeSum}(iC)$
$niC = \text{normalise}(ciC)$
$r = \text{random}(\text{uniform}, (0, 1])$
$i = \text{last index of } niC \text{ wherefore } r \leq niC_i$
$\text{new solution} = ns_i$

Algorithm 7.1: Inverse decision rule of choosing a new neighbour.

7.3.5 Solutions

In total we take five different start solutions and we will use the inverse and linear decision rule for a new neighbour. Afterwards, we do hill climbing. In order not to mix-up the solutions in subsection 7.3.2 and the solutions in this section, we start counting with five.

Our first start solution is everywhere the same speed, namely 60 km/h, the second start solution is the best solution we found in subsection 7.3.2, the third, fourth and fifth are random. We have

$$
\begin{aligned}
x_5 &= (60,\dots,60),\\
x_6 &= (90,90,60,60,60,60,60,60,60,60,60,60,50,\dots,50,30,30,50,50)\\
x_7 &= (70,80,60,70,60,90,90,60,60,60,80,80,50,1,20,50,20,20,40,40,40,50,50,20,50,20,\\
&\qquad 20,1,1,20,50,20,60,30,60,50,30,70)\\
x_8 &= (70,90,60,70,60,80,70,60,70,80,60,80,40,50,1,40,30,30,20,60,30,50,60,20,60,60,\\
&\qquad 1,40,1,50,30,1,60,50,60,60,1,30)\\
x_9 &= (70,90,60,70,90,90,90,60,90,90,90,80,20,1,20,60,1,1,60,40,30,30,50,20,30,30,50,\\
&\qquad 30,40,50,20,1,20,60,40,1,60,60)
\end{aligned}
$$

There is a little side mark with solution x_6, in subsection 7.3.2 we grouped together on types which means that we divided the arterial roads and the collector roads. In practice both road types are most of the time 50 km/h, therefore we have not looked at this types but grouped on the function of the roads in the city. Therefore the arterial roads and collector roads are mixed in the groups in this subsection. In Equation (7.8) we see that the best speed limits differ for the arterial and collector roads. We have chosen 50 km/h as speed limit, because this is the state of practice in the original situation.

The optima we get from this start solutions are

$$
\begin{aligned}
\tilde{x}_{5,\text{inv}} = (&90,90,90,60,60,70,60,60,60,60,60,60,50,40,60,60,50,60,50,60,60,60,40,60,\\
&60,50,50,50,60,60,50,40,40,50,40,40,\{1,20,30,40,50\},70),\\
\tilde{x}_{5,\text{lin}} = (&80,90,80,70,60,70,60,80,70,60,70,60,50,40,60,60,50,50,50,60,50,50,1,60,\\
&60,50,50,50,60,60,40,60,30,50,30,50,\{1,20,30,40,50\},70),\\
\tilde{x}_{6,\text{inv}} = (&90,90,60,60,60,70,60,60,60,60,60,60,50,50,60,50,50,60,60,60,50,50,\\
&\{1,20,30,40,50\},50,60,50,50,50,20,30,50,40,30,50,30,30,50,50),\\
\tilde{x}_{6,\text{lin}} = (&80,90,80,60,60,70,70,60,60,60,60,60,40,50,60,60,50,50,40,50,40,30,1,60,\\
&40,60,60,50,60,30,50,50,30,50,20,50,50,80),\\
\tilde{x}_{7,\text{inv}} = (&90,90,60,70,60,70,70,60,60,60,90,60,50,30,30,60,1,50,60,40,40,50,1,50,\\
&60,30,20,30,60,60,60,50,60,40,30,50,\{1,20,30,40,50\},80),\\
\tilde{x}_{7,\text{lin}} = (&90,90,70,60,60,70,70,60,60,60,60,60,40,40,50,60,50,50,50,40,40,40,1,50,\\
&40,50,40,20,40,60,60,40,60,40,30,50,\{1,20,30,40,50\},60),\\
\tilde{x}_{8,\text{inv}} = (&90,90,70,70,60,70,70,60,70,70,70,60,50,40,50,50,30,60,50,50,50,40,1,30,\\
&60,40,30,50,40,60,50,50,50,50,20,40,\{1,20,30,40,50\},50),\\
\tilde{x}_{8,\text{lin}} = (&90,90,80,60,60,90,60,60,70,90,70,70,30,50,40,50,50,50,60,50,40,50,60,60,\\
&40,30,20,60,60,60,50,40,60,40,40,50,\{1,20,30,40\},40),\\
\tilde{x}_{9,\text{inv}} = (&90,80,90,80,70,70,80,30,90,90,80,60,20,20,30,60,30,20,50,50,30,30,50,40,\\
&30,20,50,40,50,30,30,20,20,50,20,20,\{1,20,30,40,50\},70),\\
\tilde{x}_{9,\text{lin}} = (&90,90,60,70,80,60,90,80,70,90,80,60,20,20,30,60,20,50,60,50,60,40,30,30,\\
&40,40,40,20,50,60,30,40,30,30,20,30,\{20,30,40,50\},50).
\end{aligned}
$$

Again if for an x^i we write a set, the optimum is reached on more solutions. The fitness and costs of the optima are shown in Table 7.7. As we can see, the fitness of $\tilde{x}_{9,\text{inv}}$ is higher than its total costs, which means that $\tilde{x}_{9,\text{inv}}$ does not satisfy our constraints.

The speed limits are shown in Figure 7.12 and the difference between the original distribution and the distribution according to $\tilde{x}_{5,\text{lin}}$ is shown in Figure 7.13.

The best solution we found is $\tilde{x}_{5,\text{lin}}$, which costs over thirty years €$2.9283 \cdot 10^9$. This means that we can save €$2.9960 \cdot 10^9$ − €$2.9283 \cdot 10^9$ = €$6.77 \cdot 10^7$, which is approximately 2.26% of the original costs.

Again we can make pictures of the internal and external traffic, which are respectively shown in Figure 7.14 and Figure 7.15. Again we see that the external traffic is more or less the same. Only on small parts in cities it takes another route for a short time. The internal traffic on the other hand have a big change of route when they travel between Enschede and Hengelo. First they

solution	**fitness**	C_{total}	C_{muni}	C_{prov}	C_{RWS}
$\tilde{x}_{5,\text{inv}}$	$2.9460 \cdot 10^9$	€$2.9460 \cdot 10^9$	€$2.2367 \cdot 10^8$	€$8.4020 \cdot 10^7$	€$2.6383 \cdot 10^9$
$\tilde{x}_{5,\text{lin}}$	$2.9283 \cdot 10^9$	€$2.9283 \cdot 10^9$	€$2.2355 \cdot 10^8$	€$8.3692 \cdot 10^7$	€$2.6210 \cdot 10^9$
$\tilde{x}_{6,\text{inv}}$	$2.9518 \cdot 10^9$	€$2.9518 \cdot 10^9$	€$2.2374 \cdot 10^8$	€$8.4041 \cdot 10^7$	€$2.6441 \cdot 10^9$
$\tilde{x}_{6,\text{lin}}$	$2.9368 \cdot 10^9$	€$2.9368 \cdot 10^9$	€$2.2377 \cdot 10^8$	€$8.3987 \cdot 10^7$	€$2.6291 \cdot 10^9$
$\tilde{x}_{7,\text{inv}}$	$2.9737 \cdot 10^9$	€$2.9737 \cdot 10^9$	€$2.2370 \cdot 10^8$	€$8.3934 \cdot 10^7$	€$2.6661 \cdot 10^9$
$\tilde{x}_{7,\text{lin}}$	$2.9425 \cdot 10^9$	€$2.9425 \cdot 10^9$	€$2.2372 \cdot 10^8$	€$8.3834 \cdot 10^7$	€$2.6350 \cdot 10^9$
$\tilde{x}_{8,\text{inv}}$	$2.9371 \cdot 10^9$	€$2.9371 \cdot 10^9$	€$2.2371 \cdot 10^8$	€$8.4078 \cdot 10^7$	€$2.6293 \cdot 10^9$
$\tilde{x}_{8,\text{lin}}$	$2.9767 \cdot 10^9$	€$2.9767 \cdot 10^9$	€$2.2377 \cdot 10^8$	€$8.3962 \cdot 10^7$	€$2.6689 \cdot 10^9$
$\tilde{x}_{9,\text{inv}}$	$3.3243 \cdot 10^9$	€$3.0046 \cdot 10^9$	€$2.2376 \cdot 10^8$	€$8.4104 \cdot 10^7$	€$2.6967 \cdot 10^9$
$\tilde{x}_{9,\text{lin}}$	$2.9930 \cdot 10^9$	€$2.9930 \cdot 10^9$	€$2.2372 \cdot 10^8$	€$8.4101 \cdot 10^7$	€$2.6852 \cdot 10^9$

Table 7.7: The solutions, fitnesses and costs of the solutions.

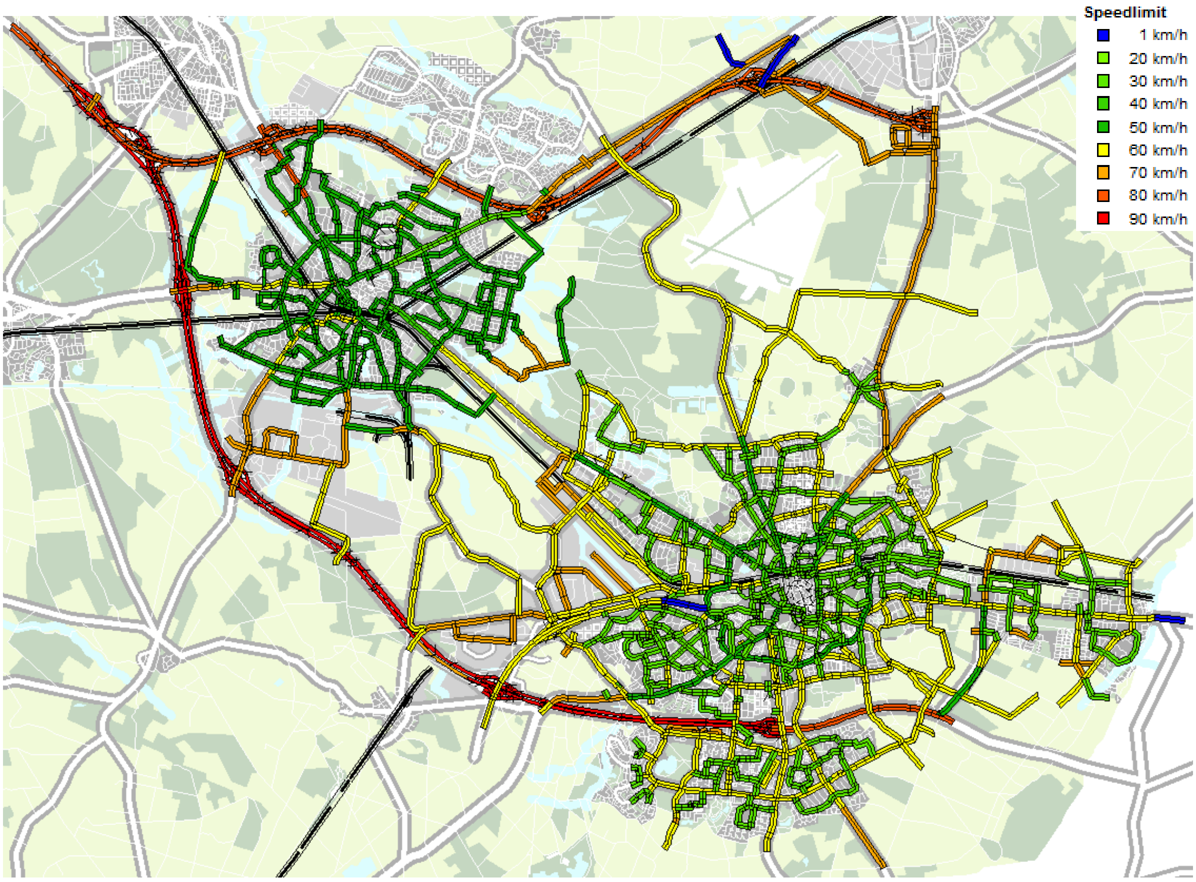


Figure 7.12: The speed limits in the network of Enschede and Hengelo according to $\tilde{x}_{5,\text{lin}}$.

would travel on the Hengelose- and Enschedesestraat and with the speed limits of $\tilde{x}_{5,\text{lin}}$ they will drive on the smaller 60 km/h road south of it.

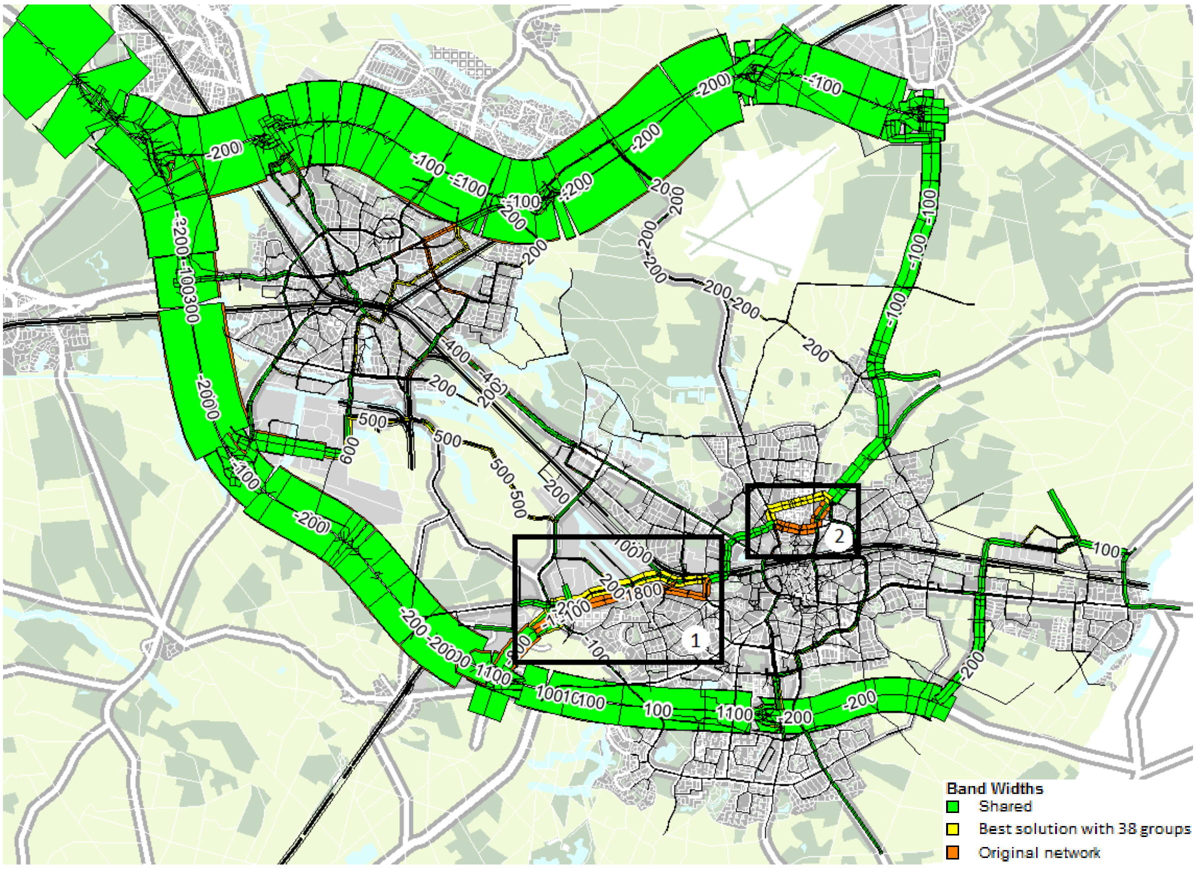


Figure 7.13: The distribution of the traffic according to $\tilde{x}_{5,\text{lin}}$ in comparison with the original situation. The rectangles are in more detail in Figure 7.16 and Figure 7.17.

7.3.6 Discussion

Our best solution found is

$$\tilde{x}_{5,\text{lin}} = (80,90,80,70,60,70,60,80,70,60,70,60,50,40,60,60,50,50,50,60,50,50,1,60,\\ 60,50,50,50,60,60,40,60,30,50,30,50,\{1,20,30,40,50\},70),$$

which gives us a profit €67.7 million, which is €26 million more when we calculate with nine groups and a 2.26% saving on the original costs. Unless it is a big amount of money, it is just a small percentage, we can see this in the traffic flows, they do not differ a lot with the original flows.

With more groups, we have a longer running time, which let us not run a high number of iterations. Therefore we also cannot say which algorithm of choosing a neighbour is better and we do not know if we have found the global minimal costs.

The problem with the slip road next to the superhighway we solved with this grouping. We can see that the speed limits in Hengelo are nearly everywhere lower or equal than 50 km/h, while Enschede has a lot of roads with a 60 km/h speed limit. These roads are not on the (inner) ring

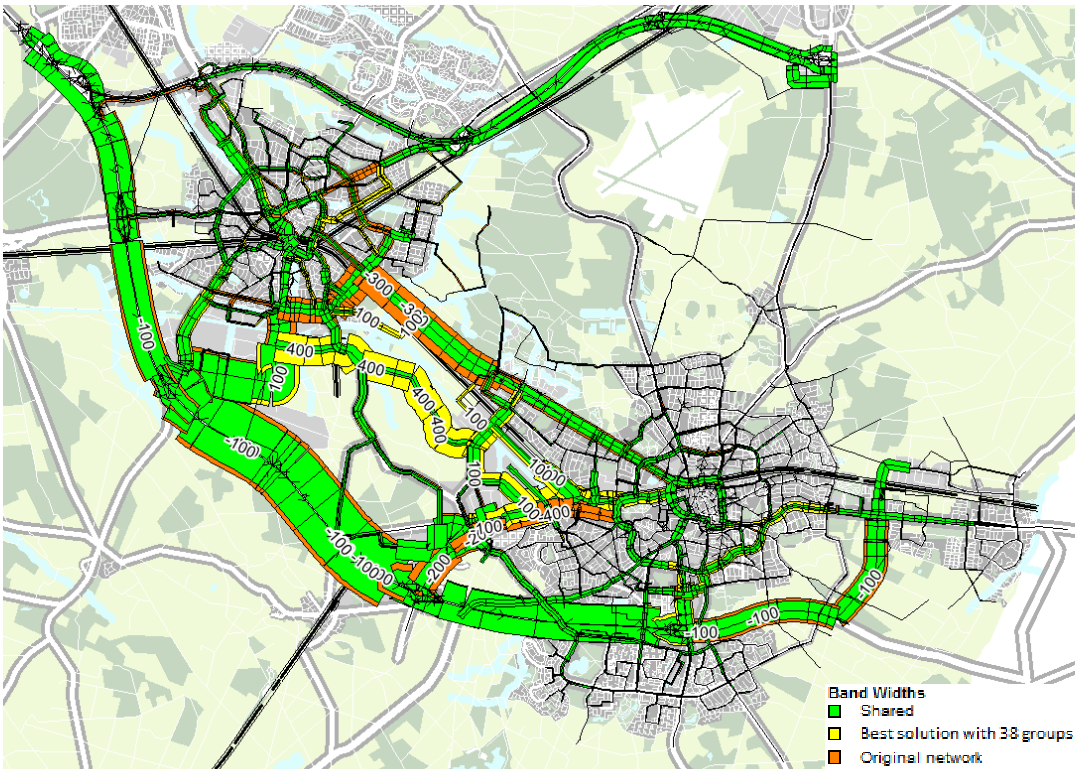


Figure 7.14: The distribution of the internal traffic of $\tilde{x}_{5,\text{lin}}$ in comparison with the current situation. With the bandwidth 10 times as high as in other pictures.

road and on most of the veins, but the rest group of 50 km/h roads in Enschede. This only causes a slightly different route choice, lorries where first driving on the north part of the ring road to take the Oldenzaalsestraat, but with this speed limit solution, they are taking the Roomweg, see Figure 7.16. Also a difference can be seen by the superhighway the Westerval, Figure 7.17. First lorry drivers drove on the Westerval to reach the ring road, with $\tilde{x}_{5,\text{lin}}$ as speed limits, they will drive on the street north of it, the Hendrik ter Kuilestraat. It is easy to explain why this gives a reduction in costs, the Hendrik ter Kuilestraat and the Westerval are next to each other and approximately the same length, but the Hendrik ter Kuilestraat is a collector road, which is smaller than a superhighway, so it has less asphalt to maintain. We also see this phenomena with the internal traffic, instead of driving on the motorway or an 80 km/h road, it is cheaper to let them drive on the smaller 60 km/h roads.

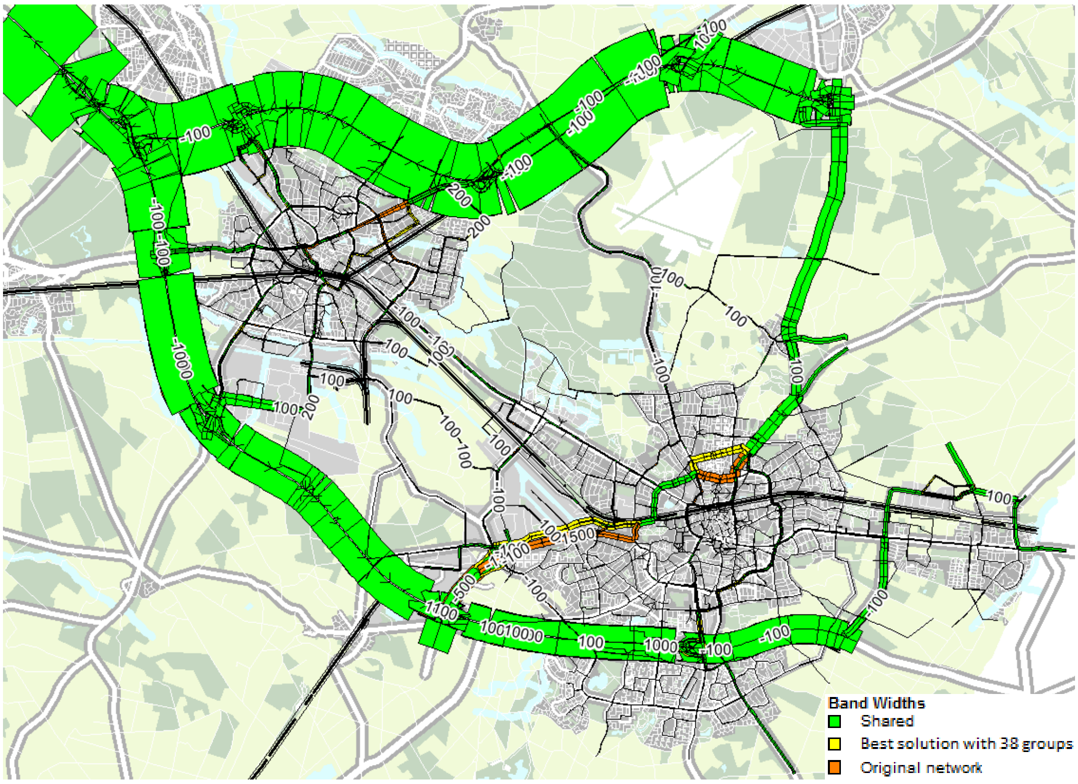


Figure 7.15: The distribution of the external traffic of $\tilde{x}_{5,\text{lin}}$ in comparison with the current situation.

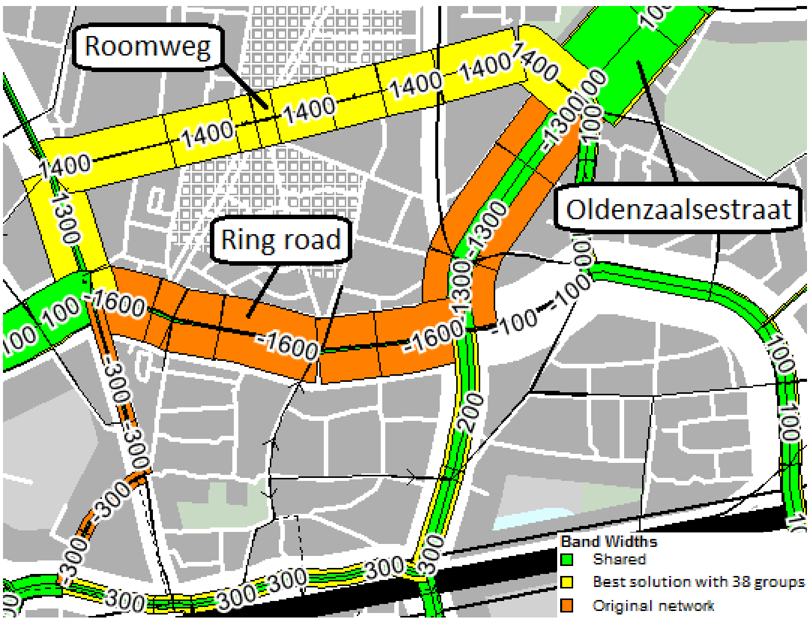


Figure 7.16: The distribution of the traffic of $\tilde{x}_{5,\text{lin}}$ in comparison with the current situation on the north part of the ring road. Picture (2) in Figure 7.13.

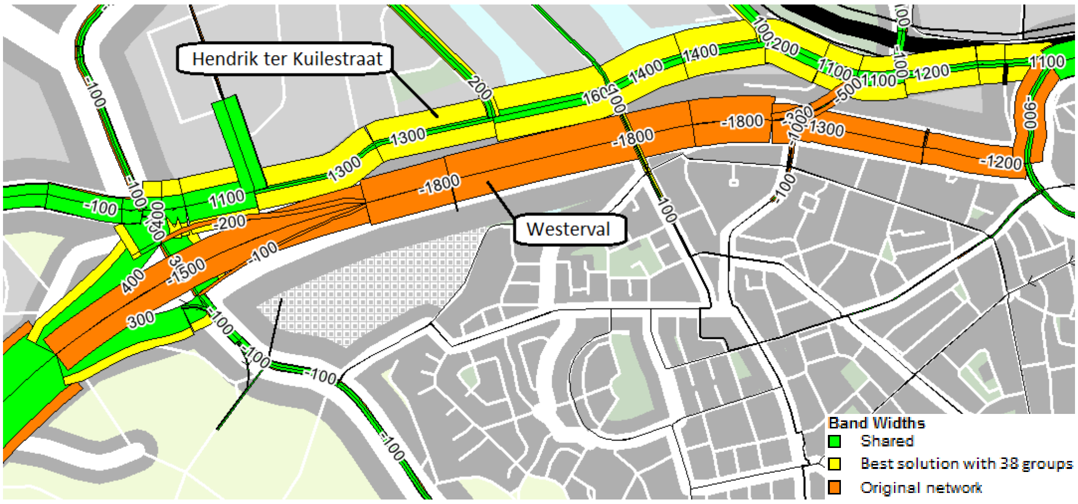


Figure 7.17: The distribution of the traffic of $\tilde{x}_{5,\text{lin}}$ in comparison with the current situation on the Westerval. Picture (1) in Figure 7.13.

CHAPTER 8

Conclusion and further research

In this chapter we will give a short summary of the research done in this thesis. Next we will present our conclusion and give some suggestions for further research.

8.1 Conclusion

We found out that the maintenance costs really can be influenced by the management of the traffic. With the assigning of other speed limits to the network than currently, we could reach a saving of approximately 2.3%.

The deterioration of the pavement caused by cars turns out to be insignificant in comparison to the deterioration caused by lorries. Therefore we made a model that only depends on the number of lorries. To get executable solutions, we cannot just minimise the costs, extra constraints are needed. We added the constraint that none of the parties (the nation, the province and the municipality) has to pay more that they do in the original situation. Our best solution found was made with a division of 38 groups of roads. The road groups are ring roads, urban veins and through roads. We found the maximal saving of approximately €67.7 million (2.3% of the original costs) for the area of Enschede and Hengelo over a period of thirty years.

We found different factors which influence the deterioration of the road. The one we took into account was the number of lorries. The maintenance costs therefore were only dependent of the variable factor of the number of traffic, the static factors are road length and width.

Traffic will distribute themself over the network according to Wardrop's first principle: no one can reduce its costs by choosing another route. Lorry drivers do not have this behaviour, they will take their shortest route as if there was no one else on the road.

The model we made is for general use. It can be used for every road network. Also the algorithm with grouping on road type can be taken over. The algorithm for another grouping needs a small change by hand, because it differ per case which road numbers are in the ring roads and in urban veins.

With all we found, we can say that we can reduce the maintenance costs for a road network over time by managing the traffic with speed limits. However, before applying the solution to a road network it has to be verified if it is sensible and will not cause unsafe situations.

8.2 Further research

For further research, one can look at the model of the rut depth and the computation time of the local search algorithm. It may also be interesting to look at other measures than speed limits.

In this research we used a deterioration model of the Present Serviceability Index. This index did not get below the value of 4.75, while we needed it that the PSI would reach values of approximately two, according to the literature. We solved this problem by multiplying the rut depth with a certain factor. Future research on the rut depth can possibly solve this problem.

We used a local search algorithm to look for the minimal costs. This algorithm is not very quick. To get better results, it is a good idea to develop a quicker algorithm, which will reach a better solution in the same time.

It will also be interesting to look at other measures than speed limits, to manage the traffic. For example extra lanes and/or roads or to forbid turns on junctions.

APPENDIX A

NOMENCLATURE

In this section, the used symbols and formulas are listed.

A.1 Symbols

The next symbols are used in this writing.

b, b'	Constants
c	Capacity of road r
f_r	Flow/Intensity on road section r
g_v	Growth factor of vehicle type v
$n_{v,r}$	Number of vehicles of type v on road r
$q_{r,t}$	State of the pavement of road r on time t
r	Road section
t	Time
v	Vehicle type
x	Speed limit vector
x^i	i^{th} element of the vector x
α	Constant
$\beta_0, \ldots, \beta_{13}$	Constants
δ	Discount factor
$\theta_{r,t}$	Decision variable of maintaining/replacing road r on time t
μ	Constant
ρ	A possible route in the road network
$\tau_{r,v}$	Travel time of vehicle type v on road r

ADF	Axle Damage Factor, a factor to compare the damage one axle brings in comparison to a standard axle
B, B_1, B_2	Replacement levels
C	Cost function per square meter asphalt
C_r	Cost function for road section r
CR	Cracks in the pavement (m/1000m^2)
F_{lanes}	Correction factor for the number of lanes of a road
L_r	Length of road r
$LT_{r,\{1,2,3\}}$	Layer thicknesses of road r
$\Delta\tilde{N}'_v$	Standardised damage a vehicle of type v brings to a road
PSI	Present Serviceability Index, an index to indicate the service the road gives to the user
R	Set of roads in the network
RD	Rut depth (mm)
$RD_{r,0}$	Initial rut depth of road section r.
$\Delta RD^{AC}_{r,t}$	Rut depth of the asphalt concrete layer of road r caused in the interval before t
$\Delta RD^{U}_{r,t}$	Rut depth of the under layer of road r caused in the interval before t
S_{single}	Load on a single axle
S_{std}	Standard axle load
S_{tandem}	Load on a tandem axle
S_{tridem}	Load on a tridem axle
SV	Slope variance (Rad^2)
T	Number of timeslots
$T_{[w_i,w_j],v}$	Number of trips between OD-pair $[w_i, w_j]$ of vehicle type v
$T_{[w_i,w_j],v,R}$	Number of trips between OD-pair $[w_i, w_j]$ of vehicle type v via route R
V	Set of vehicle types
W_r	Width of road r

A.2 Formulas

The next formulas are used in this writing.

f_r	$= n_{v_{\text{car}},r} + 1.8 n_{v_{\text{lorry}},r}$
$N'_{r,t}$	$= \sum_{s=1}^{t} \Delta N'_{r,s}$
$N_{r,t}$	$= \sum_{s=1}^{t} \Delta N_{r,s}$
$\Delta N'_{r,t}$	$= \sum_{v \in V} n_{r,v} g_v^{t-1} \Delta \tilde{N}'_{r,v}$
$\Delta N_{r,t}$	$= \sum_{v \in V} n_{r,v} g_v^{t-1} \Delta \tilde{N}_{r,v}$
$\Delta \tilde{N}'_v$	$= n_{\text{single}^v} \left(\frac{S_{\text{single}}}{S_{\text{std}}}\right)^{\beta_4} + n_{\text{tandem}^v} 2 \left(\frac{S_{\text{tandem}}}{2S_{\text{std}}}\right)^{\beta_4} + n_{\text{tridem}^v} 3 \left(\frac{S_{\text{tridem}}}{3S_{\text{std}}}\right)^{\beta_4}$
$\Delta \tilde{N}'_v$	$= n_{\text{single}^v} \left(\frac{S_{\text{single}}}{S_{\text{std}}}\right)^{\beta_1} + n_{\text{tandem}^v} \left(\frac{S_{\text{tandem}}}{\beta_3 S_{\text{std}}}\right)^{\beta_2} + n_{\text{tridem}^v} \left(\frac{S_{\text{tridem}}}{\beta_{12} S_{\text{std}}}\right)^{\beta_{13}}$
PSI_{flexible}	$= 5.03 - 1.9 \log(1 + SV) - 2.14 \cdot 10^{-3} RD^2 - 0.01\sqrt{0.3048CR + PA}$
PSI_{corr}	$= 5.05 - 2.57 \cdot 10^{-2} RD^2$
$RD_{r,t}$	$= RD_{r,0} + \sum_{s=1}^{t} \left(\Delta RD^{AC}_{r,t} + \Delta RD^{U}_{r,t}\right)$
RN	$= \beta_9 LT_{r,1} + \beta_{10} LT_{r,2} + \beta_{11} LT_{r,3}$
$\Delta RD^{AC}_{r,t}$	$\mu e^{b' N'_{r,t}} \Delta N'_{r,t}$
$\Delta RD^{U}_{r,t}$	$\alpha e^{b N_{r,t}} \Delta N_{r,t}$

Appendix B

Municipality Enschede

At the 15th of March 2017 I have spoken with Paul Koekkoek, road adviser at the municipality of Enschede. We talked about how the municipality manages their road maintenance. In this appendix a summary of this conversation is given.

The municipality of Enschede uses the instruction manual of CROW, a knowledge partner for municipalities in the scope of road maintenance. According to this manual, they view every year all the roads in the municipality and give it a score. Afterwards another person will view all the roads which should be maintained according to the manual. He or she will decide which road will be maintained according to the guidelines of the Enschede municipality.

Mr. Koekkoek indicates that a model for the road pavement without measurements will be difficult to make. On average, a road can be used 10 years. This is because the road status strongly depends on the weather. If in one period the temperatures will fluctuate around the freezing point the ice, water and salt will damage open asphalt a lot. Therefore when such a period happens, there is a very big chance that quality of the pavement will decrease rapidly. Also the quality of the bitumen mixture can differ. This means that some pavements are in a poor state after two years and other pavements are still very good after 15 years. This is also the reason why the municipality of Enschede does not have a model for the current state of the pavements.

The most occurring damage on open asphalt is disconnected stones. On other often used roads, rutting is the most occurring damage.

Included is the document *Kosten t.a.v. onderhoud deklagen / asfaltconstructie(s).* (Dutch). This views the costs of the maintenance and the replacing of the asphalt. We get this prices from Renée ter Meer from the municipality of Enschede, therefore it is only available in Dutch. The prices are approximations, because every road is different. The second column is the maintenance of replacing the top layer. The third column is replacing the whole pavement. The forth column is patching, which is not used in this paper. We only use the prices of the *AC 11* pavement.

Kosten t.a.v. onderhoud deklagen / asfaltconstructie(s)				
prijs per m2				
Soort	Deklaag (laagdikte, 35 mm)	Constructie [a] (3 lagen(135 mm)) (excl. Fundering)	Reparatie (bakken) (totaal 100 m2, laagdikte, 35 mm)	Opmerkingen
	prijs per m2			
AC 11	€ 14,50	€ 41,00	€ 51,00	[a] inclusief vervangen gootlagen.
SMA 11	€ 16,00	€ 42,00	€ 52,00	[a] inclusief vervangen gootlagen.

Uitgangspunten:

- Bedragen zijn inclusief eenmalige kosten, uitvoeringskosten, algemene kosten en winst & risico en klein percentage voor onvoorziene kosten.
- Exclusief verkeersmaatregelen, kosten voor de eigen organisatie.
- Geen rekening gehouden met teerhoudend asfalt.
- Wij hebben binnen de gemeente Enschede wat minder ervaring met geluidreducend asfalt en ZOAB.
- ZOAB wordt ook in mindere mate in binnenstedelijk gebied toegepast.
- Geluidreducerend asfalt wordt ook niet veelvuldig toegepast in Enschede.
- De geluidreducerende asfaltlagen hebben over het algemeen wat intensiever onderhoud nodig en hebben een kortere levensduur.
- Elke 2 à 3 jaar moet de laag worden gereiningd om de geluidreducerende eigenschappen te optimaliseren.
- Bovendien ligt de levensduur van geluidreducerende deklagen zo tussen de 6 à 7 jaar. Bij 'normale' deklagen ligt de levensduur gemiddeld tussen de 15 à 20 jaar.

Appendix C

Global inspection card

The following inspection card is used by the municipalities Deventer and Enschede and is set up by CROW. The inspection card is only available in Dutch.

GLOBALE INSPECTIE KAART

CROW-publicatie 146b 2013

				Ernst				
Asfaltbeton		Omvang		L	M	E		
Textuur	Rafeling asfaltbeton en oppervlakbehand. (in %)	≥5 – <30%	1	>5 – ≤20%	>20 – ≤50%	>50%	% van de maatgevende m² vertoont schade	Niet in verh.rand
		≥30 – <50%	2				Alléén steenfractie > 2 mm	
		≥ 50%	3					
	Rafeling zoab (in %)	≥5 – <30%	1	>5 – ≤10%	>10 – ≤20%	>20%		Niet in verh.rand
		≥30 – <50%	2					
		≥ 50%	3					
Vlakheid	Dwarsonvlakheid (m/100 m)	≥5 – <15 m	1	>10 – ≤20 mm	>20 – ≤30 mm	>30 mm	mm hoogteverschil	Niet in verh.rand
		≥15 – <35 m	2					Alleen > 5 m
		≥ 35 m	3					Beide sporen 1x
	Oneffenheden (st./100 m)	≥3 – <8 st.	1	>5 – ≤15 mm	>15 – ≤30 mm	>30 mm	mm hoogteverschil	Niet in verh.rand
		≥8 – <15 st.	2				Opm. Fietspaden licht	Alleen < 5 m²
		≥ 15 st.	3				vermelden onder matig	
Samenhang	Scheurvorming (m/100 m)	≥5 – <25 m	1	ja	nee	nee	Gevulde scheuren	
				≤10 mm	>10 – ≤15 mm	>15 mm	Lengtescheuren, hoogteverschil in mm	
		≥25 – <50 m	2	≤5 mm	>5 – ≤10 mm	>10 mm	Scheurwijdte in mm	
	Let op : Dwarsscheuren bij opmerkingen			nee	ja	nee	Meerdere parallel lengtescheuren	
		≥ 50 m	3	nee	ja	nee	Lengtescheuren met zijtakken	
				nee	nee	ja	Lengte- en dwarsscheuren verbonden (craquelé)	
Verhardingsrand (facultatief)	Randschade (m/100 m)	≥5 – <25 m	1	Afzonderlijke schades dwarsonvlakheid, oneffenheden en scheurvorming die in de verhardingsrand voorkomen				
		≥25 – <50 m	2					
		≥50 m	3					
Elementen		Omvang		L	M	E		
Vlakheid	Dwarsonvlakheid (m/100 m)	≥5 – <15 m	1	>10 – ≤25 mm	>25 – ≤40 mm	>40 mm	mm hoogteverschil	Tot aan goot
		≥15 – <35 m	2					Alleen > 5 m
		≥35 m	3					Beide sporen 1x
	Oneffenheden (st./100 m)	≥3 – <8 st.	1	>5 – ≤15 mm	>15 – ≤30 mm	>30 mm	mm hoogteverschil	Alleen < 5 m²
		≥8 – <15 st.	2				Opm. Fietspaden licht	< 3 st. E K.O.
		≥15 st.	3				vermelden onder matig	< 3 st. M Niets
Samenhang (facultatief)	Voegwijdte maatvast (in %)	≥5 – <15%	1	>5 – ≤10 mm	>10 – ≤20 mm	>20 mm		
		≥15 – <30%	2					
		≥30%	3					
	Voegwijdte niet maatvast (in %)	≥5 – <15%	1	>10 – ≤15 mm	>15 – ≤20 mm	>20 mm	Opmerking: houd rekening met grilligheid natuursteen	
		≥15 – <30%	2					
		≥30%	3					
Cementbeton		Omvang		L	M	E		
Vlakheid	Oneffenheden (st./100 m)	≥2 – <5 st.	1	>5 – ≤10 mm	>10 – ≤15 mm	>15 mm	mm hoogteverschil	
		≥5 – <10 st.	2				Opm. Fietspaden licht	
		≥10 st.	3				vermelden onder matig	
Samenhang	Scheurvorming (aantal platen/100 m)	≥1 – <3 st.	1	≤3 mm	>3 – ≤10 mm	>10 mm	Scheurwijdte in mm	Langs- en dwarssch.
		≥3 – <8 st.	2	≤5 mm	>5 – ≤10 mm	>10 mm	Hoogteverschil in mm	
		≥8 st.	3	nee	≤50 mm	>50 mm	Afbrokkeling scheurrand	
				ja	nee	nee	Gevulde scheuren	
Waterdichtheid	Voegvulling (m/100 m)	≥10 – <30 m	1	n.v.t.	deels	geheel	Verlies hechting a.d. rand	
		≥30 – <50 m	2	n.v.t.	deels	geheel	Noodzaak vervanging	
		≥50 m	3	n.v.t.	ja	n.v.t.	Plaatselijke uitstulping	
				n.v.t.	nee	ja	Voegvulling ontbreekt	
Divers - facultatief		Omvang		L	M	E		
Zetting	(in m)	n.v.t.		≤0,20	>0,20 – ≤0,40	>0,40	Opm.: Gelijkmatig of ongelijkmatig	

Veelgebruikte afkortingen t.b.v. asfalt- en elementenverhardingen

AI: aansluiting inspectieput
AK: aansluiting kunstwerk
DS: dwarsscheuren
BG: boorgaten
BP: bezweken plek
BW: boomwortelopgroei
PO: plaatselijke ophoging
PV: plaatselijke verzakking
RV: ribbelvorming

Extra t.b.v. cementbetonverhardingen

DS: hoogteverschil bij dwarsscheuren
DV: hoogteverschil dwarsvoegen
LV: hoogteverschil langsvoegen
BP: bewegen betonplaat

Afkortingen

Benaming wegvakonderdeel

RB: Rijbaan
RBL: Rijbaan Links
RBR: Rijbaan Rechts
FP: Fietspad
VP: Voetpad
PS: Parkeerstrook
L: Links
R: Rechts

Weersinvloeden

Niet inspecteren:
- Asfalt en cementbeton bij nat wegdek

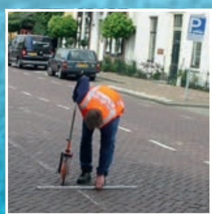
Let op bij:
- Scheurvorming bij asfalt- en cementbeton bij opdrogend wegdek
- Plasvorming
- Lichtinval

Inspectieuitrusting

- 'Handleiding globale visuele inspectie'
- rei, 1,20 meter lengte met waterpas
- wig (met schaalverdeling)
- (metalen) stiften met diameter 3, 5, 10, 15 en 20 mm doorsnede
- meetwiel
- meetlat
- veiligheidsvest of -jas
- eventueel een fotocamera
- bord 'WEGINSPECTIE', goed zichtbaar op de auto
- oranje zwaailicht
- eventuele vergunningen en ontheffingen
- wegafzetting of pijlenwagen, afhankelijk van de verkeerssituatie

Inspectiekaart globale inspectie (versie febr. 2013)

C.R.O.W



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