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An empirical study of the revisions to the Internal models approach for market
risk under the Fundamental Review of the Trading Book

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M.Sc. Thesis

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Abstract:

In a world where due to globalization and interaction a single distress can lead to a global financial crisis, risk management systems, supervision and setting regulations by a third party is crucial to ensure a healthy banking system. Therefore, the Basel Committee on Banking Supervision published its new standards to determine the minimum capital requirements for banks to prevent situations such as the global mortgage crisis to happen again in the future. This thesis studies the different underlying risk measures to determine market risk namely the Value-at-Risk and Expected Shortfall and examine the impact of new regulations set by the Basel Committee on Banking Supervision on banks.

Key-Words:

Expected Shortfall (ES); Value-at-Risk (VaR); Basel; FRTB; Minimal capital Requirements; Market Risk

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List of Abbreviations

| | |
|--------|--|
| Avg | Average |
| AWHS | Age-Weighted Historical Simulation |
| BCBS | Basel Committee on Banking Supervision |
| CRR | Capital Requirement Regulations |
| CVaR | Conditional Value-at-Risk |
| DRC | Default Risk Charge |
| ES | Expected Shortfall |
| EtI | Expected tail loss |
| HS | Historical Simulation |
| IMA | Internal Models Approach |
| m_c | Multiplication factor |
| MCR | Minimal Capital Requirements for market risk |
| m_s | Stressed multiplication factor |
| NMRF | Non-model able Risk Factors |
| Portf. | Portfolio |
| SA | Standardised Approach |
| VaR | Value-at-Risk |
| X_t | change of the portfolio's value |
| Lh | Liquidity horizon |
| n | Observation days |

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1 Introduction

In recent years, it has been observed that risk control and risk management departments of credit institutions are playing a much more prominent role in how these companies operate. Due to massive trading losses on the stock market and significant credit losses caused by corporate insolvency, it became apparent that the old risk management system could no longer sufficiently protect credit institutions against illiquidity. As a result, governmental institutions responded and developed, with the consultation papers of the Basel Committee, a mixture of laws, directives and recommendations, which will assist credit institutions with improving risk management. Let us look at the causes of the necessity of these new guidelines for a simplified banking balance. However, before it is important to know that the banking balance is different to a balance sheet of an industrial company. The positions within the balance are sorted to its liquidity and each position needs to be allocated into either the banking book or trading book whereas different capital requirements are necessary for these books. Shortly, assets held in the trading book are financial instruments held with trading intent or to hedge market risk. All instruments which are not assigned to the trading book by means of the Capital Requirements Regulation (CRR) are held in the banking book.

| Application of funds | Source of funds |
|-----------------------------|-----------------|
| Cash reserve | Saving deposits |
| Account receivables | Bonds |
| Credits to Capital acquirer | Equity |
| Investments | |

Table 1 Simplified illustration of a banks statement of financial position

The problem up to now is that individual loans and deposits do not correspond to their amount or regarding their maturity. For example, a long-term million-dollar loan can face many short-term savings deposits.

The bank must ensure that the different structures on the assets and liabilities side do not lead to liquidity problems so that the bank is always able to pay back the customer's deposits. Liquidity problems can arise when either many savers suddenly withdraw their deposits or the income from the banks' investments are less than predicted. These yield problems can arise in both the credit and investment sectors. The risk in the credit sector is that the given loans may lose profitability due to a change in interest rates or, in a worst-case scenario, a borrower may go into bankruptcy which would make it impossible to pay back the given loan¹. In the investment area, unfavorable Market conditions may lead to a performance which does not meet expectations. This is known as market risk. The four main market risk factors are commodity risk, equity risk, currency risk and interest rate risk. For each market risk, the underlying problem is that stock prices, interest rates, foreign exchange rates or commodity prices may unfavorable change. If this volatility renders certain credit institutions' predictions false, these institutions may then face a liquidity crisis. To prevent such risks, the funds used in the investment must be hedged via equity capital. At this point, the Basel Committee on Banking Supervision (BCBS) has set up

¹ In this thesis only the market risk is considered, so that counterparty risk is disregarded.

regulations to secure proper risk management procedures for internationally active banks. Regulations established by the BCBS are broad supervisory standards and recommendation statements for best practices in banking supervision and guidelines to ensure a healthy global banking system. The risks must be hedged through equity capital, whereby higher risks are met with higher equity requirements.

Risk Management, the practice of understanding risks and handling them appropriately to survive in the market environment, has received considerable attention from banks and the Basel Committee on Banking Supervision (BCBS), after the global financial crisis in 2008 showed a high undercapitalization of trading book exposures (Baptista et al., 2012, p.249). Risk awareness of the financial sector has increased and risk management has become a major area of focus for both banks and the BCBS. As a short-term response to the 2008 crisis, the BCBS implemented the Basel 2.5 regulations in 2009. As a long-term solution for Market Risk, the Basel Committee published its standards for minimum capital requirements for market risk in January 2016, which is a consolidation paper of the Fundamental review of the Trading Book (FRTB).

The FRTB includes several fundamental changes for banks. The main purpose of the FRTB is to ensure that banks have sufficient capital to withstand a potential market crash. Banks must fully implement the changes stated within the recent FRTB document (Basel Committee, 2016) concerning capital requirements for market risks until the end of 2019. The changes encompass three main topics. Firstly, a revised standardized approach (SA), secondly a revised boundary between the trading book and banking book and lastly, the revised internal models approach (IMA).

The revised SA framework is designed to be more risk-sensitive and to address the shortcomings of the current SA framework. Furthermore, the new SA Framework is expected to become more important to financial institutions since it serves as a floor and fallback to the revised IMA Approach. In addition to the SA Approach, the IMA Approach calculates the minimum capital requirements for market risk of Financial Institutions. The revised boundary between trading and banking book has discouraged regulatory arbitrage. Whereas trading book refers to assets held by a bank that are regularly traded, banking book refers to assets on a bank's balance sheet with a long holding expectation. This means assets held for trading are put in trading book and assets held to maturity put in the banking book. The assets within the trading book are marked-to-market daily whereas the assets in banking book are accounted for using the historical cost method. This new boundary restricts the current course of action for banks, reallocating financial instruments between trading and banking book which results in a positive advantage regarding banks capital requirements. Any deviation is now subject to supervisory approval. Since the revised SA and boundary between banking and trading book are not part of the main focus of my thesis, I refer to the BCBS Document 352 for more details.

The focus of this thesis is centered around the Internal models approach. Instead of using the VaR as in Basel 2.5 due to several weaknesses (BCBS, 2013, p.3), the revised IMA makes use of the 97.5%, (one-tailed confidence level) Expected Shortfall (ES) metric that improves the measurement of tail-risks and must be computed daily and bank widely to calculate the market risk. Yamai and Yoshida (2002) defined the Expected Shortfall as a measure of "how much one can lose on average in states beyond the Value-at-Risk level." The ES provides insights on how high the potential loss might be if the low probability case occurs (Acerbi & Tasche, 2002).

Furthermore, new liquidity horizons have been introduced. Other than in Basel 2.5, which adjusted all instruments to a liquidity horizon² of 10-days regardless of how risky the asset is, banks must adjust the assets to different liquidity horizons from 10 -120 days' depending on their riskiness. As in Basel 2.5, the MCR has been calculated by adding the most current VaR and a Stressed VaR component, which is the VaR within a period of stress. The MCR under FRTB is the liquidity adjusted stressed Expected Shortfall, which will be explained in greater detail in Chapter 3. To calculate the VaR or ES, different observation periods³ can be used such as 250 days, 500 days or 1000 days. Nevertheless, this should not be confused with the historical data needed. Under Basel 2.5 the most current year of data history plus a year of financial stress such the years 08/09 of the mortgage financial crisis (using an observation period of 250) was enough to calculate the MCR. To calculate the MCR under the FRTB we need a data history of at least ten years or even more if the observation period is larger than 250 days. For this reason, banks mainly used an observation period of 250 days. The BCBS allows under both regulations various modeling methods (e.g., historical simulation, variance-covariance analysis or Monte Carlo simulation) to model VaR and ES respectively and different observation periods. Thus, the credit institutions have a degree of choice regarding their underlying model. However, a model failing backtesting, meaning that the real losses exceed the hypothetical losses, penalizes the bank in the form of a higher capital requirement. The penalty factor will be determined using the "Basel traffic light" approach (BCBS, 1996). The "Basel traffic light" approach will be explained in more detail in chapter 2.6.1.

According to the result of the interim impact analysis of its fundamental review of the trading book conducted by the BCBS, these regulations will result in a median capital increase of 22% and a weighted-average capital increase of 40% (BCBS, November 2015).

However, there is still little information on the impact of the revised IMA for the banking sector and which simulation model will lead to a more risk-averse MCR. Even though a vast amount of papers in the academic field have been dedicated to the development of VaR and ES models, its examination and validation (Jorion 2007) and the fact that a lot of literature is related to the different backtesting approaches, literature is lacking in a validation of former mentioned increase in MCR from Basel 2.5 to FRTB and which components have the most influence on this increase. Therefore, the goal is to investigate the effects of the Revised IMA for banks and to conduct an empirical study about changes of the minimum capital requirements for banks based on historical data. For this purpose, the following research question will be asked:

- I. What is the effect of the chosen observation period on the amount of both risk measures VaR 99% and ES 97.5% ?
- II. Is there a significant difference, between the both risk measures VaR 99% and ES 97.5% ?
- III. What effect has the choice of risk measure, and observation period on the number of exceptions and how is this reflected by the Basel traffic light?
- IV. What effect has the change from Basel 2.5 to FRTB regulations, on the minimum capital requirements?

² "the time required to execute transactions that extinguish an exposure to a risk factor, without moving the price of the hedging instruments, in stressed market conditions" (BCBS, 2016)

³ The observation period is the number of days taken into consideration to calculate the VaR and ES.

- V. What is the effect of the choice of the calculation method on the preference order within both periods, of a decision maker who a) decides only based on risk measure and b) only on the MCR or c) the Sharpe ratio?

In short, we found out that the length of the observation period has little to no influence on the amount of MCR since there is no significant difference between 250, 500 and 1000 days in the value of the ES or Var. Moreover, the change from VaR (99%) and ES (97.5%) is not as big as expected and their values are almost equal or only little. Even though the difference is significant, it can be neglected because there is only a small effect size of the paired t-test. The real increase is due to the implementation of the different liquidity horizons and the implementation of the new risk metric. Moreover, we can support the impact study of the BCBS (BCBS, 2015) and confirm that under the new regulations the MCR is larger which is attributed to the adjustment to different liquidity horizons. However, in times of financial stress, the change from the Value at Risk to the Expected Shortfall is advantageous since it better reflects the losses.

This thesis contributes both to the academics as well as practical user such as it gives banks a first guiding principle on how much the new regulation might affect the minimum capital requirements for market risk, which observation period leads to the smallest amount of the risk measure and it can support the main idea of the BCBS, to provide more risk-averse regulations, evident with how much more risk averse the new regulations are. From a theoretical perspective it comprises regulatory, mathematical and technical concepts such as theoretical backgrounds to the underlying concepts of Value-at-Risk and Expected Shortfall additionally it theoretically describes how to calculate the MCR under Basel 2.5 and FRTB and subsequently describe how to execute the calculations using Excel.

After a rough picture of what can be expected in this thesis, the structure of this thesis is as follows. Firstly, chapter 2 starts with a theoretical framework of concepts including risk measurement, axioms of coherence, Value at Risk and Expected Shortfall. Chapter 3 provides a literature review and an outline of the different Basel accords (Basel I – III) to give an overview and a better understanding of the topic and the evolution of risk management regulations by the BCBS. The different documents from the BCBS will be presented in chronological order. In Chapter 4 presents the different calculation and simulation methods to compute the VaR, ES and consequently the minimum capital requirements. Chapter 5 presents the hypotheses and covers the methodology by firstly describing the data and underlying statistics. Afterward a description is given on how the Data has been used and how the calculation has been conducted. Chapter 6 empirical answers the former mentioned research questions and deeper exemplify the hypotheses. Chapter 7 gives a conclusion and discussion of the results from the previous chapters and provides guidelines for further research.

2 Literature Review

2.1 Risk Measure

A risk measure quantifies risk and is therefore a method to summarize the uncertainty of future returns by a single value. These methods are conducted by financial institutions, within a regulatory framework, to determine the needed size of their capital reserves, thereby ensuring that their market risk exposure is acceptably low enough for the regulators. The underlying risk measures of this thesis are the VaR and the ES.

2.2 Loss Distribution

The profit-and-loss (P&L) distribution is defined as the distribution of changes in a portfolio's value. Mathematically this changes in value are expressed as $V_{t+1} - V_t$, where V_t is the value of the Portfolio at time t and V_{t+1} is the portfolio's value at $t+1$. According to Dowd (2007, p.38), it is more convenient to deal with loss and profit (L&P) distribution for VaR and ES which is equivalent to P&L-distribution except changed signs:

$$-V_{t+1} + V_t.$$

2.3 Axioms of Coherence

According to Artzner et al. (1999) a risk measure should satisfy the following properties to be considered coherent: (1) monotonous (2) sub-additive, (3) positively homogeneous and (4) translation invariant. These properties are called the axioms of coherence. If one of these axioms is not fulfilled that risk measure is incoherent. In the following descriptions of these properties a mathematical equation is given by Artzner et al. (1997) who defines a set of sensible criteria that a measure of risk, $p(X)$ where X is a set of outcomes with V predefined as a set of real-valued random variables and is an element of \mathbb{R} .

Monotonous means that if a specific portfolio's value X_1 is always better than the value of another Portfolio X_2 , most likely the risk measure of portfolio X_1 is more likely to be lower than the risk measure of Portfolio X_2 . If $X_1 < X_2$ then $p(X_1) > p(X_2)$, with $X \in V, X \geq 0$

Sub-additive means that the risk is reduced via diversification implying that the risk measure of two individual portfolios is less than the sum of their risk measure after they have been merged. In other words, aggregating two individual portfolios will result in the risk measure either decreasing with a correlation smaller one or remaining unchanged having a correlation of 1. It can be written as $p(X_1 + X_2) \leq p(X_1) + p(X_2)$, with X_1, X_2 and $(X_1 + X_2) \in V$. The main principle behind it is the idea of diversification. The latter axiom was one of the reasons for the BCBS to take into consideration a new regulatory risk measure since the VaR could not always satisfy the axiom of subadditivity (Acerbi & Tasche, 2001) (BCBS, 2012). This will be more deeply elaborated in the next section.

Positively homogenous means that the proportions of risk measure and the number of positions within a portfolio are constant. Meaning that if the number of positions in a portfolio increases, the risk measure also increases. Or in other words, if you double your portfolio then you double your risk. It can mathematically for all $X > 0$ be written as $p(XX) = Xp(X)$ with $X \in V$

Translation invariance for all constant c with a guaranteed return implies that the additional risk-free amount of capital within a portfolio reduces its risk by the same amount and is mathematically expressed as $p(X + c) = p(X) - c$ with $X \in V$

2.4 Value at Risk (VaR)

The VaR is a concept for measuring risk and is yet widely used within risk management due to the recent risk regulations of Basel III. The VaR is a measure of the risk of investments and is defined by Longin (2001) as “the worst expected loss of the position over a given period, at a given confidence level.” Another definition is given by Pérignon et al. (2008) they define VaR as “expected maximum loss over a target horizon (e.g. ,1 day, 1 week) at a given confidence level (e.g. ,95%, 99%).” However, these definitions are potentially misleading since the VaR measure does not consider an asset’s maximum potential loss, and as a result understated the true totality of the risk. It may be the case that a potential loss is far above that found with the Var. This is known as tail risk, which is not considered within the VaR (Embrechts, Frey & McNeil, 2005). According to Embrechts, Frey & McNeil (2005), the maximum loss is dependent on the heaviness of the tail of the loss distribution. A mathematical definition is given by Ziegel (2013) with α (Confidence level) $\in (0,1)$:

$$Var_{\alpha}(Y) = -\inf\{x \in \mathbb{R} | F_Y(x) \geq \alpha\} \quad 2.1$$

Within this equitation, the VaR is defined as “greatest lower bound (infimum) on the cumulative distribution function F of any financial position Y , expressed as a real-valued, random variable” (Chen, 2014).

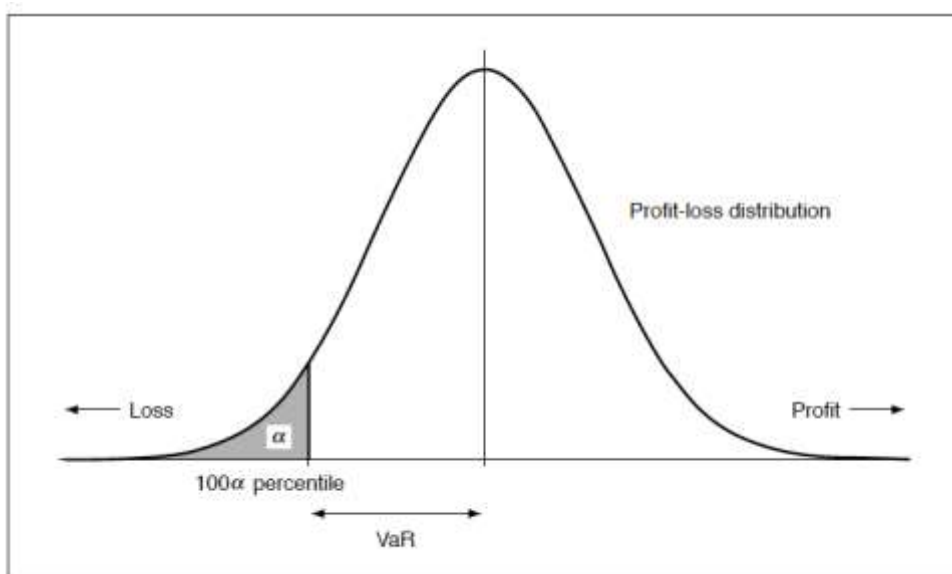


Figure 1 Profit-loss distribution and VaR (Yamai & Yoshiba, 2005)

Due to its contextual simplicity, ease of computation and applicability, it has become a standard approach for measuring risk (Yamai & Yoshida, 2005). There are three different approaches to compute VaR. First, it can be calculated using a Variance-Covariance method in which assumptions about the return distributions of market risk are made. Secondly, using a historical method where hypothetical portfolios run through historical data, or thirdly, using a Monte Carlo simulation. Banks are given the freedom to choose which simulation method they take as a basis within the IMA as long as it is confirmed through backtesting (BCBS 2016, p.54 (g)). However, the second method will be used within this thesis because there is no need for making assumptions regarding probability distribution of risk factors. Furthermore, no correlation numbers need to be assumed.

2.4.1 Limits of the VaR

Acerbi & Tasche (2002) argue that the VaR method is not a coherent measure of risk due to its lack of sub-additivity, which is a desirable property for a risk measure because sub-additivity implies that the combination of two independent products “does not create extra risk” (Artzner et al., 1999), it reduces it. As mentioned before the main principle behind this property is diversification. In other words, non-subadditivity implies that diversification will not lead to a risk reduction. In 1999 Artzner et al. introduced the concept of a coherent risk measure and its properties. As shown above, from a mathematical point of view sub-additivity can be written as $p(X + Y) \leq p(X) + P(Y)$, with X, Y and $(X + Y) \in V$. Since the subadditivity “...ensures that the diversification principle of modern portfolio theory holds...” (Danielsson et al., 2005) it could be the case that a violation of this axiom leads to a situation where the risk of a diversified portfolio could be greater than the its individual non-diversified sub-portfolio. Furthermore, banks, which are built up by several individual branches with their own activities, may have a scenario where each branch calculates its own risk, but the VaR does not fully reflect the risk of the entire bank due to the lack of sub-additivity. It might be that the aggregate risk of the branches is higher than the overall capital required adequacy requirements. In other words, the bank does not hold enough capital to cover the risk because the VaR is used to calculate it. For a better understanding of the non-subadditivity of VaR, consider the example illustrated by Danielsson et al. (2005) “Sub-additivity re-examined: the case for Value-at-Risk” in Chapter two “Sub-additivity” another good demonstration of the non-subadditivity is given by Acerbi et al., (2001) in his paper “Expected shortfall as a tool for financial risk management”. Furthermore, tail risk is a big drawback of the VaR model because it does not tell us anything about the potential losses above the maximum loss in 95% of the cases (with a 95% confidence level) when the returns are not normally distributed. The non-normal distribution might show the same VaR one that is normally distributed, however, the loss exceeding the VaR level may be significantly different. Furthermore, Yamai and Yoshida (2002) disclosed that investors could manipulate the Profit & Loss distributions by using assets which have large but infrequent losses, thereby making the tail become fat and the sides thin. These empirical drawbacks are also presented in the Consultative report of the Fundamental review of the trading book (BCBS, May 2012, pp.53-55).

2.5 Expected Shortfall (ES)

The Expected Shortfall also known as “conditional VaR”, “beyond VaR”, “tail VaR” or “Expected tail loss (ELT)” is the “conditional expectation of loss given that the loss is beyond the VaR level” (Yamai & Yoshida, 2002). The ES describes the average loss to be expected for the case, that the actual loss is bigger than the VaR (Yamai & Yoshida, 2005). Therefore, the ES defined by Yamai & Yoshida (2005) as: $ES_{\alpha}(X) = E [X|X \geq VaR_{\alpha}(X)]$, is a conservative measure of risk since it looks beyond the VaR and hence the ES does not lead the investor to take risky positions. (Yamai & Yoshida, 2002). For calculating the Expected Shortfall, the same method as in the VaR can be used. The following equation describes the ES with $X \in L^0$ (Embrechts & Wang, 2015):

$$ES_{\alpha}(X) = \frac{1}{\alpha} \int_0^{\alpha} VaR_{\alpha}(X) d\alpha \quad 2.2$$

A visual comparison of the former Presented methods is given by Yoshida & Yamai (2002)

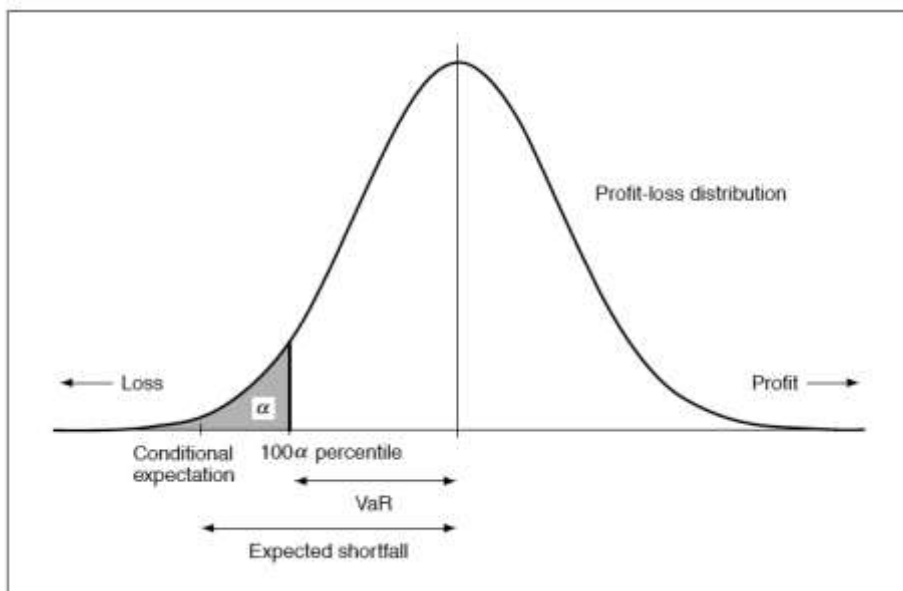


Figure 2 Profit-loss distribution, VaR and Expected Shortfall (Yamai & Yoshida, 2005)

The key property which distinguishes it from the VaR is that the ES is subadditive and, consequently, coherent which, according to Acerbi & Tasche (2002), is “the most important property of ES [...]”. Proof of subadditivity is given by Embrechts & Wang (2015) in their paper “Seven Proofs for the subadditivity of Expected Shortfall”. Furthermore, Acerbi & Tasche (2002) indicate that the VaR is sensitive to minute changes in the confidence level whereas the ES will not change dramatically. Summing up both methods, it can be said that VaR is the best worst x% losses whereas Expected Shortfall is the average of the worst x% losses. The ES model has multiple advantages over the VaR model including the described tail sensitivity and that is a coherent measure of risk. How to specifically estimate the risk under each of the considered estimation models will be presented in the following chapters.

2.6 Backtesting

"VaR is only as good as its backtest". When someone shows me a VaR number, I don't ask how it is computed, I ask to see the backtest." (Brown, 2008, p.20)

For reliability and accuracy reasons, it is crucial to backtest VaR and ES models when using them to measure risk. Backtesting is a robustness-test for the underlying risk model. Due to the recent turmoil of the 2008 financial crash, accuracy and reliability are more important than ever. The BCBS (2016) defines backtesting as "The process of comparing daily profits and losses with model-generated risk measures to gauge the quality and accuracy of risk measurement systems." It is a set of statistical methods to check if the forecasts of the VaR and ES are in line with real losses (Jorion, 2007). In other words, backtesting is a comparison of the observed P&L to VaR and ES forecasts. If the real loss exceeds the VaR, it is referred to as an exception. For example, in a sample with an observation size of 1000 and a confidence level of 99%, we would assume 10 exceptions to exist. If more than 10 exceptions exist, we have an unsuccessful model. If there are more exceptions than we expect, the underlying model underestimates losses and is rejected. On the other hand, if the backtest yields fewer exceptions than expected, it may be a sign that the model overestimates risk, and that the credit institutions using this model have allocated too much money to cover the non-existent risk. Hence, the model should be recalibrated to capture the true risk. If a recalibration is necessary, we first need to calculate the failure rate, x/N where x denotes the number of exception days and N the number of days in the sample. According to Jorion (2007 p.131-133), the failure rate should harmonize around p , which is the confidence level. To test if the failure is close to p , we need to calculate x . To do so, the literature proposes several backtesting methods including the coverage tests of Paul Kupiec (1995), Percentile test by Crnkovic and Drachmann (1996), Christoffersen's interval forecast test (1998), Basel traffic light framework (1996) or the loss function by Lopez (1999). According to Haas (2001), decent results should always be confirmed with another test. These methods which are proposed for VaR estimations are quite simple because the VaR is "elicited by the weighted absolute error scoring function" (Emmer, Katz and Tasche, 2015). For further information see Thomson (1979), Saerens (2000) or Gneiting (2011). Backtesting the ES is more difficult because we cannot compare the empirical return distribution with the $x\%$ worst case. There is even doubt as to whether the ES model is back testable at all (Gneiting 2011) because of its in-elicibility. For further explanation of elicity, I refer to Gneiting (2011). However, Kerkhof and Melenberg (2004) argue "[...] contrary to common belief, ES is not harder to backtest than VaR [...] furthermore, the power of the test for ES is considerably higher". Since the Basel Committee (2016) already recognized this problem, they proposed to backtest the ES with VaR 99% and 97.5%. If one of them fails the backtest, the ES can also be rejected. However, Costanzino and Curran (2015) even propose a traffic light test for Expected Shortfall. As already mentioned there are advanced methods (conditional tests) which consider the independence of exceptions. However, the simplest form, such as the Basel Committee traffic-light approach, solely focus on the number of exceptions which will be used within this thesis.

2.6.1 The Basel traffic light approach

The traffic light approach not only gives us information about the reliability of the underlying model but also has a multiplication factor add-on. As we will see later in chapter 3, banks must calculate a multiplication factor M_c

to Calculate the minimum capital requirements for market risk under both, Basel 2.5 and FRTB. This multiplication factor is a minimum of 3 plus an add-on depending on the backtesting result. Exactly how high this add-on will be is determined by using the “Basel traffic light “. The model is categorized into different zones depending on the number of exceptions that occur within it. According to the Basel Committee (1996), there are 0 to 4 exceptions within the green zone, 5 to 9 in the yellow and 10 or more in the red zone with a 99% confidence level and a period of 250 trading days (see table 2).

| zone | exceptions | Multiplication factor add-ons |
|-------------|------------|-------------------------------|
| Green zone | 0 – 4 | 0 |
| | 5 | 0.4 |
| Yellow zone | 6 | 0.5 |
| | 7 | 0.65 |
| | 8 | 0.75 |
| | 9 | 0.85 |
| Red zone | 10 | 1 |

Table 2 BCBS,1996 Basler traffic light- zone and multiplication factors for n=250 and a confidence level of 99%

However, it should be stressed that certain exceptions can be disregarded if the institute proves that the exception is not due to a lack of predictability in the risk model. In the red area, the model is inadequate and further use is prohibited. The following example explains how the corresponding factor is assigned to the respective exception number. At a confidence level of 99%, outliers should only occur with a frequency of 1% or 2.5 times. However, if 7 exceptions are observed for a sample of 250 days, the VaR would, in principle, only be based on a confidence level of $1 - 7/250 = 97.2\%$. The aim is to find a multiplication factor which can be used to scale the VaR to the required level of 99%. One must take the normal distribution assumption for the change in the portfolio and calculate the factor from the corresponding quantiles of the standard normal distribution (2.326 for 99% and 1.911 for 97.5%). The VaR number should be multiplied by a factor of $2.326 / 1.911 * 3 = 3.65$. This means that for the multiplier M, the constant 3 is increased 0.65. The factors for any other number of exceptions from table 1 can be calculated equivalently. If one wants to apply the Basel traffic light method for other confidence levels and other sample sizes, the critical number of exceptions which the different zones will correspond to can be recalculated. For a mathematical explanation, let X_t be the change in the portfolio’s value and N the size of the sample with:

$$A_t = \begin{cases} 1 & \text{for } X_t < -\text{VaR} \\ 0 & \text{for } X_t \geq -\text{VaR} \end{cases} \quad \text{for } t = 1, \dots, N$$

In the following, it is assumed that the individual A_t is independent of each other. Then $\sum A_t$ is binomially distributed with the parameters N, α and k (number of exceptions):

$$B(k, N, \alpha) = \sum_{i=0}^N \binom{N}{i} \alpha^i (1 - \alpha)^{N-i} \tag{2.3}$$

As already said under these settings, the daily returns are expected to exceed the VaR estimation by 2.5 times on average. Since the yellow zone begins at the cumulative probability of 95% and the red zone at a probability of 99.99%, we can calculate the exception values for these zones for different confidence levels using the binomial distribution. The assessment of the prognosis quality of the internal model is based on the results obtained Values of $\sum A_t$. Using the comparison of $\sum A_t$ with a single threshold number would classify the risk model as accurate or imprecise and is therefore confronted with a problem that is inherent in all statistical procedures. You can basically make two types of errors: (1) An inaccurate internal model is mistakenly classified as accurate (type 1) and (2) An accurate risk model is erroneously classified as inaccurate (2nd type error). These two errors cannot be minimized at the same time so that in many statistics (Hypothesis tests) only the test probability of the "worse" error type 1 is tested. The probability for the error type 2 can be reduced by increasing the sample size, this error type 2 is generally unrestricted. If the first type of error would occur, all the objectives linked to the use of the risk model are failed. Since a faulty model is neither suitable for risk control nor for risk monitoring. Error type 2 would lead to a higher capital adequacy. The Basel Committee has avoided this problem by simply using different Zones, which is limited by the number of exceptions. The definition of the three zones are defined as follows: (1)The yellow zone begins at the point where the probability of the specified number k or a lower number of exceptions, equal or is greater than 95% and (2) The red zone begins at the point where the probability of the specified number k or a lower number of exceptions, equal or is greater than 99.9%. Table 3 shows which zone (green, yellow, red) consist of how many exceptions under 97.5% confidence level and 99.0 % are allowed and table 3 shows the binominal distribution for 97.5% and 99% respectively.

| | F (250,0.025, X) | F (250,0.01, X) |
|------|------------------|-----------------|
| X≤0 | 0.0018 | 0.0811 |
| X≤1 | 0.0132 | 0.2858 |
| X≤2 | 0.0497 | 0.5432 |
| X≤3 | 0.127 | 0.7581 |
| X≤4 | 0.2495 | 0.8922 |
| X≤5 | 0.404 | 0.9588 |
| X≤6 | 0.5657 | 0.9863 |
| X≤7 | 0.7103 | 0.996 |
| X≤8 | 0.8229 | 0.9989 |
| X≤9 | 0.9005 | 0.9997 |
| X≤10 | 0.9485 | 0.9999 |
| X≤11 | 0.9753 | 1 |
| X≤12 | 0.989 | 1 |
| X≤13 | 0.9954 | 1 |
| X≤14 | 0.9982 | 1 |
| X≤15 | 0.9994 | 1 |
| X≤16 | 0.9998 | 1 |

Table 3 Binominal distribution retrieved

This means that the cumulative binomial distribution (formula 2.3) From k = 1 to N the probability that $\sum A_t \leq k$. Considering that k is an integer, we take the value at which this probability is 95% for the first time, as a limit value between green and yellow zone. The transition value from the yellow to the red zone is the value k for

which this probability is 99.9% for the first time. The table below (table 4) accounts for different observation periods and different target probabilities (α -values) and the distribution of the zones.

| n | Zone | $\alpha=2.5\%$ | $\alpha = 1\%$ |
|------|--------|----------------|----------------|
| | green | 0 to 9 | 0 to 4 |
| | yellow | 10 to 16 | 5 to 9 |
| 250 | red | 17 and above | 10 and above |
| | green | 0 to 18 | 0 to 8 |
| | yellow | 19 to 25 | 9 to 14 |
| 500 | red | 26 and above | 15 and above |
| | green | 0 to 33 | 0 to 15 |
| | yellow | 19 to 41 | 16 to 22 |
| 1000 | red | 42 and above | 23 and above |

Table 4 Basel traffic light exceptions

The green zone indicates an accurate model. However, zero exceptions might be an indicator of risk overestimation and should also be considered when checking a model's accuracy because it may mean the credit institutions hold too much capital. However, overestimation of risk is a problem unique to the green zone, as yellow and red zones will never face this issue. The yellow zone indicates that the model may have some accuracy problems, while the red zone indicates a problem with the underlying risk model. A point of criticism towards the Basel traffic light is that only the number of outliers, but not their height, are considered.

In summary, it can be said that backtesting as a form of model validation provides important feedback about model accuracy for Value at Risk and Expected Shortfall models. An accurate model needs to satisfy two equally important aspects. Firstly, the expected exceptions must be in line with the confidence level and secondly, these exceptions must be serially independent of each other.

3 The different Basel accords

This chapter deals with key documents published by the Basle Committee on Banking Supervision (BCBS) also known as banking supervision accords (recommendations on banking regulations) which was established in 1974 by the central bank governors of the Group of Ten (G-10) countries⁴. These accords have been published by the BCBS to achieve the main objectives of the BCBS namely: (1) Strengthening the banks' capital, (2) improving the quality of their capital, (3) strengthening the banks' transparency, (4) improving market discipline and (5) improving the banking sector's ability to absorb shocks. Therefore, the main goal of this chapter is to outline

⁴ As of June 1, 2017, the Committee consists of representatives of the following members: Argentina, Australia, Belgium, Brazil, Canada, China, France, Germany, Hong Kong, India, Indonesia, Italy, Japan, Korea, Luxembourg, Mexico, the Netherlands, Russia, Saudi Arabia, Singapore, South Africa, Spain, Sweden, Switzerland, Turkey, the United Kingdom and the United States.

important steps in the evolution of the Basle accords and highlight its deficiencies. Focus will be centered around the most recent documents by the BCBS. For a better understanding, I give a brief explanation of market risk. Market risk is the fluctuation of returns that result from macroeconomic factors affecting all risky assets (Acemoglu, Ozdaglar, & Tahbaz-Salehi, 2015). Market risk affects the entire market segment, not just a certain industry or stock, and is both unavoidable and unpredictable. Market risk can be mitigated using the right strategy of asset allocation or through hedging as opposed to diversification. Market risk occurs if a bank or other financial institution holds equity, commodity, FX or income positions. Therefore, several types of market risks can be identified. Among the major types of market risk is the interest rate risk. This type of risk involves a reduction in the value of a security once the interest rate rises, although many different types of exposures can arise in complex portfolios. Another type of market risk is the equity price risk which arises from volatility in prices of stock (Acemoglu, Ozdaglar, & Tahbaz-Salehi, 2015). Regarding the systematic risks, equity price risks may result from general market factors and affect the entire industry. Another type of systematic risk is the foreign exchange risk which is a result of changes or fluctuations in currency exchange rates. A company may be exposed to foreign exchange risks in its day to day operations because of imperfect hedges or unhedged positions (Acemoglu, Ozdaglar, & Tahbaz-Salehi, 2015). Commodity price risk is another type of systematic risk that may result due to unexpected alterations in commodity prices, for example, the price of oil, electricity or other resources used in the production process in the company. This thesis solely takes into consideration the equity risk which can be divided into idiosyncratic and systematic risk. Whereas idiosyncratic risk represents the default risk, systematic risk deals with market-wide risks including stock market volatility. Lastly, the credit spread risk is important to mention. Credit spread risk is the risk of the underlying issuer's credit rating changing. Hence it arises from possible changes in credit spreads and may influence the financial instruments' value. Credit spread is often understood as "compensation for credit risk" (Amato & Remolona, 2003).

3.1 Basel I

In 1988, the BCBS published the first internationally recognized capital requirements for banks known as Basel I after a discussion among bankers of the Bank of England and the Federal Reserve Bank. This accord was implemented in 1992 and mainly focused on credit risk (default risk), which is the risk of counterparty failure. This implementation was necessary due to the increasing investments of banks into off-balance-sheet products as well as loans to third world countries. The heretofore minimum capital requirement standard became insufficient. In paragraph 3 it states that "these are, firstly, that the new framework should serve to strengthen the soundness and stability of the international banking system; and, secondly, that the framework should be fair and have a high degree of consistency in its application to banks in different countries with a view to diminishing an existing source of competitive inequality among international banks". Within the Basel I accord borrowers were divided into several classes regarding their riskiness, where each group had different capital requirements. For example, banks which acted internationally needed to hold capital equal to 8% of their risk-weighted-assets (RWA). However, this classification was often illogical and did not illustrate the true amount of risk of each group. Moreover, market risk was completely neglected in this consideration. Furthermore, the Basel I accord lacked flexibility, and keeping up with market innovations were difficult. A step forward was made by

publishing the Basel I amendment in 1998 which tackled for the first time the problem of ignoring market risk. However, risks such as liquidity risk and operational risk were still ignored, even though they are important sources of insolvency exposures for banks. Banks could choose between two methods, the standardized approach (SA) and internal models approach (IMA), to calculate the amount of capital needed to cover their exposures to market risk. Whereas banks using the SA approach needed to follow the rules constituted in the Basel I amendment paper, the approach of the IMA was more open and banks did not have to use the VaR approach. Nevertheless, banks needed to meet qualitative standards using the IMA approach including the backtesting program, stress testing program etc. Additionally, the credit risk computation was revised in the 1996 published amendment, but this will not be evaluated since it is not a topic of this thesis. Banks using the Internal models approach needed to calculate the capital requirements using “the higher of (i) its previous day's Value-at-Risk number (Var_{t-1}) and (ii) an average of the daily value-at-risk measures on each of the preceding sixty business days (Var_{avg}), multiplied by a multiplication factor m_c ” (Basel Committee on Banking Supervision 2005, p. 41). The multiplication factor m_c is a value between 3 which is the minimum and 4 depending on the models' ex-post performance in the past. Mathematically it can be expressed as:

$$MCR = \max\{VaR_{t-1}, m_c * VaR_{avg}\} \tag{3.1}$$

3.2 Basel II

In 2004, the BCBS released the second accord, Basel II was introduced to improve the regulations of Basel I which was seen as imperfect mainly because of several reasons connected with credit risk (Crouhy et al., 2006). These drawbacks lead to a situation where a bank would “modify its behavior so that it incurs lower capital charges while still incurring the same amount of actual risk” (Crouhy et al., 2006) a so-called “regulatory arbitrage”. This arbitrage situation developed when the banks bent the rules by using securitization (e.g. mortgage-backed securities) and credit derivatives. To overcome the former mentioned shortcomings, the so-called “three pillars” capital regulation framework was developed (see figure 3)

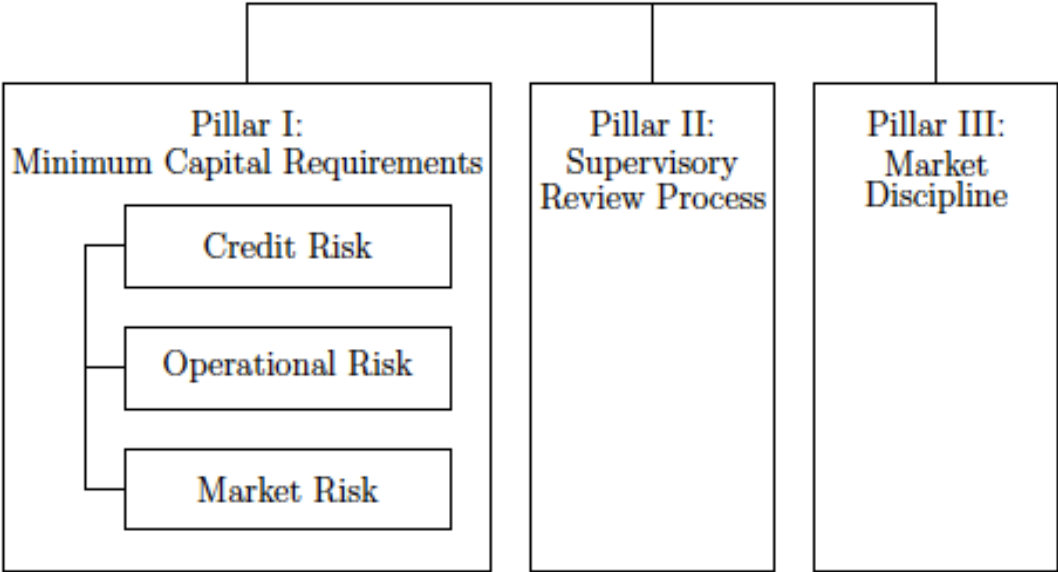


Figure 3 Basel II three pillar framework (Source: Basel Committee on Banking Supervision,2006)

Pillar I's main objective was to revise the Basel I accord for calculating the Minimum Capital Requirements. The 1998 accord only took into consideration credit risk. Later in the Basel I amendment, market risk was added. Basel II went a step further and implemented operational risk which is defined as risk of loss resulting from internal processes, human errors and system failures as well as external events. The second Pillar was created to ensure that banks measured their risk correctly and to dispose of regulatory arbitrage. The third pillar deals with disclosure requirements to investors. To calculate credit risk financial institutions could choose between three different approaches. The Standardized Approach and two internal rating-based approaches (IRB). The Market risk approach is the same as that presented in the 1996 amendment. Even though the Basel II was an improvement to the Basel I accord, a lot of literature such as Danielsson et al., (2001) criticized its ability to ensure a stable global financial system.

3.2.1 Basel 2.5

The financial crisis from 2007 to 2008 revealed several shortcomings in the Basel II accord and, as a reaction to the significant financial losses due to these shortcomings, the BCBS suggested several improvements in the "Revisions to the Basel II risk framework" (2009b). The capital framework introduced in the 1996 amendment was unsatisfactory and the new regulations required banks to hold capital against default and migration risk for un-securitized credit products, the so-called Incremental Risk Charge (IRC). The reason why migration risk wasn't included within the regulations was that most of the losses in the trading book arose from a reduction in creditworthiness rather than defaults. Furthermore, a stressed Value-at-Risk (SVaR) has been included, which makes it mandatory for banks to calculate additionally to the VaR a risk measure which is based on one-year data from a period of significant financial stress. The VaR in stressed market conditions for general market risk is based on a 10-day, 99th percentile, one-tailed confidence interval VaR measure with data from a continuous 12-month period of financial stress for example 2008/2009 (BCBS, July 2009).

The design of the Internal Models Approach under Basel 2.5 to calculate the minimum capital requirements for banks is the sum of the 1996 amendment introduced VaR plus the new Stressed VaR component, which is "the higher of (i) latest available stressed Value-at-Risk number ($SVaR_{t-1}$) and (ii) an average of the stressed Value-at-Risk numbers over the preceding sixty business days ($SVaR_{avg}$), multiplied by a multiplication factor (m_s)" (Basel Committee on Banking Supervision 2009b, p. 15). Mathematically it can be expressed as:

$$CA = \max\{VaR_{t-1}, m_c * VaR_{avg}\} + \max\{SVaR_{t-1}, m_s * SVaR_{avg}\} \quad 3.2$$

The multiplication factor m_c and m_s is set by supervisory authorities and are subject to a minimum of 3. A "plus", which ranges from 0 to 1, is added to these factors based on the ex-post performance of the model. (BCBS, July 2009). The Stressed VaR purpose is intended to "replicate a VaR calculation if the relevant market factors were experiencing a period of stress; and should be therefore based on the 10-day, 99th percentile, one-tailed confidence interval VaR measure of the current portfolio, with model inputs calibrated to historical data from continuous 12-month period of significant stress to the bank's portfolio" (Basel Committee on Banking Supervision 2009b, p. 14). For calculating the 10-day VaR, the Basel committee allows the following formula:

$$10 - day VAR = \sqrt{10} x 1 - day VaR \quad 3.3$$

Although the Basel 2.5 was an improvement compared to Basel 2, some structural problems of the framework remain (BCBS, May 2012). The BCBS identified problems regarding the risk measurement methodologies and shortcomings regarding the IMA (BCBS, May 2012, pp. 53-56). The inability to capture credit risk was admittedly identified with the introduction of the Market Risk amendment in 1996. However, 20 years ago exposures related to credit instruments in the trading book were just a small portion of all risky instruments. Unfortunately, financial innovations surpassed the regulations and massive losses have been caused due to traded credit. Another shortcoming of the Basel 2.5 accord was its inability to capture market liquidity risk. Banks lacked in their ability to hedge or exit their positions during the 2008 crisis in the short run due to an illiquid market. Furthermore, the regulations were inadequate to capture basis risk, meaning that correlations often haven't been estimated based on normal market data which did not hold true in a stressed period. This led to a situation where expecting hedging benefits did not materialize. Lastly, the individual risk assessments of banks haven't been enough to capture risk from the perspective of the banking system. Each bank assumed to be able to exit or hedge its position quickly using the IMA. Unfortunately, if all the banks have a similar exposure at the same time, the market may turn illiquid.

3.3 Basel III

In 2010, the BCBS introduced Basel III which attempted to fix the shortcomings of the previous accords (BCBS, 2010). As stated before the Basel 2.5 accord was an initial response to the financial crisis. The aim of Basel III was to strengthen the banking industry's resilience and create a more shock resistant banking industry from economic and financial stress and the spillover risk from the financial sector to the real economy. Hence Basel 3 is a more elaborated and thorough framework built upon Basel I and Basel II given by the BCBS. Its objective is to improve the banks' transparency its disclosures to investors to enhance the ability to absorb shocks arising from financial and economic stress. Although the three pillars of Basel II norms (see figure 1) remained the same, the Basel III accord proposed major changes to the Basel 2.5 regulations. To accomplish the aforementioned objectives, the BCBS enhanced multiple areas of the Basel Framework. Firstly, it introduced a much stricter definition of capital. This led to a higher loss-absorbing capacity for banks, resulting in credit institutions better able to withstand stress. Secondly, the BCBS introduced a 2.5 % conservation buffer requirement which ensures that banks maintain a capital reserve and that they can use them to absorb losses during a stressed period. Moreover, a countercyclical buffer has been introduced. The goal of this countercyclical buffer is to increase capital requirements in a good economic state and decrease it in bad ones. This leads to a slowdown of banking activity in good times and encourages lending in bad times. This buffer consists of common equity or other fully loss-absorbing capital and ranges from 0% to 2.5%. Additionally, the minimum common equity and Tier I capital requirements have been increased. Furthermore, a leverage ratio, the relative amount of capital to total assets (not risk-weighted), has been introduced. The purpose of this leverage ratio is to secure the swelling of leverage in the banking sector. Lastly, a framework for liquidity risk management has been introduced. However, the Market risk framework has not been tackled in 2010 published Basel 3 accord and remains as proposed in Basel 2 for the SA and as regulated in Basel 2.5 for the IMA. Therefore, a comparison of the IMA between Basel 2.5 and FRTB as presented in Chapter 3.4.1 will be conducted.

3.3.1 FRTB

The Fundamental Review of the Trading Book (FRTB), a consultative paper of the Basel 3 accord, published in January 2016 by the BCBS, sets new standards and rules on how banks assess minimal capital requirements in the trading book to take countermeasures of the Basel 2.5 shortcomings as presented above in chapter 2.3. The new framework tackles both the Basel II untouched SA but also the Basel 2.5 (2009) revised IMA, and additionally it introduces a new boundary between the trading and banking book. The goal of the BCBS is to fully implement these rules by the end of 2019. As the name suggests, these rules make an incremental change on how minimal capital requirements will be calculated for the Trading Book and have been subject to an intense industry-wide debate. The BCBS expect that these rules will change the financial markets in a significant way. These FRTB rules are known as minimum capital requirements for market risk. As already mentioned, the key objectives of the FRTB can be summarized by the following aspects: (1) Reducing banks' capital arbitrage abilities by transferring transactions between banking book and trading book, (2) a re-designed standardized approach and (3) a revised internal models approach. Only the last objective is important for this thesis and therefore the revised internal model approach will be illustrated in more detail. The revised IMA replaces the current 99%, 10-day VaR/stressed- VaR approach with the new 97,5% stressed Expected Shortfall (ES). This measure will be implemented to improve the measurement of tail risk by averaging tail losses. It accounts for market liquidity by varying liquidity horizons and recognizes stressed correlations through restraints on diversification benefits. Of course, there are more changes, but the most important have been illustrated above and the others can be placed into one of these three main categories. This thesis' purpose is to determine the effect of minimum capital requirements for market risk by the change from the VaR/stressed VaR approach in Basel 2.5 to the stressed expected shortfall. A description of how to calculate the stressed ES under FRTB regulations is given in the following paragraphs. The ES must be calculated daily (BCBS 2016, p.52 (a)) with a 97.5% one-tailed confidence level for each trading desk. An appropriate liquidity horizon needs to be used for scaling up an ES from the base horizon of 10 days. It is not possible to scale up the ES from the horizon shorter than the base horizon. The following Formula needs to be used to calculate the stressed (calibrated) ES:

$$ES = ES_{R,S} * \frac{ES_{F,C}}{ES_{R,C}} \quad 3.4$$

Where $ES_{R,S}$ is the Expected Shortfall based on a stressed observation period using a reduced set of risk factors. $ES_{F,C}$ is the Expected Shortfall based on the most recent 12-month observation period with a full set of risk factors and $ES_{R,C}$ is the Expected Shortfall for the most recent 12-month period with a reduced set of risk factors. The Ratio $ES_{F,C}, ES_{R,C}$ is floored at 1 (BCBS, 2016). The stressed observation is that point in time where the portfolio experienced the largest loss over a period of 10 years. The reduces risk factors must explain a minimum of 75% of the P&L variance and full historical data (10 years) must be available. These reduces risk factors are specified by banks "that are relevant for their portfolio and for which there is a sufficiently long history of observation" (BCBS,2016). Due to this reduced risk factor component, it is possible to have a portfolio which also consist of instruments which do not have a financial history of 10 years. Hence, three different ES namely $ES_{R,S}$, $ES_{F,C}$ and $ES_{R,C}$ needs to be calculated. As already mentioned, the ES needs to be scaled up to the inherent liquidity horizon. The liquidity horizon is the length for which the expected maximum loss is valid. It is also referred to as

the holding period (Dutta & Bhattacharya, 2008). The ES or VaR is usually smaller for a liquidity horizon of one day than for a month, which can be interpreted as meaning that larger deviations in the portfolio's value are more likely over a long period than in a short one (Dutta & Bhattacharya, 2008). Over the liquidity horizon, the portfolio composition is assumed to be static for ES and VaR. According to Christoffersen et al. (1998), the determination of an adequate liquidity horizon is contingent upon whether you measure from a regulatory or private perspective. Other factors to bear in mind while determining an adequate liquidity horizon are trading activity and the liquidity⁵ of assets (Khindanova and Rachev, 2000). Even though the liquidity horizon can vary between one trading day and some years, the BCBS obligates financial institutions to make use of a 10-day liquidity horizon to calculate VaR (Basel Committee, 2006). However, even this is seen to be inadequate for illiquid and frequently traded assets by Khindanova and Rachev (2000., The VaR will be calculated with a 10-day liquidity horizon. In their newest document (Basel Committee, 2016), the BCBS requires a liquidity horizon of 10 days for major interest rates markets and listed large-cap equities, and up to 120 days for exotic credit spreads to calculate the Expected Shortfall (see Table 5).

⁵ degree to which an asset or security can be bought or sold

| Risk factor category | Lh | Risk factor category | Lh |
|---|-----|---|-----|
| Interest rate: specified currencies - EUR, USD, GBP, AUD, JPY, SEK, CAD and domestic currency of a bank | 10 | Equity price (small cap): volatility | 60 |
| Interest rate: – unspecified currencies | 20 | Equity: other types | 60 |
| Interest rate: volatility | 60 | FX rate: specified currency | 10 |
| Interest rate: other types | 60 | FX rate: currency pairs ⁶ | 20 |
| Credit spread: sovereign (IG) | 20 | FX: volatility | 40 |
| Credit spread: sovereign (HY) | 40 | FX: other types | 40 |
| Credit spread: corporate (IG) | 40 | Energy and carbon emissions trading price | 20 |
| Credit spread: corporate (HY) | 60 | Precious metals and non-ferrous metals price | 20 |
| Credit spread: volatility | 120 | Other commodities price | 60 |
| Credit spread: othertypes | 120 | Energy and carbon emissions trading price: volatility | 60 |
| Equity price (large cap) | 10 | Precious metals and non-ferrous metals price: volatility | 60 |
| Equity price (small cap) | 20 | Other commodities price: volatility | 120 |
| Equity price (large cap): volatility | 20 | Commodity: other types | 120 |

Table 5 liquidity buckets and corresponding liquidity horizons: (BCBS, January 2016)

The BCBS requires banks to scale up the ES from the base horizon of 10-days to the corresponding liquidity horizon (see table 5). The following Formula (3.5) needs to be used to adjust the ES to the corresponding liquidity horizon:

$$ES = \sqrt{(ES_T(p))^2 + \sum_{j \geq 2} (ES_T(p, j) \sqrt{\frac{(LH_j - LH_{j-1})}{T}})^2} \quad 3.5$$

where

- ES is the regulatory liquidity-adjusted expected shortfall;
- T is the length of the base horizon, i.e. 10 days;
- $(ES_T(p))$ is the expected shortfall at horizon T of a portfolio with positions $P = (p_i)$ with respect to shocks to all risk factors that the positions P are exposed to

⁶ USD/EUR, USD/JPY, USD/GBP, USD/AUD, USD/CAD, USD/CHF, USD/MXN, USD/CNY, USD/NZD, USD/RUB, USD/HKD, USD/SGD, USD/TRY, USD/KRW, USD/SEK, USD/ZAR, USD/INR, USD/NOK, USD/BRL, EUR/JPY, EUR/GBP, EUR/CHF and JPY/AUD.

- $ES_T(P, j)$ is the expected shortfall at horizon T of a portfolio with positions $P = (p_i)$ with respect to shocks for each position p_i in the subset of risk factors $Q(p_i, j)$, with all other risk factors held constant;
- LH_j is the liquidity horizon j.

For a better understanding of this formula (see formula 3.6) suppose you have a pure currency portfolio invested in different currencies. From the table above we know that currency risk factors will be subject to liquidity horizons of 10 days and 20 days This means that $ES_{10}^{10\ 20}$ is calculated by shocking all the risk factors with its liquidity horizon. Based on this, the formula can be written as:

$$ES = \sqrt{(ES_{10}^{10\ 20})^2 + (ES_{10}^{20})^2 * \frac{(20 - 10)}{10}} \quad 3.6$$

The obligation to calculate the stressed (calibrated) ES (formula 3.5) and the additionally adjustment to the different liquidity horizons stress the high computational demand of the calculation method. The capital requirements (C_A) are then the max of “the higher of (1) its previous day’s aggregate minimum capital requirements for market risk; and (2) an average of the daily capital measures in the preceding 60 business days’ times” times a multiplication factor based on the previous day backtesting result (BCBS, 2016). IMCC is equal to the Stressed ES since we have for each instrument a full data history. Therefore, the minimum capital requirements for market risk are calculated using the following equation:

$$MCR = \max(IMCC_{t-1} + SES_t; mc * IMCC_{avg} + SES_{avg}) \quad 3.7$$

3.4 Summary of Important Regulations

For a better comprehensibility, I will shortly provide a summarized comparison of the most relevant changes for the IMA between Basel 2.5 and FRTB

| | <u>Basel 2.5:</u> | <u>FRTB:</u> |
|-------------------|---|---|
| Risk Measure | VaR + Stressed VaR component | Stressed Expected shortfall |
| Confidence Level | VaR 99 % | ES 97,5% |
| Liquidity Horizon | 10-day returns to calculate VaR or scale a 1-day VaR to 10-days | Basis liquidity horizon of 10-days which needs to be scaled by mapping each risk factor to one of the risk categories |

Table 6 Comparison of Basel 2.5 and FRTB

4 Simulation methods

According to Linsmeier and Pearson (1996), the three most common methods for calculating the VaR are the variance-Covariance method, a Monte Carlo simulation and historical simulation. I only speak about VaR calculation methods. However, it is important to mention that each method is also applicable for the ES since the ES or Conditional VaR builds upon the VaR as shown in Chapter 2. In addition to the methods mentioned above, other methods exist to calculate VaR such as the Orthogonal Garch model see for example Poon and Granger (2003) or Floros (2007) which are often used to forecast the volatility, the Extreme Value Theory (EVT)

(Frey, 2000), the conditional Autoregressive Value at Risk (CAViAR) (Engle and Manganelli, 2004) method and the Exponential Weighted Moving Average method (EWMA) which was developed by Zangari (1994). However, the BCBS suggest historical simulation (HS) and Monte Carlo (MC) simulation as estimation methods for calculating the VaR and ES respectively. Both, the HS and MC approaches are used to generate an empirical distribution function of changes in value. However, Pérignon and Smith (2010) reported, that 73% of banks which disclose their methodology to calculate VaR use historical simulation, and therefore the historical simulation will be used for this thesis. The literature came up with different improvements to the traditional historical simulation such as the age-weighted historical Simulation and volatility-weighted HS. From now on, the change in value will be represented as it is accepted in the literature (Linsmeier and Pearson, 1996) as log returns $\ln(P/P_{t-1})$. Furthermore, another term will be defined, the observation period (n), which determines the length of the historical data on which the simulation methods build up to. The observation period is therefore the size of the sample. Before the simulation methods will be used, this chapter will explain the underlying architecture, mathematical background as well as advantages and disadvantages of the HS.

4.1 Historical Simulation (HS)

The historical simulation uses historical data from financial price changes to compute the VaR and ES. The main idea behind the historical simulation is that a prediction for the future can be made based on historical performance and does not make use of distribution assumptions. According to Dowd (2007, p. 39), the “simplest way to estimate VaR is by means of historical simulation (HS)”. Linsmeier and Pearson (1996) gave the following terminology of historical simulation: *“The distribution of profits and losses is constructed by taking the current portfolio, and subjecting it to the actual changes in the key factors experienced during each of the last α periods [...]. Once the hypothetical mark-to-market profit or loss for each of the last α periods have been calculated, the distribution of profits and losses and the value-at-risk can then be determined.”* In other words, historical simulation makes use of the Monte Carlo method but differs in that the Monte Carlo simulation makes use of a pseudo-random realization of a sample while the historical simulation constructs it from distributions derived from historical data. It is therefore assumed that any return observed in the past with the probability of $1/m$ can also occur in the future. The historical simulation is thus based directly on historically observed realizations expressed in the form of the P & L time series. A prerequisite for the implementation of the HS is a suitable data preparation and data selection. All positions of the credit institution must be recorded and uniformly valued by the same system. There should be a company-wide central database. Equally important is the quality assurance of the historical time series used, since the accuracy of the results depends decisively on the quality of the time series. For example, missing values for some market parameters (such as a new equity issue) or many outliers due to a lack of data quality or exogenous shocks (such as currency conversions) are problematic. These problems must be identified and corrected by approximation and "conversion." historical simulation is the most used method for calculating value-at-risk. As already mentioned Pérignon & Smith (2010) report that 73% of banks which disclose their methodology make use of historical simulation to calculate VaR in 2005. According to Pérignon and Smith (2010) this popularity of the historical simulation is attributed to the size and complexity of banks' trading positions. Since banks make use of thousands of risk-factors, the parametric methods are difficult

to conduct. Additionally, banks make use of this method due to the smoothness of its risk forecast (Pérignon & Smith, 2010b), meaning that historical simulation only reacts slowly to changes in volatility and hence the VaR and ES estimates don't change from day to day. The historical simulation is a method with easy calculations (Best 1996), without the need to make estimations of statistical parameters (Bohdalova, 2007). Furthermore, it captures the real distribution of the factor and no assumption must be made (Stambaugh, 1996). Another important aspect is that the historical simulation is easy to explain to a person unfamiliar with computing risk (Penza and Bansal, 2001). According to the study of Finger (2006) risk managers respond as follows to the question, "Why they make use of the historical simulation:" "It is easy to explain", "It is conservative", it is "assumption-free", "It captures fat tails⁷" and "It gives insights into what could go wrong." A general description of calculating VaR for a portfolio using the historical simulation method is given by Linsmeier and Pearson (1996). Since the ES is mainly equal to the VaR, this approach also holds for the ES:

Step 1: First the basic market factors for the observation period N will be recognized, the price determining parameter. In this thesis case, these parameters are the daily exchange rates of the different currencies. Obtain the historical values of assets within the portfolio for the last N periods.

Step 2: Calculate daily return or price changes of all the assets in the portfolio for the pre-defined observation period N. For calculating the daily return, the following method will be used: Lognormal of today price divided by yesterday price written as:

$$\ln\left(\frac{P_t}{P_{t-1}}\right) \quad 4.1$$

Step 3: Calculate the portfolio's return (Profit & Loss). The P/L is calculated as follows:

$$P/L_t = \sum_{i=1}^N w_i r_{i,t} \quad 4.2$$

With w_i is the weight invested in asset i and $r_{i,t}$ is the return on asset i in period t.

Step 4: Generate Hypothetical Values by multiplying, for example, the change between the first and the second day of the selected period with today's actual value of the Portfolio. This will, in turn, generate hypothetical Portfolio values. (Linsmeier and Pearson, 1996)

Step 5: Sort the simulated Profit & Loss (P&L) of the portfolio from the lowest to the highest

Step 6: Identify the corresponding value to the desired confidence level. "To yield a 99 percent confidence interval the largest loss would instead be named the VaR" (Linsmeier and Pearson, 1996) if you have a 100 observations period. For a one-year observation period (252 days) the VaR would be the $(252 * 0.01 = 2.52)$ third largest loss. Another method determining the VaR, which is preferable using the Age-weighted historical simulation, is to start from the lowest return and keep accumulating the weights until 99% is reached (Boudoukh

⁷ The likelihood of a loss is greater than would be implied by the normal distribution

et al., 1997). To calculate the ES using historical simulation, all steps are the same except that you calculate the average of the simulation according to the confidence level's obligated largest losses. A similar approach is described by Crouhy et al. (2006), which stresses the correctness of this approach. It is important to consider that both the VaR and ES will always be negative. Since within Step 2, the price changes from day t-1 to date t are calculated and since it isn't realistic, that a financial instrument will constantly rise without any decrease in value, the ES and VaR are always negative. For a better understanding, I will give a short example of the steps 1-4 on historical data with a portfolio consisting of 3 currency pairs (EUR/USD, EUR/GBP and EUR/CAD) each with the same weight within the portfolio of 1.000 € hence a weight of 1/3. The Steps 5 and 6 are not be included in this example since for a significant result there are not enough data provided within the example. This whole procedure from step 1 up to step 6 is conducted within excel.

Step 1:

| Date | Exchange rates | | |
|----------------|----------------|---------|----------|
| | EUR/USD | EUR/GBP | EUR/CAD |
| March 23, 2017 | 1.07865 | 0.86306 | 1.438078 |
| March 22, 2017 | 1.08 | 0.86593 | 1.443604 |
| March 21, 2017 | 1.07816 | 0.86826 | 1.437543 |
| March 20, 2017 | 1.07516 | 0.86807 | 1.434349 |

Step 2: The daily return was calculated using the $\ln\left(\frac{P_t}{P_{t-1}}\right)$ Method.

| Date | Daily return in % | | |
|----------------|-------------------|----------|----------|
| | EUR/USD | EUR/GBP | EUR/CAD |
| March 23, 2017 | -0.1251% | -0.3320% | -0.3835% |
| March 22, 2017 | 0.1705% | -0.2687% | 0.4207% |
| March 21, 2017 | 0.2786% | 0.0219% | 0.2224% |

- $EUR/USD (March 23,2017) = \ln(1.07865 / 1.08) = -0.001251$

Step 3: For simplicity, the portfolios weights are equal of 1000€. So, each portfolio got a weight of 1/3.

| Date | Portfolio change | | |
|----------------|------------------|-----------|-----|
| | 33% | 33% | 33% |
| | Portfolio P/L | | |
| March 23, 2017 | | -0.28020% | |
| March 22, 2017 | | 0.10751% | |
| March 21, 2017 | | 0.17432% | |

- $Portfolio (March 23,2017) = (-0.1251%)*(1/3) + (-0.3320%)*(1/3) + (-0.3835%)*(1/3) = -0.28020%$

Step 4: Calculation of the Portfolio Values (Profit & Loss) by multiplying by the total value of the portfolio.

| Date | Portfolio (Profit & Loss) |
|----------------|---------------------------|
| | € |
| March 23, 2017 | -8.41 |
| March 22, 2017 | 3.23 |
| March 21, 2017 | 5.23 |

- *Portfolio P/L (March 23.2017) = (-0.28020%)*3000*

4.1.1 Advantages and shortcomings of the historical simulation

As has already been shown, the often-accepted normal distribution assumption is not consistent with practice. For this reason, it is favorable that no assumption of distribution is necessary for the implementation of the HS, but the real values that have occurred in the past should be assumed one-by-one for the future as well. Parameters such as volatility or correlations need not be estimated because they are implicitly considered, so that certain statistical problems, such as the elaborate and difficult calculation of large covariance matrices, can be circumvented from the outset. In addition, historical interconnection effects (for example, the simultaneous inclusion of interest rate and currency risk) can be recorded relatively easily. The risk potential of any desired aggregation level can be determined by simply adding the gains and losses of the same dates (Meyer, 1999 p.194). This applies both within and between different risk classes, risk types and sub-portfolios. However, the high level of acceptance in practice (for example, in the senior management of credit institutions) is mainly due to the simple understanding of the method. It is intuitive and easily reproduced and is therefore very suitable for reports and presentations.

The simple assumptions of the HS, also have disadvantages. Firstly, predictions are based on historical data, but markets can rapidly change, thereby rendering historical data unsuitable for future predictions. Factors that cause a change in the market are for example new technologies, new regulations, changed perceptions due to a scandal or a crisis or simply economic decline or growth. Therefore, historical data may be outdated and may not correctly correspond to future outcomes. (Penza and Bansal,2001). Secondly, Stambaugh (1996) argues that it is difficult to choose period's and their length. A longer period could generate more accurate results due to fewer sampling errors, but it could also question the validity of it. Thirdly, a high computing effort. For a daily VaR calculation, the complete portfolio must be included daily for all positions found therein are re-measured m times. For a very large financial institution, this can mean that the HS is not feasible for certain Portfolios (e.g., the overall portfolio). Lastly, each days' return is assigned with equal weights, regardless of age, market volatility

and the value it takes (e.g., whether it is extreme) (Dowd 2007, p. 65). Proof that this is not realistic is given by Shimko et al. 1998 who gave the example of gas prices which are, according to Shimko et al. (1998), usually more volatile in the winter months. These results, when giving each historical observation the same weight, can lead to “underestimation of risk in the winter, and overestimate them in the summer” (Dowd 2007, p. 64). This disadvantage is also demonstrated in a portfolio held during the 1987 equity crash by Pritsker (2006). Furthermore, having equal weights might result in a situation where major events, such as stock market crashes, have no effect on the VaR even though they have a very high confidence level. So, there might be a situation where risk increased, but the risk measure did not. Only after a continued downward trend in the upcoming days would this increased risk show within the risk measure VaR. However, the last-mentioned effect only applies to the VaR. Major events immediately show up in the ES estimation, which is a good example of why the BCBS sets the ES as the new risk measurement standard for calculating the IMA. Another aspect to consider is the distortion possibility and ghost effect (Dowd 2007, p. 66). Suppose there is a high single loss observation which leads to an unduly high-risk measure. These high-risk measures remain high until n-days have passed and this high loss observation will not be part of the sample anymore. At this point in time the risk measure will fall however, this is only a ghost effect created by “the weighting structure and the length of sample period used” (Down 2007, p.66) because the results of the VaR and ES are totally dependent on the underlying data set. This dependency may lead to different problems including a distortion of results because of unusually quiet or volatile periods which lead to estimates which are too low or high respectively (Dowd, 2007 p.72). Additionally, Pritsker (2006) discovered that recent historical data is more important for future returns than historical data further back in time, and therefore the former should get more weight. To overcome these drawbacks, the literature came up with adjusted methods which are based on the unweighted historical simulation. The BCBS makes clear that it is permissible for banks to use models based on historical simulation (BCBS, 2009 p.11). In the following sub-chapters, I will explain methods based on the unweighted historical simulation.

4.2 Weighted historical simulation

To incorporate the shortcomings of equal weighted returns, the literature came up with methods which weight the data to reflect their relative importance. These methods are called Weighted Historical Simulation. Dowd (2007) presents various methods including the (1) Age-weighted Historical Simulation (AWHS) and the (2) Volatility-Weighted Historical Simulation (VWHS). These methods will be presented in the following paragraphs for a thorough overview of the different historical simulation methods. However, they will not be part of this thesis. Calculations will be conducted using unweighted historical simulations because, while there are more sophisticated historical simulation methods, this thesis will follow the approach used by Cabedo and Moya (2003) in their study which made use of the traditional historical simulation approach to estimate oil price VaR.

4.2.1 Age-weighted Historical Simulation (AWHS)

A method to weight the data to their relative importance is proposed by Boudoukh, Richardson and Whitelaw (1997), who weight recent data more by using a decay factor as time weighting mechanism. This is reasonable

since recent past returns are better qualified for making predictions on the future than returns from the distant past (Damodaran, A.,2014). Under the Age-weighted Historical Simulation, each return is assigned a to a probability weight based on its newness, rather than being weighted equally (i.e., $1/n$ with n =number of observations), as within the unweighted historical simulation approach. The weight of observation t is given by:

$$w(t) = \frac{\lambda^{t-1}(1-\lambda)}{1-\lambda^n}, \text{ with } \lambda \in (0,1) \quad 4.3$$

Note that the constant $[(1-\lambda)/(1-\lambda^n)]$ ensures that weights sum to 1. The Factor λ reflects “the exponential rate of decay in the weight or value given to an observation as it ages: a λ close to 1 indicates a slow rate of decay, and a λ far away from 1 indicates a high rate of decay” (Dowd, 2007, p.66). This technique was demonstrated by Boudoukh et al. (1997) by computing the VaR immediately before and after the crash 1987 with an observation period of 250 days using the unweighted historical simulation and the Age-weighted Historical Simulation. With the former simulation method, there was no change the day after the crash, however under the latter simulation the VaR quickly adjusted to the new circumstances. Boudoukh et al. (1998) recommend using $\lambda = 0.98$. In fact, the simple historical simulation is a special case with $\lambda=1$ (Dowd, 2007, pp.66-76). To implement this method, assign to each period (t) its weight using formula (6). Then follow steps 5 and 6 as presented by Linsmeier and Pearson (1996) above.

4.2.2 Volatility-weighted Historical Simulation (VWHS)

Another possibility is to weight the data by volatility. Hull & White (1998) suggested taking into consideration the changes in volatility for returns. Hence, incorporating volatility into historical simulations is referred to as Volatility-weighted Historical Simulation (Dowd 2007, pp 67-69). According to Boudoukh et al. (1998), this method outperforms both the Historical simulation and the age-weighted historical simulation for currencies, however it performs just a bit better on a 1-day horizon for different equity indices. The underlying idea of the WHS is to adjust the returns to recent changes in volatility. For example, today’s volatility is 2% and it was 3% a month ago, then the latter data overstated the changes we can expect tomorrow and vice versa. Let $r_{t,i}^*$ the volatility weighted historical return with $r_{t,i}^*$ being the historical return at time t for asset i where $t < T$, $\sigma_{T,i}$ denotes the volatility estimate for tomorrow for asset i and $\sigma_{t,i}$ the historical estimate for time t for asset i .

$$r_{t,i}^* = \sigma_{T,i} * \frac{r_{t,i}}{\sigma_{t,i}} \quad 4.4$$

To implement this method, substitute the original return data ($r_{t,i}$), with the adjusted returns ($r_{t,i}^*$) calculated with equation (4.1) to calculate P&L (Dowd 2007 p. 94-95) and then proceed to calculate the VaR and ES way as shown in the unweighted historical simulation. The volatility $\sigma_{t,i}$ will be estimated using a Generalized AutoRegressive Conditional Heteroscedasticity (GARCH) model. In this study, the GARCH (1, 1) model will be used which is according to Mun (2010) a model which is frequently used to estimate volatility in traded and liquid assets. The GARCH (1.1) model looks like

$$\sigma_{t+1} = \omega + \alpha_1 \sigma_t^2 + \beta_1 \sigma_t^2 \quad 4.5$$

Where σ denotes variance, σ^2 the squared residual and t the period. The constants ω, α, β are modeled for every period using maximum likelihood estimations. The equation above tells us tomorrows variance consist of the

following three components: (1) today's squared residual, (2) today's variance and (3) weighted average long-term variance

This approach has some advantages compared to the equal weighted and age-weighted approaches (Dowd, 2007, p.68). However, as already mentioned, solely the unweighted historical simulation will be used for my thesis. The main assumption behind this method is that price change behavior repeats itself over time. This implies that future prices can be derived from past price changes (Hendricks, 1996). Moreover, using the unweighted historical simulation was been used within other papers such as Cabedo & Moya (2003).

5 Methodology

In the preceding chapters, all the theoretical foundations were discussed which are necessary for the calculations and to answer the research questions as already stated above. First the underlying data set is presented together with its distribution and parameters followed by a description of the appropriate portfolio structures. This chapter is finalized by instructions of the exact procedure on how to conduct the necessary calculations. For all calculated risk numbers, it is assumed that the liquidity horizon is one day and the portfolio remains constant during this period. The calculations are carried out using historical simulation. To derive to proper answers of the research questions, as already presented in the introduction, two hypotheses are formulated: (1) *There is a significant difference between VaR or ES with $n=1000$, VaR or ES with $n=500$ and VaR or ES with $n=250$ and (2) There is a significant difference between HS VaR (99%) and HS ES (97.5%).* In both hypotheses, the dependent variable is market risk charge for a specific portfolio, which is described by the independent variable, the two related risk measures ES and VaR. A further and more in depth elaboration of these hypotheses is conducted in the course of chapter 6, the empirical results section.

5.1 Data

5.1.1 Sample Period

The Historical observation period describes the time length of the data sample and can differ from 100 business days up to 5 years (Best,1998). Khindanova and Rachev (2000), arguing that the longer history of the historical returns is, the more accurate are the forecasts. Also, problems of data period length itself bear problems so could it be difficult to collect long enough data especially data from new or emerging market instruments (Penza and Bansal, 2001). However, according to Hendricks (1996) the regulatory one-year historical observation period produces rather accurate forecasts when using common volatility models. The BCBS sets a one-year minimum observation period plus a one-year stress period to calculate minimum capital requirements for market risk (Basel Committee, 2009) and a 10-year history of returns for minimum capital requirements for market risk under FRTB regulations (Basel Committee, 2016).

5.1.2 Sample

The financial data used for calculating VaR and ES could either be a single financial instrument or portfolios. The performance of the VaR and ES will be tested on different portfolios consisting of the exchange rates of currency Pairs. Hence this thesis is examining currency risk which expresses the risk of losses that occur if you have money invested in a specific currency A and not within another currency B and the rate of currency B in contrast to currency A improves. The data was retrieved from www.oanda.com⁸. The sample consist of the following

⁸ EUR= EURO, USD=US-Dollar, GBP=British Pound, CZK=Czech Krone, DKK=Danish Krone, NOK=Norway Krone, CHF=Swiss Franc, JPY=Japanese Yen, PLN=Polish zloty, RUB=Russian Ruble

currencies' Pairs: EUR/USD, EUR/GBP, EUR/CAD, EUR/CZK, EUR/DKK, EUR/NOK, EUR/CHF, EUR/JPY, EUR/PLN and EUR/RUB. All currencies are expressed in relation to €, meaning the value express for example how much USD you get for 1€. The sample consists of 3920 Historical time series of daily exchange rates. As has already been shown, the VaR and ES calculations are not the exchange rates, but their daily changes are relevant. We need the daily log-returns of the exchange rates. This will be calculated using formula (4.1). Furthermore, it is important to recall table 4 and the underlying liquidity horizons of the different currency pairs. The following pairs have a liquidity horizon of 10 days: EUR/USD, EUR/JPY, EUR/GBP and EUR/CHF. So, all other pairs need further liquidity adjustments as presented in formula 3.5.

5.1.3 Distribution and parameter of the risk factors:

To construct adequate portfolios and being able to interpret the results, the distribution and underlying parameters of the daily change of the different currencies have been determined. In Table 6 the distribution of the individual changes in exchange rates are displayed. The most important parameters such as mean value, Variance, median, minimum and maximum are indicated. For each currency, the sample size is 3920.

| | Mean | Median | Standard Deviation | Sample Variance | Minimum | Maximum |
|---------|-------------|-------------|--------------------|-----------------|-------------|------------|
| EUR/USD | -0.00003101 | 0.00000000 | 0.00427913 | 0.00001831 | -0.02525064 | 0.03457261 |
| EUR/GBP | 0.00007307 | 0.00000000 | 0.00380726 | 0.00001450 | -0.02912687 | 0.04507072 |
| EUR/JPY | -0.00004873 | 0.00000000 | 0.00563706 | 0.00003178 | -0.04946311 | 0.05185636 |
| EUR/PLN | 0.00002860 | 0.00000000 | 0.00449999 | 0.00002025 | -0.03733949 | 0.03346750 |
| EUR/RUB | 0.00018031 | 0.00000789 | 0.00830709 | 0.00006901 | -0.09687875 | 0.10389568 |
| EUR/CHF | -0.00009340 | -0.00000606 | 0.00375313 | 0.00001409 | -0.11657664 | 0.04091262 |
| EUR/NOK | 0.00003111 | -0.00000253 | 0.00351602 | 0.00001236 | -0.03855767 | 0.02446007 |
| EUR/DKK | -0.00000071 | 0.00000000 | 0.00013113 | 0.00000002 | -0.00145455 | 0.00157609 |
| EUR/CZK | -0.00001351 | 0.00000000 | 0.00270053 | 0.00000729 | -0.02765781 | 0.04294752 |
| EUR/CAD | -0.00001316 | 0.00000000 | 0.00429152 | 0.00001842 | -0.03390987 | 0.02858862 |

Table 7 Descriptive Statistics log-returns

From the analysis of the distribution of the exchange rates, it becomes clear that the value of the currencies USD, JPY, CHF, DKK, CZK and CAD against the euro has improved on average over the period under review (negative expected value⁹). The currencies with the largest exchange rate fluctuations are RUB (largest variance). The most stable currency against the euro is DKK (smallest variance), followed by CZK and NOK. The largest one-time exchange rate depreciation occurs at CHF followed by RUB and JPY. This huge depreciation of the CHF can be explained due to the days around the 15.01.2015 in which huge changes have been recognized due to the drop of the minimum level of 1.20 CHF for 1€. From one day of the other a decrease of 35% in the value of the Euro meaning if you invested into CHF you made huge losses.

⁹ average return

5.1.4 Portfolio composition

The calculations will be conducted for 6 different portfolios. For a better demonstration, it will be assumed that the investor has 100 million Euro's which can be invested in 10 different currencies. The structure of the first portfolio will have the structure, only slightly different, of a real existing fund¹⁰. Of course, the fund volume is much bigger but the allocation of the individual currencies within the portfolio are the same. In addition, five fictional portfolios' will be constructed which leads to a total of 6 portfolios which present very different structures.

Portfolio A

The first portfolio consists of a real existing fond. The "3 BANKEN WÄHRUNGSFONDS (T) AKTUELLER KURS". The allocation is as follows 52,31 % is held back in Euros, 28,87% is invested in USD, 9,63% into CZK, 4,87% in PLN and 4,32% in NOK. This fond has a 6-month performance of -3.07%, a 1-year performance of -0.8% but a 3-year performance of 5.08%. His one-year volatility is 2.29% and a one-year sharpe ratio of 2.25. The portfolio allocation can change in the future since holding different currencies is a way to make money by re-selling the currency if a favorable development took place, meaning you get more euros for selling it as you paid.

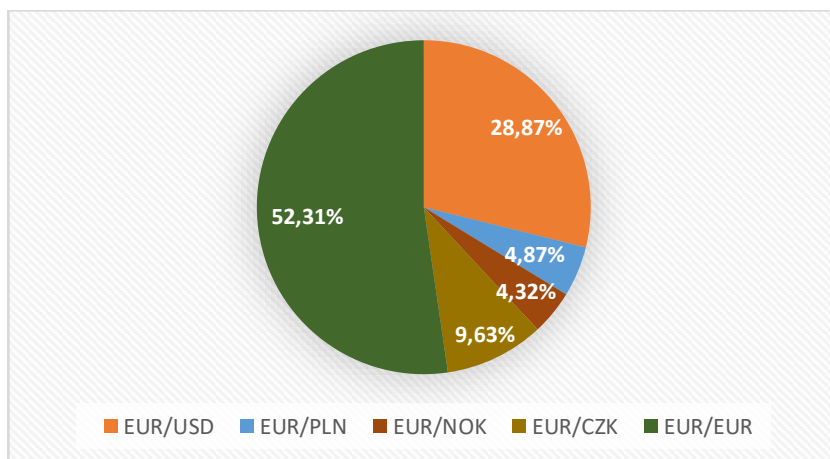


Figure 4 Portfolio A: Real existing portfolio

¹⁰ http://www.finanzen.net/fonds/3_banken_waehrungsfonds_t

Portfolio B

Portfolio B is a fictitious portfolio, in which all positions are of the same size. The assets were divided into 10 different currencies at equal shares of EUR 10 million each. No money was held back in Euro.

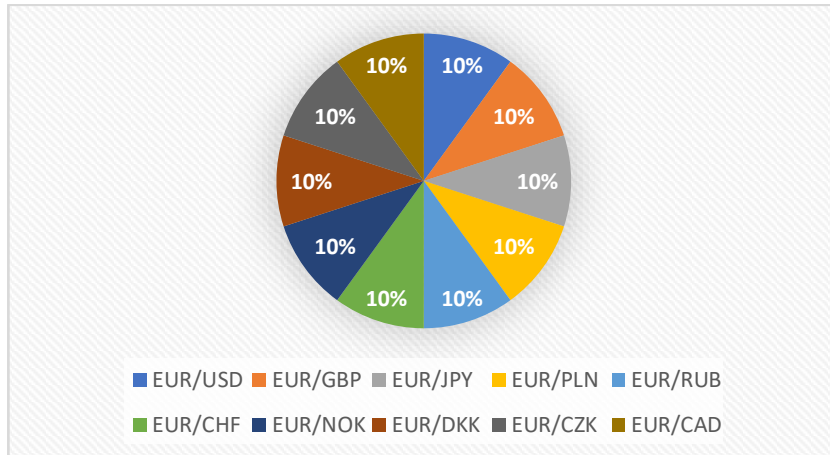


Figure 5 Portfolio B: Equal distribution

Portfolio C

Portfolio C consist only of currency pairs which have a liquidity horizon of 10 days. Which are the USD, G BP, JPY and CHF. In each currency 25 million € are invested.

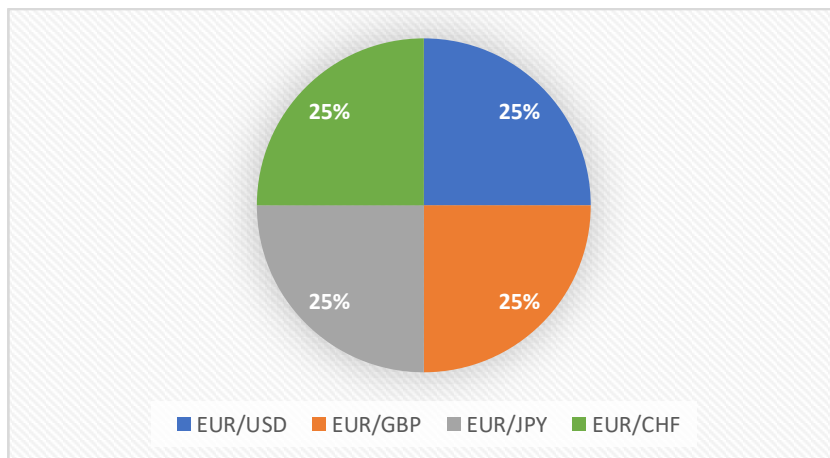


Figure 6 Portfolio C: Liquidity horizon 10d

Portfolio D

Portfolio D consist only of currency pairs which have a liquidity horizon of 20 days. Which are PLN, RUB, NOK, DKK, CZK and CAD. In each currency 16.67 million are invested.

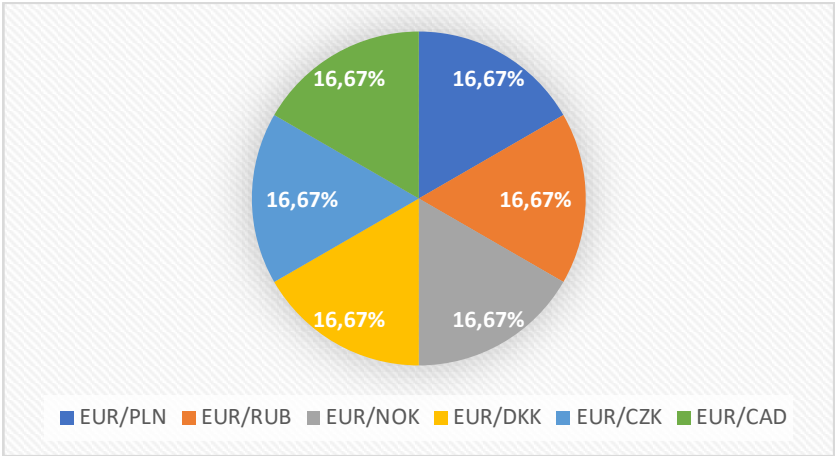


Figure 7 Portfolio D: Liquidity horizon 20d

Portfolio E

Within portfolio E euros have been only invested in currencies which are volatile (high variance). In each currency 25 million has been invested each.

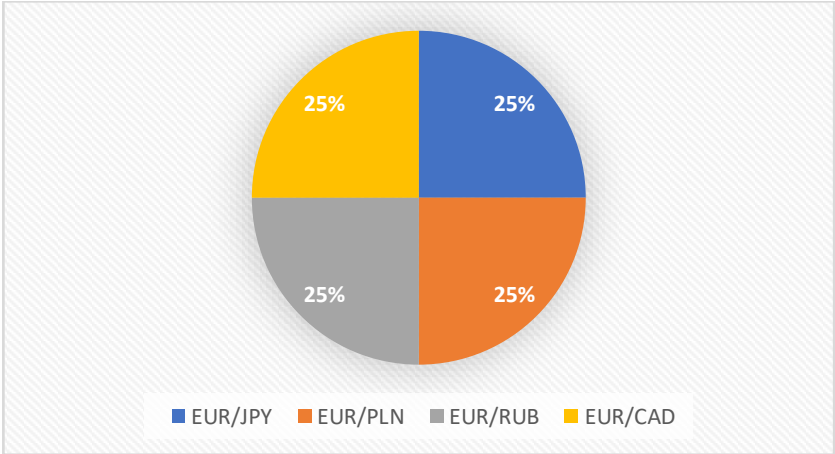


Figure 8 Portfolio E: High variance currencies

Portfolio F

Within portfolio F euros have been only invested in currencies which are relatively stable against the Euro (low variance). In each currency 25 million has been invested.

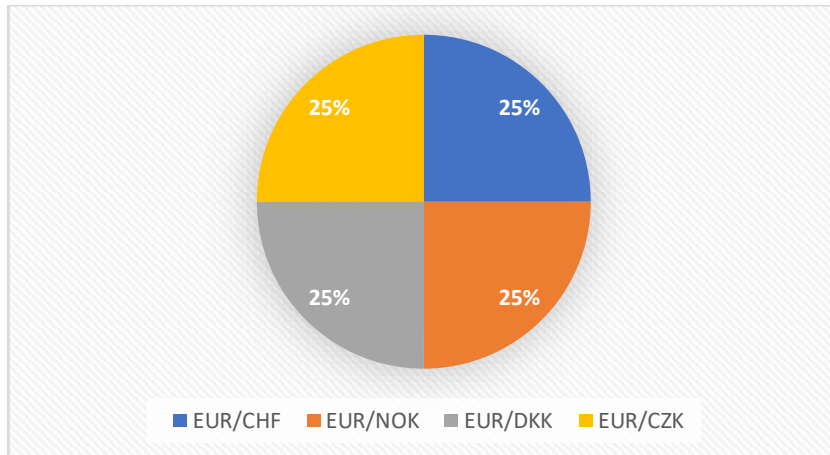


Figure 9 Portfolio F: Low variance currencies

5.2 Execution of the calculation

The goal of the thesis is to determine the VaR and ES for each portfolio for an observation period of $n=250$, $n=500$, and $n=1000$ with a confidence level of 99% and 97.5%. Furthermore, to calculate the minimum capital requirements for market risk under the old Basel 2.5 rules and the new FRTB regulations. The MCR will only be calculated for an observation period of 250 days. How the HS on the basis of this, the VaR and ES as well as backtesting and calculation of the minimum capital requirements for market risk under its underlying regulations are generally performed has already been explained in detail. Now the procedure for this thesis specific case is explained.

5.2.1 Historical simulation to determine VaR and ES

As already mentioned before is the sample size is important since it has an influence on the allowed size of backtesting exceptions. We calculate the VaR and ES for each day for 10 years on a rolling window with the three different observation periods (250 days, 500 days and 1000 days). First, the daily log-returns as presented in chapter 4 are calculated for each asset on each day. Then we compute according to the portfolio the portfolio returns by taking the weight of asset one, within the corresponding portfolio times its log-return of this asset at that day plus the weight of asset two within that portfolio times its log-return of asset two at that day and so on since you summed up each asset within portfolio time its underlying log-return. Then we calculate the VaR by taking the according to its confidence Level (99%) k -smallest log-return within its observation periods times the total portfolio Value. Here we need to remember that we have a different k depending for on the observation period. For an observation period of 250 days we must find the third smallest ($k=0.01*250=2.5$ rounded =3) log-

return¹¹ for an observation period of 500 the 5th smallest log-return and for an observation period of 1000 days we need to take the 10th smallest. Since we want to get the VaR on a rolling window the VaR will be calculated for each day by going 1 down but keeping the observation period constant. So, the most recent log-return is dropped and a new log-return will be considered. Next, we determine the ES with a confidence level of 0.975. The calculation of the ES follows almost the same procedure than for the VaR only with the difference that we want to determine the average of the k-smallest log-returns¹². Since the confidence level is 0.975 for the ES the k-factors are different. For an observation period of 250 days we need the average of the 6-smallest log-returns, for 500 days k equals 13 and for 1000 days k is equal to 25. For each a stressed VaR and ES will be determined which is simply the minimum of all the obtained results.

5.2.2 Backtesting

For each portfolio, a backtest will be conducted to check if the risk will be underestimated using that approach. We want to know how often the real negative total value change is greater than the hypothetical changes based on historical data. For this reason, we take the real P&L and compare it with the calculated VaR or ES of the previous day. If the real P&L is bigger than the VaR or ES an exception takes place¹³. As stated before backtesting the ES is difficult since we compare a Value with an average of values. However, to make them comparable with the VaR measure we assume and follow the thought of Kerkhof and Melenberg (2004) that the ES can be backtested in the same way as the VaR and by a simple comparison of real values and the hypothetical ES value.

5.2.3 Minimal capital requirements

To determine the amount of the minimum capital requirements for the market risk we calculate with real obtained numbers and two simplifying assumptions will be stated. These refer to the both formulas 3.2 and 3.7. First, we assume that the other variables other than the VaR 99% and ES 97.5% are constant and therefore they will be neglected. Secondly, since the values consist of the average of all obtained VaR/ES, the minimum capital requirement for market risk is not determined with the maximum of yesterday's VaR and the 60-day average, but directly with the calculated VaR 99% / ES 97.5%. The simplified formula to determine the MCR under Basel 2.5 (Formula 5.1) and FRTB (Formula 5.2) will be as follows:

$$\text{Average MCR BASEL 2.5} = (VaR_{t-1} + SVaR_{t-1}) \quad 5.1$$

$$\text{Average MCR FRTB} = \{IMCC_{t-1}\} \quad 5.2$$

¹¹ = (SMALL(C9: C258,3)) * Start!\$B\$23

¹² = AVERAGE(SMALL(C9: C258,{1,2})) * Start!\$B\$23

¹³ = SUMPRODUCT(--(((\$B\$4: \$B\$253) - (C4: C253) < 0))

Recalling from Chapter 3 the IMCC equals the liquidity adjusted stressed ES. To calculate the MCR we need a 10-day shifting for VaR and ES. For this reason, the help sheet “10d” has been created. We copy the real values into the corresponding cells. For portfolio A we copy the real obtained average¹⁴ VaR into Cell „C4“ and multiply these times sqrt 10. We repeat this for SVaR¹⁵ and ES¹⁶. However, for stressed VaR we do not shift it with sqrt10. To calculate the MCR under FRTB we adjust the Stressed ES 97.5%, with the corresponding liquidity factors using formula 2.4. We use the stressed ES because $ES = ES_{R,S} * \frac{ES_{F,C}}{ES_{R,C}}$ with the ratio $ES_{F,C}, ES_{R,C} = 1$ because for FX we have a full history for each currency of 10 years. Recalling from table 5 that FX some currency pairs need a liquidity adjustment of 20 days from the base horizon of ten days we calculate the MCR for HS.

¹⁴ File “Summary”, Sheet “RealAveragesValues”, Cell “B7”

¹⁵ File “Summary”, Sheet “StressedValues”, Cell “B20”

¹⁶ File “Summary”, Sheet “StressedValues”, Cell “C20” & “D20”

6 Empirical results

In the following, the results of the simulation will be presented, focus lies on the risk metrics as demanded by the BCBS in Basel 2.5 and FRTB namely VaR 99% and ES 97.5%, the exceptions occurred in the most current year and a year of significant stress and the minimum capital requirements for market risk.

- I. What is the effect of the chosen observation period on the amount of both risk measures (VaR 99% and ES 97.5%)

The following table (table 8) illustrates for ES 97.5% and VaR 99% the different values. All these values are average values of the last 10 years. Interesting is the fact that the VaR and ES of portfolio E, which is a volatile portfolio, lies under the VaR and ES of portfolio F, which is only composed of “stable” currencies (low variance). The lowest VaR can be found within portfolio A under an observation period of 250 days with a value of -322,066.79. The greatest VaR is within portfolio F under an observation period of 1000 days with a value of -812,730.80 €. The smallest ES is also within portfolio A of with an observation period of 250 days with a value of -340,746.83 and the highest ES can be found with a Value of - 869,455.40€ within portfolio F and an observation period of 1000 days.

| Portf. | n=250 | | n=500 | | n=1000 | |
|--------|---------------|---------------|---------------|---------------|---------------|---------------|
| | VaR (99%) | ES (97.5%) | VaR (99%) | ES (97.5%) | VaR (99%) | ES (97.5%) |
| A | -322,066.79 € | -340,746.83 € | -342,248.54 € | -350,873.30 € | -357,725.84 € | -372,085.93 € |
| B | -544,444.11 € | -575,411.61 € | -576,617.65 € | -594,617.92 € | -566,370.62 € | -616,294.83 € |
| C | -740,796.91 € | -786,943.59 € | -794,476.59 € | -800,315.35 € | -799,841.06 € | -826,280.28 € |
| D | -562,967.09 € | -589,191.90 € | -588,918.82 € | -609,462.34 € | -591,154.03 € | -624,022.71 € |
| E | -488,535.07 € | -528,037.04 € | -537,985.45 € | -549,791.15 € | -553,514.19 € | -581,055.46 € |
| F | -780,034.03 € | -826,358.76 € | -836,430.75 € | -855,491.73 € | -812,730.80 € | -869,455.40 € |

Table 8 Comparison of ES 97.5% and VaR 99% within the first period for different observation periods

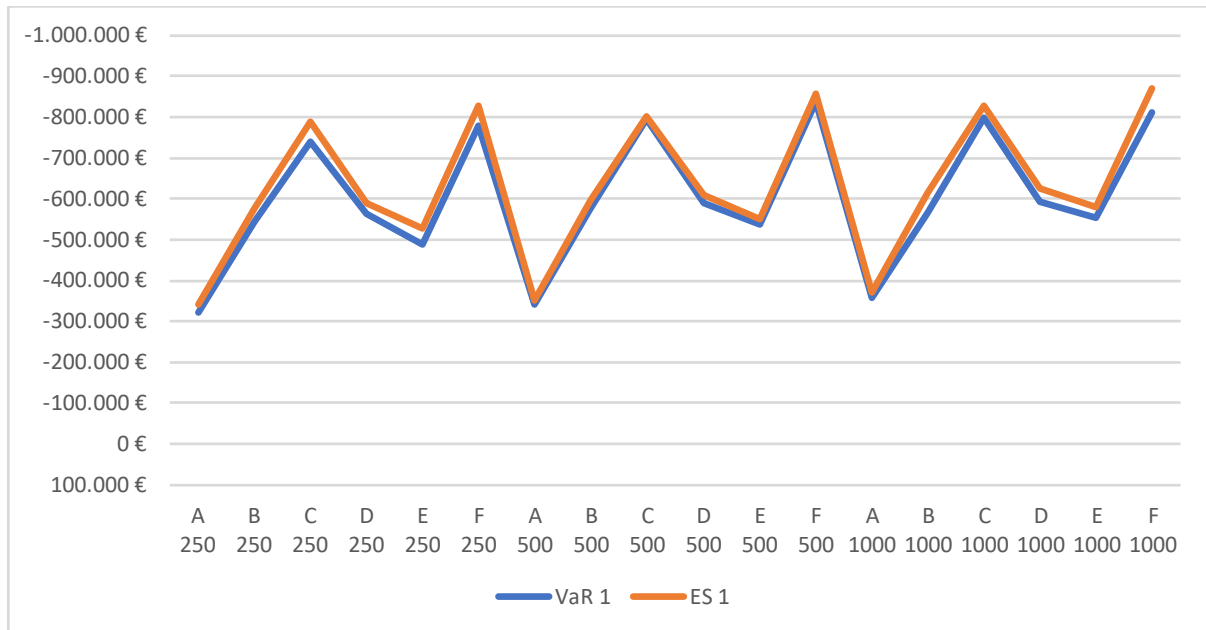


Figure 10 VaR 99% and ES 97.5%

Comparing the values of the same confidence level and calculation method, we can observe that there is a difference between the three sample sizes. Whereas the values with an observation period of 250 tend to be below 500 days and 500 days tend to be below 1000 days. This is probably because extremes occur less frequently within 250 days than within 500 and less frequently within 500 days than within 1000 days. However, the difference is small since with increasing observation period the value we use to calculate the VaR or ES increase as well as shown in table 8. Therefore, the VaR's and ES's keep relatively constant.

| | VaR | ES |
|-----------|--------------------------------|-----------------------------------|
| 250 days | 2 nd smallest loss | Average of the 6 smallest losses |
| 500 days | 5 th smallest loss | Average of the 12 smallest losses |
| 1000 days | 10 th smallest loss | Average of the 25 smallest losses |

Table 9 VaR and ES for the different observation periods

However, to be sure that there is no significant difference between the different days of observation (n) for conducting the HS simulations we state the following Hypothesis.

H1: There is a significant difference between VaR or ES with n=1000, VaR or ES with n=500 and VaR or ES with n=250.

$$H_0: \text{VaR}_{(n=1000)} = \text{VaR}_{(n=500)} = \text{VaR}_{(n=250)} \text{ or}$$

$$\text{ES}_{(n=1000)} = \text{ES}_{(n=500)} = \text{ES}_{(n=250)}$$

$$H_1: \text{VaR}_{(n=1000)} \neq \text{VaR}_{(n=500)} \neq \text{VaR}_{(n=250)} \text{ or}$$

$$\text{ES}_{(n=1000)} \neq \text{ES}_{(n=500)} \neq \text{ES}_{(n=250)}$$

To test this Hypothesis, we make use of ANOVA, but before conducting the ANOVA we need to determine the dependent and independent variables and test if the following required assumptions are fulfilled:

- a) We need to make sure the dependent variables are normally distributed. In this case, the dependent variables are the risk metrics VaR 99%, VaR 97%, ES 99% and ES 97.5%. The dependent variable is the length of the sample size. To test normality, we make use of the Shapiro-Wilk Test of Normality. For this purpose, we use SPSS. Transfer the variables that need to be tested for normality into the Dependent List and independent factor into the Factor list. By doing so we get the following Output:

| Test of Normality | | | | |
|-------------------|------------------|------------------|-----------|----------|
| | Observation Days | Shapiro-Wilk | | |
| | | <i>Statistic</i> | <i>df</i> | <i>p</i> |
| ES | 250 | .930 | 6 | .581 |
| | 500 | .929 | 6 | .573 |
| | 1000 | .931 | 6 | .590 |
| VaR | 250 | .932 | 6 | .592 |
| | 500 | .920 | 6 | .507 |
| | 1000 | .941 | 6 | .670 |

Table 10 Test of Normality

The dependent variable is normally distributed if $p > 0.05$. Since this is the case for all independent variables, we can conclude that the first assumption is fulfilled.

- b) The second assumption is that there is homogeneity of variances. This means that the population variances are equal in each group. This assumption is also fulfilled
- c) The third assumption is the independence of the group. This assumption is also fulfilled due to the design of the study. Each "group" is individual since the groups are calculated using different calculation methods and/or a different confidence level.

So, by knowing that all assumptions for both different time periods are fulfilled we conduct the ANOVA analysis with SPSS. After assigning the dependent and independent variables we get the following output:

| ANOVA | | | | | |
|-------|----------------|----------------|----|------|------|
| | | Sum of Squares | df | F | p |
| Var | Between Groups | 1.494E+9 | 2 | .030 | .970 |
| | Within Groups | 3.693E+11 | 15 | | |
| | Total | 3.708E+11 | 17 | | |
| ES | Between Groups | 570449423 | 2 | .011 | .989 |
| | Within Groups | 3.836E+11 | 15 | | |
| | Total | 3.842E+11 | 17 | | |

Table 11 Output ANOVA Analysis

The null Hypothesis is rejected if the p-value < 0.05. This means that there is no statistically significant difference between the different sample sizes for VaR (99%), ES (97.5%) as determined by one-way ANOVA with (F(2,15)=.030 p=.970) for VaR (99%) and (F(2,15)=.011 p=.989) for ES (97.5%). That means the null hypothesis can't be rejected and there is no significant difference between n=250, n=500 and n=1000 for both risk metrics, which means that a difference is just likely by chance. This implies that the length of the observation period is not as important as assumed. So, we can conclude that it does not matter which observation period will be used to determine the VaR and ES since there is only by chance a difference between these three observation periods.

- II. Is there a significant difference, between the both risk measures VaR 99% and ES 97.5%?

Since the BCBS demands to calculate the MCR to use the VaR (99%) within Basel 2.5 and ES (97.5%) within the FRTB regulations it is interesting to see if there is a significant difference between them. As long the P&L are normally distributed the VaR 99% and ES 97.5% should be almost equal according to the literature. By just looking at the values and figure 11 it is observable that the ES (97.5%) is always bigger than the VaR (99%) for each Portfolio but the both lines lies almost upon each other so they are similar. Therefore, we state the following Hypothesis:

H2: There is a significant difference between HS VaR (99%) and HS ES (97.5%)

$$H_0: ES (97.5\%) - VaR (99\%) = 0$$

$$H_1: ES (97.5\%) - VaR (99\%) \neq 0$$

To test these hypotheses, we make use of the paired-sample test using SPSS. After choosing the independent variables, we get the following output:

| Paired Samples Statistics | | | |
|---------------------------|-----|------------|----------------|
| | | Mean | Std. Deviation |
| Pair 1 | VaR | -569244.82 | 147692.451 |
| | ES | -594295.62 | 150328.674 |

Table 12 Paired Samples Statistics

| Paired Samples Correlations | | | |
|-----------------------------|----|-------------|------|
| | N | Correlation | Sig. |
| Pair 1 VaR & ES | 18 | .995 | .000 |

Table 13 Correlation between VaR and ES

| Paired Samples Test | | | | | | |
|---------------------|------------|---|----------|-------|----|----------------|
| | | 95% Confidence Interval of the Difference | | | | |
| | Mean | Lower | Upper | t | df | Sig (2-tailed) |
| Pair 1 VaR - ES | 25050.7980 | 17552.7520 | 32548.84 | 7.049 | 17 | .000 |

Table 14 Paired-Samples Test

The both risk measures VaR (99%) and ES (97.5%) are highly positively correlated ($r=.995$, $p<.001$). Furthermore there was a significant average difference between these two-risk metrics ($t_{17}=7.049$, $p<.001$). On average, the VaR is 9315.88 € higher than the ES (95% CI [17552.7520,32548,8439]). So, we can reject the null hypothesis and conclude that there is a statistically significant difference however, the difference is small as seen in figure 11. The both values for the different portfolios and observation periods lies almost upon each other therefore the effect size will be determined which determine the strength of the significance. Cohen (1992), differentiate between weak effect ($d=.20$), medium effect ($d=.50$) and strong effect ($d=.80$). To calculate the correlation coefficient (r) of Pearson we use the following formula where $M1$ is the mean of VaR, $M2$ the Mean of ES, $SD1$ the standard deviation of VaR and $SD2$ the standard deviation of ES:

$$Cohens'd = \left| \frac{M1 - M2}{\sqrt{[(SD1^2 + SD2^2)/2]}} \right| \quad 6.1$$

Inserting the values from table 12 into equation 6.1 we get the following equation:

$$Cohens'd = \left| \frac{(-569244) - (-594295.62)}{\sqrt{[(147692.451^2 + 150328.674^2)/2]}} \right| = 0.1681076801311331 \quad 6.2$$

This means that the effect size according to Cohen (1998) is small and therefore the difference is trivial even though it is statistically significant. This supports what the literature says. According to the literature, the VaR with a confidence Level of (99%) is almost equal to the ES with 97.5% as long the returns are normally distributed.

III. What effect has the choice of risk measure and observation period on the number of exceptions and how is this reflected by the Basel traffic ?

As we know from Chapter 3 the MCR is calculated by adding a multiplication factor. This multiplication is the minimum of three plus an additional penalty factor which will be derived by compiling the “Basel traffic light” with the help of the exceptions. Table 15 shows for the last current year the different exceptions and zones which occurs due to exceeding the real values according to the allowed exceptions as presented in chapter 2.1.1. It shows the occurred exceptions for the 6 portfolios using different observation periods (250 days, 500 days and 1000 days). Whereas table 16 shows the exceptions and zones for a period of significant stress from 28.05.08 – 28.05.09. We only consider the exceptions for an observation period length of 250 days and 500 days within the period of stress since an observation period of 1000 days would be biased simply because we do not have enough data history to backtest these observations correctly within on a rolling window. However, most important is the observation period of 250 days since banks make also just use of the minimum required observation period of 250 days. It can be assumed that the exceptions would increase the longer the observation period simply because a backtest is conducted not for 250 days but for 500 or 1000 days but as shown in chapter 2.1.1 the allowed exceptions also increase and therefore it should fall within the same zone.

| Portf. | n=250 | | n=500 | | n=1000 | |
|--------|-----------|------------|-----------|------------|-----------|------------|
| | VaR (99%) | ES (97.5%) | VaR (99%) | ES (97.5%) | VaR (99%) | ES (97.5%) |
| A | 2 | 2 | 4 | 3 | 17 | 14 |
| B | 2 | 2 | 1 | 1 | 18 | 13 |
| C | 3 | 1 | 3 | 2 | 14 | 12 |
| D | 2 | 2 | 0 | 0 | 16 | 15 |
| E | 4 | 2 | 3 | 1 | 10 | 11 |
| F | 4 | 3 | 0 | 0 | 22 | 14 |
| Sum | 17 | 12 | 11 | 7 | 97 | 79 |

Table 15 Occurred Exceptions for the most current year

| Portf | n=250 | | N=500 | |
|-------|-----------|------------|-----------|------------|
| | VaR (99%) | ES (97.5%) | VaR (99%) | ES (97.5%) |
| A | 5 | 5 | 15 | 17 |
| B | 3 | 3 | 18 | 16 |
| C | 7 | 5 | 11 | 12 |
| D | 7 | 4 | 13 | 11 |
| E | 7 | 7 | 10 | 11 |
| F | 5 | 6 | 9 | 6 |
| Sum | 34 | 30 | 76 | 73 |

Table 16 Occurred Exceptions for a period of significant stress

As seen in Table 15 the Sum of the exceptions for the most current year decrease from 250 days up to 500 days and then increase again. This can be explained due to the fact that under an observation period the VaR or ES respectively is very high under an observation period of 500 days since the discontinuation of the exchange rate locking of the CHF to the Euro is considered within an observation period of 500 days but not within 250 days and hence this led to a high-risk metric and hence fewer exceptions takes place. The exceptions increased than again for an observation period of 1000 days since much exceptions took already place before that event and when the event comes into consideration it will not affect the risk metric in the way as it does under an observation period of 500 days simply because the loss we are interested in is not the 5th smallest anymore but 10th smallest for the VaR and not the average of the 13 smallest losses for the ES but the average of the 25 smallest losses. Looking at the different exceptions it attracts attention that the number of exceptions of the VaR is greater than the occurring exceptions of the ES. Which supports the implementation of the ES since it better reflects possible losses and extreme losses. However, for a normal period (see table 9) we can observe that all portfolios for each risk measure is in the green zone which indicates a good model. But for a period of stress (see table 16) we can see using the Value at Risk as risk metric might be inappropriate since 4 out of 6 portfolios lie within the yellow zone or even within the red zone for an observation period of 500 days. By the change from the VaR to the ES the BCBS improved the situation since only green zone occurs for ES (97.5%). So, it can be concluded that the move from VaR to ES supports the idea of the BCBS that it will lead to a more risk-averse model and banks would not have been penalized. For the period of stress, the former mentioned assumption holds true that the bigger the observation period the greater the exceptions this is simply due to the fact, that the stressed period is more volatile but do not have such great exceptions as in the normal period and hence the risk metric will not be getting that big and exceptions can occur more easily. Hence it can be concluded, that for a normal period the change is trivial since we know from chapter 6.2 that the ES is indeed significant greater than the VaR but only with weak effect. But as we can see by having a look at the exceptions the ES will provide a better model for a stressed period since it better reflects the loss as already mentioned within the theoretical part of this thesis.

IV. What effect has the change from Basel 2.5 to FRTB regulations, on the minimum capital requirements

As we have seen above the length of the observation period n has no great influence on the calculated values. They are almost the same. Therefore, we calculate the MCR with an observation period of $N=250$ and as demanded under Basel 2.5 and FRTB with VaR (99%) and ES (97.5) for the most current year. How the MCR was calculated was explained in chapter 5.7. Table 11 shows the different MCR for the different Portfolios. This calculation is conducted using the real observed average values. No additional multiplication factor need to be added since each Portfolio lies within the green zone (see table 15).

| Portf. | Basel 2.5 | FRTB | Sensitivity |
|--------|-----------------|-----------------|-------------|
| A | -1,666,440.30 € | -2,273,923.26 € | 1.364539288 |
| B | -2,596,820.84 € | -4,628,955.42 € | 1.782547086 |
| C | -3,499,019.08 € | -4,784,648.01 € | 1.367425527 |
| D | -2,531,663.56 € | -5,340,911.96 € | 2.109645233 |
| E | -2,537,584.91 € | -4,293,115.67 € | 1.691811633 |
| F | -3,614,461.14 € | -8,196,354.95 € | 2.267656127 |

Table 17 MCR n=250

As shown in table 17 the amount a bank needs to have in reserve for a specific portfolio against possible market risk is for each portfolio higher under the new regulations. This supports the main idea behind these new regulations to force the banks to back up the Investments with more equity capital to withstand potential market risks. This means for Portfolio A for a total investment of 100,000,000 € the bank require additional capital of 1,666,440.30€ under Basel 2.5 and 2,273,923.26€ under FRTB regulations. To see which portfolio is the most or less sensitive to the new rules of FRTB we calculated for each portfolio its sensitivity by dividing the FRTB value by the Basel 2.5 value (Portfolio A: $-2,273,923.26 / -1,666,440.30 = 1.364539288$). As we can see in table 17 Portfolio A and C have almost the same sensitivity of 1.36 and are therefore the less sensitive portfolios to the new rules. The most sensitive portfolio with a sensitivity of 2.26 is portfolio F. The second most sensitive portfolio with a sensitivity of 2.10 is portfolio D. It is obvious that portfolio C is among the less sensitive portfolios since within portfolio C only currency pairs have been selected which have a liquidity horizon of 10 days and therefore we shift it only with 10 days for both Var and ES. Since we already know from research question 2 that there is a significant (table 14) but meaningless difference (Cohen's $d=0.16$) between VaR (99%) and ES (97.5%) and no significant difference between the different observation periods we can conclude that the difference of the MCR according to the new in FRTB implemented adjustment to different liquidity horizons. As in Basel 2.5 no matter how risky an instrument was, all instruments have been adjusted to a liquidity horizon of 10 days. On the other hand, under FRTB regulations, the instruments were adjusted from 10-120 days depending on their riskiness from the base horizon of 10 days.

- V. What is the effect of the choice of the calculation method on the preference order within both periods, of a decision maker who a) decides only based on risk measure and b) only on the MCR c) the Sharpe ratio?

To give an answer to this question, a hypothetical decision maker of a bank must decide in which portfolio the bank should invest into. The decision maker solely decides on the basis of the underlying risk and not on the yield the portfolio might generate. In other words, the decision maker wants to minimize the amount of the risk measure or the amount of MCR, respectively. He therefore always chooses first the portfolio which has the lowest amount. In Table 18 and 19 the preference order is illustrated. Table 18 ranks the risk measure VaR and ES and Table 19 ranks the MCR.

| | | VaR | ES | |
|---|---|---------------|---------------|---|
| 1 | A | -322,066.79 € | -340,746.83 € | A |
| 2 | E | -488,535.07 € | -528,037.04 € | E |
| 3 | B | -544,444.11 € | -575,411.61 € | B |
| 4 | D | -562,967.09 € | -589,191.90 € | D |
| 5 | C | -740,796.91 € | -786,943.59 € | C |
| 6 | F | -780,034.03 € | -826,358.76 € | F |

Table 18 Preference order just considering the Risk Measure Var and ES

| | | BASEL 2.5 | FRTB | |
|---|---|-----------------|-----------------|---|
| 1 | A | -1,666,440.30 € | -2,273,923.26 € | A |
| 2 | D | -2,531,663.56 € | -4,293,115.67 € | E |
| 3 | E | -2,537,584.91 € | -4,628,955.42 € | B |
| 4 | B | -2,596,820.84 € | -4,784,648.01 € | C |
| 5 | C | -3,499,019.08 € | -5,340,911.96 € | D |
| 6 | F | -3,614,461.14 € | -8,196,354.95 € | F |

Table 19 Preference Order just considering the MCR

For both periods, the preference order using VaR99 % and ES 97.5% is equal. This is not surprising since the VaR and the ES are highly correlated. The decision maker would choose Portfolio A first regarding the risk measure and the MCR. This is not surprising since this is a real existing portfolio. However, a little bit surprising is the fact that portfolio E, the volatile portfolio, would be chosen second and the “stable” Portfolio F last. Which might be due to extreme outliers which pushed up the value of the VaR and ES and hence overestimate the risk. On the other hand, it could be, that the risk of Portfolio E is underestimated. Considering the MCR. Next it is interesting if this order is the same if the decision maker also considers the possible return of the Portfolios. For this reason, the Portfolio Sharpe ratio for the most current year is calculated, which adjust the performance of an investment by adjusting for its risk. The formula is as follows:

$$S(x) = (rx - Rf)/StdDev(x) \quad 6.3$$

Where: x is the investment, rx is the average rate of return of x, Rf is the best available rate of return of a risk-free security (i.e. T-bills) and StdDev(x) is the standard deviation of rx. The daily risk-free rates have been calculated taking daily-10-year risk-free rates of a German bond and dividing them by 3650 which resulted in the daily-daily risk-free rates. The following table (Table 20) shows the different sharpe ratios for the different Portfolios:

| Portfolio | Sharpe Ratio |
|-----------|--------------|
| A | 0.090312147 |
| B | 0.057099779 |
| C | 0.078873468 |
| D | 0.02375688 |
| E | 0.060001013 |
| F | 0.015926365 |

Table 20 Sharpe ratios of the different Portfolios

As we can see, the decision maker would choose in the following order: A, C, E, B, D, F. So, we can conclude that taking the yield into consideration the decision maker would choose a different preference order than just considering the risk metric or MCR. But as we see the decision maker would still choose portfolio A first and portfolio F last which indicates that a low or high-risk metric and MCR is a good indicator for making first indications. However, just the sharpe ratio will give a correct ranking of the portfolios.

7 Conclusions

The goal of this thesis was to examine the effect on minimal capital requirements for banks after implementing the new standard for market risk as demanded by the BCBS in the Fundamental review of the trading book. For this purpose, both risk metrics, Value-at-Risk and Expected Shortfall, have been theoretically and practically compared to different portfolios and different confidence levels. Furthermore, the historical simulation has been introduced and carried out for the observation periods of $n=250$, $n=500$ and $n=1000$ to determine a distribution function. The Historical simulation, which has some drawbacks in determining a distribution function, has been compared in theory with the adjusted simulation method and the Age-weighted historical simulation. Lastly, the new minimum capital requirements as required by the FRTB have been compared to the minimum capital requirements for market risk as required in Basel 2.5 for each portfolio with an observation period of 250 days. To answer the research questions as stated in the introduction two hypotheses have been developed. Although this study attempted to produce a valid and reliable research, it has some limitations. First, this thesis is about a very broad and complex topic for which not much literature is available since banks do not follow the FRTB rules yet. Banks yet try to figure out best practice solutions and hence a real comparison how they conduct it is missing. Which made it hard just by having the plain rules by the BCBS to conduct the calculations and not having any example of how it could be performed. Moreover, these rules are sometimes vague. For many open questions which have been raised by banks the BCBS has published a Q&A but not every topic question is yet answered. Furthermore, it is hard to get access to a data history of 10 years as demanded by the BCBS which makes it very complicated to conduct an empirical study additionally you need even longer history if you want to determine the VaR and ES with an observation period of 1000 days within a period of stress such as 08/09 Another, limitation is the constellation of the Portfolio. Banks usually have portfolios which are much more diversified and which buy and sell assets. So, a portfolio will never be stable but changes very often. However, to reproduce this, a huge calculation power is necessary which most individuals do not have and therefore a simplified currency portfolio has been chosen in which only two simulations for each portfolio were necessary. Of course, the carried-out investigations in this work are only a small part of the wide field of risk management. It could be possible that different portfolio structures, different risk factors (e.g. change in interest rates), different calculation methods (e.g. variance-covariance simulation, Monte-Carlo simulation) could lead to different results. A consideration of all these different factors would be too much for this thesis. Therefore, the following statements will only hold for the examined modeling of factors and the most important insights of this analysis will no be summarized once again. For banks, the minimum capital requirements are necessary to hedge against the different risks they encounter. On the other hand, this hedge is also a burden and banks try to hold capital requirements as small as possible, oftentimes by underestimating risk. Because the capital required to hedge investments will rise proportional to the high of the underlying risk metrics, banks desire to hold the risk metric as small as possible. Even though the risk metric is pre-defined by the BCBS, banks can still decide the length of the observation period and which simulation method they use and hence manipulate the MCR in the desired direction. As seen in this thesis for $n=250$, the result of the risk metric for VaR and ES are lower compared to a observation period of $n=500$ and $n=1000$. Even though this difference is small and is not significant the bank can slightly influence the value by choosing a small observation period since the smaller the observation period the

less extreme values might occur. Furthermore, the possibility of choosing a different simulation method, is questionable since the original goal was to determine minimum capital requirements for market risk to hedge investments as accurately as possible and not to state it as small as possible. Even though the BCBS did already have standards to calculate the minimum capital requirements for market risk, the incentive to choose the method which leads to the smallest MCR must be eliminated. It has been shown that the BCBS already implemented a system, the Basel traffic light, to penalize a calculation method with additional capital which leads to too many exceptions and, as shown under the new regulations, the minimum capital requirements for market risk is much higher. Furthermore, with the introduction of the Expected Shortfall, which better considers tail risks the BCBS demand banks to use a risk metric which is more sophisticated within stressed times. However, sometimes the MCR is even higher when using a method which leads to a small risk metric than using a method which may lead to a higher risk metric, but no additional penalty capital needs to be added since the risk is better estimated and fewer exceptions occurred especially in times of financial stress the ES better reflect the potential losses. Therefore, the main goal of banks should be to use a method which estimates the risk as precisely as possible to make use of the trade-off between a low VaR/ES and low additional penalties. That's most possible the reason why the BCBS allows banks to choose the method to calculate the VaR within Basel 2.5 and still the ES under FRTB regulations. As shown in the theoretical part of this thesis, the VaR has several drawbacks such as no sub-additivity and not taking into consideration extreme losses. The ES on the other hand incorporates these shortcomings which is the reason why the BCBS pre-defined the risk metric and moved from the 99% VaR as used under Basel 2.5 to the 97.5% ES. But we found out that the different risk measure is not the real driver of the change for the MCR since VaR (99%) and ES (97.5%) are almost equal in their value even though the difference significant it can be neglected due to its small effect size of 0.16. Nevertheless, it can be said as central result of this thesis, that the new regulations always lead to a higher MCR and that currency portfolios react with a quit sensitive to these changes with a max of 2.267 and a minimum of 1.36. Therefore, the new regulations are already a step in the right direction but still with several freedoms for banks to manipulate their MCR into the direction they prefer, which is most likely always a small MCR. Since this thesis solely calculated the amount of the VaR and ES using historical simulation for different portfolios and observation periods it is interesting to see if these results will also hold for other simulation methods, including the age-weighted historical simulation, volatility-weighted historical simulation, monte carlo simulation or variance co-variance method. Furthermore, the observed sample only consists of one risk class (currency risk) it is interesting to see what happens if there are portfolios with other risk classes such as equity risk, interest rate risk or commodity risk, or to even have a mixed portfolio. Furthermore, it would be interesting to compare the MCR calculated by the Internal models approach with the MCR calculated by the Standardized approach. Moreover, it could be interesting to research if it is possible to mitigate the impact of the FRTB by portfolio re-optimization. Summing up it can be said that the implementation of these new rules will challenge not only banks but also academics to find best practice solutions which will on the one hand satisfy the underlying rules and on the other hand reliable and cost-saving implementations.

8 References

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