Improving stock levels in retailers' stores by proposing a model based on literature and historical data

Thymen van der Poll
S1472208
BSc. Thesis 2017

Supervision
University of Twente
Dr. M.C. van der Heijden

Technische Unie
Ir. N. de Haan

UNIVERSITY OF TWENTE.

Technische Unie

Preface

It is with great pride and joy that I can finally write the preface of this project. After months of working hard, learning and personal development I have finished the project. It has been an exciting period for me. In this preface, I want to give the credit to those who have been instrumental in successfully finishing this project.

First of all, I would like to thank my parents and sisters for their daily interest in me. They have been all ears for me whenever I needed that. They have helped me reflect on my thinking processes and in interaction with others. I should also mention my housemate. He has helped me write and review code and we shared a great deal of thoughts about the project.

Secondly, I would like to thank my girlfriend. She has given me much needed attention and positive energy on daily basis, cared for me on some moments, even without me asking. We became closer and better friends than ever during this period. I really appreciate that.

Next, I would like to thank Technische Unie, and in particular my supervisor at Technische Unie, Nicole de Haan, for giving me the space and time to finish this project even though it has taken up more than expected time. The perfectionist in me has enjoyed that. It helped me both develop my professional skills and broader my knowledge on the topic of inventory management.

I would also like to thank Matthieu van der Heijden. My supervisor at the University of Twente. Many thoughts were shared which pointed me in the right direction and gave me confidence in my thinking processes.

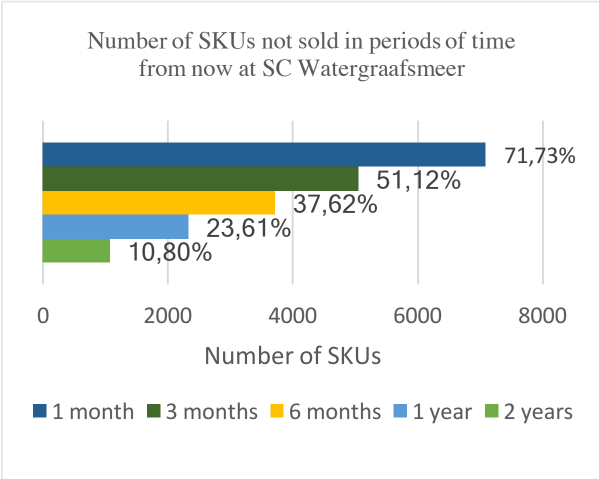
Hereby, I present my Bachelor thesis at Technische Unie. I am proud of it. I hope it can be of great use to Technische Unie, and future readers.

Kind regards,

Thymen van der Poll
Enschede, September 2017

Executive summary

The aim of this project was to lower the inventory value of the Service Centers of Technische Unie. This has been carried out by finding out what kind of demand patterns are observed at the Service Centers of Technische Unie and how to cope with these kinds of demand patterns according to literature. Sample data of Service Center Watergraafsmeer is used.



First, an analysis of sales data was done, leading to the following results:

- Almost 24% of the SKUs have not been sold during the last year, 38% the last 6 months.
- 50% of the SKUs have a coefficient of variation in lead time demand of at least 10, implying highly unstable demand.
- 18% and 24% of the SKUs that were sold in Watergraafsmeer, weren't sold in Nijmegen, Middelburg respectively, implying observed regional difference.

Then literature showed that the first important steps to follow are to use appropriate analytical methods to model the replenishment lead time demand (rLTD) distribution. Then demand can be classified in three groups, being fast moving products. These items have a rLTD of at least 10 units. In this group, the Gamma distribution is used when the standard deviation-to-mean ratio of rLTD is larger than 0.5, otherwise the Normal distribution is used. Literature sets the threshold at 0.5 since the Normal distribution does consider negative demand (unrealistically) whereas the Gamma distribution does not.

The second group consists of slow moving products which have a rLTD lower than 10 units. This group has a variance-to-mean ratio of rLTD of at most 1.1. For these items, the Poisson distribution is appropriate.

The last group consists of slow moving products with a variance-to-mean ratio of rLTD higher than 1.1, for which the demand arrival process is assumed to be independent of the demand size. The demand arrival process still follows a Poisson distribution, but the demand size in this project distributed uses its empirical probability. This group is called lumpy and the distribution is called compound Poisson.

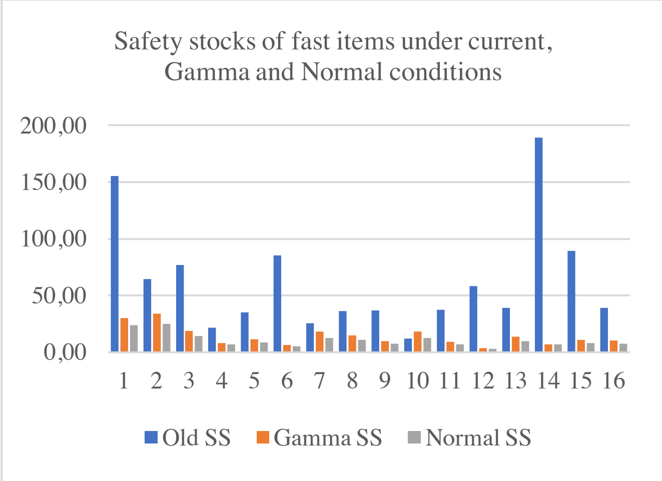
The three groups contain the following number of SKU's:

Fast:	16
Slow:	2813
Lumpy:	5424

Since the fast group is relatively small, the reorder point under both the Gamma and Normal distribution was determined although literature regards the Gamma distribution the most appropriate. After taking a sample of each group and determining the reorder point for these samples under a target fill rate of 98%, the expected stock levels are compared to the average stock levels as observed every last month during the last 2.5 years and the analytical model using the current reorder points. This comparison leads to the following results:

Fast

For this group, the safety stock (ss) is roughly 6,4 times (compared to Gamma) too high on average if the goal is to have a fill rate of 98%. Clearly, the current parameters are not based on a mathematical method in literature. More importantly, after determining the new ss, the cycle inventory takes up 82% of the total inventory of these items. It is however not known whether the lot sizes, which is the most important factor of the cycle inventory, are economical. It is expected that the total inventory can be decreased by a factor 1.54 (Gamma), however depends on the lot size due to that the service level measure is the fill rate and fill rates depend on lot sizes. It is recommended to start using the analytical methods presented for this group, using a Gamma probability distribution.



Slow
The sample for this group showed that the inventory also is too high for items in this group on average. It is expected that the inventory can decrease with a factor 1.8. Again, it is not known whether the lot sizes are economical, and since so many items are really slow moving, this should definitely be researched. Also for this group it is recommended to start using the models presented for this group, incorporating a Poisson probability distribution.

Lumpy
This is not only the largest group, but also the only group for which the inventory level is actually too low if one wants to achieve a target fill rate of 98%. Currently, the inventory is a factor 1.2 too low. There are two important issues. Firstly, there is much difference between the SKU's in this group. Some are relatively fast, others very slow. What's more, is that this means that to control such a large group, there is more work needed. The compound Poisson probability distribution used to construct the rLTD distribution can still be used, but a strategic decision should be taken here and this decision is yet to be researched: should TU invest in inventory to reach a higher fill rate, or should the fill rate be lowered for some products so that the costs are cut. Apart from that, the compound Poisson probability distribution method can still be used to determine the reorder points for items in this group.

Future research
Beside these groups, the following areas of future research within TU are needed:

- ***Figure out how economical the current lot sizes are.***
  In the estimations of reorder points, the current lot sizes are used. These lot sizes might not at all be economical, but they take up a large share of the total inventory. Since the fill rate depends on the lot size it is important that first the economical lot size is determined using the correct and then determine the reorder point.
- ***Plan for regional difference by means of appropriate forecasting methods.***
  The observed regional difference means that items can be sold in one service center, but not at all in the other. This means that a product needs its own forecast for each service center. Which forecasting methods to use is something that needs to be researched.
- ***Figure out what to do with non-moving items.***
  The large number of products that do not seem to be moving at all requires us to ask the question whether these items should be on stock at all. The criteria and thresholds for these criteria (for instance at least sold three items during the last year to date) need to be determined. By doing this, their CODP will move one stage upstream the supply chain.
- ***Research the option to pool inventory regionally.***
  Within TU, there have been voices that ask for more specialized service centers in the sense that they want SCs to sell a specific range of products and other SCs to sell a different range of products. In a generalized idea, one service center may for instance sell product range A whereas the other may sell product range B. Technische Unie can then take advantage of the risk pooling effect: the demand will be aggregated over multiple SCs and their variance will decrease. Therefore, less safety inventory is needed. This comes at the price of a higher lead time and increased ordering costs. Therefore, this has to be researched.

Table of Contents

PREFACE 2

EXECUTIVE SUMMARY 3

TABLE OF CONTENTS 5

LIST OF TERMS AND ABBREVIATIONS 7

1. INTRODUCTION 8

1.1 COMPANY DESCRIPTION 8
1.2 RESEARCH MOTIVATION 9
1.3 PROBLEM DESCRIPTION 9

2. RESEARCH DESIGN 13

2.1 BOUNDARY CONDITIONS AND SCOPE 13
2.2 RESEARCH QUESTIONS 13
2.3 RESEARCH METHODOLOGY 14
2.4 READING GUIDE 15

3. INVENTORY MANAGEMENT: CURRENT SITUATION 16

3.1 ORDER AND REPLENISHMENT PROCESSES 16
3.2 EFFECTS: ANALYSIS OF HISTORICAL DATA 17
3.2.1 ABC-ANALYSIS 17
3.2.2 FREQUENCY OF SALES 18
3.2.3 STABILITY OF DEMAND 18
3.2.4 REGIONAL DIFFERENCE 20
3.3 CONCLUSIONS 21

4. THEORETICAL FRAMEWORK 22

4.1 THEORETICAL PERSPECTIVE 22
4.2 MODELLING DEMAND AS A STOCHASTIC PROCESS 23
4.3 INVENTORY CLASSIFICATION MODELS 27
4.4 MULTI-PERIOD INVENTORY MODELS IN RETAIL 29
4.4.1 LOT SIZING 30
4.4.2 REORDER POINT AND SAFETY INVENTORY 30
4.5 DISCUSSION 33
4.6 CONCLUSION 34

5 INVENTORY MODEL 35

5.1 SKU CLASSIFICATION 35
5.2 FAST MOVING 36
5.2.1 NORMAL DISTRIBUTION 36
5.2.2 GAMMA DISTRIBUTION 39
5.2.3 RESULTS OF CATEGORY FAST 40
5.3 SLOW MOVING 42
5.4 LUMPY ITEMS 44
5.5 CONCLUSION 48

6 RECOMMENDATIONS AND FUTURE RESEARCH 49

7 BIBLIOGRAPHY 51

APPENDIX A: TOOLS FOR ETL OF THE DATA 53

APPENDIX B: BISECTION SEARCH ROOT FINDING ALGORITHM IN VBA FOR G(Z) 54

APPENDIX C: SCATTER PLOT WITH OUTLIERS 55

APPENDIX D: JAVA COMPOUND POISSON REORDER POINTS 56

D.1 JAVA CODE FOR CALCULATING COMPOUND POISSON REORDER POINTS 56
D.2 THE POOR MAN'S FACTORIAL ALGORITHM. 59

APPENDIX E: RESULTS OF CALCULATING REORDER POINTS FOR SLOW MOVING PRODUCTS 61

APPENDIX F: GAMMA DISTRIBUTION SEARCH ALGORITHM 62

List of terms and abbreviations

Word or abbreviation	Meaning
ABC classification	SKU classification with the criterion being annual currency value
CDF	Cumulative distribution function
CSL	Cycle service level
DBV	Distribution by value
DCA or DCS	Distribution Center Alphen aan den Rijn/Strijen
DV0	Products only in stock in DCs, not in Service Centers
DV4	Products on stock in Service Centers and Distribution Centers
DV7	Products not on stock but specifically ordered for customer
EOQ	Economic order quantity
ESC	Expected shortage per replenishment cycle
ETL	Extract, Transform, Load, well-known computer science process for data-warehousing
Fr	Fill rate
FSN classifications	SKU classification with the criterion being product consumption rate
MSIMM	Major study of Inventory Management Method
PDF	Probability density function
PMF	Probability mass function
Presentation stock	The minimal amount of stock for a shelf to look ‘presentable’ in a store.
SC	Service Center
SKU	Stock-keeping unit
SLA	Service level agreement
SLM	Service level measure
SNIS	Sonepar Netherlands Information Services
SS	Safety stock
Task stock	Some items are often bought in certain quantities such as table legs, which will often be bought in quantities of 4. Literature calls this the minimal order quantity.
TU	Technische Unie
XYZ classification	SKU classification with the criterion being variation of demand

1. Introduction

The next chapter contains an introduction to the company and the subject of research. Before reading the proposal, I would like the reader to realize and remember that the company is called Technische Unie (TU). Please do not confuse Technische Unie, or its abbreviation, with Technische Universiteit, University of Twente. In the short past, this has caused confusion and I want to make sure this will not happen by writing this short introduction. Section 1.1 provides a short introduction to the company and its activities, section 1.2 is about the motivation for research and section 1.3 describes the problem that is being researched in this project.

1.1 Company description

Technische Unie was founded in 1880 by H.C. Heybroek and is the biggest technical wholesaler in the Netherlands. The company sports more than 2 million products and more than 700 suppliers in the field of electrical technology, lighting, tools, (luxury) sanitary, heating and climate technology. Their customers are active in the installation sector (housing and utility), construction, industry, government and retail.

Technische Unie is a daughter of Sonepar. The French company is active in over 40 countries and has over 43.000 employees. Sonepar has an annual revenue of 20.2 billion euro (Annual report, Sonepar 2015).

The mission statement of Technische Unie consists of two important components:
1. Supporting customers with efficiently and effectively organizing their installation process. Advising them and helping them choose the necessary products for their installation activities. Delivering these products in time and completely at the right place.
2. Unburdening the customers; offering services with which make their activities easier and allows them to deliver a higher quality.

To achieve that, Technische Unie uses the following vision: We make agreements on how we interact with each other, customers and suppliers. We have summarized these agreements in the following words, which we call our values: collaboration, involvement, respect, ease, consistency, leading, total solution, continuity.

The organization of Technische Unie consists of four components: sales offices, transshipment locations, distribution centers and a head office. A Sales office is a regional office which is responsible for demand and processes in its region. Customer-specific orders are picked up by the customer at the sales office. Sales offices can also contain a Service Center. This is a store within the sales office. Technische Unie has approximately 2000 employees distributed over 37 sales offices, 2 distribution centers in Alphen aan den Rijn and Strijen and a head office in Amstelveen. The company has an annual revenue of over €1 billion (Technische Unie, 2016). The revenue has been growing with 8% the last two years.

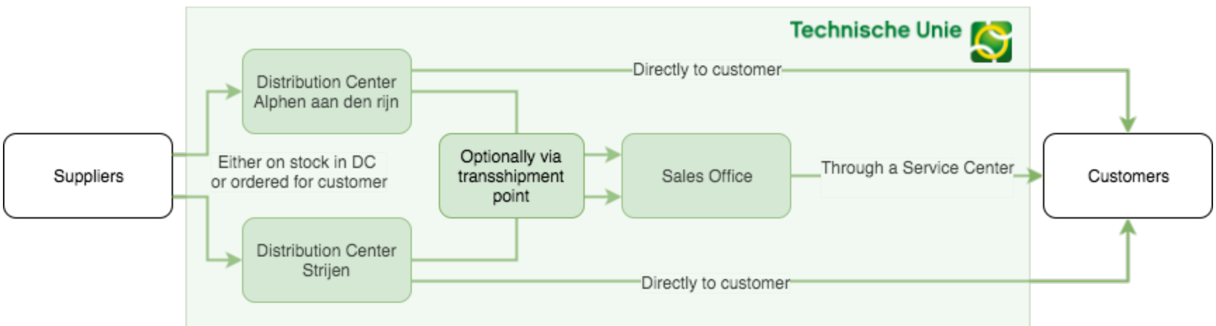


Figure 1.1. The logistics structure of Technische Unie

Trucks from the distribution centers to the sales offices arrive every morning around 6.00 am at the sales office, 5 days a week. The customer-order lead-time is 24 hours, and the replenishment lead-time is 48 hours.
Trucks line up in the evening at the distribution centers and drive through night to replenish the sales offices. This is essential for Technische Unie since they aim to score high at service. The company aims to act proactively to changes in planning. For example, if a truck is in a traffic jam, the customer is notified of the possible delay.

About 100.000 products of the total assortment are on stock in the distribution centers. There is also stock in the service centers at the sales offices. About 8 to 10% of the stock in the distribution centers is also available in the service centers, this comes down to a bit more than 10.000 products. This stock is not controlled by the distribution centers. A simplified logistics structure is given by figure 1.1.

In 2017, a large program with multiple areas of improvement is running. Technische Unie asks me to propose a new model for controlling stock in the service centers, based on (historical) data and a literature review.

1.2 Research motivation

Currently, the responsibility for inventory is divided. On one side there is the department inventory management which is responsible for monitoring the inventory levels within the assortment of articles on stock. On the other hand there is much freedom within the sales offices for maintaining the inventory level in the service centers due to the focus on service and giving the sales office the opportunity to focus on their customer needs. The purchasing department is responsible for the decision of which products are actually in assortment and which products in assortment are actually kept in stock. These are the DV0 and the DV4 product codes. DV0 is kept on stock only in the distribution centers while DV4 products are also kept in stock in the Service centers.

This divided responsibility is a source of sub-optimality due to the fact that stock in the service centers is not controlled using business rules, but is often based on the feeling of the person that sets the safety stock and order quantity for a certain item at a service center.

Technische Unie mostly operates in a business-to-business environment. They make use of the advantage of the risk-pooling effect. TU will have the spare part of a machine that 10 companies use for instance, with the alternative being that every company has their own spare part inventory. This results in sales offices having a relatively high inventory level for their stock keeping units[1] (SKUs) since the focus on service, serves as the decision to have enough of every SKU to make sure there is never a stock-out on a crucial moment.

This was also an issue in the distribution centers. Therefore, a project with the aim of controlling inventory better has been carried out in 2016 and the results of this project are a major success. Therefore, Technische Unie wants to do the same in the service centers.

1.3 Problem description

Together with the manager R&D logistics, the inventory manager and a logistics advisor the problems have been discussed. They were also involved in the 6-stage model project, that reduced the inventory in Distribution center Alphen aan den Rijn, DCA.

Wrong products on stock

Figure 2.5 shows the product categories Technische Unie alongside the number of products in this category. It is interesting to observe that there are more than 5000 items of product categories D and E

[1] A stock-keeping unit (SKU) is a distinct type of item for sale, such as a product or service and all attributes associated to the item type that distinguish it from other item types (Sawaya & Giaunque, 1986)

on stock – these are the DV4 items – but almost 400 of the A and B categories are not on stock, the DV0 items.

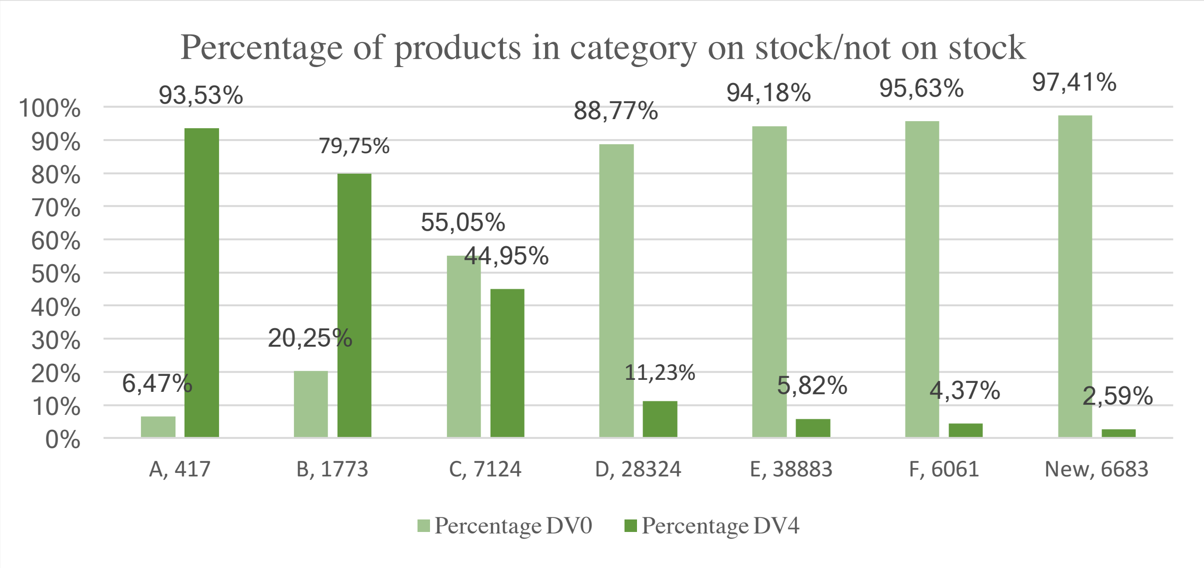


Figure 1.2. Product groups divided in 'on-stock' (DV4) and 'on-order'(DV0)

Should there be that many C, D, E and F products on stock is a question that the inventory manager underlines, and, what are the criteria for products to be DV0 or DV4.

It can also be seen by looking at the Inventory Turnover, this is now 2,5 meaning that TU sells its inventory twice and a half a year on average. The items that are on stock now are not sold enough. These items lie on stock relatively long before they are sold, and therefore there should be less stock, or the product should not be on stock at all. An extreme case is observed in the service center of sales office Utrecht: of an industrial component, there are three, already by the sun decolorized, white packages, and behind it are two green packages. The white packages haven't been used anymore for 5 years. This means that there could lie more than 10 years of stock.

That there is too much in stock can be seen by looking at the service level. Technische Unie aims to achieve a service level according to figure 1.3. Roughly speaking, Technische Unie says this comes down to a target of 98% of order lines completely delivered to customers on average.
98,5% is achieved on average. The inventory manager says that it is not a problem for fast-moving SKU's, but it is for slower moving SKU's.

Product categories at Technische Unie

A	B
- First 25% of orderliness - 417 products - Target service level: 99%	- Second 25% of orderliness - 1773 products - Target service level 98%
C	D, E, F and new
- Third 25% of orderliness - 7124 products - Target service level: 97,5%	- Last 25% of orderliness - 79951 products

Figure 1.3. Product categorization at Technische Unie

A commonly heard complaint within Technische Unie is that there are relatively many products ordered specifically for a customer. These are either emergency orders via TU's logistics service agency (LSB) or specific, manual orders (DV0). This is done because the 48-hour lead time is perceived as too long by the Service Centers. These items could have been sold via stock. This problem has two underlying problems. Firstly, again this item could have been on stock so the problem is that wrong items are on stock. Secondly, there could be a stock out, for which a problem might be that there is not enough on stock for this item on average. Both are caused by the core problem.

Core problem

Following the managerial problem solving methodology (MPSM) of (Heerkens & Van Winden 2012), which is discussed in section 2.3, the core problem is the problem that has no known cause and can be influenced by the researcher and should be stated as a - by the problem owner observed - discrepancy between a norm and reality. The statement of a core problem as a discrepancy between a norm and reality is not straightforward for the design of a stock and replenishment model. For some products, stock may be too high and for others stock may be too low. One can then not simply state in the core problem that stock is too high, or too low. It is known that both occur at Technische Unie. An underlying problem for too high or too low stock is the existing model for stock and replenishment, caused by the current inventory and replenishment control models. Therefore, together with Technische Unie, I have stated the problem in the following way:

At the service centers of Technische Unie, the current model for inventory- and replenishment-control is deficient to satisfy the desire to lower the inventory value of €16,7 million.

We realize that this statement does not include the core of the problem cluster '*Current inventory control parameters are not calculated based on business rule*s' but the core problem now also has an implied goal: the inventory value should be lower.

The 6-stage-model has proven that inventory value can be decreased. Even though the model that is used in the distribution centers is not applicable one-to-one to the service centers, TU can control stock in the service centers in a similar, thorough way.

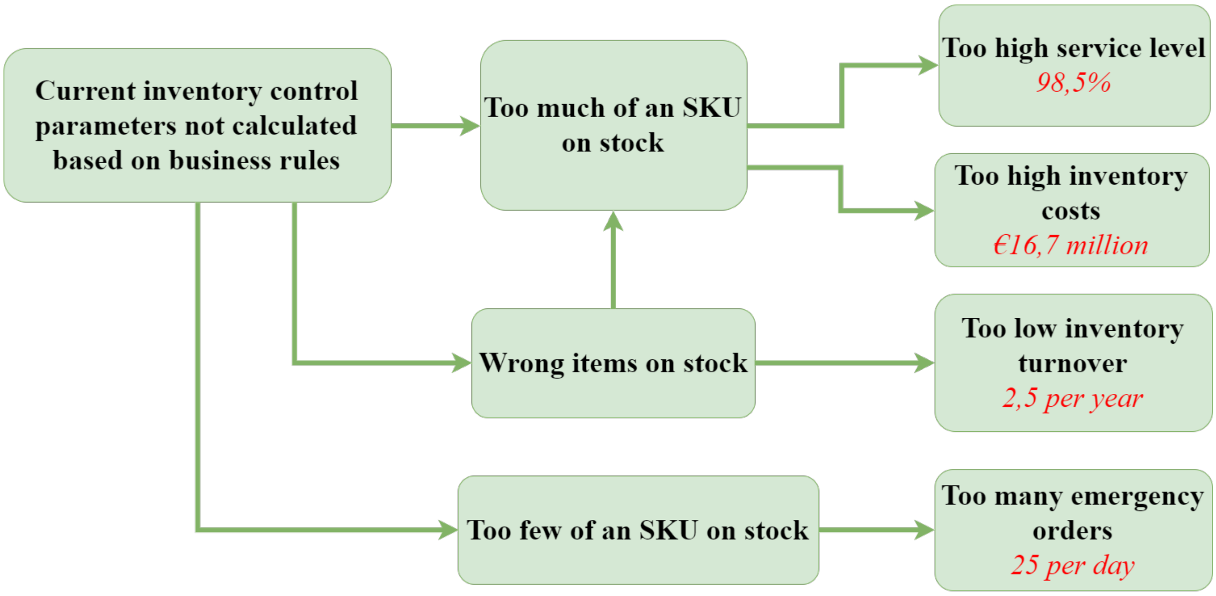


Figure 1.4 Cluster of problems occurring at Technische Unie

Norm and Reality

(Heerkens & Van Winden, 2012) emphasize in the MPSM that in a problem identification process, there should be a clear stating of a standard, or a norm, and a reality. Technische Unie told me that the company does not have a clear norm. This makes it difficult to identify by how much Technische Unie wants to improve. However, the results of the 6-stage model project provide some support:
The scale of the SC's is smaller than the scale of the DCs but they do provide an idea of how much can be saved.
Currently the inventory value of the service centers is €16,7 million. The aim of the research is therefore to propose an inventory control model that lowers this amount.

As indicators for the problem, I use:

- Service level, Technische Unie aspires to have an order line fill rate of 98% on products that are on stock. SC Watergraafsmeer achieved an average service level of 98,5%. This means that there was too much on stock on average.
- Inventory turnover, TU has an inventory turnover of about 2,5 in the SC's. Of the products with service level of 98,5% in SC Watergraafsmeer, a lot of items were slow moving items: The wrong items were on stock.
- Percentage of products on stock that are not sold in the last 15 months. this is 16% right now.
- Percentage of products that are sold once or less per month. This is 63% now.

Note that these indicators were measured in SC Watergraafsmeer in 2015 and could therefore not be representative for all the 37 service centers. However, it still gives an impression.

2. Research design

After the problem identification, this chapter will focus on the research itself, discussing the boundary conditions and scope of the research in section 2.1. Then, the research questions in 2.2, and finally the research methodologies I use in this research are presented in section 2.3.

2.1 Boundary conditions and scope

- **Service centers:** The project will focus on the stock in the service centers. The distribution centers currently replenish the service centers, but are out of the scope of the project. The project is part of a larger project. This project runs the whole year.

- **Forecasting:** Existing averages and standard deviation of demand for determining for instance the safety stock will be used. Demand is assumed to be stationary, so level and seasonality are not included. It is however measured that this is not the case.

- **Generalization** to product categories is part of the scope. Since it is not yet clear what the boundary conditions and scope of the overall project will be. The generalization focusses on the products that are on stock in the SC's (DV4).

- **Time horizon:** of the project is roughly 10 weeks.

- **Implementation:** Implementation of the model will not be part of the project.

- **Backordering vs lost sales:** The backordering methods will be used since for many products, there are not that many fast alternatives, meaning that the product will likely be backordered. This is of course not a reliable approach for all items.

2.2 Research questions

The aim of the project is to lower the inventory value by designing a new method for controlling stock in the service centers based on the analysis of (historical) data and a literature review. To achieve that, the topic is divided into subtopics. These subtopics, the stakeholders and the resources and requirements are discussed in this paragraph. The core problem identified in the problem identification phase is:

At the service centers of Technische Unie, the current model for inventory- and replenishment-control is deficient to satisfy the desire to lower the inventory value of €16,7 million.

This raises the following question: What model for inventory and replenishment control should be used to lower the inventory value? This question implicitly also asks what products should be on stock. To answer the question, this paragraph contains sub questions to get better understanding of the problem, and more importantly, to answer the question in a thorough way.

Current situation

1. How does Technische Unie currently manage inventory in the service centers?
   a) How does Technische Unie's replenishment system work?
   b) How are the safety stocks determined?
   c) What is the effect of the current way of managing inventory?
   d) Can regional differences be observed when comparing data from different service centers?

In this section, I research and describe insights on the current inventory management approach based on historical sales and experiences within Technische Unie. This provides guidelines and perspectives that support decisions later in the research, for instance the choice of an appropriate perspective of inventory classification that is discussed in chapter 4.

Literature

2. What product categorization models for retail are available in literature?
3. What inventory- and replenishment models are known in literature?
4. What methods for modelling demand are known in literature?

Literature should provide models that are applicable to the situation at Technische Unie. Important aspects of demand and the logistics structure of TU, described at Chapter 3, are taken into account. Considering both this aspects and the models provided by literature, models that seem appropriate are chosen which will be the underlying theory for chapter 5.

Inventory model

5. How should Technische Unie control their inventory?
   a) How should the SKUs be classified?
   b) What is the optimal level of safety inventory?

Results

6. What are the expected results of using the new inventory control methods and what does this imply for Technische Unie?

2.3 Research methodology

I follow a combination between two methodologies

- *'Major study of Inventory Management Method'* (MSIMM) (Silver, Pyke & Thomas, 2016)
- *'Managerial Problem Solving Method'* (MPSM) (Heerkens 2016).

The methodology of (Silver et al., 2016) focusses on the field of inventory management and is therefore very useful, but the MPSM is very concise and clear. I follow the structure of the MPSM, but I will use the content of the MSIMM.

For explanatory purposes, I will discuss the main differences and similarities and connect the MSIMM to the MPSM in figure 2. The last phases are implementation and evaluation, but since these are not part of the scope of the project, these phases are not discussed.

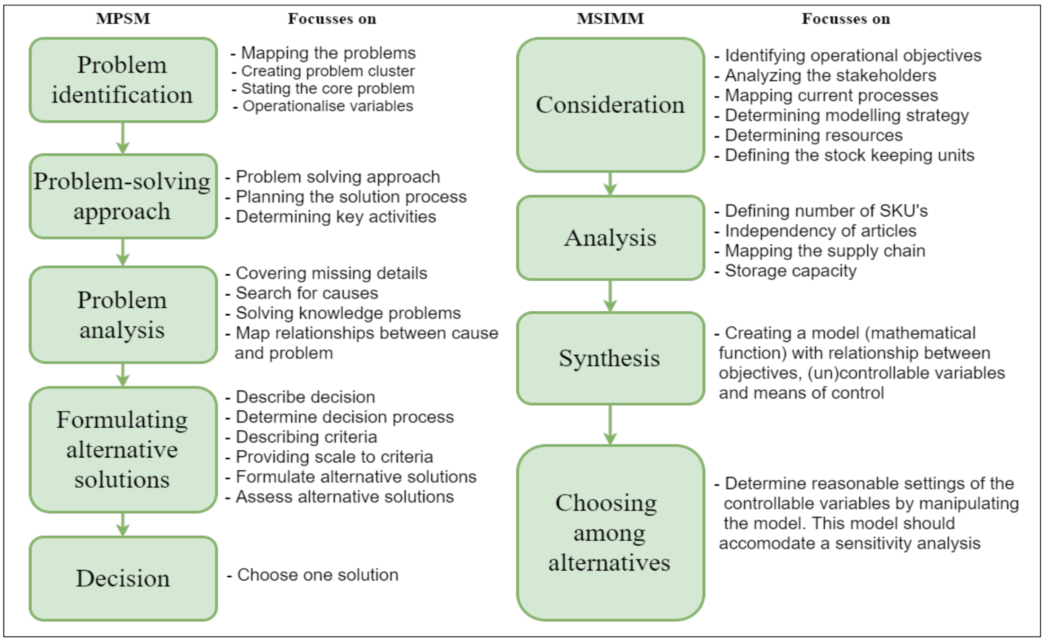


Figure 2.1 The MPSM vs the MSIMM

Note that figure 2.1 depicts only simplified versions of the MPSM and the MSIMM.

2.4 Reading guide

The report is structured according to the research questions. Chapter 3 analyses the current situation to which the improvement proposals will be compared. It discusses the demand characteristics in the service centers at the moment.
Concluded from this chapter will be the patterns and characteristics of Technische Unie's demand currently. For instance, how level is demand for their products or does it vary a lot.

Chapter 4 proposes a theoretical perspective and it discusses the results of the literature study in line with the structure of the research questions. First, SKU classification methods are reviewed through literature. Then inventory and replenishment models are discussed. Finally, methods in literature for modelling demand are reviewed and discussed. Concluding, criteria and thresholds will be presented by which inventory will be classified.

In chapter 5, the results of the literature study will be used to propose a new method for controlling inventory in the service centers. The safety stock is calculated after the SKUs have been classified. This model will use the methods concluded from chapter 4.

Chapter 6 discusses the results of the research and presents an advice to Technische Unie. It regards recommendations and future research.

3. Inventory management: current situation

This chapter tries to answer the question "*How does Technische Unie currently manage inventory in the service centers*". This is done by dividing the question into three categories. The current replenishment system is the first topic and is discussed in section 3.1. Section 3.2 discusses the effect of this inventory management methods. Section 3.3 concludes the chapter.

3.1 Order and replenishment processes

Items can be bought both directly at the service centers and ordered specifically for the customer, when items are not in stock at the service center for instance. Figure 3.1 depicts the structure and information flow of the replenishment process and information flow.

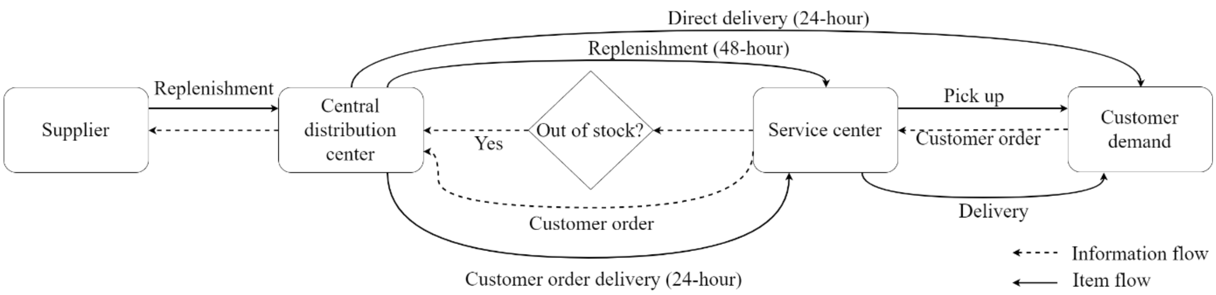


Figure 3.1, The sales logistics structure

The items in the service centers are of course sold out of stock. For these items, Technische Unie currently replenishes using a (s, Q) – model[2]. The system uses the following parameters:

- Minimum stock level.
- Order quantity.

The minimum stock level is the reorder point, and the order quantity, as the name suggests, is the order quantity in the (s, Q) – model. Every four weeks, the minimal stock level is calculated based on the forecast, the current stock level and the replenishment lead time this process is depicted in figure 3.2.

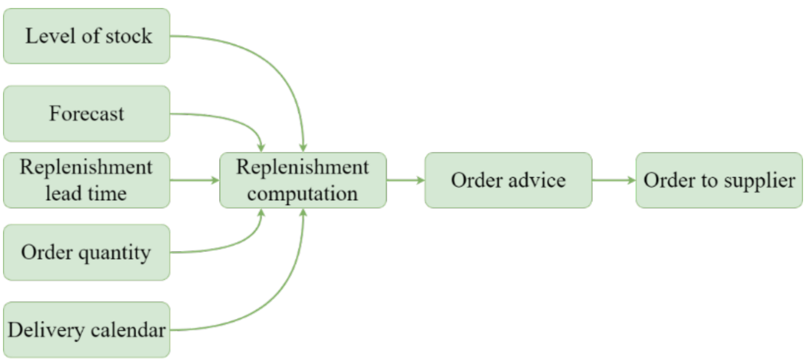


Figure 3.2, the replenishment process

[2] In (Winston, 2004) this model is called the (r, Q) – model and it has many names in literature. It is often referred to as the two-bin system for instance.

After every day, replenishment orders with products that are ordered from the same supplier are aggregated. The supplier can then try to deliver multiple orders in one truck. By doing this, Technische Unie tries to minimize the transport cost.

Currently, every sales office can determine for themselves whether to accept the newly determined safety stock. Most of them put no logic at all behind the determination. Some leave the parameter at the initial level, others make the value either higher or lower. This results in sub optimality:

- There are products for which the order quantity is set to 0, meaning that no products will be reordered if the reorder point is met.
- There are products for which the reorder point is set to 0, meaning that the replenishment time and the stochastic behavior of demand is not accounted for by means of safety inventory and the product faces a stock out of minimal 48 hours.

The target service level is based on the product categories, which are based on the product groups depicted in figure 1.3.

3.2 Effects: analysis of historical data

Many sales data are available. I will broadly discuss the extract, transform and load (ETL) process.
The tools of TU only allow for a download of sales data over a specified period with a maximum of roughly 1 month, because the . To cope with this amount of data, the data has first been put into separate text files. These text files have been bulk-loaded into a MySQL database using a shell script. For reproduction purposes, this script has been included in Appendix A[3].
After the bulk-load into the MySQL database, the appropriate sales data needed to be downloaded to a work-friendly MS-Excel file. For this purpose the SQL query in Appendix A was used. There has been put a lot of time and effort into cleaning and validating the data. In the Excel file, a pivot table was created and the data analysis could commence.

3.2.1 ABC-analysis

To get more insight in the percentage of consumption value over the percentage of items, a quick ABC-analysis has been done by multiplying the total number of sales of an item by its price and writing this number as a percentage of the sum of all prices. Putting these percentages next to the percentage of all sold SKU's responsible for these percentages results in figure 3.3. The thresholds for the classes A, B and C are arbitrary since the aim of the analysis is to get an insight in the distribution of inventory value, not to use the analysis for inventory management.

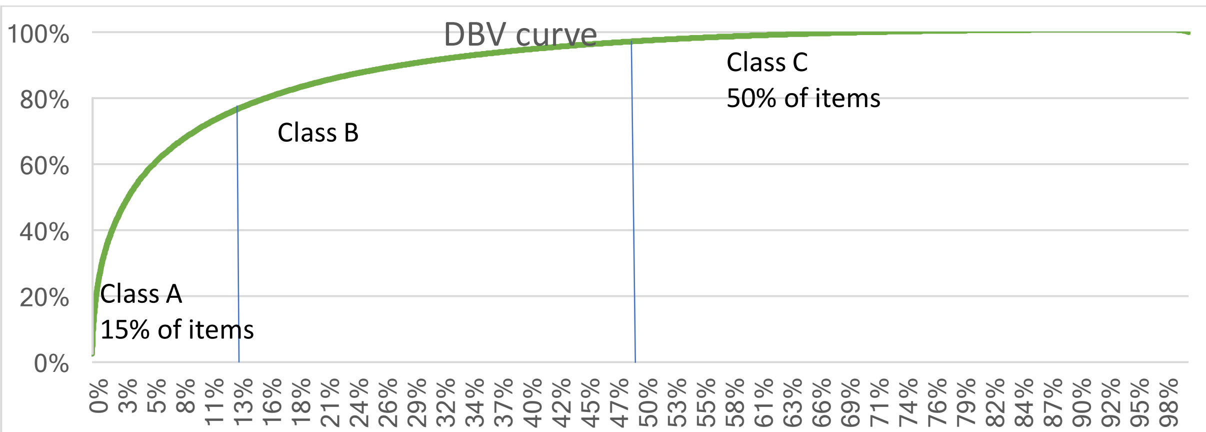


Figure 3.3, the result of an ABC-analysis depicted using a DBV curve

---

[3] Note that this was done on Unix (Darwin 16.4.0) so windows users cannot use this script.

3.2.2 Frequency of sales

Figure 3.4 shows the items that are ordered a specified period from now, followed by the percentage of total sold products. Obviously, the shorter the period, the longer the number of unsold SKU's. Remarkably, 11% of the products hasn't been issued in the last 2 years. These products are probably not being sold at all in the service centers. This raises the question whether these products should be on stock. Companies use different thresholds for calling an item a non-moving item. Some use half a year, others use 2 years. If half a year would be the threshold, 38% of the stock would be non-moving stock. More than 70% of the items are not sold even once during June 2017. This doesn't say a lot necessarily, but if this was on average, then it says something about the frequency of sold items.

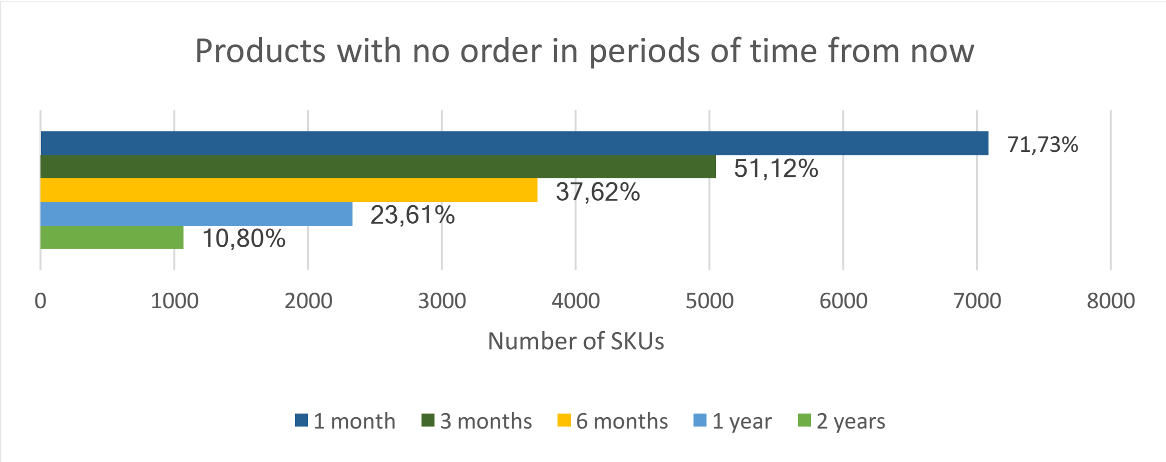


Figure 3.4, items that weren't issued in specified period from now

The frequency of sales seems remarkably low. Therefore, all sold SKU's of 2.5 years of data was analyzed on their average demand interval, or, in other words, the average period of time between two consumptions. Two and a half years of data seems appropriate since it discards data of items that were in their introduction period two years ago, or products that are in their end of their lifecycle. Of course it does not fully account for these.

3.2.3 Stability of demand

A way to look at the stability of demand for products is by means of the variance-to-mean ratio of replenishment lead time demand (SCV). This coefficient divides the variancce in lead time demand by the average lead time demand. The lower the coefficient of variation, the more stable the demand for an item is. The use of this measure is discussed more thoroughly in section 4.2. In figure 3.5 it can be seen that for many products demand is not stable at all. The figure should be read as follows: roughly 750 SKU's have a CV between 14 and 15.

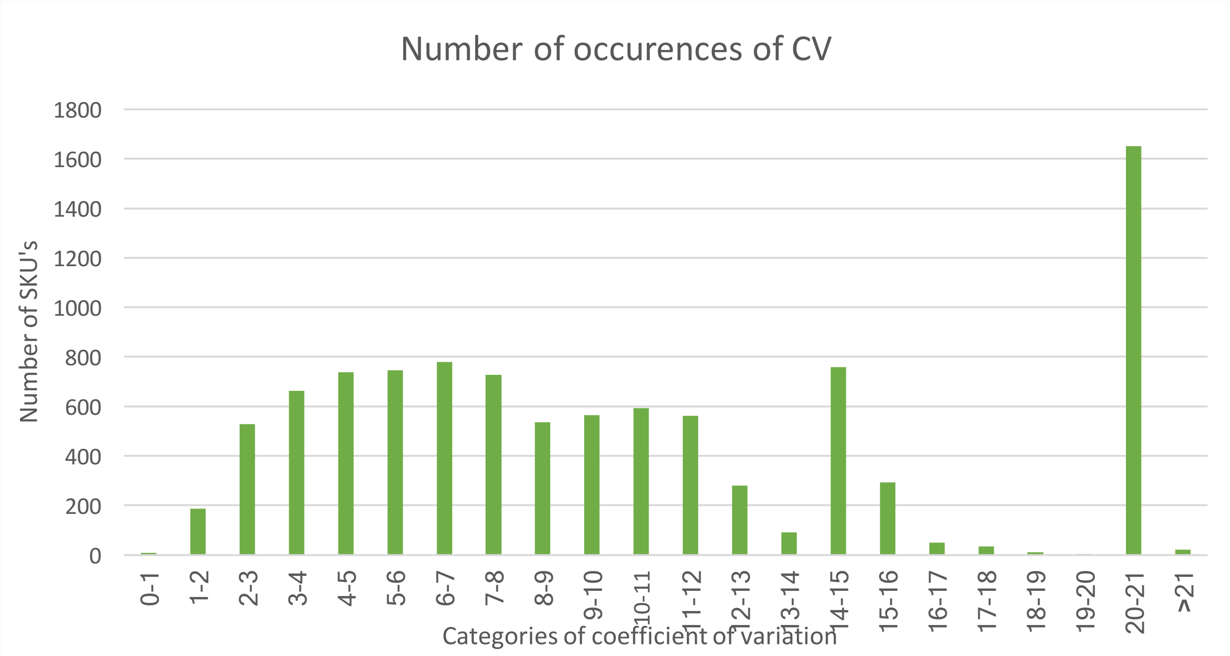


Figure 3.5, the number of occurrences of the coefficient of variation

A CV higher than 1 is considered unstable according to literature, which is also discussed in section 4.2. One of the reason for such unstable products in this measure, is the relatively short lead time which is used when the measure is determined. When using longer lead times or larger time buckets, such as weeks or months, one gives an SKU the opportunity to get sold more retroactively which will lead to higher safety stock levels.

The most occurring CV is between 20 and 21. It is interesting to know whether demand for an SKU is only erratic, or whether intermittency is also a factor. Therefore, figure 3.6 is constructed, which is a scatter plot based on the average interval of demand occurrences in days (ADI) over the squared CV of nonzero demand. Items that had only one sales in period are not taken into consideration: there is no interval of demand when there is only one sale. In literature, this method is used for classifying demand patterns.

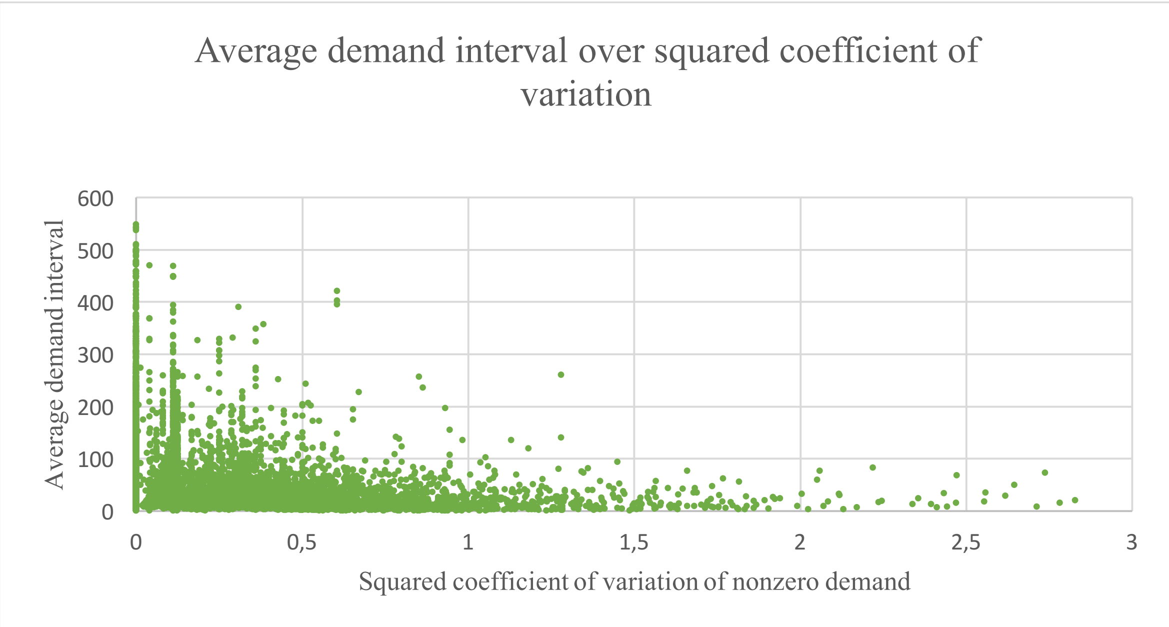


Figure 3.6. Scatter plot of SKU's based on average demand interval and SCV

This scatter plot is not of all SKU's, but some outliers have been removed since these values make the figure less easy to read. Therefore, the figure with outliers is provided in appendix C.
There is one value around SCV 18. This SKU has relatively few time between consecutive sales but the order size varies a lot. Many items seem to have no variation in demand size. This is obvious for the products with a high ADI since there are only few sales records for these items, meaning that if there were only 2 sales records of both 1 item, the variance in nonzero demand is 0, and therefore the SCV is 0. This explains why so many values seem to have no variation in nonzero demand size. These SKU's are the smooth and the intermittent demand groups. When there is more variation in demand – literature will again explain more – there are two other groups. First of both, the erratic demand. These items are sold on frequent basis, but show a high variance in demand size. The last group contains the most difficult items to plan for; lumpy demand are items that have high variance in their demand size, and also are not sold on frequent basis.

3.2.4 Regional difference

There have been voices within TU to give the sales offices more freedom in choosing the assortment in compliance with regional demand. At the time, all demand is aggregated and the forecast is done over all aggregated demand of the 37 service centers. We have analyzed one year of sales data of ales office Watergraafsmeer, sales office Nijmegen and sales office Middelburg. We decided to go for Watergraafsmeer and Middelburg since they looked the most different from each other in a demographical and geographical perspective. Watergraafsmeer lies in Amsterdam. Being a large city, the demand of for instance fence wire is likely to be lower than for instance in the city of Middelburg which has roughly 50.000 inhabitants at the time speaking. Nijmegen has been chosen for comparison with both. One would expect Nijmegen, more comparable to Amsterdam, to have sold more of the same SKU's than Middelburg.

Sales office	Number of sold SKU's sold
Sales office Watergraafsmeer	7116
Sales office Nijmegen	6900
Sales office Middelburg	6240
Total unique SKUs sold	**8532**

Figure 3.7. Number of SKU's sold in service center.

First, the total number of unique SKU's have been identified. Figure 3.7 shows us that there is a significant difference in the regions. Nijmegen sold 81% of the total number of unique SKU's for instance. More importantly is to look at how different the three Sales offices are compared to each other. It could be that the 6900 sold SKU's at Nijmegen are all also sold at Watergraafsmeer. This would mean that 8532 – 7116 SKU's were sold only by Middelburg. Therefore, figure 3.8 provides a different perspective. It shows the results of comparing two sales offices to each other.

		Looking at		
		Watergraafsmeer	**Nijmegen**	**Middelburg**
Compared to	**Watergraafsmeer**	-	84,77%	86,23%
	Nijmegen	82,21%	-	84,97%
	Middelburg	75,63%	76,84%	-

Figure 3.8. Comparing the SKUs sold in 2 sales offices

Remarkable is that of all SKU's that were sold in Watergraafsmeer, only 76% was also sold in Middelburg. Since Watergraafsmeer sold more SKU's, the other way around this is 86%. It seems to be true that Nijmegen is somewhat in the middle of the two. I would like the reader to keep in mind that this analysis leaves much to be desired. Only 3 of the 37 service centers were analyzed. The percentages of all sold SKU's can therefore only decrease. What's more, since we showed that for many SKU's demand is not really high, we cannot conclude that the observed difference is solely due to what we call regionality. It could also be due to randomness.

3.3 Conclusions

First conclusions when looking at data is that demand seems unstable for many SKU's. Very high coefficients of variation raise the question whether intermittence and slow-moving items are common within the service centers. The answers can be quite short since the remainder of the report has quite some overlap with these questions and the methods that are used are not very sophisticated.

The research question that has been answered in this chapter is

How does Technische Unie currently manage inventory in the service centers?

The sub-questions that belong to this question are:

How does Technische Unie's replenishment system work?
Technische Unie replenishes through a continuous (s, Q) system, using a fixed lot size Q and reorder point s. The lot size is not determined on an EOQ, but set to a level that seems acceptable to the manager of these parameters at the Service Center.

How are the safety stocks determined?
Safety stock are not explicitly determined mathematically or based on literature, but again, the reorder point (and thereby the safety stock) is set to a point that seems fair to the responsible at the Service Center. It could be that some Service Centers have responsibles that use methods from literature since it remains their decision.

What is the effect of the current way of managing inventory?
The results of the current ways of managing inventory are that many products are on stock that are almost never sold. Whether the stock levels are either too high or too low is determined later in the project, after the incorporation of literature.

Can regional differences be observed when comparing data from different service centers?
Regional differences between service centers are observed. Roughly 15% of the assortment differs regionally however this number also includes some randomness. Every service center should therefore have its own forecasts. Due to the time issues the scope is set to SC Watergraafsmeer in the remainder of the report. We expect this service center to be an appropriate representation of all the service centers.

4. Theoretical framework

In this chapter, a theoretical perspective is taken and theoretical models are provided. The decision for the theoretical perspective is discussed. The chapter starts with a theoretical model in which the literature review is presented in a clear way. Then the modelling of demand as a stochastic process is discussed. Section 4.3 continues with demand classification methods and to compile the first sections to be of use to inventory management, section 4.4 presents inventory control models that will be used to control safety inventory and the replenishment process. This chapter also includes methods for how to calculate the reorder points using the stochastic processes discussed in section 4.2. The last section concludes the chapter.

Figure 4.1 shows the theoretical model which was used in this project. Initially, two research questions would be answered through literature, being which inventory models exist in literature and which inventory classification models exist in literature. However, since there are so many models in literature, reading literature raised another question. How should demand be modelled? Which pattern demand follows can actually make a huge difference. Therefore, this has also been included.

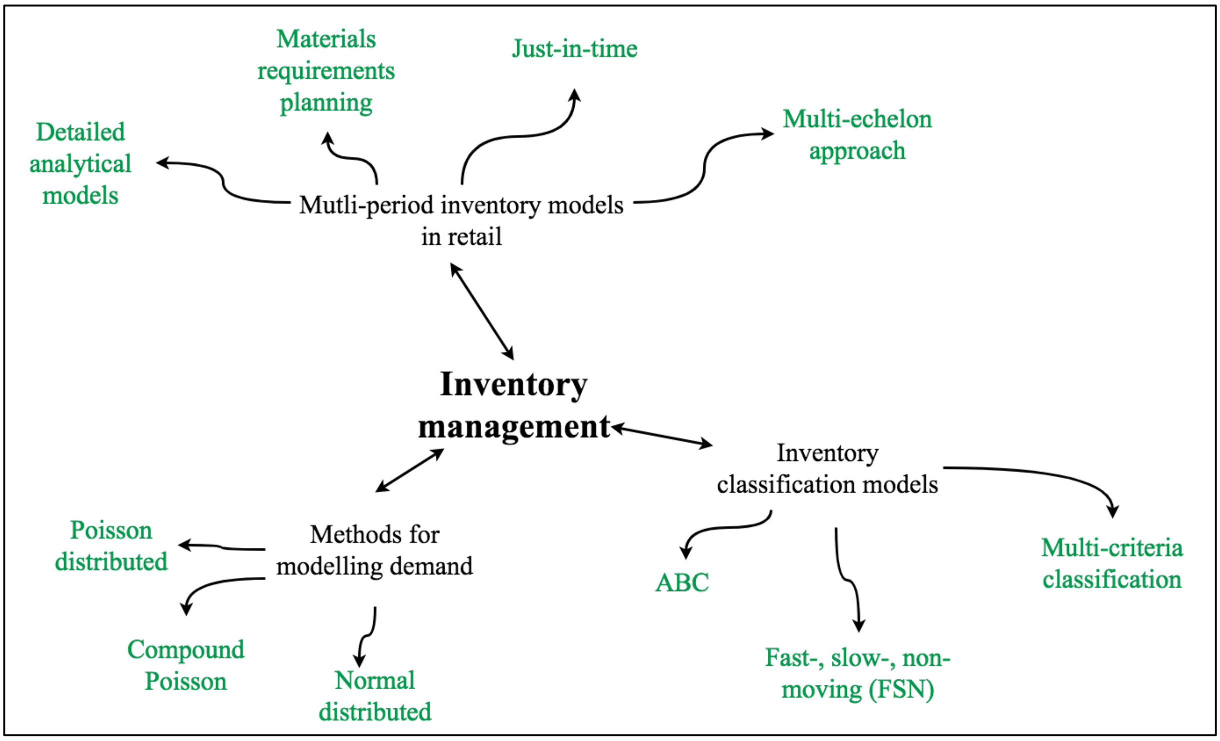


Figure 4.1, Theoretical model of considered literature

4.1 Theoretical perspective

In this project, the operations research perspective of inventory management is taken. Detailed analysis of processes is done and mathematical techniques are used as argumentation for decisions. An example of a technique used by this perspective is the EOQ.

Other perspectives are for instance the Materials Requirements Planning (MRP)- approach, the Just-In-Time (JIT)-approach (Silver et al., 2016). MRP is broader than the detailed approach and does not necessarily make use of mathematical models. The approach has less aim towards optimization. Since optimization is the aim of the project, this approach is not used. The aim of JIT is to minimize inventories without the use of mathematical models, by reducing wastes. This does not seem as the right approach since Technische Unie might want to hold inventories on stock that do not directly contribute value such as presentation stock. Another reason for using the detailed analysis perspective is that I am more familiar with this perspective.

Another approach to inventory modelling is the multi-echelon approach**.** As the name suggests, in models using this approach, there is looked more holistic to the supply chain. Stores and warehouses are not deciding separately in these models anymore, but for both it is identified what would be an ideal level of stock using for instance risk pooling: An aggregated forecasted demand and standard error of the forecast over multiple stores that are supplied by one warehouse can decrease the safety stock for instance; if an item is not sold in store 1, it might be sold in store 2. This decreases risk. (Chiang & Monahan 2005) suggest a queuing model/Markov chain approach to multi-echelon inventory management using a mathematical and operations research perspective.

The dual-echelon approach of (Chiang & Monahan 2005) is arguably the most comparable to the processes of Technische Unie. TU operates in a dual echelon structure: distribution center – service center. However, since the DCs are not in the scope of this project, I will not use the multi-echelon approach. Other argumentation for not using a multi-echelon approach includes the current way of working at Technische Unie: The responsibility is divided and it would cause a lot of reorganization and resistance within TU when one department now would be responsible for the control of all stock. In the future, it might be interesting for Technische Unie to research a more holistic view of their inventory management. For instance, lateral relationships between the service centers.

4.2 Modelling demand as a stochastic process

One of the most important concepts in inventory management is safety inventory. This inventory is kept planning for unpredictable variability in demand. When demand would be deterministic, or the variability would be totally predictable, safety inventory wouldn't be necessary since one could plan for this variability. However, demand is usually stochastic in retail and its variability can be very hard to predict. Since the sales of items depends on different aspects, there are many different patterns to be found. The right methods of forecasting demand based on historical data depends on which pattern demand takes.

Statistical functions can be used to model demand. A lot of models assume demand to be normally distributed over time. This can be a good approach, but doesn't necessarily have to be the best solution. (Janssen & Ramaekers 2008) write *"a different demand shape can increase average inventory with more than 100%, given the coefficient of variation ratio"*. This of course also means that the inventory can halve. The coefficient of variation is defined as the standard deviation of lead time demand divided by the expected lead time demand.

$$Coefficient\ of\ variation = \frac{\sigma_L}{\widehat{x_L}} \qquad (4.1)$$

in which

$$\widehat{x_L} = expected\ value\ of\ lead\ time\ demand$$
$$\sigma_L = standarddeviation\ of\ lead\ time\ demand$$

These values can easily be determined using the following equations
Average lead time demand

$$\widehat{x_L} = L * E(D)$$

Which obviously leads to

$$\widehat{x_L} = 2 * E(D)$$

Since only so few trucks do not arrive on time, the variance in lead time is regarded negligible. The variance and standard deviation in lead time demand can then be calculated using the following formulas.

Variance in lead time demand:

$$Var(X_l) = E(D)^2 * var(L) + E(L) * var(D)$$

Due to the law of total variance and since we neglect the lead time variance, this simply becomes

$$Var(X_l) = E(L) * var(D) = 2 * var(D)$$

It should be clear that the standard deviation is simply the square root of the variance. These formulas have been presented in literature extensively. From the standard deviation in lead time demand, one can target a service level such as a target fill rate or a cycle service level. We will discuss only the fill rate since that is the used service level measure of Technische Unie. This is done in section 4.4

In fact, the coefficient of variation seems to be an often-used ratio in literature. (Winston, 2004) for instance state that a squared CV > 0.2 makes demand *"too irregular to justify the use of an EOQ model*." The methods proposed by (Janssen & Ramaekers, 2008) assume that only the mean and standard deviation of the distribution of demand during the lead time are known. It makes use of the Pearson chart and use a compound Poisson distribution. They conclude that one should either use a J-shaped Beta-distribution or a unimodal Beta-distribution when using a Poisson distribution to model the frequency of orders and demand size is one of the distributions that the paper examines.

(Silver et al., 2016) discuss the use of a Nonnormally distributed lead time demand. They provide a rule of thumb using the CV. The stated rule of thumb says that one should consider a Nonnormally distributed lead time demand when the CV > 0.5. Of course, when the ratio CV < 0.5, the normal distribution should provide an adequate approximation. This is underlined by (Janssen & Ramaekers, 2008) who state that only when a variance is low with respect to the mean, which implies a low CV, the Normal distribution is appropriate. (Silver et al., 2016) write that one should use a Poisson distribution only when $\widehat{x_L} < 10$ on average. One should only use the Poisson distribution when $\sqrt{\widehat{x_L}}$ is within 10% of σ_L, or, in other words $0.9 \leq \frac{\sigma^2}{\widehat{x_L}} \leq 1.1$ since a property of the Poisson distribution is that its mean λ is the same as its variance.

$$\lambda = E(x) = var(x)$$

If this is not the case then one should use a more complicated distribution such as the Gamma distribution, or a compound Poisson distribution in which a "sales" event is modelled by the Poisson distribution and the size of the order is modelled by another probability distribution such as the geometric distribution (Silver et al., 2016). However, (Axsäter, 2006) writes that in general one can use the Poisson distribution for items for which $\frac{\sigma^2}{\widehat{x_L}} < 0.9$ even though this means the variance is overestimated.

The value for λ is the average of demand occurrence over a time t. Note that when calculating this value, one should count the sales events, not the sold quantities.

Compound Poisson demand
If demand follows a compound Poisson process, and we assume that demand occurrences (often called arrivals) are independent of the size of their order. Then the demand occurrences is a Poisson process with intensity parameter λ and average number of customers λt over time period t. Then the probability for a specified discrete number of customer arrivals is:

$$P(k) = \frac{(\lambda t)^k}{k!} \qquad k = 0, 1, 2, \dots \qquad (4.1)$$

Demand size distribution

In demand that occurs according to a Compound Poisson process, the probability of the demand size is often described by another probability distribution. This is called the compounding distribution. (Axsäter, 2006) describes the use of either a logarithmic compounding distribution or a geometric compounding distribution. When demand shows high variations of demand sizes, this can be a good approach since it prevents the calculation very high convolutions as in equation 4.4 However this is not discussed in this project.

Another possibility is to use the discrete empirical distribution. This is appropriate when demand does not follow a general probability distribution such as the geometric distribution. One then needs to find the distribution of demand size.

Let f_j be the probability for total demand size *j where* $(j = 1, 2, 3, \dots)$

We assume demand is always an integer larger than 0 (if demand occurs, the demand is always (1,2,3, …)). We also assume that demand varies, and is not a multiple of some integer larger than 1, such as 5, 10, 15, 20. This is of course always true if the probability of demand for more than one unit is larger than 0, $f_1 > 0$.

Then

$$f_j^k \textit{ is the probability of total demand } j \textit{ when } k \textit{ customers occur}$$
$$D(t) \textit{ is the demand during time interval } t$$

f_j^k can be found recursively using the following recursive function

$$f_j^k = \sum_{i=k-1}^{j-1} f_i^{k-1} f_{j-i} \qquad k = 2, 3, 4, \dots \qquad (4.2)$$

taking into account that

$f_0^0 = 1$ because when there are no customers, the probability of demand 0 is 1.

$f_j^1 = f_j$ because when there is only 1 customer, the size of the customer's order is equal to the total demand.

Then combining 4.1 and 4.2 leads to

$$P(D(t) = j) = \sum_{k=0}^{\infty} \frac{(\lambda t)^k}{k!} e^{-\lambda t} f_j^k \qquad (4.3)$$

for 0 to infinity customers.

Gamma distribution

Under the section about the Poisson distribution we wrote that the Poisson distribution gives an acceptable approximation of demand during the lead time when the CV of demand during the lead time is considerably less than 1, even though this means that the variance is overestimated. On the other hand, when one uses the Normal distribution with a relatively high variance, for instance with a CV of demand during the lead time that is not considerably less than 1, we have to be careful. This is because we then can have a significant probability of negative demand. This is of course an unrealistic scenario and this affects the safety stock. Of course, if we expect fewer demand due to a significant probability for negative demand, the safety stock that is needed is approximated lower. In this case, one should apply a probability distribution that runs on the interval [0, ∞) such as the Gamma distribution. The Gamma distribution will always lead to positive demand. (Axsäter, 2006) also state that the probability for very high demand will be larger when the Gamma distribution is used. Alternatives are the Weibull or the lognormal distribution for instance. They can be found in (Tijms, 1994). Recall that other reasons for why the normal distribution cannot be regarded a good fit for an arbitrary lead time demand include slow moving products for which a discrete distribution may be a better fit or many periods of zero demand which we call intermittent demand, in which case we independently model demand occurrence and demand size (Compound Poisson for instance).

Figure 4.2 gives a representation of what this means in practice and why we should be careful when the CV is relatively high.

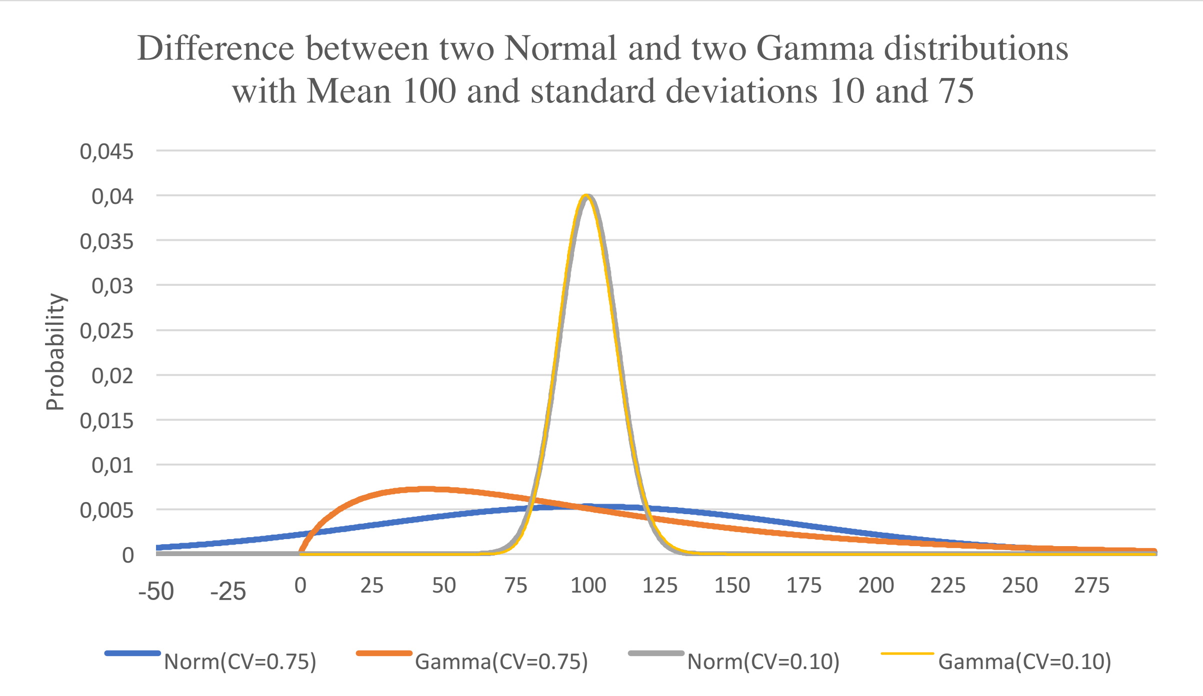


Figure 4.2, The plot of two Normal and two Gamma PDFs for mean 100 and standard deviation 10 and 75

Following the orange line, with a CV of 0.75, it is immediately visible that the probabilities using the Gamma distributions start at 0. Furthermore, while a bit difficult to see, the orange line overtakes the blue line (Normal distribution, CV 0.75) around 250 in terms of highest probability. This is what (Axsäter, 2006) means with larger probability for higher demands.
Now following the yellow and grey line with CV 0.1 for the Gamma and Normal distribution respectively, we see a much smaller difference. The question that remains is what do we set as a threshold. This threshold should be such that we use the normal distribution when the probability of negative demand can be negligible.

Literature uses a CV of 0.5 as the criterion. (Silver et al., 2016) write that the Gamma distribution should then at least be considered. Then the probability of negative demand is 0.0275 using the Normal distribution.

The Gamma PDF is formulated as follows in (Silver et al., 2016):

$$f_{\alpha,\beta}(x) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{-\frac{x}{\beta}}, \quad x \geq 0$$

In which α and β are the shape and scale parameters respectively.

$$\alpha = \frac{\hat{x}_L^2}{\sigma_L^2} \quad and \quad \beta = \frac{\sigma_L^2}{\hat{x}_L}$$

Unfortunately, no closed-form expression for determining values of this distribution exists. Excel provides a function that can determine the values quite easily using GAMMA.DIST(x; α; β; $TRUE$)[4]. We will use this function later to determine reorder points for target fill rates.

Further evidence found in literature

(Matheus & Gelders, 1998) write that one of the downsides of using a discrete probability distribution instead of a continuous probability distribution, is that it is more memory-consuming with respect to input-data even though the formulas are easy to program. Therefore, the use of discrete distributions is a better idea for more critical items because continuous distributions give an accurate approximation.

(Kocer & Tamer, 2011) have done a case study in which they model demand using various methods. They compare the current inventory management model with an empirical, a Poisson, a Croston's and a Bootstrap model. Only when lead time demand < 2 has the Croston's method provided any good results. Both the used empirical model and the Poisson model showed great promise with an increased average service level of approximately 27% along with a reduced inventory cost of approximately €480 over 7 SKUs.

Furthermore, (Lucia, Schiraldi & Varisco, 2016) compare Poisson distributed demand with Normal distributed demand. They showed that when the target fill rate is lower than 98.1% the use of the Normal distribution overestimates the target service level a bit. This leads to a higher stock than when using the Poisson distribution which they claim to be up to 100% or 50% for an SKU with a fill rate set to 1 or 2 pieces.

Conclusion

Concluding this section, there is much evidence for the use of specific statistical functions in specific context. In general, literature regards the Normal distribution a good approximation for faster moving items, while the Poisson distribution is often used in practice when demand is relatively slow ($\widehat{x_L} < 10$) and the demand size is almost always just 1. In cases in which demand is still relatively slow but the demand size is often higher than 1, separating demand occurrence and demand size, a compound Poisson process seems the right approach but only when the square root of the mean demand is more than 10% lower than the variance. When the square root of the mean is more than 10% higher than the variance, still the Poisson distribution will be used. Last, for fast items that have a CV higher than 0.5, the Gamma distribution will be used to be sure that the probability of negative demand is negligible.

4.3 Inventory classification models

Technische Unie has to cope with 2 million products in their assortment. Not all of these are kept on stock. Some are kept on stock in the distribution centers, some are also kept on stock in the service centers. It takes a lot of time to set a target service level for every item, and to control and observe. To control such a complex process, most inventory models make use of product categories. TU uses the product categories depicted in figure 1.3 for the whole assortment. This model is only partly appropriate for the service centers. The products are mainly classified by evaluating how important they are with respect to the total number of order lines. For example, the group of A-products, which contains about 417 products, is responsible for the first 25% of order lines. This is based on the aggregated demand of all products and can change over time.

Literature provides a lot of models for classifying inventory all with their own aim. Many methods are single criterion classification methods. (Pandya & Thakkar 2016) present a literature study in which

[4] In Dutch excel packages this function is GAMMA.VERD(x; α; β; $WAAR$)

they list many single criterion classification methods together with their criterion and aim. Based on this study we created table 4.3 which gives a neat representation of the single-criterion classification methods in which methods that focus on procurement and spare parts management rather than sales have been left out intentionally since the scope of the research is more to sales rather than procurement or spare parts management.

Category	Criteria	Aim
ABC classification	Annual currency usage	Focus on important items that contribute a lot to revenue
XYZ analysis, generally used in combination with another method (e.g. XYZ-ABC, -FSN)	Variability in demand of SKU	To reduce stock level by separating demand distributions
High, medium, low (HML)	Unit price	Control and focus on high cost items
Fast, slow, non-moving (FSN)	Rate at which items sell	To dispose non-moving items whilst fast movers should be kept in high level

Table 4.3, Single criterion inventory classification methods

Arguably the best-known method is the ABC-classification method. This method uses the annual currency usage of items and an arbitrary percentage combination, usually the 80-20 rule saying that 20% of the items are responsible for 80% of the revenue. It is easy to understand that these products are the most important for a company and therefore inventory management must look closely to these items whilst often the C category are for instance 50% of the items but only available for 5% of the revenue, meaning that focus of inventory management should not be on this category.

Other single criterion inventory classification models include the (Fast-, Slow-, Non-moving)-classification (FSN) which focusses on the rate with which an item is sold on average. This method is not discussed in (Silver et al., 2016) but they do mention business rules based on the rate of demand as in Section 4.2.

High, medium low classification uses the price of the unit as a criterion. This is comparable to ABC-analysis but the reasoning now is that high value items contribute more to the inventory costs and therefore a company should focus on keeping these items as low as possible.

The XYZ-classification[5] method uses the coefficient of variation. Items are classified with respect to the steadiness of demand of this item. Very often, XYZ is used in combination with another classification method. (Devrajan & Jayamohan, 2015) use XYZ analysis in combination with FSN and acknowledge the necessity of more than one classification scheme due to that *"various external factors like lost or delayed sales orders and supplies can influence the analysis"*. They conclude that companies should sub-categorize inventory and create an appropriate inventory control system for each category and provide business rules.

Both (Pandya & Thakkar 2016) and (Scholz-Reiter, Heger, Meinecke & Bergmann, 2012) analyze the XYZ classification in combination with the ABC classification method. The former conclude that when demand is very high and fluctuates a lot, XYZ analysis is preferable while the combined study of ABC and XYZ provides better results in inventory management.

(Scholz-Reiter et al., 2012) go more into detail. They found that there is a lot of deviation between two consecutive yearly ABC-classifications in 2007 and 2008: *"the comparison shows that the items were classified identically in only 60 percent of the cases"*. Therefore, they reclassified the SKU's once a month and presented the percentage of monthly deviation in classification. They distinguish two cases. The first case extends the base period from 12 to 13 months. In this case, the SKU-classification changes

[5] Confusingly the name 'XYZ'-method is also used in literature for classification based on the costs/storage value of an SKU.

with 4.24% every month on average. The second case defers the base period by one month. In this case, the SKU's change with 6.5% every month on average. What's more, they acknowledge that future trends should be incorporated in the classification and that the two cases of classification do not allow for this incorporation when only historical data of consumption is used. Therefore, they propose a model that makes partly use of historical consumption data, while the other part consists of forecasted data. The downside to forecasting is that generally its quality decreases over time. However, they found that, after comparing to the classification with actual consumption data, the models that include forecasting performed better than the classification only considering historical data. Their optimum lies around 5 classification months of forecasted data with a correspondence of 92% correspondence with the control group while the ABC-XYZ classification of the last 12 months only has a correspondence of 75%.

Conclusion
Concluding this section. The FSN classification method will be used, but the threshold values between the fast and slow products are chosen at an expected demand of 10 units during the lead time. The motivation for this is that it then is possible to make a difference between which probability distributions is appropriate for what product as described in section 4.2. Non-moving products are not considered as the strategy of Technische Unie is based on delivering service, and while it might be cost-effective to pool these products at the distribution centers, this is something that should be researched better. Therefore, it is out of the scope of this project.

4.4 Multi-period inventory models in retail

Inventory itself is intentionally necessary because of differences in demand and supply (Chopra & Meindl, 2013). A supplier could manufacture in batches larger than its demand since that could be cheaper because of setup costs for instance. A retailer may order more than its demand because of the anticipation of future demand. However, costs are made by holding inventory. Therefore, it is beneficial to minimize these costs. That is not easy because inventory has many factors that complicate decision making. Hence, Inventory models are extensively researched topic in literature.

Most models in literature are either very basic, or very specific. Many books and studies provide the same group of models. (Silver et al., 2016) and (Winston, 2003) provide the following basic models and make decision rules around these models, or implementable in the models.

	Periodic review	**Continuous review**
Fixed order quantity	(R, s, Q)	(s, Q)
Variable order quantity	(R, s, S)	(s, S)

(Boylan, Syntetos & Karakostas, 2008) use an (s, Q) model to test the performance of the classification scheme of (Syntetos et al., 2005). In their study, the focus is on very slow-moving items. However, but they state that they do not expect significant difference when using other stock policies. This is also underlined by (Sami & Kingsman, 1997). In their research, they use empirical data to show that there are no significant differences in control policies when they are used to manage intermittent demand for instance. The main advantage of the (s, S) model when compared to an (s, Q) model is that its lot size is automatically set to a level that fulfills the minimum stock level. If a slow product is bought in different quantities, the lot size automatically is set to a level that satisfies the minimum stock level, while in an (s, Q) model, the lot size needs to be determined. The main advantage of the (s, Q) model then is to have the possibility to incorporate a cost-effective lot size, using the EOQ for instance.

A periodic review policy allows for the aggregation of orders from the same supplier to create a cost-effective shipping load for instance. Since all the SC's are replenished from the same DC's, and customer orders would still be shipped if there were no replenishment orders, the aggregation of orders does not necessarily seem advantageous.

(Silver et al., 2016) propose formulas to calculate the reorder point, lot size and safety inventory Since the replenishment lead time of Technische Unie is the deterministic value of 48 hours (2 days), these will be described in the following section.

4.4.1 Lot sizing

There are many lot sizing methods available in literature. Arguably the best-known lot sizing method is the Economic Order Quantity (EOQ). The EOQ-formula finds an optimum between ordering/setup costs and holding costs. There are many additions to the EOQ such as quantity discounts and including production batches. These are not applicable to the project and therefore not discussed. The EOQ is discussed since fill rate formulas depend on it. Since the optimum between ordering/setup costs is a single variable convex function, an optimum can easily be found by equaling the derivative to 0.

$$Q^* = \sqrt{\frac{2SD}{hC}}$$

In which,
Q : Order quantity
S : Setup or ordering costs
D : Demand
H : Holding costs as percentage of unit price
C : Unit price

Other methods include using the EOQ as a time supply, dynamic programming and heuristics and many more. Both (Winston, 2003) and (Silver et al., 2016) recommend the use of dynamic programming or heuristics when the squared coefficient of variation exceeds 0.2. For instance, the Silver-Meal heuristics or the Wagner Whitin algorithm.

EOQ as time supply

Using the EOQ as a time supply is a way to cope with the variability in demand. It uses a determined time of stock instead of an actual number. A downside to this approach is that the fill rate depends on the lot size as we will see in the next paragraph.

Since there are so many products that are expected to be issued less than during per replenishment lead time, we will not put much effort on lot sizing in this project. When the probability for an item is relatively low, we can just order a discrete number of units.

Since the fill rate is based on the lot size and we want to compare the current safety stock with a new safety stock, the current lot sizes will be used to determine safety stocks. Whether the lot size is economical is something that has yet to be researched. What should be kept in mind is that since the fill rate depends on the lot size, the reorder points need to be determined after the determination of the lot size. There are examples of joint determination of the reorder point and the lot size. These are found for instance in (Axsäter, 2006) but will not be discussed further.

4.4.2 Reorder point and safety inventory

First, we define the fill rate as the fraction of demand satisfied directly from on-hand inventory. Then we define the fill rate mathematically also as

$$Fill\ rate = 1 - \frac{Expected\ shortage\ at\ end\ of\ replenishment\ cycle}{Expected\ demand\ in\ a\ replenishment\ cycle} = 1 - \frac{ESC}{Q}$$

Of course, when ordering Q per replenishment cycle, Q is the expected demand. The ESC is more difficult to find; however, literature shows that when regarding normally distributed demand then:

$$ESC = \sigma_l * G(z) = \sigma_l \, [\varphi(z) - z * 1 - \Phi(z)]$$

In the case of complete backordering according to the scope of the project.

In which,
G(z) : Standardized normal loss function
$\varphi(z)$ *: Normal probability density function*
$\Phi(z)$ *: Cumulative distribution function of the normal distribution.*

Combining these using simple algebra leads to

$$G(z) = \frac{(1 - target\ fill\ rate) * Q}{\sigma_l}$$

Using this function, the safety factor z can be found. Unfortunately, there is no formal inverse of the standardized normal loss function G(z), meaning that a value of z cannot be found exact. Therefore, there is a bisection search root-finding algorithm in Appendix B in VBA. The root finding algorithm approaches z with a freely settable tolerance. This tolerance can be adjusted to find an optimum between computation time and accuracy. The algorithm finds a z value always between:

target fill rate + tolerance

After the determination of the safety factor, the reorder point and safety stock level can be determined.

The reorder point is easily the demand during replenishment lead time added to safety inventory that makes sure the target fill rate is met according to the safety factor out of the standardized normal loss function. For example, for the (s, Q) – model this is:

$$Safety\ stock = z * \sigma_l$$

$$Reorder\ point = lead\ time\ demand + safety\ stock$$

As discussed in section 4.2, demand may not follow a normal distribution at all, therefore many models do not assume demand to be normally distributed but rather assume a compound Poisson or compound Bernoulli demand, or some other probability distribution. The compound Poisson demand appears more in literature than the compound Bernoulli demand because of the "*comparative simplicity and their theoretical appeal since they may result in standard statistical distributions*" (Babai, Jemai & Dallery, 2011). Therefore, compound Poisson demand is discussed and further used instead of normal demand.

Compound Poisson reorder points
In (Axsäter, 2006), the fill rate is expressed as follows under compound Poisson demand:

Let $P(IL = j)$ be the distributed probability of inventory level (IL) j, then the fill rate is defined as follows:

$$E(fill\ rate) = \frac{\sum_{k=1}^{\infty} \sum_{j=1}^{\infty} \min(j,k)\, f_k\, P(IL = j)}{\sum_{k=1}^{\infty} k\, f_k} \qquad (4.4)$$

In which

$$P(IL = j) = \frac{1}{Q} \sum_{k=\max\{R+1,j\}}^{R+Q} P(D(L) = k - j) \qquad j \leq R + Q \qquad (4.5)$$

This is of course the ratio between expected satisfied quantity and the expected total demand quantity. When the nonzero demand size is k and the inventory level = j, the delivered (or sold) number of products is *min(j, k)* since one cannot deliver more than the number of items in stock. Notice that if demand is Poisson distributed, then $f_l = 1$. The distribution of demand size $P(D(l) = j)$ will be discussed in the next section.

The Poisson variant can be used to find the value of S for example in the (S-1, S)-model. This model places a new order immediately when demand occurs. This model is not uncommon in systems in which demand is relatively low and it is assumed that demand follows a Poisson distribution.

Then:

$$P(IL = j) = P(D(L) = S - j) \quad j \leq S \qquad (4.6)$$

To find the reorder point R one can just increase R one by one. The reorder point is then $min\ R$ that satisfies $E(fill\ rate) \geq target\ fill\ rate$.

At last, if one wants to find the expected inventory level, one can use 4.7 as for any discrete probability distribution given that the inventory level cannot be negative.

$$\sum_{j=0}^{\infty} jP(IL = j) \qquad (4.7)$$

In practice, this can never exceed reorder point plus lot size since the probability for an inventory level higher than reorder point plus lot size would never occur.

In appendix D, a Java[6] program can be found that can approximate the reorder points under compound Poisson demand. The algorithm can easily be adjusted to be able to calculate reorder points under Poisson demand by setting $f_l = 1$. It doesn't make sense to calculate this sum to infinity of course as for slow moving items the number of customers is relatively low, and therefore the probability of many customers will be very low. This can be seen intuitively by realizing that $k!$ for k=1000 is so large that the Poisson component of the equation 0. In the algorithm, the is set as a constant. In other words, the formula is likely to converge.

Gamma distribution

Finding reorder points for the gamma distribution is easy. The ESC is formulated as follows in (Silver et al., 2016)

$$ESC = \int_{r}^{\infty} (x - r) \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{-\frac{x}{\beta}}\, dx \qquad (4.8)$$

in which r is the reorder point. One can see this as area under the right '*tail'* of the Gamma distribution under which the probability of demand x lies that could not be satisfied from inventory given reorder point r, multiplied by x – r, which can be seen as the actual part of demand that could not be served from inventory.

In (Silver et al., 2016) the ESC is formulated using algebra

$$ESC = \alpha\beta\left[1 - F_{\alpha+1,\beta}(r)\right] - r\left[1 - F_{\alpha,\beta}(r)\right] \qquad (4.9)$$

[6] The first algorithm was written in VBA but VBA was not good at large factorials. The Java program depends on the poor man's factorial calculation algorithm, which can be found here: http://www.luschny.de/math/factorial/csharp/FactorialPoorMans.cs.html

This expression can be used to find the ESC quite easily. In excel for instance we can determine $F_{\alpha+1,\beta}(r)$ using the function GAMMA.DIST(r, $\alpha + 1; \beta;\ TRUE$) and accept the r for which the target fill rate is met. As was the case with normal demand, the target fill rate can be found from the ESC as follows in the case of complete backordering

$$fill\ rate = 1 - \frac{ESC}{Q}$$

Since the Gamma distribution is a continuous probability distribution the values of r can be incremented slightly. One could create a bisection search algorithm just as Appendix B to determine the ESC for the Gamma distribution and then instead of finding a G(z), decrease the value of r with a number smaller than or equal to the tolerance. Then the accepted value of R will at most be the tolerance higher than the actual value of r. in Appendix F one will find the used algorithm of this project. Other possibilities include Excel's solver.

4.5 Discussion

Inventory management is a widely researched topic in literature. Many different models have been regarded in this literature research. From SKU classification based on intermittent demand, to classification more based on sales such as the FSN classification method. Different probability distributions for modelling demand during the lead time and methods to calculate safety stock, reorder points and lot sizes are provided. The remaining question is what route to take.

There are two strategic approaches towards what to do with the inventory at the Service Centers. The first is to cut the assortment, to determine the SKUs that are sold frequently and to stop stocking the products that are almost never sold. This strategy seems out of line with the corporate strategy Technische Unie tries to offer, which is based on service. However, if an item is not sold for one year, then the item hasn't provided any service to customers the last year. For this approach, it might be the best idea to make use of Section 4.4 and classify the inventory based on one of these methods.

The other strategic approach that can be followed is to control the stock in the service centers better, and by doing so cut the costs. In other words, to treat the stock with appropriate models.

Considering the results of analyses in chapter 3 and the service based operations strategy that Technische Unie follows, it seems most appropriate and useful to look at the literature and use models that are found appropriate for the cases that occur the most at Technische Unie. Technische Unie's decision to keep most of the stock in the service centers might be for strategic reasons. The best way to cut the costs of the currently sold SKUs is to control these items adequately. This route is therefore taken in the remainder of the report.

Because of time and scope issues, demand is chosen to be stationary. Trend and seasonality are not considered and the assortment will be classified in 3 groups according to (Silver et al. 2016). Only safety stock is considered. This comes down to the following categories.

Fast moving items
- Items with a lead time demand larger than or equal to 10
- Demand is assumed to be distributed normally when the CV < 0.5
- Demand is assumed to have a Gamma distribution when the CV ≥ 0.5

Slow moving items.
- Items with a lead time demand lower than 10
- Items for which the standard deviation of the lead time demand is at most 10% higher of the square root of the lead time demand.
- Demand is assumed to be distributed according to a Poisson process.

Lumpy items

- Items with a lead time demand lower than 10
- Items for which the standard deviation of the lead time demand is at least 10% higher than the square root of the lead time demand.
- Demand is assumed to be distributed according to a compound Poisson process.

4.6 Conclusion

1. ***What product categorization models for retail are available in literature?***

There are many models for classifying demand. The ABC classification is arguably used the most. Other models focus on the importance of the product in terms of sales and stability. Used threshold values are not really useful for Technische Unie since most products exceed these values. We decide that product categorization is not very useful for this project, therefore a classification method is used that complies with the most appropriate distribution of demand during the lead time. This method should provide the first steps in managing the inventory better.

2. ***What inventory- and replenishment models are known in literature?***

Most models consider two dimensions; periodic versus continuous review, and fixed versus variable order quantity. The periodic review is regarded inappropriate due to that its most important advantage, the aggregation of orders, is not really an advantage for TU. A fixed order quantity allows for a cost-effective ratio between replenishment and stock costs. A variable order quantity and an order up to level allows for an effective handling of strict safety stocks, however for very few demand, the model will be quite the same as an (reorder point, order quantity) model. Since literature often recommends the use of an (S-1, S) order up to level model for slow moving/Poisson distributed items, the (reorder point, order up to level) model is used in the remainder of the project.

3. ***What methods for modelling demand are known in literature?***

Demand modelling focusses on two main areas; Forecasting and creating a lead time demand distribution. Forecasting usually delivers the parameters for the lead time demand distribution and plays an important role towards cost effective inventory management. It is not considered in the project but is recommended to use an appropriate method to account for intermittency, seasonality and trend. This is later discussed in chapter 6.

The other area, the lead time demand distribution, is considered. It is one of the focal points of inventory management. So many distributions are researched, all with their specific advantages. As a first step, the groups Normal, Poisson and compound Poisson are distinguished. These distributions are extensively discussed and recommended in literature.

5 Inventory model

In this chapter, literature and historical data will be combined. Models in literature will be applied to the demand patterns that have been found at Technische Unie. Section 5.1 will focus on the characteristics that the products at the service centers show.

From each group into which the SKUs are classified, a sample of SKUs is taken. For these SKUs, the safety stock is determined using the 98% target fill rate, the current lot size, and the appropriate probability distribution for this group according to literature. This safety stock is checked against historical stock to get an idea of the performance of the new and the old inventory control policies.

5.1 SKU classification

As discussed in chapter 4, the assortment is classified in four groups, being stable fast moving, unstable fast moving, slow moving and slow-moving items shipped in irregular quantities namely lumpy items. Table 5.1 shows the classification based on 2.5 years of sales data of SC Watergraafsmeer. It should be noted that the product lifecycle is difficult to incorporate when looking at data. There was a case of a product that has been introduced in April 2017 and would be considered a fast mover based on its sales from that moment, but the zeroes it showed in the demand data (items were assumed not to be sold when they weren't even in the assortment, since not for every product it was known when they were introduced) before the introduction of the product made it a slow mover. Therefore, in production, one should carefully consider each product individually.

Category	*Number of products*	*Demand frequency*	*(Squared) coefficient of variation*[7]	*Distribution*
Fast - Normal	*16*	$X_L \geq 10$	$CV < 0.5$	*Normal*
Fast - Gamma	*16*	$X_L \geq 10$	$CV \geq 0.5$	*Gamma*
Slow	*2813*	$X_L < 10$	$SCV \leq 1.1$	*Poisson*
Lumpy	*5424*	$X_L < 10$	$1.1 < SCV$	*Compound Poisson*

Table 5.1, SKU classification criteria and the number in each class

The items that will be included in the samples have to be somewhat representative. After all, how much does an average of demand for an item say when the item was only sold once or twice in the last year. This is specifically important for items in the lumpy demand class since for these items a representative demand size distribution is needed. This is further explained in the paragraph 5.3. For products that are considered to be fast movers, there are actually enough sales records since the demand during the lead time is at least 10.

If there were for instance only 5 demand occurrences, with largely varying demand sizes, does it really make sense to then use the distribution of the demand sizes, or would that cause a very high stock level for an item for which the probability of the demand during the lead time is not that reliable at all? On the other hand, what is the alternative? One could rely on the managers' sense and feeling with the product. While that might be a right solution for some products, the performance of the decisions is in that scenario difficult to predict. For new products – for which sales records do not exist of course – one often used approach is to look at similar products and to use that as an outline.

[7] Recall that when we write about the (squared) coefficient of variation CV and SCV, in this case we mean the standard deviation and variance of lead time demand to mean lead time demand ratio: $CV = \frac{\sigma_l}{\hat{x}_l}$ $and\ SCV = \frac{\sigma_l{}^2}{\hat{x}_l}$

5.2 Fast moving

Since the lead time is only two days, there are only so few items that are considered fast. This means that for a product for which the Normal distribution seems appropriate, the item needs to be sold more than 5 times a day on average.

Remarkable enough, no fast products show a CV lower than 0.5. Literature then asks to consider the Gamma distribution, leaving the Normal distribution out in the open. However, the Normal distribution is well known and relatively easy to use. Although it can also be argued that the Gamma distribution is easy to use, the same fast products have been analyzed using both the Gamma distribution and the Normal distribution. This also future-proofs the research as there can be a moment in the future when the Normal distribution is more appropriate. Next, this is also in line with (Silver et al., 2016) who ask to consider the Gamma distribution. In the end of the section about Fast products, the Gamma and Normal distributed inventory policies will therefore be compared to each other as to the current policy for these items.

The fast-moving products are items that are somewhat general and needed for a lot of projects. Such items include PVC tubes through which wires run, wall sockets, junction boxes and power switches. The slower moving products vary in what they are. From relatively expensive boilers to relatively cheap earth leakage circuit breakers.

There are only 16 SKUs for which the demand pattern is considered to follow either a Normal or a Gamma distribution. For these items, there are many sales data available. The products that are presented in table 5.2. Together with their average observed lead time demand, the observed standard deviation of lead time demand, and their current lot size and reorder point.

Item	Item number	$\widehat{x_L}$	σ_L	Total sales	Lot size	Reorder point
Fast 1	406017	44,79	37,43	11040	300	200
Fast 2	2833283	35,46	32,12	8741	200	100
Fast 3	9307044	23,12	23,85	5700	200	100
Fast 4	513861	18,57	14,41	4577	150	40
Fast 5	6328991	14,79	13,07	3645	100	50
Fast 6	2832319	14,43	13,08	3557	150	100
Fast 7	2832327	14,41	14,86	3552	80	40
Fast 8	2832798	13,62	14,72	3357	100	50
Fast 9	521617	13,20	12,14	3254	100	50
Fast 10	3791852	12,83	16,16	3162	100	25
Fast 11	6329064	12,53	11,88	3089	100	50
Fast 12	7703960	11,53	11,08	2842	150	70
Fast 13	514802	11,08	10,40	2730	50	50
Fast 14	406025	10,87	17,13	2680	200	200
Fast 15	7703978	10,83	12,57	2669	100	100
Fast 16	514539	10,72	11,06	2643	80	50

Table 5.2, Items, item numbers and product characteristics

5.2.1 Normal distribution

Let's first consider the Normal distribution. This paragraph is a short recap of the literature and then uses the functions of literature to determine the reorder point for these fast movers. Fast 1 is the first product that was analyzed.

Fast 1
We use the functions of section 4.4.2 to model demand. Recall that the fill rate is expressed as the fraction of total demand served from stock immediately and that

$$Fill\,rate \;=\; 1 - \frac{Expected\ shortage\ at\ end\ of\ replenishment\ cycle}{Expected\ demand\ in\ a\ replenishment\ cycle} = 1 - \frac{ESC}{Q} \qquad (5.1)$$

In the case of complete backordering. Therefore, is the fraction of demand that could not be sold from stock immediately, which we call the expected shortage per cycle, or ESC. Thus, for a target fill rate of 98% we are trying to find an ESC of:

$$(1 - 0.98) * 300 = \; 6$$

Using the mathematical expression and simple algebra, we try to find *G(z).* Recall again from section 4.4.2

$$ESC \;= \sigma_l * G(z) = \; \sigma_l\,[\varphi(z) - z * 1 - \; \Phi(\mathrm{z})] \qquad (5.2)$$

Then

$$G(z) = \frac{ESC}{\sigma_l} = \frac{6}{37.43} = 0.16 \qquad (5.3)$$

Then using the VBA code of Appendix B

$$if\ G(z) = 0.16\ then\ z \approx 0.63$$

Using

$$Safety\ stock = z * \sigma_l \qquad (5.4)$$
$$Reorder\ point \;= safety\ stock + demand\ during\ lead\ time \qquad (5.5)$$

$$Safety\ stock = 0.63 * 37{,}43 = 23{,}58$$
$$Reorder\ point \;= 23{,}58 + 44{,}79 = 68{,}37$$

Or, rounding up to the nearest integer, 24 and 69 respectively. One should note that since the fill rate depends on the lot size and the standard deviation of the lead time demand – the higher the ratio Q/σ_l the higher the fill rate – we expect a higher safety factor when looking for a product that has a smaller Q/σ_l. Therefore, for Fast 13 the safety stock and reorder point are both determined.

In an (reorder point, lot size) system as is currently used by Technische Unie, the reorder point should be set to 24 for Fast 1. In a (reorder point, order up to Level) system, the parameters would be Reorder point 24 and order up to level S = s + Q = 24+ 300 = 324. This is of course as well the highest inventory level in a (reorder point, lot size) system.

The current average inventory level of Fast 1 over the last 2.5 years was 359,2 units with the reorder point set at 200. Now working back with equations 5.5 and 5.4 respectively, the safety stock is then

$$200 – 44.79 = 155.21$$

and the according safety factor

$$\frac{155.21}{37.43} = 4.15$$

Then using equations 5.2 and 5.1 respectively, the expected fill rate is

$$1 - \frac{(37.43 \ * \ G(4.15))}{300} = \ 0.999 \ldots$$

For this product, the safety stock could be more than 6 times smaller than the current safety stock while still maintaining the target fill rate of 98%. This will decrease the fill rate with less than 2%.

Fast 13

Fast 13 has a lead time demand of 11,08, a standard deviation of lead time demand of 10,40 and a lot size 50. Then using equation (5.1) the ESC is 1. Then, using equation (5.3) we are trying to find G(z) of 0.096. The VBA algorithm determines that z is approximately 0.92. As expected and only logical the safety factor is higher.

The safety stock is then 9,61 or 10 when rounding to the nearest integer and the reorder point is 20.7, or 11 when rounding.

Results

Now that for two products it was shown how the safety stock can be calculated based on observed statistics and mathematics, the rest of the fast items are analyzed. This is done to get an idea of how much can be saved for this group. The results are as well presented in table 5.3. One can see that on average, the safety stock in this group is about 7,5 times too high.

Q items are bought during a cycle. Safety stock will usually be issued only partly. Then the expected inventory value when using the new parameters is determined as in equation (5.6)

Item	Old ROP	Old ss	New ROP	New SS	Old ss / new ss	Observed average inventory	Expected new inventory
Fast 1	200	155,21	68,41	23,62	6,57	359,2	173,62
Fast 2	100	64,54	60,52	25,06	2,58	137,3	125,06
Fast 3	100	76,88	37,52	14,40	5,34	170,2	114,40
Fast 4	40	21,43	25,30	6,73	3,19	109,5	81,73
Fast 5	50	35,21	23,40	8,62	4,09	68,5	58,62
Fast 6	100	85,57	19,71	5,28	16,21	77,8	80,28
Fast 7	40	25,59	27,22	12,81	2,00	61,4	52,81
Fast 8	50	36,38	24,35	10,74	3,39	77,1	60,74
Fast 9	50	36,80	20,67	7,46	4,93	71,9	57,46
Fast 10	25	12,17	25,49	12,67	0,96	86,3	62,67
Fast 11	50	37,47	19,67	7,14	5,25	75,9	57,14
Fast 12	70	58,47	14,73	3,20	18,25	111,6	78,20
Fast 13	50	38,92	20,68	9,61	4,05	107,4	34,61
Fast 14	200	189,13	17,57	6,70	28,22	292,2	106,70
Fast 15	100	89,17	18,82	8,00	11,15	117,9	58,00
Fast 16	50	39,28	18,38	7,66	5,12	89	47,66
				Average	7,58	*Average savings factor*	1,61

Table 5.3, fast moving products with old and new safety inventory parameters

$$expected\ inventory\ value = \frac{Q}{2} + safety\ stock \qquad (5.6)$$

(5.6) is a conservative approximation of the average inventory as it does not incorporate the issued safety stock.

When using historical data and mathematical formulations of the fill rate, there can be much saved on fast moving products. Remarkable is that Fast 10 has an old ss to new ss ratio of 0.96. This means that the old ss is lower than the new, however the difference is only 0.04. For all the other items, the stock is at least 2 times as high as it could be when target fill rate of 98% was the sole criterion at stake. Interesting are item Fast 6, 12 and 15. These items have an ss that is at least 13 times higher.
For Fast 12, the average inventory level was 111,6 over the last 3 years for a product expected to be sold only 12 times during replenishment. Especially a reorder point of 70 during the lead time demand of 12 seems too high.

5.2.2 Gamma distribution

Again, a short recap of literature is presented, and then using the functions that literature provided, the reorder point is determined. Let's start with the same product; fast 1

Fast 1
Since we use the same expressions for the fill rate and ESC under Normal demand, we can use the values of these. Thus, we need to find an ESC ≤ 6 using expression (4.8), which leads to expression (4.9):

$$ESC = \alpha\beta\left[1 - F_{\alpha+1,\beta}(r)\right] - r\left[1 - F_{\alpha,\beta}(r)\right] \qquad (5.7)$$

In which α and β are the shape and scale parameters respectively:

$$\alpha = \frac{\hat{x}_L^2}{\sigma_L^2} \quad and \quad \beta = \frac{\sigma_L^2}{\hat{x}_L}$$

This means that

$$\alpha = \frac{44.79^2}{37.43^2} = 1.43 \quad and \quad \beta = \frac{37.43^2}{44.79} = 31.29$$

Now we can use (4.9) and the excel function GAMMA.DIST as described in section 4.4.2. This leads to a reorder point of 75.1, meaning a safety stock of 30.31. As expected, this is slightly higher than the safety stock of 23,52 under Normal demand. These differences will be discussed in the next section. First, the total results of the fast products are presented in table 5.4. The reorder points are determined using the algorithm in Appendix F. Again, the expected inventory levels were approximated using (5.6).

It is immediately visible that the reorder points are still relatively high when compared to the current situation. Just as expected, there is lots to be saved on safety inventory, however slightly less than using Normal demand. This leads to a discussion about which probability distribution truly describes the demand process and when to change distributions. One thing that can for sure be said is that both distributions are a better fit to the safety stock than the current model but more on that in the next section.

Item	Old ROP	Old SS	New ROP	New SS	Old SS / New SS	Observed average inventory	Expected new inventory
Fast 1	200	155,21	75,1	30,31	5,12	359,2	180,31
Fast 2	100	64,54	69,56	34,10	1,89	137,3	134,10
Fast 3	100	76,88	41,93	18,81	4,09	170,2	118,81
Fast 4	40	21,43	26,67	8,10	2,65	109,6	83,10
Fast 5	50	35,21	26,03	11,24	3,13	68,5	61,24
Fast 6	100	85,57	20,63	6,20	13,80	77,8	81,20
Fast 7	40	25,59	32,8	18,39	1,39	61,4	58,39
Fast 8	50	36,38	28,48	14,86	2,45	77,1	64,86
Fast 9	50	36,80	22,85	9,65	3,81	71,9	59,65
Fast 10	25	12,17	31,25	18,42	0,66	86,3	68,42
Fast 11	50	37,47	21,75	9,22	4,06	75,9	59,22
Fast 12	70	58,47	14,95	3,42	17,09	111,6	78,42
Fast 13	50	38,92	24,8	13,72	2,84	107,4	38,72
Fast 14	200	189,13	17,75	6,88	27,50	292,2	106,88
Fast 15	100	89,17	21,56	10,73	8,31	117,9	60,73
Fast 16	50	39,28	21,09	10,37	3,79	89,0	50,37
				Average	**6,41**	***Average savings factor***	**1,54**

Table 5.4, Determining reorder points using a Gamma distribution compared to current inventory levels

5.2.3 Results of category fast

It is definitely clear that the current safety stocks are too high. By how much? That depends on the product. Figure 5.1 provides an idea about how much can be saved. On the topic of choosing the appropriate lead time demand distributions, the Gamma distribution is a bit more conservative in its estimations.

Recall that this has two main reasons. First of all, the Gamma distribution does not consider negative demand, whereas the Normal distribution does. The other reason is that the Gamma distribution returns more probability to higher demands. It must be clear that this leads to a higher estimate of the lead time demand, and therefore the safety stock. It is therefore recommended to use the Gamma distribution as it is likely that this distribution describes the demand process best.

This is also according to, and recommended by literature, especially by these products, that all have a CV > 0.5. However, the difference with the Normal distribution is quite small as compared to the Gamma distribution.

When reading figure 5.1, one must realize that the numbers belonging to the bars are of secondary importance. The main function of the figure is to get a feeling of how much the safety inventory is actually too high.

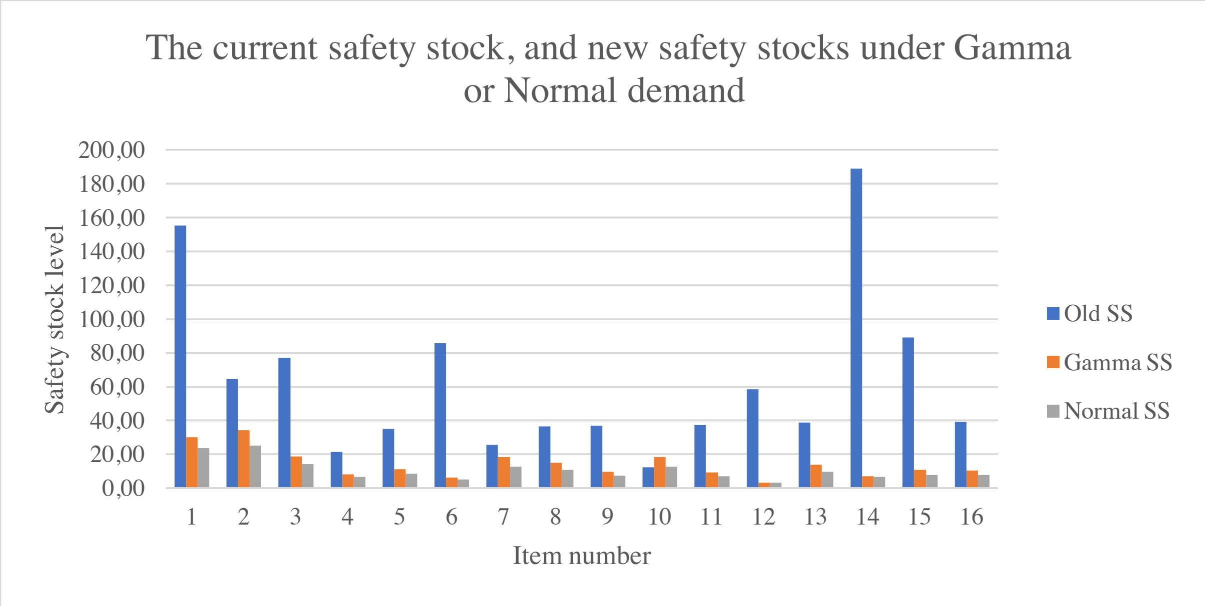
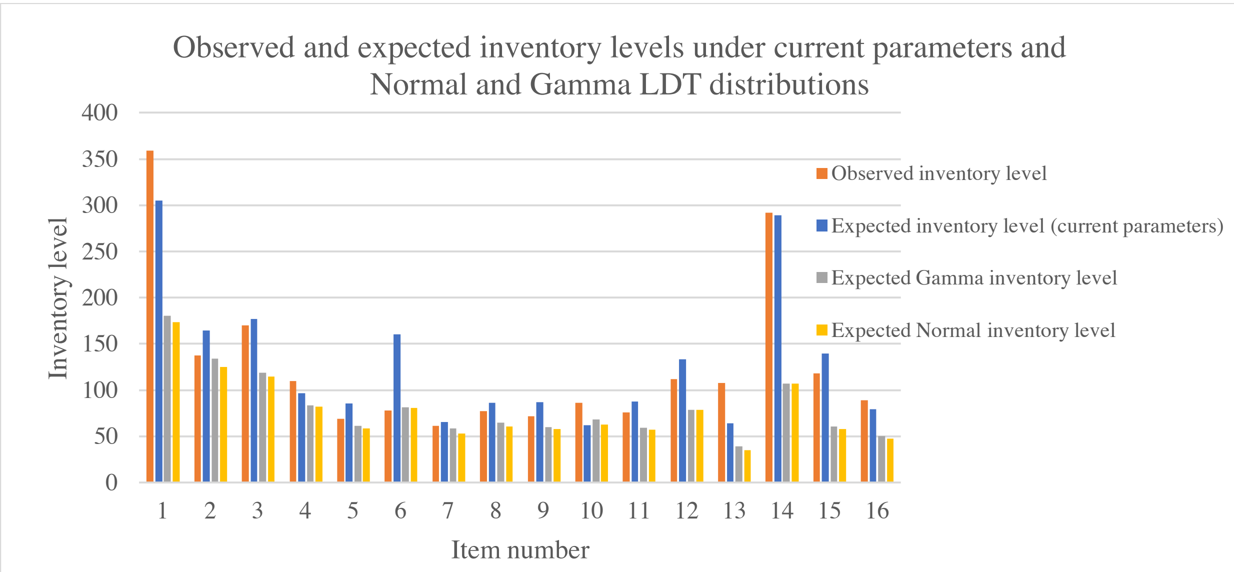


Figure 5.1, A comparison of current safety stocks and under other lead time demand distributions

When determining the expected inventory levels, one observes a much smaller relative difference between the observed value and the expected value as compared to the safety stocks. This is partly due to that the cycle inventory now takes up the lion's share of the inventory: roughly estimated on 82% of the inventory under Gamma LDT. The second reason is quite obvious: the observed inventory levels are only measured at the end of each month, it is to be argued how reliable this observation is. Figure 5.2 therefore also provides a theoretical estimate of the expected inventory level under the current parameters and expression (5.6).



What the more reliable view on the current inventory is, is for the reader to decide. We believe that the expected estimate is the more reliable perspective. In figure 5.2, when looking to Fast 6, one sees a large difference between the observed and current expected inventory level however the differences are not that extreme for most items yet both confirm that the inventory level can be decreased on these items.

Whether these lot sizes are economical is something that has to be researched. While ordering large quantities may be interesting in terms of ordering costs, the cycle stock costs is something that has to be considered. As mentioned before, since the fill rate depends on the lot size, the reorder points need to be recalculated after determining the lot size.

5.3 Slow moving

Since there are so many products that are considered to be slow moving a selection is made. An example of the sample is presented in table 5.5.

Item	Item number	Rate lambda	Current lot size	Current reorder point
Slow 1	2518009	0,185	2	4
Slow 2	8877606	0,093	1	2
Slow 3	1527969	0,229	4	5

Table 5.5, the used product characteristics of some slow-moving products

Slow 1

For product Slow 1, the reorder point and safety stock will now be determined so that it can be reproduced.

Recall that under Poisson demand, the probability of demand size 1 is 1. The fill rate is as in equation 4.4.

$$E(fill\,rate) = \frac{\sum_{k=1}^{\infty}\sum_{j=1}^{\infty}\min(j,k)\,f_k\,P(IL=j)}{\sum_{k=1}^{\infty}k\,f_k} \qquad (5.8)$$

Note that since

$$f_k = \begin{cases}1, & k=1\\ 0, & else\end{cases}$$

We only need to calculate

$$\sum_{j=1}^{\infty} P(IL=j)$$

For which the infinity sign changes to R + Q due to the condition of the following equation. This is because in practice, the inventory level can never be higher than R + Q

$$P(IL=j) = \frac{1}{Q}\sum_{k=\max\{R+1,j\}}^{R+Q} P(D(L)=k-j) \qquad j \le R+Q \qquad (5.9)$$

This is the equation in which we increase our reorder point R.

Next, consider the compound Poisson function for P(D(t) = j)

$$P(D(t)=j) = \sum_{k=0}^{\infty}\frac{(\lambda t)^k}{k!}e^{-\lambda t}f_j^k \qquad (5.10)$$

And consider f_4^3

$$f_j^k = \sum_{i=k-1}^{j-1} f_i^{k-1}f_{j-i} = \sum_{i=2}^{3} f_2^2 f_2 + f_3^2 f_1 = 0 + f_3^2 = 0 + f_2^1 = f_2 = 0 \qquad (5.11)$$

This function will always return 0 when $k \neq j$ and 1 when $k = j$. (This can be interpreted as 1 customer always ordering 1 unit). Then, combining (5.10) and (5.11), the lead time demand distribution can be written as (5.12)

$$P(D(t) = j) = \sum_{k=0}^{R+Q} \frac{(\lambda t)^k}{k!} e^{-\lambda t} \qquad (5.12)$$

Again, the infinity sign changes to R + Q, this time since demand can never exceed the inventory level. Then, the expected fill rate for this product is the combination of (5.9) and (5.12)

$$expected\ fill\ rate = \sum_{j=1}^{R+2} \frac{1}{2} \sum_{k=Max(R+1,j)}^{R+2} \frac{(0.185 * 2)^{k-j}}{(k-j)!} e^{-0.185*2}$$

And using the old lot size Q = 2.
Increasing R from –Q (we know that the fill rate is 0 at reorder point -Q since the maximum inventory R+Q would then theoretically be –Q + Q = 0) until the expected fill rate was higher than 0.98 leads to:

R	***Expected fill rate***
-2	*0*
-1	*0.345*
0	*0.819*
1	*0.970*
2	*0.996*

Table 5.6, the increasing expected fill rate when incrementing the reorder point

Thus, the reorder point for this product is set to 2. Since the safety stock is the reorder point minus the lead time demand, the safety stock is 2 – 0.37 = 1.63. However, this is not an integer and therefore not very useful in practice. Another thing that should be noted is that if one wants to use an (S-1, S) replenishment system, the Q will be 1 in practice most of the time. This factor should then be used for the determination of the reorder point. Then approximately S = r + Q and s = r. We do neglect the undershoot in that case.

This process has been automated using the java function of appendix D. A small sample of the results – it wouldn't make sense to store 540 rows of table data, the procedure of how to obtain these reorder points is much more important - can be found in appendix E.

Results of category Slow
To see whether the observed average inventory levels are representative for the group, its average was compared against the theoretical average expected inventory when using the old reorder point.
The average observed inventory was 2.67 while the average expected inventory using the same reorder point was 3.55. The observed average is lower. This can be explained by that the averages are taken of data that were only measured the last month during the last 3 years, meaning that the average is based little data. It could also be that the reorder point or order quantity have changed over time. It should be clear that this has effect on the average inventory level.
Lastly, the approximation of the reorder point assumes a *'steady state*'. It can be that many inventory levels aren't reached due to that the steady state is not 'measured' by the fact that demand occurrence rates are so low.
If a product is only expected to be demanded once a year for instance, and its inventory level was 10 while the reorder point is 5 and its order quantity is 20, the average over the last 3 years is expected to be around 8 or 7 while the expected steady state average would be more in the region of 15. These factors all play their part in the reliability of the expected decrease of inventory when implementing the Poisson distribution.

Group	Average ROP	Inventory level
Old	2.054	2.67
Theoretical old		3.55
New	0.496	1.98
Factor (Observed old/new)	4.138	1.34
Factor (Theoretical old/new)		1.78

Table 5.7, the expected decrease in inventory when implementing the Poisson distribution

When looking at the sample of 541 SKU's for which the new reorder point was determined, the results of table 5.7 occur. It is expected that the average inventory decreases by a factor 1.8. We believe that comparing the theoretical inventory levels under the old reorder point and new reorder point provides the most accurate estimate of how much could be saved. This is because the average observed inventory leaves a lot to be desired regarding the quality of this measurement and we cannot incorporate factors that change over time such as the reorder point and order quantity.
Still, we recommend implementing a method incorporating the Poisson distribution for which we expect a savings of roughly 1.8. Realize that implementing a Poisson distribution in combination with a lot-for-lot (also known as 1 by 1) ordering strategy, this sets the lot size to 1. This means that the reorder points need to be determined using Q=1.

5.4 Lumpy items

Lumpy items have two important factors. Again, the demand arrival rate lambda is important. It is actually the same lambda as in pure Poisson demand. In practice, lambda could never exceed 5 as that would mean that the product should be classified as a fast mover. The second important factor is the demand size distribution. In this project the observed, empirical distribution is used. It is calculated by dividing the number of occurrences of a demand size by the total number of orders. For an arbitrary SKU, this is as in table 5.8.

Demand size	1	2	3	4	5
Number of occurrences	20	10	0	5	4
Probability for demand size	0.5	0.25	0	0.125	0.125

Table 5.8, Probability of demand sizes

These numbers were constructed intentionally to let a practical problem occur. The probability for demand size 3 is 0 in this case while in practice it might happen. In this setting, it might be better to use a compounding distribution that allows for a probability for order line size 3, for instance a logarithmic compounding distribution (Axsäter, 2006). Another important advantage, as explained earlier, is that it only needs to consider the average demand size and demand size variance instead of the full empirical distribution.

Now for a lumpy item, it is shown how the reorder point is determined. The determination of the reorder point is quite similar to the determination of the reorder point under pure Poisson demand, however the order line size now plays an important role.

First, the empirical distribution of the demand size is constructed in accordance with table 5.8. For product *'lumpy 508'*, this looks as follows:

Demand size k	1	2	3	4	4 <	Total orders
Frequency	53	18	10	1	0	82
Probability f_k	0.646	0.220	0.122	0.012	0	

Table 5.9, demand size distribution for product 'Lumpy 508'

Now that the demand size distribution is determined, the rate lambda needs to be known. This is the same as in pure Poisson demand. For product Lumpy 508, the lambda is 0.1826. the Q of the product is 4.

Recall that the expected fill rate is defined as follows

$$E(fill\ rate) = \frac{\sum_{k=1}^{\infty}\sum_{j=1}^{\infty}\min(j,k)\, f_k\, P(IL=j)}{\sum_{k=1}^{\infty} k\, f_k}$$

Let's first determine the denominator. Since $f_k = 0, k > 4$ it wouldn't make sense to compute the value to infinity. The other f_k are listed in table 5.9.

$$\sum_{k=1}^{\infty} k\, f_k = \sum_{k=1}^{4} k\, f_k = f_1 + 2\, f_2 + 3\, f_3 + 4\, f_4 = 1.5$$

This was the easy part, now the numerator, in which R again is incremented one by one from -Q until it fulfills the requirement expected fill rate ≥ target fill rate.

$$\sum_{k=1}^{4}\sum_{j=1}^{R+Q}\min(j,k)\, f_k\, P(IL=j)$$

In which

$$P(IL=j) = \frac{1}{Q}\sum_{t=\max\{R+1,j\}}^{R+Q} P(D(L)=t-j) \qquad j \le R+Q$$

In which

$$P(D(t)=j) = \sum_{k=0}^{\infty}\frac{(\lambda t)^k}{k!}e^{-\lambda t} f_j^k$$

And lastly the recursive function

$$f_j^k = \sum_{i=k-1}^{j-1} f_i^{k-1} f_{j-i}$$

Let's calculate the probability convolution f_4^3, meaning that 3 customers demanded 4 products. Intuitively it is clear that then one customer had a demand size of 2.

$$f_4^3 = \sum_{i=2}^{3} f_i^2 f_{4-i} = f_2^2 f_2 + \ f_3^2 f_1 = f_2 f_1^1 f_1 + f_1 \sum_{i=1}^{2} f_i^1 f_{3-i} = 3(f_1 f_1 f_2) = 0.275$$

The step-by-step calculation of this recursive function is not documented as this can easily consist of millions of operations. 50 iterations seem like a good compromise between computing efficiency and approximation accuracy. When using an incredibly high arrival rate of 10, the Poisson probability for maximum number of expected customers of 50 (cutting of the loop size) is $1{,}49273 * 10^{-19}$. This adds such little probability to the summation that it is a good cutoff value. A computer is far more efficient in calculating this but even then, this comes down to 1.6 million operations for an arbitrary testing product with rate 0.8 and 10 demand sizes. Then the new reorder point is 3. The current reorder point is 4.

Results of category lumpy

For 898 lumpy-classified SKU's, the reorder point and expected inventory level were determined. The results of the fast and slow-moving products showed that the inventory level is too high for most products and that the average inventory could most definitely decrease, for the largest group, the compound Poisson distributed group, the results were actually quite opposite. It seems that the average inventory for these products is actually too low by a little bit. The inventory seems to perform quite well for these items. The average observed inventory level was 3.88 while the average expected inventory level using the current parameters is 4.62 which comes down to a factor 0.84. This means that there is quite a difference in the observed and expected inventory level. The same as for slow moving items, it could be that the average observed inventory is again not accurate due to the fact that the level was only checked once a month, or the parameters could have changed over time.

Discussion of category Lumpy

What conclusions can be drawn from this? The sample gives the idea that a large group of these items, the reorder is not large enough to maintain the required fill rate. There are multiple reasons for this.

- Mathematically, for many products, the probability of having a demand size of for instance 4 is then estimated quite large, since the empirical probability can be quite large if it has occurred in the historical data, and there were only a few demand occurrences. Then if one wants to maintain a fill rate of 98%, the safety inventory needs to be quite large. Nevertheless, the opposite is also true. If a product has had a maximum inventory level of 5 over the time in which the sample was taken, the demand size 6 will never be measured.
- In literature, there are a lot of different thresholds for when demand should be considered fast, slow or non-moving, or any other category such as intermittent or lumpy. For instance, (Syntetos et al. 2005) use an arbitrary number of 1.32 as a threshold for the average demand interval. The ADI is dependent on the size of the time bucket that is used. When using weekly time buckets, one could get completely different ADI's as compared to months. In this report, the classification is based on when it is appropriate to use a certain lead time demand distribution. Thus, the lead time demand classification as used in section 5.1 is very relative in the sense that many items could well be considered distributed normally when an item has a lead time demand relatively close to the threshold of 10 units. Items that are considered lumpy but have very low demand occurrences can behave much differently. The main point is that this group, consisting of 66% of all the sold SKU's in the last 2 years, contains differences in itself. It contains items that are almost fast, but also items that are almost nonmoving. This leads to the conclusion that this group needs to be researched itself. An ABC classification method can be a solution, with different target fill rates for each of the groups.

Figure 5.2 shows us that there is yet to be a lot to researched in this group. It shows that the average is an average of larger values by showing the deviation between the current and the new expected inventory level.

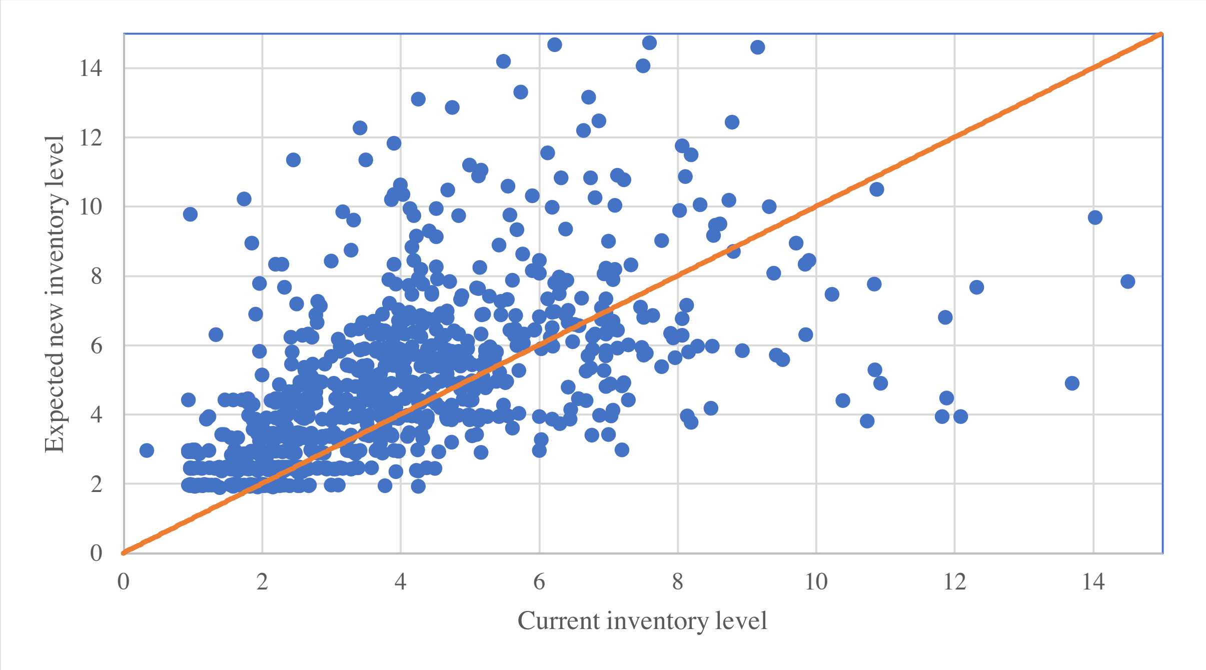


Figure 5.2, Scatter plot of Lumpy SKU's current observed average and expected new inventory level

What the figure should point out, is that there is much deviation among the SKUs. In an ideal situation, all the points lie on the orange line. If an SKU lies higher than the orange line, this means that its expected new inventory level is higher than its observed inventory level, and the other way around when SKU's lie under the orange line. Thus, if all the products would lie on the orange line, the expected inventory level is the same as its current inventory level, meaning that the products wouldn't need much change to achieve the fill rate of 98%. What this also points out is that we want products to be as close to the orange line as possible. We see that the deviation is quite large however, and that there is both understocking as overstocking with quite high differences between the observed and expected inventory level. With other words, this group of products need more research.

One last point that should be noted is that when the decision would be to increase the reorder points to maintain a fill rate of 98%, this means investing in inventory. This decision should be taken carefully as it has many implications for the Service Centers such as a need for larger shelves.

5.5 Conclusion

The research question that was answered in this chapter was

How should Technische Unie control their inventory?

To answer this question the following sub questions were answered.

How should the SKUs be classified?

SKU's should be classified based on their sales frequency, since by doing this, the appropriate lead time demand distribution can be used. Technische Unie should divide their assortment in three groups. As section 5.4 points out, if the SKU's are divided in three groups, the group with lumpy demand has quite some difference within the group. This means that for this group, there could for instance be another sub-classification method. Therefore, it is beneficial to research this group more.

What is the optimal level of safety inventory?

The optimal level of safety inventory depends on many factors that are different for each SKU. The level depends on the variance in demand, the order quantity and the target fill rate. This means that there is not a straight forward answer to this question. However, the optimal level of safety inventory for set constraints can be determined per SKU using the functions of chapter 4. Using this approach, the inventory can be optimized quite a bit. For fast items, it is estimated that the inventory can be decreased with a factor 1.54, for slow items this comes down to a factor 1.78. For lumpy items, it is beneficial to re-determine the reorder point using a Compound Poisson distribution. However, this comes down to an increase of inventory. Since this group consists of roughly 60% of all the SKU's, and there is much difference between the SKU's, it should be researched when to use the Compound Poisson distribution and when to use other methods. What also needs to be researched is whether 98% is a viable fill rate for all the items in this group. Sub-classification within this group based on for instance intermittency, item value and level of uncertainty might lead to an approach in which different fill rates are set for each sub group. It should be noted that one is then "forcing" the SKU's in an analytical model such as reorder points using a Compound Poisson distribution while it is clearly a strategic decision whether to go for a lower fill rate (cost based decision) or invest in inventory and try to reach the 98% fill rate (service based decision).

Wrapping up the chapter, it feels like there is a strategic decision to be taken on the items that are sold in the service centers. Apart from this decision, the analytical models that are presented in this chapter can clearly improve the inventories in the service centers. After taking a strategic decision and parameters as a target fill rate are set according to the strategy, the models can still be incorporated in a computer system (there are probably some suppliers of such systems) and clearly improve the current manual approach of setting the reorder points.

6 Recommendations and future research

This is the final chapter. The first section covers the recommendations for Technische Unie that became clear after the research and it discusses what should be researched to achieve this.

Implement analytical methods
The performance of the analytical methods that were found in literature and presented in chapter 4, was discussed in chapter 5. For the groups Fast and Slow, it is expected that the Gamma and Poisson models will reduce the inventory with a factor 1.54 and 1.78 respectively, while still maintaining the target fill rate of 98%. For these two groups, it is recommended to start using the presented methods. The Normal distribution could also be used as the difference in safety stocks between the Gamma and Normal distribution is quite small, but, it could well be that the Normal distribution underestimates the actual lead time demand which leads to lower fill rates. While this could also be true for the Gamma distribution, the Gamma distribution is a bit more conservative and since literature considers the Gamma distribution a better fit than the Normal distribution for items with a CV > 0.5 it is recommended to use the Gamma distribution.

The 'Lumpy' group requires more attention. For this group, the average inventory is too low right now. For this group, it is recommended to do more research. Find out for what items the target fill rate of 98% is appropriate, or alternatively what a better target fill rate would be. Since there is much difference within this group, it might be an idea to sub-classify this inventory and set new fill rates for each group.

While the mathematical functions, algorithms and control methods presented in this thesis are a good starting point in implementing the methods, there question remains whether it wouldn't be a better idea to implement an already existing software package. (it could well be that for instance SlimStock does already have implementations)

Figure out how economical current lot sizes are
Currently, the determination of reorder points using the samples used in chapter 5 use the current lot sizes. It is yet to be determined whether the current manual lot sizes are economical. The fill rate depends on the lot size (recall that the higher the lot size, the higher the fill rate) and in many lot size determination models such as an EOQ, the lot size depends on total demand (then again it should be recalled that the EOQ is not regarded appropriate for many SKUs at TU). Whether the current lot sizes are economical is yet to be determined.

Plan for regional difference by means of appropriate forecasting methods
In chapter 3, a regional difference of roughly 15% to 20% was observed in the data. Obviously, a factor from which regional difference can be read, is sales frequency. An item that is sold many times in one service center, but never in another should have a different reorder point and order quantity. Often, in one of the analytical models presented in chapter 5, the input factor *'expected demand'* is determined using forecasting. For instance, time series are often used. It is therefore recommended for these analytical models to work correct, to incorporate an appropriate forecasting method. Some methods are a moving average or exponential smoothing. If one wants to incorporate trends or seasonality – both have been observed for some items - one could use a Holt- or Winter-model. Many SKU's require a method that is more connected to their characteristics such as sales frequency, demand uncertainty or other characteristics. When demand is highly intermittent for instance – which occurs often at Technische Unie – the Syntetos Boylan Approximation-method or other methods based on this method are more appropriate. What models are appropriate for Technische Unie is yet to be researched, and this paragraph can be used as a starting point in that research.

Figure out what to do with non-moving items
What this research does not point out, is what to do with so many SKU's that can be considered non-moving. Section 3.2.2 points out that 23% of the SKU's hasn't been sold for over a year, while almost 40% hasn't been sold during the last 6 months. In a dynamic inventory control system, there should be a criterion that converts a DV4 item to a DV0 item (recall that a DV0 is an item that is only sold at the Distribution Centers via a customer order) and the other way around. There are many items can be regarded non-moving, meaning that they should not be on stock. The thresholds for such a system is something that is to be researched.

To make a long story short, Figure 6.1 is a visualization of what to do with non-movers.

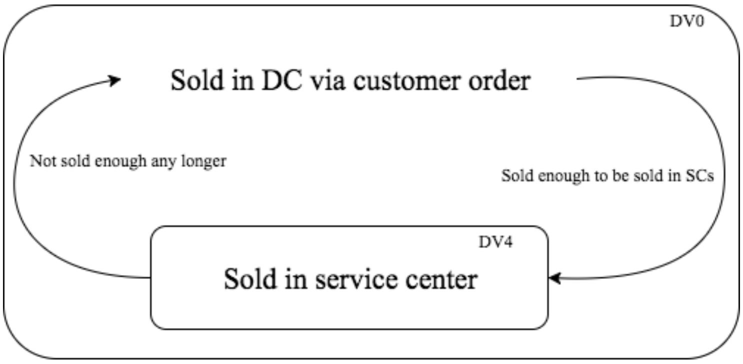


Figure 6.1, A visualization of how to cope with non-moving products

Thresholds for the two item flows need to be set. It is in practice useful to use some kind of "*buffer*" for the thresholds, so that unique events are not considered: If a supplier was somehow not able to deliver its products and therefore the product is not sold, this does not mean that the demand for the product was dropped. Regarding longer period can then be a solution. This has to be considered. In this case the customer order decoupling point is one stage upstream the supply chain. For the customer, this means a one day longer lead time, but for TU this means an aggregation of demand over some, or all the service centers which means a lower needed safety stock due to a lower variance of lead time demand.

Research regional inventory pooling
Within TU, there have been voices that ask for more specialized service centers. Specialized in the sense that they want the service centers to sell a specific range of products and other service centers to sell a different range of products. One service center may for instance sell electric components whereas the other may have more expertise in heating and cooling systems (bear in mind that this is of course a very generalized idea). Technische Unie can then take advantage of the risk pooling effect: the demand will be aggregated over multiple service centers and the variance will decrease. Therefore, less safety inventory is needed. This comes at the price of a higher lead time and increased ordering costs. Therefore, this has to be researched.

7 Bibliography

Axsäter, S. (2006). *Inventory Control.* Springer International PU.

Boylan JE, Syntetos AA, Karakostas GC (2008) *Classification for forecasting and stock control: a case study*. J Operations Research Society 59:473–481

Babai, M. Z., Jemai, Z., & Dallery, Y. (2011). *Analysis of order-up-to-level inventory systems with compound Poisson demand*. European Journal of Operational Research, 210(3), 552-558. doi:10.1016/j.ejor.2010.10.004

Caribara Communication. (2016). *2015 Panorama*. Retrieved on April 8, 2017, from http://www.sonepar.com/sites/default/files/2015%20PANORAMA%20SONEPAR%20EN.pdf

Chiang, W. K., & Monahan, G. E. (2005). *Managing inventories in a two-echelon dual-channel supply chain.* European Journal of Operational Research, 162(2), 325-341. doi:10.1016/j.ejor.2003.08.062

Chopra, S., & Meindl, P. (2013). *Supply chain management: strategy, planning, and operation.* Boston: Pearson.

Croston, J., (1972). *Forecasting and stock control for intermittent demands*. Operations Research Quarterly. 23, 289–304.

Divya devrajan, M S Jayamohan, (2015) *"stock control in chemical firm: Combined FSN XYZ analysis"* Inter. Conf. on Emerging trend in Engg. Sci and Tech (ICETEST), Procedia Technology 24(2016) 562-567.

Durlinger, P. (2014). *WAT IS MIJN VOORRAAD EIGENLIJK WAARD? 5 financiële evaluatie-methodes*. Retrieved on April 10, 2017, from http://www.durlinger.nl/files/artikelen/Wat-is-mijn-voorraad-eigenlijk-waard.pdf

Guvenir, H. A., & Erel, E. (1998). *Multicriteria inventory classification using a genetic algorithm*. European Journal of Operational Research, 105(1), 29–37.

Heerkens, H., & Winden, A. V. (2012). *Geen probleem: een aanpak voor alle bedrijfskundige vragen en mysteries: met stevige stagnatie in de kunststoffabriek*. Nieuwegein: Van Winden Communicatie.

Löwik S. (2017). *Presentation on literature review.* retrieved on April 12, 2017, from https://blackboard.utwente.nl/bbcswebdav/pid-1028481-dt-content-rid-2434349_2/courses/2016-201500021-2A/Presentation%20on%20literature%20review-BB.pdf

Lucci, G., Schiraldi, M. M., & Varisco, M. (2016). *Fashion luxury retail supply chain: Determining target stock levels and lost sale probability.* Paper presented at the Proceedings of the Summer School Francesco Turco, , 13-15-September-2016 192-197.

Matheus, P., & Gelders, L. (1998). *The (R, Q) inventory policy subject to a compound Poisson demand pattern.* International Journal of Production Economics, 68(3), 307-317. doi:10.1016/s0925-5273(99)00110-3

Pandya, B., & Thakkar, H (2016). *A Review on Inventory Management Control Techniques: ABC-XYZ Analysis.* REST Journal on Emerging trends in Modelling and Manufacturing Vol:2(3),2016 REST Publisher ISSN: 2455-4537

Prestwich, S., Tarim, A., Rossi, R., & Hnich, B., (2014). *Intermittency and obsolescence: A Croston method with linear decay.* 03-september-2014. https://arXiv :1409.1609v1

Ramaekers, K., & Janssens, G. K. (2008*). On the choice of a demand distribution for inventory management models*. European J. of Industrial Engineering*,2*(4), 479. doi:10.1504/ejie.2008.018441

Ramanathan, R. (2006). *ABC inventory classification with multiple-criteria using weighted linear optimization*. Computers and Operations Research, 33(3), 695–700. https://doi.org/10.1016/j.cor.2004.07.014

Rego, J. R., & Mesquita, M. A. (2015). *Demand forecasting and inventory control: A simulation study on automotive spare parts*. International Journal of Production Economics, 161, 1-16. doi:10.1016/j.ijpe.2014.11.009

Romeijnders W, Teunter R, van Jaarsveld W (2012). *A two-step method for forecasting spare parts demand using information on component repairs*. European Journal of Operational Research 220 (2): 386–393.

Sani B, Kingsman BG (1997) *Selecting the best periodic inventory control and demand forecasting methods for low demand items*. J Operations Research Society 48:700–713

Sawaya, W. J., & Giauque, W. C. (1986). *Production and operations management*. San Diego: Harcourt Brace Jovanovich.

Scholz-Reiter B., Heger J., Meinecke C., Bergmann J., (2012), *Integration of demand forecasts in ABC/XYZ analysis: practical investigation at an industrial enterprise*, International Journal of Productivity and Performance Management, Vol. 61, Issue 4, p. 445 – 451

Silver, E. A., Pyke, D. F., & Thomas, D. J. (2016). *Inventory and production management in supply chains*. Boca Raton, FL: CRC.

Syntetos, A., Boylan, J., (2005). *The accuracy of intermittent demand estimates*. Int. J. Forecast. 21 ,303–314.

Technische Unie (2016). *Over ons.* Retrieved April 8, 2017, from https://www.werkenbijtechnischeunie.nl/ontdek-TU/Over-ons.html

Teunter, R. H., & Duncan, L. (2009). *Forecasting intermittent demand: a comparative study*. Journal of the Operational Research Society, 60(3), 321 - 329.

Tijms, H. C. (1998). *Stochastic models: an algorithmic approach.* Chichester: Wiley.

U.U. Kocer, S. Tamer (2011), ''*Determining the inventory policy for slow-moving items: a case study*'' WCE 2011: Proceedings of the World Congress on Engineering

Winston, W. L. (2004). *Operations research (ISE): applications and algorithms*. Belmont, CA: Duxbury.

Zhou, P., & Fan, L. (2007). *A note on multi-criteria ABC inventory classification using weighted linear optimization*. European Journal of Operational Research, 182(3), 1488–1491. https://doi.org/10.1016/j.ejor.2006.08.052

Appendix A: Tools for ETL of the data

Shell script for batch-uploading data to MySQL database running on Unix (Darwin 16.4.0) running in a terminal. Note that the corresponding SQL file should have the same table structure as the text file.

```
1.#!/usr/bin/env bash
2.  cd desktop/Afstudeeropdracht/vk41data
3. for t in *.TXT
4. do
5.        mysql --user="root" --database="verkoopgegevens_tu" --execute="LOAD DATA
6. LOCAL INFILE '"$t”’INTO TABLE vkdata FIELDS TERMINATED BY '\t' LINES
7. TERMINATED BY '\n' STARTING BY '' IGNORE 1 LINES"
8. done
```

Line 2 should be the location of the TXT files
Line 3 is the specification of the loop
Line 5 is an MySQL command using an SQL-query in which:

- FIELDS TERMINATED BY '\t'
  Is for tab delimited files (\t)
- LINES TERMINATED BY '\n'
  Lines end with \n means that new records start after a new line in textfile
- IGNORE 1 LINES
  Ignore the table headers that are on line 1

SQL-query for selecting only sales at the service center

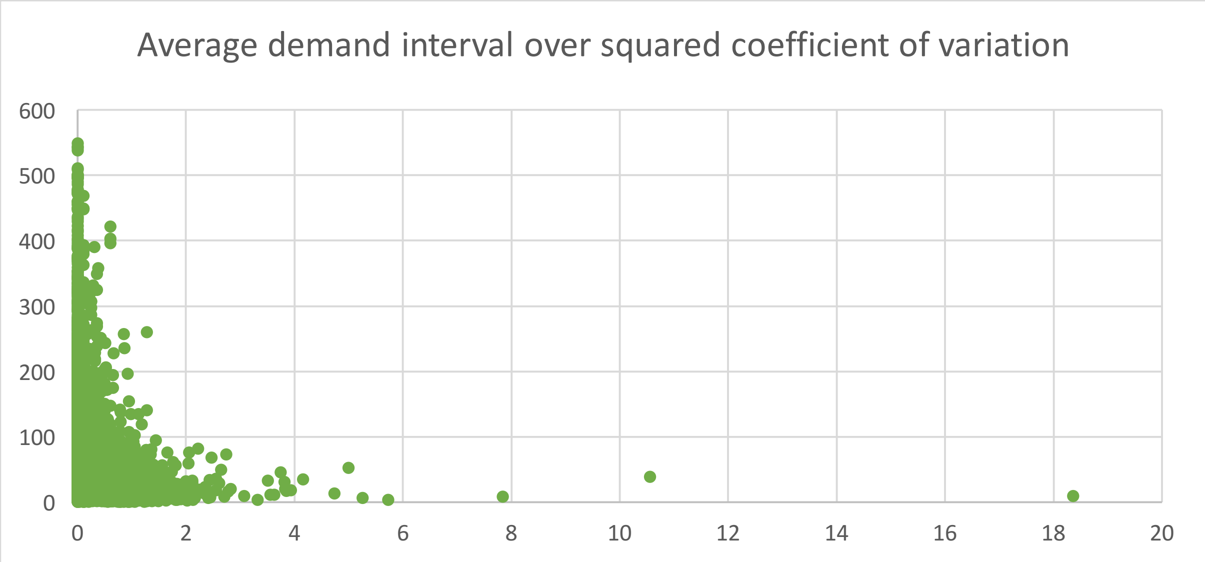
SELECT * FROM vkdata WHERE `Order,medewerker` LIKE ‘BAL%’ OR `Order,medewerker’ LIKE ‘SCAN%’;

The LIKE is used so that the database know to look for a character sequence from which the start of the word should be ‘SCAN’ or ‘BAL’ resulting in all records that have scan or bal in their `Order, medewerker` column. (‘SCAN18’ for instance)

Appendix B: Bisection search root finding algorithm in VBA for G(z)

```
Option Explicit
Function findRootOfLossfunction(x1, x2, TARGET)
'REQUIRES double x1, x2 != 0
'ENSURES findRootOfLossFunction BETWEEN TARGET +/- TOLERANCE
'Probably x1 negative, x2 positive won't work, realize this
Dim xMid, xOld, TOLERANCE As Double
TOLERANCE = 0.0001
xOld = x1

If x1 < x2 Then
x1 = x2
x2 = xOld
End If

Do
    xMid = (x1 + x2) / 2
    If Abs(f(xMid) - TARGET) < TOLERANCE Then
        Exit Do
    End If

    If f(xMid) < TARGET Then
        x1 = xMid
    Else
        x2 = xMid
    End If

Loop

findRootOfLossfunction = xMid
End Function

Sub unitTestFindRoot()
Dim x, y As Double
x = 10
y = -10
Sheets("Blad2").Cells(1, 1) = findRootOfLossfunction(x, y, 0.29)

End Sub

Function f(x)
    'Standard Normal Loss Function
   f = WorksheetFunction.Norm_S_Dist(x, False) - x * (1 - WorksheetFunction.Norm_S_Dist(x, True))
End Function
```

Appendix C: Scatter plot with outliers



Appendix D: Java compound Poisson reorder points

D.1 Java code for calculating compound Poisson reorder points

Note that the file paths in this class are all UNIX-based.

```
package FillrateCalculator;

import java.io.*;

/**
 *
 * This little application calculates reorder points for demand following a compound poisson
demand and fill rate
 * constraints. One should note that orderline size distribution is discrete and empirical
 *
 */
public class DetermineReorderPoint {

    private static final int MAXLOOPSIZE = 50;
    private int Q, j, leadTime;
    private double lambda;
    private FactorialPoorMans fpm = new FactorialPoorMans();
    private double[] theOrderProbabilities, poissonParams;
    private BufferedReader in;
    private PrintWriter convolutionWriter;

    //Constructor
    public DetermineReorderPoint(int Q, double lambda, int leadTime, double[]
theOrderProbabilities, PrintWriter convolutionWriter) {
        this.Q = Q;
        this.leadTime = leadTime;
        this.lambda = lambda * leadTime;
        this.theOrderProbabilities = theOrderProbabilities;
        this.convolutionWriter = convolutionWriter;
    }

    public DetermineReorderPoint() {
        this.readCSV();
    }

    //Should return the probability for inventory level j P( IL= j)
    private double ProbInventLevel(int r, int j) {
        double sumInventLevel = 0;
        for (int i = Math.max(r+1,j); i <= r + Q; i++) {
            sumInventLevel += ProbLeadtimeDemand(i-j);
        }
        return sumInventLevel/Q;
    }

    // We don't want all the convolutions under pure Poisson demand
    private double purePoisson(int r) {
        double pureSum = 0;
        for (int i = 1; i <= r+Q; i++) {
            for (int k = Math.max(r+1,i); k <= r + Q; k++) {
                pureSum += Math.pow(lambda,k-i) * Math.exp(-lambda) / fpm.factorial(k-i);
            }
        }
        return pureSum/Q;
    }

    //Should return the expected fill rate under reorder point r
    private double expectedFillrate(int r) {
        if (theOrderProbabilities[1] == 1) {
            return purePoisson(r);
        }

        double sumFillrate = 0, demand = 0;
        for (int k=1; k <= MAXLOOPSIZE; k++) {
            for(int j=1; j <= r+Q + 1; j++) {
```

```
                sumFillrate += Math.min(j,k) *  totalDemand(k) * ProbInventLevel(r,j);
            }
            demand += k * totalDemand(k);
        }
        System.err.println("sumFillrate " + sumFillrate + " Demand " + demand );
        return  sumFillrate/demand;
    }

    //Should return the probability of demand during a time interval t P(D(l) = t)
    private double ProbLeadtimeDemand(int j) {
        double sumLeadTimeDemand = 0;
        try {
            //convolutionWriter.println("K J FJK");

        for (int i = 0; i <= MAXLOOPSIZE; i++) {
            double pcd = ProbCustomerDemand(i,j);
            sumLeadTimeDemand += Math.exp(-lambda) * pcd * Math.pow(lambda,i) /
fpm.factorial(i);
            //convolutionWriter.println(i + " " + j + " " + pcd);
        }
        }
        catch (Exception e) {
            System.err.println("File could not be read");
        }
        return sumLeadTimeDemand;
    }

    //Should return fjk, the j-fold convolution of fj recursively
    private double ProbCustomerDemand(int k, int j) {
        double sumProbCustomerDemand = 0;
        if (k==0 && j == 0) {
            return 1;
        }
        if (k==0 && j > 0) {
            return 0;
        }
        if (j < k) {
            return 0;
        }
        if (k>1) {
            for (int i = k-1; i <= j-1; i++) {
                sumProbCustomerDemand += ProbCustomerDemand(k-1,i) * totalDemand(j-i);
            }
            return sumProbCustomerDemand;
        }
        else  {
            return totalDemand(j);
        }
    }

    //Should return the empirical probability of total demand j, fj
    //For pure poisson demand, it should return 1 for k=1, else 0
    private double totalDemand(int k) {
        if (k < 0) return 0;
        else if (k < theOrderProbabilities.length) {
            return theOrderProbabilities[k];
        }
        else return 0;
    }

    private double averageInvLevel(int r) {
        double averageInvLevel = 0;
        for (int i = 1; i <= r+Q; i++) {
            averageInvLevel += (ProbInventLevel(r, i)*i);
            System.out.println("P(IL="+i+")= "+ProbInventLevel(r, i));
        }
        return averageInvLevel;
    }

    private Double[] readCSV(){
        String location = "/Users/Thymen/Documents/PoissonCSV.csv";
        String line = "";
        String splitter = ";";
        String lineSeparate = "";
```

```
        try {
            PrintWriter writer = new
PrintWriter("/Users/Thymen/Documents/ROPExpectedInvPoisson.txt", "UTF-8");
            in = new BufferedReader(new FileReader(location));
            int lt = 2;
            int NrOfDemandSizes = 2;
            double[] OP = new double[NrOfDemandSizes];
            OP[0] = 0;
            OP[1] = 1;
            int count = 0;
            writer.println("Item ItemCode Rate Current_ROP Current_Q CurrentInvlevel NewRop
ExpectedInv ExpectedInvOldParams ExpectedFillrate Achievedfillrate");
            while ((line = in.readLine()) != null) {

                String[] poissonParamsString = line.split(splitter);
                System.err.println(line);
                int Quant = Integer.parseInt(poissonParamsString[3]);
                int OldROP = Integer.parseInt(poissonParamsString[2]);
                double lambda = Double.parseDouble(poissonParamsString[1]);
                //for (int i = 1; i<16;i++) {
                 //   OP[i] = Double.parseDouble(poissonParamsString[i+4]);
                //}
                DetermineReorderPoint drRop = new DetermineReorderPoint(Quant,lambda,lt, OP,
convolutionWriter);
                double EFR = 0;
                int r = -Q;
                double Achievedfillrate = 0;
                while(EFR < 0.98) {
                    EFR = drRop.expectedFillrate(r);
                    if (r== OldROP) {
                        Achievedfillrate = EFR;
                    }
                    System.out.println(EFR);
                    r += 1;
                }
                int Rop = r - 1;
                String eInv =  String.valueOf(drRop.averageInvLevel(Rop));
                String eOldInv = String.valueOf(drRop.averageInvLevel(OldROP));
                System.out.println( "Reorder point = " + Rop);
                System.out.println("Expected inventory level = " + eInv);
                count += 1;
                writer.println("Slow "+ count + " " + poissonParamsString[0] + " " +
poissonParamsString[1] + " " + poissonParamsString[2] + " " +
                        poissonParamsString[3] + " " + poissonParamsString[4]  +  " "  + Rop +
" " +  eInv + " " + eOldInv + " " + EFR + " " +  +Achievedfillrate);

            }
            writer.close();
        }
        catch (FileNotFoundException e) {
            System.err.println("file could not be found");
            e.printStackTrace();
        }
        catch (IOException e) {
            System.err.println("Could not read this file");
        }
        return new Double[0];
    }

    public static void main(String[] args) {
        System.out.println("Welcome... the program uses the following argument structure");
        System.out.println("target_fillrate leadTime lambda order_quantity Order_size
Prob_orderSize_1 Prob_orderSize2(don't fill if 0 and last probability) ");
        /*
        try {

            double target_fillRate = Double.parseDouble(args[0]);
            int leadTime = Integer.parseInt(args[1]);
            double lambda = Double.parseDouble(args[2]);
            int Q = Integer.parseInt(args[3]);
            double[] theOrderProbabilities = new double[args.length-3];

            for(int i = 3; i< args.length; i++) {
                theOrderProbabilities[0] = 0;
                theOrderProbabilities[i - 3] = Double.parseDouble(args[i]);
```

```
            }
            PrintWriter convolutionWriter = new
PrintWriter("/Users/Thymen/Documents/Convolutions.txt", "UTF-8");
            DetermineReorderPoint derRop = new DetermineReorderPoint(Q,lambda,leadTime,
theOrderProbabilities, convolutionWriter);
            double EFR = 0;
            int r = -Q;
            while(EFR < target_fillRate) {
                EFR = derRop.expectedFillrate(r);
                System.out.println("rop according is " +  (r));
                System.out.println("The expected fillrate is: " + EFR);
                r += 1;
            }
            System.out.println("The expected fillrate is: " + EFR);
            System.out.println("The according Reorder point is: " + (r-1));
            System.out.println("The expected inventory level is: " + derRop.averageInvLevel(r-
1));
            convolutionWriter.close();

        }
        catch (Exception e) {
            System.err.println("There was an error in the used arguments, please try again");
            e.printStackTrace();
        }*/

        DetermineReorderPoint drop = new DetermineReorderPoint();
        drop.readCSV();
    }
}
```

D.2 The poor man's factorial algorithm.

Link to algorithm: http://www.luschny.de/math/factorial/csharp/FactorialPoorMans.cs.html
Algorithm has the slight change that it returns a double instead of a string

```
package FillrateCalculator;
public class FactorialPoorMans {
              public FactorialPoorMans() {
              }

              private long N;

              public Double factorial(int n) {
                  if (n < 0) {
                      throw new ArithmeticException("Factorial: n has to be >= 0, but was " +
n);
                  }

                  if (n < 2) {
                      return 1.0;
                  }

                  DecInteger p = new DecInteger(1);
                  DecInteger r = new DecInteger(1);

                  N = 1;
                  int h = 0, shift = 0, high = 1;
                  int log2n = (int) Math.floor(Math.log(n) / Math.log(2));

                  while (h != n) {
                      shift += h;
                      h = n >> log2n--;
                      int len = high;
                      high = (h & 1) == 1 ? h : h - 1;
                      len = (high - len) / 2;

                      if (len > 0) {
                          p = p.multiply(product(len));
                          r = r.multiply(p);
                      }
                  }

                  r = r.multiply(DecInteger.pow2(shift));
                  return Double.parseDouble(r.toString());
```

```
        }

        private DecInteger product(int n) {
            int m = n / 2;
            if (m == 0) {
                return new DecInteger(N += 2);
            }
            if (n == 2) {
                return new DecInteger((N += 2) * (N += 2));
            }
            return product(n - m).multiply(product(m));
        }
    } // endOfPoorMansFactorial

    class DecInteger {

        private final long mod = 100000000L;
        private int[] digits;
        private int digitsLength;

        public DecInteger(long value) {
            digits = new int[]
                    {(int) value, (int) (value >>> 32)};
            digitsLength = 2;
        }

        private DecInteger(int[] digits, int length) {
            this.digits = digits;
            digitsLength = length;
        }

        static public DecInteger pow2(int e) {
            if (e < 31) {
                return new DecInteger((int) Math.pow(2, e));
            }
            return pow2(e / 2).multiply(pow2(e - e / 2));
        }

        public DecInteger multiply(DecInteger b) {
            int alen = this.digitsLength, blen = b.digitsLength;
            int clen = alen + blen;
            int[] digit = new int[clen];

            for (int i = 0; i < alen; i++) {
                long temp = 0;
                for (int j = 0; j < blen; j++) {
                    temp = temp + ((long) this.digits[i]) * ((long) b.digits[j]) +
digit[i + j];
                    digit[i + j] = (int) (temp % mod);
                    temp = temp / mod;
                }
                digit[i + blen] = (int) temp;
            }

            int k = clen - 1;
            while (digit[k] == 0) {
                k--;
            }

            return new DecInteger(digit, k + 1);
        }

        @Override
        public String toString() {
            StringBuilder sb = new StringBuilder(digitsLength * 10);
            sb = sb.append(digits[digitsLength - 1]);
            for (int j = digitsLength - 2; j >= 0; j--) {
                sb = sb.append(Integer.toString(digits[j] + (int) mod).substring(1));
            }
            return sb.toString();
        }
    }
```

Appendix E: Results of calculating reorder points for slow moving products

Short sample of the SKU's for which the reorder point and expected inventory level was determined

Item	Item Code	Rate	Current ROP	Current Q	Current Inventory level	New reorder point	Expected Inventory level	Expected Inventory old Parameters
Slow1	409417	0.604462	5	10	9.387097	2	5.998203	8.992593
Slow2	505735	0.403651	3	5	4.83871	2	3.749234	4.747024
Slow3	3315942	0.245436	5	5	4.451613	2	3.897435	6.897036
Slow4	1553445	0.294118	2	2	1.677419	2	2.358423	2.358423
Slow5	1527969	0.229209	5	4	5.709677	2	3.409683	6.409299
…	…	…	…	…	…	…	…	…
Slow536	9136999	0.002028	1	1	1.045455	0	0	0.999992
Slow537	3846662	0.002028	2	2	2	0	0.499996	2.499992
Slow538	443291	0.008114	1	1	0.935484	0	0	0.99987
Slow539	2568277	0.004057	3	2	3.741935	0	0.499984	3.499967
Slow540	560128	0.002028	1	1	1.064516	0	0	0.999992

Appendix F: Gamma distribution search algorithm

```
Option Explicit

Sub ExpectedShortagePerReplenishmentCycleGammaDist()

Dim i, j, k, l As Integer, r, alpha, beta, ESC, x, tolerance As Double
tolerance = 0.01

For i = 5 To 20
    r = 0
    alpha = Sheets("Gamma.Normal").Cells(i, 20)
    beta = Sheets("Gamma.Normal").Cells(i, 21)
    ESC = GammaDistr(r, alpha, beta)
    x = (1 - 0.98) * Sheets("Gamma.Normal").Cells(i, 10)
    Do While ESC > x
       r = r + tolerance
       ESC = GammaDistr(r, alpha, beta)
    Loop
    Sheets("Gamma.Normal").Cells(i, 22) = r - tolerance

Next i

End Sub


Function GammaDistr(r, alpha, beta)
    GammaDistr = (alpha * beta) * (1 - WorksheetFunction.GammaDist(r, alpha + 1, beta,
True)) - (r * (1 - WorksheetFunction.GammaDist(r, alpha, beta, True)))
End Function
```