# Analysis of flow processes at the downstream side of various river measures using 1D- and 2D flow models

Master Thesis Michiel Clements August 2016

ARCADIS



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**UNIVERSITY OF TWENTE.** 

## Preface

In this report the result of the research project that I did during 6 months at Arcadis can be found. During this time I have learned many different subjects regarding river flow modeling, river flow processes and clarifying these complicated subjects in a report. Furthermore, I have learned about what it is like to work in the civil engineering field and more specifically to work at Arcadis. I have enjoyed my time very much and I therefore want to thank the company for giving me this opportunity.

I also want to thank Arjan Tuijnder as my daily supervisor at Arcadis. Thank you for being there almost every day to answer my questions. I have learned a lot from you. You always required to formulate complicated questions as clear as possible. This has helped me a lot in writing the report. Thank you Pepijn for your help during this period too. You were always willing to help and check formulas if I asked to. We were both situated in a different location so our communication was mainly by email. And yet your response was always very quick which is very much appreciated. Thank you Bart for your help. You always were able to clearly explain questions regarding complicated subjects such as turbulence or nonhydrostatic flow. I want to thank dhr. Ribberink. Your feedback was very constructive and has helped to reshape my report in a structured way. Your flexibility regarding the research process is very much appreciated. I finally want to thank my lovely wife Jacomijn for her support. Especially in the last stage of my research almost my whole life was devoted to it. And still you always helped and supported me and never complained about me doing my work. I'm very grateful and astonished by the way you have helped me through.

I want to thank you all very much. Above all I want to thank God for His help. He has helped and supported me day after day. I have always wanted to do a research about rivers because these creations have always been very intriguing to me. Therefore, my greatest desire is to adore the most amazing river there ever will be, which is the river in heaven:

Revelation 22:10. Then the angel showed me the river of the water of life, as clear as crystal, flowing from the throne of God and of the Lamb

Michiel Clements August, 2016

# Summary

After implementation of river measures in sub-critical flow, the water level upstream of the measure decreases. However, at the downstream boundary of these measures the water level tends to increase. This is observed as a local peak. In this research the existence of this peak is investigated.

This is undertaken by introducing two schematized river measures: a widening measure and a deepening measure. The focus was mainly on the downstream side of these measures.

It is shown that the flow processes that cause the peak at the downstream boundary of the both measures can be explained by the Bernoulli equation. Bernoulli states that when the flow decelerates, the water level increases. In sub-critical flow, the effects of the river measures are only experienced upstream. Implementing the deepening measure, no changes thus occur far downstream of this measure. When going from that point in upstream direction, the flow decelerates since the river profile expands. Bernoulli explains that due to this deceleration the water level increases in upstream direction.

A similar process occurs at the downstream side of the schematized widening measure. However, at the downstream boundary of the measure, the lateral change of the river profile causes lateral flow velocities towards the axis of the river. This has a significant influence on the shape of the peak. Furthermore, turbulence tends to be an important flow process and has an increasing effect on the peak.

There is also an analysis undertaken on the used grid size in flow models. Regarding the deepening measure, using a smaller grid size then length of the step in the bed, no significant changes occur. However, using a larger grid than this length results in a smaller peak and should therefore not be used. Regarding the lateral flow constriction, when a grid size is used that is smaller than 1 \* db/dx, no significant change occurs. When using a larger grid size the peak is overestimated.

Besides the schematized flow constrictions, the real world case 'Scheller- and Oldeneler Buitenwaarden' near Zwolle is used. This project concerns the construction of a side-channel. Using the analysis of the schematized flow constrictions, it is attempted to reduce the peak downstream of the side channel. It is shown that adapting the mouth of the side channel such that the lateral flow velocities are reduced also reduces the size of the peak. It is furthermore shown that increasing the roughness that causes a local decrease of the water (the Bernoulli effect in opposite direction) only has an enlarging effect on the peak.

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# **1** Introduction

## 1.1 Background

The Netherlands is located in the middle of a delta, therefore the Dutch have had to deal with water for many centuries. They have to fight and work with water coming from upstream countries in the east as well as water from the sea in the west. Furthermore, as is shown in Figure 1, the country is subsiding while the level of the sea is rising causing many parts of the Netherlands to be below sea level. This has caused many floods such as in the 'Zuiderzeegebied' (1916) and in Zeeland (1953), for example. The latter led to the establishment of the 'Delta-committee' in 1958 which was assigned to come up with measures necessary to preventing further floods, especially of the same magnitude.



Figure 1; Schematic representation of subsidence and sea level rise throughout the years (Verhallen, 2001)

Since the establishment of this committee, many measures have been applied to protect the country. There have been many developments near the coast to prevent the inundation of sea water, and inland likewise for river water. Some examples of these measures are shown in Figure 2.



Figure 2; examples of measures in rivers

In order to decide what measure is best to apply on the river system, it is important to be able to understand and predict the flow processes in the river. Through many centuries, scientists have tried to accomplish this by developing idealized and process-based flow models. During the computer age, researchers have been able to use more complex models which has led to an increase in the number of

flow processes included in calculations. This, in turn, has led to more predictions and a better understanding of flow behavior in river systems. However, despite the advantages of using advanced models, the overview of all processes in the model can be obscured. This might mean some effects in the model output are much less easily understood.

## **1.2** The downstream peak

Most flow models used to analyze the effect of river measures are based on the Shallow Water Equations, or Saint Vanant equations. After the implementation of various river measures in these flow models, a peak occurs at the downstream side of the river measure in the results of these models. This occurs after examining the implementation of various measures such as constructing a side-channel (Herik & Rooy, 2007), widening measures (Diermanse, 2004b) and deepening measures (Linde, 2008).

All these measures have one common characteristic: the peak exists at the location where the flow is constricted, either horizontally or vertically. An example of this peak is shown in Figure 3.



Figure 3; water level difference after implementing side channel

In this Figure the difference of the water level is shown between the situation with and without the implementation of the side channel. In this case, the side channel flows into the river at rkm 980. It is clearly visible that a peak occurs at rkm 980 which is exactly at the constricted location. The peak is thus defined as the increased part of the water level after implementation of a river measure compared to the situation without this measure. In this report, the situation without the measure is called the *reference situation*.

## **1.3** Policy regarding the downstream peak

The downstream peak is limited by the 'Rivierkundig beoordelingskader' (RBK) or 'hydraulics assessment framework' (Kroekenstoel, 2014). This policy document is decisive when river interventions are implemented in the Netherlands. According to this document, the increase of water level after implementing measures is only allowed if:

- The design is 'optimized', which means that the most optimal design is chosen in order to reduce the peak as much as possible without significantly changing the decreasing effect of the water level of the intervention.
- The surface of the increased triangle is much smaller than the decreased triangle (see Figure 4). In practice this means that the decreasing effect must be 50 to 100 times larger than the increasing effect.
- The peak does not reach outside the boundaries of the intervention.



Figure 4; increased triangle and decreased triangle

## **1.4** Research motivation and questions

As stated above, the peak occurs in the result of flow models. However, until now it has been uncertain whether this peak is accurately represented. The flow models are based on several simplifications. For example, there are certain assumptions about the way turbulence due to friction or changes in the river profile are modeled. It is also assumed that the pressure distribution is hydrostatic in all circumstances. Furthermore, the analytical equations which are the basis of these flow models are discretized using a specific step size. In practice, the minimum step size is 20 m, which might leave out detailed information about the dimensions of the river measure. Until now it is uncertain to what extent this influences the way the peak is represented. Therefore, a more detailed research on these processes and their relation to the peak is necessary.

## 1.4.1 **Problem definition**

Considering the different aspects above, the problem definition is as follows: It is uncertain whether a peak occurring in the result of flow models at the downstream side of river measures is accurately represented since, on the one hand, these flow models are based on several simplifications and, on the other hand, spatial detailed information of the river measures is left out.

## 1.4.2 Objective and Research questions

The objective of this study is to gain insight into the relation between flow processes around the downstream side of river measures and the downstream peak, and also how this can be used to reduce the dimensions of the peak in the preliminary design phase of constructing a side channel.

For this research, two research questions are examined. The first question focuses on the cause of the peak and the second question focuses on different ways to reduce the dimensions of the peak.

- 1. Is the downstream peak accurately represented in the flow models that are applied to design river measures?
  - a. What flow process(es) are of importance at the downstream side of various river measures?
  - b. What is the effect of the step size in the discretized equations on the representation of these flow processes?
- 2. What can be done to reduce the dimensions of the peak in the preliminary design phase of a side channel?

## 1.5 Research approach

## 1.5.1 Schematized river measures

According to several reports, the downstream peak appears at the downstream boundary of different river measures i.e. at the location where the flow is constricted (see introduction, subsection 1.2). In order to examine the flow at the location of the constriction, two basic types of measures are introduced: a widening measure and a deepening. In both measures, the flow is constricted at the downstream boundary.

The analysis of this research is done for a river section with typical dimensions of a river in the Netherlands. These dimensions are inspired by the Waal river, which discharges 2/3 of the discharge entering the river Rhine near Lobith. Since the design-discharge is  $16000 \text{ m}^3$ /s at Lobith, the discharge of the Waal in that scenario is  $10667 \text{ m}^3$ /s. Furthermore, the average width of the Waal-river is approximately 360 m. The typical bed slope of rivers in the Netherlands is 1 m / 10 km. Furthermore, a Chézy roughness value of  $50 \text{ m}^{1/2}$ /s is chosen as a typical roughness value for the river Waal.

The deepening measure concerns the deepening of the river bed of 1 m over a section of 1000 m (see Figure 5), while the widening measure concerns an expansion of the summer bed of 20 meters at both sides of the river over a section of 1000 m (see Figure 6).



Figure 5; Schematization of a deepening measure



Figure 6; Schematization of a widening measure

## 1.5.2 Analysis of flow processes around the downstream side of various river measures

For this research, three different model configurations are introduced. Each model configuration represents a set of different terms of the Reynolds Averaged Navier Stokes equations. By subsequently applying each model configuration to the schematized river measures, it is analyzed what terms and therefore what flow processes are of importance at the downstream side of these measures. The model configurations that are examined are as follows:

- Model configuration I: 1D Bernoulli
- Model configuration II: 1D Bélanger
- Model configuration III: 2D modeling

#### Model configuration I: 1D - Bernoulli

In this research, effects on the flow around sudden changes of the river profile are examined. These sudden changes cause the water flow to accelerate locally. This might cause the acceleration term to be more important than the bottom friction term. If this is the case, the flow is described by the Bernoulli equation without head loss. This is applied to both schematized river measures.

#### Model configuration II: 1D – Bélanger

In model configuration II the influence of the friction term is examined. This is done by adding the friction term to the previous configuration. The flow is then described with the Bélanger equation or 'Back-water curve method'. After applying this to both schematized river measures, the model results are compared to the results of the previous configuration. This way it is examined whether the Bernoulli equation is still the right way of describing the flow at the schematized river measure (and more importantly around the downstream side), or that the friction term plays an important role and may thus not be neglected.

#### Model configuration III: 2D modeling

In configuration III the effect of adding a second dimension to the flow model is studied. Instead of using a 1D model a 2D model is applied.

For the deepening measure two effects are studied: 1) the influence of turbulence and 2) the influence of the hydrostatic pressure assumption

First the effect of turbulence is examined. This is done by introducing the vertical k – epsilon turbulence model. By comparing the results with k- epsilon model to the results of the previous configuration, conclusions will be drawn on the influence of turbulence. Secondly, it is examined whether the hydrostatic assumption is valid. When vertical velocities are large, the pressure distribution is non-hydrostatic. It is uncertain what the magnitudes of the vertical flow velocities at the step in the bed are. Therefore it is also uncertain whether the hydrostatic pressure assumption is still right. It is thus examined what the behavior of the flow is when non – hydrostatic effects are also included. Both the effect of vertical mixing and the non – hydrostatic effect will be analyzed in a 2DV model by introducing space steps in x – direction and several layers in z – direction.

Regarding the widening measure, two effects are studied 1) the influence of lateral flow velocities and 2) the influence of the horizontal mixing term.

First the effect of the lateral flow component is examined. This is done by including both the longitudinal as well as the lateral flow velocities in a 2DH flow model with a grid with cells of 2 m x 2 m. As will be shown in the report, with this grid size the results of the flow are converged, i.e. using a smaller grid size does not have a significant influence on the results anymore. By comparing these results to the results generated with the 1D model, conclusions will be drawn on the influence of the lateral flow velocities.

Secondly, the influence of the horizontal mixing term is examined. This is done in two ways. First a spatially variable horizontal eddy viscosity is used, namely the Horizontal Large Eddy Simulation model (HLES). This is the most advanced way of estimating the horizontal eddy viscosity in this research. Comparing the results of the model run with HLES included to the model run without turbulence model thus gives insight into the effect of turbulence. However, in practice, performing a HLES simulation is very computationally intensive. Therefore often a spatially constant horizontal eddy viscosity is used. This is also done in this research and this is the second way of examining the mixing term. The magnitude of the constant horizontal eddy viscosity is based on an estimation presented by Madsen et al (1988). By comparing these results to the results obtained with HLES, it is investigated how well the turbulence is represented by the second turbulence model.

By considering the flow configurations mentioned above, insight is obtained into the important flow processes around the downstream side of the river measures and into the flow processes that cause the peak at the downstream side of the river measures. An overview of the model configurations and the corresponding dimensions, used turbulence model and the investigated flow process is shown in table 1 for the widening measure and in table 2 for the deepening measure

Deepening measure			
Model Configuration	Dimensions taken into account	Turbulence model (vertical)	Investigated subject
I	1D	-	Bernoulli
II	1D	-	Belanger
	Quasi - 2DV	k - epsilon	Influence of turbulence
111	Full - 2DV	k - epsilon	Non - hydrostatic effects

Table 1; over view of model configurations applied to the deepening measure

Widening measure			
Model Configuration	Dimensions taken into account	Turbulence model (horizontal)	Investigated subject
I	1D	-	Bernoulli
=	1D	-	Belanger
	2DH	Constant, $v^H = 0 \text{ m/s}^2$	Influence of lateral flow
	2DH	HLES	Influence of turbulence
111	2DH	Constant $v^H = 0.6 \text{ m/s}^2$ , estimated with Madsen et al (1988)	Reliability of using a spatially constant hori- zontal eddy viscosity

Table 2; overview of model configurations applied to the widening measure

## 1.5.3 Analysis of grid size in flow models

For the analysis of the flow processes, a grid size of 2 m x 2 m is used. In this research it is investigated whether the size of the grid has an influence on the representation of the flow processes that are of importance at the downstream side of the river measure. This is done as follows: 1) by decreasing the grid size and 2) by increasing the grid size used.

First the grid size is decreased. This is done until the results generated with the flow model converges. This converged solution is compared to the solution generated with using a grid size of 2 m x 2 m. This way conclusions are drawn on the influence of the grid size on the representation of the flow processes.

Second, the grid size is increased to 10 m x 10 m and to 40 m x 40 m. The latter grid size is sometimes used in practice when river flow in the Netherlands is modeled. This is thus the worst – case scenario in which the fewest details are taken into account. By comparing these results to the results with the analysis of 2 m x 2 m, conclusions are drawn on the effect of using a large grid size.

## 1.5.4 Reducing the dimensions of the peak

With the results of the previous subsections it is investigated what the cause of the peak is. Subsequently is it examined how the dimensions of the peak can be reduced in the preliminary design phase of a side channel. This is done by focusing on the flow processes that cause a peak in a real world case.

The real world case concerns a side channel constructed in the Ijssel, close to Zwolle. This project is part of the 'Room for the River' project, which is a series of measures at 30 different locations with the main goal to give the river more room in order to manage high water events better. An overview of the project considered in this research is shown in Figure 7.



Figure 7; Project 'Scheller and Oldeneler Buitenwaarden' near Zwolle

It is shown that the side-channel is constructed in the floodplain of the IJssel. The flow direction is from South East to North West. At the downstream side of the side channel, the channel is connected to the main stream of the IJssel, and at the upstream side the channel is closed off with a small dike. Only with high water events the water flows over this dike into the side channel. This is to prevent the side channel from sedimentation, which occurs at low flow velocities (The Open University, 1999).

The summer bed of the IJssel river is approximately 160 m wide, and the winterbed 100 m wide. The winterbed at the location where the side channel is constructed is much wider, i.e. it is 600 m at the east side. The length of the wide winterbed is approximately 3.5 km. The floodplains are 5 meters above the summer bed.

The length of the side channel is approximatlely 2 km and the width is 100 m. The bed level of the side channel is 3 m below the bed level of the summer bed.

The design discharge is 16000 m<sup>3</sup>/s at lobith which corresponds to a dicharge of 1800 m<sup>3</sup>/s of the IJssel. The bed slope is approximately 1 m / 10 km.

## 1.5.5 Summary of research approach

An overview of how the research questions are examined is shown in Figure 8. By examining research question 1A, the important flow processes at both downstream side of the deepening and widening measure are determined. The mathematical equations representing these important flow processes is the input for research question 1B. In this research question the grid sizes in the discretized solution of the mathematical equations is changed. This gives insight into the representation of these flow processes when both grid sizes used in practice and smaller grid sizes are used. Both the results of research question 1A and research question 1B give insight into what flow processes cause the peak at the downstream side of the river measures and with what grid size these flow processes are well represented.



#### Figure 8; Summary of research approach

This is input for research question 2. Here some measures are proposed to reduce the dimensions of the peak.

## **1.6 Outline report**

This report is outlined in Table 3. The first chapter considers the background to the mathematical equations describing water flow. It demonstrates how specific assumptions result in the Bernoulli and Bélanger equation as well as the equations describing both 2DH and 2DV flow. It considers what the processes are behind both the k – epsilon and the HLES model.

The second chapter conveys the methodology used (either numerically and/or analytically) for both schematized river measures. It is shown how the solutions are implemented in the software used (Matlab, WAQUA and SWASH).

Furthermore, Chapter 3 presents the results of the analysis of the various flow models. These show whether the important flow processes at the downstream side of the river measures are accurately represented by the flow model used in practice, which accordingly answers research question 1.

Chapter 5 builds on the results presented in Chapter 4, reducing the dimensions of the peak. This is applied to the real-world case. Here the second research question is examined.

Chapter 6 draws the conclusions from the results and the discussion and provides recommendations.

Chapter	Research question
	examined
<ol><li>Flow theory and governing equations</li></ol>	
3. Solution method and model set up	1
4. Results	
5. Practical Application	2
6. Conclusions, Discussion	
and Recommendations	

Table 3; Overview of chapters and corresponding research questions

# 2 Flow Theory

This chapter examines flow in general and more specifically how water flow is modeled. In order to do that, first the Navier-Stokes equations are introduced. However, performing direct computations using these equations is very computationally intensive. The existence of a general solution of these equations is even one of the Millennium Problems. (Fefferman, 2002). Therefore, it is shown how Reynolds applied an averaging procedure to the equations making calculations feasible. This procedure is shown resulting in momentum equations for flow in general. After that, it is shown how the resulting equations are applied to water flow specifically, by introducing key assumptions regarding water flow.

## 2.1 Navier-Stokes equations

Throughout many centuries researchers have tried to understand and predict the behavior of fluid flow. One of the most and comprehensive mathematical descriptions used in order to obtain this is introduced by Navier (1785-1836) and Stokes (1819-1903) by introducing a balance of momentum of a fluid, the so called incompressible Navier-Stokes equations:

$$\frac{\frac{\partial u}{\partial t}}{1} + \underbrace{\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right]}_{2} + \underbrace{\frac{1}{\rho}\frac{\partial p}{\partial x}}_{3} = \underbrace{\frac{\mu_d}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)}_{4} - \underbrace{g_x}_{5}$$
(2.1)

$$\frac{\frac{\partial v}{\partial t}}{1} + \underbrace{\left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{y} + w\frac{\partial v}{\partial z}\right]}_{2} + \underbrace{\frac{1}{\rho}\frac{\partial p}{\partial y}}_{3} = \underbrace{\frac{\mu_d}{\rho}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)}_{4} - \underbrace{\frac{g_y}{5}}_{5}$$
(2.2)

$$\frac{\frac{\partial w}{\partial t}}{1} + \underbrace{\left[u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right]}_{2} + \underbrace{\frac{1}{\rho}\frac{\partial p}{\partial z}}_{3} = \underbrace{\frac{\mu_d}{\rho}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)}_{4} - \underbrace{g_z}_{5}$$
(2.3)

Where

Term 1: the temporal acceleration Term 2: the spatial acceleration Term 3: the pressure force Term 4: the viscous forces Term 5: the gravity force

With

- g gravitational acceleration [m/s<sup>2</sup>]
- u velocity in x-direction [m/s]
- t time [s]
- v velocity in y-direction [m/s]
- w velocity in z-direction [m/s]
- $\mu_d$  moleculair viscosity [kg/(m s)]
- $\rho$  density of water [kg/m<sup>3</sup>]
- p pressure  $[kg/(m s^2)]$

## 2.2 Reynolds averaged Navier-Stokes equations

A typical phenomenon of turbulent flow is the fluctuating character of the velocity in a point as shown in Figure 9. This is a sketch of the fluctuating character of turbulent flow over time. In this figure only the behavior of u is shown, but a similar behavior is observed for v and w (White, 2009). Solving for example the second spatial derivatives for velocities in all directions is very time consuming. Using the NS – equations to solve these complex flow regimes is therefore a computational intensive task.



Figure 9; Reynolds averaging principle

Therefore, Reynolds proposed to average the NS equations over time by introducing the instantaneous velocities, u', v' and w' as:

u = U + u'	(3.4)
v = V + v'	(3.5)
w = W + w'	(3.6)

In which:

U = time – averaged velocity defined as  $\frac{1}{T}\int u \, dt$  in x-direction u'= instantaneous velocity fluctuation in x - direction

Similar variables are defined for the variables in x and y direction. Substituting this in the NS-equations and averaging yields the Reynolds Averaged Navier Stokes equations and is defined as follows (White, 2009).

$$\frac{\partial u}{\partial t} + \left[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right] + \frac{1}{\rho} \frac{\partial p}{\partial x} = \frac{\partial}{\partial x} \left( \mu_d \frac{\partial u}{\partial x} - \rho u'^2 \right) + \frac{\partial}{\partial y} \left( \mu_d \frac{\partial u}{\partial y} - \rho u' v' \right) + \frac{\partial}{\partial z} \left( \mu_d \frac{\partial u}{\partial z} - \rho u' w' \right) - g_X$$
(2.7)

$$\frac{\partial v}{\partial t} + \left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right] + \frac{1}{\rho}\frac{\partial p}{\partial y} = \frac{\partial}{\partial x}\left(\mu_d\frac{\partial v}{\partial x} - \rho u'v'\right) + \frac{\partial}{\partial y}\left(\mu_d\frac{\partial v}{\partial y} - \rho v'^2\right) + \frac{\partial}{\partial z}\left(\mu_d\frac{\partial v}{\partial z} - \rho v'w'\right) - g_y$$
(2.8)

$$\frac{\partial w}{\partial t} + \left[u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right] + \frac{1}{\rho}\frac{\partial p}{\partial z} = \frac{\partial}{\partial x}\left(\mu_d\frac{\partial w}{\partial x} - \rho u'w'\right) + \frac{\partial}{\partial y}\left(\mu_d\frac{\partial w}{\partial y} - \rho v'w'\right) + \frac{\partial}{\partial z}\left(\mu_d\frac{\partial v}{\partial z} - \rho w'^2\right) - g_z \tag{2.9}$$

These equations form the basis of computational flow dynamics in various academic fields such as natural sciences (Sinha et al, 1998) and aerospace engineering (Potturi & Edwards, 2012). In this research the focus is on water flow in rivers, which means that certain assumptions are done to simplify the equations. However, before focusing on shallow water flow, first it is shown how the shear stresses are modeled.

In equation 2.7,  $\rho u'^2$ ,  $\rho u'v'$  and  $\rho u'w'$  are called the turbulent stresses, and the gradients in al velocity directions are the laminar stresses. For the x-direction this yields (White, 2009):

$$\tau_{xx} = \mu_d \frac{\partial u}{\partial x} - \rho u'^2 = \tau_{lam} + \tau_{turb,xx}; \ \tau_{xy} = \mu_d \frac{\partial u}{\partial y} - \rho u'v' = \tau_{lam} + \tau_{turb,xy}; \ \tau_{xz} = \mu_d \frac{\partial u}{\partial x} - \rho u'w'$$
(2.10)

The turbulent stresses are unknown a priori and must be related by experiment to geometry and flow conditions. In 1887, Boussinesq proposed an eddy viscosity model of these stress terms as follows:

$$\tau_{turb,xx} = \rho u'^2 = -\mu_t^H \frac{\partial u}{\partial x}, \ \tau_{turb,xy} = \rho u'v' = -\mu_t^H \frac{\partial u}{\partial y} \text{ and } \tau_{turb,xz} = \rho u'w' = -\mu_t^V \frac{\partial w}{\partial z}$$
(2.11)

With  $\mu_t^H$  [m<sup>2</sup>/s] the horizontal eddy viscosity and  $\mu_t^V$  [m<sup>2</sup>/s] the vertical eddy viscosity. The horizontal and vertical eddy viscosities are much larger than the dynamic viscosity  $\mu_d$ , therefore  $\mu_d$  is negligible. Furthermore, the spatial resolution of the grid and time steps are much larger than the typical length and time scales of turbulent motions, so the eddy viscosities  $\mu_t^H$  [m/s<sup>2</sup>] and  $\mu_t^V$  [m/s<sup>2</sup>] are both assumed to be constant (Uittenbogaard, 1992).

A similar procedure as for the momentum in x-direction can be done for the momentum equations in y and z direction. Furthermore  $g_x=g_y=0$ , since the axis are chosen such that the z-axis is in the same direction as  $g_z$ . With this, the 3D momentum equations for incompressible water flow in x, y and z direction are defined as follows:

$$\frac{\partial u}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial x} + \left[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right] = \frac{\partial^2 u}{\partial x^2} \mu_t^H + \frac{\partial^2 u}{\partial y^2} \mu_t^H + \frac{\partial^2 u}{\partial z^2} \mu_t^V$$
(2.12)

$$\frac{\partial v}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial y} + \left[ u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right] = \frac{\partial^2 v}{\partial x^2} \mu_t^H + \frac{\partial^2 v}{\partial y^2} \mu_t^H + \frac{\partial^2 v}{\partial z^2} \mu_t^V$$
(2.13)

$$\frac{\partial w}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial z} + \left[ u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right] = \frac{\partial^2 w}{\partial x^2} \mu_t^H + \frac{\partial^2 w}{\partial y^2} \mu_t^V + \frac{\partial^2 w}{\partial z^2} \mu_t^V - g_z$$
(2.14)

Applying the Boussinesq assumption, the complexity of the RANS equations is thus reduced significantly.

## 2.3 Shallow water flow: the assumption of shallowness and steadiness

The RANS equations can be further reduced to the specific case of shallow water flow, such as in river systems. In shallow water, the fluid motions are predominantly horizontal. Therefore, the vertical acceleration is very small and is therefore negligible. Under that assumption, the vertical momentum equation is reduced to the following equation:

$$\frac{1}{\rho}\frac{\partial p}{\partial z} + g_z = 0$$

This is also known as the hydrostatic pressure distribution. By integrating this equation over z and assuming  $p_0 = 0$  (atmospheric pressure), the pressure distribution is as follows:

(2.15)

$$p(z) = \rho g z$$

In here z is the distance from the water level  $z_w$  to a reference point (see figure 10).



Figure 10; Definition of water level compared to reference point

The vertical pressure distribution is then defined as follows:

 $p(z) = \rho g(z_w - z)$ 

Substituting the hydrostatic pressure distribution into the RANS equations and averaging over the depth gives the Shallow Water Equations or Saint Vanant Equations. For more detailed information, see Appendix C where it is shown how the SWE are derived.

$$\frac{\partial u}{\partial t} + g \frac{dh}{dx} + g \frac{dz_b}{dx} + \left[ \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} \right] + \frac{\partial^2 u}{\partial x^2} v_t^H + \frac{\partial^2 u}{\partial y^2} v_t^H + \frac{g |\vec{u}| \bar{u}}{hC^2} = 0$$
(2.17)

$$\frac{\partial v}{\partial t} + g \frac{dh}{dy} + g \frac{dz_b}{dy} + \left[ \bar{u} \frac{\partial \bar{v}}{\partial x} + \bar{v} \frac{\partial \bar{v}}{\partial y} \right] + \frac{\partial^2 v}{\partial x^2} v_t^H + \frac{\partial^2 v}{\partial y^2} v_t^H + \frac{g |\vec{u}| \bar{v}}{hC^2} = 0$$
(2.18)

Furthermore, besides the hydrostatic pressure distribution, it is also assumed that the change of momentum over time is zero, i.e.  $\frac{\partial}{\partial t} = 0$ . This means that the flow in all model configurations is assumed to be steady.

## 2.4 Model configuration I: 1D - Bernoulli

The most simplified model configuration considered in this research is the Bernoulli equation without head loss. The assumptions on the RANS equations are as follows:

Assumption 1: All derivatives in lateral and vertical direction are zero, i.e.  $\frac{\partial}{\partial y} = \frac{\partial}{\partial z} = 0$ .

Assumption 2: The lateral velocity v [m/s] and vertical velocity w [m/s] are much smaller than the velocity in direction of the flow u [m/s], and therefore v=w=0

Assumption 3: Both the horizontal and the vertical turbulence is neglected

The flow is also assumed to be frictionless, since no eddy viscosities are considered. The reduced momentum equations are then as follows:

$$u\frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} * \frac{1}{\rho} = 0$$
(2.19)
$$\frac{1}{\rho}\frac{\partial p}{\partial z} + g_z = 0$$
(2.20)

Substituting equation 2.20 in equation 2.19 and integrating over x gives the Bernoulli equation (see appendix A for the derivation of Bernoulli). This is a depth averaged flow and thus a 1D configuration:

 $\frac{1}{2}\bar{u}^2 + gh + gz_b = Constant$ 

Where

$\bar{u}$ [m/s]	velocity in x-direction
$g_{z}  [m^{2}/s]$	gravitational acceleration
h [m]	water depth
z <sub>b</sub> [m]	bed level

The result is an equation which represents a relation between the flow velocity, the water depth, water level and a constant value along a streamline.

(2.21)

(2.16)

## 2.5 Model Configuration II: 1D - Bélanger

The subsequent model configuration considered in this research is described by Belanger, in which the friction term is included in the mathematical description. The assumptions on the RANS equations are as follows:

Assumption 1: All derivatives in lateral direction are zero, i.e.  $\frac{\partial}{\partial v} = 0$ .

Assumption 2: The lateral velocity v [m/s] and vertical velocity w [m/s] are much smaller than the velocity in direction of the flow u [m/s], and therefore v=w=0

Assumption 3: The horizontal turbulence is neglected

The RANS equations are then reduced to the following momentum equations:

$$u\frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} * \frac{1}{\rho} = v_t^V \left(\frac{\partial^2 u}{\partial z^2}\right)$$
(2.22)

$$\frac{1}{\rho}\frac{\partial p}{\partial z} + g_z = 0 \tag{2.23}$$

In this configuration, the flow is averaged over the depth and thus is this a 1D model. In depth averaged mode, the vertical eddy viscosity  $v_t^{V}$  is related to the friction as is shown in appendix C. Substituting this in equation 2.22 yields:

$$\bar{u}\frac{\partial\bar{u}}{\partial x} + g\frac{\partial h}{\partial x} + g\frac{\partial z_b}{\partial x} = -\frac{g}{h}\frac{\bar{u}^2}{c^2}$$
(2.24)

Where

C [m <sup>1/2</sup> /s]	Chézy coefficient
h <sub>0</sub> [m]	Water depth
i <sub>b</sub>	Bottom slope

Furthermore, the following continuity relation is defined:

$$\bar{u} = \frac{Q}{A} \tag{2.25}$$

Substituting equation 2.25 in equation 2.24 and assuming that the change of width over x is zero, i.e.  $\frac{db}{dx} = 0$  gives the Bélanger equation. The derivation of this equation is shown in appendix D.

$$\frac{dh}{dx} = \left[\frac{h^3 - h_{\theta}^3}{h^3 - h_{c}^3}\right] i_b \tag{2.26}$$

With

$$h_e = \left[\frac{q}{c\sqrt{ih}}\right]^{\frac{2}{3}} = \text{equilibrium depth}$$
(2.27)

$$h_c = \left[\frac{q^2}{g}\right]^{\frac{1}{3}} = \text{critical depth}$$
(2.28)

Where

q [m<sup>2</sup>/s] Specific discharge

 $h_0$  [m] Water depth  $i_b$  Bottom slope

## 2.6 Model configuration III – 2D modeling

In all previous configurations, the flow is considered in 1 dimension. In this model configuration the 2D effects are examined. At the deepening measure the effects in vertical direction are examined and at the widening measure the effects in lateral direction are examined.

## **Deepening measure**

The 2D effects that are investigated around the downstream side of the deepening measure concerns the influence of the vertical mixing term and the effect of vertical flow. First the assumptions on the RANS equations are shown for the situation in which the influence of turbulence is considered:

Assumption 1: All derivatives in lateral direction are zero, i.e.  $\frac{\partial}{\partial y} = 0$ .

- Assumption 2: The lateral velocity v [m/s] and vertical velocity w [m/s] are much smaller than the velocity in direction of the flow u [m/s], and therefore v=w=0
- Assumption 3: The horizontal turbulence is neglected since there are no lateral changes of the river profile and wall friction is neglected

The momentum equation in y-direction is then assumed to be 0, while the momentum equation in z- and x-direction are respectively as follows.

$$\frac{1}{\rho}\frac{\partial p}{\partial z} + g_z = 0$$
 (hydrostatic situation) (2.29)

$$u\frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} * \frac{1}{\rho} = \left(\frac{\partial^2 u}{\partial z^2}\right)v^{\nu}$$
(2.30)

Again, the hydrostatic pressure distribution is substituted in the equation representing momentum in x – direction:

$$u\frac{\partial u}{\partial x} + g\frac{\partial h}{\partial x} + g\frac{\partial z_b}{\partial x} = \left(\frac{\partial^2 u}{\partial z^2}\right)v^{\nu}$$
(2.31)

Differently from the previous model configuration, the vertical mixing term is only related to Chézy at the boundary condition located at the bottom, since multiple layers are defined (thus non-depth averaged). Instead, the flow is averaged laterally. It is thus solved in quasi - 2DV mode, in which longitudinal differences in vertical direction are taken into account, but in which the vertical flows are neglected.

For the situation in which the effect of the vertical flow is considered, the assumptions are as follows:

- Assumption 1: There is no horizontal turbulence since there is no wall friction and lateral changes in the river profile
- Assumption 2: All lateral velocities can be neglected since no lateral changes occur in the river profile, so v = 0 and  $\frac{\partial}{\partial v} = 0$

Therefore, the governing equations are as follows:

$$\frac{1}{\rho}\frac{\partial p}{\partial x} + \left[u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z}\right] = \frac{\partial^2 u}{\partial z^2}\mu_t^V$$
(2.32)

$$\frac{1}{\rho}\frac{\partial p}{\partial z} + \left[u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z}\right] = \frac{1}{\rho}\frac{\partial^2 w}{\partial y^2}\mu_t^V + \frac{\partial^2 w}{\partial z^2}\mu_t^V + g_z$$
(2.33)

These set of equations represent thus momentum in x and momentum in z direction. The flow is averaged laterally in which vertical and longitudinal flow velocities are taken into account. This is thus a full-2DV analysis (instead of quasi – 2DV represented by equation 2.31). Differently from the previous configurations, the hydrostatic flow regime is not valid anymore. With this set of equation, non – hydrostatic effect are investigated where the flow is constricted at the downstream boundary of the deepening measure.

#### Widening measure

Regarding the downstream boundary of the widening measure, the assumptions on the RANS equations are as follows:

Assumption 1: The velocity w [m/s] is much smaller than the velocity in direction of the flow u [m/s] and v [m/s], and therefore w = 0.

Applying these assumptions, the following mathematical description is obtained. The momentum equations are as follows:

$$g\frac{dh}{dx} + g\frac{dz_b}{dx} + \left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] + \frac{\partial^2 u}{\partial x^2}\mu_t^H + \frac{\partial^2 u}{\partial y^2}\mu_t^H + \frac{\partial^2 u}{\partial z^2}v_t^\nu = 0$$
(2.34)

$$g\frac{dh}{dy} + g\frac{dz_b}{dy} \left[ u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} \right] + \frac{\partial^2 v}{\partial x^2} \mu_t^H + \frac{\partial^2 v}{\partial y^2} \mu_t^H + \frac{\partial^2 v}{\partial z^2} v_t^v = 0$$
(2.35)

$$\frac{1}{\rho}\frac{\partial p}{\partial z} + g_z = 0 \tag{2.36}$$

In this configuration the velocity is averaged over the depth, and therefore the vertical diffusion term is related to the friction. With this depth – averaging procedure, the so called Shallow Water Equations or Saint Venant Equations are obtained (see Appendix C) (Randall, 2006):

$$g\frac{dh}{dx} + g\frac{dz_b}{dx} + \left[\bar{u}\frac{\partial\bar{u}}{\partial x} + \bar{v}\frac{\partial\bar{u}}{\partial y}\right] + \frac{\partial^2 u}{\partial x^2}\mu_t^H + \frac{\partial^2 u}{\partial y^2}\mu_t^H + \frac{g|\bar{u}|\bar{u}}{hc^2} = 0$$
(2.37)

$$g\frac{dh}{dy} + g\frac{dz_b}{dy} + \left[\bar{u}\frac{\partial\bar{v}}{\partial x} + \bar{v}\frac{\partial\bar{v}}{\partial y}\right] + \frac{\partial^2 v}{\partial x^2}\mu_t^H + \frac{\partial^2 v}{\partial y^2}\mu_t^H + \frac{g|\vec{u}|\,\bar{v}}{hC^2} = 0$$
(2.38)

These equations represent a 2DH set of equations in which the lateral and longitudinal flow velocities both differ in longitudinal as lateral direction, but not in vertical direction.

#### Introduction to turbulence modeling

The effect of turbulence is modeled by the diffusivity terms. Diffusion is the movement of matter at high concentration to a region of lower concentration. This also applies to momentum: When there is a region with high concentration of momentum and a region of low concentration, the effect of diffusivity is that it reduces the gradient. This behavior is also observed when turbulence is present in the flow. The flow is then more mixed and differences in momentum are reduced. Therefore, including the diffusivity terms is a way of including the behavior of turbulence.

The diffusivity terms are included in the mathematical description either via the horizontal eddy viscosity coefficient  $\mu_t^H[m^2/s]$  or the vertical eddy viscosity coefficient  $\mu_t^V[m^2/s]$ . The higher these coefficients, the larger the diffusivity terms and thus the larger the effect of turbulence. In order to determine the right magnitude of the eddy viscosities, the k – epsilon model is used for modeling the vertical eddy viscosities and the HLES turbulence model is used to model horizontal eddy viscosities.

#### The *k* – epsilon turbulence model

The model used for determining vertical eddy viscosities is the so called  $\kappa - \epsilon$  turbulence model (Rodi, 1984). This model consists of two transport equations. The first transported variable determines the energy that is available in the turbulence and is called the specific turbulent kinetic energy  $\kappa \text{ [m}^2/\text{s}^2\text{]}$ . The other transported variable is the dissipation  $\epsilon \text{ [m}^2/\text{s}^3\text{]}$  which determines the rate of dissipation of the turbulent kinetic energy  $\kappa \text{ [m}^2/\text{s}^2\text{]}$ .

The energy balance of the turbulent kinetic energy is as follows (Rodi, 1984):

$$\frac{\frac{\partial \kappa}{\partial t}}{1} + \underbrace{\frac{\partial (\kappa u)}{2}}_{1} = \underbrace{\frac{\partial}{\partial z} \left[ \frac{v^{v}}{\sigma_{k}} \frac{\partial \kappa}{\partial x_{j}} \right]}_{3} + \underbrace{\left[ P_{k} + B_{k} \right]}_{4} - \underbrace{\epsilon}_{5}$$
(2.39)

And for the dissipation energy  $\epsilon$  (Rodi, 1984):

$$\frac{\frac{\partial \epsilon}{\partial t}}{1} + \underbrace{\frac{\partial (\epsilon u)}{2}}_{1} = \underbrace{\frac{\partial}{\partial x_{i}} \left[ \frac{\nu^{\nu}}{\sigma_{k}} \frac{\partial \epsilon}{\partial x_{j}} \right]}_{3} + \underbrace{C_{1\epsilon} \frac{\epsilon}{\kappa} \left[ P_{k} + (1 - c_{\epsilon 3}) B_{k} \right]}_{4} - \underbrace{C_{2\epsilon} \frac{\epsilon^{2}}{\kappa}}_{5}$$
(2.40)

With:

$$\nu^{\nu} = c_{\mu} \frac{\kappa^2}{\epsilon} \left[ \mathrm{m}^2 / \mathrm{s} \right] \tag{2.41}$$

Equation 2.39 and 2.41 each consists of five terms. The terms represent the following processes.

Term 1:	rate of change of $\kappa$ or $\epsilon$ over time
Term 2:	transport of $\kappa$ or $\epsilon$ by convection
Term 3:	transport of $\kappa$ or $\epsilon$ by diffusion
Term 4:	rate of production of $\kappa$ or $\epsilon$
Term 5:	rate of destruction of $\kappa$ or $\epsilon$

The production term consists of a Buoyance term  $B_k$  which represents the exchange between turbulent kinetic energy and potential energy and is defined by:

$$B_k = \frac{v^v g}{\sigma_c \rho} \frac{\partial \rho}{\partial z}$$
(2.42)

This term is only of importance in case of density differences in e.g. estuaries. Since there are no density differences incorporated in this research this Buoyance term is set to 0.

Furthermore, the production term consists of a term  $P_k$ , which represents the production of turbulent kinetic energy and is defined by:

$$P_k = v^{\nu} \left[ \left( \frac{\partial u}{\partial z} \right)^2 + \left( \frac{\partial v}{\partial z} \right)^2 \right]$$
(2.43)

Besides the production term, many coefficients are also included. These coefficients are determined empirically, and are shown below (Rodi, 1984):

 $\begin{array}{l} \sigma_{\epsilon} = 1.3 \\ c_{\epsilon 2} = 1.92 \end{array}$  $\begin{aligned} \sigma_c &= 0.5 \\ c_\mu &= 0.09 \end{aligned}$  $\sigma_{\kappa} = 1$  $c_{\epsilon 1} = 1.44$  $c_{\epsilon 3} = \begin{cases} 1 \ if \ B_k < 0 \\ 0 \ if \ b_k > 0 \end{cases}$ 

For the use of the k – epsilon model, the Nikuradse Roughness height  $k_s$  is introduced as the representation of bottom friction. By introducing this, the influence of the friction is dependent on the water depth, which therefore gives a more accurate way of the influence of the bottom friction on the water flow. The relation between the Chézy value and the Nikuradse height is shown in the White-Colebrook relation:

$$C = 5.75 \sqrt{g} \log\left(\frac{12R}{k}\right) \tag{2.44}$$

Where

- gravitational acceleration [m/s<sup>2</sup>] Chézy coefficient [m<sup>1/2</sup>/s] g
- С
- Hydraulic radius [m] R
- k Nikuradse Roughness height [m]

#### The HLES turbulence model en the estimation of Madsen et al (1988)

The model used for generating the horizontal eddy viscosities is the Horizontal Large Eddy simulation model. This model is based on two major components: the physical turbulence that is present and the sub-grid eddy viscosity. The way turbulence is modeled is based on the Prandtl-Kolmogrov model:

$$v_{t,Prandtl-Kolmogrov}^{H} = c_{\mu}' L \sqrt{\kappa} \left[ m^{2} / s \right]$$
(2.45)

(a .--)

Where  $\kappa$  [m<sup>2</sup>/s<sup>2</sup>] is the kinetic energy of the turbulent motion, L [m] the length scale of the geometry and  $c_{\mu}$ is an emperical constant. The eddy viscosity is further simplified as the Elder viscosity, where  $c'_{\mu} = \frac{1}{6}\kappa$ ,  $\sqrt{k} = u_*, v_*$  [m /s] and L = H [m] (Uittenbogaard, 1992):

$$v_{t,Elder}^{H} = \frac{1}{6}k(u_{*}, v_{*})H \ [m^{2}/s]$$
(2.46)

Where

k = von Karman constant (=0.41) [-] $(u_*, v_*)$  friction velocity at the bottom ( =  $\frac{\sqrt{g}}{c_{u,v}}$  u [m/s]) H is the water depth [m]

Another phenomenon that is taken into account is numerical viscosity. This becomes important when a grid size is used with an order of magnitude larger than the eddies that would occur in reality. In not taking this into account, the energy dissipated in the model is less than in reality. To solve this problem, a subgrid eddy viscosity  $v_{xx}$  [m/s<sup>2</sup>] is introduced, which takes both the magnitude of  $\Delta x$  [m] as  $\Delta y$  [m] into account. For more detailed information on the calculation and the mathematical background, reference is made to (Uittenbogaard, 1992).

Summarizing, the HLES model is introduced as follows:

$$v^{H} = v_{h0} + v_{xx} + v_{Elder} \text{ [m}^{2}\text{/s]}$$

The advantage of the HLES model is that it includes by default a horizontal eddy viscosity coefficient based on the local presence of turbulence and the locally used grid size. This is especially interesting in cases where a varying grid size is used. The disadvantage of this model is that it is more computationally intensive. Alternatively, a constant eddy viscosity coefficient can be used. An estimation of a constant horizontal eddy viscosity is introduced by Madsen et al (1988) as follows:

(2.47)

$$v^{H} = 0.15 \,\Delta x * u \,[\text{m}^{2}/\text{s}] \tag{2.48}$$

This constant is only based on the numerical eddy viscosity. The disadvantage of using a constant horizontal eddy viscosity is that it does not assess the effect of the sub – grid eddy viscosity and the presence of turbulence on a local basis, but that it is estimated for the entire domain. It may therefore overestimate the influence of the mixing term at locations where there is less turbulence or sub – grid viscosity present and under estimate this at locations with large turbulence or sub –grid viscosity.

## 3 Model set up and software used

This chapter demonstrates how the mathematical equations represented in the previous chapter are applied to both schematized river measures. Furthermore it is shown how the equations can be solved either numerically or analytically and how they are implemented in the software used.

## 3.1 Model configuration I: 1D - Bernoulli

## **Deepening measure**

For the application of the Bernoulli equation on the deepening measure, first a relation between the velocity and the bottom level is introduced:

$$\bar{u} = \frac{Q}{b} * \frac{1}{h} = \frac{q}{h} = \frac{q}{z_w - z_b}$$
(3.1)

Where

q  $[m^3/s]$ specific dischargeh [m]water depth (=  $z_w - z_b$ )

Substituting this in equation 2.19 yields:

$$\frac{1}{2}\left(\frac{q}{h}\right)^2 + gh + gz_b = Constant \tag{3.2}$$

Furthermore, the Froude-number is included. The change of water level over a distance x in relation to the Froude-number is as follows (see appendix B for the derivation):

$$\frac{dz_w}{dx} = \frac{dz_b}{dx} (Fr^2 - 1)^{-1} + \frac{dz_b}{dx}$$
(3.3)

This equation is discretized as follows:

$$z_w(x + \Delta x) = z_{w_0} + \frac{\Delta z_b}{Fr_0^2 - 1} - \Delta z_b$$
(3.4)

Where

<i>z<sub>w</sub></i> [m]	water level along the river
z <sub>wo</sub> [m]	water level at x =0
Fr <sub>0</sub> [-]	Froude number at $x = 0$
$\Delta z_b$ [m]	vertical distance of step in river bed

Equation 3.4 is implemented in Matlab to describe the longitudinal variation of the water level as function of the vertical change of the river bed.

## Widening measure

For the application of the Bernoulli equation to the widening measure, a relation between the discharge and the river width is included. For this situation, instead of a constant with b [m], a constant bed level  $z_b$  [m] is applied. The velocity is then defined as follows.

$$\bar{u} = \frac{Q}{bh}$$

Q [m<sup>3</sup>/s]discharge b [m] width of river profile h [m] water depth

Substituting this in the Bernoulli equation yields:

(3.5)

$$\frac{1}{2}\left(\frac{Q}{bh}\right)^2 + gh + gz_b = Constant$$
(3.6)

Since the Bernoulli equation is constant along a streamline, the solution is obtained by setting the constant at the boundary conditions equal to the constant elsewhere in the domain. Equation 3.6 is then solved numerically as follows:

$$\frac{Q^2}{2 g b (x + \Delta x)^2 z_w (x + \Delta x)^2} + z_w (x + \Delta x) = \frac{Q_0^2}{2 g b_0^2 z_{w_0}^2} + z_{w_0}$$
(3.7)

In here, Q  $[m^3/s]$  and g  $[m/s^2]$  are constants and so are  $z_{w_0}$  [m] and  $b_0$  [m]. With this equation the longitudinal variation of the water level is described as function of the change of the river width. The software used is Matlab.

Furthermore, the change of the water level is related to the Froude number as follows (see appendix B for the derivation):

$$\frac{dz_w}{dx} = \frac{\left(Fr^2 \frac{z_w}{b} \frac{db}{dx}\right)}{1 - Fr^2}$$
(3.8)

## 3.2 Model Configuration II: 1D - Bélanger

#### 3.2.1 Deepening measure

The mathematical description proposed by Bélanger can be solved analytically and numerically. In this research both methods are demonstrated. The equation is solved analytically by means of the 'approximation according to Bresse' (Putnam, 1948). However, the analytical approximation contains the equilibrium depth  $h_e$  (see equation 2.25). This equilibrium depth is proportional to  $1/\sqrt{i_b}$ . At the upstream boundary of the deepening measure  $i_b$  becomes negative. This means that  $h_e$  becomes imaginary and therefore the analytical solution is not used for the deepening measure. Instead of defining  $i_b$ , the change of the bed level is considered to be in an infinitely small point, leaving out the effect of the changing bed. This method is also applied in practice.

When solving the equation numerically, this problem does not occur. In the numerical solution, dh/dx is proportional to  $1/i_b$  (which is shown later on in this report, see e.g. equation 3.14) and therefore the solution can still be used when  $i_b$  becomes negative. The equation is solved numerically by means of the predictor corrector method (Van Rijn, 1990).

The analytical approximation thus neglects the change of the bed slope and is used to show how the problem is solved if this solution would be used. The numerical approximation is used to show whether the change of the bed slope may indeed be neglected and what flow processes are of importance at the downstream side of the measure.

#### Analytical solution of Bélanger using the approximation according to Bresse

The Bélanger is solved analytically with use of the approximation of Bresse, which is used when  $Fr^2 << 1$ . Using the flow parameters of the both schematized river measures, the flow velocity is estimated to be around 2 m/s, and the depth to be around 15 m. The Froude-number is than:

$$Fr = \frac{u}{\sqrt{gh}} = 0.16 \tag{3.9}$$

Therefore  $Fr^2 \ll 1$  and the approximation method can be used. The water depth at distance from  $x_0$  is determined as follows:

$$h = h_e + (h_0 - h_e) \left(\frac{1}{2}\right)^{-\left(\frac{x - x_0}{L_1}\right)}$$
(3.10)

Where

$$L_{\frac{1}{2}} = \frac{0.24 h_e}{i_b} \left(\frac{h_0}{h_e}\right)^{\frac{1}{3}} \text{ [m] half adaption length}$$
(3.11)

The 'half adaption length' indicates at which location the water depth is exactly in between the water depth at the downstream location and the equilibrium depth.

#### Implementation method in Matlab

For the implementation of the analytical solution in Matlab, the computation is divided into three different sections: the section downstream of the measure, at the measure and upstream of the measure (see Figure 11). At each section the equilibrium depth changes, and so does the adaption length. For the analysis, first the situation downstream of the measure is considered, with  $x_0$  at the far left end of the domain. When the h(x) is computed for the downstream section, the analysis is done for the section at the measure.  $x_0$  is changed to the downstream boundary of the measure, i.e. to x = 1000 m, and a new equilibrium depth (and therefore adaption length) is computed. With this, h(x) for the section at the measure is determined. The same procedure is used for the calculation of h(x) at the upstream section of the measure.



Figure 11; model set up in Matlab for analytical solution of Bélanger

#### Numerical approximation of Bélanger using the predictor-corrector method

The Bélanger equation is approached numerically as follows. First, the water depth h as a function of x is discretized using a forward Taylor expansion in space, giving a second order accuracy in  $\Delta x$ :

$$h(x + \Delta x) = h(x) + \Delta x \frac{dh}{dx}\Big|_{x} + O(\Delta x^{2})$$
(3.12)

Using a numerical notation and leaving out the higher order terms this yields:

$$h_{j+1} = h_j + \Delta x \left. \frac{dh}{dx} \right|_x \tag{3.13}$$

Furthermore,  $\frac{dh}{dx}$  is the Bélanger equation (which is derived in appendix D) and is defined as follows:

$$\frac{dh}{dx} = \left[\frac{h^3 - \frac{q^2}{C^2 i_b(x)}}{h^3 - \frac{q^2}{g}}\right] i_b(x)$$
(3.14)

For the discretization of the bed slope, both the forward and backward Taylor expansion in space are defined respectively as follows:

$$z_b(x + \Delta x) = z_b(x) + \Delta x \frac{dz_b}{dx}\Big|_x + \frac{1}{2} (\Delta x)^2 \frac{dz_b^2}{d^2 x}\Big|_x + O(\Delta x^3)$$
(3.15)

$$z_b(x - \Delta x) = z_b(x) - \Delta x \frac{dz_b}{dx}\Big|_x + \frac{1}{2} (\Delta x)^2 \frac{dz_b^2}{d^2 x}\Big|_x + O(\Delta x^3)$$
(3.16)

Subtracting the forward Taylor expansion from the backward Taylor expansion yields the central difference scheme of the bed level, with a third order accuracy in space.

$$\frac{z_b(x+\Delta x)-z_b(x-\Delta x)}{2\Delta x} = \frac{dz_b}{dx}\Big|_x + O(\Delta x^3)$$
(3.17)

Using a numerical notation and leaving out the higher order terms, the bed slope is discretized as:

$$\frac{z_{b_{i+1}} - z_{b_{i-1}}}{2\Delta x} = \frac{dz_b}{dx}\Big|_x = i_b$$
(3.18)

Substituting equation 3.18 and equation 3.17 in equation 3.13 gives a discretized function for h as function of x.

$$h_{i+1} = h_i + \Delta x \left[ \frac{h_i^{3} - \frac{q^2}{c^2 \left(\frac{z_{b_{i+1}} - z_{b_{i-1}}}{2\Delta x}\right)}}{h_i^{3} - \frac{q^2}{g}} \right] * \left( \frac{z_{b_{i+1}} - z_{b_{i-1}}}{2\Delta x} \right)$$
(3.19)

This numerical approach has a second order accuracy. That means that the following term is neglected:

$$\frac{1}{2}\Delta x \frac{dh^2}{d^2 x}\Big|_{x} + O(\Delta x^3)$$
(3.20)

Using this term, an accuracy check is performed on the proposed numerical approach shown in equation 3.19. For this, the higher order terms are left out, since that error is always smaller than the second order term. The second order term is therefore a good representation of the error made in this model. The value for this error is estimated as follows:

$$\epsilon = \frac{1}{2}\Delta x \frac{dh^2}{d^2 x} \approx \frac{1}{2}\Delta x \frac{\left(\frac{dh}{dx}\right|_{i+1} - \frac{dh}{dx}\right|_i}{x_{i+1} - x_i}$$
(3.21)

For the analysis, a  $\Delta x$  of 0.1 m is chosen. It turns out that the maximum value for the error is then in the order of decimeters.

Since this is a large error when considering water depths in the order of meters, a more accurate approach is defined. One way of increasing the accuracy is by introducing a predictor-corrector method, which uses equation 3.19 as a predictor for the calculation of  $h_{j+1}$  and corrects the obtained value by means a corrector. The predictor in this case is:

$$h_{i+1}' = h_i + \Delta x \left(\frac{dh}{dx}\right)_{i,pred}$$
(3.22)

With

$$\left(\frac{dh}{dx}\right)_{i,pred} = \left[\frac{h_i^{\ 3} - h_{e,i}^3}{h_i^{\ 3} - h_c^3}\right] * i_b \tag{3.23}$$

The corrector is then as follows:

$$h_{i+1,cor} = h_i + \Delta x * \frac{1}{2} \left[ \left( \frac{dh}{dx} \right)_i + \left( \frac{dh}{dx} \right)_{i,pred} \right]$$
(3.24)

With

$$\left(\frac{dh}{dx}\right)_{i} = \left[\frac{h_{i+1}{}^{3} - h_{e,i}^{3}}{h_{i+1}{}^{3} - h_{c}^{3}}\right] * i_{b}$$
(3.25)

When performing the same check on the second order accuracy, the error is now in the order of magnitude of cm which is considered to be accurate enough. Furthermore, in models used in practice such as WAQUA, a second order accuracy level assumed to be reliable for real world calculations (Rijkswaterstaat, 2012). Therefore this predictor corrector method complies to the requirements of the second accuracy level at the one side, but also gives a more reliable result for the water depth than merely using equation 3.19 on the other side.

#### Implementation method in Matlab

In order to do calculations, first the points in x – directions are determined. As stated above, a  $\Delta x$  of 0.1 m is chosen (see Figure 12).



Figure 12; model set up in Matlab for numerical solution of Bélanger

## 3.2.2 Widening measure

For the lateral widening measure, three solution methods are used. The first is the analytical approximation of Bresse, and the others are two different numerical approximations. The first numerical approximation is similar to the predictor corrector method which is also used for the widening measure. For this solution, the dh/dx proposed by Bélanger is solved (so with equilibrium depth, critical depth, etc.) However, one assumption that is made by Bélanger is that the lateral changes of the river width are negligible. In the second numerical solution the mathematical equation is redefined by assuming that the change in width cannot be neglected.

By comparing the analytical solution of Bresse and the numerical solution applied to the equation proposed by Bélanger, conclusions can be drawn on the accuracy of the analytical approximation. By comparing the numerical solution obtained with Bélanger and the numerical solution in which it is assumed that the lateral change of the width cannot be neglected, conclusions can also be drawn on whether this is indeed a valid assumption.

#### Solution method of the analytical approximation and implementation in Matlab

The analytical approximation is shown in the previous subsection for the deepening measure. The software used is Matlab and it is implemented as is shown in Figure 13.



#### Figure 13; Model set up of analytical solution of the widening case

## Numerical approximation I

The first numerical approximation is by using the mathematical equation proposed by Bélanger. The specific discharge q  $[m^2/s]$  is defined as a function of the discharge Q  $[m^3/s]$  and the river width b [m]:

$$q = \frac{Q}{b} \tag{3.26}$$

Both the critical depth and equilibrium depth are defined as follows:

$$h_e(x) = \left[\frac{\frac{Q}{b(x)}}{c\sqrt{i_b}}\right]^{\frac{2}{3}} = \text{equilibrium depth}$$
(3.27)

$$h_c(x) = \left[\frac{\frac{Q}{b(x)}}{\frac{Q}{g}}\right]^3 = \text{critical depth}$$
(3.28)

And the change of water depth over x is defined as:

$$\frac{dh}{dx} = \begin{bmatrix} \frac{h_0^3 - h_e^3}{h_0^3 - h_c^3} \end{bmatrix}$$
(3.29)

This way, the change of depth over x is obtained as a function of the cross-section averaged flow velocity.

The numerical solution is similar to that shown in equation 3.19. However, in this situation the bed slope does not change. Therefore, the bed slope of the entire domain  $i_b$  is implemented instead of the discretized version i.e.  $\left(\frac{z_{bk+1}-z_{bk-1}}{2\Delta x}\right) = i_b$ . Furthermore, the value of the specific discharge is no longer constant as the width varies, so the discretized specific discharge is defined as follows:

$$q_k = \frac{Q}{b_k} \tag{3.30}$$

Implementing both the definition of the bed slope and the specific discharge in equation 3.29 gives the numerical equation for the change of the water depth over x.

$$h_{j+1} = h_j + \Delta x \left[ \frac{h_j^3 - \frac{\left(\frac{Q}{b_j}\right)^2}{-C^2(i_b)}}{h_j^3 - \frac{\left(\frac{Q}{b_j}\right)^2}{g}} \right] * i_b$$
(3.31)

#### Numerical approximation II

In the second numerical approximation it is assumed that the lateral changes of the river width cannot be neglected. In order to examine this, the mathematical description for dh/dx is redefined. The change of the water depth over x as a function of the change of the river width b [m] is then as follows (see appendix D for the derivation of this equation).

$$\frac{dh}{dx} = \frac{\left(A\,i_b + \frac{Q^2}{C^2Ah} - \frac{Q^2}{gb}\frac{db}{dx}\right)}{A - \frac{Q^2b}{gA^2}}$$

Where

Surface area
Discharge
Water depth
River width
Gravitational acceleration
Chézy coefficient

(3.32)

 $\begin{array}{ll} h_0 \left[ m \right] & \mbox{Water depth} \\ i_b & \mbox{Bottom slope} \end{array}$ 

Compared to the mathematical equation proposed by Bélanger, an extra term is added, i.e.  $\frac{Q^2}{gb} \frac{db}{dx}$ . This means that  $\frac{db}{dx}$  needs to be discretized. In order to discretize this derivative, CD-scheme is used. For this, both the forward and backward Taylor expansion in space are defined respectively as follows:

$$b(x + \Delta x) = b(x) + \Delta x \frac{db}{dx}\Big|_{x} + \frac{1}{2} (\Delta x)^{2} \frac{db^{2}}{d^{2}x}\Big|_{x} + O(\Delta x^{3})$$
(3.33)

$$b(x - \Delta x) = b(x) - \Delta x \frac{db}{dx}\Big|_{x} + \frac{1}{2} (\Delta x)^{2} \frac{db^{2}}{d^{2}x}\Big|_{x} + O(\Delta x^{3})$$
(3.34)

Subtracting the forward Taylor expansion from the backward Taylor expansion yields the central difference scheme of the width b [m] with a third order accuracy in space.

$$b(x - \Delta x) = b(x) - \Delta x \frac{db}{dx}\Big|_{x} + \frac{1}{2} (\Delta x)^{2} \frac{db^{2}}{d^{2}x}\Big|_{x} + O(\Delta x^{3})$$
(3.35)

Using a numerical notation and leaving out the higher order terms, the width is discretized as:

$$\frac{b_{i+1}-b_{i-1}}{2\Delta x} = \frac{db}{dx}\Big|_{x}$$
(3.36)

Substituting equation 3.36 in equation 4.43 gives a discretized equation for h as function of x.

$$h_{i+1} = h_i + \Delta x \frac{\left(b_i h_i i_b + \frac{Q^2}{C^2(b_i h_i)^2 h_i} - \frac{Q^2}{g(b_i)^2} \left(\frac{b_{i+1} - b_{i-1}}{2\Delta x}\right)\right)}{b_i h_i + -\frac{Q^2 b_i}{g(b_i h_i)^2}}$$
(3.37)

#### 3.2.3 Implementation in Matlab

Both numerical approximations are solved in Matlab with a step size of 0.1 m as is shown in Figure 14. The change of the river width at both the upstream and downstream boundary is computed for each space step.



Figure 14; Model set up of numerical solution of widening case

## 3.3 Model Configuration III: 2D modeling

In the third configuration the effect of the mixing term in the equations is taken into account. Vertical mixing at the step in the bed downstream of the deepening measure is analyzed with the k – epsilon model whilst horizontal mixing at the downstream side of the widening measure is analyzed using two different ways of estimating the horizontal eddy viscosity. The first is a spatially constant horizontal eddy viscosity estimated by Madsen et al (1989). The average flow velocity is assumed to be 2 m/s, and the grid size used in 10 m x 10 m. The constant horizontal eddy viscosity is then estimated to be 3 m/s<sup>2</sup>. The second is a spatially variable horizontal eddy viscosity computed with the HLES turbulence model.

## 3.3.1 Deepening Measure

## 2DV modeling with k - epsilon model

The equations of the 2DV analysis with the k – epsilon model are solved numerically using the staggered grid principle in WAQUA. The derivation of the numerical approach of the mathematical equations is outside the scope of this research and can be found in the Technical Report of WAQUA (Rijkswaterstaat, 2012). The analysis is done by using a grid of  $\Delta x = 10$  m and  $\Delta y = 10$  m, but is locally refined at x = 1000 and x = 2000 (see Figure 15). In the z-direction, 10 layers are used. As grid cells are used in all directions it is technically a 3D model. However, since there are no lateral changes in the river profile, no lateral velocities occur, so it is more accurately described as a 2DV analysis in a 3D model set up.


Figure 15; Grid used in 1D analysis of deepening measure

Furthermore, the bed level is modeled uniformly in lateral direction, with a slope of 1 m / 10 km in longitudinal direction (see figure 16). The bed is deepened in between 1000 m and 2000 m along the river with 1 m.



Figure 16; bed level in WAQUA for deepening measure

#### 2DV modeling with k - epsilon model in a non - hydrostatic situation

A model is used here which also includes vertical flow velocities, differing from the hydrostatic situation. The methodology is as follows.

In order to model the effect of a non-hydrostatic flow regime, the full RANS momentum equations have to be solved. However, this requires a large amount of computation time. Therefore, an approximation of the non-hydrostatic effect is used, which is solved in the flow model SWASH. This is a tool for simulating unsteady, non-hydrostatic, free-surface flow and transport phenomena in coastal waters and river systems (Zijlema & Stelling, 2005). Non-hydrostatic flow is modeled by splitting the pressure distribution into a hydrostatic and a non-hydrostatic part:

$$p = \rho g(z_w - z_b) + q \tag{3.38}$$

Where  $g(z_w - z_b)$  represents the hydrostatic part and q the non-hydrostatic part. Substituting equation 3.38 in the pressure term  $\frac{\partial p}{\partial x} * \frac{1}{\rho}$  gives an extra pressure term representing the non-hydrostatic part:  $\frac{\partial q}{\partial x} * \frac{1}{\rho}$ . This way, the RANS equations become as follows:

$$\frac{\partial u}{\partial t} + g \frac{dz_w}{dx} - g \frac{dz_b}{dx} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \mu_t^H + \frac{\partial^2 u}{\partial z^2} \mu_t^V + \frac{\partial q}{\partial x} * \frac{1}{\rho}$$
(3.39)

$$\frac{\partial v}{\partial t} + g \frac{dz_w}{dy} - g \frac{dz_b}{dy} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \mu_t^H + \frac{\partial^2 v}{\partial z^2} \mu_t^V + \frac{\partial q}{\partial y} * \frac{1}{\rho}$$
(3.40)

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = \left(\frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 w}{\partial z^2}\right) \mu_t^H + \frac{\partial^2 w}{\partial z^2} \mu_t^V + \frac{\partial q}{\partial z} * \frac{1}{\rho}$$
(3.41)

In this analysis, only equation 3.39 and equation 3.40 are of importance since no lateral flow velocities occur. These equations are solved with the k – epsilon model. Accordingly, the results are compared to the situation with hydrostatic pressure and the k – epsilon model and conclusions are thus drawn on the influence of assuming a hydrostatic pressure distribution.

This set of equations are all solved numerically using grid cells in x- and y- direction and several layers. The exact discretization of the momentum equations is outside the scope of this research; more detailed information can be found in Zijlema & Stelling (2005). Both the grid and the way the bed level is implemented mirrors the implementation in WAQUA (see Figure 15 and Figure 16).

#### 3.3.2 Widening measure; 2DH modeling

For the implementation of the widening case in WAQUA, a grid is also used of  $\Delta x = 1 \text{ m}$  and  $\Delta y = 1 \text{ m}$  (see Figure 17). The original width of the river is 360 m, which is in-between 0 m and 1000 m and inbetween 2000 m and 3000 m along the river.



#### Figure 17; Grid used in 2DV calculation of the widening measure

Furthermore, the bed level is modeled uniformly in lateral direction, with a slope of 1 m / 10 km in longitudinal (see figure 18).



Figure 18; Bed level for widening case

For the computation with the constant eddy viscosity estimated with Madsen et al (1988), a value of  $v^{H} = 0.6 m^{2}/s$  is used since the flow velocity in the river is approximately 2 m/s.

Similar to the numerical solutions in the 1D analysis, the water depth is discretized using a Froward Euler scheme, using a staggered grid. For more detailed information about this solution, refer to the technical report of WAQUA (Rijkswaterstaat, 2012).

## 3.4 Boundary Conditions

For both river measures, the Chézy value is set to 50 m<sup>1/2</sup>/s at the bottom. It is assumed that there is no friction at the walls, so the Chézy value is 0 m<sup>1/2</sup>/s at the walls. At x = 0 the discharge is set to Q = 10667 m<sup>3</sup>/s. At x = 3000 the water depth is set to 15.2 m which is equal to the equilibrium depth.

## 4 Results

## 4.1 Model configuration I: 1D - Bernoulli

#### **Deepening measure**

First the result of the deepening measure is shown in Figure 19. The term  $\bar{u}$  [m/s] equals the depth and time averaged flow velocity,  $z_w$  [m] is the water depth and  $z_b$  [m] is the bed level. The term  $\bar{u}_0$  and  $z_{h,0}$  equals the incoming velocity and water depth respectively and  $z_{b,0}$  equals the bed level at x = 0. Furthermore, the energy is conserved in the domain since no energy is dissipated by sources such as bottom friction or bed slope. It is shown that in this case the decrease of the bed level by 1 m causes the water depth to be 1.0226 meters and therefore an increase in the water level of 2.26 cm.

The increase in water level as the bed level decreases is as expected, since the Froude number in this situation is lower than 1, indicating sub-critical flow. Equation 3.3 shows that when the Froude number is lower than 1, the water depth increases faster than the bed level decreases, indicating that the water level is increasing.



Figure 19; water level of the deepening measure when applying Bernoulli

#### Widening measure

The result of the widening measure is shown in figure 20. Instead of a changing bed level, the width changes from 360 m to 400 m. This causes the total water level to increase with 3.76 cm.



Figure 20; water level of the widening measure when applying Bernoulli

This is what is expected from the equation describing the change of the water level as function of the Froude number (see equation 3.8). When the Froude number is smaller than 1,  $\frac{dz_w}{dx}$  is positive, but when the Froude number is larger than 1,  $\frac{dz_w}{dx}$  becomes negative. Therefore only in the case of sub critical flow the water level rises.

#### Concluding remarks

In this configuration a situation is considered in which the friction term is neglected. Both at the downstream side of the deepening and widening measure, the water level increases at the location where the river profile expands or the flow decelerates, and it decreases to its original water level at the location where the river profile is constricted or accelerates. This is only the case for sub-critical flow; the opposite occurs for super-critical.

## 4.2 Model configuration II: 2D – Bélanger

#### 4.2.1 Deepening measure

#### Analytical solution

The result of the 1D analysis is shown in Figure 21. It is described mathematically according to the analytical solution of Bresse. In this situation, the equilibrium depth is equal to 15,200 m. Since there is subcritical flow, the lowering of the bed has no effect downstream of the intervention and the water depth at the section x = 2000 m to x = 3000 m is equal to the equilibrium depth. At the section in between x =1000 m and x = 2000 m, the bed level is lowered by 1 m. The equilibrium depth remains the same ( $h_e =$ 15.200 m), however, due to the lowering of the bed, the water depth at point x = 2000 m is 16.200 meter. The water depth develops towards its equilibrium depth when going further upstream to point x = 1000 m. The 'half adaption length' at this section is ca. 40 km and very large compared to the 1000 m meter over which the bed is decreased. Therefore the lowering of the bed by 1 meter over 1000 meter length causes the water level to lower by only 18 mm. At the section x = 1000 m to x = 0 m, the bed level is increased with 1 meter again. The depth right after x = 1000 m is 16.182 meters, however, due to the increase of the bed of 1 meter the depth at point x = 1000 m is 15.182 meters. When going further upstream, the water depth develops back to the equilibrium depth. The half adaption length of the section in between x = 0 m and x = 1000 m is ca. 36 km. This is again very large compared to the 1000 m section that is considered, and the water is increased over this section by only 1 mm. The water depth at x = 0 m is thus 15.183 meter.

In the whole domain,  $h_e > h_c$ , so there is a mild slope. At section x = 0 to x = 1000 m,  $h_c < h < h_e$ , therefore there is a M2 curve at this section. At section x = 1000 m to x = 2000 m,  $h_e > h$  and  $h_c < h$ . Therefore at this section a M1 – curve occurs (de Vries, 1985).



Figure 21; Solution of Bélanger according to approximation of Bresse applied to the deepening measure

Furthermore, in Figure 22 the difference in water level is shown between the situation with intervention and the situation without intervention. For the situation without intervention, the water depth is equal to the equilibrium depth over the whole domain, i.e.  $h = h_e = 15.2$  m. This Figure shows the water level as depicted in Figure 21 minus the water level of the reference situation. It can be seen that the water level is lowered by ca. 18 mm at x = 1000 m and by 17 mm at x = 0.



Figure 22; Difference between situation with and without intervention

#### Numerical solution

The result of the numerical solution can be seen in Figure 23, showing that in between x = 3000 m to x = 2000 m the water depth is the same as the equilibrium depth, i.e. h = 15.200 m. However, from x = 1990 to x = 2000, the water level increases in upstream direction. At this region, the bed level is negative, and since  $h_e \propto \frac{1}{\sqrt{i_b}}$ ,  $h_e$  becomes undefined. Due to the negative slope and  $h_c < h$ , the water level decreases according to an A2 curve. From x = 1990 m to x = 1010 m, the water level decreases according to a M1 curve, since  $h > h_e$ . From x = 1010 m to x = 1000 m, due to the sudden increase of the slope, the equilibrium depth drops significantly from 15.2 m to 1.9 m. Therefore  $h_c > h_e$  so the water level to drops according to a S1 curve. From x = 1000 m to x = 0 m the equilibrium depth is 15.2 m and the water level increases towards the equilibrium depth according to a M2 curve as  $h < h_e$ .



Figure 23; Detailed figure of the situation with deepened bed with using the numerical solution of Bélanger

In Figure 24 the difference between the situation without any intervention and the result of the numerical solution is conveyed. At section x = 2000 m to x = 3000 m, there is no difference, being downstream of the intervention. At the location of the intervention, i.e. at x = 1000 m to x = 2000 m, the water level has increased over the entire section compared to the original. In this model, the maximum increase of the water level is 23.6 mm at x = 2000 m. At x = 0 to x = 1000 m, the water level is decreased over the entire section. The maximum decrease is 18 mm at x = 1000 m and the decrease at x = 0 m is 17 mm.



Figure 24; Difference between water level without deepening of the bed and a situation with deepening of the bed

The sudden change in water level at both boundaries of the measure can also be observed in the previous model configuration. The change of the water level at both boundaries was 26.0 mm. In model configuration II the change at the upstream boundary is 23.6 mm, and the change of the water level at the downstream boundary is 23.8 mm. Both these values are thus close to what is observed in the previous configuration.

#### Magnitude of the terms in PDE around downstream side of measure

In order to understand what terms of the PDE around the downstream side of the measure are of importance, the magnitude of each term is shown (Appendix E shows how each term is computed). Figure 25 shows the size of the terms of the whole domain, i.e. from x = 0 m to x = 3000 m.



It is observed that the friction term  $\frac{g u^2}{c^2 h}$  has a relative constant value throughout the domain, i.e. an order of magnitude of 10<sup>-4</sup>. The size of the other terms change both at x = 1000 m and at x = 2000 m, which is exactly at locations where the bed slope changes. In Figure 26, the magnitude of each term of the PDE is shown around x = 1000 m.



Figure 26; magnitude of terms around x = 1000

It is observed that the relative importance of the friction term is decreased and may be neglected. The terms that are left over are thus the temporal acceleration term  $\frac{du}{dx}$ , the slope of the water depth  $g\frac{dh}{dx}$  and the bed slope term  $g\frac{dz_b}{dx}$ . This is exactly what is considered as the Bernoulli equation without head loss. The sudden change in bed slope is thus described by Bernoulli and called 'the Bernoulli effect' in this report.

Furthermore, the magnitude of each of the terms is shown for both the river section that is deepened, i.e. from x = 1010 m to x = 1990 m (Figure 27) and the upstream part of the river section, i.e. from x = 2000 m to x = 1000 m (Figure 28). The magnitude of the terms from x = 1 m to x = 1000 m are the same as from x = 1010 m to x = 1990 m, therefore the Figure which representing the terms of the deepened section represents also the magnitude of terms in the upstream section.



Figure 27; magnitude of terms of deepened section, i.e. from x = 1010 to x = 1990



Figure 28; magnitude of terms of downstream section, i.e. from x = 2000 to x = 3000

The magnitude of all terms are decreased at the deepened section, except for the friction term. The friction term cannot therefore be neglected anymore in this section. Since all terms are of importance in this section, the flow is described with the back water curve method of Bélanger. At the downstream section, it is shown that the spatial acceleration term  $u \frac{du}{dx}$  and the term describing the slope of the water depth  $g \frac{dh}{dx}$  have decreased and are therefore negligible compared to the other terms. This means that the following terms are left over:

$$g \frac{dz_b}{dx} = \frac{gu^2}{C^2 h}$$

(4.1)

When substituting the bed slope  $i_b = dz_b/dx$  into the equation, the Chézy equation is obtained:

$$u = C \sqrt{i_b h} \tag{4.1}$$

Therefore the flow is described as follows. Since the flow is in a sub-critical regime, any measures implemented in the river only have an upstream effect. Therefore, at the downstream boundary, i.e. at x =3000 m the flow is still at the same depth of the reference situation (which is the equilibrium depth in this case) after implementing the measure. When going in the upstream direction, i.e. from x = 3000 m to x =2000 m, the flow is described by the Chézy equation and the depth remains at the equilibrium depth. At the location where the bed level changes, i.e. from x = 2000 m to x = 1990 m, the flow is described by the Bernoulli equation. The bed slope term becomes negative and the spatial acceleration term becomes negative (thus a deceleration, since the flow is considered in upstream direction!). Considering this, the term  $g \frac{dh}{dx}$  becomes positive, indicating an increase of the water depth in upstream direction. In Figure 23 it is shown that the increase of the water depth is larger than the decrease of the bed slope, thus the water level increases. This increase of the water level is therefore caused by the interaction between the bed slope term and the spatial acceleration term and causes thus a peak in the water level. This peak starts exactly at the location where the bed slope increases, i.e. at x = 2000. When going further upstream, i.e. from x = 1990 m to x = 1010 m, the flow is described by the back water method introduced by Bélanger. Since the adaption length is much larger than the length of the measure, the water level is not increased to below the equilibrium depth at this section (see also Figure 23). From x = 1010 m to x = 1010 m the bed slope increases, and the spatial acceleration term increases as well. Therefore,  $g \frac{dh}{dx}$  becomes negative and the water level decreases. The peak ends therefore at x = 1000 m. From x = 1000 m to x = 0 m, the back water curve method of Bélanger is again applied and the water level develops towards the equilibrium depth in upstream direction.

#### 4.2.2 Sensitivity analysis

The cause of the peak is now explained. In this subsection the influence of various parameters of the deepening measure on the size of the peak is investigated. For this sensitivity analysis, two parameters are investigated: the depth of the step in the bed D [m] and the length of the step in the bed L [m].

First the sensitivity of the step in the bed D [m] is investigated. Besides the depth of 1 m used in the previous analysis, a smaller depth is considered of 0.5 m. The depth is gradually increased to a maximum of 4 m. The result is shown in Figure 29. At x = 2000 m, it is clearly visible that the larger the decrease of the bed level, the higher the increase of the water level becomes. Downstream of x = 1000 m, it is visible that the larger the decreased bed level the more the water level decreases.

The larger the increased bed, the larger the effect on the water level becomes. The reason for this is that a larger step in the bed level also causes a smaller slope both at the upstream and downstream boundary of the intervention. At x = 2000 m, i.e. at the downstream boundary of the intervention, a step of D = 1 m caused a maximum water level of 2.4 cm. In Figure 29 it is shown that a depth D of 4 m causes a maximum water level of 7 cm. Since the difference between the actual depth and the equilibrium depth (which is in all situations the same) is largest for this situation, the slope in between x = 1000 m and x = 2000 m is largest too. At the downstream boundary of the intervention, i.e. at x = 1000 m, the bed slope is largest again and therefore the decrease of the water level is also largest, which is 4.8 cm.



Figure 29; influence of size of decreased bed level in between x = 1000 and x = 2000

Considering the smallest step, i.e. D = 0.5 m, the largest increase of the original water level is 1.2 cm, and the largest decrease is 0.8 cm. Therefore changing the step in the bed level in the order of meters has an influence on the water level in the order of cm.

Besides the depth D [m], also the length of the step L [m] is considered. The result is shown in Figure 30. This length is varied from 1 m to 100 m. Again, at the upstream section from x = 2000 m to x = 3000 m, no changes are observed since there is a sub critical situation (for that reason only the result until x = 2250 m is shown). At section x = 1000 m to x = 2000 m, a different behavior is observed for the different used lengths. Using a length of 1 m shows the largest increase of the water level, which is 2.4 cm, using a length of 100 m shows the smallest increase of the water level, which is 2.3 cm. Using a length of 1 m shows the largest decrease at the upstream boundary of the intervention (x = 1000 m), which is a decrease of 1.8 cm. Using a length of 100 m, the smallest decrease is observed which is 1.6 cm. Therefore, changing the length of lowering the bed has an influence on the water level in the order of mm.



Figure 30; influence of size of length of the step in the bed level between x = 1000 and x = 2000

The larger the length, the less the water level changes at the upstream and downstream boundary. This is similar to what is observed when the sensitivity of the depth of the bed is considered: the smaller the slope of the step in the bed, the smaller the changes of the water level. The reason for this is that when the slope is smaller, the flow accelerations are smaller too. As is shown before, the changes of the water level at the boundaries of the river measure is described by Bernoulli. According to this description, smaller flow accelerations cause smaller changes in the water level.

#### Concluding remarks

In this analysis, two different solutions are examined for the application of Bélanger to the deepening measure, i.e. an analytical and a numerical solution. Both solutions show no water level differences upstream of the measure. A significant difference between the numerical and the analytical solution is observed at the location where the bed level changes, i.e. in between x = 2000 m and x = 1990 m and in between x = 1010 m and x = 1000 m. The analytical solution does not take the relatively large bed slope into account, while the numerical solution does. It can therefore be concluded that the analytical solution is less reliable since it does not include the effect of the bed slope changes.

It is demonstrated that these bed slope changes result, firstly, in an increase of the water depth and water level at the upstream boundary of the measure and, secondly, a decrease of the water depth and water level at the downstream boundary of the measure. This is described by the Bernoulli equation. The magnitude of the increase is similar to what is observed in configuration I in which the flow is analyzed only by means of the Bernoulli equation. It can therefore be concluded that the peak is caused by the Bernoulli – effect.

Furthermore, the flow of both the regions where the river bed is deepened, i.e. from x = 1010 m to x = 1990 m, and the upstream region, is described by the back-water curve method. The flow of the region downstream of the deepened river section is described by the Chézy equation. Configuration I therefore does not correctly describe the flow at these regions, since the incorporation of the bottom friction term cannot be neglected for them. Accordingly, in this model configuration, the friction term is important for the description of flow for the whole measure, excluding the sections where the bed level is changed.

Furthermore, when the slope of the steps in the bed level is small, the change in the water level is small too. A smaller slope causes smaller flow accelerations and, according to the description of Bernoulli, a smaller change in the water level. The depth D [m] over which the bed level is decreased also has a large effect. The smaller the step, the lower the peak becomes.

#### 4.2.3 Widening measure

#### Analytical solution

For the widening measure, first the results of the analytical solution is shown, see figure 31. The water depth is higher than the critical depth  $h_c$  in the entire domain, indicating sub critical flow. From x = 2000 m to x = 3000 m, the equilibrium depth  $h_e$  = 15.2 m and the critical depth  $h_c$  = 4.5 m. Since the flow is in equilibrium in this section, the actual water depth is equal to the equilibrium depth.



Figure 31; Result of analytical solution of Bélanger for widening measure

From x = 2000 m to x = 1980 m, the width of the river changes from 360 m to 400 m. Both the equilibrium and the critical depth change at this point since they are directly related to the width (see equation 3.27). The equilibrium depth changes from 15.2 m to 14.17 meters and the critical depth changes from 4.5 m to 4.17 m. This also causes the actual water depth to decrease in upstream direction according to a M1curve, since  $h > h_e$ . However, this change is so small that at x = 1990 m, the water depth is still at 15.2 m. From x = 1990 m to x = 1010 m, the water depth decreases again according to a M1 curve from 15.200 m to 15.181 m. From x = 1010 m to x = 1000 m, the width of the channel changes back from 400 m to 360 m, and both the equilibrium as the critical depth change back to the original values. The increase of the actual water level is very limited, therefore the actual water depth at x = 1000 m is still 15.181 m. From x = 1000 m to x = 0 m the water depth increases again in upstream direction according to a M2 curve, since h < h<sub>e</sub>. At x = 0, the water depth is increased to 15.1812 m. In Figure 32 the difference between the result of the analytical solution of the widening case and the original situation is shown.



Figure 32; Difference in water level for analytical solution of Bélanger

It is shown that from x = 2000 m to x = 3000 m no changes occur. From x = 1000 m to x = 2000 m the water level decreases with a maximum water level drop of 19.2 mm at x = 1000 m. From x = 1000 m to x = 0 m, the water level increases back towards its original equilibrium depth. The total water level drop at x = 0 m from the original situation is 18.8 mm.

#### Numerical solution I (in which db/dx = 0)

The result of the first numerical solution is shown in Figure 33. Also this solution shows a decrease of the water depth from x = 2000 m to x = 1000 m according to a M1 curve. The M1 curve occurs as long as  $h_e$ <h. Around x = 1000 m, h and  $h_e$  become equal since  $h_e$  suddenly increases. From that point to x = 0 m, the water level increases again according to a M2 curve.



Figure 33; Result of numerical solution of Bélanger for widening measure

Furthermore, the difference between a situation with the widening measure and the original situation is displayed in Figure 34. In this Figure, also the result of the analytical solution is shown. In the result of the numerical solution, the water level has decreased with a maximum of 19.2 mm at x = 1000 m and that the decreasing effect at x = 0 equals 18.7 mm. The result of both the numerical and the analytical solution are both similar. It is shown that both solutions are exactly the same from x = 3000 m to x = 2000 m. The solution starts to deviate more from x = 2000 m to x = 0 m, with the largest difference at x = 0 m. This maximum difference equals 0.1 mm.



Figure 34; Result of both the analytical as the numerical solution

#### Numerical solution II ( in which $db/dx \neq 0$ )

The result of the numerical equation in which the changes of width are included in the equation is shown in Figure 35. The situation in which db/dx is not equal to 0 only yields in between x = 1000 m to x = 1010 m and in between x = 1990 m and x = 2000 m, since the width changes at these locations from 360 m to 400. In the other sections in the domain, db/dx equals 0, which means that the original Bélanger equation is obtained.



Figure 35; Result of numerical solution of Bélanger for the widening measure

The actual depth is equal to the equilibrium depth from x = 3000 m to x = 2000 m. From x = 2000 m to x = 1990 m, the water depth increases from 15.2 m to 15.2375 m. From x = 1990 m to x = 1010 m, the water depth decreases according to an M1 curve since  $h > h_e$ . From x = 1010 m to x = 1000 m, the water depth decreases to 15.1827 m. From x = 1000 m to x = 0 m the water depth increases again according to a M2 curve since  $h < h_e$ .

The reason for the sudden increase in upstream direction at x = 2000 m and the decrease at x = 1000 m is due to the change of the river width, i.e. db/dx = 2 m/m. This change of width over x is directly related to db/dx as follows:

$$\frac{dh}{dx} \propto \frac{h Q^2}{gA} \frac{db}{dx}$$
(4.2)

Therefore, when db/dx increases, dh/dx increases as well and vice versa. Furthermore, the effect of widening the river is shown in Figure 36, which presents the result of the widening case compared to the reference situation.



Figure 36; difference in water level for situation with arbitrary cross-section

When focusing on the section where the width of the river changes, i.e. from x = 2000 m to x = 1990 m the water level suddenly increases with 37.7 mm. From x = 1010 m to x = 1000 m the water level decreases with 35.8 mm. This flow behavior is also observed in flow configuration I. The change in water level at this configuration is 37.6 mm, which is close to both the increase as the decrease in flow configuration II. At x = 0 the water level is decreased with 16.8 mm.

In comparison with the previous numerical solution where db/dx is assumed to be 0, the result is the same from x = 3000 m to x = 2000 m (see Figure 37). At the section where the bed level changes, i.e. from x = 2000 to x = 1990 m, the water level increases in the solution in which db/dx  $\neq 0$ , but decreases in the solution in which db/dx = 0. At x = 1000 m, the water level of both solutions are almost the same, and so are the water levels at x = 0 m. The difference in water level between the two solutions at this location is 1.7 mm.



Figure 37; Result of both the 1D solutions in which db/dx = 0 and db/dx  $\neq$  0

#### Magnitude of the terms in PDE around the downstream side of the measure

In order to understand what terms of the PDE are of importance, the magnitude of each term of the PDE considered is shown (see Figure 38). The bed slope is constant along the whole domain whilst the friction term is relatively constant, i.e. it has an order of magnitude of  $10^{-4}$ . At the boundaries of the measure which is at x = 2000 m and x = 1000 m, the size of the terms in the PDE's change.



Figure 38; magnitude of each term in PDE

This is shown in more detail in Figure 39, which has zoomed in to x = 1000 m. The temporal acceleration term and the slope of the water depth are the most important. These two terms again result in the Bernoulli equation without head loss.



Figure 39; order of magnitude of terms in PDE for x =1000

Therefore, both at the upstream and downstream boundary of the widening measure, the flow processes are described with the Bernoulli equation without head loss.

#### Concluding remarks

In this analysis, first the analytical and a numerical 1D solution is considered in which the changes in width are not taken into account. It is shown that the results of these solutions are very similar.

The numerical solution in which  $db/dx \neq 0$  shows a significant difference from the analytical solution and the solution in which db/dx = 0, i.e. an increase at the downstream boundary and a decrease at the upstream boundary. Thus the solution in which  $db/dx \neq 0$  approaches reality more closely as it also takes into account the effects of a varying width.

These changes in the water level are described with Bernoulli, the flow at the downstream section is described with Chézy, while both the flow at the widened section (x = 1010 m to x = 1990 m) and the upstream section are described with Bélanger. Therefore, configuration I is not the right way of describing the flow processes at these regions since the friction term tends to be of importance at these regions.

However, at the upstream and downstream boundary of the measure, the flow can be described with the Bernoulli equation without head loss. Therefore, at these sections, the flow can be described with configuration I and the bottom friction does not have a significant influence.

# 4.3 Model configuration III: 2D modeling with non – longitudinal flow and/or turbulence

#### 4.3.1 Deepening measure: vertical mixing effects in a quasi 2DV flow model

In the deepening measure, there are two components inducing turbulence: the bed friction and the two steps in the bed. In this subsection, the effect of bottom friction on the turbulence is discussed first and then the effect of the change of the bed slope on the turbulence.

#### Bottom friction

The result of using the k – epsilon model is shown in Figure 40. This Figure shows the vertical eddy viscosities generated by the turbulence model as function of the water depth.



The eddy viscosity approaches  $0 \text{ m/s}^2$  near the bed and that it increases towards the middle and decreases towards the top of the water column. This indicates that the most eddies are present in the middle. The reason for this is that in the middle there is free space for eddies to occur, while the eddies are limited near the bed and at the surface. Therefore the vertical eddy viscosities in the middle are higher than near the bed and water surface.

#### Changes of the bed slope

The effect of the deepening measure of the river bed on the vertical eddy viscosity is shown in Figure 41. In this Figure the vertical eddy viscosities are shown for each layer. The drop in the river bed is thus not visible, since only the layers are shown. Behind the deepening of the bed, from x = 1000 m to x = 1500 m the eddy viscosities are larger and behind the downstream step in the bed, from x = 2000 m to x = 2500 m the eddy viscosities are smaller. This can be observed in the upper layers of the flow, i.e. from layer 6 to 10.



Figure 41; vertical eddy viscosities for deepening case

The reason for the higher eddy viscosities from x = 1000 m to x = 1500 m, is because the flow decelerates in this region. This can be seen in Figure 42, where the velocity is shown for layer 6 to layer 10. The velocity in between x = 1000 m to x = 1500 m decreases, indicating deceleration. When flow decelerates, more turbulence occurs, and therefore the vertical eddy viscosity increases. From x = 2000 m to x = 2500m the flow velocity increases indicating acceleration of flow. When flow accelerates, turbulence is suppressed and therefore lower eddy viscosities occur. At the regions x = 0 m to 500 m, x = 1500 m to x = 2000 m and from x = 2500 m to 3000 m, the flow velocity changes are limited, which indicates that there is small acceleration and deceleration. Therefore, in these regions the eddy viscosity is the same as is presented in Figure 39.



Figure 42; velocity per layer



The effect of turbulence on the water level compared to the reference situation is shown in Figure 43. In here also the result of the previous configuration is plotted, with  $v^{\nu} = 0$  (which is the same as the Bélanger equation).

Figure 43; water level difference with and without use of vertical eddy viscosities compared to reference situation

At x = 2000 m the water level with use of the k- epsilon turbulence model is higher and at x = 1000 m the water level is lower compared to the situation where Bélanger was applied. At x = 2000 m, the water level is 24.0 mm higher with the k – epsilon model and 23.6 mm with Belanger. The reason for the higher water level at x = 2000 m is that the envelope of the water level from x = 2500 m to x = 2000 m increases. This is exactly at the locations where the eddy viscosities decrease (see Figure 41). Since the water level upstream of the step (at x = 2000 m) is higher with use of the k – epsilon model, the water level at x = 2000 m is higher too. When going further upstream, from x = 1500 m to x = 1000 m the water level starts to decrease more with use of the k – epsilon model. This is exactly at the location where the eddy viscosities increase (see Figure 41). Since the water level at x = 1000 m to the k – epsilon turbulence model is lower, the water level at x = 1000 m is lower too. At x = 0 m, the water level compared to the reference situation is 18 mm when Bélanger is applied and 19 mm lower when k – epsilon is applied.

Therefore, compared to the flow in the previous configuration (the application of the Bélanger equation or the situation with  $v^{\nu} = 0$ ), the water level also increased x = 2000 m and decreased at x = 1000 m in

upstream direction. The largest differences are downstream of both x = 1000 m and x = 2000 m, however these differences are small and may therefore be neglected.

#### Concluding remarks

The deepening measure is examined with the k – epsilon model, generating vertical eddy viscosities at locations where turbulence occurs. Compared to when Bélanger is applied, this results in a decrease of the water level at the upstream boundary of the measure and a decrease of the water level downstream of the measure. However, these differences are very small.

Therefore, it can be concluded that turbulence does not contribute to the size of the peak. It is thus an insignificant flow process at the schematized deepening measure considered. The flow upstream of the measure and in between the two steps in the bed level is described with the back-water curve method or Bélanger equation. Downstream of the river measure, the flow is described with the Chézy equation and at the boundaries of the measure the flow is described with Bernoulli.

#### 4.3.2 Deepening measure: non – hydrostatic effects in a full 2DV flow model

In all previous configurations a hydrostatic flow regime was assumed. This is only valid when vertical flow velocities are negligible. However it is uncertain whether this is indeed the case for the deepening measure, because a sudden vertical change in the bed is present. Therefore, the deepening measure is considered with a flow model in which non-hydrostatic effects are taken into account.

The result is shown in Figure 44, where the difference with the reference situation is highlighted for both the hydrostatic and the non-hydrostatic calculation (including both the k- epsilon and turbulence models). Here it can be seen that the largest differences between the hydrostatic and the non-hydrostatic calculations are located at the upstream and downstream boundary of the measure. The largest difference is at x = 1990 m. In the hydrostatic calculation a difference of 2.4 cm is observed and in the non-hydrostatic calculation a difference between the hydrostatic calculation and non-hydrostatic and non-hydrostatic calculation is thus 1.0 mm.



Figure 44; water depth at upstream step for both non-hydrostatic and hydrostatic calculation

Furthermore, the deviation of the pressure distribution of the non-hydrostatic calculation from the hydrostatic calculation is presented. Since the largest difference in the water depth upstream of the step is at x = 1990 m and downstream of the step at x = 2000 m, the deviation at these cross-sections is shown. Figure 45 shows the pressure deviation at the cross-section of x = 1980 m, and Figure 46 shows the deviation from the hydrostatic pressure at x = 2020 m. Both Figures show that the largest deviation is close to the bed. This is because the layer closest to the bed experiences the vertical velocities caused by the sudden vertical step at x = 2000 m more than the layers further away from the bed. As was shown before, when vertical velocities are of importance, the hydrostatic pressure assumption does not stand. Therefore, the more the vertical velocities are of influence, the less the pressure distribution becomes hydrostatic.





Figure 45; deviation from hydrostatic pressure distribution at cross-section x = 1980, upstream of step



The cross-section upstream of the step in the bed shows a negative deviation from the hydrostatic pressure distribution, while the cross-section downstream of the step in the bed shows a positive deviation. The negative deviation causes the water depth to be lower, and the positive deviation causes the water depth to be higher. However these differences are very limited, and thus the non-hydrostatic effects can be neglected.

#### Concluding remarks on configuration IV for the deepening measure

In this configuration, the non-hydrostatic effects have been examined. There is some effect both upstream and downstream of x = 1000 m and x = 2000 m. It is shown that an increase of the pressure results in a decrease of the water level and vice versa.

However, the difference between the hydrostatic and non – hydrostatic calculation is very small. In conclusion, the non – hydrostatic effects are thus limited and do not contribute significantly to the increase of the water level at the downstream boundary of the river measure.

#### 4.3.3 Widening measure: turbulence effects in a 2DH flow model

#### No turbulence model

For the widening measure, the results when no turbulence model at all is used, i.e. when  $v^H = 0$ , will be considered first. The analysis was undertaken with a grid size of 2 m x 2 m. As will be shown later in this report, the results of the flow model no longer vary when a smaller grid size is used. Therefore this result is considered as the converged result and is used for the analysis of the effects of lateral flow. The water depth is shown in Figure 47. It is observed that both upstream of x = 1000 m and x = 2000 m, the water depth shows lateral differences.



Figure 47; Water depth in 2DV analysis of widening measure

Besides the lateral difference in water depth, there is also a lateral velocity component. Figures 48 and 49 show that there are lateral flow components at x = 1000 m and x = 2000 m respectively. This indicates that there is a lateral momentum flux in the lateral direction at both boundaries of the widening measure.









These lateral effects also affect the water depth in the axis of the river (see Figure 50). In this Figure, both the result of the 2D model run and the 1D solution in which  $db/dx \neq 0$  are displayed. 5 lines can be distinguished. Line 1 represents the downstream water level in which the water depth is equal to the equilibrium depth. Line 2, demonstrates a clear change compared to the 1D analysis for which the line is much steeper. This is also observed for Line 4.



Figure 50; 2DH analysis with grid size 2m x 2m and 1D analysis compared to reference situation of widening measure

The increase in the 2D analysis is thus smaller than in the 1D analysis. The longitudinal flow component present in the 2D analysis is the reason for this. The lateral flow components cause a momentum flux towards the center of the river. This causes the longitudinal flow velocities to increase. The larger the lateral momentum flux, the larger the longitudinal flow velocities thus become. Because the lateral flow velocities occur over the range shown in Figure 51, the change of the longitudinal flow velocities (and thus acceleration) is also present over that range. In the 1D analysis, the change of the water level only occurred at the section where db/dx changes. Since the longitudinal range of these lateral flows is larger than the longitudinal range over which the width changes, the change of the water level in this configuration stretches out over a larger longitudinal distance compared to the 1D analysis.



Figure 51; lateral flow velocities in 2DH analysis

Furthermore, the slope and also the location of Line 3 in both the 1D and 2D analysis is almost the same. However, since this line in the 2D analysis starts much further upstream from the step in the bed (at x = 2000), and the slope of line 2 is smaller, the maximum increase is smaller: the maximum water level in the 1D analysis is 38 mm and in the 2D analysis this is 33.3 mm.

It is furthermore observed that line 5 in the 1D analysis is lower than in the 2D analysis. Summarizing, the largest difference between the 1D and the 2D analysis is that the lateral flow components cause the increase and decrease at the boundaries of the river measure to be stretched out, which thus has a significant change on the shape of the peak.

#### Magnitude of the terms in PDE around the downstream side of the measure

Also for the 2DH analysis, the magnitude of the terms in the PDE can be seen in Figure 52. Here, only the terms in x-direction are displayed, being of the most interest.



Figure 52; magnitude of terms of momentum equation in x-direction

It can be seen that both the bed slope and the friction are relatively constant over the whole domain and that the magnitudes of each term change at the locations where the river width changes, i.e. at x = 2000 m

and x = 1000 m. In order to examine what flow processes are of importance at these locations, a more detailed Figure is shown in Figure 53, which zooms in at x = 2000 m.



Figure 53; magnitude of terms in PDE from x = 1700 to x = 2300

In this Figure, the magnitude of each term is shown from x = 1700 m to x = 2300 m. The relative importance of each term changes over a long distance. The friction term has an order of magnitude of  $10^{-4}$  over the whole domain and the bed slope has an order of magnitude of  $10^{-3}$  over the whole domain. Both the term representing the slope of the water depth and the term representing the longitudinal spatial acceleration become more important and have an order of magnitude of  $10^{-2}$  at x = 2000 m. Furthermore, the magnitude of the lateral spatial acceleration term also increases towards x = 1000 m, however the magnitude of this term representing the bed slope are both likewise relatively small and are also neglected. The longitudinal acceleration term and the term representing the slope of the water depth remain and represent the Bernoulli equation without head loss. The longitudinal flow is thus described with the Bernoulli equation, however the lateral flow components have an important influence on the way the longitudinal flow accelerates.

#### HLES and constant turbulence model

The 2DH analysis is undertaken with the HLES turbulence model. This turbulence model takes both numerical turbulence and physical turbulence into account. The result is shown in Figure 54 where the horizontal eddy viscosity per grid cell is presented (the x- and y – axis show the number of the grid cell!).



Figure 54; Values of horizontal eddy viscosity coefficients for all meshes

The eddy viscosities are evidently highest at both the upstream and downstream boundary of the measure. The maximum eddy viscosity is at the downstream boundary (at x = 1995 m) and is 0.76 m/s<sup>2</sup>. The result of the use of the HLES turbulence model on the difference of the water level is shown in Figure





Figure 55; water level difference for situation with HLES, constant and no turbulence model compared to reference situation

It is demonstrated here that the water level difference is larger at the downstream boundary when a turbulence model is used. The maximum increase of the case with the no turbulence model is 33.3 while the maximum increase of the case with the HLES turbulence model is 42 mm, which is thus a difference of about 9 mm. The reason of the increase is that introducing the horizontal eddy viscosity, the diffusivity terms  $\frac{\partial^2 u}{\partial x^2}$ ,  $\frac{\partial^2 u}{\partial y^2}$ ,  $\frac{\partial^2 v}{\partial x^2}$  and  $\frac{\partial^2 v}{\partial y^2}$  are introduced in the mathematical description. Diffusivity is the process by which the momentum in lateral and longitudinal direction is mixed in the flow. This causes the velocities to be more evenly distributed in the domain. The average longitudinal flow velocity is therefore increased and therefore also the acceleration. This causes the change of the water level to increase (since the longitudinal flow is described by Bernoulli). With the constant eddy viscosity, the water level increase is smaller at x = 1995 m compared to the use of the HLES – turbulence model. This is because the constant eddy viscosity is smaller compared to the eddy viscosity at the upstream boundary that was generated with HLES. Thus the mixing effect in the case with constant eddy viscosity is smaller too.

#### **Concluding remarks**

In this analysis, it is demonstrated that in the 2DH analysis the increase and decrease at the two boundaries of the measure are explained by the Bernoulli equation without head loss. It is also shown that these changes in water level occur over a distance in the order of 100 m, while in the 1D computation the change only occurred at a distance of 10 m. This is caused by lateral momentum fluxes.

It can therefore be concluded, that at the locations where no lateral flow velocities occur, the flow can be described identically to the previous flow configuration. This means that far upstream of the measure and also in the middle of the measure, the flow is described by Bélanger, and that far downstream the flow is described by Chézy. Around the locations where the width changes, the flow is still explained by the Bernoulli equation without head loss, however, due to the lateral flow velocities the longitudinal range of the change of the water level is larger. In conclusion, since the lateral flow velocities cause significant changes on the water level around the up – and downstream boundary of the measure, the lateral flows cannot be neglected.

Furthermore, two different horizontal turbulence models are used, the HLES turbulence model and a constant horizontal eddy viscosity of 0.6 m/s<sup>2</sup>. Regarding the results of the flow model with use of the HLES turbulence model, it is evident that the increase at the downstream boundary is significantly larger. Therefore turbulence has a significant effect on the size of the peak in the schematized widening measure. By using a constant horizontal eddy viscosity the result is more closely aligned to the HLES turbulence model than that using no turbulence model at all. Therefore it can be concluded that that it is better to use the constant eddy horizontal viscosity estimated by Madsen et al (1988) than no turbulence model at all.

## 4.4 Overview of obtained results

In the previous subsections, detailed results are shown of all model configurations. In order to compare the results of each configuration to each other, a global overview of all results is shown in this subsection. Figure 57 presents the most important results of the deepening measure: the analytical solution of Bélanger, the numerical solution of Bélanger, the semi-2DV analysis with vertical mixing and the full 2DV analysis with vertical mixing and a non – hydrostatic pressure distribution. The analytical solution of Bélanger clearly shows a significant difference compared to the other results. Furthermore, neither the addition of vertical mixing nor the analysis with non-hydrostatic flow show significant differences from the numerical solution of Bélanger. The size of the peak is thus described by Bélanger and the vertical mixing and the non – hydrostatic effects have no significant influence on the flow.



Figure 56; overview of results obtained for the deepening measure

The overview of the most important results of the widening measure are shown in Figure 58. This yields the results of the following model configurations: the 1D Bélanger analysis, the 1D analysis with the assumption that  $db/dx \neq 0$ , the 2DH analysis with HLES – turbulence model and the 2DH analysis without turbulence model. There is clearly a significant difference between the 1D analysis of Bélanger and the 1D analysis with the assumption that  $db/dx \neq 0$ . Therefore, the solution of Bélanger does not describe the flow at the upstream and downstream boundary of the river measure well. Furthermore there is an evident difference between the 2DH analysis and the 1D analysis. The shape of the peak changes significantly at both the upstream and downstream boundary. This is due to the lateral flow component as demonstrated in the previous subsections. There is still clearly a difference between the situation in which a HLES model is used and the situation without turbulence model. Therefore turbulence has a significant effect on the river measure.



Figure 57; overview of the results obtained for the widening measure

### 4.5 Influence of grid size

In the previous subsections several configurations with various flow processes were introduced. With this, it has been investigated whether these flow processes result in a peak at the downstream side of the considered schematized river measures. Furthermore, the mathematical equations of these flow configurations have been discussed, particularly how they are approached, either numerically or analytically. In general, simplified mathematical equations are approached analytically while more complex mathematical equations are approached numerically. By undertaking the mathematical equations numerically, certain assumptions are made on the step size of the discretization. For example, in practice a grid size of 40 m is used. In this subsection it will be investigated whether the size of the grid has an influence on the representation of the flow processes that are of importance at the downstream side of the schematized river measures. This is undertaken by analyzing the step size used to solve the discretized equations of the important flow processes at the downstream side of the river measures.

According to the analysis of the deepening measure, the most important flow processes are analyzed with the Bélanger equation. This model is relatively easy to solve, therefore multiple step sizes are used varying from 0.1 to 100 m. The widening case is analyzed with a grid size of 1 m x 1 m, 10 m x 10 m and a grid size of 40 m x 40 m.

#### Deepening measure

The result for the deepening measure is presented in Figure 59, showing the difference of the water level compared to the reference situation. Evidently the smaller the grid size, the steeper the slope above the step and the smaller the increase at the downstream boundary. Furthermore, the larger the grid size, the smaller the decrease at x = 0 m, and thus the greater reduction of the effect at the upstream side of the measure.



Figure 58; Difference in water level compared to reference situation using various step sizes

The dimensions of the increase at the downstream boundary are also displayed in Figure 60, where the maximum water level increase can be seen for each grid size. There is a clear distinction between grid sizes smaller than 10 m and grid sizes larger than 10 m. The model runs with a smaller grid size than 10 meter do not show significant differences compared to the run with a grid size of 10 meter, while the runs with a larger grid size show larger differences. The reason for this is that the step in the bed is 1 meter in depth over 10 meters in longitudinal direction. The bed slope is represented well when a grid size of the same size or smaller is used. However, when using a larger grid size, the bed slope is decreased to 1 meter in depth over the size of the step size in longitudinal direction. As shown in the previous subsections, the change of the water level at the downstream boundary is related to the acceleration of the flow. When a smaller bed slope is modeled compared to the bed slope in reality, the accelerations are smaller too, and so is the change of the water level.



Figure 59; maximum water level increase at the downstream side of the measure

#### 4.5.1 Widening measure

A grid size of 2 m x 2 m was used for the analysis of the effect of the lateral flow processes. In this subsection it is shown that using a smaller grid size, i.e. 1 m x 1 m does not produce significant differences and that the analysis with the grid size of 2 m x 2 m thus gives a reliable result. The effect on the model results when larger grid cells are used is also demonstrated. The widening is 20 m in the y – direction and 20 m in the x – direction. At first, grid cells are used of 10 m x 10 m, and thus the widening at x = 1000 m and the constriction at x = 2000 m is represented by 2 grid cells (see Figure 61). The grid cells thus fit perfectly in the measure but are larger than the grid size of the converged result. The effect of using this grid size can be seen. Furthermore, often in practice even larger grid cells are used than the dimensions of the expansion at x = 1000 m and the constriction at x = 2000 m. In order to investigate this effect, grid cells of 40 m x 40 m are introduced (see Figure 62). The widening measure in this case is thus represented by half a grid cell.



Figure 60; Widening measure with grid size of 10 m x 10 m



Figure 61; Deepening measure with grid size of 40 m x 40 m

In order to examine the flow when using smaller grid sizes, 2 different grid sizes are used: a grid size of 1 m x 1 m and a grid size of 2 m x 2 m. It was decided not to use even smaller grid sizes because, as it will be shown, the results do not change significantly when using a smaller grid size of 2 m x 2 m and it is also very computationally intensive.

The result is shown in Figure 63. Using a grid size of 40 m x 40 m results in an increase of 6.223 cm, for the grid size of 10 m x 10 m the maximum water level increase is 4.65 cm and for the grid size of 1m x 1m and 2 m x 2 m this yields 3.31 cm. In all cases the no turbulence model is used, i.e. the horizontal eddy viscosity is set to 0 m/s<sup>2</sup>. The reason that the peak is significantly higher when larger grid cells are used is that the lateral velocities penetrate further towards the axis of the river. The lateral flow velocities thus have a larger effect, and the flow in the longitudinal direction is more greatly accelerated. Since larger flow accelerations and decelerations causes larger water level changes, the effects on the water are greater.


Figure 62; Difference in water level compared to reference situation using various step sizes

Another effect of the large grid cells is that the widened section is over estimated. While the actual increase of the width is 40 m, in the model this increase is 80 m. Therefore the effect of the river measure is much larger than in reality.

It is furthermore interesting that the upstream effect of the grid size with 10 m x 10 m is substantially decreased compared to the two smaller grid sizes. It would be expected that likewise the peak at the downstream side of the measure, the upstream effect would also be in between the analysis of 2 m x 2 m and 40 m x 40 m. The reason for this is uncertain, because all parameters used were the same in the flow models except for the varied grid sizes. A possible reason could be the fact that in the analysis with grid sizes of 40 m x 40 m the widening measure is in fact over designed (the widening has increased to double its size). Therefore the analysis with grid sizes smaller than the size of the widening can be compared, but the analysis with a grid size larger than the widening is a totally different case and may therefore not be compared. However, this reason is still not convincing and more research should be undertaken for a more clear reason.

#### Concluding remarks

Regarding the deepening measure, using a smaller grid size than the length of the step in the bed evidently does not produce significant differences. Using a larger grid size than the step does however show significant differences, thus reducing the accuracy.

Regarding the widening measure, when using grid cells with a dimension of 1 \* db/dx, the result is clearly converged. Using larger grid cells, i.e. 5 \* db/dx, the increase of the water level is overestimated thus reducing accuracy.

In conclusion, the grid size matters for both the deepening and the widening measure. The optimal grid size for the widening measure is the grid size which is 1 \* db/dx and for the deepening measure this is exactly the length of the step, as this is still accurate but the least computationally intensive.

# **5** Practical application

# 5.1 Introduction

In the previous chapters, flow processes around two measures were analyzed. The focus was both on the flow processes themselves and on the numerical representation of these processes. It was demonstrated that the changes of the water level at both the upstream and downstream side of the schematized river measures are mainly caused by flow accelerations: the larger the flow acceleration the larger the change of the water level.

In this chapter, instead of a schematized measure, a measure that approaches a real world case is considered. With the knowledge obtained in the previous subsections, a reduction of the flow accelerations - and therefore also the changes of the water level both at the upstream and downstream side of the measure – will be attempted, without significantly altering the effect obtained upstream of the measure.

The outline of this chapter is as follows. Firstly, the project introduced in subsection 1.4 will be schematized. Then the effect on the water level after constructing the side channel will be discussed. It turns out that a peak occurs at the mouth of the side channel. In the subsequent subsection it will be demonstrated how the knowledge of the previous chapters is applied to reducing the peak. Following that, the result of applying this knowledge will be presented and in the last subsection some conclusions are drawn.

# 5.2 Schematization of the real world case

The project is schematized with a straight channel according to the dimensions as follows (see Figure 64). The total length of the river is 20 km in order to visualize the upstream effects of the measure. The total length of the wide floodplain is 3.5 km and the total width is 640 m. The total width of the small floodplains is 100 m. The width of the summer bed is 160 m and the depth is 5 m below the winter bed. The slope of the bed is 1 m / 10 km.



Figure 63; Overview of schematization of real world case

The roughness of the winter bed is higher than in the summer bed as more vegetation is present in the winter bed. The roughness of the summer bed is modeled with a Nikuradse Roughness height of 0.15 m and the roughness of the winter bed is modeled with a Roughness height of 0.5 m.

The construction of the side channel is shown in Figure 65. The width of the side channel is 100 m and is located 325 m from the summer bed. The depth of the side channel is 3 m below the summer bed. Between the summer bed and the main channel a small threshold with a height of 0.5 meter is modeled. This threshold is constructed from the upstream border of the wide winter bed to 2/3 the length of the wide winter bed.



Figure 64; Construction of side channel in winter bed

### 5.3 Results of constructing the side channel

The first result presented is that of the change of the water level after constructing the side channel (see Figure 66) The water level is shown to increase by 17.2 mm near the mouth of the side channel and reduce the water level upstream by 44 mm.



Figure 65; Difference of water level compared to reference situation after construction of side channel

It is interesting to see that there is a clear upward peak at the downstream boundary, but no downward peak at the upstream boundary of the side channel as was seen at both the deepening and widening measure. The reason for this is that there is a bar constructed from the upstream boundary of the widened flood plain to approximately 1/3 downstream of the flood plain (see Figure 65). This causes the water to flow into the widened floodplain in two stages: the first is at the upstream boundary of the bar when there is high water in the river and the second is downstream of the bar when the water level is below the level of the bar but higher than the level of the flood plains. In the situation considered, the water level is higher than the bar, so it flows into the wide flood plane at both these points. The downward peak is therefore also visible at these two locations.

# 5.4 Flow processes causing the downstream peak

In the previous Chapters, it was asserted that the peak at the downstream side of the river measures can be explained by the Bernoulli equation. This was analyzed for both a deepening and a widening measure. This case is slightly different, since the real world case regards it a side channel. An important flow parameter at the mouth of the side channel is the discharge Q. Since the flow of the main channel and the side channel meet each other downstream of the measure, the discharge Q changes significantly locally. In order to clarify the interaction between this change of the discharge and the change of the water depth, Q is substituted in the Bernoulli equation:

$$\left(\frac{Q}{A}\right)\frac{\partial\left(\frac{Q}{A}\right)}{\partial x} + g_z\frac{\partial z_b}{\partial x} + g_z\frac{\partial h}{\partial x} = 0$$
(5.1)

The change of the bed level is just the bed slope and therefore  $g_z \frac{\partial z_b}{\partial x}$  does not change in the domain. Considering the flow in the main channel near the mouth of the side channel,  $\left(\frac{Q}{A}\right)\frac{\partial \left(\frac{Q}{A}\right)}{\partial x}$  increases as Q increases. This causes  $g_z \frac{\partial h}{\partial x}$  to decrease. The water depth thus decreases and so does the water level. It was shown in the previous chapters that this causes a peak when there is sub – critical flow in the river. This is explained as follows: since there is sub-critical flow, implementing a measure affects water level in the upstream direction. This is also the case in this situation, and for simplicity it is assumed that the water depth at x = 0 is at the equilibrium depth (so there is no other measure downstream). When going in the upstream direction towards the mouth of the side channel, the water depth remains at the equilibrium depth. At the mouth of the side channel, the discharge Q **de**creases in the upstream direction. Therefore, compared to the situation without a side channel, the water level increases at the downstream side of the measure.

In order to reduce the peak, the focus will be first on the change of the discharge over x:  $\frac{\partial Q}{\partial x}$ . A decrease of this term will be attempted, whereby the magnitude of the increase of the water level in the upstream direction is reduced, thus decreasing the peak. As well as  $\frac{\partial Q}{\partial x}$ , attention will be payed to  $\frac{\partial u}{\partial x}$ . As is shown in Chapter 4, this term also contributes to the peak at the downstream side of the measure. When this term increases, the magnitude of the decrease of the water level increases too. This effect is used in practice to reduce the peak; increasing this term near the peak, a larger decrease of the water level locally is obtained and thereby a local Bernoulli effect. This could thus reduce the size of the peak (this is more extensively explained in the subsequent subsections). As stated before, this is sometimes applied in practice. Whether this is indeed applicable will be addressed in this report.

#### 5.5 Reduction of the peak by decreasing dQ/dx

The first method of reducing the peak is to focus on the way the flow of the side channel enters into the main channel. When this happens at one specific point, dQ/dx is locally high. When this happens over a longer distance, dQ/dx is lower at each point and therefore dh/dx is lower at each point. This method is applied to the real world case by expanding the mouth of the side channel from 200 m to 400 m in downstream direction (see Figure 67). Furthermore, a second case where the mouth is expanded with 200 m in upstream direction and 200 m in downstream direction is considered (see Figure 68). However, this affects the distribution between the discharge in the side channel and the main channel. In order to retain the original distribution, two different cases are considered. The first case is the situation where extra friction is added to the side channel. Instead of a roughness height of 0.5 m in the side channel, a roughness height of 0.56 m will be used (see Figure 68).



Figure 66; expansion of mouth of side channel at west side



Figure 67; expansion of mouth at both west and east side



Figure 68; Increased Chézy values in case of widened mouth of side channel

The second case is the situation where a slope is introduced in the mouth of the side channel such that the cross-section in the entire side channel remains the same. Figure 70 shows the slope for the case with expansion in downstream direction. The side channel is increased to double the original size. The depth of the side channel is decreased from 3 m to 1.5 m, so it is decreased to half the original size. This way the surface of the cross-section remains the same (600 m<sup>2</sup>) and therefore also the distribution between the main and side channel is similar to the original case.



Figure 69; Slope in side channel in case of expansion of side channel at west side

Figure 71 show the slope for the case with expansion in both downstream and upstream direction. The mouth is expanded from 200 m to 600 m. The mouth of the side channel is thus expanded by three times its original size. The depth is decreased three times its original size too (from 3 m to 1 m). This way the surface remains the same ( $600 \text{ m}^2$ ) and so does the distribution between the discharge in the side channel and the main channel.



Figure 70; slope in side channel in case of expansion at both west and east side

#### Result of first case: expansion of mouth of side channel at the west side

The result of the first case with extra roughness in the side channel is shown in Figure 72. This Figure shows the difference between the original case and the case with the adjusted side channel both compared to the reference situation. The height of the peak for the situation with the adjusted channel mouth is 15.1 mm, and is thus reduced by 2.1 mm.



Figure 71; Result of situation with expanded mouth of the side channel and higher roughness in the side channel

The reason for the smaller peak near the river mouth is that dQ/dx is indeed lower. The change of the discharge over x for both the situation with and without the adjusted mouth of the side channel is shown in Figure 73. dQ/dx near the mouth of the side channel is indeed clearly smaller in the case of the adjusted channel mouth compared to the original situation. The reason for this is that the flow is better distributed over a larger distance than in the case of no increase of the width. Therefore the momentum of the flow is less concentrated at one point. Another reason is that the angle between the flow in the main channel and the flow in the side channel is smaller. Therefore magnitude of the lateral flow components are smaller, which causes a smaller acceleration in longitudinal direction (this is also explained in chapter 4). This causes the peak to be smaller.



Figure 72; dQ/dx along the river

The result of the case in which the slope of the mouth of the side channel is adjusted is shown in Figure 74. It is observed that the peak is 15.1 mm, and that the decrease effect upstream of the measure is 44 mm. This means that the peak is decreased by 2.1 mm compared to the original side channel.



Figure 73; result of analysis with expanded side channel mouth in combination with more roughness in the side channel

It is furthermore observed that the decrease of the peak in both circumstances is the same: 2.1 mm.

**Result of second case: expansion of the mouth of the side channel at both west and east side** The result with increased friction is shown in figure 75. The peak in this case is 16.3 mm, while in the original situation the peak was 17.2 mm. The peak is thus reduced with 0.9 mm.



Figure 74; result of analysis with expanded mouth of side channel at both east and west side in combination with extra friction in the side channel

The result with the slope in the mouth is shown in figure 76. The maximum increase in this case is 15.1 mm, which is thus a reduction of 2.1 mm compared to the original situation.



Figure 75; result of analysis with expanded mouth of side channel at both east and west side in combination with a slope in the side channel

Expanding the side channel to the west thus has a decreasing effect of 2.1 mm for both situations with a slope in the mouth and an increased friction in the side channel. Expanding the mouth also to the east does not have larger decreasing effect on the peak and can even result in a larger peak compared to only expanding to the west (see figure 75).

### 5.6 Reduction of the peak by increasing du/dx

In this subsection another method of reducing the size of the peak is demonstrated which is sometimes applied in practice. Instead of reducing du/dx, this term is locally increased. This works as follows. By adding friction in the winter bed, the flow will slow down locally at the section where the friction is added. Upstream of this area of high friction, the flow will be forced towards the center of the river. At that location du/dx is thus locally increased (see Figure 76). Increasing this term means a decrease of dh/dx and thus a decrease of the water level. Upstream of the section where the friction is added, the friction is decreased again, increasing the flow velocity locally. This causes the water to go from the center of the river to the wall, decreasing du/dx locally. At this location the water level thus rises again. Figure 78 shows a sketch of the water level in the center of the river. It can be seen that the water level decreases upstream of the area with friction and increases downstream of this area. The consequence of this trick is that the water level in upstream direction increases, thus reducing the upstream effect of the river measure. In practice (and also in this research) this is accepted. Adding friction in the floodplains thus causes a downward peak (see Figure 78), while the construction of the side-channel causes an upward peak (see Figure 79 for a sketch, and Figure 65 which shows the peak which was found in the result of the flow model). The combination of these two effects could thus reduce the upward peak; adding friction such that the water level is decreased at the location where the upward peak is at its maximum. This investigation addresses whether this is indeed applicable.



Figure 76; schematization of effect of adding friction in floodplains of the river



Figure 77; Upward peak caused by side channel

Figure 78; Downward peak caused by friction

The extra friction is obtained by planting trees at different locations around the inlet of the side channel. The trees that are planted have a Nikuradse Roughness height of 10 m. These are applied to two different locations.

The first is in the winter bed at the south side of the measure (see Figure 79). It is located from x = 2800 to x = 5000. The peak starts around x = 4000 and ends around x = 3000. This way, the decrease of the water level is obtained downstream of where the peak starts and the increases again around the upstream side of the peak.



Figure 79; Extra friction located at the south side of the measure

Figure 80 shows the water level of the floodplain (exact location is depicted in figure 79). The reference situation (without extra friction) is displayed in this figure too. As was expected, the water level is increased at the upstream boundary of the area with extra friction and is decreased at the downstream boundary.



Figure 80; water level at the floodplain after increasing friction

Figure 81 shows the water depth in the axis of the river around the area where the friction is added (see figure 79 for exact location of the cross-section). It is shown that the water depth decreases locally indeed at the upstream boundary and increases again at the downstream boundary. That means that the downward peak of figure 78 is obtained and so was the upward peak (as was the peak in figure 65).



Figure 81; water depth in axis of the main channel as a result of the addition of friction in the winter bed at the south side

The final result on the peak is shown in Figure 82. The increase of the friction only has a negative effect: the peak is increased and the upstream effect is reduced compared to the original situation. The peak in this case is 23.9 mm, which is 6.7 mm larger than the original situation.



Figure 82; water level difference in axis of river compared to reference situation for both the original situation and the situation with increased friction at the south side

The second place where the friction is increased is at a smaller area at the south of the river. In this case the focus is not on the entire peak itself as was covered in the previous situation, but on the area where the peak is at its maximum which is around x = 3500. Decreasing the water depth in front of the peak where it is at its maximum is attempted. The friction is increased from x = 3350 to x = 3850 (see Figure 86).



Figure 83; Extra friction located at the location where the peak is at maximum

The consequence of increasing the friction in the floodplain is that the water level in the flood plain increases. This is indeed the case and is depicted in Figure 84 (for the location of the cross-section, see Figure 83).



Figure 84; water level in floodplain after increasing the friction

The result of the water depth in the axis of the main channel (see figure 83 for the exact location) is as shown in Figure 85. It is observed that the water depth decreases at the upstream boundary of the area with increased friction. However, it is difficult to observe where the water depth increases again. The reason for this is uncertain.



Figure 85; result of water depth in the axis of the main channel after increasing the friction at the south side of the main channel

The result of the final case is shown in figure 88. The maximum water level increase is 20.6 mm which is 3.4 mm larger than in the original situation. Furthermore, the upstream effect is reduced significantly.



Figure 86; water level difference in axis of river compared to reference situation for both the original situation and the situation with increased friction at the location where the peak is at its maximum

Thus clearly increasing the friction in the floodplains does indeed decrease the water depth locally. Therefore, the downward peak as is sketched in Figure 79 is obtained. However, despite this, at the location where the peak occurs, du/dx is increased too. This causes the upward peak as sketched in Figure 78 to be larger as well. Increasing du/dx causes a larger increase of the upward peak than a decrease of the downward peak. The result of this is a net increase of the water level and thus a larger peak. Furthermore by obtaining the downward peak, an increase in upstream direction occurs too. This causes the reduction in the upstream effect. Therefore, in all cases the increase of friction has a bad effect in two ways: it increases the peak and decreases the upstream effect. The peak is thus not reduced by increasing du/dx.

### Concluding remarks on practical application

In this chapter, some practical measures are shown that are deployed to reduce the dimensions of the peak. The first focus was on the way the mouth of the side channel is designed. It is shown that widening the mouth of the side channel by twice its original size the peak is reduced. However, this also causes a change in the distribution of the discharge in the side and main channel. This is solved in two ways: (i) by increasing the roughness in the side channel and (ii) by introducing a slope in the mouth of the side channel. It is shown that in both cases the peak is reduced by 14%. It is shown that widening the river mouth in the opposite direction of the flow does not have an additional reducing effect on the peak.

Furthermore, it was investigated whether it is possible to create an extra Bernoulli effect by adding friction in the winter bed. It was demonstrated that in all circumstances, this effect only causes the peak to be larger and the reducing effect upstream of the measure to be smaller.

In conclusion, when the focus is on reducing the peak, it is best to both widen the mouth of the side channel and construct a slope, as it causes the flow accelerations to be lower. When focusing on a larger decrease of the water level upstream without enlarging the peak, it is best to only widen the mouth and not to construct the slope. Furthermore, the idea of adding friction only increases the acceleration and therefore the peak.

# 6 Conclusion, Discussion and Recommendations

# 6.1 Conclusion

The goal of this study was to gain insight into the relationship between flow processes around the downstream side of various river measures and the downstream peak and how these insights can be used to reduce the dimensions of the peak in the preliminary design phase of constructing a side channel.

This goal was pursued by examining the following research questions:

# 1. Is the downstream peak accurately represented in the flow models that are applied to design river measures?

In order to examine this research question, two basic types of river measures were introduced: deepening measure and a widening measure. The focus of this research was on the downstream side of both measures where the flow is constricted.

# a. What flow process(es) are of importance at the downstream side of various river measures?

The widening measure was considered at first. This was analyzed to begin with by the 1D description introduced by Bernoulli, demonstrating that the water level increases due to flow deceleration and that the water level decreases due to acceleration.

Furthermore, in the subsequent model configuration, the friction term was added. This configuration was described by Bélanger. This term was shown to be of importance over the entire domain, except for the steps in the bed level at the up- and downstream boundary of the measure. For both upstream of the river measure and in the middle, the flow is described by Bélanger, whilst downstream of the measure the flow is described by Chézy. Despite the addition of the bottom friction, the flow at the two steps in the bed level is still described by the Bernoulli equation without head loss. It was discovered that the change in bed slope causes the flow to accelerate or decelerate, thus causing the water level to change. Therefore (compared to the reference situation without any measure), a peak occurs in the positive direction at the upstream boundary of the deepening measure and a peak occurs in the negative direction at the downstream boundary of the deepening measure.

The influence of turbulence on the peak was likewise investigated. This was undertaken by introducing several layers and the k – epsilon turbulence model. It was shown that the difference between the analysis performed with Bélanger and that with the k – epsilon turbulence model was negligible. Therefore it was concluded that turbulence is an unimportant flow process for this schematized river measure. Furthermore, the effect of vertical flow velocities on the peak was also examined by considering a non-hydrostatic pressure distribution. These effects were also shown to be very small and that the non-hydrostatic effects can be neglected too.

Therefore, for the deepening measure, it can be concluded that the flow at the downstream side of the deepening measure is explained by Bernoulli, and that the peak occurs due to the interaction between the acceleration caused by the steep slope in the bed and the change of the water level. It was shown that an estimation of the vertical size of the peak can be computed with the Froude number and the change of the bed slope at the boundaries of the measure.

Summarizing, it can be concluded that the most important flow process at the downstream side of the deepening measure is described by Bernoulli. With Bernoulli the reason that a peak occurs at this boundary can be explained.

Secondly the widening measure was examined. The flow was first analyzed by means of the Bernoulli equation in configuration I. It was shown that the decelerating flow at the upstream boundary causes the water level to increase and that the accelerating flow at the downstream boundary causes the water level to decrease.

In configuration II the friction term was added. The friction term was shown to be of importance in the entire river section except for the section where the river width changes. At the location where the river width changes, the flow is still described by Bernoulli. This thus causes the water level to increase in the upstream direction at the downstream boundary, which is observed as a peak. Downstream of the constriction, the flow is described by Chézy, and in the middle and upstream of the measure the flow is described by the back-water curve method of Bélanger.

Furthermore, the effect of lateral flows was examined in configuration III. It was determined that the changes of the water level both at the upstream and downstream side of the river measure are still described by Bernoulli, but that the lateral flow velocities influence the way the flow is longitudinally accelerated. This evidently causes the increase in upstream direction to be stretched out over a longer longitudinal distance. This effects a decrease in the slope of the water level, and therefore the maximum water level decreases as well. In conclusion, the inclusion of lateral flow components causes the shape of the peak to change and is therefore an important component of the flow processes at the boundaries of the widening measure.

Additionally, the effect of turbulence was considered. This was undertaken in two ways: 1. By using the HLES turbulence model and 2. By using a constant horizontal eddy viscosity which was estimated by the formula proposed by Madsen et al (1988). It was established that including HLES shows a significant difference compared to using no turbulence model. Therefore it can be concluded that turbulence is a significant flow process at the two boundaries of the widening measure. Using the constant horizontal eddy viscosity clearly underestimates the increase of the water level at the upstream boundary of the measure and the decrease of the water level at the downstream boundary. However, the results with the constant horizontal eddy viscosity are still closer to the results obtained with the HLES model and it can therefore be concluded that for this schematization it is better to use the estimation of Madsen et al (1988) than using no horizontal turbulence model at all.

Summarizing, it can be concluded that the peak at the downstream side of the widening measure can be explained with Bernoulli. Furthermore, it can be concluded that the peak is significantly influenced by both lateral flow and turbulence and therefore the lateral flow becomes a significant component and likewise the turbulence a significant flow process.

# b. What is the effect of the step size in the discretized equations on the representation of these flow processes?

Besides the analysis of the important flow processes around the downstream side of the river measures, the influence of the size of the grid cells on the dimensions of the peak was also addressed. For the deepening measure, it was demonstrated that by using a smaller  $\Delta x$  than the longitudinal size of the step in the bed, the differences of the peak become negligible. However, when using a larger  $\Delta x$  than the step in the bed, the accuracy is reduced.

For the widening measure using a smaller grid size than 1 \* db/dx was proved to give an accurate result. However, using a larger grid size reduces the accuracy of the flow processes around the downstream side of the river measure.

#### Conclusion of research question 1:

In conclusion, it was determined that the peak exists in reality, and that it can be described by Bernoulli. The interaction between changes in the water level and accelerations or decelerations caused by sudden changes of the river profile effect an increase in the water level at the downstream side of the river measures. Furthermore, for the widening measure, turbulence and the lateral flow are deemed important.

All these flow processes are in principle correctly represented by the flow models that are applied to design river measures. However, this only stands if the measure is correctly implemented in these flow models. For the deepening measure it is viable if the grid size around the upstream and downstream side is not larger than the length of the step in the bed, and for the widening measure likewise only if the grid size upstream and downstream of the measure is not larger than 1 \* db/dx.

# 2. What can be done to reduce the dimensions of the peak in the preliminary design phase of a side channel?

In order to propose some measures that reduce the dimensions of the peak, a real world case was schematized. This concerned the 'Schellener and Oldeneler Buitenwaarden' near Zwolle.

The means of reducing the peak involved looking at both reducing dQ/dx and increasing du/dx. Decreasing dQ/dx was achieved by focusing on the mouth of the side channel, as the peak occurs at the location where the water of the side channel flows into the main channel. Widening the mouth of the side channel in combination with either a slope in the mouth of the side channel or increasing the friction in the side channel itself was shown as an effective way to reduce dQ/dx and thus the size of the peak.

Increasing du/dx was undertaken by increasing the roughness in the floodplains at three different locations. This evidently causes a downward peak at the whole section where the friction is added. However, it was also determined that the increase of the upward peak is larger than the decrease of the downward peak. Therefore the peak increases in all circumstances and the upstream effect is reduced in all circumstances.

#### **Conclusion of research question 2:**

The conclusion of research question 2 is thus that the peak is reduced by ensuring a gentle inflow of the water from the side channel into the main channel such that dQ/dx is reduced and accordingly dh/dx is also reduced. The peak is not reduced by increasing the roughness of the floodplains which only increases the peak.

#### Objective

The objective of this study was to gain insight into the relation between the important flow processes around the downstream side of river measures and the downstream peak. For both the measure considered, it was demonstrated that the important flow processes are explained by Bernoulli. For the deepening measure, momentum in the longitudinal direction is predominant. For the widening measure, momentum both in the longitudinal and in the lateral direction are predominant and turbulence is an important flow process. Furthermore, the objective was to gain insight into how the dimensions of the peak can be reduced in the preliminary design phase of the river measures. It has been shown that reducing dQ/dx has a significant effect. Therefore, the objective has been reached.

# 6.2 Discussion

#### Application of Bernoulli in rivers

In river dynamics, there are several situations in which Bernoulli can be applied. This mathematical description is used for constructions in rivers such as bridges, weirs (Castro-Orgaz & Chanson, 2009) and groynes (Proust et. al, 2002). Furthermore much research has shown that it is also applicable in some circumstances at river bends (Kalkwijk & de Vriend (1980) and Niesten, 2016). This research has determined that it is also applicable at river measures where the river profile suddenly changes, i.e. both at the upstream and downstream side of a deepening and widening measure and of a side channel.

#### Non analyzed flow processes

In this research, several flow processes are examined. For the widening measure, horizontal turbulence was deemed to be important. Thus the effect of spatial differences in longitudinal and lateral direction was investigated. However, it is also likely that at the widening measure vertical differences in the lateral components occur. For example, Fischer (1973) has explained that this is an existing phenomenon in open channel flow in general. This process was not taken into account and may be of importance too.

Furthermore, for the deepening measure non-hydrostatic effects are examined. These effects are not addressed for the widening measure. Since there vertical changes are observed in the water level, vertical flow may occur. It is uncertain whether these are available and whether these lead to non-hydrostatic effects.

#### Uniformity of schematized flow river measures

Furthermore, the river measures considered are, except for the downstream and upstream boundary, all spatially uniform. Also the real case scenario is schematized such that it is spatially uniform at many sections. In reality, the rivers are more non-uniform. The effect of this is not taken into account in this research.

#### Sensitivity analysis

In this research, the sensitivity of some parameters was not investigated. The sensitivity of using various grid sizes was discussed, however the dimensions of the river measures were not addressed. Furthermore, only one type of friction was used for the schematized river measures, wherefore the effect of using more or less friction is uncertain. Also the other parameters such as the discharge, bed slope, etc. were not considered.

#### Real world experiment

In this research, the flow is examined by means of various flow models. These flow models are all an approximation of reality. Despite concluding that the peak exists, the uncertainties of the models were not considered and how they might affect the model results.

#### Approximation of 2DV PDE's

The partial differential equations for the analysis of the non-hydrostatic effects at the deepening measure were not solved directly. Instead, they were approached by means of a layer model. This approximation is a reduction of reality and some differences might occur when these equations are solved directly in a more advanced model.

# 6.3 Recommendations

In this research it was shown that what grid size is used is of importance. For the deepening measure it is recommended not to use a larger grid size than the step of the bed level, since this reduces the accuracy of the peak. For both the deepening and widening measure, it is also recommended not to use larger grid sizes than the constriction, as this will amplify the peak. For the widening measure, it is recommended to even use a grid with a significantly high resolution at both the upstream and downstream boundary, because the peak is very sensitive to excessively large grid sizes.

Regarding the deepening measure, it was demonstrated that the flow is described by Bernoulli, and that no other flow processes are of significant importance at the downstream side of the measure. It was shown that the magnitude of the peak can be estimated by only the size of the step in the bed and the Froude number. This might therefore also be used in practice to give an estimation of the peak that occurs when implementing a deepening measure. However, the analysis was undertaken on a strongly schematized river measure and therefore more research is recommended on this before using it in practice.

For the widening measure, turbulence was shown to be an important flow process. In practice, turbulence is included in the calculations by a constant horizontal eddy viscosity. However, there is no standard for this: users of the flow models are free to choose what eddy viscosity is used, whilst the actual size of the peaks that occur in the model results are sensitive to the eddy viscosity. It is therefore recommended to design a standard for this. In this research it is shown that the estimation of Madsen et al (1988), gives a reasonably good estimation of the right constant eddy viscosity. However, the research of Madsen et al (1988) takes limited processes into account, being a calibration study. Improving the way the constant horizontal eddy viscosity is computed might thus improve the accuracy of the peak.

Furthermore, an analysis was undertaken with various flow models. Each flow model is an approximation of reality. Although the flow processes contributing to the existence of the peak have been investigated, greater certainty would be obtained by confirming the results in a flume experiment. Therefore a real world experiment is recommended, comparing to the results of this research.

In this research the sensitivity analysis was limited. Therefore, in further research one should focus on the sensitivity of various flow parameters. One important parameter whose influence may be large is friction. Furthermore, the dimensions of the measures have clearly shown to be important as well. However, to what extend this influences the behavior of the flow around the downstream boundary has been limited investigated.

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# Appendix A – Derivation of the Bernoulli equation

$$u\frac{\partial u}{\partial x} + \frac{\partial p}{\partial x} * \frac{1}{\rho} = 0 \tag{A.1}$$

With no vertical flow velocities, the momentum in z-direction yields:

$$\frac{\partial p}{\partial z} * \frac{1}{\rho} = -g_z \tag{A.2}$$

Where p is defined as the hydrostatic pressure distribution as:

$$p = \rho g_z (\mathbf{z}_{\mathbf{w}} - \mathbf{z}) \tag{A.3}$$

Where  $z_w$  – z is the distance from the free surface to a reference point. Substituting A.3 into A.1 gives:

$$u\frac{\partial u}{\partial x} + g_z \frac{\partial z_w}{\partial x} = 0$$

Defining  $z_w = z_b + h$  and substituting gives:

$$u\frac{\partial u}{\partial x} + g_z\frac{\partial z_b}{\partial x} + g_z\frac{\partial h}{\partial x} = 0$$
(A.5)

(A.4)

Integrating equation A.5 over x gives the Bernoulli equation:

$$\int_{x=0}^{x} \left( u \frac{\partial u}{\partial x} + \rho g_z \frac{\partial h}{\partial x} + \rho g_z \frac{\partial z_b}{\partial x} \right) dx = \frac{1}{2} \bar{u}^2 + \rho g_z h + \rho g_z z_b = C$$
(A.6)

# Appendix B – Relation between Bernoulli equation and changing river profile

#### Changing bottom level

Bernoulli equation:

$$\frac{1}{2}\bar{u}_{1}^{2} + gh_{1} + gz_{b,1} = \frac{1}{2}\bar{u}^{2} + gh + gz_{b}$$
(B.1)

Substituting the discharge yields:

$$\frac{Q_1^2}{2gb_1^2h_1^2} + h_1 + z_{b_1} = \frac{Q^2}{2gb^2h(x)^2} + h(x) + z_b(x)$$
(B.2)

Differentiating to x yields:

 $\frac{-Q^2}{2 g b^2 h(x)^3} \frac{dh}{dx} + \frac{dh}{dx} + \frac{dz_b}{dx} = 0$ (B.3)

The Froude number is defined as follows:

$$Fr^2 = \frac{u^2}{gh} = \frac{Q^2}{gb^2h^3}$$
(B.4)

Substituting this definition into equation B.3 yields:

$$\frac{dh}{dx} = \frac{dz_b/dx}{\left[\frac{Q^2}{db^2 h^3 - 1}\right]} = \frac{dz_b/dx}{Fr^2 - 1}$$
(B.5)

Substituting  $h = z_w - z_b$  into the equation give a relation of the change of bed slope and Froude number as a function of  $z_{w:}$ 

$$\frac{dz_w}{dx} = \frac{dz_b/dx}{Fr^2 - 1} + \frac{dz_b}{dx}$$
(B.6)

#### Changing river width

For a changing width, the variables are slightly different:

$$\frac{Q_1^2}{2 g b_1^2 h_1^2} + h_1 + z_{b_1} = \frac{Q^2}{2 g b(x)^2 h(x)^2} + h(x) + z_b$$
(B.7)

#### Differentiating to x yields:

$$\frac{-Q^2}{2g b^2 h(x)^3} \frac{dh}{dx} - \frac{Q^2}{2g b^3 h(x)^2} \frac{db}{dx} + \frac{dh}{dx} = 0$$
(B.8)

Substituting the Froude number:

$$\frac{dh}{dx} = \frac{\left(\frac{Q^2}{gb^3h^2dx}\right)}{1 - \frac{Q^2}{gb^2h^3}} = \frac{\left(Fr^2\frac{hdb}{bdx}\right)}{1 - Fr^2} \tag{B.9}$$

Because the water depth is the same as the water level for the lateral flow constriction,  $h = z_w$ :

$$\frac{dz_w}{dx} = \frac{\left(Fr^2 \frac{z_w db}{b \ dx}\right)}{1 - Fr^2} \tag{B.10}$$

# Appendix C – Derivation of Shallow Water Equations

Starting with the momentum equations in x - and y - direction:

$$\frac{\partial u}{\partial t} + g \frac{dz_w}{dx} - g \frac{dz_b}{dx} + \left[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] = \frac{1}{\rho} \left[ \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} \right]$$
(C.1)

$$\frac{\partial v}{\partial t} + g \frac{dz_w}{dy} - g \frac{dz_b}{dy} + \left[ u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] = \frac{1}{\rho} \left[ \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} \right]$$
(C.2)

The shear stresses are defined as follows:

$$\tau_{xx} = \mu_d \frac{\partial u}{\partial x} - \rho u'^2 = \tau_{lam} + \tau_{turb,xx}; \ \tau_{xy} = \mu_d \frac{\partial u}{\partial y} - \rho u' v' = \tau_{lam} + \tau_{turb,xy}; \ \tau_{xz} = \mu_d \frac{\partial u}{\partial x} - \rho u' w' \tag{C.3}$$

The Boussinesq assumption yields:

$$\tau_{turb,xx} = \rho u'^2 = -\mu_t^H \frac{\partial u}{\partial x}, \ \tau_{turb,xy} = \rho u' v' = -\mu_t^H \frac{\partial u}{\partial y} \text{ and } \tau_{turb,xz} = \rho u' w' = -\mu_t^V \frac{\partial w}{\partial x}$$
(C.4)

Averaging over the depth of the flow velocities u and v yields:

$$\bar{u} = \frac{1}{h} \int_{z_b}^{z_w} u \, dz \qquad \qquad \bar{v} = \frac{1}{h} \int_{z_b}^{z_w} v \, dz \qquad (C.5)$$

Integrating the Left Hand Side of equations C.1 and C.2 and substituting the hydrostatic pressure distribution yields:

$$\int_{z_b}^{z_w} \left[ \frac{\partial u}{\partial t} + g \frac{dz_w}{dx} - g \frac{dz_b}{dx} + \left[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right] \right] dz = \frac{\partial \overline{u}}{\partial t} h + g \frac{dz_w}{dx} h - g \frac{dz_b}{dx} h + \left[ \overline{u} \frac{\partial \overline{u}}{\partial x} h + \overline{v} \frac{\partial \overline{u}}{\partial y} h \right]$$
(C.6)

$$\int_{z_b}^{z_w} \left[ \frac{\partial v}{\partial t} + g \frac{dz_w}{dy} - g \frac{dz_b}{dy} + \left[ u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right] \right] dz = \frac{\partial \bar{v}}{\partial t} h + g \frac{dz_w}{dy} h - g \frac{dz_b}{dy} h + \left[ \bar{v} \frac{\partial \bar{u}}{\partial x} h + \bar{u} \frac{\partial \bar{v}}{\partial y} h \right]$$
(C.7)

Integrating  $\frac{\partial \tau_{XZ}}{\partial z}$  of equations C.1 and C.2 yields:

$$\int_{z=z_b}^{z_w} \frac{\partial \tau_{zx}}{\partial z} dz = \tau \left( z = z_{w,x} \right) - \tau \left( z = z_{b,x} \right) = \tau_{w,x} - \tau_{b,x}$$
(C.8)

Where:

 $\tau_{w,x}$  = shear stress in x direction induced by wind  $\tau_{b,x}$  = shear stress in x direction induced by friction of the bed

The wind shear stress is assumed to be 0. The bed shear stress is defined as:  $\tau_b = \rho \frac{g \|\vec{u}\| \|\vec{u}\|}{C^2}$ 

(C.9)

With

C Chézy coefficient [m<sup>1/2</sup>/s]

Integrating  $\frac{\partial \tau_{xy}}{\partial y}$  and substituting the Boussinesq assumptions yields:

$$\int_{z=z_b}^{z_w} \frac{\partial \tau_{xy}}{\partial y} dz = \frac{\partial \tau_{xy}}{\partial y} h = -\mu_t^H \frac{\partial^2 u}{\partial y^2} h$$
(C.10)

The same can be done for  $\frac{\partial \tau_{xx}}{\partial x}$  and  $\frac{\partial \tau_{yy}}{\partial y}$ . Doing this and substituting gives the shallow water equations:

$$g\frac{dh}{dx} + g\frac{dz_b}{dx} + \left[\bar{u}\frac{\partial\bar{u}}{\partial x} + \bar{v}\frac{\partial\bar{u}}{\partial y}\right] + \frac{\partial^2 u}{\partial x^2}\mu_t^H + \frac{\partial^2 u}{\partial y^2}\mu_t^H + \frac{g|\bar{u}|\,\bar{u}}{hC^2} = 0$$
(C.11)

$$g\frac{dh}{dy} + g\frac{dz_b}{dy} + \left[\bar{u}\frac{\partial\bar{v}}{\partial x} + \bar{v}\frac{\partial\bar{v}}{\partial y}\right] + \frac{\partial^2 v}{\partial x^2}\mu_t^H + \frac{\partial^2 v}{\partial y^2}\mu_t^H + \frac{g\,|\vec{u}|\,\bar{v}}{hc^2} = 0 \tag{C.12}$$

# Appendix D – Derivation of Bélanger equation with db/dx = 0 and db/dx $\neq$ 0

Starting with the momentum equation in x – direction:

$$u\frac{du}{dx} + g\frac{dh}{dx} + g\frac{dz_b}{dx} = -\frac{gu^2}{C^2h}$$
(D.1)

The velocity  $u [m/s^2]$  is a function of the discharge  $Q[m^3/s]$  and the surface area A  $[m^2]$ :

$$u = \frac{Q}{A} \tag{D.2}$$

Substituting equation 2 in equation 1 and rewriting yields:

$$Q\frac{d}{dx}\left(\frac{Q}{A}\right) = -gA\frac{dh}{dx} - gA\frac{dz_b}{dx} - \frac{gQ^2}{C^2Ah}$$
(D.3)

Q is defined as a constant value, and A is defined as a variable with variable height h[m] and width b [m]. Differentiating gives:

$$Q^2 \frac{d}{dx} \left(\frac{1}{bh}\right) = Q^2 \left[\frac{-h\frac{db}{dx} - b\frac{dh}{dx}}{b^2 h^2}\right] = -\frac{Q^2}{A^2} \left(h\frac{db}{dx} + b\frac{dh}{dx}\right)$$
(D.4)

Assuming db/dx = 0Substituting  $\frac{db}{dx} = 0$  gives:

$$Q^2 \frac{d}{dx} \left(\frac{1}{bh}\right) = -\frac{Q^2}{A^2} \left(b \frac{dh}{dx}\right)$$

Substituting this in equation 3 yields:

$$-\frac{Q^2}{A^2} \left( b \frac{dh}{dx} \right) = -gA \frac{dh}{dx} - gA \frac{dz_b}{dx} - \frac{gQ^2}{C^2Ah}$$
(D.5)

This equals:

$$-u^{2}\left(\frac{dh}{dx}\right) + gh\frac{dh}{dx} = -gh\frac{dz_{b}}{dx} - \frac{gu^{2}}{c^{2}}$$
(D.6)

With this, a relation for the slope of the water level is obtained, by setting  $\frac{\partial z_b}{\partial x} = i_b$ :

$$\frac{\partial h}{\partial x} = \frac{g i_b - \frac{g}{h} \frac{u^2}{c^2}}{g - \frac{u^2}{h}} \tag{D.7}$$

Multiplying by 
$$\frac{\frac{1}{g}}{\frac{1}{g}}$$
 yields:

$$\frac{\partial h}{\partial x} = \frac{gi_b - \frac{g}{h}\frac{u^2}{C^2}}{g - \frac{u^2}{h}} * \frac{\frac{1}{g}}{\frac{1}{g}} = \frac{i_b - \frac{1}{h}\frac{u^2}{C^2}}{1 - \frac{u^2}{hg}}$$
(D.8)

Multiplying by 
$$\frac{i_b}{i_b}$$
 yields:

$$\frac{\partial h}{\partial x} = \frac{gi_b - \frac{g}{h}\frac{u^2}{c^2}}{g - \frac{u^2}{h}} * \frac{\frac{1}{g}}{\frac{1}{g}} = \left[\frac{1 - \frac{1}{h}\frac{u^2}{c^2i_b}}{1 - \frac{u^2}{hg}}\right] i_b \tag{D.9}$$

Substituting  $u = \frac{q}{h}$  finally yields the Bélanger equation:

$$\frac{\partial h}{\partial x} = \frac{g i_b - \frac{g}{h} \frac{u^2}{c^2}}{g - \frac{u^2}{h}} * \frac{\frac{1}{g}}{\frac{1}{g}} = \left[\frac{1 - \frac{q^2}{c^2 i_b h^3}}{1 - \frac{u^2}{hg}}\right] i_b \tag{D.10}$$

Equation D.10 can also be expressed as:

$$\frac{dh}{dx} = \begin{bmatrix} \frac{h_0^3 - h_e^3}{h_0^3 - h_c^3} \end{bmatrix}$$
(D.11)

With

$$h_e = \left[\frac{q}{c\sqrt{i_b}}\right]^{\frac{2}{3}} = \text{equilibrium depth}$$
(D.12)  
$$h_c = \left[\frac{q^2}{g}\right]^{\frac{1}{3}} = \text{critical depth}$$
(D.13)

# Assuming db/dx $\neq$ 0

When db/dx  $\neq$  0, equation D4 is substituted in equation D.3. Rewriting yields:

$$-\frac{dh}{dx}\left(\frac{Q^2b}{A^2} - gA\right) = -gA\frac{dz_b}{dx} - \frac{gQ^2}{C^2Ah} + \frac{Q^2}{A^2}h\frac{db}{dx}$$

The changing water depth over x as function of the changing width is then as follows:

$$\frac{dh}{dx} = \frac{\left(A\,i_b - \frac{Q^2}{C^2Ah} + \frac{h\,Q^2}{gA}\frac{db}{dx}\right)}{A - \frac{Q^2b}{gA^2}}\tag{D.14}$$

# Appendix E – Discretization of terms in PDE for analysis of magnitude of terms

The coordinates are as shown in figure 87:



Figure 87; coordinates of each axis

 $\Delta y$ 

The discretized momentum equations in x-direction are as follows:

$$u\frac{\partial u}{\partial x} = u_i \frac{(u_{i+1} - u_i)}{\Delta x}$$
(E.1)

$$v\frac{\partial u}{\partial y} = v_j \frac{(u_{j+1} - u_j)}{\Delta y}$$
(E.2)

$$g\frac{\partial\zeta}{\partial x} = g\frac{(z_{w_{i+1}} - z_{w_i})}{\Delta x}$$
(E.3)

$$gu\frac{u}{c^2(h+\zeta)} = \frac{gu_i^2}{c^2*h_i}$$
(E.4)

$$v_t^H \left(\frac{\partial^2 u}{\partial x^2}\right) = v_t^H \frac{\left(\frac{(u_{i+1}-u_i)}{\Delta x} + \frac{(u_i-u_i-1)}{\Delta x}\right)}{\Delta x}$$
(E.5)

$$v_t^H \left(\frac{\partial^2 u}{\partial y^2}\right) = v_t^H \frac{\left(\frac{(u_{j+1}-u_j)}{\Delta y} + \frac{(u_j-u_{j-1})}{\Delta y}\right)}{\Delta y}$$
(E.6)

The discretized momentum equations in y-direction are as follows:

$$u\frac{\partial v}{\partial x} = u_i \frac{(u_{i+1} - u_i)}{\Delta x}$$
(E.7)

$$v\frac{\partial v}{\partial y} = v_j \frac{(u_{j+1} - u_j)}{\Delta y}$$
(E.8)

$$g\frac{\partial\zeta}{\partial y} = g\frac{\left(z_{wj+1}-z_{wj}\right)}{\Delta y}$$
(E.9)

$$gv \frac{v}{c^2(h+\zeta)} = \frac{gv_i^2}{c^2*h_i}$$
(E.10)

$$v_t^H \left(\frac{\partial^2 v}{\partial x^2}\right) = v_t^H \frac{\left(\frac{(v_{i+1}-v_i)}{\Delta x} + \frac{(v_i-v_i-1)}{\Delta x}\right)}{\Delta x}$$
(E.11)  
$$v_t^H \left(\frac{\partial^2 v}{\partial y^2}\right) = v_t^H \frac{\left(\frac{(v_{j+1}-v_j)}{\Delta y} + \frac{(v_j-v_j-1)}{\Delta y}\right)}{\Delta y}$$
(E.12)