DEVELOPING AND EVALUATING SIMM FORECASTING MODELS

PRIS, M (MIRAN)
RABOBANK INTERNATIONAL

List of essential abbreviations

CCR	Counterparty Credit Risk, specific type of risk arising from exposure to a counterparty by means of derivative transactions.
CIR	Cox-Ingersoll-Ross model for modelling the term structure, based on short rates.
CSA	Credit Support Annex, a legal document that regulates derivative transactions and its credit support (like collateral). It is one of the four elements of an ISDA Master Agreement.
DEV	Displaced Exponential-Vašíček, a method for modelling short rates, which is an extension of the Exponential-Vašíček method with the possibility for negative simulated rates.
EaD	Exposure at Default, a risk measure used for regulatory capital calculations.
EC	Economic Capital, amount of capital that an organization should hold in order to ensure it stays solvent given its risk profile.
EV	Exponential-Vašíček, a method to model short rates
FRA(*t, T, S, τ, N, K)*	Forward-Rate Agreement with valuation date t, expiry date T, maturity date S, year fraction τ between T and S, notional amount N, and fixed-rate K.
IA	Independent Amount. A precursor of IM, which is static (constant) throughout the lifetime of a bilateral netting set.
IM	Initial Margin, type of collateral that is bilaterally exchanged, regardless of moneyness of counterparties and segregated. The collateral is supposed to cover 99% of exposure during the MPOR.
IMM	Internal Model Method, an institution's specific internal framework. To be able to use the IMM for collateral calculation, the institution needs to obtain approval from regulators.
(P-/R-)IRS(*t, T, τ, N, K*)	(Payer-/Receiver-) Interest Rate Swap with valuation date t, a vector of payment dates T, tenors τ, notional amount N, and fixed-rate K. The final payment date and maturity date coincide.
LIBOR	London Interbank Offered Rate
MPOR	Margin Period of Risk, the time period between the last margin posting and the unwinding of the counterparty portfolio in case the counterparty defaults. In this thesis the MPOR is assumed 14 calendar days (10 business days).
MTA	Minimum Transfer Amount, a threshold that determines at which minimum level (exposure) the collateral should be posted to the other party.
MtM	Mark-to-Market, a fair-value measure of an instrument or asset. It is also known as the replacement value of the instrument or asset.
OTC	Over-The-Counter
PFE	Potential Future Exposure, measure for exposure based on a quantile of the MtM-VM distribution.
RC	Regulatory Capital, the amount of capital an organization should hold in order to survive any difficulties, such as market or credit risks.
SA	Standardized Approach, a standardized 'add-ons'-based model for calculating the amount of t_0-IM needed for a netting set.
SIMM	Standard Initial Margin Model, a standardized sensitivities-based model developed by ISDA for the calculation of t_0-IM
T-Bill	Treasury Bill.

VM Variation Margin, a type of collateral that is held only by the in-the-money counterparty in order to reduce exposure.

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Management Summary

Regulations
According to new regulatory requirements, financial market participants (financial institutions and NFC+s) are obliged to exchange collateral margins (Variation Margin and Initial Margin) when trading OTC derivatives. They have two choices to transact such trades (and post corresponding collateral): either by clearing through central clearinghouses (CCPs) and by posting collateral to these central parties or by transacting bilaterally and posting corresponding collateral to each other. Our research focuses on the bilateral margining of OTC transactions.

Variation Margin (VM) is used to cover daily changes of the bilateral netting set's value (replacement value, marked to market). In essence, when a counterparty is in the money on the bilateral netting set, then that party receives VM. This type of collateral is posted unilaterally. On the other hand, regulators introduce Initial Margin (IM), which is intended to cover the change of netting set value between the last exchange of VM and the unwinding of the bilateral netting set. This type of collateral is needed, because a counterparty may go into default and from that moment on, this defaulting counterparty will not post VM anymore to the non-defaulting counterparty. From that default moment until the unwinding of the netting set (this time period is also called the Margin Period of Risk or MPOR), the non-defaulting party is exposed. Different from VM, IM is posted by both parties to a third party segregated account.

Regulators allow for three ways to calculate IM at t_0:

- By using the Standardized Approach (SA), which is an add-on based model.
- By using an initial margin model, developed by one or both counterparties, or by a third party. The requirements for the own-developed model is that the IM should be calculated as a 99% VaR of netting set value movements over an MPOR of minimally 10 business days.
- The Standard Initial Margin Model (SIMM), which is an initiative by ISDA to standardize IM calculation based on transaction sensitivities (Greeks).

The Rabobank assesses that SIMM will become the market standard and therefore SIMM is the preferred choice for calculating IM at t_0.

Research
Next to calculating IM to cover a period between the counterparty's default today and the unwinding of the netting set following this default (we can term this as the t_0-IM), the Rabobank is interested in forecasting IM for future time points (i.e., covering the MPORs if the counterparty were to default at a future time point, we can term this future IM). The reason for forecasting IM is for future capital management purposes and product pricing purposes. We can define exposure as:

$$Exposure_t = MtM_t - VMR_t - IM_t + VMP_t,$$

where MtM_t represents the Marked-to-Market value of the netting set at time t, VMR_t represents the received VM at time t, IM_t represents the received IM at time t, and VMP_t represents the posted VM at time t.

Rabobank has a Monte Carlo simulation engine that is able to determine MtM of the netting set across time periods. Also, the Rabobank has a Brownian Bridge method that is able to accurately estimate future VM. Therefore, our research focused on developing a method to forecast future IM. As Rabobank is interested in using SIMM for t_0-IM calculations (regulatory IM calculations), the specific focus was to forecast future SIMM.

Because of the complexity of forecasting sensitivities at future time points for each simulation scenario, the Rabobank is interested in developing a method for forecasting SIMM, which performs well on approximating the true future SIMM and which is easily implementable into current Rabobank systems. However, for our research a benchmark was implemented that is based on forecasting future sensitivities and we term this method the Dynamic SIMM. The purpose is benchmarking the approximation methods only.
Subsequently, three alternative future SIMM methods were developed:

- Weighted Notional method, which approximates future SIMM by taking into account the ageing of trades within the bilateral netting set.
- Weighted MtM method, which approximates future SIMM by taking into account the ageing of trades and the MtM dynamics of the bilateral netting set.
- American Monte Carlo (AMC) method, which is based on the Longstaff-Schwartz method and approximates future SIMM by implying MtM movement distributions from local volatilities.

These three methods are benchmarked against the Dynamic SIMM method. To compare the methods and determine how well the three approximations perform, representative portfolios were used (representative for Rabobank's entire portfolio product composition).

Findings

The results for the three methods are summarized in the table below, where the plus-sign represents good performance and the minus-sign lesser performance.

	Weighted Notional	Weighted MtM	AMC
Accuracy	--	-+	+
Netting set dynamics	--	-+	+
Computational requirements	++	++	-
Implementation	++	-+	-

Our findings show that the accuracy of the three approximation methods is high when a bilateral netting set contains only interest rate receiver swaps or only interest rate payer swaps, no matter how large the bilateral netting set is or how long the residual maturity of the trades in the netting set is. The reason is that such netting sets have relatively simple dynamics (the future IM profile is mainly driven by the ageing of trades). As the Dynamic SIMM is generally seen to overcollateralize a bilateral netting set, we see that in such cases, residual exposure as measured by the Potential Future Exposure (PFE) measure is reduced to negligible levels under each IM approximation method. We see this from the top sub-figure in Figure 1-1.

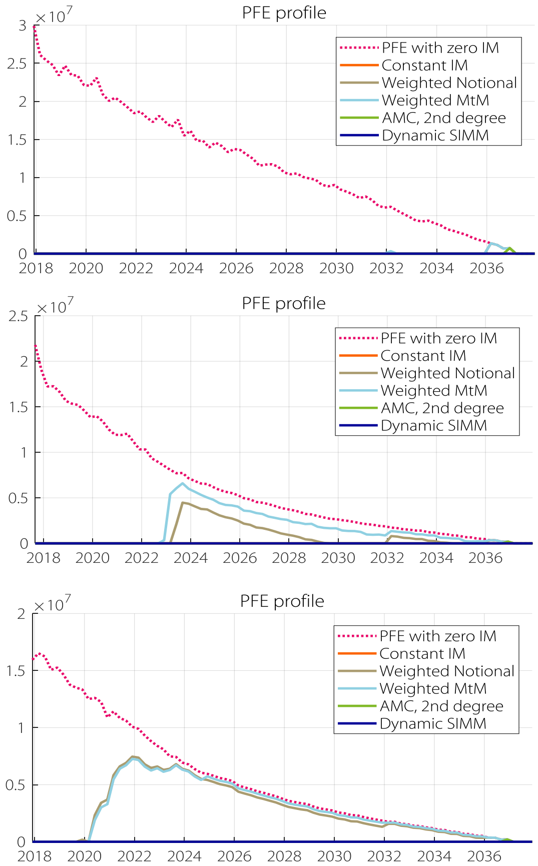

Figure 1-1. Impact of IM approximation method on Potential future Exposure (PFE) for an example bilateral netting set. From top to bottom the sub-figures show an increasing complexity of sensitivity dynamics within a bilateral netting set. We see this development for each experimental netting set.

As soon as the dynamics of the bilateral netting set become more complex (i.e., possibly offsetting sensitivities), the less the two weighted methods are able to track the benchmark, due to their incapability of capturing these dynamics well. Consequently, we see that the PFE profiles show increasingly residual exposure (the middle and bottom sub-figures in Figure 1-1). From a regulatory perspective, the Rabobank is more conservative when a non-zero residual exposure is assumed.

Under the more complex sensitivity dynamics conditions, the Weighted MtM method performs slightly better than the Weighted Notional method, due to the incorporation of the net-to-gross MtM ratio in the Weighted MtM method. This ratio has the capability to incorporate the netting set dynamics to some extent. However, for the effect of this ratio to become more profound and subsequently for the Weighted MtM method to track the Dynamic SIMM better, development of a multiplier on this net-to-gross MtM ratio should be considered. Also, some improvements on the used add-ons could be performed in order to improve performance of the Weighted MtM method. With these improvement possibilities, the Weighted MtM method is recommended for implementation within Rabobank.

The AMC method on the other hand has potential to track the benchmark almost perfectly under more complex netting set dynamics as well. It must be noted, however, that the AMC method was scaled with a scaling function that improves the performance of the AMC significantly. However, due to the specific Brownian Bridge that the Rabobank has currently, there should be some adaptions in order to implement the AMC method. This puts a burden on the implementation procedure and therefore, the AMC method is less suited for the Rabobank with their current systems.

1 Introduction

The weaknesses of the global financial system were exposed during the financial crisis of 2007-2009. Especially the financial linkages and interdependence between large financial institutions were highlighted negatively. The underlying risk in these interconnections was mostly posed by the over-the-counter (OTC) derivatives market. The domino-effect that regulators feared, was almost realized after large counterparty exposures accumulated between market participants. Additionally, the risk stemming from such opaque and extremely complex financial interconnections was subject to risk management practices that were not mature enough to deal with this level of complexity. It went unnoticed that such constructs allowed for a build-up of extreme leverages. After September 2008, a different view towards counterparty credit risk and opaque multilateral constructs assisted by OTC-derivatives, was taken. The rest of this chapter explains briefly the regulatory reforms of the OTC derivatives market as a response to the financial crisis events and the outcomes of these reforms, which are shaped mainly into central counterparty clearing obligations and collateral margin regulatory frameworks.

1.1 Regulatory reforms

Some regulatory measures taken before the financial crisis prevented to a large extent that the feared domino-effect became reality. After the bankruptcy of Lehman Brothers, many of its counterparties were protected against the default due to collateral that Lehman Brothers posted under the regulatory requirements at that time. Additional collateral was posted by Lehman Brothers for the clearing of more than 60,000 OTC derivative contracts at LCH.Clearnet, thereby further dampening the financial default shock wave. After events like the Lehman bankruptcy, the attitude towards counterparty credit risk changed as the financial community realized the importance of this type of risk and the mitigation thereof.

The potential for systemic risk generated by the OTC derivatives market raised the interest of regulatory bodies and led to discussions at the G20 Summit in Pittsburgh on September 26th 2009 and subsequently to the development of regulation towards minimizing this type of risk. From the end of 2012 onwards, regulators required financial market participants to trade standardized OTC derivative contracts on exchanges or electronic trading platforms and that such transactions should be cleared through central counterparties. Additionally, OTC derivative contracts should be reported to trade repositories. Other derivative contracts, which are not centrally cleared became subject to higher capital requirements (European Union, 2012).

From that point on, the OTC derivatives market has been faced with two general requirements as set out by regulators. This main principle is shown in Figure 1-1. The **first requirement** is that as much as possible of the OTC derivatives contracts should be cleared by central counterparty clearing houses (CCPs). *A CCP is an independent legal entity that interposes itself between the buyer and the seller of a derivative security* (Cechetti *et al.*, 2009, pp. 45-46). When an OTC derivatives contract is cleared through a CCP, it is split into two contracts. Namely, between the CCP and each of the two initial counterparties. Instead of having each other as a counterparty, the buyer and seller now have the CCP as their counterparty. There are three benefits related to this structure (Cechetti *et al.*, 2009): the first benefit is that counterparty credit risk becomes better managed. Secondly, the CCP is able to manage the payments and perform multilateral netting of exposures. From these two benefits, the third benefit arises in form of an increased transparency on market activity and exposures. These benefits serve both the public and the regulators. However, the CCPs must be well-capitalized in order to clear OTC derivative contracts and be able to net exposures. This required capitalization comes from the exchanging contract parties and is posted to the CCPs in the form of initial and variation margin.

The **second general requirement** imposed by regulators is related to non-centrally cleared OTC derivative contracts (i.e., contracts for which it is not possible to clear them through a CCP, because of their specific character). For such non-centrally cleared OTC derivative contracts, counterparties are required to post initial and variation margins to each other in order to replicate the effect of CCPs' risk exposure management and the netting of exposures. This is termed bilateral margining, which is the focus of this thesis.

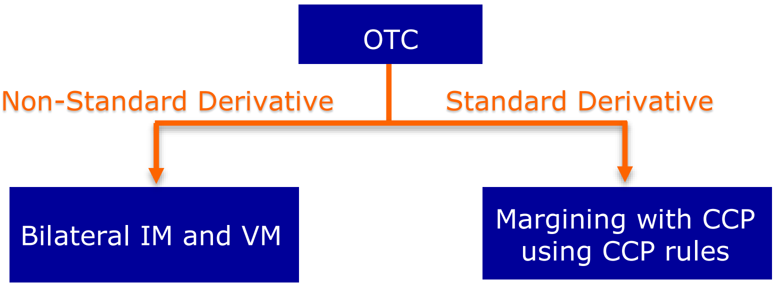

Figure 1-1. The main principle of the new regulations regarding OTC derivatives. The OTC derivatives should be cleared as much as possible through CCPs. This is possible for standard OTC derivatives. When the transaction concerns a non-standard derivative contract, then the transaction shall be bilaterally collateralized. This figure is adapted from O'Kane (2016).

1.2 Problem setting

Within this new regulatory context, Rabobank is in need of modelling future initial margin (IM) for bilaterally collateralized OTC trade netting sets to be able to determine counterparty credit risk over the entire lifetime of a bilateral netting set. Because Rabobank assesses that ISDA's upcoming Standard Initial Margin Model (SIMM, a sensitivity-based model) will become the market standard for calculating IM, the future IM forecast should be based on SIMM. Therefore, the main problem statement is:

How can the Rabobank forecast future IM based on SIMM in the most effective way?

This problem can be broken down into the following sub-questions:

- *RQ1) What are the characteristics and regulatory requirements related to bilateral collateral exchange?*
- *RQ2) What are the challenges for Rabobank in implementing a SIMM forecasting methodology?*
- *RQ3) How is Potential Future Exposure defined?*
- *RQ4) What are alternatives for modelling future SIMM?*
- *RQ5) What is the most effective model?*

1.3 Scope of research

Our research will focus on developing an effective method for forecasting future SIMM. The scope of research regarding this goal is to first develop a benchmarking model that will forecast SIMM based on future sensitivities (this future sensitivities model will not be implemented due to its computational infeasibility and serves merely as a benchmark, see discussions in the next chapters) and to subsequently develop approximation models for future SIMM from which the performance will be benchmarked against the benchmark model. From the approximation methods, the most effective method based on performance will be selected as a potential candidate for implementation at Rabobank. In determining the effectiveness of the forecasting methods, the impact of each method on the potential future exposure (PFE) will be used as graphical representation of the method performance. Rabobank uses the PFE for limit management purposes (i.e., to control counterparty future exposures).

In order to be able to perform future SIMM simulations in an isolated environment (isolated from Rabobank's daily production environment), the author created a Matlab experimental environment for this research that is capable of simulating netting set MtM valuations in the same way as Rabobank's current Monte Carlo simulation engine. As the main product in Rabobank's counterparty portfolios is the vanilla interest rate swap, the scope of our research will be experimental netting sets containing only this linear product. The Matlab experimental environment is therefore also confined to vanilla interest rate swaps (and also forward rate agreements[1]). Furthermore, the experimental bilateral netting sets will be single-currency (EUR), for simplicity purposes facilitating analysis.

Finally, the thesis focuses mainly on the regulatory frameworks and regulations that apply to the EU (EMIR). As mentioned, the focus of this work are the requirements related to non-centrally cleared OTC derivative transactions. Transactions that clear through CCPs are out of scope.

1.4 Document Structure

The document structure is as follows. Chapter 2 discusses the characteristics of bilateral margining and some main regulatory points related to this type of margining, thereby answering *RQ1*. Chapter 3 treats the challenge related to forecasting future IM and focuses on answering *RQ2*. Chapter 4 discusses how exposure is modelled and how collateral plays a role in it. By defining PFE in this chapter, we are answering *RQ3*. Chapter 5 outlines the benchmark model and the three approximation models and the reasoning behind the models to answer *RQ4*. In Chapter 6, we provide the research setup and the experimental portfolios, which are employed in our Matlab environment to come up with the research results provided in Chapter 7. Ultimately, we answer research question *RQ5* and our overall research problem in Chapters 8 (Discussion) and 9 (Conclusions) and we finalize the thesis with future research recommendations in Chapter 10.

[1] Interest rate swaps can be seen as a cascade of multiple forward rate agreements. Therefore, being capable of simulate MtM valuations for interest rate swaps implies being capable of simulating MtM valuations for forward rate agreements. See also Section B in the Appendix.

2 Bilateral Margining and Regulations

The Working Group on Margin Requirements (WGMR) -a joint initiative of the Basel Committee on Banking Supervision (BCBS) and the International Organization of Securities Commission (IOSCO)- have put efforts into developing a framework that addresses the margin requirements related to non-centrally cleared OTC derivative contracts (ISDA, n.d.).This framework initiative is being translated into law by the individual regulatory authorities across jurisdictions. At this point, the US and EU authorities are implementing regulatory acts for these margins. In the US, such margin requirement regulations fall under the Dodd-Frank Act Title VII and are developed by the US Commodity Futures Trading Commission (CTFC) (Miller & Ruane, 2013). In the EU, these regulations are established by the European Market Infrastructure Regulation (EMIR) (O'Kane, 2016).

As mentioned above, many counterparties were protected to some extent from the Lehman Brothers' bankruptcy due to the fact that there were already some margin requirements present at that time. Requirements regarding the initial margin were existent as so-called independent amounts (IA) and fell under the Credit Support Annexes (CSAs), which were part of ISDA Master Agreements applicable to OTC derivative contracts. ISDA Master Agreements are the legal foundation of the OTC derivatives market and a CSA defines the overall amount of collateral that counterparties must deliver to each other, based on the specifics of the trade contract. The IAs were already part of these CSAs since *the earliest days of the collateralized OTC derivative market, which dates back to the late 1980s* (ISDA, 2010, p. 6). Although more formally applicable to exchange traded derivatives than OTC derivatives, requirements on the variation margin were always part of the Credit Support Balance in the CSAs.

So, if the margin requirements already existed in some form, why was the bankruptcy of Lehman Brothers leading to financial distress at their counterparties and at the counterparties of their counterparties? The main issue at that time was that collateral received could be rehypothecated by the receiving party, as is the case with variation margin (discussed below). When that party defaults, counterparties who (over-) collateralized their exposure to the defaulting party will then see a recovery rate on their posted collateral of (significantly) less than 100%. This leaves the counterparties unsecured, potentially putting them at financial distress and triggering a domino-effect of defaults. Regulators now focus on redefining the variation margin collateral and on this rehypothecation issue and pose that next to a variation margin, an initial margin should be exchanged (bilaterally) for non-centrally cleared OTC derivative transactions and this collateral must be segregated at a third party account, thereby limiting rehypothecation to very specific conditions.

In the EU, shaping of the procedures for bilateral collateralizing of OTC trades comes from the BCBS and IOSCO initiative (Basel Committee on Banking Supervision, 2015) that led to the establishment of Article 11(15) of Regulation (EU) No. 648/2012 (EMIR)[2] and the subsequent framework that implements that Article (EIOPA, EBA & ESMA, 2016). As mentioned in the Introduction, the focus of our research is on the OTC market reforms' collateral requirements. One of the key principles and requirements under the new regulations is that "*all financial firms and systemically important non-financial entities ("covered entities") that engage in non-centrally cleared derivatives must exchange initial and variation margin as appropriate to the counterparty risks posed by such transactions*" (Basel Committee on Banking Supervision, 2015, p. 5). The scope of the new margining requirements are financial counterparties and so-called NFC+ (non-financial counterparties with outstanding derivative transactions with a gross notional amount exceeding €1 billion for credit and equity derivatives or €3 billion for interest rate, FX, and commodity transactions). Under the new EMIR collateral requirements, the NFC+ counterparties have the same obligations as the financial counterparties. Below, the specifics of regulations for the two types of collateral will be discussed for the EU jurisdiction.

[2] Regulation (EU) No. 648/2012 of the European Parliament and of the Council of 4 July 2012 on OTC derivatives, central counterparties and trade repositories.

2.1 Variation Margin

Under the new regulations the goal of VM is to provide the in-the-money party with sufficient collateral to cover the contract loss in case the out-the-money counterparty defaults, under the assumption that the defaulting counterparty provides a recovery rate of zero. VM is exchanged with a relatively high frequency (e.g., daily). First, netting is applied over all the contracts between the same two counterparties. Then, the netting value is determined based on market valuation. If this contract netting market value makes a counterparty in-the-money, then that party is protected. Thus, the VM must cover the maximum (end-of-day) loss that an in-the-money party can face due to the default of the counterparty. The mechanics related to VM are then such that the in-the-money counterparty is always holding the VM collateral and is allowed to rehypothecate this collateral. As soon as the market value of the contract shifts in favour of the other party (i.e., the in-the-money counterparty becomes out-the-money), then the VM collateral changes party.

The amount of VM collateral depends on factors such as the creditworthiness of parties (through Credit Valuation Adjustments (CVAs) on the mutual contract portfolios) and mutual contract portfolio discounting factors. Also, there may be subjectivity in determining the market value of some trades in the netting set, due to their character (like exotics or illiquid trades). This all may lead to a different valuation of the netting contracts by the two counterparties involved, thereby providing potential for disputes. For such dispute purposes, BCBS and IOSCO state that covered entities "*should have rigorous and robust dispute resolution procedures in place with their counterparty before the onset of a transaction*" (Basel Committee on Banking Supervision, 2015, p. 15).

Furthermore, a minimum transfer amount (MTA) can be agreed between counterparties. The regulators set the maximum MTA at €500,000 to enhance operational practicality. There is also a possibility for a threshold that allows for a maximum amount of VM uncollateralized exposure. This threshold can be agreed upon in the CSA between the counterparties. No VM has to be posted until the threshold amount is reached. When the exposure exceeds this threshold, the difference between the exposure and the threshold is posted to the in-the-money counterparty. However, upcoming regulations for VM require a zero threshold. Because of the one-way posting of VM, the overall market liquidity is not affected. The liquidity shifts from the posting counterparty to the receiving counterparty, creating a liquidity zero sum.

The receiving party does not have to segregate the received VM and can rehypothecate it in any way it likes. The possibility of rehypothecation of VM poses some risk to the VM posting counterparty when the counterparty that has received VM, defaults. As the netting set value increases from the non-defaulting party's perspective following the default, the only way for the non-defaulting counterparty to get its posted VM back is to enter into a bankruptcy claim process. The non-defaulting counterparty now faces a recovery rate on getting back the VM. That recovery rate is likely to be less than 100%.

There is a phase-in time period for the VM regulatory requirements. From 1 September 2016, the VM requirements apply to covered entities that have an aggregated month-end average notional amount of more than €3.0 trillion in non-centrally cleared derivatives for the months March, April, and May 2016. This applies only if the counterparty meets those criteria as well. From 1 March 2017, the new VM regulatory requirements apply to all covered entities, regardless of the average aggregated notional amounts of non-centrally cleared derivatives in the netting set. The phasing of VM is summarized in Table 2-1.

Table 2-1. The phasing of VM regulatory requirements.

Date of implementation	VM specifics
1 September 2016	Every non-cleared OTC contract that is executed after 1 September 2016 between two covered entities, both with an average notional amount exceeding €3.0 trillion in non-centrally cleared transactions across March, April, and May 2016, is under VM margining requirements.
1 March 2017	From this date onwards, all covered entities are required to exchange VM.

2.2 *Initial Margin*

The IM collateral, on the other hand, is used to cover losses in the case the non-defaulting party faces a replacement cost of the joint contracts that is higher than the latest contract valuation (and thus exceeds received VM) that took place before the default of the counterparty. So, in the time between the latest valuation (and receipt of VM) and default of the counterparty, market movements may have increased exposure of the non-defaulting party and now this higher potential loss is uncovered by previously posted VM collateral. The time period between the last receipt of VM and the closing out of the bilateral netting set is called the Margin Period of Risk (MPOR). In contrast to VM, IM is posted two-way. Whether the non-defaulting party is in-the-money or out-the-money, it has received IM. When a counterparty defaults, it must return received IM to the non-defaulting party in full. This is possible, because IM is segregated. In case the loss occurs to the defaulting party, both counterparties receive back their IM collateral. The principle here is that the defaulting party pays the contract replacement costs (O'Kane, 2016).

IM collateral is posted by both counterparties to a third-party, custodian account so that IM is segregated and can only be rehypothecated under specific conditions. These conditions occur in the case when a corporate customer or non-financial company that post IM give permission to the IM receiving party that the collateral can be rehypothecated and only for hedging purposes of their joint netting set. In such cases, the receiving party with the rehypothecation permission, must inform their customer that the IM is rehypothecated and the amount that has been rehypothecated. Even when permission for rehypothecation is granted by the customer, the counterparty is responsible for protecting the rights of their customer.

Furthermore, there is a threshold for IM posting, set out by regulators. The maximum IM uncollateralized exposure that is allowed, is set at €50 million. When exposure builds up to this amount, no IM has to be posted. When the threshold is exceeded, the difference between the exposure and the threshold is posted as IM. Counterparties may agree a different threshold in the CSA, as long as it does not exceed €50 million. Next to the IM threshold, there is an IM minimum transfer amount. This amount is set at a maximum of €500,000. Again, the counterparties are able to agree on a different minimum transfer amount, as long as it does not exceed €500,000. To avoid the possibilities of counterparties creating new entities in order to use the thresholds multiple times and build excessive uncollateralized exposures, the regulators have set the thresholds to apply on a consolidated group basis.

An important remark to make on the scope of transactions that fall under the IM margining requirements is that some foreign exchange (FX) transactions are treated differently from the other non-centrally cleared derivatives. One type of product that is exempted from the margining requirements are physically settled FX transactions without optionality. This means that FX forwards and swaps exchanging notional amounts do not fall under the margining requirements. Additionally, cross-currency swaps are also treated differently. Because a cross-currency swap can be theoretically decomposed into a physical FX transaction and a series of interest rate payments, the FX leg of the trade does not fall under the IM margining requirements.

As with the VM requirements, there are phase-in time periods for IM collateral requirements. The phase-in is related to the outstanding notional amount of non-centrally cleared transactions. The timeline for IM introduction is given in Table 2-2. It should be noted that the outstanding notional amounts of FX transactions, which are exempted from IM margining, are considered when determining the thresholds for phase-in of the IM requirements. Unlike with VM, there is no liquidity zero sum with IM posting. Exchange of IM collateral will have an impact on the overall liquidity of the financial markets.

Table 2-2. The phasing of IM regulatory requirements.

Date of implementation	IM specifics
1 September 2016 – 31 August 2017	All covered entities with outstanding non-centrally cleared transactions with a monthly notional amount exceeding €3.0 trillion are required to exchange IM.
1 September 2017 – 31 August 2018	All covered entities with outstanding non-centrally cleared transactions with a monthly notional amount exceeding €2.25 trillion are required to exchange IM.
1 September 2018 – 31 August 2019	All covered entities with outstanding non-centrally cleared transactions with a monthly notional amount exceeding €1.5 trillion are required to exchange IM.
1 September 2019 – 31 August 2020	All covered entities with outstanding non-centrally cleared transactions with a monthly notional amount exceeding €0.75 trillion are required to exchange IM.
1 September 2020 – 31 August 2021	All covered entities with outstanding non-centrally cleared transactions with a monthly notional amount exceeding €8.0 billion are required to exchange IM.

2.2.1 *Calculating Initial Margin at t_0 (the regulatory IM)*

Even though the focus of our research is on forecasting future IM collateral (based on SIMM), the first step is to determine IM at t_0 (with which the forecasted IM at t_0 should reconcile, more on this in subsequent chapters). For calculating this regulatory IM at t_0, regulations (EIOPA, EBA & ESMA, 2016) allow three approaches:

- The Standardized Approach (SA), which is based on add-ons.
- Own developed initial margin model (based on Internal Model Method) which is developed by one or both counterparties, or by a third party. This model shall be based on a one-tailed 99 percent confidence interval over an MPOR of at least 10 business days. Furthermore, the calibration of the model should use at least three years of data and not exceed five years of data, from which at least 25% should be stressed data.
- The Standard Initial Margin Model (SIMM), which is an initiative by ISDA to provide financial market participants with a standardized sensitivity-based initial margin model. This model is based on a one-tailed 99 percent confidence interval over an MPOR of 10 business days and is calibrated by using three years of data from which one year is stressed data.

As mentioned, Rabobank's preferred choice of the three models for calculating IM at t_0 is SIMM[3]. At this point, the SIMM as a method for calculating IM is still under review in Europe. In the US and Japan, the method went live. Because of these two globally large financial powers and global trading, it will be a matter of time before the SIMM becomes a global standard for initial margin calculation. Being at the forefront and getting the SIMM implementation right can be a major business opportunity for financial institutions.

2.2.2 Allowed collateral types

For VM and IM collateral to reduce counterparty credit risk during adverse times, it is necessary for the collateral to be stable in value even during times of financial stress. Regulators want to avoid that the collateral used is correlated to the default of a counterparty. To reduce such correlation risk, the collateral should be the most highly liquid assets. However, there is a trade-off between the liquidity of collateral and the overall market liquidity. Due to the fact that IM is segregated after posting, the overall liquidity of the financial markets is reduced because large numbers of highly liquid assets are taken out of circulation. To reduce such a strain in liquidity, regulators have determined a variety of eligible collateral. Compensating for liquidity of the eligible collateral, so-called collateral haircuts to the asset's value are introduced for each type of collateral. The haircuts are presented in Table 2-3 (Basel Committee on Banking Supervision, 2015). The table shows that it is possible to hold assets as collateral with a different underlying currency than the one of the trades in the netting set. In such cases, the haircut is increased with 8% of the asset's market value (last row in Table 2-3).

Table 2-3. Standardized haircut schedule.

Asset Class	Haircut (% of market value)
Cash in same currency	0
High-quality government and central bank securities: residual maturity less than one year	0.5
High-quality government and central bank securities: residual maturity between one and five years	2
High-quality government and central bank securities: residual maturity greater than five years	4
High-quality corporate\covered bonds: residual maturity less than one year	1
High-quality corporate\covered bonds: residual maturity greater than one year and less than five years	4
High-quality corporate\covered bonds: residual maturity greater than five years	8
Equities included in major stock indices	15
Gold	15
Additional (additive) haircut on asset in which the currency of the derivatives obligation differs from that of the collateral asset	8

[3] A comparison between calculations for IM at t_0 under SIMM and Standardized Approach is presented in Appendix A for an example netting set.

3 Challenges for Rabobank

Rabobank has a comprehensive risk management philosophy that has led to an integrated market and counterparty credit risk policy, control and reporting framework. Within that integrated framework, Rabobank has a Counterparty Credit Risk (CCR) sub-framework that is geared towards the analysis and quantification of counterparty credit risk. Within CCR, Monte Carlo analysis is the essential technique to analyse and quantify market (risk) factor movements dynamically and their impact on the financial derivatives portfolio that the bank holds. Based on these movements, Rabobank is able to compute:

1. The Potential Future Exposure (PFE), which is a 97.5 percentile of the distribution of exposures[4] at a given time point. This risk measure is used to check whether a trade with a counterparty can be executed within set limits (i.e., limit management).
2. Exposure at Default (EaD), which is a risk measure used for regulatory (RC) and economic capital (EC) calculations.

As mentioned in the first chapter, a graphical representation of performance of the SIMM forecasting methods will be provided by showing the impact of the methods on the PFE profile. At this point, the Rabobank's CCR framework includes a precursor of IM (Independent Amount), which is modelled as a linear balance throughout the lifetime of a bilateral netting set. In order to see the impact of new IM regulations on exposure profiles, more accurate modelling is needed than a linear model.

3.1 Forecasting future IM

Figure 3-1 shows the difference between IM at t_0 (the new regulatory requirement) in subfigure A, and the future IM in subfigure B if we were to use the IMM to model future initial margin (as mentioned previously, we not use IMM, but we extend SIMM. The reason for showing the figure is merely to illustrate the complexity when IMM would have been used).

In Figure 3-1A, we see a portfolio value movement over the MPOR period depicted as scenarios and starting at t_0. The 99th percentile Value-at-Risk (VaR) of the portfolio value movement over the 10 business day MPOR period, based on these value movement scenarios, represents the regulatory IM (i.e., the IM that our counterparties should post to us in order to cover our 99th percentile value change of the portfolio at t_0). In Figure 3-1B, we see that the MPOR period does not start at t_0, but at a future time point t. This means that we are forecasting IM in the case the counterparty defaults at time point t. With this approach a situation would be introduced where nested scenarios arise.

Simulating thousands of value movement scenarios for large and complex portfolios burdens computational resources, let alone simulating nested scenarios for such cases. Simulating future SIMM accurately would require simulation of future sensitivities within the mentioned nested model. Due to the complexity and computational infeasibility, Rabobank has decided not to base their SIMM forecasting on IMM, but rather to use an approximation method. The challenge lies in developing an approximation method that is performing well in terms of precision of the future SIMM forecast, while at the same time being an easy to implement method. The challenge can be summarized as an accuracy versus complexity trade-off.

[4] Exposure is defined as:
$\text{Exposure} = \max(MtM, 0)$, see next chapter.

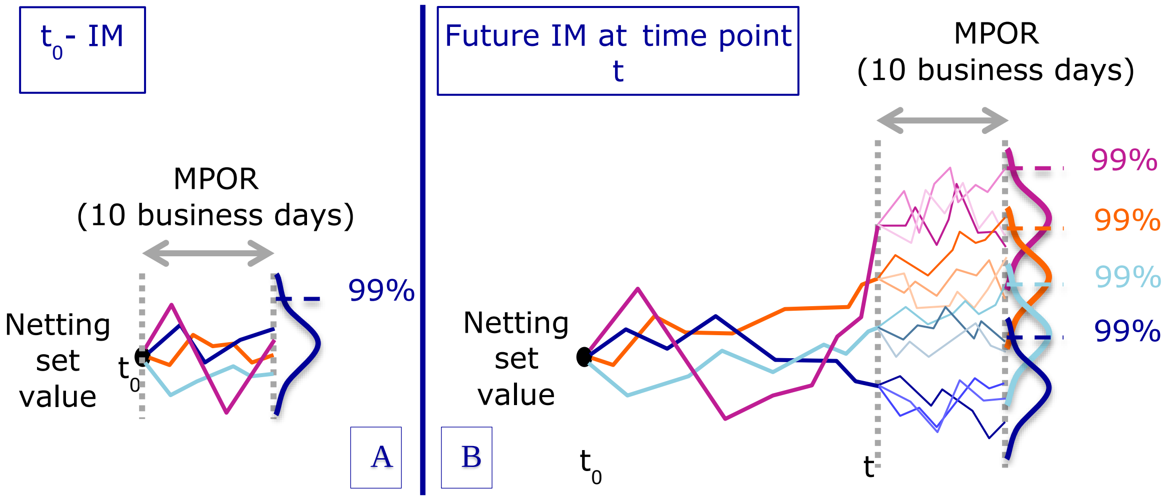

*Figure 3-1. **A**. Netting set market value movement over the MPOR period when MPOR starts at t_0 (i.e., when the counterparty defaults at t_0. This is the regulatory IM).*
***B**. Netting set market value movement between t_0 and t and the subsequent market value movement during the MPOR period when the MPOR period starts at t (i.e., the counterparty defaults on time point t).*

4 Modelling Exposure

In this chapter, the process of modelling counterparty exposure through time will be outlined. Before we proceed to the modelling process, the exposure and how it is defined will be explained first.

4.1 Potential Future Exposure (PFE)

Generally, exposure can be defined as:

$$Exposure = \max(MtM, 0)$$

It is the positive value of the netted trade values in the counterparty portfolio or the so-called *netting set*. So, to be able to quantify exposure, the MtM of each individual trade in the portfolio throughout time has to be simulated first and then netted. The positive part of the resulting distribution of netted MtM represents the exposure. Figure 4-1 shows the future exposure based on simulated evolutions (scenarios) of a netting set. For our research, the exposure will be quantified by a 97.5% Potential Future Exposure (PFE). The 97.5% PFE level is also shown in Figure 4-1. The formal definition of the uncollateralized PFE is:

$$PFE_{t,x\%} = (\max(MtM_t, 0))_{x\%}$$

where MtM_t is the fair value of the netting set and x represents the quantile level (in this case 97.5%). With this level (quantile) of the PFE, it means that the PFE will be exceeded in 2.5% of the scenarios at each time point that the distribution is considered. However, the size of the loss within that 2.5% can be infinite.

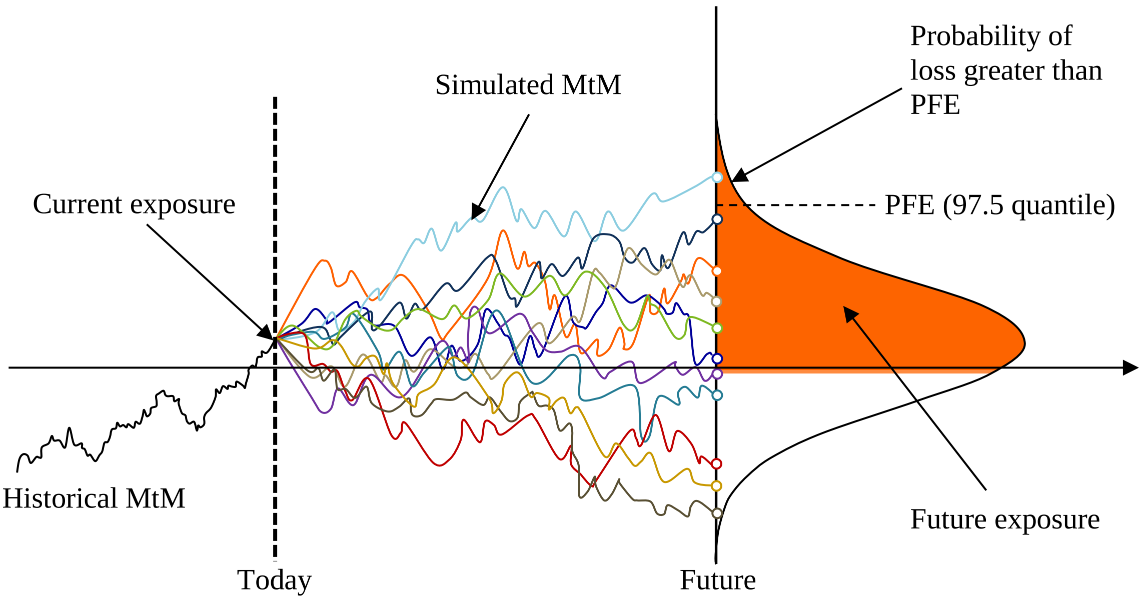

Figure 4-1. Showing a graphical representation of the simulated MtM future evolutions (scenarios) of a netting set and the resulting future exposure (shaded area on the distribution graph. Also, the 97.5 quantile PFE is shown.

Received collateral (like VM) is used to reduce the exposure. The *collateralized* PFE is expressed as:

$$PFE_{t,x\%} = (\max(MtM_t - C, 0))_{x\%}$$

where C represents the collateral balance, existing of VM and IM. To be able to measure future exposure, it is necessary to model future collateral. An important aspect in modelling collateral is the Margin Period Of Risk (MPOR).

4.2 Margin Period Of Risk (MPOR)

Between a margin call (a call to the counterparty for posting collateral) and the actual receiving of the collateral, some time may pass due to a dispute for example. Disputes related to collateral posting may arise, because it is very likely that both parties have different methods of calculating the collateral, thereby increasing the probability for a mismatch between the margin call and the amount of collateral the called-upon party believes it should post. During this time between the last posting of collateral and the dispute settlement, the party that is entitled to collateral may see a rise of the portfolio value at their side and thereby face an increased exposure. This settlement time period is also termed MPOR. Another example that may invoke an MPOR is a default of the counterparty. When the counterparty goes into default there will be an MPOR between their last collateral posting and the re-hedging of the portfolio by the non-defaulting counterparty. The MPOR can be depicted graphically as is done in Figure 4-2. In line with the Rabobank's Counterparty Credit Risk (CCR) framework, the length of the MPOR is defined to be 14 calendar days (or 10 business days). For the remainder of this thesis the 14-day MPOR will be used.

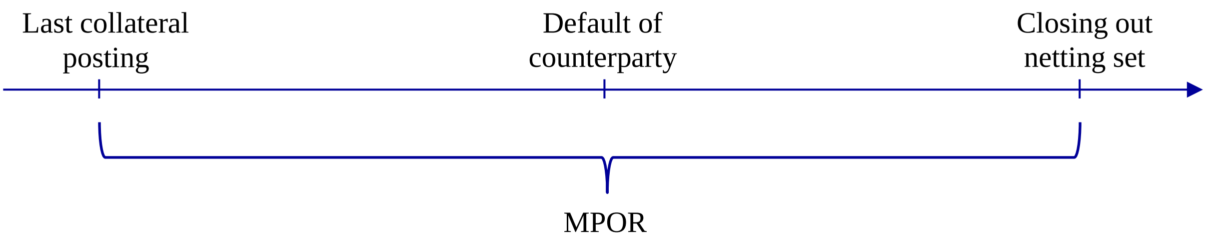

Figure 4-2. Timeline representing the Margin Period Of Risk (MPOR).

4.3 Collateral Modelling

Generally, the collateral balance at time t is a function of MtM at time t-$MPOR$. Furthermore, to increase operational convenience there are two types of thresholds related to collateral posting, which are taken up in the CSAs between counterparties.

4.3.1 Exposure threshold

The first type of threshold is a limit on how much unsecured counterparty exposure can be built up before collateral posting is required. For example, if the threshold is set at 5 million by a counterparty, the exposure of 4 million at that party will not lead to a margin call from that party to its counterparty. Only when the exposure has accumulated to a value equal to or larger than 5 million, collateral calls will be made. In practice, it is very likely that two counterparties have different thresholds.

4.3.2 Minimum Transfer Amount (MTA)

The second type of threshold is a minimum limit on the amount of collateral to be posted when the exposure threshold has been reached. This threshold is called the Minimum Transfer Amount (MTA). There are some rounding rules applicable to the MTA, which are specified in CSAs. Like the exposure threshold discussed above, the MTA can be different for both counterparties.

4.3.3 Collateral balance

The collateralized counterparty exposure can generally be denoted as:

$$Exposure = \max(MtM - VM - IM, 0)$$

where *IM* represents the received IM. From this the 97.5% PFE at each time point *t* follows:

$$PFE_{t,97.5\%} = (\max(MtM_t - VM_t - IM_t, 0))_{97.5\%}$$

So, to find the exposure we need to find the evolution of the collateral balance $VM_t + IM_t$ throughout time. This expression makes again clear the purpose of modelling future IM; otherwise it is not possible to obtain the PFE. At this point, Rabobank does not have a model to be able to model IM throughout time (future IM). As part of our research, specific alternatives are presented in Chapter 5.

4.3.4 Obtaining the MtM at non-simulation dates

Next to the IM_t-term, the $MtM_{t\text{-}MPOR}$-term in Equation **Error! Reference source not found.** is not directly available by sampling the MtMs at the simulation grid dates. In other words, the $MtM_{t\text{-}MPOR}$-term is not a simulated point, unless the simulation grid is extremely granular. This is not possible due to computational restraints. With a coarsely grained simulation grid, the $MtM_{t\text{-}MPOR}$-term has to be interpolated between the simulated points in some way. The interpolation technique used is the Brownian Bridge. The Brownian Bridge gives a conditional probability distribution of a Wiener process $W(t)$, for which values at the points $t=0$ and t are set. We can envision these points to be the points between $t=0$ and the simulation date t. Then, the Brownian Bridge provides a normal distribution from which we are able to sample the $MtM_{t\text{-}MPOR}$-terms.

5 Future SIMM forecasting model alternatives

This chapter will discuss considered alternative models for forecasting future SIMM. Generally, a distinction can be made between three types of models that have potential to this end (Gregory, 2015) mentions these types for quantification of credit risk exposure, so they are equally well applicable to forecasting future SIMM):

- Parametric approaches (for example, add-on based approximations).
- Semi-analytical methods (approximating distributions based on some conditions, like the values of MtM at specific points or the values of risk drivers at specific points).
- Monte Carlo simulation of future SIMM (the most complex and computationally heavy methods, but highly accurate).

It is clear from the three model categories that there is a trade-off between high computation and time consumption, and lower accuracy in forecasting. To be able to evaluate the performance of the developed alternatives for forecasting future SIMM, it is necessary to first develop a benchmark to which the performance of the approximation models can be compared. To obtain the best possible accuracy, the benchmark model will be one of the Monte Carlo simulation category. The benchmark model is not considered for implementation at the Rabobank. It is merely used for development and evaluation of the approximation models. Next to the benchmark model, we develop two models that use a parametric approach to forecast future SIMM, and one model that fits the semi-analytical methods category.

Additionally, there are some requirements for the alternative IM forecasting models (Anfuso. *et al.*, 2017):

Req1. The IM at t_0 should reconcile with the regulatory required IM at t_0 as obtained by the SIMM method (i.e., $IM(path_s, t_0) = SIMM(t_0)\ \forall s$, see Chapter A for the t_0-IM calculation).

Req2. The calculated IM should segregate trades from different asset classes, so there should be no netting benefit across asset classes (i.e., $IM(path_s, t) = \sum_{k=1}^{K} IM_k(path_s, t)\ \ \forall k\ and\ s$, where k represents the asset class) (Basel Committee on Banking Supervision, 2015).

Req3. To minimize computational burden, the future SIMM forecasting model should use as inputs the simulation outcomes from currently available systems.

5.1 Dynamic SIMM – the benchmark model

The first model to be treated is the model that will be used as the benchmark for the approximations. This model is based on SIMM. The difference between the t_0-value obtained from SIMM and the future-t SIMM-values lies in the use of dynamic sensitivities. So, where the t_0 SIMM-value is obtained by using today's sensitivities, we plug in simulated sensitivities at future time points into SIMM to obtain the future SIMM forecast. This model will be called the Dynamic SIMM throughout the rest of this thesis.

Since we have established that the IRS is the scope of analysis for our research, the sensitivities of interest will be deltas. To obtain the dynamic sensitivities, the zero curve is shocked at 9 different tenor points by 1 basis point. The shock is applied alternately at each tenor point (i.e., a nonparallel shift is performed) and the corresponding change in trade value is quantified. The shocking procedure of the zero curve is depicted in Figure 5-1. This procedure was first described by (Reitano, 1992) and the partial sensitivity is called the partial duration. In the figure it can be seen that the 5Y tenor point is shifted by 1 basis point. From this shock we obtain the partial duration of a trade for the 5Y tenor point on the zero curve. Then, the DV01 of a trade at each future time point, for each scenario can be obtained from the partial durations through sommation over all tenor points (Hull, 2015).

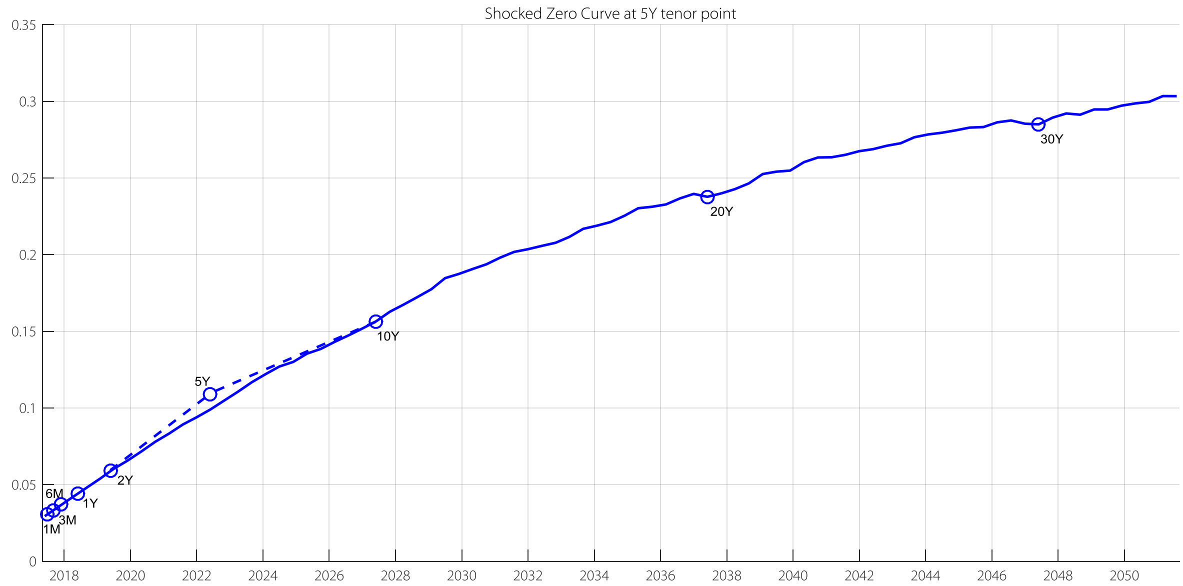

Figure 5-1. Graphical representation of a simulated zero curve, shocked at the 5Y tenor point.

The sensitivities that SIMM uses as input for interest rate products are partial durations for the following tenor points (ISDA, 2017): 2W, 1M, 3M, 6M, 1Y, 2Y, 3Y, 5Y, 10Y, 15Y, 20Y, and 30Y. As seen in Figure 5-1, the simulated tenor points differ from SIMM's requested input tenor points. To transform our tenor grid to that for SIMM, linear interpolation will be used on our partial durations (obtained from simulation). This interpolation procedure is in line with the ISDA's Risk Data Standards for SIMM (ISDA, 2017).

5.2 Weighted Notional Model

In order to establish practical approximations, models based on simulation of future SIMM from dynamic deltas cannot be considered, as these are impractical due to the computational burden. We need to come up with alternative models that have the potential to approximate the Dynamic SIMM, while not using simulated deltas and at the same time satisfy requirements Req1-Req3. In order to satisfy requirement Req1, we use the t_0-IM value as obtained by SIMM, rather than SA. Because we use Dynamic SIMM as the benchmark model, it is natural to use t_0 IM values as obtained by the t_0 SIMM. Henceforth, all the t_0 IM values will be obtained by the t_0 SIMM (i.e., by plugging in t_0 sensitivities into SIMM).

After obtaining the t_0 IM value, the next step is to determine how this value is to be amortized over the lifetime of a counterparty portfolio. For this, we consider the ageing of the trades within the portfolio. Correspondingly, we need to determine some sort of value associated with the still alive trades in the portfolio at each future time point, while at the same time not creating any computational burden and using the outputs from the currently available systems (requirement Req3). The simplest values we can come up with are the notional amounts of the trades. These are available in the current system. To capture the ageing of trades we can simply use (regulatory) add-ons. With the proposed elements, we propose the following simple IM amortization scheme, based on weighting the notional with add-ons:

$$IM_{WN}(t) = \sum_k IM_k(t_0)\left[\frac{\sum_{i\epsilon k}|Notional_i(t)| \cdot AddOn_i(t)}{\sum_{i\epsilon k}|Notional_i(t_0)| \cdot AddOn_i(t_0)}\right] \quad 5\text{-}1$$

where *WN* indicates the amortization method (Weighted Notional), *t* indicates time, *k* indicates the asset class, and *i* indicates the trades within a single asset class, subject to IM requirements. By separating the trades and t_0-IM per asset class, we satisfy requirement Req3.

5.3 Weighted MtM Model

With the next model, we aim to capture portfolio dynamics better than is possible with Equation 5-1. The simplest dynamic that we can capture additionally is the netting benefit of trades within a portfolio. Therefore, we extend the Weighted Notional Method by introducing MtM into Equation 5-1. Future MtM simulations of trades are also available from the currently used system at Rabobank, so we are still satisfying requirement Req3. To include the effect of the netting benefit by means of MtM, we mimic the Standardized Approach, as described in Chapter A. This means that the netting benefit can be up to 60%. With the discussed addition, we propose the following amortization method:

$$IM_{WM}(t,s) = \sum_k IM_k(t_0)\left[\frac{\sum_{i\in k}|Notional_i(t,s)|\cdot AddOn_i(t)\cdot\left(0.4+0.6\cdot\frac{[\sum_i MtM_i(t,s)]^+}{\sum_i[MtM_i(t,s)]^+}\right)}{\sum_{i\in k}|Notional_i(t_0)|\cdot AddOn_i(t_0)\cdot\left(0.4+0.6\cdot\frac{[\sum_i MtM_i(t_0)]^+}{\sum_i[MtM_i(t_0)]^+}\right)}\right] \quad 5\text{-}2$$

where *WM* indicates the amortization method (Weighted MtM), *t* indicates time, *s* indicates senarios, *k* indicates the asset class, and *i* indicates the trades within a single asset class, subject to IM requirements. Comparing Equation 5-2 to Equation 5-1, we see that the Weighted MtM method is scenario-dependent, whereas the Weighted Notional method is not. The reason is that Rabobank's system simulates MtM at different time points as well as for different scenarios. Just like the Weighted Notional method, the Weighted MtM method satisfies requirement Req2.

5.4 American Monte Carlo

Here, we propose a model that is of the semi-analytical type, which is based on the seminal publication from Longstaff & Schwartz (2001). The method has been known as American Monte Carlo and it is based on least-squares regression. This method is proposed by Anfuso *et al.* (2017) to forecast future SIMM.

The American Monte Carlo (AMC) method for forecasting future SIMM is based on determining the local volatilities via least-squares regression with a polynomial basis. Generally, the AMC methodology reuses the MtM simulation paths at each time point *t* to determine the local distribution of ΔMtM(t, t+MPOR, $path_s$). The IM can then be expressed as:

$$IM_k(t,s) = Q_q(\Delta MtM(t, t+MPOR, path_s)|\mathcal{F}_t) \quad 5\text{-}3$$

where *k* represents the asset class, *Q* is the quantile and *q* represents the confidence level, *t* represents time, *s* represents the scenario, and $\boldsymbol{\mathcal{F}}_t$ is all the information available at time *t* (we assume a (Ω, $\boldsymbol{\mathcal{F}}$, P) probability space and $\boldsymbol{\mathcal{F}}_t$ is a filtration such that conditions are adapted to it). Equation 5-3 states that the IM at a specific time point and specific scenario is represented by its local ΔMtM distributions, conditional on the available information at that specific time point. This means that we approximate nested MtM paths at a specific future time point *t* and specific scenario *s*. So, the nested character arises because we assume a distribution of ΔMtMs at each time point and each scenario. Rather than simulating the nested distributions, we approximate them by determining the distribution parameters. The assumption made henceforth is that the nested distributions are normal and can be determined by two parameters: μ and σ. The following assumption is that the drift parameter μ of the nested distributions is negligible over the MPOR horizon and can therefore be set equal to zero (i.e., μ=0). It remains that we need to determine the σ parameter of the nested distributions. By following the

Longstaff-Schwartz argumentation working towards the situation in Equation 5-3, we can express the second moment of the nested distributions as (only if μ=0):

$$\sigma^2(t,s) = \langle \left(\Delta MtM(t,s)\right)^2 | MtM(t,s) \rangle = \sum_0^n a_{\sigma,n} MtM(t,s)^n \qquad 5\text{-}4$$

where *ΔMtM(t,s)* is short for *ΔMtM(t, t+MPOR, path$_s$)*. The right hand side of the Equation can be obtained by least-squares regression with a polynomial basis as shown by Longstaff and Schwartz (2001) and *n* represents the polynomial degree. It has been established that a second-degree polynomial gives the best results for determining the local variances (Caspers *et al.*, 2017), so n=2 in our case. Here, we have chosen our filtration $\boldsymbol{\mathcal{F}}_t$ to consist of conditional value in terms of MtMs, but we can also adapt the risk drivers to the filtration. Anfuso *et al.* (2017) and Caspers *et al.* (2017) argue that this is a more precise approach. However, for simplicity we use conditioning on MtM. Now, we can forecast future IM by:

$$IM_{R,k}^U(t,s) = \Phi^{-1}(0.99, \mu = 0, \sigma = \sigma(t,s)) \qquad 5\text{-}5$$

Where $\Phi^{-1}(x,\mu,\sigma)$ represents an inverse cumulative normal distribution, evaluated at confidence level *x*, and *R* indicates received IM (on purpose we make a distinction between received IM and posted IM, because of asymmetry of the distribution). We are only interested in the received IM in the rest of this thesis, so we disregard the posted IM and refer to the received IM when mentioning IM. In Equation 5-5, The *U* in the subscript indicates that the future IM profile is unnormalized. In order to satisfy requirement R1, we need to normalize the function in Equation 5-5 with the following normalization function:

$$\alpha_0 = \sqrt{\frac{MPOR}{10d}} \cdot \frac{IM_{t=0}}{\sum_{k=1}^K Q_{0.99}\left(\Delta MtM_k(0, MPOR)\right)} = \frac{\sum_{k=1}^K SIMM_{t=0,k}}{\sum_{k=1}^K IM_{t=0,k}^U} \qquad 5\text{-}6$$

With Equation 5-6, we are able to reconcile our unnormalized model from Equation 5-5 with the SIMM-value that we obtain for t=0, in the following way:

$$SIMM(t,s) = \alpha_0 \sum_{k=1}^K IM_k^U \qquad 5\text{-}7$$

Anfuso *et al.* (2017) further propose in their publication that we use the following function to normalize Equation 5-5:

$$\alpha(t) = \left(1 - h(t)\right) \cdot \sqrt{\frac{10d}{MPOR}} \cdot [\alpha_\infty + (\alpha_0 - \alpha_\infty) \exp -\beta(t) \cdot t] \qquad 5\text{-}8$$

where *β(t)>0* and *h(t)<1*, with *h(t=0)=0* are functions to be calibrated (in our case two functions, but usually these represent four functions: each function is calibrated for receiving IM as well as posting IM). The α_∞ parameter is a constant and α_0 is also a constant as defined in Equation 5-6. The α_∞ parameter accounts for the long-term scaling level as t→∞. *β(t)* is a mean-reversion speed function that determines the transition from *α(t=0)* to α_∞. The *h(t)* function can be used to reduce back-testing exceptions (Anfuso *et al.*, 2017) and make to IM forecasting model more conservative. Including the *α(t)* function, this gives us the final SIMM forecast from the AMC method as:

$$SIMM_{AMC}(t,s) = \alpha(t) \sum_{k=1}^K IM_k^U \qquad 5\text{-}9$$

Calibration methods for the elements of the function in Equation 5-8 are discussed by Anfuso F. *et al.* (2017) and Caspers *et al.* (2017). In order to satisfy requirement Req3, an adaption has to be made to the currently used systems at Rabobank: the values of MTM(t+MPOR) have to be obtained for each scenario by adjusting the Brownian Bridge. From the AMC method Equations it can be seen that we have also satisfied requirement Req2.

6 Research Setup

We select three representative bilateral netting sets as a starting point to obtain our results and perform the analysis. Representative means in this case that the three netting sets cover the characteristics seen in counterparty portfolios consisting of IRS trades only and have EUR as the only underlying currency, together with the characteristic that all the trades in the EUR bucket are exposed to the same yield curve (i.e., EUR 3M LIBOR).

The selection consists of two large netting sets (>250 trades), but with different compositions of receiver and payer IRSs and difference in the time to maturity, and a smaller netting set (<100 trades). Below, these three netting sets will be discussed in more detail. Also, we alter the ratios of payer to receiver swap within these netting sets later in this section to obtain a higher, more representative diversity of netting sets.

We have selected these three netting sets as a starting point for our analysis, because they have similar characteristics to most of Rabobank's bilateral netting sets: single-currency, single underlying yield curve, and in which 80%-100% of the trades are IRSs. By choosing these sets and altering the payer to receiver swap ratios in an experimental setting, we are able to generalize the outcomes of our analysis over all such Rabobank bilateral netting sets.

6.1 Portfolio 1

The first netting set (to be called Portfolio 1 throughout the remainder of this thesis) is a portfolio which contains 264 IRS trades. The ageing of trades and the composition of payer and receiver swaps within this portfolio is shown in Figure 6-1. From the right-hand graph in the figure, it can be seen that the t_0 composition of this counterparty portfolio contains 197 receiver swaps and 67 payer swaps.

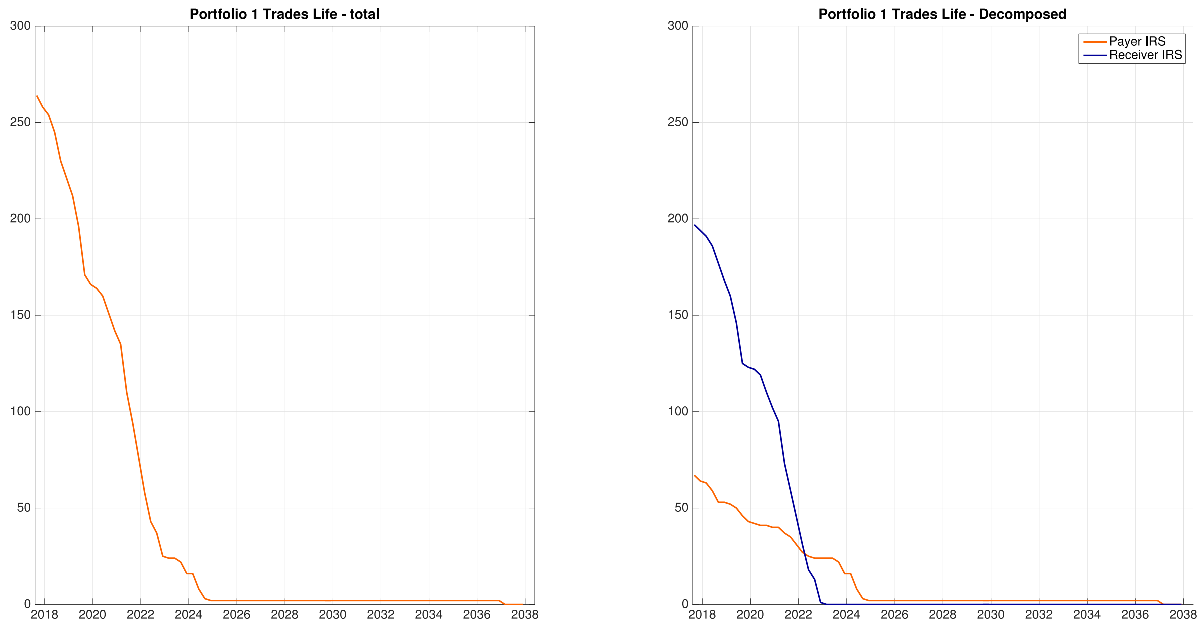

Figure 6-1. Portfolio 1 evolution of trades through time. There is a total of 264 trades in the Portfolio at t_0.

Figure 6-2 shows the composition of Portfolio 1 in terms of time-to-maturity at t_0 (residual maturity at t_0). We can see from the figure that most of the trades have a relatively short residual maturity (0-5 years). The portfolio has a low number of trades with a longer residual maturity as well (5-10 years) and even a few long residual maturity trades (10-20 years), but its number is negligable.

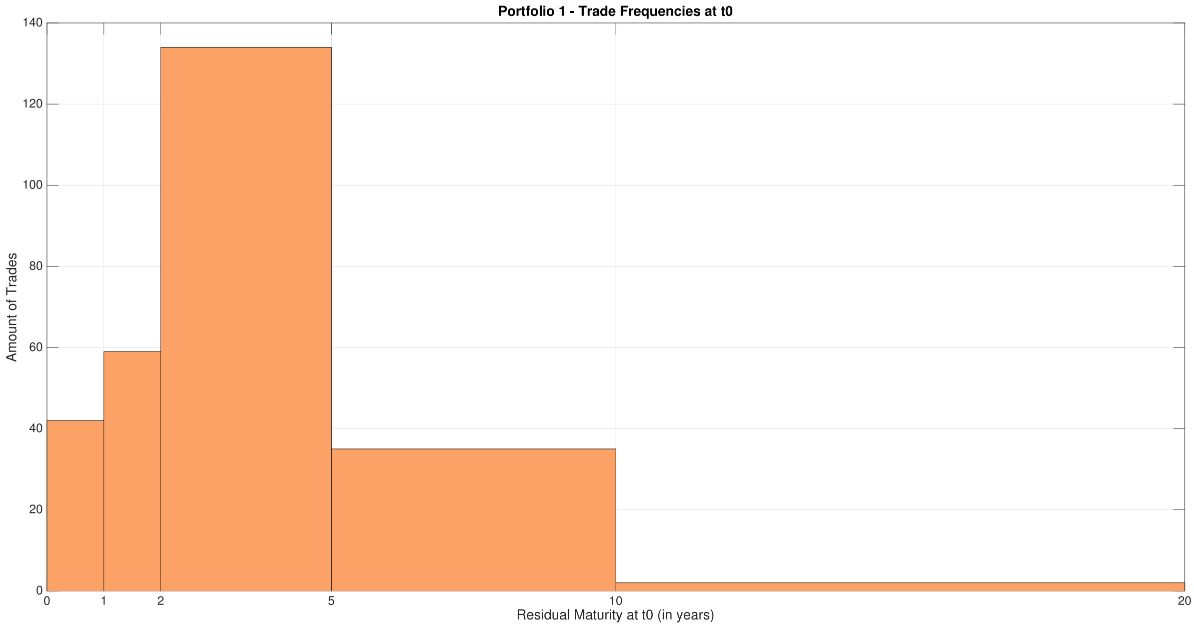

Figure 6-2. The number of trades in Portfolio 1 per residual maturity (at t_0) category. The largest portion of the trades in the counterparty portfolio have a residual maturity at t_0 between 0 and 5 years.

6.2 Portfolio 2

The second counterparty portfolio (to be called Portfolio 2 throughout the remainder of this thesis) is a Portfolio with 275 IRS trades. Figure 6-3 shows the ageing of the trades and the composition of the portfolio in terms of receiver and payer swaps. We can see that this counterparty portfolio is an all-receiver swap portfolio.

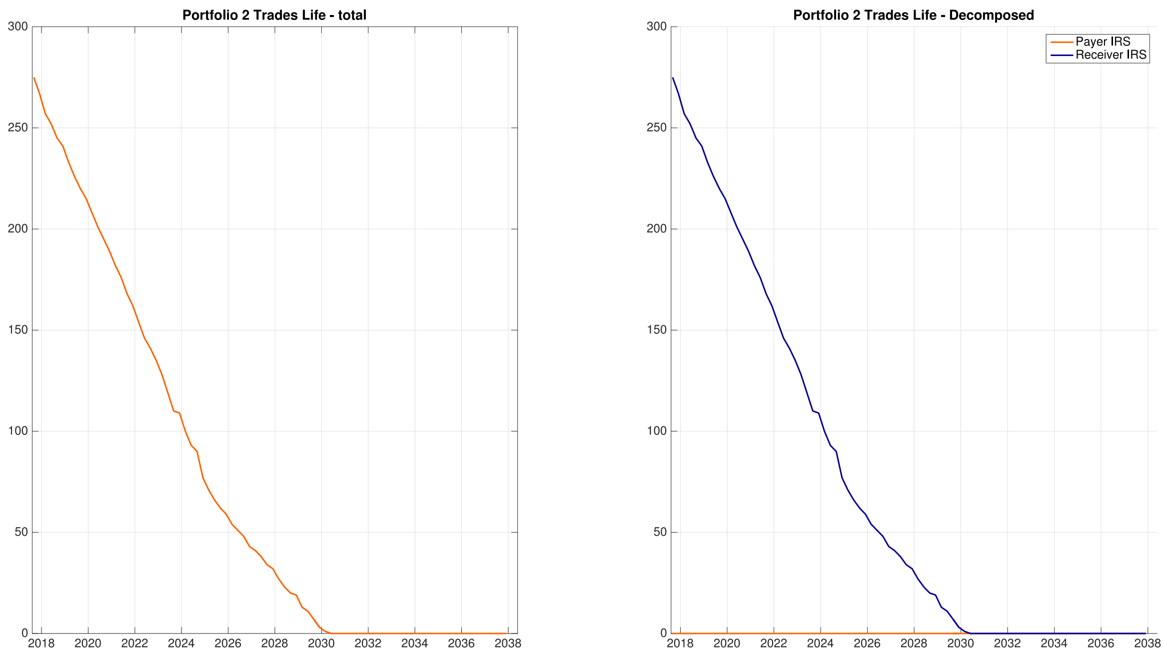

Figure 6-3. Portfolio 2 evolution of trades through time. There is a total of 275 trades in the Portfolio at t_0.

Figure 6-4 shows the composition of the portfolio in terms of residual maturity at t_0. Compared to Figure 6-2, Portfolio 2 has more trades in the mid- to long residual maturity region.

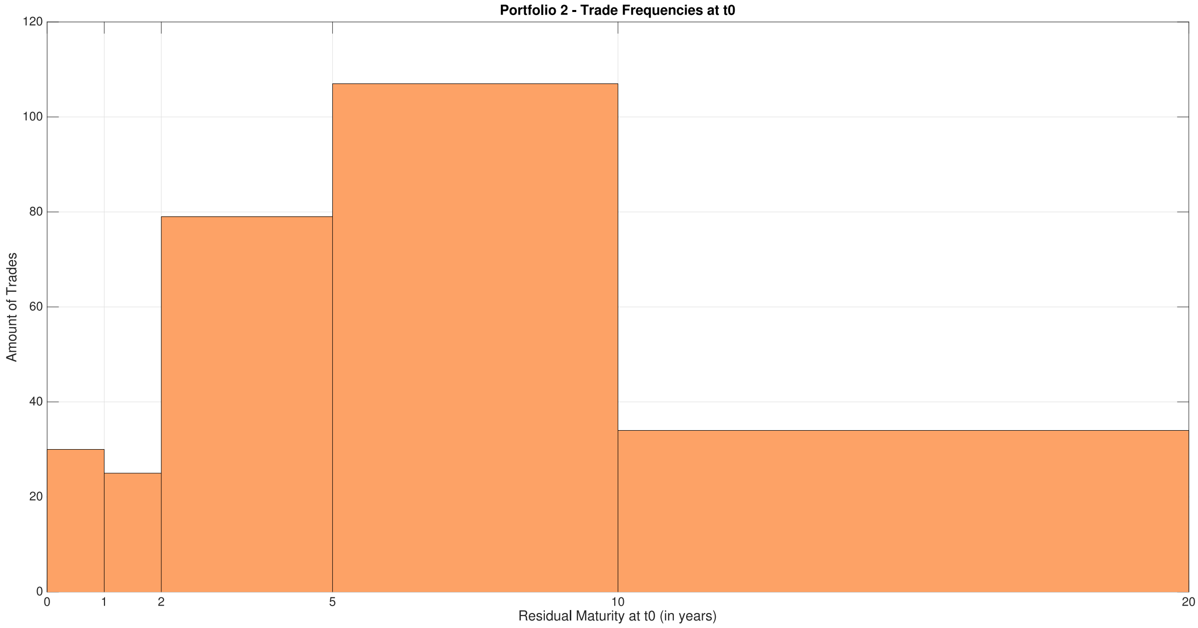

Figure 6-4. The number of trades in Portfolio 2 per residual maturity (at t_0) category. The largest portion of the trades in the counterparty portfolio have a residual maturity at t_0 between 2 and 20 years.

6.3 Portfolio 3

Portfolio 3 is the smallest counterparty portfolio of the three selected portfolios and contains 72 IRS trades, from which all are receiver swaps (Figure 6-5) and the largest portion of the trades has a residual maturity at t_0 in the mid-region (2-10 years)(Figure 6-6).

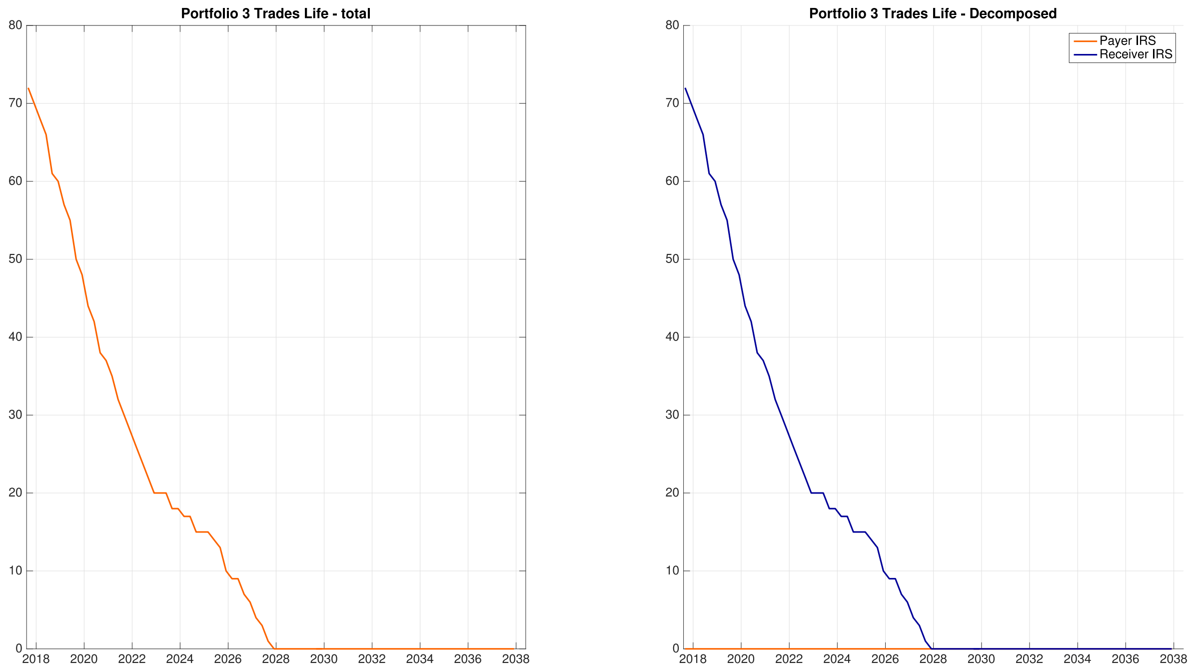

Figure 6-5. Portfolio 3 evolution of trades through time. There is a total of 72 trades in the Portfolio at t_0.

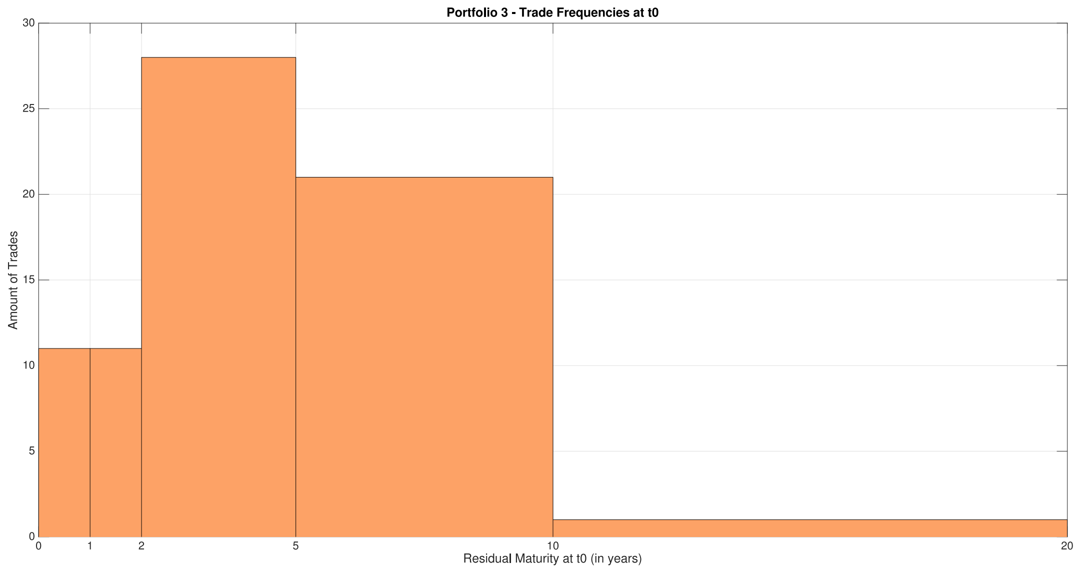

Figure 6-6. The number of trades in Portfolio 3 per residual maturity (at t_0) category. Most of the trades in the counterparty portfolio have a residual maturity at t_0 between 2 and 10 years.

6.4 Changing the payer-to-receiver swap ratio

The above-mentioned three counterparty portfolios have a limited range of payer-to-receiver swap ratios (i.e., Portfolios 2 and 3 are all-receiver swap portfolios and only Portfolio 1 has a non-zero amount of payer swaps). Changing the ratio allows for variation in delta profiles through time, as a higher payer-to-receiver swap ratio may create a more delta-offsetting position within a single portfolio. Therefore, we introduce in a systematic way some more portfolios to be able to capture the effects of delta exposure on the alternative future SIMM models.

The approach to expand the experimental portfolios systematically is to create new portfolios from the three portfolios in the way as described in the table below. All portfolio names for further reference are stated in each cell between parentheses.

Table 6-1. Experimental portfolio setup

	All-receiver swap	**25% payer swaps**	**50% payer swaps**
Large portfolio, short-mid residual maturity	Portfolio 1 where all the payer swaps are converted into receiver swaps *(Portfolio 1A)*	Unaltered Portfolio 1 *(Portfolio 1B)*	Portfolio 1 where 65 of the 197 receiver swaps are converted into payer swaps *(Portfolio 1C)*
Large portfolio, mid-long residual maturity	Unaltered Portfolio 2 *(Portfolio 2A)*	Portfolio 2 where 69 of the 275 receiver swaps are converted into payer swaps *(Portfolio 2B)*	Portfolio 2 where 138 of the 275 receiver swaps are converted into payer swaps *(Portfolio 2C)*
Small portfolio, mid res. maturity	Unaltered Portfolio 3 *(Portfolio 3A)*	Portfolio 3 where 18 of the 72 receiver swaps are converted into payer swaps *(Portfolio 3B)*	Portfolio 2 where 36 of the 72 receiver swaps are converted into payer swaps *(Portfolio 3C)*

The various experiment portfolios are obtained by converting the IRS trades from receiver swaps to payer swaps and vice versa. The trades to be converted are chosen randomly with a seeded random generator (for reproducability). The way in which the trades are converted is by simply multiplying the value of the trade by -1 (i.e., multiplying Equation 11-15 by -1):

$$\begin{aligned}\mathbf{P}-\mathbf{IRS}(t,T,\tau,N,K) &= -1\cdot\mathbf{R}-\mathbf{IRS}(t,T,\tau,N,K)\\ &= N\sum_{i=\alpha+1}^{\beta}\tau_i P(t,T_i)[F(t;T_{i-1},T_i)-K]\end{aligned} \tag{6-1}$$

Portfolios with 25% payer swaps give similar outcomes in terms of netted MtM and netted delta as portfolios with 25% receiver swaps would give. Similarly, the outcomes for netted MtM and netted delta of an all-receiver swap and an all-payer swap are closely related. So, for keeping the experiment and the analysis more concise without loosing out on generalization of the results later on, portfolios with 75% payer swaps and all-payer swap portfolios are left out of consideration.

6.5 Experimental environment

To perform our experiments and test our models, we have set up an experiment environment in Matlab. The experimental counterparty portfolio data are obtained from Rabobank's front-office system. The experimental environment mimics the Rabobank's Monte Carlo simulation engine.

After loading in the data, the yield term structure is built as outlined in Chapter A. At each future time point a new term structure is modelled. The modelling process incorporates a multidimensional Brownian motion to simulate the correlations between the different tenors of the term structures. From the yield term structures, the valuation of the IRS trades is performed as outlined in Chapter B for each future time point. Exposure (PFE), without IM (including only VM) is then modelled by using the Brownian Bridge. The future delta sensitivities are obtained by shocking the term structures at each future time point for each tenor. With all these elements, the developed IM amortization models and the Dynamic SIMM are calculated as outlined in Chapter 5. Then, the new PFE is obtained by including the future SIMM profiles. The characteristics of the experimental environment, including some employed assumptions and simplification, are:

- No thresholds and MTAs are implemented for VM, nor for IM, these are assumed to be zero for simplification (i.e., for VM in Equation **Error! Reference source not found.**, the CTA, CMTA, and UTA are set to zero and for IM these are not taken up in the Equations). Also, no haircuts are implemented.
- With Monte Carlo simulations, simulations over 2000 scenarios are conducted.
- No concentration threshold is implemented for Dynamic SIMM. The reason is that the total delta exposure of the portfolio has to be €250 million per basis point[5] for the concentration penalization to arise and this in highly unlikely to be reached with the counterparty portfolios at the Rabobank.
- In our test model, there is no correlation implemented between different yield curves (OIS, LIBOR1M, LIBOR3M, etc.) for Dynamic SIMM. For simplicity it is assumed that all the IRS trades in the sample counterparty portfolios have the same underlying yield curve. With the selected counterparty portfolios this happens to actually be the case. The current experimental environment can be extended with correlation between yield curves.
- The experimental environment has the possibility to only simulate correlations between the 9 tenors for a single currency. Therefore, there is no correlation implemented for different currency buckets that can be supported by Dynamic SIMM. The experimental environment can

[5] For more information on the concentration threshold and its mechanics within SIMM, see Appendix A.2.2.

be extended to support multicurrency portfolios. For our research, the selected portfolios showed exposure to the EUR currency only.

- The correlations between tenors for the EUR currency in the experimental environment come from the Rabobank's internal systems. The correlations necessary for the Dynamic SIMM come from ISDA. The add-ons used for the weighted IM amortization methods are the same as the interest rate product add-ons from the Rabobank's approved (IMM): residual maturity ≤1, add-on 0.0; 1< residual maturity ≤5, add-on 0.005; residual maturity >5, add-on 0.015.
- For the AMC IM amortization method, a Brownian Bridge was used to obtain the valuation date minus MPOR MtM values for each future time point and each scenario.
- The MPOR is set at 14 days (10 business days).

6.6 Delta profiles

Now, we are going to look into the dynamic deltas as simulated by our experimental environment for the counterparty portfolios. Because the dynamic deltas play an essential role in our benchmark model, we discuss their behaviour and briefly review whether they behave as we would expect, as an initial robustness check for our experimental environment.

Figures 6-8 to 6-10 show the delta exposure profiles for the different compositions of Portfolios 1-3, respectively. The figures show the total delta (aggregated over all tenors and averaged over all the simulation scenarios). In all three figures, comparing the total delta profile (for each ratio, the bottom graph) and especially its y-axis range, we see that when we increase the number of payer swaps in the portfolio until there is a balance in the number of payer swaps and receiver swaps (all C portfolios) we see a decrease in the range of delta exposure. This makes sense, because we are essentially offsetting receiver swap deltas with payer swap deltas. Our method of randomly selecting receiver swaps and converting them to payer swaps in order to decrease the total delta seems to work for these selected portfolios. We mentioned above that the 75% payer swap portfolios and the all-payer swaps portfolio compositions were disregarded. The main reason was that if we were to keep increasing the number of payer swaps by randomly converting receiver swaps into payer swaps, we would be 'going back' more or less from the bottom graph delta situation (50% payer swap) towards the top graph, all-receiver swap delta situation for each portfolio, because we are creating an increasingly unbalanced situation, where the trades in the portfolio are less and less offsetting each other's delta exposures. This was the motivation to exclude those ratios and keep the analysis clear and concise, while maintaining generalizability.

Furthermore, comparing the all-receiver swap situation in Figure 6-8, with the 25% payer swap situation in Figure 6-9 (which is actually unaltered Portfolio 2, see again Table 8-1 above) with the all-receiver situation in Figure 6-10, we see that the exposures at t_0 represent the portfolio compositions at t_0 as given in Figures 6-2, 6-4 and 6-6. For instance, with the given portfolio composition at t_0 for Portfolio 1, we would expect to see that the delta exposure for this (all-receiver) portfolio would be highest to tenors 2Y and 5Y at t_0. This is actually the case. Similarly, we would expect the delta exposure at t_0 for Portfolio 2B to be the highest to tenors 5Y and 10Y, based on the composition of the portfolio at t_0. We see that this is the case. Finally, for Portfolio 3 all-receiver swap situation, we expect to see the delta exposure at t_0 to be higher for tenors 2Y, 5Y, and 10Y than for the other tenors, based on Figure 6-6 and this expectation is matched in Figure 6-10.

In this case the delta exposure at t_0 shows what the residual maturity at t_0 more or less predicted. However, this should not always be the case. For instance, it could be that only a handful of trades within a portfolio show such a high delta exposure to a particular tenor that it does not match the residual maturity profile anymore. Regardless, a higher number of trades with a longer residual maturity at t_0 is more likely to show higher delta exposure to longer tenors at t_0, but it is not a perfect predictor. In this regard, our experimental environment seems to operate as would be expected.

Moreover, we see that the shorter tenor deltas (i.e., the 1M and the 3M tenors in the top graphs for each portfolio composition situation) show a less smooth line and more of a zigzag pattern when compared to the longer tenor deltas (i.e., the 2Y, 5Y, 10Y, 20Y, 30Y in the middle graphs for each portfolio composition situation). This can be explained by Figure 6-7. The reason is that the displacements of shorter tenor points affect a smaller region of the zero curve than the displacements of longer tenor points, thereby causing more abrupt changes in the zero curve when the basis point displacement is employed. This is reflected in the corresponding delta profile and is caused by the distance between tenor points. The figure shows this phenomenon for a shock of the 5Y tenor point (top graph in the figure) and a shock of the 20Y tenor point (bottom graph in the figure). Because the absolute value of the total delta profile (bottom graphs for each ratio in Figures 6-8 to 6-10) is the absolute value of the sum of all the tenor deltas, we see that the zigzag behaviour ripples through into the total delta exposure profile. As the absolute value of the total delta exposure decreases, the lower tenor zigzag effect becomes more visible in the total delta profile, because the shorter tenor delta exposure may account now for a larger portion of the total delta exposure.

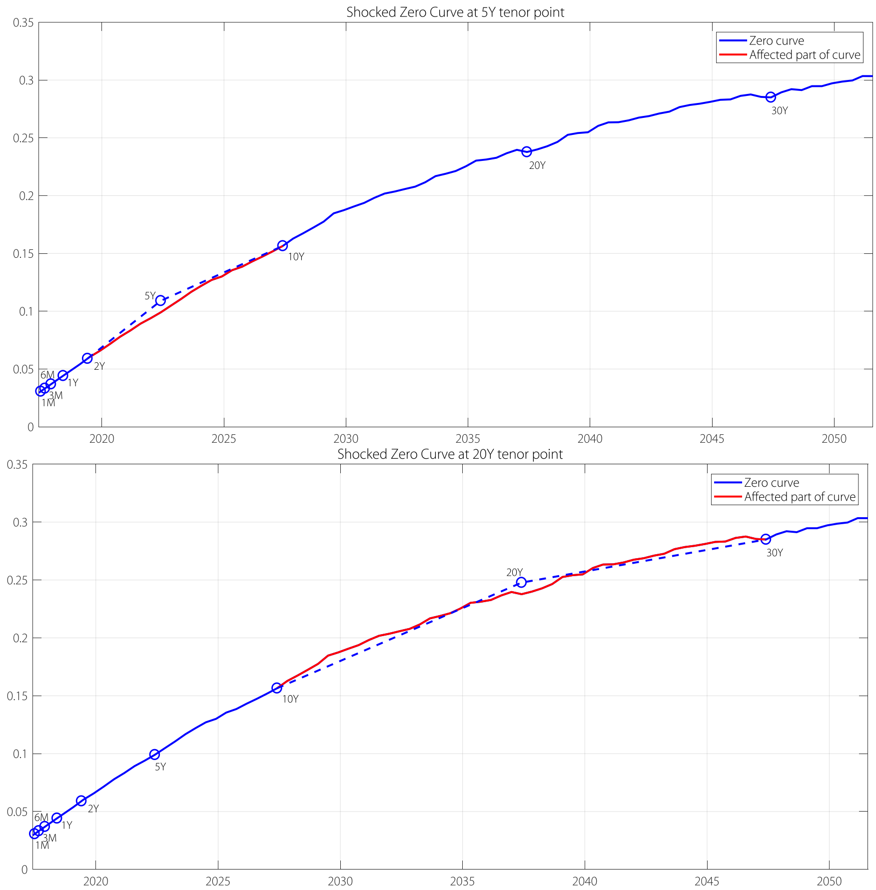

Figure 6-7. Graphical representation of the 1 basis point displacement (shock) of the 5Y tenor point of the zero curve (top graph) and the 20Y tenor point (bottom graph). The red part shows the affected part of the curve when the shock takes place.

Another aspect of the deltas is the positive delta exposure that we see in Figure 6-9 for the all-receiver portfolio. As the counterparty portfolio consists of receiver swaps only, it is remarkable to see positive delta exposure (as the deltas of receiver swaps usually have negative deltas). The reason we see this occurrence comes from the negative interest rate environment. Even though the present value of the swap is calculated by discounting under negative interest rates we still need to have a negative relation between the shock and the present value of the swap. This implies a positive delta in a negative interest rate environment for receiver swaps.

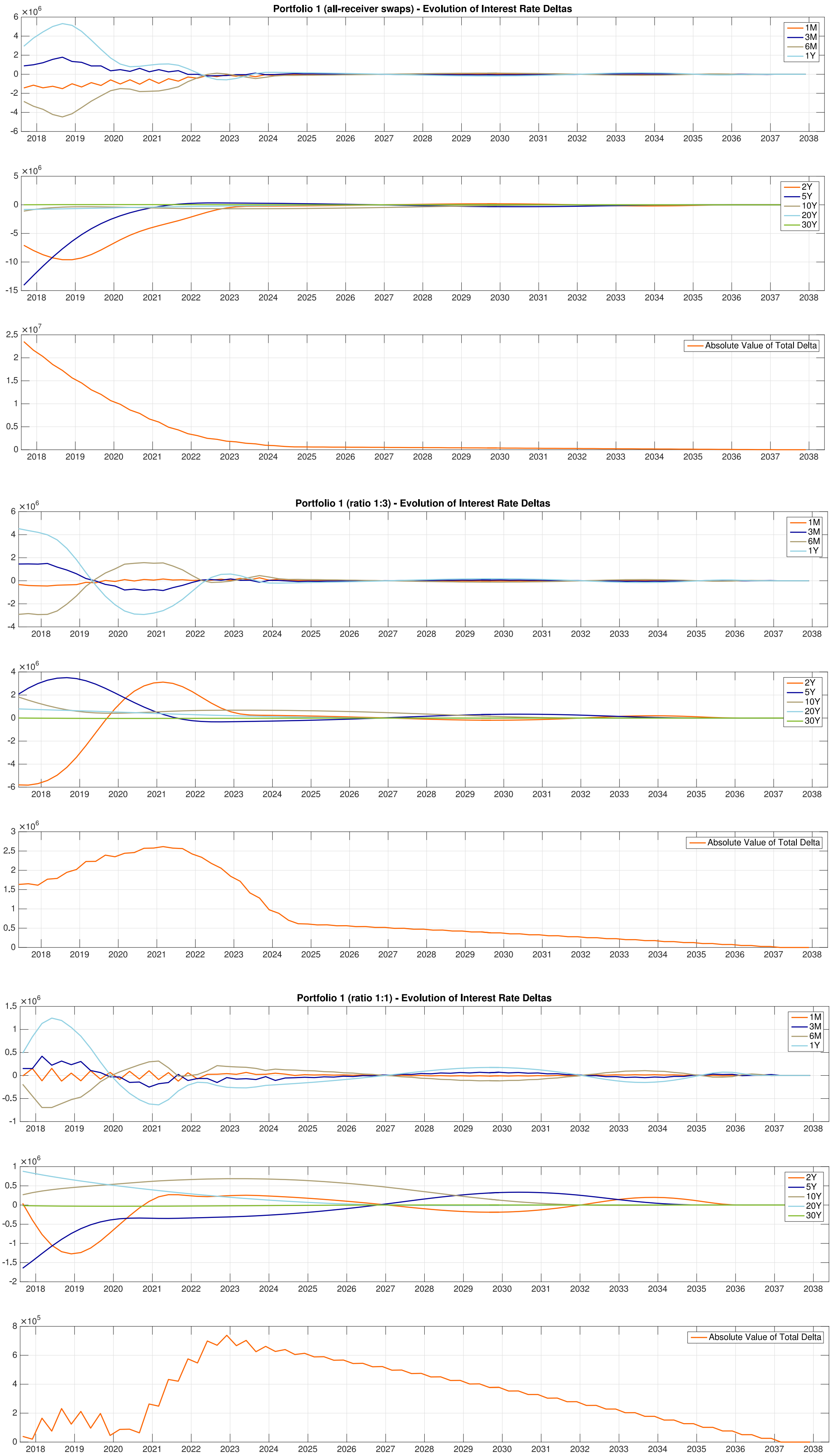

Figure 6-8. Delta exposure for Portfolio 1, with different compositions (ratios of payer-to-receiver swaps)

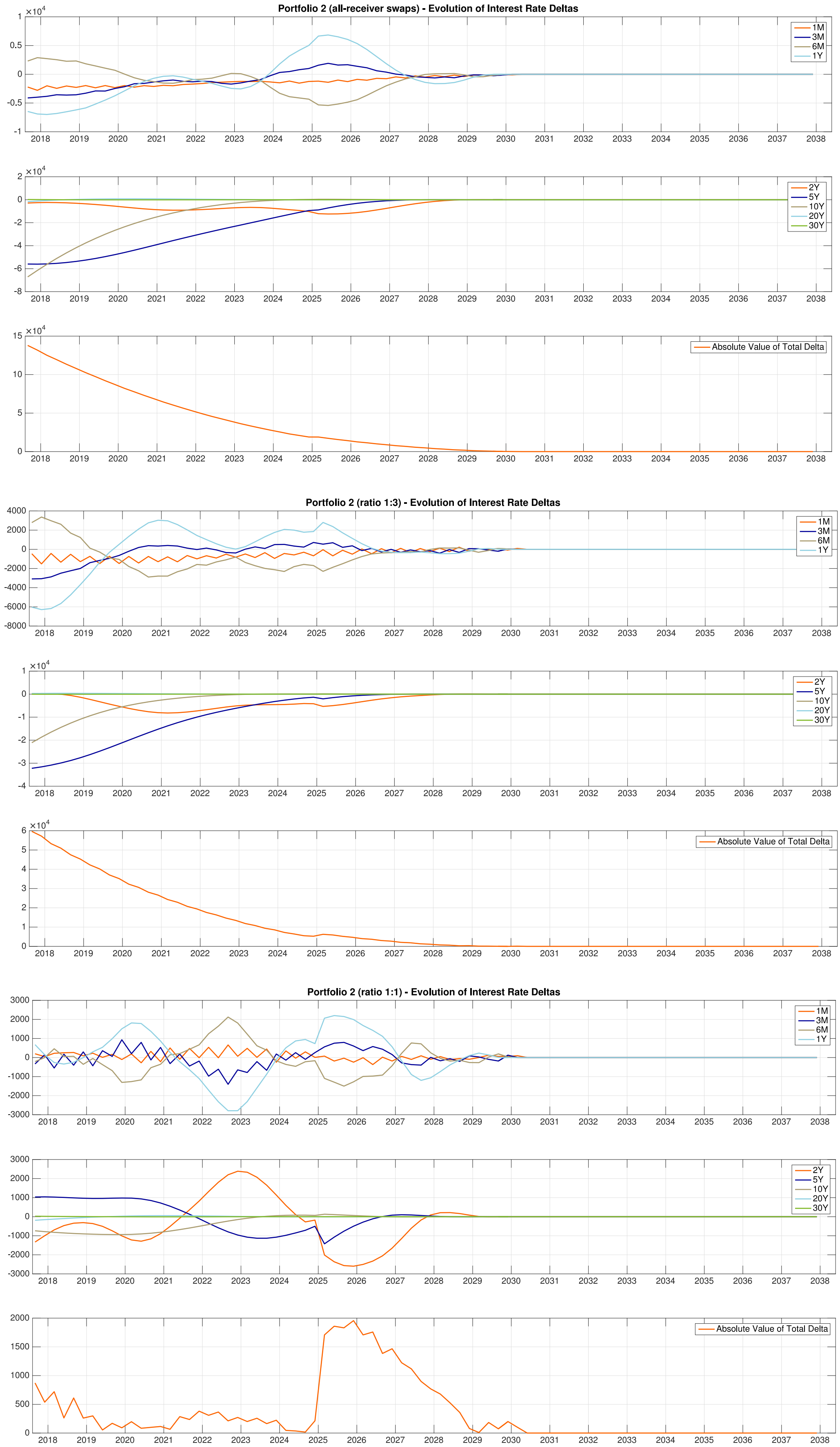

Figure 6-9. Delta exposure for Portfolio 2, with different compositions (ratios of payer-to-receiver swaps)

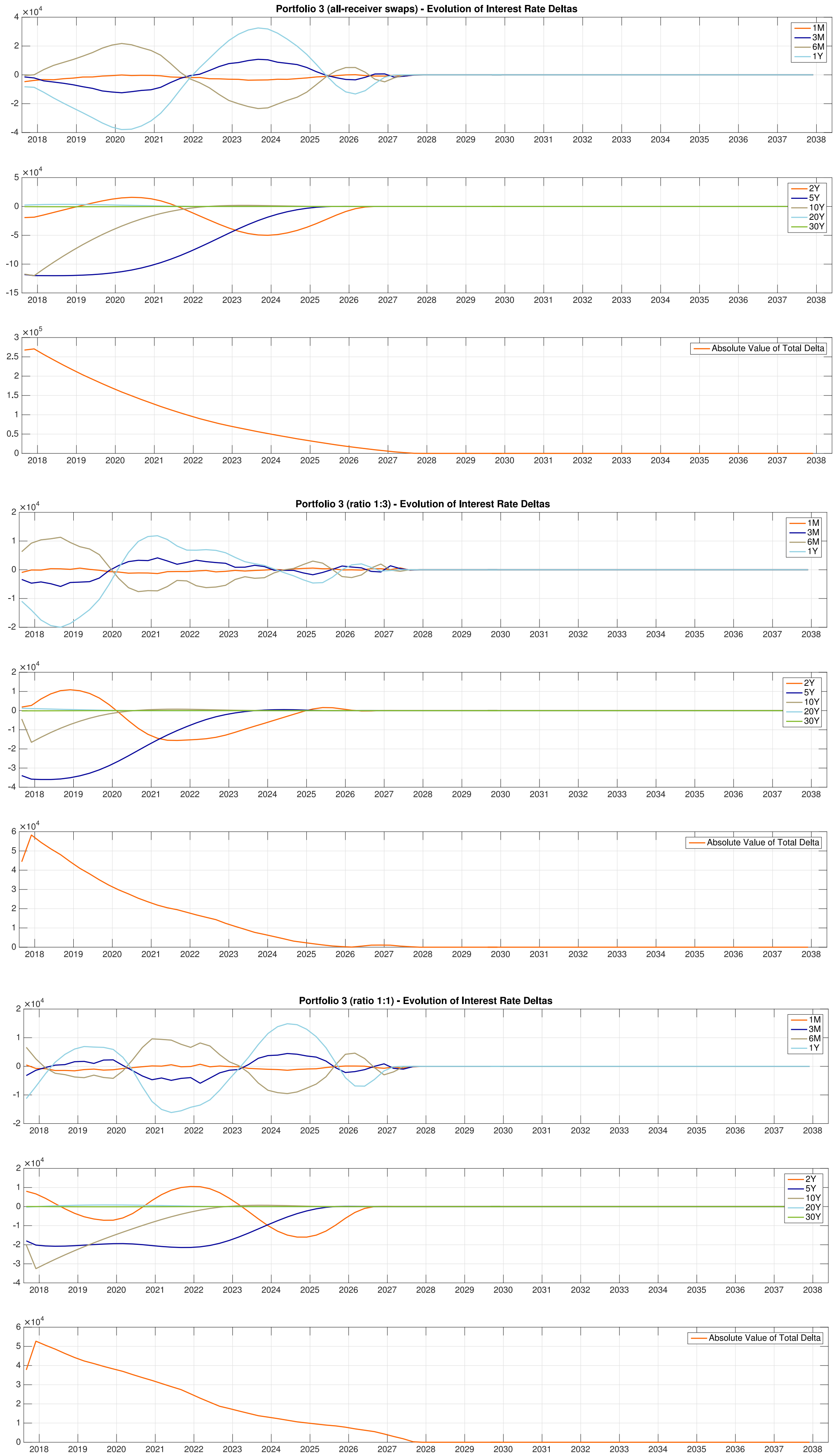

Figure 6-10. Delta exposure for Portfolio 3, with different compositions (ratios of payer-to-receiver swaps)

7 Results

We split the results section to first show and analyse the results from the Weighted Notional and Weighted MtM IM amortization models and their comparison to the Dynamic SIMM. Subsequently, we show and analyse the results of the American Monte Carlo regression-based method and its comparison with the Dynamic SIMM. This will be done for all the nine experimental portfolios. After the separate analyses of the models, the combined analysis on all the models and their effect on the PFE will be performed.

7.1 Weighted Methods and Dynamic SIMM

The results from the comparison of the Weighted MtM versus Weighted Notional versus Dynamic SIMM IM amortization are shown in Figures 7-1 to 7-3 for Portfolios 1-3, respectively. The left-hand graphs in the figures show the Weighted Notional and Weighted MtM methods as defined in Equations 5-1 and 5-2, respectively for each ratio of the portfolio. The right-hand graphs show the Weighted Notional and Weighted MtM methods as defined below in Equations 7-1 and 7-2, respectively:

$$IM_{WN}(t) = \sum_k IM_k(t_0)\left[\frac{\sum_{i\epsilon k} AddOn_i(t)}{\sum_{i\epsilon k} AddOn_i(t_0)}\right] \qquad 7\text{-}1$$

$$IM_{WM}(t,s) = \sum_k IM_k(t_0)\left[\frac{\sum_{i\epsilon k} AddOn_i(t)\cdot\left(0.4+0.6\cdot\frac{[\sum_i MtM_i(t,s)]^+}{\sum_i [MtM_i(t,s)]^+}\right)}{\sum_{i\epsilon k} AddOn_i(t_0)\cdot\left(0.4+0.6\cdot\frac{[\sum_i MtM_i(t_0)]^+}{\sum_i [MtM_i(t_0)]^+}\right)}\right] \qquad 7\text{-}2$$

The difference between Equations 5-1, 5-2 and Equations 7-1, 7-2 is that we left out the notional ratio, as defined by the expression:

$$Notional\ ratio(t,s) = \frac{\sum_{i\in k}|Notional_i(t,s)|}{\sum_{i\in k}|Notional_i(t_0)|}$$

The way the Weighted Notional method is defined in Equation 7-1 we might just as well call it the Weighted Add-Ons method, as the notional does not enter the Equation anymore, but we keep calling the method the Weighted Notional for its potential to include the portfolio notional dynamics, in case the notionals are not assumed constant through time and across scenarios.

We see from Figures 7-1 to 7-3 that the Weighted Notional and Weighted MtM methods perform better (are closer to the Dynamic SIMM) when the notional ratio is left out of the equation (i.e., when they are defined as in Equations 7-1 and 7-2). The reason is that we assume constant notionals (constant through time for each trade, as long as the trade is alive) and constant across scenarios. With such an assumption, the notional ratio becomes a decreasing function (due to the maturing of trades). Multiplying the decreasing notional ratio with the other terms in square brackets in Equations 7-1 and 7-2 ensures that the overall amortization methods are normalized by just another ratio, thus decreasing the amortized values at each time point. Because we assume constant notionals throughout time and across scenarios, we use the definitions from Equations 7-1 and 7-2 for the Weighted Notional and Weighted MtM methods henceforth, as these give better results. Also, various maxima for the netting benefit were evaluated (maximally 20%, 40%, 60%, and 80% netting benefit by adapting Equation 7-2 accordingly). The best results were obtained with the maximum of 60% netting benefit (the present form of Equation 7-2), so only these results are shown below.

The right-hand graphs in Figures 7-1 to 7-3 give the overview of the results for the newly defined Weighted Notional versus Weighted MtM methods versus Dynamic SIMM IM amortization for counterparty Portfolios 1 to 3, respectively. The figures have three rows, one for each ratio, as outlined in Table 6-1. What is noticeable directly from the figures is that, as the counterparty portfolio composition becomes increasingly balanced (i.e., the counterparty portfolio compositions go from all-receiver swap portfolios to equal payer and receiver amounts), the less the trajectories of the weighted methods resemble the Dynamic SIMM forecast. Based on the three counterparty portfolios, this appears to be independent of the residual maturity of the trades (also see Figures 6-2 to 6-6) within the portfolios and appears to be related to the delta exposure of the portfolios (see also Section 6.6).

Another directly noticeable phenomenon is that Dynamic SIMM fluctuates more strongly to an extent of sharp peaks when the portfolio composition moves towards the 50% payer swap situation. This has to do with the total delta profiles. As delta exposure offsetting occurs with an increasing number of payer swaps, we see a decreasing total delta exposure. However, some parts of the total delta exposure evolution curve see highly reduced total delta exposure, whereas other parts (where deltas exposure offsetting is not so prominent) see residual delta peaks. This has to do with the fact that we do not obtain perfect delta exposure offsetting over the entire curve, due the payer and receiver swaps present in the counterparty portfolios, because the trades that have a delta-offsetting effect on each other within the portfolios may have different delta dynamics throughout time. Such differences can cause some parts of the delta evolution curve to be offset almost perfectly, while other parts show significant residual exposure. Because we have decreased the level of the total delta exposure in such situations, the relative differences between the almost perfectly offset delta and less well offset parts of the delta evolution curve become large and show peaks of residual delta exposure.

Generally, we see that the Dynamic SIMM follows the absolute value of the total delta exposure (review the bottom graphs in Figures 6-8 to 6-10). This of course makes sense as the delta sensitivities are input for the Dynamic SIMM method. When the delta exposure evolution graph shows sharp peaks in its profile, the Dynamic SIMM amplifies these delta peaks by the risk weights underlying the calculation procedure. A great example of this is Portfolio 3C. As the counterparty portfolio nears its end-of-life, the exposure to the short tenors becomes dominant (see also Figure 6-10). With the SIMM methodology, the shorter tenors have larger risk weights (review the risk weights of SIMM for the EUR currency in Chapter A). For well-trades currencies, these risk weights increase as the length of the tenor decreases (only after the 15 years tenor the risk weights start to increase again a bit, but we do not have significant exposure to those larger tenors). That is why we see increasing peaks with the Dynamic SIMM as we move towards the end-of-life of Portfolio 3C.

We see that the Dynamic SIMM moves away further from the weighted methods as the portfolio compositions become more balanced. The weighted methods are not able to replicate the more strongly fluctuating behaviours of the Dynamic SIMM due to their constraint to the small number of steps present to quantify the trade ageing process (there are only 3 steps of add-ons).

Furthermore, the figures show difference between the Weighted MtM and Weighted Notional methods. What is noticeable from the results is that the Weighted MtM method increasingly has future SIMM values above the Weighted Notional values when the counterparty portfolios become increasingly balanced with payer and receiver swaps. We need to explain what exactly drives this difference for our counterparty portfolios.

7.1.1 Weighted MtM versus Weighted Notional

We start the comparison of the two weighted methods by decomposing them mathematically. Looking at Equation 5-2, we can see that the Weighted MtM amortization method can be decomposed in the Weighted Notional method and a scaling factor. The scaling factor can be defined as:

$$Scaling\ Factor = \frac{0.4 + 0.6 \cdot \frac{[\sum_i MtM_i(t,s)]^+}{\sum_i [MtM_i(t,s)]^+}}{0.4 + 0.6 \cdot \frac{[\sum_i MtM_i(t_0)]^+}{\sum_i [MtM_i(t_0)]^+}} \qquad 7\text{-}3$$

Only when the right-hand side of Equation 7-3 is equal to 1, is the Weighted MtM method equal to the Weighted Notional method for time points t at which the equality holds. Otherwise, the two IM amortization methods follow a different path. Their paths can be seen from the resulting Figures 7-4 to 7-6, where we have added the scaling factor evolutions for each counterparty portfolio (right-hand graphs in the figures). Comparing the graphs for the Weighted Notional and the Weighted MtM IM amortization with the scaling function evolution in the top row of Figure 7-4, we can explain why the Weighted MtM and Weighted Notional graphs only show the same values at the range close to t_0 and at the end-of-life of the portfolio. The reason is that the scaling function has a value equal to 1 at those time points. For the rest of the time points, we see from the scaling function that the Weighted MtM is below the Weighted Notional graph.

When we compare the middle row graphs in Figure 7-4 (zoomed in rectangle illustrates the area in which the Weighted MtM is above the Weighted Notional graph on the left hand-side and the arrow shows the corresponding range at which the scaling function has a value above 1 on the right-hand side), we can see that there are situations that cause the Weighted MtM IM amortization values to be larger than the Weighted Notional IM amortization values. Generally, we see that the scaling function has increasingly larger ranges where its value is above 1 when the counterparty portfolio has a more balanced composition of payer-to-receiver swaps (i.e., all Portfolios B and C), meaning that the Weighted MtM method increasingly has values above the Weighted Notional when the portfolio composition moves from all-receiver to equal number of both type of swaps. This is a general picture that we see occurring at all the three counterparty portfolios.

The main reason for an increasing proportion of the scaling function showing values larger than 1 is that the net-to-gross ratio at t_0 is lower in value than the net-to-gross ratio for an increasing number of future time points. Because all Portfolios B and C show an increasingly balanced number of payer and receiver swaps *at* t_0 (!), the net-to-gross ratio at t_0 is more likely to have a lower value than the net-to-gross ratio at future time points at which the balances are not 25% payer swap and 50% payer swap anymore due to trades expiring.

Overall, when we look at figures showing the scaling factor (net-to-gross MtM ratio), we see that their shapes do not show any resemblance to the delta profiles or the Dynamic SIMM profile (i.e., clearly distinguishable characteristics, like peaks or other characteristic behaviour at specific time points). With this observation, we can state that the net-to-gross terms in their current form in Equation 7-2 are not a well-performing predictor of future delta exposure.

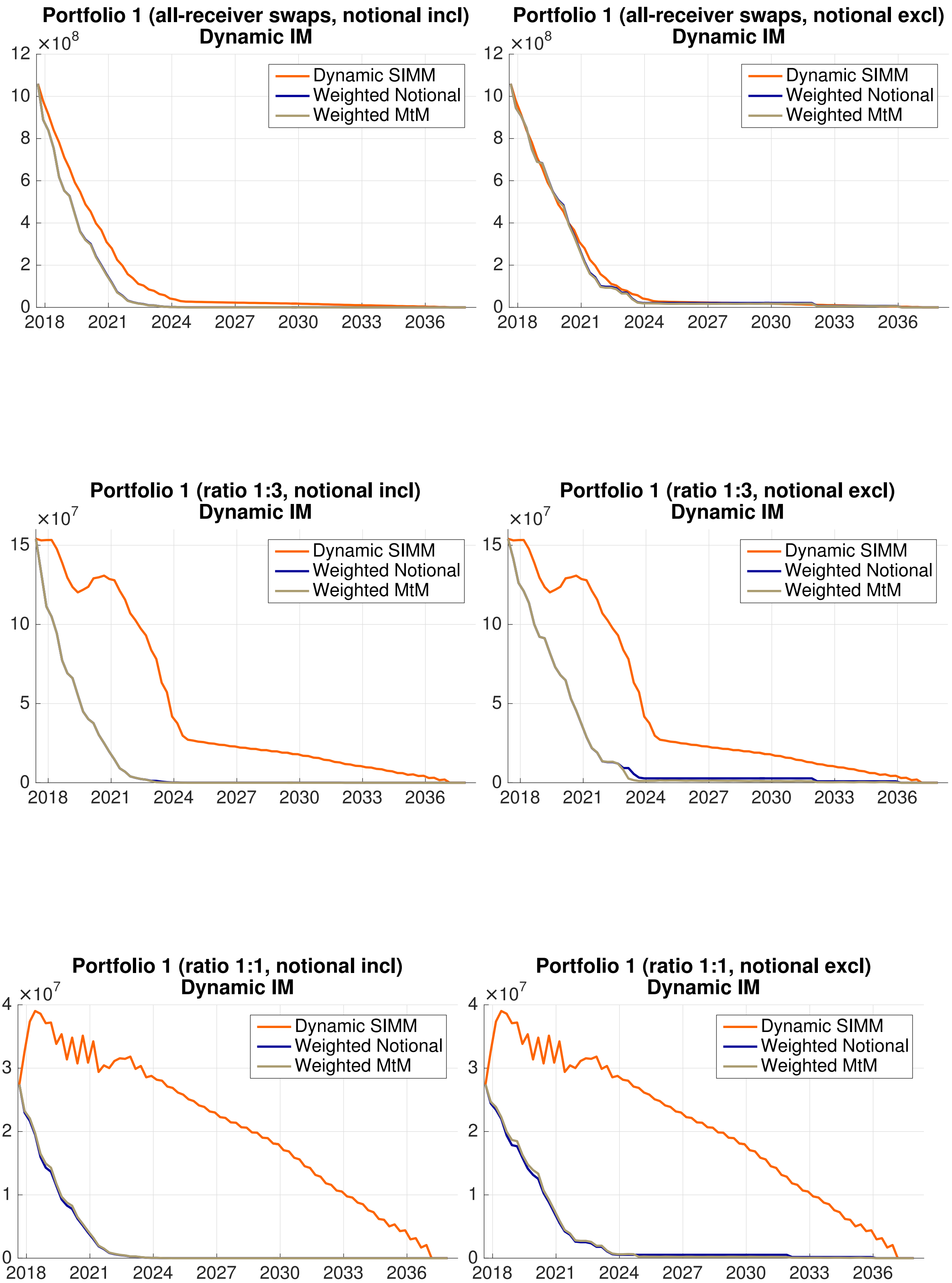

Figure 7-1. Weighted MtM versus Weighted Notional versus Dynamic SIMM for Portfolio 1 at the different experimental ratios. For each ratio, the left figure shows the Weighted Notional and Weighted MtM as defined in Equations 5-1 and 5-2, respectively. The right figures show the Weighted Notional and Weighted MtM as defined in Equations 7-1 and 7-2 , respectively.

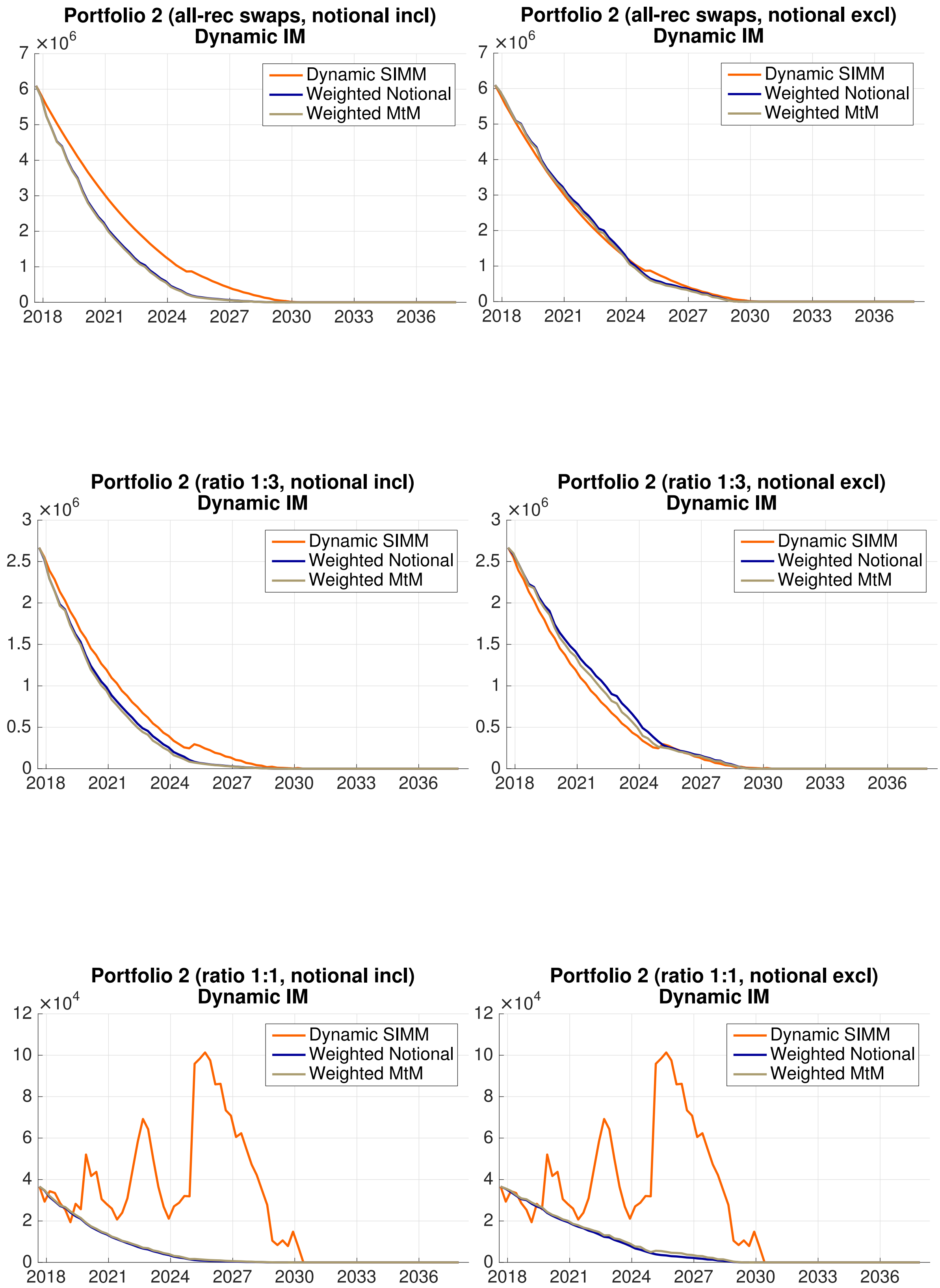

Figure 7-2. Weighted MtM versus Weighted Notional versus Dynamic SIMM for Portfolio 2 at the different experimental ratios. For each ratio, the left figure shows the Weighted Notional and Weighted MtM as defined in Equations 5-1 and 5-2, respectively. The right figures show the Weighted Notional and Weighted MtM as defined in Equations 7-1 and 7-2, respectively.

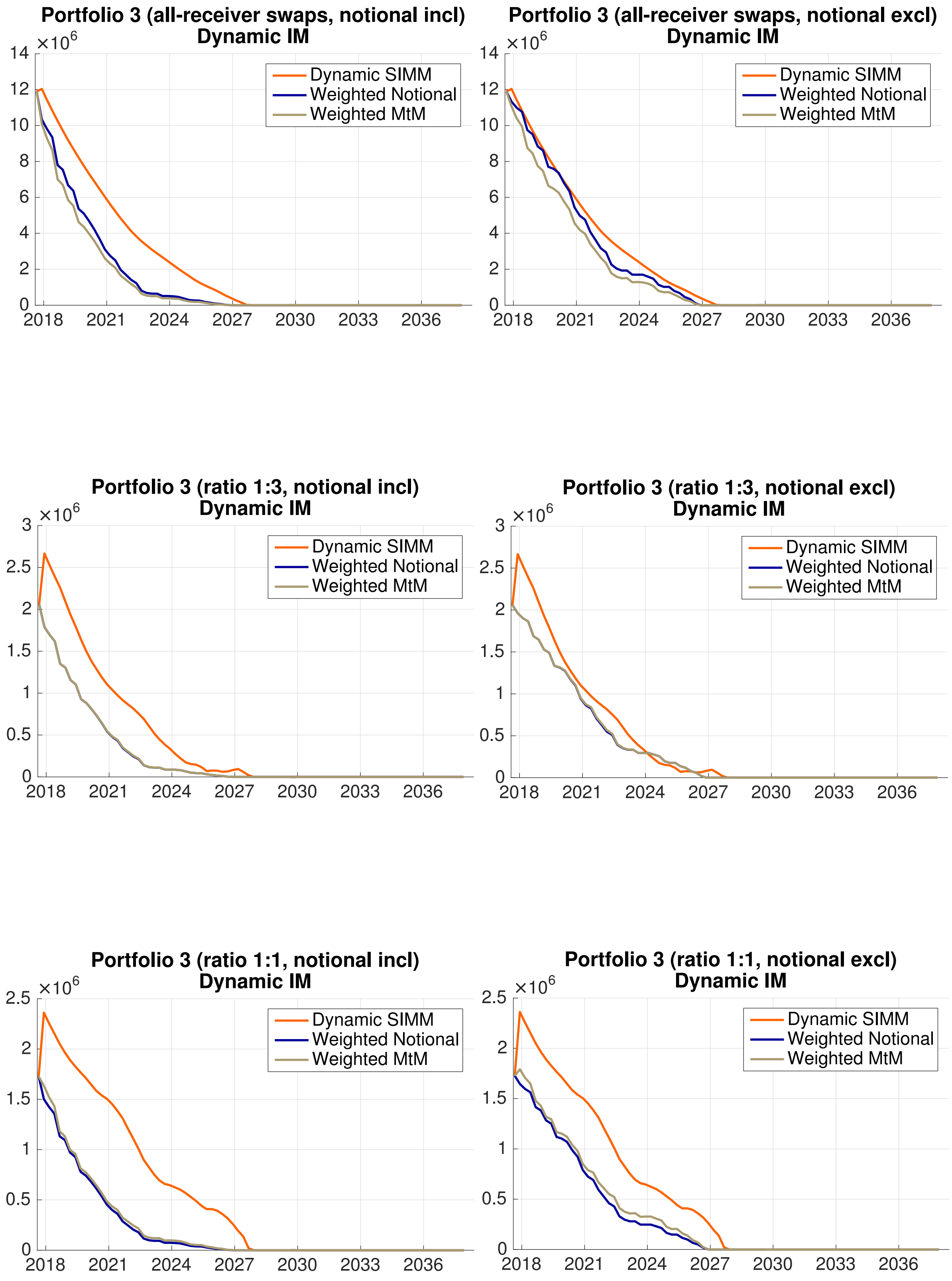

Figure 7-3. Weighted MtM versus Weighted Notional versus Dynamic SIMM for Portfolio 3 at the different experimental ratios. For each ratio, the left figure shows the Weighted Notional and Weighted MtM as defined in Equations 5-1 and 5-2, respectively. The right figures show the Weighted Notional and Weighted MtM as defined in Equations Equations 7-1 and 7-2, respectively.

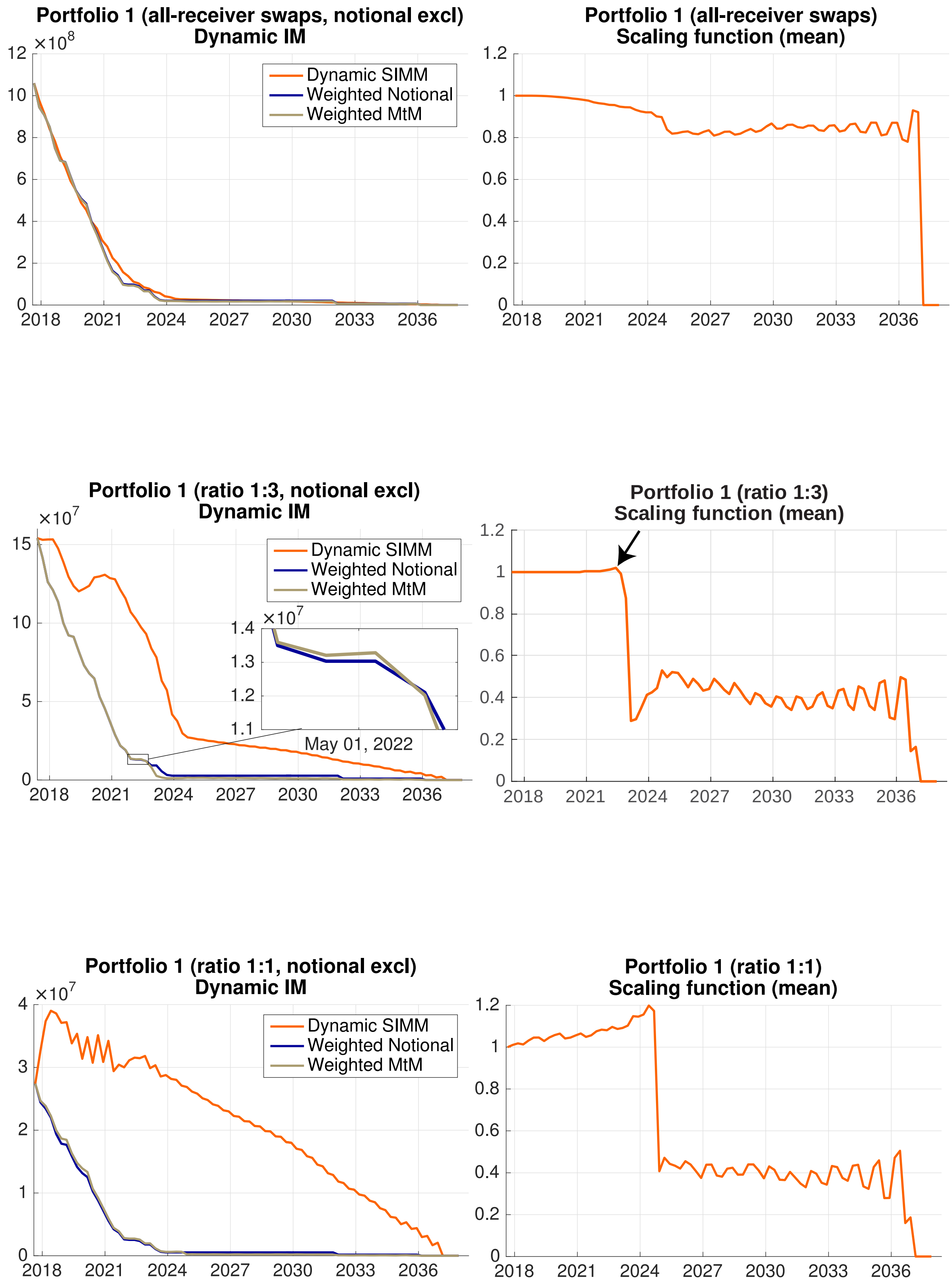

*Figure 7-4. **Left:** Comparison of the Weighted MtM and Weighted Notional with the Dynamic SIMM for Portfolio 1 at the different ratios: A). All receiver swaps, B). 25% payer swap, C). 50% payer swap. **Right:** The mean of the scaling function corresponding to the Weighted MtM in the left-hand graphs.*

The middle two graphs show the situation when the Weighted MtM is above the Weighted Notional IM amortization profile (zoomed-in part of the left-hand graph): in that case, the mean of the scaling function has a value larger than 1 (indicated by the arrow in the right-hand graph over the corresponding time range).

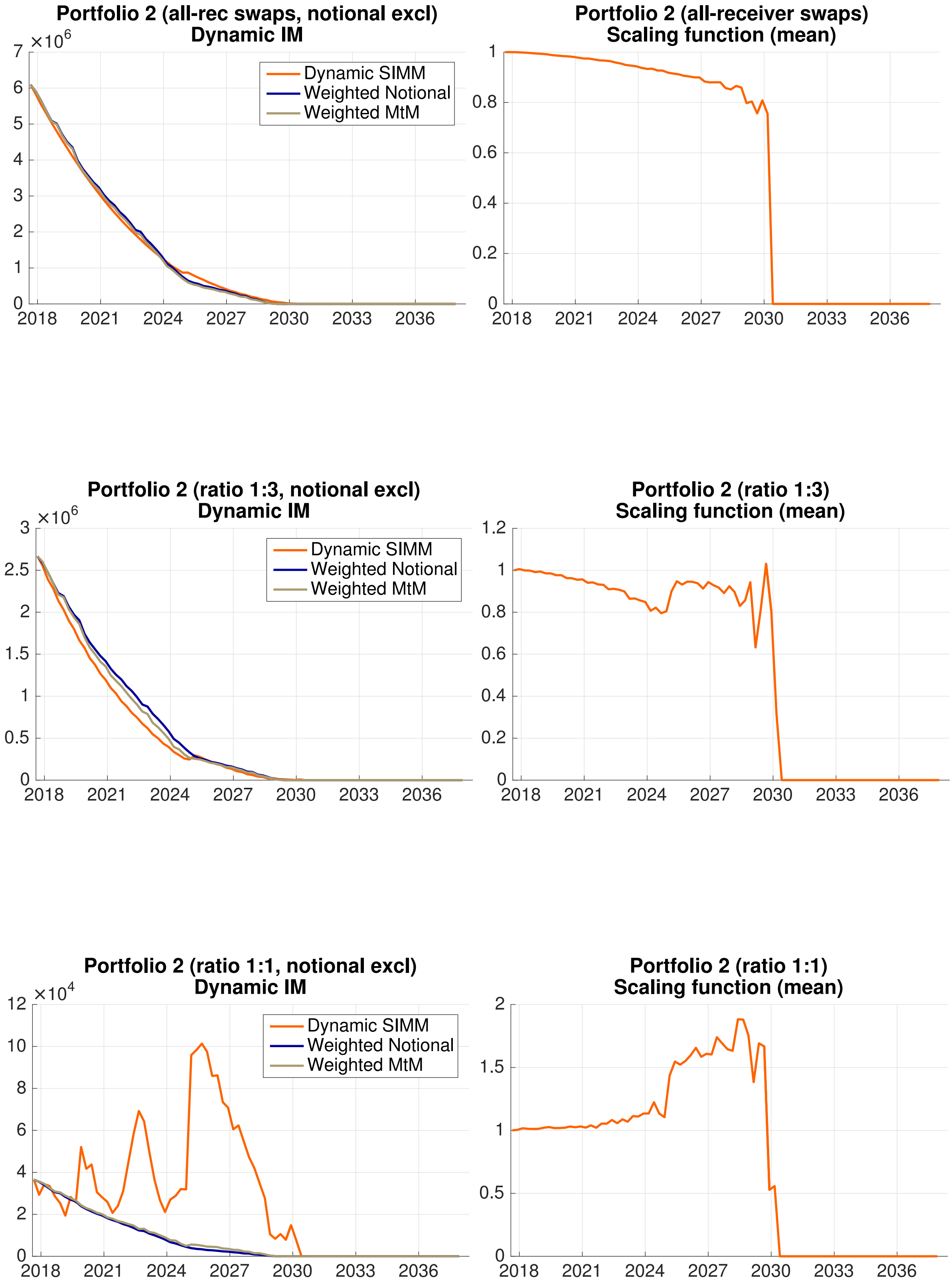

*Figure 7-5. **Left:** Comparison of the Weighted MtM and Weighted Notional with the Dynamic SIMM for Portfolio 2 at the different ratios: A). All receiver swaps, B). 25% payer swap, C). 50% payer swap. **Right:** The mean of the scaling function corresponding to the Weighted MtM in the left-hand graphs.*

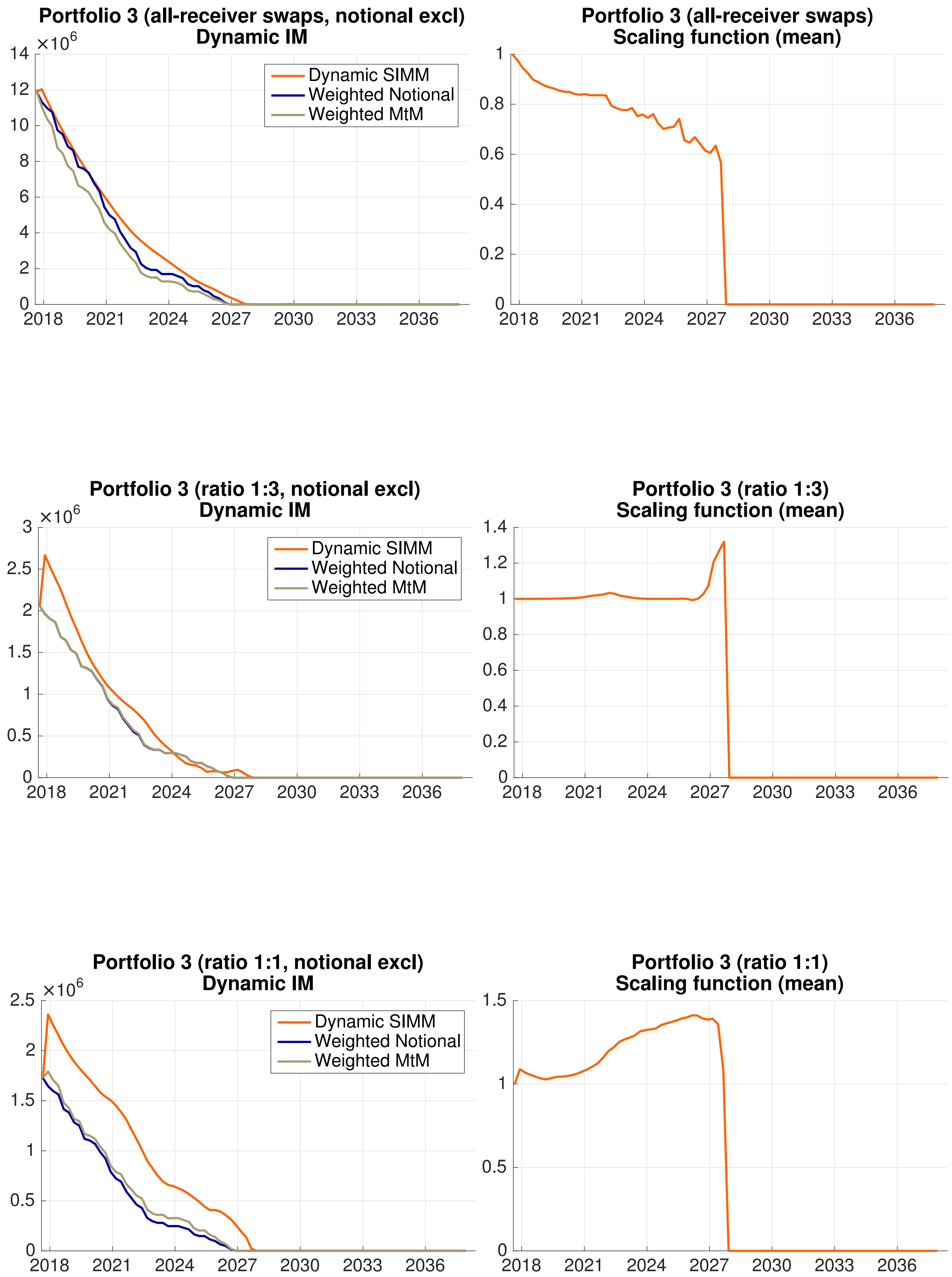

*Figure 7-6. **Left:** Comparison of the Weighted MtM and Weighted Notional with the Dynamic SIMM for Portfolio 3 at the different ratios: A). All receiver swaps, B). 25% payer swap, C). 50% payer swap. **Right:** The mean of the scaling function corresponding to the Weighted MtM in the left-hand graphs.*

7.1.2 Improving the weighted methods

With our weighted methods, we have used the following add-ons:
residual maturity ≤1, add-on 0.0; 1< residual maturity ≤5, add-on 0.005; residual maturity >5, add-on 0.015. We have seen the performance of the weighted methods with these add-ons compared to Dynamic SIMM in Figure 7-4, middle row. Now, we focus on trying to improve the add-ons to see whether the weighted methods are able to better mimic the Dynamic SIMM profile. In our example, we do this for the unaltered counterparty Portfolio 1.

To capture the Dynamic SIMM movement better, we focus on some of the characteristics of the Dynamic SIMM profile. This focus is performed in Figure 7-7. The figure shows three points of the Dynamic SIMM profile that show distinct characteristics: the local minimum at time point 01 June 2019, the local maximum at time point 01 September 2020, and the bend at time point 01 September 2024. For these time points we have made histograms showing the frequency of trades that have the residual maturity with bins according to our current add-ons: residual maturity ≤1, 1< residual maturity ≤5, and residual maturity >5 (in years).

Now we want to derive new add-ons in a simple fashion. The process is summarized in Table 7-1. First we start with three Equations (second column). The Equations are derived by looking at the histograms in Figure 7-7. For the first Equation, for example, we have 112 trades that are multiplied by the first unknown add-on x_1, 152 trades that are multiplied by unknown add-on x_2, and 8 trades that are multiplied by unknown add-on x_3. The denominator represents t_0. At t_0, we have 27 trades with a residual maturity of equal to or less than 1 year, 202 trades with a residual maturity between 1 and 5 years, and 43 trades with a residual maturity larger than 5 years. So, at each time point, we are calculating the add-on ratio from Equations 7-1 and 7-2:

$$Add - on\, ratio = \left[\frac{\sum_{i\epsilon k} AddOn_i(t)}{\sum_{i\epsilon k} AddOn_i(t_0)}\right]$$

Then we derive the target values (third column in Table 7-1). These will be the values that we want to approach as closely as possible with our derived Equations. The target values are basically the normalized SIMM values at the three time points, where the denominator is the t_0 SIMM-value. Now it becomes an optimization problem, where the x's are equal to, or larger than zero.

Table 7-1. Calculating the Dynamic SIMM characteristics for the three time points for counterparty Portfolio 1. The resulting Equations are used as input for the optimization problem.

Date point	**Equations**	**Target (Dynamic SIMM value/SIMM t_0 value)**
01 June 2019	$\frac{112 \cdot x_1 + 152 \cdot x_2 + 8 \cdot x_3}{27 \cdot x_1 + 202 \cdot x_2 + 43 \cdot x_3}$	$\frac{12.022 \cdot 10^7}{15.409 \cdot 10^7} = 0.7802$
01 September 2020	$\frac{178 \cdot x_1 + 92 \cdot x_2 + 2 \cdot x_3}{27 \cdot x_1 + 202 \cdot x_2 + 43 \cdot x_3}$	$\frac{13.080 \cdot 10^7}{15.409 \cdot 10^7} = 0.8489$
01 September 2024	$\frac{270 \cdot x_1 + 2 \cdot x_3}{27 \cdot x_1 + 202 \cdot x_2 + 43 \cdot x_3}$	$\frac{2.716 \cdot 10^7}{15.409 \cdot 10^7} = 0.1763$

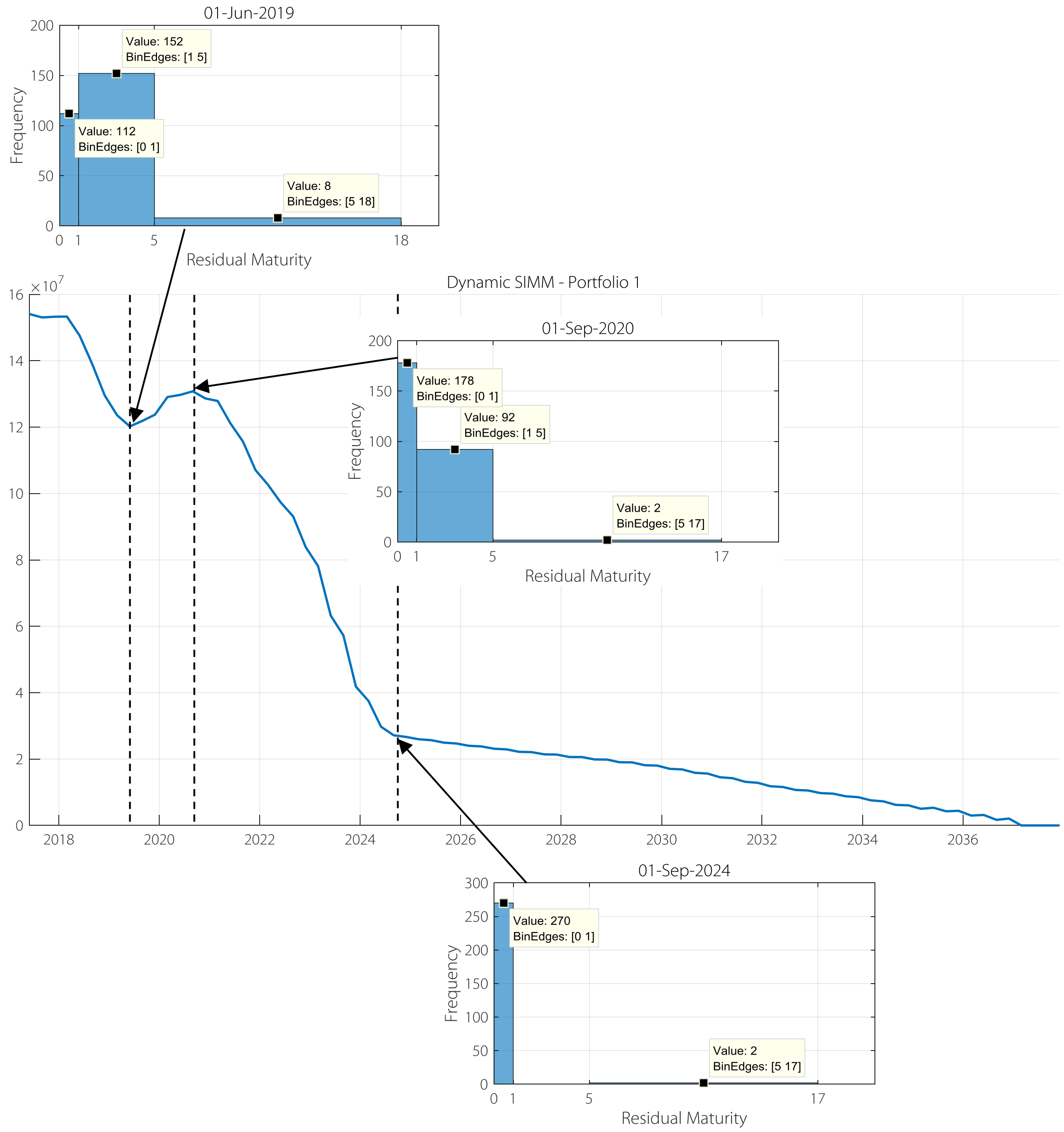

Figure 7-7. Dynamic SIMM for unaltered Portfolio 1 with the added counts of trades with their residual maturity at points: 01 June 2019, 01 September 2020, and 01 September 2024.

We start our optimization by bounding the possible add-ons between 0.0 and 0.5. The algorithm takes steps of 0.001 and calculates whether the total error function has decreased. The error function that we use is the mean squared error (MSE):

$$MSE = \frac{1}{n}\sum_{i=1}^{n}\left(\hat{Y}_i - Y_i\right)^2$$

where $\hat{\boldsymbol{Y}}$ represents the vector of target values and $\boldsymbol{Y}$ represents the vector of observed values. We can also apply different weights to each observation. For example, we apply a weight of 2 to the first observation and weights equal to 1 to the second and third observation. This gives us the add-ons as provided in Table 7-2. The weighted schemes with the new add-ons are shown in Figure 7-8.

Table 7-2. New add-ons by optimizing the problem from Table 7-1 with weight 2 applied on Observation 1 and weights 1 applied on Observations 2 and 3.

Residual Maturity (in years)	**Old add-ons**	**New add-ons**
≤1	0.0	0.0610
Between 1 and 5	0.005	0.2920
>5	0.015	0.0

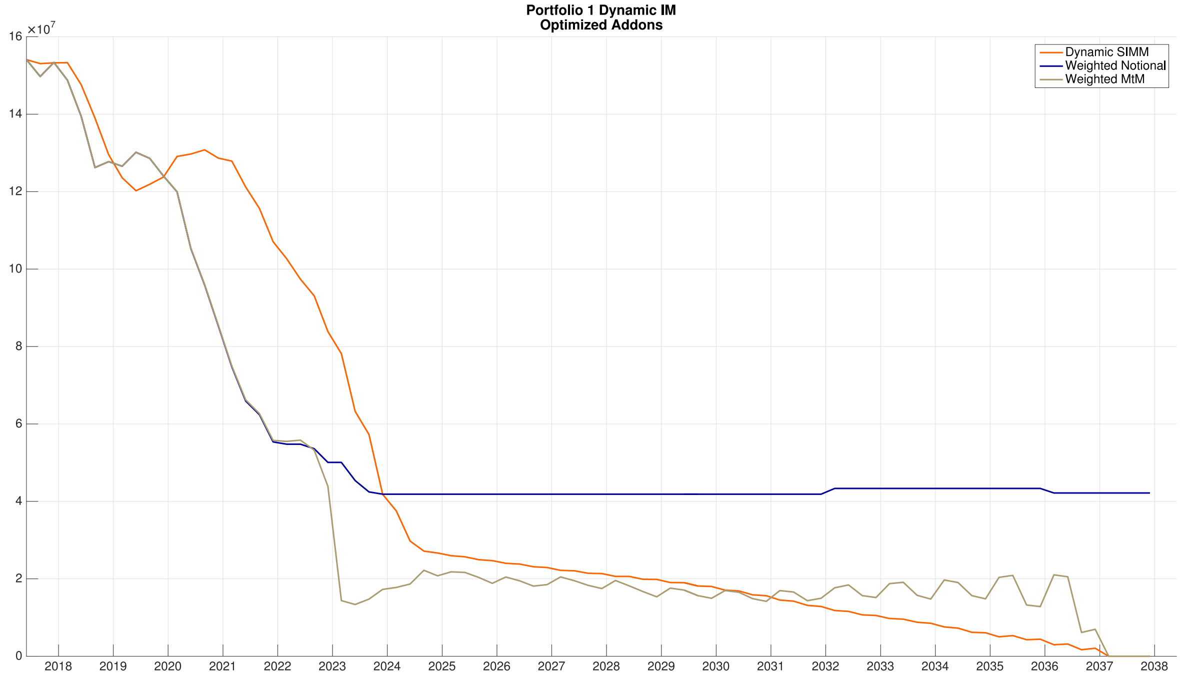

Figure 7-8. Comparing the Dynamic SIMM with the weighted amortization methods, where the new add-ons (as listed in Table 7-2) are implemented, for Portfolio 1.

In Figure 7-8, we see that we have improved the performance of the weighted methods up to year 2024. Around that year, we see that the two weighted methods starts to diverge. The Weighted Notional amortization method overstates the required future SIMM significantly (this has to do with the add-on for the shortest residual maturity; as this add-on is not zero anymore, the IM forecast at the end of the portfolio's life is significantly overestimated), whereas the Weighted MtM method stays closer to the Dynamic SIMM from 2024 onwards. So far, the net-to-gross ratio of MtM has not made a really significant difference when the Weighted Notional and Weighted MtM methods were compared. However, because of the net-to-gross ratio of MtM and its profile near the end for this portfolio (again see the scaling function in Figure 7-4, middle row), we are allowed to use larger than zero add-ons for the short residual maturity (i.e., residual maturity ≤1 year, this had a zero add-on with the old add-ons scheme) without extremely overstating the required IM towards the portfolio's ending, because the net-to-gross ratio of MtM normalizes the sum of add-ons, making the total value of future SIMM lower.

We can only use non-zero add-ons for the short residual maturity when the net-to-gross MtM ratio is below 1 toward the portfolio's end-of-life (then we end up with lower values of required future SIMM than is the case if we just sum the add-ons, as with the Weighted Notional). However, when looking at the scaling functions in Figure 7-5 bottom row and Figure 7-6 middle and bottom rows, we see that the net-to-gross ratio of MtM is significantly larger than 1 near the end of the portfolio. For such cases, we cannot use non-zero add-ons for the short maturity, because the sum of the add-ons would be amplified, cause an extreme overstatement of future SIMM at the end of the portfolios. As we have mentioned previously, the net-to-gross ratio of MtM is not a well-performing predictor of future delta sensitivities, so when the add-ons are determined, this should be kept in mind.

With the Weighted Notional amortization method, we see that larger than zero add-ons for the short residual maturity leads to extreme overestimation of required future SIMM at the portfolio's ending, due to the unnormalized sum of add-ons. This will be the case for each portfolio. In the same way, we could obtain new add-ons for the other portfolios or across portfolios.

A final remark on updating the add-ons, is that the obtained add-ons are likely not going to give a good performance for the other portfolios. In order to develop common add-ons, we should add multiple time points across a large variety of portfolios and then perform the optimization. Additionally, we could set the boundaries of the add-on bins differently than we have done in the example and we could make the add-on scheme more granular by creating a scheme that has more than three add-ons. Making the add-on scheme more finely granulated would also allow us to force the short residual maturity add-on to be zero to improve the performance of the Weighted Notional model at the portfolio end-of-life. Setting the short-residual maturity add-on to zero in a three-add-on-scheme has too much impact on the entire weighted method profile. With a more finely granulated add-on scheme, we could isolate the impact of the short residual maturity add-on to the final time points of the portfolio. However, increasing granularity of the add-on scheme and introducing the possibility to alter add-on bins in the optimization problem significantly increases the complexity of determining new add-ons. The goal of this section was to show that a simple improvement of a three-add-ons-scheme can increase performance of the weighted methods (on some sections of the) IM amortization profile. Developing a single set of add-ons that perform well for all representative portfolios is out of scope for our research.

7.2 American Monte Carlo and Dynamic SIMM

In this section, we show some results for the AMC method and its comparison to the Dynamic SIMM. We first start at the trade level to show the performance of the AMC method for two different trades with different residual maturity. In literature, it is noted that the AMC method performs less when the time horizon is larger (Caspers *et al.*, 2017). We see this occurring in Figure 7-9. It is a general picture that we saw when tested across several trades. The divergence of the AMC method from Dynamic SIMM becomes even more when the trades have a residual maturity longer than 10 years. The explanation for this observation is that the risk factor dynamics for the AMC method and the Dynamic SIMM differ. For instance, the Monte Carlo simulation (which is dominant in the AMC model) uses a different correlation table for the correlations between the different tenors, which has no direct relation with the SIMM tenor correlation table. As the trades become more exposed to longer tenors, the correlations in the tables start to differ more and more. Therefore, it is expected that the AMC method will perform less when counterparty portfolios contain trades with a longer residual maturity.

Here, we would like to add another comment regarding the weighted methods. The step-wise character of the weighted methods is clearly distinguishable with the single trades in Figure 7-9. In the left-half of the figure, we see that there is only one step: the trades ages from the 1 year < residual maturity ≤5 year – add-on to the residual maturity ≤1 year – add-on (which is zero). The right-half of the figure shows one more step in the amortization function of IM, because of the longer residual maturity of the trade and the corresponding multiple add-ons. Furthermore, we see a difference in the right-half of the figure between the Weighted Notional and Weighted MtM methods. The Weighted MtM method is zero during the entire lifetime of the trade, whereas the Weighted MtM shows a non-zero step function. The reason for this is that the MtM of the trade is negative up to the trade's end-of-life. This negative MtM causes the denominators in Equation 7-2 to be zero. With our current implementation of the Weighted MtM methods, this gives a zero value for the amortization.

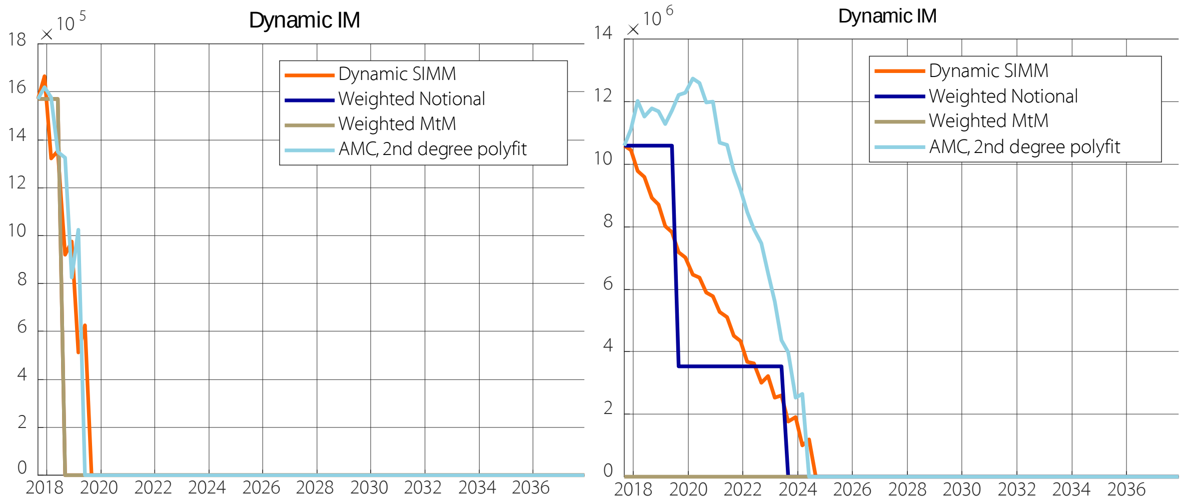

*Figure 7-9. Comparison of the different IM forecasting models for two receiver trades: **left**: residual maturity is approximately 2 years, **right**: residual maturity is approximately 6 years.*

Furthermore, in the above example at trade level, we have not used any scaling on the AMC method (except for α_0, the scaling function was discussed in Section 5.4). Generally, the AMC method performs less well than the weighted IM amortization methods when no scaling is applied. Only after applying scaling, the values of the AMC method become comparable with the Dynamic SIMM and we can overcome the divergence between the two methods for the longer time horizons. An overview is given in Figures 7-10 to 7-12 to show the improvements in forecasting performance by using a scaling function. The AMC is scaled according to the scaling function in Equation 5-8. The three portfolio-corresponding α^0, α^∞, and β parameters are defined in the figures, above each graph. The three 4-step discontinuous h(t) functions are defined as:

Portfolio 1

$$h(t) = \begin{cases} 0, & 0 \le t < 1 \\ -0.1, & 1 \le t < 11 \\ -0.28, & 11 \le t < 24 \\ 0.42, & t \ge 24 \end{cases}$$

Portfolio 2

$$h(t) = \begin{cases} 0, & 0 \le t < 4 \\ -0.15, & 4 \le t < 7 \\ -0.3, & 7 \le t < 29 \\ 0, & t \ge 29 \end{cases}$$

Portfolio 3

$$h(t) = \begin{cases} 0, & 0 \le t < 1 \\ -0.3, & 1 \le t < 2 \\ -0.5, & 2 \le t < 16 \\ 0, & t \ge 16 \end{cases}$$

We have performed the scaling of the AMC method by keeping the α^0, α^∞, and β parameters constant through time and selecting a relatively simple h(t) function. In a similar way, we can find (more complex) h(t) functions to scale the AMC for all the portfolio ratios in order to better replicate the Dynamic SIMM. The left-hand graphs in Figure 7-13 show the scaled AMC method for counterparty Portfolio 1, for each portfolio composition. The other two portfolios show the same precision of replication of the Dynamic SIMM, even with the most extreme case in Portfolio 3C, it is possible to almost exactly replicate the Dynamic SIMM forecast. The h(t) functions used for the three composition-different portfolios are presented in the right-hand graphs in Figure 7-13. Although, the h(t) functions differ, they share the well-shape (albeit under different orders of magnitudes and widths), with the used α^0, α^∞, and β parameters. The general picture was that the width of the well-shape in the h(t) functions corresponds to the observed width of the bumps in the Dynamic SIMM, because the width of the Dynamic SIMM indicates the range for which the AMC profile needs to be adjusted by the h(t) function.

Here, we came up with different α^∞, $\beta(t)$, and $h(t)$ profiles for nearly each experimental portfolio situation. The reason for this approach is that we are not trying to calibrate single value scaling parameters, because such a task would require a research in itself. Rather, we are merely trying to show the potential of the AMC method with these scaling parameters and more specifically with relatively simple step functions for the h(t) scaling parameter. In practice, it is required to define single α^∞, $\beta(t)$, and $h(t)$ scaling parameters, which are calibrated so that they provide a single (best approximation to Dynamic SIMM) solution for all possible portfolio compositions and show insensitivity to volatility, to some extent (more on this later in this chapter). Then, it would not be recommendable to try to fit the AMC profile closely to a noisy Dynamic SIMM, because then the probability of overfitting the AMC method to this single counterparty portfolio would be significant, thereby creating a loss of generalization potential for the AMC method over other, less noisy Dynamic SIMM profiles.

Compared to the weighted methods, for the AMC method to perform well under a great variety of counterparty portfolios, it requires a complex procedure of calibrating the parameters, as there is more to consider. In the demonstration, the α^∞ and $\beta(t)$ parameters were constant through time. It is also possible, just like with the $h(t)$ function, to make the α^∞ and $\beta(t)$ parameters functions of time. All in all, the requirement for a complex calibration procedure puts the AMC method at slight disadvantage, compared to the weighted methods, but the precision with which the AMC can replicate the Dynamic SIMM IM forecast provides an incentive to consider engaging in the complex calibration process.

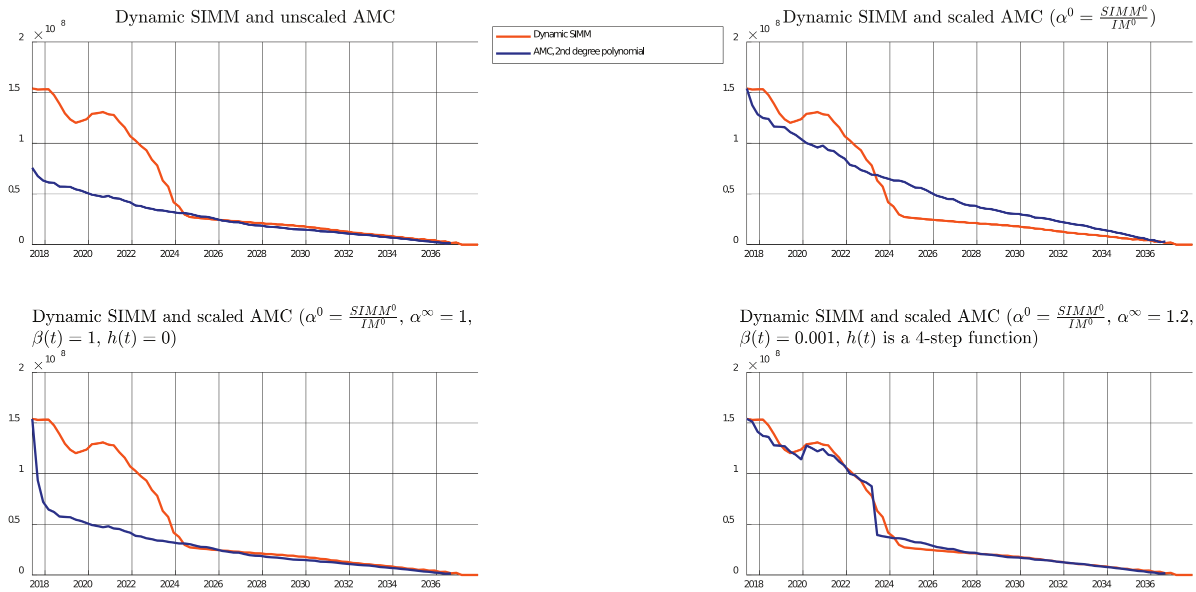

Figure 7-10. Unaltered counterparty Portfolio 1 with: ***left-top corner****, the unscaled AMC;* ***right-top corner****, t_0-value scaled AMC;* ***left-bottom corner****, scaled AMC by defining all the parameters, except for the h(t) parameter;* ***right-bottom corner****, AMC scaled by defining all the parameters, including the h(t) function as a 4-step discontinuous function.*

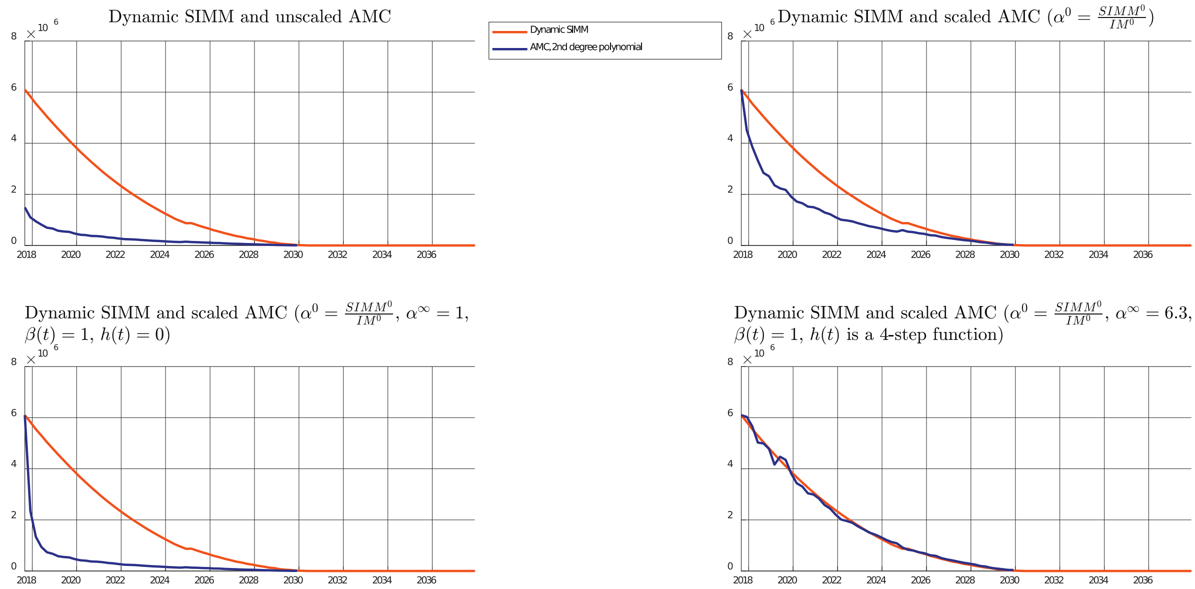

*Figure 7-11. Unaltered counterparty Portfolio 2 with: **left-top corner**, the unscaled AMC; **right-top corner**, t_0-value scaled AMC; **left-bottom corner**, scaled AMC by defining all the parameters, except for the h(t) parameter; **right-bottom corner**, AMC scaled by defining all the parameters, including the h(t) function as a 4-step discontinuous function.*

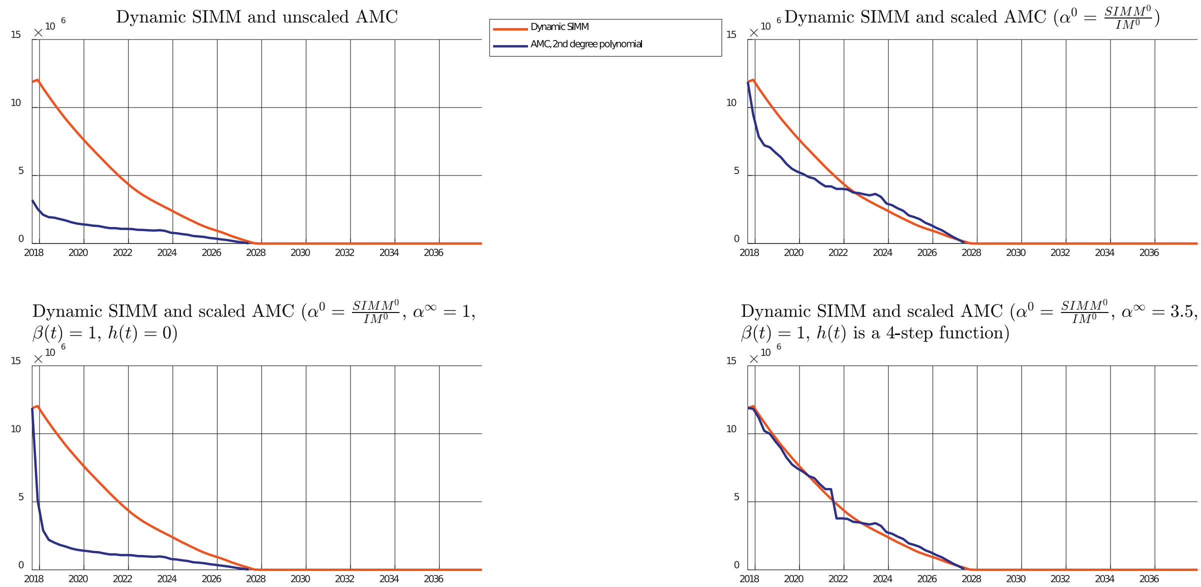

Figure 7-12. Dynamic SIMM and AMC methods, for unaltered counterparty Portfolio 3.

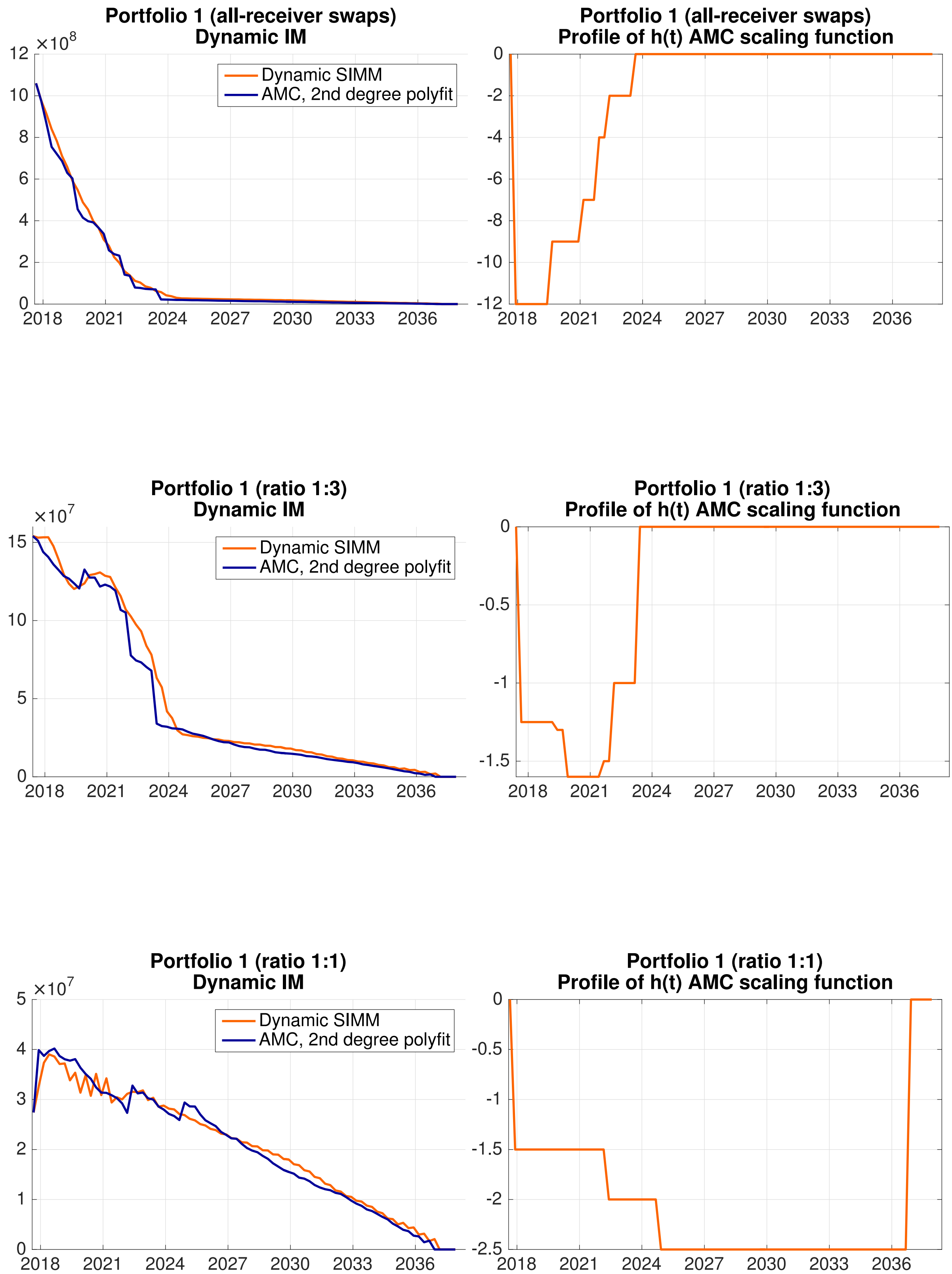

Figure 7-13. The left-hand graphs show a comparison of the AMC method with the Dynamic SIMM for Portfolio 1 at the different ratios: **A)**. *All receiver swaps ($\alpha^\infty = 1, \beta = 10$),* **B)**. *25% payer swap ($\alpha^\infty = 1, \beta = 10$),* **C)**. *50% payer swaps ($\alpha^\infty = 0.3, \beta = 10$).*

The right-hand graphs show the corresponding profile of employed h(t) scaling functions for each portfolio composition.

7.3 Impact of volatility

In the analysis, we have not yet considered the impact of market volatility on the various IM forecast models. That is the focal point in this section. Figure 7-14 shows the impact on counterparty Portfolio 1. This is a general image that we see with the other portfolios as well (and the various ratio portfolios). We can see that the Dynamic SIMM is hardly affected by the volatility level (one has to enlarge the figure extremely to see that the curves almost overlap). The reason for this is that SIMM is calibrated to data, from which a large proportion is stressed data (ISDA has calibrated the model against a 1 year stressed period and a 3 year normal period). The result in Figure 7-14 is therefore in line with the non-procyclicality criterion identified by the ISDA SIMM Committee.

Moving to the Weighted Notional model, we see that the model is not affected by market volatility. This makes sense, off course, since the model does not include any opportunity for volatility to be taken into account, as only add-ons are used. The Weighted MtM model, on the other hand does show some movement when the volatility is changed. However, the movement is negligible. The main reason is that the Weighted MtM model uses ratios (the net-to-gross-ratio of MtM at time t and at time t_0). These ratios have the capability to offset the volatility impacts to a high degree. As the volatility may impact the overall level of the MtM, the relative difference is quite stable.

The only tested IM forecasting model that shows high sensitivity to market volatility is the AMC method. This is because the model has not been calibrated to any market data and it does not use any ratios to absorb the volatility changes.

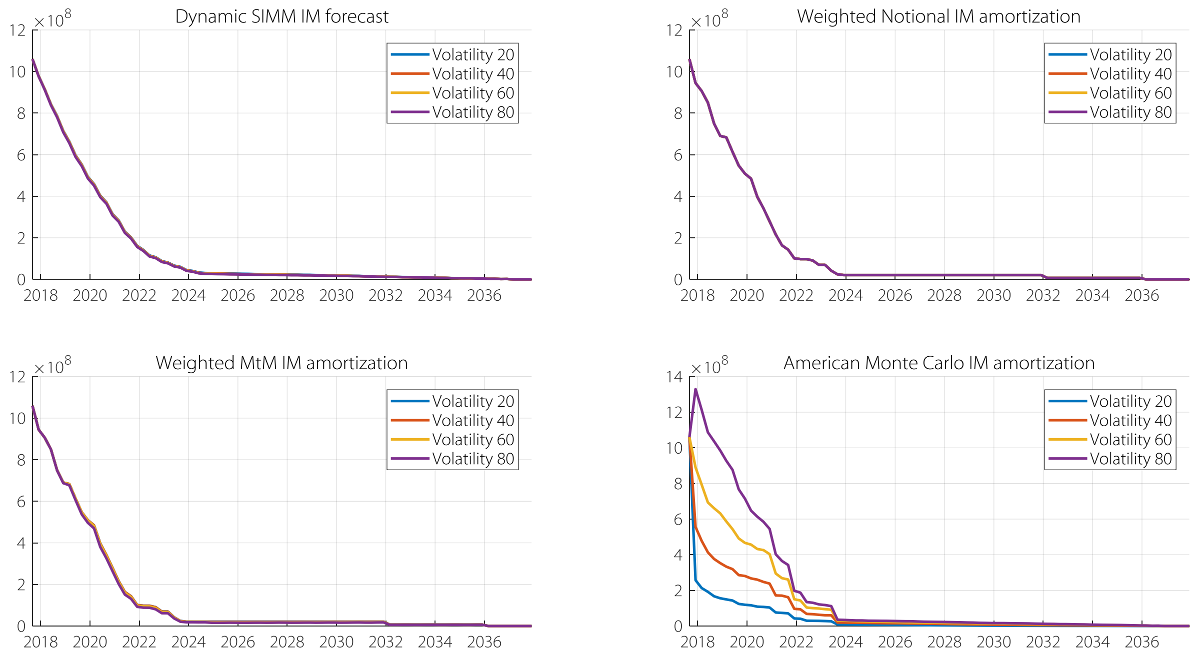

Figure 7-14. The impact market volatility on the IM forecasting methods on unaltered counterparty Portfolio 1.

7.4 Impact of IM Methods on PFE

The main reason why we develop these models it to decrease counterparty exposure. To assess how well the models perform in decreasing the exposure, we need to see their impact on the PFE profile. This is shown in Figures 7-15 to 7-17 for the three counterparty portfolios and their ratios of payer-to-receiver swaps. The top graphs in each subfigure show the range between the 2.5 and 97.5 percentiles of the MtM evolution. The MtM evolution is the driver of the VM through the employed Brownian Bridge. The bottom-right graphs (PFE profile) in each subfigure show the residual exposure after the VM is exchanged (the dashed magenta graph). We also term this PFE profile as the zero-IM PFE. The zero-IM profile is reduced with the forecasted IM methods, for which the profiles are shown in the bottom left graphs of each subfigure. We have also introduced a constant IM profile, the size of t_0-SIMM value. This is done to replicate the so-called Independent Amount (IA) that is used in collateral agreements between counterparties and part of Rabobank's current collateral modelling. With the AMC methods in Figures 7-15 to 7-17, we have used the individually optimized α(t) functions. The weighted methods have their original (non-improved) add-ons.

Generally, the IM does what it is intended to do and that is to reduce the exposure, regardless of IM forecasting method. However, the Dynamic SIMM (our benchmark) and the matched-performance AMC method are seen to reduce the PFE most. Also, we see from the figures that, as the composition of the counterparty portfolios becomes more balanced between payer and receiver swaps, there is a higher residual exposure (after the VM and IM have been reduced), especially when the weighted methods are used to forecast future SIMM. The reason we see this is that the delta offsetting (as discussed in Section 6.6) reduces the t_0-SIMM value. As this value is the starting point for all the IM forecasting methods, the overall level of the forecasted IM is reduced. The Weighted Notional and the Weighted MtM in their current form, never provide a profile where a future SIMM value is larger than the t_0-IM value as these methods show a rather simple, decreasing profile. This is inherent to their normalization, driven by the ageing of trades. So, a lower t_0-IM value causes the entire profile of the weighted methods to be reduced. However, the balance of the two types of trades in a portfolio does not affect the total MtM to the same extent, as does the delta profile. This is the reason that the weighted methods show increasing residual exposure in the PFE figures as the portfolio contains a more balanced ratio between payer and receiver swaps.

However, the Dynamic SIMM method and the improved AMC method are able to show forecasted IM values that exceed the t_0-IM value. As the profile of the delta exposure shows more fluctuation when payer and receiver delta offsetting occurs, the fluctuation becomes such that the total delta exposure is higher at some future time points than it is at t_0. Correspondingly, the Dynamic SIMM and matched AMC method are able to show higher future SIMM values than the t_0-IM. We have seen that there is a strong delta reduction over the entire portfolio lifetime (not only at t_0) due to delta exposure offsetting when the number of payer swaps is increased in the portfolios to match the receiver swaps. As such an offsetting is not reflected to the same extent in the MtM (and correspondingly in the tightly related VM), the Dynamic SIMM and the matched-performance AMC method forecast a future SIMM profile that is increasingly below the zero-IM PFE as the trade composition is more balanced within the portfolios. A best-case example for this is the situation in Figure 7-16, 25% payer swap. When we compare the reduction in delta exposure from Figure 6-9 to the total MtM reduction in Figure 7-16, we see that there is no direct relationship between the two and that the offsetting in delta is relatively higher than the offsetting in MtM when the portfolio composition is adapted to attain a more balanced portfolio. Because the total delta exposure levels (and the t_0 delta exposure) are so low in this situation, we see that there is not one IM forecasting model that reduces the PFE to zero during the entire lifetime of the portfolio.

Another observation that we can make comes from Figures 7-15A, 7-16A, 7-16B, and 7-17A. We see that over the entire lifetime of the portfolios the PFE is reduced to zero, except by the weighted methods for the last year of the portfolios' life. This is due to the add-ons we are using in the weighted methods as described in Section 6.5. With the used add-ons, every trade with a residual maturity of less than or equal to 1 has an add-on of zero (for the interest rate risk class) and thus no risk is assigned to those trades. Therefore, the final year of a portfolio's lifetime has a zero IM forecast under the weighted methods and no PFE reduction takes place in that final year under these two methods.

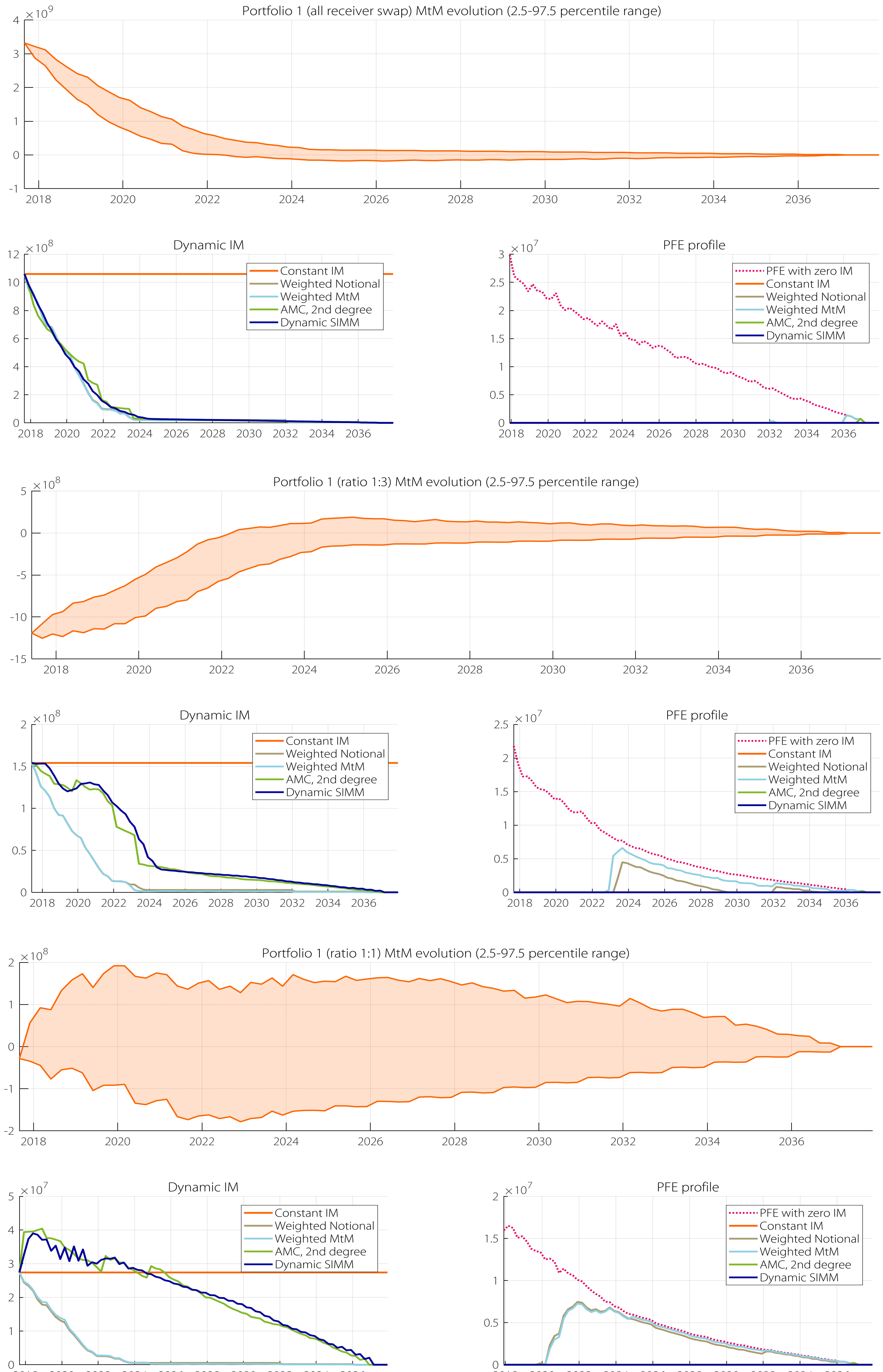

Figure 7-15. MtM, Dynamic IM, and PFE profiles for Portfolio 1 at the different ratios: A). All receiver swaps, B). 25% payer swap, C). 50% payer swap.

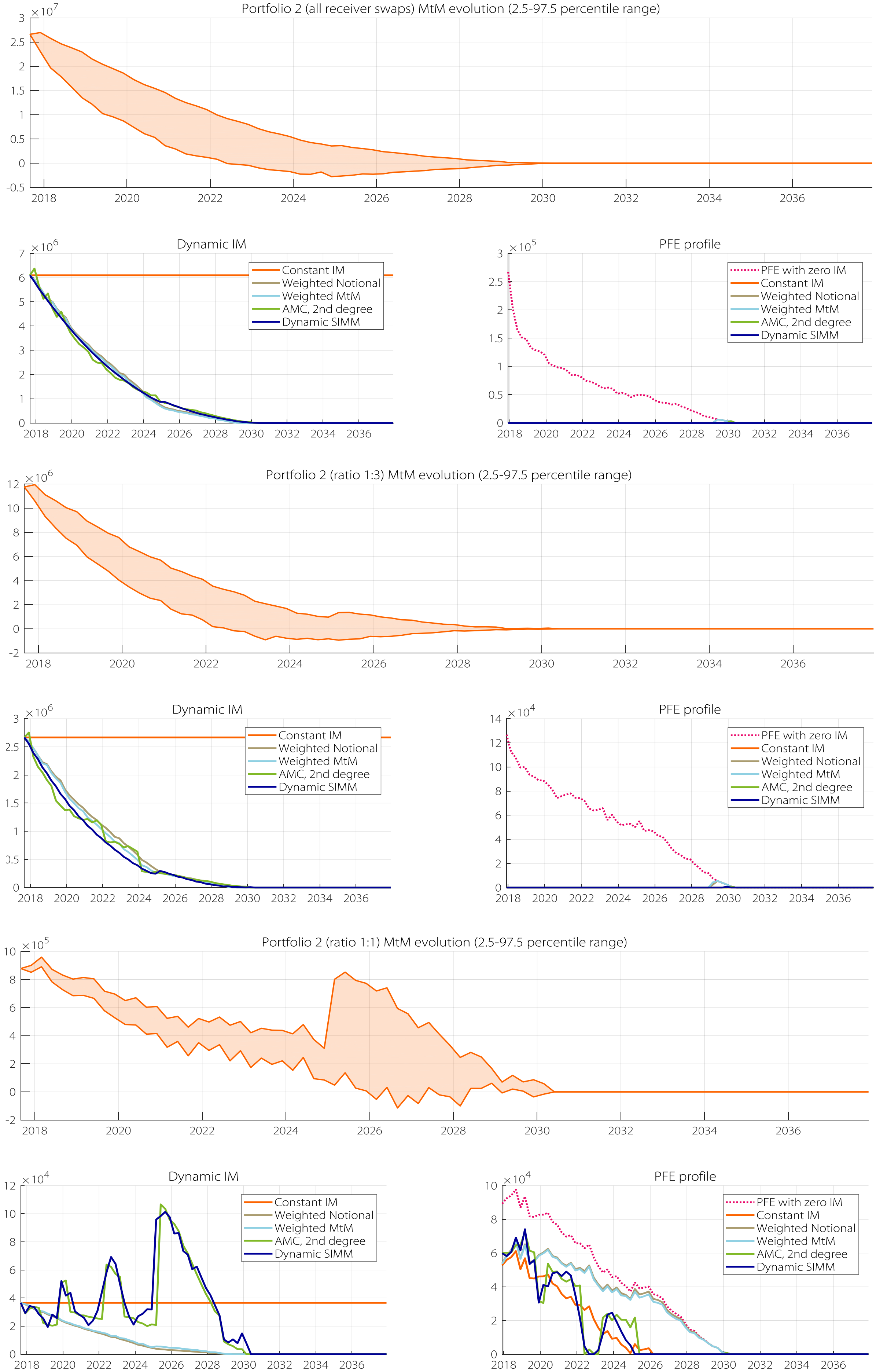

Figure 7-16. MtM, Dynamic IM, and PFE profile for Portfolio 2 at the different ratios: A). All receiver swaps, B). 25% payer swap, C). 50% payer swap.

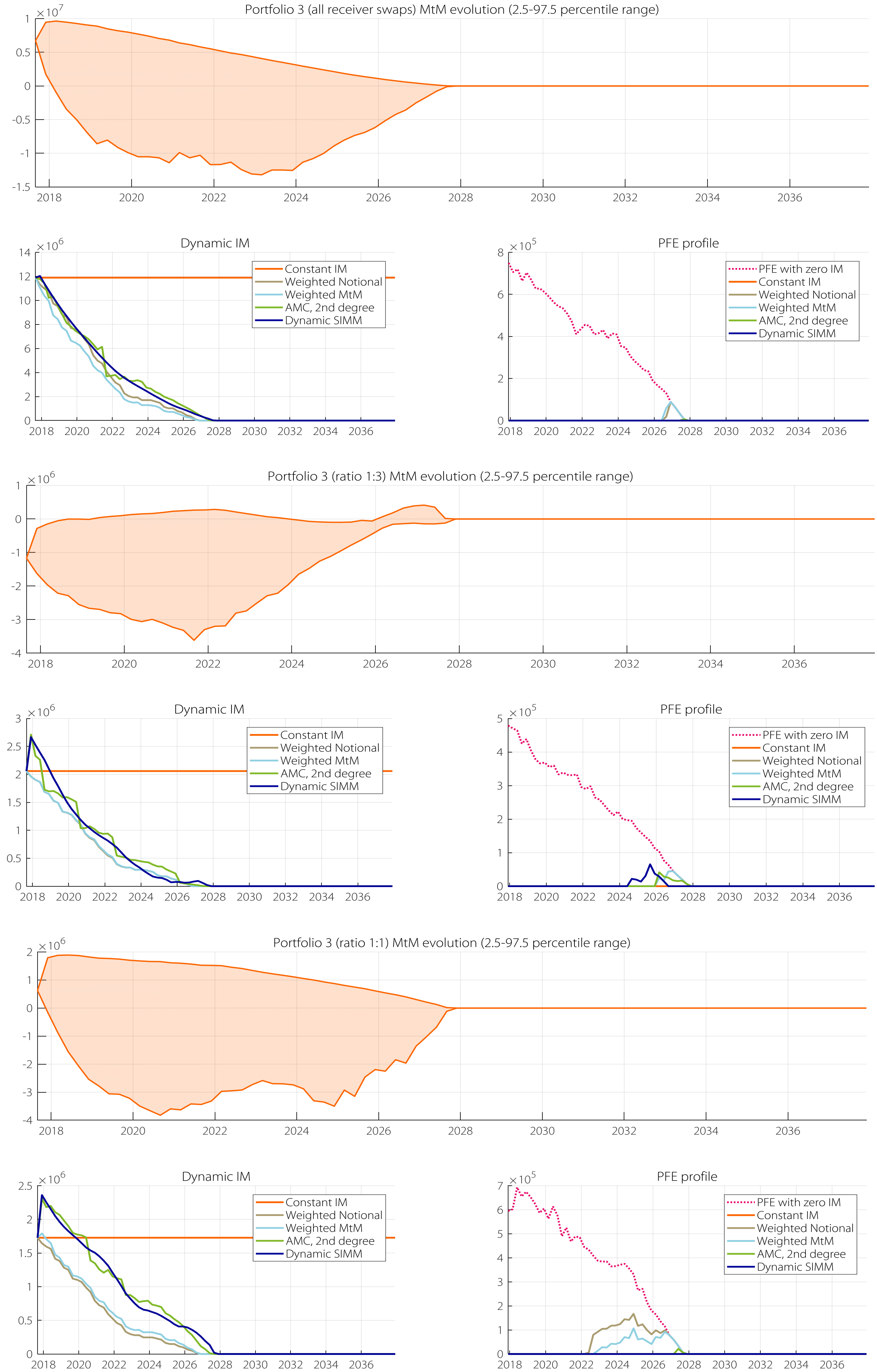

Figure 7-17. MtM, Dynamic IM, and PFE profile for Portfolio 3 at the different ratios: A). All receiver swaps, B). 25% payer swap, C). 50% payer swap.

We need to address one more thing related to the PFE profiles in Figures 7-15 to 7-17. We need to keep in mind that the PFE profiles obtained do not include any thresholds or minimum transfer amounts. The levels of such (transfer) thresholds may influence the entire level of the PFE. The situation depicted in our figures may give a more optimistic view for the PFE reduction than would be the case when the (transfer) threshold would have been applied.

8 Discussion

8.1 Performance and implementation

From our results, we briefly outline the observations:

- The weighted IM amortization methods perform best when the Dynamic SIMM has a simple, decreasing profile. This is the case when a counterparty portfolio consists of (mainly) a single type of vanilla swap (i.e., all-receiver or all-payer swap portfolios). The Dynamic SIMM shows a simple, decreasing profile for such portfolios because the delta exposures from the trades within an all-receiver or all-payer swap portfolios are not significantly offsetting each other, like in the case of mixed receiver and payer swap portfolios. The dominating driver of the portfolio then becomes the ageing of the trades and this is something that is captured well by the weighted methods. As soon as there is significant offsetting of delta exposures by receiver swaps and payer swaps in a counterparty portfolio, the dynamics of the delta offsets become the main driver for the future SIMM profile and the weighted methods fail to capture these delta dynamics.
- The Weighted MtM method in its current form has a rather simple, decreasing IM profile, but shows slightly more complexity than the IM profile provided by the Weighted Notional method. In low delta environments, caused by offsetting payer and receiver swaps, the Weighted MtM method performs slightly better (profile closer to Dynamic SIMM profile) than the Weighted Notional method, due to the netting of MtM of the offsetting trades. Thereby the Weighted MtM method captures slightly more of the portfolio dynamics than is the case with the Weighted Notional method.
- The Weighted Notional method in its current form has a simple, decreasing IM profile that is driven only by the aging of trades within the portfolio and the method is insensitive to trade composition of the portfolios (i.e., to whether the portfolio is an all-receiver swap portfolio or it consists of half payer swaps, half receiver swaps). Therefore, this method is the least capable of all the proposed methods of reflecting portfolio dynamics.
- We are able to improve the performance of both weighted methods under trade mixture portfolios by finding add-ons that better reflect the portfolio composition. Also, the number of bins to which the trades can be categorized (their residual maturity being the categorical) can be enhanced to show better performance
- The AMC method is capable to mirror the Dynamic SIMM to a high degree of accuracy by employing the scaling function. Within the scaling function, parameters can be set to capture the speed at which the IM model reverts to its long-term IM value. As the mean-reversion parameter can be function of time, rather than a constant, it is possible for the AMC method to better capture the dynamics of a group of portfolios with common characteristics than is possible with the weighted methods.
- The Dynamic SIMM method, as our benchmark, is best capable of capturing the delta dynamics of the portfolios, because of its reflection of the dynamic sensitivities. We have seen that the absolute value of the total delta is a good predictor for the Dynamic SIMM profile.
- The evolution of the portfolio MtM is not a good predictor of the delta exposure evolution. As the number of payer and receiver swaps becomes increasingly balanced, all the methods become less able to cover the PFE exposure.

These observations are summarized in Table 8-1 below. Additionally, we have added the performance elements *computational requirements* and *implementation* along which we compare the proposed IM forecasting models. These two elements are related.

The calculation underlying the Weighted Notional method uses as inputs the outputs of Rabobank's systems. All the necessary elements are already present in the current systems and can be used with nearly no adaption needed. The only element that is not directly provided by the current system is the add-on ratio, but it is expected that outputting this ratio from the current system will not provide a significant challenge to implement, as the regulatory add-ons are already in use.

The weighted MtM method has the additional net-to-gross ratio that needs to be calculated. It is possible to calculate this ratio with the current systems at Rabobank, however, it needs an adjustment to the input provided to the simulation engine. At this point, a netted MtM and a sum of gross MtMs for a portfolio can be obtained by manually adjusting the input parameters for the simulation engine. This would require an automatic calculation procedure from a single run to determine the netted and grossed MtMs. Automating this process will provide a minor challenge.

The AMC method uses a Brownian Bridge to determine the valuation date + MPOR MtM values for each scenario, which are used in combination with the valuation date MtMs as inputs for the regression. Rabobank has set up a Brownian Bridge to obtain the MtM at non-simulation dates. This Brownian Bridge has a pre-set grid that does not coincide with the valuation date + MPOR. So, in order to obtain these values, the current system that the Rabobank employs has to be adjusted. This puts a burden on the simulation engine and increases computational requirements. Furthermore, the regression calculation adds to the implementation challenge and computational requirements.

Table 8-1. Summary for the IM forecasting models

	Weighted Notional	Weighted MtM	AMC
Accuracy	--	-+	+
Portfolio dynamics	--	-+	+
Computational requirements	++	++	-
Implementation	++	-+	-

8.2 Getting ready for t_0-SIMM

Finally, we comment on the implementation of the t_0-SIMM, which has been the cornerstone of our IM forecasting methods. To implement the t_0-SIMM, Rabobank faces a complex data and system optimization challenge, because the format and granularity of the sensitivities required for SIMM does not coincide with the format and granularity that is currently output by Rabobank's systems. A mapping procedure will be required with the additional streamlining of various fragmented internal systems. As Rabobank operates multiple systems for the various asset types and various trading books, the first challenge will be to consolidate the dispersed trades into one system. During our relatively short period of research, the difficulties of fragmented systems were already felt due to the fact that different systems assign different trade IDs to the same trades. For example, the front-office system that calculates the sensitivities uses a different transaction ID than does the simulation engine for the same trades. Matching the sensitivities for the t_0-SIMM with the simulated MtMs needed to amortize the t_0-IM with the Weighted MtM method under the current IT landscape at Rabobank will require a complex automation procedure.

The next challenge lies in the fragmentation of the front-office risk management systems and the collateral management systems. As the front-office systems provide the sensitivities for the initial IM calculation, there needs to be a streamlined process between the front-office systems and the collateral management systems in order to translate the initially calculated IM to collateral requirements. Another challenge arising is the processing of disputes related to IM exchange.

It will become imperative for the Rabobank to reconcile the front-office systems at the group level. In order to reconcile the fragmented systems to calculate IM under SIMM, the Common Risk Interchange Format (CRIF) will provide some guidance. This is a standard format for risk sensitivities developed by ISDA (ISDA, 2017) to standardize the input for margin calculations. Adopting this format at each system will allow the fragmented systems to communicate the sensitivities across system interfaces and facilitating the sensitivities reconciliation.

9 Conclusions

We have proposed three different approximation methods to forecast IM and benchmarked them against the Dynamic SIMM. We have created experimental portfolios, consisting of only IRSs to test our methods. We have excluded the notional ration from the weighted methods. As the assumption was that the notional is constant throughout time and across scenarios, it only captures the ageing of the trades, which is already done by the add-on ratio.

The results show that the AMC method has the greatest potential to forecast IM, as it has the highest accuracy of tracking the Dynamic SIMM profile. Under conditions in which the portfolio consists of a single vanilla swap type (i.e., only receiver or only payer swaps are included in the counterparty portfolio), the AMC method as well as the Weighted MtM method are able to track the Dynamic SIMM profile to a high degree. When a portfolio has a mixture of payer and receiver swaps, we see that the weighted methods start to perform badly, but the Weighted MtM performs slightly better than the Weighted Notional, because it is able to capture the portfolio dynamics better than the Weighted Notional method, which is insensitive to whether a trade is a payer swap or a receiver swap. The only drivers that change the profile of the Weighted Notional method under an all-receiver portfolio and the various ratio portfolios, is the t_0-IM value and the ageing of the trades.

Although the AMC method is able to track the Dynamic SIMM to a high degree of accuracy, regardless of the portfolio composition (this was only the case when the α(t) scaling function was applied), this method requires a significant change in the way the simulation engine of Rabobank functions. Moreover, the method would require additional computational requirements in order to perform the regression.

An improvement of the Weighted MtM was seen when the add-ons were optimized for a single portfolio. In a similar way, the add-on scheme could be improved by considering an optimization over multiple portfolios with the most varying characteristics. Considering the possibility for such improvements and the ease of implementation of the Weighted MtM method, we recommend to implement the Weighted MtM method and investigate further its potential related to improvements (for example, developing a multiplier for the MtM net-to-gross ratio element to enhance its effect). An additional improvement for the Weighted MtM could entail implementing a function similar to the α(t) scaling function used for the AMC method.

The MtM net-to-gross ratio did not predict the delta exposure evolution well. Therefore, the net-to-gross ratio in the Weighted MtM method is not able to capture the delta dynamics fully. In order to better capture some of the delta dynamics by the Weighted MtM method, it should be considered to implement a net-to-gross MtM ratio multiplier. Additionally, we advise to create an add-on table that is optimized to implied add-ons from SIMM. One could determine the add-ons that would be implied by SIMM and base the add-ons table for the Weighted MtM method on those implied add-ons. With these improvement possibilities and taking into account its ease of implementation in current systems, we recommend the Weighted MtM method for Rabobank.

10 Future Research

We have focused on only IRS portfolios, disregarding non-linear products. As SIMM has two more components additional to the *DeltaMargin* (i.e., *VegaMargin* and *CurvatureMargin*), it will provide more dynamics for portfolios with non-linear products. Also, multicurrency portfolios should be considered. Our research could be extended by including such portfolios and comparing the performance of the proposed methods under such conditions in order to enhance generalizability of results across a larger variety of portfolio types.

We have proposed some optimizations for the analyzed methods in our research. The way the optimizations were carried out was not robust and was merely done for the sake of showing the models' potential. Therefore, an extension to our research could be to investigate robust methods to determine improved add-ons or optimize the AMC $\alpha(t)$ scaling function by accepted calibration methods.

As mentioned, the IT landscape of financial institutions must undergo some significant changes with the implementation of t_0-IM requirements. In order to determine an IT roadmap between the as-is situation and the to-be situation under IM requirements, it is necessary to build on our research from an enterprise architectural view. This means that gap analysis should be performed and a clear e-strategy should be developed to make the smoothest possible transition from the current state of IT systems towards the state capable of streamlining IM calculation with pricing and capital management. Inventory of system capabilities and mapping is an initial step.

For our research, the focus was forecasting IM for capital management purposes. However, there is a pricing aspect related to IM. IM will introduce a new term, the MVA, which is the present value of the future funding cost of IM. This impacts pricing and profitability. It is stated that the MVA will reduce the CVA significantly due to the fact that MVA also implies the possibility of counterparty default. A natural extension to our research would therefore be to perform analysis on how exactly the MVA will impact pricing and its implications.

When implementing SIMM and the standardized CRIF format, the collateral exchange process could be fully automated (for example by using blockchain technology in combination with CRIF), reducing grounds for disputes and being able to communicate the calculated sensitivities directly with the counterparty and automatically determine where the mismatches are between the two parties' calculated sensitivities. This would be an interesting topic to research, as its outcomes could transform the way collateral is exchanged, making the entire collateral exchange process more streamlined, adding to the bottom-line.

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A t_0-IM calculation

Even though the focus of our research is to calculate the t_0-IM by using SIMM, in this appendix section we discuss and outline an example calculation of the other standardized t_0-IM method as well: the Standardized Approach (SA). The reason for showing an SA calculation is purely for comparison purposes. The SA model will not be considered as a t_0 reconciliation for future IM forecasting.

The SA model, developed by the WGMR, is part of the SA-CCR framework and it uses standardized add-ons as provided in Annex IV of the Regulatory Technical Standards on Risk Mitigation Techniques (EIOPA, EBA & ESMA, 2016). The SIMM is a more comprehensive model, developed by ISDA, which uses transaction sensitivities.

Standardization of IM calculation is an important step in providing financial institutions some support in handling the increased complexity for handling collateral requirements in a multi-faceted regulation framework. Furthermore, by standardizing the calculation process for IM, financial institutions will face less disputes when it comes to the amount of collateral that should be posted.

For the discussion of the two models, two example trades are used. Those trades are within one single netting set. The two trades are summarized in Table 11-1 below. As we are focussing on IRSs in this thesis, we take the IRSs only in the calculation examples. We take today's date as 28th of April 2017.

Table 11-1. Example trades for calculation of IM at t_0 in the subsequent sections.

Trade	**Type**	**End Date**	**Residual Maturity**	**EUR notional**	**EUR MtM at t_0**
1	IRS	8th of May 2019	2.03 years	29,174,733.00	2,940,349.20
2	IRS	7th of October 2025	8.45 years	11,000,000.00	(3,037,202.06)

A.1 Standardized Approach (SA)

The Standardized Approach (SA) is an 'add-ons'-based model. IM is calculated by applying weights (add-ons) to the notional of the trade. In each netting set, the trades are categorized according to their asset class. This categorization is based on the trade's primary risk factor. Then, a sub-categorization is performed based on the residual maturity of the trades to determine the appropriate add-on. Table 11-2 summarizes the categorization and the corresponding add-ons. So, for example, if a trade has interest rate as its primary risk factor, then that trade should be categorized in the '*FX and interest rate*' asset class. Furthermore, if that trade today has a residual maturity of 3 years, then it should be sub-categorized in the '*2 – 5 year residual maturity*' subcategory. According to the table, the weight applied to that trade towards calculating IM should then be 0.02.

Table 11-2 – Add-ons for IM calculation with the SA as provided by WGMR. The add-ons depend on the asset class and the residual maturity.

Asset Class	**Subcategory**	**IM add-on**
Credit	0 – 2 years residual maturity	0.02
	2 – 5 years residual maturity	0.05
	more than 5 years residual maturity	0.10
Commodity		0.15
Equity		0.15
FX and interest rate	Foreign exchange	0.06
	0 – 2 years residual maturity	0.01
	2 – 5 years residual maturity	0.02
	more than 5 years residual maturity	0.04
Other		0.15

From the add-on procedure above it becomes clear that some maturing trades move to buckets in descending order of weight. As time to maturity decreases for some trades, these trades will be bucketed in a lower residual maturity bucket and a lower add-on will be applied. However, not all the asset classes roll into different buckets as time progresses. Trades that have a primary risk factor coming from *Commodity*, *Equity*, or *Other* risk exposures will face a constant add-on, independent of residual maturity. Another note to make is that when the primary risk factor for a trade cannot be clearly identified, then the highest add-on should be applied.

The IM calculation procedure under SA is as follows. First, the Gross Initial Margin (GIM) is calculated, taking into account the IM of all the trades separately and summing them:

$$GIM = \sum_{i=1}^{N} AddOn_i * N_i \qquad \text{11-1}$$

where N_i represents the notional of the trade *i*. At this point, netting is only allowed for trades that have identical feature and only have a differing notional. In that case *N* should be taken as the net values of those contracts.

After calculating the gross initial margin, the model attempts to capture the netting benefit of diversification in the netting set by transforming the gross initial margin into a net initial margin, which corresponds to the actual IM amount that should be posted to the counterparty. The Net Initial Margin (NIM) calculation is performed as follows:

$$NIM = (0.4 + 0.6 * NGR) * GIM \qquad \text{11-2}$$

where *NGR* is the so-called net-to-gross ratio that takes into account the netting set diversification benefits and is defined as

$$NGR = \frac{\left|\sum_{i=1}^{N} MtM_i\right|}{\sum_{i=1}^{N} |MtM_i|} \qquad \text{11-3}$$

where MtM_i is the replacement value, or Mark-to-Market value of trade *i*. The net IM is calculated for each asset class separately. This means that this model does not allow for cross-asset class diversification. From Equation 11-2 we can see that the *NGR* allows the net IM to reduce to 40% of the gross IM (*within a single asset class!*). The process for our two example trades is summarized in Table 11-3 below. The total amount of IM we should receive for these two trades is €409,397.86. As discussed, the IM is posted bilaterally, unlike VM. Even though we are out-of-the money with this two-trade netting set, we still receive IM.

Table 11-3. Calculation of t_0 IM for the two example trades under SA.

<table>
<tr><th>Trade</th><th>Residual Maturity</th><th>EUR notional</th><th>Add-On</th><th>EUR trade GIM</th><th>EUR Total GIM</th><th>NGR</th><th>EUR NIM</th></tr>
<tr><td>1</td><td>2.03 years</td><td>29,174,733.00</td><td>0.02</td><td>583,494.66</td><td rowspan="2">1,023,494.66</td><td rowspan="2">0</td><td rowspan="2">409,397.86</td></tr>
<tr><td>2</td><td>8.45 years</td><td>11,000,000.00</td><td>0.04</td><td>440,000.00</td></tr>
</table>

A.2 SIMM

The SIMM model is a sensitivity-based model. It uses sensitivities (Deltas and Vegas) as inputs and calculates IM based on correlation tables. The model is comprehensive as it tries to capture Delta, Vega, SIMM categorizes the instruments within a netting set according to their corresponding asset class. There are four asset classes recognized by SIMM:

- Interest Rates and Foreign Exchange (RatesFX).
- Credit.
- Equity.
- Commodity.

Every trade is categorized as one of the four asset classes and IM is calculated per asset class. This means that the netting is isolated to the asset classes and no cross-asset class netting is allowed, just as is the case with SA. To calculate the IM per asset class, the product is decomposed into its risk factors according to the six risk classes:

- Interest Rate.
- Credit (Qualifying).
- Credit (Non-Qualifying).
- Equity.
- Commodity.
- FX.

For each of the six risk classes, IM consists of the following elements:

$$IM_X = DeltaMargin_X + VegaMargin_X + CurvatureMargin_X + BaseCorrelationMargin_X$$

where *X* represents the risk class and the *BaseCorrelationMargin*-term applies only to the Credit (Qualifying) risk class.

So, different from the SA, a single trade can have more than one (primary) risk factors. For example, SIMM allows for recognition of interest rate risk as well as equity risk for equity derivatives. To aggregate the IM for an asset class, the following formula is used:

$$SIMM_{product} = \sqrt{\sum_r IM_r^2 + \sum_r \sum_{s \neq r} \psi_{rs} IM_r IM_s}$$

where *product* represents one of the four asset classes, *r* and *s* represent risk classes, and ψ_{rs} represents a correlation matrix for the different risk classes. All the $SIMM_{product}$ terms are summed to obtain the overall IM (i.e., the IM that should be received by the counterparty).

A.2.1 SIMM Calculation Example

In this section, we provide an example on how to calculate IM for the two-trade example netting set by using SIMM. For this, we use ISDA's SIMM Methodology document (ISDA, 2017), which provides us with the elements necessary for the calculation. Table 11-4 and Table 11-5 show the sensitivities for the two trades. All sensitivities are expressed in Euros.

The tables provide a typical sensitivities grid, but this sensitivities grid is not identical to the requirements for ISDA's SIMM model. First, we need to perform linear interpolation mapping to ISDA's required risk vertices and indices grid. Table 11-6 below shows the remapped grids for the delta sensitivities of the two trades. Since both trades are exposed to the EUR6M curve, we can map them directly to the LIBOR 6M curve in SIMM's yield grid (SIMM offers the following yield curve grid: OIS, LIBOR 1M, LIBOR 3M, LIBOR 6M, LIBOR 12M and only for US: PRIME). Now, we can net the delta exposures per yield curve, which is only the LIBOR 6M in our case. The netting is shown in the rightmost column.

Table 11-4. Trade 1 delta sensitivities

Risk Measure Tenor	EUR6MCurve
1d	8.88
1w	31.37
1m	
2m	
3m	
6m	16.53
9m	(16.24)
12m	(0.33)
15m	0.24
18m	75.26
21m	(0.22)
2y	(5,952.19)
3y	(16.20)
4y	
5y	
6y	
7y	
8y	
9y	
10y	
12y	
15y	
20y	
25y	
30y	
35y	
40y	
45y	
50y	
TOTAL	**(5,852.91)**

Table 11-5. Trade 2 delta sensitivities

Risk Measure Tenor	EUR6MCurve
1d	(1.34)
1w	
1m	
2m	
3m	(129.53)
6m	(355.48)
9m	1.03
12m	4.75
15m	(19.45)
18m	(13.67)
21m	77.42
2y	(51.84)
3y	4.06
4y	(1.39)
5y	3.90
6y	(10.04)
7y	1.73
8y	5,291.27
9y	3,934.88
10y	
12y	
15y	
20y	
25y	
30y	
35y	
40y	
45y	
50y	
TOTAL	**8,736.28**

Table 11-6. Remapped trade sensitivities from Rabobank's system grid to ISDA SIMM grid. All sensitivities are expressed in EUR.

ISDA SIMM Tenors	Trade 1 LIBOR 6M	Trade 2 LIBOR 6M	**Total LIBOR 6M**
2W	40.25	(1.34)	38.91
1M			
3M		(129.53)	(129.53)
6M	8.19	(354.95)	(346.76)
1Y	30.04	3.81	33.85
2Y	(5,915.18)	(6.11)	(5,921.29)
3Y	(16.20)	3.36	(12.84)
5Y		2,899.68	2,899.68
10Y		6,321.35	6,321.35
15Y			
20Y			
30Y			
TOTAL	**(5,852.91)**	**8,736.28**	**2,883.37**

With this cleaned input, we can start calculating the DeltaMargin for the interest rate class. With our two-trade netting set of IRSs, we have only one asset class and risk class to consider: RatesFX and Interest Rate, respectively. Furthermore, since we do not have any optionality (IRSs are linear products), we have no Vega and curvature risk, and so when we have determined the DeltaMargin for the interest rate class, we have our final IM value.

A.2.1.1 DeltaMargin

To obtain the delta margin for this trade, we need to assign risk weights to the sensitivities and determine the concentration risk associated with the delta sensitivities. This will give use the delta weighted sensitivities *WS*:

$$WS_{k,i} = RW_k s_{k,i} CR_b \tag{11-4}$$

where $s_{k,i}$ are the delta sensitivities as outlined in Table 11-6. The subscript k indicates the ISDA SIMM tenors (2W, 1M, etc.) and the subscript i represents the index name of the sub yield curve (e.g. LIBOR 3M, LIBOR 6M, etc.). RW is the tenor-dependent and currency-dependent risk weight. CR_b is the concentration risk factor for currency b and is defined as

$$CR_b = \max\left(1, \sqrt{\frac{\sum_{k,i} s_{k,i}}{T_b}}\right) \tag{11-5}$$

where T_b is the currency-dependent concentration threshold parameter. This parameter is provided by ISDA and has a value of \$250 million / basis point for well-traded currencies with regular volatility, like the EUR currency. What this means is that the weighted sensitivity WS is defined as a multiplication of risk weight RW and the delta sensitivity as long as the total delta exposure to that specific currency is under \$250 million per basis point. When the total delta exposure to a single currency exceeds this threshold, the weighted sensitivity is calculated as a multiplication of risk weight, delta sensitivity, and a penalizing factor larger than 1 for the fact that there is a large concentration exposure to a single currency. More on the CR_b-term in Section A.2.2 below.

For the EUR currency, the SIMM input table format with our example deltas would something like Table 11-7. In the second column we have included the risk weights (RW), as provided by ISDA for well-traded currencies with a regular volatility (the currency category to which the EUR belongs according to ISDA). Each currency category has its own risk weights RW. We refer to the methodology document again (ISDA, 2017).

Table 11-7. ISDA SIMM input table format for EUR.

Risk Vertices	**RW**	**OIS**	**LIBOR 1M**	**LIBOR 3M**	**LIBOR 6M**	**LIBOR 12M**
2W	77	-	-	-	38.91	-
1M	77	-	-	-	-	-
3M	77	-	-	-	(129.53)	-
6M	64	-	-	-	(346.76)	-
1Y	58	-	-	-	33.85	-
2Y	49	-	-	-	(5,921.29)	-
3Y	47	-	-	-	(12.84)	-
5Y	47	-	-	-	2,899.68	-
10Y	45	-	-	-	6,321.35	-
15Y	45	-	-	-	-	-
20Y	48	-	-	-	-	-
30Y	56	-	-	-	-	-

As the total delta exposure in Table 11-7 does not exceed \$250 million, there is no additional concentration risk penalization, so the CR_b-term equals 1 and we can further disregard this term from Equation 11-5 for our example calculation. Thus, by multiplying each column in Table 11-7 by the risk weights column (RW), we obtain the weighted sensitivities $WS_{k,i}$ as provided in Table 11-8.

Table 11-8. Weighted sensitivities for our example trades.

Risk Vertices	**OIS – WS**	**LIBOR 1M – WS**	**LIBOR 3M – WS**	**LIBOR 6M – WS**	**LIBOR 12M – WS**
2W	-	-	-	2,996.07	-
1M	-	-	-	-	-
3M	-	-	-	(9,973.81)	-
6M	-	-	-	(22,192.64)	-
1Y	-	-	-	1,963.30	-
2Y	-	-	-	(290,143.21)	-
3Y	-	-	-	(603.48)	-
5Y	-	-	-	136,284.96	-
10Y	-	-	-	284,460.75	-
15Y	-	-	-	-	-
20Y	-	-	-	-	-
30Y	-	-	-	-	-

After scaling the deltas to obtain the weighted sensitivities, we now have to aggregate the obtained weighted sensitivities within each currency bucket. The aggregation is called K by ISDA and is expressed as:

$$K = \sqrt{\sum_{i,k} WS_{k,i}^2 + \sum_{i,k} \sum_{(j,l)\neq(i,k)} \varphi_{i,j} \rho_{k,l} WS_{k,i} WS_{l,j}}\ , \qquad \phi_{i,j} = 0.98\ \forall\ i,j \qquad 11\text{-}6$$

where $\rho_{k,l}$ is the correlation between the tenor weighted sensitivities within the same sub-yield curve column and $\varphi_{i,j}$ is the correlation factor between the tenor weighted sensitivities across sub-yield curve columns. The ISDA-provided ρ-matrix is given in Table 11-9, below. The $\varphi_{i,j}$-factor equals 0.98, which means that we have to multiply Table 11-9 by 0.98 in order to get the correlation matrix between the WS terms across sub-yield curve columns. Since we have deltas in only one column in our example, we do not need to use the $\varphi_{i,j}$-factor and can proceed with by only using Table 11-9, as is. This gives us the aggregated $WS_{k,i}$-terms in Table 11-10.

Table 11-9. The tenor correlation matrix A, as provided by ISDA

Tenor Correlations

	2W	1M	3M	6M	1Y	2Y	3Y	5Y	10Y	15Y	20Y	30Y
2W	1	1	1	0.782	0.618	0.498	0.438	0.361	0.27	0.196	0.174	0.129
1M	1	1	1	0.782	0.618	0.498	0.438	0.361	0.27	0.196	0.174	0.129
3M	1	1	1	0.782	0.618	0.498	0.438	0.361	0.27	0.196	0.174	0.129
6M	0.782	0.782	0.782	1	0.84	0.739	0.667	0.569	0.444	0.375	0.349	0.296
1Y	0.618	0.618	0.618	0.84	1	0.917	0.859	0.757	0.626	0.555	0.526	0.471
2Y	0.498	0.498	0.498	0.739	0.917	1	0.976	0.895	0.749	0.69	0.66	0.602
3Y	0.438	0.438	0.438	0.667	0.859	0.976	1	0.958	0.831	0.779	0.746	0.69
5Y	0.361	0.361	0.361	0.569	0.757	0.895	0.958	1	0.925	0.893	0.859	0.812
10Y	0.27	0.27	0.27	0.444	0.626	0.749	0.831	0.925	1	0.98	0.961	0.931
15Y	0.196	0.196	0.196	0.375	0.555	0.69	0.779	0.893	0.98	1	0.989	0.97
20Y	0.174	0.174	0.174	0.349	0.526	0.66	0.746	0.859	0.961	0.989	1	0.988
30Y	0.129	0.129	0.129	0.296	0.471	0.602	0.69	0.812	0.931	0.97	0.988	1

Table 11-10. Aggregated $WS_{k,i}$ terms and K-value for the EUR currency.

Risk Vertices	OIS – WS	LIBOR 1M – WS	LIBOR 3M – WS	LIBOR 6M – WS	LIBOR 12M – WS
2W	-	-	-	(125,449,749.78)	-
1M	-	-	-	-	-
3M	-	-	-	417,617,735.52	-
6M	-	-	-	820,499,601.43	-
1Y	-	-	-	(12,428,336.08)	-
2Y	-	-	-	(7,610,167,731.92)	-
3Y	-	-	-	(40,428,546.80)	-
5Y	-	-	-	17,103,011,380.88	-
10Y	-	-	-	51,827,919,637.50	-
15Y	-	-	-	-	-
20Y	-	-	-	-	-
30Y	-	-	-	-	-
TOTAL				**62,380,573,990.75**	
	$K_{EUR} = \sqrt{62{,}380{,}573{,}990.75} = 249{,}761.03$				

The bold value in Table 11-10 is the result of $WS^T \cdot A \cdot WS$, where WS is the vector of weighted sensitivities, as provided in Table 11-8, and A is the tenor correlation matrix from Table 11-9. The other entries in Table 11-10 are obtained in Excel by using the SUMPRODUCT-function[6]. The K-value for the EUR currency is also given in Table 11-10.

Finally, we can obtain the *DeltaMargin* by using the following expression:

$$DeltaMargin = \sqrt{\sum\nolimits_{i,k} K_b^2 + \sum\nolimits_b \sum\nolimits_{c \neq b} \gamma_{bc} g_{bc} S_b S_c} \qquad 11\text{-}7$$

where

$$S_b = max\left(min\left(\sum\nolimits_{i,k} WS_{k,i}, K_b\right), -K_b\right) \qquad 11\text{-}8$$

and $g_{bc} = \frac{min(CR_b, CR_c)}{max(CR_b, CR_c)}$, ∀ currencies b and c.

Equation 11-7 calculated the *DeltaMargin* by aggregating all the K-values over all the currencies. The combination terms are obtained by correlating for each currency combination the S-term as obtained in Equation 11-8. This Equation expresses the summed weighted sensitivity values within a single currency bucket, capped by the currency's K-value and floored by its $-K$-value (i.e., $S_b \in [-K_b, K_b]$). The γ_{bc}-term is equal to 0.27 for the interest rate risk class, regardless of the currency combination.

Since we have only the EUR as underlying currency, our *DeltaMargin* is equal to the K_{EUR}-value. Furthermore, our IM for this netting set has no *VegaMargin* and no *CurvatureMargin*. This means that we need an IM of €249,761.03. Under the SA, an IM of €409,397.86 required for this same netting set (again see end of Section A.1 above).

[6] We can obtain Table 11-10 easily in Excel by using the SUMPRODUCT function over the WS-column and the correlation-matrix column (CTRL-ALT-ENTER command after selection).

Under SIMM the required IM is significantly less than under the SA for this example netting set. The reason for this lies in the higher precision of calculation of required IM under SIMM by taking into account more aspects of the trades within a netting set. By taking into account multiple aspects, SIMM allows for more diversification benefits across these aspects. The add-ons of the SA approach are a rather crude method of determining the required IM and are therefore more conservative, because less elements have to take into account all the trades' characteristics and less diversification benefits are possible. If we were to have multiple underlying currencies and multiple sub-yield curves to which a netting set's trades are exposed, there is even more diversification potential under SIMM.

A.2.2 Concentration risk under SIMM

SIMM also takes concentration risk into account. As mentioned above, this is expressed with the CR_b-term in Equation 11-5. In this subsection, it will be shown briefly and graphically what this term means for calculation of the weighted sensitivities in the interest rate risk class. Let us assume a hypothetical example with the following EUR t_0-delta exposures to the 12 SIMM tenors:

SIMM tenors	**EUR t_0-deltas (in millions)**
2W	15.225
1M	16.225
3M	17.225
6M	18.225
1Y	19.225
2Y	20.225
3Y	21.225
5Y	22.225
10Y	23.225
15Y	24.225
20Y	25.225
30Y	26.225
TOTAL	**248.7**

It is clear that this is a hypothetical case, as the amount of delta exposure to each tenor is unlikely huge. For us to hit the EUR threshold as set out by ISDA and show the mechanics of the concentration risk term, we need to use extreme values. The EUR threshold T_b equals 250 million/basis point (expressed in USD by ISDA, for simplicity we assume USD and EUR to be of equal worth at this point). If we start increasing the t_0-delta exposure to the 30Y SIMM tenor only, so that the total t_0-delta exposure goes beyond the threshold, we see that the concentration risk term CR_b 'kicks in' (i.e., becomes larger than 1) and each weighted sensitivity *WS* term (for each SIMM tenor) gets penalized, not only the weighted sensitivity for the SIMM tenor for which we have increased the t_0-delta exposure. This situation is graphically shows in Figure 11-1 below.

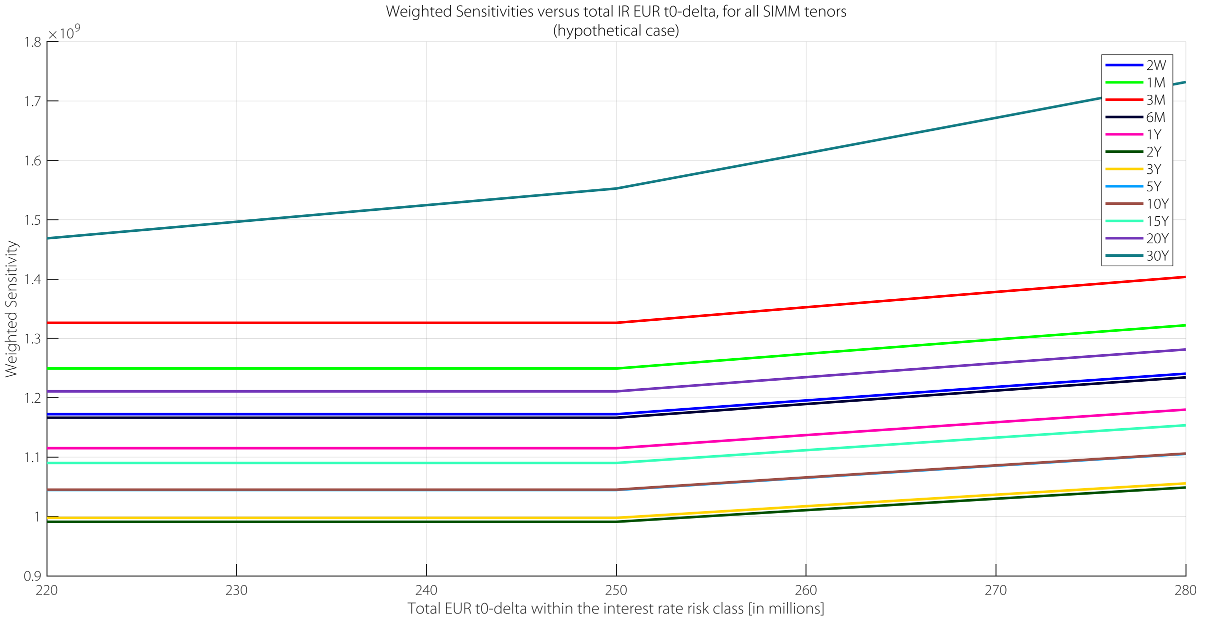

Figure 11-1. The response of WS-terms at each SIMM tenor to increase of total t_0-delta exposure for the interest rate risk class, within the EUR currency bucket. Even though only the t_0-delta exposure to the 30Y SIMM tenor is increased, all SIMM tenor WS terms are penalized when the total t_0-delta exposure exceeds the 250 million/bp threshold.

We see in the figure that the t_0-delta exposure to the 30Y SIMM tenor increases until the total t_0-delta within this EUR currency bucket within the interest rate risk class reaches the threshold. Correspondingly, the *WS*-term for this SIMM tenor increases. The other *WS*-terms remain constant. However, when the threshold is exceeded, we see from Figure 11-1 that also the other *WS*-terms start to increase.

B Interest Rate Swaps

Because the majority of the Rabobank transactions are interest rate swaps (Rabobank is an interest rate swap market maker), the analysis and modelling in subsequent chapters will revolve mainly around this type of financial product. Therefore, this chapter will treat the basics of plain vanilla interest rate swaps.

B.1 Characteristics

Interest rate swaps (IRSs) have a payoff that depends on the level of interest rates. It is a contract between two parties to exchange streams of future interest rate payments, based on an agreed upon notional (the contract's nominal value) and over a set period of time. Generally, the IRS is an exchange of a floating interest rate for a fixed interest rate (receiver swap), or the other way around (payer swap). However, it is also possible to exchange one floating rate for another floating rate (float-to-float swap). In that case the swap is called a (single currency) basis swap. With a basis swap it is possible to exchange for example a 1-month LIBOR for a 3-month LIBOR, or for a T-Bill rate, or a federal funds rate. Generally with IRSs, the notional is not exchanged and the payments are netted per payment, so that only the in-the-money party receives payment. The motivation for exchange of interest rates is diverse and might be for portfolio management, risk management, or speculation on interest rate changes.

IRSs are instruments which are OTC-traded and hence fall under the rules for bilateral margining. Figure 11-2 shows the growth of the OTC interest rate derivatives market. From the figure it can be seen that the IRSs comprise the majority of the interest rate market. By the end of 2014, the single-currency interest rate derivatives represented 80% of the total OTC derivatives market and over 60% was represented by IRSs (Bank for International Settlements, 2015), indicating their importance within financial markets. Commercial banks and investment banks with high credit ratings are interest rate swap market makers, providing their counterparties with fixed and floating payment streams. The counterparties are typically other financial institutions, corporations, or investors.

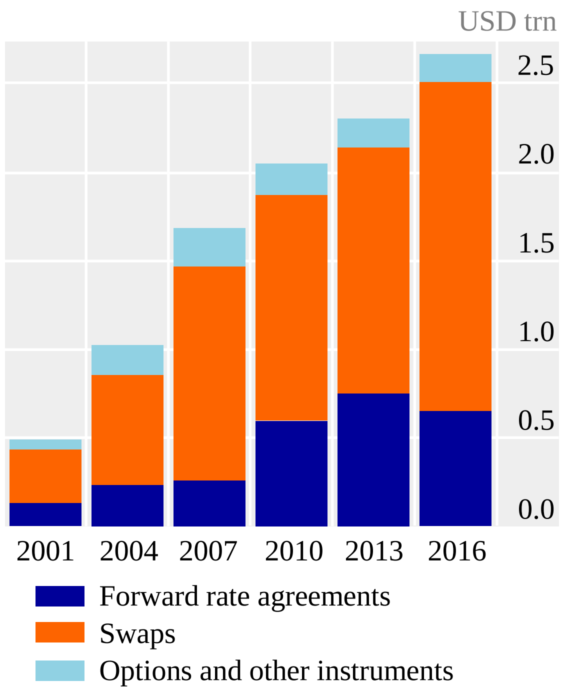

Figure 11-2. OTC interest rate derivative instruments market growth, expressed in trillion USD. It can be seen from the figure that IRSs are increasingly dominating the interest rate derivatives market. Adopted from (Ehlers & Egemen, 2016).

B.2 Valuation

The payoff from an IRS depends on the forward rates. A forward rate is the interest rate between time instant T and S, determined at time t, with $t \leq T \leq S$. In other words, a forward rate is an interest rate that is locked in today (at time t) for an investment in a future time period between time T and S, $T \leq S$. The forward rate can be generally denoted as $F(t;T,S)$. Forward rates are deduced from the term structure of discount rates as it is at time t (now). The term structure of discount rates defines the curve of discount factors at different maturities T, applicable towards today. In its turn, the term structure of discount rates is obtained from the zero-coupon curve (or zero-curve), which provides a current relationship between maturities T and the corresponding interest rates, based on market data of interest rates. The notation and mathematical derivation in the remainder of this subchapter are based on (Brigo & Mercurio, 2006).

B.2.1 Deriving an expression for the forward rate and FRAs

The forward rate can be mathematically expressed by setting up the model for a forward-rate agreement (FRA)[7] and using the no-arbitrage arguments. FRAs are very similar to IRSs and are actually a sub-element of IRSs; an IRS can be seen as a cascade of FRAs. A FRA is a contract between two parties to exchange a fixed for floating rate at the contract's maturity S, based on the notional N, or vice versa. The fixed rate can be denoted as K and the floating rate is the spot rate that resets at time T and with maturity S, where $t \leq T \leq S$, and can be denoted as $L(T,S)$ (which represents a LIBOR rate). This spot rate is a simply-compounded rate and can be expressed as:

$$L(T,S) = \frac{1 - P(T,S)}{\tau(T,S)P(T,S)} \qquad 11\text{-}9$$

In the above Equation $P(T,S)$ is time T's value of one unit of currency received at time S and $\tau(T,S)$ is the year fraction of the time period between time T and time S. So, for a FRA which gives the holder the right to exchange a simply compounded spot rate $L(T,S)$ for a fixed rate K, the contract value can be written as the sum of receiving $\tau(T,S)KN$ units of currency at time S and paying $\tau(T,S)L(T,S)N$ units of currency at time S. Formally, the value of the FRA at time S can then be expressed as:

$$N\tau(T,S)(K - L(T,S)) \qquad 11\text{-}10$$

By replacing $L(T,S)$ by the expression given in Equation 11-9, we can rewrite the expression for the FRA value in time S as:

$$N\tau(T,S)\left[K - \left(\frac{1 - P(T,S)}{\tau(T,S)P(T,S)}\right)\right] = N\left[\tau(T,S)K - \frac{1}{P(T,S)} + 1\right] \qquad 11\text{-}11$$

From the above Equation, the value of $\frac{1}{P(T,S)}$ at time S is equal to $P(T,S)\left(\frac{1}{P(T,S)}\right) = 1$ (or one unit of currency) at time T. The value of one unit of currency at time T is equal to $P(t,T)$ at time t. So, we can replace the term $\frac{1}{P(T,S)}$ by $P(t,T)$ in Equation 11-11. Furthermore, the value of $\tau(T,S)K + 1$ at time S is equal to $P(t,S)(\tau(T,S)K + 1)$ at time t. So, we can rewrite the value of the FRA contract in Equation 11-11 as:

$$\mathbf{FRA}(t,T,S,\tau(T,S),N,K) = [P(t,S)\tau(T,S)K - P(t,T) + P(t,S)] \qquad 11\text{-}12$$

The reason for deriving an expression for the value of FRA was to obtain an expression for the forward rate $F(t;T,S)$ by using the no-arbitrage argumentation. To do that, we have to determine K, for which

[7] FRAs are also OTC-traded, see **Error! Reference source not found.** for an overview of the size of interest rate derivative market apitalization by FRAs.

the value of the FRA contract has zero value at time t (i.e., no arbitrage). That will give us the simply-compounded forward rate. The following expression gives the resulting solution:

$$F(t;T,S) = \frac{1}{\tau(T,S)}\left(\frac{P(t,T)}{P(t,S)} - 1\right) \qquad \text{11-13}$$

Now, we can formally write the value of the FRA contract as:

$$\mathbf{FRA}(t,T,S,\tau(T,S),N,K) = NP(t,S)\tau(T,S)[K - F(t;T,S)] \qquad \text{11-14}$$

For completion: this is the value of an FRA contract that gives the holder the right to pay a floating LIBOR rate and receive a fixed rate K, based on the notional and a *simply-compounded* $L(T,S)$ LIBOR spot rate, prevailing at time t. If we wanted to use a differently compounded rate, we should replace the spot rate in Equation 11-10 with an expression that corresponds to the differently compounded rate and repeat the procedure.

B.2.2 *Valuing IRS through FRAs*

As mentioned above, the IRS is a cascade of FRAs. Where the FRA has a resetting date T, payment date S and a single payment, the IRS has a vector of floating-leg resetting dates $[T_\alpha, T_{\alpha+1}, \ldots, T_{\beta-1}]$, vector of payment dates $[T_{\alpha+1}, T_{\alpha+2}, \ldots, T_\beta]$, and accordingly a stream of payments. Another consequence of cascading FRAs is that an IRS has a vector of τ's, which can be defined as $[\tau_{\alpha+1}, \tau_{\alpha+2}, \ldots, \tau_\beta]$. Defining τ_i as the year fraction between time point T_{i-1} and T_i, the fixed leg has a payoff of $N\tau_i K$ and the floating-leg has a payoff of $N\tau_i L(T_{i-1}, T_i)$, where $L(T_{i-1}, T_i)$ being the rate that resets at T_{i-1} and with maturity T_i.

With the definitions as stated, we can obtain the value of a receiver IRS (R-IRS) by cascading the FRAs as expressed in Equation 11-14:

$$\mathbf{R-IRS}(t,T,\tau,N,K) = \sum_{i=\alpha+1}^{\beta} \mathbf{FRA}(t,T_{i-1},T_i,\tau_i,N,K) = N\sum_{i=\alpha+1}^{\beta} \tau_i P(t,T_i)[K - F(t;T_{i-1},T_i) \qquad \text{11-15}$$

This is the expression for the value of a receiver IRS, based on simply compounded LIBOR rates $L(T_{i-1}, T_i)$. It is also possible to use different compounding. Also the payment dates of the floating-leg match the payment dates of the fixed leg (i.e., the payment frequencies are equal for both legs). This is usually not the case in practice as the floating-leg typically has a higher payment frequency, but for simplicity it is assumed to be the case in this chapter. Also, the day count convention of the two legs is assumed to be the same. This is also not always the case in practice.

Finally, the derived expression was based on cascading FRAs. It is also possible to derive the expression through bond prices. Derivation of the above expression is then obtained by decomposing the R-IRS into a long position in a fixed bond-rate bond and a short position in a floating-rate bond (Brigo & Mercurio, 2006; Hull J., 2012).

C Term structure modelling

In the above value derivation for the FRA and subsequently for the IRS, there was no in-depth analysis on how to model the forward-rate. It was only mentioned briefly that the forward rates are derived from discount rates, which in turn are derived from the zero-curve. Here, some more attention will be given on how forward-rates can be modelled without going into too much detail.

In principle, instruments with a maturity up to one year can be valued by using the observed LIBOR zero-curve prevailing at the valuation date in combination with some interpolation, if needed. If the maturity is between 2 and up to 5 years, it is also possible to extend the LIBOR zero-curve by using Eurodollar futures (Hull J., 2012). However, to value instruments with longer maturities, more complex modelling has to be performed. Generally, there are two types of models for modelling the forward-rate; short-rate models, and LIBOR market models

C.1 Short-rate models

Short-rate models use the instantaneous spot rate *r(t)* (also known as short-rate), as the basic stochastic variable. The instantaneous spot rate is the rate that prevails at time *t*, when the time period is taken to be infinitesimal. Short-rate models are also termed one-factor models as they only model one factor (the short-rate). These models output the term structure of rates, rather than taking it as an input. That is why these models are also called endogenous term structure models (Brigo & Mercurio, 2006). Three of the most common short-rate models will be briefly discussed in this section: the Vašíček model, the Exponential-Vašíček (EV) model, and the Displaced Exponential-Vašíček (DEV) model.

C.1.1 Vašíček Model

The Vašíček model defines the evolution of the short-rate under the risk-neutral measure Q, as (Vašíček, 1977):

$$dr_t = \underbrace{k(\theta - r_t)dt}_{\text{Drift rate}} + \underbrace{\sigma}_{\text{Diffusion rate}} \underbrace{dW_t}_{\text{Brownian motion}}, \qquad r(0) = r_0 \tag{11-16}$$

where r_0, k, θ, and σ are positive constants. The Vašíček model is an *Ornstein-Uhlenbeck* process. The Vašíček model has mean-reversion characteristics. The short-rate reverts back to level θ, with rate k. In the model, σ is the volatility parameter and it is a constant. The explicit solution to Equation 11-16 is:

$$r_t = r_0 e^{-kt} + \theta\left(1 - e^{-kt}\right) + \sigma\sqrt{\frac{1 - e^{-2kt}}{2k}}\,\epsilon \tag{11-17}$$

where ε is a standard normally distributed variable. The mean is given by:

$$\langle r_t \rangle = r_0 e^{-kt} + \theta\left(1 - e^{-kt}\right)$$

and variance:

$$\langle r_t^2 \rangle = \frac{\sigma^2}{2k}\left(1 - e^{-2kt}\right).$$

The Vašíček model can get deep into negative interest rates, which are unrealistic market outcomes.

C.1.2 Exponential-Vašíček (EV) Model

The outcome of the Vašíček model is a normal distribution of rates. It may be preferred to use a model with a resulting skewed distribution for interest rates, like the χ^2 or the log-normal distribution. With the EV model, we assume that the logarithm of r follows an Ornstein-Uhlenbeck process y under the risk-neutral measure Q:

$$dy_t = k(\theta - y_t)dt + \sigma dW_t, \qquad y(0) = y_0 \tag{11-18}$$

where we define r_t=exp(y_t) and where θ, k, and σ are positive constants. This model gives the following expression for the process of the short-rate:

$$d\ln r_t = k(\theta - \ln r_t)dt + \sigma dW_t \tag{11-19}$$

with the explicit solution:

$$r_t = \exp\left\{\ln(r_0)e^{-kt} + \theta\left(1 - e^{-kt}\right) + \sigma\sqrt{\frac{1 - e^{-2kt}}{2k}}\epsilon\right\} \tag{11-20}$$

The EV model results in positive short rates evolution and has a resulting log-normal distribution of r_t.

C.1.3 Displaced Exponential-Vašíček (DEV) Model

The DEV model better suits the current low-interest environment, because it builds on the EV model, but with the additional introduction of a deterministic shift to account for the possibility of negative interest rates that do not get deeply negative. The DEV model is described by the following SDE:

$$d\ln(r_t + \gamma) = k(\theta - \ln(r_t + \gamma))dt + \sigma dW_t, \tag{11-21}$$

where γ is the shift. The analytic solution is given by:

$$r_t = exp\left\{\ln(r_0 + \gamma)e^{-kt} + \theta\left(1 - e^{-kt}\right) + \sigma\sqrt{\frac{1 - e^{-2kt}}{2k}}\epsilon\right\} - \gamma \tag{11-22}$$

The above list is not a complete overview of the models for short-rates. For a more extensive overview, please refer to Brigo and Mercurio (2006) and Hull J. (2012).

C.2 Alternatives to short-rate models

Short-rate models are relatively easy implementable and can be used with Monte-Carlo simulation techniques and the dynamics allow for historical estimation of parameters. Furthermore, the models have the potential to provide accurate valuations of the most common non-standard interest rate derivatives. There are some drawbacks to the models, however. For one, most of the models consider a single driver of the modelling process. Secondly, the volatility parameter is represented by a constant. This gives little freedom in selecting the volatility structure (Hull J, 2012). To accommodate for these shortcomings, there are two acclaimed models: the Heath-Jarrow-Morton (HJM), and the Brace-Gatarek-Musiela (BGM) model. These models are only mentioned in this chapter for completeness. They are out of scope for our research and the interested reader is referred to Heath *et al.* (1992) and Brace *et al.* (1997) and the subsequent publications on this topic.