UNIVERSITY OF TWENTE.

Faculty of Electrical Engineering,
Mathematics & Computer Science

Analyzing and reducing switching frequency ripple of class D amplifiers by active filtering

Eva J. Knol

B.Sc. Thesis
July 2018

Supervisors:
C.E. Lokin, MSc
dr. ir. R. vd Zee
dr. ir. A.B.J. Kokkeler

Integrated Circuit Design Group
Faculty of Electrical Engineering,
Mathematics and Computer Science
University of Twente
P.O. Box 217
7500 AE Enschede
The Netherlands

Abstract

A class D amplifier creates, next to the desired signal, also a PWM frequency ripple at the output. This ripple can radiate from the speaker wires as electromagnetic radiation, which is undesirable. Feedback models are designed to reduce the amplitude of the output ripple and the magnitude at 620 kHz (lowest ripple frequency). In this report, two feedback models are analyzed and optimized. One feedback model from the output voltage to a controlled current source and another feedback model from the inductor voltage to a controlled current source. The feedback models use a first or second order high pass filter. Both options are analyzed and optimized for each feedback model.

The optimal parameter values of the feedback models are found by restricting the magnitude of the audio frequencies and the characteristics of the frequency response of the feedback model. The characteristics were found by analyzing the transfer function of the feedback model via a method that reduces the size of the transfer function at specific frequencies, to find the characteristics easier.

The results were that the feedback model from the output voltage with a first order high pass filter did not reduce the voltage ripple, however the other feedback models did. The model with feedback from the inductor voltage with a second order high pass filter reduced the output voltage ripple amplitude from 60.8 mV to 4.65 mV (reduction of 92.4%) and the magnitude at 620 kHz (lowest PWM frequency) from -53.7 dB till -87.8 dB (decrease of 64.7%). The inductor feedback model with a first order filter reduced the ripple from 60.8 mV to 6.21 mV (reduction of 89.8%) and the magnitude at 620 kHz from -53.7 dB to -73.3 dB (decrease of 37.5%). The inductor feedback model with a second order filter reduced the ripple from 60.8 mV to 3.29 mV (reduction of 94.6%) and the magnitude at 620 kHz from -53.7 dB to -78.4 dB (decrease of 47.1%). The best feedback model is the model with feedback from the inductor voltage with a second order high pass filter.

The conclusion is that the output feedback model with the second order filter reduces the ripple the most, as it has the lowest magnitude at 620 kHz.

Contents

Introduction

A class D amplifier consists of a PWM modulator, a switch and a second order low pass filter, see Figure 1. An audio signal is modulated by the PWM modulator to a high-frequency pulse signal with a varying duty cycle, depending on the audio signal. The pulse signal switches the switch between ground and a voltage supply to enlarge the pulse signal. The low pass filter recovers the original signal by filtering out the switching frequency. A second order low pass filter consists of an inductor and a capacitor, where the inductor is connected to the output of the switch [1]. The switch changes the voltage over the inductor. The current through the inductor is the integral of the inductor voltage ($V_L = L \cdot \frac{di_L}{dt}$), which means that a triangular inductor current will be created. The current increases when the inductor voltage is positive and decreases when the inductor voltage is negative. The inductor current contains a signal current and a ripple current. The signal current flows through the load and the ripple current divides between the capacitor and the load. The final current through the load is shown in Figure 2 (blue). The high frequency ripple can radiate from the speaker wires as electromagnetic radiation [2]. This is undesirable.

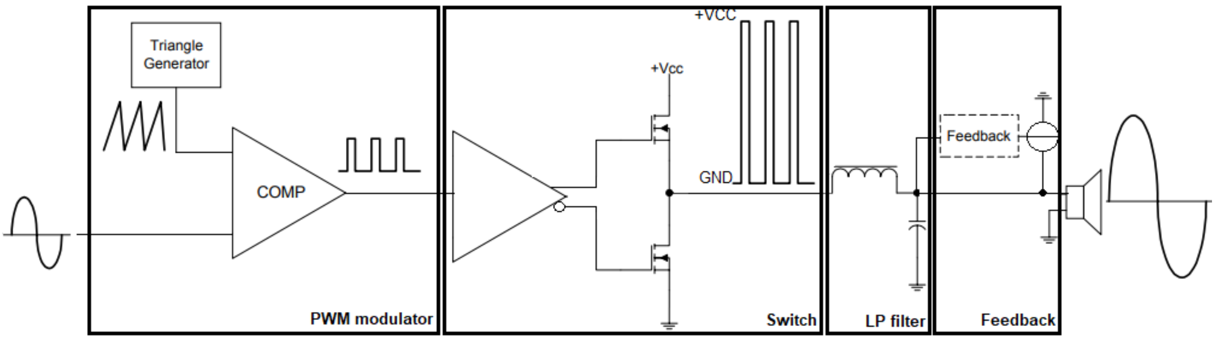

Figure 1: Class D amplifier with an active filter feedback

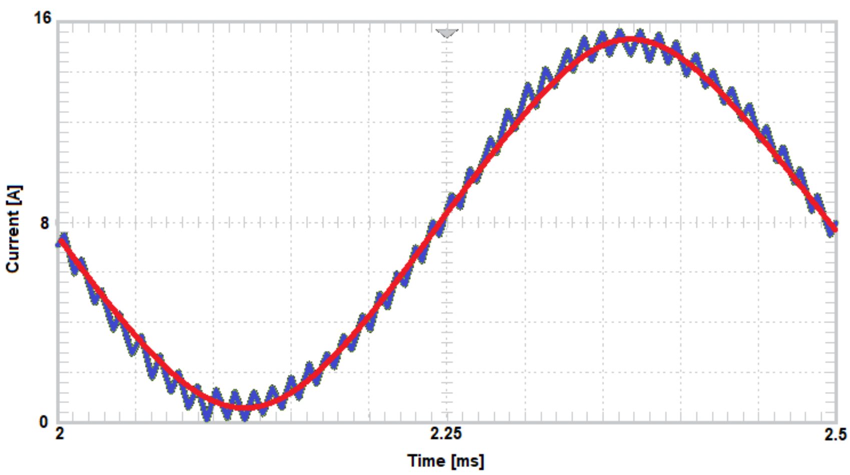

Figure 2: *Red*: signal current. *Blue*: signal current with the ripple current.

There are several solutions to reduce the ripple current flowing through the speaker wires. One solution is to compensate the ripple with an active feedback filter, which is a solution with relatively low cost and small board space. Last year, Lucas Timmermans tried to reduce the ripple by using active filters. The solutions were promising, but they can be improved [3].

The goal of this report is to find the limitations and improve the solutions of Lucas Timmermans. The limitations will be found by a more thorough analysis: by looking at the transfer function, stability, and characteristics and limitations of the frequency response. The optimal parameters values will be determined from the findings of the limitations.

This paper has been divided into four chapters. It begins by analyzing a class D amplifier without feedback. It will then go on with analyzing the first solution of Lucas Timmermans, a feedback loop from the output voltage to a controlled current source at the output node. The output voltage is filtered and amplified in the feedback loop. Chapter three analyzes a solution with a feedback loop from the inductor voltage to a controlled current source at the output node. In the feedback loop, the inductor voltage signal is transformed to a inductor current signal, and then filtered and amplified. The final chapter compares the findings of the research, focusing on the reduction of the output ripple voltage.

Chapter 1

Basic model of a class D amplifier

This chapter begins by analyzing the basic model of a class D amplifier, shown in Figure 1.1. The analysis is done by looking at the transfer function, frequency response and stability. During the analysis, the parameter values of the basic model are determined. The parameters values will be summarized in a table. At the end, the output voltage will be analyzed in frequency domain, and the output ripple voltage in time domain.

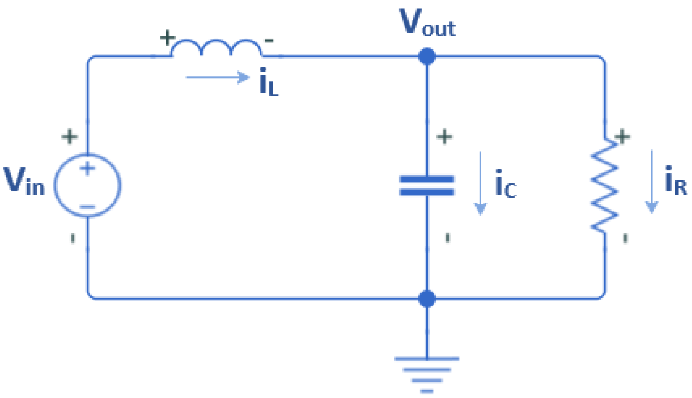

Figure 1.1: Circuit model of a class D amplifier

1.1 Model analysis

This section starts by deriving the transfer function from the circuit model, shown in Figure 1.1. The transfer function gives several insights, which determines the parameter values of the second order filter. Next, the frequency response and stability of the circuit model will be visualized in a Bode and pole-zero plot.

1.1.1 Transfer function

The voltage transfer function is derived from the circuit model that represented the class D amplifier. Equation 1.1[a] shows the voltage transfer function $H(s)_{basic}$, which represent the relationship between the output and input voltage. The transfer function gives insights into the steepness of the decrease in dB after the cut-off frequency, the cut-off frequency, quality factor, poles and zeros. The respective expressions are shown in Equations 1.2 till 1.5[b] (note that this transfer function does not have zeros).

$$H(s)_{basic} = \frac{V_{out}}{V_{in}} = \frac{R}{s^2LCR + sL + R} \tag{1.1}$$

$$-20 \cdot n \; dB/decade \tag{1.2}$$

$$f_c = \frac{w_c}{2\pi} = \frac{1}{2\pi\sqrt{CL}} \tag{1.3}$$

[a] see A.1 for the derivation
[b] see A.2 for the derivation

$$Q = \sqrt{\frac{CR^2}{L}} \tag{1.4}$$

$$poles = \frac{-L \pm \sqrt{L^2 - 4CLR^2}}{2CLR} \tag{1.5}$$

The dependent parameters of Equations 1.2 till 1.5 are the order, load-resistor, inductor value and capacitor value. The order of the transfer function is determined by the highest exponent of s, so the order is two [4]. The load resistor is the resistor in a speaker, normally 4 Ω [1]. The value of the inductor and capacitor can be chosen such that the right cut-off frequency, quality factor and poles are achieved. The cut-off frequency must be above 20kHz, because the human hearing range is between 20 Hz and 20 kHz [5]. The amplitude of the PWM frequency must be as low as possible, such that the undesirable ripple is as small as possible. This means that the cut-off frequency must be as low as possible. The quality factor must be in the range between 0.6 and 0.8 to avoid underdamped or overdamped behavior [6]. The real value of the poles must be lower than zero, such that the system is stable [7]. Concluding, the facts and requirements indicates that the order is 2, load-resistor 4 Ω, inductor 32 μH and capacitor 1 μF .

Results of Equations 1.2 till 1.5:

- roll-off: $-40dB/decade$
- $f_c = 28kHz \rightarrow w_c = 176krad/sec$
- $Q = 0.707$
- $poles = -125000 \pm j123527$

1.1.2 Frequency response

The frequency response of the basic model is visualized in the Bode plot of Figure 1.2. The steepness after the cut-off frequency is -40dB/decade, the cut-off frequency is around 25kHz and Q around 0.7. These values are approximately the same as the calculated values.

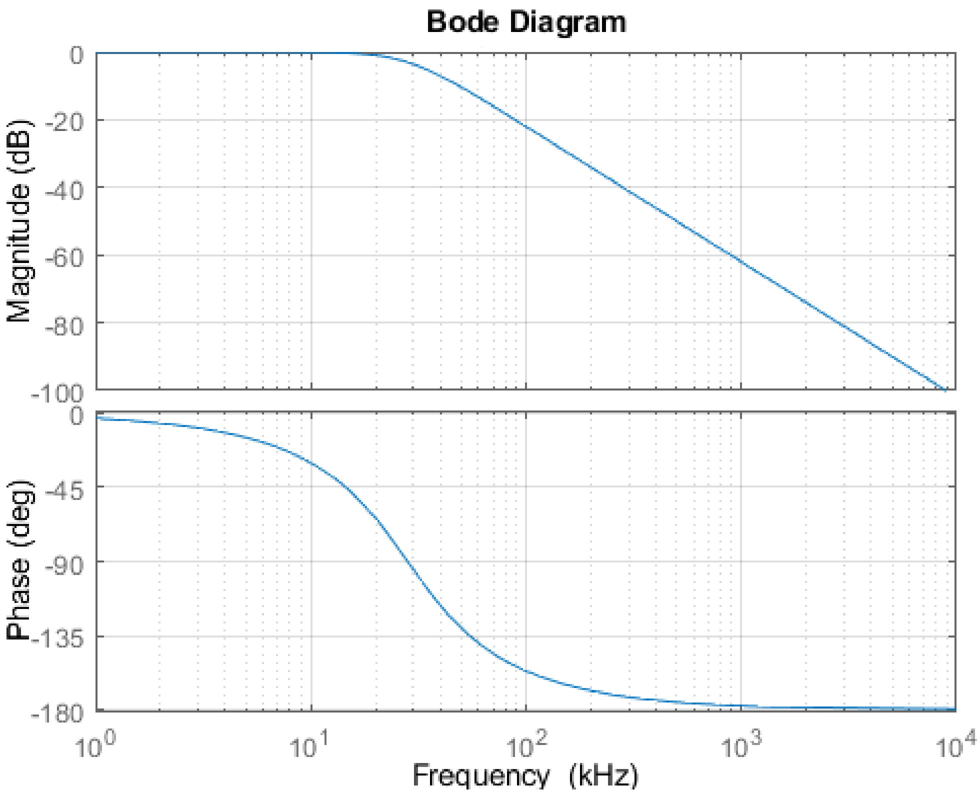

Figure 1.2: Bode plot of the transfer function of the model in Figure 1.1

1.1.3 Stability

The system is stable if all the poles are at the left side of the pole-zero plot (the real pole values are lower than zero). The pole-zero diagram for the basic model is shown in Figure 1.3. The poles of the diagram are -1.25e5+1.25e5i and -1.25e5-1.25e5. Approximately the same as the calculated poles. All poles are at the left side, so the system is stable [7].

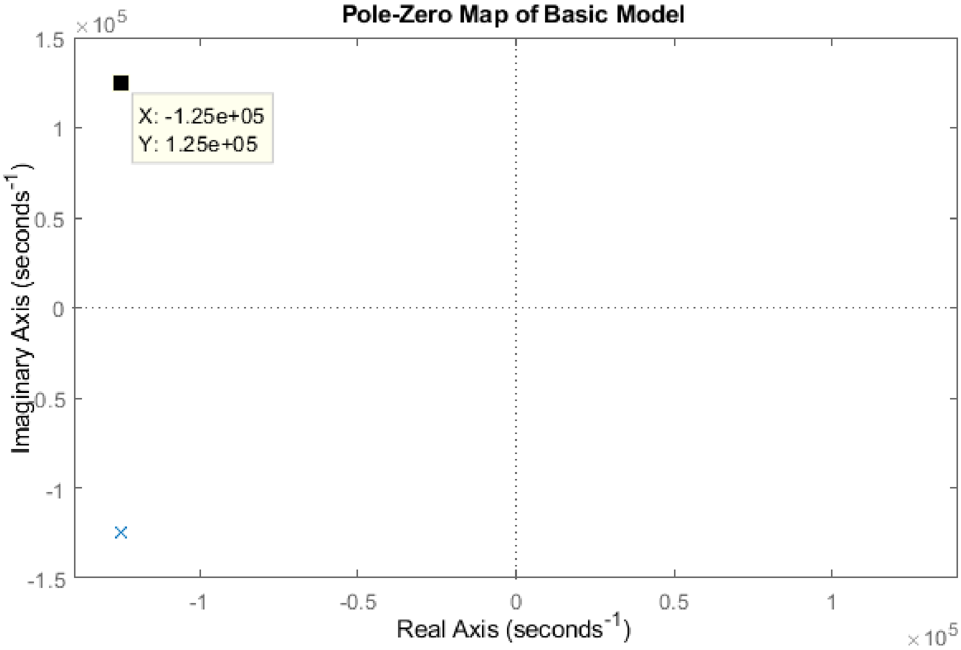

Figure 1.3: Pole-Zero plot of the model in Figure 1.1 (represented in angular frequency)

1.2 Overview parameters

The parameters of the circuit are summed up in Table 1.1. These parameters are explained in Appendix E.2. All other models use the same parameter values.

Table 1.1: Parameters of the basic model

Parameter	Symbol	Value
Input sine amplitude	A_{in}	1 V
Input frequency (between 20 -20000 Hz)	f_{in}	5 kHz
Sample frequency	f_s	44.1 kHz
PWM carrier frequency (16 bit)	$f_{pwm-carrier}$	$f_s \cdot 16 = 706$ kHz
PWM amplitude	A_{pwm}	1 V
Gain of system	G_{sys}	30
Inductor	L	32 μH
Capacitor	C	1 μF
Load-resistor	R	4 Ω

1.3 Output voltage analysis

In this section, the output ripple voltage of the basic model is analyzed in the time domain, to get the voltage amplitude of the ripple signal. Furthermore, the output voltage of the basic model is analyzed in the frequency domain, to find the lowest PWM frequency.

1.3.1 Output ripple voltage in time domain

The voltage amplitude of the output ripple signal can be found from a time domain graph that shows the output ripple voltage. The voltage amplitude of the ripple signal will be used to easily compare to the ripple amplitudes of the feedback model, and see if the amplitudes are reduced. The output ripple voltage of the basic model is found by subtracting the ideal output from the real output, see Equation 1.6. The output ripple voltage in time domain is shown in Figure 1.4. The ripple does not have a constant amplitude, because the duty cycle in the PWM generator changes and the inductor ripple current is related to the duty cycle [8]. The ripple looks like an AM signal. For other input frequencies, the amplitude is the same. Only the shape is more stretched out (lower then 5 kHz) or compressed (higher then 5 kHz). The time domain figure shows that the maximum voltage amplitude is 60,8 mV.

$$V_{ripple} = V_{out} - V_{ideal} = V_{out} - V_{in}G_{sys}H(s)_{basic} \tag{1.6}$$

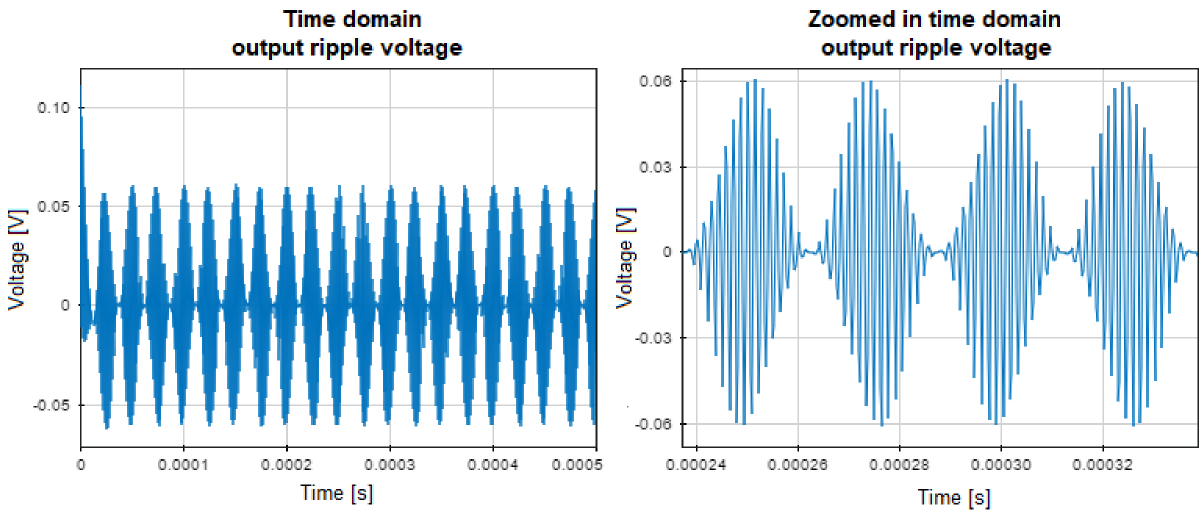

Figure 1.4: Time domain plot of the output ripple voltage with input frequency 20 kHz. Shows that the maximum amplitude of the output ripple voltage is 60.8 mV

1.3.2 Output voltage in frequency domain

The lowest PWM frequency (ripple frequency) can be determined from the frequency domain of the voltage output. To identify from which moment the Bode plot has to be as low as possible, the lowest PWM frequency is needed. Figure 1.5 shows the output voltage in frequency domain for different input frequencies in the range of the human ear. Note that the figure is zoomed in at the y-axis, otherwise the ripple magnitude would be too small to recognize. The amplitude of the fundamental frequency (input frequency) is shown at the top of each diagram. For high frequencies, the amplitude of the fundamental frequency is lower than 30V, because the magnitude of the filter for these frequencies is below 0 dB. The sum of all ripple magnitude around the center frequency of the ripple stays the same, which results in the total harmonic distortion (THD) being higher for high frequencies. In addition, the figure shows that the input frequency and the ripple frequency are more spread around there center frequency when the input frequency is high. The PWM frequency has a maximum range around the PWM carrier of 85 kHz. This means that the lowest PWM frequency is 620 kHz.

It is interesting to know how large the magnitude of the lowest PWM frequency is. The Bode plot of Figure 1.2 shows that the magnitude of the lowest PWM frequency is -53.7 dB.

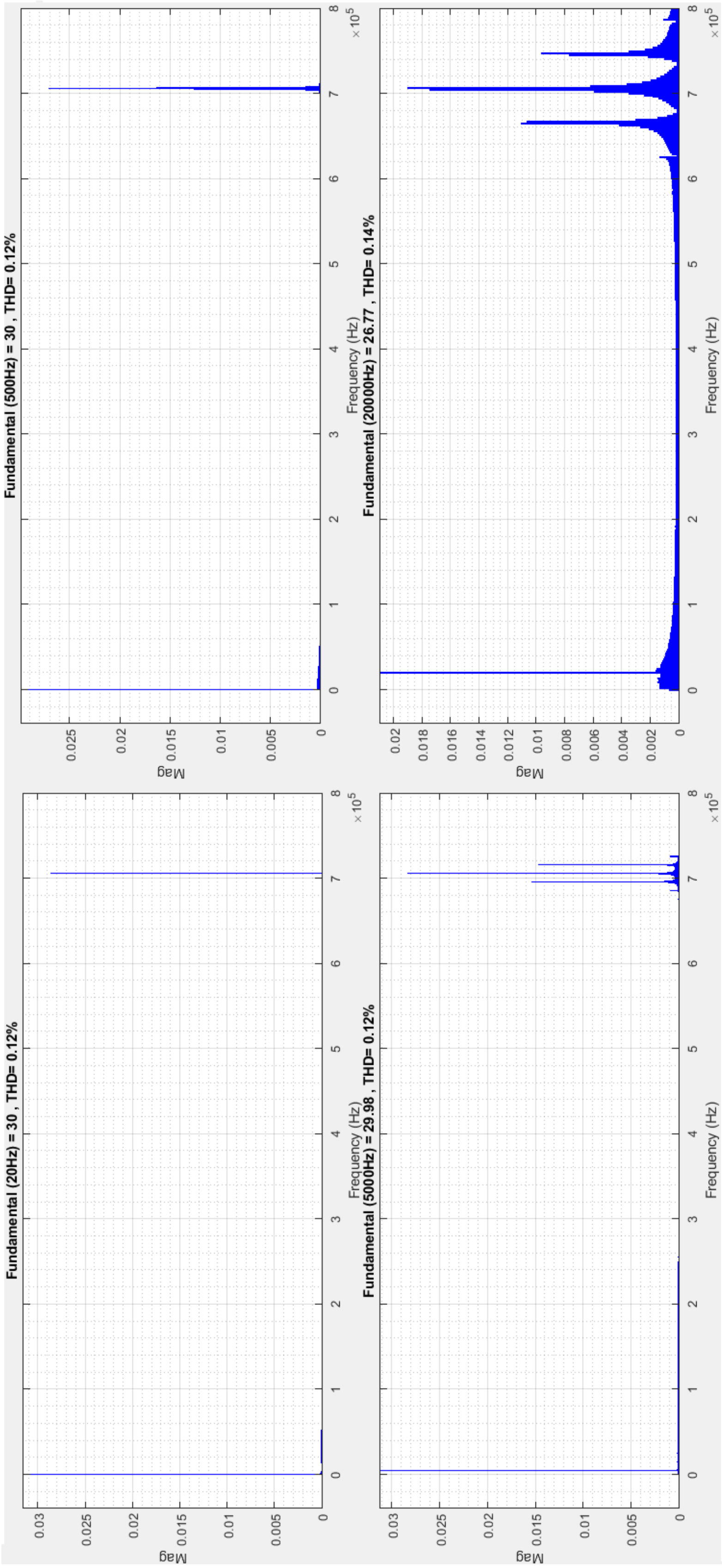

Figure 1.5: Frequency domain of the output voltage with input frequency 20, 500, 5000 and 20000 Hz. Shows that the lowest PWM frequency is 620 kHz

Chapter 2

Feedback from the output voltage

In this chapter, the design with feedback from the output voltage to a controlled current source, shown in Figure 2.1, is analyzed and optimized. In the feedback loop, the output voltage is high pass filtered, such that the PWM frequencies are left over, amplified with gain g, and then amplified with a transconductance g_m. For this report, two kinds of high pass filter are used, a first and a second order. The feedback model with a first and second order feedback filter are separately analyzed and optimized. The analysis is done by looking at the transfer function, frequency response and stability. From the analysis, the characteristics can be found, which will help to find the limitations. The limitations show how the parameters values influence the characteristics. With this knowledge, the optimal parameters can be determined. At the end, the optimal output and ripple voltage will be analyzed.

The feedback model is designed to reduce the output ripple voltage as follows. It is desired to make the controlled current source equal to the ripple current, such that the ripple current of the inductor does not flow to the load-resistor. That way, the output voltage will not contain a ripple voltage. The controlled current source is controlled via the feedback loop from the output voltage. The output voltage is filtered such that the PWM frequencies (ripple frequencies) are left over. The left over ripple frequency is amplified with a gain g, such that the source current is equal to the ripple current. This g must be chosen, such that the feedback system is still stable. Note that the feedback signal is also amplified with a transconductance g_m of the controlled current course, which is chosen to be 1 A/V.

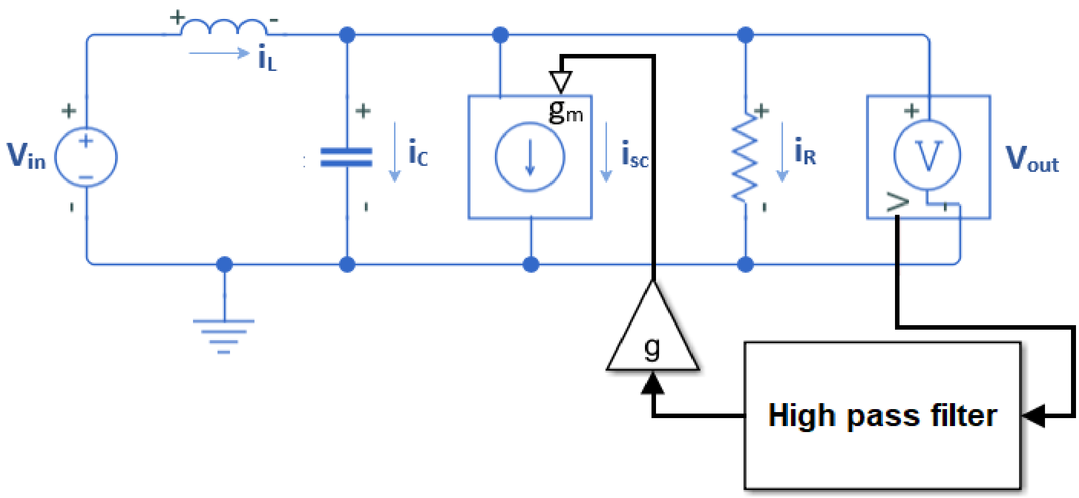

Figure 2.1: Circuit model of design with feedback from the output voltage

2.1 First order high pass filter feedback

2.1.1 Analysis

In this section, the feedback from the output voltage with a first order high pass filter is analyzed. Starting with describing the first order high pass filter via a transfer function and determining the first order high pass filter parameters. Next, the total feedback model is analyzed by determining the transfer function, checking the stability and visualizing the frequency response. Finally, the frequency response will be characterized such that the limitations of the model will be found.

Describing the first order high pass filter

The transfer function of the first order high pass filter is shown in Equation 2.1, which depends on τ. From Section 1.3.2, it is known that the lowest frequency of the PWM is 620 kHz. This means that the cut-off frequency of the high pass filter must be lower than 620 kHz. In addition, the cut-off frequency must be as high as possible, because the output signal will be attenuated as well. The cut-off frequency is chosen to be 200 kHz. Now a question may rise: why is the cut-off frequency so low? It was chosen to have the magnitude at 620 kHz above -0.5 dB, such that gain g does not have to be increased to compensate. Which makes $\tau = 800ns$[a].

$$H(s)_{1^e HP} = \frac{s\tau}{s\tau + 1} \tag{2.1}$$

Analysis of total feedback model

The analysis is done by looking at the transfer function, Bode plot and pole-zero plot. The transfer function is derived from the Kirchhoff's rules and Laplace functions, see Appendix C.1 for the derivation. The transfer function of the circuit model with a first order high pass filter, $H(s)_{out-1^e HP}$, is shown in Equation 2.2. The Bode and pole-zero plots with different feedback gains are shown in Figures 2.2 till 2.4. The Bode plot shows that the cut-off frequency and Q factor is changed because of the feedback. Besides, the steepness changes from -40 dB/decade to -20 dB/decade to -40 dB/decade. The phase is also changed; the phase is quicker steep, steeper and it has a hill at the end. The hill is caused by the zero and pole at the real axis being farther apart, see pole-zero plot. The farther away, the higher the hill. The pole-zero plot shows that the feedback model with different gains g is stable; all real pole values are at the left side of the plot [7].

$$H(s)_{out-1^e HP} = \frac{V_{out}}{V_{in}} = \frac{sR\tau + R}{s^3 LCR\tau + s^2(LCR + L\tau + LR\tau g g_m) + s(L + R\tau) + R} \tag{2.2}$$

[a] see B for the Bode plot

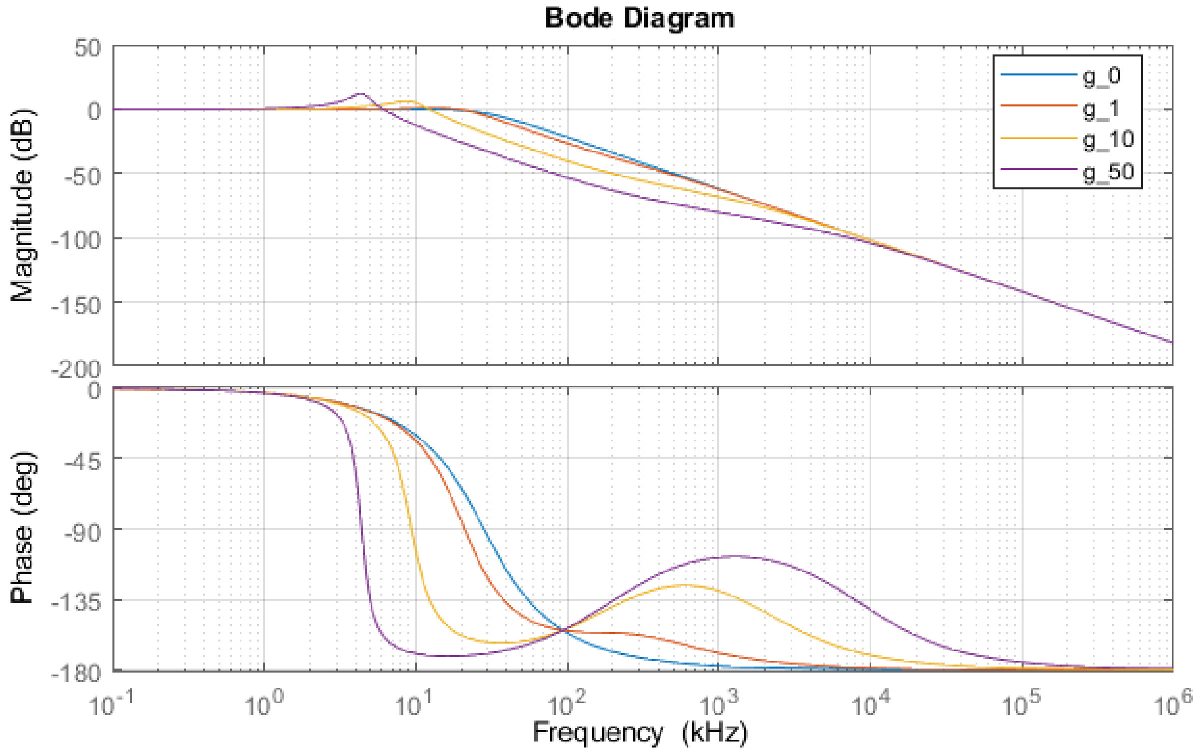

Figure 2.2: Bode plot of transfer function of model 2.1 with a first order filter and different feedback gains 0,1,10 and 50

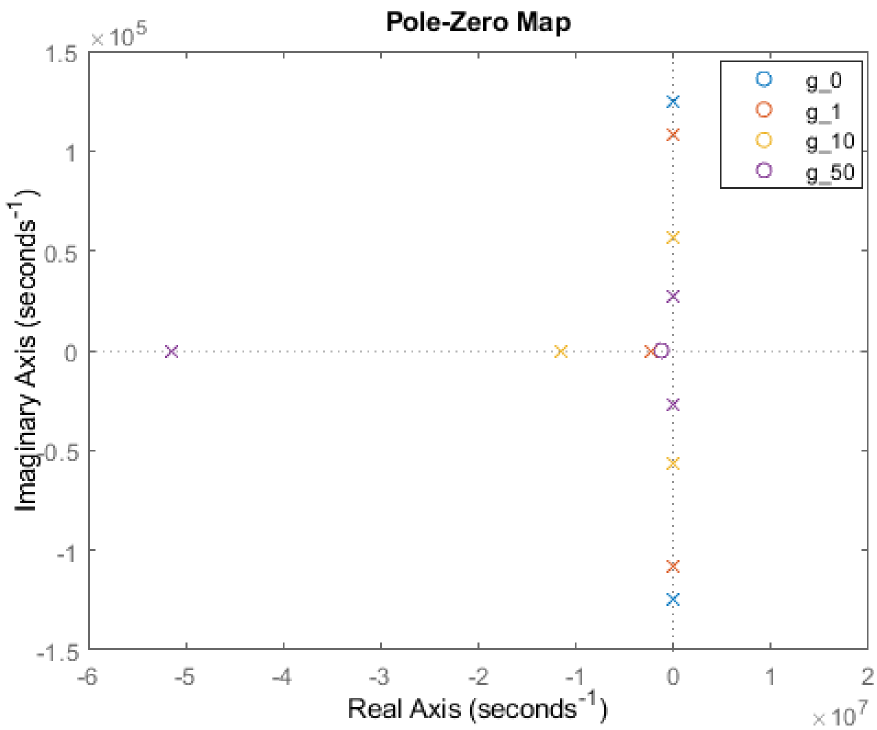

Figure 2.3: Pole-zero plot of the model in Figure 2.1 with first order filter and different feedback gains 0,1,10 and 50 (represented in angular frequency)

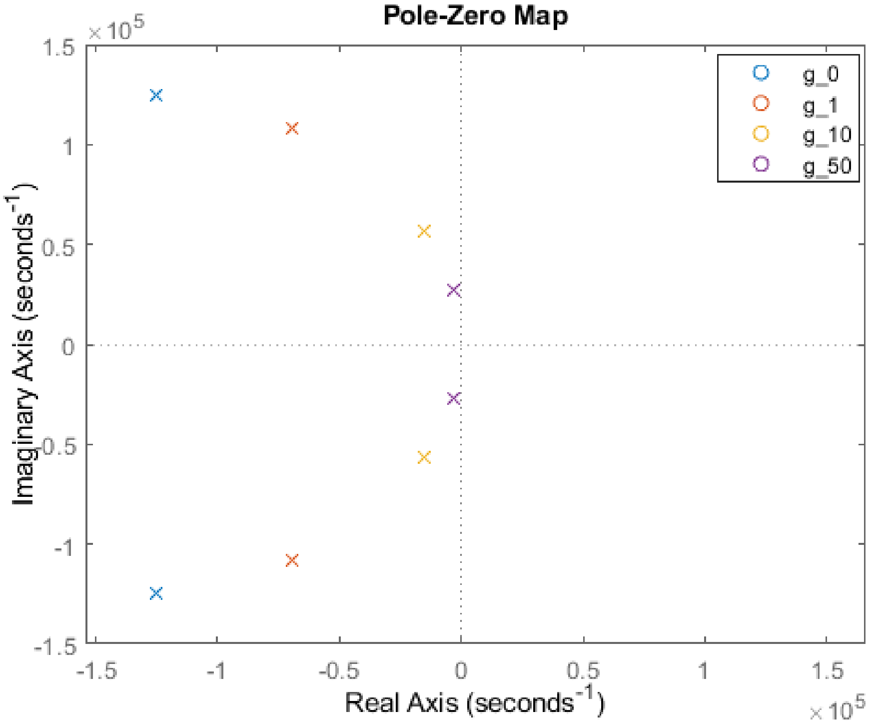

Figure 2.4: Zoomed in pole-zero plot of the model in Figure 2.1 with first order filter and different feedback gains 0,1,10 and 50 (represented in angular frequency)

Characteristics

The dominant pole method was tried to determine the characteristics of the frequency response [9]. This method did not work for both feedback models with a second order filter. The new transfer function did not have the same frequency response as the total frequency response. To overcome this problem, it was chosen to use a method which based on a video from Prof. Dr. Hanspeter Schmid (Professor for Microelectronics and Signal Processing, Institute of Microelectronics FHNW). The video is called Hanspeter's Tools of the Trade [10]. It was chosen to use the same method throughout the whole report to avoid confusion about the approach to find the characteristics. The method is explained below.

If a transfer function has a order higher than two, the transfer function is difficult to characterize. This is the case with the transfer function of Equation 2.2. To make the characterization easier the transfer function can be reduced to several smaller transfer functions that describe the transfer function of Equation 2.2 at certain frequencies. The smaller transfer functions are derived by identifying which parts of the denominator and numerator are dominant at certain frequencies. Parts are dominant if they are at least ten times bigger than the other parts [11]. Some of the smaller transfer functions can be neglected, because an other smaller transfer function describes the same frequency response at those frequencies. For example a transfer function $H(s) = 10 + 0.1s + 0.00001s^2 + 0.000001s^3$ consist of the following parts: "10"; "$0.01s$"; "$0.00001s^2$"; "$0.00001s^2$";"$0.000001s^3$". Around 1 rad/s the part "10" is dominant, which forms a transfer function of $H1(s) = 10$. Around 10 rad/s the parts "10" and "$0.01s$" are dominant, which forms a transfer function of $H2(s) = 10 + 0.01s$. Note that the transfer function H2 has the same characteristics around 1Hz as H1. The transfer function H1 can be neglected, because H2 describes the characteristics of H1. The residual smaller transfer functions can characterize the transfer function of Equation 2.2 if they have less than three poles or zeros.

The transfer function of Equation 2.2 is characterized by the smaller transfer functions of Equation 2.3. See Appendix F.1 for all smaller transfer function and why other smaller transfer functions are neglected. The Bode plot of the smaller transfer functions which describe the transfer function of Equation 2.2 is shown in Figure 2.5. The residual smaller transfer functions can characterize the transfer function of Equation 2.2 if they have less than three poles or zeros. This is the case, so the smaller transfer functions can characterize the transfer function of Equation 2.2.

$$G3 = \frac{sR\tau + R}{s^2(LCR + L\tau + LR\tau g g_m) + s(L + R\tau) + R} \tag{2.3a}$$

$$G6 = \frac{sR\tau}{s^3 LCR\tau + s^2(LCR + L\tau + LR\tau g g_m)} = \frac{R\tau}{s(sLCR\tau + LCR + L\tau + LR\tau g g_m)} \tag{2.3b}$$

The interesting characteristics of the total frequency response are circled in the Bode plot, shown in Figure 2.5. These characteristics are interesting, because after these positions the steepness of the frequency response changes. After the first cut-off the steepness change from 0 dB/decade to -60 dB/decade, after the second the steepness changes to -20 dB/decade and after the third to -40 dB/decade.

The moment these changes take place are crucial to get the magnitude at the lowest PWM frequency (620 kHz) as low as possible. The circled characteristics are determined from Equation 1.1, if gain g is zero, or from Equation 2.3, if gain g is greater than zero. The circled characterisitcs are shown in Equation 2.4. Note that the second and third cut-off frequency and Q factor do not exist when gain g is zero.

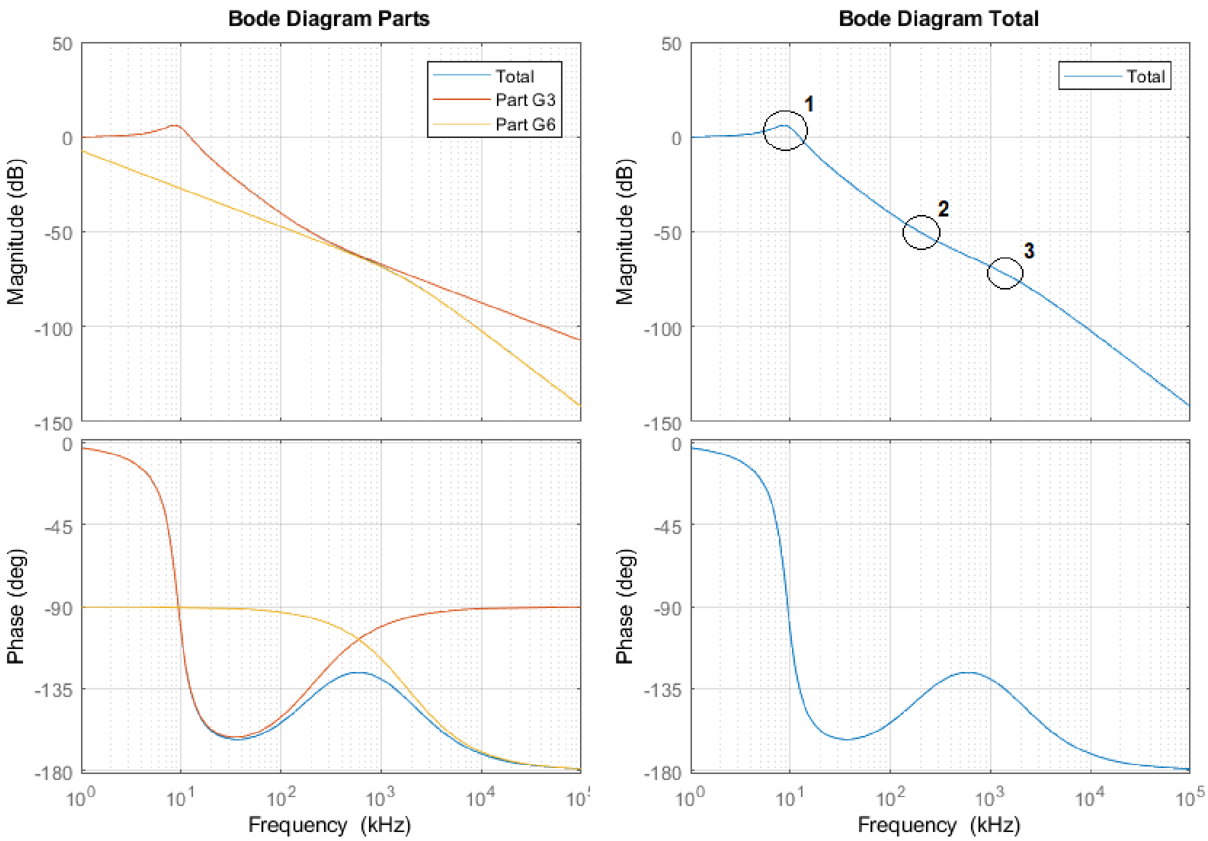

Figure 2.5: Bode plot of smaller transfer function which describe the transfer function of Equation 2.2(total) with a gain g of 10

$$w_{c1} = \begin{cases} \frac{1}{\sqrt{CL}} & g = 0 \\ \sqrt{\frac{R}{LCR+L\tau+LR\tau g g_m}} & g > 0 \end{cases} \tag{2.4a}$$

$$Q_1 = \begin{cases} \sqrt{\frac{CR^2}{L}} & g = 0 \\ \sqrt{\frac{R(LCR+L\tau+LR\tau g g_m)}{(L+R\tau)^2}} & g > 0 \end{cases} \tag{2.4b}$$

$$w_{c2} = \begin{cases} - & g = 0 \\ \frac{1}{\tau} & g > 0 \end{cases} \tag{2.4c}$$

$$w_{c3} = \begin{cases} - & g = 0 \\ \frac{CR+\tau+R\tau g g_m}{CR\tau} & g > 0 \end{cases} \tag{2.4d}$$

Limitations

The limitations can be found with the knowledge of the found characteristics. Table 2.1 shows how the parameters influence the characteristics. If the parameters increase,the value of the characteristics is this displayed with "+", otherwise with "$-$". If the characteristics do not change, the cell is empty. To illustrate what kind of effect a parameter change has on the frequency response, the Bode plot of Figure 2.5 is used. The first cut-off frequency is below 20 kHz at that moment. This is undesirable, because the input signal is in the range of 20 till 20000 Hz. By decreasing the capacitor, the first cut-off frequency is increased, the Q factor decreases and the third cut-off frequency increases. The increase of the third cut-off frequency is undesirable, because this implicates that the steepness after the second cut-off frequency stays for an extend period -20 dB/decade which means that the magnitude of the lowest PWM frequency is higher.

Table 2.1: Parameters influencing characteristics of the model in Figure 2.1 with first order filter

	Characteristics g=0:				Characteristics g>0:			
	w_{c1}	Q_1	w_{c2}	w_{c3}	w_{c1}	Q_1	w_{c2}	w_{c3}
Increase:								
C	-	+			-	+		-
L	-	-			-	-		
τ					-	-	-	-
g					-	+		+

2.1.2 Optimal result

To find the optimal values of the variables that minimize the transfer magnitude at the lowest PWM frequency (620 kHz), a model is created and solved using an solver tool. The solver uses a "NEOS using Bonmin" algorithm. This algorithm is a heuristic algorithm: an approximated solution. This algorithm is chosen, because it can solve this complex non-linear problem [12].

The solver model consists of an objective, parameters, variables and restrictions. The objective is to minimize the transfer magnitude at 620 kHz. The parameters are predefined inputs to the solver model. The predefined inputs for this solver model are the load-resistance of 4 Ω and the transconductance g_m of 1 A/V. The values of the variables are changed by the solver to find the objective. The variables are the inductor value L, the capacitor value C, τ, the first cut-off frequency, the magnitude at 16 kHz and the magnitude at 20 kHz. The restrictions define the variables and the characteristics of the frequency response. The restrictions for the inductor value L, capacitor value C and τ are based on common values for these variables.

For gain g,the restriction is chosen such that the feedback model is stable. The first cut-off frequency must be higher than 20 kHz. It was chosen that the magnitude at 16 kHz must be between -0.5 dB and 0.5 dB, and the magnitude at 20 kHz between -1 dB and 1 dB. The range of the magnitude is chosen to ensure that audio signal is correctly passed through. Without these restrictions the outcome would not be feasible. Once the model is fully defined, the solver uses the aforementioned algorithm to find the values of the variables that minimize the objective. The output of the solver are the values of the variables that together minimize the transfer magnitude, as well as the value of the objective itself: the minimum transfer magnitude at 620 kHz [13].

The optimal result is -55.7dB. The respective values and restrictions for the model are shown in Table 2.2. Note that the value τ does not matter, because g is zero. This feedback loop is not used. This is logical, because if gain g is not zero the cut-off is lower and the Q factor is increased (see Table 2.1). Thefore, the other parameters have to change such that the cut-off frequency is greater and the restrictions of the magnitude at the 20 kHz are satisfied. If τ is lowered, the second and third cut-off frequency would be greater (see Table 2.1). It could be seen as if the frequency response with a gain g of 10 in Figure 2.2 is shifted to the right till the restrictions of the magnitude at 20 kHz are satisfied. It will show that the magnitude at 620 kHz would be lower. If τ stayed the same and other parameters would decrease, the third cut-off frequency would be greater, which means that the steepness stays longer -20 dB/decade after the second cut-off frequency. The magnitude at 620 kHz would be lower. With all other options, the magnitude at 620 kHz would be lower. This can be seen in Table 2.1. The optimal solution is to not use the feedback loop.

The optimal Bode and pole-zero plot are shown in Figures 2.6 and 2.7. It is shown that the magnitude at the frequency 620kHz is the magnitude -55.7 dB and that the model is stable. The output ripple voltage can be found by subtracting the ideal output (input signal times the system gain and feedback transfer function) from the real output. In Figure 2.8, the ripple voltage is shown in time domain. The amplitude of the ripple voltage is 48.9 mV.

Concluding, the magnitude at 620 kHz changes from -53.7 dB to -55.7 dB and the output voltage ripple amplitude is reduced from 60.8 mV to 48.9 mV. Note that the feedback loop is not used. The basic model parameters are optimized, such that the output ripple voltage is reduced.

Table 2.2: Optimal values and restrictions of the model in Figure 2.1 with first order filter

Symbol	Restriction	Optimal values
R		4 Ω
g_m		1 A/V
L	1 nH $\leq$ L $\leq$ 10 mH	31.4 μH
C	1 pF $\leq$ C $\leq$ 10 mF	1.28 μF
g	0 $\leq$ g $\leq$ 1000	0
τ	100 ns $\leq \tau \leq$ 1 s	-
f_{c1}	$f_{c1} >$ 20 kHz	28kHz
$20log(\|H(j20000)\|)$	-1 dB $\leq 20log(\|H(j20000)\|) \leq$ 1 dB	-0.557 dB
$20log(\|H(j16000)\|)$	-0.5 dB $\leq 20log(\|H(j16000)\|) \leq$ 0.5 dB	0.111 dB

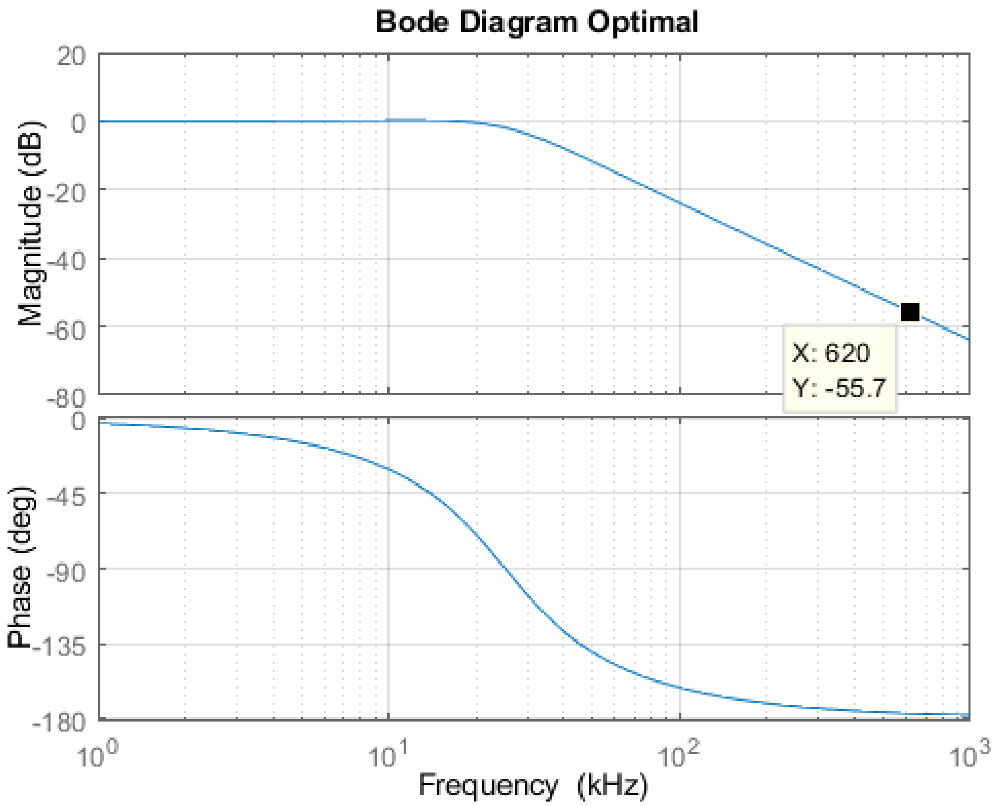

Figure 2.6: Optimal Bode plot of the model in Figure 2.1 with first order filter. The magnitude at 620 kHz is decreased from -53.7 dB till -55.7 dB .

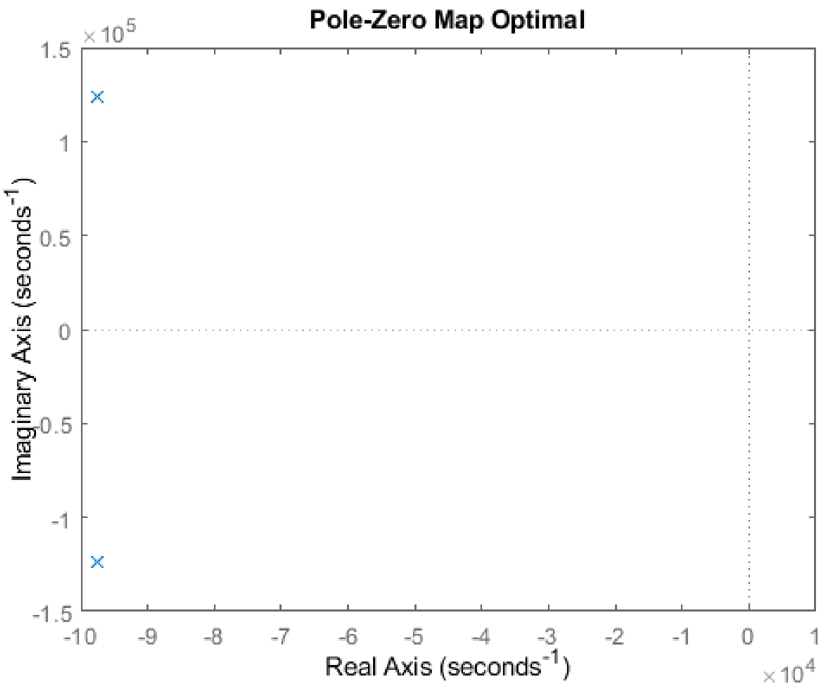

Figure 2.7: Optimal pole-zero plot of the model in Figure 2.1 with first order filter (represented in angular frequency)

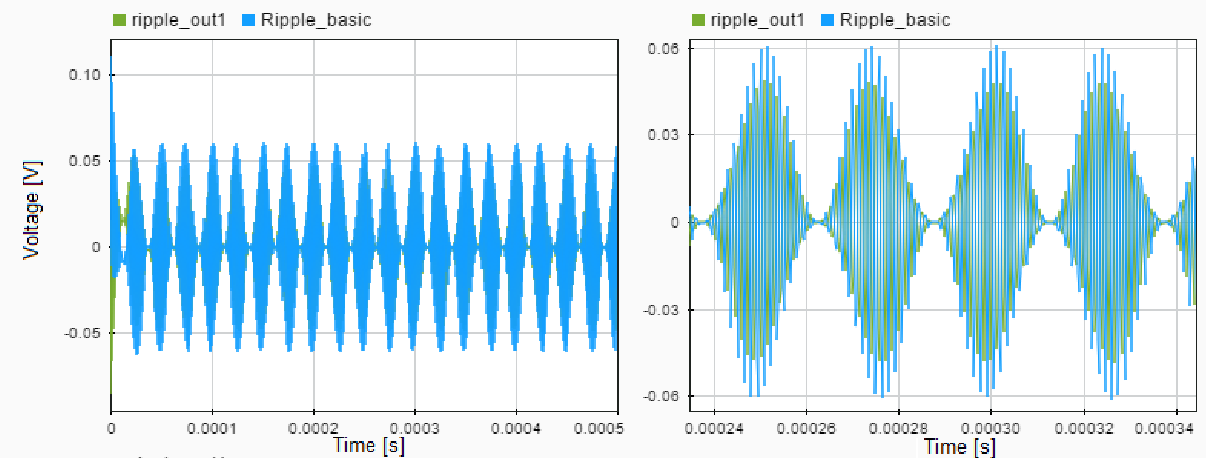

Figure 2.8: Optimal time domain plot of the model in Figure 2.1 with first order filter. The output voltage ripple amplitude is reduced from 60.8 mV (ripple_basic) till 48.9 mV (ripple_out1) .

2.2 Second order high pass filter feedback

2.2.1 Analysis

In this section, the feedback from the output voltage with a second order high pass filter is analyzed. The structure from the first order high pass filter is used in this section: describing second order filter, analyzing total feedback model, characterization of frequency response, finding the limitations and analyzing the optimal solution of the feedback model.

Describing the first order high pass filter

The transfer function of the feedback second order high pass filter is shown in Equation 2.5. The cut-off frequency must be below 620 kHz and as high as possible, see argumentation in section 2.1.1. The Q factor between 0.6 and 0.8 to avoid over- or underdamping. It was chosen to make $w_c = 2Mrad/s$ (cut-off frequency of 320 kHz) and $Q = 0.707$ [b].

$$H(s)_{2^eHP} = \frac{s^2}{s^2 + s\frac{w_c}{Q} + w_c^2} \tag{2.5}$$

Analysis of total feedback model

The analysis is done by looking at the transfer function, Bode plot and pole-zero plot. The transfer function of the circuit model with a first order high pass filter, $H(s)_{out-2^eHP}$, is shown in Equation 2.6. The derivation is derived from the Kirchhoff's rules and Laplace functions, see Appendix C.1 for the derivation. The Bode and pole-zero plots with different feedback gains are shown in Figures 2.10 till 2.9. The Bode plot has the same characteristics, but more extreme than the Bode plot of the first order filter, Figure 2.2. The pole-zero plot shows that the feedback model with different gains g is stable. Except for g is 50, some poles are in the right half plane [7].

$$H(s)_{out-2^eHP} = \frac{V_{out}}{V_{in}} \quad \Rightarrow$$
$$H(s)_{out-2^eHP} = \frac{s^2R + s\frac{w_c}{Q}R + w_c^2R}{s^4LCR + s^3(LCR\frac{w_c}{Q} + L + LRgg_m) + s^2(LCRw_c^2 + L\frac{w_c}{Q} + R) + s(w_c^2L + R\frac{w_c}{Q}) + Rw_c^2} \tag{2.6}$$

[b]see B for the Bode plot

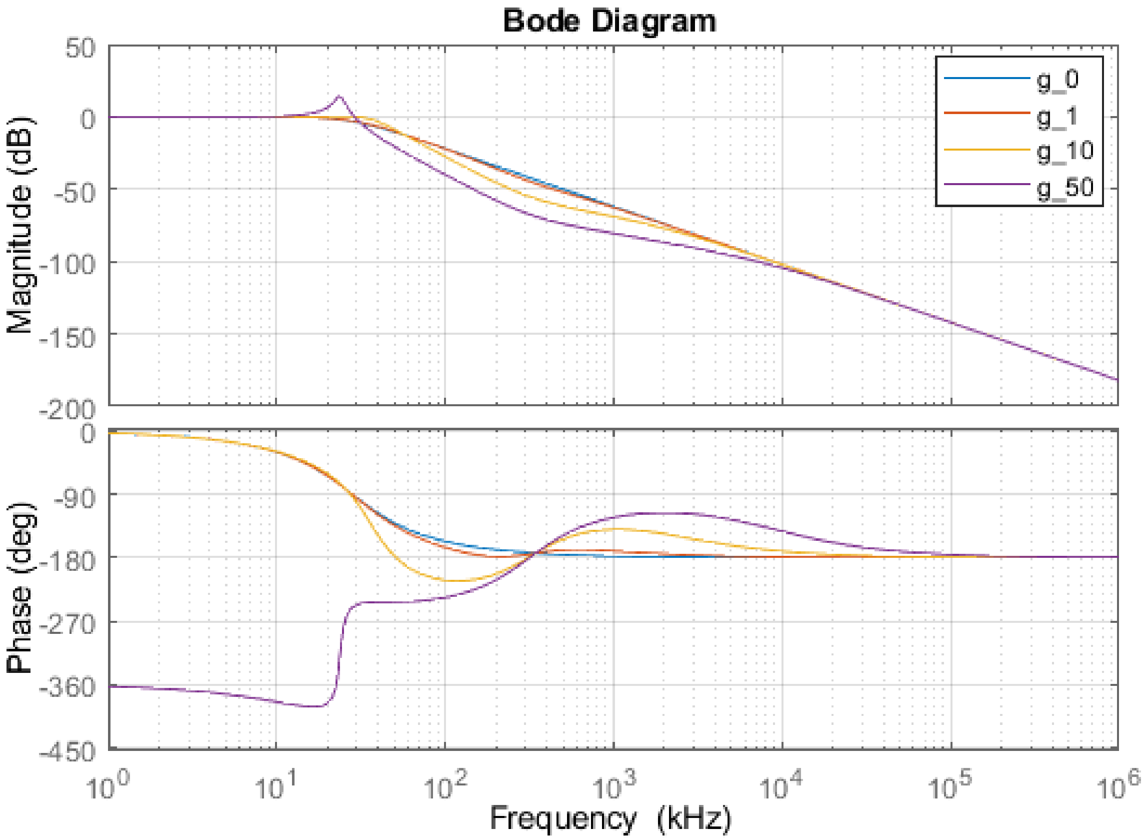

Figure 2.9: Bode plot of transfer function of model 2.1 with a second order filter and different feedback gains 0,1,10 and 50

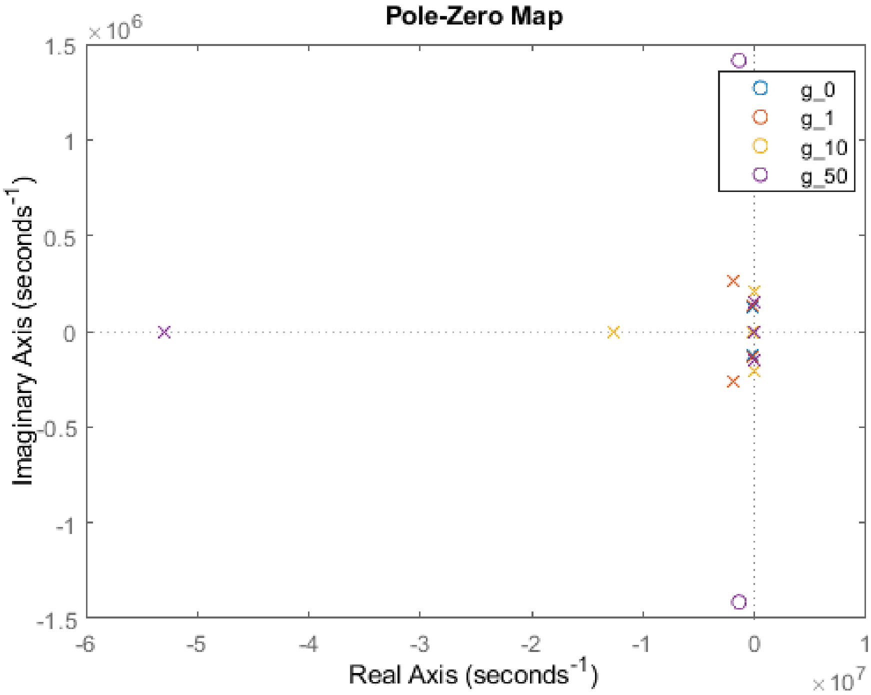

Figure 2.10: Pole-zero plot of the model in Figure 2.1 with second order filter and different feedback gains 0,1,10 and 50 (represented in angular frequency)

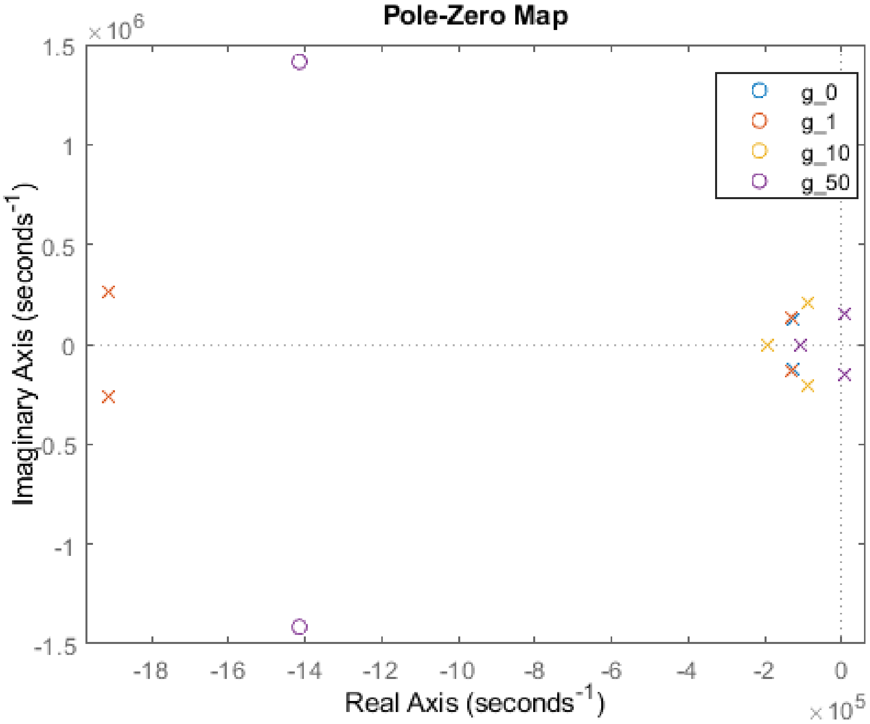

Figure 2.11: Zoomed in pole-zero plot of the model in Figure 2.1 with second order filter and different feedback gains 0,1,10 and 50 (represented in angular frequency)

Characteristics

The characterization uses the method from Section 2.1.1. The transfer function is reduced to smaller transfer functions such that it can be characterized. The smaller transfer functions derivation and Bode plot are shown in Equation 2.7[c] and Figure 2.12. Note that the denominator of H3 has three poles, so the transfer function of Equation 2.6 can not be totally characterized. The Bode plots shows four interesting characteristics. After the first cut-off frequency the steepness is -60 dB/decade, second -40 dB/decade, third 0 dB/decade and fourth -40 dB/decade. The interesting characteristics are shown Equation 2.8. It is unknown what the first cut-off frequency and Q factor are when gain g is not zero.

$$H3 = \frac{s\frac{w_c}{Q}R + w_c^2 R}{s^3(LCR\frac{w_c}{Q} + L + LRgg_m) + s^2(LCRw_c^2 + L\frac{w_c}{Q} + R) + s(w_c^2 L + R\frac{w_c}{Q}) + Rw_c^2} \tag{2.7a}$$

$$H4b = \frac{s^2 R + s\frac{w_c}{Q}R + w_c^2 R}{s^4 LCR + s^3(LCR\frac{w_c}{Q} + L + LRgg_m) + s^2(LCRw_c^2 + L\frac{w_c}{Q} + R)} \tag{2.7b}$$

[c]See Appendix F.2 for the detailed description and all results at specific frequencies.

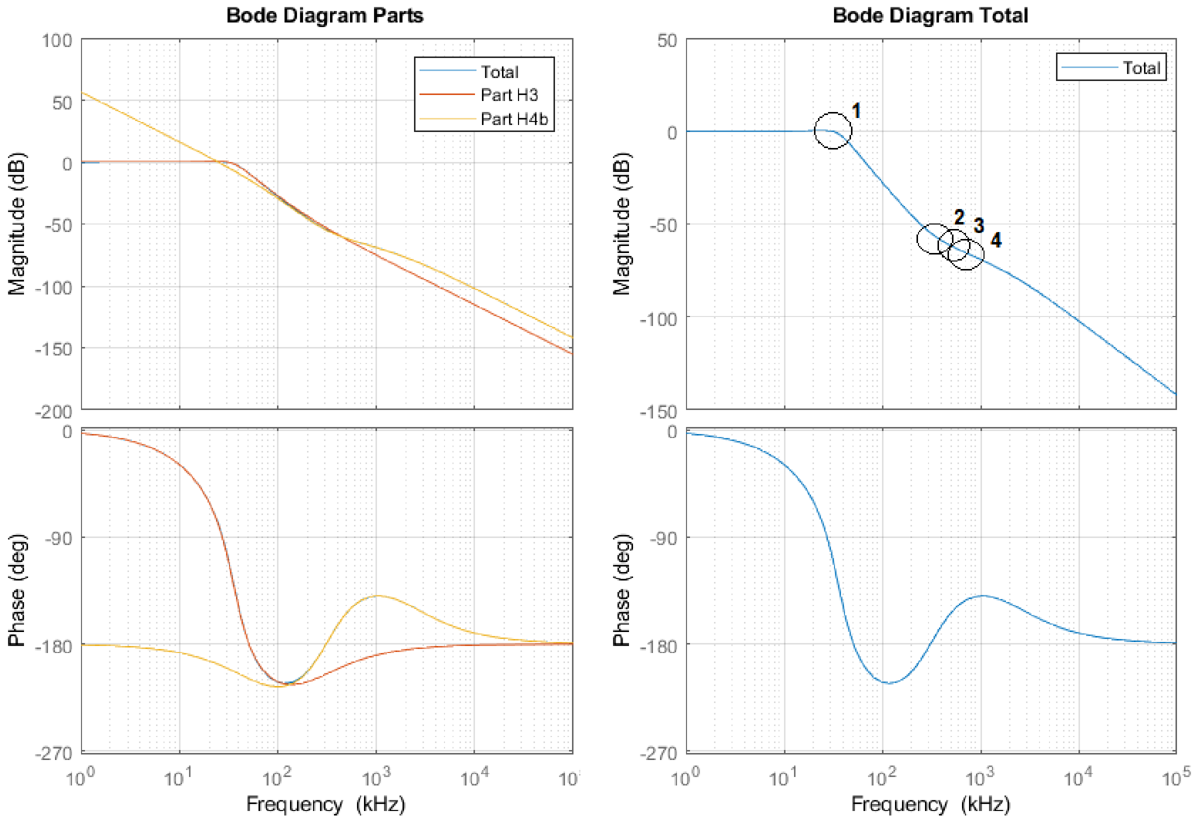

Figure 2.12: Bode plot of smaller transfer function which describe the transfer function of Equation 2.6(total) with a gain g of 10

$$w_{c1} = \begin{cases} \frac{1}{\sqrt{CL}} & g = 0 \\ X & g > 0 \end{cases} \tag{2.8a}$$

$$Q_1 = \begin{cases} \sqrt{\frac{CR^2}{L}} & g = 0 \\ X & g > 0 \end{cases} \tag{2.8b}$$

$$w_{c2} = \begin{cases} - & g = 0 \\ Qw_c & g > 0 \end{cases} \tag{2.8c}$$

$$w_{c3} = \begin{cases} - & g = 0 \\ w_c & g > 0 \end{cases} \tag{2.8d}$$

$$Q_3 = \begin{cases} - & g = 0 \\ QR & g > 0 \end{cases} \tag{2.8e}$$

$$w_{c4} = \begin{cases} - & g = 0 \\ \sqrt{\frac{LCRw_c^2 + Lw_c/Q + R}{LCR}} & g > 0 \end{cases} \tag{2.8f}$$

$$Q_4 = \begin{cases} - & g = 0 \\ \sqrt{\frac{(LCRw_c^2 + Lw_c/Q + R)LCR}{(LCRw_c/Q + L + LRgg_m)^2}} & g > 0 \end{cases} \tag{2.8g}$$

Limitations
The characteristics of the frequency response are dependent on the parameter values of the model. Table 2.3 shows how the parameters influence the characteristics. ” ” means that the characteristic will not change, ”+” means that the characteristic is increased, ”-” means that the characteristic is decreased and ”x” means that the characteristic is unknown. Note that the first cut-off frequency and Q factor are unknown, when g is above zero. However, the Bode plot of Figure 2.9 shows that gain g influence the first cut-off frequency and Q factor. If gain g increases, the cut-off frequency decreases and the Q factor increases. There is not enough evidence to describe the limitations for the total frequency response.

Table 2.3: Parameters influencing characteristics of feedback output voltage second order

	Characteristics g=0:							Characteristics g$>$0:						
	w_{c1}	Q_1	w_{c2}	w_{c3}	Q_3	w_{c4}	Q_4	w_{c1}	Q_1	w_{c2}	w_{c3}	Q_3	w_{c4}	Q_4
Increase:														
C	-	+						x	x				-	+
L	-	-						x	x				-	-
w_c								x	x	+	+		+	-
Q								x	x	+		+	+	-
g								-	+					-

2.2.2 Optimal result

To find the optimal values of the variables that minimize the transfer magnitude at the lowest PWM frequency (620 kHz), the method using the solver is applied (Section 2.1.2). The objective is again to minimize the transfer magnitude at 620 kHz. The used variables, parameters and restrictions are the same as in Section 2.1.2. Only the restriction of gain g is smaller, otherwise the system is not stable. Also, τ is not a variable, and w_c and Q are a variable.

Figures 2.13 till 2.15 show: the magnitude at the 620 kHz is -87.8 dB; the model is stable; the amplitude of the output ripple voltage is 4.65 mV. Table 2.4 shows the optimal parameter values and restrictions of the model. It is difficult to check whether the algorithm finds the correct values, because the formula for the first cut-off frequency is not known. However, Figure 2.9 shows that the gain g will not be zero, because when gain g is 10 frequency response satisfies the restrictions and the magnitude at 620 kHz is lower than the original basic model.

Table 2.4: Optimal values and restrictions of the model in Figure 2.1 with second order filter

Symbol	Restriction	Optimal values
R		4 Ω
g_m		1 A/V
L	1 nH $\leq$ L $\leq$ 10 mH	31.4 μH
C	1 pF $\leq$ C $\leq$ 10 mF	1.28 μF
g	0 $\leq$ g $\leq$ 20	0
w_c	$1 \cdot 10^6 \leq w_c \leq 1 \cdot 10^{15}$	-
Q	0.1$\leq w_c \leq$10	-
$20log(\lvert H(j20000)\rvert)$	-1 dB $\leq 20log(\lvert H(j20000)\rvert) \leq$ 1 dB	-0.557 dB
$20log(\lvert H(j16000)\rvert)$	-0.5 dB $\leq 20log(\lvert H(j16000)\rvert) \leq$ 0.5 dB	0.111 dB

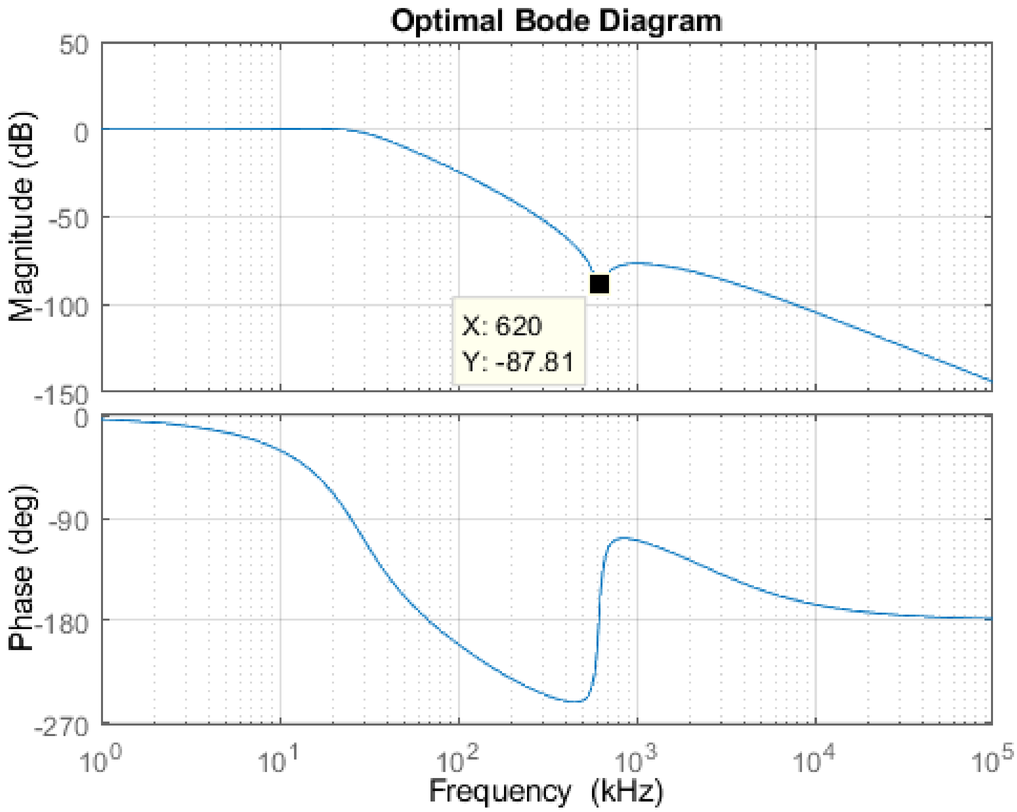

Figure 2.13: Optimal Bode plot of the model in Figure 2.1 with second order filter. Magnitude at 620 kHz is decreased from -53.7 dB till -87.8 dB.

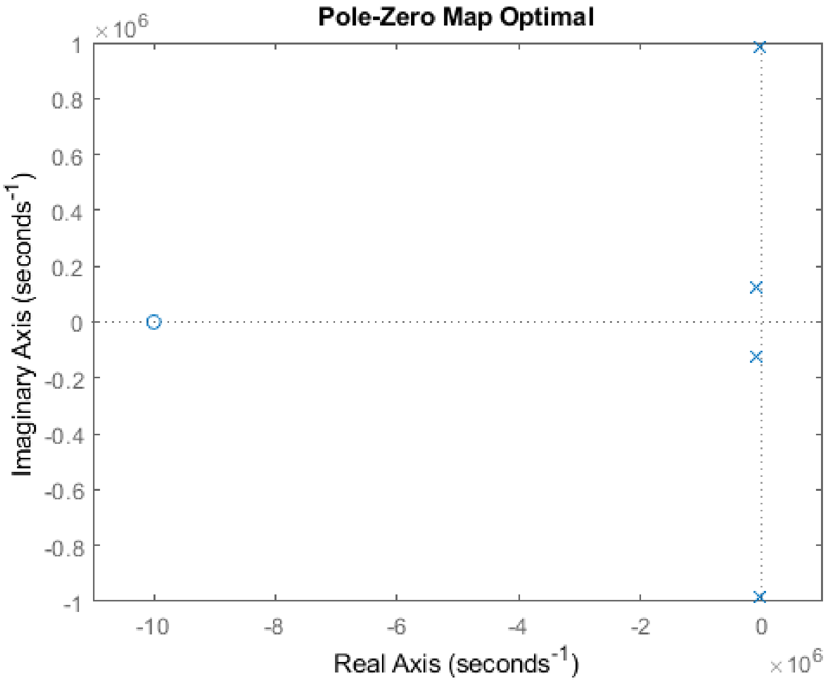

Figure 2.14: Optimal pole-zero plot of the model in Figure 2.1 with second order filter (represented in angular frequency)

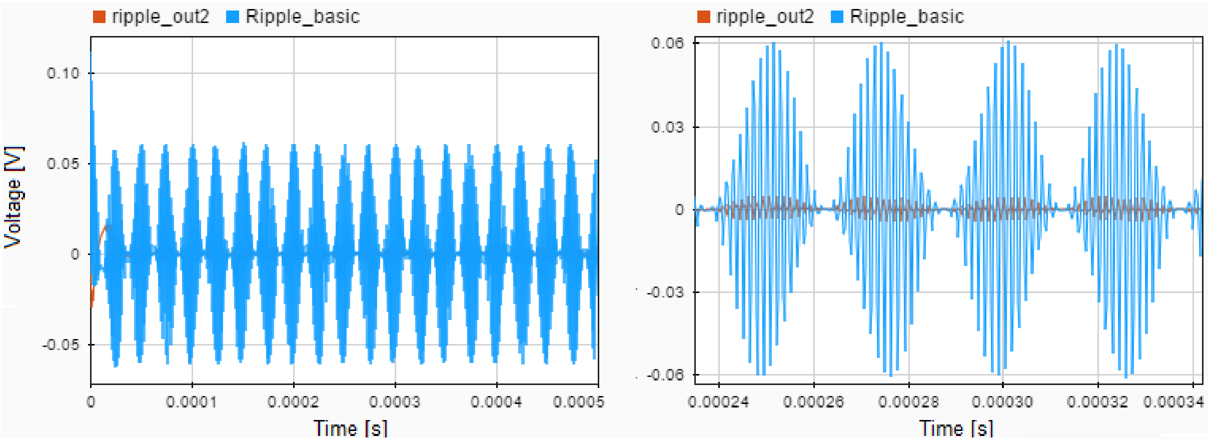

Figure 2.15: Optimal time domain plot of the model in Figure 2.1 with second order filter. Output ripple voltage amplitude is reduced from 60.8 mV (ripple_basic) till 4.65 mV (ripple_out2).

Chapter 3

Feedback from the inductor voltage

This chapter has the same structure as Chapter 2. The feedback model is the basic model with feedback from the inductor voltage to a controlled current source, shown in Figure 3.1. In the feedback loop the inductor voltage is transformed to a current signal high pass filtered (such that only the PWM frequencies are passed through) and amplified with gain g, and then amplified with a current source amplifier g_{cs}. This model will be analyzed by looking at the transfer function, frequency response, stability, characteristics and limitations. Finally, the optimal results will be determined and analyzed. The analysis and optimization is done for the model with a first and second order high pass filter.

The feedback model is designed to reduce the output ripple voltage as in Chapter 2. The difference is that the current source is controlled via a feedback loop from the inductor voltage. The inductor voltage is transformed to a current signal by integrating and dividing the signal with the inductor. Then the inductor current is filtered such that the ripple frequencies are left over. The same first and second order high pass filters as in Chapter 2 are used: τ is 800 ns, w_c is 2Mrad/sec and Q 0.707. After the filtering, the signal is amplified with a gain g. The optimal gain g value will be probably one, because the current ripple is directly found via the feedback model. However, the dividing with the exact inductor value is hard. To make it more realistic the gain g will vary around 1. After amplifying with gain g, the feedback signal is amplified with a current source amplifier g_{cs} of the controlled current course, which is chosen to be 1.

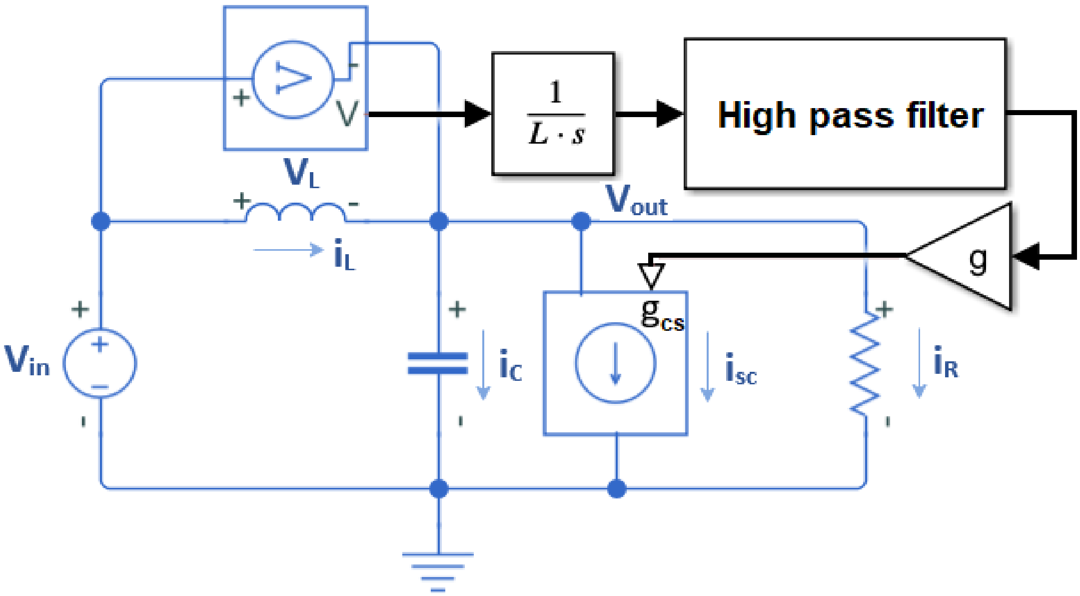

Figure 3.1: Circuit model of design with feedback from the inductor voltage

3.1 First order high pass filter feedback

3.1.1 Analysis

This section analyzes the feedback from the output voltage with a first order high pass filter by determining the total feedback transfer function, visualizing the frequency response with a Bode plot and checking the stability via pole-zero plot. Next, the characteristics will be identified, which will help to find the limitations of the model.

Analysis of total feedback model

The total feedback model is analyzed by looking at the transfer function, Bode plot and pole-zero plot. The transfer function of the circuit model with a first order high pass filter, $H(s)_{L-1^eHP}$, is shown in Equation 3.1, see Appendix D.1 for the derivation. Figures 3.2 and 3.3 show the Bode and pole-zero plot with different feedback gains around one. The Bode plot shows that the optimal gain is one, because the inductor voltage is transformed to the current by integrating and dividing with the inductor value ($i_L = V_L/(sL)$), so the controlled current source takes the total ripple current. The Bode plot also shows that the cut-off and Q factor stay the same for different gains, only the steepness changes from -40 dB/decade to -60 dB/decade. The pole-zero plot shows that the feedback model is stable for different gains g; all real pole values are at the left side of the plot [7].

$$H(s)_{L-1^eHP} = \frac{V_{out}}{V_{in}} = \frac{sR\tau(1-gg_{cs})+R}{s^3CRL\tau + s^2(L\tau + CRL) + s(L + R\tau - R\tau g g_{cs}) + R} \tag{3.1}$$

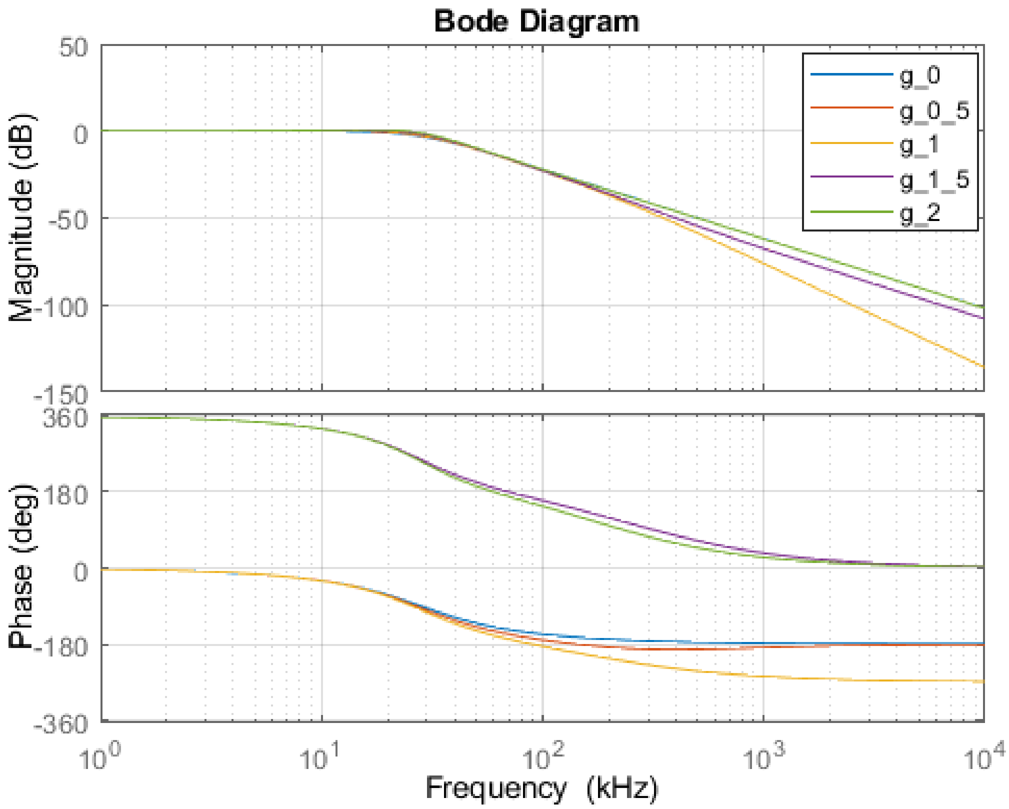

Figure 3.2: Bode plot of the transfer function of the model in Figure 3.1 with first order filter and different feedback gains g 0, 0.5, 1, 1.5 and 2

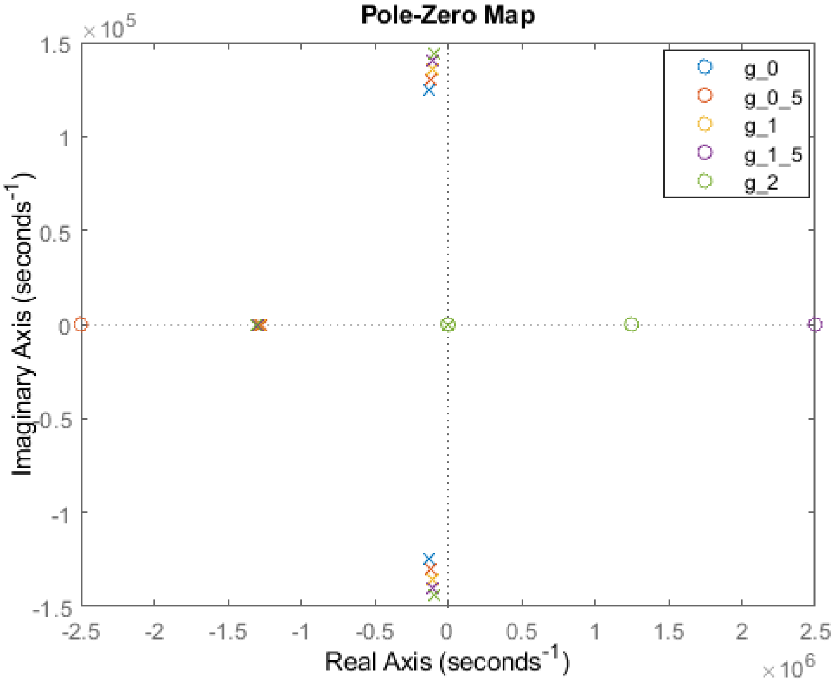

Figure 3.3: Pole-zero plot of the model in Figure 3.1 with first order filter and different feedback gains 0,1,10 and 50 (represented in angular frequency)

Characteristics

The characterization uses the method from Section 2.1.1. The transfer function is reduced to multiple smaller transfer functions such that it can be characterized. The Bode plot of the smaller transfer functions that describe the transfer function of Equation 3.1 are shown in Figure 3.4[a]. Interesting characteristics are circled in the Bode plot, because after these positions the steepness of the frequency response changes. After the first cut-off the steepness changes from 0 dB/decade to -40 dB/decade, after the second the steepness changes to -60 dB/decade and after the third it depends on whether the gain g is 1 or not. If gain g is 1, the steepness stays -60dB/decade and if gain g is not 1 the steepness changes to -20 dB/decade. The smaller transfer function has the same first order function in the numerator. This is why the steepness is increased by +40 dB/decade, instead of +20 dB/decade. In the Bode plot of Figure 3.4, the third cut-off frequency is not visible because gain g is 1. The interesting characteristics are shown in Equation 3.3.

$$K2b = \frac{sR\tau(1 - gg_{cs}) + R}{s^2(L\tau + CRL) + s(L + R\tau - R\tau g) + R} \tag{3.2a}$$

$$K4 = \frac{sR\tau(1 - gg_{cs}) + R}{s^3CRL\tau + s^2(L\tau + CRL)} \tag{3.2b}$$

[a]See Appendix F.3 for the detailed description and all results at specific frequencies.

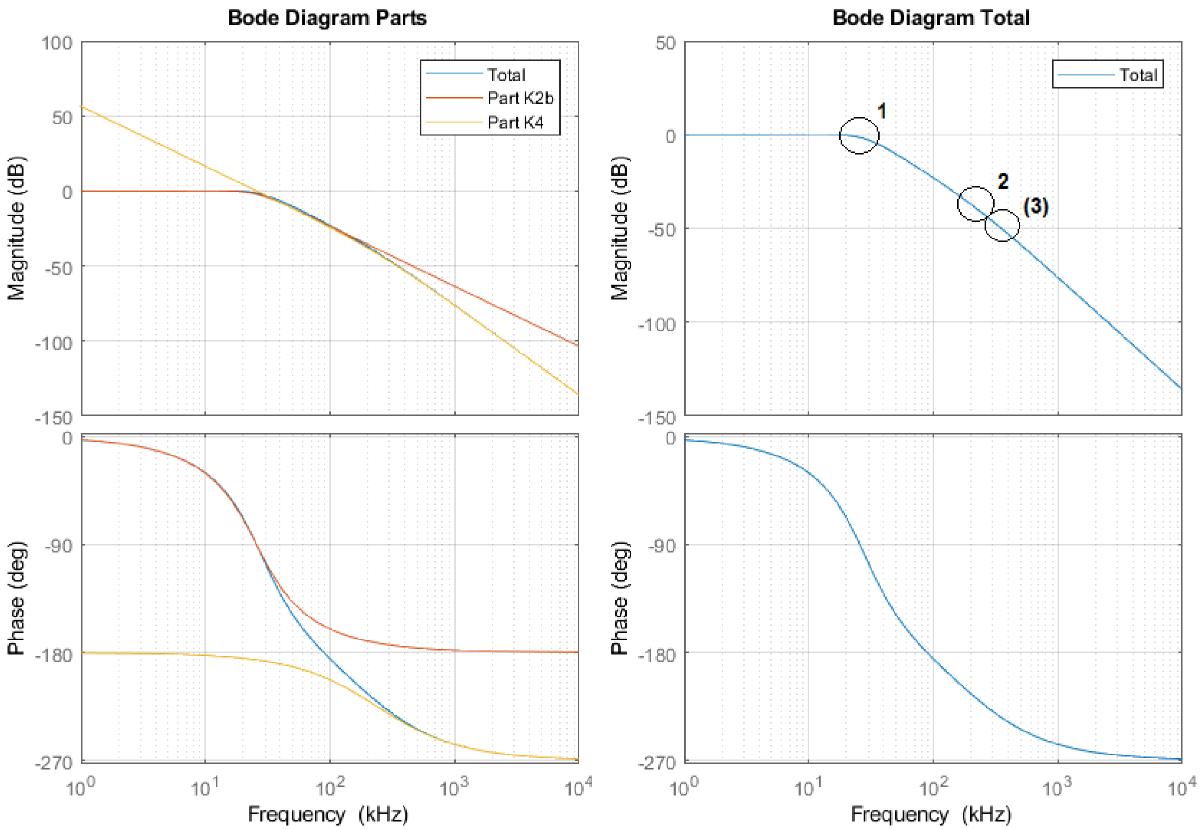

Figure 3.4: Bode plot of smaller transfer function which describe the transfer function of Equation 3.1(total) with a gain g of 1

$$w_{c1} = \begin{cases} \frac{1}{\sqrt{CL}} & g = 0 \\ \sqrt{\frac{R}{L\tau + CRL}} & g > 0 \end{cases} \tag{3.3a}$$

$$Q_1 = \begin{cases} \sqrt{\frac{CR^2}{L}} & g = 0 \\ \sqrt{\frac{R(L\tau + CRL)}{(L + R\tau(1 - gg_{cs}))^2}} & g > 0 \end{cases} \tag{3.3b}$$

$$w_{c2} = \begin{cases} - & g = 0 \\ \frac{\tau + CR}{CR\tau} & g > 0 \end{cases} \tag{3.3c}$$

$$w_{c3} = \begin{cases} - & g = 0 \\ - & g = 1 \\ \frac{1}{|\tau(1 - gg_{cs})|} & else \end{cases} \tag{3.3d}$$

Limitations
The insights in the characteristics, which are shown in Equation 3.3, help to find the limitations of the characteristics. Table 3.1 shows the influence of parameter values on the characteristics. ”+” means that the characteristic is increased, ”-” means that the characteristic is decreased and ” ” means that the characteristic will not change.

Table 3.1: Parameters influencing characteristics of the model in Figure 3.1 with first order filter

	Characteristics g=0:				Characteristics g>0:			
	w_{c1}	Q_1	w_{c2}	w_{c3}	w_{c1}	Q_1	w_{c2}	w_{c3}
Increase:								
C	-	+			-	+	-	
L	-	-			-	-		
τ					-	- (+ if g=1)	-	- (” ” if g=1)
g						+ (” ” if g=1)		- (+ if g>2)

3.1.2 Optimal result

The optimal parameter values of the feedback model that minimize the transfer function at the lowest PWM frequency (620 kHz)are found using the method with the solver, as described in Section 2.1.2. The objective is to minimize the transfer magnitude at 620 kHz.

The optimal parameter values and restrictions of the feedback model are shown in Table 3.2. It is logical that the gain g is one, this is the best value as shown in Figure 3.2. If the gain g is one, the third cut-off frequency do not exist. Only the first and second cut-off frequency and Q factor are adjusted such that the minimum magnitude at 620 kHz is achieved.

The minimum magnitude of the lowest PWM frequency is -69.2 dB. The Bode and pole-zero plot are shown in Figures 3.5 and 3.6. The figures shown that the magnitude of the model is -73.3dB and the model is stable. The output ripple voltage can be found by subtracting the ideal output (input signal times the system gain and feedback transfer function) from the real output. The amplitude of the output ripple voltage can be read from the time domain plot in Figure 3.7, 6.21 mV.

The conclude, the magnitude at 620 kHz is decreased from -53.7 dB to -73.3 dB and the output voltage ripple amplitude is reduced from 60.8 mV to 6.21 mV. The output ripple is reduced by the feedback model.

Table 3.2: Optimal values and restrictions of the model in Figure 3.1 with first order filter

Symbol	Restriction	Optimal values
R		4 Ω
g_{cs}		
L	1 nH $\leq$ L $\leq$ 10 mH	38 μH
C	1 pF $\leq$ C $\leq$ 10 mF	714 nF
g	0 $\leq$ g $\leq$ 2	1
τ	100 ns $\leq \tau \leq$ 1 s	2.85 μs
f_{c1}	$f_{c1} >$ 20 kHz	24.4 kHz
$20log(\|H(j20000)\|)$	-1 dB $\leq 20log(\|H(j20000)\|) \leq$ 1 dB	-0.608 dB
$20log(\|H(j16000)\|)$	-0.5 dB $\leq 20log(\|H(j16000)\|) \leq$ 0.5 dB	-0.199 dB

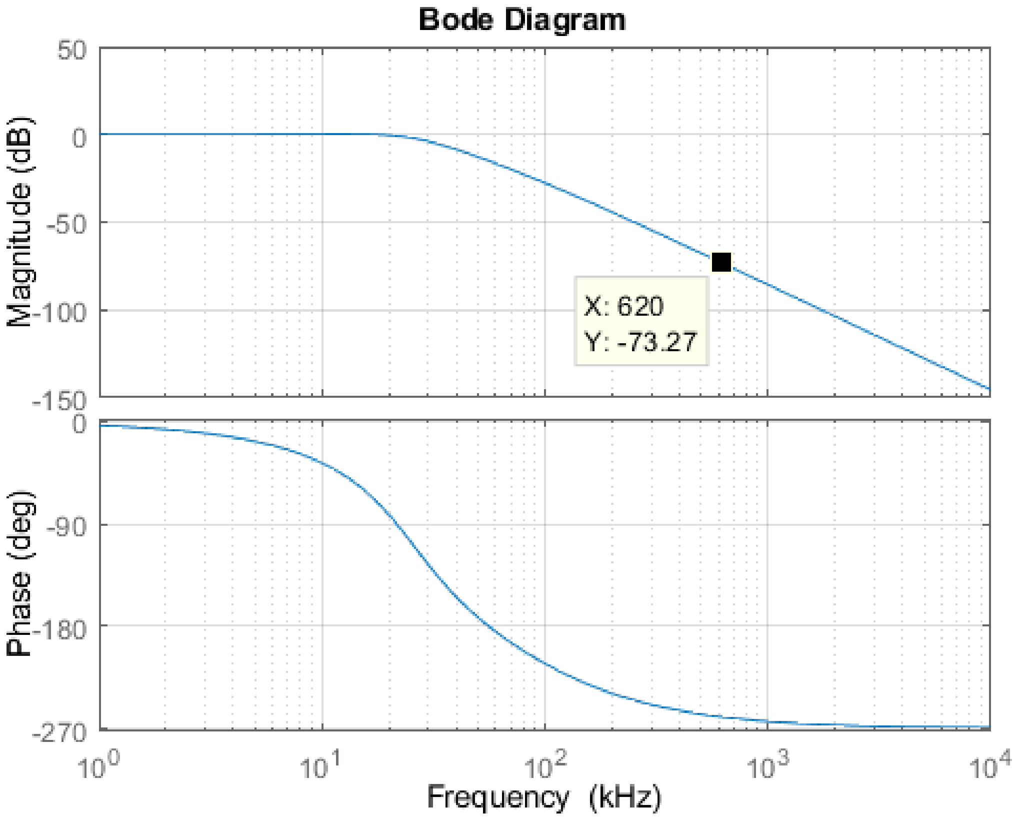

Figure 3.5: Optimal Bode plot of the model in Figure 3.1 with first order filter. The magnitude at 620 kHz decreases from -53.3 dB to -73.3 dB .

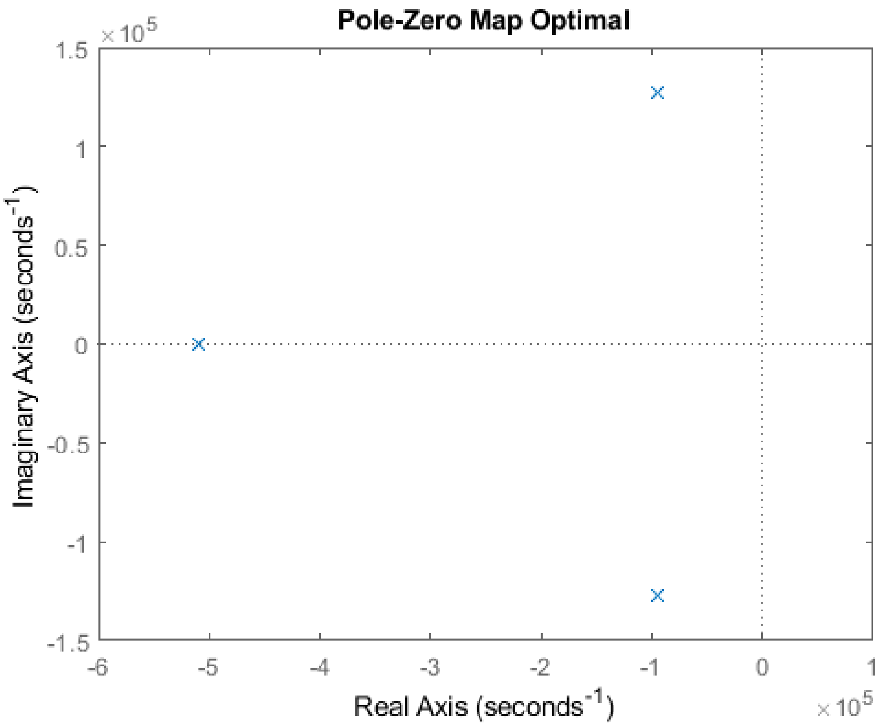

Figure 3.6: Optimal pole-zero plot of the model in Figure 3.1 with first order filter (represented in angular frequency)

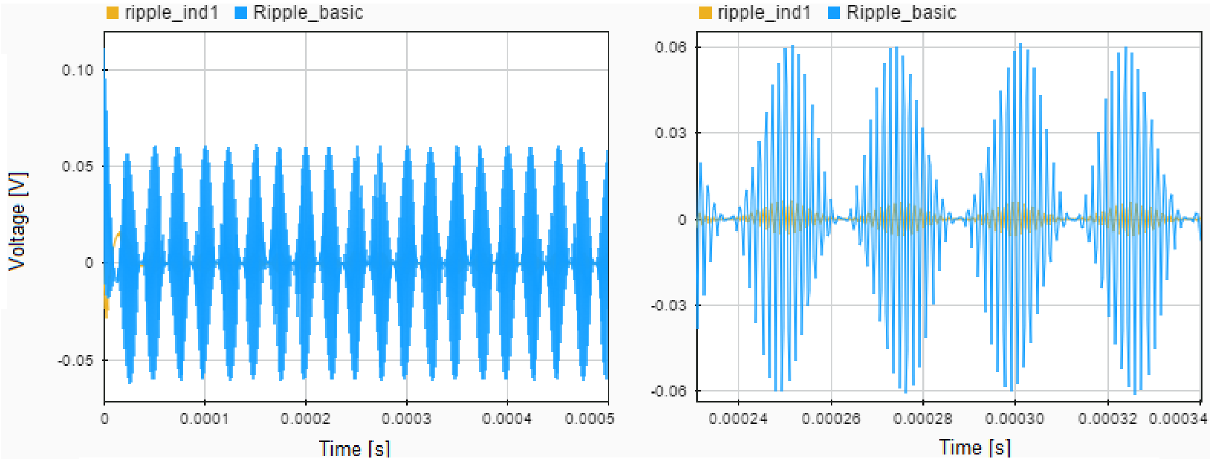

Figure 3.7: Optimal time domain plot of the model in Figure 3.1 with first order filter. The output voltage ripple amplitude is reduced from 60.8 mV (ripple_basic) to 6.21 mV (ripple_ind1) .

3.2 Second order high pass filter feedback

3.2.1 Analysis

In this section,the feedback from the inductor voltage with a second order high pass filter is analyzed. Starting by analyzing the total feedback model by determining the transfer function, checking the stability and visualizing the frequency response. Then, the frequency response will be characterized. At the end, the limitations of the model will be given.

Analysis of total feedback model

The analyzing is done by looking at the transfer function, Bode plot and pole-zero plot. The transfer function of the circuit model with a second order high pass filter, $H(s)_{L-2^eHP}$, is shown in Equation 3.4[b]. The Bode and pole-zero plot with different feedback gains around 1 is shown in Figures 3.8 and 3.9. From the Bode plot it can been seen that the optimal gain is 1 (as in Section 3.1.1), and the cut-off and Q factor stays the same for different gains g, only the steepness changes from -40 dB/decade to -60 dB/decade. The pole-zero plot shows that the feedback model is stable for different gains g [7].

$$H(s)_{L-2^eHP} = \frac{V_{out_{L-2^eHP}}}{V_{in}} \quad \Rightarrow$$
$$H(s)_{L-2^eHP} = \frac{s^2R(1-gg_{cs})+sR\frac{w_c}{Q}+Rw_c^2}{s^4CRL+s^3(L+CRL\frac{w_c}{Q})+s^2(L\frac{w_c}{Q}+R+CLRw_c^2-gg_{cs}R)+s(Lw_c^2+R\frac{w_c}{Q})+w_c^2R} \tag{3.4}$$

[b]see Appendix D.1 for the derivation

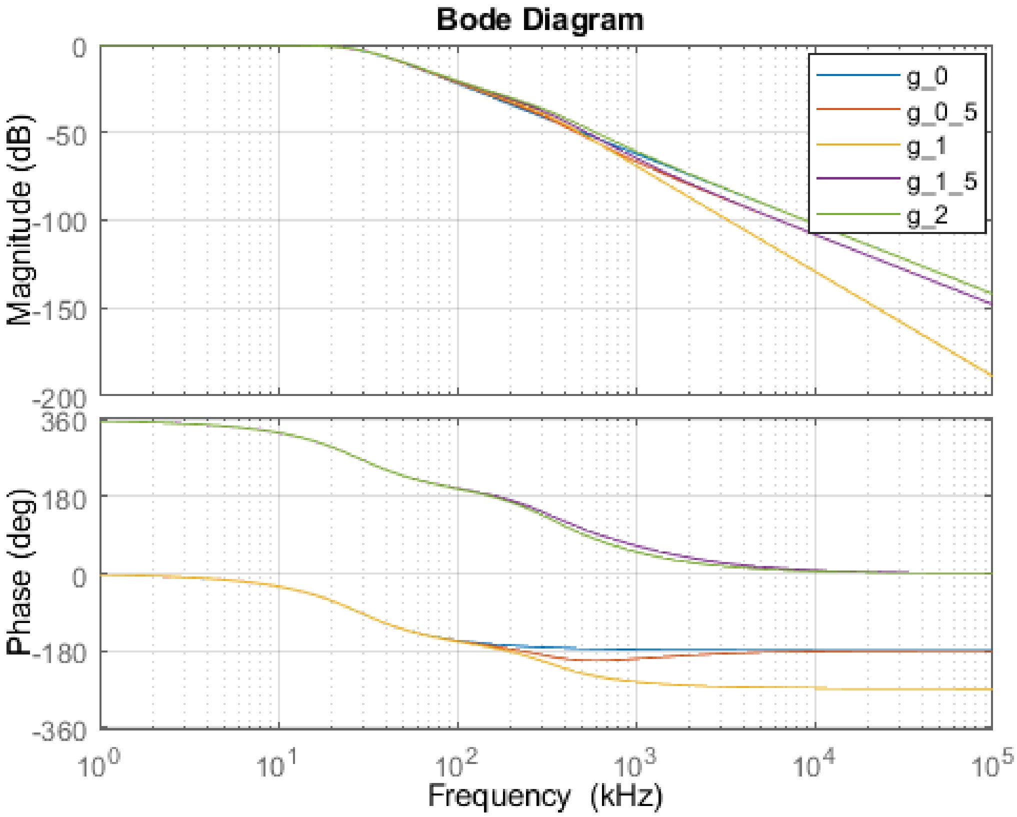

Figure 3.8: Bode plot of the transfer function of the model in Figure 3.1 with second order filter and different feedback gains g 0, 0.5, 1, 1.5 and 2

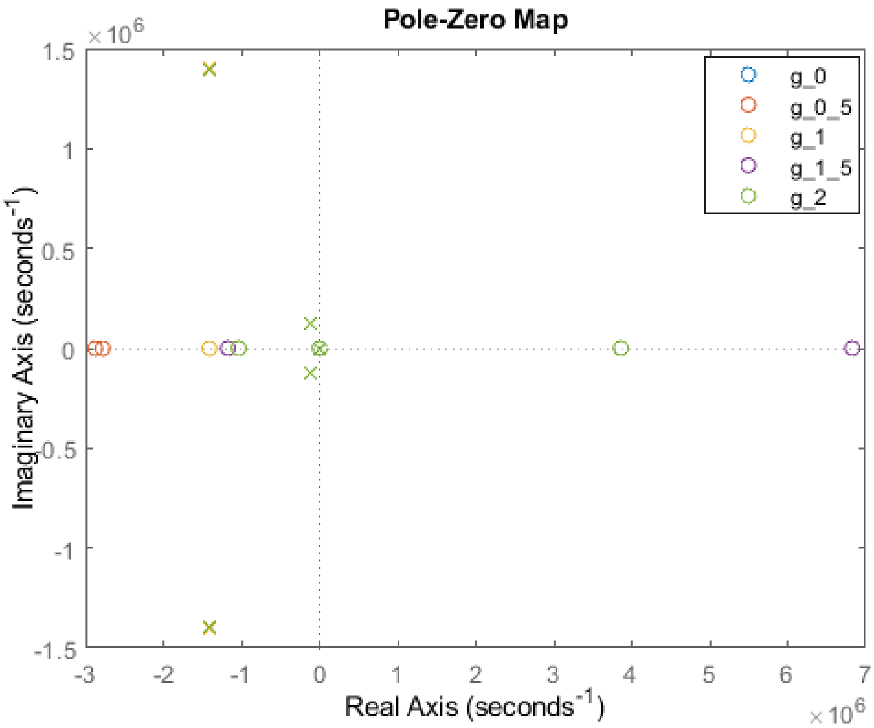

Figure 3.9: Pole-zero plot of the model in Figure 3.1 with second order filter and different feedback gains 0,1,10 and 50 (represented in angular frequency)

Characteristics
The characterization of the frequency response is done by reducing the transfer function of Equation 3.4 into several smaller transfer function, see Section 2.1.1 for the detailed description. The smaller transfer functions that describe the transfer function of Equation 3.4 are shown in Figure 2.5. The smaller transfer function are shown in Equation 3.5[c]. Figure F.8 shows the Bode plot of the smaller transfer functions and transfer function of Equation 3.4. From Equation 3.5 it is shown that there are four interesting characteristics. Both smaller equation have in the numerator and denominator a higher order than one, which indicates that there are four cut-off frequencies. The interesting characteristics are circled in the Bode plot. After the first cut-off, the steepness change from 0 dB/decade to -40 dB/decade. After the second cut-off frequency the steepness changes to -20 dB/decade because gain g is 1, which implicates that the fourth cut-off is at the same cut-off frequency as the second. The third cut-off frequency changes the steepness to -60 dB/decade. If gain g was not 1, the steepness would change after the second cut-off frequency to -40 dB/decade, the third to -80 dB/decade and the fourth to -40 dB/decade. In Equation 3.6 the characteristics are shown.

$$L3 = \frac{sR\frac{w_c}{Q} + Rw_c^2}{s^2(L\frac{w_c}{Q} + CLRw_c^2 + R(1-gg_{cs})) + s(Lw_c^2 + R\frac{w_c}{Q}) + w_c^2 R} \tag{3.5a}$$

$$L4b = \frac{s^2R(1-gg_{cs}) + sR\frac{w_c}{Q} + Rw_c^2}{s^4CRL + s^3(L + CRL\frac{w_c}{Q}) + s^2(L\frac{w_c}{Q} + CLRw_c^2 + R(1-gg_{cs}))} \tag{3.5b}$$

$$w_{c1} = \begin{cases} \frac{1}{\sqrt{CL}} & g = 0 \\ \sqrt{\frac{w_c^2 R}{L\frac{w_c}{Q} + CLRw_c^2 + R(1-gg_{cs})}} & g > 0 \end{cases} \tag{3.6a}$$

$$Q_1 = \begin{cases} \sqrt{\frac{CR^2}{L}} & g = 0 \\ \sqrt{\frac{w_c^2 R(L\frac{w_c}{Q} + CLRw_c^2 + R(1-gg_{cs}))}{(Lw_c^2 + R\frac{w_c}{Q})^2}} & g > 0 \end{cases} \tag{3.6b}$$

$$w_{c2} = \begin{cases} - & g = 0 \\ w_c Q & g > 0 \end{cases} \tag{3.6c}$$

$$w_{c3} = \begin{cases} - & g = 0 \\ \sqrt{\frac{L\frac{w_c}{Q} + CLRw_c^2 + R(1-gg_{cs})}{CRL}} & g > 0 \end{cases} \tag{3.6d}$$

$$Q_3 = \begin{cases} - & g = 0 \\ \sqrt{\frac{CRL(L\frac{w_c}{Q} + CLRw_c^2 + R(1-gg_{cs}))}{(L + CRL\frac{w_c}{Q})^2}} & g > 0 \end{cases} \tag{3.6e}$$

$$w_{c4} = \begin{cases} - & g = 0 \\ w_c Q & g = 1 \\ \sqrt{\left|\frac{w_c^2}{(1-gg_{cs})}\right|} & else \end{cases} \tag{3.6f}$$

$$Q_4 = \begin{cases} - & g = 0 \\ - & g = 1 \\ \sqrt{\left|\frac{w_c^2 R^2(1-gg_{cs})}{(R\frac{w_c}{Q})^2}\right|} = \sqrt{|Q^2(1-gg_{cs})|} & else \end{cases} \tag{3.6g}$$

[c] see Appendix F.4 for the detailed description and all results at specific frequencies

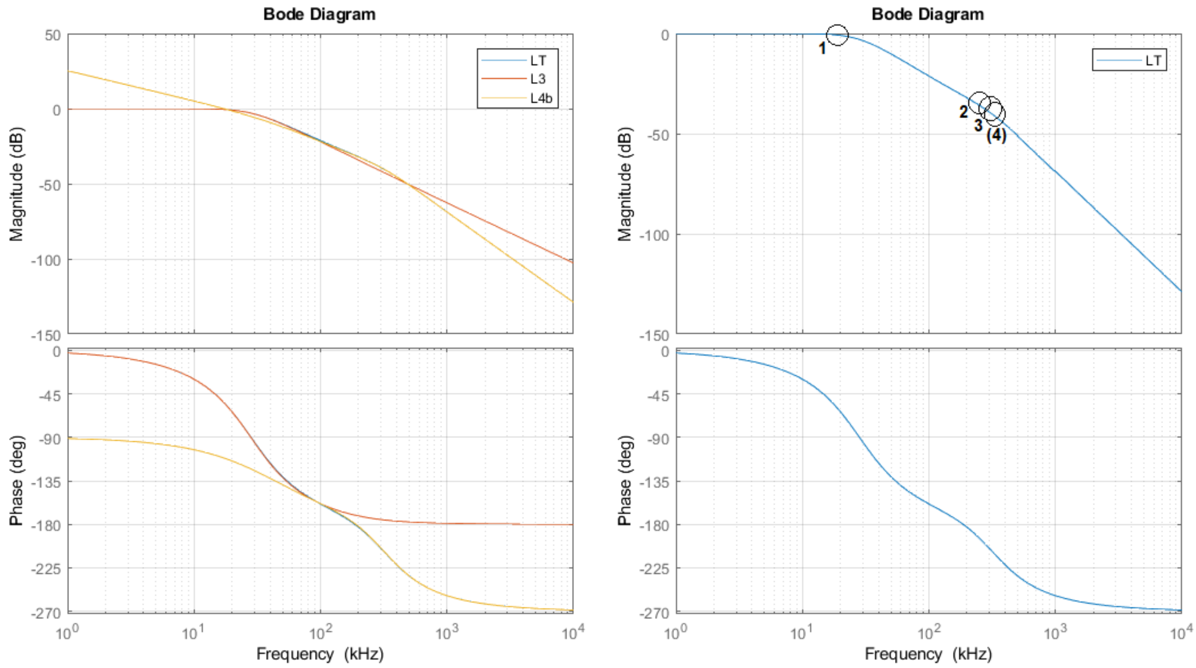

Figure 3.10: Bode plot of smaller transfer function which describe the transfer function of Equation 3.4(total) with a gain g of 1

Limitations

The limitations of the characteristics are found with the help of Equation 3.6. Table 3.3 shows the influence of parameter values on the characteristics. "+" means that the characteristic is increased, "-" means that the characteristic is decreased and " " means that the characteristic will not change. Note that when g is 1 or larger than 2 the some influence changes. To illustrate how changing parameters influences the frequency response, w_c is decreased. The decrease increases the third cut-off frequency, which causes the change of the steepness, from -20dB/decade to -60dB/decade, to start earlier. This makes the magnitude at the lowest PWM frequency lower (620 kHz). Unfortunately, the first cut-off frequency also decreases. This causes the class d amplifier to be unable to amplify the total audible frequency range, which is undesirable.

Table 3.3: Parameters influencing characteristics of the model in Figure 3.1 with second order filter

	Characteristics g=0:						
	w_{c1}	Q_1	w_{c2}	w_{c3}	Q_3	w_{c4}	Q_4
Increase:							
C	-	+					
L	-	-					
w_c							
Q							
g							
	Characteristics g>0:						
	w_{c1}	Q_1	w_{c2}	w_{c3}	Q_3	w_{c4}	Q_4
Increase:							
C	-	+		-	-		
L	-	-		-	-		
w_c	+	-	+	+	-	+	
Q	-	+	+	-	+	(+ if g=1)	+ (" " if g=1)
g	+	-		-	-	+ (- if g>2)	- (+ if g>2)

3.2.2 Optimal result

The optimal parameter values of the feedback model that minimize the transfer function at the lowest PWM frequency (620 kHz)are found using the method with the solver, as described in Section 2.1.2. The objective is to minimize the transfer magnitude at 620 kHz.

The used optimal values and restrictions are shown in Table 3.4. It is logical that the gain g is one, otherwise the fourth cut-off frequency and Q factor exist, which changes the roll-off at the end -40 dB/decade, instead of -60 dB/decade. Furthermore, the third cut-off frequency will be minimized, such that the steepness of -20 dB/decade after the second cut-off frequency is as small as possible or does not exist. Figure 3.11 shows that the steepness does not exist.

Figures 3.11 till 3.14 show:the magnitude at 620 kHz is decreased from -53.7 dB to -78.4 dB; the system is stable;the ripple amplitude is reduced from 60.8 mV to 3.29 mV. The feedback model reduces the output ripple.

Table 3.4: Optimal values and restrictions of the model in Figure 3.1 with second order filter

Symbol	Restriction	Optimal values
R		4 Ω
g_{cs}		
L	1 nH $\leq$ L $\leq$ 10 mH	32μH
C	1 pF $\leq$ C $\leq$ 10 mF	1.3 μF
g	0 $\leq$ g $\leq$ 2	1
w_c	$1 \cdot 10^6 \leq w_c \leq 1 \cdot 10^{15}$	1Mrad/sec
Q	0.3$\leq w_c \leq$10	10
f_{c1}	$f_{c1} > 20kHz$	28 kHz
$20log(\|H(j20000)\|)$	-1 dB $\leq 20log(\|H(j20000)\|) \leq$ 1 dB	-0.872 dB
$20log(\|H(j16000)\|)$	-0.5 dB $\leq 20log(\|H(j16000)\|) \leq$ 0.5 dB	0.109 dB

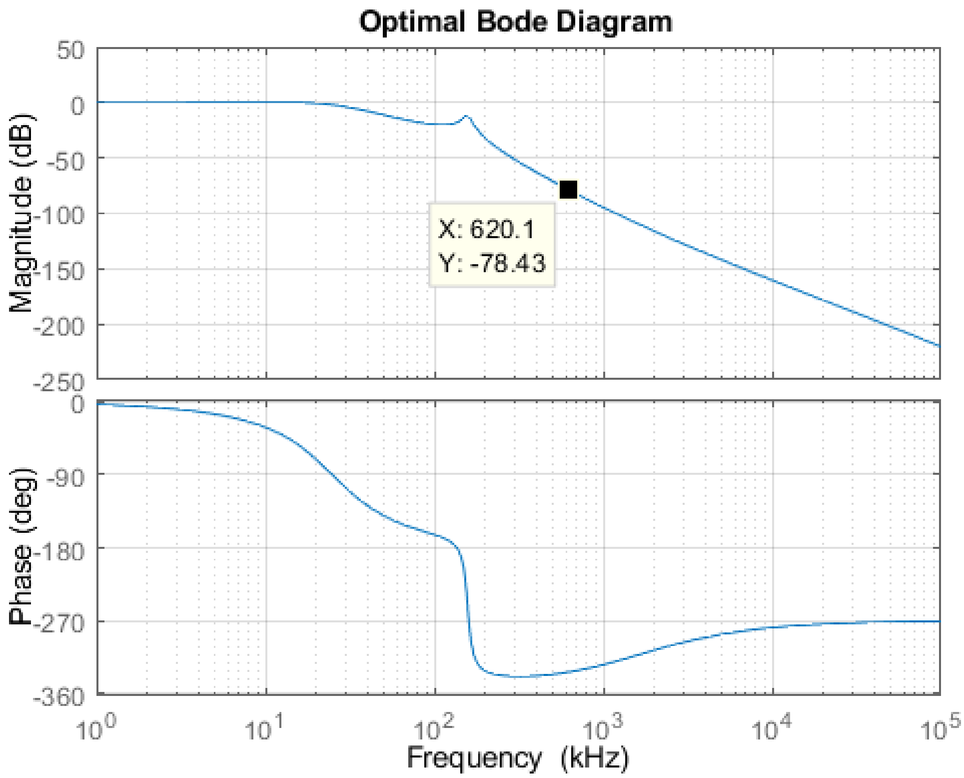

Figure 3.11: Optimal Bode plot of the model in Figure 3.1 with second order filter. The magnitude at 620 kHz is decreased from -55.3 dB to -78.4 dB .

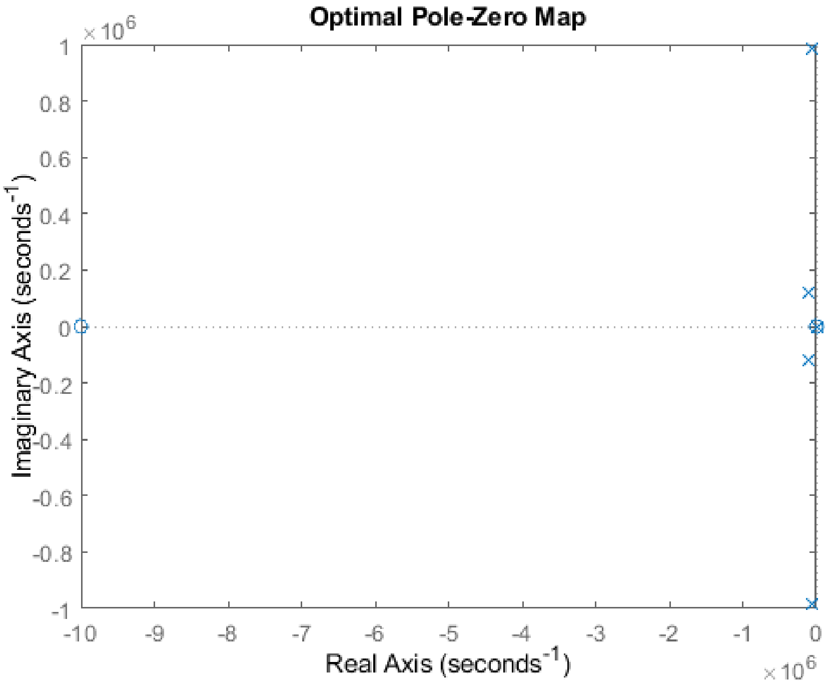

Figure 3.12: Optimal pole-zero plot of the model in Figure 3.1 with second order filter (represented in angular frequency)

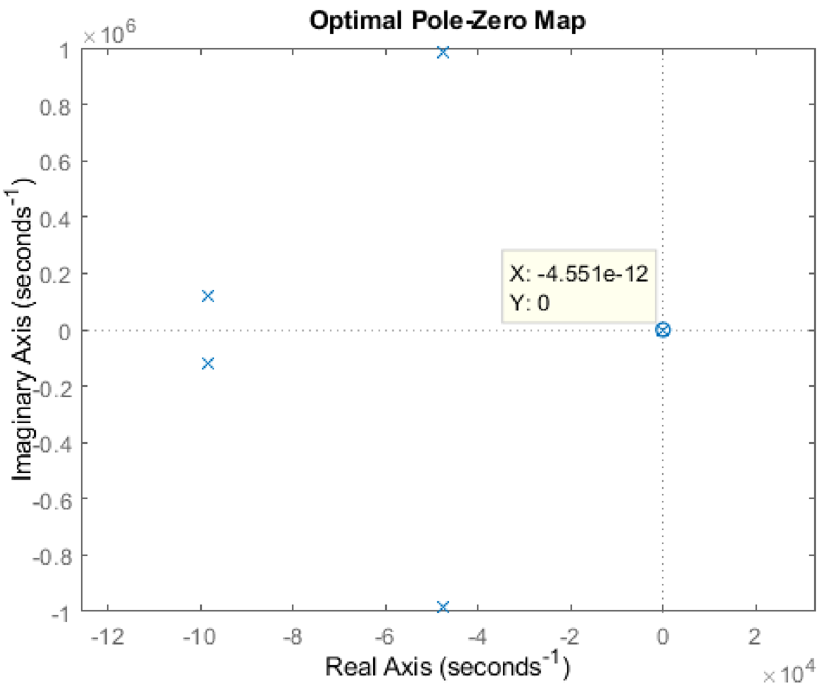

Figure 3.13: Optimal pole-zero plot of the model in Figure 3.1 with second order filter zoomed in (represented in angular frequency)

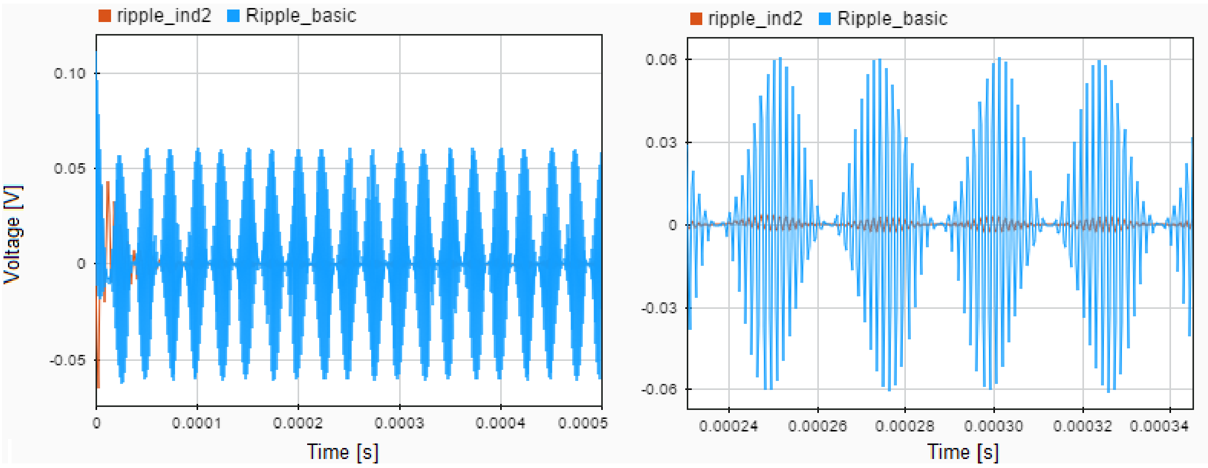

Figure 3.14: Optimal time domain plotof the model in Figure 3.1 with second order filter. The output voltage ripple amplitude is reduced from 60.8 mV (ripple_basic) and 3.29 mV (ripple_ind2)

Chapter 4

Compare designs

In this chapter, all models are compared in a Bode plot and an overview of the results is given in a table.

4.1 Bode plot

In Figure 4.1, all optimal frequency responses are shown in a Bode plot. The frequency response of feedback output voltage with a first order filter and basic model lie almost at the same line. This is because the magnitude of the lowest PWM frequency (620 kHz) is the lowest when the feedback loop is not used, gain g is zero. The difference is that the feedback model from the output voltage with a first order filter has the optimal basic parameter values. The Bode plot shows that the feedback model from the output voltage with a second order filter has the lowest magnitude of the lowest PWM frequency.

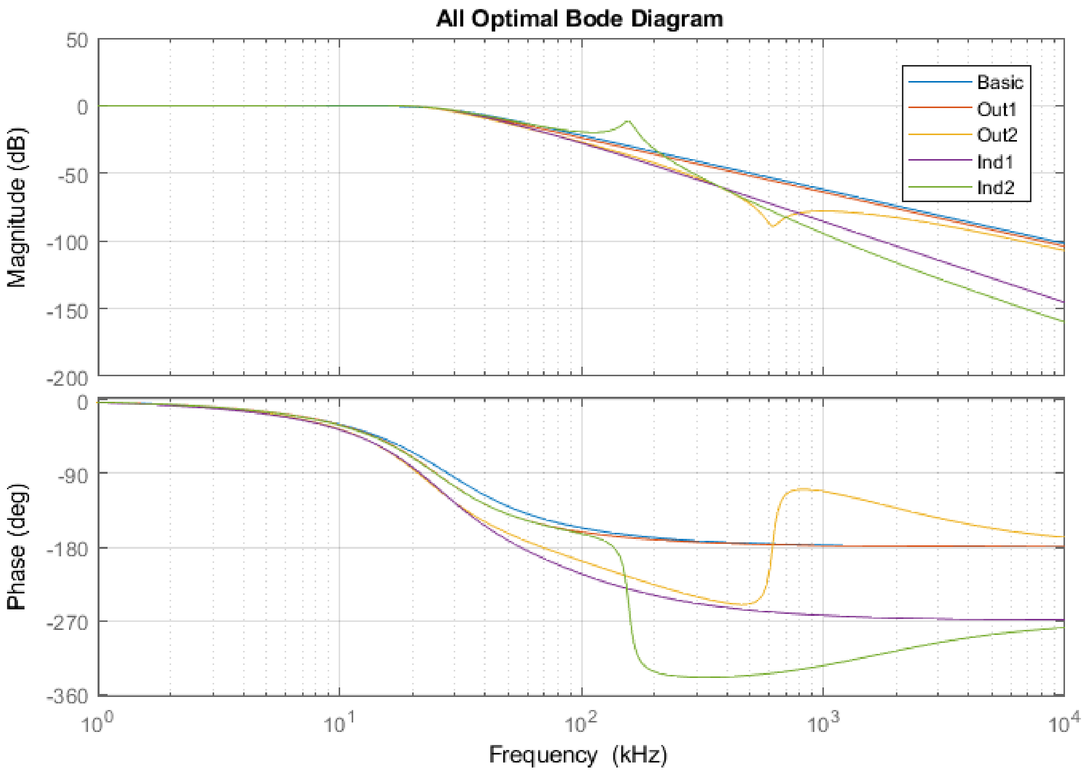

Figure 4.1: Optimized frequency responses. Basic = basic model. Out1 = model with feedback from the output voltage with first order filter. Out2 = model with feedback from the output voltage with second order filter. Ind1 = model with feedback from the inductor voltage with first order filter. Ind2 = model with feedback from the inductor voltage with second order filter.

4.2 Overview results

Table 4.1 gives an overview of the results: the magnitude of the lowest PWM frequency from the Bode plot; percentage of decrease of the magnitude; output ripple amplitude; percentage of reduction of the amplitude. The percentage of decrease of the magnitude and reduction of the amplitude are calculated with Equations 4.1 and 4.2.

The amplitude of the output ripple is higher for Out2 than for Ind2. Figure 4.2 shows both output ripples. Ind2 looks like an AM signal and Out2 looks like an AM signal with peaks. These peaks create the higher amplitude. Although the feedback model Out2 has a higher output ripple amplitude, the feedback model is the best feedback model, as it has the lowest magnitude at 620 kHz.

$$percentage = 100\% \cdot (\frac{Mag_{feedback}}{Mag_{Basic}} - 1) \tag{4.1}$$

$$percentage = 100\% \cdot (1 - \frac{A_{ripple-feedback}}{A_{ripple-Basic}}) \tag{4.2}$$

Table 4.1: Overview results

Model	Magnitude [dB]	Percentage decrease of magnitude [%]	Ripple amplitude [mV]	Percentage reduction of amplitude [%]
Basic	-53.7	0	60.8	0
Output voltage first order filter (Out1)	-55.7	4.5	48.9	19.6
Output voltage second order filter (Out2)	-87.8	64.7	4.65	92.4
Inductor voltage first order filter (Ind1)	-73.3	37.5	6.21	89.8
Inductor voltage second order filter (Ind2)	-78.4	47.1	3.29	94.6

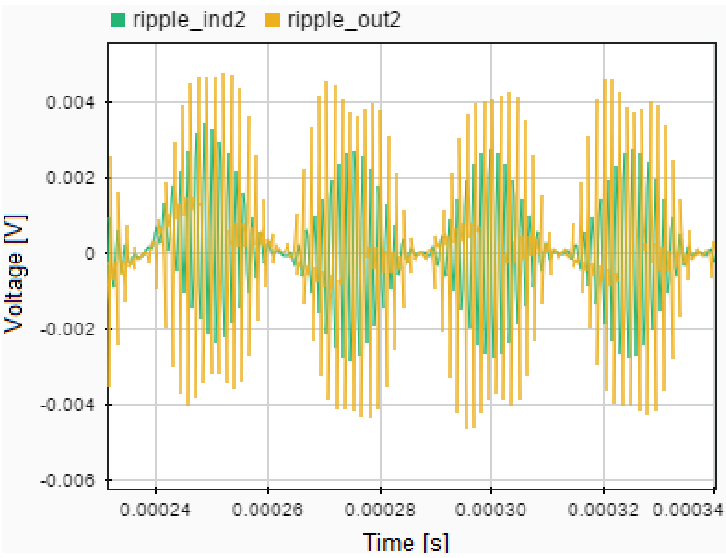

Figure 4.2: Output voltage ripple of Out2 and Ind2

Conclusions and recommendations

Conclusions

The goal of the this report was to find the limitations and improve the feedback models of Lucas Timmermans. These feedback models are designed to reduce the output voltage ripple. The limitations were found via the characteristics of the feedback model transfer function. Using these limitations, the model parameters were optimized, such that the model was improved. The model with feedback from the output voltage with a first order filter did not reduce the ripple. From the results, it seemed like the feedback model reduced the output voltage ripple amplitude from 60.8 mV to 48.9 mV (reduction of 19.6%) and the magnitude at 620 kHz (lowest PWM frequency) from -53.7 dB till -55.7 dB (decrease of 4.5%). However, this was not the case, because the gain g in the feedback loop was zero. The model with feedback from the inductor voltage with a second order high pass filter reduced the output voltage ripple amplitude from 60.8 mV to 4.65 mV (reduction of 92.4%) and the magnitude at 620 kHz (lowest PWM frequency) from -53.7 dB till -87.8 dB (decrease of 64.7%). The inductor feedback model with a first order filter reduced the ripple from 60.8 mV to 6.21 mV (reduction of 89.8%) and the magnitude at 620 kHz from -53.7 dB to -73.3 dB (decrease of 37.5%). The inductor feedback model with a second order filter reduced the ripple from 60.8 mV to 3.29 mV (reduction of 94.6%) and the magnitude at 620 kHz from -53.7 dB to -78.4 dB (decrease of 47.1%). The best feedback model is the model with feedback from the inductor voltage with a second order high pass filter, as it has the lowest magnitude at 620 kHz: -87.7 dB.

Recommendations

The first recommendation of improvement is the method of characterization of the frequency response. This method is not a peer-reviewed method, however, it does generate valid outcomes. The biggest uncertainty of the method is the impact of neglecting one or more smaller transfer functions. Smaller transfer functions can be neglected if they have overlap with other smaller transfer functions. Furthermore, in one case the residual smaller transfer function had an order higher than two in the denominator, which meant that the characteristics of the total frequency response could not be fully characterized. It was tried to use a known method to determine the characteristics of the frequency response: the dominant pole method. However, this method did not work for both feedback models with a second order filter, because the new transfer function did not have the same frequency response as the original frequency response. The recommendation is to validate the used method or find a peer-reviewed method that works for these feedback models.

The second recommendation is regarding the optimization of the feedback models. Optimization is a specialized mathematical field, which means that someone with more expertise in this area could find an algorithm and restrictions that potentially are satisfied a better solution than the one presented here.

This research used a sine input. For further studies, it would be interesting to analyze what would happen to the output ripple when an audio signal is used as input instead.

Another interesting study is to increase the order of the high pass filter. Especially the high pass filters of the model with feedback from the inductor current. Theoretically it seems like that the magnitude of the transfer function at 620 kHz would decrease if the order of the filter increases. This would be interesting to analyze.

A question that comes to mind is how much the efficiency of the class D amplifier with the feedback model from the inductor voltage decreases. The class D amplifier is popular because of its high efficiency. If the implementation of the feedback model decreases the efficiency too much, the model would lose its practicality.

If the feedback models are efficient enough, it would be interesting to construct and simulate the feedback model with non-ideal components,such that performance in practice can be predicted more accurately. If with non-ideal components the model remains promising, the feedback could be constructed physically, to verify whether the model works in practice.

Bibliography

[1] J. Honda and J. Adams, “Class d audio amplifier basics,” International Rectifier, Aug. 2005. [Online]. Available: http://data.eefocus.com/08-03/27_1204618041/File/1205141140.pdf

[2] A. S. Limjoco, “Measuring output ripple and switching transients in switching regulators,” Analog Devices, 2013. [Online]. Available: http://www.analog.com/media/en/technical-documentation/application-notes/AN-1144.pdf

[3] L. Timmermans, “Reduction of output ripple of a class-d amplifier using active filtering,” Ph.D. dissertation, University of Twente, 2018.

[4] R. Lidl and H. Niederreiter, *Introduction to Finite Fields and their Applications*. Cambridge University Press, 1994, ISBN: 9780521460941.

[5] A. R. Moller, *Hearing: Anatomy, Physiology, and Disorders of the Auditory System*. Academic Press, 2006, ISBN: 9780080463841.

[6] J. Widder and Y. T. Zhao, “Understanding output filters for class-d amplifiers,” STMicroelectronics, Sep. 2008. [Online]. Available: https://www.eetimes.com/document.asp?doc_id=1274877

[7] C. Basso, *Designing Control Loops for Linear and Switching Power Supplies: A Tutorial Guide*. Artech House, 2012, ISBN: 9781608075577.

[8] B. Hauke, “Basic calculation of a buck converter’s power stage,” Texas instruments, p. 8, Dec. 2011. [Online]. Available: http://www.ti.com/lit/an/slva477b/slva477b.pdf

[9] P. R. GRAY, P. J. HURST, S. H. LEWIS, and R. G. MEYER, *Analysis and design of analog integrated circuits*. Don Fowley, 2009, ISBN: 9780470245996.

[10] H. Schmid, “Hanspeter’s tools of the trade,” Jul. 2017. [Online]. Available: https://tube.switch.ch/videos/6942d1cd

[11] A. Veloni and A. Palamides, *Control System Problems: Formulas, Solutions, and Simulation Tools*. CRC Press, 2016, ISBN: 9781439868515.

[12] P. Bonami and J. Lee, “Bonmin users’ manual,” p. 24, Aug. 2007. [Online]. Available: https://projects.coin-or.org/Bonmin/browser/stable/0.1/Bonmin/doc/BONMIN_UsersManual.pdf?format=raw

[13] K. Aardal, “Modelling linear and linear integer optimization problems - an introduction,” TuDelft, p. 21, Oct. 2015. [Online]. Available: https://leovaniersel.files.wordpress.com/2015/08/modelling.pdf

[14] J. Caldwell, “Analog pulse width modulation,” Texas Instruments Incorporated, 2013. [Online]. Available: http://www.ti.com/lit/ug/slau508/slau508.pdf

Appendix A

Math derivations of basic model

A.1 Transfer function

The transfer function of the basic model, Figure A.1, is derived from the Kirchhoff's rules, Equations A.1, and Laplace functions, Equations A.2. By combining the equations is the output voltage derived, shown in Equation A.3. The transfer function, shown in Equation A.4, is derived by dividing the ouput voltage with the input voltage.

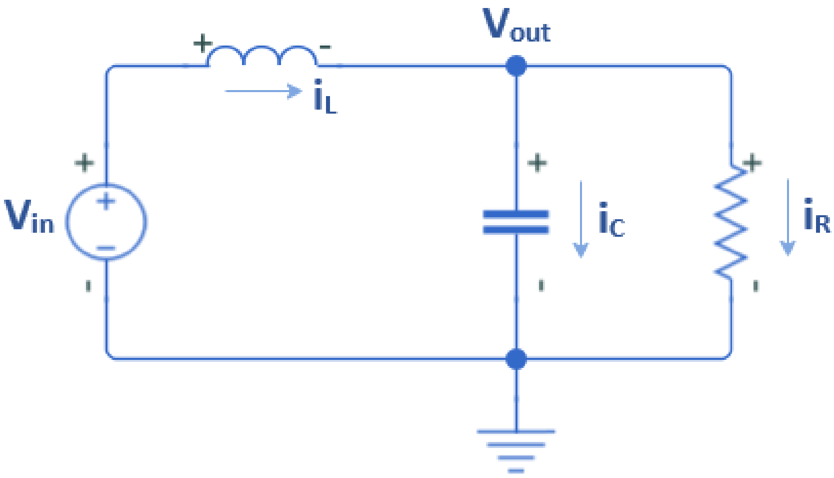

Figure A.1: Circuit model of a class D amplifier

$$i_L = i_C + i_R \quad \text{(A.1a)}$$
$$V_C = V_R = V_{out} \quad \text{(A.1b)}$$
$$V_{in} = V_L + V_C \quad \text{(A.1c)}$$

$$si_L = \frac{V_L}{L} = \frac{V_{in} - V_{out}}{L} \quad \Rightarrow \quad i_L = \frac{V_{in} - V_{out}}{sL} \quad \text{(A.2a)}$$
$$sV_C = \frac{i_C}{C} = sV_{out} \quad \Rightarrow \quad i_C = sV_{out}C \quad \text{(A.2b)}$$
$$V_{out} = Ri_R \quad \text{(A.2c)}$$

$$\begin{aligned} &V_{out} = Ri_R = R(i_L - i_C) = R(\tfrac{V_{in}-V_{out}}{sL} - sV_{out}C) \quad \Rightarrow \\ &V_{out}(1 + \tfrac{R}{sL} + sCR) = V_{in}\tfrac{R}{sL} \quad \Rightarrow \\ &V_{out} = V_{in}\tfrac{R}{sL(1+\frac{R}{sL}+sCR)} = V_{in}\tfrac{R}{s^2LCR+sL+R} \end{aligned} \quad \text{(A.3)}$$

$$H(s)_{basic} = \frac{V_{out}}{V_{in}} = \frac{R}{s^2LCR + sL + R} \quad \text{(A.4)}$$

A.2 Cut-off frequency, quality factor, poles and zero's

From the standard low pass filter transfer function the cut-off frequency and quality factor could be determined, Equation A.5. The zeros are the roots of the numerator of the transfer function. The poles are the roots of the denominator of the transfer function. The poles and zeros are determined in Equation A.6. Note that the numerator doesn't have roots, so no zeros.

$$H(s) = \frac{w_c^2}{s^2 + s\frac{w_c}{Q} + w_c^2} = \frac{R}{s^2LCR + sL + R} = \frac{R}{s^2LCR + sL + R} = \frac{\frac{1}{LC}}{s^2 + s\frac{1}{CR} + \frac{1}{LC}} \tag{A.5a}$$

$$w_c^2 = \frac{1}{LC} \quad \Rightarrow \quad w_c = 2\pi f_c = \frac{1}{\sqrt{LC}} \quad \Rightarrow \quad f_c = \frac{1}{2\pi\sqrt{LC}} \tag{A.5b}$$

$$\frac{w_c}{Q} = \frac{1}{CR} \quad \Rightarrow \quad Q = w_c CR = \frac{CR}{\sqrt{LC}} = \sqrt{\frac{CR^2}{L}} \tag{A.5c}$$

$$N(s) = R \tag{A.6a}$$

$$D(s) = s^2LCR + sL + R \quad \Rightarrow \quad s = \frac{-L \pm \sqrt{L^2 - 4CLR^2}}{2CLR} \tag{A.6b}$$

Appendix B

Feedback filters

The Bode plots of the first and second order feedback filter are shown in Figure B.1 and Figure B.2.

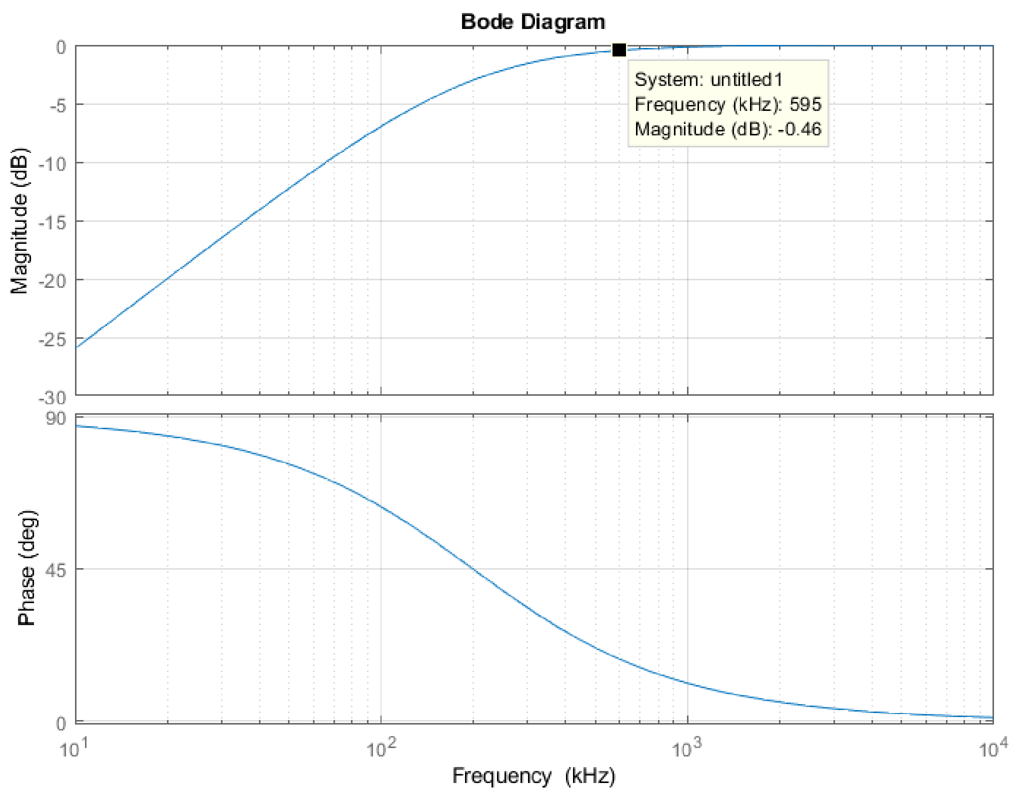

Figure B.1: Bode plot of first order filter

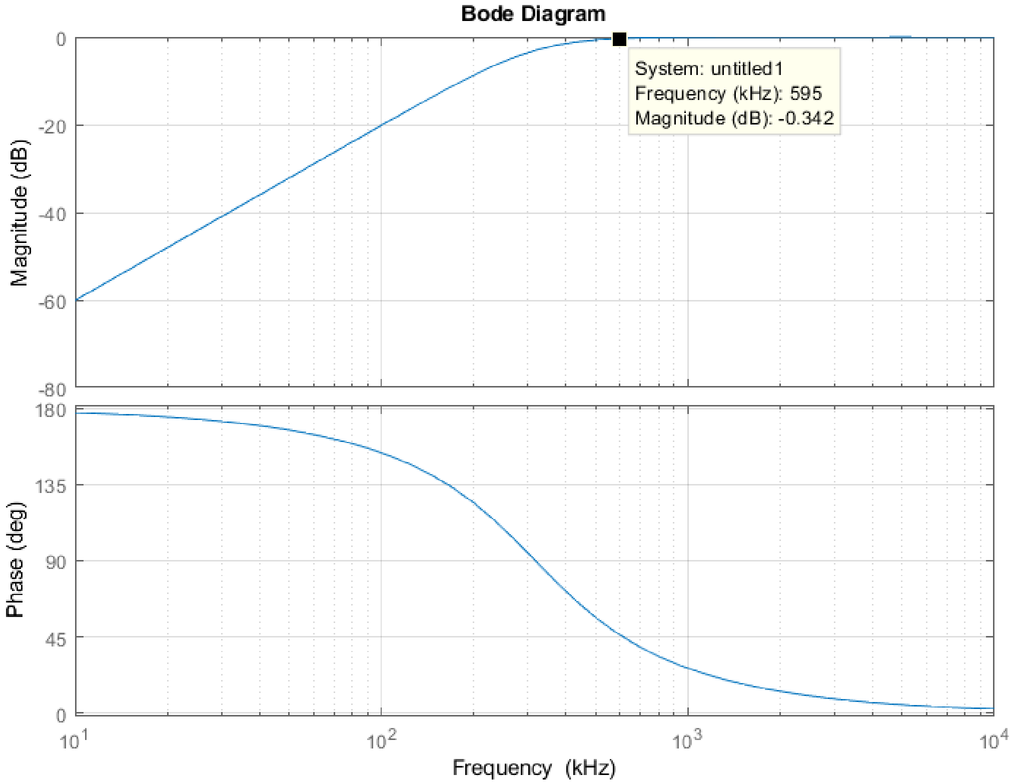

Figure B.2: Bode plot of second order filter

Appendix C

Math derivations of output feedback

C.1 Transfer function

The transfer function of the model with feedback from the output voltage, Figure C.1, is derived with the following equations; Kirchhoff's Equation C.1, Laplace Equation C.2 and current source Equations C.3 and C.4. Note that the current of the current source is different for the first and second order high pass filter. Combining the equations solves the equation for the output voltage, Equations C.5 and C.6. The transfer functions are the output voltage divided with the input voltage, shown in Equations C.7 and C.8.

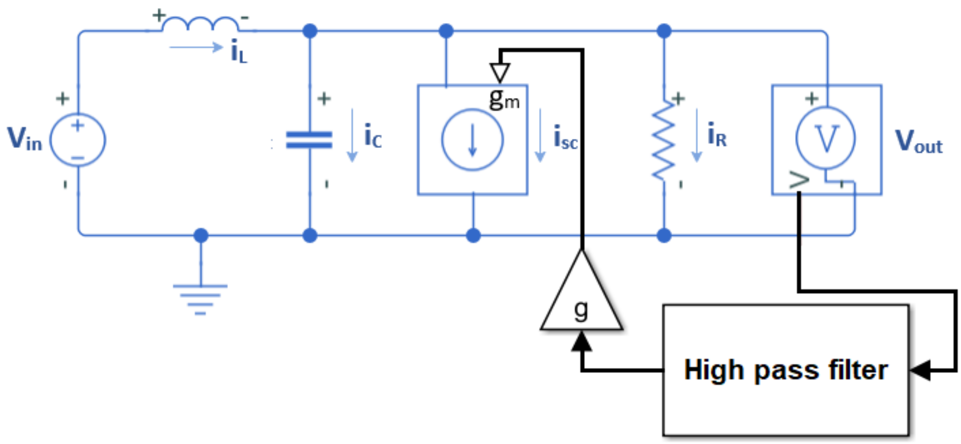

Figure C.1: Circuit model of design 1

$$i_L = i_C + i_R + i_{sc} \tag{C.1a}$$
$$V_C = V_R = V_{out} \tag{C.1b}$$
$$V_{in} = V_L + V_C \tag{C.1c}$$

$$si_L = \frac{V_L}{L} = \frac{V_{in} - V_{out}}{L} \quad \Rightarrow \quad i_L = \frac{V_{in} - V_{out}}{sL} \tag{C.2a}$$
$$sV_C = \frac{i_C}{C} = sV_{out} \quad \Rightarrow \quad i_C = sV_{out}C \tag{C.2b}$$
$$V_{out} = Ri_R \tag{C.2c}$$

$$i_{sc_{1eHP}} = V_{out}\frac{\tau s}{\tau s + 1} g g_m \tag{C.3}$$

$$i_{sc_{2eHP}} = V_{out}\frac{s^2}{s^2 + s\frac{w_c}{Q} + w_c^2} g g_m \tag{C.4}$$

$$
\begin{aligned}
V_{out_{1^e HP}} &= Ri_R = R(i_L - i_C - i_{sc_{1^e HP}}) && \Rightarrow \\
V_{out_{1^e HP}} &= R(\tfrac{V_{in}-V_{out_{1^e HP}}}{sL} - sV_{out_{1^e HP}}C - V_{out}\tfrac{s\tau g g_m}{\tau s+1}) && \Rightarrow \\
V_{out_{1^e HP}}&(1+\tfrac{R}{sL}+sCR+\tfrac{sR\tau g g_m}{\tau s+1}) = V_{in}\tfrac{R}{sL} && \Rightarrow \\
V_{out_{1^e HP}} &= V_{in}\tfrac{R}{sL(1+\frac{R}{sL}+sCR+\frac{sR\tau g g_m}{\tau s+1})} = V_{in}\tfrac{R(\tau s+1)}{sL(\tau s+1)+R(\tau s+1)+s^2LCR(\tau s+1)+s^2LR\tau g g_m} && \Rightarrow \\
V_{out_{1^e HP}} &= V_{in}\tfrac{sR\tau+R}{s^3LCR\tau+s^2(LCR+L\tau+LR\tau g g_m)+s(L+R\tau)+R}
\end{aligned}
\tag{C.5}
$$

$$
\begin{aligned}
V_{out_{2^e HP}} &= Ri_R = R(i_L - i_C - i_{sc_{2^e HP}}) && \Rightarrow \\
V_{out_{2^e HP}} &= R(\tfrac{V_{in}-V_{out_{2^e HP}}}{sL} - sV_{out}C - V_{out_{2^e HP}}\tfrac{s^2 g g_m}{s^2+s\frac{w_c}{Q}+w_c^2}) && \Rightarrow \\
V_{out_{2^e HP}}&(1+\tfrac{R}{sL}+sCR+\tfrac{Rs^2 g g_m}{s^2+s\frac{w_c}{Q}+w_c^2}) = V_{in}\tfrac{R}{sL} && \Rightarrow \\
V_{out_{2^e HP}} &= V_{in}\tfrac{R}{sL(1+\frac{R}{sL}+sCR+\frac{Rs^2 g g_m}{s^2+s\frac{w_c}{Q}+w_c^2})} && \Rightarrow \\
V_{out_{2^e HP}} &= V_{in}\tfrac{R(s^2+s\frac{w_c}{Q}+w_c^2)}{s^2LCR(s^2+s\frac{w_c}{Q}+w_c^2)+s(s^2+s\frac{w_c}{Q}+w_c^2)L+R(s^2+s\frac{w_c}{Q}+w_c^2)+s^3LRg g_m} && \Rightarrow \\
V_{out_{2^e HP}} &= V_{in}\tfrac{s^2R+s\frac{w_c}{Q}R+w_c^2R}{s^4LCR+s^3(LCR\frac{w_c}{Q}+L+LRg)+s^2(LCRw_c^2+L\frac{w_c}{Q}+R)+s(w_c^2L+R\frac{w_c}{Q})+Rw_c^2}
\end{aligned}
\tag{C.6}
$$

$$
H(s)_{out-1^e HP} = \frac{V_{out_{1^e HP}}}{V_{in}} = \frac{sR\tau+R}{s^3LCR\tau+s^2(LCR+L\tau+LR\tau g g_m)+s(L+R\tau)+R} \tag{C.7}
$$

$$
\begin{aligned}
H(s)_{out-2^e HP} &= \frac{V_{out_{2^e HP}}}{V_{in}} && \Rightarrow \\
H(s)_{out-2^e HP} &= \frac{s^2R+s\frac{w_c}{Q}R+w_c^2R}{s^4LCR+s^3(LCR\frac{w_c}{Q}+L+LRg g_m)+s^2(LCRw_c^2+L\frac{w_c}{Q}+R)+s(w_c^2L+R\frac{w_c}{Q})+Rw_c^2}
\end{aligned}
\tag{C.8}
$$

Appendix D

Math derivations of inductor feedback

D.1 Transfer function

The transfer function of the model with feedback from the inductor voltage, Figure D.1, is derived from the Kirchhoff's Equation D.1, and Laplace Equation D.2 and current source Equations D.3 and D.4. The current of the current source is different for the first and second order high pass filter. The output voltage is derived by combining Equations D.1 till D.4, shown in Equations D.5 and D.6. Dividing the output voltage with the input voltage gives the transfer function, Equations D.7 and D.8.

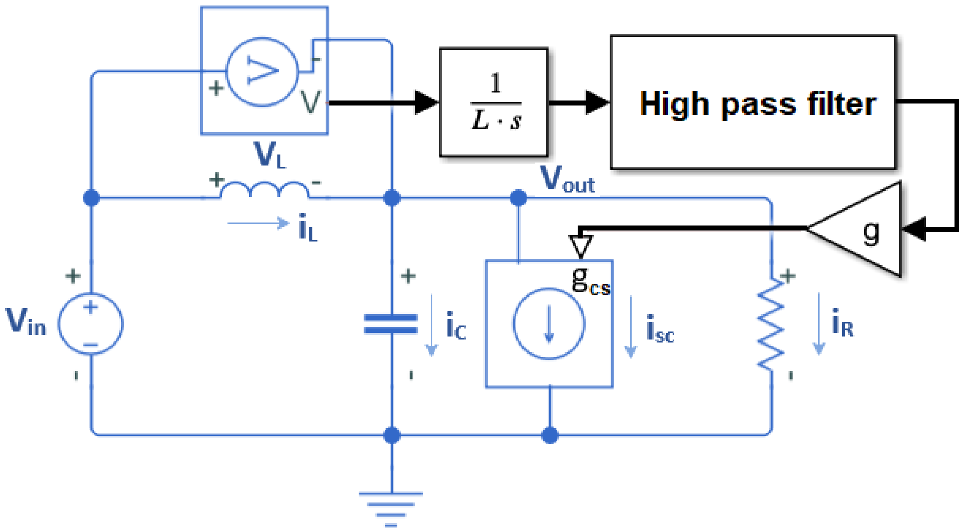

Figure D.1: Circuit model of design 4

$$i_L = i_C + i_R + i_{sc} \tag{D.1a}$$
$$V_C = V_R = V_{out} \tag{D.1b}$$
$$V_{in} = V_L + V_C \tag{D.1c}$$

$$si_L = \frac{V_L}{L} = \frac{V_{in} - V_{out}}{L} \quad \Rightarrow \quad i_L = \frac{V_{in} - V_{out}}{sL} \tag{D.2a}$$
$$sV_C = \frac{i_C}{C} = sV_{out} \quad \Rightarrow \quad i_C = sV_{out}C \tag{D.2b}$$
$$V_{out} = Ri_R \tag{D.2c}$$

$$i_{sc_{L-1^e HP}} = (V_{in} - V_{out})\frac{1}{sL}\frac{\tau s}{\tau s + 1} g g_m = \frac{\tau(V_{in} - V_{out}) g g_m}{L(\tau s + 1)} \tag{D.3}$$

$$i_{sc_{L-2^eHP}} = (V_{in} - V_{out})\frac{1}{sL}\frac{s^2}{s^2 + s\frac{w_c}{Q} + w_c^2}gg_m = \frac{sgg_m(V_{in} - V_{out})}{L(s^2 + s\frac{w_c}{Q} + w_c^2)} \quad \text{(D.4)}$$

$$\begin{aligned}
V_{out_{L-1^eHP}} &= Ri_R = R(i_L - i_C - i_{sc_{L-1^eHP}}) &\Rightarrow\\
V_{out_{L-1^eHP}} &= R(\tfrac{V_{in}-V_{out_{L-1^eHP}}}{sL} - sV_{out}C - \tfrac{\tau(V_{in}-V_{out_{L-1^eHP}})gg_m}{L(\tau s+1)}) &\Rightarrow\\
V_{out_{L-1^eHP}}(1 + \tfrac{R}{sL} + sCR - \tfrac{\tau Rgg_m}{L(\tau s+1)}) &= V_{in}(\tfrac{R}{sL} - \tfrac{\tau Rgg_m}{L(\tau s+1)}) = V_{in}\tfrac{R(\tau s+1-s\tau gg_m)}{sL(\tau s+1)} &\Rightarrow\\
V_{out_{L-1^eHP}} &= V_{in}\tfrac{R(\tau s+1-s\tau gg_m)}{sL(\tau s+1)(1+\frac{R}{sL}+sCR-\frac{\tau Rgg_m}{L(\tau s+1)})} &\Rightarrow\\
V_{out_{L-1^eHP}} &= V_{in}\tfrac{R(\tau s+1-s\tau gg_m)}{sL(\tau s+1)+R(\tau s+1)+s^2CRL(\tau s+1)-s\tau Rgg_m} &\Rightarrow\\
V_{out_{L-1^eHP}} &= V_{in}\tfrac{sR\tau(1-gg_m)+R}{s^3CRL\tau+s^2(L\tau+CRL)+s(L+R\tau-R\tau gg_m)+R}
\end{aligned} \quad \text{(D.5)}$$

$$\begin{aligned}
V_{out_{L-2^eHP}} &= Ri_R = R(i_L - i_C - i_{sc_{L-2^eHP}}) &\Rightarrow\\
V_{out_{L-2^eHP}} &= R(\tfrac{V_{in}-V_{out_{L-2^eHP}}}{sL} - sV_{out}C - \tfrac{sgg_m(V_{in}-V_{out_{L-2^eHP}})}{L(s^2+s\frac{w_c}{Q}+w_c^2)}) &\Rightarrow\\
V_{out_{L-2^eHP}}(1 + \tfrac{R}{sL} + sCR - \tfrac{sgg_mR}{L(s^2+s\frac{w_c}{Q}+w_c^2)}) &= V_{in}(\tfrac{R}{sL} - \tfrac{sgg_mR}{L(s^2+s\frac{w_c}{Q}+w_c^2)}) &\Rightarrow\\
V_{out_{L-2^eHP}}(1 + \tfrac{R}{sL} + sCR - \tfrac{sgg_mR}{L(s^2+s\frac{w_c}{Q}+w_c^2)}) &= V_{in}\tfrac{R(s^2+s\frac{w_c}{Q}+w_c^2)-s^2Rgg_m}{sL(s^2+s\frac{w_c}{Q}+w_c^2)} &\Rightarrow\\
V_{out_{L-2^eHP}} &= V_{in}\tfrac{R(s^2+s\frac{w_c}{Q}+w_c^2)-s^2Rgg_m}{(1+\frac{R}{sL}+sCR-\frac{sgg_mR}{L(s^2+s\frac{w_c}{Q}+w_c^2)})sL(s^2+s\frac{w_c}{Q}+w_c^2)} &\Rightarrow\\
V_{out_{L-2^eHP}} &= V_{in}\tfrac{R(s^2+s\frac{w_c}{Q}+w_c^2)-s^2Rgg_m}{sL(s^2+s\frac{w_c}{Q}+w_c^2)+R(s^2+s\frac{w_c}{Q}+w_c^2)+s^2CRL(s^2+s\frac{w_c}{Q}+w_c^2)-s^2gg_mR} &\Rightarrow\\
V_{out_{L-2^eHP}} &= V_{in}\tfrac{s^2R(1-gg_m)+sR\frac{w_c}{Q}+Rw_c^2}{s^4CRL+s^3(L+CRL\frac{w_c}{Q})+s^2(L\frac{w_c}{Q}+R+CLRw_c^2-gg_mR)+s(Lw_c^2+R\frac{w_c}{Q})+w_c^2R}
\end{aligned} \quad \text{(D.6)}$$

$$H(s)_{L-1^eHP} = \frac{V_{out_{L-1^eHP}}}{V_{in}} = \frac{sR\tau(1-gg_m)+R}{s^3CRL\tau+s^2(L\tau+CRL)+s(L+R\tau-R\tau g)+R} \quad \text{(D.7)}$$

$$\begin{aligned}
H(s)_{L-2^eHP} &= \tfrac{V_{out_{L-2^eHP}}}{V_{in}} &\Rightarrow\\
H(s)_{L-2^eHP} &= \tfrac{s^2R(1-gg_m)+sR\frac{w_c}{Q}+Rw_c^2}{s^4CRL+s^3(L+CRL\frac{w_c}{Q})+s^2(L\frac{w_c}{Q}+R+CLRw_c^2-gg_mR)+s(Lw_c^2+R\frac{w_c}{Q})+w_c^2R}
\end{aligned} \quad \text{(D.8)}$$

Appendix E

Matlab and simulink

E.1 Measured models and used tools

The basic and feedback models (Figure E.1 till E.5) are simulated as follows:

- Bode and pole-zero plot with the tool Linear Analysis.
- Time domain ripple plot by subtracting the output with the ideal output (amplified transfer function ($G_{pwm}H(s)$) of circuit model). The plot is shown and compared with previous measurements in Simulation Data Inspector.
- Frequency domain output plot is shown with the Continuous Powergui tool FFT Analysis.

The transfer function of each model is checked via the tool linear Analysis. The input perturbations are placed at the input of the transfer function and at the output of the PWM generator block. The output measurement are placed at the output of the transfer function and at output of the PS-Simulink converter Vout. If the Bode plots are equal, the transfer function is correct.

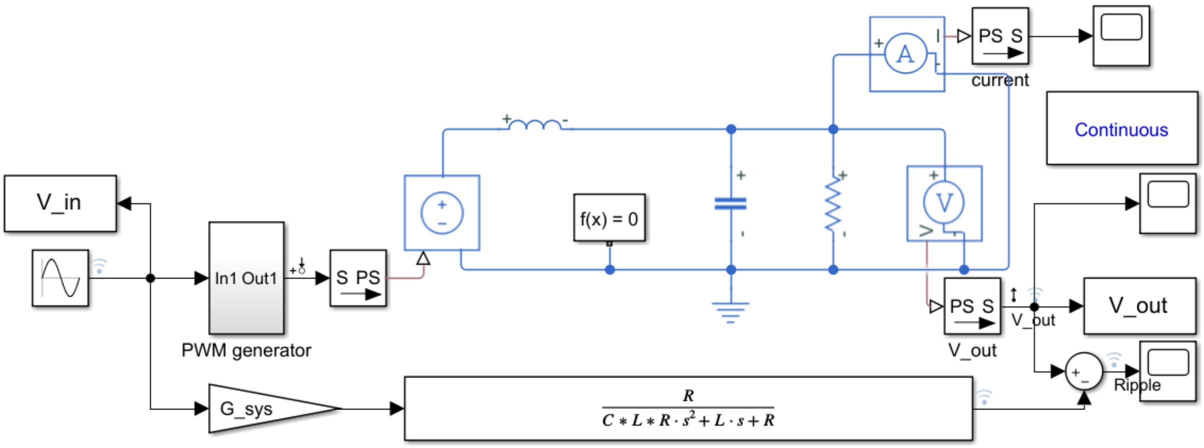

Figure E.1: Measured circuit model of a basic class D amplifier

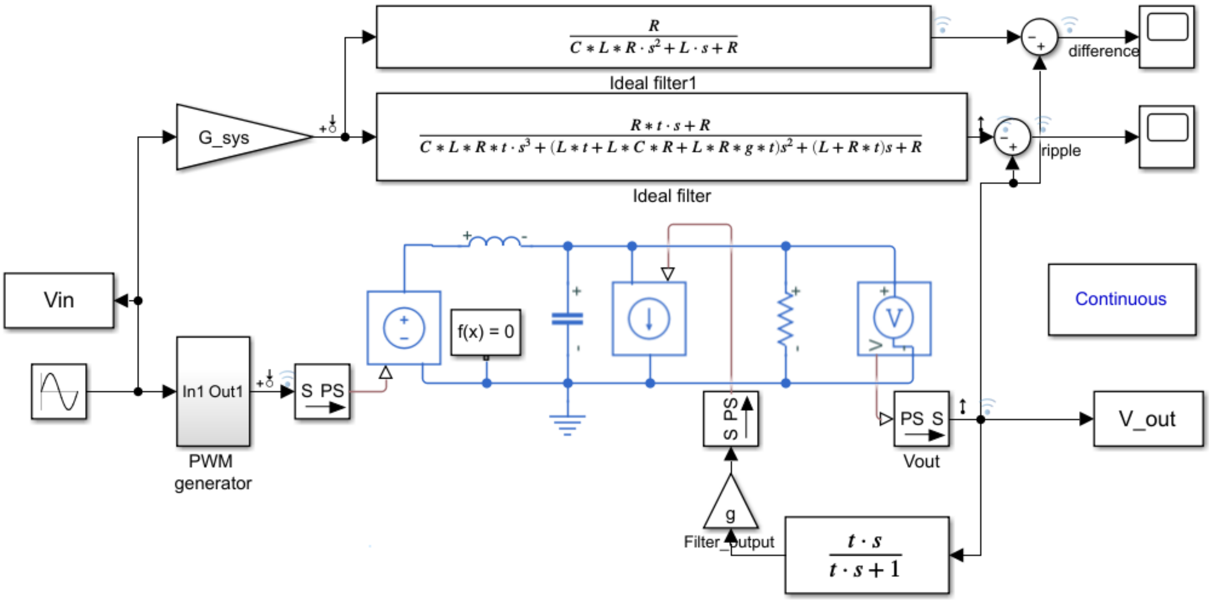

Figure E.2: Measured circuit model of first order output feedback

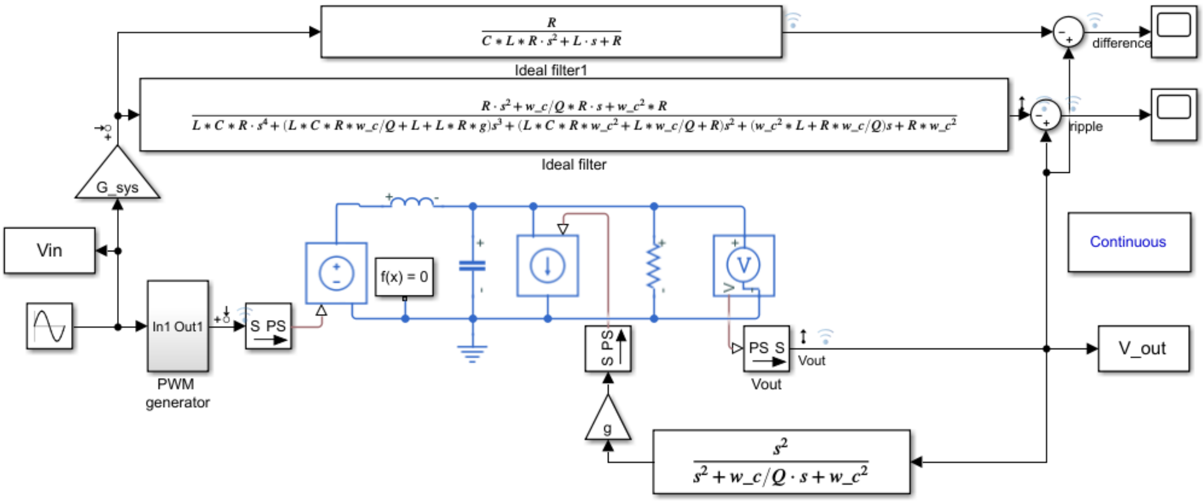

Figure E.3: Measured circuit model of second order output feedback

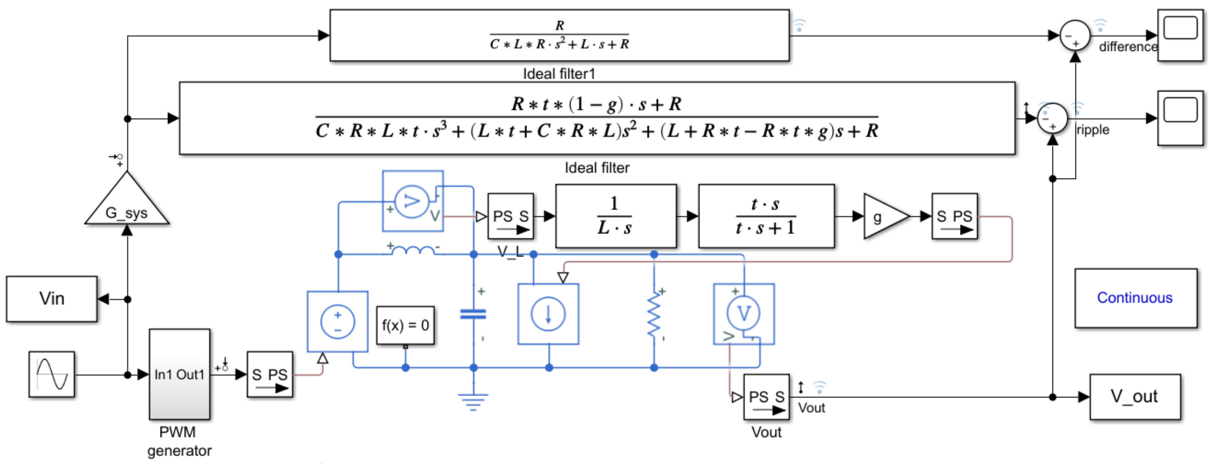

Figure E.4: Measured circuit model of first order inductor feedback

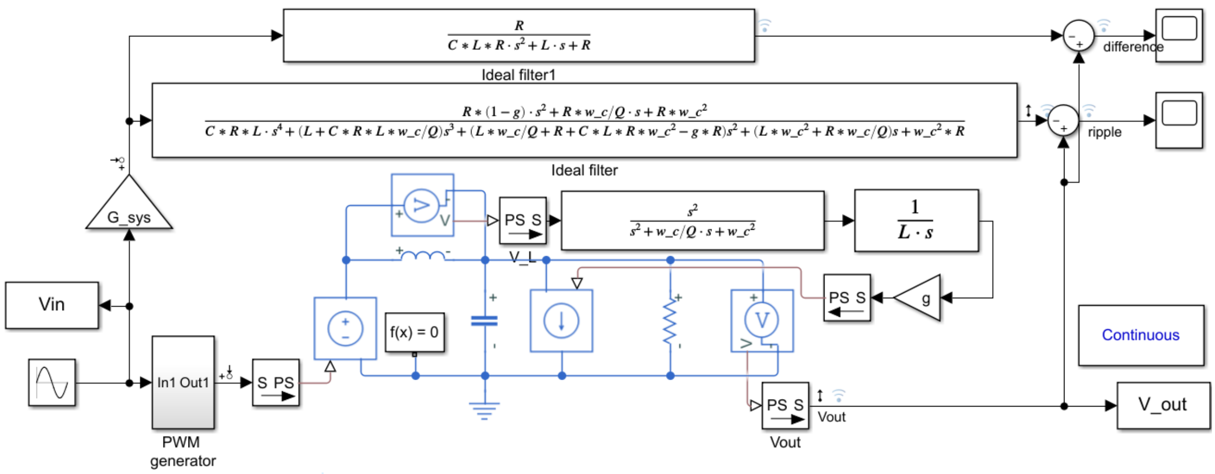

Figure E.5: Measured circuit model of second order inductor feedback

E.2 PWM generator and basic model values explained

The basic model, shown in Figure E.1, uses several components and blocks in Simulink. One of the blocks is a PWM generator. The PWM generator will be explained in this section. Furthermore, all input values will be discussed.

E.2.1 Explanation of PWM generator block

How a PWM generator works is shown with Figure E.9. If the triangle signal is above the input signal, the PWM signal is low. When the triangle signal is below the input signal, the PWM signal is high. Inside the PWM generator block is the input signal compared with a triangle wave, shown in Figures E.6 and E.7. The triangle wave is made from a pulse generator. The pulse generator creates a pulse from zero to $2 \cdot A_{pwm}$ (A_{pwm}=PWM amplitude). The pulse signal is subtracted with A_{pwm}, multiplied with $f_{pwm} \cdot 4$ and integrated. The initial condition starts at $-A_{pwm}$. Such that the right triangle will be made, Equation E.1. The comparator creates the PWM signal of the input signal, shown in Figure E.8. This is done by subtracting the input and triangle, multiplying the outcome with a big value (1000), limit the signal to the upper (A_{pwm})

and lower ($-A_{pwm}$) saturation values, and multiply the limited signal with the PWM amplitude. The multiplying with the big value must be big enough, such that the signal will be limited in the saturation component which create the PWM signal.

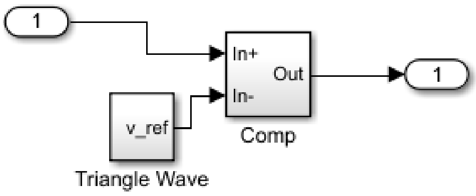

Figure E.6: PWM generator

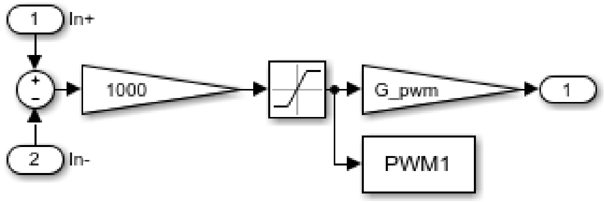

Figure E.8: Comparator

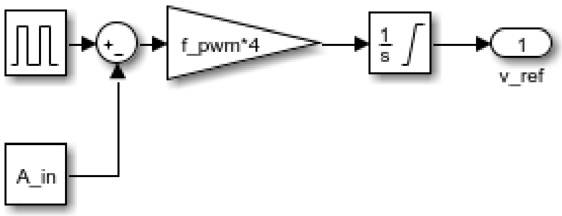

Figure E.7: Triangle wave

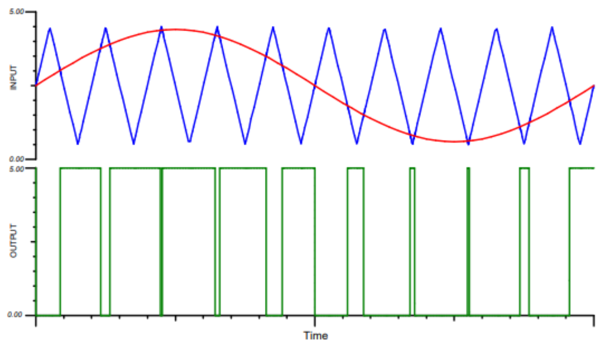

Figure E.9: How a PWM generator works shown with signals [14]

$$A_{\wedge\wedge_k} = \begin{cases} -A_{in} + \int_{0+\frac{k}{f_{pwm}}}^{t} 4A_{in}f_{pwm} & , 0 + \frac{k}{f_{pwm}} \leq t \leq \frac{1}{2f_{pwm}} + \frac{k}{f_{pwm}} \\ A_{in} - \int_{\frac{1}{2f_{pwm}}+\frac{k}{f_{pwm}}}^{t} 4A_{in}f_{pwm} & , \frac{1}{2f_{pwm}} + \frac{k}{f_{pwm}} \leq t \leq \frac{1}{f_{pwm}} + \frac{k}{f_{pwm}} \end{cases} \tag{E.1a}$$

$$A_{\wedge\wedge_k} = \begin{cases} -A_{in} + 4A_{in}f_{pwm}t - 4A_{in}k & , 0 + \frac{k}{f_{pwm}} \leq t \leq \frac{1}{2f_{pwm}} + \frac{k}{f_{pwm}} \\ 3A_{in} - 4A_{in}f_{pwm}t + 4A_{in}k & , \frac{1}{2f_{pwm}} + \frac{k}{f_{pwm}} \leq t \leq \frac{1}{f_{pwm}} + \frac{k}{f_{pwm}} \end{cases} \tag{E.1b}$$

E.2.2 Basic values

All input values of the basic model are the input amplitude A_{in}, input frequency f_{in}, PWM carrier frequency f_{pwm}, PWM amplitude A_{pwm}, system gain A_{sys}, inductor L, capacitor C and load-resistor R. The input is a sine with an amplitude A_{in} and frequency f_{in}. The amplitude is chosen to be 1V, because this is a common value. The frequency must be a frequency between 20 and 20000 Hz, human hearing range [5]. So there is chosen to have a input frequency of 5 kHz. If a PWM generator with 16 times oversampling, then is the PWM frequency $16 \cdot f_s$. A common sample frequency f_s for audio signals is 44.1 kHz. This makes that the PWM carrier frequency is 706 kHz. The PWM amplitude (is used for the pulse and triangle height, and PWM components) must be higher or equal to the input amplitude, otherwise the PWM generator produce a wrong signal. It is chosen to make the PWM amplitude 1V. The system gain is how much the input signal will be amplified. It is chosen to make this gain 30. The inductor and capacitor are derived in Section 1.1: 32 μH and 1 μF. A common load-resistor is 4 Ω.

Appendix F

Transfer function expression reduction

F.1 Feedback output voltage first order

The total transfer function of the feedback model from the output voltage with a first order filter, shown in Equation F.1, has a too high order to characterize easily. The dominant parts of the transfer functions at certain frequencies are shown in Table F.1. The dominant parts form the smaller transfer functions, Equation F.2. The Bode plot of all smaller transfer functions and total transfer function is shown in Figure F.1. Not all small parts are needed, some can be neglected. The smaller transfer function G1 and G2 will be neglected, because transfer G3 describes the same characteristics at their specific frequency. Transfer G3 and G6 together describe the same characteristics as G4 and G5. The numerator as G3 till G5 is the same and the combination as G3 and G6 have the same denominator. So the smaller transfer functions G4 and G5 can be neglected. This makes that G3 and G6 together describe the total transfer function. The Bode plot of the used smaller transfer function and total transfer function is shown in Figure F.2. The characteristics of the smaller transfer functions are the same characteristics of the total transfer function, shown in Equation F.3.

$$H(s)_{out-1^eHP} = \frac{sR\tau + R}{s^3LCR\tau + s^2(LCR + L\tau + LR\tau gg_m) + s(L + R\tau) + R} \tag{F.1}$$

Table F.1: Dominant pole zero values of feedback output voltage first order filter

	f(kHz)						
x(s)	10^0	10^1	10^2	10^3	10^4	10^5	10^6
R	4	4	4	4	4	4	4
$sR\tau$	$5.09 \cdot 10^{-4}$	$5.09 \cdot 10^{-3}$	$5.09 \cdot 10^{-2}$	$5.09 \cdot 10^{-1}$	5.09	$5.09 \cdot 10^{1}$	$5.09 \cdot 10^{2}$
R	4	4	4	4	4	4	4
$s(L + R\tau)$	$5.60 \cdot 10^{-3}$	$5.60 \cdot 10^{-2}$	$5.60 \cdot 10^{-1}$	5.60	$5.60 \cdot 10^{1}$	$5.60 \cdot 10^{2}$	$5.60 \cdot 10^{3}$
$s^2(LCR + L\tau$ $+LR\tau gg_m)$	$(3.89+$ $2.59g)10^{-6}$	$(3.89+$ $2.59g)10^{-4}$	$(3.89+$ $2.59g)10^{-2}$	$(3.89+$ $2.59g)$	$(3.89+$ $2.59g)10^{2}$	$(3.89+$ $2.59g)10^{4}$	$(3.89+$ $2.59g)10^{6}$
$s^3LCR\tau$	$4.13 \cdot 10^{-10}$	$4.13 \cdot 10^{-7}$	$4.13 \cdot 10^{-4}$	$4.13 \cdot 10^{-1}$	$4.13 \cdot 10^{2}$	$4.13 \cdot 10^{5}$	$4.13 \cdot 10^{8}$

$$G1 = \frac{R}{R} \tag{F.2a}$$

$$G2 = \frac{R}{s^2(LCR + L\tau + LR\tau gg_m) + s(L + R\tau) + R} \tag{F.2b}$$

$$G3 = \frac{sR\tau + R}{s^2(LCR + L\tau + LR\tau gg_m) + s(L + R\tau) + R} \tag{F.2c}$$

$$G4 = \frac{sR\tau + R}{s^3LCR\tau + s^2(LCR + L\tau + LR\tau gg_m) + s(L + R\tau)} \tag{F.2d}$$

$$G5 = \frac{sR\tau + R}{s^3LCR\tau + s^2(LCR + L\tau + LR\tau gg_m)} \tag{F.2e}$$

$$G6 = \frac{sR\tau}{s^3LCR\tau + s^2(LCR + L\tau + LR\tau gg_m)} = \frac{R\tau}{s(sLCR\tau + LCR + L\tau + LR\tau gg_m)} \tag{F.2f}$$

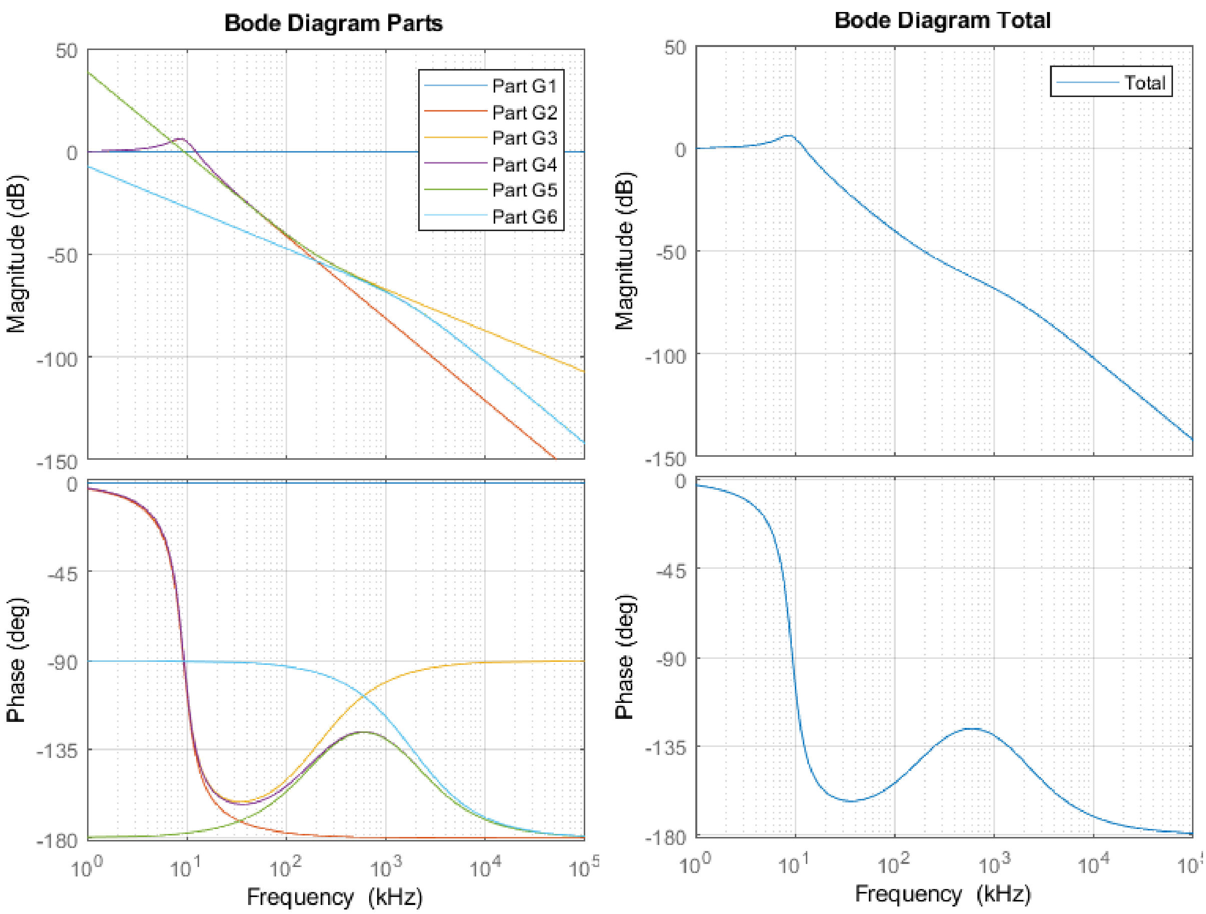

Figure F.1: Bode plot of all smaller transfer function of model 2.1 with a first order filter and gain g of 10

$$w_{c1} = \begin{cases} \frac{1}{\sqrt{CL}} & g = 0 \\ \sqrt{\frac{R}{LCR+L\tau+LR\tau g g_m}} & g > 0 \end{cases} \tag{F.3a}$$

$$Q_1 = \begin{cases} \sqrt{\frac{CR^2}{L}} & g = 0 \\ \sqrt{\frac{R(LCR+L\tau+LR\tau g g_m)}{(L+R\tau)^2}} & g > 0 \end{cases} \tag{F.3b}$$

$$w_{c2} = \begin{cases} - & g = 0 \\ \frac{1}{\tau} & g > 0 \end{cases} \tag{F.3c}$$

$$w_{c3} = \begin{cases} - & g = 0 \\ \frac{CR+\tau+R\tau g g_m}{CR\tau} & g > 0 \end{cases} \tag{F.3d}$$

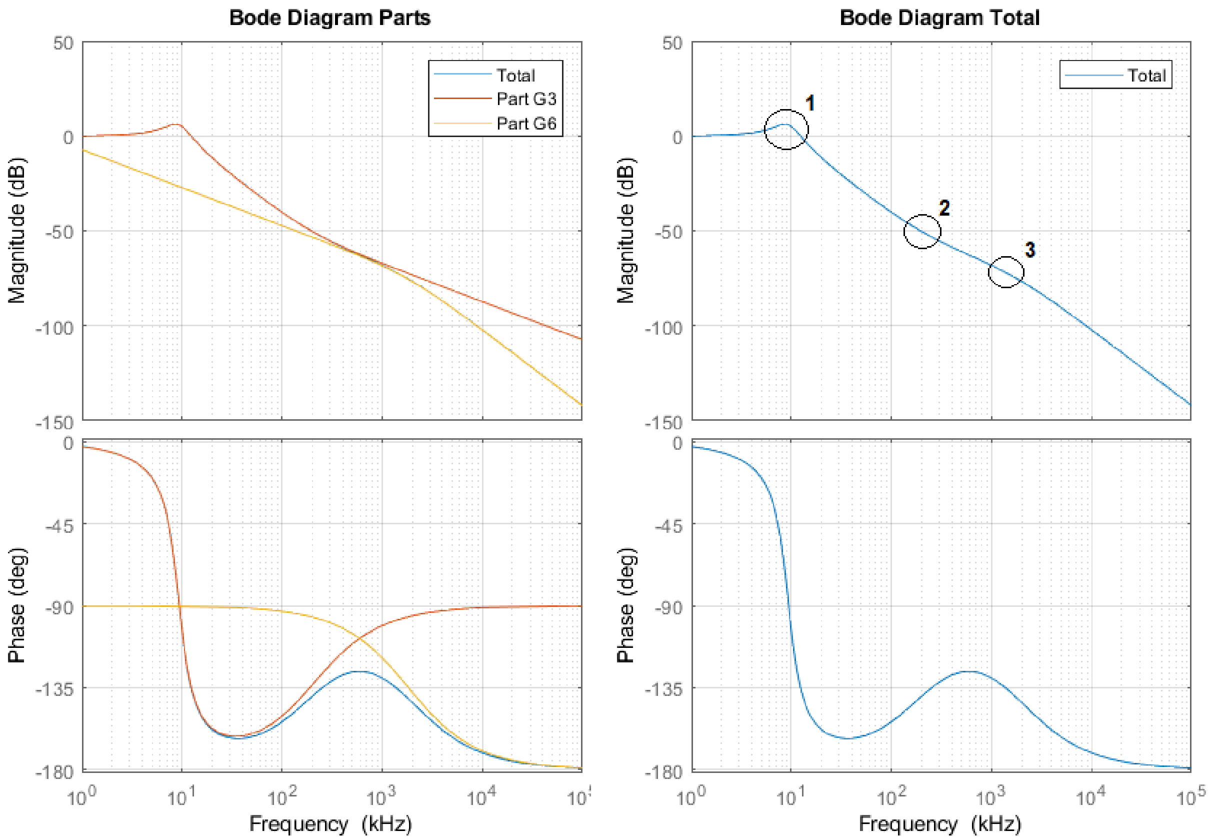

Figure F.2: Bode plot of final smaller transfer function of model 2.1 with a first order filter and gain g of 10

F.2 Feedback output voltage second order

The total transfer function of the feedback model from the output voltage with a second order filter, Equation F.4, has a too high order to characterize easily. Table F.2 shows which parts of the transfer functions at certain frequencies are dominant. The dominant parts form the smaller transfer functions, shown in Equation F.5. The Bode plot of all smaller transfer functions are shown in Figure F.3. Not all small parts are needed, some can be neglected. The smaller transfer function H1 and H2 will be neglected, because transfer H3 describes the same characteristics at their specific frequency. Transfer H3 and H4 together describe the same characteristics as H5 and H6. But note that H3 and H4 both have a too high order in the denominator to characterize easily. H4 can be made smaller to H4b, shown in Equation F.6. H4b is determined by looking at the dominant frequency between 10 MHz and 100 MHz. The smaller transfer function is checked if it still describes the higher frequencies of the total transfer function via a Bode plot graph, shown in Figure F.4. H3 can not be made smaller, a smaller transfer function between with the dominant parts between 10 kHz and 100kHz which had a order two or lower did not describe the total transfer function around 20kHz correct. This means that the total transfer function can be described by the smaller transfer function H3 and H4b. Only H4b can be characterized, so the total transfer function is partly characterized, see Equation F.7.

$$H(s)_{out-2^eHP} = \frac{s^2R + sR\frac{w_c}{Q} + Rw_c^2}{s^4LCR + s^3(LCR\frac{w_c}{Q} + L + LCgg_m) + s^2(LCRw_c^2 + L\frac{w_c}{Q} + R) + s(Lw_c^2 + R\frac{w_c}{Q}) + Rw_c^2} \tag{F.4}$$

Table F.2: Dominant pole zero values of feedback output voltage second order filter

	f(Hz)						
x(s)	10^3	10^4	10^5	10^6	10^7	10^8	10^9
Rw_c^2	$1.6 \cdot 10^{13}$	$1.6 \cdot 10^{13}$	$1.6 \cdot 10^{13}$	$1.6 \cdot 10^{13}$	$1.6 \cdot 10^{13}$	$1.6 \cdot 10^{13}$	$1.6 \cdot 10^{13}$
sRw_c/Q	$1.8 \cdot 10^9$	$1.8 \cdot 10^{10}$	$1.8 \cdot 10^{11}$	$1.8 \cdot 10^{12}$	$1.8 \cdot 10^{13}$	$1.8 \cdot 10^{14}$	$1.8 \cdot 10^{15}$
s^2R	$1.01 \cdot 10^5$	$1.01 \cdot 10^7$	$1.01 \cdot 10^9$	$1.01 \cdot 10^{11}$	$1.01 \cdot 10^{13}$	$1.01 \cdot 10^{15}$	$1.01 \cdot 10^{17}$
Rw_c^2	$1.6 \cdot 10^{13}$	$1.6 \cdot 10^{13}$	$1.6 \cdot 10^{13}$	$1.6 \cdot 10^{13}$	$1.6 \cdot 10^{13}$	$1.6 \cdot 10^{13}$	$1.6 \cdot 10^{13}$
$s(Rw_c/Q + Lw_c^2)$	$2.22 \cdot 10^{10}$	$2.22 \cdot 10^{11}$	$2.22 \cdot 10^{12}$	$2.22 \cdot 10^{13}$	$2.22 \cdot 10^{14}$	$2.22 \cdot 10^{15}$	$2.22 \cdot 10^{16}$
$s^2(R + RCLw_c^2$ $+ Lw_c/Q)$	$1.54 \cdot 10^7$	$1.54 \cdot 10^9$	$1.54 \cdot 10^{11}$	$1.54 \cdot 10^{13}$	$1.54 \cdot 10^{15}$	$1.54 \cdot 10^{17}$	$1.54 \cdot 10^{19}$
$s^3(LCRw_c/Q$ $+ L + LRgg_m)$	$(1.59+$ $0.52g)10^3$	$(1.59+$ $0.52g)10^6$	$(1.59+$ $0.52g)10^9$	$(1.59+$ $0.52g)10^{12}$	$(1.59+$ $0.52g)10^{15}$	$(1.59+$ $0.52g)10^{18}$	$(1.59+$ $0.52g)10^{21}$
s^4LCR	$8.21 \cdot 10^{-2}$	$8.21 \cdot 10^2$	$8.21 \cdot 10^6$	$8.21 \cdot 10^{10}$	$8.21 \cdot 10^{14}$	$8.21 \cdot 10^{18}$	$8.21 \cdot 10^{22}$

$$H1 = \frac{w_c^2R}{Rw_c^2} \tag{F.5a}$$

$$H2 = \frac{w_c^2R}{s(w_c^2L + R\frac{w_c}{Q}) + Rw_c^2} \tag{F.5b}$$

$$H3 = \frac{s\frac{w_c}{Q}R + w_c^2R}{s^3(LCR\frac{w_c}{Q} + L + LRgg_m) + s^2(LCRw_c^2 + L\frac{w_c}{Q} + R) + s(w_c^2L + R\frac{w_c}{Q}) + Rw_c^2} \tag{F.5c}$$

$$H4 = \frac{s^2R + s\frac{w_c}{Q}R + w_c^2R}{s^4LCR + s^3(LCR\frac{w_c}{Q} + L + LRgg_m) + s^2(LCRw_c^2 + L\frac{w_c}{Q} + R) + s(w_c^2L + R\frac{w_c}{Q})} \tag{F.5d}$$

$$H5 = \frac{s^2R + s\frac{w_c}{Q}R}{s^4LCR + s^3(LCR\frac{w_c}{Q} + L + LRgg_m) + s^2(LCRw_c^2 + L\frac{w_c}{Q} + R) + s(w_c^2L + R\frac{w_c}{Q})} \tag{F.5e}$$

$$H6 = \frac{s^2R}{s^4LCR + s^3(LCR\frac{w_c}{Q} + L + LRgg_m)} \tag{F.5f}$$

$$H4b = \frac{s^2R + s\frac{w_c}{Q}R + w_c^2R}{s^4LCR + s^3(LCR\frac{w_c}{Q} + L + LRgg_m) + s^2(LCRw_c^2 + L\frac{w_c}{Q} + R)} \tag{F.6}$$

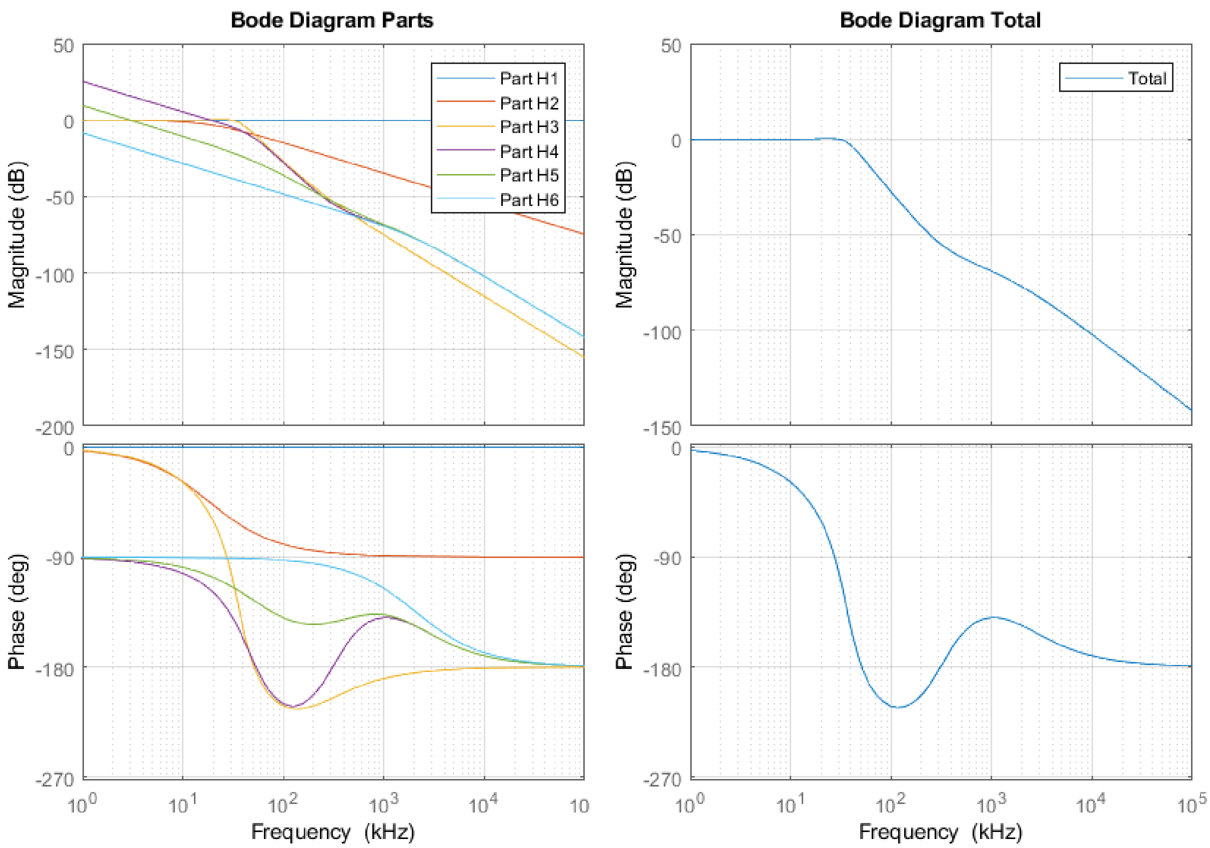

Figure F.3: Bode plot of all smaller transfer function of model 2.1 with a second order filter and gain g of 10

$$w_{c1} = \begin{cases} \frac{1}{\sqrt{CL}} & g = 0 \\ X & g > 0 \end{cases} \tag{F.7a}$$

$$Q_1 = \begin{cases} \sqrt{\frac{CR^2}{L}} & g = 0 \\ X & g > 0 \end{cases} \tag{F.7b}$$

$$w_{c2} = \begin{cases} - & g = 0 \\ Qw_c & g > 0 \end{cases} \tag{F.7c}$$

$$w_{c3} = \begin{cases} - & g = 0 \\ w_c & g > 0 \end{cases} \tag{F.7d}$$

$$Q_3 = \begin{cases} - & g = 0 \\ QR & g > 0 \end{cases} \tag{F.7e}$$

$$w_{c4} = \begin{cases} - & g = 0 \\ \sqrt{\frac{LCRw_c^2 + Lw_c/Q + R}{LCR}} & g > 0 \end{cases} \tag{F.7f}$$

$$Q_4 = \begin{cases} - & g = 0 \\ \sqrt{\frac{(LCRw_c^2 + Lw_c/Q + R)LCR}{(LCRw_c/Q + L + LRgg_m)^2}} & g > 0 \end{cases} \tag{F.7g}$$

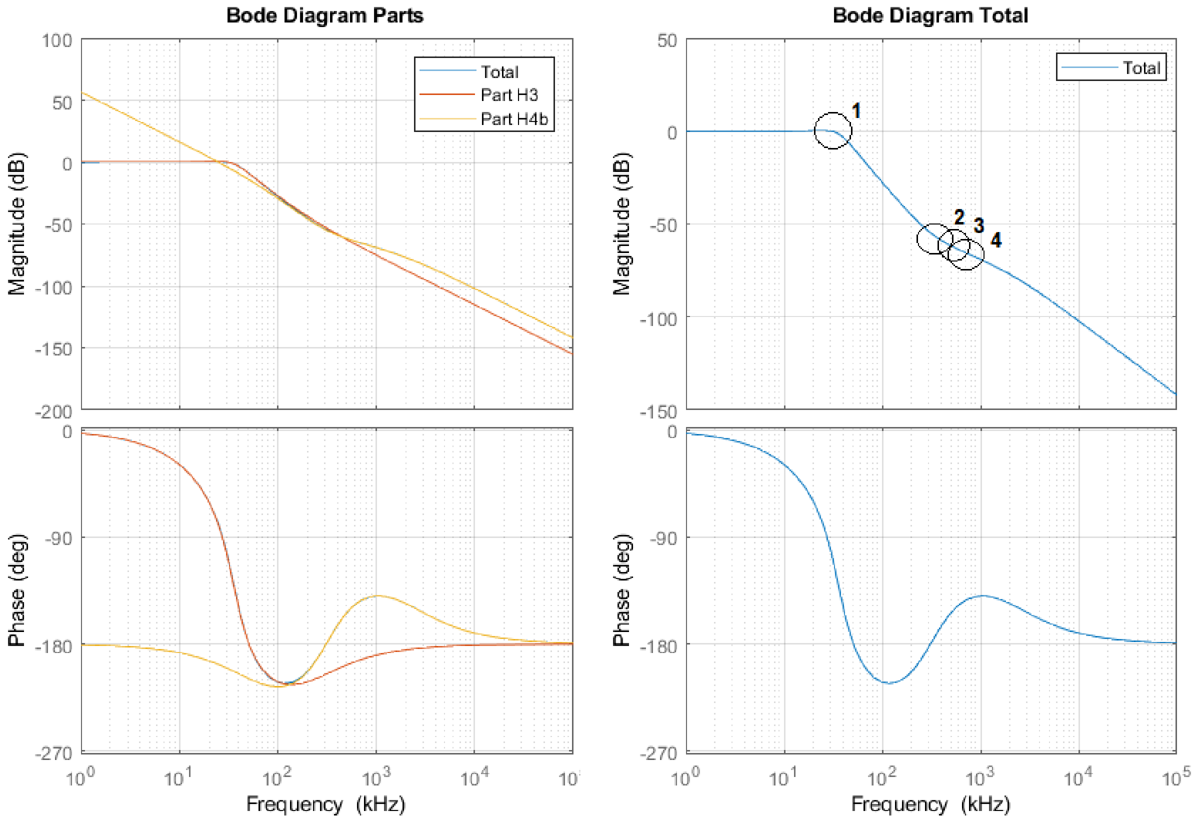

Figure F.4: Bode plot of final smaller transfer function of model 2.1 with a second order filter and gain g of 10

F.3 Feedback inductor voltage first order

The total transfer function of the feedback model from the inductor voltage with a first order filter, Equation F.8, has a too high order to characterize easily. Table F.3 shows which parts of the transfer functions at certain frequencies are dominant. The dominant parts form the smaller transfer functions, shown in Equation F.9. The Bode plot of all smaller transfer functions and total transfer function is shown in Figure F.5. Not all small parts are needed, some can be neglected. The smaller transfer function K1 and K2 will be neglected, because transfer K3 describes the same characteristics at their specific frequency. Transfer K3 and K4 describes the same characteristics as transfer K5. But note that K3 has a too high order in the denominator to characterize easily. H3 can be made smaller to K2b, shown in Equation F.10. K2b is determined by looking at the dominant frequency between 100 kHz and 1 MHz. The smaller transfer function is checked if it still describes the higher frequencies of the total transfer function via a Bode plot graph, shown in Figure F.6. Both transfer functions K2b and K4 have two or lower order denominator and numerator, which means that the total transfer function can be characterized by the smaller transfer functions, see Equation F.11.

$$H(s)_{L-1^eHP} = \frac{sR\tau(1-gg_m)+R}{s^3CRL\tau + s^2(L\tau + CRL) + s(L + R\tau - R\tau g) + R} \tag{F.8}$$

Table F.3: Dominant pole zero values of feedback inductor voltage 1^e order filter

	f(kHz)						
x(s)	10^0	10^1	10^2	10^3	10^4	10^5	10^6
R	4	4	4	4	4	4	4
$sR\tau(1-gg_m)$	$5.09(1-g)$ 10^{-4}	$5.09(1-g)$ 10^{-3}	$5.09(1-g)$ 10^{-2}	$5.09(1-g)$ 10^{-1}	$5.09(1-g)$	$5.09(1-g)$ 10^{1}	$5.09(1-g)$ 10^{2}
R	4	4	4	4	4	4	4
$s(L+R\tau$ $-R\tau gg_m)$	$(0.56-$ $5.09g)10^{-2}$	$(0.56-$ $5.09g)10^{-1}$	$(0.56-$ $5.09g)$	$(0.56-$ $5.09g)10^{1}$	$(0.56-$ $5.09g)10^{2}$	$(0.56-$ $5.09g)10^{3}$	$(0.56-$ $5.09g)10^{4}$
$s^2(LCR+L\tau)$	$3.89\cdot10^{-6}$	$3.89\cdot10^{-4}$	$3.89\cdot10^{-2}$	3.89	$3.89\cdot10^{2}$	$3.89\cdot10^{4}$	$3.89\cdot10^{6}$
$s^3LCR\tau$	$4.13\cdot10^{-1}$0	$4.13\cdot10^{-7}$	$4.13\cdot10^{-4}$	$4.13\cdot10^{-1}$	$4.13\cdot10^{2}$	$4.13\cdot10^{5}$	$4.13\cdot10^{8}$

$$K1 = \frac{R}{R} \tag{F.9a}$$

$$K2 = \frac{R}{s(L+R\tau-R\tau g)+R} \tag{F.9b}$$

$$K3 = \frac{sR\tau(1-gg_m)+R}{s^3CRL\tau+s^2(L\tau+CRL)+s(L+R\tau-R\tau g)+R} \tag{F.9c}$$

$$K4 = \frac{sR\tau(1-gg_m)+R}{s^3CRL\tau+s^2(L\tau+CRL)} \tag{F.9d}$$

$$K5 = \frac{sR\tau(1-gg_m)}{s^3CRL\tau} \tag{F.9e}$$

$$K2b = \frac{sR\tau(1-gg_m)+R}{s^2(L\tau+CRL)+s(L+R\tau-R\tau g)+R} \tag{F.10}$$

$$w_{c1} = \begin{cases} \frac{1}{\sqrt{CL}} & g=0 \\ \sqrt{\frac{R}{L\tau+CRL}} & g>0 \end{cases} \tag{F.11a}$$

$$Q_1 = \begin{cases} \sqrt{\frac{CR^2}{L}} & g=0 \\ \sqrt{\frac{R(L\tau+CRL)}{(L+R\tau(1-gg_m))^2}} & g>0 \end{cases} \tag{F.11b}$$

$$w_{c2} = \begin{cases} - & g=0 \\ \frac{\tau+CR}{CR\tau} & g>0 \end{cases} \tag{F.11c}$$

$$w_{c3} = \begin{cases} - & g=0 \\ \frac{1}{\tau(1-gg_m)} & g>0 \end{cases} \tag{F.11d}$$

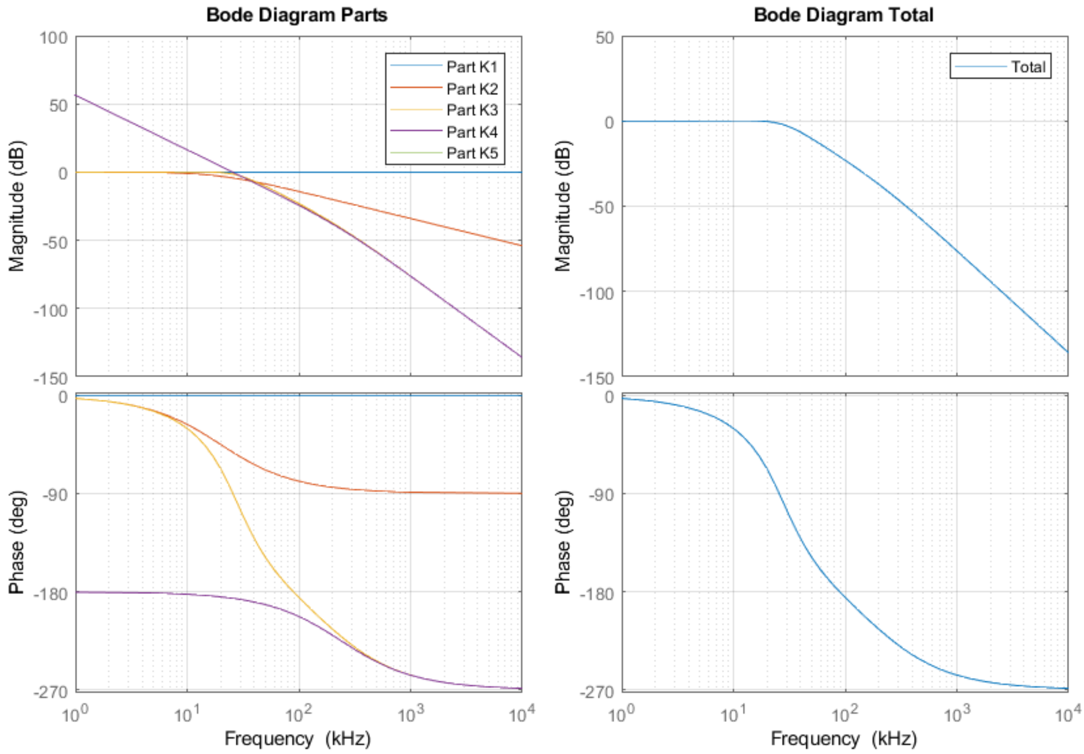

Figure F.5: Bode plot of all smaller transfer function of model 3.1 with a first order filter and gain g of 1

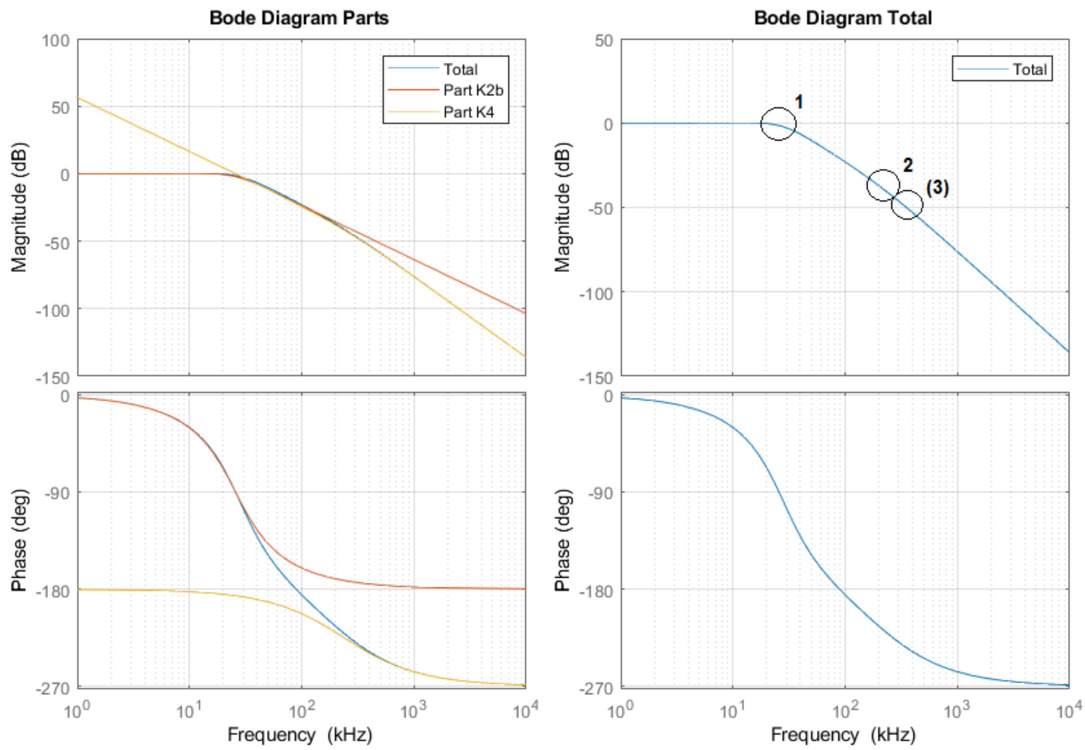

Figure F.6: Bode plot of final smaller transfer function of model 3.1 with a first order filter and gain g of 1

F.4 Feedback inductor voltage second order

The total transfer function of the feedback model from the inductor voltage with a second order filter, Equation F.12, has a too high order to characterize easily. Table F.4 shows which parts of the transfer functions at certain frequencies are dominant. The dominant parts form the smaller transfer functions, shown in Equation F.13. The Bode plot of all smaller transfer functions and total transfer function is shown in Figure F.7. Not all small parts are needed, some can be neglected. The smaller transfer functions L1 and L2 will be neglected, because transfer L3 describes the same characteristics at their specific frequency. Transfer L3, L5 and L6 will be also neglected because the transfer function L4 describes their characteristics. Note that L4 has a too high order in the denominator to characterize easily. L4 can be made smaller to L4b, shown in Equation F.14. L4b is determined by looking at the dominant frequency between 10 MHz and 100 MHz. The smaller transfer function is checked if it still describes the higher frequencies of the total transfer function via a Bode plot graph, shown in Figure F.8. Both transfer functions L3 and L4b have two or lower order denominator and numerator, which means that the total transfer function can be characterized by the smaller transfer functions, see Equation F.15.

$$H(s)_{L-2^eHP} = \frac{s^2R(1-gg_m) + sR\frac{w_c}{Q} + Rw_c^2}{s^4CRL + s^3(L + CRL\frac{w_c}{Q}) + s^2(L\frac{w_c}{Q} + CLRw_c^2 + R(gg_m) + s(Lw_c^2 + R\frac{w_c}{Q}) + w_c^2R} \tag{F.12}$$

Table F.4: Dominant pole zero values of feedback inductor voltage second order filter

	f(kHz)						
x(s)	10^0	10^1	10^2	10^3	10^4	10^5	10^6
Rw_c^2	$1.6 \cdot 10^{13}$	$1.6 \cdot 10^{13}$	$1.6 \cdot 10^{13}$	$1.6 \cdot 10^{13}$	$1.6 \cdot 10^{13}$	$1.6 \cdot 10^{13}$	$1.6 \cdot 10^{13}$
sRw_c/Q	$1.8 \cdot 10^{9}$	$1.8 \cdot 10^{10}$	$1.8 \cdot 10^{11}$	$1.8 \cdot 10^{12}$	$1.8 \cdot 10^{13}$	$1.8 \cdot 10^{14}$	$1.8 \cdot 10^{15}$
$s^2R(1-gg_m)$	$1.01(1-g)$ 10^5	$1.01(1-g)$ 10^7	$1.01(1-g)$ 10^9	$1.01(1-g)$ 10^{11}	$1.01(1-g)$ 10^{13}	$1.01(1-g)$ 10^{15}	$1.01(1-g)$ 10^{17}
Rw_c^2	$1.6 \cdot 10^{13}$	$1.6 \cdot 10^{13}$	$1.6 \cdot 10^{13}$	$1.6 \cdot 10^{13}$	$1.6 \cdot 10^{13}$	$1.6 \cdot 10^{13}$	$1.6 \cdot 10^{13}$
$s(Rw_c/Q$ $+Lw_c^2)$	$2.22 \cdot 10^{10}$	$2.22 \cdot 10^{11}$	$2.22 \cdot 10^{12}$	$2.22 \cdot 10^{13}$	$2.22 \cdot 10^{14}$	$2.22 \cdot 10^{15}$	$2.22 \cdot 10^{16}$
$s^2(R(1-gg_m$ $+CLw_c^2)$ $+Lw_c/Q)$	$(1.54-$ $0.01g)10^7$	$(1.54-$ $0.01g)10^9$	$(1.54-$ $0.01g)10^{11}$	$(1.54-$ $0.01g)10^{13}$	$(1.54-$ $0.01g)10^{15}$	$(1.54-$ $0.01g)10^{17}$	$(1.54-$ $0.01g)10^{19}$
$s^3(LCRw_c/Q$ $+L)$	$1.59 \cdot 10^{3}$	$1.59 \cdot 10^{6}$	$1.59 \cdot 10^{9}$	$1.59 \cdot 10^{12}$	$1.59 \cdot 10^{15}$	$1.59 \cdot 10^{18}$	$1.59 \cdot 10^{21}$
s^4LCR	$8.21 \cdot 10^{-2}$	$8.21 \cdot 10^{2}$	$8.21 \cdot 10^{6}$	$8.21 \cdot 10^{10}$	$8.21 \cdot 10^{14}$	$8.21 \cdot 10^{18}$	$8.21 \cdot 10^{22}$

$$L1 = \frac{Rw_c^2}{w_c^2 R} \tag{F.13a}$$

$$L2 = \frac{Rw_c^2}{s(Lw_c^2 + R\frac{w_c}{Q}) + w_c^2 R} \tag{F.13b}$$

$$L3 = \frac{sR\frac{w_c}{Q} + Rw_c^2}{s^2(L\frac{w_c}{Q} + R + CLRw_c^2 - gg_m R) + s(Lw_c^2 + R\frac{w_c}{Q}) + w_c^2 R} \tag{F.13c}$$

$$L4 = \frac{s^2R(1 - gg_m) + sR\frac{w_c}{Q} + Rw_c^2}{s^4CRL + s^3(L + CRL\frac{w_c}{Q}) + s^2(L\frac{w_c}{Q} + R + CLRw_c^2 - gg_m R) + s(Lw_c^2 + R\frac{w_c}{Q})} \tag{F.13d}$$

$$L5 = \frac{s^2R(1 - gg_m) + sR\frac{w_c}{Q}}{s^4CRL + s^3(L + CRL\frac{w_c}{Q}) + s^2(L\frac{w_c}{Q} + R + CLRw_c^2 - gg_m R)} \tag{F.13e}$$

$$L6 = \frac{s^2R(1 - gg_m)}{s^4CRL + s^3(L + CRL\frac{w_c}{Q})} \tag{F.13f}$$

$$L4b = \frac{s^2R(1 - gg_m) + sR\frac{w_c}{Q} + Rw_c^2}{s^4CRL + s^3(L + CRL\frac{w_c}{Q}) + s^2(L\frac{w_c}{Q} + R + CLRw_c^2 - gg_m R)} \tag{F.14}$$

$$w_{c1} = \begin{cases} \frac{1}{\sqrt{CL}} & g = 0 \\ \sqrt{\frac{w_c^2 R}{L\frac{w_c}{Q} + CLRw_c^2 + R(1-gg_m)}} & g > 0 \end{cases} \tag{F.15a}$$

$$Q_1 = \begin{cases} \sqrt{\frac{CR^2}{L}} & g = 0 \\ \sqrt{\frac{w_c^2 R(L\frac{w_c}{Q} + CLRw_c^2 + R(1-gg_m))}{(Lw_c^2 + R\frac{w_c}{Q})^2}} & g > 0 \end{cases} \tag{F.15b}$$

$$w_{c2} = \begin{cases} - & g = 0 \\ w_c Q & g > 0 \end{cases} \tag{F.15c}$$

$$w_{c3} = \begin{cases} - & g = 0 \\ \sqrt{\frac{L\frac{w_c}{Q} + CLRw_c^2 + R(1-gg_m)}{CRL}} & g > 0 \end{cases} \tag{F.15d}$$

$$Q_3 = \begin{cases} - & g = 0 \\ \sqrt{\frac{CRL(L\frac{w_c}{Q} + CLRw_c^2 + R(1-gg_m))}{(L + CRL\frac{w_c}{Q})^2}} & g > 0 \end{cases} \tag{F.15e}$$

$$w_{c4} = \begin{cases} - & g = 0 \\ w_c Q & g = 1 \\ \sqrt{\frac{w_c^2}{(1-gg_m)}} & else \end{cases} \tag{F.15f}$$

$$Q_4 = \begin{cases} - & g = 0 \\ - & g = 1 \\ \sqrt{\frac{w_c^2 R^2(1-gg_m)}{(R\frac{w_c}{Q})^2}} = \sqrt{Q^2(1 - gg_m)} & else \end{cases} \tag{F.15g}$$

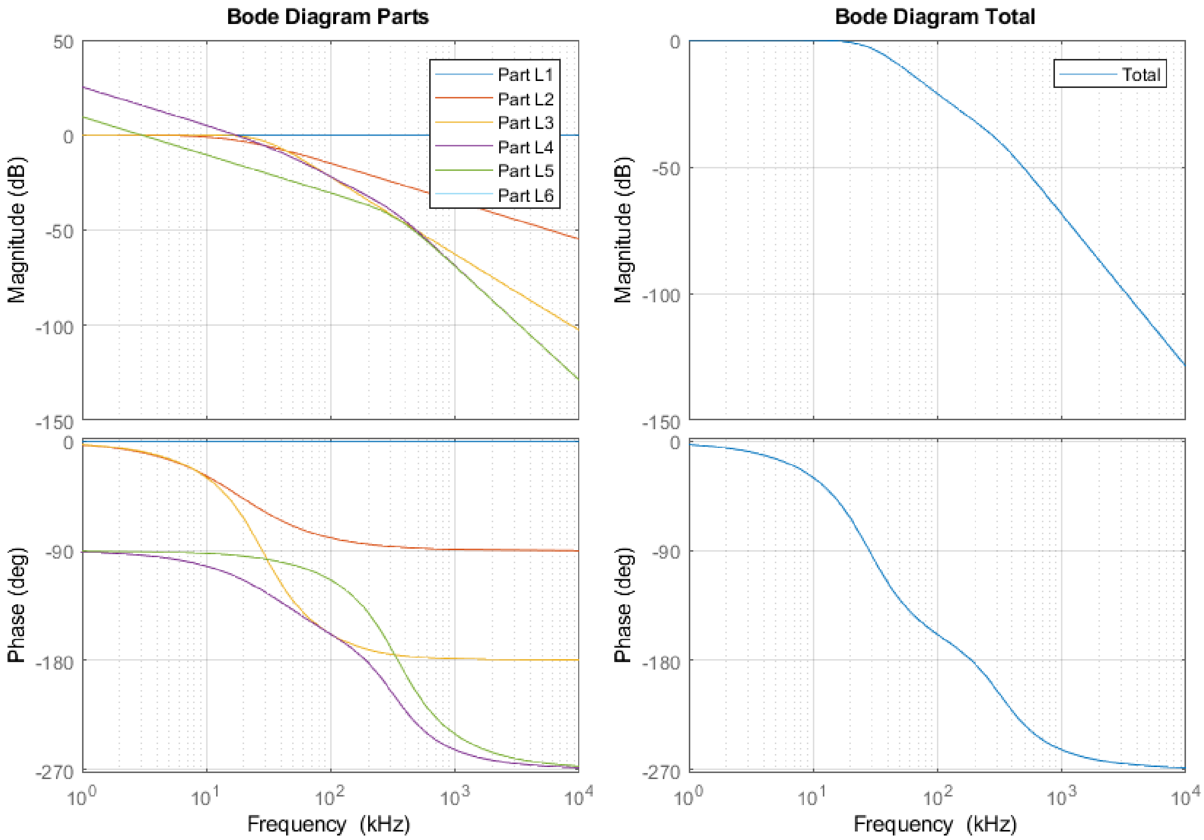

Figure F.7: Bode plot of all smaller transfer function of model 3.1 with a second order filter and gain g of 1

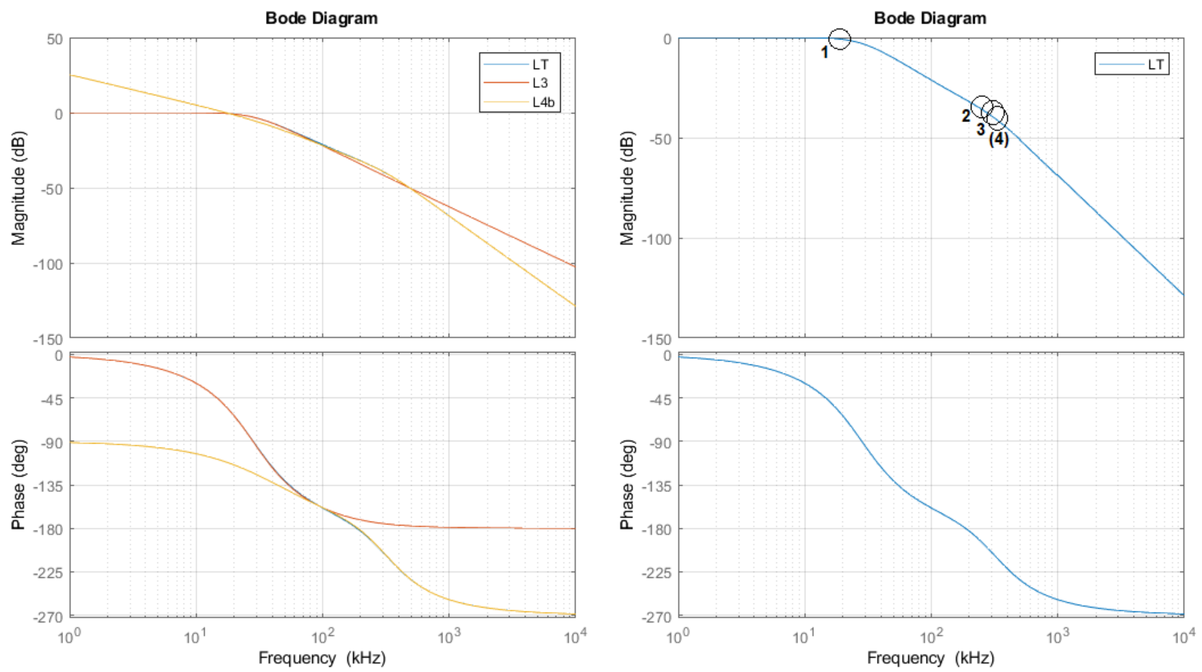

Figure F.8: Bode plot of final smaller transfer function of model 3.1 with a second order filter and gain g of 1