Multiple versions of Tetris

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Abstract

This paper investigates different versions of the game Tetris on their playability. It does so by utilising an automated Tetris controller 'TetNet' that uses Genetic Algorithm to create an evolving Tetris AI (Artifical Intelligence). Using 'TetNet' several hypotheses will be formed on the playability of different Tetris games. By making use of existing theorems and newly defined theorems, a handful of proofs will be given for the hypotheses formed earlier. This will be done by creating infinitely playable strategies for each version of a Tetris game, and drawing out cycles according to the playing strategies.

Introduction

Tetris is one of the oldest and most classic video games out there, designed and programmed by Russian game designer Alexey Pajitnov in 1984. The game is played with seven different tetrominoes (seen in Figure 1) that are each four blocks large. The game itself takes place on a 20x10 grid (20 rows, 10 columns), and each time a random tetromino starts in the top row, middle column. Immediately afterwards the player is able to see the next tetromino as well. The current tetromino in play can be rotated and moved by the player, when the current tetromino is set in place the next tetromino will appear at the top. Whenever a row is fully filled with blocks, the row gets cleared. Any blocks above the destroyed row will fall down and the game continues. The goal of the game is to destroy as many rows as possible. When there is no more space for any new tetrominoes to be placed in the top row, middle column, the game ends. In this article I will be looking at which tetrominoes can be used in a Tetris game to allow the game to be played infinitely long.

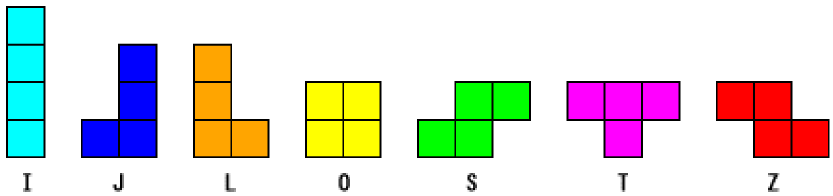

Figure 1: The 7 tetrominoes

Related works

First there are several articles on automating the Tetris playing process that I would like to mention. 'Approximate Modified Policy Iteration and its Application to the Game of Tetris' by B. Scherrer, M. Ghavamzadeh, V. Gabillon, B. Lesner, M. Geist [2] makes use of approximate modified policy iteration to build a Tetris controller, this method yields great results (average 51.000.000 lines cleared) but is a fairly new method. Another method used for building a Tetris controller is the cross-enthropy method utilised in 'Learning Tetris Using the Noisy Cross-Entropy Method' by István Szita and András Lörincz [6] with a performance of around 300.000 lines average. The final method is genetic algorithm used in 'Evolving and discovering Tetris gameplay strategies' by Somnuk Phon-Amnuaisuk [7], this is a fairly common method used in Tetris controllers and is said to yield a performance of roughly the same as the cross-enthropy method.

Furthermore there are some articles regarding the playability of Tetris that are crucial to the construction of this article. In Heidi Burgiel's article 'How to lose at Tetris', it is proven that in a Tetris game where Z and S tetrominoes are given to the player in an alternating sequence, the maximum amount of moves one could make before losing is 70000 [4]. Similary research has been done in the area of whether Tetris is winnable with certain pieces. In the thesis 'Can you win at Tetris' [3] by John Brzustowski it is proven that any Tetris game with only a single tetromino is infinitely playable. Some combinations of two different tetrominoes turned out to be infinitely playable as well. The exact known results will be specified later.

Research question

Which tetrominoes allow a Tetris game to be played infinitely, and how do you prove it?

Approach

Before formulating any proofs on which tetrominoes can be played infinitely long, an automated Tetris controller 'TetNet' will be used to run several different games with different tetrominoes. This is to mostly get a feel for how "playable" each game is for the chosen tetrominoes. Using these results it is possible to form hypotheses on which tetrominoes may lead to an infinitely playable game and which ones may not. In the next sections a thorough explanation will be provided for the controller, and afterwards proofs will be given for some combinations of tetrominoes.

Tetnet

In this article I will be using TetNet [5] as a basis, a program that uses a genetic algorithm to build an AI (Artificial Intelligence) that is capable of playing Tetris. The reason I chose to work with a controller that is built on genetic algorithm instead of other algorithms is because genetic algorithm is widely used, and the evaluation function used by genetic algorithm is very general and thus easy to manipulate for use in non-standard Tetris games. With this program I will be testing out various tetrominoes combinations to evaluate their individual performances. TetNet generally functions mostly the same as the one described in the article by Somnuk Phon-Amnuaisuk [7], the difference being that TetNet uses a slightly different fitness function and evolution process.
Genetic algorithms work by creating a population of ‘genomes’ that have multiple ‘genes’, representing parameters for the algorithm. TetNet makes use of the following genes for its genomes.

id	The unique identifier for the genome
rowsCleared	The weight of each row cleared by the given move
weightedHeight	the absolute height of the highest column to the power of 1.5
cumulativeHeight	The weight of the sum of all the columns’ heights
relativeHeight	The weight of the highest column minus the lowest column
Holes	The weight of the sum of all the empty cells that have a tetromino above them
Roughness	The weight of the sum of absolute differences between the height of each column. ($\sum_{i=0}^{9}$ | height column i+1 - height column i |)

Table 1: TetNet Genes

The base program works with a population size of 50 genomes for each generation, with each genome evaluated over a maximum of 500 moves or until failure. After this, the generation of genomes will be evolved and go to the next generation with generally better performing genomes.

Fitness Function

The fitness function used in the TetNet program is a way to evaluate a genome’s performance. It is simply the total score achieved by a genome with more score implying a fitter genome, in this case the score is the total amount of lines cleared.

Evolution Process
The fittest genome is directly taken to the next generation. Every generation the least fit 50% of the total population is dropped, the other genomes are put through a process where two of them are selected randomly each time as parents. The child will have a random combination of characteristics of the parents, and each of those characteristics has a chance to mutate.

Whether these changes are beneficial or not is hard to tell, but since the methods used by Somnuk Phon-Amnuaisuk and TetNet are highly similar it is unimaginable that one performs a lot better than the other.
First of all the fitness function in TetNet is score, whilst the fitness function used by Somnuk Phon-Amnuaisuk is a weighed sum of holes in the playfield. These two are inversely correlated since having a lot of holes means having little lines cleared. One could say that a genome that clears a lot of lines is also a genome that has few holes. So in that regard the resulting fitness of a genome in both cases is very similar.
As for the evolution process, the principle in both programs is the same. The fitter genomes are seen as more favorable. In Somnuk Phon-Amnuaisuk's article the way a generation is evolved is by directly taking the top 10% fittest genomes to the next generation, and the rest will go through a process of selection and mutation. This is mostly the same as the method used by TetNet, aside from slight differences in the numbers.

Self edits
Whilst TetNet was a great program for automating the Tetris playing process, it did lack several features that were essential to test different Tetris games with. First is the scoring system used by the game, this scoring system is according to the old retro Tetris rules from the NES (Nintendo Entertainment System) [1] and makes it so that clearing multiple lines in a single move is worth more points than clearing the same amount of lines with more moves. However such a scoring system is irrelevant to the goal of this article, as the only thing that matters here is the amount of lines cleared. So the scoring system is changed to only represent the amount of lines cleared. Furthermore TetNet makes use of the 'bag', which means that in the case of a game with seven different tetrominoes, all seven tetrominoes will be dealt in a random order repeatedly. The 'bag' is removed in the game that I will be testing, to allow for random distribution of the tetrominoes. A few other things that are changed for convenience during testing are, customized tetrominoes, customized playfield, highscore recording and custom move limit per genome.

Results
The modified TetNet program produced quite some results see table 2, what is worth noting is that a game without the S and Z tetrominoes performed

Table 2: Generation 40 score of various Tetris games

Amount of tetrominoes	Tetromino combinations tested	Highscore
7	Normal game	400
6	I+L+J+O+T+S	400
5	I+J+L+O+T	5000
	S+J+L+I+T	1600
	S+J+L+I+O	1400
	S+L+I+O+T	950
	S+J+I+O+T	300
	S+J+L+O+T	150
3	L+J+O	250
	S+O+T	70
	S+T+Z	70
2	I+O	150,000+
	L+J	500
	S/Z alternating	40

significantly better than a game with all 7 tetrominoes, and that a game with only S and Z tetrominoes alternating performed extremely bad. This is in line with what has been found so far; S and Z tetrominoes together in a game will cause the game to end in a finite amount of moves. The game with only the Z tetromino left out performs roughly the same as a full game with 7 tetrominoes, that gives the suspicion that there are other combinations of tetrominoes that cause the game to end in a finite amount of moves. By looking at every single five-piece game that does not contain both the S and Z tetromino, one could tell that the games without the S or Z tetromino still perform better than those with an S tetromino. when looking at the three worst performing five-piece games, the one common combination amongst them is S+O+T. This combination does not occur in the top 3 performing five-piece games and also contains the S tetromino, after testing the S+O+T game out in a simulation the perfomance only yielded a highscore of 70, around the same as the already known hard game S+T+Z. For that reason it is suspected that the S+O+T game is not infinitely playable. Other suspects that might not be infinitely playable are any games containing a single S piece, because of the massive performance drop (highscore of 5000 to 1600) from the best five-piece game, to the best five-piece game containing an S tetromino. The next section will be used to proof some of these combinations.

Playing forever

In order to define a set of blocks that are able to be played infinitely, let's define the opposite first.

Theorem 1. *A game of Tetris cannot be played infinitely long if there exists a sequence of tetrominoes that cause the game to end in a finite amount of moves.*

Proof. Any sequence of tetrominoes has a chance of appearing. If any of these sequences cause the game to end in a finite amount of moves, it means that the Tetris game cannot be played infinitely long. □

With the help of this theorem, a lemma follows.

Lemma 2. *Suppose a game exists that consists of a set of N different blocks $B = \{a_1, a_2, .., a_N\}$, then it is infinitely playable if and only if there does not exist a combination of blocks in B that cause the game to end in a finite amount of moves; any game that consists of blocks $\tilde{B} \subseteq B$ makes is infinitely playable as well.*

Proof. Follows from Theorem 1. □

So e.g. it is proven that the sequence of alternating S and Z-tetrominoes will terminate in a finite amount (70.000) of moves [4]. This implies that any Tetris game containing both the S and Z-tetromino is not infinitely playable. Following this train of thought one could draw out a tree like in Figure 2, with each node not infinitely playable causing each of its parents ending in a finite amount of moves as well. Note however that even if every node is in fact infinitely playable, it does not mean that their parent is also infinitely playable.

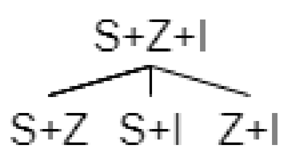

Figure 2: Example of the bottom part of the tree

Since every one-piece game has proven to be able to played infinitely long [3], the next games that need prove are the two-piece games. With 7 tetrominoes in total, there are $\binom{7}{2} = 21$ possible combinations for two-piece games. But considering some combinations of pieces are symmetrical to others (e.g. S+I is the same as Z+I but mirrored), the total amount of unique games is displayed as below in Table 3:

O	S	T	L	I	
S	Z	J	L	T	I
T	J	I			
J	L	I			

Table 3: Unique two-piece games

So far all the combinations marked in green have been proven to either be infinitely playable or not. The game with S+Z tetrominoes is proven to end in a finite amount of moves [4], and all the other ones in green (O+L, O+I, J+I, J+L) are proven to be infinitely playable [3]. Out of these, I+L and J+L require the lookahead function to prove.
To formulate proofs on games with a Z tetromino, it is handy to know the exact placement of the Z tetromino. Theorem 3 tells us that Z-tetrimonoes may only be placed with their left most cell in an uneven column, and that they must be placed vertically.

Theorem 3. *In a Tetris game in which only Z-tetrominoes are presented to the player, no more than 120 Z-tetrominoes can be placed either vertically with their leftmost cells in an even-numbered column or horizontally in any column without losing.*

Proof. Proof of this theorem can be found in the article "How to lose at Tetris" by Heidi Burgiel[4]. □

Theorem 4. *A Tetris game is infinitely playable if and only if there exists a cycle, a set of game states that the game stays in indefinitely.*

Proof. Let a Tetris game be infinitely playable, since the game platform is only 20x10 large there can only exist a finite amount of game states N. Thus the game is forced to stay within N game states no matter how many moves the player makes; the game has a cycle.
If a Tetris game always stays within a cycle, it means that the game may never leave the cycle. A cycle does not contain any losing states, so the game is infinitely playable. □

In the case that a two-piece game is infinitely playable, a cycle may serve as a form of proof. We will be making use of these theorems in the following sections.

Two-piece games

In this section several different two-piece games will be looked atFigure, and accommodated with proofs on whether they are infinitely playable or not.

Figure 3: Z and I lane

Z+I

The first combination that was looked at is the Z+I game. As mentioned earlier in the table for unique two-piece games, so far there is no information about whether the Z+I game is infinitely playable or not. To see whether Z+I is infinitely playable, the following playing strategy has been constructed:

1. Always play the Z tetromino vertically, with its left most block in an uneven column. This follows from Theorem 3.
2. Always play the I tetromino vertically.
3. Count the amount of Is played, ones that are uneven numbered are played in the lowest even column, even numbered I following afterwards is played directly to the left of the previous I's placement.
4. Always play the Z tetromino on the Z lane. Unless the Z lane is 2 or more higher than the I lane.

The Z lane and I lane are as shown in Figure 3.
The thought behind this strategy is explained as such. Step 1 is simply derived from Theorem 3. Step 2 is put in place because one can not play only I tetrominoes horizontally forever in a 10 wide Tetris game, it is not hard to see as horizontal Is can at most fill 8 blocks per row and thus cannot ever clear a row. Step 3 and 4 follow more from intuition. Because of the way that a Z tetromino is shaped, it is much better to place the Z tetromino on top of an even column I rather than an uneven column I as seen in Figure 4. That's why its desirable to place the I in an even column first, followed by an uneven column. Step 4 in combination with step 3 makes it such that whenever you play a Z tetromino in an I lane, the Z is immediately cleared. I will be using this strategy to prove the Z+I game for 4 blocks wide as a basis for the full game of 10 blocks wide, as the 10 blocks wide game is too difficult to prove.
For the 4 blocks wide Z+I game there are 4 different opening plays, see Figure 5. Each of these opening plays causes the game to have a different cycle. These cycles are drawn out in the section 'Appendix', Figure 10 to 15. As seen in these cycles, the worst sequence that could possibly occur is by starting up

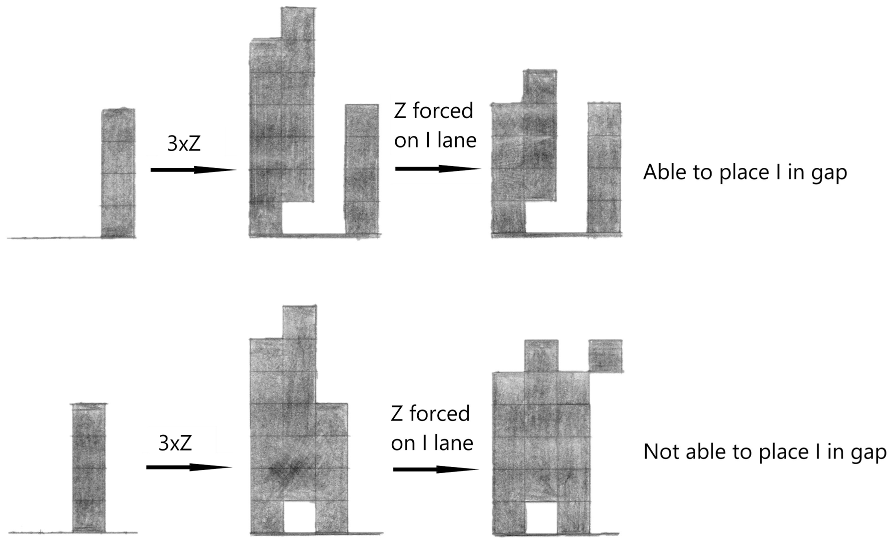

Figure 4: Good and bad I placement

with a 2xI+Z opening and eventually going into the double Z opening cycle which could lead to a maximum column height of 13. Assuming a playfield of 20 columns high, this 4 blocks wide Z+I game is thus proven to be infinitely playable.

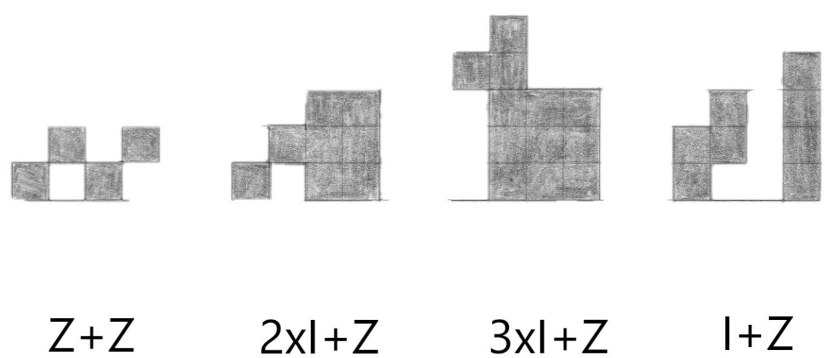

Figure 5: All openings for the Z+I game on a 4 wide playfield

Z+O

For the prove of Z+O a similar approach was used. Start by defining a general playing strategy, followed by a cycle. The playing strategy is a follows:

1. Always play the Z tetromino vertically, with its left most block in an uneven column.
2. Always play the Z tetromino on a Z lane, unless all Z lanes present are at least 3 higher than the lowest O lane. In that case play Z in the lowest O lane.
3. Always play the O tetromino on a O lane, unless all O lanes present are at least 2 higher than the lowest Z lane. In that case play O in the lowest Z lane.

The Z lane and O lane are defined as in Figure 7. Again step 1 is derived directly from Theorem 3. Step 2 and 3 are put in place to make sure that whenever you have to play a Z on an O lane or vice versa, the next time a row/rows is cleared the Z and O placed on opposite lanes are at least cleared as much as possible. An example of this is seen in figure.

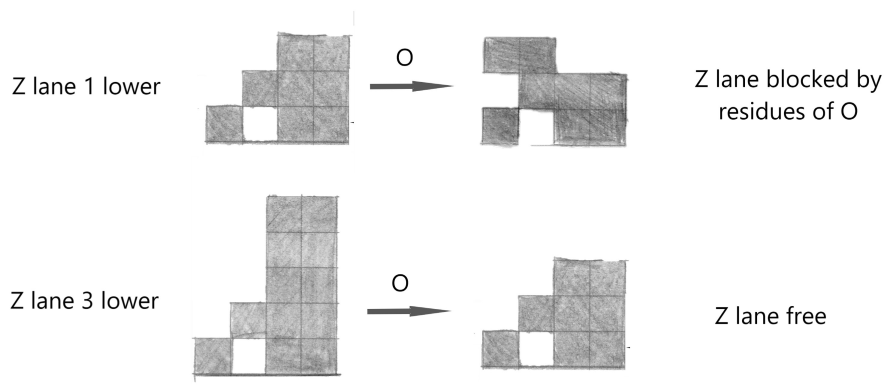

Figure 6: placing an O on a Z lane 1 lower than the lowest O lane vs placing an O on a Z lane 3 lower than the lowest O lane

Figure 7: Z lane and O lane

Again I will be using this strategy to prove that the Z+O game is playable on a 4 blocks wide playfield. For this particular Z+O game there are two different openings that each have a unique cycle, shown in Figure 8. Their respective cycles are shown in Figure 9. As you can see the Z+O opening cycle has a chance of going into the Z+Z opening cycle, but with an increased height of 1. This means that the worst case scenario for this particular game is a maximum column height of 6. This is stil well within the playfield boundaries, so the Z+O game is proven to be playable on a 4 blocks wide playfield.

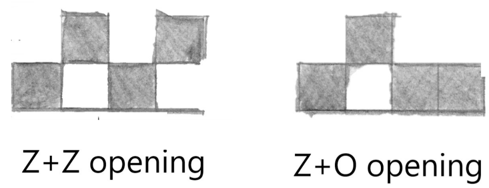

Figure 8: the possible openings for a Z+O game on 4 blocks wide

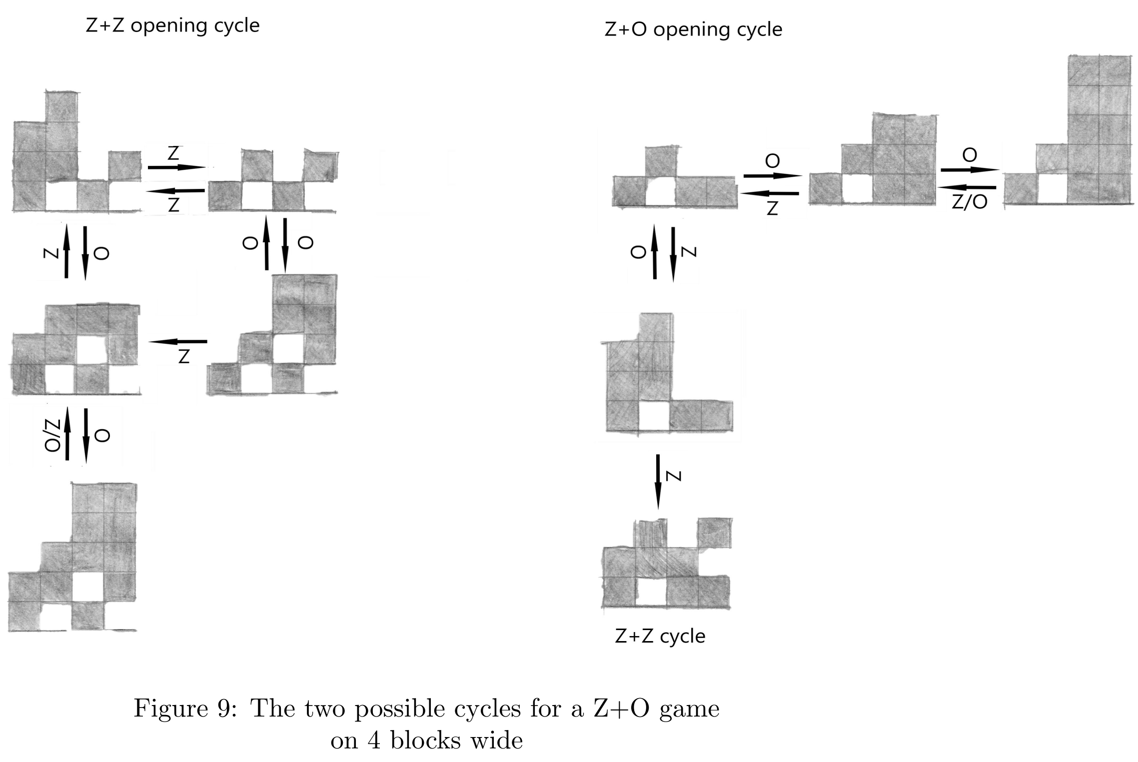

Figure 9: The two possible cycles for a Z+O game on 4 blocks wide

Unlike the proof of the Z+I game, this proof involves much simpler cycles. It is thus possible to extend this proof to a 10 blocks wide game. Recall the Z lane and O lane defined earlier in Figure 7, and follow the same playing strategy as before. For the ten blocks wide Z+O game there are three different scenarios that may occur.

1. Scenario 1: Tetrominoes are handed out to the player in such a way that the player can place every Z tetromino on a Z lane and every O tetromino on an O lane. In this the game stays in the same cycle.

2. Scenario 2: The player is forced to play Z tetromino(es) on O lanes, this will cause the game to enter a new cycle with more Z lanes. Each time the row is cleared where a Z tetromino(es) is played on O lane(s), the minimum height of the new cycle is one higher than the minimum height of the old cycle. When this process happens with only O lanes present, the mimimum height of the new cycle is increased by 2.

3. Scenario 3: The player if forced to play O tetromino(es) on Z lanes, this does not change the cycle that the game is in.

To get an insight on why the game enters a new cycle during scenario 2, Theorem 5 is introduced. The game enters a new cycle since the requirement for staying in the same cycle is that it is possible to get to every state in the cycle, but the Z tetromino(es) played on top of O lanes can never be fully cleared.

Theorem 5. *In the game with Z and O tetrominoes, once a Z tetromino has been played on an O lane, it can never be fully cleared.*

Proof. When a Z tetromino is played on an O lane (must be vertically, left most block uneven column), a hole is created on top of the right block of the O lane. That hole can never be filled with Z or O tetrominoes, thus making the Z tetromino never fully clearable. □

Any action that the player performs has different consequences depending on which scenario is currently happening.

Actions and results
(results after a line(s) is cleared)

1. Scenario 1:
   Z on Z lane ⇒ Z lane same height
   O on O lane ⇒ O lane same height or 2 higher

2. Scenario 2:
   Z on Z lane ⇒ Z lane 1 higher

Z on O lane ⇒ Z lane 2 higher
O on O lane ⇒ O lane 1 or 3 higher

3. Scenario 3:
Z on Z lane ⇒ Z lane 1 higher
O on Z lane ⇒ O lane 1 higher
O on O lane ⇒ O lane 1 or 3 higher
Only Os handed out ⇒ O lanes possibly 2 higher

One might wonder why playing Os on O lane can cause the lane to be an additional 2 higher as well (1 or 3 higher), that is because the given playing strategies allows us to place 2 Os on top of an O lane without clearing any lines. The same however does not apply to playing O on Z lane or Z on Z lane.
For two Os to be played on a Z lane, at least one of the five things must occur after the first O is played:

1. It is lower than the lowest O lane
2. It is the same height as the lowest O lane
3. It is lower than the lowest Z lane
4. It is the same height as the lowest Z lane
5. It is one higher than the lowest Z lane

The cases 1 and 5 do not naturally occur when following the playing strategies. Cases 2, 3 and 4 do occur, however these cases will result in full row(s), thus clearing lines. So no state exists where one can play two Os on a Z lane without clearing any lines after the first O.
The same pricinple can be applied to playing Zs on any lane. In order to play 2 Zs on top of a lane, at least one of the six things must occur after playing the first Z:

1. It is the same height as the lowest Z lane
2. It is lower than the lowest Z lane
3. It is two higher than the lowest O lane
4. It is one higher than the lowest O lane
5. It is the same height as the lowest O lane
6. It is lower than the lowest O lane

Cases 3 and 6 do not occur when following the playing strategies, and all the other cases will cause a row(s) to be full. So again it is not possible to play a second Z on the same lane before clearing any lines.

Worst case performance Z+O

The game stays indefinitely in a final cycle once every O lane has been converted to a Z lane, since then scenario 2 can no longer happen. By going through all of the aforementioned scenarios, the worst case occurs when scenario 2 is repeated the maximum amount of times. In the case of a 10 blocks wide playfield, scenario 2 can be repeated a maximum of 5 times starting on an empty board, causing the minimum height of the final cycle to be 6. The final cycle of 5 Z lanes has an additional maximum height of 6. So by following the playing strategy introduced earlier, the worst case scenario is a column height of 12. What is worth noting is that for every 2 columns wider (an extra lane), the worst case scenario is 2 higher. That happens since for every lane increase, scenario 2 may be performed one more time, and the resulting final cycle has an additional maximum height of 1 higher.

Conclusion

With the help of 'TetNet' is was possible to roughly see which games performed well, and which did not. It is known that any game containing the combination of Z and S tetrominoes is not infinitely playable. Aside from the games containing both the Z and S tetromino, games containing the S+O+T combination of tetrominoes are suspected to be similarly hard to play. In this paper two new games have been proven to be infinitely playable by using certain playing strategies and drawing out cycles. The two-piece games that are succesfully proven to be infinitely playable are the Z+I game on a 4 blocks wide playfield and the Z+O game on a normal playfield. Whilst these results alone might not be enough to complete the proof for the entire tree of Tetris games, they do form the basis for future continuation. For those interested in doing future research in this area, it is suggested to expand my proof of the Z+I game to a normal playfield. By further proving every other unique two-piece game, one could advance very far into the tree of Tetris games (taking advantage of the fact that any game not infinitely playable will cause its parents to have the same property). Any games containing more than two tetrominoes are extremely hard to prove (the amount of states in the cycles explode), but some of those proofs are required to succesfully proof the entire Tetris games tree. Thus I still feel like it is definitely worth it in future work to extend the proofs given in this paper to those games, or perhaps even find new methods to formulate proofs.

Discussion

there are several points I would like to address here.

1. The automated Tetris controller used in this paper 'TetNet' uses Genetic Algorithm, and as mentioned before it is not the best performing method to build a controller. This causes the simulation to result in lower scores achieved compared to other algorithms. However this should not be too

big of a problem, since the importance lies in the relative performances of different games.

2. For testing games with TetNet I only ran each game for 40 generations, this could causes a low highescore achieved and also possible causes the game have a higher deviation in its performance. The reason for running a low amount of generations is because of horrendous simulation times, as running a game for only 40 generations already takes a whole day.

3. The proof given for the 10 blocks wide Z+O game could perhaps be shown in a better way. Originally a similar approach was planned for this proof (showing it through cycles), but I quickly realised that it is going to be too many states to draw out. For this reason everything is explained in words, but of course it also means that it is harder to follow as a reader.

4. As mentioned earlier in the conclusion, a lot of other Tetris games are still missing proofs. The original goal was to prove the entire Tetris games tree, but that quickly turned out to be impossible in the time given.

Acknowlegdement

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Appendix

Z+I 4 blocks wide game cycle

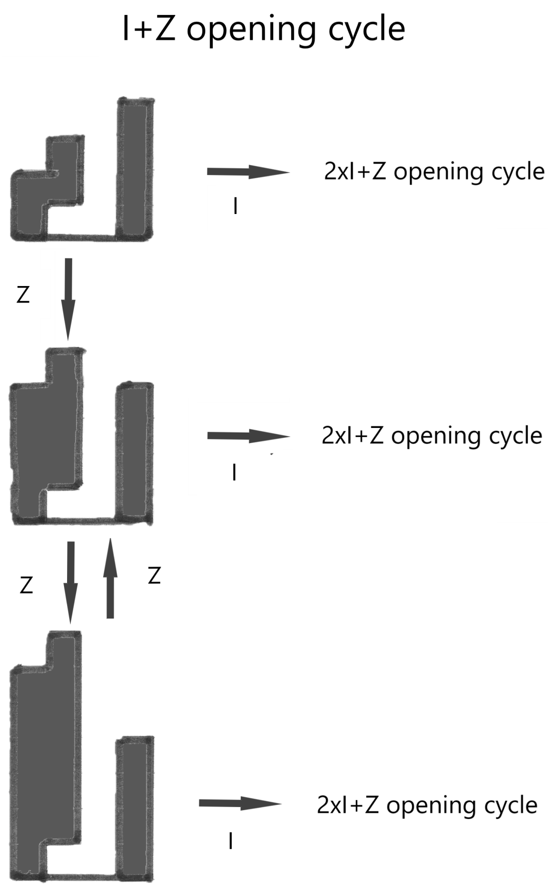

Figure 10

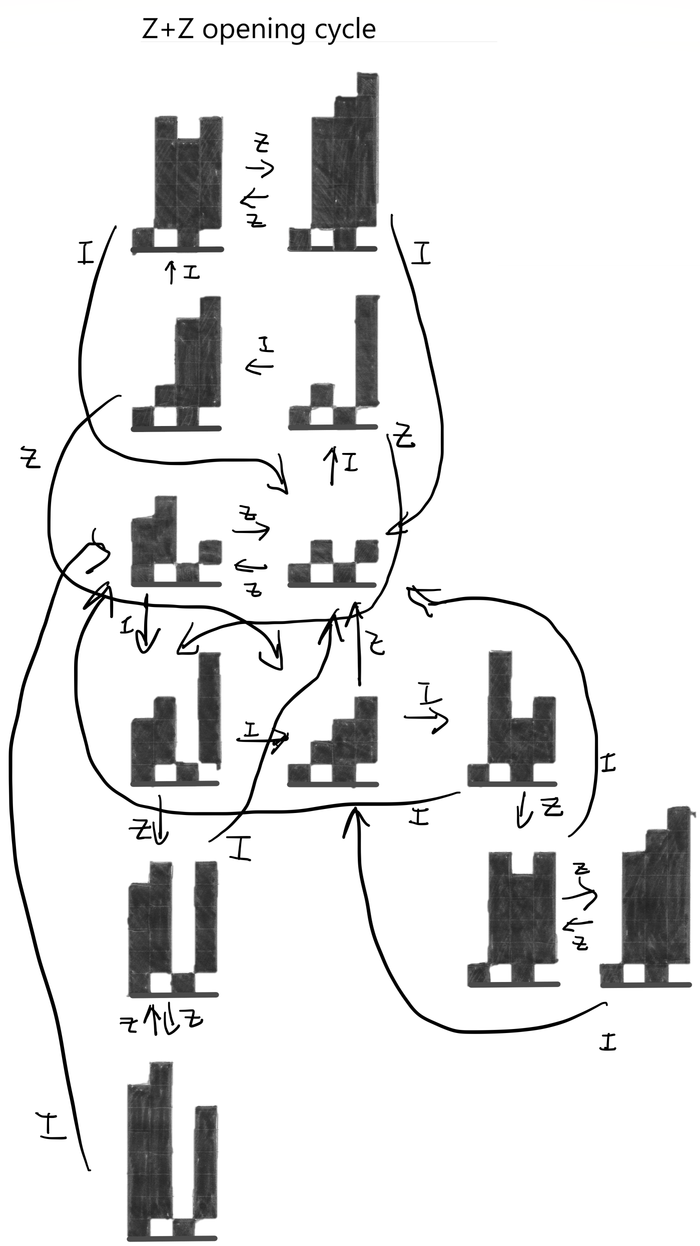

Figure 11

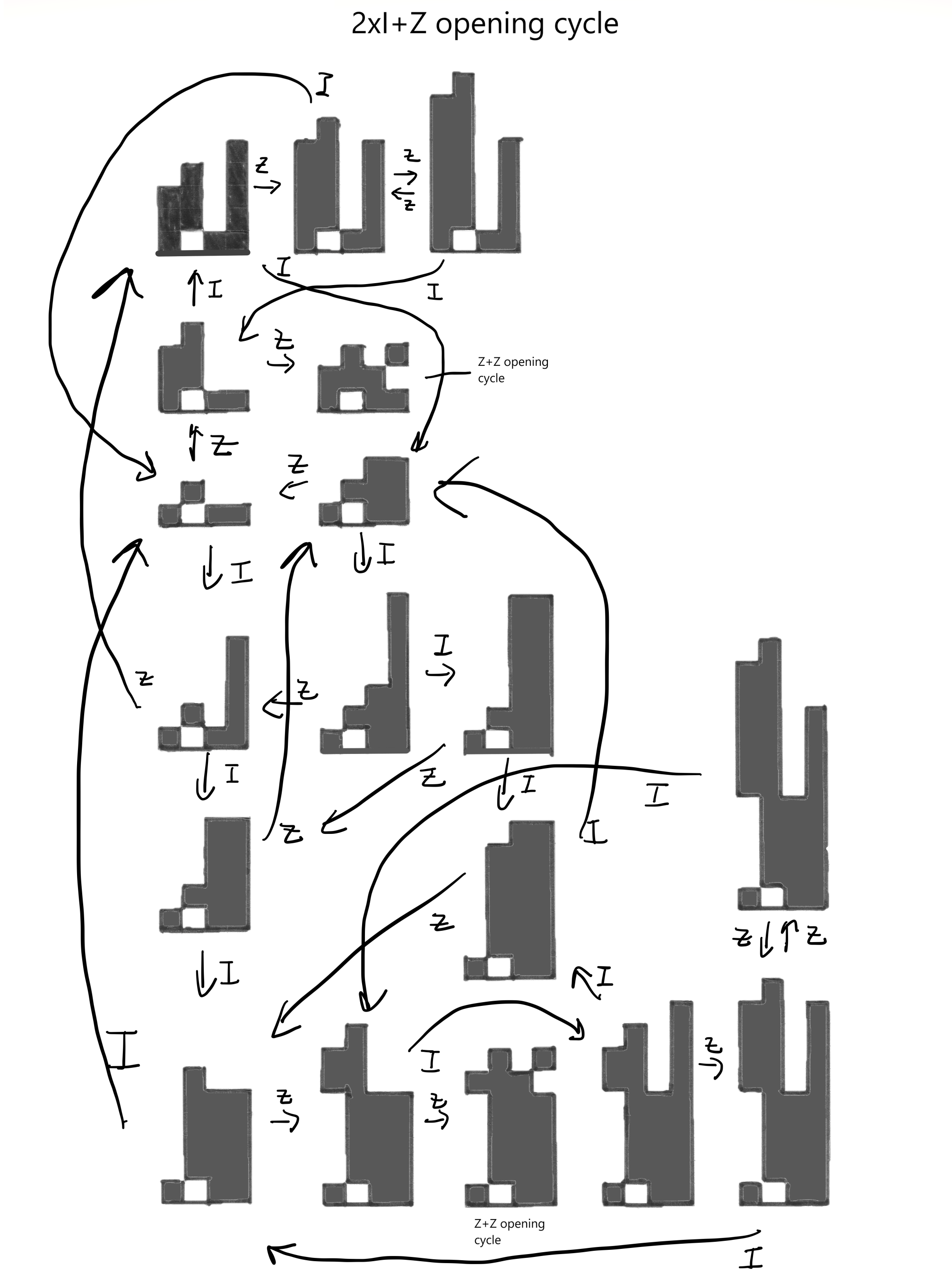

Figure 12


3xI+Z opening cycle
(bottom 3 rows left out)
3xI only
Continued
Z/I

Figure 13

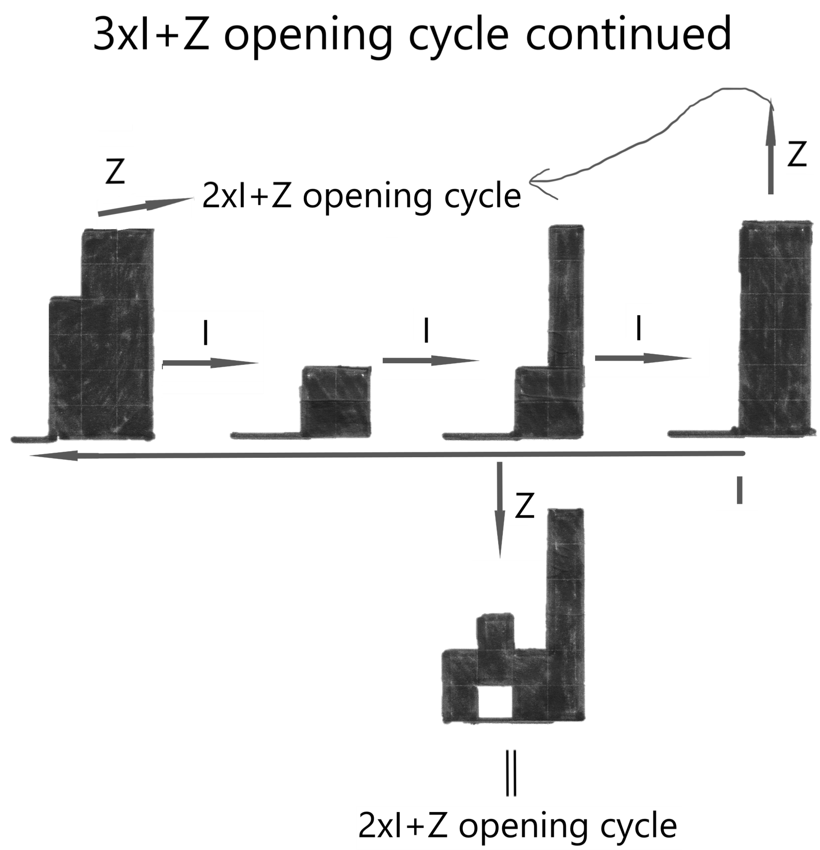

Figure 14: 3xI+Z opening cycle continued

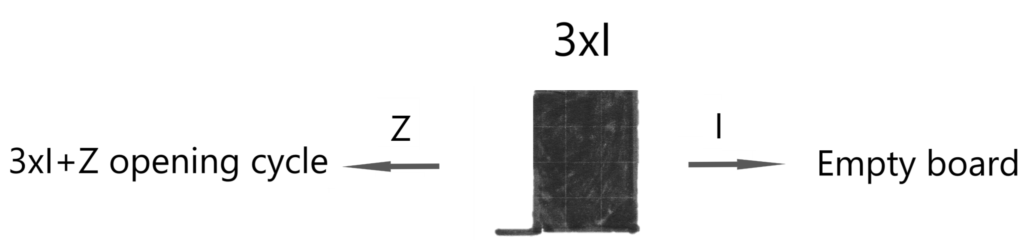

Figure 15: 3xI