An inventory and queueing model for a hospital uniform hand out system

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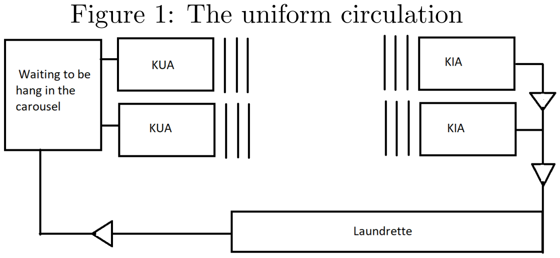
Abstract: This paper is written as a bachelor assignment of Applied Mathematics for the University of Twente. In this paper two problems of the uniform hand out system in the ZGT hospital are analysed. First, a model for an inventory problem is used to determine the optimal number of uniforms in the system. The results point to an influence of the behaviour of the employees on the system. Second, a queueing model is presented to analyse waiting times for the employees at the hand out servers. For multiple arrival rates, the required number of servers is determined. Both models are based on data of the two hospitals of ZGT: location Hengelo and location Almelo.

Keywords: Inventory, circulation, demand, queueing, poisson process

Introduction

The hospitals in Almelo and Hengelo have a automatic uniform hand out system such that a clean uniform is available at the beginning of each shift for each employee. After usage, the uniforms get picked up by the laundrette to be cleaned. This system is used to ensure clean clothes for all employees, which is very important for the hygiene in the hospital. On both locations, the system consists of two units that provide clean uniforms (KUA) and two units that receive used uniforms (KIA) to send them to the laundrette. After cleaning in the laundrette, the uniforms arrive at the hospital and are hung into the storage carousel. Uniforms in the carousel can again be handed out by the KUA. This circulation is shown in figure 1.

Figure 1: The uniform circulation



After a personel card is recognized by the KUA, the KUA hands out an uniform. This personel card is linked to an uniform size and type. By rules of the hospital, employees have to take a new uniform before each shift and have to return this uniform afterwards. Because of the limited credits on a personel card, employees can only get a limited number of uniforms at the same time. When returning an uniform, the uniform is recognized by the KIA. On several moments in a week, used clothes are brought to the laundrette and clean clothes return from the laundrette.

The uniform system has two main operational challenges: Employees indicate that the waiting times at the KUAs can be very long. Also, there is a chance that an employee scans his card and finds that there is no uniform available that fits his personel uniform preference.

In this paper we take a look at the influence of the inventory and the number of KUA's to the service level of the system.

Some employees try to keep their own small stock of clothing in their locker. This is against the rules of the hospital. In this way, a percentage of the uniforms is out of the system. According to the managers, this behaviour is a big influence on the service level of the system.

Due to changes in the services of both hospitals, employees are gradually shifted from Hengelo to Almelo. The uniform system managers want to know the required number of KUAs depending on the number of employees such that the system serves well.

Many factors are relevant in this hospital cleaning system. These multiple factors make the problem complicated and require a mathematical analysis. By examining the minimal number of KUA's and the minimal inventory size, an overview for the hospital can be built.

For both problems a mathematical model is formulated. The first part of this paper considers an inventory problem. By analyzing data, average return times can be found. These will be used to determine the uniform circulation and the required inventory.

The second part of the paper considers the waiting times in queue at the KUA. This is a queueing model, in which waiting times and queue length will be determined with different arrival rates. We take a look at the M/M/n model in case of a stable system, and use a simulation to determine waiting times in case the system is not stable.

Literature review inventory

Determining the number of uniforms that are needed, can be formulated as an inventory problem in which all uniforms circulate in a closed system. The inventory problem of the hospital is a model with uncertainties and a decision vector wich is the optimal inventory. Inventory models are described by Arrow, Harris and Marschak [1]. The reordering point that is described in this paper, is simular to the moment in the hospital in which new clothes are hung in the carousel and used clothes are taken to the laundrette. The hospital model differs from business models, because the system in the hospital is closed. The inventory in the hospital is determined by looking at the total number of uniforms that are in the closed system (in use, out of use or being cleaned).

An inventory problem

A shortage of uniforms can lead to big problems in the uniform system. We want to know whether the hospital has currently an inventory such that it can reach their desired service level. The current inventory is known. Data of 1 year of running the cleaning system is available and includes timestamps of 3 activities:

1 The moments of handing out an uniform at the KUA

2 The moments of handing in an uniform at the KIA

3 The moments of hanging in an uniform in the carousel.

First, we will take a look at the number of uniforms needed per type, size and per day. We call the needed number of uniforms the demand. To determine the demand we use the hand in moment. It is known that some people take an uniform out of the KUA and leave it in their locker untill they need it some days later. Using the hand in moments will be a good indicator of what was needed the hours before.
By analyzing data in R, the average demand per day per uniform sort and size is obtained. This demand is the basis for the inventory problem.

By additional analysis is determined what the total inventory is, that satisfies this demand. This inventory depends on the speed of circulation of the system. So, the speed of washing, bringing clothes to and from the hospital and hanging the clothes in the system.
We divide the circulation of an uniform into two parts: The using part (this includes handing out, wearing the uniform and handing it in) and the cleaning part (this includes travelling from and to the laundrette, the cleaning and the hanging in the carousel). Before determining the circulation speed of the system and the corresponding inventory some assumptions and simplifications are made:

- Employees hand in their uniforms right after their shift.
- When returning from the laundrette, uniforms are hung in the carousel instantly.
- We focus on the circulation of location Almelo only
- Clothes are taken from the hospital to the laundrette at 12:00 AM. Clean clothes are also returned at this time.
- The day is divided into two parts: before 12:00 AM and after 12:00 AM.
- 95% of the uniforms are returned to the hospital within 90 hours. Uniforms that are not returned within 90 hours are not taken into account in this model (those are outliers).

The data of the uniforms that are used at least 60 times a year, is used to determine the circulation speed. The data is collected and sorted on day of use and returning time (before or after 12:00 AM). Collecting all the data and sorting them gives us averages of each day and day part. These averages are plotted in a histogram with breaks every 12 hour.

Figure 2: Circulation Tuesday

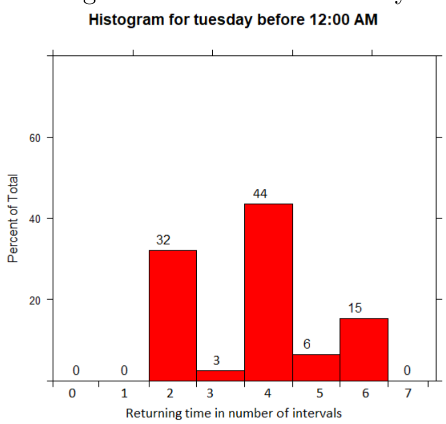
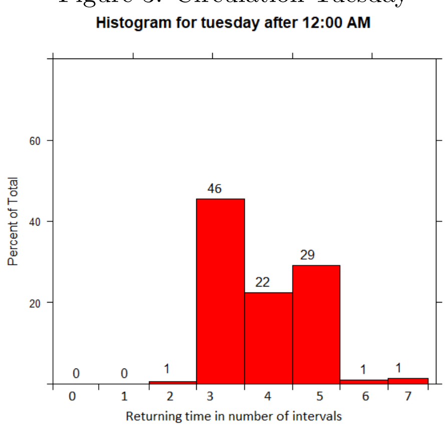


Figure 3: Circulation Tuesday



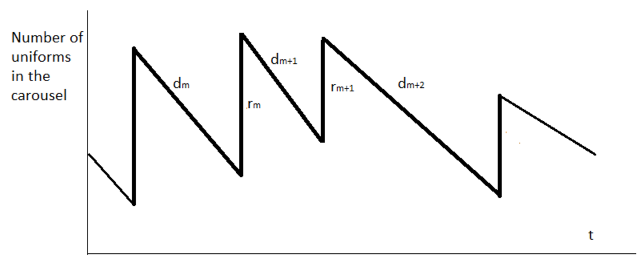
Handing in, in the morning or afternoon makes a big difference for the circulation speed and the corresponding chances per returning time, as can be seen in Figure 3 and Figure 4. 14 time intervals are defined where each time interval takes 12 hours ($t = 1, 2, ..., T = 14$). The returning times of all histograms are put in a matrix M. We call this matrix the circulation matrix. $M \in \mathbb{R}^{T \times T}$ includes all chances of returning in the hospital on a time interval depending on the time interval the uniform was handed out. Each column represents a time interval of handing out. Each row represents a time interval of returning in the system in the hospital.

We introduce the demand vector: $d \in \mathbb{R}^T$ which contains the demand on each time interval. Multiplying row t of M with d gives us the number of uniforms that will be returned on time interval t: r_t:

$$M_t d = r_t$$

This demand and returning vector represent the change in number of uniforms that are in the carousel as shown in Figure 4.

Figure 4: Demand and use



Recall that the inventory is the total number of uniforms that circulates in the whole cleaning system (so both in the carousel, laundrette and in use of employees). To determine the inventory, a critical moment must be found. The critical moment is when a lot of uniforms are in the cleaning part, and the demand is high. The inventory I can therefore be examined as the maximum number of uniforms in the cleaning part together with the demand:

$$I = \max_t (C_t + d_t)$$

C_t represents the total number of uniforms that is in the cleaning part at time interval t. A formula for C_t can be made by taking a look at matrix M.

The uniforms that are in the cleaning part, are the ones that are not yet in the carousel. At moment t, uniforms that are not yet in the carousel are uniforms that will be hung in the carousel within the next 90 hours. These uniforms will be returned today or in one of the next 7 intervals.In the circulation matrix we have to add up the next 7 rows, from the previous

7 time columns. This means that from the previous 7 time-intervals, everything that is not yet hung in, is summed up. We use
$q^{\pm} = (t \pm 7) mod(14)$ to define the C_t:

$$C_t = \sum_{j=t}^{q^-} \sum_{i=t}^{q^+} M_{ij} d_j$$

$t = 1, 2, ..., 14$
Filling this in in the expression for the inventory gives us a general expression for the inventory. This formula depends only on the demand vector d and the circulation matrix M. This formula is used to determine the inventory in the next section.

$$I = \max_t \left\{ \left(\sum_{j=t}^{q^-} \sum_{i=t}^{q^+} M_{ij} d_j \right) + d_t \right\}$$

Note that this inventory is determined by using an average circulation and an average demand/ Therefore, this inventory is an optimistic approach.

To get a better indication of the optimal inventory, other approaches are done. By modification of the circulation matrix a worst case circulation is obtained. The modified circulation matrix $\tilde{M}$ has te following entries:

$$\begin{aligned} M_{ij} &= 1 \quad j = i + 6 \\ M_{ij} &= 0 \quad j \neq i + 6 \end{aligned}$$

This means that after handing in on time interval t, it takes 72 to 84 hours before the uniform is hung in. This worst case circulation can be caused by delay in the laundrette but also by behaviour of the employees. Employees that leave their uniforms in their lockers will delay the circulation as well.

Just as the circulation matrix includes average values, the demand that was used so far is also an average. We assume that the demand per time interval follows a normal distribution. By constructing 95%-intervals of the demand per time interval, a maximum demand $\tilde{d}$ is obtained. This means that the demand is in 95% of the cases less than $\tilde{d}$. The inventory for this demand $\tilde{d}$ is obtained with both the circulation matrix M and can be found in the results.

Results inventory

The following results are inventories for the hospital in Almelo. Dameshes size M in Almelo is an uniform that is often used in the hospital. The current inventory in the hospital is 718 uniforms. For Dameshes M the demand vector is detemined (Table 1). The demand vector that is used, is obtained by setting the demand at time interval t as the number of uniforms that is handed in in time interval $t + 1$.

Table 1: Demand Dameshes M location Almelo

Day and time	Demand
Monday AM	79
Monday PM	18
Tuesday AM	80
Tuesday PM	18
Wednesday AM	81
Wednesday PM	19
Thursday AM	78
Thursday PM	19
Friday AM	76
Friday PM	17
Saturday AM	78
Saturday PM	18
Sunday AM	80
Sunday PM	19

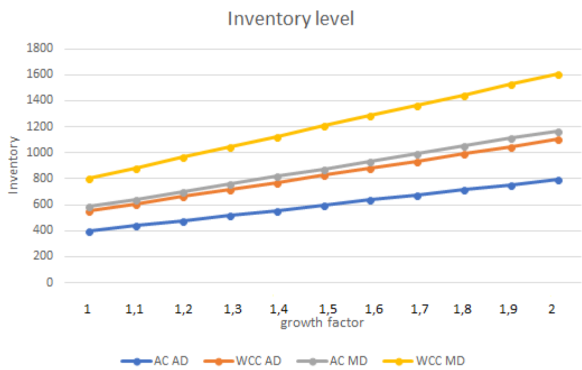
The inventories determined with the four approaches can be found in Table 2:

Table 2: Inventory Dameshes M

AC AD	398
WCC AD	551
AC MD	584
WCC MD	803

When the number of employees in Almelo increases, the demand will increase. We want to know the influence on the inventory if the number of employees grows. In figure 5 the results for dameshes M are shown (AC means average circulation, WCC means worst case circulation, AD means average demand and MD means maximun demand). It can be concluded that the inventory grows linearly depending on the demand.

Figure 5: Inventory increasing demand Dameshes M



For other uniforms, the inventory is determined with the same four approaches. Some uniforms with a high demand, and some with a lower demand are taken. The results can be found in Table 5 in the appendix.

Discussion inventory

The current inventory is in most cases at least as big as the first, second and third approach. As a standard inventory level, the AC MD seems to strike a good balance between reliability and overstocking.

In some cases, the current inventory is problematic (smaller than the first, second or third approach). Based on our results, we advice the hospital to increase the inventory.

Possibly, the circulation is dependent on uniform type. This could be due to different working times by different types of employees. Taking this dependence into account could be an improvement for our model.

Other improvements on this inventory model can be done by investigating the delay depending on the day of usage. Employees in the hospital say that they prefer to receive their uniform just before the weekend, to use it on monday. The delay in the weekend is possibly worse than during the week.

When determining the inventory, the capacity of the carousel (4200 uniforms) should be taken into account. When the number of employees in Almelo increases, the carousel is possibly to small. Installing a new carousel or KUA will have a big influence on the waiting times that are described in the next chapters.

Conclusion inventory

From the results of multiple approaches, we can conclude several things.The first approach can be seen as a minimal inventory and the fourth approach can be seen as a maximal inventory. If the fourth approach is not enough in practice, this is probably due to delay by employees and the hospital should commence action or change their inventory.

An constant increase in demand will lead in each approach to a constant increase of the needed inventory. We can conclude that the inventory grows linearly depending on the demand.

When we take a look at the results in the appendix and compare them with the current inventory, we find some remarkable conclusions:

For the Dameshes in both sizes, the results seem to fit the current inventory quite well: the first, second and third approach are always under the current inventory so we can assume that this uniform is not problematic in both sizes.

For the Damesbroek 3/4 M we see that every approach is much smaller than the current inventory. If the hospital still experiences problems with this uniform, some further research should be done in wich the behaviour of employees might play a role. If the hospital does not experience problems with this uniform, they should reduce the inventory such that enough space in the carousel is left for other uniforms.

For Damesbroek 3/4 L holds that all approaches are bigger or equal then the current inventory. We expect that the current inventory is too big.

Unisexbroek M gives us some very problematic results because only the first approach is smaller then the current inventory. This means that the hospital will probably experience problems if delay in cleaning or returning by employees happens. We would advise the hospital to increase the inventory for this uniform.

Uni. D. Jas KM S is a uniform that is currently in the hospital 489 times. Using our approaches we find that not any approach gave an inventory bigger than 143, which is a lot smaller than the current inventory. We can not say wherther the hospital has good reasons to keep this inventory as high as it is now. Still if delay happens, the hospital could reduce this inventory to make more space for other uniforms in the carousel.

The hospital currently has some high inventory levels, but could make them more dependent of uniform type and size such that enough space is left in the carousel when an increase of employees takes place. The hospital has some inventories that seem too low. Our advice to the hospital is to take a closer look at these uniforms because they might cause problems.

Literature review queueing

Queueing theory was introduced by A.K.Erlang when he created models to describe the Copenhagen telephone exchange [2]. Our model is based on the $M/M/n$ queues that have exponential service times, n servers and arrivals via a poisson proces. We use the $M/M/n$ model to determine waiting times in the case of different number of service, different arrival rates and several service rates. In this case the system must be stable. We doubt the fact that the system is always stable and therefore a simulation is made to determine the waiting times in a instable queueing system.

Waiting times probabilities for $M/M/n$ queues are described by Wayne L. Winston.[3]. Note that these probabilities can only be used when the system is stable.

A queueing problem

The KUA's in the hospital are located next to each other. On certain time intervals it can be very busy at the KUA. An approach of the behaviour of the queue is made by doing some important assumptions:

- The service times at the KUA differ per uniform depending on the place of the uniform in the carousel. (This place in the carousel is random).

  Because of this unpredictable service times, we assume that the service times are distributed exponential with a service rate μ per minute.

  (Note that the service time of handing out a uniform is independent of the service time of the previous uniform because of the random place in the carousel of each uniform)
- Employees arrive at the KUA according to a poisson process with a time-dependent arrival rate λ per minute. (Arrivals can often be modeled as poisson processes [3])
- The two KUA's share one queue
- We assume a service level such that 95% of the requests is served within t minutes.
- Employees get their uniform by a first in first out principle (FIFO)
- We focus on location Almelo only (because most problems are experienced here)

The queue of the KUA can be seen as a $M/M/n$ queueing system. We define n as the number of KUA's, λ as the arrival rate per minute and μ as the rate of the KUA per minute (the number of uniforms that can be handed out per minute per KUA on average). In a $M/M/n$ system: The service rate is as follows:

$\mu_{total} = n * \mu_{1KUA}$

μ_{1KUA} is not known and λ is dependent of the time. We now use the following $\rho = \frac{\lambda}{\mu_{total}} = \frac{\lambda}{n*\mu_{1KUA}}$ such that ρ depends only on μ_{1KUA} and λ. Only if $\rho = \frac{\lambda}{n*\mu_{!KUA}} < 1$, the system is stable.

in $M/M/1$ the probability of waiting time in queue less then t is: $p(w \leq T) = 1 - e^{-\mu(1-\rho)T}$ [3]

The hospital wants to give good service. We say that 95% employees should not have to wait too long (so only 5% will have to wait too long). Waiting too long means waiting for more than t minutes, where t has to be set by the hospital. The waiting time in queue, with a probility less then 5% that can be reached is:

$$\begin{aligned} p(w > T) &< 0.05 \\ e^{-\mu(1-\rho)T} &< 0.05 \\ -\mu(1-\rho)T &< \ln(0.05) \\ T &< \frac{\ln(0.05)}{-\mu(1-\rho)} \end{aligned}$$

An upperbound for the waiting time in queue with a probability of 95% is the following:

$$T(n, \lambda) = \frac{\ln(0.05)}{-n\mu_{1KUA}(1 - \frac{\lambda}{n*\mu_{1KUA}}))}$$

Results of this upperbound are plotted in the next section.

The number of KUA's has to be enough on the busiest moments. The hospital points to two busy moments: From 6.45 AM till 8:00 Am and from 2:30 PM till 4:00 PM. The arrival rate on these moments are determined by analyzing data. The rate is detemined for location Almelo only, because most problems take place at this location. Determining the arrival rate for location Hengelo could be done in exact the same way. A few assumptions were needed to determine the arrival rate:

- The rate is based on current arrival behaviour of the employees.
- The arrival moments are based on the hand out moments and waiting time in queue is not taken into account here

The arrival rate is determined by counting the number of arrivals in time intervals of 15 minutes. We find the following results of average number of arrivals and the corresponding arrival rate per minute:

Table 3: Arrival rate KUA Almelo

Time	Average per interval	λ per minute
6:45 AM untill 7:00 AM	85	5.7
7:00 AM untill 7:15 AM	53	3.5
7:15 AM untill 7:30 AM	20	1.3
7:30 AM untill 7:45 AM	32	2.1
7:45 AM untill 8:00 AM	34	2.3
2:30 PM untill 2:45 PM	28	1.9
2:45 PM untill 3:00 PM	61	4.1
3:00 PM untill 3:15 PM	37	2.5
3:15 PM untill 3:30 PM	18	1.2
3:30 PM untill 3:45 PM	26	1.7
3:45 PM untill 4:00 PM	29	1.9

As can be seen in the table above, the arrival rate differs broadly per time interval. We can use these results in combination with the M/M/n model to determine the number of KUA's that is needed at each time. In that case we have to assume that the system is stable on every moment. But if the arrival rate is high and the number of KUA's is low the system might nog be stable for at least a couple of minutes.

It is very likely that the system is not stable when the arrival rate is high early in the morning, but that the system is stable after 15 minutes. In this case we can not use the M/M/n model. To analyse this problem a simulation is needed.

The simulation that is made, focusses on the busy moment in the morning. Because of the high arrival rate in the morning we expect more problems here than in the afternoon. Some important assumptions are made:

- The queue at the KUA is empty at 6:45 AM
- Employees do not leave the queue

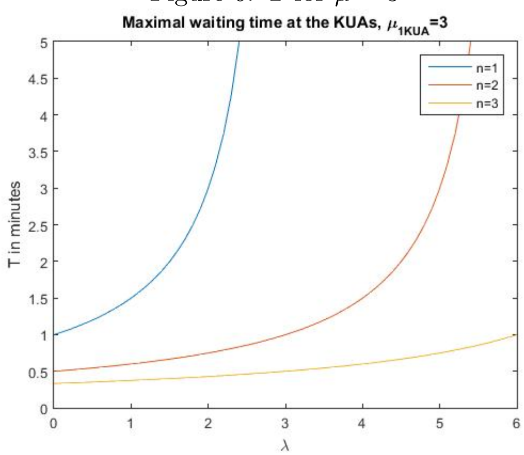
- The arrival rate increases with the same factor as the number of employees.
- Average waiting times per morning are distributed normally

The simulation runs 100 independent mornings. At the beginning of each run, the arrivals of that morning are generated with the arrival rates per 15 minutes. Waiting times of each employee are determined by substracting the arrival time from the leaving time. An average waiting time per morning is stored at a list. In the end, the computer builts 95% confidence interval such that the maximum waiting time in the morning is obtained. Results of this simulation can be found in the next section.

Results queueing

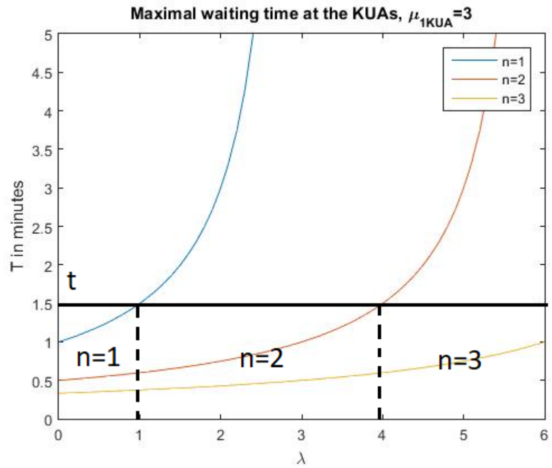
For $n = 1, 2, 3$ and $\mu_{1KUA} = 3$ the maximal waiting times determined with the $M/M/n$ model (with a certainty of 95%) are plotted:

Figure 6: T for $\mu = 3$



By setting a maximal accepted waiting time and making a horizontal line at height t the number of KUA's needed (depending on λ) is detemined. For $\mu = 3$ and maximum accepted waiting time $t = 1,5$ minute the number of KUA's is determined. The results are shown in Figure 7.

Figure 7: Example for $\mu = 3$ and $t = 1,5$



Comparing the results of Figure 7 with the average arrival rates on busy moments, gives us that 3 KUAs will be needed if the hospital sets the maximum waiting time on 1.5 minutes. For other values of t the waiting times can be determined with the same method.

We take a look at the number of KUAs that is required based on our simulation model. For the current arrival rate, and service rate $\mu = 3$ we find the maximum waiting times with 95% certainty in the morning:

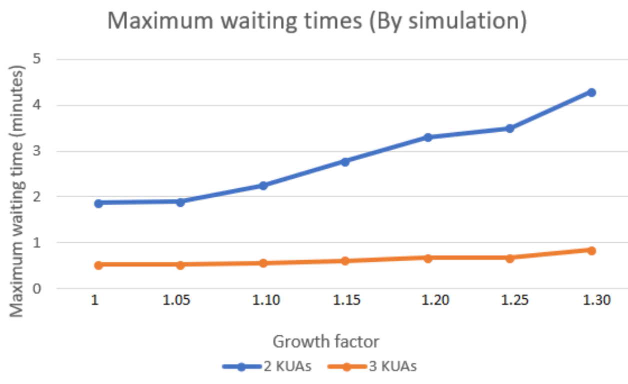
Table 4: Maximum waiting times, $\mu = 3$

Number of KUAs	Maximum waiting time
n=1	16.28 minutes
n=2	1.68 minutes
n=3	0.50 minutes

The waiting times for only one KUA are very high. We can assume that minimal 2 KUAs are needed. We therefore determine the waiting times for 2 and 3 KUAs when the number of employees increases only.

As can be seen in Figure 8, the maximum waiting times for 2 KUAs increase rapidly when the arrival rate increases. The waiting times for 3 KUAs increase as well but not so fast.

Figure 8: Maximum waiting times in Almelo, increasing arrival rate, $\mu = 3$



Discussion queueing

We have determined the maximum waiting times in two ways. Both ways are based on average arrival rates. Firstly, further research could be done using a worst case arrival rate. Second, we assumed the service rate of one KUA to be equal to 3. This service rate could be determined more precisely. When both service and arrival rate are determined, it could be determined whether the system is stable or not. Although the rates may, the method stays the same.

The distribution of the waiting times per morning could be investigated more precisely. More research could be done on this distribution to provide better 95% confidence intervals.

We expect that there is an upperbound for the service times. This upperbound could be used as a maximum deterministic service time. Research on $M/D/c$ queues could be done to model the queue. Note that this model can only be used when the system is stable. Maximum waiting times and steady state distributions for the $M/D/c$ queue are described in[4].

Conclusion queueing

We can conclude that waiting times increase when the arrival rate increases.

Both models show that a maximum waiting time of less then 1.5 minutes in the morning can not be reached with 2 KUAs and a service rate of 3 services per minute.

For other values of t, the number of KUAs can be determined in the same way.

Overall conclusion

A lower bound and an upperbound for the inventory are determined. A programm is available to determine this inventory for each uniform type and size.

Two models for the queue at the KUA are built and give the required number of KUAs that is needed to serve on a certain level.

Appendix

Table 5: Inventory for different uniform types and sizes (location Almelo)

Uniform	Current inventory	1. Inv. AC AD	2. Inv. WCC AD	3. Inv. AC MD	4. Inv. WCC MD
Dameshes M	718	398	551	584	803
Dameshes XS	52	30	41	51	72
Damesbroek 3/4 M	416	101	137	168	224
Damesbroek 3/4 L	204	92	129	148	204
Unisexbroek M	433	388	543	564	798
Uni. D. Jas KM S	489	59	77	109	143

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