Temperature model of heat dissipation in the ground

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Abstract

Heat dissipation into the ground is modelled as part of a model that has to help decrease production time and carbon dioxide emission at Tata-Steel. Finite difference methods are used to create a one dimensional model that is used to approximate surface temperatures. The output of the model is compared with available data from Tata-Steel. A slightly deviation is observed, due to comparison of two different models. However, the average error is small and therefore we conclude that a one dimensional model is accurate enough for the purpose Tata-Steel has with it.

Keywords: Ground temperature model, heat dissipation, finite difference method

1 Introduction

At Tata-Steel, a Dutch firm that produces steel, the question exists if it is possible to approximate the temperature of all the steel slabs that they make, with the aid of a model, in order to know slab temperatures before putting the slabs in the oven. Then, it will be possible to sort slabs by temperature - instead of by age - in order to reduce production time and carbon dioxide emission. This reduction is caused by the fact that the coldest slab determines the tempo of the oven, in which slabs are reheated before being rolled. Grouping slabs at their temperature will make sure the oven can work as fast as possible.

In order to construct a correct model, it is needed to take into account all the different stacks of slabs that are on two slab yards. A schematic representation of such a slab yard can be found in figure 1. From a previous study, in which a start is made in modelling the slab yards, is known that neglecting the ground surface temperature causes the model to be less accurate (De Laat, 2017). The part of the problem this paper addresses, is therefore building a model which predicts heat dissipation into the ground, in order to calculate the needed surface temperatures.

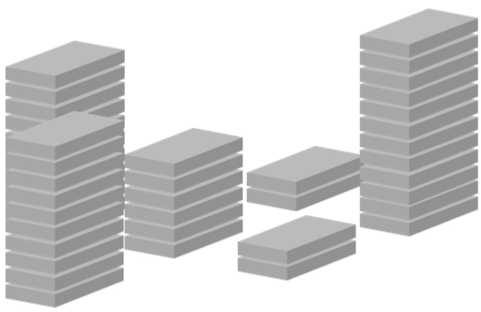


Figure 1: Simple schematic representation of the slab yard

To make sure the model can run faster than real time, the model will be one-dimensional, instead of the more accurate three-dimensional. Validation has to show whether this choice is sufficient for the required accuracy of the model.

In this paper, there will first be proposed some equations that are used to model the problem. Those equations will be made using finite difference methods and equations that describe conduction, convection and radiation. Secondly, some stability requirements are discussed. Subsequently, the model is verified and validated by use of at Tata-Steel known data. At last, the results are discussed and several recommendations are made to improve the model in future.

2 The heat equation and its boundary condition

2.1 General heat equation

To model the ground, we can use the partial differential equations that are used to model heat flow inside objects. The equation looks as follows:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial z}\right)=\rho C_p\frac{\partial T}{\partial t}. \tag{1}$$

The lefthandside represents the heat transfer within a body through conduction in three directions, with thermal conductivity k, which may depend on the coordinates x, y, z and time t. The temperature T also depends on x, y, z and t. The righthandside represents the rate of increase of internal energy of a solid, with the density of steal ρ and the specific heat capacity C_p.

2.2 Boundary condition at the surface

For the problem we are dealing with, there exist two boundary conditions, one at the surface level and one at the bottom of the ground. We assume that the ground has a flat surface which therefore has its normal vector in the vertical direction. The boundary condition at the surface level then looks as follows:

$$k\frac{\partial T(0,t)}{\partial z}=\sigma\mathcal{F}\left[(T_0)^4-T_R^4\right]+h\left[T_0-T_\infty\right]. \tag{2}$$

In this equation, the lefthandside represents the heat flux through the surface. The righthandside can be separated into two parts.
The first part is $\sigma\mathcal{F}\left[(T_0)^4-T_R^4\right]$, which represents the radiation from the surface to the roof. The roof represents either a slab that lays on top of the ground, or the atmosphere in case there is no slab on the ground. The parameter σ represents the Stefan-Bolzmann constant, whereas F is a combined factor which contains the emissivity (ϵ) and a number of view-factors, thus $F=\Sigma_{j=1}^{N}\epsilon_j\cdot\text{view-factor}[j]$. In section 3.3.1, we will say more about the case that there is a slab on top. In case there is no slab on top, the parameter becomes a combination of emissivities times view-factors.
The second part, $h\left[T_0-T_\infty\right]$, represents the convection, in which h is the heat transfer coefficient through convection and T_∞ is the ambient temperature.

2.3 Boundary condition at the bottom of the ground

At the bottom of the ground, the temperature always stays the same, as long as we choose the bottom deep enough. There are now two possible options to choose as a boundary condition.

2.3.1 Constant temperature

In case we say that the temperature at the bottom of the ground always stays the same, the boundary condition will look as follows:

$$T(x, y, L, t) = T_{\text{bottom}}. \tag{3}$$

The variable L represents the depth in the ground in meters and T_{bottom} the bottom temperature in Kelvin. This boundary condition only 'works' if the chosen depth is big enough.

2.3.2 No heat flux

It is, however, also sufficient to state that there will be no heat flux through the bottom of the ground. In that case, the boundary condition will look as follows:

$$\frac{\partial T(x, y, L, t)}{\partial z} = 0. \tag{4}$$

In this case, it is also necessary for a good result to choose the bottom of the ground at a sufficient depth. To verify which of these boundary conditions is best to choose, we will compare them later on in section 5.2.

2.4 One-dimensional version of the problem

The above describes a third-dimensional case. However, the model that is made for TATA-Steel is a one-dimensional model. In that case, one assumes the slab on top of the surface is infinite in both horizontal directions. This is illustrated in figure 2. A one-dimensional model takes only one direction into account. As direction for the one-dimensional model for the ground, it does not make sense to choose a horizontal direction, that is not what we are interested in. Therefore, the direction of the one-dimensional model will be the vertical one, as pictured in figure 2.

2.4.1 Equations

The equations stated above are valid for a three-dimensional case. In order to make them valid for a one-dimensional case, we neglect the y and z direction and take x as the vertical direction. Hereby, we obtain the following equation:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) = \rho C_p \frac{\partial T}{\partial t}, \tag{5}$$

with boundary conditions

$$k\frac{\partial T(0, t)}{\partial x} = \sigma \mathcal{F}\left[(T_0)^4 - T_R^4\right] + h\left[T_0 - T_\infty\right], \tag{6a}$$

$$T(L, t) = T_{\text{bottom}} \tag{6b}$$

or

$$k\frac{\partial T(0, t)}{\partial x} = \sigma \mathcal{F}\left[(T_0)^4 - T_R^4\right] + h\left[T_0 - T_\infty\right], \tag{7a}$$

$$\frac{\partial T(L, t)}{\partial x} = 0. \tag{7b}$$

Of course, this model outputs an approximation of reality. However, it is useful to determine ground temperatures below the middle of a slab. In that case, the one-dimensional heat spread is similar to the three-dimensional case, as can be observed in figure 3, in which an artistic representation of a cross-section of the ground below a hot slab is shown, in which is represented how the warm slab influences the temperature of the ground.

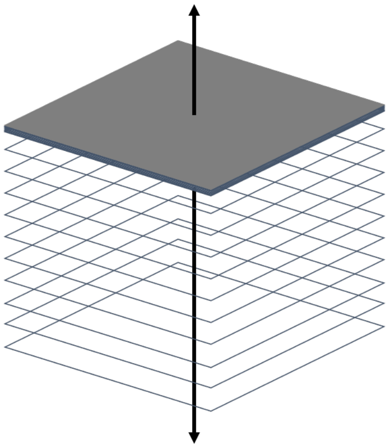


Figure 2: Visualisation of the one-dimensional case, in which both horizontal directions are neglected.

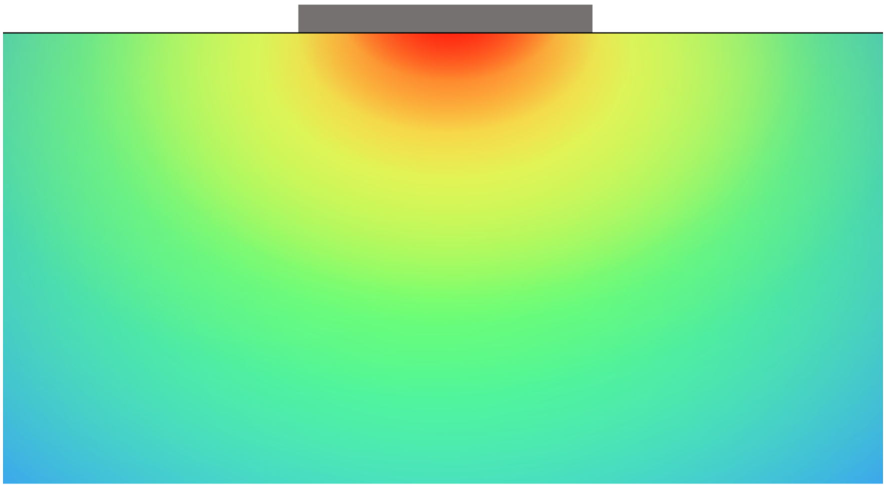


Figure 3: Artistic representation of heat distribution in the ground caused by a hot slab on on top of the ground.

3 Discretization of the heat equation and the boundary condition

In order to model the given partial differential equations, we used finite difference methods to discretize the equation and the boundary conditions on a uniform grid.

3.1 The heat equation

We started with equation (5). Approximating the derivative to time using the forward finite difference method, gives the following result:

$$\frac{\partial T_i^n}{\partial t} = \frac{T_i^{n+1} - T_i^n}{\Delta t}. \tag{8}$$

Due to the temperature dependence of k, using the central difference method to discretize the derivative with respect to x, gives the following:

$$\frac{\partial}{\partial x}\left(k(T(x,t))\frac{\partial T}{\partial x}\right) = \frac{k\left(T_{i+\frac{1}{2}}^n\right)\left(T_{i+1}^n - T_i^n\right) - k\left(T_{i-\frac{1}{2}}^n\right)\left(T_i^n - T_{i-1}^n\right)}{\left(\Delta x\right)^2}. \tag{9}$$

Substituting these equations into the heat equation gives

$$\rho \cdot C_p \cdot \frac{T_i^{n+1} - T_i^n}{\Delta t} = \frac{k\left(T_{i+\frac{1}{2}}^n\right)\left(T_{i+1}^n - T_i^n\right) - k\left(T_{i-\frac{1}{2}}^n\right)\left(T_i^n - T_{i-1}^n\right)}{\left(\Delta x\right)^2}, \tag{10}$$

$$T_i^{n+1} = \frac{\Delta t}{\rho \cdot C_p} \cdot \frac{k\left(T_{i+\frac{1}{2}}^n\right)\left(T_{i+1}^n - T_i^n\right) - k\left(T_{i-\frac{1}{2}}^n\right)\left(T_i^n - T_{i-1}^n\right)}{\left(\Delta x\right)^2} + T_i^n \tag{11}$$

as a discritization of the heat equation, with k depending on coordinate x and time t.

Because of the fact that we do not exactly know $T_{i+\frac{1}{2}}^n$ and $T_{i-\frac{1}{2}}^n$, we have to find an approximation for both $k\left(T_{i+\frac{1}{2}}^n\right)$ and $k\left(T_{i-\frac{1}{2}}^n\right)$ in terms of T_{i+1}^n, T_i^n and T_{i-1}^n, which are values that we do know. It is possible to approximate the values with

$$k\left(T_{i+\frac{1}{2}}^n\right) = k\left(T_i^n\right) \tag{12}$$

or

$$k\left(T_{i+\frac{1}{2}}^n\right) = k\left(T_{i+1}^n\right), \tag{13}$$

but a better way to approximate $k\left(T_{i+\frac{1}{2}}^n\right)$ is

$$k\left(T_{i+\frac{1}{2}}^n\right) = \frac{1}{2}\left(k\left(T_{i+1}^n\right) - k\left(T_i^n\right)\right), \tag{14}$$

because of the fact that this equation is, as also is equation (11), a second order accurate approach. Similarily, we can approach $k\left(T_{i-\frac{1}{2}}^n\right)$ with

$$k\left(T_{i-\frac{1}{2}}^n\right) = \frac{1}{2}\left(k\left(T_i^n\right) - k\left(T_{i-1}^n\right)\right). \tag{15}$$

3.1.1 Constant thermal conductivity

In order to simplify the problem, we shall first model the case in which the thermal conductivity k does not depend on x and t. We observe that if we take k as a constant, it holds that $k\left(T_{i+\frac{1}{2}}^n\right) = k\left(T_{i-\frac{1}{2}}^n\right)$. Therefore, in case of a constant thermal conductivity, the discretized heat equation becomes

$$T_i^{n+1} = \frac{k \cdot \Delta t}{\rho \cdot C_p} \cdot \frac{T_{i+1}^n - 2T_i^n + T_{i-1}^n}{\left(\Delta x\right)^2} + T_i^n. \tag{16}$$

We took $\frac{k\Delta t}{\rho \cdot C_p \cdot (\Delta x)^2}$ as R, and thereby gained

$$T_i^{n+1} = R\left(T_{i+1}^n - 2T_i^n + T_{i-1}^n\right) + T_i^n \tag{17}$$

as a discetization of the heat equation.

As one can see, R must be dimensionless, to make sure the dimensions do not get messed up. We observed that R is indeed dimensionless by taking a closer look to the dimensions of $\frac{k}{\rho C_p}$. Thermal conductivity has dimensions J$\cdot$s$^{-1}\cdot$m$^{-1}\cdot$K^{-1}. The dimensions of density are kg$\cdot$m^{-3} and the dimensions of specific heat are J$\cdot$kg^{-1}. Therefore, the dimensions of $\frac{k}{\rho \cdot C_p}$ are m$^2\cdot$s^{-1}, while the dimensions of $\frac{\Delta t}{(\Delta x)^2}$ are s$\cdot$m^{-2}. As we observe, the dimension of R is 1, which means R is dimensionless, as necessary.

3.2 The boundary conditions

3.2.1 The surface

The boundary condition that applies to the surface of the ground equals

$$k\frac{\partial T(0,t)}{\partial x} = \sigma \mathcal{F}\left[(T_0)^4 - T_R^4\right] + h\left[T_0 - T_\infty\right]. \tag{18}$$

In order to discretize the surface boundary condition, we used the central finite difference method,

$$\frac{\partial T_i^n}{\partial x} = \frac{T_{i+1}^n - T_{i-1}^n}{2\Delta x}, \tag{19}$$

and substituted this into the boundary condition, obtaining

$$k \cdot \frac{T_1^n - T_{-1}^n}{2\Delta x} = \sigma \mathcal{F}\left[(T_0^n)^4 - T_R^4\right] + h\left[T_0^n - T_\infty\right], \tag{20}$$

$$T_{-1}^n = -\frac{\sigma \mathcal{F}\left[(T_0^n)^4 - T_R^4\right] + h\left[T_0^n - T_\infty\right]}{k} \cdot 2\Delta x + T_1^n. \tag{21}$$

Substituting this into

$$T_0^{n+1} = T_0^n + R\left(T_1^n - 2T_0^n + T_{-1}^n\right), \tag{22}$$

gives

$$\begin{aligned}
T_0^{n+1} &= T_0^n + R\left(T_1^n - 2T_0^n - \frac{\sigma \mathcal{F}\left[(T_0^n)^4 - T_R^4\right] + h\left[T_0^n - T_\infty\right]}{k} \cdot 2\Delta x + T_1^n\right), \\
&= T_0^n - \frac{2R\Delta x \sigma \mathcal{F}[(T_0^n)^4 - T_R^4]}{k} - \frac{2R\Delta x h\left[T_0^n - T_\infty\right]}{k} + 2RT_1^n - 2RT_0^n, \\
&= T_0^n - \frac{2R\Delta x \sigma \mathcal{F}\,(T_0^n)^4}{k} + \frac{2R\Delta x \sigma \mathcal{F} T_R^4}{k} - \frac{2R\Delta x h T_0^n}{k} + \frac{2R\Delta x h T_\infty}{k} + 2RT_1^n - 2RT_0^n, \\
&= T_0^n\left(1 - \frac{2R\Delta x \sigma \mathcal{F}}{k}\,(T_0^n)^3 - \frac{2R\Delta x h}{k} - 2R\right) + \frac{2R\Delta x h T_\infty}{k} + \frac{2R\Delta x \sigma \mathcal{F} T_R^4}{k} + 2RT_1^n,
\end{aligned}$$

$$= T_0^n\left(1 - 2R\left[\frac{\Delta x \sigma \mathcal{F}}{k}\,(T_0^n)^3 + \frac{\Delta x h}{k} + 1\right]\right) + \frac{2R\Delta x h T_\infty}{k} + \frac{2R\Delta x \sigma \mathcal{F} T_R^4}{k} + 2RT_1^n. \tag{23}$$

3.2.2 The bottom

As stated above, there are two possible boundary conditions that can be used to describe the behaviour of the bottom of the ground.

The first one,

$$T(L,t) = T_{\text{bottom}}, \tag{24}$$

can be implemented in the model directly. In order to get a good result, one has to make sure that the bottom node is chosen deep enough.

The second one,

$$\frac{\partial T(L,t)}{\partial x} = 0, \tag{25}$$

can be discretized using the forward difference method

$$\frac{\partial T(L,t)}{\partial x} = \frac{T_i^n - T_{i-1}^n}{\Delta x}, \tag{26}$$

which gives

$$T_{i+1}^n = T_{i-1}^n \tag{27}$$

as a boundary condition. For this boundary condition it also holds that the bottom node has to be chosen deep enough.

3.3 Emissivity and the heat transfer coefficient

3.3.1 Emissivity

To finish the equations, we now have to define F and h from equation (23). Because of the fact that there will be some reflection between the ground and the slab, we need to use an expression for F that takes this into account. According to Anthony Mills (1999), we should use

$$F = \frac{1}{\frac{1}{\epsilon_{\text{ground}}} + \frac{1}{\epsilon_{\text{slab}}} - 1}. \tag{28}$$

The parameter ϵ_{ground} represents the emissivity of the ground, the parameter ϵ_{slab} the emissivity of the slab. However, in case there is a slab on the ground, according to an earlier studies done at Tata-Steel, only 95% of the heat will travel by radiation (De Laat, 2017). Therefore, the final equation that we use for radiation, in case there is a slab on top of the ground, will be

$$F = \frac{0.95}{\frac{1}{\epsilon_{\text{ground}}} + \frac{1}{\epsilon_{\text{slab}}} - 1}. \tag{29}$$

In case there is no slab, we assume that the ground does not radiate towards the atmosphere, and therefore we will use $F = \epsilon_{\text{ground}}$ in that case.

3.3.2 Heat transfer coefficient

According to the same study, 5% of the total amount of heat is transferred by conduction, in case there is a slab on the ground (De Laat, 2017). Hence we will use $0.05 \cdot h$ instead of just the heat transfer coefficient in case there is a slab on the ground.

4 Stability

To ensure that the output of the model is stable, one can not choose to big time steps. Because of the linearity of the equation that is used to update the interior points, one gains stability by making sure that the coefficient of T_i^n is greater or equal to zero. The equation that is used to update the interior points is the following:

$$\begin{aligned} T_i^{n+1} &= R\left(T_{i+1}^n - 2T_i^n + T_{i-1}^n\right) + T_i^n, &(30)\\ &= T_i^n\left(1-2R\right) + R\left(T_{i+1}^n + T_{i-1}\right). &(31)\end{aligned}$$

Thus the coefficient of T_i^n equals $1-2R$. As a result we gain that R should be less or equal to $\frac{1}{2}$ to make sure that $1-2R \geq 0$. In order to find which time step is allowed, we look at

$$\begin{aligned} R &= \frac{k\Delta t}{\rho \cdot C_p \cdot (\Delta x)^2} \leq \frac{1}{2}, &(32)\\ \Delta t &\leq \frac{1}{2}\frac{\rho \cdot C_p \cdot (\Delta x)^2}{k}, &(33)\\ \Delta t &\leq \frac{(\Delta x)^2}{2\alpha}, &(34)\end{aligned}$$

in which the parameter α equals $\frac{k}{\rho \cdot C_p}$, which is the thermal diffusivity of sand.

We now have to investigate whether this restriction also applies to the boundary condition. However, due to the non-linearity of the boundary equation, it is not enough to state that the coefficient of T_0^n in the boundary condition should be equal to zero. In order to figure out what the maximum time step is to have stability at the boundary, one has to look to the derivative of T_i^{n+1} to T_i^n . This derivative must be greater or equal to zero in order to obtain stability (Milton and Goss, 1973).

Setting the derivative greater or equal to zero gives

$$\begin{aligned} \frac{\partial T_i^{n+1}}{\partial T_i^n} &= 1 - 2R\left(1 + \frac{\Delta x \sigma \mathcal{F}}{k} \cdot 4\left(T_0^n\right)^3 + \frac{\Delta x h}{k}\right) \geq 0, &(35)\\ R &\leq \frac{1}{2\left(1 + \Delta x\left[\frac{\sigma \mathcal{F}}{k} \cdot 4\left(T_0^n\right)^3 + \frac{h}{k}\right]\right)}, &(36)\\ \Delta t &\leq \frac{(\Delta x)^2}{\alpha} \cdot \frac{1}{2\left(1 + \Delta x\left[\frac{\sigma \mathcal{F}}{k} \cdot 4\left(T_0^n\right)^3 + \frac{h}{k}\right]\right)}. &(37)\end{aligned}$$

Comparing the two conditions for Δt shows that the condition for the boundary is the one that restricts Δt the most. This means that the maximum allowed time step is determined by the surface temperature T_0^n.

There are now two possible choices. The first one is to set a maximum temperature that will never be reached in the model, to determine a maximum time step that will always guarantee stability. The other possibility, perhaps a more efficient one, is to determine the maximum time step again before each update of temperatures. In this way, the model stays always stable, and runs as quickly as possible.

5 Analysis of the model

5.1 Data used to verify and validate

In order to be able to validate the model, we use data that is generated at Tata-Steel. The used model for this is normally used to model hotboxes, certain buildings at the terrain of Tata-Steel in which fragile slabs are stored that are not allowed to cool down too quickly. The reason that this model is not suitable for

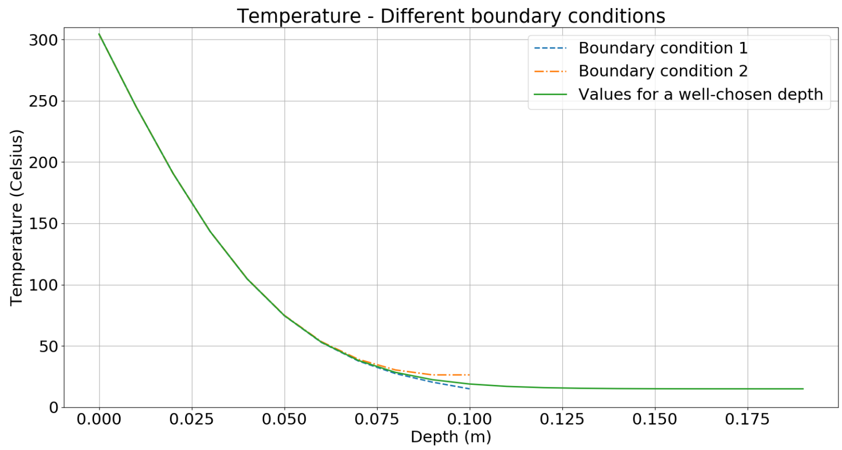


Figure 4: Surface temperature two hours after a slab with a constant temperature of 600 degrees Celsius arrives. Two different boundary conditions are used to simulate. In two cases, the bottom node is not chosen deep enough, in the third case it is.

the whole slab yard is that it runs way to slow for such a huge job. It is however suitable for calculating temperatures of up to 144 slabs. Also the ground temperature is calculated by this model, up to a depth of one meter.

The used model, which we shall call the Tata-Steel model, is a Finite Element Model. It uses the same method that is used to build the one-dimensional model this paper addresses, however, the already existing Finite Element Model is a three-dimensional one. The model adjusts time steps to the smallest possible. In other words, if temperatures increase or decrease fast, the model takes small time steps. If, however, temperatures changes slowly, the model takes large time steps.

In order to make sure that deviation in the outputs of our model and the Tata-Steel model does not occur by the fact that different parameters are used to simulate, we will use the same values that are used in the Tata-Steel model, unless indicated otherwise. The values are however confidential and therefore not displayed in this paper.

5.2 Bottom boundary condition

In section 3.2.2, we found two possible ways to model the boundary condition that applies to the bottom of the ground. In case we choose the bottom of the ground deep enough, using a different boundary condition will not result in another output. If, however, we pick the bottom of the ground at a point where it will warm up due to a heat source on top of the ground, we see a difference. In figure 4, the output is shown for a simulation in which we simulated two times the situation that a slab with a constant temperature of 600 degrees Celsius lays on top of the ground. The first run uses the first boundary condition (equation (25)), the second run the second possible boundary condition (equation (27)). We picked the bottom of the ground at 10 centimetres depth, which is not deep enough. In the same figure, we also displayed the result of a run where we choose the bottom deep enough.

We observe that boundary condition one estimates the ground too cold at the bottom of the ground, which will cause the overall output to be too low. Boundary condition two however, predicts a temperature that is too high, resulting in too high surface temperatures. Also, we see that the simulation with boundary condition two starts to deviate from the green line as first. The purpose of the model is to predict surface temperatures of the ground such that slab temperatures can be predicted in order to group slabs at their temperature. A cold slab in a group of hot slabs will cause that the oven can heat up slower, though a hot slab in a group of cold slab does not influence the tempo of the oven. Therefore, it is better to predict some slabs too low, and hence we choose to use boundary condition one (equation (25)) as a boundary condition for the bottom. In other words, we choose a constant temperature for the bottom node.

The best output, however, is achieved by choosing a sufficient depth for the ground, in which case it does not matter which of the two boundary conditions is used.

5.3 Verification

5.3.1 Extreme values

To see if the ground model behaves logical, we put an slab on top of the ground that has a constant temperature of 600 degrees Celsius. If we differ some parameters, the model should give an output that corresponds with our intuition. We will discuss the behaviour of the model in the next section.

Thermal conductivity
Choosing different values for the thermal conductivity should influence the speed at which heat spreads through the ground. In figures 5 and 6 is shown how the model reacts to a thermal conductivity of 0.0001 $W \cdot m^{-1} \cdot K^{-1}$ and 100 $W \cdot m^{-1} \cdot K^{-1}$ respectively.

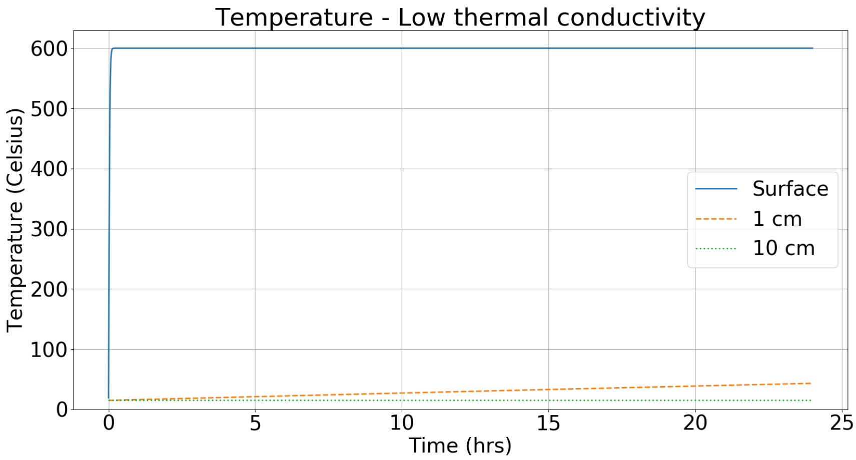


Figure 5: Temperature distribution with a thermal conductivity of 0.0001 $W \cdot m^{-1} \cdot K^{-1}$ and a slab with a constant temperature of 600 degrees Celsius on top.

In figure 5, we observe that the surface heats up due to radiation, in contrast to the rest of the ground, which hardly heats up. In one day, only the first few centimetres of the ground heat up like 40 degrees Celsius, though there lays a slab of 600 degrees Celsius on top.

Choosing a high thermal conductivity will, however, cause the ground to heat up more equal, almost linear at the top.

Emissivity
Choosing different values for the emissivity of the ground should influence the amount of radiation that is absorbed by the ground. We let the model run with an emissivity of 0.0001 and 1, which is shown in figures 7 and 8 respectively.

As we observe, the difference in emissivity does not make a big difference in the output of the model, however it does effect the speed at which the ground heats up.

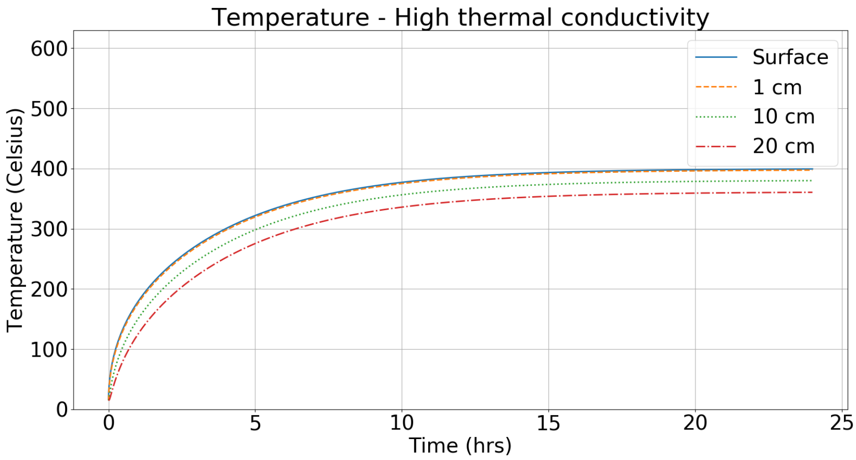


Figure 6: Temperature distribution with a thermal conductivity of 100 $W{\cdot}m^{-1}{\cdot}K^{-1}$ and a slab with a constant temperature of 600 degrees Celsius on top.

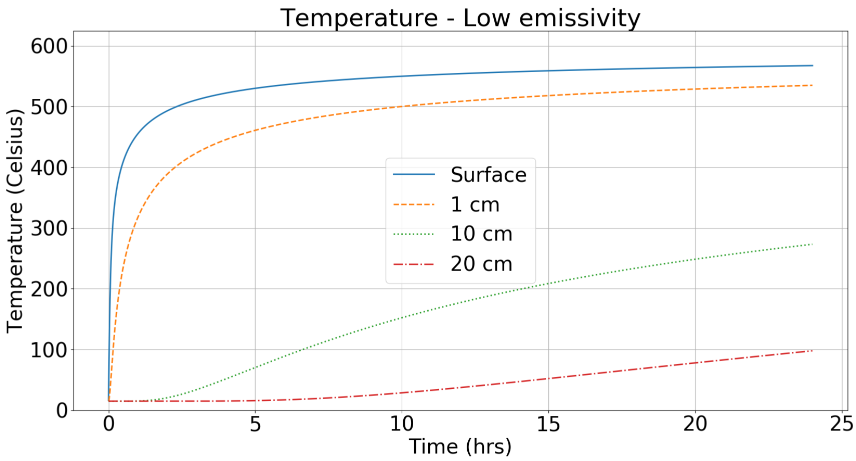


Figure 7: Temperature distribution with an emissivity factor of 1 and a slab with a constant temperature of 600 degrees Celsius on top.

5.3.2 No slab on the ground

At the slab yard, there are also pieces of ground that have no slab on top and sometimes a slab is taken way, creating an empty piece of ground. In cases like this, the model should behave correctly, to ensure the model outcomes stay trustworthy. In figures 9 and 10, the output of the model is shown in case there is no slab on top and the temperature of the atmosphere is minus 50 or plus 50 degrees Celsius respectively. We observe that the temperature of the atmosphere influences the temperature of the ground. However, also the ambient temperature plays a role in the final surface temperature.

5.3.3 Alternating slab and no slab

There is another thing that has to be modelled correctly, namely the reaction of the ground to the whole process of slabs that are coming, cooling down and leaving, after which there will come a new slab and the process starts all over again. We will model the following process, using data simulated by the model

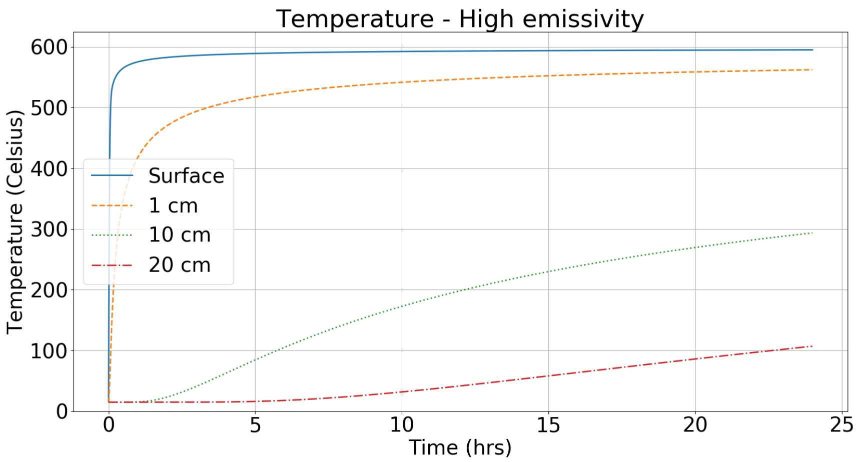


Figure 8: Temperature distribution with an emissivity factor of 0.0001 and a slab with a constant temperature of 600 degrees Celsius on top.

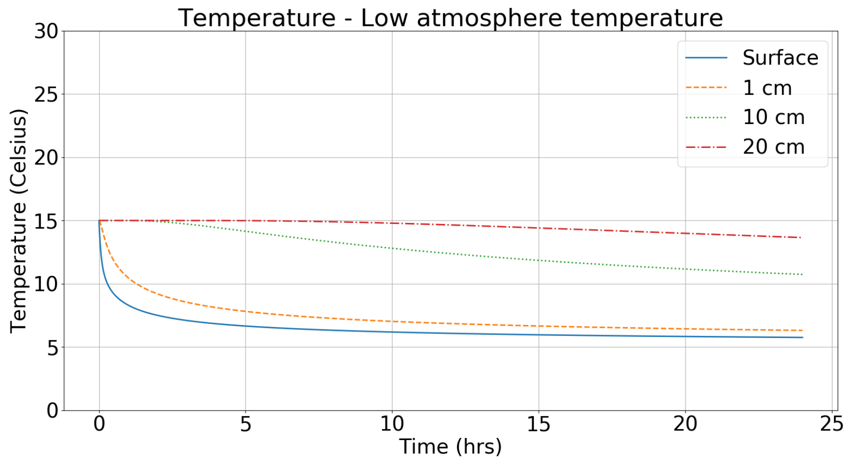


Figure 9: Temperature distribution in case there is no slab on top with an atmosphere temperature of -50 degrees Celsius.

that already exists at Tata-Steel on how a single slab cools down if it lays on top of the ground. We used the following time scheme, and took as an initial temperature for new slabs 600 degrees Celsius and as atmosphere temperature 10 degrees Celsius. The ambient temperature is set at 20 degrees and the initial ground temperature at 15 degrees Celsius.

00:00 - 05:00	A single slab is put onto the ground
05:00 - 05:30	The slab is removed
05:30 - 12:00	A new single slab arrives and starts to cool down
12:00 - 12:15	Also this slab is removed
12:15 - 13:00	Another single slab arrives
13:00 - 16:00	The slab on top of the ground is removed
16:00 - 24:00	A final new slab arrives

This scenario gives the output as shown in figure 11.

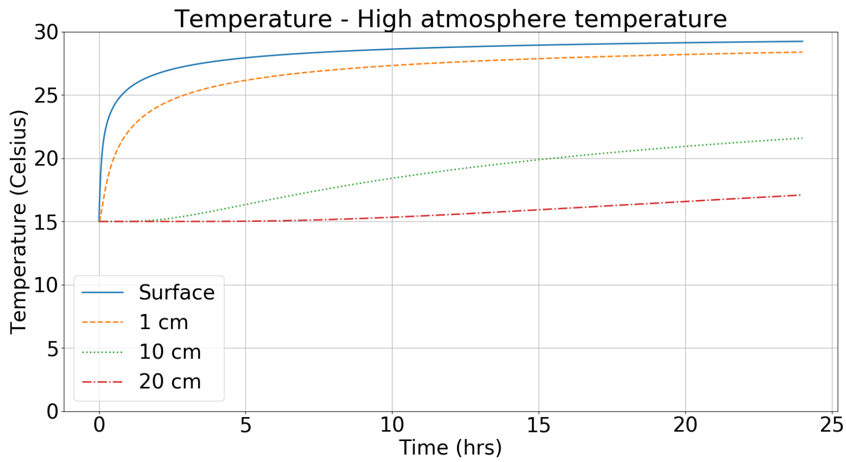


Figure 10: Temperature distribution in case there is no slab on top with an atmosphere temperature of 50 degrees Celsius.

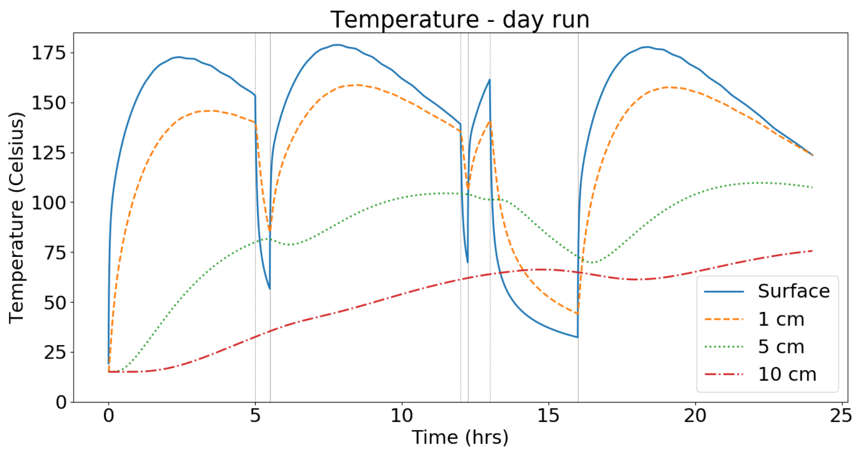


Figure 11: Heat spread in the ground as a result of hot slabs that are put onto the ground, cool down and are removed again. The used data for the surface temperatures of slabs has been obtained using another model that is available at Tata-Steel.

5.3.4 Convergence and error estimate

To check whether the output of the model is convergent, we run the model several times on different grids and check whether the following holds if x goes to 0:

$$Q = \frac{T(4x) - T(2x)}{T(2x) - T(x)} \approx 4. \tag{38}$$

In this formula, x equals the space between two nodes and $T(x)$ is the temperature at a certain point at the grid. If we keep doubling the number of nodes in the grid, the values of the temperature should converge to a certain value. To check if the temperatures converge, we look at the value of T, 10 centimetres below the surface, after 20 minutes of running time, if we put a slab on top of the ground with a constant temperature of 600 degrees Celsius. The initial temperature of the ground is 15 degrees Celsius. To get a consistent result, we use time steps of 1 second. The values that follow are shown in table 1. We can use the values to estimate a more precise temperature after 20 minutes, at a depth of 10 centimetres, in combination with

Table 1: Temperature estimation T with error after 20 minutes at a depth of 10 centimetres in case there is a slab on top with a constant temperature of 600 degrees Celsius.

Δx	T at 10 centimetres depth	Q	T-estimate with error
0.1 m	310.040938449471 °C		
0.05 m	298.701763366761 °C		295 ± 4 °C
0.025 m	293.544434129424 °C	2.19	292 ± 2 °C
0.0125 m	291.828173355366 °C	3.00	291.3 ± 0.6 °C
0.00625 m	291.353364947677 °C	3.61	291.2 ± 0.2 °C
0.003125 m	291.233076527944 °C	3.95	291.19 ± 0.04 °C

an estimation of the error. The estimates that are displayed in table 1 are made by extrapolating with the following formulas:

$$y_{\text{better}}(h) = \frac{2^p}{2^p - 1} y(h) - \frac{1}{2^p - 1} y(2h), \tag{39}$$

with error

$$\text{error} = |y_{\text{better}}(h) - y(h)|\,. \tag{40}$$

As we can observe in table 1, taking a grid with spatial steps of 0.0125 meter, will give almost the same result as taking spatial steps that are four times as small. We suggest to use spatial steps of 0.01 meter, with the reason that smaller steps take way more running time, though the result is not much more accurate. Also, taking steps of 0.01 meter allows one to calculate values for every centimetre, which is not achieved by taking steps of 0.0125 meter.

5.4 Validation

We set up two validation scenarios, for which we calculated the surface temperature of the slab that is on top of the ground with the aid of the model that already exists at Tata-Steel, see figure 12. The output of our model compared with the Tata-Steel model is shown in figures 13 and 14. The biggest error in the simulation of scenario 1 equals approximately 57 degrees Celsius, however, this is caused by the fact that the temperatures increase quickly at the start of the simulation. For scenario 2, the maximum error equals almost 51 degrees Celsius, caused by the same reason. The errors at the end of the simulation are way lower, namely circa 5 degrees Celsius for scenario 1 and about 11 degrees Celsius for scenario 2.

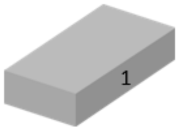
6 Discussion

6.1 Verification

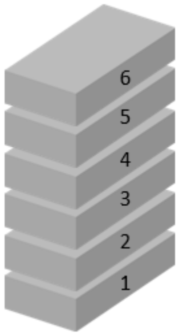
As a result of the verification that we did, we can conclude that the model behaves logical and as expected.

Thermal conductivity
It has been shown that the model reacts to a change in thermal conductivity. As expected, the ground temperature stays almost equal if the thermal conductivity coefficient is nearly zero. If, however, the thermal conductivity coefficient is chosen to be large, the ground heats up almost in a linear way, due to a lot of conduction.

Emissivity
A change in emissivity also caused the model to give different outputs. Modelling with a low emissivity caused that the ground heated up slower then when a high emissivity was used. The final temperature was however almost the same in both cases, as we saw in figures 7 and 8.



(a) Scenario 1 - A single slab on top of the ground



(b) Scenario 2 - A stack of 6 slabs on top of the ground

Figure 12: Two validation scenario's that will be used to validate the model

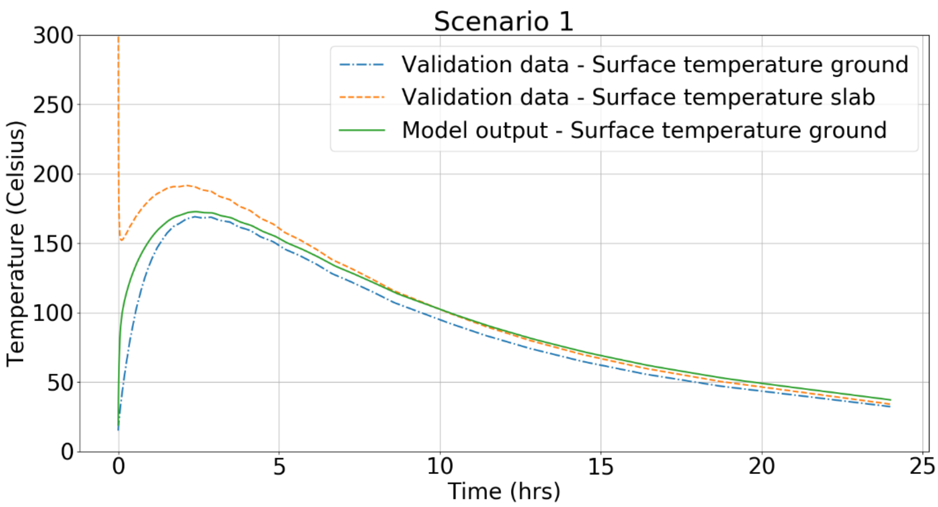


Figure 13: Model output for scenario 1, in which one single slab with an initial temperature of 600 degrees Celsius lays on top of the ground and cools down. The data is provided by a model at Tata-Steel.

No slab on top

In case there is no slab on top of the ground, we expected that the temperature of the ground would be mainly determined by the ambient temperature. In case of both simulations of the ground without a slab on top, this temperature equals 20 degrees Celsius. However, as we can also observe from the results of the simulations, the atmosphere temperature will influence the temperature of the ground, as is logical to expect. Because of the assumption we made that the ground does not reflect any radiation, the models shows a bit more influence of the atmosphere temperature on the ground then we had expected.

Alternating slab and no slab

As we observe in the graph pictured in figure 11, the model reacts to the fact that slabs cool down and are removed. It is however not possible to check if the model gets the right values with the aid of the validation data, because of the fact that this data is only valid if the initial ground temperature is 15 degrees Celsius everywhere. However, if we look at the graphs, we clearly see that the same pattern comes back every time a slab arrives. We assume this indicates that the model behaves in the right way. To ensure this, it is however needed to perform measurements at a real test scenario.

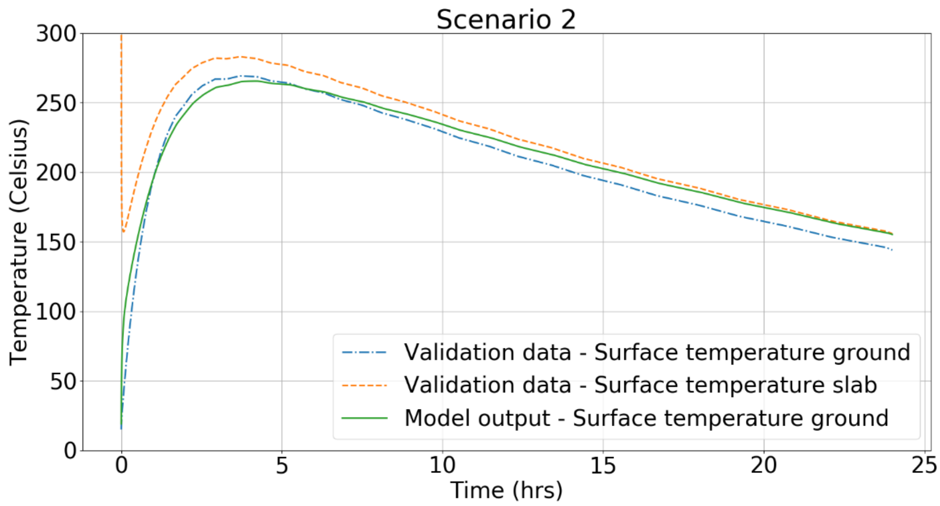


Figure 14: Model output for scenario 2, in which a stack of six slabs with an initial temperature of 600 degrees Celsius lays on top of the ground and cools down. The data is provided by a model at Tata-Steel.

6.2 Validation

As we saw in the validation, the model approximates the validation data. As we already mentioned, the maximum errors for the two scenarios are still high, caused by the fact that the ground heats up fast. In case the temperature of the ground changes slowly, the error that the model makes becomes lower and steady. The error that still occurs, can be caused by the fact that the validation data is made by another model, which models scenarios different, due to the fact that this model is a three-dimensional model. As one can observe in figure 3, the closer we get to the edge of a slab, the lower the surface temperatures get. The validation data takes the average temperature of all the simulated surface temperatures below the slab, which will result in a slightly lower final output for the surface temperature.
We made sure that the error is not caused by the fact that different parameters are use, by using the same parameters for our model as the Tata-Steel model does.

6.3 Non-uniform grid

In order to improve a one-dimensional model, it is in some cases possible to use a non-uniform grid. In the case of this problem however, it is not suitable to use a non-uniform model, unless one uses a grid that changes over time. This can be observed in figure 15, where the scenario from section 5.3.3 is plotted with temperature with respect to depth instead of time. This allows us to see exactly what happens inside the ground at a certain point of time, according to the model.

By observing the figure, it becomes clear that for different points of time, different non-uniform grids are needed to make sure that the right output is achieved. For example, if we would take a non-uniform grid for the temperature distribution after one hour, this will mean that there will be more nodes at the top of the ground and less towards the bottom. However, if we would use this grid in the situation that we see occurring at other points of times, we see that this will not give an accurate result. To do achieve a good result, we need another non-uniform grid.

In order to solve this problem, it is possible to define the non-uniform grid every time step again. This would however mean that there will be some new grid points that do not have data on it, so therefore the data should be interpolated every time step as well. In the future, it must be investigated whether this method takes less running time then taking a uniform grid.

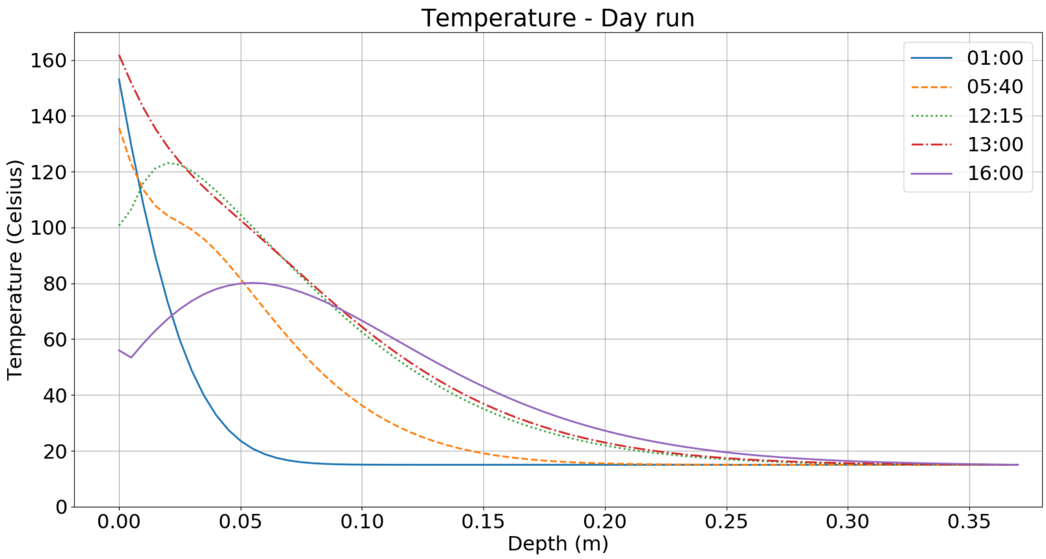


Figure 15: Heat spread in the ground as a result of hot slabs that are put onto the ground, cool down and are removed again. The used data for the surface temperatures of slabs has been obtained using another model that is available at Tata-Steel.

7 Recommendations

After this study, in which we build a one-dimensional model for the ground, there are a few things left to do.
At first, a three-dimensional model will be more accurate then a one-dimensional model. It is recommended to investigate how the extra accuracy achieved by a two- or three-dimensional model relates to the extra amount of running time. It also has to be investigated how a non-uniform grid that is changing over time influences running times.
Furthermore, we already derived equations for the case that the thermal conductivity depends on temperature. However, there are no measurements done by which we are able to know these values. Therefore, it will be a good thing to perform those measurements and implement those values in the model. Thereby, the model will perform better.
Another thing that has to be done, is implementing the composition of the soil as it is at Tata-Steel. At some spots at the slab yard, there are steel slabs in the ground that may influence the temperature of the slabs. In order to implement this correctly, it is first necessary to investigate and map where all those steel slabs are positioned exactly.

Acknowledgements

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A Code

The model that we made is written in Python. All the used parameter names are explained in table 2. Furthermore, all the formulas that are used in the code are described in the paper. In table 3 can be found in what section an equation is derived and discussed.

```python
import numpy as np

def Groundupdate(T, T_top, T_bottom, T_atmosphere, slab_top, dx, dt, SB, h_c_air, h_c_slab,
    k, epsilon_gr, epsilon_sl, alpha):

    #Fourier number
    R = alpha * dt / (dx**2)

    if slab_top:
        #slab on top
        N = 0.05 * h_c_slab / k
        F = 0.95 / (1 / epsilon_gr + 1 / epsilon_sl - 1)
        psi = k / (SB * F)
        #calculations
        T_boundary = (-(T[0] ** 4 - T_top ** 4) / psi
                      - N * (T[0] - T_top)) * 2 * dx + T [1]
    else:
        #No slab on top
        N = h_c_air / k
        F = epsilon_gr
        psi = k / (SB * F)
        #calculations
        T_boundary = (-(T[0] ** 4 - T_atmosphere ** 4) / psi
                      - N * (T[0] - T_top)) * 2 * dx + T [1]

    Tnew = [T[0] + R * (T[1] - 2 * T[0] + T_boundary)]
    for j in range(1, len(T) - 1):
        Tnew.append(T[j] + R * (T[j + 1] - 2 * T[j] + T [j - 1]))
    Tnew.append(T_bottom)
    return Tnew
```

Table 2: Used parameters in the code

T	Current temperature of the ground at all grid point
T_top	In case there is a slab on top, this is the surface temperature of the slab. If not, it is the ambient temperature
T_bottom	The temperature at the bottom of the ground
T_atmosphere	The temperature of the atmosphere
slab_top	Boolean, indicates if there is a slab on top or not
dx	Spatial step
dt	Time step
SB	Stefan-Bolzmann constant
h_c_air	Heat transfer coefficient between ground and air
h_c_slab	Heat transfer coefficient between ground and slab
k	Thermal conductivity
epsilon_gr	Emissivity factor of the ground
epsilon_sl	Emissivity factor of the slab
alpha	Thermal diffusivity

Table 3: References to the paper for all equations in the code

R	See section 3.1.1
N	See section 3.3.2
F	See section 3.3.1
T_boundary	See section 3.2.1
Tnew	See section 3.1.1