Valuation of Mortgages Without Prepayment Penalties

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Summary

The main contribution of this research is a simulation model to assess mortgage refinancing risk in the absence of prepayment penalties. Here, *refinancing* refers to the prepayment of a mortgage by replacing it with a new loan. If the mortgage rate at the moment of refinancing is lower than the initial contract rate, then Dutch mortgagors usually pay a prepayment penalty for the purpose of compensating the lender's loss. We examine how this interest rate risk can be priced into the mortgage contract rate directly instead of charging the penalty in the event of refinancing. In short, we evaluate the refinancing option in penalty-free mortgages. Here, the option value is measured in terms of a risk premium that the lender charges on top of the annual rate of a regular mortgage with prepayment penalties.

Relevance

The penalty-free mortgage could constitute an attractive opportunity for borrowers who value high flexibility, as well as risk-tolerant lenders. However, there is no applicable literature that can directly be used to analyze such a product in the context of the Dutch residential mortgage market. Furthermore, models for option-theoretic mortgage valuation are often either not sufficiently transparent, or difficult to control. Therefore, it is necessary to derive a new methodology.

Methods

The first research question is *"How can borrowers' refinancing strategies be characterized?"*. In Chapter 3, we derive a new mortgage rate model from two existing frameworks for stochastic interest rate development. Subsequently, we use the new interest rate model to define the economic value of mortgages as a function of interest rate and time. Based on this theoretical framework, the borrowers' optimal refinancing strategy is approximated in terms of an interest rate threshold. Here, the underlying idea is that borrowers who wish to maximize their expected profit should refinance as soon as interest rate drops below the threshold. Since real borrowers' refinancing decisions likely deviate from the optimum our theoretical approach suggests, we consider several behavioral and other practical implications for the exercise boundary in Chapter 4.

The second and third research questions are *"What is the value of a fair risk premium on a mortgage without prepayment penalties?"* and *"Given the fair risk premium, what is the downside risk for the lender?"*. In Chapter 5, we present a Monte Carlo simulation model to calculate respective answers. Here, the appropriate premium is calculated such that the lender's expected payoffs for a regular and a penalty-free mortgage are equal to each other, while downside risk is measured in terms of expected shortfall.

Results

The simulation results are reported in Chapter 6. We find that a risk premium of 30 basis points is appropriate under assumptions we consider likely (see base case in Chapter 6.2). That is, the annual interest rate of a penalty-free mortgage should be 0.3% higher than the rate of a regular mortgage loan with prepayment penalties. In the 5% of simulation instances that are most beneficial for the borrower, the expected loss for the lender amounts to about 8%. This corresponds to 40 basis points lower annual yield than a comparable regular interest-only mortgage with penalties. However, these results are highly sensitive to assumptions on future mortgage rate development and behavioral aspects.

Discussion and Conclusions

In this research, we define a new interest rate model, characterize borrowers' refinancing behavior explicitly, and present a simulation model to evaluate the worth of the penalty-free refinancing option. However, we find that assumptions on future mortgage rate development and behavioral aspects alter the simulation outcome considerably.

The fact that refinancing losses depend on interest rates, which are equally available to *all* borrowers, leads to a lack of diversification in portfolio context. Consequently, the realized instance of loss due to refinancing is potentially not close to the charged risk premium. Even if the lender was able to eliminate all uncertainty regarding the model validity and choice of input parameters, the premium might not be able to compensate the losses in a bad-case scenario. To avoid this problem, lenders could hedge against low interest rates. Alternatively, the mortgage contract conditions could be changed such that severe losses are prevented. For example, an interest rate floor could be introduced, where the borrower cannot refinance without penalty if mortgage rate is below floor value. However, major contract re-designs are a topic for future research. Before launching a penalty-free mortgage, it is also important to estimate how many Dutch mortgage providers will soon start offering the product and study the implications on borrowers' refinancing behavior. Finally, ethical issues might arise, because the charged risk premium might exceed the true expected loss due to refinances.

Preface

This report is the result of my graduation project for the *Financial Engineering and Management* specialization of the *Industrial Engineering and Management* Master's degree at the University of Twente. When I started six months ago, I knew little about mortgages and next to nothing about prepayment risk, but my level of understanding has skyrocketed! Writing this thesis was an exciting experience and I would like to sincerely thank everyone who contributed to the success of this project.

First of all, I would like to express my gratitude to my lead supervisor Dr. Berend Roorda, who kept giving me critical help when I needed it most, and my second supervisor Dr. Reinoud Joosten, whose close attention to detail enabled me to improve my work considerably. I conducted the project in the framework of an internship at the Dutch Mortgage Funding Company (DMFCO) in The Hague. DMFCO welcomed me with open arms and provided all the support and freedom I needed. I thank them for the great time! I am especially grateful to my daily supervisor Edo van de Burgwal, who enthusiastically guided the project through all phases. I also thank Willem Moerkens for many helpful remarks.

Last, but definitely not least, I would like to thank my parents. Even during the difficult times, they always believed in me, and their moral support was invaluable for getting the education I wanted. I dedicate this thesis to them.

Saskia Caspari — Enschede, November 2018

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Notation

$B(r,t)$	Mortgage value without embedded options (equivalent to value of a bond)
$c(t)$	Continuous payment rate of the mortgage contract
$\delta(r,t)$	Standard deviation of the Îto process which models interest rate
dz	Wiener process
$f(t)$	Fixed-rate risk premium
$F(t,T)$	Forward rate
ζ	Threshold that distinguishes two interest rate domains
i	Initial interest rate differential for refinancing. That is, $i = r_0 - r^*(0)$
κ	Mean-reversion rate of the mortgage interest rate
m	Mean deviation from the theoretical refinancing threshold
$\mu(r,t)$	Drift of the Îto process which models interest rate
μ_T	Expected return of the mortgage at maturity
$M(r,t)$	Mortgage value if the mortgage has a refinancing option
$O(r,t)$	Value of the mortgage's refinancing option
p	Risk premium that compensates for penalty-free refinancing
r_0	Initial mortgage rate (excluding risk premium)
$r(t)$	Mortgage interest rate
$r^*(t)$	Refinancing threshold / exercise boundary of the refinancing option
$R(t,T)$	Term structure of interest rates
s	Standard deviation of the normally distributed refinancing threshold
σ	Standard deviation parameter of interest rate models
τ	Expected time until the borrower refinances in number of months
t	Measure for time in number of months
T	Mortgage maturity in number of months
θ	Long-term mean of the mortgage interest rate
X	Normally distributed variable with mean m and standard deviation s
Y	Categorical variable defining the mortgage type (linear, annuity or interest-only)

Chapter 1

Introduction

1.1 Prepayment Risk

A residential mortgage constitutes a loan which is issued to individuals for the purpose of privately purchasing real estate and typically repaid in monthly intervals over the course of 30 years. Here, the lender has the right of ownership of the underlying real estate object until the loan is paid back. The borrower is referred to as *mortgagor*, because he or she grants the lender conditional ownership of the property, while the lender is called mortgage *originator*, *mortgagee* or mortgage *initiator*. If the mortgagor does not fulfill his or her payment obligations, the initiator has the right to sell the underlying real estate object to limit or avoid losses.

Mortgagors sometimes decide to repay a higher amount than the contract demands in some months or even repay the full loan early, which is a practice referred to as prepayment. At the moment of mortgage origination, the lender assumes that a fixed income is locked in for the duration of the fixed-rate period. However, if the loan is repaid earlier than the contractual payment schedule demands, then the initiator is forced to reinvest the prepaid amount at a new, potentially lower interest rate. As a consequence, prepayment can lead to losses for the lender and is a serious risk in the context of mortgage portfolio management.

We illustrate the impact of prepayments and our method to quantify respective losses with an example: Consider a 30-year fixed-rate mortgage which is originated at a 2.5 percent interest rate. Suppose, the market interest rate drops from 2.5 percent to 2 percent within the first two years of the contract. Next, the mortgagor decides to prepay the full loan and the initiator consequently has to reinvest the prepaid amount at the new rate of 2 percent. For an interest-only type of loan with a principal of €$300,000$, the initiator would miss out on €125 per month for the remainder of the fixed-rate period. Discounting these cash flows at 2 percent over a period of the corresponding 28 years yields an approximate total loss of €$32,200$ for the lender. In other words, the economic value of the mortgage at the two-year mark is €$332,200$, but the borrower only pays €$300,000$ in exchange for a waive of future mortgage payments.

Compensating for Prepayment Risk

In the Netherlands, there are two common ways for the lender to countervail losses due to prepayments: First, the contract can include a penalty clause which obliges the borrower to pay a part of the difference between the original and current market value of the prepaid amount at the moment of prepayment. In the example above, that difference amounts to €$32,200$. Alternatively, the loss due to expected prepayments can be priced into the mortgage up-front by increasing the contractual interest rate by an appropriate risk premium. A third possibility to compensate for prepayment risk is charging an up-front fee, which is a popular practice in the United States. Those fees are commonly referred to as *points* and constitute an alternative to including an additional risk premium in the contract's interest rate. However, this method of charging compensation for prepayment risk is not popular in the Netherlands.

Among other factors, the choice between the two main ways to charge compensation for prepayment risk depends on the type of prepayment the lender faces. We distinguish between the following prepayment categories:

1. *Exceptional Events:* The mortgagor prepays, because he or she moves to a new house, faces bankruptcy, the fixed-rate period ends or a similar special event occurs. In line with Dutch regulations, a prepayment penalty[1] is not charged in these cases and prepayment risk is compensated by adding a risk premium to the interest rate instead.

2. *Refinancing:* The mortgagor prepays the full loan by replacing it with a new mortgage. In this case, it is common practice to charge a penalty over 90% of the loan amount.

3. *Other Prepayments:* The mortgagor prepays the loan partially or fully using his or her own resources such as bonus salary, inherited money or similar means. Some Dutch mortgage providers do and some do not charge a penalty for this type of prepayment.

Chapter 2.2 will consider a more detailed break-down of prepayment types and their implications.

[1] In this research, the term *(prepayment) penalty* is used exclusively to describe the fee a borrower has to pay at the moment of refinancing. For example, the risk premium integrated in the mortgage rate is not referred to as a penalty. In Chapter 2.2.1, we will provide a more detailed definition of the terminology.

1.2 Research Formulation

1.2.1 Goal

DMFCO is an independent asset manager providing institutional investors with access to the Dutch mortgage market and originating residential mortgages under the label of MUNT Hypotheken. Prepayments that belong to the refinancing category are currently penalized by MUNT Hypotheken in order to compensate for potential losses, while the other two categories are penalty-free and the losses are priced into the mortgage interest rate in form of the risk premium.

DMFCO's ambition is to offer fair, transparent, and flexible mortgages, and a potential, new product that fits this objective is a mortgage loan that does not include any prepayment penalties at all. That is, mortgagors would be able to prepay any amount at any time without paying a compensation fee, even if the prepayment is financed using a new mortgage. For this new contract type, the expected loss due to refinances has to be assessed correctly and priced into the mortgage interest rate in form of a risk premium. Estimating the premium's fair value is is the main goal of this research.

1.2.2 Research Questions

To achieve the research goal, we first consider under which circumstances borrowers likely refinance their mortgage. Subsequently, we calculate the expected losses due to refinancing to determine an appropriate risk premium. Furthermore, due to the lack of diversification of refinancing risk in portfolio context, we would like to asses how high the losses can become when mortgage rate drops unexpectedly low. The comprehensive formulation of the research questions is:

1. *How can borrowers' refinancing strategies be characterized?*
2. *What is the value of a fair risk premium on a mortgage without prepayment penalties?*
3. *Given the fair risk premium, what is the downside risk for the lender?*

Since MUNT Hypotheken products already offer the possibility to prepay without penalty as long as this payment is not financed by a new mortgage, the valuation needs to assess losses due to refinancing only. The appropriate risk premium is calculated for mortgages with fixed-rate periods of 30 years, but differentiated by mortgage type (linear, annuity, and interest-only).

1.2.3 Approach

The thesis is structured as follows: In Chapter 2, we present a literature summary on the topic of prepayment modeling and analyze the prepayment option embedded in a mortgage contract.

In Chapters 3 and 4, we introduce relevant option theory and behavioral aspects of mortgage refinancing, respectively. The goal of these chapters is to estimate the interest rate boundary that mortgagors use to make their refinancing decisions. The findings in Chapters 3 and 4 are based on a mixture of proven methods and new contributions, and they will enable us to answer the first research question.

Since the remaining two research questions are interconnected, a single simulation study is conducted to address them. Here, the refinancing threshold derived in Chapters 3 and 4 serves as one of the model's underlying assumptions. Chapters 5 and 6 present the simulation methodology and results. Respective results are calculated under several different interest rate and behavioral scenarios. The simulation model gives the prospective user much transparency and control about underlying assumptions, as opposed to most existing models in literature.

Chapter 2

Background

2.1 Prepayment Modeling

2.1.1 Overview

The purpose of prepayment models is predicting mortgage cash flows in the presence of prepayments and quantifying respective change in mortgage value. The biggest challenge in this field is determining when borrowers decide to refinance their mortgage. There are two fundamental ways to approach the problem:

1. *Endogenous models* view prepayment decisions as a function of mortgage rate, housing price, and time, where borrowers simply prepay whenever they expect the largest financial gain. In other words, mortgagors are assumed to act according to a set of rules implied by the respective framework, and no exogenous factors influence the prepayment decision.

2. *Empirical models* on the other hand include behavioral and other practical components in the prepayment decision. Empirical methods usually constitute statistical frameworks based on analysis of historical prepayment data. Empirical models can either be stand-alone or extensions of a theoretical framework.

The following two sections explore existing literature on each approach.

2.1.2 Endogenous Models

A simple rule mortgagors could consider is to refinance whenever the amount of remaining debt becomes smaller than the sum of the outstanding payments discounted at the current market rate. In theory, this happens as soon as the current market rate drops below the contract mortgage rate. However, the realized profit also depends on the level prepayment penalties and refinancing costs such as notary fees. Furthermore, waiting for the interest rate to drop even further might be better than prepaying immediately, which makes the calculation of good refinancing decisions challenging. Endogenous frameworks attempt to define the critical interest rate threshold at which refinancing is optimal. The working principle of endogenous frameworks will be treated in detail in Chapter 3. In this chapter on the other hand, we focus on reviewing research on the subject.

There has been a growing body of literature since the 1980s that employs option pricing models in the context of stochastic interest rate development in order to estimate the optimal refinancing strategy. In these models, the prepayment option on a mortgage is considered equivalent to an American call option on a bond. Furthermore, the option to default, which is the option to sell the underlying security, is often viewed as put option on a bond and its value depends on housing prices. However, we assume that credit risk is exceptionally low in the Netherlands (Mastrogiacomo and van der Molen, 2018). Therefore, we always view the mortgage as default-free callable bond when considering the option-theoretic approach.

Evolution of Option-Theoretic Prepayment Modeling

Option-theoretic prepayment modeling is derived from option-pricing methodology by Bachelier (1900), who introduced continuous-time models into the field of finance, as well as Black and Scholes (1973) and Merton (1973) who defined the Black-Scholes-Merton model for option-pricing. Dunn and McConnell (1981) were among the first to apply option-theory to mortgage valuation and their model was later extended by many authors. For example, Schwartz and Torous (1989) added a random component based on macro-economic factors to the refinancing decision as defined by the Dunn and McConnell model.

Stanton (1995) on the other hand introduced variability in refinancing costs across borrowers. The purpose of his research was to capture hidden or unexpected costs at the moment of prepayment, such as the borrower's time and effort to gather information and required documents. Deng et al. (2000) used an altered version of the Stanton model to show that prepayment behavior is indeed not homogeneous among mortgagors. According to their research, there are significant differences in individuals' decision making. This result becomes especially important in empirical prepayment modeling which we will discuss in Section 2.1.3.

There are many more papers that present different versions of the original model of Dunn and McConnell. For example, Dunn and Spatt (2005) took into account that not only one, but multiple calls are possible until maturity. However, in this brief review we restrict ourselves to the in our opinion most only the most notable breakthroughs.

Interest Rate Modeling

The decision to prepay depends on interest rate development, which implies that appropriate frameworks to model mortgage rate are essential. Here, it is common practice to view the mortgage rate as stochastic mean-reversion process. Cox et al. (1985) developed a one-factor equilibrium model (CIR model), which is widely used for option-theoretic prepayment modeling (e.g. Dunn and McConnell (1981), Kau et al. (1992), Sharp et al. (2009), Hung et al. (2012) and Wu et al. (2017)). Vasicek (1997) suggested a different interest rate process whose most notable difference to the CIR framework is the fact that it allows negative interest rates. Several theoretical prepayment models use the Vasicek process (e.g. Jiang et al. (2005), Xie et al. (2017) and Wu et al. (2018)).

Most option-theoretic prepayment models work either with the CIR or with the Vasicek framework, but exceptions occur. For example, Kimura and Makimoto (2008) argue that a single model with certain parameters cannot describe interest developments over long periods of time equally well at any time. The authors address the possibility that the stochastic interest rate process might change over time by implementing a model with different interest rate regimes. A precise description of the working principles of different relevant interest rate models is given in Chapter 3.

Solution Methods

Option-theoretic models require solving a set of partial differential equations. Usually, this is done numerically by stepping either forward in time by means of Monte Carlo Simulation or backwards in time using finite differences or decision trees. Some recent papers that apply Monte Carlo Simulation to value mortgages under prepayment risk are presented by Zheng et al. (2012) and Xie et al. (2017), while the articles by Rigatos and Siano (2018) and Hilliard et al. (1998) are examples of solutions using the finite difference method and decision trees, respectively.

On the other hand, Agarwal et al. (2013) derived a closed-form solution to calculate the mortgage value in the presence of prepayment option. A more recent contribution of a closed-form solution is presented in Followill and Olsen (2015). However, an analytic solution requires many highly debatable simplifications such as the assumption that the risk-free rate follows a simple zero-drift random walk instead of a mean-reversion process.

2.1.3 Empirical Prepayment Models

Mortgagors could choose a set of assumptions and compute their optimal refinancing threshold with one of the available endogenous approaches. However, one would expect that not all borrowers follow the same set of rules that determines when they should refinance. Exogenous factors such as personal circumstances are likely highly relevant as well. Empirical models for predicting prepayment behavior attempt to assess the impact of those factors by analyzing historical mortgage data. The data consequently serve as input to statistical frameworks, where the most popular approach is the proportional hazard model. For example, Green and Shoven (1986) applied the proportional hazard model to model mortgage prepayment, and many authors followed by adopting similar frameworks (eg., Follain et al. (1992), Sugimura (2002), and Liang and Lin (2014)).

But how large is the proportion of mortgagors who do not refinance in the way popular theoretical models suggest? A recent empirical study by Keys et al. (2015) used the closed-form solution to the mortgage-valuation problem as presented in Agarwal et al. (2013) to calculate whether a household should refinance their mortgage. They find that 20% of households that held an active mortgage in December 2010 had not refinanced when it would have been profitable according to the model. Agarwal et al. (2016) used the same closed-form solution as reference and considered a data set of mortgages that were originated between 1998 and 2011. They found that 57% of households either deviate at least 0.5% from the computed optimal refinancing rate or wait longer than six months after the calculated optimal refinancing date until they refinance.

François and Pardo (2015) developed a model that computes the optimal refinancing threshold for callable bonds. The framework is based on the CIR interest rate model and the authors report that 88 percent of borrowers deviate more than three months from their optimal refinancing date. Although the study does not treat mortgage prepayments in particular, we assume that we can apply the results to mortgage prepayment modeling, because prepayment options on mortgages and call options on bonds are similar to each other. This equivalence will be treated in more detail in Chapter 3.

The presented studies show that the refinancing rules mortgagors use to make decisions are not easily determined and certainly not described accurately by an endogenous solution method. On the other hand, it is also debatable whether historical data are a good measure for the future development. Nevertheless, many researchers use historical data to gain more insight into the exogenous factors that are associated with prepayment decisions. In the following section, we summarize their findings.

Explanatory Factors

Empirical studies show that factors which are correlated with prepayment behavior include seasoning, seasonality, type of real estate, age of the mortgagor, transaction costs, house price, expected tenure in the house, loan-to-value (LTV) ratio, tax laws, and several macroeconomic factors (e.g. Jacobs et al. (2005), Deng et al. (2000), Charlier and van Bussel (2003), Hung et al. (2012), Chernov et al. (2017), Harding (2000)). Here, seasoning refers to the age of the mortgage contract, seasonality corresponds to different times in a year and the type of real estate represents whether the mortgagor lives in a house or apartment.

Chernov et al. (2017) recently presented evidence that relevant factors also include changes in the credit risk of the institution guaranteeing timely payments and changes of the mortgage liquidity. Alink (2002), who developed a logistic regression model for mortgage prepayments in the Netherlands, finds the following additional factors that influence prepayment behavior of Dutch mortgagors: distribution channel, legal mortgage rank, urbanization, geographic region, and several general properties of the interest rate.

Applicability of Literature

Much empirical research has been conducted in the field of prepayment risk modeling, but there is no clear consensus on the way different factors influence borrowers' prepayment decisions. For example, the Stanton model, which was introduced in Section 2.1.2, was used by many researchers, but Green and LaCour-Little (1999) showed that the hidden transaction costs, a central characteristic of the Stanton framework, have negligible impact on prepayment decisions.

Furthermore, the majority of results is not based on the Dutch mortgage market. Instead, underlying data often originate from American portfolios which are subject to different culture, laws and policies. Daniel (2010) applied the Stanton model to the Australian mortgage market and found significant differences in the prepayment behavior between Americans and Australians. This outcome suggests that empirical evidence based on U.S. data is not automatically applicable to mortgage markets in other countries.

Our next concern regarding applicability regards the format of used data. Many studies are not exclusively based on individual residential mortgages, but include commercial mortgages, mortgage-backed securities or highly aggregated data. These data formats make an assessment on individual loan level challenging. For example, consider a single collective interest rate, which condenses the interest rates of a large number of mortgages into one single value. If that value is compared against the current market rate, it is nearly impossible to draw meaningful conclusions about the difference between a single mortgage's rate and market rate.

Another factor that limits the applicability of results in literature for our purposes is the fact that the models are built to predict prepayments as a whole instead of refinancing in particular. Furthermore, a vast majority of research has been conducted before the 2008 financial crisis, which caused changes in the economic environment. It is difficult to asses the effects these changes had on the validity of prepayment models.

In summary, while we can use relevant empirical studies as reference point for our purposes, we cannot assume that the results are applicable to the mortgage portfolio of a Dutch residential mortgage provider.

2.2 The Prepayment Option

2.2.1 Reviewing Important Concepts

In order to gain deeper understanding of important concepts and to complete our conceptual framework, in this section we address the following question: *What constitutes the prepayment option and what is the role of prepayment penalties?*

A *prepayment option* refers to the borrower's right to pay more than the mortgage contract's prepayment schedule demands. A *prepayment penalty* is the fine the borrower has to pay to the lender in the case of prepayment. While the term prepayment *penalty* suggests that a borrower is *penalized*, the Dutch regulatory authority does not allow mortgage providers to charge more than a mere compensation for their losses. Therefore, it would be more appropriate to refer to a prepayment *compensation*. However, since the respective fines are commonly called *penalties* in practice and literature, we choose to adopt this terminology.

Whether a prepayment penalty may be charged or not is also regulated by Dutch authorities. For example, when the borrower moves and sells the house, then a prepayment penalty may not be charged. Furthermore, mortgage originators are never allowed to charge more than the economic loss they face due to the underlying prepayment. This motivates our previous remark that the fee constitutes a compensation rather than a penalty. Regulatory implications are discussed in more depth at a later point in this section.

Next, we explore how the value of a risk premium is connected to the magnitude of prepayment penalties. Suppose the prepayment penalty exactly compensates the lender for losses whenever the borrower prepays. In this case, a mortgage contract with prepayment penalties is associated with zero prepayment risk for the lender. However, if the lender decides to introduce a second type of mortgage contract which does not include prepayment penalties, then the losses have to be compensated by adding a risk premium to the new mortgage's interest rate. The value of the risk premium must be chosen in such a way that the payoff for the mortgage contract with penalties is exactly equal to the expected payoff of the penalty-free contract. In the long run, the risk premium *exactly* compensates for losses due to prepayments. The main purpose of this research is to find out what the fair value of the respective premium should be.

2.2.2 Penalties Versus Prepayment Behavior

We would expect that high prepayment penalties are associated with low prepayment rates, which has been confirmed by Beltratti et al. (2017), who analyzed Italy's 2007 reform that reduced prepayment penalties on existing mortgages and banned them from newly issued ones. However, if borrowers are given a choice between a regular and penalty-free product, then only part of the borrower population would opt for the penalty-free contract type. Would the prepayment behavior of this group differ from the prepayment behavior of the group who would choose a regular contract including prepayment penalties?

To give an indicative answer to this question, we turn to the United States, where both mortgage types are offered in the market. Research suggests that the risk-profile of mortgagors is connected to their behavior regarding choosing an option to enter a penalty-free contract or not. For example, Brueckner (1994) and Bian and Yavas (2013) analyze the respective self-selection process of mortgagors and find that those borrowers who are most likely to refinance would choose the option for a penalty-free contract. Also Stanton and Wallace (1998) show that borrowers with different likelihood to refinance tend to choose different types of mortgage contracts, respectively. We leave details about profiling mortgagors to Chapter 4.

2.2.3 The Worth of the Prepayment Option

This thesis revolves around the idea that omission of prepayment penalties is associated with a certain loss for the lender due to more refinances, which can be compensated by adding an appropriate risk premium to the interest rate. To assess the order of magnitude of the premium, we consider existing studies on prepayment option assessment and subsequently evaluate if any of their results can be used in this research.

Non-Dutch Studies: LaCour-Little and Holmes (2008) studied a set of American mortgages active between 1997 and 2003, where some mortgages were penalty-free while others included a penalty on 80% of all prepayments due to moving to a new house and refinancing. The authors found that the prepayment option is worth around 15 basis points of interest rate difference. A similar study by Courchane and Giles (2002) (as cited in LaCour-Little and Holmes (2008)) investigated the Canadian mortgage market and concluded that the prepayment option is worth between 45 and 80 basis point difference in interest rate.

Kalotay et al. (2004) studied the worth of the prepayment option by developing an option-based theoretical model and testing it using data on mortgage-backed securities from 2003. The authors found that the difference between regular and penalty-free mortgages amounts to 40 basis points.

The research of Beltratti et al. (2017), which was already mentioned in the previous section, compared Italian mortgage rates from 2005 and 2009. Within this time period, prepayment penalties had been banned in Italy. The authors find that the omission of penalties resulted in an interest rate increase of 80 basis points.

Furthermore, several studies come to the general conclusion that a lower prepayment penalty is associated with higher interest rate. For example, Liu and Roca (2015) studied Australian mortgages between 1996 and 2011 and found that lower up-front fees (points) come with a higher interest rate and vice versa.

Dutch Studies: The studies mentioned so far are not based on Dutch data. Since this might restrict the applicability for our purposes, we proceed by considering Dutch research.

Alink (2002) used the prepayment model he developed in his PhD dissertation to estimate the worth of the prepayment option at 22 basis points. However, this result was based on the analysis of one particular mortgage with a fixed-rate period of ten years, a loan-to-foreclosure-value of 80% and a mortgagor of age 45. Alink's goal was merely to demonstrate how the model he had developed could be used. Therefore, he completely omitted a sensitivity analysis and we cannot be sure whether the worth of a prepayment option is approximately equal across several mortgages with different properties.

Kuijpers (2007) estimated the fair mortgage value without prepayment penalties, but the author did not specify the difference between mortgages with and without penalties. Kuijpers merely states that the interest rate is "considerably" higher when including a penalty-free prepayment option.

The Nederlandse Vereniging van Banken estimated the risk premium of mortgages without prepayment penalties around one percent, as cited by Hoekstra (2018). However, their methodology is not reported.

Table 2.1 lists the appropriate risk premia as suggested by literature. There is a considerable difference across results. Risk premia can have values as low as 15 or as high as 100 basis points. Since the presented studies are based on different - sometimes unclear - assumptions and they usually do not distinguish between refinancing and other prepayments, it is barely possible to judge their applicability for our purposes. These issues show the necessity to develop new methodology for Dutch mortgage refinancing in this thesis.

2.2.4 DMFCO's Options Regarding Penalties

The purpose of prepayment penalties is reducing or ideally entirely avoiding the initiator's losses due to prepayments. At the same time, penalties heavily restrict

Table 2.1: Risk premia found in research

Authors	**Risk Premium**
Hoekstra (2018)	100
Beltratti et al. (2017)	80
LaCour-Little and Holmes (2008)	15
Kalotay et al. (2004)	40
Courchane and Giles (2002)	45-80
Alink (2002)	22

the borrowers freedom to deviate from the payment plan determined in the mortgage contract. The Autoriteit Financiële Markten (AFM) is the Dutch regulatory authority for financial services. It defines a set of rules for Dutch mortgage originators which aims to balance the risk mitigation possibilities for the lender and the rights of the borrower. In the following list, we analyze each of the prepayment types that were briefly introduced earlier with regards to penalties.

1. **Prepaying less than ten percent in a single year**: If the prepayment amounts to less than ten percent of the remaining outstanding loan, then the lender may not charge a prepayment penalty.

2. **Refinancing**: In case of refinancing with a new loan, the mortgage provider may charge a penalty over 90% of the total prepaid amount. That is, the borrower prepays 10% without penalty, but the mortgage provider is permitted to charge compensation for the losses over the remaining 90% of the total loan amount. As mentioned previously, respective penalty may not exceed the present value of the economic loss.

3. **Exceptional Events**: Borrowers always have the right to move to a new house and in the course of moving prepay the old property's mortgage without any penalties. Similar policies exist for other special events, such as bankruptcy or demise. Therefore, mortgage originators may not induce penalties in these cases.

4. **Other Prepayments**: When the borrower prepays partly or in full using his or her own resources, then the same regulations stated for the case of refinancing apply. In the Netherlands, some mortgage providers do and some do not charge a penalty over 90% of the prepaid amount. DMFCO's policy allows borrowers to prepay using own resources without obligation to pay a penalty.

Figure 2.1 shows a summarizing diagram of the discussed prepayment categories. As shown in the figure, the primary distinction is made between prepayments which amount for less than ten percent of the loan's face value and those

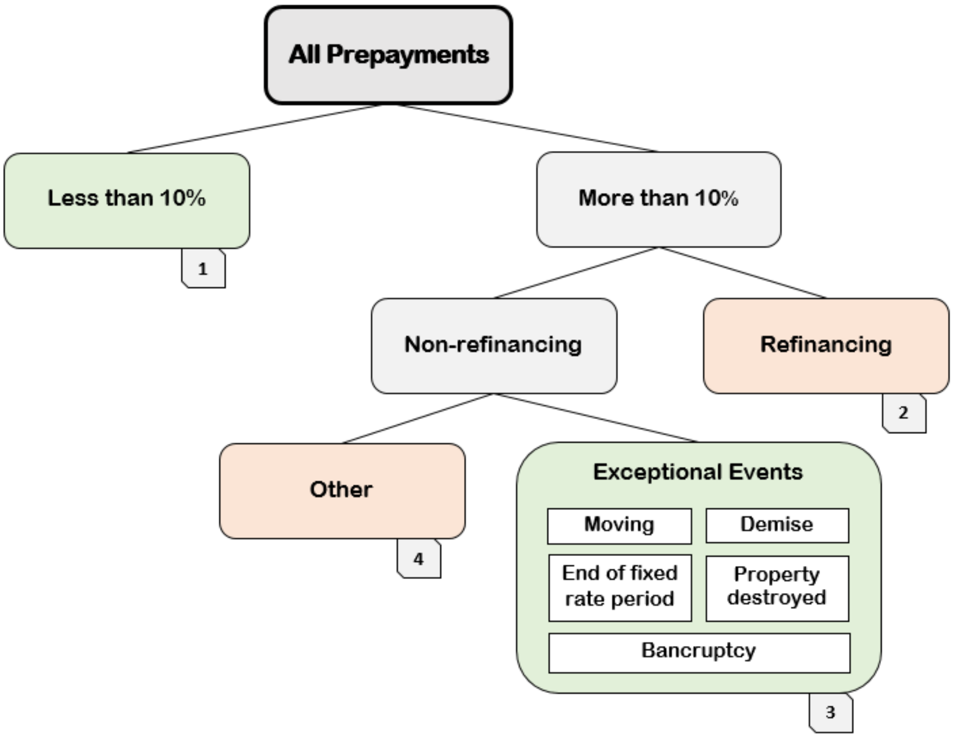


Figure 2.1: Categorization of prepayment types. Green classes indicate that a prepayment penalty is never charged and red categories indicate that it is common to charge a penalty if the borrower prepays. The numbers attached to some elements represent the category number that we assign to the prepayment type in the framework of this thesis.

amounting for more than that. The latter category is divided in a refinancing and non-refinancing component, respectively. Non-refinancing prepayments above ten percent are again split into exceptional events and other prepayments.

In this research, it is important to be aware of the prepayment type that is currently under consideration. In the remainder of this thesis, the term *prepayment* generally refers to the *"All Prepayments"* box in Figure 2.1. Therefore, *prepayment* includes all prepayment types combined. When explicitly using the term *refinancing* on the other hand, we exclusively refer to the respective category as indicated in the diagram.

2.2.5 Contract Design

We recall that the new contract type enables the borrower to refinance the mortgage at any point in time without prepayment penalty. However, given sufficiently low transaction costs, lower interest rates would then result in frequent - possibly monthly - refinancing decisions by mortgagors, and the mortgagee would be fully exposed to interest rate risk. To avoid this problem, refinancing could be restricted by means other than charging a penalty. Examples of such restrictions are:

- The refinancing option could be converted to a cliquet-type option which can only be exercised in certain intervals during the fixed-rate period - for example every five years in an otherwise 30-year fixed-rate contract.

- The mortgage interest rate has a floor value and if the market rate falls underneath the floor, then penalty-free refinancing is not possible anymore.

- The refinancing option is the equivalent of a barrier option, which means that penalty-free prepayment is only possible if the interest rate fell below a certain threshold in a certain period of time.

Many more possibilities to structure the prepayment option similar to different types of exotic options exist. However, we argue that most borrowers likely view their mortgage as funding for their home rather than an investment opportunity. Therefore, the terms of the contract should be kept as simple as possible. A contract for a residential mortgage should not be turned into a more complicated product than it already is. In summary, the borrower should have the feeling of having *fewer* conditions and restrictive rules instead of *more* of them.

Consequently, we choose the format of the restriction to be as simple as possible: The mortgagor may refinance at any time and any interest rate, but the option can only be used once. The most important implication of this *one-time-only* restriction is the fact that borrowers are required to refinance externally, as opposed to getting a new penalty-free mortgage with MUNT Hypotheken.

Finally, we illustrate the working principle of this contract with an example: Suppose a client closes a penalty-free mortgage at an interest rate of 3 percent, while a regular mortgage with prepayment penalties is currently priced at 2.5 percent. The difference between the two prices is arbitrarily chosen at 50 basis points for the sake of this example, but the real price difference will of course depend on the results in this thesis. If the interest rates fall to 2 percent for a regular and 2.5 percent for a penalty-free contract, then the mortgagor might want to refinance the mortgage at the new interest rate. However, in this case he or she is not allowed to close another penalty-free contract at 2.5 percent, but instead is only permitted to use a regular mortgage contract with 2 percent interest rate. That means, if DM-FCO is the only mortgage originator offering the penalty-free contract type, then a mortgagor has a one-time opportunity to refinance without penalty during the duration of the fixed-rate period.

2.3 Relevance

The fact that mortgagors pay penalties when refinancing is often seen as a natural property of the Dutch mortgage market and we know of no evidence of attempts to change the situation. For example, Hoekstra (2018) writes about the possibility of a penalty-free mortgage, but concludes that borrowers likely would not accept the respective risk premium on their mortgage rate. Groot and Lejour (2018) state that "it is often not attractive to refinance the mortgage [because of the penalty]" and that "the Dutch situation does not allow for refinancing". Although Kuijpers (2007) observes that refinances are common in the Netherlands, he says that "prepaying more than allowed is never optimal".

In summary, the existence of prepayment penalties is often perceived as a given situation and possibilities regarding penalty-free products have barely been explored. However, omitting prepayment penalties could provide borrowers with a new, high degree of flexibility that has not been offered before on the Dutch mortgage market. Furthermore, the new product could address potential mortgage lenders with a certain appetite. Finally, Dutch regulation has been changed to provide borrowers with more flexibility in the past. Therefore, it is possible that the penalties will be banned from the entire Dutch market at some point in the future. It is favorable to have done comprehensive analysis on penalty-free mortgages before this case occurs.

However, it is difficult to put numbers on refinancing risk. Consider the fact that MUNT Hypotheken generally offers the possibility to prepay the mortgage in full as long as the borrower uses his or her own funds. Why is the risk associated with borrowers using this option so different from refinancing risk? In short, prepayment risk - excluding refinancing - is unsystematic, because it mostly depends on variables that differ among borrowers such as income or wealth. As a consequence, most types of prepayment risk are reasonably diversified when put in portfolio context. Losses due to refinancing on the other hand are likely depend heavily on mortgage rate, which is equal for *all* borrowers: If interest rate drops, then all mortgagors have a strong incentive to refinance. In the worst thinkable scenario, the entire mortgage portfolio is liquidized within few months without any compensation for the lender. Consequently, correctly assessing the appropriate risk premium to counteract losses due to refinances is essential for the analysis of the potential new penalty-free product.

Chapter 3

Theoretical Model

3.1 Introduction

In this chapter, we derive an option-based risk-neutral continuous-time framework for mortgage valuation, where the refinancing option on a mortgage is treated equivalently to a call option on a bond. The goal is to determine the theoretical exercise boundary that borrowers should use to refinance. Here, the term *"theoretical"* expresses that we assume that the respective boundary approximates the optimal refinancing threshold under the model. The purpose of this chapter is laying a foundation for answering the first research question: *How can borrowers' refinancing strategies be characterized?* The final answer to this question will be provided in Chapter 4, where we will add behavioral components to the theoretical threshold.

This chapter is structured as follows: Section 3.2 describes similarities and differences between mortgages and bonds. Subsequently, Section 3.3.1 derives a model for mortgage rate development which we will require for the process of mortgage valuation. In Section 3.4, we express mortgage value as a function of interest rate and time to complete the theoretical framework. Finally, in Section 3.5 we use our insights to estimate an optimal exercise boundary for borrowers.

3.2 Mortgages and Bonds

Option-theoretic mortgage valuation approaches view mortgages with a refinancing option as callable bonds. A look at mortgages' and bonds' cash flows immediately reveals similarities between the two products. For example, the interest-only mortgage consists of fixed monthly interest rate payments and a single repayment of the principal at the end of the mortgage contract. These cash flows by themselves are indistinguishable from a bond's coupons and its principal's repayment at maturity.

An American call option on a bond gives the holder the right to take a long position in the bond contract at or before maturity. The strike price is a predefined interest rate, which is usually close to the value of the bond's interest rate at the moment the option was written. A refinancing option on a mortgage is similar to a call option on a bond: The holder has the right to buy his or her current mortgage loan at a strike price equal to the remaining debt.

Despite the similarities in terms of cash flows, there are also important differences between the product types: Once the mortgage's refinancing option is exercised, there is no possibility to cash in the value difference in the market. Instead, the holder continues to be exposed to interest rate risk for the remainder of the mortgage lifespan. Second, the most notable property of the mortgage loan is the fact that it is granted for a specific purpose and associated with an underlying security in form of real estate. A bond on the other hand is not issued for a particular purpose. Finally, bonds are typically instruments used by investors, while mortgagors do not necessarily consciously seek the real estate investment opportunity, but rather simply fund a place to live in. Therefore, the profiles of a typical bond holder and a mortgagor might differ, which can result in different behaviors regarding exercising the option. However, we leave behavioral implications to Chapter 4.

3.3 Interest Rate Model

The most important property of interest rates that we consider in this research is the short-rate, which is the instantaneous interest spot rate. This section presents the short-rate models that are are most frequently used in literature on theoretical refinancing models. Here, we pay close attention to the advantages and downsides of particular models, as an appropriate interest model is essential for the quality of our refinancing model.

3.3.1 The Mortgage Rate

The mortgage rate determines how much interest a borrower pays on the mortgage's face value. However, before diving into the theory of interest rate models, we wish to understand the composition of the mortgage rate and which factors drive its development over time.

Mortgage rate consists of two main components: The price of a risk-free interest rate swap and a fair risk premium. The former component simply constitutes the risk-free interest rate that is available on the market at the time the mortgage is originated. The risk premium on the other hand compensates for the expected losses due to different risk factors, such as prepayment, credit, liquidity, and operational risk. It is common to differentiate mortgage rates by loan-to-value ratio and fixed-rate period for the purpose of determining the fair risk premium. For example, a high loan-to-value ratio results in a higher mortgage rate, because of the increased exposure to credit risk. Furthermore, mortgages with longer fixed-rate periods typically have higher rates as well, since interest rate risk (e.g. prepayments) increases.

In practice however, there are several more factors driving mortgage rate: Depending on the type of organization, the spread on top of the risk free rate does not only have to cover several risks, but also additional costs such as operations and funding costs. Furthermore, the actual mortgage rate that is charged may vary significantly based on factors such as choice of profit margin, secondary product conditions, demand in the market, or marketing strategy.

This circumstance leads to the fact that several mortgage providers in the Dutch market might offer the same mortgage at significantly different rates. For example, 30-year fixed-rate mortgage with 100% loan-to-value is currently available at different rates between 2.83% and 4.45%.[1] To achieve an accurate prediction of refinancing behavior, information about all offers in the Dutch mortgage market should be considered. This concerns the offered mortgage rate, but also secondary conditions such as customer service quality. However, creating a meaningful model of the respective differences regarding spread and other conditions is out of scope of this thesis. Therefore, we make several simplifying assumptions about the mortgage rate:

1. All mortgage providers offer the same interest rate and secondary conditions, such that a mortgagor is indifferent between providers. In other words, there is no arbitrage.
2. The mortgage rate is not viewed as combination of risk-free rate, risk premia and other costs, but modeled as a whole instead. This enables us to describe mortgage rate with an one-factor stochastic process.
3. We model the annual rate of a mortgage with a 30-year fixed rate period and very high loan-to-value.

3.3.2 Short-Rate Models

The temporal development of the mortgage short rate depends on a larger number of exogenous variables. For example, different indicators for the economic status of Europe such as unemployment-rate, economic output, population growth or inflation rate can provide an incentive to the European Central Bank to make changes to the risk-free rate. For background information on the drivers of interest rate in

[1] Source: https://www.hypotheekrente.nl/rente/30-jaar-rentevast/100/#overzicht , accessed: 10-09-2018

the European economic area, the reader is referred to Bonam et al. (2018).

Even attempting to develop a statistical framework that models all those variables for the purpose of determining mortgage rate is far out of scope of this thesis. Instead, we assume that interest rate moves up and down in a random manner and may be described in terms of a stochastic differential equation (SDE). We define the one-factor process that describes interest rate behavior with the SDE

$$dr = \mu(r,t)dt + \delta(r,t)dz \tag{3.1}$$

which is known as Markov diffusion or Itô process. Here, r is the interest rate which depends on time, $\mu(r,t)$ is the drift, and $\delta(r,t)$ is the standard deviation of the rate. Finally, dz denotes the Wiener process. The functions μ and δ may, but do not have to, depend on interest rate and time. In fact, equilibrium models, which make up a large portion of popular interest models for option pricing, assume that these variables are independent of time. In the following sections, we discuss some specific choices for $\mu(r,t)$ and $\delta(r,t)$ for the sake of modeling the interest rate.

Random Walk

The simplest possible process constitutes a plain random walk. The process is described by the SDE

$$dr = \sigma dz$$

where σ is the volatility of the interest rate. However, it is not common to use a simple random walk for option pricing. Researchers usually assume that the real interest rate behavior is more accurately described by a mean-reverting process. The frameworks in the following subsections are all frequently used for the mean-reversion modeling of interest rates.

Cox-Ingersoll-Ross Model

The Cox-Ingersoll-Ross (CIR) model (Cox et al., 1985) assumes that interest rate change behaves according to a square root mean-reversion process. That is, the interest rate follows

$$dr = \kappa(\theta - r)dt + \sigma\sqrt{r}dz \tag{3.2}$$

Here, κ is the mean-reversion rate, which is the speed at which r reverses towards the long-term equilibrium mean θ and is a measure of sensitivity of the interest rate with respect to time. While a random walk approaches (plus or minus) infinity when running for a long period of time, mean reversion processes - as their name suggests - revert around their equilibrium θ.

The special characteristic of the CIR model is that the rate's volatility depends on the square root of the interest rate. As consequence, the standard deviation of the stochastic process becomes smaller as the interest rate decreases. Furthermore, interest rate cannot be negative under this model, because the square root of the rate would not yield a real solution. To ensure that r does not jump to a negative value when using a discrete version of this model, it is common to set $2\kappa\theta \geq \sigma^2$. The larger the difference between $2\kappa\theta$ and σ^2, the lower the probability that interest rates comes close to zero.

Vasicek Model

Another popular mean-reversion process for modeling interest rates was first proposed by Vasicek (1997). It is similar to the CIR model, but the square root factor of the random process is omitted. Therefore, volatility does not decrease when the interest rate becomes smaller, which makes negative interest rates possible. The following SDE describes the corresponding process:

$$dr = \kappa(\theta - r)dt + \sigma dz \tag{3.3}$$

Hull-White Model

To understand the Hull-White model, we must first take a look at the term structure of interest rates $R(t, T)$, which is also known as yield curve. The term structure describes the relationship between returns of the same underlying security for different times to maturity. In the case of risk-neutral mortgage valuation, it is defined as the rate of return on a mortgage with maturity T at time t. That means that the risk-neutral price of the mortgage is given by $e^{-R(t,T)(T-t)}$ and therefore the term structure is:

$$R(t, T) = -\frac{1}{T - t}\, ln\; E[e^{-\tilde{r}(T-t)}] \tag{3.4}$$

Here, $\tilde{r}$ is the average interest rate during the term $T - t$ and $E[e^{-\tilde{r}(T-t)}]$ is the expected value of the expected return at time t.

The CIR and Vasicek SDEs are equilibrium frameworks, which means that they attempt to model market supply and demand of the underlying security. Therefore, equilibrium models do not necessarily fit the actual market prices and are hence not arbitrage-free. In the CIR and Vasicek SDEs, both drift and volatility depend only on the interest rate and not on time, which means that the term structure $R(t, T)$ merely is an output of the model. For the Hull-White model on the other hand, the long-term mean does depend on time which makes today's term structure an input for calculating the interest rate. In other words, Hull and White (1990) suggest a no-arbitrage extension of both the CIR and the Vasicek model. However the Vasicek extension is more commonly used in literature on mortgage valuation, and it is defined as follows:

$$dr = \kappa(\frac{\theta(t)}{\kappa} - r)dt + \sigma dz \tag{3.5}$$

The main difference to Vasicek's original model is the definition of the long-term mean, which now depends on time and is defined as

$$\theta(t) = \frac{\partial F}{\partial t}(0,T) + \kappa F(0,T) + \frac{\sigma^2}{2\kappa}(1 - e^{-2\kappa t}) \tag{3.6}$$

where $F(0,T)$ is the instantaneous forward rate. In Hull and White's original Vasicek extension, the mean-reversion rate also depends on time, but this version of the framework is barely used in practice.

Other Models

There are many other processes such as log-normal, multi-factor or multi-regime frameworks that potentially describe interest rate movements, but we omit reviewing all of them.

3.3.3 Model Selection

Which framework should we choose for simulating mortgage interest rate paths for the purpose of this research? Before answering this question, we must clarify which qualities we seek in an interest rate model.

- Since modeling mortgage rate is only a small part of this thesis, the resources to do so are fairly limited. Therefore, we seek an interest model that is generally simple to understand and implement.
- Second, we are looking for a framework that enables us to interpret and control its parameters intuitively, because we will simulate several interest rate scenarios.
- Third, we would like to achieve a certain degree of accuracy. However, the objective for high accuracy is not our primary concern: As mentioned previously, mortgage rate depends on a multitude of exogenous factors and the assessment of many such factors are out of our capabilities. For example, we will make very limited use of historical data to estimate interest rate parameters. The ability to control scenarios is in our case more important than whether or not these scenarios accurately match future reality.

Table 3.1 shows a brief comparison between the four models that were introduced in the previous paragraphs. Although the random walk is by far the easiest framework to use, it is not suitable for our purposes, since it is not possible to control the long-term trend at all.

Table 3.1: Comparison of interest rate models. Here, *RW* is the random walk and *HW* is Hull-White.

	RW	**CIR**	**Vasicek**	**HW**
Easy to use	✓	✓	✓	
Control		✓	✓	
No arbitrage				✓
Mean-reverting		✓	✓	✓
Negative rate possible	✓		✓	✓
Variability adjusts to interest rate level		✓		✓
Aims to match real market rate				✓

Since Hull-White has the potential to match actual market prices well and is furthermore unbiased in terms of historical mortgage rate, it first seems like a good choice for our purposes. However, fitting the Hull-White model to the current mortgage yield curve might result in over-fitting parameters. Furthermore, the parameters are dynamically intertwined in a way that is not easy to understand. For instance, a scenario with a certain assumed long-term mean - say three percent - is not easily implemented, because the long-term mean under Hull-White depends on the mean reversion rate and its development over time is not easy to control. This does not fit into our objective of conducting a scenario analysis. Furthermore, unlike the other models, Hull-White cannot be represented by a simple normal distribution. In conclusion, we decide against the high complexity and accuracy ambitions of Hull and White.

By now we have narrowed down the range of choices for appropriate interest rate models to CIR and Vasicek. Both are equilibrium models whose parameters are simple to understand and control. Furthermore, both have one critical characteristic that the other one lacks, respectively: Under CIR, volatility dynamically adjusts to the current interest rate level. However, negative interest rates are not possible under CIR, as the volatility simply approaches zero as interest rate approaches zero. Volatility under Vasicek on the other hand is constant and negative interest rates are possible. To choose between the two models, we must examine their two key differences more closely:

Negative interest rates: Until recent years it was common to assume that interest rates can generally not be negative, but today we know that rates below zero are in fact possible.[2] However, can mortgage rates become negative as well? To answer this question, we recall our knowledge about the two main components of the mortgage rate: the risk-free component and a spread. As we

[2]Depositing at the European Central Bank (ECB) currently yields -40 basis points and the rates are expected to remain on this level until at least mid 2019: `https://www.ecb.europa.eu/press/pr/date/2018/html/ecb.mp180726.en.html` Accessed: 26-09-2018

have just established, the risk-free rate can be negative, but we do not know where the floor value is. We further assume that a negative spread on top of the risk-free rate is not possible: In case of a spread below zero, it is always be preferable to invest in a risk-free asset instead of a mortgage. Thus, we draw two conclusions: First, the risk-free rate can be negative. Second, the mortgage rate is always above the risk-free rate. As we do not know how low the risk-free rate can sink and we cannot assess the minimum spread size other than making the assumption that it must have a positive value, we cannot find any evidence that negative mortgage rates are impossible.

Dynamic volatility: It is a common assumption that low interest rates are less volatile than high rates and the CIR model accounts for this circumstance, while Vasicek has no such mechanism.

The description of the two aspects shows that both CIR and Vasicek model have characteristics that we require to fit mortgage rate development to our view of reality. Therefore, we propose to use two interest rate domains which are modeled in a different manner, respectively. If interest rates are low, they follow a Vasicek process with sufficiently low volatility, while high rates are modeled using the CIR approach. As a consequence, volatility dynamically adjusts to the current level of mortgage rate and at the same time negative rates are not excluded. Formally:

$$dr = \kappa(\theta - r)dt + \sigma\sqrt{max(r,\zeta)}\,dz \tag{3.7}$$

Here, ζ is the threshold below which the interest rate is described in terms of the Vasicek model and above which it is modeled using CIR. Consequently, ζ becomes an additional parameter in the framework of the interest rate model. The values of the four parameters κ, θ, σ, and ζ will be determined in Chapter 5, where we present the methodology for our simulation study. Furthermore, Appendix A.1 provides an in-depth technical analysis of the model induced by Equation 3.7.

3.4 Expressing Mortgage Value

3.4.1 Defining Value

In short, the mortgage value at time t is equal to the net present value of all future cash flows until maturity T. Economic value of a mortgage determines the borrowers refinancing behavior, where the difficulty regarding the valuation procedure lies in the fact that the future cash flows are uncertain in the presence of a refinancing option. The goal of this section is understanding mortgage value in the presence of a penalty-free refinancing option.

3.4.2 The Refinancing Option

Let $B(r,t)$ be the net present value of the remaining cash flows of a mortgage without prepayment (or any other kind of) option, which is equivalent to the value of holding an option-free bond. Therefore, we assume that the underlying mortgage has no components that could bring uncertainty into the cash flows other than the refinancing option. Furthermore, let $O(r,t)$ be the value of the refinancing option. Then the overall value of the mortgage to the borrower is given by:

$$M(r,t) = B(r,t) - O(r,t) \tag{3.8}$$

The value of the refinancing option $O(r,t)$ consists of the following components:

- The *intrinsic value* of the option is represented by the difference between the contract interest rate and the current market interest rate. That is, the mortgage's future cash flows are determined using the contract and current market rate, respectively, and the difference between the net present values of the two cash flow streams constitutes the mortgage's intrinsic value.
- The *extrinsic value* (or *time value*) of the option refers to the benefit the option holder could gain when waiting with exercising instead of doing so immediately. The extrinsic value is an important component of our model, because it incorporates the fact that the option is American. Furthermore, the fact that the borrower may use the refinancing option only once majorly influences the option's time value. For example, if the mortgagor could refinance arbitrarily often, then the time value would vanish in the absence of refinancing costs and similar inefficiencies.

Figure 3.1 shows the mortgage value without refinancing option (B) and the mortgage value including the refinancing option (M). Therefore, M represents the function which values the mortgage for our purposes. The figure shows mortgage value at one particular point in time. The area that represents the option's time value is relatively large in the beginning of the mortgage contract, but decreases over time. For example, just before maturity the extrinsic value of the option - which reflects probability that one can refinance in an even more profitable manner in the future - is almost zero.

3.4.3 Formalizing Value

Let $M(r,t)$ with (r,t) in $[r^*, \infty) \times [0,T]$ be the value of the mortgage that we aim to determine, where r^* is the critical interest rate, below which refinancing is considered beneficial. The remainder of this section derives the PDE that describes M and the following section on boundary conditions provides an in-depth analysis of the properties of the valuation function.

To obtain the corresponding PDE, we use the equilibrium condition and Itô's Lemma, respectively, to define expressions for the expected return of the mortgage at maturity $\mu(r,T)$, which we will denote as μ_T. Since the resulting PDE is similar to the well-known Black-Scholes-Merton differential equation for option-pricing,

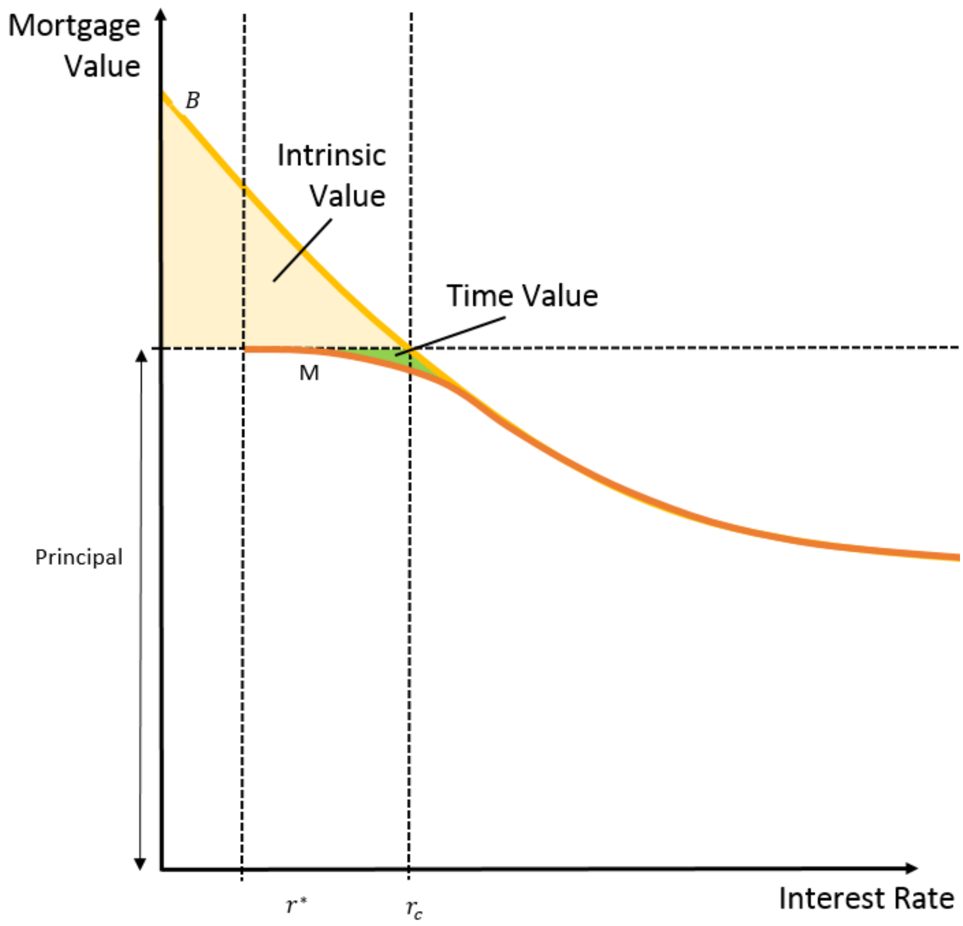


Figure 3.1: Illustration of the mortgage value, where the value of the refinancing option is subtracted from the option-free mortgage value (B) in order to get the value of the mortgage with refinancing option (M). r_c is the contractual interest rate and r^* is the critical interest rate, below which refinancing is considered favorable. The graph is a representation of an interest-only loan. If the underlying mortgage is linear or an annuity, then its maximum value is bound by the remaining debt instead of the principal.

we restrict ourselves to a brief, informal analysis without working out each step along the way. For an in-depth derivation of a bond-pricing PDE, the reader is referred to Brennan and Schwartz (1977).

Equilibrium Condition

The equilibrium condition constitutes the point where the risk-neutral expected value of the mortgage, which is the amount of remaining principal, is exactly equal to the mortgage's future cash flows discounted at the current interest rate. If the remaining principal is smaller than the current market value, then refinancing is profitable and if it is larger, then refinancing results in a loss. Returning to the notation that we introduced in the preceding sections, the equilibrium PDE that describes the value of the mortgage is:

$$\mu_T M = r(t)M \tag{3.9}$$

In words, the above equation states that the mortgage discounted at the expected interest rate, which is the contract rate, equals its current market value.

Itô's Lemma

Itô's lemma is the second argument we use to derive an expression for mortgage value. According to Itô's lemma, the mortgage value follows the stochastic process

$$dM = (\frac{\partial M}{\partial t} + \mu(r,t)\frac{\partial M}{\partial r} + \frac{1}{2}\delta(r,t)^2\frac{\partial^2 M}{\partial r^2})dt + \frac{\partial M}{\partial r}\delta(r,t)dz \tag{3.10}$$

where the drift rate is the expected total return over the mortgage at maturity. That is, it is given by the change in mortgage value over time ($\frac{\partial M}{\partial t}$), to which we add the value difference with changing interest rate ($\mu(r,t)\frac{\partial M}{\partial r}$). Finally, we take the non-linearity of the M curve into account by adding the second-order term of the Taylor expansion as well ($\frac{1}{2}\delta^2(r,t)\frac{\partial^2 M}{\partial r^2}$). Furthermore, we assume that the mortgage contract may be evaluated like a bond with continuous payment rate $c(t)$, which we add to the return. Here, $c(t)$ is defined by the contractual payment schedule. In summary, we evaluate the expected return on the mortgage using Itô's lemma as follows:

$$\mu_T M = \frac{\partial M}{\partial t} + \mu(r,t)\frac{\partial M}{\partial r} + \frac{1}{2}\delta(r,t)^2\frac{\partial^2 M}{\partial r^2} + c(t) \tag{3.11}$$

Assembling the PDE

Setting the two derived expressions for expected mortgage return equal to each other yields:

$$\frac{1}{2}\delta^2(r,t)\frac{\partial^2 M}{\partial r^2} + \mu(r,t)\frac{\partial M}{\partial r} + \frac{\partial M}{\partial t} - r(t)M + c(t) = 0 \tag{3.12}$$

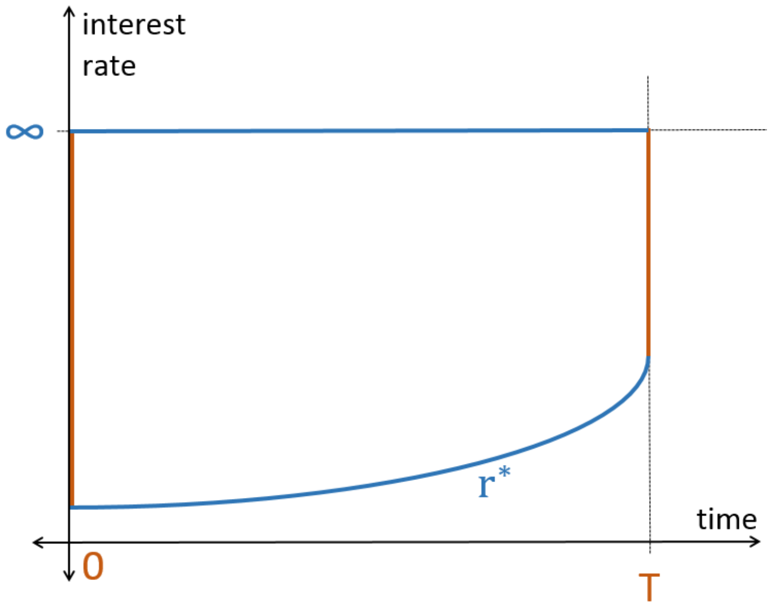


Figure 3.2: Illustration of the boundaries of the mortgage valuation PDE

The above PDE describes the characteristics of mortgage value, when it is dependent on interest rate and time. Using our interest rate model from Chapter 3.3.3, the PDE that describes the mortgage value M is given by:

$$\frac{1}{2}\sigma^2 max(r,\zeta)\frac{\partial^2 M}{\partial r^2} + \kappa(\theta - r(t))\frac{\partial M}{\partial r} + \frac{\partial M}{\partial t} - r(t)M + c(t) = 0 \tag{3.13}$$

Equation 3.13 describes the properties of the mortgage value M from Figure 3.1. However, to fully define the value of a mortgage with an prepayment option, fixing appropriate boundaries is required additionally. We recall that we defined the mortgage value in the box of (r,t) in $[r^*,\infty)\times[0,T]$. In order to find the shape of M, the mortgage value should be constrained in at least three directions, one in the time domain and two in the interest rate domain. The following section elaborates on the valuation function's behavior near the several boundaries.

3.4.4 Boundary Conditions

Equation 3.13 describes the general properties of $M(r,t)$, but in order to define our particular model, we also need to find the PDE's boundary conditions. Figure 3.2 illustrates the box in which the function is defined. The exact shape of the critical rate $r^*(t)$ will be defined in Section 3.5. In the following sections, we explore the three relevant boundaries.

Temporal Condition: Contract Maturity

Mortgage value diffuses backwards in time, so that the temporal condition is fixed at maturity T. The boundary condition in time is given by

$$M(r, T) = 0 \tag{3.14}$$

Equation 3.14 states that the value of the mortgage at contract maturity is always zero, because the principal has been fully repaid by that time.

Upper State Boundary: High Interest Rate

To define the first boundary condition in the interest rate domain we assess mortgage value under high interest rate. Mortgage value equals the net present value of future cash flows based on the current short-rate. Therefore, an increase in interest rate causes a decrease in mortgage value. For example, if the current short-rate became extremely high, then future cash flows would be worth extremely little. Therefore, the value of the mortgage becomes zero as the interest rate approaches infinity:

$$\lim_{r_{max} \to \infty} M(r_{max}, t) = 0 \tag{3.15}$$

However, using the concept of infinity is a weak formulation of a boundary condition.

Lower State Boundary: Low Interest Rate

The final boundary is located at the equilibrium interest rate below which refinancing is desirable. This critical interest constitutes the lower boundary r^* in Figure 3.2 and will be referred to as *refinancing threshold* for the remainder of this thesis. If the interest rate rises above the equilibrium point, then the mortgage value is described by the function M. If it is below the equilibrium point, then the borrower refinances and effectively resets the mortgage value to the principal amount.

Due to the option's time value, the refinancing threshold differs over time, which makes it a free boundary. More formally: The equilibrium call rate is bound at $M(r^*(t), t)$, where $r^*(t)$ is the refinancing threshold at that particular point in time. Referring to the shape of the mortgage value function in Figure 3.1, the final (free) boundary condition is:

$$\frac{\partial M}{\partial r}(r^*, t) = 0 \tag{3.16}$$

If chosen correctly, the equilibrium refinancing threshold $r^*(t)$ defines the mortgage rates which borrowers theoretically require to refinance at different times. The following section explores the methods to find the correct threshold over time.

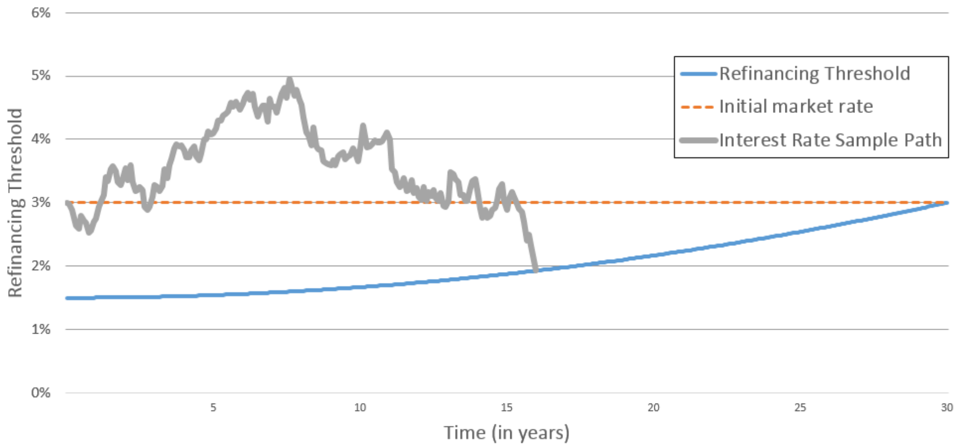


Figure 3.3: A sample interest rate path and the refinancing threshold

3.5 The Refinancing Threshold

3.5.1 Overview

We are now ready to draw conclusions about the exercise boundary to answer the first research question. Figure 3.3 shows an example of a potential threshold function. The idea is the following: Whenever the current market rate drops below the refinancing threshold, then the borrower refinances. Consequently, the function $r^*(t)$ fully defines the borrower's refinancing behavior. The example in Figure 3.3 shows a mortgage that is refinanced after 16 years and one month. If we knew the critical refinancing rates r^* for all points in time, then we knew the borrowers refinancing decisions and all that was left for us to do is calculating the lender's losses based on the resulting cash flows. However, we *do not* know which refinancing thresholds borrowers use.

Problem Statement

We recall the equilibrium rule, which states that a borrower should refinance whenever the current market value of the mortgage becomes smaller than the net present value of the active mortgage's future cash flows. Sometimes, waiting for even lower interest rates might be even more beneficial, even if the equilibrium rule is met. This incentive to wait is represented by the option's time value. In other words, if we wanted to know the critical point of refinancing, then we had to calculate the present value of future cash flows, which includes the appropriate time value, and compare the result to the outstanding principal. There are two major problems when attempting to solve this problem:

1. **Cyclic dependence:** The refinancing threshold depends on the net present value of future cash flows and simultaneously the net present value of future

cash flows depends on the refinancing threshold. That means, at the moment we wish to evaluate the mortgage, we do not know when the borrower chooses to refinance and hence we do not know if the future mortgage repayments are described accurately by the contractual terms or not. In short, at the moment of evaluation we do not know the future cash flows that are necessary to determine the refinancing threshold

2. **Failed reality check:** The model includes exactly one state variable, which is the interest rate r. Calculating the refinancing threshold as implied by the model is equivalent to assuming that mortgagors' refinancing decision only depend on interest rate and no other factors such as personal preferences have any influence on the decision. However, that is not true, as we learned in the literature review in the introductory chapter. Even when neglecting behavioral aspects for the moment, practical implications such as tax benefits and refinancing costs such as notary and advisory costs are not included in the model yet.

On Solving the Problems

Literature provides solutions on the problem of cyclic dependence. The issue could be resolved by simply calculating the model backwards: That is, we find the solution at maturity first and then work backwards through time until arriving at time zero. This way, at each point in time where we wish to evaluate the mortgage, we already know exactly what the future cash flows are. However, whether or not the borrower refinances at a certain point in time not only depends on the future cash flows, but also on the past refinancing decisions, as the latter influence the contractual interest rate at the moment of evaluation. In other words, the evaluation of the mortgage value at any point in time is interest-path-dependent. To solve this additional problem, the function M should thus now not only depend on interest rate and time, but also on the interest rate history.

Numerical procedures such as the decision tree concept or the finite difference method usually are suitable to solve the problems at hand, but would introduce a great deal of complexity into our model. Furthermore, it would not be able to address behavioral components. Therefore, we choose to define the approximate optimal refinancing threshold ourselves. This is done in the following paragraphs.

3.5.2 Endogenous Shape

Since solving the model is a lengthy task that requires additional assumptions and is out of scope for this research, we rely on results from other research to get a first indication of the correct general shape of the refinancing threshold. Figure 3.4 shows the relationship between the refinancing differential and time based on theoretical models similar to the framework proposed in Chapter 3. Examples of sources that confirm the shape of the theoretical refinancing threshold function are

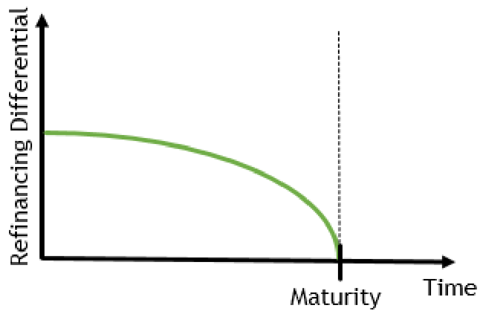


Figure 3.4: Relationship between the optimal refinancing differential and time

Figure 4 in Brennan and Schwartz (1977), Figure 4 in Harding (2000), Figure 17 in Zheng et al. (2012), and Figure 2b in Xie et al. (2017), who all solved the model on a purely theoretical basis.

The corresponding graph clearly suggests that the refinancing differential decreases over time. To find out why, we lean on our understanding of an option's time value, which was introduced in Chapter 3.4.2: In the beginning of a contract, the probability that there will be a very profitable refinancing opportunity in the future is relatively large. Therefore, the borrower requires a fairly high refinancing differential to be triggered to refinance immediately after start of the contract. However, at the end of the fixed-rate period the borrower knows that there is not a large chance anymore that the differential increases a lot - simply because there is not enough time for interest rate to make significant movements. Therefore, he or she accepts a relatively low differential at that point in time.

More formally, the time value of the option often outweighs the intrinsic value at the beginning of the mortgage contract. However, towards the end of the contract the time value approaches zero and thus becomes less significant when compared to the intrinsic value. Since the intrinsic value is positive as soon as the current interest rate falls just below contract rate, the required refinancing differential approaches zero at maturity.

Expressing the Shape as Function of Time

In order to be able to use the refinancing threshold, it has to be formalized into a function. As discussed, we merely know the approximate shape, but not which explicit expression represents it well. Agarwal et al. (2013) assumed that interest rate follows a random walk and subsequently derived an analytical expression for the refinancing threshold. However, the authors also assume that refinancing is possible as many times as the borrower desires. The purpose of their solution is providing a way to incorporate taxes, refinancing costs, and other prepayment into the refinancing threshold, while the time value is neglected. Since our very goal is capturing time value over time, the result of Agarwal et al. (2013) cannot be used.

Instead, we choose that the shape of the underlying threshold function that we observe in literature can be approximated by one quarter of an ellipse. The threshold function $r^*(t)$ for a 30-year mortgage with initial refinancing differential i basis points and initial market rate r_0 (excluding risk premium) is thus respresented by the following equation:

$$\frac{t^2}{T^2} + \frac{(r^*(t) - r_0)^2}{i^2} = -1$$

Here, time t represents the number of months, i is the initial differential that the borrower requires and T is the maturity of the fixed-rate period in months. Here, the value of i depends on several interest rate parameters, as well as the mortgage type Y. Solving for $r^*(t)$ yields:

$$r^*(t) = -i\sqrt{1 - \frac{t^2}{T^2}} + r_0 \tag{3.17}$$

The initial differential i depends on interest rate parameters and mortgage type. Its value will be determined in Chapter 5.8.

3.5.3 Fixed-rate risk premia

Theoretical models evaluate the mortgage value in a risk-neutral world, but we have to account for real-world risks associated with mortgages. Most importantly, mortgage initiators charge risk premia for long fixed-rate periods. For example, we assume a 30-year fixed-rate mortgage to have a higher rate than a ten-year fixed-rate mortgage at a fixed point int time, since committing for 30 years is more risky than committing for ten years. These differences have to be included into the refinancing threshold. The corresponding methodology is presented in the remainder of this section.

In Section 3.3.2, we chose to model the interest rate of a mortgage with a fixed period of 30 years. We now assume, that the rates of mortgages with other fixed-rate periods can be derived from the 30-year rate by subtracting a certain fixed-rate risk premium, respectively. These fixed-rate risk premia between mortgage rates with different fixed-rate periods are important because they determine what rate is available to a borrower who decides to refinance. For example, if a mortgagor decides to refinance a 30-year fixed-rate loan after 15 years, then he or she likely continues with a 15-year fixed-period mortgage in order to keep the overall life-span at 30 years. However, 15-year mortgages are less risky and thus the mortgage rates are generally lower than these of 30-year mortgages.

Table 3.2: Fixed-rate risk premia $f(t)$, where t is the time of refinance assuming a 30-year contract period. Each value in the right column corresponds to the difference between the 30-year rate and the rate available to the borrower at the corresponding time of refinance.

t (in years)	f (in bps)
$[0,5)$ years	-5
$[5,10)$ years	-15
$[10,15)$ years	-30
$[15,20)$ years	-60
$[20,25)$ years	-100
$[25,30)$ years	-100

Table 3.2 is based on the analysis in Appendix B and shows the fixed-rate risk premia compared to the original 30-year mortgage that we assume for the remainder of this thesis. For example, if a 30-year fixed-rate mortgage is refinanced after 21 years, then the mortgage rate that is available for the borrower to refinance is one percent lower than the 30-year rate at the time of refinancing. Figure 3.5 shows the theoretical refinancing threshold based on the refinancing differential from Figure 3.4, as well as the effects of the fixed-rate premia on the threshold. The refinancing threshold now becomes

$$r^*(t) = -i\sqrt{1 - \frac{t^2}{T^2}} + r_0 + f(t) \tag{3.18}$$

where $f(t)$ represents the fixed-rate premium.

3.6 Conclusion

In this chapter, we defined a mean-reversion process for the purpose of modeling mortgage rate development, we derived an option-theoretic framework for mortgage valuation, and finally approximated the optimal refinancing threshold for borrowers under the model. Respective threshold is given by Equation 3.18 and will be referred to as *theoretical refinancing threshold* for the remainder of this thesis. It serves as a starting point for the behavioral implications that Chapter 4 will introduce.

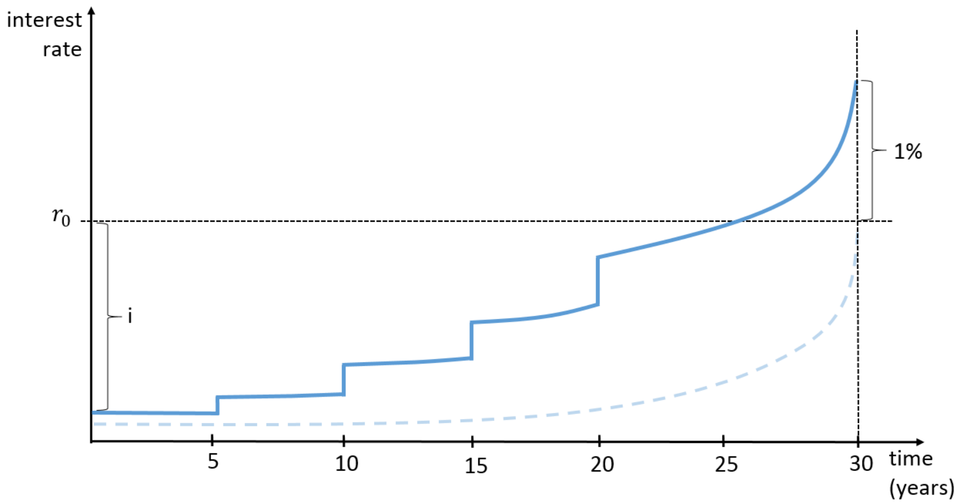


Figure 3.5: Effect of fixed-rate premia on refinancing threshold. Here, steps corresponding to the values in Table 3.2 are added to the theoretical threshold at the appropriate points in time. The initial refinancing differential is illustrated as parameter i and differs when interest rate parameters change.

Chapter 4

Behavioral Framework

4.1 Introduction

The theoretical refinancing threshold derived in Chapter 3 is based on mortgage rate and time, and it neglects any other factors that might influence the borrower's decision making process. The purpose of this chapter is the analysis of respective behavioral and other practical aspects to answer the first research question: *How can the borrowers' refinancing strategies be characterized?*

Our approach is structured as follows: In Section 4.2 the effects of refinancing costs and tax benefits are considered. In Section 4.3, we introduce several interest rate properties and how they might influence the borrower's refinancing behavior. In Section 4.4, we address the possibility that mortgagors with certain refinancing behavior might self-select into the penalty-free mortgage. Next, we consider the impact of having multiple refinancing opportunities instead of only one in Section 4.5. The implications on the refinancing threshold are summarized in Section 4.6.

4.2 Financial Considerations

Financial considerations that borrowers make besides mortgage payments include tax benefits and refinancing costs such as notary fees. Although these factors are not strictly *behavioral*, they are treated in this chapter because they merely change the mortgage value from the borrowers perspective and do not affect the profits or losses for the lender. In other words, while Chapter 3 considered a zero-sum game between borrower and lender, the financial considerations that we include in the behavioral model merely change the borrower's payoff. The fact that these costs differ among borrowers due to various reasons is another motivation for including tax benefits and refinancing costs in the behavioral instead of the theoretical framework.

4.2.1 Tax Benefits

First, we consider the Dutch tax system: If the mortgage is of linear or annuity type, then interest payments as well as prepayment penalties are tax deductible. Consequently, refinancing a mortgage due to lower mortgage rates likely leads to a reduction of tax advantage.

The applicable deduction rate varies among mortgagors: A mortgagor with a high income can deduct at a high rate, while lower income comes along with a lower deduction rate. Furthermore, house owners pay property tax in the Netherlands, which influences the deductible amount as well. We demonstrate the working principle of the relevant aspects of the Dutch tax system with a simplified example: A borrower owns a house with a value of €$300,000$ and pays €$2,000$ of property tax annually. Furthermore, the house has an outstanding mortgage value of €$300,000$, on which three percent interest rate is charged by the mortgage initiator, amounting to €$9,000$. Subtracting the property tax from the interest payment yields a final tax deductible amount of €$7,000$ annually. Applying the (future) maximally applicable 37.05% interest rate results in a net tax benefit of €$2,593.50$ on an annual basis.

The effective tax benefit decreases yearly along with outstanding debt until eventually reaching zero after typically about twenty years. Therefore, the tax system leads to a lower refinancing threshold, but the effect decreases over time and vanishes completely after about twenty years.

In conclusion, the a general expression for tax benefit is difficult to determine, because it depends on the borrower's income, the value of the underlying house, and remaining debt over time. Furthermore, the Dutch government is changing the maximally applicable deduction rate from 50% to 37.05% over the course of the coming years. Besides the obvious fact that this might change the refinancing behavior, this circumstance also indicates the presence regulatory uncertainty.

4.2.2 Refinancing Costs

Refinancing a mortgage requires effort and comes with costs. An example constellation of refinancing costs is: Advice (€300), notary documents (€800), closing costs (€900), tax report (€400), taxation reports (€800), bank guarantee (one percent of remaining debt). For instance, when the outstanding debt is €$150,000$, then the refinancing costs amount to €$4,700$, when using our example cost constellation. Refinancing costs influence the refinancing threshold more severely in later stages of the mortgage contract, because they become relatively large compared to the benefit of refinancing.

Another way refinancing costs can influence the exercise boundary is an affordability barrier: Even if interest rates became extremely low, it is possible that a part of the borrowers does not refinance because the refinancing costs are too high for them.

4.3 Interest Rate Properties

When considering the interest rate as a variable connected to refinancing behavior, we are used to taking the instantaneous short-rate - the current level of mortgage interest rate - into account. However, Alink (2002) suggests several properties of interest rate development that might influence the borrower's decision to prepay as well. These additional properties include the interest rate trend, the yield curve shape and the current rate in historical context. In the following paragraphs, we analyze the respective factors.

Trend: If the interest rate is falling, then borrowers tend to wait in the hope of even lower rates. On the other hand, rising interest rates increases the probability that mortgagors prepay quickly, given a sufficiently low overall interest rate level. To account for this circumstance, the refinancing threshold must not only depend on time, but also on historical mortgage rate.

Yield Curve: If the yield curve is steep, then interest rates are expected to rise in the future, which is for instance confirmed by Abraham and Theobald (1995). This circumstance makes borrowers eager to refinance today according to the proposition we made in the explanation of the *trend* variable. Also Alink shows that a steeper yield curve is connected to a higher prepayment rate, but does not find the correlation to be high. Furthermore, the option-theoretic work of Kau and Keenan (1995) shows that the term structure is an important variable for refinancing activity in particular. However, in this research we assume a constant yield curve which is observed in DMFCO's current mortgage rate (see Appendix B). Therefore, the yield curve is not included as factor influencing the refinancing threshold.

Historical Context: When interest rates are historically low, they receive increased media attention, which leads to higher prepayment rates. Just like the mortgage rate trend, the historical context implies the threshold's dependency on historical mortgage rate.

4.4 Profiling

Since the new penalty-free mortgage co-exists with regular mortgages with lower interest rates, we generally assume that borrowers who opt for the penalty-free product do so consciously. In other words, the group of mortgagors who hold a penalty-free mortgage have actively chosen to participate in holding the refinancing option in exchange for a certain risk premium and plan to exercise it when profitable. In the Netherlands, it is mandatory to consult a professional adviser before being allowed to enter a mortgage contract. A mortgagor who is not aware of the way refinancing might affect the mortgage's cash flows should not be advised to choose the penalty-free mortgage over the cheaper regular product.

Therefore, an individual who does not plan to participate in the refinancing option component of his or her mortgage is better off choosing the regular product with refinancing penalty, and we assume that every prospective mortgagor is aware of this fact.

4.5 Multiple Refinancing Opportunities

Suppose the new penalty-free mortgage is launched into the market and soon afterwards more mortgage originators start offering the same product to their customers. Borrowers suddenly have the opportunity to refinance not only once, but multiple times. The corresponding changes in borrowers' refinancing behavior are the focus of this section. For this purpose, we consider the implications of multiple refinancing opportunities in a single mortgage contract.

First, the time value of the option introduced in Chapter 3.4.2 would essentially disappear: Borrowers simply refinance whenever the available mortgage rate becomes lower than their contract rate, when neglecting refinancing costs and similar aspects for the moment. This is the equivalent to a variable-rate mortgage with the difference that variability only applies downwards and never upwards. The obvious implication for the lender in the described situation is potential full exposure to every single interest rate drop. However, refinancing costs, missing out on tax advantages, the effort that is required to refinance one's mortgage, and behavioral aspects are likely to prevent a borrower from refinancing too frequently in practice.

Furthermore, the characteristics of the borrower group opting for penalty-free mortgages might change. If only few providers offer the penalty-free mortgage, borrowers are able to self-select into the contract. Therefore, only those customers buy the product who either believe that interest rates will drop or who simply value the freedom of refinancing. However, AFM has majorly changed rules and guidelines in the past and we have to consider the possibility that prepayment penalties could be abolished from the Dutch market due to changes in regulation. In the latter case, mortgagors could not self-select into the product anymore, which potentially changes the refinancing behavior considerably.

4.6 Behavioral Refinancing Threshold

The following list summarizes the effects that we expect the discussed behavioral components to have on the refinancing threshold:

- *Refinancing costs* lower the threshold, especially in later stages of the contract.
- *Tax benefits* lower the threshold for linear and annuity mortgages, especially in early stages of the contract.
- *Mortgage rate trend* can cause the threshold to become higher or lower, depending on the trend's direction. Furthermore, the threshold now depends on historical mortgage rate.
- *Yield curve* shape can cause the threshold to become higher or lower.
- *The borrower's awareness*, which represents his or her willingness to use the refinancing option, can lower the threshold. Here, the effect is larger with low levels of awareness.
- *Other exogenous factors* can increase or decrease the threshold, but this effect is not easily determined without empirical data.

Since the level of uncertainty regarding refinancing behavior is very high, we cannot structurally adjust the refinancing threshold $r^*(t)$. Instead, we introduce a $N(m, s^2)$ distributed random component X into the theoretical threshold:

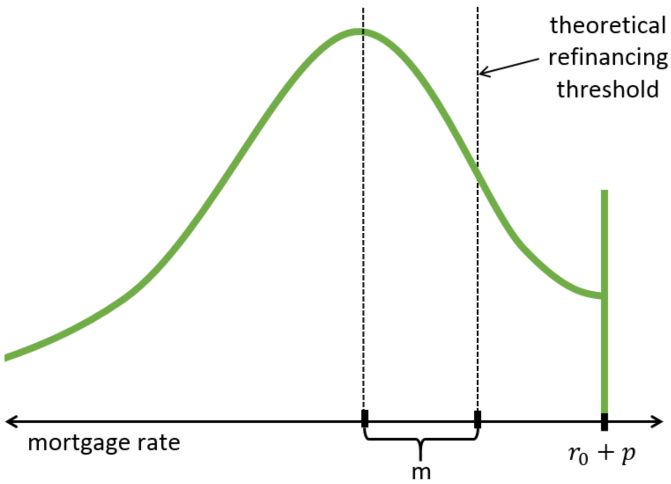
$$r^*(t) = -i\sqrt{1 - \frac{t^2}{360^2}} + r_0 + f(t) - X \tag{4.1}$$

In words, the refinancing threshold is now normally distributed with a mean equal to the theoretical refinancing threshold minus m and with standard deviation s. However, we do introduce the restriction that the borrower cannot refinance at a rate that is higher than the original contract rate $r_0 + p$. If the latter is the case, we simply set the refinancing threshold to the initial contract rate $r_0 + p$. The shape of the resulting distribution is illustrated in Figure 4.1. A mechanism to account for mortgage rate trend, which makes the refinancing decision dependent on historical rate, is left for future research.

4.7 Conclusion

We considered several behavioral and other practical implications that influence the difference between the theoretical threshold from Chapter 3 and mortgagors' real refinancing behavior. Since the behavioral component of the refinancing threshold is highly uncertain, we introduced a random component to the threshold. In conclusion, Equation 4.1 is the formalization of all considerations from Chapters 3 and 4 and is a quantitative representation of our answer to the first research question.

Figure 4.1: Distribution of the refinancing threshold. The standard deviation of the distribution is defined by the model parameter s. The left-hand side of the distribution is cut at the initial contract rate r_0+p.



Chapter 5

Simulation Methods

5.1 Introduction

The first research question was answered in Chapters 3 and 4. The remaining two research questions are *"What is the value of a fair risk premium on a mortgage without prepayment penalties?"* and *"Given the fair risk premium, what is the downside risk for the lender?"*. Using the insights of the previous chapters, we now introduce the simulation methodology that calculates the fair risk premium and potential losses. Specifically, we utilize the interest rate model from Chapter 3.3.3 and the definition of the refinancing threshold from Chapter 4.6 to simulate the lender's losses due to refinancing.

This chapter is structured as follows: In Section 5.2, we describe the idea behind the simulation methodology using examples and graphical illustrations. Next, we list our central assumptions in Section 5.3. In Section 5.4, the working principle of the algorithm is described in depth. In the following sections, we describe several different components of the simulation framework in detail. Finally, the model's limitations are addressed in Section 5.9.

5.2 Overview

To motivate our approach for solving the research problems, we first review important aspects of our model from Chapters 3 and 4. In Chapter 3, we formalized the mortgage valuation problem into the following PDE and indicative boundary conditions:

As shown in Chapter 3, the mortgage value is described by

$$\frac{1}{2}\sigma^2 max(r,\zeta)\frac{\partial^2 M}{\partial r^2} + \kappa(\theta - r(t))\frac{\partial M}{\partial r} + \frac{\partial M}{\partial t} - r(t)M + C(t) = 0$$

with

1. Temporal condition: $M(r,T) = 0$
2. Upper state boundary: $\lim_{r_{max}\to\infty} M(r_{max}, t) = 0$
3. Lower state boundary: $\frac{\partial M}{\partial r}(r^*, t) = 0$

The equations above roughly describe the properties of mortgage value, but the model is not strong enough to fully define the problem. It is possible to find stronger formulations for the boundary conditions and subsequently solve the set of equations by finite differences or binomial decision trees. Here, simulation approaches are rather complicated because of the mortgage value's dependence on the interest rate path. More specifically, the position of the lower state boundary usually depends on future mortgage values and future mortgage values depend on the boundary position. To simplify the matter at hand, we opted to estimate the refinancing threshold ourselves:

As derived in Chapter 4, the refinancing threshold is estimated as follows:

$$r^*(t) = -i\sqrt{1 - \frac{t^2}{360^2}} + r_0 + f(t) - X$$

with $X \sim N(m, s^2)$

The explicit definition of the shape of the refinancing threshold simplifies the solution procedure significantly: In the purely theoretical model, we must use the expression for mortgage value *after subtracting the option value* in order to calculate losses. In the model presented above on the other hand, the refinancing decision is determined by the predefined threshold function $r^*(t)$, which already includes the time value of the option. In other words, we merely have to find the intrinsic option value in order to calculate losses. Therefore, the model is solvable using a simple simulation approach. We use a Monte Carlo method to evaluate the mortgage at several points in time and several interest rates. The results give the mortgage value at time zero, which can directly be used to derive the loss.

Defining Profit and Loss

The goal of the simulation study is to find the premium for which the expected loss due to refinancing risk is zero. As explained in the introductory chapter, *loss* is

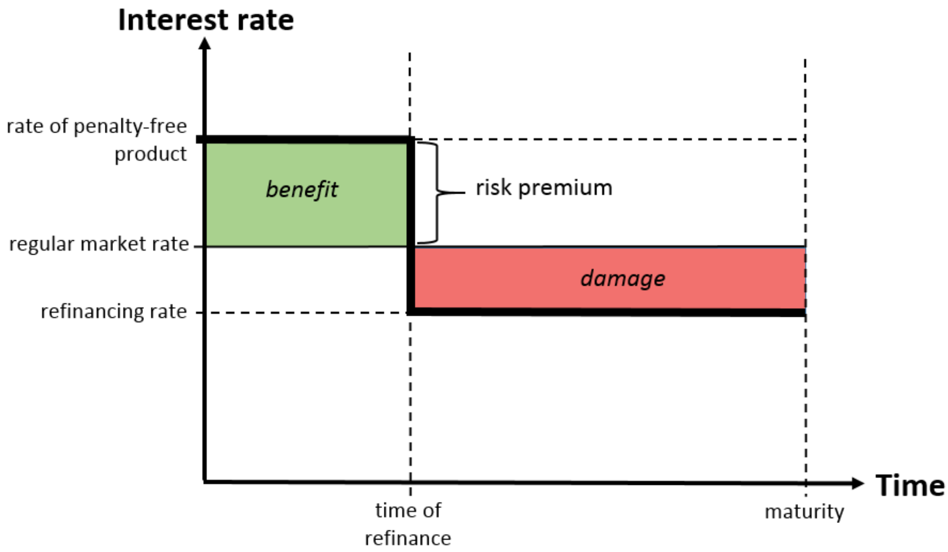


Figure 5.1: The green and red areas show the profit and loss of a penalty-free mortgage compared to a regular product, respectively.

defined as difference between the economic value of future cash flows including refinances and the economic value of the future cash flows of a regular mortgage with prepayment penalties. The latter is typically equal to the amount of outstanding debt. In the context of our simulation model, *economic value* is determined by discounting all cash flows at the initial mortgage rate for regular mortgages. That rate thus excludes the risk premium. We will frequently use the term *profit* as well, which is simply defined as negative loss.

Due to the risk premium charged, the cash flows before refinancing are higher for the new product than for a regular mortgage with prepayment penalties. However, the cash flows after refinancing might be lower than the payments of a regular mortgage, which results in damage for the lender. Figure 5.1 shows an example of the (non-discounted) benefit and damage of a penalty-free product compared to a regular mortgage. A fair risk premium is chosen in such a way, that the expected benefit is equal to the expected damage. For the remainder of this thesis, the terms *profit* and *loss* refer to the sum and negative sum of the benefit and damage shown in Figure 5.1, respectively.

Example Calculation

We use an example to illustrate the calculation procedure: Suppose we wish to evaluate the total profit or loss from an interest-only mortgage with a face value of €$100,000$ that matures in six months. The annual market interest rate for a regular mortgage is 3.1%, to which a risk premium of 0.5% is added. This implies monthly

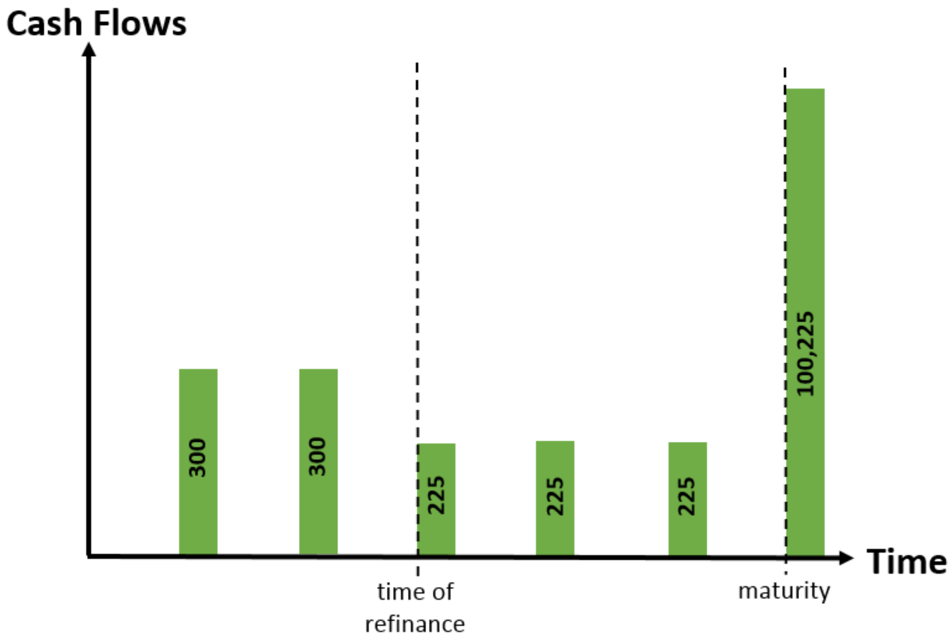


Figure 5.2: Cash flow illustration of a mortgage which is refinanced in the third month.

interest payments of €300. Suppose we also know that the borrower refinances at a market rate of 2.2% in the third month, which means that the lender originates a new mortgage at 2.7% at that point in time. That yields monthly interest of €225 for the remainder of the mortgage's life-span. The cash flows that correspond to this example are shown in Figure 5.2.

Discounting the cash flows from Figure 5.2 at 3.1%, the initial market rate of a regular mortgage, and summing up the results yields a mortgage value of €99, 951.22. As this is €49.78 lower than the mortgage's face value, the risk premium is close, but still not high enough for this particular instance of refinancing time and rate. In other words, the cash flows that the borrower pays to the lender over the course of six months are worth slightly less than the face value of €100, 000 that the lender pays to the borrower in the beginning of the contract. The profit in this example amounts to $-\frac{49.78}{100{,}000} = -0.05\%$.

Simulation Inputs and Outputs

We recall that the goal of the simulation is answering the second and third research questions by calculating the value and downside risk of the refinancing option. Option value is measured in terms of a risk premium p that the lender should charge on top of the regular mortgage rate. Downside risk is measured with the 95% expected shortfall $ES_{95\%}$, which is given by average loss based on the worst 5% of

Table 5.1: Inputs and outputs of simulation model

Type	Parameter	Definition	Minimum	Maximum
Input	Y	Mortgage Type	-	-
Input	θ	Long-term mean	$0\,\%$	$6\,\%$
Input	κ	Mean-reversion rate	$0.5\,\%$	$1.5\,\%$
Input	σ	Volatility	$0\,\%$	$1.2\,\%$
Input	r_0	Initial mortgage rate	$2\,\%$	$4\,\%$
Input	m	Deviation from theoretical threshold	$0\,\%$	$2.5\,\%$
Input	s	Standard deviation of $r^*(t)$	$0\,\%$	$2.5\,\%$
Input	i	Initial differential	$0\,\%$	-
Output	p	Risk premium	-	-
Output	$ES_{95\%}$	95% expected shortfall	-	-
Output	τ	Average time of refinance	2 months	T

simulated samples. Both outputs depend on the mortgage type Y, starting rate r_0, interest rate parameters θ, κ, and σ, as well as the behavioral parameters m and s.

Table 5.1 summarizes all input and output parameters of the simulation model and our choice of their respective minimum and maximum possible values. These ranges do not necessarily cover the full range of possibilities in the real world, but are important for the sensitivity analysis in Chapter 6. Besides reporting the risk premium and expected shortfall, the average time to refinance τ is included in the outputs as well.

5.3 Assumptions

Before presenting the in-depth working principle of the simulation model, we state our key assumptions:

- We work with risk-neutral probabilities and the model calculates the profit distribution rather than real probabilities. However, for the purpose of interpreting the simulation results, we assume that we can derive real probabilities from the profit distribution. For example, we assume that the 95% expected shortfall represents the expected loss with a probability of 5%.
- The interest model as defined in Chapter 3.3.3 describes the 30-year mortgage rate for a loan between 90% and 100% loan-to-value ratio. The mortgage rates for other fixed-rate-periods can be derived by subtracting respective fixed values from the 30-year rate (see Table 3.2). The domain threshold ζ has a constant value of 3% and does not change over simulation runs. The other three interest rate parameters can be subject to change and will be defined in Section 5.7.

- For the sake of simplicity, the only option in the mortgage contract is the refinancing option. Other prepayments and defaults are neglected.
- The model is discretized to time periods of the length of one month. Specifically, we assume that the mortgage rate only changes once a month. Furthermore, the execution of the call has a lag of one month, because we consider it improbable that a borrower can arrange necessary paperwork on the same day he or she decides to refinance.
- Suppose the borrower refinances at time t. To calculate losses, we assume that the lender is able to immediately re-invest the prepaid amount into a mortgage with a life-span of $T - t$ at a rate of $r(t) + p - f(t)$. In Section 5.5 we elaborate the reasoning behind this choice for the refinancing rate.
- Each mortgagor uses a deterministic refinancing threshold. That is, one instance of the random component of the refinancing threshold (see Equation 4.1) is valid for the entire duration of the mortgage contract. A new instance of refinancing threshold is only defined if we start a new simulation run with a new interest rate sample.

5.4 Algorithm Logic

One iteration of the simulation calculates the combined benefit and damage as shown in Figure 5.1 for a certain interest rate scenario. The general steps within one iteration are:

1. Generate a random interest rate scenario
2. Find the time of refinance based on the refinancing threshold
3. Calculate the profit or loss for the mortgage initiator

These steps are repeated several times and the average of the resulting profits equals the *expected profit*. We choose that the average profit resulting from the simulation should deviate less than 5% from the actual expected loss under the model at a confidence level of more than 95%. Appendix C shows why a number of 1,000 iterations per simulation run fulfills this requirement.

After 1,000 iterations, we have obtained the expected profit or loss for the lender for a certain risk premium. This procedure is repeated for several different risk premia in order to find the premium that results in an expected loss of approximately zero. To achieve this goal, we use a simple optimization approach, which is described in the remainder of this section.

First, the loss is calculated for zero risk premium. That is, we check whether a risk premium must be charged at all in order to avoid losses. If the profit has a negative value, then we increase the risk premium by 0.5% and calculate the new profit. This process is repeated until the profit is positive, which indicates that we stepped over the equilibrium risk premium. Next, the step size is decreased in order to search within the range of width 0.5% where we expect the equilibrium. This process is repeated until we find the appropriate risk premium, where the following step sizes are used, respectively:

Stage 1: Increase premium in steps of 0.5% until profit is positive.

Stage 2: Decrease premium in steps of 0.25% until profit is negative.

Stage 3: Increase premium in steps of 0.05% until profit is positive.

After finishing the necessary iterations on all three stages, the result is a risk premium that results in an expected profit of zero or just above zero. To clarify the logic of the algorithm, Appendix D presents a respective flow chart. The following sections elaborate on important components of the simulation in more detail.

5.5 Refinancing Rate

As stated in the list of assumptions in Section 5.3, we use a mortgage rate of $r(t) + p - f(t)$ to calculate cash flows after the moment of refinancing. This implies that the lender re-invests prepaid mortgages into the penalty-free product, and we neglect the possibility that the newly originated mortgage is refinanced again. Intuitively, this is problematic, because throughout this research we usually assume that the borrower does in fact use the refinancing option if it is beneficial for him or her.

Alternatively, we could let the lender could re-invest at a rate of $r(t) - f(t)$. This is true in either of the following cases: 1. The lender invests prepayments in regular mortgages with prepayment penalties or 2. the lender re-invests in penalty-free mortgages, where the premium p exactly compensates the loss due to refinancing of the new mortgage. However, the premium p compensates refinancing risk under the initial model parameters, but not under the parameters at the moment of refinancing. For example, the new starting rate r_0 could be lower than the original starting rate. Furthermore, the fixed-rate premium $f(t)$ might have changed over time. In conclusion, while the true refinancing rate is likely somewhere between the values of $r(t) - f(t)$ and $r(t) + p - f(t)$, we cannot know for sure. For this simulation implementation, we pick $r(t) + p - f(t)$ as refinancing rate from the lender's perspective. However, we stress that any other choice within the discussed range would have been feasible as well.

Table 5.2: Scenarios for the interest rate environment. The mean-reversion rate and volatility are given for the time period of one month, respectively.

Case	Central Assumption	κ	θ	σ
Historic	Historic rates are a good future predictor	1%	5.9%	0.645%
Expected	The long-term mean is moderately low	1%	4%	0.645%
Negative	The long-term mean is low	1%	2%	0.645%
Worst	The long-term mean is extremely low	1%	0%	0.645%

5.6 Interest Rate Distribution

The interest rate scenario is modeled using the framework presented in Chapter 3.3.3. Here, the interest rate in month t is normally distributed with expectation

$$E[r(t)] = r(0)e^{-\kappa t} + \theta(1 - e^{-\kappa t})$$

and variance

$$Var[r(t)] = \begin{cases} \frac{\zeta^2\sigma^2}{2\kappa}(1 - e^{2\kappa t}), & \text{if } r(0) < \zeta \\ \frac{\sigma^2 r(0)}{\kappa}(e^{-\kappa t} - e^{-2\kappa t}) + \frac{\theta\sigma^2}{2\kappa}(1 - e^{-\kappa t})^2, & \text{otherwise} \end{cases}$$

The derivation of these expressions is presented in Appendix A.1.

5.7 Interest Rate Base Scenarios

We choose to simulate four different interest rate scenarios, where a scenario represents certain underlying assumptions about future mortgage rate development. It is defined by the values of the three modeling parameters κ, θ, and σ. As mentioned in the assumptions, the domain threshold ζ is fixed at 3% and not considered a variable. Table 5.2 shows our specific parameter choices per scenario and the following list elaborates on each of the four cases.

Historic Case: The historic case represents our best guess for the future interest rate environment based on historical mortgage data between 1999 and 2018. To understand why we consider these particular parameters to represent the historic case, the reader is referred to Appendix A. The respective appendix contains a technical analysis of the interest rate model, a description of the historical data used, and finally an estimation of interest rate parameters.

Expected Case: This scenario assumes that the long-term mean is around 4%, while all other parameters are equal to the estimation in Appendix A. As mentioned previously, the ECB is expected to increase their rates in the medium-term. Therefore, the new choice for the long-term mean of 4% approximately equals the current value for a mortgage with a 30-year fixed-rate period of 3% after adjusting it slightly to the upper side to account for the expectation of rising rates. Consequently, the initial upward "pull" of mortgage rate is less strong than in the historic case. We refer to this scenario as *expected case* because we assume that it has a high likelihood of occurring.

Negative Case: The negative scenario represents the middle ground between the expected and worst case, where the long-term mean is assumed to be 2%.

Worst Case: Since scenarios with very low interest rates are of particular interest to our case, we also create a *worst case* scenario, where the long-term mean of the 30-year mortgage rate is zero. This choice of parameters helps assessing the downsides of the new mortgage product. When the 30-year mortgage rate with high loan-to-value ratio is zero, then the risk-free rate is likely far below zero. Since we expect that the probability of this scenario is extremely low, we treat it as absolute lower threshold for future interest rate development.

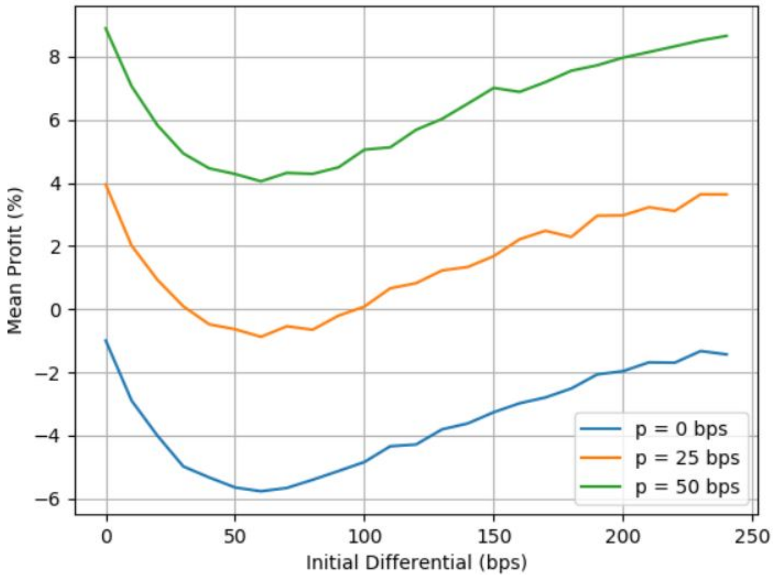
To illustrate possible mortgage rate developments, Appendix A.4 shows two examples of interest paths over 30 years under different scenarios.

5.8 Initial Differential of Exercise Boundary

The initial differential i measures the difference between the initial refinancing threshold and the initial regular mortgage rate r_0, which excludes the risk premium p. Although the general shape of the refinancing threshold is known (see Figure 3.5), the initial refinancing differential i still has to be calibrated. It depends on the mortgage type, starting rate, and all parameters of the interest rate model, since those factors influence the best choice for the theoretical refinancing threshold. In conclusion, the initial differential must be calculated for each interest rate scenario and mortgage type, where a correctly chosen refinancing threshold minimizes the profit for the lender.

Figure 5.3 shows the expected profits for different values of i at the example of a interest-only mortgage under the expected interest rate scenario from Section 5.7. As shown the figure, the correct initial differential equals 60 basis points. Table E.1 in Appendix E shows which values for i minimize the expected profit for all combinations of interest rate scenario and mortgage type. An additional differentiation based on the starting rate r_0 is omitted, because the starting rate has a known value in the beginning of a contract. The initial differential i should be re-calibrated in future implementations of the simulation if the starting rate has changed significantly.

Figure 5.3: Calibration of initial differential i at the example of a linear mortgage and the expected interest rate scenario. The vertical axis shows the average profit based on $1,000$ interest rate instances. The tested values for i are discretized at steps of 10 basis points.



5.9 Limitations

In this section, we discuss limitations of the simulation model, as well as possibilities to improve future implementations.

Refinancing Threshold

Our daring assumption about the shape of the refinancing threshold is a major concern regarding validity. Namely, we must wonder whether the assumed threshold in the shape of a quarter arch is a strong approximation of the implied solution of the complete option-theoretic free boundary problem. While the function approximates theoretical and empirical shapes we found in literature, we cannot validate our choice for the quarter arch from a theoretical perspective. Furthermore, the steps in the threshold are based on the current yield curve of MUNT's mortgage rates, which by no means must stay constant in the future: both risk-free yield curve and mortgage spreads can be subject to change.

Modeling Interest Rate

To claim that mortgage rate is well represented by a mean-reversion stochastic process constitutes a questionable assumption, as explained in Chapter 3.3.2. A better model for mortgage rate - for example one including macro-economic variables - has the potential to greatly improve the validity of the simulation method.

Different Fixed-Rate Periods

The simulation is based on a mortgage with 30 years fixed interest rate period, while contracts with 10 years or 20 years fixed-rate periods are very popular in practice as well. Once a lender chooses a specific simulation methodology with input parameters, he or she should be able to calculate the correct premium for other fixed-rate periods as well. However, the simulation currently merely provides indications for fair risk premia for 30-year fixed-rate periods. Furthermore, the damage after refinancing as shown in Figure 5.1 is calculated using the rate $r(t) + p - f(t)$. This implies that risk premia must be equal across different fixed-rate periods under the model and makes it difficult to calculate other fixed-rate periods separately.

Technical Improvements

The risk premium is calculated in discrete intervals of five basis points and based on only 1,000 samples. There is much potential to obtain better results by simply decreasing the minimum basis point interval and running more samples. However, this would increase the computational cost of running the simulation considerably. Since both the interest rate model and the refinancing threshold function are merely approximations to begin with, the simulation outcome can merely provide indications of the correct premium and downside risk. Therefore, we decided to not increase the sample number or decrease the risk premium intervals in this thesis. However, this could be done in future research.

Furthermore, the current model's search methods are discrete and relatively inefficient: the initial differential is estimated graphically and the risk premium is calculated by a simple step method. Both the calibration of the initial differential and the subsequent optimization of the risk premium can likely be achieved much quicker using a better optimization algorithm. However, since we take samples rather than calculate exact solutions, the functions are not strictly unimodal. This makes the implementation of a simple algorithm, such as a golden-section search, challenging. Defining an efficient optimization algorithm that can deal with small errors in the solution is thus a subject for further research.

5.10 Conclusion

We introduced a simulation model for calculating the fair risk premium on a penalty-free mortgage, as well as its downside risk. Therefore, we created the tool that will enable us to answer the second and third research question. The simulation results are presented in the following chapter.

Chapter 6

Results

6.1 Introduction

This chapter answers the research questions *"What is the value of a fair risk premium on a mortgage without prepayment penalties?"* and *"Given the fair risk premium, what is the downside risk for the lender?"*. For this purpose, the simulation derived in Chapter 5 is executed for several different sets of input parameters. In Section 6.2, we define and simulate a base scenario, while we provide a sensitivity analysis with respect to all input parameters in Section 6.3. We proceed by considering uncertainty of input parameters in Section 6.4. Finally, we interpret the results to answer the research questions in Section 6.5.

6.2 Base Scenario

We define the following base case regarding simulation input parameters:

- The interest rate parameters correspond to the expected case, as defined in Chapter 5.7. That is, $\theta = 4\%$, $\kappa = 1\%$, and $\sigma = 0.645\%$.

- The starting mortgage rate is $r_0 = 3\%$. This value approximately equals the tariff of a mortgage with 30 years fixed-rate period and a loan-to-value ratio of more than 90% as of mid-2018.

- The loan is interest-only. While annuities are far more popular than interest-only mortgages, the impact of refinancing is largest for the interest-only type. We consider the resulting risk premium an upper bound for all other mortgage types. In practice, the interest-only part of a mortgage is restricted to 50% of the collateral's value.

- Borrowers use the theoretical threshold. That is, $m = 0$ and $s = 0$. These parameter settings result in upper bounds for the appropriate risk premium as well. In other words, the base case assumes that borrowers act *"optimally"* under our model from Chapter 3.

Table 6.1: Theoretical simulation results with $m = 0$, $s = 0$ and $r_0 = 3\%$. As described in Chapter 5.7, different interest rate scenarios are characterized by different values of the long-term mean θ. The simulation calculates the risk premium p, the expected shortfall $ES_{95\%}$, and the expected time until refinancing takes place, denoted as τ.

Interest Rate Scenario	Mortgage Type	i (bps)	p (bps)	ES (%)	τ (years)
Historic	Linear	10	10	1.99	11.16
Historic	Annuity	10	15	1.58	10.28
Historic	Interest-Only	20	15	3.99	17.18
Expected	Linear	50	25	3.70	12.20
Expected	Annuity	50	25	4.40	12.32
Expected	Interest-Only	60	30	8.05	14.17
Negative	Linear	120	60	5.62	8.96
Negative	Annuity	120	65	5.70	8.79
Negative	Interest-Only	150	85	10.58	10.82
Worst-case	Linear	210	110	6.75	7.40
Worst-case	Annuity	210	115	6.90	7.36
Worst-case	Interest-Only	270	150	13.44	10.23

Simulating the base scenario yields an appropriate risk premium of 30 basis points, where the 95% expected shortfall is 8.05%. The latter value is equivalent to approximately 40 basis points lower annual yield than a regular interest-only mortgage with penalties.[1] The expected time of refinance is in the 15th year.

6.3 Sensitivity Analysis

6.3.1 Long-Term Mean and Mortgage Type

Table 6.1 shows the simulation results assuming a starting market rate of $r_0 = 3\%$, no structural behavioral deviation from the theoretical refinancing threshold ($m = 0$), and no variability in the refinancing threshold ($s = 0$). The initial differential i of the exercise boundary is calibrated for each simulation run as described in Chapter 5.8. Since borrowers minimize mortgage value, these results represent upper bounds for the risk premium that is required to compensate refinancing risk in the respective interest rate environment. The following paragraphs provide additional description of the impact that the long-term mean and mortgage type have on the results.

[1] Discounting the cash flows of a 2.6% interest-only loan at 3% yields a present value of 92%.

Mortgage Type: When all other input variables are held constant, the fair risk premium of a linear mortgage is always smallest, because its payment schedule reduces the outstanding debt (and thus exposure to refinancing risk) fastest. The risk premium of annuity mortgages is slightly higher, but in our discrete model often has the same value as a linear mortgage's premium. The premium of an interest-only mortgage on the other hand is around $30 - 40\%$ higher than that of an annuity or linear loan. The expected shortfall of an interest-only mortgage is approximately twice as high than that of a linear or annuity loan. Finally, interest-only mortgagors refinance later than others, because the high remaining debt ensures profitability even in later stages of the mortgage contract.

Interest Rate Scenario: Since the predefined interest rate scenarios only differ with respect to the equilibrium mean θ, Table 6.1 shows that higher θ generally yields higher risk premium. Specifically, we find that the risk premium increases from 15 basis points for $\theta = 5.9\%$ to 110, 115 or 150 basis points for $\theta = 0\%$, depending on the mortgage type. Moreover, higher values of θ are associated with shorter times until refinancing.

Next, we assess the meaningfulness of the expected time until refinance τ. Appendix G shows histograms of the time until refinance for different interest rate parameters. If the mortgage rate is expected to rise (high θ), then it is very likely that the borrower refinances either in the beginning of the contract or not at all (see Figure G.1). However, the lower the long-term mean theta, the more likely it becomes that the borrower refinances in the middle of the contract, close to the value of τ in Table 6.1.

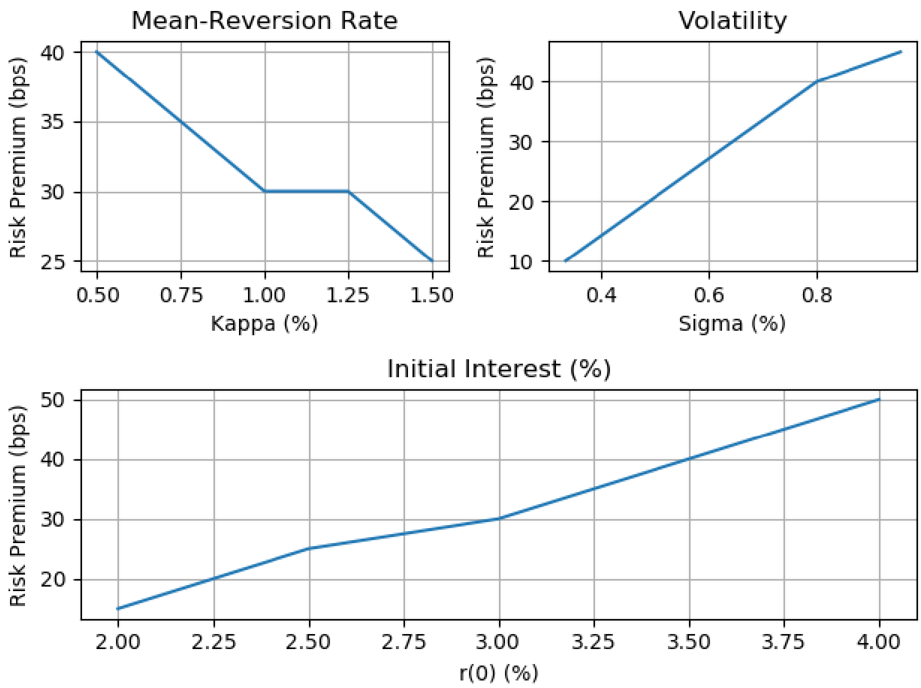
6.3.2 Volatility, Mean-Reversion Rate, and Starting Rate

Next, we explore the result's sensitivity with respect to the volatility parameter σ, mean-reversion rate κ, and starting rate r_0. Figure 6.1 shows the sensitivity of the fair risk premium with respect to these parameters.[2] The following paragraphs briefly evaluate the results.

Starting Rate: The risk premium falls from 50 basis points at $r_0 = 4\%$ to only 15 basis points when $r_0 = 2\%$. Therefore, the starting rate has a relatively large influence on the fair risk premium. However, unlike the other model parameters, the starting rate is certain at the point of originating a mortgage. Therefore, the high sensitivity does not pose a major problem: In practice, the risk premium for new penalty-free mortgages will simply have to be reevaluated over time. Furthermore, the loss is diversified over time when mortgages are originated at different levels of market rate.

[2] Note that these results are less accurate than the outcomes presented in Table 6.1, because the initial differential i is not newly calibrated for every single parameter change anymore. Instead, an initial differential of $i = 0.6\%$ is used for all simulation runs.

Figure 6.1: Sensitivity of required risk premium with regards to several model parameters



Mean-Reversion Rate: The faster mortgage rate reverts around the long-term mean, the lower the required risk premium, because there is more certainty about mortgage rate rising from 3% to the 4% level. Of course, if the relationship between the long-term mean and the starting rate changes, the resulting sensitivity graph for mean-reversion rate is subject to change.

Volatility: The premium increases with increasing the model's volatility parameter: For $\sigma = 0.335\%$, the risk premium amounts 10 basis points and increases up to 45 basis points for $\sigma = 0.955\%$. The effect constitutes the opposite to the mean-reversion rate, because higher volatility increases uncertainty in the level of future rates.

6.3.3 Behavioral Parameters

Finally, the respective impact of the behavioral parameters m and s is assessed. We recall from Chapter 4 that these variables characterize the deviation from the theoretical refinancing threshold. Table F.1 in Appendix F shows simulation results for different values for m and s. Increasing both m and s from 0% to 1% cause the risk premium to decrease by 15 basis points. While behavioral components seem not to have a large effect on the risk premium and expected shortfall, the expected time until refinance becomes longer with larger m and shorter with larger s, where the effect of m weights more heavily.

6.4 Uncertainty of Input Parameters

Expected shortfall as calculated in the previous section does not capture the losses that might occur when the model parameters have been misjudged. For instance, if we expect the input parameters to correspond to the base case, then the previous simulation results suggest that a risk premium of 30 basis points should be charged. However, if the future development of interest rates takes place in a different environment than expected - for example in the worst-case scenario or in a high-volatility environment - then the expected shortfall calculated previously is not applicable anymore.

In this section, we assess the effect of uncertainty in the long-term mean θ, volatility σ, and the two behavioral parameters m and s. The sensitivity of expected profit and expected shortfall with respect to these parameters is shown in Figures 6.2 and 6.3. Here, all remaining input parameters correspond to the base case from Section 6.2. This includes setting the initial differential to $i = 0.6\%$ and the risk premium to $p = 0.3\%$. The following paragraphs describe the results.

Long-Term Mean: Figure 6.2a shows the expected profit and shortfall as function of the long-term mean θ. The expected profit is small for low long-term means θ and becomes larger for high θ's in an s-shaped curve. Furthermore, the spread between expected profit and expected shortfall increases with larger long-term mean. The graph shows that losses can amount to more than 9% under the worst-case interest scenario.

Volatility: Figure 6.2b shows the expected profit and shortfall as function of volatility σ. Here, larger σ's causes higher losses. In the extreme case of $\sigma = 1.2\%$, the expected shortfall amounts to more than 10%.

Reduction of theoretical refinancing threshold: Figure 6.3a shows losses when reducing the theoretical refinancing threshold by m. The expected profit rises with larger m. The expected shortfall on the other hand becomes maximal at a reduction of the refinancing threshold of about 60 basis points, before it starts to move closer towards the expected profit. These extreme losses can amount to more than 12.5%.

Variability of refinancing threshold: Figure 6.3b shows profits as function of variability s in the refinancing threshold. The expected profit behaves similarly to the graphs in Figure 6.3a. The expected shortfall on the other hand moves much more slowly towards a positive profit for the lender. Even when the refinancing threshold has a standard deviation of 2.5%, the expected shortfall is still more than 6%.

6.5 Conclusion

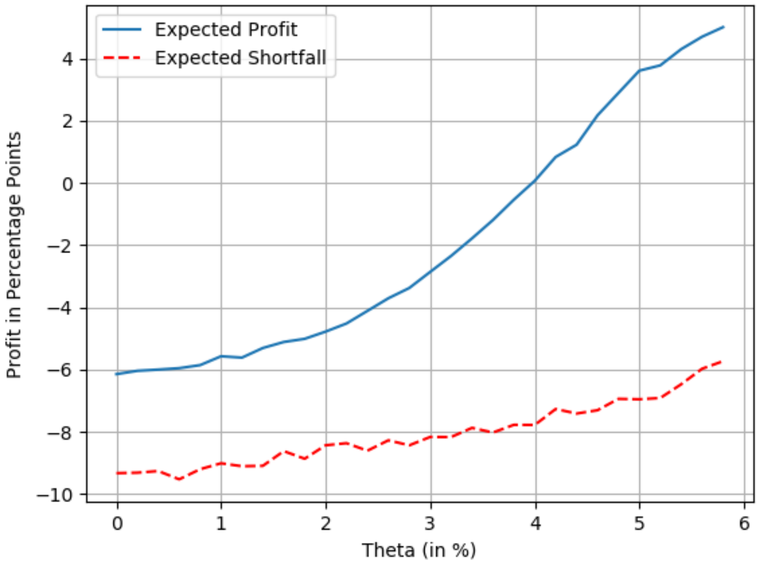
This chapter presented the results of the simulation study and addressed the result's sensitivity with respect to the underlying assumptions.

Risk Premium

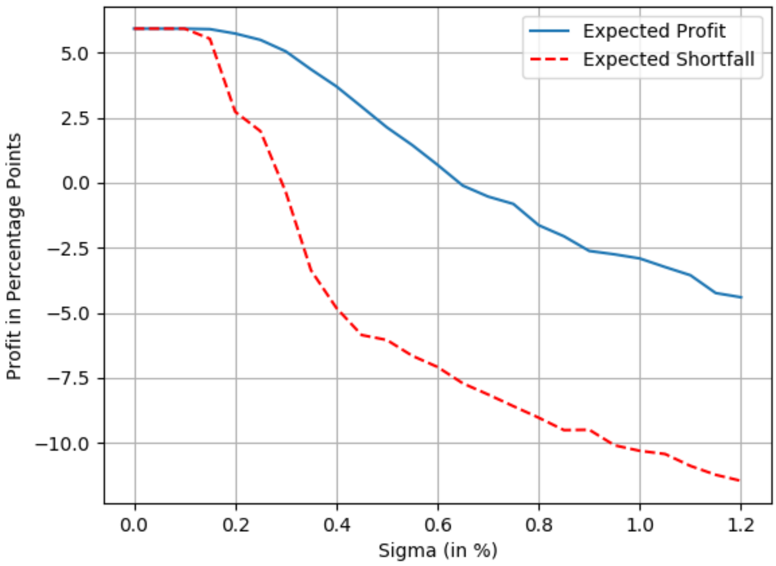
The second research question is *"What is the value of a fair risk premium on a mortgage without prepayment penalties?"*. Using our model, premia as small as 10 or as large as 150 basis points can be appropriate, depending on the payment schedule, the starting rate, assumptions about future mortgage rate development, and how borrowers deviate from the deterministic part of the refinancing threshold. However, the risk premium under the base case is 30 basis points.

Expected Shortfall

The third research question is *"Given the fair risk premium, what is the downside risk for the lender?"*. We measure worst-case losses in terms of the 95% expected shortfall and find that the downside risk is around 8% under the base case and with a risk premium of 30 basis points. As comparison, the profit of this mortgage in

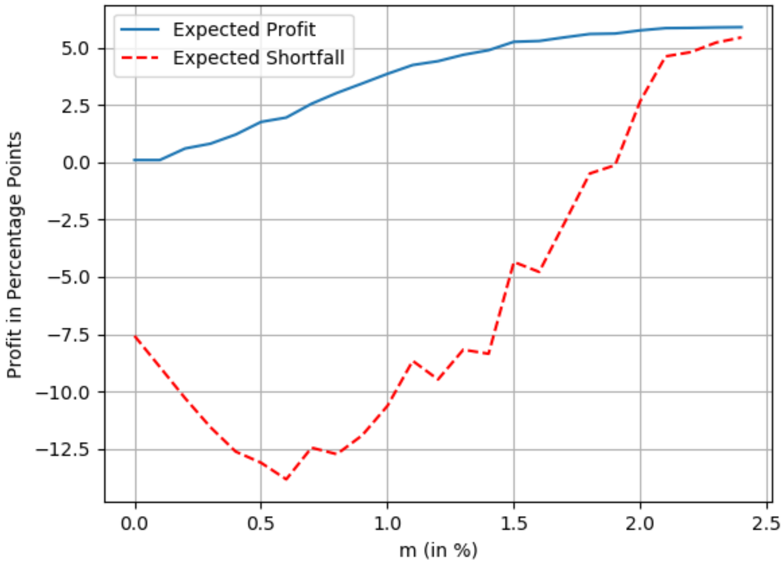


(a) Profit measures as function of θ

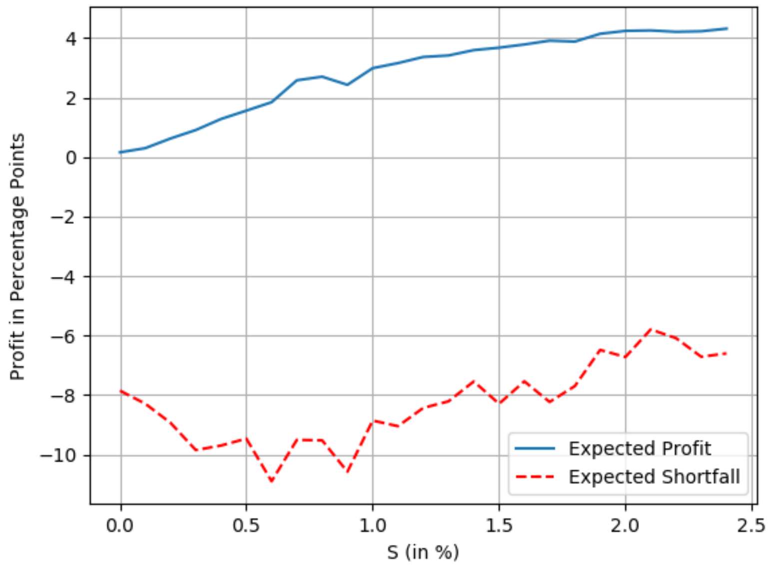


(b) Profit measures as function of σ

Figure 6.2: Expected profit and shortfall as a function of long-term mean and volatility, where a risk premium p of 30 basis points an an initial differential i of 60 basis points are used. The remaining parameters are set according to the base case.



(a) Profit measures as function of reduction m in theoretical refinancing threshold



(b) Profit measures as function of variability s of theoretical refinancing threshold

Figure 6.3: Expected profit and shortfall as a function of behavioral parameters, where a risk premium p of 30 basis points an an initial differential i of 60 basis points are used. The remaining parameters are set according to the base case.

case of no refinancing is 5.9%.[3] In the worst-case scenario we simulated expected shortfalls up to 13.4%. On average, the losses for linear and annuity mortgages amount to approximately half of the losses of interest-only loans, given their appropriate risk premium.

Uncertainty

Finally, we considered uncertainty in assumptions about interest rate parameters: When choosing a risk premium of 30 basis points for an interest-only mortgage, the expected shortfall becomes more than 9%, if the long-term mean of mortgage rate turns out to be 0% (see Figure 6.2a). Unexpectedly large values for the volatility parameter σ have a similarly severe impact on the expected shortfall. Deviating from the theoretical threshold by introducing behavioral parameters m and s on the other hand do not have a negative effect on the expected profit. The expected shortfall on the other hand can become higher when setting m and s to positive values (see Figure 6.3).

[3] Discounting the cash flows of a 3.3% interest-only mortgage at 3% yields a net present value of 105.9%.

Chapter 7

Conclusions

7.1 Contributions

In this research, we provide methodology to evaluate the refinancing option of Dutch residential mortgages without prepayment penalties. The main contributions are summarized as follows:

1. *Development of a new interest rate model.* To overcome the shortcomings of existing short-rate processes, we merged the Cox-Ingersoll-Ross and Vasicek frameworks into a new model for the purpose of describing mortgage rate development.

2. *Explicit approximation of the interest rate threshold that mortgagors use to refinance.* We used results from literature, as well as our own extensive analysis to define the exercise boundary of the refinancing option.

3. *A simulation model that evaluates the refinancing option of a penalty-free mortgage and the lender's downside risk.* Here, the corresponding option value is expressed in terms of a risk premium that the lender should charge on top of the regular annual mortgage rate, while downside risk is expressed in terms of expected shortfall. Furthermore, the wide range of flexible input parameters gives the user much control over assumptions about interest rate environment and behavioral aspects.

7.2 Simulation Results

Under the base scenario for input parameters, we calculated a fair risk premium of 30 basis points, while the downside risk for the lender is given by a potential loss of economic value of 8%. This loss is approximately equivalent to reducing the annual yield of a comparable regular 30-year mortgage by 40 basis points. When altering the set of underlying assumptions in appropriate ranges, the fair risk premium can obtain different values between 10 and 150. These are approximately in line with the range of risk premia found in literature, as presented in Table 2.1. Since the results are highly sensitive to the assumptions about future interest rate development and behavioral aspects, the exact value of the refinancing option of a penalty-free mortgage cannot be determined.

7.3 Recommendations

In this section, we assess whether the penalty-free mortgage should be offered in the Dutch residential mortgage market and if so, at what price. For this purpose, we consider potential ethical issues and general product attractiveness.

A penalty-free mortgage comes with the possibility for lenders to charge any risk premium they find appropriate. However, a central characteristic of the current penalty system is the fact that the fine in the event of refinancing cannot exceed the lender's economic loss. However, when integrating the same refinancing risk into the mortgage rate, the amount the lender charges in exchange for the refinancing option becomes much less transparent. The lender could now charge more than the true expected economic loss. This could be a big issue when a single mortgage provider offers a regular mortgage in parallel to a penalty-free loan, where the latter has a higher mortgage rate. In future research, it should be examined how the Dutch mortgage market should deal with this potential problem.

Furthermore, a potentially successful mortgage product must be interesting to both investors (lenders) and the mortgagor market. In other words, there must be borrowers who are willing to pay the premium in exchange for the refinancing option and there must be investors who accept the increased return in exchange for refinancing risk.

Borrower's Perspective: The penalty-free mortgage is attractive to borrowers who believe that interest rates will decrease in the future. Furthermore, the product is interesting to mortgagors who simply value the flexibility and feeling of freedom it can provide, even if they do not have a vision on future price development.

Lender's Perspective: While the borrower is the equivalent to a buyer of a call option, the lender is the option writer and thus bears all the risk. There is a lack of upside potential, while downside risk is (theoretically) unlimited. Putting it in simple words, the lender's payoff scheme of a penalty-free mortgage is: *Either win a little or lose a lot*. Since there are severe shortcomings in terms of diversification, believing that the probability of winning is much larger than the probability of losing cannot be a decisive argument to start originating penalty-free mortgages. Therefore, the lender's perspective requires further analysis in the following paragraphs.

The premium is calculated for risk-neutral originators, but since mortgages are widely considered conservative investments, lenders might tend to be more risk-averse than risk-seeking. This leads to the problem, that the theoretical risk premium might be insufficient to compensate mortgage investors. If that was the case, the calculated risk premium had to be adjusted to account for low willingness to take risk. In other words, a *"risk-aversion premium"* could be part of the price on top of the risk-neutral premium. However, the resulting total risk premium would not be fair towards the borrower anymore.

Another possibility to solve the problem for the lender is hedging against low interest rate. However, approaching the problem from a hedging-perspective is a potential topic for future research. On the other hand, if respective risk cannot be hedged, then more research into the future development of interest rates is required before recommending investing in penalty-free mortgages. If both solutions are not possible, the contract conditions themselves could be re-designed in order to limit losses.

Multiple Refinancing Opportunities

Finally, we recall that borrowers have to refinance externally in order to prevent that the refinancing option can be used more than once. Therefore, a mortgage provider's market share could be seriously exposed, depending on the number of originated penalty-free mortgages. However, this issue has less impact, if more mortgage providers offer a penalty-free mortgage. Furthermore, in this case the behavioral component of refinancing behavior gains importance, while the development of mortgage rate becomes less relevant. Consequently, the losses are more diversified in portfolio context and the downside risk becomes less severe.

Finally, we address the price that lenders should charge to compensate for their refinancing risk. The base case from Chapter 6.2 represents assumptions that are in our opinion most likely to approximate circumstances in future reality. Therefore, we recommend to charge a risk premium of 30 basis points. However, as mentioned several times, the level of uncertainty in this result is very high.

7.4 Suggestions for Future Research

The topics for potential future research can roughly be divided into two directions: First, the model that is presented in this thesis can be improved in terms of accuracy and validity. Second, practical issues that follow from our findings can be addressed.

The two in our opinion most meaningful ways to improve the simulation model are addressing the interest rate framework and the refinancing threshold:

Mortgage rate model: As discussed in Chapter 5, the mortgage rate model leaves much room for improvement. For example, it could be modeled using a two-factor framework, where one process for risk-free rate is combined with a second one that describes the spread between risk-free and mortgage rate. On the other hand, perhaps it is not even appropriate to model mortgage rate using a stochastic process.

Refinancing Threshold: Another focus of future research could be the validity of the refinancing threshold. For example, the PDE derived in Chapter 3.4 could be solved instead of simply assuming the exercise boundary. Furthermore, there is much room for exploring borrower's real refinancing behavior. For example, while modeling heterogeneity, we assumed that the refinancing threshold becomes (approximately) normally distributed. However, empirical research should be conducted to decide whether a normal distribution really represents the probabilities of the exercise threshold accurately. Another example of improving the threshold is the proper incorporation of different fixed-rate periods instead of assuming 30 years fixed-rate. Finally, a proper implementation of the Dutch tax system instead of the superficial analysis in Chapter 4 could help predicting the refinancing behavior in our opinion significantly.

Furthermore, there are some practical concerns that future research could address:

Ethics: As explained in Section 7.3, lenders might have the possibility to charge more than their true expected economic loss. This is especially true if the penalty-free mortgage is offered as more expensive product next to regular mortgages. In this case, also legal issues could arise. Before launching the penalty-free mortgage, one should thus examine how the regulatory authorities would view the penalty-free mortgage.

Contract specifications: The underlying contract conditions of the simulated mortgages could be altered. For example, the contract could include other option designs, such as a cliquet option over multiple fixed-rate periods, or a barrier option with an interest rate floor. Alternatively, the expected losses could be compensated in form of an upfront fee, or the prepayment penalty could be a fixed amount. In all these cases, the simulation model must be modified to include the new contract conditions, because the appropriate risk premium might change.

Perception of the borrower: We could study how mortgagors will perceive the new product. More specifically: Will the feeling of newly obtained freedom be substantial? The research can be brought to a broader level by studying what exactly provides mortgagors with increased satisfaction regarding product conditions. For example, factors that borrowers likely value are flexibility, simplicity and transparency. However, we do not know which of these aspects is more valuable than the others. The results could be connected to the research about different product conditions that was described in the previous paragraph. In conclusion, understanding the borrower's attitude can help deciding how the new mortgage contract should be designed exactly.

Correlation between prepayment types: Our assumption that prepayments can be split into independent categories (see Figure 2.1) is not necessarily true. First of all, we neglect the 10% that can currently be refinanced without prepayment penalty. Furthermore, we did not pay attention to the interconnection between the several prepayment categories. For example, it is possible that borrowers who decide to refinance under the current system simply move to a new house in order to avoid a penalty. Those prepayments due to house sales would migrate to the refinancing category under penalty-free conditions. Future research is necessary to clearly define the interconnection between different prepayment types.

Predicting market development: The future research topic with arguably most practical significance is the assessment of a Dutch market that widely offers penalty-free mortgages. We have touched upon the possibility of multiple mortgage providers offering the new product, but at this point we cannot fully understand the implications. Furthermore, it is important consider the way mortgage providers will choose to shape the product. For example, the premium could be priced into the current market rate, such that there is no obvious price difference between a regular and penalty-free mortgage. Potentially, this has a large impact on borrowers' refinancing behavior. For instance, if there is no visible price difference, more mortgagors who do not plan to use the refinancing option might opt for the penalty-free mortgage.

Concluding Remarks

While improving the interest rate model or refinancing threshold can be interesting topics for future research, addressing practical issues has in our opinion much more relevance for assessing the penalty-free mortgage in the context of the Dutch mortgage market. Therefore, the next big challenge is the re-design of the contract conditions, such that the payoff for the lender becomes less risky, while the borrower perceives the product in a positive manner and no ethical boundaries are violated.

Appendices

Appendix A

Estimating Interest Rate Parameters

We recall that we assume that mortgage rate follows the process given by

$$dr(t) = \kappa(\theta - r(t))dt + \sigma\sqrt{max(r(t), \zeta)}\, dz \tag{A.1}$$

The purpose of this Appendix analyzing the interest rate model quantitatively to derive the short-rate's probability distribution and subsequently estimating appropriate values of the four parameters κ, θ, σ, and ζ.

A.1 Quantitative Analysis of the Model

Explicit Solution

First, we attempt to find an explicit solution for the interest rate at time t. Multiplying both sides of Equation A.1 with $e^{\kappa t}$ yields:

$$e^{\kappa t}\, dr(t) = e^{\kappa t}\kappa(\theta - r(t))dt + e^{\kappa t}\sigma\sqrt{max(r(t), \zeta)}\, dz$$
$$\Leftrightarrow e^{\kappa t}\, dr(t) + e^{\kappa t}\kappa r(t)dt = e^{\kappa t}\kappa\theta dt + e^{\kappa t}\sigma\sqrt{max(r(t), \zeta)}\, dz$$

The left-hand side of the above equation can be rewritten as $d(e^{\kappa t}r(t))$. Thus:

$$d(e^{\kappa t}r(t)) = e^{\kappa t}\kappa\theta dt + e^{\kappa t}\sigma\sqrt{max(r(t), \zeta)}\, dz$$

Integrating both sides from 0 to t yields:

$$\int_0^t d(e^{\kappa t}r(t)) = \int_0^t e^{\kappa s}\kappa\theta ds + \int_0^t e^{\kappa s}\sigma\sqrt{max(r(s), \zeta)}\, dz$$
$$\Leftrightarrow e^{\kappa t}r(t) - r(0) = e^{\kappa t}\theta - \theta + \int_0^t e^{\kappa s}\sigma\sqrt{max(r(s), \zeta)}\, dz$$

After rewriting the above expression, we can express the interest rate at time t as follows:

$$r(t) = r(0)e^{-\kappa t} + \theta(1 - e^{-\kappa t}) + e^{-\kappa t} \int_0^t e^{\kappa s} \sigma \sqrt{max(r(s), \zeta)}\, dz \tag{A.2}$$

Distribution

Since the stochastic component, which is given by $\int_0^t e^{\kappa s}\sigma\sqrt{max(r(s),\zeta)}dz$, depends on a Wiener process, the interest rate $r(t)$ is normally distributed. To clarify the further calculation process, we define:

$$r(t) = X + Y$$

where

$$X = r(0)e^{-\kappa t} + \theta(1 - e^{-\kappa t})$$
$$Y = e^{-\kappa t} \int_0^t \sigma \sqrt{max(r(s), \zeta)} e^{\kappa s} dz$$

Here, X is the deterministic component while Y is stochastic. Further, we note that $E[Y] = 0$, because the mean of the Wiener process is zero. Since $E[X + Y] = X + E[Y] = X$, the mean of the distribution is given by the deterministic part of Equation A.2. That is:

$$E[r(t)] = r(0)e^{-\kappa t} + \theta(1 - e^{-\kappa t})$$

Unlike the expectation, the expression for variance cannot intuitively be derived from Equation A.2. Furthermore, the variance is defined differently for the two interest rate domains, respectively. Therefore, the following sections will show the calculation of variance in detail.

A General Expression for Variance

We start with a basic definition:

$$Var[r(t)] = E[r(t)^2] - E^2[r(t)]$$

Since we know that $E[r(t)] = X$:

$$\begin{aligned} Var[r(t)] &= E[(X + Y)^2] - X^2 \\ &= 2X \cdot E[Y] + E[Y^2] \\ &= E[Y^2] \\ &= E[(e^{-\kappa t} \int_0^t \sigma \sqrt{max(r(s), \zeta)} e^{\kappa s} dz)^2] \end{aligned}$$

Since the expression is integrated with respect to a Wiener process, we can use Itô isometry, a property of stochastic integrals, to rewrite the problem:

$$\begin{aligned} E[r(t)^2] &= E[e^{-2\kappa t}\int_0^t \sigma^2\ max(r(s),\zeta)e^{2\kappa s}ds] \\ &= e^{-2\kappa t}\int_0^t \sigma^2\ max(r(s),\zeta)e^{2\kappa s}ds \\ &= \sigma^2 e^{-2\kappa t}\int_0^t max(r(s),\zeta)\ e^{2\kappa s}ds \end{aligned}$$

To solve the integral, we have to distinguish between two cases: The interest rate can either be larger or smaller than ζ.

Variance for $r < \zeta$ (Vasicek)

Setting $\delta(r(s)) = \sigma\sqrt{\zeta}$ yields a variance of:

$$\begin{aligned} Var[r(t)] &= \zeta\sigma^2 e^{-2\kappa t}\frac{1}{2\kappa}(e^{\kappa t} - e^{\kappa\cdot 0}) \\ &= \zeta^2\sigma^2 e^{-2\kappa t}\frac{1}{2\kappa}(e^{\kappa t} - 1) \\ &= \frac{\zeta\sigma^2}{2\kappa}(1 - e^{-2\kappa t}) \end{aligned}$$

Therefore, the interest rate at time t is normally distributed with mean $r(0)e^{-\kappa t} + \theta(1 - e^{-\kappa t})$ and variance $\frac{\zeta\sigma^2}{2\kappa}(1 - e^{2\kappa t})$. Of course, this term for variance is only valid when interest rate is lower than ζ. Taking the limit to infinity, the long-term mean is indeed given by θ, while the long-term variance is $\frac{\zeta^2\sigma^2}{2\kappa}$.

Variance for $r > \zeta$ (CIR)

Second, we consider the case of the interest rate being larger than the parameter ζ. In that case, we have to solve:

$$\begin{aligned} Var[r(t)] &= \sigma^2 e^{-2\kappa t}\int_0^t r(s)\ e^{2\kappa s}ds \\ &= \sigma^2 e^{-2\kappa t}\int_0^t E[r(s)]\ e^{2\kappa s}ds \\ &= \sigma^2 e^{-2\kappa t}\int_0^t (r(0)e^{-\kappa s} + \theta(1 - e^{-\kappa s}))\ e^{2\kappa s}ds \end{aligned}$$

When solving the integral and rearranging the terms yields:[1]

$$Var[r(t)] = \frac{\sigma^2 r(0)}{\kappa}(e^{-\kappa t} - e^{-2\kappa t}) + \frac{\theta\sigma^2}{2\kappa}(1 - e^{-\kappa t})^2$$

[1] To avoid a lengthy derivation, several simple calculation steps are skipped here. The reader is referred to literature (e.g. Jafari and Abbasian (2017)) to understand all steps that lead to the final expression for variance.

This term gives the variance of the normally distributed interest rate in the cases where interest rate is higher than ζ.

Conclusion

We showed that the explicit solution of the SDE that models stochastic interest rate is given by:

$$r(t) = r(0)e^{-\kappa t} + \theta(1 - e^{-\kappa t}) + e^{-\kappa t} \int_0^t e^{\kappa s} \sigma \sqrt{max(r(s), \zeta)}\, dz$$

The probability distribution of the interest rate at time t is normal. Its mean is given by

$$E[r(t)] = r(0)e^{-\kappa t} + \theta(1 - e^{-\kappa t})$$

while the variance is:

$$Var[r(t)] = \begin{cases} \frac{\zeta^2\sigma^2}{2\kappa}(1 - e^{2\kappa t}), & \text{if } r(0) < \zeta \\ \frac{\sigma^2 r(0)}{\kappa}(e^{-\kappa t} - e^{-2\kappa t}) + \frac{\theta\sigma^2}{2\kappa}(1 - e^{-\kappa t})^2, & \text{otherwise} \end{cases}$$

A.2 Data

The goal of this appendix is the estimation of parameters, which can potentially be used as starting point for generating interest rate scenarios. Some historical data is required as reference, but MUNT did not start production until September 2014. As the spread on top of the risk-free rate can change significantly over time, it is also not an option to use Euro Swaps or different European government bonds as reference for the risk-free rate and simply add a spread in order to approximate mortgage rate. Fortunately, the mortgage provider Obvion published historical mortgage data from 1999 onwards.[2] Figure A.1 shows Obvions historical mortgage rate on a monthly basis, which will be used for the estimation process in the following section.

A.3 Parameter Values

Next, we would like to estimate the interest rate parameters in order to get a feeling for appropriate values for the interest rate scenarios. We are aiming for a rough approximation rather than a precise calibration and the introduction of the term $\sqrt{max(r, \zeta)}$ would cause the estimation model to become quite complicated. Therefore, we assume that $\sigma\sqrt{max(r, \zeta)}$ has a constant value of δ_0 for now. Then,

[2] https://www.obvion.nl/Hypotheekrente/Hypotheekrentehistorie.htm Accessed: 27-09-2018

Figure A.1: Obvion's historical mortgage rates, where the fixed-rate period is 20 years and the loan-to-value ratio is above 90%.

linear least square autoregression can be used to estimate the respective parameters, where we follow the approach of van den Berg (2011). The method works as follows: We assume that the mortgage rate next month may be approximated by the following linear equation:

$$r_{t+1} = ar_t + b + \epsilon$$

Here, ϵ is a normal distribution and serves as measure for variability. The parameters are chosen in such a way that the squared difference between the observed interest rate and the value predicted by the above equation is minimized. From the analysis in Appendix A.1 follows a particular relationship between the results and the Vasicek parameters:

$$a = e^{-\kappa \Delta t}$$
$$b = \theta(1 - e^{-\kappa \Delta t})$$
$$std(\epsilon) = \delta_0 \sqrt{\frac{1 - e^{-2\kappa \Delta t}}{2\kappa}}$$

Here, ϵ is the normally distributed standard error. Since we assume that δ_0 is a constant, $std(\epsilon)$ derives from the solution of the PDE for $r(t) < \zeta$. The Vasicek parameters can be calculated as follows:

$$\kappa = -\frac{ln(a)}{\Delta t}$$
$$\theta = \frac{b}{1 - a}$$
$$\delta_0 = std(\epsilon) \sqrt{\frac{-2ln(a)}{\Delta t(1 - a^2)}}$$

Using the data from the previous section and assuming that one time unit is one month, we estimate the parameters as follows:

$$\kappa = -0.365\%$$
$$\theta = 9.097\%$$
$$\delta_0 = 0.155\%$$

The attentive reader immediately spots severe issues with these results: A negative mean reversion rate would cause the interest rate to be repelled from θ. Furthermore, the long-term mean θ is higher than the historical rate has ever been within the respective time period. In conclusion, the available data cannot be used to calibrate a mean-reversion interest rate model in the straighforward manner that we have just attempted. The reason for this is likely that interest rates have been dropping constantly over the available time period instead of reverting around an equilibrium value. However, it is still possible to draw some conclusions about the parameters from the data. The following paragraphs estimate the interest rate parameters that will form the *historic scenario* of the future interest rate environment.

Volatility (σ)

We assume that the mentioned issues of the calibration technique do not have a severe effect on the estimation of δ_0: According to the least square method that we used above, the mortgage rate next month can be approximated by:

$$r_{t+1} = 1.00365 r_t - 0.00033 + N(0,\ 0.00155^2)$$

While there is a considerable drift compared to the error's standard deviation, we recall that we merely seek a reference point for for the value of volatility. Therefore, we consider $\delta_0 = 0.155\%$ a reasonable estimation. From the original model formulation we further know that:

$$\sigma = \frac{\delta_0}{\sqrt{max(r, \zeta)}}$$

Setting $max(r, \zeta) = 5.75\%$, which is the average interest rate from the data set, yields $\sigma = 0.645\%$. We note again, that this value is purely indicative, but helps determining the order of magnitude of appropriate values for σ.

Long-Term Mean (θ)

More than any other parameter, the long-term mean has a large influence on the general interest rate trend and is thus a key choice in terms of defining an interest rate scenario. As mentioned in Chapter 3.3.3, we do not necessarily assume that the past is a good predictor of the future. Therefore, the simulation will consider a range of different values for θ which are not necessarily based on Obvion's data. However, for the purpose of estimating the order of magnitude of the mean-reversion rate in the next paragraph, we assume a long-term mean of 5.75% for now. Similarily to the previous section, this percentage represents the average value of the available historical data. Of course, we have to keep in mind that this average corresponds to data about mortgages with 20-year fixed-rate period. When conducting the simulation at a later point, we will add 0.15% as defined in Appendix B to the average.

Mean-Reversion Rate (κ)

The mean-reversion rate determines how quickly interest rate reverts towards its equilibrium. If its value is chosen too low, then the interest rate would end up behaving similarily to a simple random walk. However, if it is chosen too high, then the rate "jumps" to the equilibrium value and subsequently cannot move away from it anymore. To get a feeling for appropriate values for κ, we turn to Figure A.2 which models the interest rate with $\theta = 5.9\%$, a starting value of 3% and and in the absence of randomness. The graphs show that a high mean-reversion rate of $\kappa = 24\%$ results in a steep increase of interest rate in the first couple of years, such that equilibrium level is quickly reached, whereas lower values for κ result in a less steep ascent towards equilibrium rate.

For the purpose of establishing a historic case, we choose a mean-reversion rate that ensures that the interest rate just reaches 5.9% within a time frame of 30-years, given a starting rate of 3%. As shown in Figure A.2, this requirement implies a mean-reversion rate of roughly $\kappa = 1\%$.

Domain Threshold (ζ)

The domain threshold determines which interest rate domains are modeled by Vasicek and CIR, respectively: For example, a low threshold means that most of the time the stoachstic interest rate process is described using CIR. Therefore, the probability that the interest rate drops below zero becomes low as well. We choose to select the domain threshold ζ according to the following rule: Given that the long-term mean equals the start rate of 3%, approximately one out of thousand instances should drop below zero at some point within 30 years.

To get a feeling for a correct threshold, we generate $5,000$ interest paths and count how many paths cross zero at some point during 30 years. Figure A.3 shows the number of paths that fell below zero at some point for different thresholds ζ. The remaining interest rate parameters are set as defined in the previous paragraphs, except that the long-term mean is $\theta = 3\%$, which is the starting interest rate. The graph shows that even for very low values of ζ below 2.75% not a single sample path dropped below zero. For ζ's between 2.75% and 3.5% on the other

Figure A.2: Interest rate paths over 30 years in the absence of randomness. The legend shows the different mean-reverion rates that have been used, respectively.

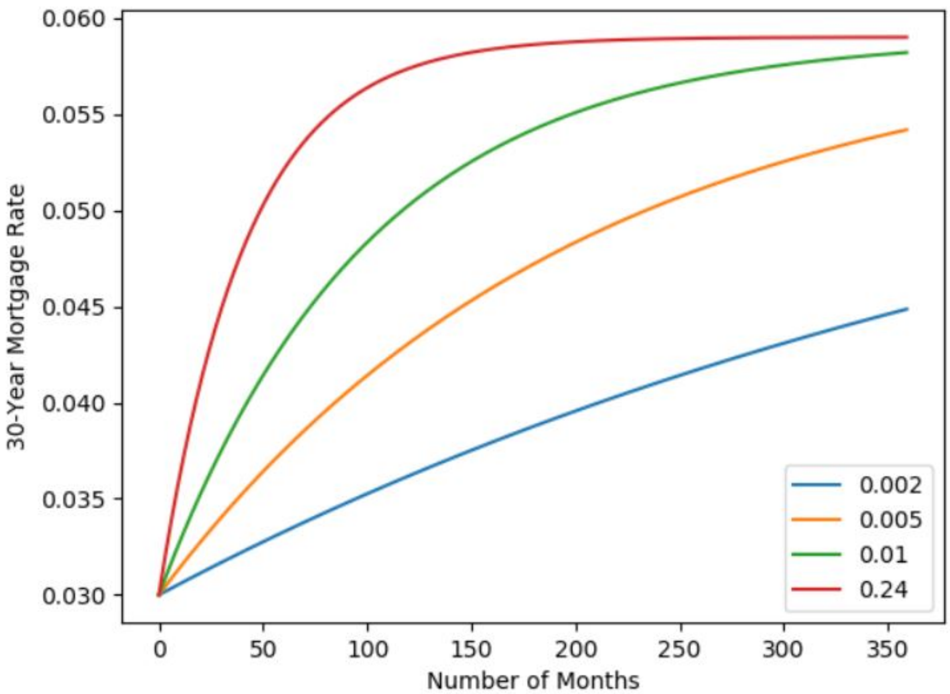
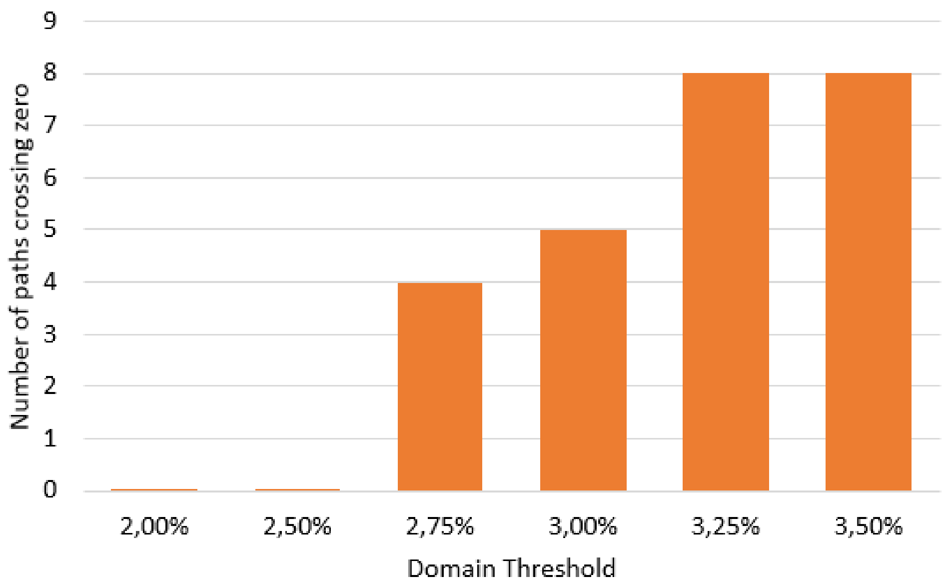


Figure A.3: Relationship between interest rate domain threshold ζ and the number of interest paths that fall below zero at some point during 30 years. For each ζ, $5,000$ interest rate paths are generated. The other parameters are set as follows: $\theta = 3\%$, $\kappa = 1\%$ and $\sigma = 0.645\%$.



hand, at least one out of thousand samples crosses zero.

Although we have to keep in mind the large margin of error due to low sample numbers and the random variability that it causes in the resulting graph, we can draw the conclusion that ζ should be on the lower and of the range between 0.05% and 0.5% if we want less than one out of hundred paths to drop below zero. This number will be different as soon as the other three interest rate parameters change, but we choose to use 0.05% as general indicative value.

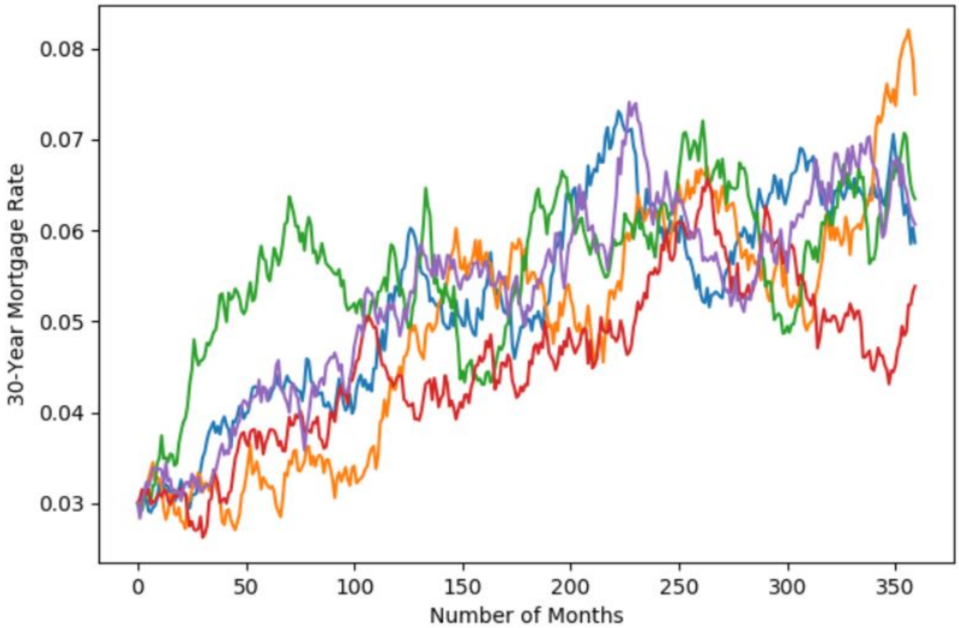
Conclusion

In conclusion, the following interest rate parameters will form the historic case of mortgage rate scenarios:

$$\theta = 5.9\%$$
$$\kappa = 1\%$$
$$\sigma = 0.645\%$$
$$\zeta = 3\%$$

These parameters correspond to a process where one time unit equals one month.

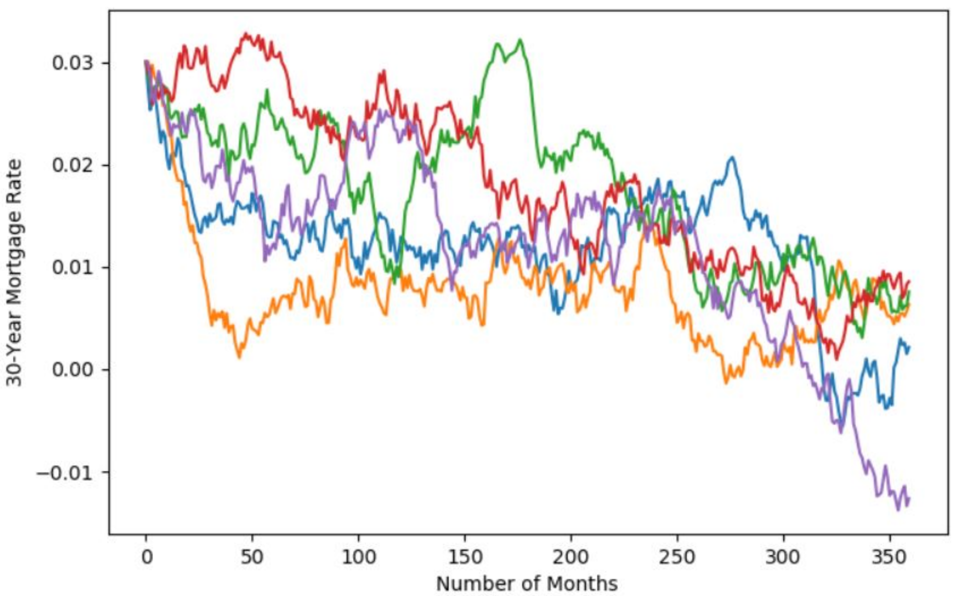
Figure A.4: Five example interest rate paths in the historic scenario. Here, the interest rate is "pulled up" over time to its equilibrium value of 5.9%.



A.4 Sample Paths

Figures A.4 and A.5 give an impression of possible interest rate developments. Figure A.4 illustrates sample paths under the model parameters estimated in this appendix, while Figure A.5 uses a long-term equilibrium of $\theta = 0\%$.

Figure A.5: Five example interest rate paths in the worst case scenario. The long-term equilibrium mean is zero.



Appendix B

Market Rate Differential

This appendix shows how we determine the difference between the 30-year fixed-rate period mortgage with a loan-to-value ratio above 90% and the mortgage rates of other buckets. Here, ”buckets” refers to categories that differ in the lenght of fixed-rate periods and loan-to-value ratio.

DMFCO’s mortgage rates of August 1st, 2018, are shown in Table B.1. Table B.2 shows the difference between the different rates and the 30-year rate for a loan-to-value of above 90%. For example, the rate of a 25-year fixed-rate mortgage is five basis points lower than the 30-year mortgage within the respective loan-to-value bucket. Only the NHG mortgage deviates from this pattern.

From the tables, we deduct the approximate rules for the yield curve as shown in Table B.3. Here, the differentials per bucket constitute the sum of the approximate differences of the right column and bottom row. The numbers in the right column and bottom row are chosen in such a way, that the real differentials from Table B.2 are represented by Table B.3 as closely as possible.

However, we cannot assume that the DMFCO rates from this particular date

Table B.1: DMFCO’s mortgage rates as of August 1st, 2018. The rows represent different fixed-rate periods (FRP), while the columns show different loan-to-value (LTV) categories.

FRP / LTV	**NHG**	**<60%**	**<80%**	**<90%**	**>90%**
Variable	1.57%	1.57%	1.72%	1.87%	1.97%
5 years	1.59%	1.59%	1.74%	1.89%	1.99%
10 years	1.96%	1.94%	1.94%	2.04%	2.29%
15 years	2.28%	2.28%	2.34%	2.49%	2.68%
20 years	2.51%	2.40%	2.50%	2.65%	2.80%
25 years	2.56%	2.50%	2.65%	2.80%	2.90%
30 years	2.68%	2.55%	2.70%	2.85%	2.95%

Table B.2: DMFCO's mortgage rate differentials based on Table B.1

FRP / LTV	**NHG**	**<60%**	**<80%**	**<90%**	**>90%**
Variable	-1.38%	-1.38%	-1.23%	-1.08%	-0.98%
5 years	-1.36%	-1.36%	-1.21%	-1.06%	-0.96%
10 years	-0.99%	-1.01%	-1.01%	-0.91%	-0.66%
15 years	-0.67%	-0.67%	-0.61%	-0.46%	-0.27%
20 years	-0.44%	-0.55%	-0.45%	-0.30%	-0.15%
25 years	-0.39%	-0.45%	-0.30%	-0.15%	-0.05%
30 years	-0.27%	-0.40%	-0.25%	-0.10%	0%

Table B.3: Structured approximation of market mortgage rate differentials

FRP / LTV	**NHG**	**<60%**	**<80%**	**<90%**	**>90%**	Difference to 30y
Variable	-1.30%	-1.40%	-1.25%	-1.10%	-1.00%	-1.00%
5 years	-1.30%	-1.40%	-1.25%	-1.10%	-1.00%	-1.00%
10 years	-0.90%	-1.00%	-0.85%	-0.70%	-0.60%	-0.60%
15 years	-0.60%	-0.70%	-0.55%	-0.40%	-0.30%	-0.30%
20 years	-0.45%	-0.55%	-0.40%	-0.25%	-0.15%	-0.15%
25 years	-0.35%	-0.45%	-0.30%	-0.15%	-0.05%	-0.05%
30 years	-0.30%	-0.40%	-0.25%	-0.10%	0%	0%
Difference to >90%	-0.30%	-0.40%	-0.25%	-0.10%	0%	

are a truely accurate representation of the market and conclude, that there is a large uncertainty in the resulting differential approximations. Therefore, we omit optimizing the numbers further and use the roughly matching differentials instead.

Appendix C

Number of Simulation Iterations

Given a certain risk premium, we would like to calculate the average loss based on the underlying model. However, if there are too few iterations, then the simulated average loss might have a large error compared to the true (unknown) mean. This appendix estimates the number of simulation runs that are appropriate to approximate the mean profit with sufficient accuracy.

Running the Simulation

The interest rate parameters are set according to the expected scenario. That is, $\kappa = 1\%$, $\theta = 4\%$, $\sigma = 0.645\%$, and $\zeta = 3\%$. Furthermore, we arbitrarily choose a linear mortgage type, as well as a risk premium of 50 basis points and an initial interest $i = 0.5\%$. We recall, that a "profit" or "loss" corresponds to the economic value that the mortgage has gained after 30-years as percentage of initial value.

Figure C.1 shows a histogram of the profits after $500,000$ iterations to give the reader an impression of the distribution. In a large number of cases, the borrower simply does not refinance and the lender makes a fixed profit according to the value of the risk premium. However, since we are more interested in the distribution in the case of refinancing, the graph in Figure C.1 "zooms" into the respective area. In other words, the frequency of the cases without refinancing exceeds the displayed bar. Moreover, we note that the shown distribution does not have to average zero, as we picked the risk premium arbitrarily. There are four small peaks just below minus 1%, below 2%, below 4% and at around 5%. These peaks are caused by the steps in the refinancing threshold, which suddenly make a mortgage available at a lower rate, as soon as certain time thresholds are exceeded.

Calculating Required Sample Number

Next, we determine the number of samples needed to obtain a reliable result for the sample mean, which is assumed to be normally distributed. We would like to be 95% confident that the sample mean deviates at most a certain percentage from

Figure C.1: Histogram of profits that a linear mortgage yields based on a risk premium of 50 basis points, an initial differential of $i = 50$ basis points, and under the expected interest rate scenario. The maximum profit occurs when the borrower does not refinance and equals 5.69%. The frequency of this special case is around $175,000$, but exceeds the shown frame. The histogram is based on $500,000$ samples. Note that horizontal axis shows the profit as percentage from the loan's face value, while the values themselves are not expressed as percentages. That is, a range between -8% and 6% profit is shown.

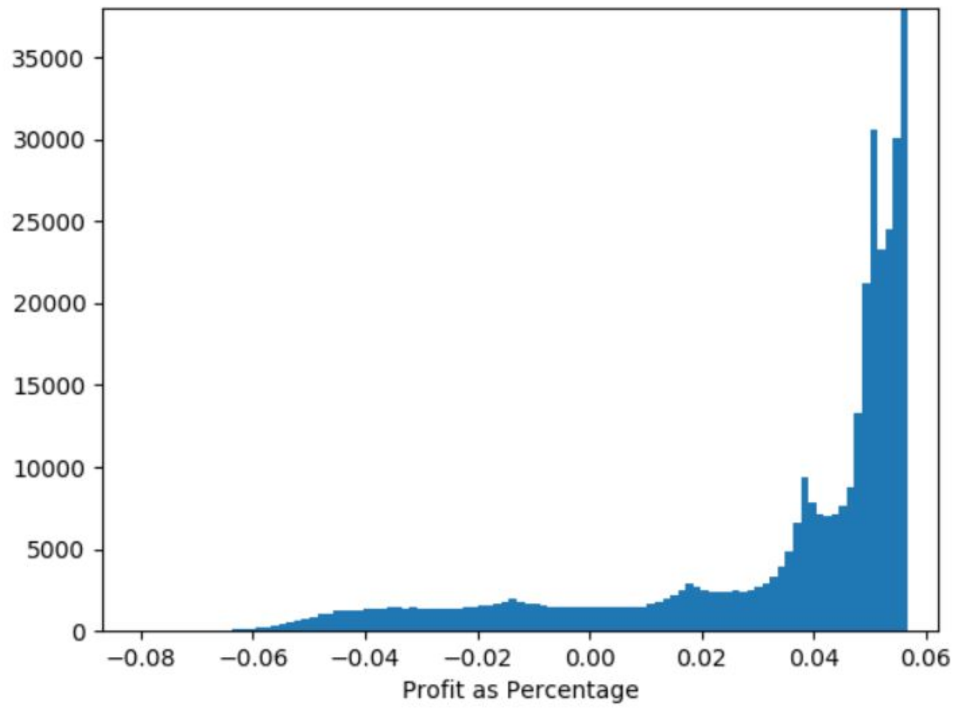
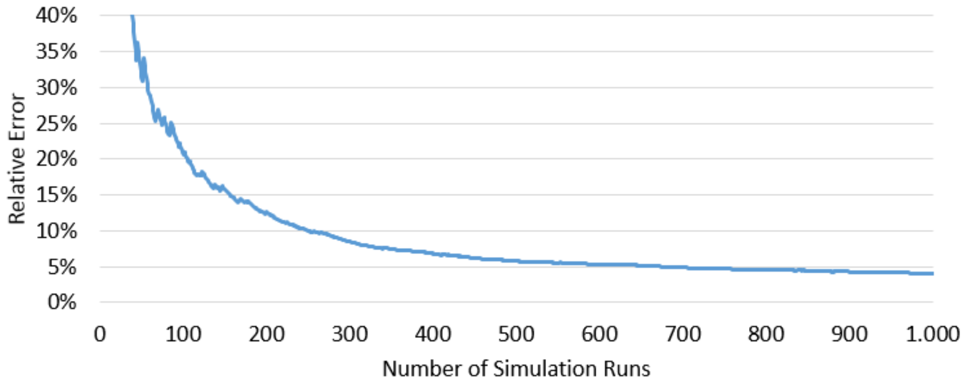


Figure C.2: Error of expected profit as a function of number of simulation runs



the true mean. That is, one half of the 95% confidence interval is given by:

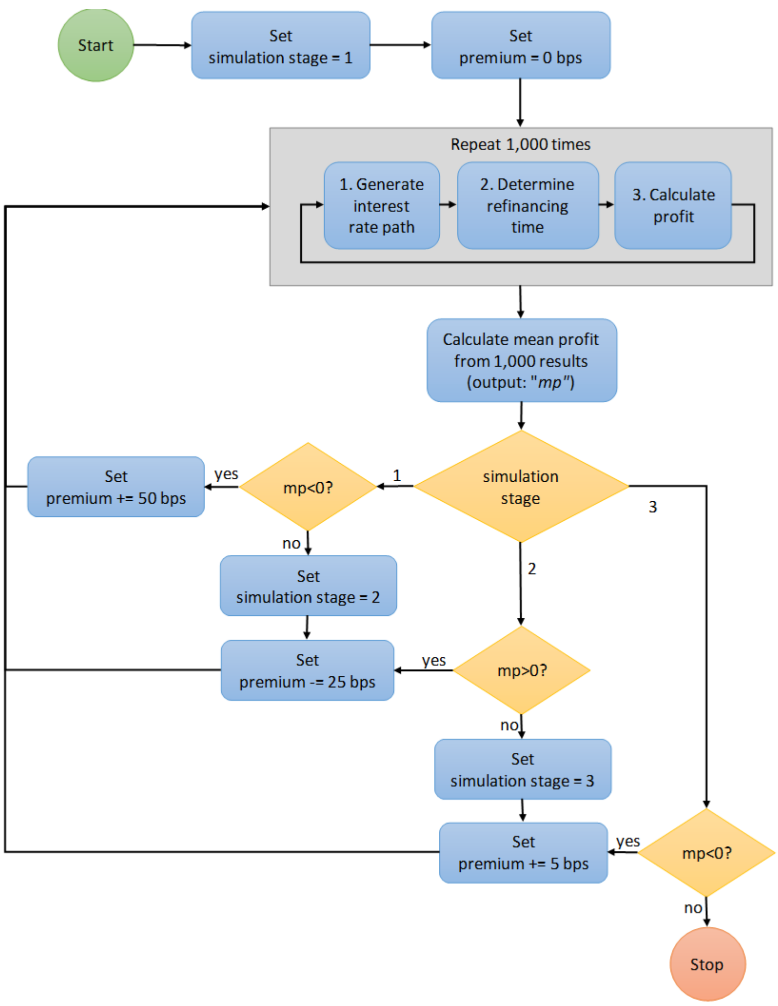
$$\frac{1}{2}CI_{95\%} = 1.96\frac{\hat{\sigma}}{\sqrt{n}}$$

Here, n is the sample size and $\hat{\sigma}$ refers to the sample's standard deviation. Figure C.2 shows that the error margin of the calculated expected profit decreases quickly at first, while the slope becomes less steep after a couple hundred runs. We choose to simulate $1,000$ instances to calculate expected profit or loss for a high chance for an error below 5%.

Appendix D

Algorithm Flow Chart

Figure D.1: Flow chart illustrating the simulation logic



Appendix E

Initial Differential Calibration

Table E.1: Calibration of initial differential i for different interest rate scenarios and mortgage types

Interest Rate Scenario	Mortgage Type	i (bps)
Historic	Linear	10
Historic	Annuity	10
Historic	Interest-Only	20
Expected	Linear	50
Expected	Annuity	50
Expected	Interest-Only	60
Negative	Linear	110
Negative	Annuity	110
Negative	Interest-Only	150
Worst-case	Linear	210
Worst-case	Annuity	210
Worst-case	Interest-Only	270

Appendix F

Behavioral Simulation Results

Table F.1: Behavioral simulation results, where all other parameters correspond to the base scenario (Y =*"interest-only"*, $r_0 = 3\%$, $\theta = 4\%$, $\kappa = 1\%$, $\sigma = 0.645\%$, and $i = 0.6\%$). Here, the theoretical threshold is decreased by $N(m, s^2)$.

m (bps)	s (bps)	p (bps)	ES (%)	τ (years)
0	0	30	8.05	14.17
0	25	30	9.69	13.96
0	50	25	11.20	13.65
0	75	20	11.08	13.69
0	100	20	11.23	13.62
25	0	30	10.59	18.63
25	25	30	10.79	18.55
25	50	25	11.51	17.24
25	75	20	11.43	16.71
25	100	20	11.21	16.61
50	0	25	14.95	22.09
50	25	25	14.12	22.01
50	50	20	12.53	21.19
50	75	20	12.63	19.78
50	100	15	12.37	18.50
75	0	20	15.23	25.12
75	25	20	14.72	24.76
75	50	20	14.29	23.50
75	75	20	13.85	22.68
75	100	15	12.72	21.29
100	0	15	15.32	27.12
100	25	15	15.00	27.08
100	50	15	13.80	25.97
100	75	15	13.35	24.65
100	100	15	13.99	22.78

Appendix G

Time Until Refinance Histograms

Figure G.1: Histogram of the time until refinance when setting input parameters corresponding to the historic interest rate scenario as defined in Chapter 5.7 and the base scenario (see Section 6.1) is used to set all other parameters. When no refinancing occurs, then the time of refinance equals 360 months.

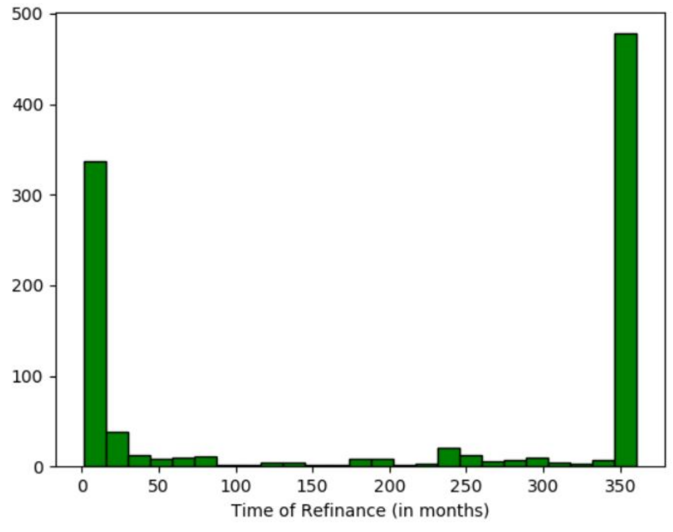


Figure G.2: Histogram of the time until refinance when setting input parameters corresponding to the base scenario defined in Section 6.1. When no refinancing occurs, then the time of refinance equals 360 months.

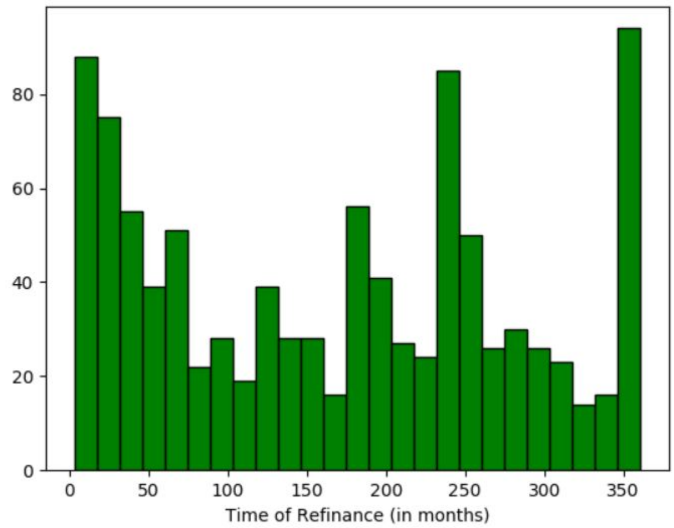


Figure G.3: Histogram of the time until refinance when setting input parameters corresponding to the negative interest rate scenario as defined in Chapter 5.7 and the base scenario (see Section 6.1) is used to set all other parameters. When no refinancing occurs, then the time of refinance equals 360 months.

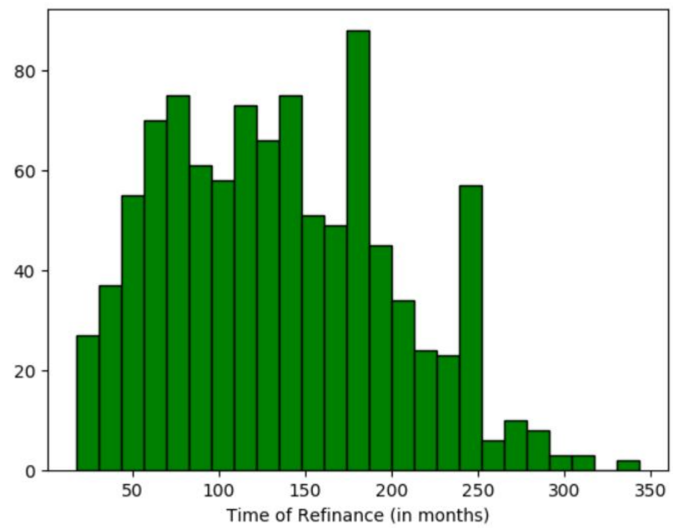
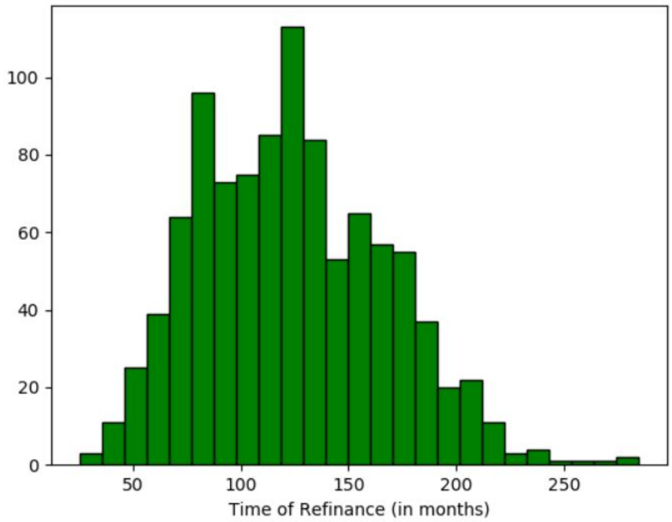


Figure G.4: Histogram of the time until refinance when setting input parameters corresponding to the worst-case interest rate scenario as defined in Chapter 5.7 and the base scenario (see Section 6.1) is used to set all other parameters. When no refinancing occurs, then the time of refinance equals 360 months.



Glossary

Annuity Mortgage One of the three common payment schedule types for mortgages. In any given month, the sum of the principal and interest add up to the same nominal amount. That is, the total monthly payment remains constant over the entire life-span of the mortgage.

Basis Point Equal to 0.01%. That is, 100 basis points sum up to one percent.

Fixed-Rate Period A mortgage contract sets the mortgage rate at the current market rate for a certain period of time, which is referred to as fixed-rate period. The most popular fixed-rate periods are 10 years, 20 years, and 30 years.

Forward Rate The interest rate of today's zero-coupon bonds implies interest rates for future time periods. This implied interest rate for the future is referred to as forward rate.

Interest-Only Mortgage One of the three common payment schedule types for mortgages. Here, the principal is not repaid over time, but paid back all at once at contract maturity. Therefore, the total monthly payment solely consists of respective interest payments.

Linear Mortgage One of the three common payment schedule types for mortgages. In the beginning of the (30-year) contract, the principal is split up into $30 \cdot 12 = 360$ equal portions to be paid back.Therefore, the remaining principal decreases linearly with an aging mortgage contract. The monthly interest and thus the total payments per month decrease over time.

Mortgage Value The economic value of a mortgage. It is given by the net present value of all future cash flows, which are discounted at the interest rate at the moment of valuation. Mortgage value depends on interst rate and time. Notation: M.

Prepayment Penalty A fee charged by the mortgage originator in the case of mortgage prepayment. Generally, the amount of the penalty is driven by interest rate and its purpose is compensating the lender for the loss caused by the respective prepayment.

Refinancing Costs Extra costs borrowers have to pay when refinancing a mortgage, such as notary costs and other fees. The term *refinancing costs* excludes potential prepayment penalties.

Refinancing Differential The difference between the refinancing threshold and the contract interest rate of a mortgage.

Refinancing Threshold Indicates the mortgage interest rate, below which a borrower would choose to refinance. The corespdonding interest rate level changes over time. Notation: r^*.

Risk Premium A risk premium compensates for the risky components of an investment. In other words, it is the amount the risk-free return should be increased in order to countervail potential losses.

Seasoning Refers to the age of a mortgage contract.

Seasonality Refers to the time of year.

Term Structure of Interest Rates Relationship between returns of the same underlying security for different times to maturities. Term may be used interchangibly with *yield curve*. Notation: R.

Wiener Process Continuous-time stochastic process, where the change in an infinitely small time interval is drawn from a normal distribution.

Yield Curve See *term structure of interest rates*.

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