High Frequency Trading – highway to financial hell, or economic salvation?

-a comprehensive review of the High Frequency Trading literature-

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Abbreviations and symbols

AHFTI	Artificial HFT Intelligence
AI	Artificial Intelligence
ATS	Automated Trading System
CDA	Continuous Double Auction
CPU	Central Processing Unit
(Super) DOT	Designated Order Turnaround
ECNs	Electronic Communication Networks
FED	Federal Reserve
FINRA	Financial Industry Regulatory Authority
FT	Fundamental Trader
GPU	Graphics Processing Unit
HFT	High Frequency Trader, High Frequency Trading
IEX	Investors Exchange
IT	Information Technology
LOB	Limit Order Book
LT	Liquidity Trader
MC	Market Clearing
MM	Market Maker
MPID	Market Participant Identifier
ms	Milliseconds
NASDAQ	National Association of Securities Dealers and Automated Quotations
NBBO	National Best Bid and Offer
NYSE	New York Stock Exchange
Reg. HTS	Regulation Alternative Trading System
Reg. NMS	Regulation National Market System
SEC	Securities and Exchange Commission
SIP	Securities Information Processor

Δ	Price notch
ϵ	External information
i	Unit of stocks that a trader wants to buy or sell
P	Price
P_{MC_1}	Market clearance price calculated at time t by MM
$\left(P_{MC_t} - \Delta_t^x\right)$	Pressurized or notched price.
Q_t^x	Quantity of stock that trader x is willing to sell or buy at time t
s	Seconds
t	Time
θ_t^x	Demand curve of player x at time t

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1. Introduction

Stock trading has been for many years a vital part of financial trade and economic development. In trading it is essential to not just make the right decision but also at the right time. This means that if a trader is too late with a buy or sell order, she might suffer the financial consequences. The vision of screaming and stressing stock traders on the stock exchange floor is familiar to everyone. With the onset of computer technology and its steady development over the past few decades, hardware and software facilitated trading have also been on the rise. As traditional stock trading done by traders involves buy and sell orders that are decided on by humans, computer aided trading was found to be much quicker since computer algorithms can analyze the movements of the market and decide on the desired course of action within a fraction of a second. Over the last decade it has become apparent that due to the speed of the decision making of algorithms they can also be used in order to influence the market and the trend of any stock. This feature of high frequency trading has been used in several high profile cases to illegally influence stock prices. However, other than that, there are very few comprehensive studies that explain and combine most of the aspects of algorithmic trading and High Frequency Traders (HFTs).

This thesis gives an overview of the development of High Frequency Trading, its influence on the market, a model of the interactions between an HFT and other traders and the possible future developments. Additionally, we present a model which an HFT might use in a realistic trading situation when squaring off against other traders. The thesis is divided in chapters that will treat the following topics: Chapter 1 introduces the thesis and provides the general outline and purpose of the study. Chapter 2 provides an overview of the history of the stock exchange from the early beginnings till algorithmic trading. Chapter 3 discusses the differences between continuous and discrete time in games that can occur and additionally argues as to which time an HFT is following. Chapter 4 focusses on the various market participants that can be found on the stock exchange. In Chapter 5 the market structure is explained, a detailed description is given of the most common types of market orders and the workings of the Limit Order Book. Chapter 6 gives an in depth overview of the various strategies that can be followed by an HFT. Chapter 7 explores various game scenarios that can happen between HFTs and the other market participants, in addition in this chapter a model for the subsequent game is introduced. In Chapter 8 the advantages and disadvantages of having an HFT present on the market are discussed. In Chapter 9 the potential future of High Frequency Trading is explored. Finally, Chapter 10 elaborates in more detail on some key discussion points mentioned throughout the thesis and gives potential topics for future study.

2. The history of the connected stock exchange and algorithmic trading

High Frequency Traders are predominantly relying on the speed of their algorithms and how quickly they can make a decision. However, equally important is the quality of the information that is being fed into the algorithm. Though the importance of information and the role it can play on the path to financial success is not a recent phenomenon, it dates back many centuries all the way to the Dutch East India Company (and trading of their stocks) or even to the ancient Roman times in which share/stock markets were also assumed to exist.

In the late 1500s and early 1600s stock exchanges were starting to emerge. Cities such as Antwerp, Frankfurt and London opened stock exchanges in order to facilitate financial growth and development of the economies of the respective countries and companies. Interestingly, at the Royal Stock Exchange in London, stock brokers were not allowed inside. They were rejected due to their rude nature and behavior. In order to circumvent this, the brokers gathered at "Jonathan's coffee house" located nearby the actual exchange. A trader by the name of John Casting started to list the prices of some commodities a few times a week on a chalkboard [1]. This board was used as an information source by the stock brokers.

Years later the industrial revolution was around the corner and railroads were being laid down. Trains and express delivery services were used to deliver notes and letters from one exchange to another. They were also used to get information to brokers in order for them to figure out what they should do with a stock of interest. Prior to this delivery of information, orders, money and everything else was even slower and more expensive. As telegraph lines were constructed in the early to mid-1800s a quicker and more secure way was found to transport information. Though it was clear that the telegraph was by far the fastest way of sharing information, it was a costly method as it required a wired network to be built. This was impossible to achieve on a worldwide or even nationwide scale. Thus, nodes of telegram stations were built where the information would be shared from one node to the other. The final recipient of the information would still have to be informed by an errand runner who still might have had to travel a considerable distance. Express companies and their errand boys were vulnerable for attack, theft, disease and fatigue. An alternative way to share information quickly and prevent loss of data through any of the aforementioned causes was to use homing pigeons. These animals would be trained to fly from one location to the other, even over lakes and overseas, with a note attached to their leg. Upon arrival the note would then be delivered to the final recipient. The pigeons were much faster than the trains and express couriers of the day and they were a relatively safe way to transport information (even though in ancient Rome homing pigeons were intercepted by hawks in order to prevent the data they might carry to arrive at the final destination). A famous example of the use of homing pigeons was the case of the Rothschild family. They owned several banks in major European cities, they used pigeons to communicate between these banks. On the 18th of June 1815 the British forces defeated Napoleon (totally unexpected) at Waterloo. As the Royal Exchange was anxiously awaiting news of the outcome of the battle, no traders were daring to make any major orders. As Napoleon was defeated, a Rothschild homing pigeon was sent to London carrying the news. The Rothschilds heard about the British victory a day earlier before the

government did and this enabled them to buy stock in massive amounts at the exchange, which were then sold for a huge profit after the victory was officially announced [2, 3].

Morse code telegraphing was at this time in its infancy and messages were being sent on a telegraph system with a different manner of transmitting the data. The so called Chappe telegraph system used visual transmission through light. Telegraph towers were built that would communicate between each other and send encoded messages according to an agreed upon alphabet. The advantage was that messages could travel at the speed of light, however the towers would have to be within each other's sight. The range of the messages could be extended by the use of telescopes with which tower workers could see messages incoming from towers further away, but still at times human carriers of the messages were needed in the form of repeaters and routers. These repeaters and routers were people that would take a message with them and bring them either to the nearest tower and relay the message there (repeater) or bring a message to the appropriate tower depending on what the final destination of the message was (router). This technique was used extensively by Napoleon and his military while the general public was not allowed to use it, as the French military was considered more important than the general common population. The Chappe telegraph was also used for day trading, due to the possibility of the message crossing a relatively large distance in a short period of time. This way profitable arbitrage opportunities could effectively be communicated and exploited. This was done for instance in the US, where Philadelphia stock exchange brokers created their own private Chappe telegraph network. With this network they were able to send a message from New York to Philadelphia (110 miles) within 10 minutes [3]. The Chappe system was able to compete with the Morse system for a while, but ultimately technological enhancements and cheaper cost of the Morse system made the rather cumbersome Chappe system fall out of grace and get replaced.

Paul Julius Reuter, founder of Reuters, in 1845 still saw the advantages of sending information by pigeon, but this time between telegraph nodes. He became the fastest information provider between the London and Paris stock exchange by using almost 200 feathered mail couriers. As telegraph stations and networks became more prevalent, pigeon based messenger services grew more obscure (though homing pigeons would still see considerable use even in World War II). Reuter, not one to be defeated, later switched fully to telegraphic data transfer to be even faster in the provision of news and information [3].

Around the turn of the century, wireless Morse telegraphy was developed and found to be able to transmit data transcontinentally, by using the so called Heaviside layer in the earth atmosphere [3]. This layer is capable to reflect medium-frequency radio waves up to transcontinental distances. This laid the groundwork for future radio wave and satellite based technology. With the onset of telephony, day time trading became even easier, and buy and sell orders could be done from anywhere at almost any time. Nevertheless, telephone based trading was expensive. The most common way to trade was still an actual physical presence of traders on the exchange floor, where they would try to make a deal. With computers getting more common and more affordable, the first computer based trading platforms also started to appear. In 1971 the National Association of Securities Dealers and Automated Quotations (NASDAQ) was deployed in which any Market Maker from anywhere could issue their buy or sell orders for any NASDAQ listed stock through the Nasdaq II workstation computer. This facilitated Market Maker competition amongst each other in order for their customers (the actual stock buyers and sellers) to get the best price possible. As the data in the Nasdaq II workstations was being processed for all Market Makers at the same speed, a small company by the name of Datek Securities developed "The Watcher", this system would run the same NASDAQ data only much faster and it would highlight any potential interesting trading

opportunities to the user. This gave the users of the Watcher system an edge over the conventional Nasdaq II users [3].

Computerization of the order flow of markets was also started in the mid-1970s with the introduction of the "designated order turnaround" (DOT) system in 1976 and later the Super DOT system in 1984 [4, 5].

In the 1980s financial markets were becoming fully electronic. Till the 1980s the New York Stock Exchange (NYSE) and NASDAQ were the main exchanges on which trade was done electronically. This changed in the 1990s as other electronic stock exchanging platforms started to appear. These platforms were known as "Electronic Communication Networks" (ECNs). These ECNs facilitated stock and currency trade outside of the traditional exchanges and were even authorized in their existence by the US Securities and Exchange Commission (SEC). The idea was that the duopoly held by the NYSE and NASDAQ was not beneficial for the market according to the commission and as such ECNs would be a favorable development [4].

ECNs quickly rose in popularity in the 90s at the expense of NASDAQ due to their many advantages over previous trading venues. Most of the trading on ECNs is done through automatic search and execution of contra-side orders, in which for instance a bid order is being matched with an ask order by a contra broker, after a subscribing investor enters an order into the network using a custom computer terminal. Algorithmic trading also got more popular due to the increasing popularity of ECNs, seeing how these essentially are using algorithms to search and execute orders. This development in the end also gave rise to HFT firms [4]. Program trading started to gain popularity among traders, program trading uses computer algorithms to buy and sell a basket of stocks. Institutional traders usually use program trading to buy or sell a portfolio of stocks over a period of time during a trading day. By using program trading the risks of simultaneous orders are minimized and the trader can take advantage of market inefficiencies.

As described previously, the early history of HFT lies in the usage of pigeons and various forms of communication for information transmission. Though HFT as we know it today really started with the passing of the Regulation Alternative Trading Systems (Reg. ATS) by the SEC. It is staggering to imagine that HFT nowadays is responsible for about 50% of all stock trades made in the USA. HFT even had a bigger impact on the market a few years ago when 60% of all trades were HFT based trades [6]. This is especially impressive if one considers that the market share of HFTs was 10% as recent as the year 2000, for all equity trades [7]. As computer and data communication technology developed and became increasingly faster, algorithmic trading became more popular. It was now becoming possible to receive information and send out orders so fast that a trader might be able to beat the normal market and information flow at the various exchanges. A number of regulatory changes were also influential to the increasing popularity of HFT in the early 2000s, in addition to the technological advancements [8]. In 2001, US stock exchanges effectively narrowed the spreads that were possible by no longer listing bid and ask prices in fractions, but in decimals. This made the spreads of stock less interesting to traders that previously were profiting from the relatively large minimum spreads ($1/6^{th}$ of a dollar). Now the spread of a stock could even be 0.01 cent, which was deemed "not worth it". However algorithmic traders stepped into this niche of the stock market and now essentially make most of their profits based on these small amounts. In addition, the SEC again passed a new regulation REG. NMS (Regulation National Market System) which promoted competition and transparency between markets by requiring trade orders be noted nationally and no longer merely locally at individual exchanges. Small price differences between two exchanges can

now be exploited by traders as long as they are fast enough and are able to use the inter-exchange movement to their benefit [8].

Nowadays HFT is not just about having a fast computer and algorithm, it is just as much about having the fastest connection. The idea is that it is no use to have a fast acting program if you are using a dial up connection with high latency to connect to the exchange. Though this would be an extreme case, nobody uses dial up connections anymore, but there is a certain level of neurosis (when it comes to speed and a level play field) at the exchanges and in trading firms. All trading firms that are co-located at the same location as the actual exchange they trade on have the same lengths of data cable attached to their terminals. This is in order to prevent any latency advantages to any firm within the location [9]. In essence, an HFT and her algorithm lives within the framework that is defined by the various regulations. Information then feeds a machine learning system capable of performing loop wise analysis, which is possible because of a specific combination of software and hardware (where the latter two are making up the algorithm). Lastly, this algorithm is capable of high speed communication with the outside world and stock markets through its network connection (see Figure 1).

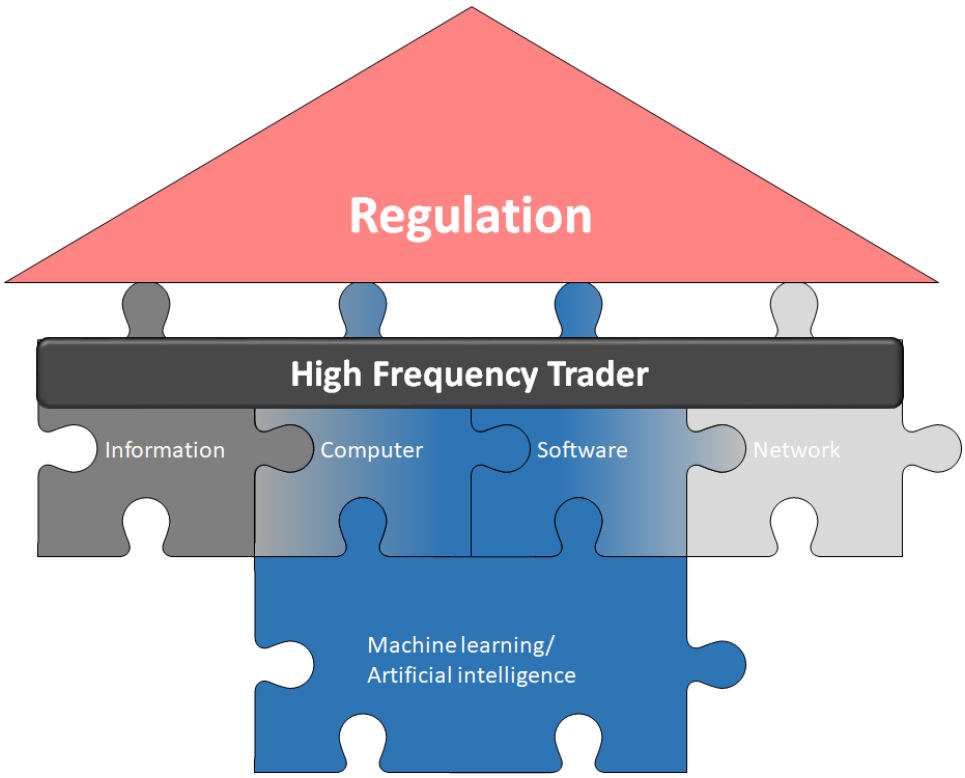

Figure 1: The building stones of an HFT algorithm. The actual hardware and software components are highlighted in blue. With the transition from information to computer and software to network in fading blue. These areas are indicative of where the algorithm begins and ends, and where it interfaces with the outside world.

2.1 What is an algorithm?

As defined by Harris and Ross [10], "an algorithm is a set of well-defined steps required to accomplish some task". Each of the steps defines an action to be taken. Often when writing an algorithm, the last step does not end the action but it sends the algorithm back to the first or second position. This way the algorithm is constantly looped for as long as it is needed to fulfill its purpose. This is very important for an HFT since she is checking the market continuously. An algorithm, even though it might be seen as a logical

set of steps to follow, is often complex and complicated to construct. There can be many ways to describe a certain action, some can be more efficient than others. For instance, a multiplication that has as the outcome the number 10 but consists of the numbers 2 and 5 can be written as an addition of the number 2 to each other 5 times, or an addition of the number 5 to each other 2 times. It is desirable when writing an algorithm to do it always as efficiently as possible. If this is not done and an algorithm is written too complex, it might be prone to bugs and it most likely will unnecessarily waste precious computing resources. In some cases finding a precise solution to a problem can be too costly and time consuming. In this situation a heuristic approach can be more appropriate. Though this might be overcome one day by the use of quantum computing [11], currently however this is not the case. Using a heuristic approach when designing a trading algorithm means that the algorithm should be written in such a way that it will try to estimate the real-world outcome using information that can be fed into it quickly. This information can be coming from media outlets, financial performance data or personal interviews and the like. However, the key of heuristics is that not all possible data and information is used in order to avoid inefficiencies. This in itself brings along a unique set of challenges. Let us use the example given by Harris and Ross [10] in which an algorithm is supposed to recognize the side on which traffic is conducted (left or right hand drive). The easiest way to achieve this would be to feed the algorithm with a list of countries and the respective side on which people drive in those counties. However, this brings a few issues with itself, the list would be relatively lengthy and most importantly the list would be completely useless if the algorithm has no way to identify which country it is analyzing at the moment. The heuristic approach to determine the side on which traffic is conducted in a certain location is to study certain characteristics that will in almost all cases give the correct outcome. In most countries it is not allowed to park a car in the opposing direction to which the traffic travels. Therefore, most people will when they parallel park their car do this in the same direction as traffic is being conducted. However, there is always “the 1 %” in the population that does not adhere to the rules or makes a mistake. Let’s assume that an algorithm was written using this heuristic principle of parking direction and this algorithm is given an image to analyze. The image shows one car which is parallel parked, what the algorithm does not know however is that this car is parked in the opposing direction to which the traffic travels. The algorithm will in this case therefore wrongly derive the driving direction [10].

A recent example of what most likely was a heuristic limitation of trading algorithms is when socialite Kylie Jenner (of “keeping up with the Kardashian’s” fame) tweeted on Twitter “sooo does anyone else not open Snapchat anymore? Or is it just me... ugh this is so sad.” [12]. The stock price of Snapchat immediately started to decline (with a slow decline on the actual day of the tweet, but a much bigger decline of almost 7% a day after [12-14]). The day after the tweet, the cosmetics firm Maybelline New York went on to ask its followers on social media whether or not it should stay active on the Snapchat platform [13]. It is assumed that trading algorithms picked up on the negative tweet when it went viral after the markets closed for the day and even the feeble attempt of Jenner to express her continuous “love” for the Snapchat platform could not revoke the downward spiral over the next few trading days. As a result, the company lost $1 billion in market value [12-14]. Though it is difficult to prove, but it is very likely that the algorithms took the tweet of Jenner as legitimate input. This in combination with the recent negative exposure Snapchat was getting for its new revised user interface most likely contributed to the algorithms decision to massively sell the stock [12].

3. Distinction between continuous time and discrete time

It is very important to know what the difference is between the two types of times that are used in game theory. Discrete time is generally described more often in literature. Continuous time differs from discrete time in this that it does not have a “last time before t” point [15]. Though continuous time can be seen as an alternative of discrete time, it differs in the key point that continuous time has an infinitely fine time grid [15]. With discrete time the players involved in the game stick to the same basic rules of the time frame. Generally speaking, the players can make their decisions of what to do in a game on certain agreed upon times or time frames. In a game which is of a continuous nature there are no real time frames or moments were actions can be undertaken, these actions can be undertaken at virtually all times. However, following the definition of Simon and Stinchcombe, strictly speaking there are time frames but with an infinitely fine amount of decision points [15].

In games that use discrete time, the players are able to account for past events from the game and evaluate their actions (and those of other players). In continuous time this is not really the case, nor is it very relevant (if the player that plays in continuous time gets pitted against a player following discrete time). The player that adheres to continuous time will have the edge of instantaneous reactivity to a dynamic situation. This also indicates that lags in decision can be truly negligible in a continuous time setting, whereas lags in a discrete time setting may also be very short but they will still span “the length of one period” [15]. With lags referring to the time between the moment of making up the next strategical move till the moment of actual execution of this move.

As Simon and Stinchcombe [15] explain, for players in continuous time based games there are sets of times that the players choose to move (whenever they see fit), whereas in discrete time the players have sets of times that players have the ability to move (and not whenever they want).

3.1 Is the HFT trader adhering to a truly continuous time game plan?

It is generally assumed in literature that an HFT is playing according to a continuous time game strategy, especially when compared to a trader that trades on an exchange in a traditional way [16]. It is certainly true that using an algorithm to follow market prices is quicker and more continuous-like, in addition it is much quicker to respond to market changes. Some consider this an unfair advantage that an HFT has. A traditional trader cannot respond as quickly to market changes because the rate at which data gets renewed in the exchange is limited by the speed at which orders can get processed by the market clearing [16]. In the meantime, an HFT can already have made several other orders to sell or purchase a certain stock, and further backlog the market and slow down the processing of all other orders. The HFT however is only as fast as its algorithm and its connection to the exchange. In recent years, line speeds have been increased through Information Technology (IT) innovations and strategic investments by banks and trading agencies. Line latency has been brought down to less than 2.6 milliseconds [17]. In fact, HFTs are even willing to pay millions of euros to be located closely to the exchange to even further bring down latency (speed) losses. This is done in order to be able to respond to the market even faster and be in front of the line when it comes to having orders cleared [18]. From the moment the data gets passed to the HFT, the algorithm then takes over and makes a decision on the course of action. The faster it can do this, the bigger the potential advantage. This by itself implies that therefore the game an HFT plays is not

truly continuous in nature. In addition, by definition a continuous time based game is characterized by not having a "last time before t" [15], however an HFT does definitely look at past events and if needed even corrects her moves from the past in order to successfully complete her strategy [18]. One reason for such adjustments would be because the HFT has recognized a trading pattern of another trader and therefore wants to adjust her strategy. A trader can for instance be intending to sell or buy a large number of stocks, but will do this in tranches or slices in order to not impact the stock price too much and to not attract any unwanted attention (more about sliced orders in Chapter 6.1.3). An HFT might recognize a pattern of this trader, like for instance that she is putting up an order every 10 minutes [19]. This shows that contrary to the definition of when continuous time is used, an HFT is actually looking at (recent) historic data and is basing her moves on the knowledge she has gained from it. Though an HFT uses a direct feed from the stock exchange, there still is a 2.6 millisecond latency/delay on these data, and then there is the delay that an HFT algorithm has to determine what to do based on the data she receives. Also, seeing how we earlier have explained that an algorithm is looping through its steps which were coded in, the looping also takes time. This means that indeed the "continuous time" an HFT trader follows can be seen as a discrete time with a finer time grid [15]. In short, an HFT keeps track of information about the stock she is interested in (News), historic stock market information and characteristics of other traders (Stock information) and other information, like company financial ratio and social media related exposure (Miscellaneous). She uses this information to "place her bet" on the stock market (see Figure 2).

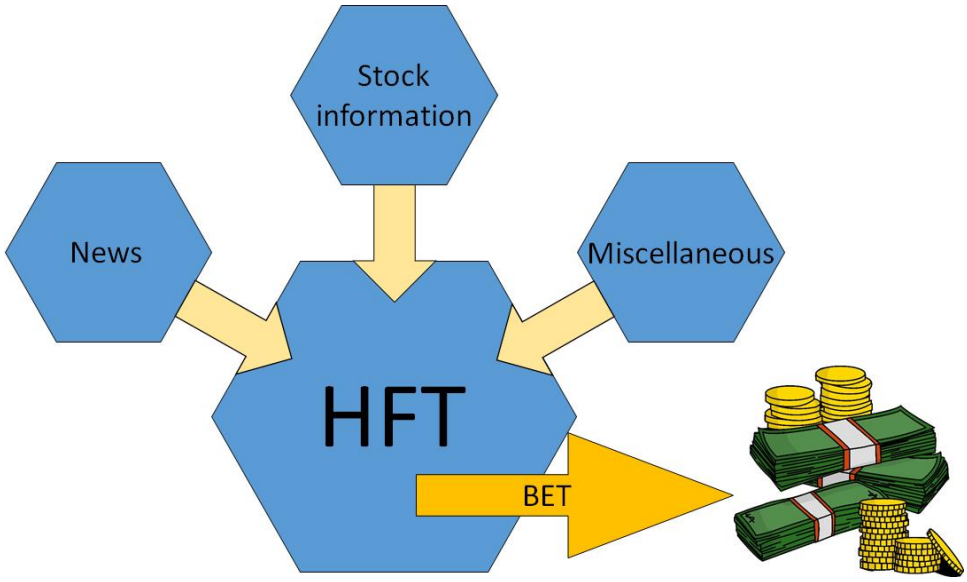

Figure 2: The various sources of information used by an HFT. Based on these sources she will decide on how to approach a stock on the stock market.

Further arguments for the fact that HFTs are playing a very continuous-like, but nevertheless not actually continuous, game are given by the theory described by Calford and Oprea [20]. In their paper they compare continuous time games to discrete time games and they find that when inertia is implemented in continuous time games the games tend to collapse back to a discrete time game, with the characteristics getting more pronounced as the inertia grows. When however the inertia is tending towards zero the game played takes on characteristics as seen in perfectly continuous time. According to Bergin and MacLeod [21], a strategy is required to satisfy an inertia condition, an agent which chooses an action at time t' must also maintain this action for some small period after t'. This applies to the HFT as

she only will respond to market changes, otherwise she will just maintain her previous action. In addition, the actions of an HFT are maintained until they are cleared by the market.

To further demonstrate, in a somewhat dramatized way, the differences and similarities between an HFT and a Fundamental Trader (FT), let us assume that they are both business executives that travel by a chauffeured vehicle (assuming that their drivers are mindless drones acting only upon input of the traders). The HFT has a company car in which she sits alone with her driver, whereas the FT has a bus filled with a few personal assistants and a driver. The HFT is able to tell directly to her driver where she wants to go and tell the driver of any immediate course changes she would like to undertake. There is some inertia (or lag) from the moment the HFT expresses her desire for a change in course or direction till the moment that the driver has processed this request and starts to undertake action. Although it seems that the actions by the driver are instantaneous. In the case of the FT she sits in the back of the bus, she indicates to one of her employees sitting close to her that she wants to undertake a change of direction. The employee takes this information and relays it to an employee sitting closer to the driver of the bus, this second employee then finally tells the driver to change course as directed. The driver then processes this instruction and starts to change the course. As one can imagine, the ordering of direction changes takes longer in the bus of the FT compared to the car of the HFT. Therefore, in case of sudden changes on the road that require immediate action, orders will be processed quicker in the HFT vehicle and the HFT will likely be able to manage the situation better. The situation in the bus however will likely be very chaotic and end in tragedy as the orders were not relayed fast enough to the driver.

Now in order to answer the question of whether or not the HFT (and FT) are following discrete or truly continuous time, imagine that the vehicles in which the traders travel are constantly on the move and the scenery (market) is constantly changing. However, due to the fact that no matter how quick the HFT is and how quick her driver can process her orders, she always for a short period of time will have to see her previous action through until it gets “replaced” by a new order. With this the inertia condition as postulated by Bergin and MacLeod [21] is satisfied and it can be said that though very continuous-like, the HFT still follows a discrete time.

4. Market participants

Making the distinction between an HFT and a "regular" trader might seem straightforward on paper, but in reality this is not the case. Traders do not necessarily identify themselves as to belonging to one group or another. They trade on the market with the intention to achieve their goals (whatever they might be). Nevertheless, in order to be able to explain the strategies for these traders, first we need to characterize the players using descriptions from previous research. This is relevant as we need to understand how these traders trade in real life situations and what their moves and strategies are. In Kirilenko et al. [2011] a good description is given of the various players that are active on the stock market. The paper further elaborates on what according to the authors was the reason behind the infamous flash crash of May 6th 2010. The general thought is that the flash crash was "caused" by HFT traders, however Kirilenko et al. [2011] deny this and show an analysis of the events which proves the HFT traders did not directly cause the flash crash nor did they exhibit any major difference in their behavior compared to other days of trade [22]. A number of traders are identified in the analysis [22]. These are: HFTs, intermediaries, fundamental buyers, fundamental sellers and opportunistic traders.

4.1 The High Frequency Trader (HFT)

HFTs are characterized by the fact that in general they are not after accumulating a significant net position and they do not deviate from their strategy but stick to it in virtually any situation. HFTs usually tend to sell off any of their bought positions quickly, their strategy of buying and selling a contract at a high speed makes HFTs seemingly unaffected by volatile situations such as the May 6th 2010 flash crash [22]. It was noticed that the net holdings of HFTs have a half life time of 137 seconds[22]. It is very possible that currently an HFT would have an even lower half time, due to increased computing power and algorithm speeds. Additionally it was noticed that an HFT buys when immediate prices are rising and they start to sell after about 10-20 seconds if the prices were still rising [18, 22].

4.2 The intermediaries

The intermediaries are generally characterized as being similar (see [18]) in behavior on the market place to the HFTs. The main difference is however that they seem to be acting and responding a bit slower to market changes compared to HFTs, these can in essence be considered as the other algorithmic traders. This is likely due to the fact that they have slower algorithms, this is also the reason why they are relatively more vulnerable to volatile market conditions when compared to HFTs. However, these traders too do not pursue obtaining high net positions [22].

4.3 Fundamental traders

Fundamental Traders are seen as non HFTs. They are also known as Liquidity Traders (LTs), they might still use algorithmic trading but generally are not speed centered and are more working according to the principles of gaining large net positions. Their strategy attempts to limit market impact, in order to minimize transaction costs [22]. Fundamental traders can even be split up further into fundamental buyers and sellers, though in order to not overcomplicate matters we will not go into further analysis of these subgroups and will treat them as a single "fundamental trader". In general, fundamental traders are not as fast as HFTs and intermediaries in anticipating on market opportunities, like buying when prices are low and selling when they are high.

4.4 Opportunistic traders

As the name implies these traders are generally capable of seizing the right moment when a profitable opportunity arises and are able to anticipate on market developments either through sheer "luck" or insight. As an example of this opportunistic skill again the flash crash case can be used. On the day of the crash a group of traders that held themselves largely out of the market entered on the moment when prices were dropping steeply, these traders then acquired net positions which were then sold when the market stabilized and prices were on the rise again. Though these traders seem to share some characteristics of HFTs and intermediaries in that they tend to exhibit mean reverting behavior, but they also seem to be establishing large net holdings similar to fundamental traders [22].

4.5 Market Makers

Market Makers are registered member firms of an exchange and appointed by the exchange [23]. Their role on the exchange is to inject liquidity and trade volume into stocks [23, 24]. Through their activities they contribute towards a fair and orderly stock market [24]. The appointment of Market Makers can be done in various ways. Broker firms are often times appointed as Market Makers by the exchange, or in some cases (trader divisions of) banks can be appointed for this task as well. But it is also possible that smaller firms become a Market Maker, this however can only be achieved if such a firm applies to become a Market Maker after it meets some (financial) standards. In case a firm wants to be a Market Maker in the USA, the initial application need to be sent to and approved by the Financial Industry Regulatory Authority (FINRA) [25]. Then the firm needs to register each of the stocks that it wants to be a Market Maker for. After the firm has been approved as a Market Maker by the exchange the firm will get a Market Participant Identifier (MPID) [25]. It is also possible for exchanges to put up tenders (much like a vacancy posting in a job classifieds ad) in which they are looking for Market Makers in a specific market of the stock exchange.

Market Makers always sell from their own inventory rather than that of others and buy stock from sellers when there is a lack of other buying parties. [23, 24, 26]. Because Market Makers are simultaneously placing bid and ask offers, they make their money by basically being paid the spread of a stock (offer – bid price) [23, 24, 26]. As an example, let us assume that the Market Maker indicated the bid-ask spread as $40 and $45. If she now buys 1000 stock and then a moment later sells it, she would have made a $5 profit per share on this turnaround as is indicated by the spread. Typically, they will buy a stock if there are many sellers of a particular stock, but not enough buyers [24]. This is done to instill confidence in the market and attract more investors [24]. Market Makers also sell stocks, specifically when an investor wants to own a certain stock as soon as possible. The Market Maker will then sell the stock to the investor at the listed price [23]. The activities of the Market Makers don't just instill confidence in the markets, but they also make the process of buying and selling stocks smoother and easier [26]. Their activities are seemingly similar to brokerage house, though they differ distinctly as Market Makers do not operate as an agent for customers and do not charge their customers a commission fee [24].

Market Makers much like HFTs do not pursue to establish a meaningful position in a stock. They only try to have a sufficient inventory (or position) of a stock in order to simply service any incoming orders. To achieve this the Market Makers adjust their prices to encourage orders (either buy or sell) to take them closer to the inventory size they are ideally pursuing. The bid ask spread of the Market Makers therefore usually increases as they move further away from their ideal inventory volume of a stock. Alternatively to

changing the quotes Market Makers might keep them the same but change the sizes that they attach to their bid and ask quotes [27].

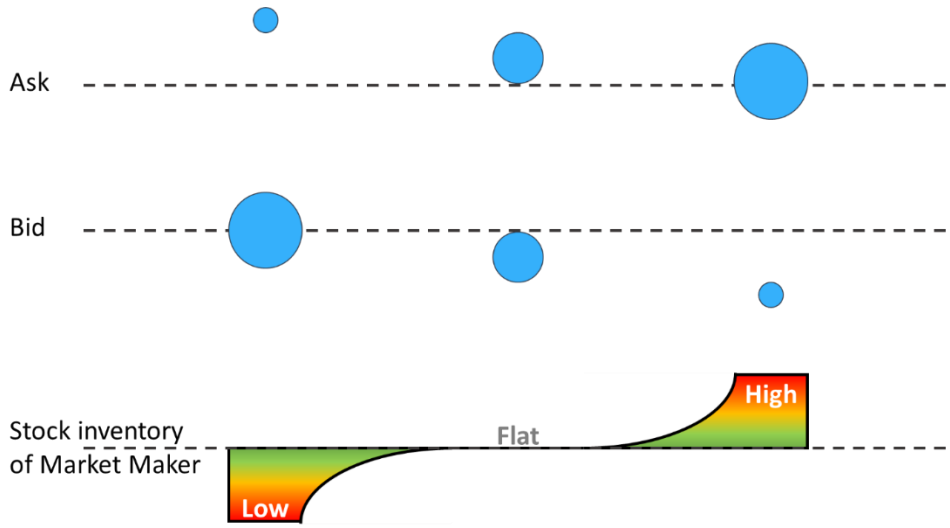

Figure 3: Schematic illustration of how a Market Maker tries to obtain and get rid of her stock inventory [27].

In Figure 3 the process is illustrated. When the stock inventory of the Market Maker is running low, she will increase the size and the price of her bid in order to entice owners of the stock to sell to her. The opposite holds for when the Market Maker has too much stock inventory. She will increase the size she offers up for sale while decreasing the ask price [27].

5. Market structure

Most of the US equity trading is done on one of the registered exchanges and in dark pools [28]. The registered exchanges need to be registered with the SEC and with that they need to meet certain requirements. The dark pools however can be seen as unofficial and much less regulated exchanges. Both registered exchanges and dark pools are run by an Automated Trading System (ATS) [28]. This ATS can receive, process and execute orders very fast. Another analogy that registered exchanges and dark pools have is that they are governed by Regulation National Market System (Reg. NMS). However, there are some key differences in the way they are regulated [28].

Originally Reg. NMS was conceptualized to create a linked national market system and to promote competition between the exchanges. Reg. NMS accomplishes this in two ways. First, a consolidated market data system was created which collects “consolidated quote data” and “consolidated trade data”. Consolidated quote data represent all of the best bids and offers from all registered exchanges, whereas consolidated trade data are the record of all trades executed (which includes dark pools and other alternative trading platforms) [28]. Dark pools are not required to report consolidated quote data and because of this investors have some form of anonymity [29] and with this they can “prevent” tipping of the market about large trades, in which also the main difference lies between registered exchanges and dark pools [28]. The data from these quote and trade feeds are then combined into one single feed which is also known as the Securities Information Processor (SIP). These data are then made available to all other market participants. The SIP calculates the National Best Bid and Offer prices (NBBO) of every listed stock across the nation [28].

Second, the order protection rule is implemented by Reg. NMS. This rule mandates that any trading venue or exchange must execute an order at the current NBBO price. If the exchange is not able to do that, because for instance not enough stocks are available on that exchange to fill the order, then the order must either be cancelled or routed to a different exchange at the best price [28]. Here also the HFT can look at what has happened in the past, as she can see that when an order gets rerouted through the SIP she can move quickly to the other exchange to anticipate on this.

5.1 Stock market order types

Each of the tradable securities has its own Limit Order Book (LOB or Centralized Limit Order Book) which is kept up to date by the exchange. The Order Book is like a log book in which all incoming orders are noted until they are filled. When an order gets filled it is removed from the book. The Order Book has 2 separations, one is dedicated to the intention of buying stocks (usually called the "Bid" side) and the other one is dedicated to the intention of selling stocks (usually called the "Ask" side). When an order comes in, the time of the order is logged together with the price and number of stocks intended to be bought or sold (this is done by either the BidSize or AskSize). In many Order Books these sizes are batched size representations where they are given in single digits, but they actually represent 100 stocks. This means that when an order comes in with the size of 1, it is actually concerning 100 stocks. The orders on the bid side are ordered on price with the highest bid price going to the highest level, whereas on the ask side the orders are ordered with the lowest price going to the highest level. This means that the orders are not sorted by the time they come in but they are sorted based on the price that they are willing to accept to either buy or sell a number of stock and then by time if the price is the same. On the bid side it represents the maximum price at which the seller is willing to buy the specified number of stock, while on the ask side it represents the minimum price at which the stock owner is willing to sell the stock. The Order Book also gives a good impression on whether or not a stock is liquid or how the bid – ask spread for the stock looks like. The spread is the difference between the top level bid and ask price. If this difference is low then this implies the stock is relatively liquid, if the difference is high then the stock is not very liquid.

A trader can issue many different types of orders that will then be processed and executed. The main ones referred to in this thesis are listed below.

5.1.2 Limit Order

A Limit Order consists of a set of information such as a stock symbol (representing the stock of interest), order direction (which specifies if the trader wants to buy or sell a stock), limit price (which specifies the maximum bid or minimum ask price a trader is willing to accept), and the number of shares or contracts that the trader wishes to buy or sell. If the limit price submitted by the trader who wants to buy stocks is not corresponding with a price on the other side of the Order Book then her order stays in the book till the moment a match is found. When it comes to a Limit Order price we can think about it as a future price. Trade intentions that are recorded in the Limit Order Book are considered liquidity providers since with their entry into the book they are indicating their willingness of trading [30].

5.1.3 Market Order

With a Market Order the trader submits to the exchange the stock symbol, order direction and the size of the contract. It is an order to buy or sell immediately for the best price in the Order Book. Trades that are Market Orders are considered to be liquidity takers as these trades are executed instantly with no questions asked and with no indication of intent [30].

5.1.4 Continuous Double Auction Order (also known as Double Auction)

In a Continuous Double Auction (CDA) Order both buyers and sellers of a stock submit their desired prices to an auctioneer. The auctioneer chooses a certain price (p) that will clear the market and finalize the transaction. In stock trading this usually works in this way that a stock is put up for sale for a certain price (S). Now another trader wants to buy this stock and the trader indicates the maximum price at which she would want to buy this stock (B). This now means that sellers that have a price S that is equal or lower than the established price p will sell at price p, and buyers that have a price B that is equal or higher than

the established price p will buy at price p [31]. A detailed example of a continuous double auction is given in Chapter 6.4.

5.1.5 Immediate-or-cancel Order

An Immediate-or-cancel Order can be used on both a Limit Order and a Market Order. This order is a conditional request that is made to the broker (or to a system) to execute a transaction immediately or to cancel it. With an Immediate-or-cancel Limit Order the transaction will be carried out immediately if the number of shares demanded are available at the desired price. If they are not then the order is canceled. With an Immediate-or-cancel Market Order the transaction will be carried out immediately if the number of shares in demand are available at the current best market price, otherwise the order is cancelled [32].

5.2 Submission of orders on the stock exchange, the Limit Order Book

To fully understand where exactly the advantage of the HFT lies we have to know how the stock exchanges process the information submitted by individual traders who wish to buy or sell a security. On stock exchanges many securities are traded daily. Most of the exchanges use an Order Book to keep track of all orders coming in.

A trader that wishes to participate in trading can do that by for instance submitting a Limit Order or Market Order. It is important to note that there are many other types of stock market orders besides the Limit Order and Market Order, however the way that these are processed by the Limit Order Book is very similar. With the following simple example the way the Limit Order Book works is explained [30].

Let us assume that the market opened at 9 o'clock in the morning and that the illustrated Order Book is all for one single stock.

The first order of the day comes in and it is a bid to buy order for 200 stocks at $29.79. The time is logged at 9:01:12 AM and the order number O_001 is given. Since this is the first order it goes to level 1 of the Bid side.

Table 1: Limit Order Book.

Level	OrderID	Time	BidSize	BidPrice	AskPrice	AskSize	Time	OrderID	Level
1	O_001	9:01:12 AM	2	$29.79					

The second bid order comes in at 9:01:23 AM for 400 stocks, but since the price of $27.77 is not besting the previous order it is slotted on the second level of the Bid side.

Table 2: Limit Order Book.

Level	OrderID	Time	BidSize	BidPrice	AskPrice	AskSize	Time	OrderID	Level
1	O_001	9:01:12 AM	2	$29.79					
2	O_002	9:01:23 AM	4	$29.77					

The third order comes in which is also a bid to buy. This time it is for 100 stocks and it is for $29.79. Even though it has the same price as O_001 it will not share the level 1 spot, but it will be slotted in level 2 since the order came later at 9:03:01 AM.

Table 3: Limit Order Book.

Level	OrderID	Time	BidSize	BidPrice	AskPrice	AskSize	Time	OrderID	Level
1	O_001	9:01:12 AM	2	$29.79					
2	O_003	9:03:01 AM	1	$29.79					
3	O_002	9:01:23 AM	4	$29.77					

The fourth order of the day comes in at 9:03:02 AM, however it is the first ask order. It is an order to sell 500 stocks for $29.83.

Table 4: Limit Order Book.

Level	OrderID	Time	BidSize	BidPrice	AskPrice	AskSize	Time	OrderID	Level
1	O_001	9:01:12 AM	2	$29.79	$29.83	5	9:03:02 AM	O_004	***1***
2	O_003	9:03:01 AM	1	$29.79					
3	O_002	9:01:23 AM	4	$29.77					

The second ask order comes in and it bests the first order with a lower ask price. Therefore, it gets slotted on level 1 on the Ask side of the book.

Table 5: Limit Order Book.

Level	OrderID	Time	BidSize	BidPrice	AskPrice	AskSize	Time	OrderID	Level
1	O_001	9:01:12 AM	2	$29.79	$29.81	3	9:03:16 AM	O_005	***1***
2	O_003	9:03:01 AM	1	$29.79	$29.83	5	9:03:02 AM	O_004	***2***
3	O_002	9:01:23 AM	4	$29.77					

Another order (O_006) comes in on the Bid side of the book. But because the order has the lowest bid amount ($29.72) it is slotted at the lowest level.

Table 6: Limit Order Book.

Level	OrderID	Time	BidSize	BidPrice	AskPrice	AskSize	Time	OrderID	Level
1	O_001	9:01:12 AM	2	$29.79	$29.81	3	9:03:16 AM	O_005	***1***
2	O_003	9:03:01 AM	1	$29.79	$29.83	5	9:03:02 AM	O_004	***2***
3	O_002	9:01:23 AM	4	$29.77					
4	O_006	9:03:55 AM	3	$29.72					

A last order (O_007) on the bid side comes in and bests the previous orders (with a price of $29.80) and therefore that order is slotted on the top position on the bid side of the book. This makes the final iteration of the Limited Order Book look as follows.

Table 7: Limit Order Book.

Level	OrderID	Time	BidSize	BidPrice	AskPrice	AskSize	Time	OrderID	Level
1	O_007	9:06:12 AM	2	$29.80	$29.81	3	9:03:16 AM	O_005	***1***
2	O_001	9:01:12 AM	2	$29.79	$29.83	5	9:03:02 AM	O_004	***2***
3	O_003	9:03:01 AM	1	$29.79					
4	O_002	9:01:23 AM	4	$29.77					
5	O_006	9:03:55 AM	3	$29.72					

Now let us assume that a Market Order comes in for buying 500 stocks. With a Market Order there is no level appointment being done, but it immediately clears as the order is filled with the listed orders in the book. In this case this would mean that the order on the Ask side of the book will be cleared as follows, 3 from order O_005 and 2 from order O_004. The price estimation per stock would therefore also be a combination of the orders for an average price of: ((300 x $29.81) + (200 x $29.83)) / 500 = $29.818. This would mean that of order O_004 still 300 stocks would remain for sale, which would then make that the

current level 1 Ask order, since order O_005 was filled and has been cleared from the book. The Limited Order Book would therefore look like as given below.

Table 8: Limit Order Book.

Level	OrderID	Time	BidSize	BidPrice	AskPrice	AskSize	Time	OrderID	Level
1	O_007	9:06:12 AM	2	$29.80	$29.83	3	9:03:02 AM	O_004	***1***
2	O_001	9:01:12 AM	2	$29.79					
3	O_003	9:03:01 AM	1	$29.79					
4	O_002	9:01:23 AM	4	$29.77					
5	O_006	9:03:55 AM	3	$29.72					

The above explained process is not exclusively valid just for HFTs but it is used by all traders on the exchange, however HFTs can use their speed to their advantage to swipe all the orders that they would be interested in from under the noses of any of the other traders before they can make their move. HFTs are able to send many hundreds of orders to the exchange in the time fundamental traders make a move. An HFT with this action can guess what the best price is for the stock in the Order Book. For the sake of clarity let's use an example of an HFT that wants to buy a stock and let us use the Order Book. Let us assume that the HFT in this example wants to figure out if there is a hidden order that has not been made public to traders. These orders do not show in the Order Book as they would do normally, but they are logged in a "hidden" way. In this example this can be seen on the ask side of the Order Book by the gray accentuated level 1 order. The HFT puts up an immediate-or-cancel order to buy stock at a price of $29.84, she does this hoping there is a hidden order that is even cheaper than the cheapest publically visible order (O_002 at $29.85). The order of the HFT however does not find a match, in which case the order gets cancelled. She then directly after this sends a new immediate-or-cancel order to buy at $29.83, but this one also does not find a match and gets cancelled immediately. She does this as long as she finally finds a match at $29.78 and starts to buy up all the stock she can at that price. She now knows that indeed this was a hidden order that was not publically visible to the other traders and that she has made a great deal compared to the best publically visible order.

Table 9: Limit Order Book.

Level	OrderID	Time	BidSize	BidPrice	AskPrice	AskSize	Time	OrderID	Level
1	O_007	9:06:12 AM	2	$29.80	$29.78	3	9:03:02 AM	O_004	***1***
2	O_001	9:01:12 AM	2	$29.79	$29.85	4	9:01:06 AM	O_002	***2***
3	O_003	9:03:01 AM	1	$29.79	$29.89	2	9:04:16 AM	O_005	***3***
					$29.92	6	9:04:32 AM	O_006	***4***

After the purchase has been completed she puts 200 units of the stock that she has just bought up for sale for a slightly higher price ($29.80) which becomes the new level 1 Ask price and matching Bid Order O_007.

Table 10: Limit Order Book.

Level	OrderID	Time	BidSize	BidPrice	AskPrice	AskSize	Time	OrderID	Level
1	O_007	9:06:12 AM	2	$29.80	$29.80	2	9:06:55 AM	O_008	***1***
2	O_001	9:01:12 AM	2	$29.79	$29.85	4	9:01:06 AM	O_002	***2***
3	O_003	9:03:01 AM	1	$29.79	$29.89	2	9:04:16 AM	O_005	***3***
					$29.92	6	9:04:32 AM	O_006	***4***

As that order clears she puts up 100 stocks for sale for $29.79, partially clearing order O_001. This would give her a total profit of ((200 x $0.02) + (100 x $0.01)) = $5.

Table 11: Limit Order Book.

Level	OrderID	Time	BidSize	BidPrice	AskPrice	AskSize	Time	OrderID	Level
1	O_001	9:01:12 AM	2	$29.79	$29.79	1	9:06:56 AM	O_009	***1***
2	O_003	9:03:01 AM	1	$29.79	$29.85	4	9:01:06 AM	O_002	***2***
					$29.89	2	9:04:16 AM	O_005	***3***
					$29.92	6	9:04:32 AM	O_006	***4***

It is important to note that market participants can submit (as we have established by now) different types of orders in the Order Book, this includes the canceling and replacing of orders. When an order is cancelled or replaced, the initial time stamp of the original order still is valid. This means that when an order is replaced by an order with a different price, size or bid/ask direction, the time stamp does not change. However, as a way of compensating for this, order sizes can only be made smaller when an order is changed but they cannot be increased [33]. Summarizing, the traders can submit multiple orders. They can also change existing orders by manipulating the price in order to move up or down in the Order Book.

6. Potential HFT strategies on the stock exchange

When it comes to strategies that specifically an HFT might use, there are a few which we can highlight. Some of these are more common than others and some are even considered to be on the borderline of being ethical or even legal. Some of the less controversial strategies are elaborated below, followed by the more controversial ones.

6.1 HFT strategies

6.1.1 Electronic Front Running

Front running (also known as insider trading) is defined as trading done with the aid of nonpublic information received or gathered by a trader. This news can be related to an order or it can be price influencing information. A trader who acquired nonpublic information is able to front run anyone else that does not have this information, and make a favorable decision about trade that will benefit her. The simplest form of front running is insider trading, meaning that a trader received nonpublic information from a person who violated confidence, trust or an embargo. Due to the fact that an HFT trades for herself and does not use any nonpublic information but rather finds hidden information in the Order Book, this "electronic front running" can therefore be considered as "legal". This is because electronic front running does not use any given information that was released ahead of time or in violation of trust like it would be the case with insider trading [34].

6.1.2 Large Block Orders

Let us assume a situation in which an investor wants to buy 150 000 of XYZ stock which is priced at $5.20 per stock. The investor submits a request to her broker to purchase the stock for her, the broker then proceeds to investigate the availability of the stock. The broker sees that there is a total of 200 000 of this stock available to buy, spread over different exchanges at the same price. For the sake of simplicity let us assume that each of the exchanges have 50 000 of the stock available. An HFT is at the same time placing large amounts of sell orders of stocks, among which is also the stock of XYZ. As the broker puts up the order to buy 150 000 stocks of XYZ she ends up buying all the available 50 000 stocks at exchange 1, part of the 50 000 stocks were also the stocks being sold by the HFT. As the order was only partially filled the remaining order gets forwarded to the other exchanges to be filled there (according to the order protection rule). The HFT knows (due to the consolidated trade data) that only 50 000 stocks were actually sold on this particular exchange. She also knows that due to the order protection rule the order will be forwarded on to the other exchanges, therefore the HFT quickly rushes to the other exchanges to buy all the available stock at the NBBO price. She then quickly puts up the stock back for sale at a slightly elevated price of $5.22. As this is now the best available price in the market, the remaining order of the broker will be filled with these higher priced stocks being sold by the HFT. Alternatively, the HFT will hold the stocks hoping that the large order will drive the price up even further [28].

6.1.3 Sliced orders

Sometimes brokers that want to buy or sell large amounts of stocks slice their order so as to not put up a large batch order to buy at once. This way they hope they will not drive up the price and (with a little delay) get the amount of stocks they want for their client at the right price, or sell them at the right price. But this does not always work out that way. HFTs still have ways to detect these types of "hidden" orders by using for instance complex pattern recognition software that informs them about trade volume, order

size and other characteristics. Alternatively, an HFT can check the latency of the broker. Latency, if studied properly and accurately recorded by the HFT, can be used to identify a broker. By using this latency data the HFT could identify which trade was done by which broker. If she sees a certain pattern (trade volume) and a certain latency it is most likely the same broker as in the past. This could help to tip the HFT off when this exact latency pops up somewhere, it could indicate a new large trade is about to happen [28]. Similarly, an HFT can also study order interval times to also anticipate on the timing of a trader's next order.

6.1.4 Pinging

The HFT can also use pinging or sniping to discover hidden sliced orders. As described by Jiangmin Xu [35], pinging (otherwise known as aggressive fleeting orders), is a strategy used by HFTs in order to find and learn about hidden orders inside the Bid/Ask spread. A hidden order is placed inside the Limit Order Book without being observable to any of the market participants. An HFT submits limit orders which she shortly thereafter cancels, as was demonstrated in Table 9. The HFT is able to send large amounts of orders with different prices within a millisecond giving her the advantage that eventually a matching order with a particular price will be cleared by the market [35]. Ping orders work practically like a sonar in the ocean, they detect a trade in the Limit Order Book. Once detected, the HFT then is able to intercept the whole order all to herself before anyone else can [34]. The pinging technique currently is considered to be legal and is allowed by many exchanges around the world [35, 36]. However some experts have the opinion that it is not a legal technique, or that it should not be, due to the fact that it violates many of the same regulations that are violated by spoofing (and other similar techniques) which is illegal [34]. When we consider the activity of pinging to our hidden sliced order example here, the HFT cycles through a wide range of orders. When the HFT however stumbles on a large hidden order she will again move in front of the traders by buying up all the stock that she feels is of interest and the HFT will again exploit the increased resale price strategy [28]. Dark pools are often thought of being more anonymous than registered exchanges. The thought is that therefore trading on a dark pool would make it much more difficult to be front run. Theoretically this is true, however several dark pools have been accused of selling access to trading data to high-frequency traders, consequently they are now facing lawsuits. HFTs nevertheless use pinging on dark pools just like they would do on registered exchanges to sniff out large orders and subsequently use the same tactics to maximize their profits [28]. As mentioned earlier, dark pools do not log consolidated quote data however they do log consolidated trade data. This fact is used by the HFT, as when an order gets cleared this information is being flashed and could be used for exploitation by the skilled HFT.

6.1.6 Manipulating the price

Let us assume that there is a stock for which an HFT has a buying order booked in the system along with some other traders. Now an FT needs liquidity and decides to sell a number of this stock for a price of $5. The HFT cancels her buy order and immediately puts up a sell order to increase the pressure and create a shift towards selling of this stock rather than buying. Because the HFT is so quick the stocks that she put up for sale are used to fulfill the buy orders of the other traders. After the inventory of the HFT is depleted the HFT quickly puts up a buy order in the system but at a price that is lower, for instance at $4.98. The FT notices that nobody is buying her stock anymore at $5.00 and decides to sell it at the price proposed by the HFT, but this can only work if the HFT is fast enough to beat the Market Maker (MM) in the queue. Because the HFT is at the front of the queue she is also able to notice first when the market balance shifts again towards buying, at which the prices will go back up (to for instance $4.99). As soon as this happens the HFT will cancel her buying order and will start to sell her bought stocks again at this slightly elevated

price. The MM will still however be buying the stocks at this new price as the HFT will continue to unload her stocks till she has no stock. The HFT therefore has made a small profit on the stocks she was able to buy at $4.98, and a loss on the stocks she might have bought at $5.00. But due to her speed she should have been able to buy the vast majority of the stocks at the lower price [18].

6.1.7 Immediate-or-cancel sell

When, for instance, positive company performance news comes out, the price of the stock will go up because market participants will respond to the good news and start buying that company's stock. HFTs, due to their sheer speed, will be able to notice non HFT buying orders popping up in the system. An HFT can then buy as many as possible of these shares and then post a so called immediate-or-cancel sell order at an elevated price. Since the stock is "sold out" other traders still interested in the stock will have to buy them from the HFT at the new higher price. If there are no takers however the HFT will cancel the order and post a new one at a lower price, this process will repeat until she discovers a price at which she finds a buyer. The HFT will then try to offload all of the stock at that price, making a profit [18]. This is similar to the pinging strategy, except here it is not the intention to find hidden orders. With the immediate-or-cancel sell strategy the HFT wants to find the price at which the other traders would want to buy the stock.

Sometimes when a trader has the intention to buy a stock, the order is flashed in the system briefly (it pops up in the logs for a short moment). HFTs can catch this happening in the system and they can act on it. If the flashed deal was a relatively small batch deal at a certain price, the HFT will then quickly go and buy all outstanding stocks concerning the flashed deal. After buying all the stock the HFT then proceeds to offload it again in order to have zero inventory. This is done in such a way that the price is high and gradually going down until the party that has initiated the deal which got flashed agrees to the price and buys the stock from the HFT [18]. With big batch deals this strategy is not so easily executed, as the HFT needs to be able to find an equally big batch of available stock at a reasonable price in order to be able to resell it at a profit.

6.1.8 Rebate strategies

Another way that HFTs can make profit is not only by buying cheap and selling high, but they can also make money by getting rebates. Exchanges offer rebates to traders that trade large amounts of stocks, they do this as an incentive to provide liquidity. This means that an HFT just by buying and selling a stock at the same price can still make a profit based on the per share rebate she might have gotten [18].

6.2 Controversial HFT strategies

An HFT has a speed advantage that can be used to manipulate the market. This advantage can also be used to manipulate prices in such a way that they might seem as genuine price movements, but in reality they are not. In this section the more controversial manipulative tactics are explained.

6.2.1 Front running by insider trading

Front running can be done both legally and illegally. The legal form involves for instance, the mining of the LOB or using published or otherwise released information obtained in a legal way. The illegal form involves information that was released or leaked ahead of time as to when it was supposed to be released. This kind of information usage can be very profitable. For instance, market moving information might heavily impact the price of gold futures or other commodities. Wednesday the 18th of September 2013 at exactly 2 PM the Federal Reserve (FED) made a surprising decision not to slow down its bond purchases [37]. The information was embargoed not to be released by anyone publically before 2 PM outside of a

lock out room in the William McChesney Martin, Jr. building in Washington DC. As the deadline approached however, journalists were given paper copies of the decision in order for them to digest and prepare for the public announcement. Fed officials instructed reporters not to send information about the decision before 2 PM as measured by the atomic clock in Colorado. And yet as the announcement was made public, it seems that a number of HFTs in Chicago have had access to this information about 7 milliseconds earlier [37]. The gold futures are traded in Chicago and a number of massive gold purchases were made a little bit earlier than it would have been possible for the Fed information to have reached the traders in Chicago. The only way this information could have reached the traders before the embargo was by a leak of some sort. It is also conceivable that a low latency news service had already preloaded the decision on their servers in Chicago and released it at precisely 2 PM together with the Fed, however the news agencies release of the decision would have reached the traders sooner as the Fed release was traveling from Washington [38]. This breaking of an embargo is illegal and can be used to make profits of gigantic proportions [34]. The traders in Chicago have either indeed used this leaked information, or just took a gamble and were very lucky (and rich afterwards).

6.2.2 Wash trading

Wash trading is the activity on the stock market that is intended to create a false impression of an active market of a certain stock, whereas in reality there is very little activity. This is done by a trader (or a group of collaborators) in order to lure in other traders to participate in the activity of that particular stock [39]. The way washing works is that one trader keeps buying and selling stocks to their own account. If there are a group of collaborators working together then they buy and sell stock to each other without the stock effectively changing ownership and/or being subjected to market risk [39]. An example of wash trading is given below.

Trader A is putting up a Limit Order to buy 10 000 stocks at $29.79. A few seconds later trader A puts up a Limit Order to sell 9 500 stocks at $29.79. The stocks essentially never change ownership as they always remain with trader A.

Table 12: Wash trading.

Level	Trader	OrderID	Time	BidSize	BidPrice	AskPrice	AskSize	Time	OrderID	Trader	Level
1	***A***	O_001	9:01:12 AM	100	$29.79	$29.79	95	9:01:16 AM	O_002	***A***	***1***

Trader A puts up several buy orders quickly after each other for a total of 20 000 stocks. A few seconds afterwards trader A puts up a sell order for 19 000 stocks. Again, the stocks still remain in the ownership of the trader, just making it look like a real trade is going on.

Table 13: Wash trading.

Level	Trader	OrderID	Time	BidSize	BidPrice	AskPrice	AskSize	Time	OrderID	Trader	Level
1	***A***	O_003	9:05:20 AM	100	$29.79	$29.79	190	9:05:27 AM	O_005	***A***	***1***
2	***A***	O_004	9:05:25 AM	100	$29.79						

Another example is for instance when trader B is collaborating with trader A (it can even be the same person/entity, just under different trader IDs). Trader A puts up a buy order for 10 000 stocks for $30.00. Trader B puts up a sell order for 9 500 stocks at $29.50. On some exchanges these seemingly close but not matching orders will still be matched as they are close enough, so they get matched with certain preconditions (like which price will then be taken for the match). Similarly, a bit later, trader B puts up a buying order for 9 500 stocks for $30.50 and trader A puts up a selling order for 10 000 for $30.00. The orders are again matched by the exchange, but effectively not much has changed in the ownership of the stocks.

Table 14: Wash trading.

Level	Trader	OrderID	Time	BidSize	BidPrice	AskPrice	AskSize	Time	OrderID	Trader	Level
1	***A***	O_010	9:50:10 AM	100	$30.00	$29.50	95	9:50:11 AM	O_011	***B***	***1***
2	***B***	O_012	9:55:00 AM	95	$30.50	$30.00	100	9:55:01 AM	O_013	***A***	***2***

In the examples above it can be seen that between an initial action of the washing party and the final action there are a few seconds. In reality the times can be much shorter and take even just a split second. Though it is possible for wash trading to be thwarted by other traders if they are fast enough in posting a matching counter order after the initial order of the washing party.

6.2.3 Spoofing and layering

In order to explain the phenomenon of spoofing let us assume that the intention of an HFT is to buy a stock. However, the HFT puts up Limit Orders to sell in the Order Book. The HFT does not want these orders to be executed, so they are placed in this way that they will beat the current best ask price. Due to the speed that the HFT has, she does not need to worry much about the fact that these orders will actually get cleared. Since the HFT will be able to monitor the trends of the market and cancel the orders in time if the market price actually will come close to the quoted ask price of the HFT. Upon quickly cancelling the order she places a new buy order at this new low price, revealing her actual intention and completing the spoof. When this action is done singularly, meaning with a onetime single order and not with a large number of consecutive orders, this is called spoofing. When the HFT puts up a sequence of limit sell orders all above the ask price in incremental fashion, sometimes even in combination with large amounts of stocks, then this is known as layering. This is done to scare other market participants and make the other traders to actually sell. This can bring the price of a stock down and it would be possible for the HFT to buy the stock for a lower price than it was just moments ago, since the HFT would in the meantime have placed a buy order for the stock and cancelled her order to sell [40].

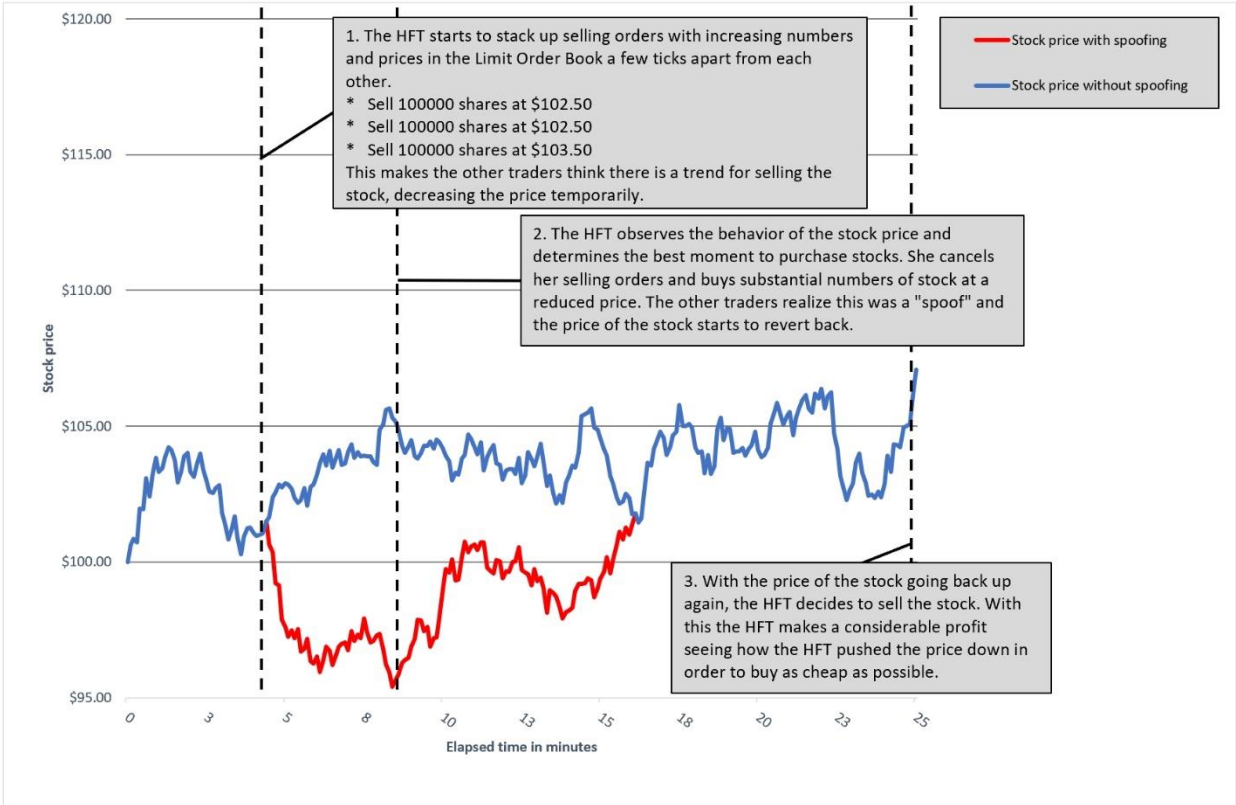

Figure 4: Spoofing.

Spoofing and layering can also be used in the opposite direction when the HFT wants to shortly beef up the stock price before she in reality is intending to sell (large numbers of) stocks.

6.2.4 Smoking

Smoking is oftentimes described as the luring of (non-)HFT traders. The HFT can do this by enticing a non-HFT with incredible prices. Let us assume that the HFT puts up a buy order for a stock which currently has the best bid offer of $100. The HFT can best this by for instance putting the bid price for 10 000 stocks on $120. An FT will see this and might want to sell her stocks at that price to the HFT by issuing a market order, as the price is just too good to pass on. However, due to the low speed of the FT the HFT has time to change the bid order in the Order Book by bringing the price down (still being however the best price, but not by nearly as much as she initially indicated). The market order will therefore get cleared for the much lower bid price. This is because the FT did not have time to respond to the revised best bid price of the HFT. This way the HFT can buy a lot of stock fairly quickly that she otherwise would not have gotten if she would have put up a more realistic bid price [40].

6.2.5 Stuffing

The tactic of (quote) stuffing by HFT traders is performed specifically to throw slower traders off their pace even further, by entering orders into the Limit Order Book and withdrawing them subsequently. This is done specifically by HFTs that put up a lot of immediate-or-cancel orders, these all need to be processed by the market (and the market participants). The putting up of the immediate-or-cancel orders is done in such fast fashion that the "buffer" of the market gets filled up, or stuffed. Because of this the participants get slowed down considerably by the fact that they first need to process all these "fake" orders. By stuffing the market system with all of these orders that the market is not able to clear in a timely manner, so called phantom orders can occur.

6.2.6 Momentum Ignition

Momentum ignition is in essence very similar to layering and spoofing. It might even be confused with the two. It does however differ in this way that the instigator will usually see through her action. She does this by selling or buying some of her shares in aggressive fashion in order to provoke, or rather ignite, an upward or downward price momentum of the stock in question. This way it can be used to the HFTs advantage [8]. The HFT essentially will know what will happen with the stock price for a short period of time. As the HFT is the instigator of the ignition, she has seen that the stock she is eyeballing tends to spike in price after an initial sign of interest that is shown by the market. The instigator starts to aggressively purchase the stock and the price goes up. Another trader notices this and as such also starts to buy it at the elevated price. Right before the price starts to decrease again the trader that initiated the spike will start to sell off her shares, the other trader also will notice the change in the pricing and will also start to sell. However, as the instigator bought the stock when it was at a lower price, she would have a larger profit compared to the second trader. This tactic can also be used if the instigator has a large number of stocks that she already owns. The HFT will then ignite this price increase so that she can unload her shares at the higher price.

6.3 HFT strategies on a single market

As described previously the Limit Order Book contains current trading prices (Bid/Ask) of a stock. Latency arbitrage allows the HFT to take an advantage over the FT by simply buying the stock before the order of the FT has even a chance to get to the Limit Order Book. Then immediately thereafter the HFT puts up a sell order. When finally, the order of the FT gets put in the Order Book it is sold to her for a higher price. An HFT maximizes her advantages by co-location, this means that even though an HFT and an FT will send their orders simultaneously the order of an HFT will arrive in the Limit Order Book first, since HFT firms are frequently co-located at the location of the exchange they most often trade at, as this minimizes their latency. If additionally, the HFT has a foreknowledge of how many of the stocks the FT wants to buy, she can make a profit from a simple round trip (a buying and immediate sale cycle). It is assumed that if an FT is using an algorithm to trade, it is relatively easy for an HFT to learn about the moves of the FT and use it to her advantage. Let's analyze this by a simple example. An HFT purchases an X number of stocks for the current Ask price. The HFT after buying the previously mentioned X number of stocks will immediately put out an order to sell her stocks for a slightly increased price. Because of this the FT has a disadvantage since she will have to buy her stocks for this higher price than that the HFT originally did. If the number of stocks that the FT wants to buy is equal to the number of stocks that the HFT is selling, this means then that the profit that the HFT generates is equal to the number of the new current Ask price multiplied by the X number of stocks.

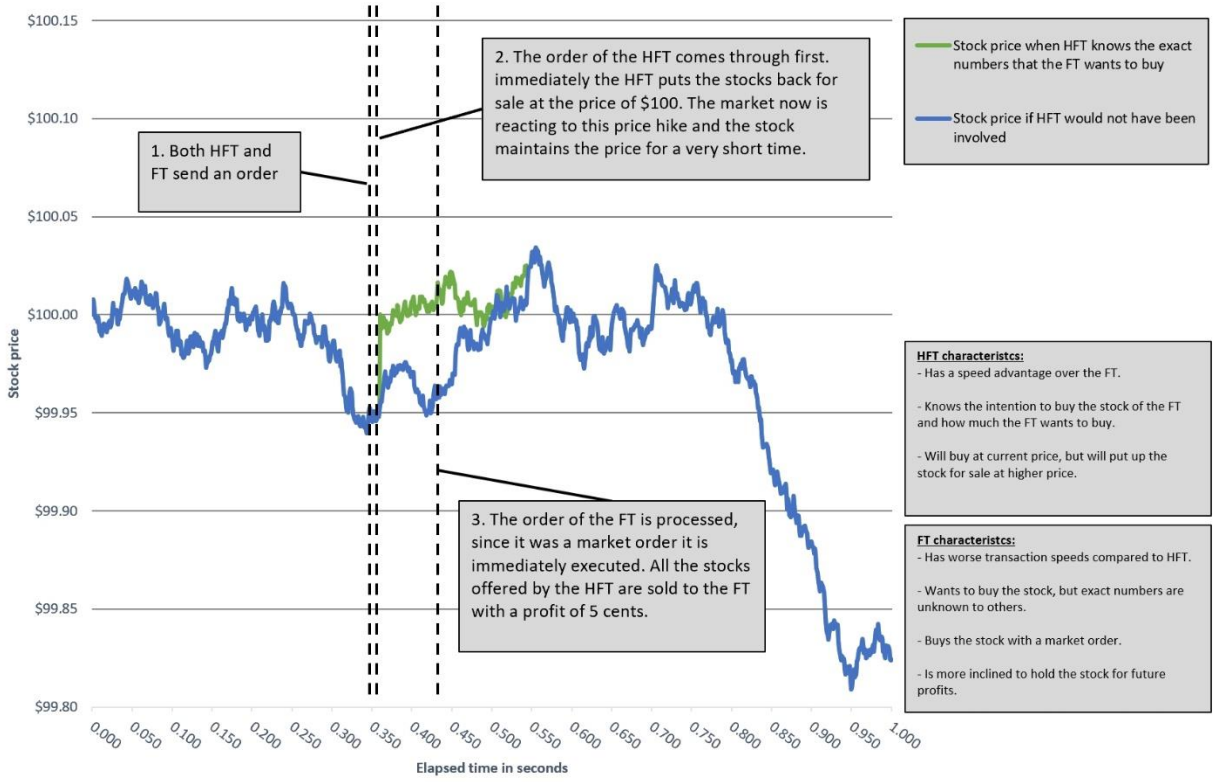

Figure 5: The HFT buys the stock before the FT can get them.

In case if the number of stocks that the FT wants to buy is lower than the number the HFT offers, then the profit will be calculated as follows: the stocks bought by the FT will be multiplied by the new current Ask price and the remaining ones most likely are going to revert to the original price and will be sold for that price to new buyers. The HFT is able to make a risk free profit, however it is limited to the number that the FT wants to buy. Everything traded outside of the amount dictated by the FT is considered risky [41].

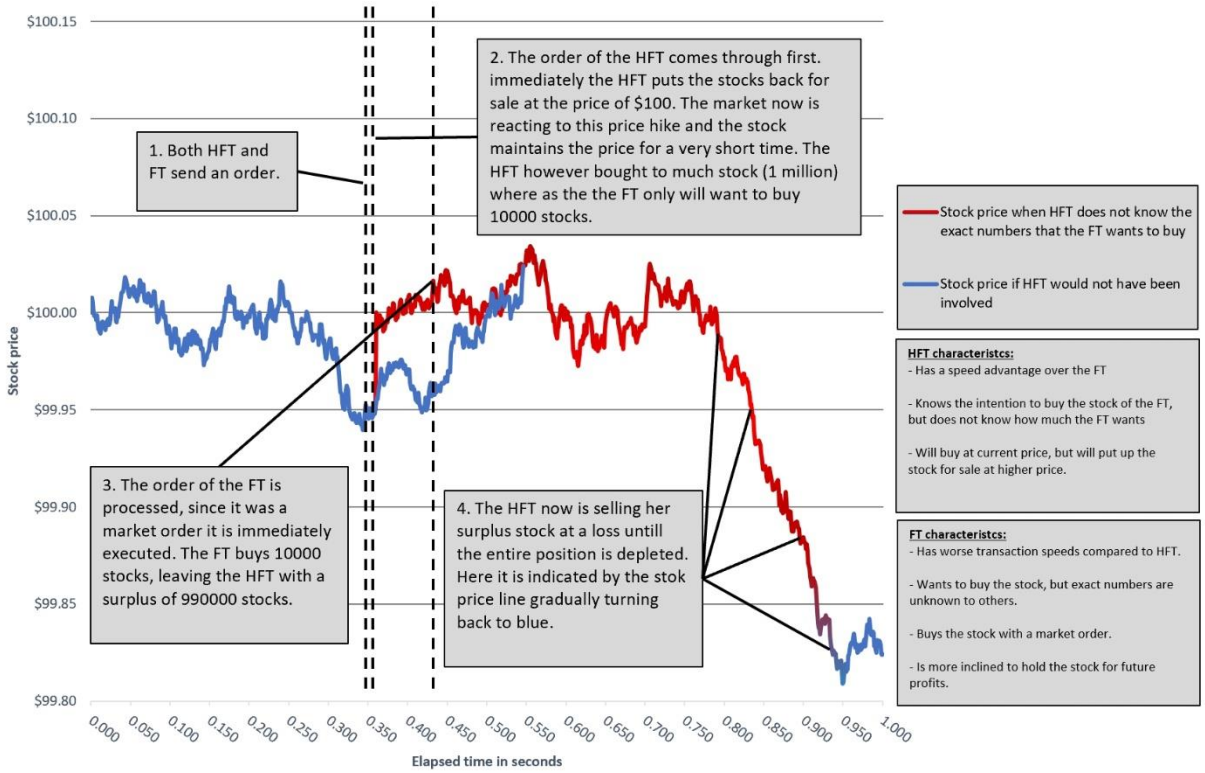

Figure 6: The HFT overestimates the number the FT wants to buy.

Though the above only really applies if the HFT is aware of the intentions of the FT (does she want to buy? And if yes how many?). The situation for the HFT becomes riskier and more unpredictable if the intentions of the FT are completely unknown. If the HFT buys stock with the intention to resell them to the FT however the FT is not interested in buying the stock, then the HFT will have to sell the stock at a loss using market orders [41]. A more detailed single market game simulation will be given later on.

6.4 Multimarket HFT strategy

HFTs in real market situations are usually active in many markets simultaneously. They do not always actively participate in market activity, but they can initially be observing these markets and await a moment when an arbitrage opportunity arises. In literature, usually single market activities are discussed or modeled. Wah and Wellman [42] show a two-market model in which an HFT is faster than the SIP. The SIP updates public price quotes and calls them the NBBO. As orders for buying and selling stream into the exchanges the SIP updates the NBBO accordingly. When the updates are done then the Reg. NMS mandates cross-market communication and order routing for the best executions. Since however the SIP updates the NBBO with some latency (δ), an HFT can beat the SIP as she analyses the market activities faster and can buy a stock for a lower price in one market and sell it on for a higher price in another market. If the SIP would have determined the NBBO without latency the HFT would not be able to act upon this arbitrage opportunity as there would have been an intermarket routing of the sell-buy order without the intervening of the HFT.

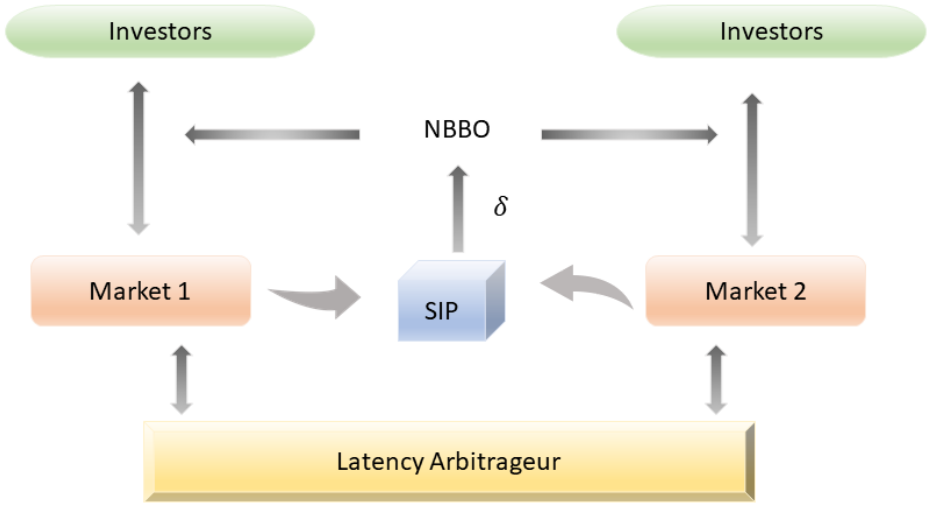

Figure 7 Wah and Wellman's [42] two-market model.

Assuming the model of Wah and Wellman where there are 2 markets in which an HFT is active. These two markets have an initial NBBO of $104 bid and $110 ask. At time t a background trader (whose primary market is market 1) wants to sell a stock at $105. At time t+1 a background trader (whose primary market is market 2) wants to buy that same stock at $109. If the SIP would have been latency free and would have updated the NBBO instantly the two orders would have been routed and linked with each other. However as this is not the case and there is latency, the NBBO does not get updated instantly. The orders of the two traders therefore stay in their respective primary markets. The HFT spots this arbitrage opportunity and buys the stock in market 1 for a price at which she knows she will get it for sure (in this case $107, but since the ask is $105 she gets it for the lower price), the HFT then immediately proceeds to put up a sell order for the stock in market 2 for $107 but she actually gets $109 since that was the current bid. With this action the HFT was able to make a $4 profit [42]. The HFT is able to do this by putting up an order which is known as a CDA. The HFT with this specifies the maximum price at which she would be willing to buy, and the minimum and which she would be willing to sell. By looking back at the example above, the HFT puts up a bid to buy for $107 as a maximum price. With this price the HFT will surely be entered at the top level of the Order Book. But what defines a CDA order is this that even if the bid to buy is at $107 the stock will still be bought by the HFT for the price defined by the seller (in our case $105). The same applies to when the HFT is reselling the stock later, the HFT indicates she would be willing to sell for at least $107. However, since the order is of this kind that if there is a better offer, the HFT will take that instead and that will be the final sale price (in our case $109) [42].

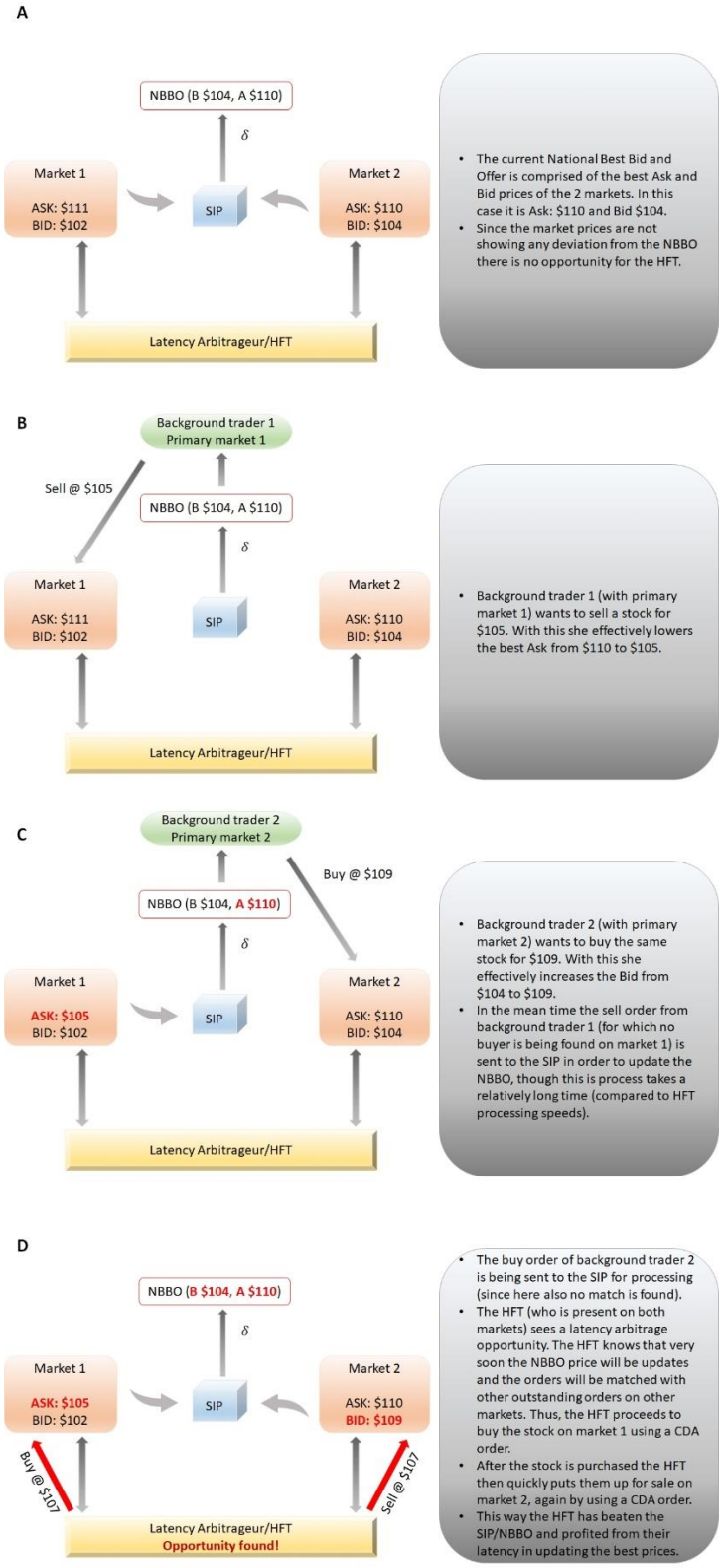

Figure 8 illustration of the discovery of an arbitrage opportunity by an HFT (latency arbitrageur) [42]. A: Shows the initial state of the 2 markets. B: Background trader 1 wants to sell at $105. C: Background trader 2 in another market wants to buy at $109. D: the HFT notices that the NBBO is not up-to-date and therefore she sees an arbitrage opportunity in which she sells the for sale stock at a profit to the buyer in the other market.

6.5 Machine learning

It is not well understood how an HFT is able to anticipate so quickly on the stock market and how she is seemingly able to predict in which direction the stock price will move. As was described earlier, one reason she is able to beat everyone to the punch is her sheer speed advantage. As the market is live and changes are happening she is able to react immediately upon them. She can instantly see when for instance the trend from selling a stock switches to buying and as this happens she can switch her order direction around virtually immediately. However sometimes it seems that the HFT is almost capable of foreseeing how some stocks are going to respond to for instance bad media exposure or to good financial results. This in combination with the fact that HFTs sometimes seem to be probing the market (with some of the earlier mentioned strategies) gives reason to believe that machine learning is used by HFTs. In this subchapter briefly the principle of machine learning is explained and how it can be used by HFTs. It is not meant to explain in-depth the detailed workings of machine learning or how it should be programmed.

Similar to algorithmic trading, machine learning has seen a rapid development over the past 2 decades. Increasing computer power in smaller and more practical workable computers has paved the way towards using computer algorithms to learn about human preferences and behavior. Machine learning can aid in making decisions or become part of a fully autonomous artificial intelligence that will be able to make decisions on its own accord.

At the moment however it is unclear how many HFTs (if any) are using machine learning or artificial intelligence (AI) [43]. Nevertheless, it is safe to assume that some form of machine learning/AI is employed by HFTs in the present day. Especially considering the fact how the before mentioned Jenner crash seems to have characteristics of machine learning or AI shortcomings [12, 13].

Machine learning can more or less be considered as a part of a fully-fledged AI. If an AI is a fully autonomous sensing, reasoning, acting and adapting algorithm, machine learning is an essential part of it. Very simplistically put, in general there are 3 levels of computerized intelligence: deep learning, machine learning and AI (see Figure 9) [44].

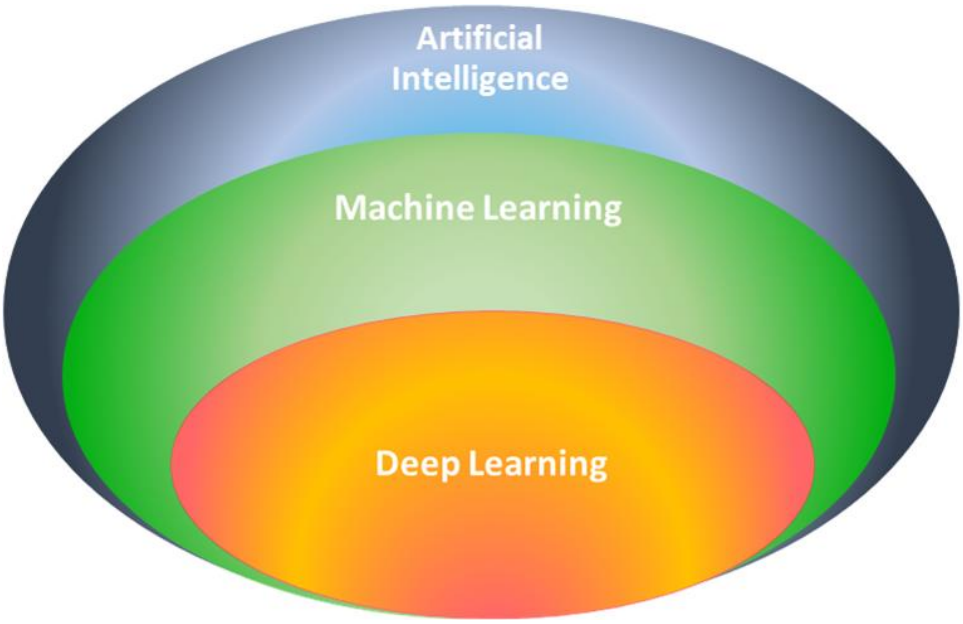

Figure 9: The 3 levels of computerized intelligence.

Deep learning is vaguely inspired by biological nervous systems in the way they process information and communication patterns. Deep learning is mostly based on an artificial neural network, though it can also be comprised of and use formulas and variables that are organized layer wise. Deep learning is most often used in speech, image recognition and natural language processing.

Machine learning algorithms are different in this that they build a mathematical model of sample data (usually referred to as training data) from which they then learn and use to make predictions or decisions without being programmed specifically for the task.

AI on the other hand is the full integration of deep learning and machine learning which would enable a machine or a system to mimic human behavior in this that it can effectively and autonomously sense, reason, act and adapt.

Machine learning has various different types of learning algorithms which can be classified as: supervised learning, unsupervised learning and reinforcement learning [45].

Supervised learning applies to algorithms that need to work with features and labels. For the sake of understanding let us assume that a friend would like to give you a chocolate drop candy. These come in 3 variations, one which is plain chocolate and weighs 1 gram, one which has a cookie center and weighs 2 grams and one which has a peanut in the center and weighs 4 grams. The algorithm can then use the input of the weight (the feature) to determine the type of the candy (the label).

Unsupervised learning does not use labels for the data. When taking the chocolate example into account again, this would mean that for every piece of given chocolate the algorithm will plot the weights of the candies and it would be able to cluster them.

Reinforcement learning is based on feedback and reward. If again we use the candy example, let us now assume that the type of chocolate candies is refined by the colors. Red is a plain chocolate one, blue is one with a cookie center and yellow is one with a peanut. If the algorithm is now given a picture of a red

candy the algorithm will try to identify it. Let us assume that the algorithm will say it is a candy with a peanut, the user will now feedback to it by saying that this is wrong and the algorithm will try again. On the second try it will say that it is a candy with a cookie center, again the user will say that this is wrong. Lastly, the algorithm "understands" that it can only be a plain chocolate one, which the user says it is correct and the algorithm will store this data and use it for its subsequent analysis.

If we now explore the possibilities of using these types of machine learning for High Frequency Trading it is possible to come up with a few examples that show how machine learning might be used.

Figure 10: Supervised learning.

How supervised learning can be used for an HFT algorithm can be illustrated in various different ways. One way can be by having the algorithm learn to connect features to a label (see Figure 10). Let us assume that the trader is teaching the algorithm to distinguish between a high and low current ratio (the label). In order to be able to do this the algorithm needs to know how to determine the ratio, it does this by dividing a company's current assents by the current liabilities (the features). After this the algorithm will be able to work out the ratio and use it in its analysis of the performance and futureproof characteristics of the company.

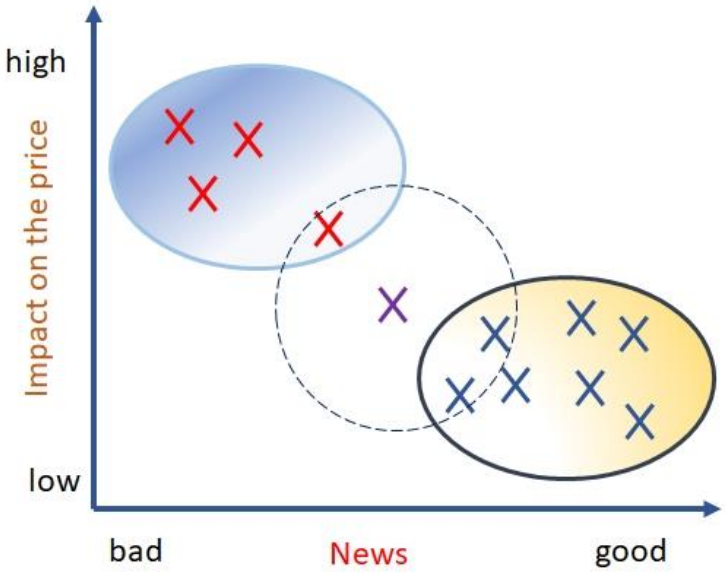

Figure 11: Unsupervised learning.

As for unsupervised learning, this can be used to learn the algorithm to cluster various companies. For instance, when it comes to the impact of either good or bad news on the stock price of a company (see Figure 11). This way the algorithm can be trained to identify companies that do not have (or have a very small) response to bad news. This might be relevant as when an algorithm learns that it does not have to sell a stock immediately upon the breaking of bad news about a company, potential pointless crashes can be prevented and millions of dollars in value will not be lost. Again, being indicative of the notion that the Jenner crash was likely caused by overreacting algorithms. However, what if the choice is not so obvious and a sample value falls right in the middle of the clusters? Then various methods can be used to determine the best possible course of action. One such method is the "k-nearest neighbor's algorithm" method. Basically, with this method, for the non-clustered sample (or feature) the algorithm determines to which clustered sample (or feature) space it should belong based on the k-NN classification (plurality vote) or regression (average values k-nearest neighbors). As is seen in the example above, the non-clustered X falls right in the middle of the 2 clusters, but due to a vote of plurality it is classified as having a low impact on the price.

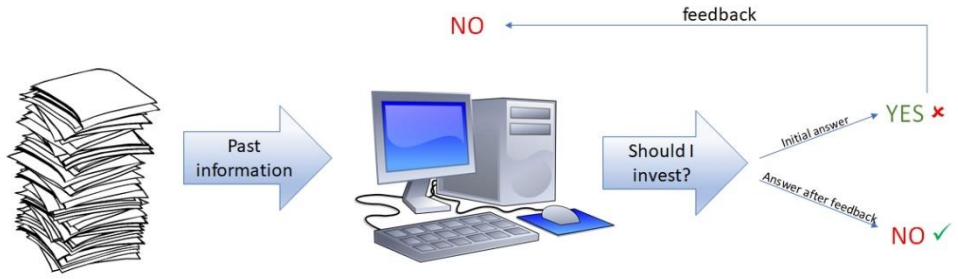

Figure 12: Reinforcement learning.

When it comes to reinforcement learning, an HFT can teach its algorithm based on historic data what it should do in a certain situation. This way the user can provide the algorithm feedback based on which it will make (hopefully) the right decision in the future (see Figure 12). For instance, let us assume that the trader is teaching the algorithm on how to act upon stock "x". The trader now asks the algorithm what it would do if a stock has a lot of negative attention in (social) media (assuming the algorithm is able to distinguish between positive and negative attention). Let us assume, the algorithm would say that it would choose to buy the stock. The trader disagrees and feedbacks to the algorithm that it should not buy the stock in this case as previously in this situation the stock has performed badly afterwards. Now the next sample case is presented in which there is a lot of positive (social) media attention concerning stock "x". Again, the trader asks what the algorithm would do. The algorithm again says that it would buy the stock. The trader now agrees and feedbacks to the algorithm that this is the right decision seeing how the stock usually fared well after positive (social) media exposure. This way the algorithm will get trained to read real situations of (social) media exposure and will based on this training be able to make the right decisions.

With these various methods of machine learning the algorithms can be trained to anticipate on any type of market situation that might occur. The training that the user of the algorithm has given it makes it able to perform real time analysis based on current and historic data. With this the algorithm can within milliseconds make decisions on what to do. Whereas even the most experienced traders not using algorithms cannot even begin to understand the developing situation in the same time. As with humans, machine learning gets better with experience (more data to work with means better decisions are being

made for the HFT). Since high frequency traders have been around for a few decades now, their algorithms have also gotten more experienced with all the data that has been fed into them. It is therefore to be expected that as time will progress the algorithms will become even smarter and less prone to (obvious) errors.

7. The playing field enabled by the stock exchange, the various games that can occur

In this section the various games are discussed that can occur at an exchange between traders. First, we will focus on a game that will indicate the role of the enabler (the stock exchange) and how the enabler benefits from it. Later on, we focus on a zero sum game. And finally, we will discuss and model a game that is more based on principles of supply and demand.

When two (or more) traders decide to buy and sell their stocks, they may use the stock exchange. The stock exchange is considered the enabler of stock trading. As an enabler obviously the stock exchange is receiving income from all the trades being done. However, there are some unique situations in which the exchange specifically can come out as the most profiting party in the trade of stocks, rather than any of the involved traders. An example of such a case is given in Chapter 7.1.

Often times when reading articles about the interaction between HFTs and other traders the impression is given that a zero sum game is being played, as the HFTs get everything and the other traders nothing. However, we have to realize that this is neither the intention of the HFT, nor is it very true. The HFT has as a principle to have no position in any stock at the end of the day. This therefore means that the other traders in the end do get the stock, they just do so with a side road that happens to be laid out by the HFT. Nevertheless, we will treat the workings of a zero sum game in Chapter 7.2.

The last game that will be described and modeled is based on principles of supply and demand. Though the game is largely based on the model previously described by Cartea and Penalva [18], nevertheless it is needed to get an understanding of the basic principles of supply and demand. In its most basic form stock trading is nothing more than the fulfilling of a demand with a supply of the good that is demanded. To be able to understand how the pricing of a stock therefore is done, we need to understand the basic concept and the origins of supply and demand. The idea of supply and demand is almost so normal that it is strange to think that it would require extensive study or thought to fully understand it, however its concept was not fully described and mathematically established until surprisingly recent times. One of the first mentions of something resembling supply and demand can be found in a collection of Tamil couplets called the Tirukkuṛaḷ supposedly authored by Thiruvalluvar some 2000 years ago. In one of the couplets it states that “if people do not consume a product or service, then there will not be anybody to supply that product or service for the sake of price” [46]. Several other religious scholars have throughout the years contributed to the understanding of basic supply and demand in similar fashion as Thiruvalluvar. Taqī ad-Dīn Ahmad ibn Taymiyyah already in the 13th century understood that “if desire for goods increases while

its availability decreases, its price rises. On the other hand, if availability of the good increases and the desire for it decreases, the price comes down" [47]. It was however not until the 18th century that the phrase "supply and demand" was coined by James Denham-Steuart and Adam Smith [48]. Smith assumed in his book (The wealth of nations) that the price of supply was fixed, but that its value would decrease as the scarcity of the supply increased [49]. French mathematician Antoine Augustin Cournot was first in developing a mathematical model of supply and demand in combination with diagrams [50]. In the 19th century the son of Auguste Walras (close friend of Antoine Augustin Cournot), Léon Walras, further developed the theory of supply and demand. His work in general and his theories have turned into what is now considered the cornerstone of all political and mathematical economic theory. He championed the idea that price is set by the subjective value of a good or item at the margin, a big difference compared to Adam Smith's thread of thought on supply price.

7.1 Prisoners' dilemma – when the third dog runs away with the bone

As electronic trading has emerged in the last few decades, traders became more and more aware of the fact that speed (and with that a competitive edge) can be gained in several ways. As we mentioned earlier, being faster as a trader has the benefit of being able to anticipate on market changes more quickly. In the beginning of the development of algorithmic trading most speed was gained in the programming of the algorithms. In more recent years however most speed gains were made in the area of networking and collocation of a trading firm at an exchange. However, today pretty much all the gains that can be had in networking speed have been exhausted. In addition it has become almost futile for a trader to continuously invest in faster computing power and algorithms since major gains there are also almost not to be had anymore within the current state-of-the-art.

If a trader already has a state-of-the-art computer system and accompanying algorithm, the only way for her to further improve is indeed to collocate at an exchange. For this collocation a trader pays the exchange to use their services and network. This is (obviously) beneficial to the exchange, since they gain an extra source of income.

Let us now assume the setting of a game in which two algorithmic traders are facing each other's competition, they both have an equally fast computer system and algorithm. The only way at which they can differentiate themselves from each other is to collocate at an exchange. Let us assume that their primary exchange of interest is indeed offering collocation services and makes both the traders an offer. The traders know that if one of them chooses to collocate and the other does not, the one who has collocated will have the benefit (δ) of a faster data feed and processing of orders, while the other trader does not (α). The trader that chooses to not invest will not only fall back compared to her competitor, but will also lose the profits to her. If neither of them chooses to collocate, then no costs were incurred by either, nor is there any gain to be had in terms of speed (γ, γ). It is clear that collocating is the dominant strategy, you have to collocate to not risk falling behind which could happen if you do not. If both traders choose to collocate, because of the risk that if one of them doesn't, the other most likely will, both will incur costs but at the end neither will have a competitive edge (β, β). The only party profiting in that situation would be the exchange that will gain a monthly service fee from both traders. It is exactly as the proverb goes, while two dogs fight for a bone, the third one runs away with it.

Table 15: Prisoners' dilemma.

		HFT 2	
		Invest in collocation	Don't invest
HFT 1	Invest in collocation	β, β	δ, α
	Don't invest	α, δ	γ, γ
α < β < γ < δ			

In reality however a lot of traders do not invest in collocation. This is mostly the case with traders that even though they are active in algorithmic trading, their algorithms are not the fastest in the market, or they are not as experienced in the field. These traders choose to not spend their money on collocation since they feel they would never gain their money back on this investment. However, traders that did invest in the progression of their speed and algorithm in the past will likely keep investing.

Let us assume that there are 2 HFTs with an equally fast algorithm, but neither of them has invested in collocation. One of these traders now decides to invest in collocation in order to minimize the effect of latency (see Figure 13). The other trader also knows this and is aware of the fact that if she does not invest in order to just keep up, she will fall behind. This phenomenon of investing to just be able to keep up with the rest and seemingly not gaining and advantage is known as the "red queen effect". Biais and Wooley [40] have described this in the context of HFTs and their constant drive to invest in progressing their algorithms, but it is imaginable that the same now holds up for HFTs trying to minimize latency by investing in collocation services. Though another dilemma is the fact that these massive investments are by some scholars considered wasteful [51]. Large amounts of money will continuously be spent for minimal (if any) progress, which would further disadvantage less technologically advanced investors [51]. Nevertheless, the peer pressure on HFTs is so high that with the current non regulated latency HFTs will keep investing in minimizing data delay and therefore keep expenses going up.

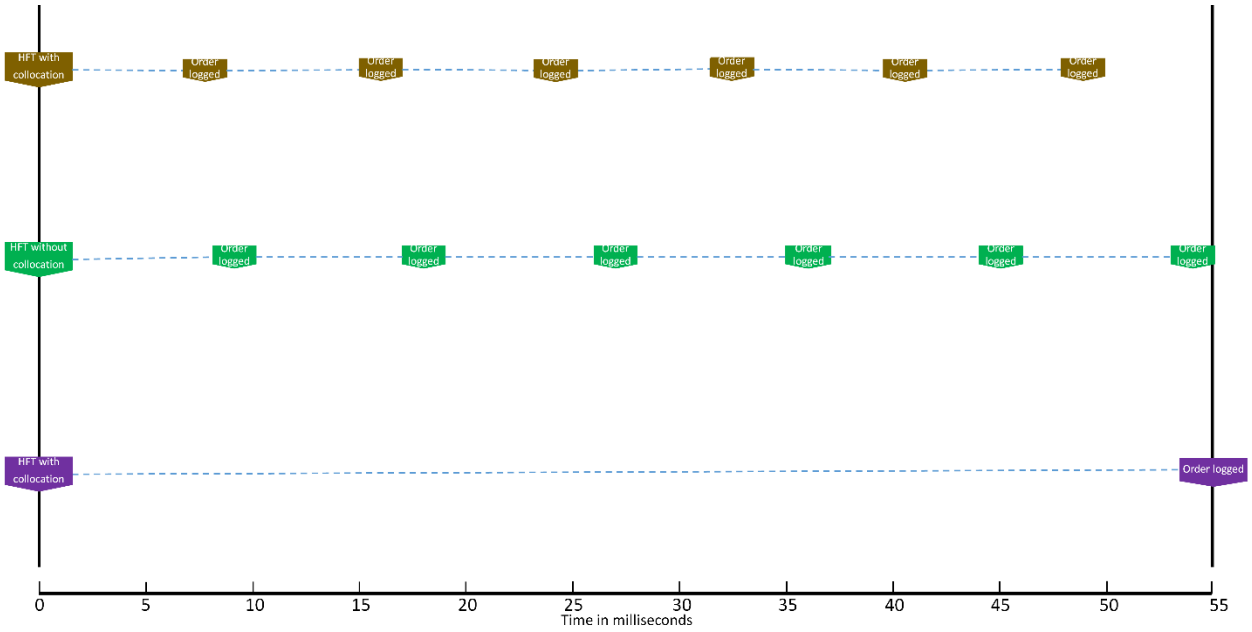

Figure 13: Difference between an HFT with collocation and an HFT without collocation.

7.2 Zero sum game theory

In a zero sum game the gain of one of the players is the loss of the other. That means that the game always has one winner that takes the other players' (anticipated) wealth and the defeated party stays with nothing. These types of games can have anywhere between 2 and millions of players. The hallmark feature of these games is thus that the net change in wealth is zero, the gains to be had are only those that are on the "table" (much like a poker game).

Let us now assume such a zero sum game in the setting of stock trading between an HFT trader and an FT. These two traders are eyeballing a stock that they want to buy and let us also assume that in this game we do not look to the long term payout of the stock, but we only look at owning the stock as the "profit". This is indicated by α, β and γ in the matrix in Table 2.

Table 16: Zero sum game theory.

		Fundamental Trader	
		Buy stock	Do nothing
High Frequency Trader	Buy stock	α,-α	β, -β
	Do nothing	-γ, γ	δ, δ
-α; -β; -γ < δ < α; β; γ			

When a trader tries to own a stock but fails to get it then this trader loses out, which is indicated by –α in the matrix. It is also possible that one trader actively tries to get the stock while the other decides (despite her interest) not to pursue it, the non-pursuing outcome is then indicated by –β and –γ, which still counts as a lost opportunity. When both traders choose not to pursue, then this stalemate situation is indicated by δ. It is important to know that in this game the decision to buy or not to buy the stock will be made at the same time and that the benefits of the HFT being faster are still valid, meaning that she is faster than the other trader in having her order executed. Later on, in the model that is introduced we focus on how an HFT is using her speed advantage over a Fundamental Trader (in the case of the model FT2). Figure 24 shows the course of action that the game takes when the HFT decides to buy the stock immediately. In order to make the game between the HFT and FT2 clearer, Figure 14 shows this same scenario but without any involvement of the MM. We can clearly see that the HFT will always get to own the stock before the FT2 would since her orders get logged faster in the Order Book. Knowing this, let us have a look at how such a game would play out.

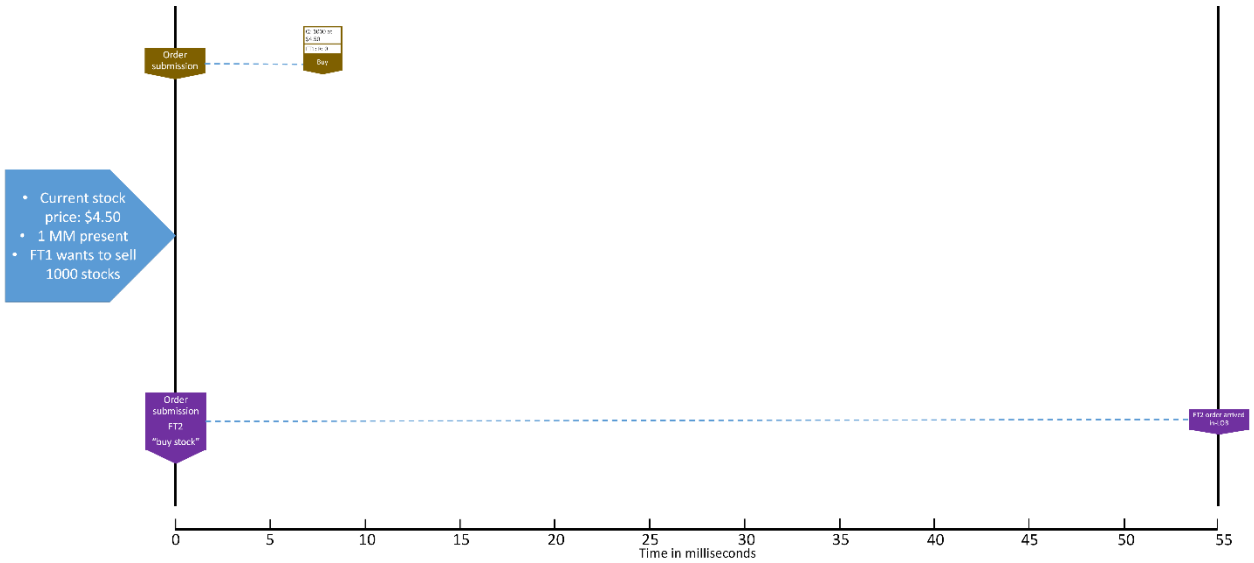

Figure 14: Zero sum game between HFT and FT2.

Let us assume that both traders decide to buy the stock, in such a case the HFT has the advantage anyway and win the game as she is faster than the FT in getting the order effected, yet the FT does not incur a loss as the money to buy the stock stays in her pocket. The only loss is her missing out on the stock (α, -α). If the HFT chooses to buy the stock and the FT does not, naturally the HFT wins the game (β, -β). If the FT however decides to buy the stock and the HFT does not buy it, then the FT wins the game (-γ, γ). Lastly, if neither of them decides to buy the stock then the game has no winner and the prize stays on the table (δ, δ). It becomes evident that the dominant strategy here is that the HFT no matter what is always better off by choosing to buy the stock, seeing how she will always win in that case. Though it has to be remembered that winning the stock does not mean that profit will be made for sure (as can be seen in Figure 24, when the HFT immediately buys the stock, resembling a zero sum game, and then proceeds to barely make a profit from this seemingly beneficial position). When it comes to the FT however she is more or less at the mercy of the HFT and will only be able to win if she buys and the HFT does not, though this is unlikely to happen if the HFT realizes what her dominant strategy is.

7.3 Subsequential game, a model for interactions between HFTs, FTs and MMs

7.3.1 Modeling the interactions of the traders

When trading is done in the real world, frequently 2 traders with a matching transaction order are not present at the same time. This means that a trader that for instance wants to sell a stock cannot always find a partner that will buy it from her. This so called “immediacy imbalance” has an influence on market liquidity and by that the price of the stock. In real life, MMs are present on the market to rectify temporarily this imbalance, they accept trades from traders who want to sell the stock and are willing to keep this stock until the arrival of another trading party that wants to buy it. The MM with this takes on to herself the market risk of price change, but she does not do it for free. The MM will buy the stock for a price lower than the current price of the stock and sell it at a slightly higher price. The pricing strategy of the MM was described in Chapter 4.5.

As is indicated above at times there is an “imbalance’’ in the market when two matching orders are not able to be actually matched. Cartea and Penalva [18] have presented a model which includes three main

types of traders: Liquidity Traders LTs which we will refer to as the Fundamental Traders FTs from now on, MMs and HFTs. Their model is based on the Grossman and Miller [52] model which illustrates the liquidity imbalance in a simple timeframe of three distinct time points but uses only two type of traders the MMs and the FTs. Cartea and Penalva [18] introduce the HFT whose speed allows her to position herself between the "matching" parties and extract surplus from that transaction. Using this model and its associated mathematical equations it is possible to calculate numbers of stocks that should be purchased or sold by the various traders as the prices are changing, as well as to calculate the relevant wealth of all the traders at each time [18]. With this model we can investigate the various ways an HFT can use order imbalances in the market to her advantage.

In this section an explanation of the Cartea and Penalva model is given [18]. It is important to understand the fundamentals of the model, especially when it comes to strategies taken by traders and how the value of the outcome of a strategy is calculated. Some of the principles and calculations will be used in our model which will map the various interactions between traders. The Cartea and Penalva model uses three dates $t \in \{1,2,3\}$. Date $t = 3$ is used as a reference point to determine the cash value of the asset. The involved traders are indicated as follows: Fundamental Trader 1 (FT1), Fundamental Trader 2 (FT2), MM and an HFT. It is assumed that the future price $P_3 = \mu + \epsilon_2 + \epsilon_3$, where μ is constant and ϵ_2, ϵ_3 are normally and independently distributed with mean zero and variance σ^2. The $\epsilon_t, t \in \{1,2,3\}$, represents the public information that is announced between the t times. At $t = 1$ the price changes from its fundamental level μ to the post trade price P_1 and at $t = 2$ the price changes from $P_1 + \epsilon_2$ to P_2. The FT1, FT2, and the MM traders are all risk averse and price-taking [18].

As is specified before [18], the HFT uses fractional decreases in price when purchasing and fractional increases in price when selling the stock. This in our model is referred to as a "price notch" which will be denoted as Δ. As Cartea and Penalva also describe, the price notch is a potentially critical determinant that might play a decisive role in how many stocks an HFT might be able to buy or sell [18]. This means that the price notches will be adjusted by the HFT in order for her to maximize her profits following her trading activity.

The FT1 order to sell an i number of stocks "X" reaches the market at time $t = 1$, whereas the FT2 order to buy an $i > 0$ number of stock "X" arrives at time $t = 2$. If both of the aforementioned orders did arrive into the Order Book at the same time, the traders would exchange the stocks at the current market price. Because of the temporary imbalance the MM is willing to buy the stock "X" from the FT1 and hold it untill the FT2 shows up, however at the slightly different price than the current price [18]. It is here where the HFT can come in and sweep the stocks up to her benefit.

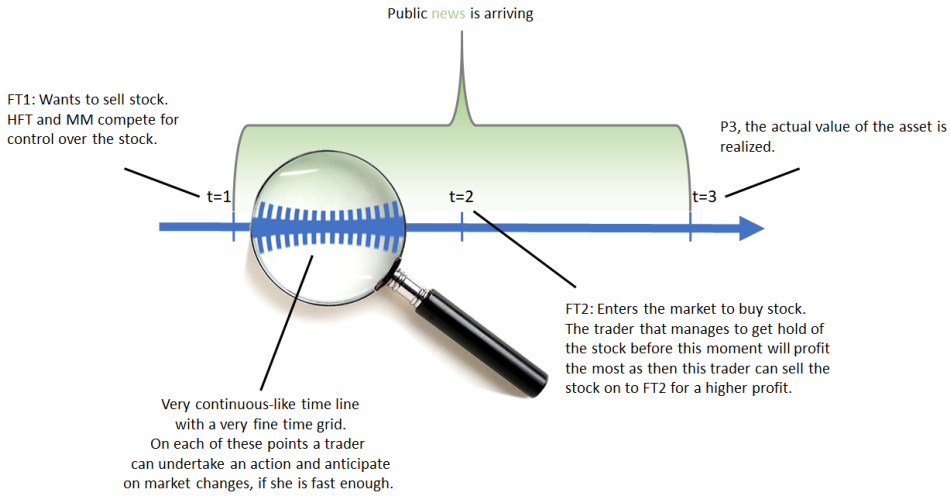

Figure 15: Illustration of the timeline.

The profits of traders are determined by the asset demands of traders. At time $t = 1$ FT1 (who wants to sell stock) realizes that she will not be able to sell her originally intended amount at the market price $(\theta_1^{FT1}(P_1) - i)$. The trader knows that she will have to pay a price notch Δ for some trades and will not be able to sell all of her stocks. The HFT will try to invoke the notch Δ by pushing the price of the stock lower by increasing the selling demand and then quickly purchasing the stock [18].

The MM is also putting up offers for the stock that is being sold by FT1. The MM sees the activity that is going on in the market. The MM uses the demand of the present traders on the market as well as her own and equalizes this to zero, in order to determine a price at which she would be willing to sell or buy the stock. Cartea and Penalva [18] refer to this as Market Clearing (MC) in their paper. In the case of our model described here we know that FT1 wants to sell stocks and that only at the moment the MM is present. In this situation the MC would look as follows [18]:

$$MC = (\theta_1^{FT1} - i) + (\theta_1^{MM}) = 0 \tag{7.1}$$

The MC price will be lower than the fundamental price, due to the fact that there is no other party that wants to buy the stock at the moment.

The HFT on the other hand, will try to push the limits of the stock price (given by MC) even lower, by applying pressure on the demand of the stock. This can be seen in Equation (7.2) in the "MC of a pressurized situation" part of the equation.

Delta in the equation below is indicated by the first part of the equation consisting of a "real" MC situation in which FT1 really holds and wants to sell her stocks. The second part consists of a situation in which in addition to the FT1's true intentions are also the "fake" intentions of the HFT who does not own any stock or want to sell it. She only has the intention to drag the price down a "notch" in order to make a profit:

$$\Delta = \overbrace{[(\theta_t^{FT1} - i) + (\theta_t^{MM})]}^{MC\ of\ a\ "real"\ situation} - \overbrace{[(\theta_{t+s}^{FT1} - i) + (\theta_{t+s}^{MM}) + (\theta_{t+s}^{HFT})]}^{MC\ of\ a\ \text{"pressurized" situation}} \tag{7.2}$$

Each of the traders will use the new price derived from the MC equation in their own demand curve formula and determine the maximum quantity that they would be willing to trade on the market. For FT1

and MM the maximum amount of stock that they are willing to retain is indicated by the following equation:

$$Q_t^x(P) = \theta_t^x(P - P_{MC}) \tag{7.3}$$

Where $Q_t^x(P)$ is the maximum quantity after the price change at the price P, θ_t^x indicates the demand curve for trader x at time t, P is the price used in their demand curve and P_{MC} is the price derived from the MC formula. Each of the other traders calculates their maximum quantities in a similar way. For the sake of understanding let us use an example. Let us assume that the demand for the trader is $5000\ (4.50 - P)$. In order to determine the new quantity when the MC price is $4.46 the equation would look as follows:

$$Q_t^x(4.46) = 5000\ (4.50 - 4.46) = 200$$

If this quantity is calculated for the FT1 who wants to sell 1000 stocks, according to the equation above it then indicates that at the price of $4.46 she would be willing to retain 200 stocks and therefore sell 800 stocks.

If the given equation above would be indicative of the MM, this would then mean that the MM is willing to buy 200 stocks.

The HFT is able to extract surplus and generate profit, as is indicated by the following equation [18]:

$$\pi = \left\{ \overbrace{(Q_1^{FT1})(P_{MC_1} - \Delta_1^{FT1})}^{Max\ Q\ sell\ by\ FT1\ at\ pressurized\ price} \right\} \Delta_1^{FT1} + \left\{ \overbrace{(Q_1^{MM})(P_{MC_1} - \Delta_1^{MM})}^{Max\ Q\ buy\ by\ MM\ at\ pressurized\ price} \right\} \Delta_1^{MM} \tag{7.4}$$

As can be seen in the equation above, the HFT is essentially applying the notch (Δ_1^{FT1}) to bring down the price when she is buying the stock from FT1. As she proceeds to sell it to the MM she applies another notch (Δ_1^{MM}) to bring the sales price up in order to maximize profit.

7.3.2 Model description

In this section a description is made of a "subsequential" game. The game between the traders represented in this model combines various aspects of different game types, like a simultaneous and sequential game. This hybrid which is formed consisting of various games in combination with set response times of the players has not been given before in literature. When the game starts it has all the traits of a simultaneous one, the traders start off at the same time and play out their intent without paying much attention to any other traders. However, as the HFT is so fast, her moves are quickly logged in the Order Book and reacted on by the MM. In essence the game becomes more sequential, but still is not fully taking over all of the characteristics belonging to that game type. It becomes something of a combination between a sequential and a simultaneous game, where (after the traders become aware of each other) the traders almost play a reactive game. The HFT enjoys the luxury of being able to respond to whatever moves have occurred on the market, whereas the other traders can only hope that the HFT will slip up and make a bad decision that could benefit them at one point. Nevertheless, it becomes clear that the HFT has the speed to even correct some apparent mistakes made at an earlier point in the game. The other traders have to go through the agony of being able to see the changes in the market but they will not be able to respond in time to effectively change or prevent things.

7.3.3 Model Ecosystem

For the model that is presented in this thesis a description has to be given about the market conditions and the players that are involved and what their envisioned demands are. In our market, which is based on the Cartea and Penalva model [18], we assume that there is an FT1 that wants to sell 1000 stocks at the current market price of $4.50, with her demand being

$$\theta^{FT1}(P) = 5000\ (4.50 - P)$$

FT1 is trying to sell the stock as soon as possible as FT1 wants to liquefy her assets. However, this does not mean that she will sell everything under any condition. She will take a given price for a transaction under consideration and calculate how much she would be willing to sell for that price. FT1 in this game is assumed to only sell her stock, she is not assumed to buy any of it back. The amounts indicated (both quantity and price) in each respective "buy" box of the other traders are the max numbers of stock that FT1 is willing to sell at the indicated price.

On the market there is also an MM whose demand is as follows

$$\theta^{MM}(P) = 45000\ (4.50 - P)$$

As can be seen the demand of the MM is 9 times higher than the demand of FT1. The boxes (see Figure 16) that come from and to the MM represent the stocks that are being bought from and by the MM, the numbers of stocks indicated represent the maximum number that the MM agrees to buy or sell at the indicated price.

The FT1 and MM are defining the market ecosystem. The other players that are involved are the ones with an interest in obtaining the stock, these are the HFT and FT2 (for whom it is assumed that they have the same demand as FT1). They are in a race to obtain the stock in order to be able to use it to execute their "usual" strategy. We are focusing on the gains and losses of these two traders in this model.

The traders have a few options as to what they can do at each time point. These options are; buy, do nothing and sell. Cancellations are also possible for a previous order given within a particular timeline, these are then indicated by an asterisk in the model. The time setting of the market modeled here is daytime trading. All decision nodes given in the model are actually the time points at which the orders get logged in the book. In addition, in the model here we assume odd lots (or half lots) as tradeable amounts, unlike most exchanges on which whole lots only are being traded as minimum amounts (100 stocks). This is because the model depicted here shows a simplified trading game with a small market that revolves around just 1000 available stocks.

In the model, after the information becomes known that FT1 wants to sell the stock, both the HFT and FT2 send out their intended orders at the same time. The HFT's orders are processed and entered into the Order Book the quickest and cleared immediately (if a matching order can be found), we assume a latency of about 8 ms for the HFT (which is kept consistent between the decisions/orders of the HFT), whereas the FT2's and MM's orders are processed slower (we assumed 12 ms for the MM and 55 ms for FT2). In the figures which represent the actions of the traders in the model, the latency is visualized in such a way that between the clearance of the various orders there is a difference along the X axis between the various traders. The HFT will always be the quickest if orders are submitted at the same time. In a real situation HFTs can also submit several orders almost simultaneously, due to their speed, but in reality each of these orders is logged individually in the LOB. This makes the process of the HFT almost look continuous but

since each of these orders have their unique submission and clearance time points, the argument again arises that an HFT is more continuous-like (and in fact following a discrete time strategy) rather than being truly continuous in her modus operandi. Also, in real markets an HFT often stacks orders to fool other traders into thinking there is a high buying or selling pressure on a stock. This is in particular effective on markets where there are a lot of active traders. In our model however, there are only 4 traders in total (if the MM is included in the picture), here therefore stacking or layering of orders isn't as effective as in a real setting. What the HFT has to realize is that when she wants to fool the other participants (mainly the MM in the model described here) she needs to exploit her own low latency. This means effectively that she needs to know when she can put up a "fake" order to fool the other traders into thinking there is an increased buying or selling pressure. She then needs to turn the order around before it would get matched with an order of a counter party and cleared by the market. If the HFT does not anticipate her own speed correctly and effectively, this could mean that an unintended order could get cleared, which would make her lose money. It has to be stressed however that model is a simplified representation of a market and does not include all of the possible scenarios that the traders could take, nor does it include all the elements of a real market setting. For instance, when the HFT makes a mistake and effectively gives away ownership of all the stock to another trader the game will end by default. In reality an HFT could attempt to correct this by increasing the selling pressure on a stock hoping to fool the other players into selling the stock off again so she can pick it back up. However, modeling this would be very complex and intricate. Similarly, when the HFT at one point is not able to sell off any stock to the MM and therefore remains holding some stock, it is assumed in our model that she will sell it off to FT2 upon her entry into the market. In reality it is more likely that the HFT will nevertheless try to get rid of the stock as soon as possible, rather than to hold it for a longer period of time (since this is considered too risky by the HFT). The model process is based on the condition that the supply does not match demand, which means that there is no buyer (FT2) at the time available when there is a seller (FT1). In the model the choice was also made to not have "external information" ϵ be of influence on the stock price, the only information that is able to influence the price is the actions of the players involved regarding the stock. Finally, the decision was also made to not have the possibility to have any traders enter the market with the exception of FT2 and the HFT.

When it comes to the representation of gains and profits in the model presented here, it was decided to represent the gains and profits made by the HFT in the same way as it was done by Cartea and Penalva. However, when it comes to the representation of the gains and profits of FT2 the decision was made to focus on gained stocks or missed opportunities (due to her lack of speed), this way the impact of the HFT becomes more obvious.

In the following section the game as it is represented in Figures 17-26 will be explained in detail from the point of view of the HFT. The decision of what the HFT will do is assumed to be made fully automatically by the algorithm and it bases its decision on the current information of the stock performance. In this model all the possible paths are shown and its consequences illustrated for the remainder of the game and the outcome on the gained profits. The game starts essentially when the sell order of the FT1 gets logged in the LOB and all the other market participants are able to react on it (FT2, MM and HFT). Considering this scenario, the order of the HFT will always be logged first, since she is the fastest regarding the latency times.

In our model representation of the interactions between the various traders, interaction boxes are used to visualize the moment of effectuation of an order, the type of order, the quantities and price involved and the consequences for the FT1 (and in some cases other traders) (see Figure 16).

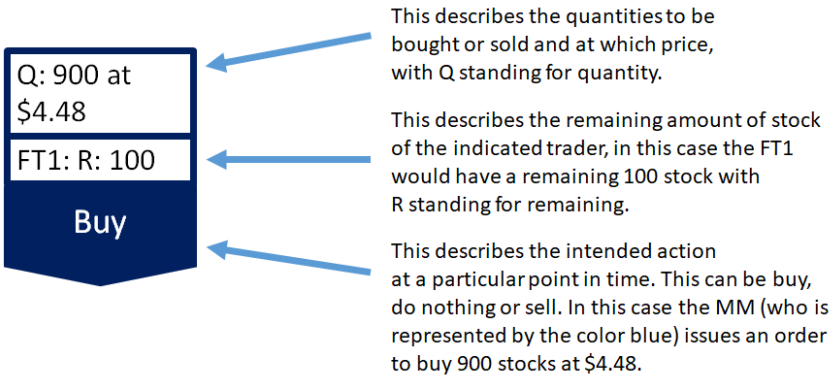

Figure 16: Explanation of the action boxes used in the visualization of the model.

Now that the workings of the game have been explained and its mechanics have been shown, we can proceed to describe the various possible moves that can be followed by the HFT and the other traders.

7.3.4 Steps followed by the traders

In the game that is modeled here the traders all submit their (initial) order at the same time. The orders then get logged as soon as they enter in the LOB. It is assumed in the model that within a "mini game" (direct square off between traders in one particular point in time) the first order that gets logged in the LOB is the "winning" order and gets effected.

As the game starts, each of the traders (as mentioned previously) have the choice of selecting buy, do nothing or sell as their action. However, depicting the full game in one single figure would be very complex and interpretation of it next to impossible. Therefore, first a schematic is given with all the possible moves by the traders when the HFT choses "sell" as her first move. It has to be noted however that the HFT most likely, before even FT1 has put up its stock for sale, has put up a number of immediate-or-cancel orders. By these the HFT is able to get a feel for how the market and the stock price will react when certain orders are logged in the LOB. This information is used by the HFT to predict what is going to happen when she puts up an actual order she intends to follow through. This means that when FT1 has put up her stock for sale and the HFT wants to obtain it, whenever she puts up an order she can predict the response of the market. Because of his she can beat the other traders in the LOB by putting up an order that will get to the highest level in the Book (meaning it will get cleared first). As can be seen in Figure 17, the possible steps make the model very complex. In order to facilitate the legibility of the model a more detailed step by step description will be given by highlighting what would be the most likely path to be chosen by the HFT.

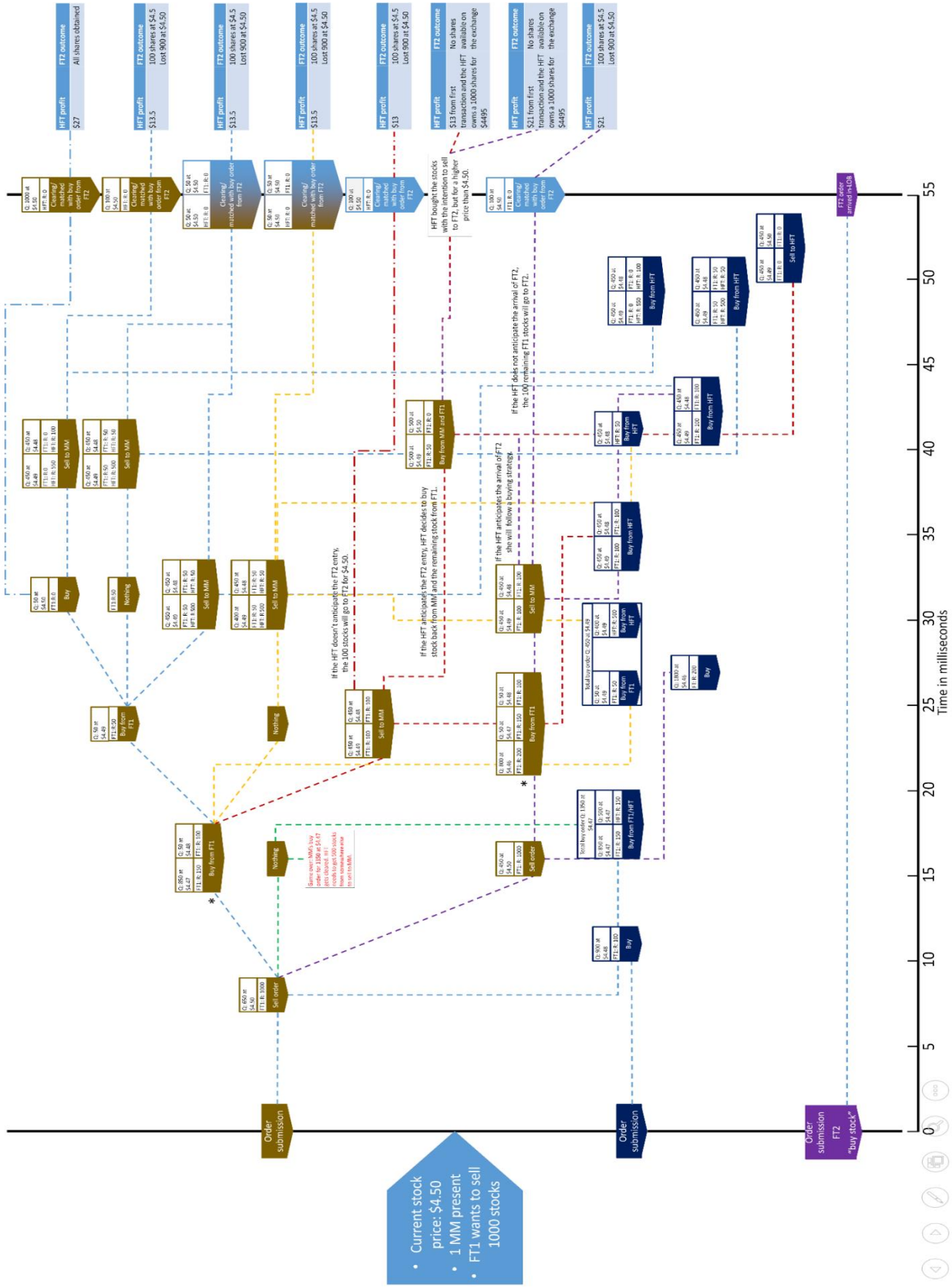

Figure 17: Possible moves by the traders when the HFT chooses to sell initially.

Now a detailed description will be given of the highlighted path (see Figure 18) in order to get an idea of the interaction of the HFT, MM and FT1 & 2. In Figure 18 the most likely path that we think will be taken by the HFT, is given. The reasoning as to why we feel this is the most likely strategy to be chosen by the HFT is further elaborated in Chapter 7.3.6. We will elaborate the various possible choices by the traders, but for the sake of readability and understanding we will zoom in on the figure below step by step.

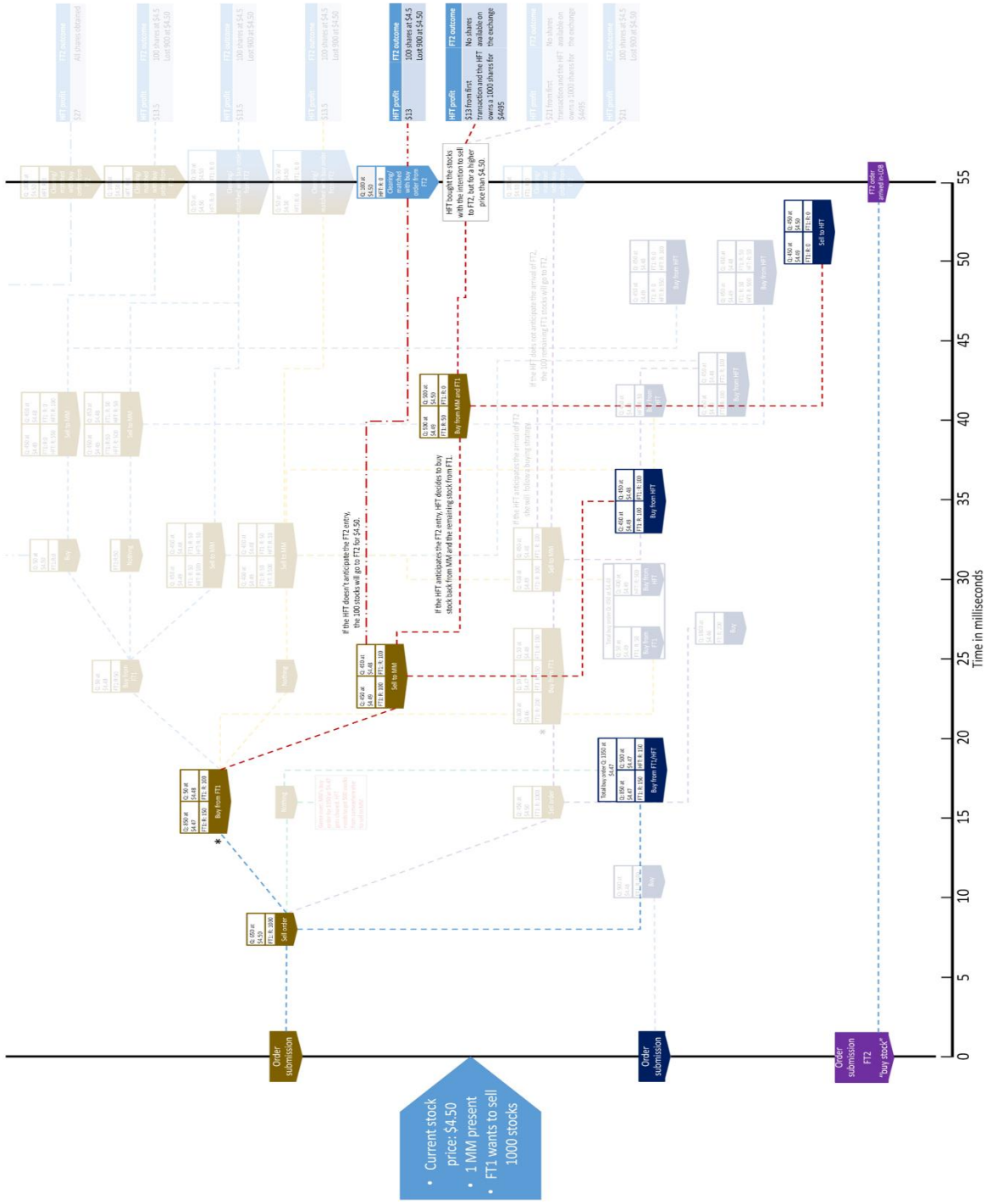

Figure 18: Most likely HFT strategy.

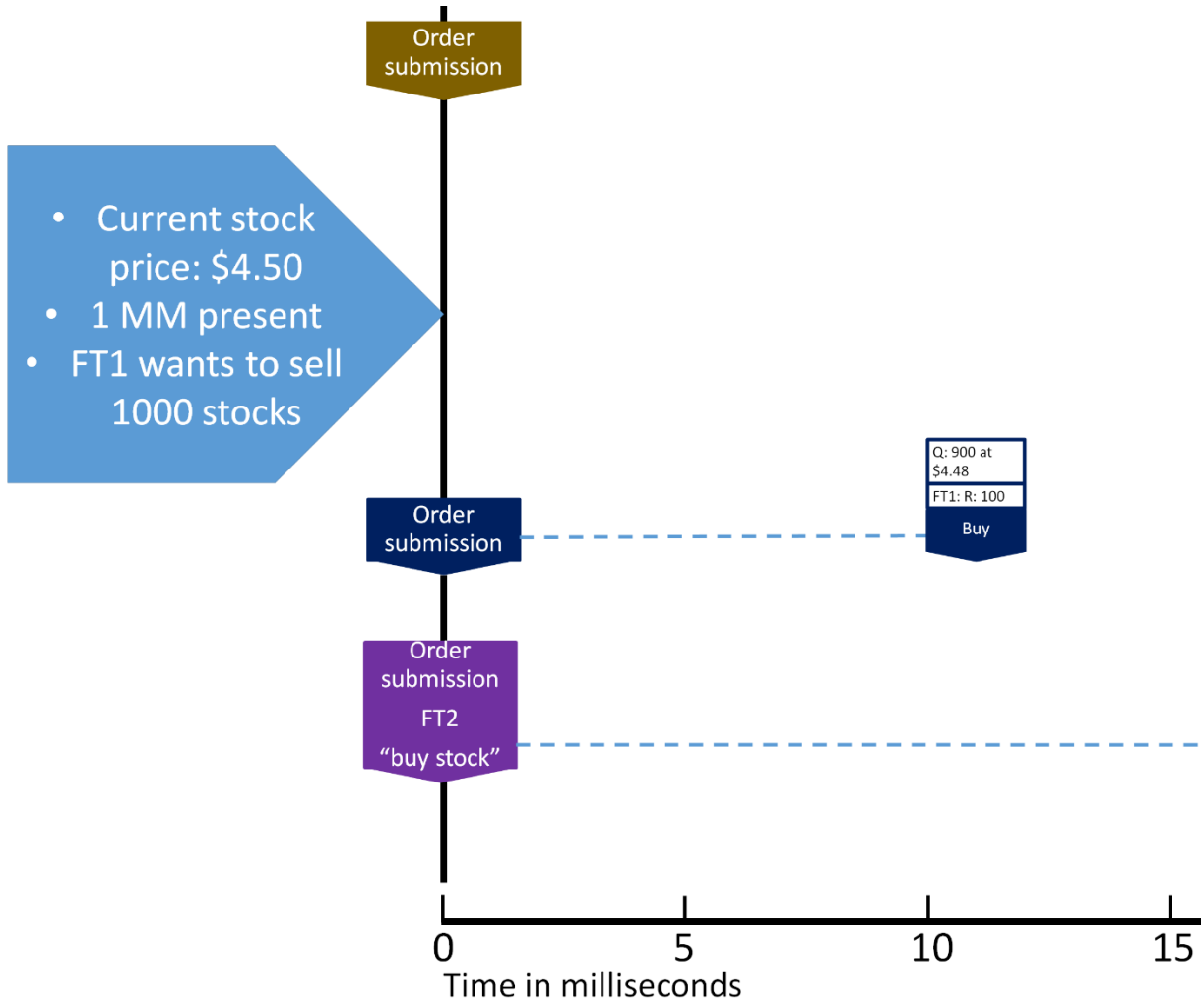

Figure 19: HFT does nothing while the MM submits a buy order.

The traders see that FT1 has put up 1000 stocks for sale at the current price of $4.50. As the traders see this order come up, they all start to crunch the numbers. The MM sees that there are no takers of the stock for that price, so the MM is making FT1 an offer at a lower price. Using Equation (7.1) the new price of the stock is derived.

$$\theta^{MM}(P)+\theta^{FT1}(P) - 1000 = 0$$

$$45000\ (4.50 - P) + 5000\ (4.50 - P) = 1000$$

$$\leftrightarrow P = 4.48$$

The MM now takes the newly determined price of $4.48 (from the market clearing equation) and enters it into her demand curve equation, in order to determine the quantity that she is willing to buy at that price. The calculation below shows that in this case the MM for the price of $4.48 is willing to buy 900 stocks according to equation (7.3).

$$Q^{MM}(4.48) = \theta^{MM}(P) = 45000\ (4.50 - 4.48) = 900$$

The same is done by the seller of the stock (FT1), she puts the new price in her demand curve equation. However, in this case the equation indicates the amount of stock that she would retain at the new price.

$$Q^{FT1}(4.48) = \theta^{FT1}(P) = 5000\ (4.50 - 4.48) = 100$$

Originally FT1 wanted to sell 1000 stocks, but as the demand curve equation indicates at the new price of $4.48 she should retain 100 stocks. This means that in the end she will sell only 1000 – 100 = 900 stocks. In Figure 13 this is visualized by the submission of the buy order by the MM at 12 ms. The blue box shows the price and quantities that concern this order and the amount which the FT1 would retain. Normally the order would clear and the MM will later be able to resell the stock at a slightly higher price to FT2 (or anyone else). However, it has to be noted, that this order will only happen if no other player beats the MM to the punch.

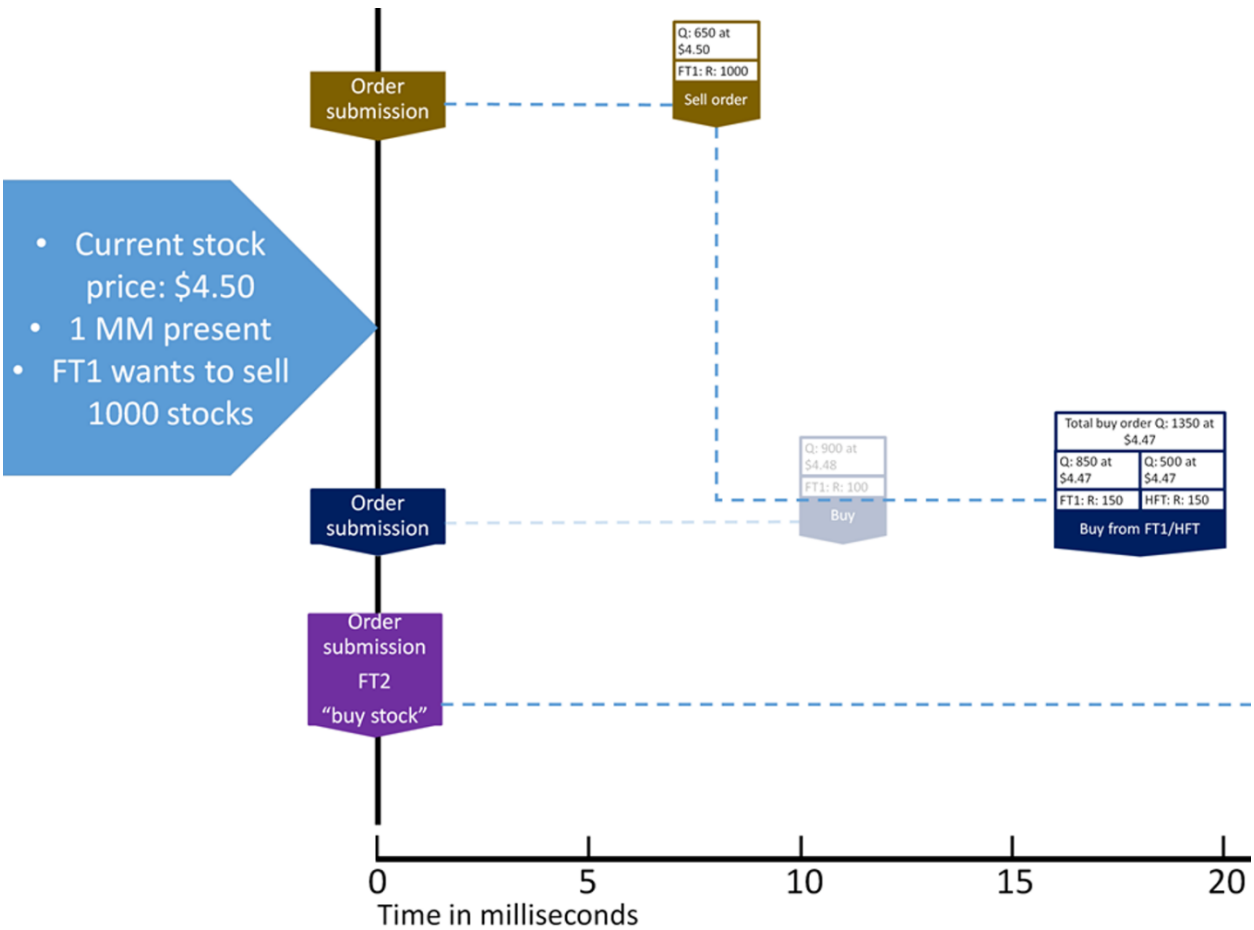

Figure 20: The HFT puts up a sell order, one which the MM responds.

However, the HFT does not let this opportunity slip by and puts up an additional sell order (additional to the one by FT1) before the MM can put her buy order into the book (see Figure 20). Even though the HFT does not own any of the stock at the moment, she by this action is trying to drag the price down so she can buy the stock a bit cheaper later on. She puts up a sell order for 650 stocks at $4.50, bringing the total stocks offered to sell to 1650.

The calculation for this order is done as follows.

$$\theta^{FT1}(P) + \theta^{HFT}(P) + \theta^{MM}(P) - 1650 = 0$$

$$5000\ (4.50 - P) + 5000\ (4.50 - P) + 45000\ (4.50 - P) = 1650$$

$$\leftrightarrow P = 4.47$$

With the new price which is derived, the new quantity can also be determined.

$$Q^{MM}(4.47) = \theta^{MM}(P) = 45000\ (4.50 - 4.47) = 1350$$

$$Q^{FT1}(4.47) = \theta^{FT1}(P) = 5000\ (4.50 - 4.47) = 150$$

With this initial sell order the HFT is faster in the LOB than anyone else and therefore as that order gets logged the MM has to respond to it. The MM refrains from her initial intention of buying 900 stocks at $4.48, instead however she is willing to buy 1350 stocks at a price of $4.47. As can be seen from the equation above, at the current price of $4.47 FT1 is willing to sell 850 and retain 150 stocks. This means that if the MM order will get cleared, the HFT will have to sell to the MM 1350 – 850 = 500 stocks.

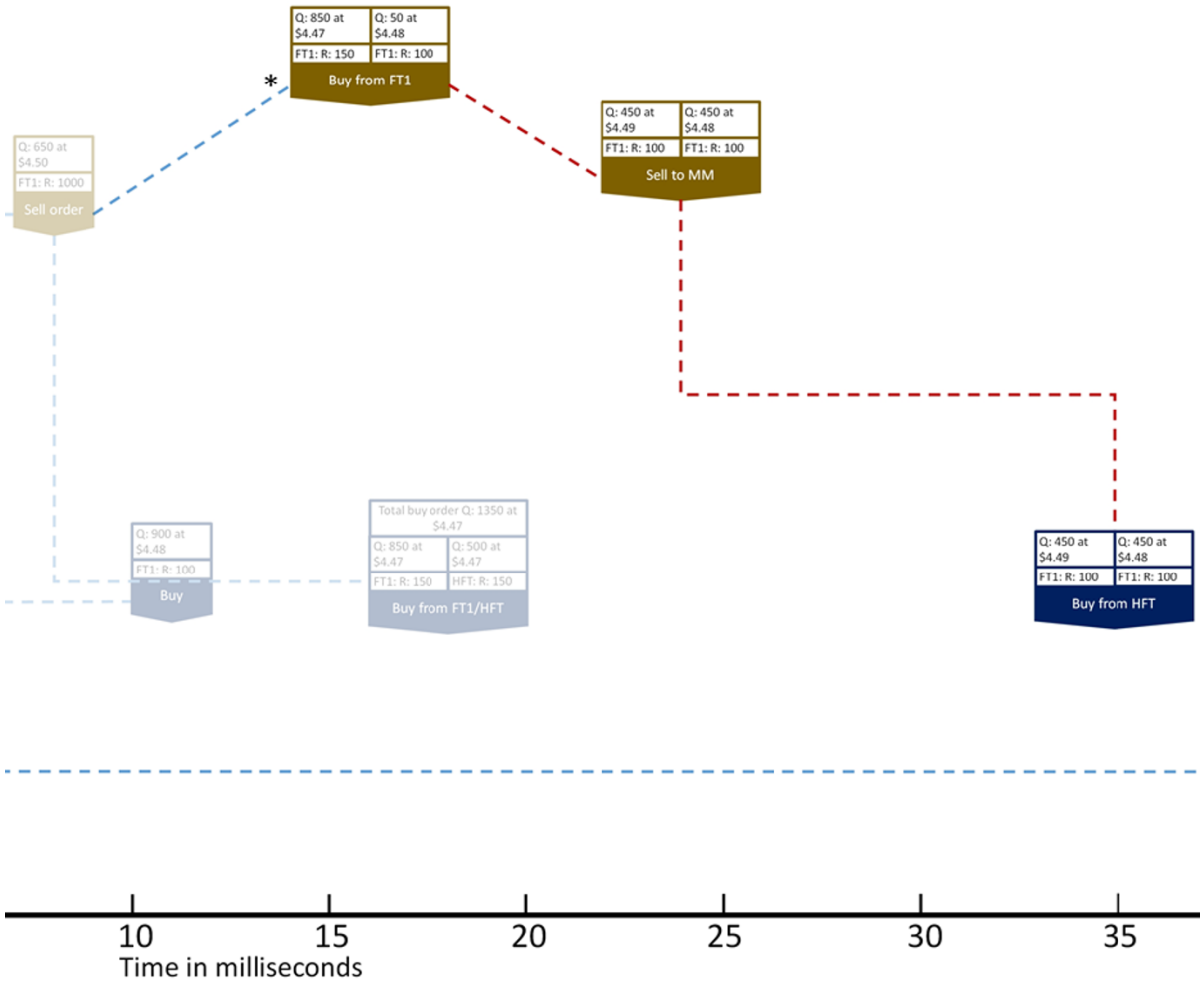

Figure 21: HFT effectively lowered the stock price.

The HFT however does not intend to sell any stock but only wants to lower the price by this action. Therefore right before the MM would match the sell order by the HFT, the HFT cancels the order (indicated by the asterisk, see Figure 21) and puts up a buy order. The HFT offers a price of $4.47 for which the FT1 is willing to sell 850 stocks (exactly like the order the MM is intending to put up), the HFT then offers a price of $4.48 at which the FT1 is willing the sell another 50 stocks as can be seen in the equation below. We have to keep in mind that FT1 sold already 850 stocks at $4.47, this made her retain 150 stocks. As the price now increased to $4.48 as derived by the demand curve, she is now willing to hold 100 stocks, making her only willing to sell 50.

$$\theta^{FT1}(P) = 5000\ (4.50 - 4.48) = 100$$

$$Q^{FT1} = 150 - 100 = 50$$

The order of the HFT gets logged in the LOB before the MM and therefore the order of the HFT is the one that gets cleared.

The HFT now decides to sell the stock to the MM (see Figure 21), she does this at $4.49 for which the MM is willing to buy 450 stocks, as indicated by the MM's demand curve. The HFT immediately after that puts up another sell order for $4.48 at which the MM is willing to buy another 450 stock.

$$Q^{MM}(4.49) = \theta^{MM}(P) = 45000\ (4.50 - 4.49) = 450$$

$$Q^{MM}(4.48) = \theta^{MM}(P) = 45000\ (4.50 - 4.48) = 900$$

$$Q^{MM} = 900 - 450 = 450$$

The reason why the HFT is selling the stock off straight away is because of the strategy that the HFT has of not having any position in the stock. As such in general, HFTs want to sell any stock they have as soon as possible.

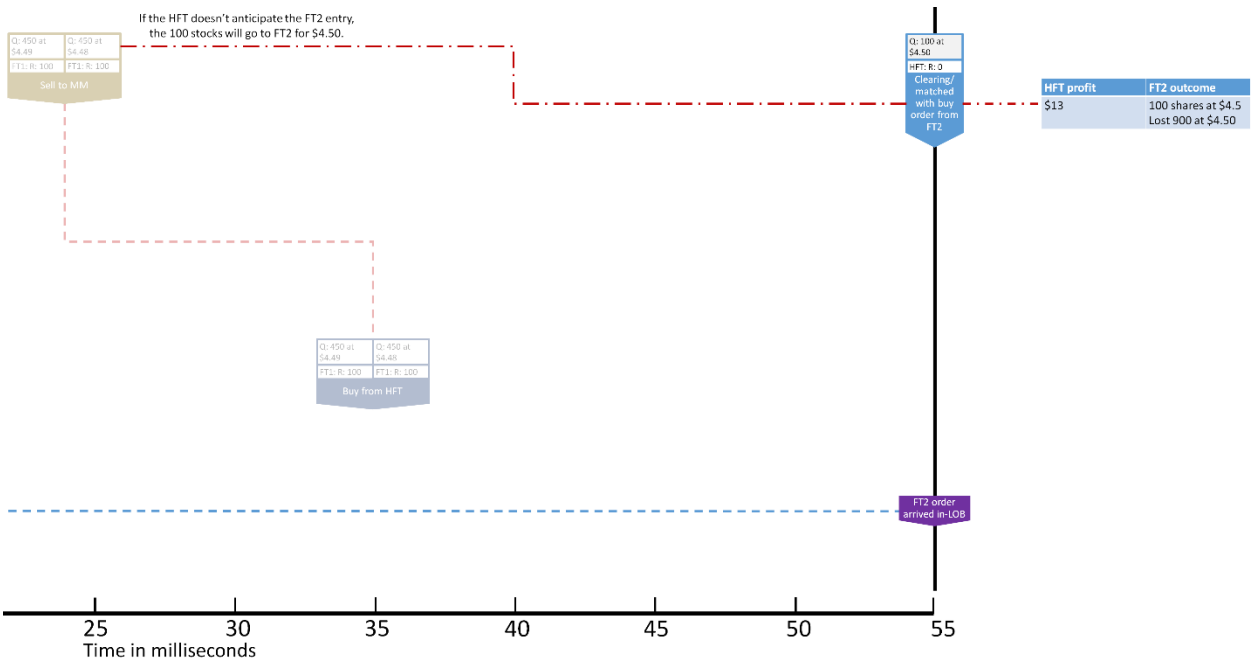

Figure 22: The HFT does not anticipate entry of the FT2.

Now if the HFT does not anticipate the arrival of FT2 the remaining 100 stock of FT1 will go to FT2 and the HFT would not have made an additional profit and would have just made $13 from her sale to the MM (see Figure 22). The profit is calculated according to Equation (7.4). The $13 profit is built up from the following transactions: the HFT bought 850 stocks for $4.47 and 50 stocks for $4.48. She then sells 450 stocks for $4.49 (Δ$0.02), another 450 for $4.48 (from which she sold 400 with a Δ$0.01 and 50 with a Δ$0.00).

$$\pi_{s-b-s-c} = [450 * (4.49 - 4.47) + 400 * (4.48 - 4.47) + 50(4.48 - 4.48)] = 13$$

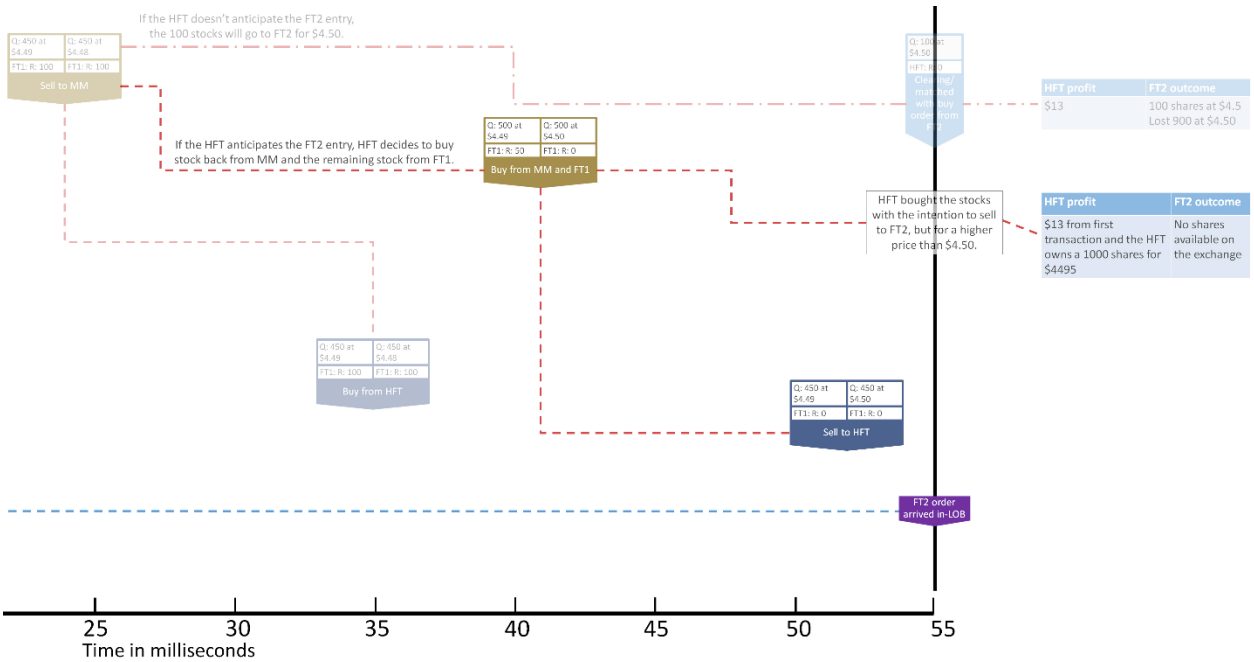

Figure 23: The HFT does anticipate entry of the FT2.

If however the HFT does anticipate on the arrival of FT2, she will buy up all the stock (including that available at the MM and the remaining FT1 stock) at $4.49 and $4.50 (both for 500 stocks) (see Figure 23). As can be seen in the demand curves given below at the prices that the HFT is offering to the MM and FT1, they agree to sell all their stock to the HFT.

$$Q^{MM}(4.49) = \theta^{MM}(P) = 45000\ (4.50 - 4.49) = 450$$

$$Q^{MM}(4.50) = \theta^{MM}(P) = 45000\ (4.50 - 4.50) = 0$$

$$Q^{FT1}(4.49) = \theta^{FT1}(P) = 5000\ (4.50 - 4.49) = 50$$

$$Q^{FT1}(4.50) = \theta^{FT1}(P) = 5000\ (4.50 - 4.50) = 0$$

As this trade gets cleared the HFT now owns all the available stock and can sell it to FT2 at a much higher price since she is the only trader that has the stock. However, this subsequent stage is not covered in this model.

This means that the profit is again $13 from the original sale to the MM, plus the fact that the HFT now owns all the stock available (which she had to buy however).

$$\pi_{s-b-s-c} = [450 * (4.49 - 4.47) + 400 * (4.48 - 4.47) + 50(4.48 - 4.48)]$$
$$= 13 + cost\ of\ buying\ 1000\ shares\ 500 * 4.49 + 500 * 4.50 = -4495$$

Figures 24 and 25 visualize the course of the game if the HFT chooses to buy and to do nothing as the initial move.

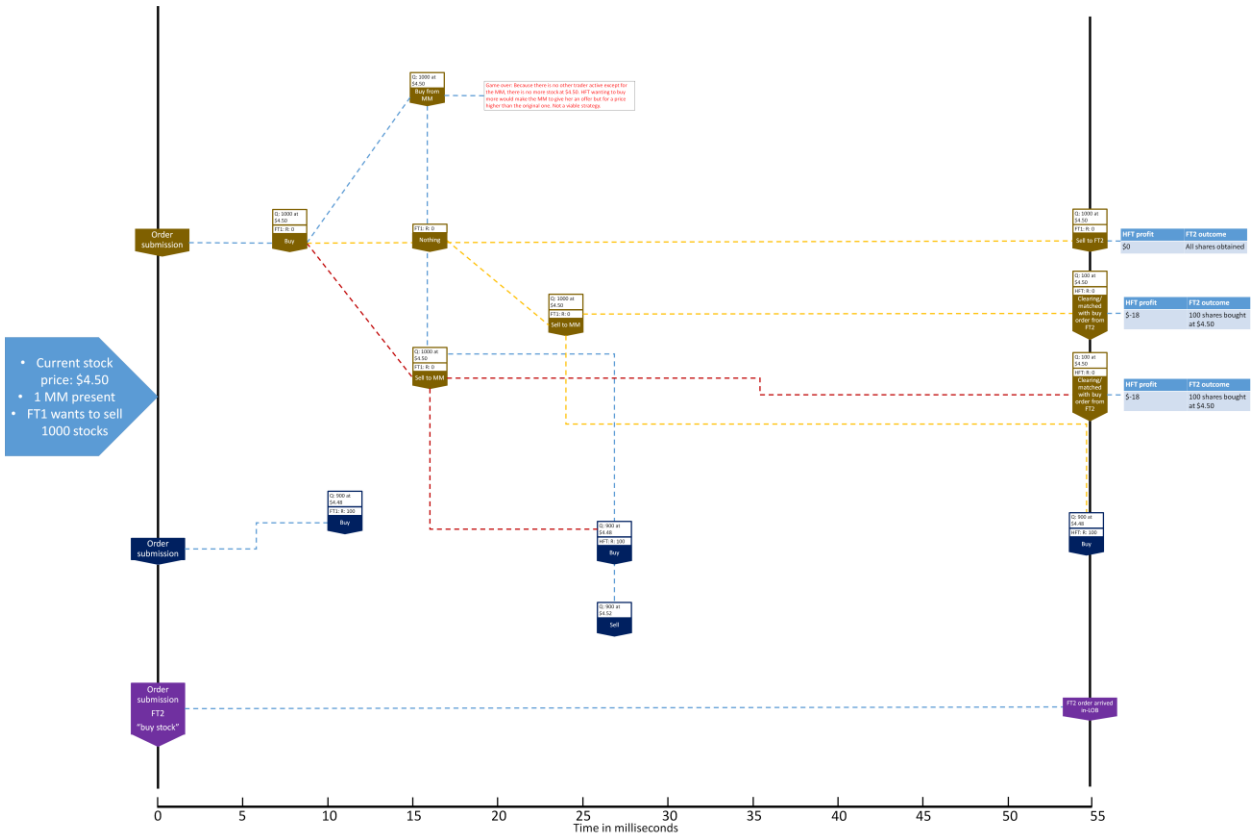

Figure 24: Possible moves by the traders when the HFT chooses to buy initially

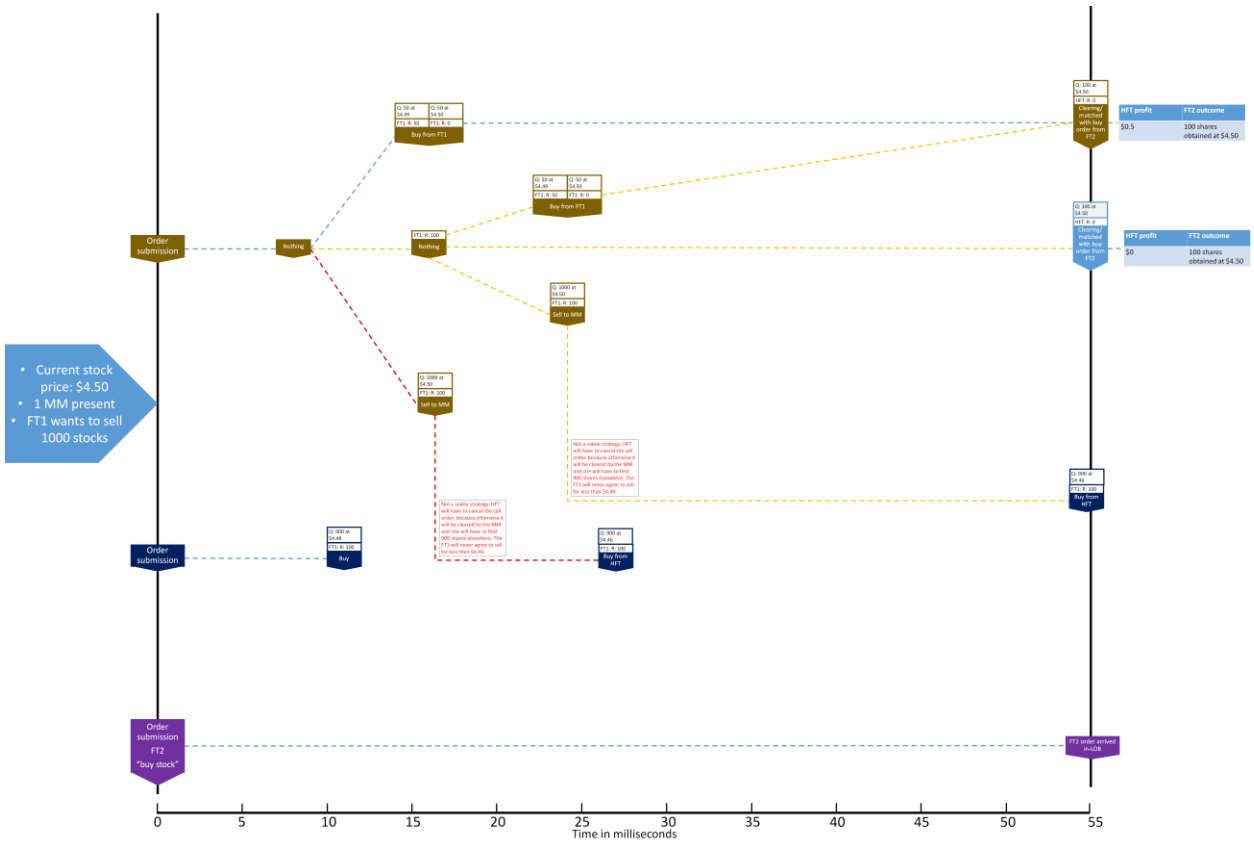

Figure 25: Possible moves by the traders when the HFT chooses to do nothing initially.

7.3.5 Mistakes that can be made by the HFT

The HFT is not always coming out on top, as we can see in our model in Figure 26, the HFT can also make a mistake. This can be because of a glitch in the algorithm or generally a slip up on the HFT's part. In the example given here the HFT brings the price down initially by putting up a sell order, but for whatever reason subsequently the HFT did not do anything. This means the initial sell order of the HFT did not get cancelled and gets realized by getting matched with the buy order of the MM. This means that the HFT now will have to actually sell the number of stock it said it would in her initial sell order, however we have to remember that the HFT does not actually have any stock. Therefore, the HFT will have to source the stock from somewhere else (potentially at a higher price), making her incur a considerable loss.

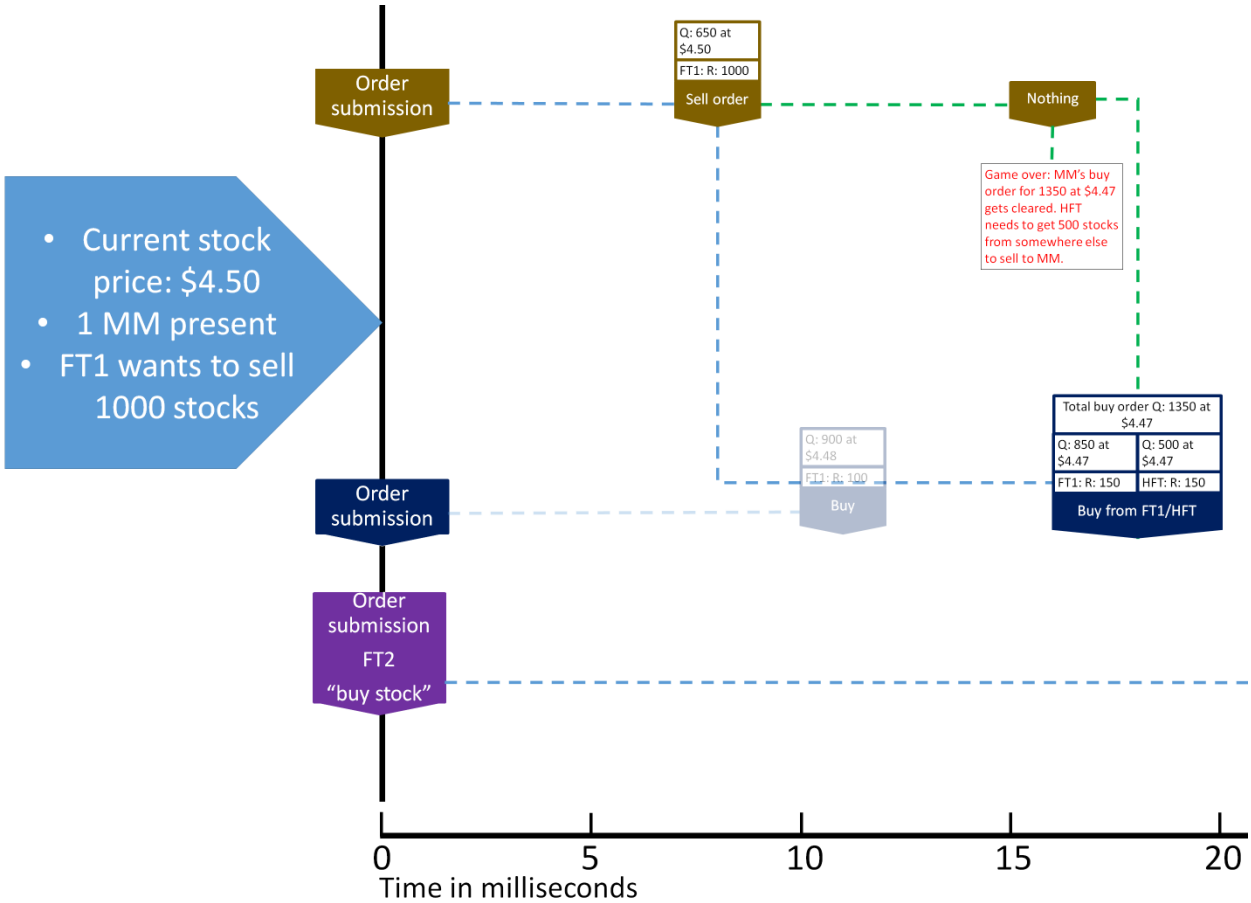

Figure 26: Mistake made by the HFT.

7.3.6 Optimum strategy outcome for the HFT

Figures 17, 18, 24 and 25 show all the possible strategies that could occur within the game. Though some strategies seem to be more profitable than the others for the HFT, some also are very unlikely to be chosen. For instance, it is very unlikely that an HFT will choose a strategy with repeated spoofs/layering of fake sale orders. Though the outcome in our model of these is not so bad ($21 dollars profit), it is very likely that an HFT in a real market setting will be recognized as a fraudulent spoofer and might thereby suffer repercussions. It is much more favorable for the HFT to choose a strategy that is not risky or one that leaves her with a position in the stock (if she reckons that another trader might be placing a buy order from which she can take an advantage). The HFT in general would choose a smaller profit but a sure one over a high profit but with also a high risk of not getting it. Figure 24 shows the strategy outcome when the HFT starts off with a buy order. It becomes evident that starting with a buy is not beneficial to the HFT as there are no profits made when the game is played out in this simplistic market representation. Therefore, it is not likely that the HFT will use any of these strategies. Figures 19 and 25 similarly show that the doing nothing initially strategy is not beneficial for the HFT as no (or negligible) profits are made. It becomes clear that if the HFT wants to maximize her chances for a high profit, she should start straight away with manipulating the price. She can do this by acting quick without missing any opportunity to keep moving the stock around as was shown in Figures 17 and 18. Though this profit again seems not to be the highest achievable one, it is as we said before the safest one. In addition, one has to realize that an HFT

can do such a strategy on several hundreds (if not thousands) of different stocks simultaneously. Which immensely increases the profit potential of such a relatively safe and conservative strategy.

8. Advantages and disadvantages of HFT

In this next section a number of advantages and disadvantages of HFT will be listed and elaborated. This is needed in order to understand the full scope of impact that HFTs can have on the market.

8.1 Advantages of HFT

8.1.1 HFTs add liquidity

Some of the more often claimed positive influences of HFTs on the markets is the fact that the bid-ask spreads have gotten narrower and that liquidity of market improves as HFT application broadens [51, 53]. This is because the time between intention to trade and the trade being done is decreased drastically. However, with spoofing there might be a lot of trade offers but only one is effected in the end. It still may be faster than an FT and decreasing the bid-ask spread, however there is a lot of doubt whether or not this is real liquidity provision or not [54]? More about that can be read in Chapter 8.2. Some others claim that especially smaller traders are benefited by HFT trading. Smaller traders no longer have to influence the market, but all they have to do is to have a smart algorithm. This algorithm should know when to buy and sell if certain trends are detected and the stock moves in a certain direction [55]. Cvitanic and Kirilenko [56] have shown that the presence of an HFT can change average transaction prices, they are more concentrated around the mean with lower volatility and prices become more predictable. Jarnecic and Snape [57] similarly have found that HFTs are not likely to make volatility worse, but that they are more likely to have a minimizing effect on it.

8.1.2 An HFT algorithm ensures that assets are priced consistently

In order to understand how algorithmic trading can benefit price consistencies between trading assets, let's focus our attention on the discrepancies between trading currencies such as dollar-euro, dollar-yen. Traders hope to find a riskless profit opportunity (such as triangular arbitrage) that might show up in the market and help them to make a riskless profit. For example, exchanging a currency into another a few times over (dollars into euros, euros into yens, yens into dollars) may result in a trader ending up with a higher amount of dollars than when she entered the transaction. This means that the trader has spotted and exploited an arbitrage opportunity. The profit is generated by simply exchanging one currency for another in hope of generating a profit due to a slight misprice in the market between the currencies involved in the exchange. That slight misprice and the riskless profit will not last forever. Once spotted, the participation rate in such a transaction will increase and as such the prices between currencies will correct themselves and the arbitrage opportunity will disappear. With the concept now explained it is clear how algorithmic trading can help in keeping the prices among traded assets consistent. Thanks to the speed of the algorithm such an opportunity would be noticed and as a result disappears within milliseconds [40]. It has to be noted however that the correction of an arbitrage opportunity comes at a

price to fundamental traders. It is an HFT who obtains the profit due to the fast processing of information by her algorithm that helps her to spot the mispriced asset.

8.1.3 HFT algorithms help to overcome market fragmentation

It is a known fact that the information about one stock is disseminated and dispersed among several trading platforms. The so-called fragmentation caused by the coexistence of many trading platforms made the speed of algorithms the epitome of how to deal with the spread of information. Market participants have to comprehend all the real-time information simultaneously from different venues. Additionally, traders have to make choices that benefit them most. Having an algorithm that deals with aforementioned information effortlessly and makes decisions within a millisecond is an advantage [40]. In Chapters 5 and 6.4 the SIP system, which matches trades between different platforms, was explained. The chapters elaborate on how the HFT uses its algorithm to get the best deal between platforms before the FT does, again profiting from what is essentially an arbitrage opportunity.

8.1.4 HFT algorithms help in dealing with humans processing information limits

Regardless of the fact if trading is centralized or fragmented, the traders deal with a flow of information which for a human is difficult to comprehend and make a cognitive decision in a timely matter. Humans stand no chance when competing with an algorithmic program that can process several streams of information and make decisions within “the blink of an eye”. Analyzing various pieces of rapidly incoming information “manually” could potentially harm FTs gains due to their slow response when making trading decisions. Algorithmic trading compensates for the limited rationality of FTs and thus improve market liquidity [40]. HFTs are able to predict an aggregated liquidity shock, during which a lot of market participants want to sell an asset and not many would like to hold it. An HFT is able to buy an asset early after the shock when the prices are relatively low and at the same time place a limit order for the same asset but this time to sell when the prices will start to rise again. This way they provide liquidity to the market and act almost like a Market Maker. The FTs are not able to comprehend the information that the market gives to them and therefore they are not able to take an advantage of it [58].

8.2 Disadvantages of HFT

8.2.1 HFT manipulations

The speed at which a decision can be made, processed, and executed gives to an HFT great power. The power that can be used to manipulate other traders. An HFT can submit within few milliseconds hundreds of various orders making it difficult for other market participant to realize what actually is happening. The various methods of manipulations such as spoofing, smoking, etc. were discussed in Chapter 6.

8.2.2 Lost opportunities for non-HFTs

As HFTs invested so heavily in their speed and algorithms they are also as a result very quick in price discovery. This means that when for instance a potentially price impacting news event comes out, the HFT is much quicker in anticipating on it. As a result, the FT is often too late in anticipating on price changing information events. This means that the HFT can therefore negatively impact spreads for the FT. To illustrate this the following example is given, an HFT is selling a certain stock for $100, which is currently the best ask. The second best ask is $100.01, posted by an FT. As long as there is no information released that might impact the price, the HFT will keep the ask up and will not retract it. Thus, if a market order comes in, she will sell her stock. However, let us assume now that “good” news is released, this means that the stock price will be impacted and make it go up. The HFT will now quickly cancel her order, as she

realizes that she might be able to sell the stock for a higher price a little bit later. The FT however is not capable of canceling her order so quickly and therefore it now becomes the best ask, which would get cleared if a market order would now be entered. This would make the FT lose potential profits [40].

8.2.3 Correlation of trades

Though algorithms differ from each other, they still are usually programmed to maximize profits and to recognize potentially profitable situations. Therefore, there can be certain market situations in which algorithms are likely to respond in the same way, making their orders correlate in one direction. This (if it goes on uncontrolled and without any countering) can exacerbate a potentially negative situation into something truly disastrous. On August 16th 2007 in the Dollar – Yen exchange market the Yen increased sharply in value within a few hours. This is mostly attributed to the undiversified orders of HFTs which decided to aggressively sell Dollars and buy Yens almost as a collective group [59], though this trend was reversed when FTs started to buy Dollars which balanced the market again. Similarly, as was seen in the flash crash of 2010. Kirilenko states that despite the fact that HFTs initially absorbed the selling order and its subsequent high price impact of an algorithm using FT, the HFTs reversed their positions and (again) almost as a group started to sell off stock which increased the general selling pressure. This further worsened the flash crash [22, 40, 59]. Also the example of Kylie Jenner which was mentioned earlier comes to mind. The HFT algorithms interpreted her tweet as “negative” and collectively started to sell Snapchat stock.

8.2.4 Fake sense of market security

Liquidity

Since the onset of HFTs on the markets, a lot of research has been done to evaluate their behavior and intentions. Some claim that HFTs are manipulators and cheaters, whereas others say that HFTs are actually good and provide additional liquidity to markets. Providing liquidity to the market surely can’t be seen as a bad thing, however it all depends on how liquidity provision is defined and when it is done. When markets are in a good state and showing promising signs of growth, liquidity provision is nice but not exactly a necessity. Nevertheless, the HFTs exactly in those times do this. However, when the markets take a turn for the worse the HFTs quickly take their liquidity and withdraw it from the market, by canceling their limit orders for example. FTs are not capable of doing this as it takes them time to cancel their orders, which usually means that they incur (bigger) loses, compared to HFTs. Also, the HFTs are not obliged to provide liquidity, so there is no guarantee they will be doing this when the markets most need it. Konczal [60] aptly compared the liquidity provided by HFTs as a car with faulty airbags, implying a sense of security even encouraging you to take risks while driving. But as the car crashes the airbags don’t deploy and therefore do not provide security.

Trading volume

One more clear change in the markets since the arrival of HFTs is that the trading volume (numbers of stocks changing ownership) definitely went up. However, it is important to realize why the trading volume went up. Let us assume that there are 2 traders in the market. One trader wants to sell and the other wants to buy a stock. The selling trader puts up a sell order at time t for 100 stocks, the buying trader at time t + s submits her buy order for 100 stocks at which time the order will clear as it gets matched with the previously posted sell order. These 100 stocks have now changed ownership. However, with HFTs getting involved the number of ownership changes have increased. This is because often times HFTs behave like MMs. The HFT will buy the stocks from the selling party and then in turn sell them on to the

trader that would normally have bought the stocks at time t + s. This is also supported by the Cartea and Penalva [18] model. Therefore, assuming that trading volume increases are a good indicator of market potential can be a flawed thread of thought. A trader has to realize that trading volume could have increased because of the more prevalent presence and activity of the new pseudo-MMs, the HFTs.

9. The future of High Frequency Trading

With the explosive growth seen around the turn of the century for algorithmic and High Frequency Trading, this growth has now turned around a bit. Recent figures suggest that around 35% of all equity trades in Europe are HFT trades and in the US this number lies at around 50%. The reason why this number seems to have dropped over the last few years is the increasing cost, competition and regulation for HFTs. Still it is expected that HFT will develop as long as it remains profitable. Developments over the next few years can go into various directions.

9.1 Regulation

The future of HFT is likely going to be paved in constant regulatory changes. In fact, the very proliferation of HFT in the 1990s and 2000s was largely caused by the various regulations that were introduced, such as the earlier mentioned Reg. ATS and Reg. NMS [8] as described in Chapter 5. Some experts say that though regulations are inevitable, the regulators need to realize that whatever they might implement could actually have negative influences [61].

9.2 Taxing of HFT transactions

A specific form of regulation is most likely going to be the taxation of HFT transactions. Some countries have already implemented such measures, like Italy and France [62, 63]. These taxations are not specifically intending to eliminate totally HFTs but they are mostly intending to prevent the spoofing and unrealistically fast cancelling of orders. In France this is done by imposing a 0.01% tax on the value of an order which was cancelled or modified within half a second of its original submission. This would mean that most of the profits of the HFT would be cancelled out by this tax and therefore the spoofing techniques (and the like) will hopefully be falling out of favor with HFTs. However, according to some of the first analysis of the data by the French government there was no revenue of tax money since the implementation of this rule, in addition the decreased activity of HFTs did not significantly impact market volatility and liquidity [62]. Therefore, further analysis of other cases of tax implementation are needed to see whether these types of interventions really do have the desired impact.

9.3 Evening out the playfield

It is generally believed that more regulation to restrain the undesired aspects of HFTs will further expand. In addition, exchanges which actively try to even out the playing field between slower traders and HFTs, are seemingly becoming more popular. One of those is the Investors Exchange (IEX), this is an exchange in which all incoming orders are artificially delayed. This way the speed advantage of HFTs are cancelled out and both FTs and HFTs have practically the same response speed. The market share of this exchange has seen an almost uninterrupted growth from its inception in 2013. As of December 27th 2018 the market

share of IEX was 2.056%, with a peak market share seen on November 1st 2018 of 3.079%. This might imply that the growth of IEX is plateauing a bit, but nevertheless it also shows that there is an interest among traders to trade on a level playing field [43]. There is also some criticism regarding the approach of IEX, the artificial delay of the incoming orders might harm investors as prices during this delay could have changed considerably, making it impossible for them to quickly anticipate on it [64]. Though it is important to note that the main criticizers of the IEX approach are the NYSE and NASDAQ, "regular" exchanges that earn money from facilitating "regular" HFTs by offering collocation services.

9.4 Artificial HFT Intelligence (AHFTI)

Though profits of HFTs have seemingly decreased in the recent few years, they did show however signs of stabilizing recently. HFT companies need to come to terms with the increasing regulatory changes and ever increasing competition. One way to do this is to have an algorithm that would be able to work fully autonomously, dynamically responding to whatever is thrown at it both from a market and a regulatory point of view. This machine learning artificial intelligence approach may offer a lot of new possibilities in finance. Current trading algorithms to a certain extent already make some decisions on their own based on their programming. It is however not exactly known whether there is any advanced Artificial HFT Intelligence, which in this thesis we coin as AHFTI, algorithms active [43]. These AHFTIs would be capable of recognizing complex market developments and tendencies and make their "best possible" decision fully autonomously by using all sorts of data input, that are not just coming from the market exchange but also from the internet as a whole for instance. It is also unknown how these AHFTIs would respond against each other [43] and will we for instance see more correlated trades with potentially disastrous results or not?

9.5 Batch trading

Some research claims that making trading more explicitly discrete in the time dimension (by introducing batch trading) can favorably turn around the negative aspect of the arms race that has emerged amongst HFTs [16, 65]. Haas and Zoican [65] in particular describe a batch cleared market in which the HFTs no longer compete just based on their speed. They claim that in a batch cleared market the HFTs would have to compete on both prices and speed. They would have to be able to process market clearing information and based on that data put up competitive price offers for whatever action they would want to take. This differs from the current "continuous" market in which the HFTs (simply) just have to anticipate on price changes in real time and they do not have to do any major data analysis [65]. This batch trading therefore would mean that the exchange would "collect" all the orders done between 2 distinct clearing time points. These orders then would be cleared simultaneously by the exchange as if they were entered into the Order Book at the same time (see Figure 27). This way the speed advantage that HFTs have (of their orders always coming in faster than those of the other traders) will have effectively been nullified. Batch clearing was conceptualized as a method to limit the advantages experienced by HFTs [16]. There are some proponents to actually implement batch trading in the future throughout the exchanges, however it would have to be done across the board or at least on most of the major exchanges in order for it to become effective. As it is not clear how the exchanges would work alongside each other if some implement it and some don't [64]. It however seems unlikely that batch trading will be implemented and therefore it will most likely not play a large role in the future of trading.

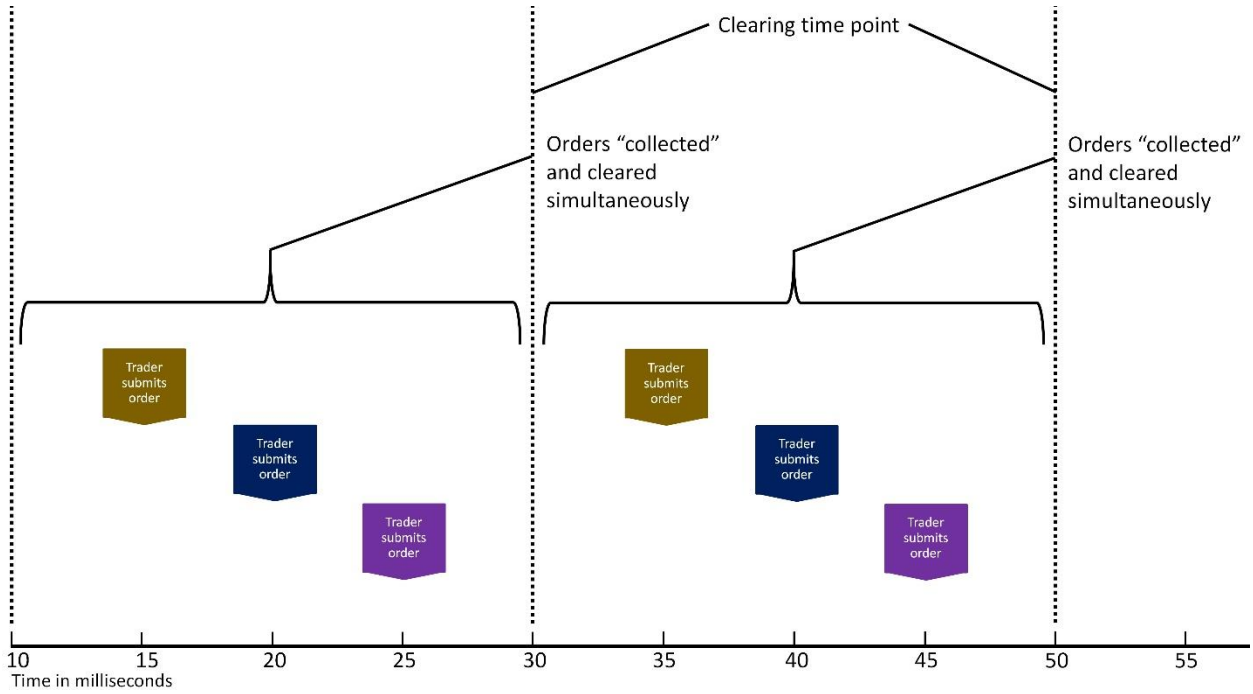

Figure 27: Batch trading.

9.6 Further increasing speed of data transfer

Clearly the development of HFT is strongly influenced by the developments of further increasing speeds of data transfer, or rather decreasing the time it takes before information reaches its destination. The most influential development of the recent few years was the introduction of fiber optic cables that connect the various exchanges with each other. In the future some other techniques might get implemented that will bring the data travel times down even further. One of these techniques is microwave transmission, which uses microwave tower nodes (not dissimilar to the Chappe telegraph). To compare with the current fastest way, the fiber optic cable, the microwave transmission method takes about 4.7 milliseconds for data to reach from Chicago to New York whereas currently it takes 8.3 milliseconds [64]. This would naturally further increase the speed advantage of HFTs, assuming no measures will be undertaken preventing them to further exploit their swiftness.

9.7 Quantum computing

Current "normal" HFT algorithms run on clustered computers that combine relatively off the shelf components like high end CPUs and GPUs. However, at a certain point physics will catch up on you and no matter how many components you cluster together, the gains in speed in processing power will not be noticeable anymore and with that Moore's law will come to an end. This is the point at which binary digital computers have reached their limit and where quantum computing comes in. Where binary digital computers use 1s and 0s in their basic coding to run and compute things one task at a time, quantum computers use the laws of physics to compute many complex things simultaneously. Unlike binary computers, quantum computers are not composed of bits and transistors but of qubits and physical components that are so small that they operate by the rules of quantum physics. These components can be for instance electrons that are captured in magnetic fields [66]. Where in binary digital computers bits can only be a 1 or 0, in quantum computers qubits can be a 1 or 0 or both simultaneously. Because of this many more calculations can be made in the same time. This makes quantum computers in theory much

more powerful than even the most powerful conventional binary digital computer. Though currently quantum computing is in its infancy, it is expected that the first practically useable quantum computers will be presented by the end of the decade [11]. But how can quantum computing be the next big thing for HFT (or algorithmic trading in general)? Current computing technology is not capable of accurately modeling portfolio problems (what should be part of a portfolio to maximize possible expected returns against a certain level of risk). To compensate for this, currently concessions are made in computing these models, by using approximations in the data [11, 67]. With quantum computers this will not be needed as they can do billions of computations more than a regular binary digital computer [68], making impossibly long calculations suddenly possible and useable. However, there are fears that as opposed to making models of outcomes more reliable, quantum computing will reinvigorate HFT popularity. This might drive more companies to base all their investments on fast algorithmic decisions with possibly more flash crash like events with potentially even worse outcomes [68]. If regulators want to prevent these unwanted events to occur, they too will likely have to employ quantum computers to keep track of all market activities without falling behind as they do now sometimes [68].

10. Discussion

As has become evident in this elaborate study into the workings of HFTs, the effect the HFTs have had on the stock exchanges has been enormous. It did not just have an impact on the speed of order submission but it has also changed the complete dynamic between the various traders. HFTs have been able to exploit their speed advantage over other traders in such a way that they are even influencing the prices of stocks in the short term. One of the dilemmas that have occurred over the last few years is the ongoing realization that the development of algorithms and connection speeds have actually grown out of control in a way. The realization that HFTs can influence the price of stocks and also cause “flash crashes” has come too late and at a cost, with previous crashes destroying a lot of market value and stock exchanges suffering from increasing volatility on the markets. Now exchanges and even governments are struggling with attempting to regulate and contain the negative aspects of HFT. Taxes are already being implemented on HFT submitted orders in France and Italy, however as first analysis have shown the effectiveness is seemingly nonexistent. This can be because either the HFTs were able to somehow circumvent the taxation rules or the HFTs have left the respective exchanges and have moved their activities elsewhere or have abandoned them altogether. A potentially more effective way of regulating the activities of HFTs would be active real time monitoring of the trades and detecting any suspicious behavior as it happens. This currently (arguably) is also done, but HFTs have the possibility to flood the buffer of the exchanges and with this can further increase their advantage of speed. With the onset of quantum computing imminent, processing speed and simulation accuracies will (again) drastically improve. Exchanges intent on leveling the play field and in favor of curbing illegal activities, should adopt quantum computing as soon as possible (after their effectiveness has been proven). This way they could speed up the processing of orders and prevent flooding of the buffer and also, they could analyze trading patterns and stock price behavior quicker, preventing large financial losses of value on the markets that might be caused by correlated trades.

Traders being natural opportunists of course are always going to try to take advantage of the rules and get the most out of any given situation. It is therefore not strange to think that HFT traders are exploiting their main advantage over the other traders, speed. A lot of criticism has been expressed towards the HFTs (that are pursuing controversial strategies). However, the unwillingness of exchanges to effectively regulate or even ban such strategies is just as questionable as the behavior of some of the HFTs. The main reason why the exchanges are, for lack of a better word, tolerating the behavior of the HFTs is because they are getting revenue from them. They facilitate the HFTs by offering them collocation services which enables the HFTs to fully exploit the speed of their algorithms. The exchanges know that if they will try to stop or alter the behavior of the HFTs, they might potentially impact one of their revenue sources considerably. However, on the other hand this should not be a limitation to the exchanges. The exchanges can keep charging for their collocation services, but as with almost any other service provider should be able to deny service to any entity that violates certain terms and conditions. If the user of the collocation services will only be faster than the other traders but not perform any illegal price manipulations, then the collocation service can continue and revenue is still to be had for the exchange.

One can argue that the exchanges that practically endorse HFT collocation (and thereby facilitate the advantage of the HFTs) are just as much "at fault" as the rogue HFT traders. Exchanges should take a more active role in banning rogue traders to which false market orders can be traced back, however it is doubtful if banning these traders would at all be effective as they can easily register a new business entity and reenter the exchange.

The model presented in this study (despite its complexity with regards to interactions between the traders) is a very simplistic representation of an exchange. Therefore, for future studies it would be useful to expand the model, this can be done in several ways. First, the model should be expanded with more traders (of different types as well, such as intermediaries). This way the interactions modeled will be able to reflect more accurately the diversity of traders seen on actual exchanges. The addition of another exchange to the model should also be considered. As this way the anticipation of the multimarket HFT on the SIP can be modeled and her speed in acting between two markets can be studied. Though modeling this would be considerably more complicated. The "delay" in intermarket data update rates for the HFT and the SIP would have to be accounted for, on top of the lag from the other traders responding to the market changes. The model as it is presented here does not incorporate the release of positive or negative information regarding the stock that the traders are interested in. This is quite a big omission as the inclusion of this would make the model possibly explain the behavior of HFTs in a more detailed way. HFTs would (if the algorithm is properly programmed) not just go by the fact whether information is positive or negative, but the algorithm should also analyze the historic behavior of the stock whenever positive or negative information was released. Some stocks have relatively weak growth after positive news has been released, whereas other stocks perform strongly even after a few days before negative news has been released. HFTs have the processing power to incorporate historic data in their decision making upon the release of stock impacting information, therefore the incorporation of this would strengthen the model.

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Appendix

<u>Initial offering of FT1</u>

$MC_{\ after\ order\ submission}$

$$45000(4.50 - S) + 5000(4.50 - S) - 1000 = 0$$

$$S = 4.48$$

Quantity offered by traders

MM $45000(4.50 - 4.48) = 900$ offers to buy for \$4.48

FT1 $5000(4.50 - 4.48) = 100$ will hold, $1000 - 100 = 900$ will sell for \$4.48

<u>Sell after the initial order of FT1</u>

$MC_{\ o-s}$

$$45000(4.50 - S) + 5000(4.50 - S) + 5000(4.50 - S) = 1650$$

$$S = 4.47$$

Quantity offered by traders

MM $45000(4.50 - 4.47) = 1350$ offers to buy for \$4.47

FT1 $5000(4.50 - 4.47) = 150$ will hold, $1000 - 150 = 850$ will sell for \$4.47

<u>Buy after sell of HFT</u> $Q_{\ o-s-b}$

Quantity offered by traders

FT1 $5000(4.50 - 4.47) = 150$ will hold, $1000 - 150 = 850$ will sell for \$4.47

FT1 $5000(4.50 - 4.48) = 100$ will hold, $150 - 100 = 50$ will sell for \$4.48

$Q_{\ o-s-b-b}$

Quantity offered by traders

FT1 $5000(4.50 - 4.49) = 50$ will hold, $100 - 50 = 50$ will sell for \$4.49

$Q_{\ o-s-b-b-b}$

Quantity offered by traders

FT1 $5000(4.50 - 4.50) = 0$ will hold, 50 will sell for \$4.50

$Q_{o-s-b-b-b-s}$

MM $45000(4.50 - 4.49) = 450$ offers to buy for \$4.49

MM $45000(4.50 - 4.48) = 900 - 450 = 450$ offers to buy for \$4.48

$Q_{o-s-b-b-n-s}$

MM $45000(4.50 - 4.49) = 450$ offers to buy for \$4.49

MM $45000(4.50 - 4.48) = 900 - 450 = 450$ offers to buy for \$4.48

$Q_{o-s-b-b-s}$

MM $45000(4.50 - 4.49) = 450$ offers to buy for \$4.49

MM $45000(4.50 - 4.48) = 900 - 450 = 450$ offers to buy for \$4.48

$MC_{o-s-b-n}$

MM $45000(4.50 - 4.49) = 450$ offers to buy for \$4.49

FT1 $5000(4.50 - 4.49) = 50$ will hold, $100 - 50 = 50$ will sell for \$4.49

$Q_{o-s-b-n-s}$

MM $45000(4.50 - 4.49) = 450$ -50 (bought from FT1) 400 offers to buy from the HFT for \$4.49

MM $45000(4.50 - 4.48) = 900 - 450 = 450$ offers to buy for \$4.48

$Q_{o-s-b-s}$

MM $45000(4.50 - 4.49) = 450$ offers to buy from the HFT for \$4.49

MM $45000(4.50 - 4.48) = 900$ – 450 already sold above = 450 offers to buy from the HFT for \$4.48

$Q_{o-s-b-s-b}$

MMs decision after purchasing 900 stocks

MM450 $(4.49 - 4.48) = 4.5\ profit$

MM450 $(4.50 - 4.49) = 4.5\ profit$

Nothing after sell of HFT

Q_{o-s-n}

MM $45000(4.50 - 4.47) = 1350$ offers to buy for \$4.47,

FT1 $5000(4.50 - 4.47) = 150$ will hold, $1000 - 150 = 850$ will sell to MM for \$4.47

HFT $5000(4.50 - 4.47) = 150$ will hold, $650 - 150 = 500$ will sell to MM for \$4.47

Sell after sell of HFT

MC_{o-s-s}

$$45000(4.50 - S) + 5000(4.50 - S) + 5000(4.50 - S) = 2100$$

$$S = 4.46$$

$$Q_{o-s-s}$$

MM $45000(4.50 - 4.46) = 1800$ offers to buy for \$4.46

FT1 $5000(4.50 - 4.46) = 200$ will hold, $1000 - 200 = 800$ will sell for \$4.4

Buy after sell after sell

$Q_{o-s-s-b}$

FT1 $5000(4.50 - 4.46) = 200$ will hold, $1000 - 200 = 800$ will sell for \$4.46

FT1 $5000(4.50 - 4.47) = 150$ will hold, $200 - 150 = 50$ will sell for \$4.47

FT1 $5000(4.50 - 4.48) = 100$ will hold, $150 - 100 = 50$ will sell for \$4.48

Sell after buy after sell after sell

$Q_{o-s-s-b-s}$

MM $45000(4.50 - 4.49) = 450$offers to buy from the HFT for \$4.49

MM $45000(4.50 - 4.48) = 900$ – 450 already sold above= 450 offers to buy from the HFT for \$4.48

Buy after order submission

Q_{o-b}

FT1 $5000(4.50 - 4.50) = 0$ will hold, 1000 will sell for \$4.50

MC_{o-b-b}

$$45000(4.50 - S) + 5000(4.50 - S) = -1000$$

$$S = 4.52$$

Q_{o-b-b}

MM $45000(4.50 - 4.52) = -900$offers to sell to the HFT for \$4.52

$MC_{o-b-n-s} = MC_{o-b-s}$

$$45000(4.50 - S) + 5000(4.50 - S) = 1000$$

$$S = 4.48$$

Quantity offered by traders

MM $45000(4.50 - 4.48) = 900$ offers to buy for \$4.48

HFT $5000(4.50 - 4.48) = 100$ will hold, $1000 - 100 = 900$ will sell for \$4.48

Nothing after order submission

MC_{o-n}

$$45000(4.50 - S) + 5000(4.50 - S) = 1000$$

$$S = 4.48$$

Quantity offered by traders

MM $45000(4.50 - 4.48) = 900$ offers to buy for \$4.48

FT1 $5000(4.50 - 4.48) = 100$ will hold, $1000 - 100 = 900$ will sell for \$4.48

$Q_{o-n-b} = Q_{o-n-n-b}$

FT1 $5000(4.50 - 4.49) = 50$ will hold, $100 - 50 = 50$ will sell for \$4.49

FT1 $5000(4.50 - 4.45) = 0$ will hold, 50 will sell for \$4.50

MC_{o-n-s}

$$45000(4.50 - S) + 5000(4.50 - S) + 5000(4.50 - S) = 2000$$

$$S = 4.46$$

MM $45000(4.50 - 4.46) = 1800 - 900\ already\ bought\ from\ FT1 = 900$ offers to buy for \$4.46

FT1 $at\ this\ moment\ holds\ 100\ shares\ and\ will\ not\ sell\ anything\ for\ less\ than\ \$4.49$

Profit calculations

<u>Sell after initial offering</u>

$$\pi_{s-b-b-b-c} = [850 * (4.50 - 4.47) + 50 * (4.50 - 4.48) + 50(4.50 - 4.49) + 50(4.50 - 4.50)] = 27$$

$$\pi_{s-b-b-b-s-c} = [450 * (4.49 - 4.47) + 400 * (4.48 - 4.47) + 50(4.48 - 4.48) + 50(4.50 - 4.49) + 50(4.50 - 4.49)] = 13.5$$

$$\pi_{s-b-b-n-s-c} = [450 * (4.49 - 4.47) + 400 * (4.48 - 4.47) + 50(4.48 - 4.48) + 50(4.50 - 4.49)] = 13.5$$

$$\pi_{s-b-b-s-c} = [450 * (4.49 - 4.47) + 400 * (4.48 - 4.47) + 50(4.48 - 4.48) + 50(4.50 - 4.49)] = 13.5$$

$$\pi_{s-b-n-s-c} = [400 * (4.49 - 4.47) + 450 * (4.48 - 4.47) + 50(4.50 - 4.48)] = 13.5$$

$$\pi_{s-b-s-c} = [450 * (4.49 - 4.47) + 400 * (4.48 - 4.47) + 50(4.48 - 4.48)] = 13$$

$$\pi_{s-b-s-c} = [450 * (4.49 - 4.47) + 400 * (4.48 - 4.47) + 50(4.48 - 4.48)] = 13 + cost\ of\ buying\ 1000\ shares\ 500 * 4.49 + 500 * 4.50 = -4495$$

$$\pi_{s-s-b-s-b-c} = [450 * (4.49 - 4.46) + 350 * (4.48 - 4.46) + 50(4.48 - 4.47) + 50(4.48 - 4.48)] = 21 + cost\ of\ buying\ 1000\ shares\ 500 * 4.49 + 500 * 4.50 = -4495$$

$$\pi_{s-s-b-s-c} = [450 * (4.49 - 4.46) + 350 * (4.48 - 4.46) + 50(4.48 - 4.47) + 50(4.48 - 4.48)] = 21$$

<u>Buy after order submission profits</u>

$$\pi_{b-n-c} = [-(1000 * 4.5) + (1000 * 4.5)] = 0$$
$$\pi_{b-n-s-c} = [-(1000 * 4.5) + (900 * 4.48) + (100 * 4.5)] = -18$$

$$\pi_{b-s-c} = [-(1000 * 4.5) + (900 * 4.48) + (100 * 4.5)] = -18$$

<u>Nothing after order submission profits</u>

$\pi_{n-b-c} = [-(50 * 4.49) + (-50 * 4.5) + 100 * 4.50] = 0.5$

$\pi_{n-n-b-c} = [-(50 * 4.49) + (-50 * 4.5) + 100 * 4.50] = 0.5$